INTERNATIONAL MONETARY POLICY ANALYSIS

WITH DURABLE GOODS

A Dissertation

by

KANG KOO LEE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2009

Major Subject: Economics

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ABSTRACT

International Monetary Policy Analysis with Durable Goods. (August 2009)

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Chair of Advisory Committee: Dr. Dennis W. Jansen

The dissertation studies a model of an economy which produces and exports durable goods. It analyzes the optimal monetary policy for such a country.

Generally, monetary policy has a bigger economic effect on durable goods relative to non-durable goods because durable goods can be stored and households get utility from the stock of durable goods. This dissertation shows that, in Nash equilibrium, the central bank of a durable goods producing country can control changes of the price level with smaller changes in the monetary policy instrument. In the cooperative equilibrium, the monetary authority of the country which imports non-durable goods and exports durable goods should raise the interest rate by more, relative to the Nash case, in response to a rise in foreign inflation. On the other hand, the monetary authority of the country which imports durable goods and exports non-durable goods should raise the interest rate by less than the other country.

To my family

TABLE OF CONTENTS

		Page
ABSTRACT		iii
DEDICATION		iv
TABLE OF CONTENTS		v
LIST OF FIGURE		vi
CHAPTER		
I	INTRODUCTION	1
II	THE MODEL	5
	1. Representative Household	6
	2. Firms and Price Setting	9
	3. Government Policy	13
	4. Uncovered Interest Parity	13
	5. Market Clearing Conditions	14
III	EQUILIBRIUM	17
	1. Steady-State Equilibrium	18
	2. Flexible-Price Equilibrium	19
	3. Sticky-Price Equilibrium	22
IV	THE WELFARE FUNCTION AND OPTIMAL MONETARY POLICY	25
	1. The Welfare Function and Optimal Subsidy Rate	25
	2. Optimal Monetary Policy	27
V	CONCLUSIONS	33
REFERENCES		34
APPENDIX A		36
VITA		69

LIST OF FIGURES

Page

Figure 1 Flow of durable goods in two countries setting 6

CHAPTER I

INTRODUCTION

"A significant portion of measured consumption spending reflects purchases of consumer durables – long-lived items such as furniture, autos, televisions, and home computers. … . If all durables were rented in perfect rental markets, the durable-nondurable distinction would be unimportant. Empirically, however, consumer purchases of durable items are substantial. … ."[1]

Many New Keynesian models neglect durable goods, such as Clarida, Gali and Gertler (1999) and Woodford (2003). This may be due to the fact that the share of the durable goods sector in the gross domestic product (GDP) is quite a bit smaller than the non-durables and service sectors. According to Erceg and Levin (2006), the share of the durable goods sector in U.S. GDP is only about 12%.

Some recent papers, however, pay attention to the role played by durable goods. Erceg and Levin (2006) and Monacelli (2008) show that durable goods spending is more sensitive to monetary shocks than non-durable goods spending. Barsky et al. (2007) explain these results as stemming from the special properties of durables. They write that durable goods have "interesting and unique properties" that are directly related to their length of life. The intertemporal elasticity of substitution for the consumption of durable goods is naturally high for durables with low depreciation rates, such as housing and

This dissertation follows the style and format of the Journal of International Economics.

[1] Obstfeld and Rogoff (1999) p. 96

business structures. This plays a much larger role in fluctuations of aggregate output and can play a large role in business cycles.

The unique feature of durables and the distinctive feature of long-lived durables with sticky prices lead to results that are different from results drawn from studies looking only at non-durables and comparing equilibria with sticky and flexible prices.

Though there are recent studies about durables in closed economy, there are few papers focusing on durables and non-durables in an international setting. This study about durable goods spending in two different countries and the resulting policy questions will, I think, prove to be a fruitful area of research in the New Open Economy Macroeconomics.

This dissertation works to provide answers to the following questions: What is the gain from monetary policy cooperation with durables? How does this differ from models with only non-durables? Which country would have more benefit from monetary policy cooperation, a country producing only durable goods or a country producing only non-durable goods?

Several recent papers have introduced durable goods into a closed-economy model. Barsky et al. (2007) examine both sticky and flexible price sectors in a closed economy model with durable and non-durable goods, and shows that the presence of durable goods can alter the transmission of monetary shocks even though durables have a small share in total spending. Particularly, if durable prices are flexible, their work shows that monetary policy is neutral. If durable prices are sticky, the model works as a sticky price model even if non-durable prices are flexible.

Campbell and Hercowitz (2005) study the role of collateral debt in a business cycle model and they conclude that lowering the interest rate induces an expanding labor supply by increasing the demand for durable goods, instead of the standard result of a contracting labor supply by reducing the price of current leisure relative to future leisure.

Monacelli (2008) studies a New Keynesian model with durable goods and argues that assumption on the degree of stickiness of durables may become irrelevant when once a collateral constraint on borrowing is introduced in the model. The reduced ability of borrowing also reduces the demand for non-durables when prices are sticky in durable goods sector. Therefore, the collateral constraint makes a complementary effect between durable and non-durable demand, while the monetary policy tightening produces a substitution effect from durables to non-durables by rising the user cost when prices are flexible.

Erceg and Levin (2006) compare the performance of several monetary policies in a sticky price model with durable and non-durable goods, and they show that the optimal policy rule can be approximated by a simple targeting rule for a weighted average of aggregate wage and price inflation when the social welfare function consists of sector-specific output gaps and inflation rates.

Han (2008) examines the relationship among heterogeneous durability of consumption goods, price indexes and monetary policy in a sticky price model, and concludes that the optimal policy is to stabilize core inflation, giving more weight to the sector producing more durable goods.

A classic issue in international monetary economics is to ask whether there is a gain from policy coordination. Our goal is to examine this issue in an international economic model that includes durable goods. Recent works in international monetary economics area have involved the development of the dynamic stochastic general equilibrium models with imperfect competition and nominal rigidities.

Clarida et al. (2002) examine international monetary policy in a two-country sticky price model, where each country faces a short run trade off between output and inflation. They show that, in the Nash equilibrium, each central bank needs to adjust its interest rate in response to only domestic inflation, while, in the coordinative equilibrium, there are gains when they should respond to foreign inflation.

Gali and Moncelli (2005) analyze the properties and macroeconomic implications of alternative monetary policy regimes in a small open economy model with staggered price-setting, and show that a domestic inflation-based Taylor rule dominates a CPI-based Taylor rule, and the latter dominates an exchange rate peg.

The rest of the dissertation is structured as the following. Chapter II presents the two-country model introducing durable goods. Chapter III describes equilibrium conditions when prices are flexible and sticky. Chapter IV studies the welfare function and optimal monetary policy in the Nash case and the case of cooperation. Concluding remarks and future works are in Chapter V.

CHAPTER II

THE MODEL

We follow the two-country model of Clarida et al. (2002). There are two regions, labeled H and F. While the final goods producer in country j supplies a single differentiated durable good, the representative household in country j is a consumer of all the goods produced in both country H and F. Thus all goods produced are traded between regions. The population of two countries is a continuum of agents on the interval [0, 1]. The population is divided by (n, 1] belongs to country H and [0, n] belongs to country F.

In this economy, there are a large number of identical and infinitely lived households. The representative household derives utility from services provided by two durable goods and disutility from supplying labor.

We introduce durable consumption goods as in Han (2008). The final durable goods are composite goods of differentiated non-durable intermediate goods. The production of intermediate goods requires labor as the only input. Intermediate goods are introduced to provide an analytically convenient mean of having sticky prices, as it is the intermediate goods producers who set prices in advance. The flow of final durable goods and intermediate goods is shown in Figure 1.

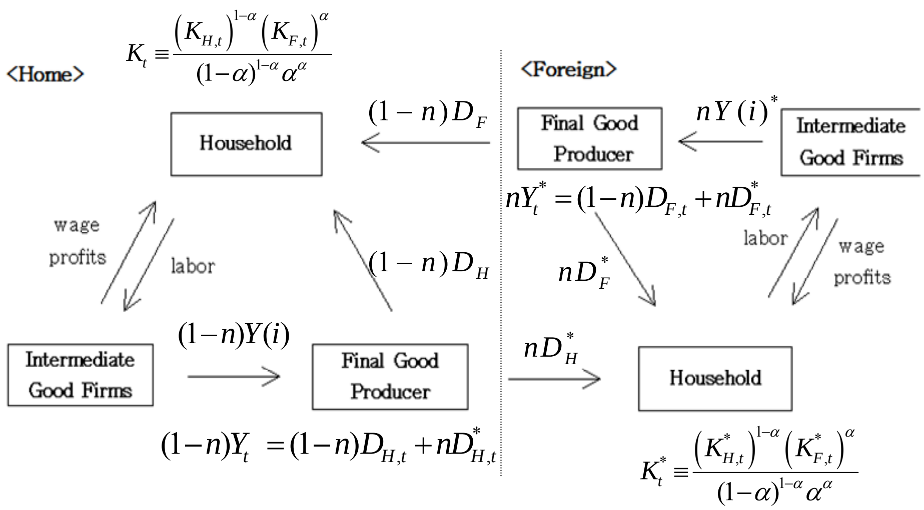

Figure 1. Flow of durable goods in two countries setting

1. Representative Household

The representative household of country H and F seeks to maximize a discounted sum of per-period utilities of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(K_t) - \int_0^1 v(N_t(i))di \right], \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[u(K_t^*) - \int_0^1 v(N_t^*(i))di \right] \tag{2.1}$$

Here E is the expectation operator, $0 < \beta < 1$ is the discount factor, and $N_t(i)$ is the quantity of labor of type i supplied in period t. Variables with superscript * indicate foreign country values.

K_t is an argument in the utility function, a Cobb-Douglas index of two final durable goods stocks :

$$K_t \equiv \frac{\left(K_{H,t}\right)^{1-\alpha}\left(K_{F,t}\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \qquad K_t^* \equiv \frac{\left(K_{H,t}^*\right)^{1-\alpha}\left(K_{F,t}^*\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} \tag{2.2}$$

where α denotes the intratemporal elasticity of substitution between $K_{H,t}$ and $K_{F,t}$. The stock per person of the final durable goods produced in each country *j* at time *t* is $K_{j,t}$, $j = H, F$. K_t is the index of consumption per person of durables in the home country, and K_t^* is the index of durable goods consumption per person in the foreign country. The stock of the final durable goods produced in each country *j* evolves according to a law of accumulation:

$$K_{j,t} = (1-\delta_j)K_{j,t-1} + D_{j,t}, \qquad K_{j,t}^* = (1-\delta_j^*)K_{j,t-1}^* + D_{j,t}^* \tag{2.3}$$

for $j = H, F$, where $D_{j,t}$ is the amount of the purchased final durables produced in country *j* per capita, and $0 < \delta_j \le 1$ is the depreciation rates of the final durable goods in country *j*. Note $\delta_j = \delta_j^*$ for j=H,F.

Let Γ_{t+1} denote the (random) payoffs in period t+1 of the financial asset portfolio held at the end of t. Note Γ_{t+1} does not refer to the quantity held of some single type of bond. As assuming complete markets, households must be able, at least in principle, to hold any of a wide selection of instruments with different state-contingent returns. However, we don't need to introduce any notation for the particular types of financial instruments that are traded internationally. Since any pattern of future state-contingent payoffs that a household may desire can be arranged for the appropriate price, we can write the household's consumption planning and wealth-accumulation problems without any explicit reference to the particular assets' quantities that a household holds.[2]

$\Omega_{t,t+1}$ denotes the corresponding stochastic discount factor for one-period-ahead nominal payoffs relevant to the domestic household.

The budget constraint of the representative household of country H and F in period t is

$$\begin{aligned} E_t\left\{\Omega_{t,t+1}\Gamma_{t+1}\right\} &= \Gamma_t + \int_0^1 W_t(i)N_t(i)di + \int_0^1 \Pi_t(i)di - P_{H,t}D_{H,t} - P_{F,t}D_{F,t} - T_t \\ E_t\left\{\Omega^*_{t,t+1}\Gamma^*_{t+1}\right\} &= \Gamma^*_t + \int_0^1 W^*_t(i)N^*_t(i)di + \int_0^1 \Pi^*_t(i)di - P^*_{H,t}D^*_{H,t} - P^*_{F,t}D^*_{F,t} - T^*_t \end{aligned} \tag{2.4}$$

where $P_{j,t}$ is the price of the final durable goods in country j, $N_t(i)$ is the quantity of labor of type i supplied, $W_t(i)$ is the nominal wage of labor of type i, $\Pi_t(i)$ is the nominal profit from sales of intermediate good i, and T_t represent net lump-sum tax collections by the government.

The first-order conditions are given by

$$\frac{u_k(K_t)}{u_k(K_{t+1})} = \frac{\beta}{\Omega_{t,t+1}} \frac{K_{H,t}/K_t}{K_{H,t+1}/K_{t+1}} \frac{P_{H,t} - (1-\delta_H)\Omega_{t,t+1}P_{H,t-1}}{P_{H,t+1} - (1-\delta_H)\Omega_{t+1,t+2}P_{H,t}} \tag{2.5}$$

Let R_t denote the gross nominal yield on a one-period discount bond. Then by taking the expectation of each side of (2.5), we have the Euler equation :

$$1 = \beta R_t E_t\left\{\frac{u_k(K_{t+1})}{u_k(K_t)}\frac{Q_t}{Q_{t+1}}\right\} \tag{2.6}$$

[2] Woodford (2003) Ch.2

$$\frac{1-\alpha}{\alpha}\frac{K_{F,t}}{K_{H,t}} = H_t^{-1} \tag{2.7}$$

$$\frac{v_N(N_t(i))}{u_k(K_t)} = \frac{W_t(i)}{Q_t} \tag{2.8}$$

for $i \in [0,1]$, where $H_t \equiv Q_{F,t} / Q_{H,t} = Q^*_{F,t} / Q^*_{H,t}$, $R_t^{-1} = E_t\left\{\Omega_{t,t+1}\right\}$ is the expected price of the discount bonds paying off one unit of domestic currency in t+1, and the user cost of durable goods consumption in country j $Q_{j,t}$ is defined as

$$\begin{aligned} Q_{j,t} &\equiv P_{j,t} - E_t\left[(1-\delta_j)P_{j,t+1} / R_t\right] \\ Q^*_{j,t} &\equiv P^*_{j,t} - E_t\left[(1-\delta^*_j)P^*_{j,t+1} / R^*_t\right] \end{aligned} \tag{2.9}$$

The cost of living index (COLI) Q_t is defined as

$$Q_t \equiv Q_{H,t}^{1-\alpha}Q_{F,t}^{\alpha}, \qquad Q^*_t \equiv Q_{H,t}^{*1-\alpha}Q_{F,t}^{*\alpha} \tag{2.10}$$

According to Cobb-Douglas preferences, the demand functions are given by

$$\begin{aligned} K_{H,t} &= (1-\alpha)(Q_t / Q_{H,t})K_t, & K^*_{H,t} &= (1-\alpha)(Q^*_t / Q^*_{H,t})K^*_t \\ K_{F,t} &= \alpha(Q_t / Q_{F,t})K_t, & K^*_{F,t} &= \alpha(Q^*_t / Q^*_{F,t})K^*_t \end{aligned} \tag{2.11}$$

2. Firms and Price Setting

2.1 Final Goods Producers

We assume final durables producers behave competitively. The technology for producing the final goods in country H and F from intermediate goods is

$$Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \qquad Y^*_t \equiv \left[\int_0^1 Y^*_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{2.12}$$

where Y_t is output per person and θ>1 is the elasticity of substitution across goods produced within a country.

Maximization of profits yields demand functions for the typical intermediate good *i* in country H and F:

$$Y_t\ (i) = \left(P_t\ (i) / P_t\ \right)^{-\theta} Y_t\ , \qquad Y_t^*(i) = \left(P_t^*(i) / P_t^*\right)^{-\theta} Y_t^* \tag{2.13}$$

The country's price index consistent with the final goods producer in each country earning zero profits respectively, is given by

$$P_{H,t} \equiv \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \qquad P_{F,t}^* \equiv \left[\int_0^1 P_t^*(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{2.14}$$

2.2 Intermediate Goods Producers

Intermediate goods producers behave as monopolistic competitors. Each of the differentiated intermediate goods uses a specialized labor input in its production. The technology for producing intermediate goods type $i \in H, F$ is

$$Y_t(i) = A_t N_t(i), \qquad Y_t^*(i) = A_t^* N_t^*(i) \tag{2.15}$$

We assume that time-varying exogenous technology factors A_t, A_t^* are identical across suppliers within a country.

The variable cost of supplying a quantity $Y_t(i),\ Y_t^*(i)$ of good i is given by

$$\begin{aligned} &W_t(i)Y_t(i) / A_t \\ &W_t^*(i)Y_t^*(i) / A_t^* \end{aligned} \tag{2.16}$$

Differentiating (2.16) with respect to the intermediate good, we find that the (nominal) marginal cost of supplying good i is equal to

$$\begin{aligned} NMC_t(i) &= W_t(i) / A_t \\ NMC_t^*(i) &= W_t^*(i) / A_t^* \end{aligned} \tag{2.17}$$

The relationship between the real marginal cost and the quantity supplied is given by

$$\begin{aligned} MC_t(i) &\equiv NMC_t(i) / P_{H,t} = \frac{v_N(Y_t(i)/A_t)}{u_k(K_t)} \frac{Q_t}{A_t P_{H,t}} \\ MC_t^*(i) &\equiv NMC_t^*(i) / P_{F,t}^* = \frac{v_{N^*}(Y_t^*(i)/A_t^*)}{u_{k^*}(K_t^*)} \frac{Q_t^*}{A_t^* P_{F,t}^*} \end{aligned} \tag{2.18}$$

The Case of Perfectly Flexible Prices

As usual in a model of monopolistic competition, it is assumed that each supplier understands that its sales depend upon the price charged for its good, according to the demand function (2.13).

If prices are perfectly flexible, optimization by the supplier of good i in country j involves setting a price $P_t(i) = \frac{\mu}{1+\tau} NMC_t(i), \quad P_t^*(i) = \frac{\mu}{1+\tau^*} NMC_t^*(i)$, where $\mu \equiv \theta/(\theta-1) > 1$ is the seller's desired markup, determined by the usual Lerner formula, and τ , τ^* is the subsidy to intermediate good production in country H and F.

It follows that each supplier wishes to charge a relative price satisfying

$$P_t(i)/P_{H,t} = \frac{\mu}{1+\tau} MC_t(i), \qquad P_t^*(i)/P_{F,t}^* = \frac{\mu}{1+\tau^*} MC_t^*(i) \tag{2.19}$$

The country's natural rates are implicitly defined by

$$\frac{P_t(i)}{P_{H,t}} = \frac{\mu}{1+\tau} \frac{v_N(Y_t / A_t)}{u_k(K_t)} \frac{Q_t}{A_t P_{H,t}}, \quad \frac{P_t^*(i)}{P_{F,t}^*} = \frac{\mu}{1+\tau^*} \frac{v_{N^*}(Y_t^* / A_t^*)}{u_{k^*}(K_t^*)} \frac{Q_t^*}{A_t^* P_{F,t}^*} \tag{2.20}$$

The Calvo Model of Price Setting

Following the Calvo style's price-setting, fractions $0<\gamma<1,\ 0<\gamma^*<1$ of intermediate goods prices in country H, F remain unchanged each period, whereas new prices are chosen for the other $1-\gamma,\ 1-\gamma^*$ of the intermediate goods in country H, F. For simplicity, the probability that any given price will be adjusted in any given period is assumed to be $1-\gamma,\ 1-\gamma^*$, independent of the length of time since the price was set and of what the particular good's current price may be.

Then, the Dixit-Stiglitz price index of country j in period t satisfies

$$\begin{aligned} P_{H,t} &= \left[(1-\gamma)P_{H,t}^{0\ 1-\theta} + \gamma P_{H,t-1}^{\ 1-\theta}\right]^{\frac{1}{1-\theta}} \\ P_{F,t}^* &= \left[(1-\gamma^*)P_{F,t}^{*0 1-\theta} + \gamma^* P_{F,t-1}^{*\ 1-\theta}\right]^{\frac{1}{1-\theta}} \end{aligned} \tag{2.21}$$

An intermediate goods supplier in country H, F that changes its price in period *t* and chooses its new price $P_t(i)$ to maximize

$$\begin{aligned} & E_t\sum_{k=0}^{\infty}\gamma^k\lambda_{t,t+k}\left[(1+\tau)P_t(i)Y_{t,t+k}(i)-W_{t+k}(i)N_{t+k}(i)\right] \\ &= E_t\sum_{k=0}^{\infty}\gamma^k\lambda_{t,t+k}\left[(1+\tau)P_t(i)-\frac{W_{t+k}(i)}{A_{j,t+k}}\right]Y_{t,t+k}(i) \end{aligned} \tag{2.22}$$

where $\lambda_{t,t+k}=\beta^k(u_k(K_{t+k})/Q_{j,t+k})/(u_k(K_t)/Q_{j,t})$,

and $Y_{t,t+k}(i)=\left(P_t(i)/P_{t+k}\right)^{-\theta}Y_{t+k}$.

The first order condition for the optimal choice of P_t^0 is

$$E_t\sum_{k=0}^{\infty}\left(\gamma\beta\right)^k\left[(\theta-1)(1+\tau)\Lambda_{t+k}P_t^0(i)-\theta v_N(Y_{t,t+k}(i)/A_{t+k})/A_{t+k}\right]Y_{t,t+k}(i)=0 \tag{2.23}$$

where $\Lambda_{t+k} = u_k(K_{t+k}) / Q_{t+k}$.

3. Government Policy

The fiscal authority chooses a subsidy rate τ that maximizes the utility of the domestic household in a zero inflation steady state. Meanwhile, the central bank determines the short-term nominal interest rate by a time-invariant commitment rule. The monetary authority supplies bonds in amounts enough to meet bond demand at the target nominal-interest rate. That means the central bank, as issuer of bonds that promise to pay units of its own bonds, can fix both the nominal interest rate on its bonds and the quantity of bonds in existence. In a cashless economy, the central bank stabilizes the price level around a desired level through adjusting the interest rate paid on central-bank balances.

The home country's government budget constraint in time t is given by

$$B_t + T_t = R_{t-1}B_{t-1} + \tau \int_0^1 P_t(i)Y_t(i)di \tag{2.24}$$

where B_t is government debt (the supply of bond) per person at end of period t.

The foreign country's government budget constraint in time t is given by

$$B_t^* + T_t^* = R_{t-1}^* B_{t-1}^* + \tau^* \int_0^1 P_t^*(i)Y_t^*(i)di \tag{2.25}$$

4. Uncovered Interest Parity

We assume the complete international financial markets. This means that real rates of return in the two countries are equal. It must satisfy that

$$\begin{aligned} r_t - E_t\pi_{t+1} &= rr_t = r_t^* - E_t\pi_{t+1}^* \\ r_t - r_t^* &= E_t\pi_{t+1} - E_t\pi_{t+1}^* = E_t\varepsilon_{t+1} - \varepsilon_t \end{aligned}$$

where rr_t is real interest rate, $\varepsilon_t = \log(P_t / P_t^*)$ from the law of one price and PPP and $\pi_t \equiv \log(P_{t+1} / P_t)$.

We introduce the nominal exchange rate and the link between prices for two goods produced by the two countries. We call $\mathcal{E}_t$ the nominal exchange rate, denoted the price of foreign currency in terms of domestic currency. We have the law of one price that goods sell for the same price in both the home and foreign countries. This requires $P_{H,t} = P_{H,t}^*\mathcal{E}_t$, $P_{F,t} / \mathcal{E}_t = P_{F,t}^*$

5. Market Clearing Conditions

Market clearing conditions

Labor market : $N_t = N_t^d = N_t^s$ (2.26)

Intermediate goods market : $Y_t(i) = Y_t$, $Y_t^*(i) = Y_t^*$ (2.27)

Final goods market :

$$\begin{aligned} (1-n)Y_t &= (1-n)D_{H,t} + nD_{H,t}^* \\ nY_t^* &= (1-n)D_{F,t} + nD_{F,t}^* \end{aligned} \tag{2.28}$$

Bond market :

$$\begin{aligned} (1-n)R_tB_t + n\mathcal{E}_t R_t^*B_t^* &= (1-n)R_tB_t^d + n\mathcal{E}_t R_t^*B_t^{*d} \\ &= (1-n)\Gamma_{t+1} + n\mathcal{E}_t \Gamma_{t+1}^* = 0 \end{aligned} \tag{2.29}$$

Walras' law holds in the economy. Suppose labor and intermediate goods markets are in equilibrium. Then combining the household's budget constraint and the profit functions of intermediate suppliers yields the bond demand function in the form[3]

$$E_t \left\{\Omega_{t,t+1}\Gamma_{t+1}\right\} = \Gamma_t - T_t + \tau\int_0^1 P_t(i)Y_t(i)di + \int_0^1 P_t(i)Y_t(i)di - P_{H,t}D_{H,t} - P_{F,t}D_{F,t} \quad (2.30)$$

Rearranging the government budget constraint, one has the bond supply function given by

$$B_t^s = R_{t-1}B_{t-1} - T_t + \tau\int_0^1 P_t(i)Y_t(i)di \quad (2.31)$$

Combining (2.30) and (2.31) gives

$$E_t \left\{\Omega_{t,t+1}\Gamma_{t+1}\right\} = \Gamma_t - \left(B_t - R_{t-1}B_{t-1}\right) + P_{H,t}Y_t - P_{H,t}D_{H,t} - P_{F,t}D_{F,t} \quad (2.32)$$

We can get foreign country's equation along with the same analogues.

$$E_t \left\{\Omega^*_{t,t+1}\Gamma^*_{t+1}\right\} = \Gamma^*_t - \left(B^*_t - R^*_{t-1}B^*_{t-1}\right) + P^*_{F,t}Y^*_t - P^*_{H,t}D^*_{H,t} - P^*_{F,t}D^*_{F,t} \quad (2.33)$$

Multiply economic scale and nominal exchange rate for country F

$$\begin{aligned}(1-n)E_t \left\{\Omega_{t,t+1}\Gamma_{t+1}\right\} &= (1-n)\left\{\Gamma_{t-1} - \left(B_t - R_{t-1}B_{t-1}\right) + P_{H,t}Y_t - P_{H,t}D_{H,t} - P_{F,t}D_{F,t}\right\} \\ n\mathcal{E}_t\, E_t \left\{\Omega^*_{t,t+1}\Gamma^*_{t+1}\right\} &= n\mathcal{E}_t \left\{\Gamma^*_t - \left(B^*_t - R^*_{t-1}B^*_{t-1}\right) + P^*_{F,t}Y^*_t - P^*_{H,t}D^*_{H,t} - P^*_{F,t}D^*_{F,t}\right\}\end{aligned} \quad (2.34)$$

From the bond market clearing condition and the zero trade balance condition, $TB = n\mathcal{E}_t\, P^*_{H,t}D^*_{H,t} - (1-n)P_{F,t}D_{F,t} = 0,$ (2.34) can be rewritten as

[3] The profit function of intermediate supplier of type *i* is given by

$$\begin{aligned}\Pi_t(i) &= (1+\tau)P_t(i)Y_t(i) - W_t(i)H_t(i) \text{ in H} \\ \Pi^*_t(i) &= (1+\tau^*)P^*_t(i)Y^*_t(i) - W^*_t(i)H_t(i) \text{ in F}\end{aligned}$$

$$
\begin{aligned}
(1-n)Y_t &= (1-n)D_{H,t} + nD^*_{H,t} \\
nY^*_t &= (1-n)D_{F,t} + nD^*_{F,t}
\end{aligned} \tag{2.35}
$$

Equation (2.35) is the final good market clearing condition. Thus, Walras' law holds in the model.

CHAPTER III

EQUILIBRIUM

For a real variable X_t, we define $\bar{x}_t = \log(\bar{X}_t / X)$, $x_t = \log(X_t / X)$, and $\hat{x}_t = \log(X_t / \bar{X}_t)$, where X and $\bar{X}_t$ are the steady state value and the natural rate of X_t, respectively.

We define X_t : the aggregate stock of the final good produced in country H per capita and X_t^* : the aggregate stock of the final good produced in country F per capita.

$$\begin{aligned}(1-n)\mathrm{X}_t &\equiv (1-n)K_{H,t} + nK_{H,t}^* \\ n\mathrm{X}_t^* &\equiv (1-n)K_{F,t} + nK_{F,t}^*\end{aligned} \tag{3.1}$$

Using good market clearing conditions and definition of $K_{j,t}$,

$$\begin{aligned}Y_t &= \mathrm{X}_t - (1-\delta_H)\mathrm{X}_{t-1} \\ Y_t^* &= \mathrm{X}_t^* - (1-\delta_F)\mathrm{X}_{t-1}^*\end{aligned} \tag{3.2}$$

$$K_t = K_t^* \qquad (3.3)[4]$$

From the demand functions (2.11)

$$\begin{aligned}(1-n)Q_{H,t}\mathrm{X}_t &= (1-\alpha)Q_t K_t \\ nQ_{F,t}^*\mathrm{X}_t^* &= \alpha Q_t^* K_t^*\end{aligned} \tag{3.4}$$

It then follows the relation of the aggregate stock of the final good produced in country H and F

[4] Proof is Appendix A.1.

$$
\begin{aligned}
\mathrm{X}_t &= \frac{1-\alpha}{1-n} H_t^{\alpha} K_t \\
\mathrm{X}_t^* &= \frac{\alpha}{n} H_t^{-(1-\alpha)} K_t^* \\
\frac{\mathrm{X}_t}{\mathrm{X}_t^*} &= \frac{n(1-\alpha)}{(1-n)\alpha} H_t \\
K_t &= \frac{1-n}{1-\alpha}\left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha} \mathrm{X}_t^{1-\alpha} \mathrm{X}_t^{*\alpha}
\end{aligned}
\tag{3.5}
$$

where $H_t \equiv Q_{F,t} / Q_{H,t} = Q_{F,t}^* / Q_{H,t}^*$

1. Steady-State Equilibrium

In the steady-state (with no inflation), from the first order necessary conditions for the household's decision problem, we have:

$$R = 1/\beta \tag{3.6}$$

$$(K_F / K_H)(1-\alpha/\alpha) = H^{-1} \tag{3.7}$$

$$\frac{v_N(Y/A)}{u_k(K\ )} = \frac{W}{Q} \tag{3.8}$$

Laws of accumulation of durable goods is

$$
\begin{aligned}
&D_j = \delta_j K_j \\
&Y = \delta_H \mathrm{X}, \qquad Y^* = \delta_F \mathrm{X}^*
\end{aligned}
\tag{3.9}
$$

Market clearing conditions (final and intermediate goods markets, labor market, bond market)

$$
\begin{aligned}
&(1-n)Y = (1-n)D_H + nD_H^* \\
&nY^* = (1-n)D_F + nD_F^*
\end{aligned}
\tag{3.10}
$$

$$Y(i) = Y\ ,\ Y^*(i) = Y \tag{3.11}$$

$$Y\ (i) = A\,N(i),\ Y^*(i) = A\,N^*(i) \tag{3.12}$$

$$(1-n)\Gamma + n\mathcal{E}\Gamma^* = 0 \tag{3.13}$$

2. Flexible-Price Equilibrium

When prices are fully flexible, prices are set as a mark-up over marginal costs, monetary policy is neutral, and thus, real variables are affected by only real disturbances (productivity shocks in our model).

In flexible-price equilibrium, quantities and prices of each differentiated good are equal:

$$Y_t(i) = Y_t,\ Y_t^*(i) = Y_t^* \tag{3.14}$$

and

$$P_t(i) = P_t,\ P_t^*(i) = P_t^* \tag{3.15}$$

The Domestic Flexible Price Equilibrium

The flexible domestic price equilibrium can be represented by the following a linearized system:

From (3.2),

$$\bar{y}_t = \frac{1}{\delta_H}\left(\bar{\chi}_t - (1-\delta_H)\bar{\chi}_{t-1}\right) \tag{3.16}$$

From (3.5),

$$\bar{\chi}_t = \bar{k}_t + \alpha\bar{h}_t \tag{3.17}$$

$$\bar{h}_t = \bar{\chi}_t - \chi_t^* \tag{3.18}$$

$$\overline{k}_t = (1-\alpha)\overline{\chi}_t + \alpha\chi_t^* \tag{3.19}$$

Define $\sigma \equiv -u_k / (Ku_{kk})$ is the intertemporal elasticity of substitution of private expenditure, and $\omega \equiv Nv_{NN} / v_N$ is the elasticity of marginal disutility of work. Note that the upper bar means the domestic flexible price equilibrium.

$$\overline{rr}_{H,t}^C = \frac{\sigma_0}{\sigma} E_t \Delta\overline{\chi}_{t+1} + \frac{\sigma_1}{\sigma} E_t \Delta\chi_{t+1}^* \qquad (3.20)^5$$

where $\sigma_0 \equiv \sigma + (1-\alpha)(1-\sigma)$, $\sigma_1 \equiv \alpha(1-\sigma)$ and $\Delta\overline{\chi}_{t+1} \equiv \overline{\chi}_{t+1} - \overline{\chi}_t$

Every final goods producing firm chooses the same price under flexible prices. It follows $P_t(i)/P_{H,t} = 1$ on (3.18). Using real marginal cost $\overline{MC}_t(i) = \dfrac{1-\tau}{\mu}$, we have the country's natural rates from (3.19)

$$\frac{\sigma_0}{\sigma}\overline{\chi}_t + \frac{\sigma_1}{\sigma}\chi_t^* + \omega\overline{y}_t + \frac{1-q_H}{q_H}\overline{rr}_{H,t}^A - (1+\omega)a_t = 0 \qquad (3.21)^6$$

where $q_H \equiv Q_H / P_H = 1 - \beta(1-\delta_H)$. The relationship between real interest rates by the acquisition approach and the user cost approach is

$$\overline{rr}_{H,t}^C = \frac{1}{q_H}\left(\overline{rr}_{H,t}^A - (1-q_H)E_t\overline{rr}_{H,t+1}^A\right) \tag{3.22}$$

Assuming the technology shocks are stationary, the natural rate of the durable good stock evolves the following process[7]:

$$\eta_1\overline{\chi}_t - \eta_2\left(\beta E_t\overline{\chi}_{t+1} + \overline{\chi}_{t-1}\right) + \frac{\sigma_1}{\sigma} q_H \chi_t^* = (1+\omega)\left(a_t - (1-q_H)E_t a_{t+1}\right) \tag{3.23}$$

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[5] Appendix A.2.
[6] Appendix A.3.
[7] Appendix A.4.

where $\eta_1 \equiv \left[\frac{\sigma_0}{\sigma} q_H + \frac{\omega}{\delta_H}\left(1+\beta(1-\delta_H)^2\right) \right]$, $\eta_2 \equiv \frac{\omega}{\delta_H}(1-\delta_H)$

For $0 < \upsilon < 1-\delta_H$,

$$\frac{\eta_2}{\upsilon}\left(-\beta\upsilon E_t \overline{\chi}_{t+1} - \upsilon\overline{\chi}_{t-1} + \frac{\eta_1 \upsilon}{\eta_2}\overline{\chi}_t \right) + \frac{\sigma_1}{\sigma} q_H \chi_t^* = (1+\omega)\left(a_t - (1-q_H)E_t a_{t+1}\right)$$

$$\frac{\eta_2}{\upsilon}\left\{ \left(\overline{\chi}_t - \upsilon\overline{\chi}_{t-1}\right) - \beta\upsilon E_t \overline{\chi}_{t+1} + \left(\frac{\eta_1 \upsilon}{\eta_2} - 1\right)\overline{\chi}_t \right\} + \frac{\sigma_1}{\sigma} q_H \chi_t^* = (1+\omega)\left(a_t - (1-q_H)E_t a_{t+1}\right)$$

$$\frac{\eta_2}{\upsilon}\left\{ \left(\overline{\chi}_t - \upsilon\overline{\chi}_{t-1}\right) - \beta\upsilon\left(E_t \overline{\chi}_{t+1} - \upsilon\overline{\chi}_t\right) \right\} + \frac{\sigma_1}{\sigma} q_H \chi_t^* = (1+\omega)\left(a_t - (1-q_H)E_t a_{t+1}\right) \tag{3.24}$$

with $\eta_1 \upsilon = \eta_2(\beta\upsilon^2 + 1)$.

The Flexible Price Equilibrium

The double upper bar means the natural level when prices are flexible worldwide.

$$\overline{\overline{y}}_t^* = \frac{1}{\delta_H}\left(\overline{\overline{\chi}}_t^* - (1-\delta_H)\overline{\overline{\chi}}_{t-1}^*\right) \tag{3.25}$$

From (3.5),

$$\overline{\overline{\chi}}_t^* = \overline{\overline{k}}_t^* - (1-\alpha)\overline{\overline{h}}_t^* \tag{3.26}$$

$$\overline{\overline{h}}_t = \overline{\overline{\chi}}_t - \overline{\overline{\chi}}_t^* \tag{3.27}$$

$$\overline{\overline{k}}_t = (1-\alpha)\overline{\overline{\chi}}_t + \alpha\overline{\overline{\chi}}_t^* \tag{3.28}$$

$$\overline{\overline{rr}}_{H,t}^C = \frac{\sigma_0}{\sigma} E_t \Delta\overline{\overline{\chi}}_{t+1} + \frac{\sigma_1}{\sigma} E_t \Delta\overline{\overline{\chi}}_{t+1}^* \tag{3.29}$$

where $\sigma_0 \equiv \sigma + (1-\alpha)(1-\sigma)$, $\sigma_1 \equiv \alpha(1-\sigma)$

$$\overline{\overline{rr}}_{F,t}^C = \frac{\sigma_2}{\sigma} E_t \Delta\overline{\overline{\chi}}_{t+1} + \frac{\sigma_3}{\sigma} E_t \Delta\overline{\overline{\chi}}_{t+1}^* \tag{3.30}$$

where $\sigma_2 \equiv (1-\alpha)(1-\sigma)$, $\sigma_3 \equiv \sigma + \alpha(1-\sigma)$

$$\eta_1 \bar{\bar{\chi}}_t - \eta_2 \left(\beta E_t \bar{\bar{\chi}}_{t+1} + \bar{\bar{\chi}}_{t-1}\right) + \frac{\sigma_1}{\sigma} q_H \bar{\bar{\chi}}_t^* = (1+\omega)\left(a_t - (1-q_H) E_t a_{t+1}\right) \tag{3.31}$$

where $\eta_1 \equiv \left[\frac{\sigma_0}{\sigma} q_H + \frac{\omega}{\delta_H}\left(1+\beta(1-\delta_H)^2\right)\right]$, $\eta_2 \equiv \frac{\omega}{\delta_H}(1-\delta_H)$

$$\eta_3 \bar{\bar{\chi}}_t^* - \eta_4 \left(\beta E_t \bar{\bar{\chi}}_{t+1}^* + \bar{\bar{\chi}}_{t-1}^*\right) + \frac{\sigma_2}{\sigma} q_F \bar{\bar{\chi}}_t = (1+\omega)\left(a_t - (1-q_F) E_t a_{t+1}\right) \tag{3.32}$$

where $\eta_3 \equiv \left[\frac{\sigma_3}{\sigma} q_F + \frac{\omega}{\delta_F}\left(1+\beta(1-\delta_F)^2\right)\right]$, $\eta_4 \equiv \frac{\omega}{\delta_F}(1-\delta_F)$

3. Sticky-Price Equilibrium

The log-linearization of the Euler equation and the equations around the zero-inflation steady-state, respectively, are given by

$$k_t = E_t k_{t+1} - \sigma\left(r_t - E_t \pi_{t+1}^C\right) \tag{3.33}$$

$$y_t = \frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) \tag{3.34}$$

$$\chi_t = k_t + \alpha h_t \tag{3.35}$$

$$h_t = \chi_t - \chi_t^* \tag{3.36}$$

where $\pi_t^C \equiv \log\left(Q_t / Q_{t-1}\right)$ is the COLI inflation rate, and the real interest rate measured by the user cost can be rewritten as

$$r_t - E_t \pi_{t+1}^C = \left(r_t - E_t \pi_{H,t+1}^C\right) - \alpha\left(E_t h_{t+1} - h_t\right) \tag{3.37}$$

$$r_t - E_t \pi_{H,t+1}^C = \frac{1}{q_H} E_t\left[\left(r_t - E_t \pi_{H,t+1}^A\right) - (1-q_H)\left(r_{t+1} - \pi_{H,t+2}^A\right)\right] \tag{3.38}$$

where $\pi_{H,t}^C \equiv \log\left(Q_{H,t} / Q_{H,t-1}\right)$, and $\pi_{H,t}^A \equiv \log\left(P_{H,t} / P_{H,t-1}\right)$.

Combining above equations, one obtains

$$\sigma_0 E_t \Delta\chi_{t+1} + \sigma_1 E_t \Delta\chi^*_{t+1} = \sigma rr^C_{H,t} \tag{3.39}$$

With (3.20),

$$\hat{\chi}_t = E_t \hat{\chi}_{t+1} - \frac{\sigma}{\sigma_0}\left(r_t - E_t \pi^C_{H,t+1} - \overline{rr}^C_{H,t}\right) \tag{3.40}$$

With (3.29),

$$\frac{\sigma_0}{\sigma}\left(E_t \hat{\hat{\chi}}_{t+1} - \hat{\hat{\chi}}_t\right) + \frac{\sigma_1}{\sigma}\left(E_t \hat{\hat{\chi}}^*_{t+1} - \hat{\hat{\chi}}^*_t\right) = rr^C_{H,t} - \overline{\overline{rr}}^C_{H,t} \tag{3.41}$$

With (3.30),

$$\frac{\sigma_2}{\sigma}\left(E_t \hat{\hat{\chi}}_{t+1} - \hat{\hat{\chi}}_t\right) + \frac{\sigma_3}{\sigma}\left(E_t \hat{\hat{\chi}}^*_{t+1} - \hat{\hat{\chi}}^*_t\right) = rr^C_{F,t} - \overline{\overline{rr}}^C_{F,t} \tag{3.42}$$

We have the Phillips curve equation, as follows[8];

$$\pi_t = \beta E_t \pi_{t+1} + \varphi(1+\omega\theta)\left\{\begin{array}{l} \left(\dfrac{\sigma_0}{\sigma} + \dfrac{\omega}{\delta_H}\right)\hat{\chi}_t - \dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_{t-1} \\ + \dfrac{1-q_H}{q_H}\left(r_t - E_t \pi^A_{H,t+1} - \overline{rr}^A_{H,t}\right) \end{array}\right\} \tag{3.43}$$

where $\varphi = \dfrac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$.

Using (3.22) and (3.40) to substitute for the output gap and interest rate terms in (3.43), the supply equation can be rewritten equivalently in the form

$$\begin{aligned} \left(\pi_t - \beta E_t \pi_{t+1}\right) &= \beta(1-\delta_H)E_t\left(\pi_{t+1} - \beta\pi_{t+2}\right) \\ &\quad + \varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}E_t\left[\left(\hat{\chi}_t - \upsilon\hat{\chi}_{t-1}\right) - \beta\upsilon\left(E_t\hat{\chi}_{t+1} - \upsilon\hat{\chi}_t\right)\right] \end{aligned} \tag{3.44}$$

[8] Appendix A.5.

For the worldwide flexible price, we have the New Keynesian Phillips Curve of Country H, as follow;

$$\pi_t = \beta E_t \pi_{t+1} + \varphi(1+\omega\theta)\left\{ \begin{array}{l} \left(\dfrac{\sigma_0}{\sigma} + \dfrac{\omega}{\delta_H}\right)\hat{\hat{\chi}}_t - \dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\hat{\chi}}_{t-1} \\ + \dfrac{\sigma_1}{\sigma}\hat{\hat{\chi}}_t^* + \dfrac{1-q_H}{q_H}\left(rr_{tH,t}^A - \overline{\overline{rr}}_{H,t}^A\right) \end{array} \right\} \tag{3.45}$$

As shown in the Appendix A.5., equation (3.45) can be simplified as below;

$$\begin{aligned} \left(\pi_t - \beta E_t \pi_{t+1}\right) &= \beta(1-\delta_H)E_t\left(\pi_{t+1} - \beta\pi_{t+2}\right) \\ &+ \varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}E_t\left[\left(\hat{\chi}_t - \upsilon\hat{\chi}_{t-1}\right) - \beta\upsilon\left(E_t\hat{\chi}_{t+1} - \upsilon\hat{\chi}_t\right)\right] + \varphi(1+\omega\theta)\frac{\sigma_1}{\sigma}q_H\hat{\hat{\chi}}_t^* \end{aligned} \tag{3.46}$$

A similar analogy is derived for the New Keynesian Phillips Curve of Country F:

$$\begin{aligned} \left(\pi_t^* - \beta E_t \pi_{t+1}^*\right) &= \beta(1-\delta_F)E_t\left(\pi_{t+1}^* - \beta\pi_{t+2}^*\right) \\ &+ \varphi(1+\omega\theta)\frac{\eta_4}{\upsilon}E_t\left[\left(\hat{\hat{\chi}}_t^* - \upsilon\hat{\hat{\chi}}_{t-1}^*\right) - \beta\upsilon\left(E_t\hat{\hat{\chi}}_{t+1}^* - \upsilon\hat{\hat{\chi}}_t^*\right)\right] + \varphi(1+\omega\theta)\frac{\sigma_2}{\sigma}q_F\hat{\hat{\chi}}_t \end{aligned} \tag{3.47}$$

CHAPTER IV

THE WELFARE FUNCTION AND OPTIMAL MONETARY POLICY

1. The Welfare Function and Optimal Subsidy Rate

Case 1: Non-Cooperative Case (Nash Equilibrium)

The domestic monetary policy authority seeks to maximize the utility of the household, taking as given the other country's policy and outcomes.

In open economy, the flexible price economy is not the optimal, because the domestic central bank has an incentive to make a surprise change of price level for advantage of a household of the domestic country. Therefore, the fiscal authority chooses a production subsidy rate that makes the natural level of output correspond to the efficient level in a zero inflation steady state, as follows[9];

$$1+\tau=\mu(1-n) \tag{5.1}$$

Note that subsidy depends only on the openness (n), not on the durability (δ).

We can get the monetary authority's objective function by taking second-order approximation of the utility function of the representative household around the domestic flexible price equilibrium. As shown in the Appendix A.7., we obtain the following quadratic objective function:

$$\begin{aligned}W_{Nash}=&-\frac{(1-\alpha)}{2}\frac{\delta_H}{q_H}\frac{\theta}{\varphi}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varphi}{\theta\delta_H}\frac{\eta_2}{\upsilon}\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)^2+\left(\pi_t\right)^2\right\}\\&+t.i.p+O\left(\left\|k,y,a\right\|^3\right)\end{aligned} \tag{5.2}$$

[9] Appendix A.6.

where $\varphi \equiv \dfrac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$

Case 2: Cooperative Case

In this section, we analyze optimal policy under cooperation. We assume that the two monetary authorities maximize a weighted sum of the objective functions of the home and foreign households.

Like the Nash equilibrium, we assume that both fiscal authorities choose the subsidy rate that maximizes objective function in the steady state. The optimal subsidy rate for domestic good is, as follows[10];

$$1+\tau = \mu \tag{5.3}$$

We can get a similar equation of the optimal subsidy rate for foreign good.

$$1+\tau^{*} = \mu \tag{5.4}$$

This means that two countries' fiscal authorities (H, F) choose the common subsidy.

$$1+\tau = \mu = 1+\tau^{*} \tag{5.5}$$

We can obtain the monetary authority's objective function by taking second-order approximation of the utility function of a weighted average by the relative economic size around the globally flexible price equilibrium. As we show in the Appendix A.9., we obtain the following quadratic objective function:

[10] Appendix A.8.

$$W_{Cooperative}=-(1-\alpha)\frac{\delta_H}{2q_H}u_kKE_0\sum_{t=0}^{\infty}\beta^t\begin{pmatrix}\frac{1}{\delta_H}\frac{\eta_2}{\upsilon}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t-\upsilon\hat{\hat{\chi}}_{t-1}\right)^2+\left(\pi_t\right)^2\\+\frac{n}{1-n}\frac{1}{\delta_F}\frac{\eta_4}{\upsilon^*}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t^*-\upsilon^*\hat{\hat{\chi}}_{t-1}^*\right)^2\\+\frac{n}{1-n}\left(\pi_t^*\right)^2\end{pmatrix}\tag{5.6}$$

$$+t.i.p+O\left(\left\|k,y,a\right\|^3\right)$$

2. Optimal Monetary Policy

Case 1 : Non-Cooperative Case (Nash Equilibrium)

An optimal monetary policy in the Nash equilibrium seeks to minimize the discounted sum of losses (5.2) subject to the sticky-price equilibrium given by equations (3.40) and (3.44). The Lagrangian for this problem is given by

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{\varphi}{\theta\delta_H}\frac{\eta_2}{\upsilon}\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)^2+\left(\pi_t\right)^2+2\phi_{1,t}\left[\hat{\chi}_t-E_t\hat{\chi}_{t+1}+\frac{\sigma}{\sigma_0}\left(rr_{H,t}^C-\overline{rr}_{H,t}^C\right)\right]\\+2\phi_{2,t}\left[\begin{array}{l}\pi_t-\beta E_t\pi_{t+1}-\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\+\varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}E_t\left[\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)-\beta\upsilon\left(E_t\hat{\chi}_{t+1}-\upsilon\hat{\chi}_t\right)\right]\end{array}\right]\end{array}\right\}\tag{5.7}$$

First order necessary conditions with respect to $rr_{H,t}^C$, $\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}$, and π_t are given by

$$\phi_{1,t}=0\tag{5.8}$$

$$\frac{\varphi}{\theta\delta_H}\frac{\eta_2}{\upsilon}\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)=\varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}\left(\phi_{2,t}-\upsilon\phi_{2,t-1}\right)\tag{5.9}$$

$$\pi_t+\phi_{2,t}-(2-\delta_H)\phi_{2,t-1}+(1-\delta_H)\phi_{2,t-2}=0\tag{5.10}$$

for each $t \geq 0$. Following Woodford (1999), the optimal policy under a timeless perspective is that the central bank implements conditions (5.9) and (5.10) for all periods. Hence, the system of the economy under an optimal policy can be represented by equations (3.44), (5.9), and (5.10) given initial conditions, $\hat{\chi}_{-1} = 0$ and $\phi_{2,-2} = \phi_{2,-1} = 0$.

Combining (5.9) and (5.10), PPI inflation and the output gap under the optimal policy satisfy

$$(\pi_t - \upsilon\pi_{t-1}) + \frac{1}{\theta\delta_H(1+\omega\theta)}(1-L)(\hat{\chi}_t - \upsilon\hat{\chi}_{t-1}) = 0 \tag{5.11}$$

where L is the lag operator.

Note that if there is no uncertainty which causes monetary policy to deviate from the optimal rule, equation (5.11) can be reduced to the optimal policy rule in a standard model given by

$$\pi_t + \frac{1}{\theta\delta_H(1+\omega\theta)}(1-L)\hat{\chi}_t = 0 \tag{5.12}$$

which implies that the optimal policy maximizing social welfare is to keep the (acquisition) price and the output gap at a constant rate that does not depend on the durability of goods.

We may combine the equation (3.40), with the above solutions to obtain an expression for an interest rate rule that implements the time-consistent policy in the Nash equilibrium:

$$\frac{1-q_H}{q_H} r_{t+1} - \frac{1}{q_H} r_t = \frac{1-q_H}{q_H} \overline{rr}^A_{H,t+1} - \frac{1}{q_H} \overline{rr}^A_{H,t} + \frac{1-q_H}{q_H} E_t \pi_{t+2} - \frac{1}{q_H} E_t \pi_{t+1} + \frac{\sigma_0}{\sigma} \frac{\theta \delta_H (1+\omega\theta)}{(1-L)} \left(E_t \pi_{t+1} - \pi_t \right) \tag{5.13}$$

The optimal rule may be expressed as the sum of three components; the domestic natural real interest rate, next period's expected domestic inflation gap, and a term indicating that the central bank adjusts the nominal rate with respect to domestic inflation gap.

We obtain a similar equation of the optimal interest rate rule for the foreign country:

$$\frac{1-q_F}{q_F} r^*_{t+1} - \frac{1}{q_F} r^*_t = \frac{1-q_F}{q_F} \overline{rr}^{*A}_{F,t+1} - \frac{1}{q_F} \overline{rr}^{*A}_{H,t} + \frac{1-q_F}{q_F} E_t \pi^*_{t+2} - \frac{1}{q_F} E_t \pi^*_{t+1} + \frac{\sigma_3}{\sigma} \frac{\theta \delta_F (1+\omega\theta)}{(1-L)} \left(E_t \pi^*_{t+1} - \pi^*_t \right) \tag{5.14}$$

Intuitively, the monetary authority needs to change the nominal interest rate less with respect to durability, because the monetary policy have more influence on durable goods than non-durable goods. The country which produces durable goods (e.g. $\delta_H < 1$) may adjust the nominal interest rate less in response to the domestic inflation rate.

Case 2: Cooperative Case

An optimal monetary policy in the Cooperation case seeks to minimize the discounted sum of losses (5.6) subject to the sticky-price equilibrium given by equations (3.41), (3.42) and (3.46), (3.47). The Lagrangian for this problem is given by

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{1}{\delta_H}\frac{\eta_2}{\upsilon}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t-\upsilon\hat{\hat{\chi}}_{t-1}\right)^2+\left(\pi_t\right)^2\\+\frac{n}{1-n}\frac{1}{\delta_F}\frac{\eta_4}{\upsilon^*}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t^*-\upsilon^*\hat{\hat{\chi}}_{t-1}^*\right)^2+\frac{n}{1-n}\left(\pi_t^*\right)^2\\+2\psi_{1,t}\left[rr_{H,t}^C-\overline{\overline{rr}}_{H,t}^C-\frac{\sigma_0}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}-\hat{\hat{\chi}}_t\right)-\frac{\sigma_1}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}^*-\hat{\hat{\chi}}_t^*\right)\right]\\+2\psi_{2,t}\left[rr_{F,t}^C-\overline{\overline{rr}}_{F,t}^C-\frac{\sigma_2}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}-\hat{\hat{\chi}}_t\right)-\frac{\sigma_3}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}^*-\hat{\hat{\chi}}_t^*\right)\right]\\+2\psi_{3,t}\left[\begin{array}{l}\left(\pi_t-\beta E_t\pi_{t+1}\right)-\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\-\varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}E_t\left[\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)-\beta\upsilon\left(E_t\hat{\chi}_{t+1}-\upsilon\hat{\chi}_t\right)\right]\\-\varphi(1+\omega\theta)\frac{\sigma_1}{\sigma}q_H\hat{\hat{\chi}}_t^*\end{array}\right]\\+2\psi_{4,t}\left[\begin{array}{l}\left(\pi_t^*-\beta E_t\pi_{t+1}^*\right)-\beta(1-\delta_F)E_t\left(\pi_{t+1}^*-\beta\pi_{t+2}^*\right)\\-\varphi(1+\omega\theta)\frac{\eta_4}{\upsilon}E_t\left[\left(\hat{\hat{\chi}}_t^*-\upsilon\hat{\hat{\chi}}_{t-1}^*\right)-\beta\upsilon\left(E_t\hat{\hat{\chi}}_{t+1}^*-\upsilon\hat{\hat{\chi}}_t^*\right)\right]\\-\varphi(1+\omega\theta)\frac{\sigma_2}{\sigma}q_F\hat{\hat{\chi}}_t\end{array}\right]\end{array}\right\} \quad (5.15)$$

First order necessary conditions with respect to $rr_{H,t}^C$, $rr_{F,t}^C$, $\hat{\hat{\chi}}_t-\upsilon\hat{\hat{\chi}}_{t-1}$, $\hat{\hat{\chi}}_t^*-\upsilon\hat{\hat{\chi}}_{t-1}^*$, and π_t, π_t^* are given by

$$\psi_{1,t}=0 \quad (5.16)$$

$$\psi_{2,t}=0 \quad (5.17)$$

$$\frac{1}{\delta_H}\frac{\eta_2}{\upsilon}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t-\upsilon\hat{\hat{\chi}}_{t-1}\right)=\varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}\left(\psi_{3,t}-\upsilon\psi_{3,t-1}\right) \quad (5.18)$$

$$\frac{n}{1-n}\frac{1}{\delta_F}\frac{\eta_4}{\upsilon^*}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t^*-\upsilon^*\hat{\hat{\chi}}_{t-1}^*\right)=\varphi(1+\omega\theta)\frac{\eta_4}{\upsilon}\left(\psi_{4,t}-\upsilon\psi_{4,t-1}\right) \quad (5.19)$$

$$\pi_t + \psi_{3,t} - (2-\delta_H)\psi_{3,t-1} + (1-\delta_H)\psi_{3,t-2} = 0 \tag{5.20}$$

$$\frac{n}{1-n}\pi_t^* + \psi_{4,t} - (2-\delta_F)\psi_{4,t-1} + (1-\delta_F)\psi_{4,t-2} = 0 \tag{5.21}$$

for each $t \geq 0$.

Combining (5.18), (5.19), (5.20), and (5.21), PPI inflation and the output gap under the optimal policy in the cooperative equilibrium can be written as follows

$$(\pi_t - \upsilon\pi_{t-1}) + \frac{1}{\theta\delta_H(1+\omega\theta)}(1-L)(\hat{\hat{\chi}}_t - \upsilon\hat{\hat{\chi}}_{t-1}) = 0 \tag{5.22}$$

$$(\pi_t^* - \upsilon\pi_{t-1}^*) + \frac{1}{\theta\delta_F(1+\omega\theta)}(1-L)(\hat{\hat{\chi}}_t^* - \upsilon\hat{\hat{\chi}}_{t-1}^*) = 0 \tag{5.23}$$

where L is the lag operator.

We may combine the equations of (3.41) and (3.42), with the above solutions to obtain an expression for an interest rate rule that implements the time-consistent policy under the cooperative case:

$$\begin{aligned}\frac{1-q_H}{q_H}r_{t+1} - \frac{1}{q_H}r_t = \frac{1-q_H}{q_H}\overline{\overline{rr}}_{H,t+1}^A - \frac{1}{q_H}\overline{\overline{rr}}_{H,t}^A + \frac{1-q_H}{q_H}E_t\pi_{t+2} - \frac{1}{q_H}E_t\pi_{t+1} \\ + \frac{\sigma_0}{\sigma}\frac{\theta\delta_H(1+\omega\theta)}{(1-L)}\left(E_t\pi_{t+1} - \pi_t\right) + \frac{\sigma_1}{\sigma}\frac{\theta\delta_F(1+\omega\theta)}{(1-L)}\left(E_t\pi_{t+1}^* - \pi_t^*\right)\end{aligned} \tag{5.24}$$

If we consider the case of $\frac{1-q_H}{q_H}\overline{\overline{rr}}_{H,t+1}^A - \frac{1}{q_H}\overline{\overline{rr}}_{H,t}^A = \frac{1-q_H}{q_H}\overline{rr}_{H,t+1}^A - \frac{1}{q_H}\overline{rr}_{H,t}^A$, so the real interest rate in the domestic flexible price equilibrium is the same as in the globally flexible price equilibrium, optimal policy in the cooperative equilibrium can be written as a linear rule of optimal policy in the Nash equilibrium and foreign inflation gap. This suggests that the central bank of the country which imports non-durable goods should raise the interest rate by more relative to the Nash case in response to a rise in foreign inflation.

A similar analogy is derived for an interest rate rule of Country F:

$$
\begin{aligned}
\frac{1-q_F}{q_F} r^*_{t+1} - \frac{1}{q_F} r^*_t = \frac{1-q_F}{q_F} \overline{\overline{rr}}^A_{F,t+1} - \frac{1}{q_F} \overline{\overline{rr}}^A_{F,t} + \frac{1-q_F}{q_F} E_t \pi^*_{t+2} - \frac{1}{q_F} E_t \pi^*_{t+1} \\
+ \frac{\sigma_2}{\sigma} \frac{\theta\delta_H (1+\omega\theta)}{(1-L)} \left(E_t \pi_{t+1} - \pi_t \right) + \frac{\sigma_3}{\sigma} \frac{\theta\delta_F (1+\omega\theta)}{(1-L)} \left(E_t \pi^*_{t+1} - \pi^*_t \right)
\end{aligned}
\tag{5.25}
$$

This suggests that the central bank of the country which imports durable goods should raise the interest rate by less relative to the Nash case in response to a rise in inflation of country H.

CHAPTER V

CONCLUSIONS

This dissertation studies a model of an economy which produces and exports durable goods. It analyzes the optimal monetary policy for such a country.

Generally, monetary policy has a bigger economic effect on durable goods relative to non-durable goods because durable goods can be stored and households get utility from the stock of durable goods. This paper shows that, in Nash equilibrium, the central bank of a durable goods producing country can control changes of the price level with smaller changes in the monetary policy instrument. In the cooperative equilibrium, the monetary authority of the country which imports non-durable goods and exports durable goods should raise the interest rate by more, relative to the Nash case, in response to a rise in foreign inflation. On the other hand, the monetary authority of the country which imports durable goods and exports non-durable goods should raise the interest rate by less than the other country.

Future work might find interesting implications from examining different values of δ, and this will probably require numerical analysis. It might also be interesting to show the behavior of exchange rates from imperfect exchange rate pass-through settings.

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APPENDIX A

A.1. Proof of $K_t = K_t^*$ (3.3)

(pf) From FOC in H $\dfrac{u_k(K_t)}{u_k(K_{t+1})} = \beta R_t E_t \dfrac{Q_t}{Q_{t+1}}$

FOC in F $\dfrac{u_k(K_t^*)}{u_k(K_{t+1}^*)} = \beta R_t^* E_t \dfrac{Q_t^*}{Q_{t+1}^*}$

Given the international tradability of state-contingent bond, the intertemporal efficiency condition is

$$\frac{u_k(K_t^*)}{u_k(K_{t+1}^*)} = \beta R_t E_t \frac{Q_t^*}{Q_{t+1}^*} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$$

The law of one price, $P_{H,t} = P_{H,t}^* \mathcal{E}_t$, $P_{F,t} / \mathcal{E}_t = P_{F,t}^*$ for all t, and the definition of the user cost (2.9) and the cost of living index (2.10), and the uncovered interest parity $r_t - r_t^* = E_t \varepsilon_{t+1} - \varepsilon_t$ yields

$$K_t = K_t^*$$

A.2. Deriving the equation of (3.20)

From (2.6),

$$1 = \beta R_t E_t \left\{ \frac{u_k(K_{t+1})}{u_k(K_t)} \frac{Q_t}{Q_{t+1}} \right\}$$

$$\frac{u_k(K_t)}{u_k(K_{t+1})} = \beta R_t \frac{K_{H,t}/K_t}{K_{H,t+1}/K_{t+1}} \frac{P_{H,t} - (1-\delta_H)P_{H,t-1}/R_t}{P_{H,t+1} - (1-\delta_H)P_{H,t}/R_{t+1}}$$
$$= \beta R_t \frac{K_{H,t}/K_t}{K_{H,t+1}/K_{t+1}} \frac{Q_{H,t}}{Q_{H,t+1}}$$

$$\frac{u_k(K_t)}{u_k(K_{t+1})} = \beta R_t \frac{K_{H,t}/K_t}{K_{H,t+1}/K_{t+1}} \frac{Q_{H,t}}{Q_{H,t+1}}$$
$$\frac{K_{H,t+1}/K_{t+1}}{K_{H,t}/K_t} \frac{u_k(K_t)}{u_k(K_{t+1})} = \beta R_t \frac{Q_{H,t}}{Q_{H,t+1}}$$

$$-(k_{H,t} - k_{H,t+1}) + (k_t - k_{t+1}) - \sigma^{-1}(k_t - k_{t+1}) = rr^C_{H,t+1}$$

Using (3.19),

$$\overline{k}_t = (1-\alpha)\overline{\chi}_t + \alpha\chi^*_t, \qquad \overline{h}_t = \overline{\chi}_t - \chi^*_t$$

$$\log(K_{H,t}) = \log(1-\alpha) + \log(K_t Q_t / Q_{H,t})$$
$$= \log(K_t) + \alpha \log(H_t)$$

$$\sigma(k_{H,t} - k_{H,t+1}) - \sigma(k_t - k_{t+1}) + (k_t - k_{t+1}) = -\sigma rr^C_{H,t+1}$$

$$\sigma\left\{k_t - k_{t+1} + \alpha\left(h_t - h_{t+1}\right)\right\} + (1-\sigma)(k_t - k_{t+1}) = -\sigma rr^C_{H,t+1}$$

$$\sigma\left\{(1-\alpha)\overline{\chi}_t + \alpha\chi^*_t - ((1-\alpha)\overline{\chi}_{t+1} + \alpha\chi^*_{t+1}) + \alpha\left(\overline{\chi}_t - \chi^*_t - (\overline{\chi}_{t+1} - \chi^*_{t+1})\right)\right\}$$
$$+(1-\sigma)((1-\alpha)\overline{\chi}_t + \alpha\chi^*_t - ((1-\alpha)\overline{\chi}_{t+1} + \alpha\chi^*_{t+1})) = -\sigma rr^C_{H,t+1}$$

$$\sigma\left\{\overline{\chi} - \overline{\chi}_{t+1}\right\} + (1-\sigma)(1-\alpha)\left\{\overline{\chi}_t - \overline{\chi}_{t+1}\right\} + (1-\sigma)\alpha(\chi^*_t - \chi^*_{t+1}) = -\sigma rr^C_{H,t+1}$$

$$\overline{rr}^C_{H,t} = \frac{\sigma_0}{\sigma} E_t \Delta\overline{\chi}_{t+1} + \frac{\sigma_1}{\sigma} E_t \Delta\chi^*_{t+1}$$

where $\sigma_0 \equiv \sigma + (1-\alpha)(1-\sigma)$, $\sigma_1 \equiv \alpha(1-\sigma)$

A.3. Deriving the country's natural rates

From (3.19)

$$\frac{P_t(i)}{P_{H,t}} = \frac{\mu}{1+\tau}\frac{v_N(Y_t \ / A_t)}{u_k(K_t)}\frac{Q_t}{A_t P_{H,t}}$$

$$1 = \frac{\mu}{1+\tau}\frac{v_N(Y_t \ / A_t)}{u_k(K_t)}\frac{Q_t}{A_t P_{H,t}}$$

$$1 = \frac{\mu}{1+\tau}\frac{(Y_t \ / A_t)^{\omega}}{(K_t)^{-\sigma^{-1}}}\frac{Q_t}{A_t P_{H,t}}$$

$$Y_t^{\omega} A_t^{-(1+\omega)} K_t^{\sigma^{-1}} \left(\frac{Q_t}{P_{H,t}}\right) = 1$$

$$Y_t^{\omega} A_t^{-(1+\omega)} \left(\frac{1-n}{1-\alpha}\left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha} \mathrm{X}_t^{1-\alpha}\mathrm{X}_t^{*\alpha}\right)^{\sigma^{-1}} \left(\frac{Q_{H,t}}{P_{H,t}}\frac{Q_t}{Q_{H,t}}\right) = 1$$

$$Y_t^{\omega} A_t^{-(1+\omega)} \left(\frac{1-n}{1-\alpha}\left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha} \mathrm{X}_t^{1-\alpha}\mathrm{X}_t^{*\alpha}\right)^{\sigma^{-1}} \left(H_t\right)^{\alpha} \left(\frac{Q_{H,t}}{P_{H,t}}\right) = 1$$

$$Y_t^{\omega} A_t^{-(1+\omega)} \left(\frac{1-n}{1-\alpha}\left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha} \mathrm{X}_t^{1-\alpha}\mathrm{X}_t^{*\alpha}\right)^{\sigma^{-1}} \left(\frac{(1-n)\alpha}{n(1-\alpha)}\frac{\mathrm{X}_t}{\mathrm{X}_t^{*}}\right)^{\alpha} \left(\frac{Q_{H,t}}{P_{H,t}}\right) = 1$$

$$Y_t^{\omega} A_t^{-(1+\omega)} \left(\frac{1-n}{1-\alpha}\left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha}\right)^{\sigma^{-1}} \left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha} \mathrm{X}_t^{(1-\alpha)\sigma^{-1}+\alpha}\mathrm{X}_t^{*\alpha\sigma^{-1}-\alpha} \left(\frac{Q_{H,t}}{P_{H,t}}\right) = 1$$

$$\frac{\sigma_0}{\sigma}\overline{\chi}_t + \frac{\sigma_1}{\sigma}\chi_t^{*} + \omega\overline{y}_t + \frac{1-q_H}{q_H}\overline{rr}_{H,t}^{A} - (1+\omega)a_t = 0$$

A.4. Analytical solution to flexible-price equilibrium

Combining $\bar{y}_t = \frac{1}{\delta_H}\left(\bar{\chi}_t - (1-\delta_H)\bar{\chi}_{t-1}\right)$

and $\frac{\sigma_0}{\sigma}\bar{\chi}_t + \frac{\sigma_1}{\sigma}\chi_t^* + \omega\bar{y}_t + \frac{1-q_H}{q_H}\overline{rr}_{H,t}^A - (1+\omega)a_t = 0$, one gets

$$\begin{aligned}
&\frac{\sigma_0}{\sigma}\bar{\chi}_t + \frac{\sigma_1}{\sigma}\chi_t^* + \frac{\omega}{\delta_H}\left(\bar{\chi}_t - (1-\delta_H)\bar{\chi}_{t-1}\right) + \frac{1-q_H}{q_H}\overline{rr}_{H,t}^A - (1+\omega)a_t = 0 \\
&\left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H}\right)\bar{\chi}_t + \frac{\sigma_1}{\sigma}\chi_t^* - \frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1} + \frac{1-q_H}{q_H}\overline{rr}_{H,t}^A - (1+\omega)a_t = 0
\end{aligned} \tag{A.1}$$

Using $\overline{rr}_{H,t}^C = \frac{1}{q_H}\left(\overline{rr}_{H,t}^A - (1-q_H)E_t\overline{rr}_{H,t+1}^A\right)$ to substitute for $\overline{rr}_t^C$ in $\overline{rr}_{H,t}^C = \frac{\sigma_0}{\sigma}E_t\Delta\bar{\chi}_{t+1} + \frac{\sigma_1}{\sigma}E_t\Delta\chi_{t+1}^*$, one gets

$$\frac{\sigma_0}{\sigma}E_t\Delta\bar{\chi}_{t+1} + \frac{\sigma_1}{\sigma}E_t\Delta\chi_{t+1}^* = \frac{1}{q_H}\left(\overline{rr}_{H,t}^A - (1-q_H)E_t\overline{rr}_{H,t+1}^A\right) \tag{A.2}$$

Updating (A.1) one period and taking expectations in period *t*, one obtains

$$\left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H}\right)\bar{\chi}_{t+1} + \frac{\sigma_1}{\sigma}\chi_{t+1}^* - \frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_t + \frac{1-q_H}{q_H}\overline{rr}_{H,t+1}^A - (1+\omega)a_{t+1} = 0 \tag{A.3}$$

Using (A.1) and (A.3) to substitute for $\overline{rr}_{H,t}^A$ in (A.2), one gets

$$\frac{1-q_H}{q_H}\overline{rr}_{H,t+1}^A = -\left\{\left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H}\right)E_t\bar{\chi}_{t+1} + \frac{\sigma_1}{\sigma}E_t\chi_{t+1}^* - \frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_t - (1+\omega)E_t a_{t+1}\right\}$$

$$\frac{1}{q_H}\overline{rr}_{H,t}^A = -\frac{1}{1-q_H}\left\{\left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H}\right)\bar{\chi}_t + \frac{\sigma_1}{\sigma}\chi_t^* - \frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1} - (1+\omega)a_t\right\}$$

$$\frac{\sigma_0}{\sigma}E_t\Delta\bar{\chi}_{t+1}+\frac{\sigma_1}{\sigma}E_t\Delta\chi^*_{t+1}$$

$$=-\frac{1}{1-q_H}\left\{\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\bar{\chi}_t+\frac{\sigma_1}{\sigma}\chi^*_t-\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1}-(1+\omega)a_t\right\}$$

$$+\left\{\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)E_t\bar{\chi}_{t+1}+\frac{\sigma_1}{\sigma}E_t\chi^*_{t+1}-\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_t-(1+\omega)E_ta_{t+1}\right\}$$

$$\beta(1-\delta_H)\frac{\sigma_0}{\sigma}\left\{E_t\bar{\chi}_{t+1}-\bar{\chi}_t\right\}+\beta(1-\delta_H)\frac{\sigma_1}{\sigma}\left\{E_t\chi^*_{t+1}-\chi^*_t\right\}$$

$$=\beta(1-\delta_H)\left\{\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)E_t\bar{\chi}_{t+1}+\frac{\sigma_1}{\sigma}E_t\chi^*_{t+1}-\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_t-(1+\omega)E_ta_{t+1}\right\}$$

$$-\left\{\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\bar{\chi}_t+\frac{\sigma_1}{\sigma}\chi^*_t-\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1}-(1+\omega)a_t\right\}$$

$$-\beta(1-\delta_H)\frac{\sigma_0}{\sigma}\bar{\chi}_t-\beta(1-\delta_H)\frac{\sigma_1}{\sigma}\chi^*_t$$

$$=\beta(1-\delta_H)\left\{\left(\frac{\omega}{\delta_H}\right)E_t\bar{\chi}_{t+1}-\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_t-(1+\omega)E_ta_{t+1}\right\}$$

$$-\left\{\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\bar{\chi}_t+\frac{\sigma_1}{\sigma}\chi^*_t-\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1}-(1+\omega)a_t\right\}$$

$$-\beta(1-\delta_H)\frac{\sigma_0}{\sigma}\bar{\chi}_t-\beta(1-\delta_H)\frac{\sigma_1}{\sigma}\chi^*_t$$

$$=\beta(1-\delta_H)\frac{\omega}{\delta_H}E_t\bar{\chi}_{t+1}-\beta\frac{\omega}{\delta_H}(1-\delta_H)^2\bar{\chi}_t-\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\bar{\chi}_t-\frac{\sigma_1}{\sigma}\chi^*_t+\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1}$$

$$+(1+\omega)\left[a_t-\beta(1-\delta_H)E_ta_{t+1}\right]$$

$$\left\{-\beta(1-\delta_H)\frac{\sigma_0}{\sigma}+\beta\frac{\omega}{\delta_H}(1-\delta_H)^2+\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\right\}\bar{\chi}_t-\beta(1-\delta_H)\frac{\sigma_1}{\sigma}\chi^*_t+\frac{\sigma_1}{\sigma}\chi^*_t$$

$$=\beta(1-\delta_H)\frac{\omega}{\delta_H}E_t\bar{\chi}_{t+1}+\frac{\omega}{\delta_H}(1-\delta_H)\bar{\chi}_{t-1}+(1+\omega)\left[a_t-\beta(1-\delta_H)E_ta_{t+1}\right]$$

$$\left\{\frac{\sigma_0}{\sigma}q_H+\frac{\omega}{\delta_H}\left(1+\beta(1-\delta_H)^2\right)\right\}\bar{\chi}_t+\frac{\sigma_1}{\sigma}q_H\chi^*_t-\frac{\omega}{\delta_H}(1-\delta_H)\left[\beta E_t\bar{\chi}_{t+1}+\bar{\chi}_{t-1}\right]$$

$$=(1+\omega)\left[a_t-\beta(1-\delta_H)E_ta_{t+1}\right]$$

$$\eta_1\bar{\chi}_t-\eta_2\left(\beta E_t\bar{\chi}_{t+1}+\bar{\chi}_{t-1}\right)+\frac{\sigma_1}{\sigma}q_H\chi^*_t=(1+\omega)\left(a_t-(1-q_H)E_ta_{t+1}\right)$$

where $\eta_1 = \left[\frac{\sigma_0}{\sigma} q_H + \frac{\omega}{\delta_H}\left(1+\beta(1-\delta_H)^2\right) \right]$, $\eta_2 = \frac{\omega}{\delta_H}(1-\delta_H)$

A.5. Analytical solution to flexible-price equilibrium

From the optimal price setting rule and the Dixit-Stiglitz price index, we can obtain log-linear approximation forms

$$\log P_t\ (i) = \gamma \log P_{t-1}(i) + (1-\gamma)\log P_t^0(i) \tag{A.4}$$

and

$$\sum_{T=t}^{\infty}\left(\gamma\beta\right)^{T-t} E_t\left[\log P_t^0(i) - \log P_T\ (i) - mc_T\right] = 0 \tag{A.5}$$

Solving this equation for $\log P_t^0(i)$, subtracting $\log P_t\ (i)$ from both sides[11], we get

$$\begin{aligned} p_t^0(i) &= \left(1-\gamma\beta\right)\sum_{T=t}^{\infty}\left(\gamma\beta\right)^{T-t} E_t\left[\sum_{S=t+1}^{T}\pi_S + mc_T\right] \\ &= \sum_{T=t}^{\infty}\left(\gamma\beta\right)^{T-t} E_t\left[\gamma\beta\pi_{T+1} + (1-\gamma\beta)mc_T\right] \end{aligned} \tag{A.6}$$

where $p_t^0(i) \equiv \log(P_t^0(i)/P_t\ (i))$ is the log relative price chosen by suppliers that revise their prices in period t.

If above equation is expected to hold in time t+1 as well as time t, it follows that

$$p_t^0(i) = \left[\gamma\beta E_t\,\pi_{t+1} + (1-\gamma\beta)mc_t\ \right] + \gamma\beta E_t\ p_{t+1}^0(i) \tag{A.7}$$

Equation (A.4) implies that the inflation rate is proportional (up to a log-linear approximation) to the log relative price chosen by suppliers that revise their prices,

[11] Woodford (2003)

$$\begin{aligned} \pi_t &= \frac{1-\gamma}{\gamma} p_t^0(i) \\ \frac{\gamma}{1-\gamma}\pi_t &= p_t^0(i) \end{aligned} \tag{A.8}$$

Using this relation, to substitute for both $p_t^0(i)$ and $p_{t+1}^0(i)$ in (A.7), we get

$$\begin{aligned} p_t^0(i) &= \left[\gamma\beta E_t\, \pi_{t+1} + (1-\gamma\beta) mc_t\right] + \gamma\beta E_t\, p_{t+1}^0(i) \\ \frac{\gamma}{1-\gamma}\pi_t &= \left[\gamma\beta E_t\, \pi_{t+1} + (1-\gamma\beta) mc_t\right] + \gamma\beta E_t\, \frac{\gamma}{1-\gamma}\pi_{t+1} \\ \frac{\gamma}{1-\gamma}\pi_t &= (1-\gamma\beta) mc_t + \gamma\beta E_t\, \frac{1-\gamma+\gamma}{1-\gamma}\pi_{t+1} \\ \pi_t &= \frac{(1-\gamma)(1-\gamma\beta)}{\gamma} mc_t + \beta E_t\, \pi_{t+1} \end{aligned} \tag{A.9}$$

From the log-linearized form of the expression for marginal cost $MC_t(i) = \frac{v_N(Y_t(i)/A_t)}{u_k(K_t)} \frac{Q_t}{A_t P_{H,t}}$,

$$\begin{aligned} MC_t &= \frac{(Y_t \;/ A_t)^{\omega}}{(K_t)^{-\sigma^{-1}}} \frac{Q_t}{A_t P_{H,t}} \\ &= Y_t^{\omega} A_t^{-(1+\omega)} K_t^{\sigma^{-1}} \frac{Q_t}{P_{H,t}} \\ &= Y_t^{\omega} A_t^{-(1+\omega)} K_t^{\sigma^{-1}} \left(\frac{Q_{H,t}}{P_{H,t}} \frac{Q_t}{Q_{H,t}} \right) \\ &= Y_t^{\omega} A_t^{-(1+\omega)} K_t^{\sigma^{-1}} \left(H_t\right)^{\alpha} \left(\frac{Q_{H,t}}{P_{H,t}} \right) \end{aligned}$$

Log-linearize,

$$mc_t = \sigma^{-1} k_t + \omega y_t + \alpha h_t + q_{H,t} - (1+\omega) a_t$$

where $(1+\omega) a_t = \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \overline{\chi}_t + \frac{\sigma_1}{\sigma} \chi_t^* - \frac{\omega}{\delta_H}(1-\delta_H)\overline{\chi}_{t-1} + \frac{1-q_H}{q_H} \overline{rr}_{H,t}^{A}$

Then we have the Phillips curve equation from (A.9),

$$\pi_t = \frac{(1-\gamma)(1-\gamma\beta)}{\gamma} mc_t + \beta E_t \pi_{t+1} \tag{A.10}$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\gamma)(1-\gamma\beta)}{\gamma} \left\{ \begin{array}{l} \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \hat{\chi}_t - \frac{\omega}{\delta_H} (1-\delta_H) \hat{\chi}_{t-1} \\ + \frac{1-q_H}{q_H} \left(r_t - E_t \pi^A_{H,t+1} - \overline{rr}^A_{H,t} \right) \end{array} \right\} \tag{A.11}$$

$$\pi_t = \beta E_t \pi_{t+1} + \varphi(1+\omega\theta) \left\{ \begin{array}{l} \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \hat{\chi}_t - \frac{\omega}{\delta_H} (1-\delta_H) \hat{\chi}_{t-1} \\ + \frac{1-q_H}{q_H} \left(r_t - E_t \pi^A_{H,t+1} - \overline{rr}^A_{H,t} \right) \end{array} \right\} \tag{A.12}$$

where $\varphi = \frac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$.

From (3.22) and (3.40),

$$\hat{\chi}_t = E_t \hat{\chi}_{t+1} - \frac{\sigma}{\sigma_0} E_t \left[\frac{1}{q_H} \left(rr^A_{H,t} - \overline{rr}^A_{H,t} \right) - \frac{1-q_H}{q_H} \left(rr^A_{H,t+1} - \overline{rr}^A_{H,t+1} \right) \right]$$

$$\frac{1}{q_H} (rr^A_t - \overline{rr}^A_t) = \frac{1}{1-q_H} \left\{ \frac{1}{\varphi(1+\omega\theta)} \left(\pi_t - \beta \pi_{t+1} \right) - \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \hat{\chi}_t + \frac{\omega}{\delta_H} (1-\delta_H) \hat{\chi}_{t-1} \right\}$$

$$\frac{1-q_H}{q_H} (rr^A_{t+1} - \overline{rr}^A_{t+1}) = \left\{ \frac{1}{\varphi(1+\omega\theta)} \left(\pi_{t+1} - \beta \pi_{t+2} \right) - \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \hat{\chi}_{t+1} + \frac{\omega}{\delta_H} (1-\delta_H) \hat{\chi}_t \right\}$$

$$\hat{\chi}_t = E_t \hat{\chi}_{t+1} - \frac{\sigma}{\sigma_0} E_t \left[\begin{array}{l} \frac{1}{1-q_H} \left\{ \frac{1}{\varphi(1+\omega\theta)} \left(\pi_t - \beta \pi_{t+1} \right) - \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \hat{\chi}_t + \frac{\omega}{\delta_H} (1-\delta_H) \hat{\chi}_{t-1} \right\} \\ - \left\{ \frac{1}{\varphi(1+\omega\theta)} \left(\pi_{t+1} - \beta \pi_{t+2} \right) - \left(\frac{\sigma_0}{\sigma} + \frac{\omega}{\delta_H} \right) \hat{\chi}_{t+1} + \frac{\omega}{\delta_H} (1-\delta_H) \hat{\chi}_t \right\} \end{array} \right]$$

$$\beta(1-\delta_H)\frac{\sigma_0}{\sigma}\left(E_t\hat{\chi}_{t+1}-\hat{\chi}_t\right)=E_t\left[\begin{array}{l}\left\{\begin{array}{l}\frac{1}{\varphi(1+\omega\theta)}\left(\pi_t-\beta\pi_{t+1}\right)\\-\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\hat{\chi}_t+\frac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_{t-1}\end{array}\right\}\\-\beta(1-\delta_H)\left\{\begin{array}{l}\frac{1}{\varphi(1+\omega\theta)}\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\-\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\hat{\chi}_{t+1}+\frac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_t\end{array}\right\}\end{array}\right]$$

$$\begin{aligned}\left(\pi_t-\beta E_t\pi_{t+1}\right)&=\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\&+\varphi(1+\omega\theta)E_t\left[\begin{array}{l}\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\hat{\chi}_t-\frac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_{t-1}\\-\beta(1-\delta_H)\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\hat{\chi}_{t+1}+\beta(1-\delta_H)\frac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_t\\+\beta(1-\delta_H)\frac{\sigma_0}{\sigma}\left(E_t\hat{\chi}_{t+1}-\hat{\chi}_t\right)\end{array}\right]\end{aligned}$$

$$\begin{aligned}\left(\pi_t-\beta E_t\pi_{t+1}\right)&=\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\&+\varphi(1+\omega\theta)E_t\left[\begin{array}{l}\left\{-\beta(1-\delta_H)\frac{\sigma_0}{\sigma}+\beta\frac{\omega}{\delta_H}(1-\delta_H)^2+\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\right\}\hat{\chi}_t\\\beta(1-\delta_H)\frac{\omega}{\delta_H}E_t\hat{\chi}_{t+1}+\frac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_{t-1}\end{array}\right]\end{aligned}$$

$$\begin{aligned}\left(\pi_t-\beta E_t\pi_{t+1}\right)&=\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\&+\varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}E_t\left[\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)-\beta\upsilon\left(E_t\hat{\chi}_{t+1}-\upsilon\hat{\chi}_t\right)\right]\end{aligned}\qquad\text{(A.13)}$$

For the worldwide flexible price,

$$\pi_t=\beta E_t\pi_{t+1}+\varphi(1+\omega\theta)\left\{\begin{array}{l}\left(\frac{\sigma_0}{\sigma}+\frac{\omega}{\delta_H}\right)\hat{\hat{\chi}}_t-\frac{\omega}{\delta_H}(1-\delta_H)\hat{\hat{\chi}}_{t-1}\\+\frac{\sigma_1}{\sigma}\hat{\hat{\chi}}_t^*+\frac{1-q_H}{q_H}\left(rr_{tH,t}^A-\overline{rr}_{H,t}^A\right)\end{array}\right\}\qquad\text{(A.14)}$$

From (3.41)

$$\frac{\sigma_0}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}-\hat{\hat{\chi}}_t\right)+\frac{\sigma_1}{\sigma}\left(E_t\hat{\hat{\chi}}^*_{t+1}-\hat{\hat{\chi}}^*_t\right)=E_t\left[\frac{1}{q_H}\left(rr_t^A-\overline{\overline{rr}}_t^A\right)-\frac{1-q_H}{q_H}\left(rr_{t+1}^A-\overline{\overline{rr}}_{t+1}^A\right)\right]$$

$$\begin{aligned}&\frac{\sigma_0}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}-\hat{\hat{\chi}}_t\right)+\frac{\sigma_1}{\sigma}\left(E_t\hat{\hat{\chi}}^*_{t+1}-\hat{\hat{\chi}}^*_t\right)\\ &=E_t\left[\begin{array}{l}\dfrac{1}{1-q_H}\left\{\dfrac{1}{\varphi(1+\omega\theta)}\left(\pi_t-\beta\pi_{t+1}\right)-\left(\dfrac{\sigma_0}{\sigma}+\dfrac{\omega}{\delta_H}\right)\hat{\hat{\chi}}_t+\dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\hat{\chi}}_{t-1}-\dfrac{\sigma_1}{\sigma}\hat{\hat{\chi}}^*_t\right\}\\ -\left\{\dfrac{1}{\varphi(1+\omega\theta)}\left(\pi_{t+1}-\beta\pi_{t+2}\right)-\left(\dfrac{\sigma_0}{\sigma}+\dfrac{\omega}{\delta_H}\right)\hat{\hat{\chi}}_{t+1}+\dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\hat{\chi}}_t-\dfrac{\sigma_1}{\sigma}\hat{\hat{\chi}}^*_{t+1}\right\}\end{array}\right]\end{aligned}$$

$$\begin{aligned}\left(\pi_t-\beta E_t\pi_{t+1}\right)&=\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\ &+\varphi(1+\omega\theta)E_t\left[\begin{array}{l}\left(\dfrac{\sigma_0}{\sigma}+\dfrac{\omega}{\delta_H}\right)\hat{\hat{\chi}}_t-\dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\hat{\chi}}_{t-1}+\dfrac{\sigma_1}{\sigma}\hat{\hat{\chi}}^*_t\\ -\beta(1-\delta_H)\left(\dfrac{\sigma_0}{\sigma}+\dfrac{\omega}{\delta_H}\right)\hat{\hat{\chi}}_{t+1}+\beta(1-\delta_H)\dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\hat{\chi}}_t\\ -\beta(1-\delta_H)\dfrac{\sigma_1}{\sigma}\hat{\hat{\chi}}^*_{t+1}\\ +\beta(1-\delta_H)\left(\dfrac{\sigma_0}{\sigma}\left(E_t\hat{\hat{\chi}}_{t+1}-\hat{\hat{\chi}}_t\right)+\dfrac{\sigma_1}{\sigma}\left(E_t\hat{\hat{\chi}}^*_{t+1}-\hat{\hat{\chi}}^*_t\right)\right)\end{array}\right]\end{aligned}$$

$$\begin{aligned}\left(\pi_t-\beta E_t\pi_{t+1}\right)&=\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\ &+\varphi(1+\omega\theta)E_t\left[\begin{array}{l}\left\{-\beta(1-\delta_H)\dfrac{\sigma_0}{\sigma}+\beta\dfrac{\omega}{\delta_H}(1-\delta_H)^2+\left(\dfrac{\sigma_0}{\sigma}+\dfrac{\omega}{\delta_H}\right)\right\}\hat{\chi}_t\\ \beta(1-\delta_H)\dfrac{\omega}{\delta_H}E_t\hat{\chi}_{t+1}+\dfrac{\omega}{\delta_H}(1-\delta_H)\hat{\chi}_{t-1}+\dfrac{\sigma_1}{\sigma}q_H\hat{\hat{\chi}}^*_t\end{array}\right]\end{aligned}$$

$$\begin{aligned}\left(\pi_t-\beta E_t\pi_{t+1}\right)&=\beta(1-\delta_H)E_t\left(\pi_{t+1}-\beta\pi_{t+2}\right)\\ &+\varphi(1+\omega\theta)\frac{\eta_2}{\upsilon}E_t\left[\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)-\beta\upsilon\left(E_t\hat{\chi}_{t+1}-\upsilon\hat{\chi}_t\right)\right]+\varphi(1+\omega\theta)\frac{\sigma_1}{\sigma}q_H\hat{\hat{\chi}}^*_t\end{aligned}\qquad\text{(A.15)}$$

A.6. The optimal rate of production subsidy in Nash equilibrium

$$\max \quad U_t = \sum_{t=0}^{\infty} \beta^t \left[u(K_t) - v(N_t) \right]$$

$$\text{s. t.} \quad K_t = \frac{1-n}{1-\alpha} \left(\frac{(1-n)\alpha}{n(1-\alpha)} \right)^{\alpha} \mathrm{X}_t^{1-\alpha} \mathrm{X}_t^{*\alpha}$$

$$\mathrm{X}_t = (1-\delta_H)\mathrm{X}_{t-1} + N_t$$

$$\Rightarrow \quad \begin{aligned} 0 &= \sum_{j=0}^{\infty} \beta^j \frac{\partial u(K_{t+j})}{\partial K_{t+j}} \frac{\partial K_{t+j}}{\partial \mathrm{X}_{t+j}} \frac{\partial \mathrm{X}_{t+j}}{\partial N_t} - \frac{\partial v(N_t)}{\partial N_t} \\ &= \sum_{j=0}^{\infty} \beta^j u_k(K_{t+j})(1-\alpha) \frac{K_{t+j}}{\mathrm{X}_{t+j}} \delta_H (1-\delta_H)^j - v_N(N_t) \end{aligned}$$

$\Rightarrow$ In steady states,

$$0 = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) - N v_N(N)$$

$$N v_N(N) = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K)$$

$$\begin{aligned} P &= \frac{\mu}{1+\tau} NMC = \frac{\mu}{1+\tau} \frac{v_N(N)}{u_k(K)} \frac{Q}{A} \\ &= \frac{\mu}{1+\tau} \frac{\delta_H}{q_H} \frac{(1-\alpha)QK}{N} = \frac{\mu}{1+\tau} \frac{\delta_H}{q_H} \frac{Q_H K_H}{N}, \\ &= (1-n)\frac{\mu}{1+\tau} \frac{Q_H}{q_H} \end{aligned}$$

which gives the optimal subsidy rate in this economy,

$$1+\tau = \mu(1-n)$$

$$\because (1-n)\mathrm{X} = K_H, A = P_H = P_F = P = 1, q_H \equiv Q_H / P_H = 1-\beta(1-\delta_H)$$

A.7. Second-order approximation of the welfare function in Nash equilibrium

The second order approximation of the utility of consumption

$$u(K_t) \approx u(K) + u_k \left(K_t - K\right) + \frac{1}{2} u_{kk} \left(K_t - K\right)^2 + O\left(\|K\|^3\right) \tag{A.16}$$

Using $\frac{K_t - K}{K} = k_t + \frac{1}{2} k_t^2$, and $\sigma^{-1} \equiv -u_{kk} / u_k \cdot K$, where $k_t \equiv \log(K_t / K)$, we obtains

$$u(K_t) = u_k K\left(k_t + \frac{1}{2}(1-\sigma^{-1}) k_t^2 \right) + t.i.p + O\left(\|k\|^3\right) \tag{A.17}$$

Using $k_t = (1-\alpha)\chi_t + \alpha \chi_t^*$

$$\begin{aligned} u(K_t) &= u_k K\left((1-\alpha)\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)\chi_t + \alpha\chi_t^*\right)^2 \right) + t.i.p + O\left(\|k\|^3\right) \\ &= u_k K\left((1-\alpha)\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)^2 \chi_t^2 + 2\alpha(1-\alpha)\chi_t^* \chi_t\right) \right) + t.i.p + O\left(\|k\|^3\right) \end{aligned}$$

(A.18)

The second order approximation of the disutility of labor

$$\begin{aligned} &v(Y_t(i)/A_t) \approx v + v_Y\left(Y_t(i) - Y(i)\right) + v_A\left(A_t - A\right) \\ &+ \frac{1}{2} v_{YY}\left(Y_t(i) - Y(i)\right)^2 + v_{AY}\left(A_t - A\right)\left(Y_t(i) - Y(i)\right) \\ &+ \frac{1}{2} v_{AA}\left(A_t - A\right)^2 + O\left(\|Y, A\|^3\right) \end{aligned}$$

where $v_Y = \partial v / \partial Y(i)$, $v_{YY} = \partial^2 v / \partial Y(i)^2$, and $v_{AY} = \partial^2 v / \left(\partial A_j \partial Y(i)\right)$

Using $Y_t - Y = Y(y_t + \frac{1}{2} y_t^2)$, $A_t - A = A(a_t + \frac{1}{2} a_t^2)$.

$$\nu(Y_t(i)/A_t) = \nu + \nu_Y Y\left(y_t(i) + \frac{1}{2} y_t(i)^2 \right) + \nu_A A\left(a_t + \frac{1}{2} a_t^2 \right)$$
$$+\frac{1}{2}\nu_{YY} Y^2 \left(y_t(i) + \frac{1}{2} y_t(i)^2 \right)^2 + \nu_{AY} AY\left(a_t + \frac{1}{2} a_t^2 \right)\left(y_t(i) + \frac{1}{2} y_t(i)^2 \right)$$
$$+\frac{1}{2}\nu_{AA} A^2 \left(a_t + \frac{1}{2} a_t^2 \right)^2 + O\left(\left\| y,a \right\|^3\right)$$

$$\nu(Y_t(i)/A_t) = \nu + \nu_Y Y\left(y_t(i) + \frac{1}{2} y_t(i)^2 \right) + \frac{1}{2}\nu_{YY} Y^2 y_t(i)^2$$
$$+\nu_{AY} A\, Y\, a_t\, y_t(i) + t.i.p + O\left(\left\| y,a \right\|^3\right)$$

$$\nu(Y_t(i)/A_t) = \nu_Y Y\left(y_t(i) + \frac{1}{2}(1+\omega) y_t(i)^2 - (1+\omega) y_t(i) a_t \right)$$
$$+t.i.p + O\left(\left\| y,a \right\|^3\right)$$

where $\omega \equiv \frac{Y\nu_{YY}}{\nu_Y} > 0$ represents the elasticity of the marginal disutility of work with respect to output, and using $\nu_{AY} A = -\nu_{YY} Y$

$$\int_0^1 \nu(Y_t(i)/A_t)di = \nu_Y Y\begin{pmatrix} E_i y_t(i) + \frac{1}{2}(1+\omega)\left[\left(E_i y_t(i) \right)^2 + \operatorname{var}_i\, y_t(i) \right] \\ -(1+\omega) a_t\, E_i y_t(i) \end{pmatrix}$$
$$+t.i.p + O\left(\left\| y,a \right\|^3\right)$$

where $E_i y_t(i)$ is the mean value of $y_t(i)$ across all diffentiated goods at time t , and $\operatorname{var}_i\, y_t(i)$ is the corresponding variance.

We use the Taylor series approximation to the CES production function ,

$y_t = E_i y_t(i) + \frac{1}{2}(1-\theta^{-1}) \operatorname{var}_i\, y_t(i) + O\left(\left\| y \right\|^3\right)$ to eliminate $E_i y_t(i)$.

Then

$$\int_0^1 v(Y_t(i)/A_t)di = v_Y Y \begin{pmatrix} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\, y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i\, y_t(i) \end{pmatrix} + t.i.p + O\left(\left\|y,a\right\|^3\right) \quad \text{(A.19)}$$

Using $\qquad N v_N(N) = (1-\alpha)\dfrac{\delta_H}{q_H} K u_k(K)$,

$$\int_0^1 v(Y_t(i)/A_t)di = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) \begin{pmatrix} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\, y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i\, y_t(i) \end{pmatrix} \quad \text{(A.20)}$$
$$+t.i.p + O\left(\left\|y,a\right\|^3\right)$$

$$W_{Nash} = u_k K E_0 \sum_{t=0}^{\infty} \beta^t \left((1-\alpha)\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)^2 \chi_t^2 + 2\alpha(1-\alpha)\chi_t^* \chi_t\right)\right)$$
$$-(1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \begin{pmatrix} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\, y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i\, y_t(i) \end{pmatrix} + t.i.p + O\left(\left\|k,y,a\right\|^3\right)$$
$$= (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \begin{Bmatrix} \frac{q_H}{\delta_H}\left(\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)\chi_t^2 + 2\alpha\chi_t^* \chi_t\right)\right) \\ -y_t - \frac{1}{2}(1+\omega)y_t^2 + (1+\omega)a_t\, y_t \\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i\, y_t(i) \end{Bmatrix}$$
$$+t.i.p + O\left(\left\|k,y,a\right\|^3\right)$$

(A.21)

By the definition of the durable good stock,

$$\mathrm{X}_t - \mathrm{X} = Y(Y_t - Y) + (1-\delta_H)(\mathrm{X}_{t-1} - \mathrm{X}) \quad \text{(A.22)}$$

Combining the formula $X_t - X = X(x_t + x_t^2/2)$ with (A.22), one obtains

$$\mathrm{X}\left(\chi_t + \chi_t^2/2\right) = \delta_H \mathrm{X}\left(y_t + y_t^2/2\right) + (1-\delta_H)\mathrm{X}\left(\chi_{t-1} + \chi_{t-1}^2/2\right) \tag{A.23}$$

Plugging $y_t^2 = \frac{1}{\delta_H^2}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right)^2$ into (A.23), we get

$$\begin{aligned} y_t &= \frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) + \frac{1}{2\delta_H}\left(\chi_t^2 - (1-\delta_H)\chi_{t-1}^2\right) - \frac{1}{2\delta_H^2}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right)^2 \\ &= \frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) - \frac{1-\delta_H}{2\delta_H^2}\left(\chi_t - \chi_{t-1}\right)^2 \end{aligned}$$

(A.24)

$$W_{Nash} = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{q_H}{\delta_H}\left(\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)\chi_t^2 + 2\alpha\chi_t^*\chi_t\right)\right) \\ -\frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) + \frac{1-\delta_H}{2\delta_H^2}\left(\chi_t - \chi_{t-1}\right)^2 \\ -\frac{1}{2}(1+\omega)\left(\frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right)\right)^2 \\ +(1+\omega)a_t \frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) \\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\left\|k, y, a\right\|^3\right)$$

Assume that the initial value of the stock of durables is equal to its steady state value.

Then we get the formula $\sum_{t=0}^{\infty} \beta^t q_H \chi_t = \sum_{t=0}^{\infty} \beta^t (\chi_t - (1-\delta_H)\chi_{t-1})$.

$$W_{Nash}=(1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{q_H}{\delta_H}\left(\chi_t+\frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)\chi_t^2+2\alpha\chi_t^*\chi_t\right)\right)\\ -\frac{1}{\delta_H}\left(\chi_t-(1-\delta_H)\chi_{t-1}\right)+\frac{1-\delta_H}{2\delta_H^2}\left(\chi_t-\chi_{t-1}\right)^2\\ -\frac{1}{2}(1+\omega)\left(\frac{1}{\delta_H}\left(\chi_t-(1-\delta_H)\chi_{t-1}\right)\right)^2\\ +\frac{(1+\omega)}{\delta_H}\left(a_t-(1-q_H)a_{t+1}\right)\chi_t\\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i)\end{array}\right\}$$

$$+t.i.p+O\left(\|k,y,a\|^3\right)$$

$$W_{Nash}=(1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{1}{2}(1-\sigma^{-1})\frac{q_H}{\delta_H}(1-\alpha)\chi_t^2+(1-\sigma^{-1})\frac{q_H}{\delta_H}\alpha\chi_t^*\chi_t\\ +\frac{1-\delta_H}{2\delta_H^2}\left(\chi_t^2-2\chi_t\chi_{t-1}+\chi_{t-1}^2\right)\\ -\frac{(1+\omega)}{2\delta_H^2}\left(\chi_t^2-2(1-\delta_H)\chi_t\chi_{t-1}+(1-\delta_H)^2\chi_{t-1}^2\right)\\ +\frac{(1+\omega)}{\delta_H}\left(a_t-(1-q_H)a_{t+1}\right)\chi_t\\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i)\end{array}\right\}$$

$$+t.i.p+O\left(\|k,y,a\|^3\right)$$

From $\eta_1\overline{\chi}_t-\eta_2\left(\beta E_t\overline{\chi}_{t+1}+\overline{\chi}_{t-1}\right)+\frac{\sigma_1}{\sigma}q_H\chi_t^*=(1+\omega)\left(a_t-(1-q_H)E_ta_{t+1}\right)$,

$$W_{Nash}=(1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{1}{2}(1-\sigma^{-1})\frac{q_H}{\delta_H}(1-\alpha)\chi_t^2+(1-\sigma^{-1})\frac{q_H}{\delta_H}\alpha\chi_t^*\chi_t\\+\frac{1-\delta_H}{2\delta_H^2}\left(\chi_t^2-2\chi_t\,\chi_{t-1}+\chi_{t-1}^2\right)\\-\frac{(1+\omega)}{2\delta_H^2}\left(\chi_t^2-2(1-\delta_H)\chi_t\,\chi_{t-1}+(1-\delta_H)^2\chi_{t-1}^2\right)\\+\frac{1}{\delta_H}\left(\eta_1\overline{\chi}_t-\eta_2\left(\beta E_t\overline{\chi}_{t+1}+\overline{\chi}_{t-1}\right)+\frac{\sigma_1}{\sigma}q_H\chi_t^*\right)\chi_t\\-\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i)\end{array}\right\}$$

$$+t.i.p+O\left(\left\|k,y,a\right\|^3\right)$$

With $(1-\sigma^{-1})\frac{q_H}{\delta_H}\alpha\chi_t^*\chi_t=\frac{\alpha(\sigma-1)}{\sigma}\frac{q_H}{\delta_H}\chi_t^*\chi_t=-\frac{\sigma_1}{\sigma}\frac{q_H}{\delta_H}\chi_t^*\chi_t$

$$W_{Nash}=(1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{1}{2}(1-\sigma^{-1})\frac{q_H}{\delta_H}(1-\alpha)\chi_t^2\\+\frac{1-\delta_H}{2\delta_H^2}\left(\chi_t^2-2\chi_t\,\chi_{t-1}+\chi_{t-1}^2\right)\\-\frac{(1+\omega)}{2\delta_H^2}\left(\chi_t^2-2(1-\delta_H)\chi_t\,\chi_{t-1}+(1-\delta_H)^2\chi_{t-1}^2\right)\\+\frac{1}{\delta_H}\left(\eta_1\overline{\chi}_t-\eta_2\left(\beta E_t\overline{\chi}_{t+1}+\overline{\chi}_{t-1}\right)\right)\chi_t\\-\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i)\end{array}\right\}$$

$$+t.i.p+O\left(\left\|k,y,a\right\|^3\right)$$

$$W_{Nash} = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2}(1-\sigma^{-1})\frac{q_H}{\delta_H}(1-\alpha)\chi_t^2 \\ -\frac{\delta_H+\omega}{2\delta_H^2}\chi_t^2 - \frac{(1-\delta_H)(\omega/\delta_H)}{\delta_H}\chi_t\,\chi_{t-1} \\ +\frac{\left(1-\delta_H\right)(1-(1+\omega)(1-\delta_H))}{2\delta_H^2}\chi_{t-1}^2 \\ +\frac{1}{\delta_H}\left(\eta_1\bar{\chi}_t - \eta_2\left(\beta E_t\bar{\chi}_{t+1} + \bar{\chi}_{t-1}\right)\right)\chi_t \\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\left\|k, y, a\right\|^3\right)$$

$$W_{Nash} = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H}\left(\begin{array}{l} -\frac{\sigma_0}{\sigma}q_H + q_H - 1 - \frac{\omega}{\delta_H} \\ +\beta\left(1-\delta_H\right)(1-(1+\omega)(1-\delta_H))/\delta_H \end{array} \right)\chi_t^2 \\ -\frac{(1-\delta_H)(\omega/\delta_H)}{\delta_H}\chi_t\,\chi_{t-1} \\ +\frac{1}{\delta_H}\left(\eta_1\bar{\chi}_t - \eta_2\left(\beta E_t\bar{\chi}_{t+1} + \bar{\chi}_{t-1}\right)\right)\chi_t \\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\left\|k, y, a\right\|^3\right)$$

$$W_{Nash} = (1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H}\left(-\frac{\sigma_0}{\sigma} q_H - \frac{\omega}{\delta_H}(1-\beta(1-\delta_H)^2)\right)\chi_t^2 \\ -\frac{(1-\delta_H)(\omega/\delta_H)}{\delta_H}\chi_t\,\chi_{t-1} \\ +\frac{1}{\delta_H}\left(\eta_1 \bar{\chi}_t - \eta_2\left(\beta E_t \bar{\chi}_{t+1} + \bar{\chi}_{t-1}\right)\right)\chi_t \\ -\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\|k, y, a\|^3\right)$$

$$W_{Nash} = -(1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H}\left[\begin{array}{l} \eta_1 \chi_t^2 - \eta_2\left(\beta E_t \chi_{t+1} + \chi_{t-1}\right)\chi_t \\ -2\left(\eta_1 \bar{\chi}_t - \eta_2\left(\beta E_t \bar{\chi}_{t+1} + \bar{\chi}_{t-1}\right)\right)\chi_t \end{array} \right] \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\|k, y, a\|^3\right)$$

$$W_{Nash} = -(1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H}\left[\begin{array}{l} \frac{\eta_2}{\upsilon}\left\{\left(\chi_t - \upsilon\chi_{t-1}\right) - \beta\upsilon\left(E_t \chi_{t+1} - \upsilon\chi_t\right)\right\}\chi_t \\ -2\frac{\eta_2}{\upsilon}\left\{\left(\bar{\chi}_t - \upsilon\bar{\chi}_{t-1}\right) - \beta\upsilon\left(E_t \bar{\chi}_{t+1} - \upsilon\bar{\chi}_t\right)\right\}\chi_t \end{array} \right] \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\|k, y, a\|^3\right)$$

$$W_{Nash} = -(1-\alpha)\frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H}\left[\begin{array}{l} \frac{\eta_2}{\upsilon}\left\{\left(\chi_t - \upsilon\chi_{t-1}\right) - \beta\upsilon\left(E_t \chi_{t+1} - \upsilon\chi_t\right)\right\}\chi_t \\ -2\frac{\eta_2}{\upsilon}\left\{\left(\bar{\chi}_t - \upsilon\bar{\chi}_{t-1}\right) - \beta\upsilon\left(E_t \bar{\chi}_{t+1} - \upsilon\bar{\chi}_t\right)\right\}\chi_t \end{array} \right] \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+t.i.p + O\left(\|k, y, a\|^3\right)$$

where $0 < \upsilon < 1-\delta_H$ is the smaller root of quadratic equation

$$\eta_1 \upsilon = \eta_2 (\beta \upsilon^2 + 1)$$

Using $\sum_{t=0}^{\infty} \beta^t \beta \upsilon \chi_{t+1} \chi_t = \sum_{t=0}^{\infty} \beta^t \upsilon \chi_t \chi_{t-1} + t.i.p.$,

$$\sum_{t=0}^{\infty} \beta^t \left\{ \left((\chi_t - \upsilon \chi_{t-1}) - \beta \upsilon (\chi_{t+1} - \upsilon \chi_t) \right) \chi_t - 2 \left((\overline{\chi}_t - \upsilon \overline{\chi}_{t-1}) - \beta \upsilon (\overline{\chi}_{t+1} - \upsilon \overline{\chi}_t) \right) \chi_t \right\}$$

$$= \sum_{t=0}^{\infty} \beta^t \left\{ \left((\chi_t - \upsilon \chi_{t-1}) k_t - \beta \upsilon (\chi_{t+1} - \upsilon \chi_t) \chi_t \right) - 2 \left((\overline{\chi}_t - \upsilon \overline{\chi}_{t-1}) - \beta \upsilon (\overline{\chi}_{t+1} - \upsilon \overline{\chi}_t) \right) \chi_t \right\}$$

$$= \sum_{t=0}^{\infty} \beta^t \left\{ \left((\chi_t - \upsilon \chi_{t-1}) k_t - \upsilon (\chi_t - \upsilon \chi_{t-1}) \chi_{t-1} \right) - 2 \left((\overline{\chi}_t - \upsilon \overline{\chi}_{t-1}) \chi_t - \upsilon (\overline{\chi}_t - \upsilon \overline{\chi}_{t-1}) \chi_{t-1} \right) \right\}$$

$$= \sum_{t=0}^{\infty} \beta^t \left\{ \left(\chi_t - \upsilon \chi_{t-1} \right)^2 - 2 \left((\overline{\chi}_t - \upsilon \overline{\chi}_{t-1}) \chi_t - \upsilon (\overline{\chi}_t - \upsilon \overline{\chi}_{t-1}) \chi_{t-1} \right) \right\}$$

$$= \sum_{t=0}^{\infty} \beta^t \left((\chi_t - \overline{\chi}_t) - \upsilon (\chi_{t-1} - \overline{\chi}_{t-1}) \right)^2$$

$$W_{Nash} = -(1-\alpha) \frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H} \frac{\eta_2}{\upsilon} \left((\chi_t - \overline{\chi}_t) - \upsilon (\chi_{t-1} - \overline{\chi}_{t-1}) \right)^2 \\ + \frac{1}{2} (\theta^{-1} + \omega) \operatorname{var}_i y_t(i) \end{array} \right\}$$

$$+ t.i.p + O\left(\left\| k, y, a \right\|^3 \right)$$

$$\sum_{t=0}^{\infty} \beta^t \operatorname{var}_i y_t(i) = \frac{\gamma \theta^2}{(1-\gamma)(1-\gamma\beta)} \sum_{t=0}^{\infty} \beta^t \left(\pi_t \right)^2$$

$$W_{Nash} = -(1-\alpha) \frac{\delta_H}{q_H} K u_k(K) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2\delta_H} \frac{\eta_2}{\upsilon} \left((\chi_t - \overline{\chi}_t) - \upsilon (\chi_{t-1} - \overline{\chi}_{t-1}) \right)^2 \\ + \frac{1}{2} (\theta^{-1} + \omega) \frac{\gamma \theta^2}{(1-\gamma)(1-\gamma\beta)} \left(\pi_t \right)^2 \end{array} \right\}$$

$$+ t.i.p + O\left(\left\| k, y, a \right\|^3 \right)$$

$$W_{Nash}=-(1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{aligned}&\frac{1}{2\delta_H}\frac{\eta_2}{\upsilon}\left(\hat{\chi}_t-\upsilon\hat{\chi}_{t-1}\right)^2\\&+\frac{1}{2}\frac{\gamma\theta(1+\omega\theta)}{(1-\gamma)(1-\gamma\beta)}\left(\pi_t\right)^2\end{aligned}\right\}$$

$$+t.i.p+O\left(\left\|k,y,a\right\|^3\right)$$

where $\varphi\equiv\dfrac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$

A.8. The optimal rate of production subsidy in the cooperation equilibrium

$$\max\quad U_t=\sum_{t=0}^{\infty}\beta^t\left[u(K_t)-(1-n)v(N_t)-nv(N_t^*)\right]$$

$$K_t=\frac{1-n}{1-\alpha}\left(\frac{(1-n)\alpha}{n(1-\alpha)}\right)^{\alpha}\mathrm{X}_t^{1-\alpha}\mathrm{X}_t^{*\alpha}$$

$$\text{s. t. } \mathrm{X}_t=(1-\delta_H)\mathrm{X}_{t-1}+N_t$$

$$\mathrm{X}_t^*=(1-\delta_F)\mathrm{X}_{t-1}^*+N_t^*$$

$$\Rightarrow\quad\begin{aligned}0&=\sum_{j=0}^{\infty}\beta^j\frac{\partial u(K_{t+j})}{\partial K_{t+j}}\frac{\partial K_{t+j}}{\partial \mathrm{X}_{t+j}}\frac{\partial \mathrm{X}_{t+j}}{\partial N_t}-(1-n)\frac{\partial v(N_t)}{\partial N_t}\\&=\sum_{j=0}^{\infty}\beta^j u_k(K_{t+j})(1-\alpha)\frac{K_{t+j}}{\mathrm{X}_{t+j}}(1-\delta_H)^j-(1-n)v_N(N_t)\end{aligned}$$

$$\begin{aligned}0&=\sum_{j=0}^{\infty}\beta^j\frac{\partial u(K_{t+j})}{\partial K_{t+j}}\frac{\partial K_{t+j}}{\partial \mathrm{X}_{t+j}^*}\frac{\partial \mathrm{X}_{t+j}^*}{\partial N_t^*}-n\frac{\partial v(N_t^*)}{\partial N_t^*}\\&=\sum_{j=0}^{\infty}\beta^j u_k(K_{t+j})\alpha\frac{K_{t+j}}{\mathrm{X}_{t+j}^*}(1-\delta_F)^j-nv_{N^*}(N_t^*)\end{aligned}$$

⇨ In steady states,

$$0 = (1-\alpha)\frac{\delta_H}{q_H} Ku_k(K) - (1-n)Nv_N(N)$$
$$Nv_N(N) = \frac{(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H} Ku_k(K) \tag{A.25}$$

$$0 = \alpha\frac{\delta_F}{q_F} Ku_k(K) - nN^* v_{N^*}(N^*)$$
$$N^* v_{N^*}(N^*) = \frac{\alpha}{n}\frac{\delta_F}{q_F} Ku_k(K) \tag{A.26}$$

$$Nv_N(N) = N^* v_{N^*}(N^*) = \frac{(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H} Ku_k(K)$$
$$\because \frac{\alpha}{n}\frac{\delta_F}{q_F} = \frac{(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H}$$

$$\begin{aligned} P &= \frac{\mu}{1+\tau} NMC = \frac{\mu}{1+\tau}\frac{v_N(N)}{u_k(K)}\frac{Q}{A} \\ &= \frac{\mu}{1+\tau}\frac{\delta_H}{q_H}\frac{(1-\alpha)QK}{(1-n)N} = \frac{\mu}{1+\tau}\frac{\delta_H}{q_H}\frac{Q_H K_H}{(1-n)N}, \\ &= \frac{\mu}{1+\tau}\frac{Q_H}{q_H} \end{aligned}$$

which gives the optimal subsidy rate for domestic good in this economy,

$$1+\tau = \mu$$
$$\because (1-n)\mathrm{X} = K_H, A = P_H = P_F = P = 1, q_H \equiv Q_H / P_H = 1 - \beta(1-\delta_H)$$

We can get a similar equation of the optimal subsidy rate for foreign good by above way.

$$\begin{aligned} P^* &= \frac{\mu}{1+\tau^*} NMC^* = \frac{\mu}{1+\tau^*} \frac{v_{N^*}(N^*)}{u_{k^*}(K^*)} \frac{Q^*}{A^*} \\ &= \frac{\mu}{1+\tau^*} \frac{\delta_F}{q_F} \frac{\alpha Q^* K^*}{nN^*} = \frac{\mu}{1+\tau^*} \frac{\delta_F}{q_F} \frac{Q_F^* K_F^*}{nN^*}, \\ &= \frac{\mu}{1+\tau^*} \frac{Q_F^*}{q_F} \end{aligned}$$

which gives the optimal subsidy rate for foreign good,

$$1+\tau^* = \mu$$
$$\because n\mathrm{X}^* = K_F, A^* = P_H^* = P_F^* = P^* = 1, q_F \equiv Q_F^* / P_F^* = 1-\beta(1-\delta_F)$$

This means that two countries' fiscal authorities (H,F) choose the common subsidy.

$$1+\tau = \mu = 1+\tau^*$$

A.9. Second-order approximation of the welfare function in the cooperation equilibrium

The second order approximation of the utility of consumption

$$u(K_t) \approx u(K) + u_k\left(K_t - K\right) + \frac{1}{2}u_{kk}\left(K_t - K\right)^2 + O\left(\|K\|^3\right) \tag{A.27}$$

Using $\frac{K_t - K}{K} = k_t + \frac{1}{2}k_t^2$, and $\sigma^{-1} \equiv -u_{kk}/u_k \cdot K$, where $k_t \equiv \log(K_t / K)$, we obtains

$$u(K_t) = u_k K\left(k_t + \frac{1}{2}(1-\sigma^{-1})k_t^2\right) + t.i.p + O\left(\|k\|^3\right) \tag{A.28}$$

Using $k_t = (1-\alpha)\chi_t + \alpha\chi_t^*$

$$u(K_t) = u_k K\left((1-\alpha)\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)\chi_t + \alpha\chi_t^*\right)^2 \right) + t.i.p + O\left(\|k\|^3\right)$$

$$= u_k K\left((1-\alpha)\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)^2\chi_t^2 + 2\alpha(1-\alpha)\chi_t^*\chi_t + \alpha^2\chi_t^{*2}\right)\right) \qquad \text{(A.29)}$$

$$+t.i.p + O\left(\|k\|^3\right)$$

The second order approximation of the disutility of labor

From (A.19),

$$\int_0^1 v(Y_t(i)/A_t)di = v_Y Y \begin{pmatrix} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\, y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{pmatrix} + t.i.p + O\left(\|y,a\|^3\right)$$

Using $Nv_N(N) = \frac{(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H}Ku_k(K)$ from (A.25),

$$(1-n)\int_0^1 v(Y_t(i)/A_t)di = (1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)\begin{pmatrix} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\, y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{pmatrix} \qquad \text{(A.30)}$$

$$+t.i.p + O\left(\|y,a\|^3\right)$$

$$\int_0^1 v(Y_t^*(i)/A_t^*)di = v_{Y^*}Y^* \begin{pmatrix} y_t^* + \frac{1}{2}(1+\omega)y_t^{*2} - (1+\omega)a_t^* y_t^* \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i) \end{pmatrix} + t.i.p + O\left(\|y,a\|^3\right)$$

Using $N^* v_{N^*}(N^*) = \frac{\alpha}{n}\frac{\delta_F}{q_F}Ku_k(K)$ from (A.26),

$$n\int_0^1 v(Y_t^*(i)/A_t^*)di = \alpha\frac{\delta_F}{q_F}Ku_k(K)\begin{pmatrix} y_t^* + \frac{1}{2}(1+\omega)y_t^{*2} - (1+\omega)a_t^* y_t^* \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i) \end{pmatrix} \qquad \text{(A.31)}$$
$$+t.i.p + O\left(\|y,a\|^3\right)$$

$$W_{Cooperative} = u_k K E_0 \sum_{t=0}^{\infty}\beta^t\left((1-\alpha)\chi_t + \frac{1}{2}(1-\sigma^{-1})\left((1-\alpha)^2\chi_t^2 + 2\alpha(1-\alpha)\chi_t^*\chi_t + \alpha^2\chi_t^{*2}\right)\right)$$
$$-(1-\alpha)\frac{\delta_H}{q_H}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\begin{pmatrix} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \end{pmatrix}$$
$$-\alpha\frac{\delta_F}{q_F}Ku_k(K)E_0\sum_{t=0}^{\infty}\beta^t\begin{pmatrix} y_t^* + \frac{1}{2}(1+\omega)y_t^{*2} - (1+\omega)a_t^* y_t^* \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i) \end{pmatrix}$$
$$+t.i.p + O\left(\|k,y,a\|^3\right)$$
(A.32)

$$y_t = \frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) - \frac{1-\delta_H}{2\delta_H^2}\left(\chi_t - \chi_{t-1}\right)^2$$

$$y_t^* = \frac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) - \frac{1-\delta_F}{2\delta_F^2}\left(\chi_t^* - \chi_{t-1}^*\right)^2$$

$$\sum_{t=0}^{\infty}\beta^t \operatorname{var}_i y_t(i) = \frac{\gamma\theta^2}{(1-\gamma)(1-\gamma\beta)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t\right)^2$$
$$\sum_{t=0}^{\infty}\beta^t \operatorname{var}_i y_t^*(i) = \frac{\gamma\theta^2}{(1-\gamma)(1-\gamma\beta)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^*\right)^2$$

Using the condition $$\begin{aligned}\frac{\alpha}{n}\frac{\delta_F}{q_F} &= \frac{(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H}\\ \alpha\frac{\delta_F}{q_F} &= \frac{n(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H}\end{aligned}$$

$$W_{Cooperative}$$

$$= (1-\alpha)\frac{\delta_H}{q_H}u_k K E_0 \sum_{t=0}^{\infty}\beta^t \begin{pmatrix} \dfrac{q_H}{\delta_H}\chi_t + \dfrac{1}{2}\dfrac{q_H}{\delta_H}(1-\sigma^{-1})\begin{pmatrix}(1-\alpha)\chi_t^2 + 2\alpha\chi_t^*\chi_t \\ +\dfrac{\alpha^2}{(1-\alpha)}\chi_t^{*2}\end{pmatrix} \\ -\begin{pmatrix} y_t + \dfrac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t y_t \\ +\dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i)\end{pmatrix} \\ -\dfrac{n}{1-n}\begin{pmatrix} y_t^* + \dfrac{1}{2}(1+\omega)y_t^{*2} - (1+\omega)a_t^* y_t^* \\ +\dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i)\end{pmatrix}\end{pmatrix}$$

$$+t.i.p + O\left(\left\|k, y, a\right\|^3\right)$$

$$
\begin{aligned}
&W_{Cooperative} \\
&= (1-\alpha)\frac{\delta_H}{q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t
\left(
\begin{array}{l}
\frac{q_H}{\delta_H}\chi_t + \frac{1}{2}\frac{q_H}{\delta_H}(1-\sigma^{-1})\left(\begin{array}{l}(1-\alpha)\chi_t^2 \\ +2\alpha\chi_t^*\chi_t \\ +\frac{\alpha^2}{(1-\alpha)}\chi_t^{*2}\end{array}\right) \\
-\left(\begin{array}{l}\left(\frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) - \frac{1-\delta_H}{2\delta_H^2}\left(\chi_t - \chi_{t-1}\right)^2\right) + \\
\frac{1}{2}(1+\omega)\left(\frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right)\right)^2 \\
-(1+\omega)a_t \frac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) + \frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i)\end{array}\right) \\
-\frac{n}{1-n}\left(\begin{array}{l}\left(\frac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) - \frac{1-\delta_F}{2\delta_F^2}\left(\chi_t^* - \chi_{t-1}^*\right)^2\right) \\
+\frac{1}{2}(1+\omega)\left(\frac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right)\right)^2 \\
-(1+\omega)a_t^* \frac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) + \frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i)\end{array}\right)
\end{array}
\right) \\
&+ t.i.p + O\left(\left\|k, y, a\right\|^3\right)
\end{aligned}
$$

$$
\begin{aligned}
&W_{Cooperative} \\
&= (1-\alpha)\frac{\delta_H}{q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t
\left(
\begin{array}{l}
\dfrac{q_H}{\delta_H}\chi_t + \dfrac{1}{2}\dfrac{q_H}{\delta_H}(1-\sigma^{-1})\left(\begin{array}{l}(1-\alpha)\chi_t^2 \\ +2\alpha\chi_t^*\chi_t \\ +\dfrac{\alpha^2}{(1-\alpha)}\chi_t^{*2}\end{array}\right) \\
-\left(\begin{array}{l}
\dfrac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) + \dfrac{\delta_H+\omega}{2\delta_H^2}\chi_t^2 + \dfrac{(1-\delta_H)\omega}{\delta_H^2}\chi_t\chi_{t-1} \\
-\dfrac{(1-\delta_H)}{2\delta_H^2}\left(1-(1+\omega)(1-\delta_H)\right)\chi_{t-1}^2 \\
-(1+\omega)a_t\dfrac{1}{\delta_H}\left(\chi_t - (1-\delta_H)\chi_{t-1}\right) + \dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i)
\end{array}\right) \\
-\dfrac{n}{1-n}\left(\begin{array}{l}
\dfrac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) + \dfrac{\delta_F+\omega}{2\delta_F^2}\chi_t^{*2} + \dfrac{(1-\delta_F)\omega}{\delta_F^2}\chi_t^*\chi_{t-1}^* \\
-\dfrac{(1-\delta_F)}{2\delta_F^2}\left(1-(1+\omega)(1-\delta_F)\right)\chi_{t-1}^{*2} \\
-(1+\omega)a_t^*\dfrac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) + \dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i)
\end{array}\right)
\end{array}
\right) \\
&+t.i.p + O\left(\left\|k, y, a\right\|^3\right)
\end{aligned}
$$

$$
\begin{aligned}
&W_{Cooperative} \\
&= (1-\alpha)\frac{\delta_H}{q_H}u_k K E_0 \sum_{t=0}^{\infty}\beta^t
\begin{pmatrix}
\frac{1}{2\delta_H}\left(q_H(1-\sigma^{-1})(1-\alpha)-\left(1+\frac{\omega}{\delta_H}\right)\right)\chi_t^2 \\
-\frac{(1-\delta_H)\omega}{\delta_H^2}\chi_t\,\chi_{t-1}+\frac{(1-\delta_H)}{2\delta_H^2}\big(1-(1+\omega)(1-\delta_H)\big)\chi_{t-1}^2 \\
+(1+\omega)a_t\,\frac{1}{\delta_H}\big(\chi_t-(1-\delta_H)\chi_{t-1}\big)-\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \\
+\frac{q_H}{\delta_H}(1-\sigma^{-1})\alpha\chi_t^*\chi_t-\frac{n}{1-n}\frac{q_F}{\delta_F}\chi_t^* \\
+\left(\frac{q_H}{2\delta_H}\frac{\alpha^2(1-\sigma^{-1})}{(1-\alpha)}-\frac{n}{1-n}\frac{\delta_F+\omega}{2\delta_F^2}\right)\chi_t^{*2} \\
-\frac{n}{1-n}\frac{(1-\delta_F)\omega}{\delta_F^2}\chi_t^*\chi_{t-1}^*+\frac{n}{1-n}\frac{(1-\delta_F)}{2\delta_F^2}\big(1-(1+\omega)(1-\delta_F)\big)\chi_{t-1}^{*2} \\
+\frac{n}{1-n}(1+\omega)a_t^*\frac{1}{\delta_F}\big(\chi_t^*-(1-\delta_F)\chi_{t-1}^*\big)-\frac{n}{1-n}\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i)
\end{pmatrix} \\
&+t.i.p+O\left(\|k,y,a\|^3\right)
\end{aligned}
$$

$$W_{Cooperative}$$

$$= (1-\alpha)\frac{\delta_H}{q_H}u_k KE_0 \sum_{t=0}^{\infty}\beta^t \left(\begin{array}{l} \frac{1}{2\delta_H}\left(\begin{array}{l} -\frac{\sigma_0}{\sigma}q_H + q_H - 1 - \frac{\omega}{\delta_H} \\ +\beta\left(1-\delta_H\right)(1-(1+\omega)(1-\delta_H))/\delta_H \end{array} \right)\chi_t^2 \\ -\frac{(1-\delta_H)(\omega/\delta_H)}{\delta_H}\chi_t\,\chi_{t-1} \\ +\frac{(1+\omega)}{\delta_H}\left(a_t - (1-q_H)a_{t+1}\right)\chi_t - \frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\, y_t(i) \\ +\frac{q_H}{\delta_H}(1-\sigma^{-1})\alpha\chi_t^*\chi_t - \frac{n}{1-n}\frac{q_F}{\delta_F}\chi_t^* \\ +\left(\frac{q_H}{2\delta_H}\frac{\alpha^2(1-\sigma^{-1})}{(1-\alpha)} - \frac{n}{1-n}\frac{\delta_F+\omega}{2\delta_F^2}\right)\chi_t^{*2} \\ -\frac{n}{1-n}\frac{(1-\delta_F)\omega}{\delta_F^2}\chi_t^*\chi_{t-1}^* + \frac{n}{1-n}\frac{(1-\delta_F)}{2\delta_F^2}\left(1-(1+\omega)(1-\delta_F)\right)\chi_{t-1}^{*2} \\ +\frac{n}{1-n}(1+\omega)a_t^*\frac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) - \frac{n}{1-n}\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\, y_t^*(i) \end{array} \right)$$

$$+t.i.p + O\left(\left\|k,y,a\right\|^3\right)$$

Using the condition $\begin{array}{l} \frac{\alpha}{n}\frac{\delta_F}{q_F} = \frac{(1-\alpha)}{(1-n)}\frac{\delta_H}{q_H} \\ \frac{\alpha}{(1-\alpha)}\frac{q_H}{\delta_H} = \frac{n}{(1-n)}\frac{q_F}{\delta_F} \end{array}$

$$W_{Cooperative}$$

$$= (1-\alpha)\frac{\delta_H}{q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t \begin{pmatrix} \frac{1}{2\delta_H}\left(-\frac{\sigma_0}{\sigma} q_H - \frac{\omega}{\delta_H}(1-\beta(1-\delta_H)^2)\right)\chi_t^2 \\ -\frac{(1-\delta_H)(\omega/\delta_H)}{\delta_H}\chi_t \chi_{t-1} \\ +\frac{1}{\delta_H}\left(\eta_1 \overline{\overline{\chi}}_t - \eta_2\left(\beta E_t \overline{\overline{\chi}}_{t+1} + \overline{\overline{\chi}}_{t-1}\right)\right)\chi_t - \frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \\ +\left(\frac{q_F}{2\delta_F}\frac{\alpha n(1-\sigma^{-1})}{(1-n)} - \frac{n}{1-n}\frac{\delta_F+\omega}{2\delta_F^2}\right)\chi_t^{*2} \\ -\frac{n}{1-n}\frac{(1-\delta_F)\omega}{\delta_F^2}\chi_t^* \chi_{t-1}^* + \frac{n}{1-n}\frac{(1-\delta_F)}{2\delta_F^2}\left(1-(1+\omega)(1-\delta_F)\right)\chi_{t-1}^{*2} \\ +\frac{n}{1-n}(1+\omega)a_t^* \frac{1}{\delta_F}\left(\chi_t^* - (1-\delta_F)\chi_{t-1}^*\right) - \frac{n}{1-n}\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i) \end{pmatrix}$$

$$+t.i.p + O\left(\|k, y, a\|^3\right)$$

$$W_{Cooperative} = -(1-\alpha)\frac{\delta_H}{q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t \begin{pmatrix} \frac{1}{2\delta_H}\begin{bmatrix} \eta_1 \chi_t^2 - \eta_2\left(\beta E_t \chi_{t+1} + \chi_{t-1}\right)\chi_t \\ -2\left(\eta_1 \overline{\overline{\chi}}_t - \eta_2\left(\beta E_t \overline{\overline{\chi}}_{t+1} + \overline{\overline{\chi}}_{t-1}\right)\right)\chi_t \end{bmatrix} \\ +\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t(i) \\ -\frac{n}{1-n}\frac{1}{2\delta_F}\begin{pmatrix} \frac{\sigma_3}{\sigma} q_F - q_F + \left(1+\frac{\omega}{\delta_F}\right) \\ -\beta\left(1-\delta_F\right)(1-(1+\omega)(1-\delta_F))/\delta_F \end{pmatrix}\chi_t^{*2} \\ -\frac{n}{1-n}\frac{(1+\omega)}{\delta_F}\left(a_t^* - (1-q_F)a_{t+1}^*\right)\chi_t^* \\ +\frac{n}{1-n}\frac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_i y_t^*(i) \end{pmatrix}$$

$$+t.i.p + O\left(\|k, y, a\|^3\right)$$

$$W_{Cooperative}=-(1-\alpha)\frac{\delta_H}{q_H}u_k KE_0\sum_{t=0}^{\infty}\beta^t\left(\begin{array}{l}\frac{1}{2\delta_H}\left[\begin{array}{l}\eta_1\chi_t^2-\eta_2\left(\beta E_t\chi_{t+1}+\chi_{t-1}\right)\chi_t\\-2\left(\eta_1\overline{\overline{\chi}}_t-\eta_2\left(\beta E_t\overline{\overline{\chi}}_{t+1}+\overline{\overline{\chi}}_{t-1}\right)\right)\chi_t\end{array}\right]\\+\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i)\\+\frac{n}{1-n}\frac{1}{2\delta_F}\left[\begin{array}{l}\eta_3\chi_t^{*2}-\eta_4\left(\beta E_t\chi_{t+1}^*+\chi_{t-1}^*\right)\chi_t^*\\-2\left(\eta_1\overline{\overline{\chi}}_t^*-\eta_2\left(\beta E_t\overline{\overline{\chi}}_{t+1}^*+\overline{\overline{\chi}}_{t-1}^*\right)\right)\chi_t^*\end{array}\right]\\+\frac{n}{1-n}\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t^*(i)\end{array}\right)$$

$$+t.i.p+O\left(\|k,y,a\|^3\right)$$

where $0<\upsilon<1-\delta_H$ is the smaller root of quadratic equation

$$\eta_1\upsilon=\eta_2(\beta\upsilon^2+1)$$

and $0<\upsilon^*<1-\delta_F$ is the smaller root of quadratic equation

$$\eta_3\upsilon^*=\eta_4(\beta\upsilon^{*2}+1)$$

$$W_{Cooperative}=-(1-\alpha)\frac{\delta_H}{q_H}u_k KE_0\sum_{t=0}^{\infty}\beta^t\left(\begin{array}{l}\frac{1}{2\delta_H}\left[\begin{array}{l}\frac{\eta_2}{\upsilon}\left\{\left(\chi_t-\upsilon\chi_{t-1}\right)-\beta\upsilon\left(E_t\chi_{t+1}-\upsilon\chi_t\right)\right\}\chi_t\\-2\frac{\eta_2}{\upsilon}\left\{\left(\overline{\overline{\chi}}_t-\upsilon\overline{\overline{\chi}}_{t-1}\right)-\beta\upsilon\left(E_t\overline{\overline{\chi}}_{t+1}-\upsilon\overline{\overline{\chi}}_t\right)\right\}\chi_t\end{array}\right]\\+\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i)\\+\frac{n}{1-n}\frac{1}{2\delta_F}\left[\begin{array}{l}\frac{\eta_4}{\upsilon^*}\left\{\left(\chi_t^*-\upsilon^*\chi_{t-1}^*\right)-\beta\upsilon^*\left(E_t\chi_{t+1}^*-\upsilon^*\chi_t^*\right)\right\}\chi_t^*\\-2\frac{\eta_4}{\upsilon^*}\left\{\left(\overline{\overline{\chi}}_t^*-\upsilon^*\overline{\overline{\chi}}_{t-1}^*\right)-\beta\upsilon^*\left(E_t\overline{\overline{\chi}}_{t+1}^*-\upsilon^*\overline{\overline{\chi}}_t^*\right)\right\}\chi_t^*\end{array}\right]\\+\frac{n}{1-n}\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t^*(i)\end{array}\right)$$

$$+t.i.p+O\left(\|k,y,a\|^3\right)$$

$$W_{Cooperative} = -(1-\alpha)\frac{\delta_H}{q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t \begin{pmatrix} \frac{1}{2\delta_H}\frac{\eta_2}{\upsilon}\left((\chi_t - \overline{\overline{\chi}}_t) - \upsilon(\chi_{t-1} - \overline{\overline{\chi}}_{t-1})\right)^2 \\ +\frac{1}{2}\frac{\gamma\theta(1+\omega\theta)}{(1-\gamma)(1-\gamma\beta)}\left(\pi_t\right)^2 \\ +\frac{n}{1-n}\frac{1}{2\delta_F}\frac{\eta_4}{\upsilon^*}\left((\chi_t^* - \overline{\overline{\chi}}_t^*) - \upsilon^*(\chi_{t-1}^* - \overline{\overline{\chi}}_{t-1}^*)\right)^2 \\ +\frac{n}{1-n}\frac{1}{2}\frac{\gamma\theta(1+\omega\theta)}{(1-\gamma)(1-\gamma\beta)}\left(\pi_t^*\right)^2 \end{pmatrix}$$

$$+t.i.p + O\left(\|k,y,a\|^3\right)$$

$$W_{Cooperative} = -(1-\alpha)\frac{\delta_H}{q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t \begin{pmatrix} \frac{1}{2\delta_H}\frac{\eta_2}{\upsilon}\left(\hat{\hat{\chi}}_t - \upsilon\hat{\hat{\chi}}_{t-1}\right)^2 \\ +\frac{1}{2}\frac{\theta}{\varphi}\left(\pi_t\right)^2 \\ +\frac{n}{1-n}\frac{1}{2\delta_F}\frac{\eta_4}{\upsilon^*}\left(\hat{\hat{\chi}}_t^* - \upsilon^*\hat{\hat{\chi}}_{t-1}^*\right)^2 \\ +\frac{n}{1-n}\frac{1}{2}\frac{\theta}{\varphi}\left(\pi_t^*\right)^2 \end{pmatrix}$$

$$+t.i.p + O\left(\|k,y,a\|^3\right)$$

where $\varphi \equiv \frac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$

$$W_{Cooperative} = -(1-\alpha)\frac{\delta_H}{2q_H} u_k K E_0 \sum_{t=0}^{\infty} \beta^t \begin{pmatrix} \frac{1}{\delta_H}\frac{\eta_2}{\upsilon}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t - \upsilon\hat{\hat{\chi}}_{t-1}\right)^2 + \left(\pi_t\right)^2 \\ +\frac{n}{1-n}\frac{1}{\delta_F}\frac{\eta_4}{\upsilon^*}\frac{\varphi}{\theta}\left(\hat{\hat{\chi}}_t^* - \upsilon^*\hat{\hat{\chi}}_{t-1}^*\right)^2 \\ +\frac{n}{1-n}\left(\pi_t^*\right)^2 \end{pmatrix} \quad \text{(A.33)}$$

$$+t.i.p + O\left(\|k,y,a\|^3\right)$$

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