APPROACHES TO THE USE OF GEOMETRY IN ARCHITECTURE:
A STUDY OF THE WORKS OF ANDREA PALLADIO,
FRANK LLOYD WRIGHT, AND FRANK GEHRY

A Thesis
by
URMILA SRINIVASAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2009

Major Subject: Architecture
APPROACHES TO THE USE OF GEOMETRY IN ARCHITECTURE:
A STUDY OF THE WORKS OF ANDREA PALLADIO,
FRANK LLOYD WRIGHT, AND FRANK GEHRY

A Thesis

by

URMILA SRINIVASAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Chair of Committee, Robert Warden
Committee Members, Vivian Paul
                                      Susan Geller
Head of Department,        Glen Mills

May 2009

Major Subject: Architecture
ABSTRACT

Approaches to the Use of Geometry in Architecture:

A Study of the Works of Andrea Palladio, Frank Lloyd Wright, and Frank Gehry.

(May 2009)

Urmila Srinivasan, B.Arch, CEPT University, India

Chair of Advisory Committee: Prof. Robert Warden

Geometry deals with form, shape, and measurement and is a part of mathematics where visual thought is dominant. Both design and construction in architecture deal with visualization, and architects constantly employ geometry. Today, with the advent of computer software, architects can visualize forms that go beyond our everyday experience. Some architects claim that the complex forms of their works have correlations with non-Euclidean geometry, but the space we experience is still Euclidean. Given this context, I have explored possible correlations that might exist between mathematical concepts of geometry and the employment of geometry in architectural design from a historic perspective. The main focus will be to describe the two phenomena historically, and then investigate any connections that might emerge from the discussion. While discussing the way geometry has been approached in architecture, I have focused on the Renaissance, Modern, and Post-modern phases as they have a distinct style and expression. Andrea Palladio, Frank Lloyd Wright, and
Frank Gehry’s works will be case studies for the Renaissance, Modern, and Post-modern phases respectively.

One of the important conclusions of this study is that architects use geometry in a more subconscious and intuitive manner while designing. Certain approaches to geometry can be determined by the way an architect deals with form and space. From the discussions of the works of Palladio, Wright, and Gehry, it can be concluded that from a two-dimensional simple approach to form and space in architecture, there has been a development of thinking about complex forms three dimensionally. Similarly, in mathematics, geometry has developed from a two-dimensional and abstract description of our surroundings to something that can capture the complex and specific nature of a phenomena. It is also shown that architects rarely come up with new concepts of geometry. Significant developments in geometry have always been in the domain of mathematics. Hence, most correlations between geometry in architecture and geometry in mathematics develop much later than the introduction of those concepts of geometry in mathematics. It is also found that the use of Euclidean geometry persists in architecture and that later concepts like non-Euclidean geometry cannot be used in an instrumental manner in architecture.
To

Sudarsan ……..

……….for all his love and patience.
ACKNOWLEDGEMENTS

I would like to thank my committee chair, Prof. Robert Warden, for helping me focus and guiding me through this vast topic. His insights always proved useful and were critical in shaping this thesis. I also want to extend my gratitude to Dr. Vivian Paul and Dr. Sue Geller for their support through the course of this research. Dr. Paul brought clarity to my writing and made me conscious of details when writing about architectural history. Dr. Geller was of immense help in developing the chapter on the history of geometry in mathematics. Discussions with her helped me understand many concepts and write about a subject that I was not very familiar with.

Thanks also goes to The Melbern G. Glasscock Center For Humanities Research for awarding me the graduate travel grant, which helped me visit some of Frank Lloyd Wright and Frank Gehry’s works. Visiting their works was an invaluable experience, which helped me understand them in a better manner. I also thank the Technical Reference Center and the Evans Library for all their support.

This thesis would not have been possible without all my friends who made my time at Texas A&M memorable. Finally, thanks to my husband and family for all their support and patience.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Significance</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Methodology</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>End Notes</td>
<td>9</td>
</tr>
<tr>
<td>II</td>
<td>GEOMETRY IN MATHEMATICS</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Euclidean Geometry</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Limitations of Euclidean Geometry</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Linear Perspective</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Projective Geometry</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Analytical Geometry</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Differential Geometry</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Descriptive Geometry</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>‘N’ Dimensions and Non-Euclidean Geometry</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Fractal Geometry</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Geometry and Computers</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>End Notes</td>
<td>39</td>
</tr>
<tr>
<td>III</td>
<td>ANDREA PALLADIO: 1508 – 1580</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Typical Characteristics of Palladio’s Villas</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>Palazzo Chiericati</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>Villa Malcontenta</td>
<td>59</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Villa Rotonda</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>Correlations</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>End Notes</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>IV FRANK LLOYD WRIGHT: 1876-1959</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>The Prairie Houses</td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>The Unity Temple</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>The Hanna House</td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>Correlations</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>End Notes</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>V FRANK GEHRY: 1926-Present</td>
<td>121</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>121</td>
<td></td>
</tr>
<tr>
<td>Early Rectilinear Works</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td>Fragmented Forms in Houses</td>
<td>133</td>
<td></td>
</tr>
<tr>
<td>The Aerospace Museum</td>
<td>141</td>
<td></td>
</tr>
<tr>
<td>The Disney Hall: The Post Computer Phase</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>Gehry’s Design Process</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td>156</td>
<td></td>
</tr>
<tr>
<td>Correlations</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>End Notes</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>VI CONCLUSION</td>
<td>163</td>
<td></td>
</tr>
<tr>
<td>Future Directions</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>End Notes</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>REFERENCES</td>
<td>172</td>
<td></td>
</tr>
<tr>
<td>VITA</td>
<td>178</td>
<td></td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The Igloo</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>The dome of St. Peter’s Cathedral</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Alberti’s system of musical proportions</td>
<td>4</td>
</tr>
<tr>
<td>1.4</td>
<td>Le Corbusier’s The Modular</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>Methodology diagram</td>
<td>6</td>
</tr>
<tr>
<td>2.1</td>
<td>The five postulates from Euclid’s Elements</td>
<td>11</td>
</tr>
<tr>
<td>2.2</td>
<td>An example of a metrical theorem</td>
<td>11</td>
</tr>
<tr>
<td>2.3</td>
<td>Musical ratios</td>
<td>13</td>
</tr>
<tr>
<td>2.4</td>
<td>The Pythagorean tuning system</td>
<td>13</td>
</tr>
<tr>
<td>2.5</td>
<td>Construction of the Golden Ratio</td>
<td>15</td>
</tr>
<tr>
<td>2.6</td>
<td>The Platonic Solids</td>
<td>15</td>
</tr>
<tr>
<td>2.7</td>
<td>The conic sections</td>
<td>17</td>
</tr>
<tr>
<td>2.8</td>
<td>Drawing by Durer</td>
<td>17</td>
</tr>
<tr>
<td>2.9</td>
<td>Perspective drawing by Alberti</td>
<td>20</td>
</tr>
<tr>
<td>2.10</td>
<td>Musical proportion in Perspective</td>
<td>20</td>
</tr>
<tr>
<td>2.11</td>
<td>A theorem in projective geometry</td>
<td>24</td>
</tr>
<tr>
<td>2.12</td>
<td>The way Kepler unified the conic sections</td>
<td>24</td>
</tr>
<tr>
<td>2.13</td>
<td>Cartesian geometry</td>
<td>26</td>
</tr>
<tr>
<td>2.14</td>
<td>An Epicycloid</td>
<td>26</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>2.15</td>
<td>A hyperbolic parabaloid</td>
<td>29</td>
</tr>
<tr>
<td>2.16</td>
<td>The three coordinate axes</td>
<td>29</td>
</tr>
<tr>
<td>2.17</td>
<td>Spherical geometry</td>
<td>33</td>
</tr>
<tr>
<td>2.18</td>
<td>A Psuedosphere</td>
<td>33</td>
</tr>
<tr>
<td>2.19</td>
<td>The Koch snowflake</td>
<td>34</td>
</tr>
<tr>
<td>2.20</td>
<td>Fractal geometry</td>
<td>35</td>
</tr>
<tr>
<td>2.21</td>
<td>An example of a Julia set</td>
<td>38</td>
</tr>
<tr>
<td>2.22</td>
<td>Computer generated forms</td>
<td>38</td>
</tr>
<tr>
<td>3.1</td>
<td>Drawing of the Vitruvian Man by Leonardo da Vinci</td>
<td>43</td>
</tr>
<tr>
<td>3.2</td>
<td>Examples of Palladio’s Villas</td>
<td>47</td>
</tr>
<tr>
<td>3.3</td>
<td>Plan types of eleven Villas</td>
<td>49</td>
</tr>
<tr>
<td>3.4</td>
<td>Geometric construction of vaults</td>
<td>51</td>
</tr>
<tr>
<td>3.5</td>
<td>Plan and elevation, Palazzo Chiericati, Vicenza</td>
<td>53</td>
</tr>
<tr>
<td>3.6</td>
<td>Front view, Palazzo Chiericati</td>
<td>54</td>
</tr>
<tr>
<td>3.7</td>
<td>The three bay plan of Palazzo Chiericati</td>
<td>54</td>
</tr>
<tr>
<td>3.8</td>
<td>Direction of vaults and movement in Palazzo Chiericati</td>
<td>56</td>
</tr>
<tr>
<td>3.9</td>
<td>Axial organization in Palazzo Chiericati</td>
<td>56</td>
</tr>
<tr>
<td>3.10</td>
<td>Dimensions in Palazzo Chiericati</td>
<td>57</td>
</tr>
<tr>
<td>3.11</td>
<td>The Pythagorean and Just tuning systems</td>
<td>57</td>
</tr>
<tr>
<td>3.12</td>
<td>Plan and elevation, Villa Malcontenta, Mira (near Venice)</td>
<td>60</td>
</tr>
<tr>
<td>3.13</td>
<td>Front view, Villa Malcontenta</td>
<td>61</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>3.14 Bays at Villa Malcontenta</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>3.15 The central hall of Villa Malcontenta</td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>3.16 The rear elevation of Villa Malcontenta</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>3.17 Dimensions in Villa Malcontenta</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>3.18 Villa Rotonda, Vicenza</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>3.19 Plan and elevation, Villa Rotonda</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>3.20 Comparison of movement in the Pantheon and Villa Rotonda</td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>3.21 External form in Villa Rotonda</td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>3.22 Dimensions in the plan of Villa Rotonda</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>3.23 Proportions suggested by Lionel March</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>3.24 Drawing of the entrance to the Palazzo Farnese</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>3.25 Alignment of doors in Villa Poiana</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>4.1 The Bauhaus, Dessau</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>4.2 The geometric transformations of a figurative seed germ</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>4.3 Sullivan’s use of geometry for ornamentation</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>4.4 Froebel’s gifts</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>4.5 Some examples of patterns created with Froebel’s toys</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>4.6 Two dimensional figurative representation of a praying monk</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>4.7 First floor plan, Robie House, Chicago, Illinois</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>4.8 Asymmetric grid in the Robie House</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>4.9 Breaking the box</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.10</td>
<td>Horizontality in the Robie House</td>
<td>90</td>
</tr>
<tr>
<td>4.11</td>
<td>The living room of the Robie House</td>
<td>91</td>
</tr>
<tr>
<td>4.12</td>
<td>The horizontal and the vertical elements in the Robie House</td>
<td>91</td>
</tr>
<tr>
<td>4.13</td>
<td>The cross axis in the Willitts House</td>
<td>92</td>
</tr>
<tr>
<td>4.14</td>
<td>The cross axis of the Robie House divide the site into four parts</td>
<td>92</td>
</tr>
<tr>
<td>4.15</td>
<td>Axial organization in the Robie House</td>
<td>93</td>
</tr>
<tr>
<td>4.16</td>
<td>Movement inside the Robie House is not along the axis</td>
<td>93</td>
</tr>
<tr>
<td>4.17</td>
<td>First floor plan, Unity Temple, Oak Park, Illinois</td>
<td>96</td>
</tr>
<tr>
<td>4.18</td>
<td>Sanctuary level plan, Unity Temple, Oak Park, Illinois</td>
<td>96</td>
</tr>
<tr>
<td>4.19</td>
<td>A view of the Unity Temple showing the identical facades</td>
<td>98</td>
</tr>
<tr>
<td>4.20</td>
<td>Relationship of the axis and movement in the Unity Temple</td>
<td>98</td>
</tr>
<tr>
<td>4.21</td>
<td>Theme of the square intercepted with the cross in the Unity Temple</td>
<td>99</td>
</tr>
<tr>
<td>4.22</td>
<td>The repletion of the theme of the plan in the Unity temple</td>
<td>99</td>
</tr>
<tr>
<td>4.23</td>
<td>Treatment of the corner at the Unity Temple</td>
<td>101</td>
</tr>
<tr>
<td>4.24</td>
<td>The complex layering dissolves the corner in the Unity Temple</td>
<td>101</td>
</tr>
<tr>
<td>4.25</td>
<td>The different grids of the Unity Temple and the Unity House</td>
<td>103</td>
</tr>
<tr>
<td>4.26</td>
<td>Dimensional relationships in the Unity Temple</td>
<td>103</td>
</tr>
<tr>
<td>4.27</td>
<td>Plan, Hanna House, Palo Alto, California</td>
<td>104</td>
</tr>
<tr>
<td>4.28</td>
<td>The living room of the Hanna House</td>
<td>107</td>
</tr>
<tr>
<td>4.29</td>
<td>Section through the living room of the Hanna House</td>
<td>107</td>
</tr>
<tr>
<td>4.30</td>
<td>Timber walls in the Hanna House</td>
<td>108</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>4.31 Interlocking of the redwood battens and boards in the Hanna House</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>4.32 The three distinct floor plans of the Hanna House</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>4.33 The repetition of hexagonal module for the chimney and the furniture</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>4.34 The thin folded walls provide support for the roof</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>4.35 The plan showing the folded walls in the Hanna House</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>4.36 The three dimensional grid of the Unity Temple</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>5.1 The fish in Gehry’s works</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>5.2 The DG Bank project, Berlin</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>5.3 Works of Claes Oldenburg</td>
<td>126</td>
<td></td>
</tr>
<tr>
<td>5.4 A sketch for the Jung institute, an unbuilt project</td>
<td>126</td>
<td></td>
</tr>
<tr>
<td>5.5 ‘To and Fro, by Ron Davis</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>5.6 Drawing, Ron Davis Studio, Malibu, California</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>5.7 Drawing, Steeves House, Brentwood, California</td>
<td>129</td>
<td></td>
</tr>
<tr>
<td>5.8 Danzinger Studio, Los Angeles, California</td>
<td>129</td>
<td></td>
</tr>
<tr>
<td>5.9 Ground floor plan, Frances Howard Goldwyn Regional Library, Los Angeles, California</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>5.10 Front view, Frances Howard Goldwyn Regional Library</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>5.11 Interior views of the Goldwyn Library</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>5.12 Treatment of the roof in Goldwyn Library</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>5.13 Drawing, Gehry’s house, Santa Monica, California</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>5.14 The falling cube on the north facing of Gehry’s house</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.15</td>
<td>A distorted glass cube on the north-east end of Gehry’s house</td>
<td>135</td>
</tr>
<tr>
<td>5.16</td>
<td>Distortion of the grid by Gehry.</td>
<td>137</td>
</tr>
<tr>
<td>5.17</td>
<td>Aerial view, Schnabel House, Brentwood, California</td>
<td>138</td>
</tr>
<tr>
<td>5.18</td>
<td>Axial organization in the plan of the Schnabel House</td>
<td>139</td>
</tr>
<tr>
<td>5.19</td>
<td>The Entrance to the Schnabel House</td>
<td>139</td>
</tr>
<tr>
<td>5.20</td>
<td>The dome in the office of the Schnabel House rests on glass pendentives</td>
<td>140</td>
</tr>
<tr>
<td>5.21</td>
<td>The office of the Schnabel House</td>
<td>140</td>
</tr>
<tr>
<td>5.22</td>
<td>The Aerospace Museum, Los Angeles, California</td>
<td>142</td>
</tr>
<tr>
<td>5.23</td>
<td>Plan, the Aerospace Museum</td>
<td>142</td>
</tr>
<tr>
<td>5.24</td>
<td>The truncated polygonal cone rests on a wedge shaped projection</td>
<td>143</td>
</tr>
<tr>
<td>5.25</td>
<td>The wall of the truncated polygonal cone cuts beyond the glazing</td>
<td>143</td>
</tr>
<tr>
<td>5.26</td>
<td>The entrance to the Aerospace Museum</td>
<td>144</td>
</tr>
<tr>
<td>5.27</td>
<td>The inside of the Aerospace Museum</td>
<td>144</td>
</tr>
<tr>
<td>5.28</td>
<td>Roof plan of the Aerospace Museum</td>
<td>146</td>
</tr>
<tr>
<td>5.29</td>
<td>The sky lights of the Aerospace Museum</td>
<td>146</td>
</tr>
<tr>
<td>5.30</td>
<td>Rear View, The Disney Hall, Los Angeles, California</td>
<td>148</td>
</tr>
<tr>
<td>5.31</td>
<td>The Disney Hall, front view</td>
<td>148</td>
</tr>
<tr>
<td>5.32</td>
<td>Model of the auditorium</td>
<td>149</td>
</tr>
<tr>
<td>5.33</td>
<td>Convex forms in the ceiling of the auditorium</td>
<td>149</td>
</tr>
<tr>
<td>5.34</td>
<td>Convex surfaces defining the seating in the auditorium</td>
<td>150</td>
</tr>
<tr>
<td>5.35</td>
<td>Plan, the Disney Hall</td>
<td>150</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.36</td>
<td>The steel structure behind the metal skin at the Disney Hall</td>
<td>153</td>
</tr>
<tr>
<td>5.37</td>
<td>Interior views of the Disney Hall</td>
<td>154</td>
</tr>
<tr>
<td>5.38</td>
<td>One of the first sketches for the Disney Hall</td>
<td>157</td>
</tr>
<tr>
<td>5.39</td>
<td>A later sketch of the Disney Hall</td>
<td>157</td>
</tr>
<tr>
<td>5.40</td>
<td>Sketch, the Sydney Opera House, Sydney</td>
<td>160</td>
</tr>
<tr>
<td>5.41</td>
<td>The Sydney Opera House</td>
<td>160</td>
</tr>
<tr>
<td>6.1</td>
<td>Choir vault, Gloucester Cathedral</td>
<td>167</td>
</tr>
<tr>
<td>6.2</td>
<td>Philips Pavilion, 1958, World’s Fair, Brussels</td>
<td>167</td>
</tr>
<tr>
<td>6.3</td>
<td>Sagrada Familia, Barcelona</td>
<td>168</td>
</tr>
<tr>
<td>6.4</td>
<td>Different views of a 3-D projection of a tesseract</td>
<td>168</td>
</tr>
</tbody>
</table>
When one thinks of geometry, images that come to the mind are of lines, points, squares, curves, circles, and other forms. Geometry deals with form, shape, and measurement and is the part of mathematics where visual thought is dominant. Since visual thought is a dominant part of architectural design, geometry is also an important part of architecture. Both design and construction in architecture deal with visualization, and architects constantly employ geometry. As a subject, geometry belongs more to mathematics where, for over two thousand years, Euclidean geometry was considered to be the only system of geometry that could be applied to reality. During the 19th century it was realized that there were other possible models of geometry, like spherical and hyperbolic, that could be applied to reality. There are also other descriptions of geometry that have come with time, like perspective, projective geometry, Cartesian geometry, trigonometry, differential geometry, topology, fractal geometry, etc. In this thesis, I intend to explore the possible correlations between mathematical concepts of geometry and the employment of geometry in architectural design.

Architects use geometry in diverse ways while designing. The form of the igloo came from an intuitive understanding of geometry and structure. Here the domical shape had the least surface area exposed to the wind, while at the same time it provided enclosure and stability.¹ [Fig.1.1] The dome in medieval architecture assumed symbolic

¹ This thesis follows the style of *Journal of Architectural Education.*
connotations; its shape symbolized heaven and was found to be appropriate for churches. Structural considerations prompted Renaissance architects to make its shape slightly elongated.² [Fig.1.2]. Renaissance architects, like Alberti, emphasized proportions derived from perspective drawing, which were also found in musical harmonies, to compose plans and elevations.³ [Fig.1.3] In contrast Le Corbusier emphasized modular proportions, using his modular, which was based on the Golden Ratio.⁴ [Fig.1.4] As geometry developed in mathematics, the use of geometry in architecture also changed.

The Bauhaus approach to design and design education, in the modern movement, was influenced by Friedrich Froebel’s emphasis on a kindergarten education that focused on developing a visual vocabulary of straight lines, curves and diagonals to describe the environment. This influence created a purist and intuitive use of geometry amongst modernists.⁵ Another significant development during the modern movement was the introduction of descriptive geometry in architectural education. Descriptive geometry provided a more systematic method of depicting objects in a manner suitable for production. Digital tools developed at the end of the 20th century employed concepts of the Cartesian co-ordinate system and descriptive geometry to generate views of objects. Today the computer is not only capable of generating models, but is also used in the construction process. The field of digital media uses algebraic concepts of geometry extensively to generate and construct form. Another use of geometry is in green building design where solar angles are used to determine the angles and spacing of shading devices. All these examples demonstrate that geometry plays an important role in architectural design and construction.
Fig. 1.1 - The Igloo. The dome shape of the igloo reduces the exposed surface area. (Norman Crowe, Nature and the Idea of a Man-Made World, 1995).

Fig. 1.2 - The dome of St. Peter’s Cathedral. The shape symbolized heaven and due to structural considerations it was slightly elongated. (http://www.pbs.org/wgbh/buildingbig/wonder/structure).
Fig. 1.3 - Alberti’s system of musical proportions. (Jay Kapraff in Kim Williams ed., Nexus, Architecture and Mathematics, Vol.1, 1996).

Fig. 1.4 - Le Corbusier’s The Modular. (http://blog.lib.umn.edu/buch0234/architecture/aa_modulor.jpg).
In most of the cases discussed above, Euclidean geometry dominated. It became possible to explore other concepts of geometry with the advent of the computer and digital tools. Many architects today try to break away from conventional forms, which are related to Euclidean geometry, in an attempt to express new concepts of geometry. They are seeking a very direct correlation between recent developments in geometry and architecture. One direct correlation between geometry in mathematics and geometry in architecture is found in architectural drawings that use orthogonal projections. The modern use of repetitive modules both in plan and elevation reflects a notion of a three-dimensional grid in space that could be correlated with the grid generated from three coordinate axes present in Cartesian geometry.

From these examples it is obvious that there are numerous ways architects employ geometry and that there are different correlations that exist between the way geometry is approached in architecture and geometry in mathematics. Therefore, in this thesis, I intend to study the ways architects employ geometry, and to investigate possible correlations that might exist between architectural design and different mathematical concepts of geometry. I will use three architects’ works, from different periods in history, as case studies to investigate various ways geometry has been approached in architecture. [Fig. 1.5] I will give a chronological overview of the various descriptions or kinds of geometries that have developed chronologically. There are: Euclidean plane geometry, perspective, projective geometry, descriptive geometry, Cartesian geometry, differential geometry, non-Euclidean geometry and fractal geometry. The Italian Renaissance (1400 to 1600), Modern (1750 – 1950) and Post-Modern (1950- to present)
CONCEPTS OF GEOMETRY

EUCLIDEAN PLANE GEOMETRY

PROJECTIVE GEOMETRY
(INCLUDING PERSPECTIVE AND DESCRIPTIVE GEOMETRY)

CARTESIAN GEOMETRY

DIFFERENTIAL GEOMETRY

NON-EUCLIDEAN GEOMETRY

FRACTAL GEOMETRY

Fig. 1.5 – Methodology diagram. (author).
phases saw significant changes in form and shape in architecture. Therefore, I will discuss the works of Andrea Palladio (1508-1580), Frank Lloyd Wright (1867-1959), and Frank Gehry (1929-present) as case studies respectively for the Italian Renaissance, Modern and Post-Modern periods.

**Significance**

Computer software helps us visualize complex forms of higher dimensions and non-Euclidean geometries, which go beyond our everyday experience. A whole new range of forms is now accessible to designers and artists, as Euclidean and non-Euclidean geometries can easily be visualized with a few moves of the computer cursor. Mathematicians consider the present to be a golden era for geometry, and architects feel it is an age when they can break new grounds in geometry and form. According to architectural critic Charles Jenks, post-modern sciences such as fractals, chaos theory, nonlinear dynamics and complexity theory reveal a creative and complex world. He claims that architects need to develop a new language that reflects this new world view. Most contemporary mainstream architecture is oriented towards generating complex forms like warped grids, curved dynamic structures and colliding cubes to represent a post-modern world view, but the space we experience still remains Euclidean. It is within this context that architects employ geometry today. The need to use complex geometric forms in relation to the recent descriptions of geometry opens questions regarding the correlations between geometry in mathematics and architecture. In this thesis I intend to understand the way geometry has been approached in architecture at different points in history and explore correlations that exist between geometry in
mathematics and architecture. In the process, I hope to set the context in which architects work today in a historical perspective.

**Methodology**

This study will primarily focus on the various descriptions of geometry that have evolved in the western context, although it is beyond the scope of this thesis to undertake an exhaustive study of the history of western geometry and architecture. Particular phases in the history of architecture, which have distinct stylistic expressions, have been selected for discussion. The Renaissance is the first historical period that is fairly well documented. Hence, it will be considered as a starting point for discussing case studies. Developments in the field of geometry will be discussed chronologically. Important kinds of geometries that have a very different description of form and space, or those that have a significant relationship to the field of architecture will be focused on. This study is not attempting to establish a cause-effect relationship between geometry in architecture and geometry in mathematics. The main focus will be to describe the two phenomena historically, and then investigate any connections that might emerge from the discussion.

This is a historical-interpretative study that assumes the Hegelian notion of a common communal spirit that explains similarities in stylistic expressions of architecture during any particular phase in history. This approach categorizes history into particular periods and assumes that within each period the spirit of that time manifests itself in all material forms. Again, this method of looking at history is used as a backdrop to discuss works of individual architects. The study of an individual architect’s works will
be used as a case study to explain approaches to geometry in architecture within a particular historical period. The particular architect’s works, which will be discussed, reflect an innovative use of geometry that was significant in defining that phase of history. Only selected works of these architects that display distinct approaches to geometry will be discussed. Literary data are the primary source of information for this study. Texts, drawings and photographs will be used to outline the various kinds of geometries that have developed in mathematics and the way geometry has been employed by architects.

End Notes

4 Ibid., 320.
9 Ibid.
CHAPTER II

GEOMETRY IN MATHEMATICS

Euclidean Geometry

Early human beings built their huts and erected their tents with an intuitive notion of geometry. Basic ideas of shape and form could have developed from observing the sky and nature, which could have been further developed for practical needs of measurement and calculation of areas. It is often claimed that geometry was the gift of the Nile. Geometry is mostly considered to have developed when rope stretchers had to measure and mark boundaries of cultivable land every year after the flooding of the Nile. Early civilizations, like Mesopotamia and Egypt, had a practical approach to geometry. They knew how to calculate the area of a triangle and rectangle, they also knew the 3, 4, 5 triangle, and had basic units of measurements. These concepts, which were based on observations and approximations, helped them measure and calculate land areas. The etymology of the word geometry (geo [earth] and measure [metron]-geometry) suggests that the Greeks referred to it as a metrical study. The Greeks went beyond just providing procedures for calculation; they were not satisfied with facts, and wanted to apply reasoning to justify things. Hence, they developed geometry as a system based on logic. They began with general postulates or assumptions, and used them to logically develop other theorems. [Fig.2.1 and 2.2] We owe our knowledge of geometry as a systematized subject to the Greeks. This process began with Thales in 6th century B.C.E, and was further developed by Pythagoras, Plato, and many others.
1. A straight line segment can be drawn by joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius, and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the parallel postulate: Given any straight line and a point not on it, there exists one and only one straight line which passes through that point and never intersects the first line, no matter how far they are extended.

Fig. 2.1 - The five postulates from Euclid’s Elements. The figure represents the fifth postulate of Euclid. The sum of the interior angles $\alpha$ and $\beta$ is less than 180°, the two straight lines, when extended, meet on that side. (Victor Katz, A History of Mathematics: An Introduction, 1998).

To construct an equilateral triangle with a line segment as one of the sides:
Let $AB$ be the finite line segment. With center $A$ and radius $AB$ draw a circle $D$. Similarly draw a circle $E$ with center at $B$. Let the two circles intersect at $C$. Join points to form lines $AC$ and $BC$. Triangle $ABC$ is equilateral.

To prove the constructed triangle is an equilateral triangle:
- $A$ is the center of circle $CDB$
- So $AC = AB$  \[ 1 \]
- $B$ is the center of circle $ACE$
- So $AB=BC$ \[ 2 \]
- From 1 and 2 – $AC=AB=BC$
- So $ABC$ is an equilateral triangle

Fig. 2.2 - An example of a metrical theorem. (Victor Katz, A History of Mathematics: An Introduction, 1998).
Pythagoras and the Pythagoreans made significant contributions to geometry and arithmetic. They developed the natural numbers and also knew about the three means: arithmetic, geometric and harmonic. Most Pythagoreans believed that numbers were closely associated with magnitude. Their belief that the universe could be reduced to basic units that could be represented by numbers led them towards the idea of proportion. They were interested in proportion as it related the part to the whole. Therefore, the integrity of the universe could be expressed through proportion. The Pythagoreans found that musical harmonies were dependent on ratios of lengths of a string. For example, on a monochord, ratios of length 1:2, 2:3, and 3:4 gave the octave, fifth, and fourth musical notes (according to modern notation), which were consonants (sound harmonious when played together). [Fig.2.3] These were further divided by Plato into five intervals of 8/9 (tone) and two intervals of 256/243 (hemitone) to give the Pythagorean tuning system. [Fig.2.4] The Pythagoreans were mystics and attributed qualities to numbers. The number one was similar to the geometrical point and represented unity. Two represented duality or opposites and was related to a line. Three represented wisdom and was related to the triangle. Four represented the square and was related to the earth. Hence, geometry and number were thought about together and were associated with specific meanings.

The Pythagoreans, while coming up with the proof for their famous theorem (the square of the hypotenuse of a right triangle is equal to the sum of squares of its other two sides), came across the incommensurability of $\sqrt{2}$. Their pursuit for perfection did not allow them to approximate such values, and they sought a solution in geometry where
Fig. 2.3 - Musical ratios. A sliding bridge on a monochord divides the string lengths to the corresponding musical notes: Unison – 1:1; Fourth – 3:4; Fifth – 2:3; Octave – 1:2. (Jay Kapraff, in Kim Williams, ed., Nexus, Architecture and Mathematics, Vol.1, 1996).

Fig. 2.4 - The Pythagorean tuning system. (modified by author).
constructions with a compass and a straight edge helped them avoid calculations.

Starting from Plato through Euclid and later Greek mathematicians, geometry was developed as a subject with great rigor. The first formal and systematically compiled version of postulates and theorems available to us is the Elements by Euclid around 300 B.C.E. It is a work that has thirteen books, and Euclid was responsible for compiling and presenting the matter in a systematic manner. The first six books dealt with two-dimensional geometry; books seven to ten dealt with the theory of numbers with emphasis on proportion, and commensurable and incommensurable magnitudes; books eleven to thirteen dealt with three-dimensional geometric objects. The Elements hardly contained any descriptive theorems; most of them were metrical theorems involving measurement or properties of shapes or objects (descriptive theorems would deal with positions). Euclid used the five postulates to prove most of these theorems, which were constructions based on the straight edge and compass. He systematically developed proofs for theorems based on the postulates and theorems proved earlier in his book. In this manner he proved various theorems of which some important ones were the Pythagorean Theorem, the fact that the internal angles of a triangle add up to 180 degrees, and other theorems on congruency and similarity of triangles. He also demonstrated the construction of the incommensurable Golden Ratio. Euclid used two methods; one used the square in a semi-circle and the second a star pentagon. The last book dealt with polyhedra. The five regular polyhedra, also known as the Platonic solids, were assigned to the five basic elements by Plato. Euclid demonstrated the reason for the existence of only five regular polyhedra and also the
Let $ABCD$ be a square inscribed in a semicircle with diameter $GH$. Let $AD = AB = 1$, therefore $AE = 1/2$.

By Pythagoras theorem:
\[
DE^2 = AD^2 + AE^2 = 1 + 1/4 = 5/4
\]
\[
DE = EG = EH = \frac{\sqrt{5}}{2}
\]
\[
BG = AH = (\sqrt{5}+1)/2 \approx 1.618 = \text{the golden ratio}.
\]

Fig.2.5 - Construction of the Golden Ratio. The square and semi-circle method. (Richard Padovan, Proportion, 1999).

Fig.2.6 - The Platonic Solids. Shown with their corresponding elements. (Mario Livio, The Golden Ratio, 2002).
Limitations of Euclidean Geometry

The Greeks were interested in defining the absolute and unchanging aspects of objects. They focused on metrical theorems, which were proven with rigorous logic, and could be considered to be true under most circumstances. This led to a static approach to form. “The Greeks measured line segments, angles, areas, and volumes. They worked out many relationships between those measurements, but their geometry paid no attention to notions such as infinity and geometrical continuity.” Until Apollonius (250-175 B.C.E), they did not know that the three conics could be generated from a single cone. [Fig.2.7] The Greeks also did not know that all conics were obtained by varying the foci distances, which is an essential concept for geometric continuity. They could not see that two intersecting straight lines were also part of the conics. Hence, the idea of geometric continuity did not exist for the Greeks.

The Greeks did not have the concept of a three-dimensional continuous and infinite space. In their geometry line segments had finite length. Hence lines could be extended to any finite length, but were not infinite and never ending. This also limited them from the idea of parallel lines meeting at infinity, which resulted in the last postulate of Euclid. Many mathematicians before Euclid had failed to prove the last postulate and this made Euclid incorporate it as a postulate. The parallel postulate was later modified in the 1800s to the way we know it today. This is an important idea as it helps prove that the internal angles of a triangle are 180 and also suggests a two-dimensional planar notion of space. For this reason Greek geometry and their approach
Fig 2.7 - The conic sections. Here they are generated by the intersections of a plane on a cone at different angles. (mathworld.wolfram.com).

Fig 2.8 - Drawing by Durer. This drawing made by Durer demonstrates the concept of perspective construction. (Morris Kline, Mathematics in Western Culture, 1953).
to form is mostly considered to be two-dimensional. While they did deal with three-dimensional objects, they were more interested in the finite limits of those objects which in turn were generated from two-dimensional shapes.

The fact that the Greeks did not utilize the concept of infinity stems from the idea that geometry for them was based on experience of touch or tactile intuitions. This is evident in the way they establish the idea of congruence. Triangles were considered congruent when they were placed over each other. The tactile awareness accounts for things that are near at hand. Euclidean theorems dealt with measurement, proportion, congruence, and parallelism, which were ideas that resulted from the experience of touch. Amongst these ideas, infinity did not find a place as it escaped the experience of handling and measurement.

**Linear Perspective**

Although there was some use of perspective during the Greek and Roman periods, it was accomplished through a trial and error method. During the Renaissance, painters wanted to give visual depth to their paintings as they sought to produce an accurate representation of reality. They were trying to represent three-dimensional objects in a two-dimensional plane. The main problem was the extent to which an object at the back of the scene had to be shorter than the one in front. The Renaissance artist-mathematicians realized that a solution for this problem lay in geometry. Filippo Brunelleschi was the first to develop an accurate method of drawing a perspective using geometry around 1425 C.E. The first written account of perspective was that of Leon Battista Alberti in his book the Della Pittura in 1435 C.E. He imagined the artist’s
canvas to be a glass screen, or a picture plane, through which the artist looked at the scene to be painted. From the eyes imaginary lines of light called projections went to various points on the scene. The impression was created by marking the points on the glass where those lines intersected. Thus, the central idea was that the picture plane was a section of the cone of vision.\textsuperscript{21} [Fig.2.8]

Using the above concept Alberti, in his book, represented a tiled plane on a canvas. [Fig.2.9] Suppose the canvas was held in a vertical position, the perpendicular from the eye met the canvas at a point called the vanishing point. The horizontal line through the principal vanishing point was called the horizon.\textsuperscript{22} The figure illustrates how the construction took place. Three assumptions were made in this construction. First was that all horizontal parallel lines that were perpendicular to the picture plane met at the vanishing point. The vanishing point was a hypothetical concept and did not coincide with anything in the scene. The second was that any set of parallel horizontal lines not perpendicular to the picture plane converge at a point along the horizon. The third was that all horizontal and vertical parallel lines, also parallel to the picture plane, remain parallel in perspective.\textsuperscript{23} Alberti’s method of drawing a perspective dealt with one point perspective, also called focused perspective. It was the basic and simplest demonstration of perspective drawing. Later artist-mathematicians like Pierro Della Francesca and Albrecht Durer explored drawing curves in perspective.

During the Renaissance, one of the main issues with proportion was that when one viewed buildings in perspective, the calculated proportions changed and the building might not appear the way it should appear. Piero Della Francesca and later Leonardo da
On the plane ABN the horizontal line NR is drawn based on the height of the eye. The point R is determined by setting NR as the viewing distance. R is the eye of the viewer. The vanishing point C is chosen at the centre of the line NR within the plane. This point in the picture is directly opposite the viewer’s eye. The line AB in the picture is divided equally, and each division point is joined to C by a line. These are lines that run perpendicular to the plane of the picture. As shown in the figure the line NB is intersected by the lines converging at R. The final step is to draw lines perpendicular to the line NB from these intersection points. These run parallel to the ground line AB of the picture. The resulting image represents a tiled floor in perspective. The spacing of the square grids gets smaller as the lines of the grid get farther away from the viewer. This is known as foreshortening.

Fig.2.9 – Perspective drawing by Alberti. He demonstrates the way a tiled plane can be constructed in perspective. (Morris Kline, Mathematics in Western Culture, 1953).

Fig.2.10 - Musical proportion in Perspective. (Rudolf Wittkower, Idea and Image: Studies in the Italian Renaissance, 1978).
Vinci tried to prove that receding objects in a perspective were similar in proportion to the one they had used in their buildings. Hence, for them objective proportion and subjective proportions of a perspective were similar. Leonardo went a step further and showed that proportions in perspective could relate to musical ratios. If four objects of same unit height were kept at one, two, three and four units distance from the eye, their distance, considering the picture plane to be a unit distance, would be 1/2, 1/3 and 1/4. These could be equated as 1: ½ or 2:1, 1: 2/3 or 3:2, or 3/4 or 4:3 which were octave, fifth and fourth. [Fig 2.10] It is evident from this that the Renaissance artists and architects invented perspective to obtain metrical relationships. Perspective established a metrical relationship between the object and the picture, and many scholars conclude that Renaissance perception was primarily tactile.

Perspective as developed during the Renaissance was the earliest known geometrical scheme of depicting objects in a three-dimensional space. This development was a significant departure from Greek geometry where there was no two way relationship between the subject and the object. The Greeks dealt with form in an absolute manner; they were interested in the properties of the object that were intrinsic to it. The Renaissance saw a two way relation between the object and observer. They were interested in the way the appearance of an object changed with the position of the observer. The intrinsic properties of an object do not change but the changes that occurred were a result of the position of the viewer. The Renaissance developed linear perspective with the hope of representing reality accurately. Whether they captured the three-dimensionality of space in a two-dimensional painting is questionable. Scholars
often believe that the way a perspective is geometrically constructed does not represent the spatial structure of our visual space. It represented a particular view of space. Perspective unified the composition in a hierarchical manner with all parallel lines converging at a notional centre, which was the vanishing point. It also assumed all lines on a scene to be parallel or perpendicular to the picture plane. This is not the way we actually see our surroundings as our visual impression is created by two eyes that are constantly moving. Focused perspective was therefore static and reduced binocular vision to a fixed point that was the apex of the cone of vision.

Projective Geometry

The basic idea in linear perspective was that of projection and section. A picture plane intercepted the cone of vision, and produced an image at points of intersection. If one were to change the angle or the position of the picture plane, the projected image would be different. In such a condition, what would be the properties of an object that did not vary under different projections? This was the main question addressed by Girard Desargues, a self-educated architect and engineer. Previously, studies were focused on the way the angles and lengths changed under projection: these were essentially metrical properties. Desargues developed projective geometry around 1639 C.E to study the properties of objects that were invariant under projection. He was the first to make a significant departure from the metrical theorems of geometry to more descriptive ones. As a part of his study, Desargues had to make some assumptions: there had to be points at infinity; every line segment could be extended on both sides indefinitely; all parallel lines intersected at infinity; every plane could be extended to
infinity, etc. From this he came to the conclusion that cones and cylinders were similar, because a cylinder could be a cone with vertex at infinity. He also came up with descriptive theorems that suggested that, if a pattern of curves or lines had certain intersections with each other, they remained unchanged with respect to their collineation, betweeness and order of points. [Fig. 2.11] These properties were invariant under any projection. He would later use these ideas to prove that all conic sections were equivalent to a circle.

It was previously known that a circle appeared as an ellipse in perspective. Johann Kepler had proved around 1621 C.E. that by continuous transformation of the distance between the foci one could obtain different conic sections. He showed that there were five conic sections: the circle, ellipse, parabola, hyperbola and line. [Fig. 2.12] Desargues was the first to unify all five conic sections through perspective construction. If one were to assume the vertex of the cone to be the eye, then as the angle of the picture plane cutting the cone changed, it would give a different conic section. Since these sections were essentially projections of a circle, and some of their properties did not change under projection, one could assume the conics to be equivalent to a circle. Desargues’ system opened the possibility of a more unified approach to various geometric objects.

**Analytical Geometry**

Analytical geometry was born around 1637 C.E of two fathers, Rene Descartes and Pierre Fermat. Both men were investigating new ways of representing and analyzing curves, and developed similar techniques of uniting algebra and geometry.
Imagine the eye at $O$ looking at a triangle $ABC$. The projection of $ABC$ is $A'B'C'$ with $A$ corresponding to $A'$, $B$ to $B'$ and $C$ to $C'$. Then when the sides are extended as shown in the figure the intersection of the corresponding sides are in a line. This is true for triangles in any plane.

Kepler assumed that all conic sections have two foci. The foci meet each other at a single point, in case of the circle and the line. The distance of the foci is greater than zero, but finite in an ellipse and hyperbola, and infinite in a parabola. The second focus of the parabola is at infinity. He, in this manner, unified the circle, the conic sections and the line. He could also conclude that the crossing lines, the asymptotes belong to sides of the cone. This reiterates the idea that a line is a circle with the centre at infinity.
There were minor differences in their method, but Descartes’ method will be discussed here as his work was published first, and was more popular. Descartes mostly focused primarily on relating geometric constructions to algebraic equations. He developed the method of using horizontal and vertical distances to locate points; this method is known as co-ordinate geometry today. It forms the basis of a lot of contemporary mathematics. Descartes wanted to find a method to define every possible curve that existed. He started with the simplest way of understanding a curve, which was in terms of a line. He generated the curves with the motion of a point P on a vertical line PG moving along a horizontal line OQ. As P moved up or down the curve was traced. Using this concept of continuous motion he derived equations with two variables for a straight line, circle, and conic sections. Each curve had an equation that uniquely described the points on the curve, and no other points. Descartes and subsequent mathematicians have used this idea to define various curves mathematically. Descartes was bothered by a many curves that were not geometrically definable or could not be reducible to any metrical relationship. He mainly experimented with two dimensional curves, but his contribution paved the way for further developments that dealt with algebraic representations of complex curves. In a contemporary context, Descartes’ coordinate system is used to locate points and define objects in computer graphics.

**Differential Geometry**

During the eighteenth century, investigation shifted towards analysis of complex curves and surfaces. Scientists wanted to represent more complex curves like free form curves in two and three dimensions more accurately. Differential geometry developed as
Fig. 2.13 - Cartesian geometry. A curve generated by the motion of a straight line segment of varying length. (Morris Kline, Mathematics in Western Culture, 1953).

Fig. 2.14 - An Epicycloid. Top: An Epicycloid is generated by moving the small circle over the circumference of the larger circle. Bottom: Different curves are obtained by varying the speed of movement. (mathworld.wolfram.com)
a branch of geometry that used calculus to study such complex surfaces and curves. Calculus was developed by Sir Isaac Newton, Gottfried Wilhelm Leibniz and Pierre Fermat to study instantaneous speed. Previously scientists had calculated average speed by dividing total distance by total time. These scientists were interested in studying the changing speed of a moving body and focused on calculating the speed of a body at a particular instant. Since at any instant in time the distance travelled is zero, it was difficult to compute such a value. The way they approached it was to calculate the average speed as intervals tend towards zero. It was possible to calculate the average speed of a moving body for a duration of one minute or less. So, they would calculate average speeds at 1/100th of a second or closer. It was this idea of breaking something into infinitely small quantities that mathematicians borrowed from calculus and applied it to the study of curves and surfaces.

Concepts of analytical geometry were used to develop equations for curves like the conic sections, cycloids, and epicycloids. [Fig.2.14] Most of these curves could be represented on a plane surface and were considered to be two-dimensional curves. Quadratic surfaces like the hyperbolic paraboloid, parabolic cylinder, elliptic cone etc., were represented as a single second degree equation in three variables. [Fig.2.15] Some surfaces of revolution could also be expressed as a single second degree equation in three variables. Differential calculus was used to calculate arc lengths, slopes and surface areas of either these two-dimensional curves or curved surfaces. For example, the arc length of a curve was calculated by assuming the curve to be made of infinitesimal lines, and integrating these small distances. It was not possible to derive a
single equation for either space curves like the helix or other complex curved surfaces. Differential geometry was used to express the geometric properties like curvature, surface area, and arc lengths of either these space curves, or surfaces.

It was especially challenging for the scientists, at the turn of the nineteenth century, to determine the curvature of some complex curved surfaces. Gauss in the early part of the nineteenth century realized that the central idea involved in the study of surfaces would be curvature. His main assumption was that curvature was a local property of a surface. Around 1827 he published works that showed the curvature of a surface at a point could be expressed in terms of two specific sections of the surface at that point. He considered two perpendicular sections at a point, and expressed the curvature of the surface as the product of the curvature of the perpendicular sections. This idea of localized curvature later led to the idea of n-dimensions and non-Euclidean geometry.

**Descriptive Geometry**

Gaspard Monge made significant contributions to systemizing various methods in geometry. He was the first to introduce the coordinate system for the three axes as we know it today. The equation of a line based on the slope and intercept variable \((y=mx+c)\) was introduced by him. He used the slope variable in the equation to determine parallel and perpendicular lines. Monge, in 1795 C.E, developed ideas put forth by Desargues, into descriptive geometry. Desargues had made significant progress in presenting a unified concept of geometry, but it was Monge who generalized and simplified this idea to make it available to a non-mathematician. In the process it became an important tool.
Fig. 2.15 – A hyperbolic paraboloid. (mathworld.wolfram.com).

Fig. 2.16 – The three coordinate axes. Left: The basic idea of intersecting planes and parallel projection in descriptive geometry. Right: The way the planes could be related to the co-ordinate axes. (Peter Gasson, Geometry of Spatial Forms, 1983).
for visualization. With descriptive geometry any object in three-dimensional space could be represented in a two-dimensional plane, from any point of view. He introduced two intersecting orthogonal planes on which the images were projected. The plan of the object was projected on the horizontal plane, and the section and the elevation of the object were projected on the two vertical planes. [Fig.2.16] He used parallel projections to represent these images, and assumed the observer to be an infinite distance. In such a method there is no change in the size of the object. Monge also proposed that the plan, section, and elevation were sufficient to give a complete idea of the object.

Representing objects in three dimensions as two dimensional plans or sections was not unfamiliar previously. A more rudimentary form of such representation was familiar to medieval stone masons; though such knowledge might have been restricted to limited shapes like voussoirs for arches. They also might have used a rule of thumb method to cut stones for ribs and staircase soffits with complex curves. Descriptive geometry became an important tool for visualization and representation in architecture and engineering schools. After being introduced in the Ecole Polytechnique in the later part of eighteenth century, it became a requirement for all schools of design and engineering world over. Today, computer softwares continue to use this method of representing objects on the screen. By introducing such a method of using geometry to construct images, Monge truly objectified visualization.

‘N’ Dimensions and Non- Euclidean Geometry

In the early part of the nineteenth century two people in different parts of the world, Nicolai Lobachevsky from Russia and Janos Bolyai in Hungary, discovered non-
Euclidean geometry. Each of them found that the fifth postulate of Euclid failed under certain conditions. Both Lobachevsky and Bolyai determined that, given a line and a point not on that line, there was more than one line through that point which was parallel to the given line. They also proved that, for the above, the sum of the internal angles of a triangle was less than 180 degrees. They suggested that the new geometry would not be a plane geometry like the Euclidean geometry, and also suggested that they did not know which geometry was closer to reality.

Riemann tried to unify these different concepts of geometry put forth by Gauss, Bolyai and Lobachevsky. He introduced the concept of an abstract mathematical space, which was called a manifold in contemporary terminology. Gauss had approached such an idea when he localized curvature. Here, one is not interested in understanding the curve as seen from outside, but is interested in the property of the curve as seen from within. It is equivalent to saying that when one sees the earth from space it seems different from the way one would see the earth when on it. Gauss did not bring his idea to this clarity. His student Riemann threw light on the subject when he introduced the idea of an abstract mathematical space with changing dimensions. So, one could have spaces that vary from one dimension to two, three, four, and so on to ‘n’ dimensions. Here a dimension is the number of variables required to define a point in a space. Riemann also proposed that a space in any dimension could be flat or curved.

Helmholtz used these ideas of Riemann to propose three possible variations of our space. If the curvature is positive, it is a spherical space where all lines will form circles when extended. There are no parallel lines in such a space, and the sum of internal angles of a
triangle is greater than 180 degrees. [Fig. 2.17] This is also known as spherical geometry. If the curvature is negative, there are an infinite number of parallel lines through a point. In such a case the sum of the internal angles a triangle is less than 180 degrees. This is the geometry of Lobachevsky and Bolyai, also known as L&B geometry or hyperbolic geometry. [Fig.2.18] When the curvature is zero, it is Euclidean geometry, where the fifth postulate of Euclid holds true, in such a condition the sum of the internal angles of a triangle is 180 degrees. Thus, Helmholtz succeeded in unifying the works of Riemann, Lobachevsky and Bolyai. For two thousand years all scientists had operated within Euclidean geometry; they assumed it to be the only explanation of reality. The works of these scientists were significant as these concepts lead to a different explanation of reality.

**Fractal Geometry**

The methods of differential geometry were very useful in the analysis of smooth curves and surfaces. Scientists thought that any curve with sharp bends or kinks could be broken to smaller curves. The first notion of a fractal was discovered in 1861 by Karl Weierstrass. He pointed out that it was not possible to define a curve that completely consisted of corners within the existing method of analysis. There was no concept to describe such a condition at that time. In 1904 Helge Von Koche defined the shape of a snowflake as an infinite sequence of similar sharp curves. [Fig. 2.19] The finished shape had a finite area but an infinite perimeter. Felix Hausdorff in 1941 proposed such shapes to have dimensions that are fractions. Imagine a line in one dimension. If one doubles it in one dimension, one gets two similar lines. [Fig.2.20] Similarly if one
Fig. 2.17 – Spherical geometry. An example where the sum of internal angles is greater than 180 degrees. (www.math.cornell.edu/~mec/mircea.html).

Fig. 2.18 – A Psuedosphere. A constant negative curvature makes it a model for hyperbolic geometry. (mathworld.wolfram.com).
Fig. 2.19 – The Koch snowflake. As one zooms out from step 1 to step 4, the outline of a snowflake emerges as an infinite sequence of self-similar shapes. (Nigel Lesmoir-Gordon et al, ed by Richard Appignanesi Introducing Fractal Geometry, 2000).
Table 1 – Fractal dimension. Table showing the relation between fractal dimension and number of copies for different shapes. (math.rice.edu/~lanius/fractals/dim.html)
doubles a square in two-dimension one gets fours squares. If a cube in three-dimension is doubled one gets eight cubes. Similarly, if one were to double a Sierpinski's Triangle in two-dimension, one would get three triangles. Therefore within a dimension the number of copies on doubling can be generalized to \( n = 2^d \) where \( n \) is the number of copies and \( d \) the dimension. So by substituting in the above formula one gets \( 3 = 2^d \) for a Sierpinski's Triangle. By using log function \( d \) is calculated to be 1.58, which is the fractal dimension of a Sierpinski's Triangle.

Benoit Mandelbrot in 1975 coined the term fractal geometry for such shapes that have infinite self-similar entities. The word fractal has its roots in the Latin term fractus, which means to break. Mathematicians of the nineteenth century thought themselves to have gone beyond the structure of nature to abstract mathematical concepts. Mandelbrot again turned mathematics towards nature to demonstrate the existence of fractals. The two main ideas behind fractals are self-similarity and scaling. Sometimes, certain shapes that appear to be composed of very small irregular units have the same feature as that of the larger one. These features are maintained as one zooms into it. Such shapes are called fractals and have a fractal dimension. Hence, “fractal geometry is the study of mathematical shapes that display a cascade of never-ending, self similar, and meandering detail as one observes them more closely. Fractal dimension is a mathematical measure of the degree of meandering of the texture displayed.” Mandelbrot showed that the fractal dimension of the British coast line was approximately 1.26, which was same as the Koch curve or snowflake. Fractals are ubiquitous in nature and could be assigned to clouds, cauliflowers, coastlines, mountains
and many other things in nature. Unlike Euclidean geometry that sought to abstract irregularities in nature to straight lines and smooth curves, fractals capture the complexity and textures of nature. There has been a renewed interest in fractal geometry since Mandelbrot’s work due to the ability to visualize fractals with the aid of Computer Graphics. Mandelbrot was one of the first to study the fractal qualities of Julia sets with the aid of computers. [Fig. 2.21] Loren Carpenter during the 1980s used fractal geometry to create software that could simulate surfaces of mountains, trees and parts of nature with texture similar to fractals. Carpenter used the idea of fractals to compose surfaces that are recursive subdivision of triangles. This method creates the best landscapes in computer graphics.

**Geometry and Computers**

Geometry has played a key role in visualization. Starting from Renaissance perspective, to Monge’s descriptive geometry, various techniques have been developed to represent reality accurately. Engineering drawing used descriptive geometry and provided an accurate way of communicating designs to the manufacturer. Until the 1980s, drawings were manually drafted with a straightedge and triangle. The first CAD (computer aided design) software in the 1960s was used only for 2-D drafting. This software program was a literal replacement of hand drafting, but was still helpful as it reduced human errors and increased the reusability of drawings. It used concepts from Euclidean geometry, such as, lines, angles, and curves to generate drawings. While the user interface had fairly simple concepts from Euclidean geometry, the software architect used complex concepts of geometry to design the software.
Fig. 2.21 – An example of a Julia set. (mathworld.wolfram.com).

Fig. 2.22 – Computer generated forms. Computer software generates complex geometries with algebraic concepts of geometry. (Peter Szalapaj, Contemporary Architecture and the Digital Design Process, 2005).
softwares for visualization and modeling have always built on an extensive use of differential geometry, vectors, determinants and matrices.

The early CAD software used a solid modeling interface or constructive solid model interface to generate three-dimensional models. This consisted of basic primitives like the cube/prism, cylinder, cone, sphere and torus on which Boolean operations like intersection, union and subtraction could be performed. It was possible to create complex models from simple geometric objects, but it was not possible to make minor changes or create organic shapes. Initially, this software was expensive and lacked good user interface. Around the 1980s a more user friendly software was released by Autodesk. With interactive systems like NURBS (Non-uniform rational b-spline), which could be changed with control points, it was easier to represent free form curves digitally. [Fig. 2.22] These tools were developed by Paul de Casteljau, Pierre Bezier, Stevens Coons and others in the 1960s, but were put to use only in the 1980s. Their work is the foundation of complex curves and surface modeling to this day. Initially, computers were used only to represent objects; with the CAD/CAM (Computer Aided Manufacturing) interface, one can directly construct objects three dimensionally from the computer model. This tool was developed in the aerospace industry to cut and shape complicated curved surfaces and has totally eliminated the need for two dimensional drawings.

End Notes

1 Moriss Kline, Mathematics in Western Culture, (New York: Oxford University Press, 1953),16.
It is not clear if Pythagoras individually came up with a lot ideas or it was his group and disciples who also made contributions.

Geometric continuity is the idea that the five conic sections: the circle, parabola, ellipse, hyperbola and a pair of intersecting lines, are variations of each other. This leads to the idea that a line could be a circle of infinite radius.

The ninth book of Euclid deals with conics. Here a cone is defined as a solid generated by rotating a right triangle about its legs. He classifies cones based on their vertical angles as acute, right, or obtuse angled. The conic sections the ellipse, parabola and hyperbola were gotten respectively from the acute, right and obtuse angled cones. Victor J. Katz, *A History of Mathematics: An Introduction*, (Mass.: Addison-Wesley, 1998), 117.
The deviation of a surface from being a curve.

Victor J. Katz.


http://www.vanderbilt.edu/AnS/psychology/cogsci/chaos/workshop/Fractals.html.

Ibid, 7.


http://www.cs.fit.edu/~wds/classes/graphics/History/history/history.html.


CHAPTER III

ANDREA PALLADIO: 1508-1580

Introduction

The Italian Renaissance (1400 C.E to 1600 C.E) was a revival of classical culture; in architecture specifically, it was a rediscovery of Roman ruins and texts. “Temples, basilicas, and baths [became] objects of study and artists’ reconstruction, and the classical language of the architectural orders [was] once again proposed as the principle element of articulation for the surfaces that define [architectural] space.”¹ A set of well defined elements borrowed from classical architecture was used to explore various compositions in plans and sections. “Architectural composition could be defined as a process capable of ordering and harmonizing [a] series of [architectural] elements defined in form and structure.”² Renaissance intellectuals believed that the human body was a creation of God, and that it reflected the cosmic order.³ The human body was considered to be well proportioned, and it became a symbol of perfection and harmony; man became the measure of all things. [Fig.3.1] Architecture had to emulate the composition of the human body to achieve perfection. Since the human body was bilaterally symmetrical, it became an important concept for composition during the Renaissance. The use of proportions in architecture became important as it helped integrate different parts of a building into a harmonious whole. Classical rules of composition, which dealt with symmetry, proportions, and orders, became important as they were tools to achieve beauty and harmony.
Fig. 3.1 – Drawing of the Vitruvian Man by Leonardo da Vinci. The human scale becomes the measure for architectural space which is symbolized by the unity of the square and the circle. (Peter Murray, Renaissance Architecture, 1985).
Brunelleschi is considered to have laid the foundation for Renaissance architecture. He appropriated the formal elements of classical architecture to develop a language that would be followed by his successors, such as Alberti and Bramante. Andrea Palladio followed the tradition of earlier Italian Renaissance architects by reiterating the principles of classical architecture that his predecessors had revived. His works are often associated with symmetrical fronts and columns of classical orders topped with a pediment. He achieved the Renaissance ideals of beauty and harmony through a rigorous application of proportion and geometry. It was the systematic application of a grammar of forms and proportions that made Palladio’s works distinct from his predecessors and contemporaries.

Count Giangiorgio Trissino and Daniele Barbaro were two influential figures in Palladio’s life. Trissino gave Andrea di Pietro della Gondola, the talented young stone mason, the Roman nick name Palladio. It was probably Trissino who introduced Vitruvius’ Ten Books on Architecture to Palladio. Marcus Vitruvius Pollio wrote the Ten Books on Architecture around 50 B.C.E. This book could have been Palladio’s main source for ideas of Classical architecture before he undertook trips to Rome with Trissino. In his book Palladio states that, “I always held the opinion that the ancient Romans, as in many other things, [had] greatly surpassed all those who came after them in building well, I [therefore] elected as my master and guide Vitruvius, who is the only ancient writer on this art.” Vitruvius “particularly admired the Hellenistic temples, which represented for him the perfect union of geometry, measure and proportion – qualities which for him mirrored the beauty found in nature and in the human body.”
The Vitruvian concept of symmetry was based on correspondence of measure. It meant something that was well balanced and was related to proportion. Renaissance architects interpreted symmetry as something reflective along an axis, such as bilateral or rotational symmetry. Barbaro became Palladio’s mentor after Trissino’s death. He was a Humanist and was responsible for the first translation and commentary of Vitruvius for which Palladio prepared the drawings of Roman buildings. Barbaro was an influential person in Venice and helped Palladio secure some important projects. Most of Palladio’s knowledge of mathematics and proportions is based on the fact that he was close to Barbaro, who was well versed in music and mathematics.

In spite of being engaged with Italian Renaissance ideas of proportion and geometry, Palladio also had a practical and functional approach to design. This is substantiated by his Four Books on Architecture written around 1570 C.E. This book is an important testament in which he sets out rules for the architectural orders, room sizes, stairs, and details. The second and third books are particularly important as they give a retrospective of buildings designed by him. Unlike treatises by other authors, Palladio, in his book, labels all the dimensions (his unit of measurement was the Vicentine foot of 357mm) of all plans and elevations. The dimensions in the drawings of his projects, published in the Four Books, do not coincide with the respective dimensions of the built project. In this thesis, I focus only on the plans published in the Four Books as it is dated later than all the built projects. Thus, is likely to reflect a refinement of his design ideas. Palladio’s Fourth Book consists of reconstructions of Roman buildings in the form of plans, sections and elevations. From these Roman Buildings he extracted principles
concerning geometric forms, plan types and elements, and skillfully adapted those to his own design needs.\(^9\)

**Typical Characteristics of Palladio’s Villa**

Palladio made significant contributions to the development of country Villas, which fulfilled a new need for countryside residences in the Veneto. These Villas did not have to be as big as a palace, but still had to work as a functioning farm, and express the power and wealth of the owner.\(^{10}\) While planning his Villas, Palladio established some planning principles from which he never deviated. The more public rooms were on the central axis while the left and right sides had symmetrical suites of rooms, which went from rectangular chambers, via square middle sized rooms, to small rectangular ones.\(^{11}\) Kitchens, store rooms, laundries and cellars were mostly on the ground floor. His approach to designing these took into account the practical needs of his clients, and function was always given priority over aesthetics. The Villas usually had a cubic exterior with minimal ornamentation. [Fig. 3.2] They were built of rough brick with a coating of stucco; the entrance porch had columns of classical orders with a pediment on top and was usually the only part that reflected a direct reference to antiquity. The pediment with columns, which was used in antiquity for temple fronts, was used by Palladio for Villas, for he believed that temples essentially developed from residential architecture.\(^{12}\) Thus, he justified his use of temple fronts for Villas.

A sample of few Villas built over fifteen years indicates that they were derived from a single geometric pattern.\(^{13}\) The Villas were mostly organized along a triple bay system within a rectangle divided by four longitudinal axes and two transverse axes.
Fig. 3.2 – Examples of Palladio’s Villas. His Villas minimal ornamentation, and a Greek temple front. The exterior forms of his Villas have a simple cubic exterior. a) Villa Cornaro at Piombino Dese b) Villa Emo at Fanzolo c) Villa Pisani at Montagnana, d) Villa Badoer at Fratta. (Branko Mitrovic, Learning from Palladio, 2004).
Palladio adapted this geometric pattern to the requirements of each commission. [Fig.3.3] His Villas had a clear hierarchy of parts where the center was very important both in plan and section. The three bay system of the plan usually had an important central bay that was wider than the other two, or formed a cross axis shaped room that reinforced the center. There was an uneven number of longitudinal sections with the center being dominant in the cross section. In elevation the center was emphasized by the portico with a pediment supported by columns. In some Villas the use of hierarchy was further emphasized by the body of the villa being dominant over the porticoes, barns, and other structures. According to Palladio, if there were a single porch, it had to be placed on the central bay, and if there were two porches, they had to be on the side bays. This rule was applied to staircases too. The resulting design seemed to grow from the centre.

Besides developing a new scheme for the functional distribution of spaces in the Villa, one of the main contributions of Palladio was to connect all the dimensions of a building into a harmonious whole. Therefore with “full control of width and depth as well as height relationships in his central bay, in his lateral bays and the interrelationship of all these to the whole aspect, both in elevation and in plan, [he created a design that] was tightly knit as an organism.” Creating numerically integrated buildings, where the numbers had their roots in proportion, was one of Palladio’s main focuses. In his Four Books he talks about seven types of rooms: “..they can be made circular, though these are rare, or square; or their length will equal the diagonal of the square of the breadth; or a square and a third; or a square and a half; or a square and two thirds; or two squares.”
Fig. 3.3 - Plan types of eleven Villas. They form a basic geometric pattern. (Rudolph Wittkower, Architectural Principles in the Age of Humanism, 1998).
These length/width ratios of the rectangular rooms that Palladio mentions in his book could be represented as $\sqrt{2}/1$, $4/3$, $3/2$, $5/3$, or $2/1$. He also adds that the closer the rooms are to a square the more they are ‘praiseworthy and practical’. It was common practice then to use vaults for the ground floors and flat ceilings for the upper floors. Palladio also gives simple geometrical constructions for obtaining vault heights from room dimensions in plan. [Fig.3.4] These geometric constructions for obtaining vault heights are the arithmetic, geometric and harmonic means of the lengths and widths of the rooms. While Palladio has explicitly stated the ratios for the room dimensions, he gives neither any calculations nor any explanation for the basis of these ratios.

Over the years, scholars have tried to offer different interpretations of Palladio’s use of proportions. For more than fifty years the standard interpretation of Palladio’s works has been Rudolph Wittkower’s study, which discusses the use musical ratios in Palladio’s works. Wittkower grounded his theory in the context of “the Renaissance beliefs that God used harmonic [or] musical proportion in the creation of the world, which implied that Renaissance architects used these proportions.” However, Palladio’s writings contain neither any direct reference to harmonic proportions, nor any detailed calculations to show that he was adept with such mathematics. It could be possible that he determined the numbers by rule of thumb or tables to generate these proportions. Above all, one of Palladio’s most famous buildings, Villa Rotunda, does not have harmonic proportions. Deborah Howard and Malcolm Longair, in a detailed study of the drawings published in the Four Books, have pointed out that only two-thirds of the room dimensions correspond to harmonic proportions. This has led to certain
1. **Square rooms**: Height of the vault will be 1/3 more than the breadth.

2. **Rectangular rooms**

   a) 
   
   ![Diagram](image1)
   
   **Calculation of Arithmetic mean:**
   - For rectangular room abcd – construct point e such that eb = ac+ab.
   - Let f be the midpoint of eb.
   - Height of the vault is bf.

   b) 
   
   ![Diagram](image2)
   
   **Calculation of Geometric mean:**
   - Mark f,e as above for room abcd.
   - With f as centre and fb as radius construct semi-circle as shown in the diagram.
   - Extend ca to intersect semi-circle at g.
   - Height of the vault is ag.

   c) 
   
   ![Diagram](image3)
   
   **Calculation of Harmonic mean:**
   - Mark e as above for room abcd.
   - Extend ec to intersect bd at f.
   - Height of the vault is df.

---

Fig.3.4 - Geometric construction of vaults. (Andrea Palladio, translated by Tavernor and Schofield The Four Books on Architecture, 1997).
amount of skepticism concerning the harmonic interpretation of proportions in Palladio’s works. Lionel March has suggested new interpretations of proportions in Palladio’s works. He traces the proportions used by Palladio in the Four Books to classical mathematics and simple geometric constructions from Euclidean geometry. He also demonstrates the way these proportions could have been used in the Villa Rotonda. In this study I have primarily focused on the musical ratios discussed by Wittkower, as the projects discussed here reflect such room ratios. Lionel March’s interpretation has been discussed for the Villa Rotonda, as musical ratios are not applicable there.

**Palazzo Chiericati**

Palazzo Chiericati, started in 1551 C.E, was one of Palladio’s exceptional urban projects in Vicenza that could be compared with his Villas meant for the country side. Though it was meant to be a palace, the presence of the loggia in front and the absence of an internal courtyard make it closer to the Villa type of buildings.\(^\text{23}\) [Fig. 3.5] This Palazzo was designed for a small lot, which resulted in a front that is wider than the depth of the building. [Fig. 3.6] This building has a three bay plan with some walls, in the side bays, being offset to accommodate staircases. [Fig.3.7] The plan is bilaterally symmetric about its central longitudinal axis, which is typical of all Palladian plans. One enters the palazzo on this axis, which also serves as the main circulation path. Unlike his Villa architecture where the entrance porch emphasized the central bay, in this project there is a more subtle emphasis of the central bay. The central part projects slightly from the plane of the loggia, and the upper level was also intended to have a pediment. In the elevation, this further differentiates the central bay from the two bays on either sides.
Fig. 3.5 – Plan and elevation, Palazzo Chiericati, Vicenza. Top: plan. Bottom: elevation. (Andrea Palladio, translated by Tavernor and Schofield The Four Books on Architecture, 1997).
Fig.3.6 – Front view, Palazzo Chiericati. (James Ackerman, Palladio, 1966).

Fig.3.7 – The three bay plan of Palazzo Chiericati. The central bay with the hall is wider than the other two bays. (author).
Due to the shape of the site, Palladio could not entirely integrate structure and function. The vaults, which had to run along the longer side of the building, are perpendicular to the circulation spine. [Fig.3.8] The façade does not reveal the internal vaulted structure anywhere. The only arched lintel is found on the sides of the loggia. Despite the fact that the central hall is not in the shape of a cross, this organization results in an overall cross axis which is also repeated in each room individually. In the central room, openings on all four sides mark the axes and in the adjoining rooms, either a window or a fire place marks the axes.²⁴ [Fig.3.9] This idea could have been borrowed from the Frigidarium in the Baths of Caracalla where the main spaces had vistas of further areas along both the cross axes.

Unlike some of his early Villas, which have a very simple application of harmonic proportion, the plan of Palazzo Chiericati has a more complex use of dimensions. The rooms are arranged according to ascending or descending order of size. Here one dimension of the previous room has been continued to the next while the other dimension changes. The smallest room is 12’ by 18’, the next is 18’ by 18’, and last one on the row is 18’ by 30’. [Fig.3.10] The ratios generated from the dimensions of these rooms are 2:3, 1:1, and 3:5. They correspond to musical harmonies which are measured lengths on a monochord: a fifth, unison, and a major sixth. The 3:5 ratio is the major sixth in the Just tuning system which is slightly different from the Pythagorean system. [Fig.3.11] The main notes like the unison, fourth, fifth and octave are the same for both the systems. The hall, loggia, and the dimensions of other rooms on the central bay are not related to musical proportions. Wittkower suggests that the hall which is 16’ by 54’
Fig. 3.8 – Direction of vaults and movement in Palazzo Chiericati. The vaults are perpendicular to the direction of movement in Palazzo Chiericati. (author).

Fig. 3.9 – Axial organization in Palazzo Chiericati. Openings, niches, and fireplaces are aligned along the cross axes in Palazzo Chiericati. (James Ackerman, Palladio, 1966).
Fig. 3.10 – Dimensions in Palazzo Chiericati. (author)

Fig. 3.11 – The Pythagorean and Just tuning systems. (Branko Mitrović, Learning from Palladio, 2004).
could have been meant to be 18’ by 54’. According to him, it could have been reduced to keep the vault height low. The 18’ by 54’ room would have a ratio of 1:3 which is a double octave. It is not clear what the room height for the hall is and one is not able to ascertain how the reduction of height would have affected the overall scheme.

As discussed earlier, the room heights were obtained as means of the room dimensions. According to Branko Mitrovic, the rooms were proportioned such that one could derive the same vault heights in a floor using Palladio’s method of deriving heights of rooms. In Palazzo Chiericati he demonstrates that the vault height for the 30’ by 18’ room could be obtained by the arithmetic mean: \((30+18)/2=24’\). The next room is a square and hence its height is 4/3 of its side: \(4/3\times18=24’\). It is not clear how a height of 24’ is achieved for the other rooms, thought Mitrovik suggests that it is possible to achieve that by using different types of means. The inner room dimensions are also coordinated with the columns spacing on the façade.

Palazzo Chiericati is considered to be one of Palladio’s first projects that reflected a more refined use of classical orders. Palladio was one of the first Renaissance architects who made the intercolumniations on the façade coincided with the spacing of the inner walls. For the first time Vitruvius’ rule of keeping the intercolumniation less than 3 times the diameter was followed. This rule was not followed on the upper floors where the columns are thinner.\(^{25}\) It must have been a challenge for Palladio to coordinate the façade dimensions generated from the orders with room dimensions (length, width and height) that have their own rules of proportion. It is evident that variations are numerous in the building, and it is a difficult to develop mathematical
rules to coordinate all dimensions in a building. Nevertheless, Palazzo Chiericati serves as a classic example of such an attempt.

**Villa Malcontenta**

Villa Malcontenta, built around 1560 C.E, has the austere form of a cube with a pyramid shaped roof. [Fig.3.12,3.13] It is a typical Palladian Villa with three bays, and follows strict symmetry about the central axis. The walls on the second and third rows are offset on both sides to accommodate staircases. This offset of walls creates a cross shaped central hall that directly determines the cross axis and the centre. The longitudinal part of the cross shaped hall is longer than the lateral part, which makes it consistent with the central axis. [Fig.3.14] There are no farm buildings attached to the Villa it, and it stands as a simple cube on an 11’elevated base. Kitchens, smaller dining rooms and similar functions are placed on the ground floor; which functions as a basement. The loggia or the entrance porch that projects out from the building is a free standing unit of ionic columns topped with a pediment. Positioned against the central bay, it emphasizes the centre and forms the central spine of circulation. The central hall has a semi-circular cross vault that is double storied in height and seems to recall the grandeur of a Roman bath. [Fig.3.15] The buildings of Palladio do not express their structure on the outside, but the rear elevation of this Villa reveals the internal structure. The semicircular window on the upper level, and three windows below it, on the rear elevation, makes the inner vault transparent. [Fig.3.16] The room ratios are 12’ by 16’, 16’ by 16’, and the largest is 16’ by 24’. [Fig.3.17] These dimensions generate ratios like 3:4, 1:1, and 2:3, which correspond to the musical fourth, unison, and fifth. The lateral
Fig.3.12 – Plan and elevation, Villa Malcontenta, Mira (near Venice). Top: plan. Bottom: elevation. (Andrea Palladio, translated by Tavernor and Schofield The Four Books on Architecture, 1997).
Fig. 3.13 – Front view, Villa Malcontenta. (Guido Beltramini and Antonio Padoan, Andrea Palladio: the complete illustrated works, 2001).

Fig. 3.14 – Bays at Villa Malcontenta. Left: The central bay with the hall is wider than the other two bays. Right: The cross shaped central hall at Villa Malcontenta emphasizes movement along the linear axes. (author)
Fig. 3.15 - The central hall of Villa Malcontenta. The scale of the vaulting is similar to that of a Roman bath. (Guido Beltramini and Antonio Padoan, Andrea Palladio: the complete illustrated works, 2001).
Fig. 3.16 - The rear elevation of Villa Malcotenta. The central bay has a semicircular window with three long windows below it. (Guido Beltramini and Antonio Padoan, Andrea Palladio: the complete illustrated works, 2001).

Fig. 3.17 – Dimensions in Villa Malcontenta. (author)
part of the cross shaped central hall is 16’ by 32’ which corresponds to an octave.\textsuperscript{29} The
length of the longitudinal part of the hall is 46-1/2’ and the width is 16’. The ratio of
these dimensions does not correspond either to any musical ratios, nor to any of the
ratios cited Palladio in his Four Books; even Wittkower is silent about these dimensions.

The intercolumniation of the columns on the front porch is less than 3 times the
diameter of the columns. The spacing of the columns reflects the positions of the inner
walls, with the middle two columns being an exception. The two middle columns are 6’
apart, which is exactly three times the diameter of the column. This spacing of the
middle columns also emphasizes the central axis. According to Palladio, the heights of
the large rooms in front were derived from the arithmetic mean of the room’s breadth
and width: \((24’+16’)/2= 20\). The central hall is as wide as it is high, which is 16’.\textsuperscript{30} Since
he does not mention anything about the heights of the smaller rooms, it is difficult to
deduce their heights. Despite the fact that there are some dimensions that are not known
or some that do not fit into any scheme, Villa Malcontenta is another example where
Palladio tries to integrate the building through its dimensions.

\textbf{Villa Rotonda}

This Villa is usually considered to be the ‘universal icon’ of all Palladian
Villas.\textsuperscript{31} In reality it is more like a suburban house that was close to the city walls, and
not typical of the Villa built away from the cities.\textsuperscript{32} The construction of Villa Rotonda
began around 1567 and was completed with some alterations by a second architect
around 1591. Here the ‘structure is planned such that the topography and the planning
emphasized the prominence of the building’.\textsuperscript{33} [Fig.3.18] The Villa Rotonda might be
considered a ‘Villa-temple’ as the form reflects an idealized idea of order and harmony. The Villa is made of simple composition of a cube and the hemi-sphere.\textsuperscript{34} The most significant characteristic of the Villa is its centralized organization. [Fig.3.19]

There are a number of precedents for centralized organization before and during the Renaissance. Alberti, Leonardo da Vinci, Bramante and Michelangelo all expressed their preference for a central plan in religious buildings.\textsuperscript{35} There were also ancient buildings that had a centralized plan, of which the Pantheon is the most famous. While the Pantheon is centrally planned it is not ideally central as the portico creates an axial movement.\textsuperscript{36} The difficulty of joining the rectangular portico to a cylindrical form is not resolved and the pediment and the dome seem to be a contrast on the façade. In the Villa Rotunda Palladio developed an effective and original solution. By inserting a circle within the square Palladio resolved the problem of combining the curve and the straight line.\textsuperscript{37} [Fig.3.20] He repeated the portico on all the four sides of the square plan to make the building symmetric on all four sides, which resulted in a perfectly centralized building.\textsuperscript{38}

The centralized plan was also suited to the location of the Villa. Located on a hill top where it is possible to turn around and view all the four directions, this Villa has porticos on all four sides which promote gazing at the scenery.\textsuperscript{39} In most other Villas the central hall was in the shape of a cross axis, but in this building the central room takes the shape of a circle. Palladio further emphasized the center by placing a dome on top, which also creates a clear hierarchy of the external form. [Fig.3.21] The dome of the building, as built, seems lower than the one published in his Four Books. The built dome
Fig. 3.18 – Villa Rotonda, Vicenza. (James Ackerman, Palladio, 1966).
Fig. 3.19 – Plan and elevation, Villa Rotonda. Top: plan. Bottom: elevation. (Andrea Palladio, translated by Tavernor and Schofield The Four Books on Architecture, 1997).
Fig. 3.20 – Comparison of movement in the Pantheon and Villa Rotonda. Left: The Pantheon, Rome. The portico leads to axial movement. Right: In Villa Rotonda the circle is inserted in a square to achieve an ideally centralized organization. (author)

Fig. 3.21 – External form in Villa Rotonda. Left: The dome emphasizes the vertical axis in Villa Rotonda. Right: Scaling of form in Villa Rotonda. (author)
has clay tiles clad along the exterior corbelled surface of the dome. The intended structure of the dome is not known, but Palladio had planned a high semi-circular dome, which was made lower by the architect who finished the building. The lower floors have vaulted basements for which the heights were determined by methods prescribed in his book.\textsuperscript{40} The only arched openings on the facades are the ones on the sides of the porticos.

From Palladio’s Four Books one is not able to determine whether the dimensions of the Villa Rotonda were intended to have any relation to harmonic proportions. None of the room dimensions comply with Wittkower’s interpretation of harmonic proportions. Branko Mitrovik suggests that the corner rooms which are 26’ by 15’ could be approximated to $\sqrt{3}/1$ ratio, but he is not able to apply this ratio to other dimensions.\textsuperscript{41} Lionel March gives a non-musical interpretation for the proportions of this Villa. He arranges the dimensions of the plan in descending order: 30, 26, 15, 12, 11 and 6.\textsuperscript{[Fig.3.22]} The sum of these numbers adds to 100, which is a third in the series of monad which starts from 1 and is then followed by 10. This series signifies unity in the plan. The first three numbers add to 71 and the last three add to 29. The numbers 71 and 29 are related as shown in the construction; they are the incommensurable ratios $\sqrt{3}/1$ and $\sqrt{2}/1$. From the first three numbers, 30 is the diameter of a circle with radius 15 and has an equilateral triangle of side 26 inscribed in it. The second series can be related to a pentagon with chord 11 and side 7, inscribed in a circle with diameter 12 and radius 6.\textsuperscript{[Fig.3.23]} In this manner these numbers also evoke primary shapes like the square, circle, equilateral triangle and regular pentagon. Lionel March discusses in depth more
Fig. 3.22 – Dimensions in the plan of Villa Rotonda. (author)

Fig. 3.23 – Proportions suggested by Lionel March. Left: The relationship between s 71 and 29. Middle: The relationship between 30 and 26. Right: The relationship between 12,11 and 6. (Lionel March, Architectonics of Humanism: essays on number in architecture, 1998).
relationships among these numbers and follows up with dimensions from the elevation and section. March does recognize that these might have not been consciously designed, but points out that these mathematical relationships were known during the Renaissance. This discussion also takes the dimensional relationships beyond restrictive musical intervals to more general mathematical relationship of square, circle, triangle, polygon and above all unity. Hence harmony is sought in a broader sense than just harmonic proportion.  

**Conclusion**

One of the most remarkable aspects of Palladio’s oeuvre is the variety of combinations he achieved with limited elements like walls, columns, pediments, flat ceilings, vaults, windows, and fireplaces. He organized these elements on a fixed orthogonal grid, and used principles like bilateral symmetry and axial organization to achieve harmony and perfection. Palladio’s buildings followed strict symmetry both in plans and elevations. His works usually have single axis reflective symmetry (bilateral symmetry), except Villa Rotunda and Villa Trissino where there is rotational symmetry. Bilateral symmetry provides smooth movement along the axis in contrast to rotational symmetry, where the centre becomes very important, and breaks the movement by creating a pause; this makes the space static. Therefore, in Villa Malcotenta the longitudinal part of the cross shaped central hall is made longer than the lateral part to maintain the linear movement along the central axis. Palladio’s works never had a wall or column built along the central axis. Qualities like the hierarchical organization of forms, the cross axis, the positioning of the loggia on the central bay, and bilateral
symmetry were developed to reiterate the centre, which was invariably the geometric centre too.

One of Palladio’s main goals was to integrate his building dimensionally. It was a challenge for Palladio to coordinate the diameter of the column, the intercolumniation, the position of inner walls, and the height of the rooms. Wittkowers’ interpretation of harmonic proportions deals only with dimensions on the plan. He is also selective about the dimensions he discusses. For instance, he does not attribute any proportional relationship to the 46-1/2’ by 16’ hall in Villa Malcoententa. Similarly, Wittkower is not very clear about which scale Palladio uses, as there were two tuning systems that were known during the Renaissance: the Pythagorean, and Just systems. For Palazzo Chiericati and Villa Malcontenta, Wittkower uses the Just tuning ratios, while at Villa Emo, Wittkower uses the Pythagorean tuning ratios. Even Lionel March’s non-musical interpretation seems to be a number game. The dimensions he attributes to simple geometric constructions are approximations, and it is very unlikely that such complicated relationships could have been used unintentionally. The next question is whether the dimensions on the plans in the Four Books coincide with Palladio’s preferred ratios: √2/1, 4/3, 3/2, 5/3, or 2/1. Howard and Longair’s study shows that only 55 percent of the ratios correspond with Palladio’s preferred ratios. Amongst these ratios, only three rooms have the ratio √2/1. With absence of wall thicknesses, and other similar dimensions, it is also not clear how Palladio related the individual room dimensions to the heights of the rooms, the overall plan and heights on the elevation. Despite the existing ambiguity regarding the use of dimensions in Palladio’s works,
scholars are intrigued to his works as his treatise suggests a dimensional relationship between the plan and elevation.

**Correlations**

In Palladio’s four books there is very little use of geometry to generate proportions. Other than descriptions of geometric constructions to generate vault heights, and the $\sqrt{2}/1$ ratio, which was one of his preferred ratios, there are not many instances where he mentions the use of geometry for his designs. Howard and Longair’s study shows that some ratios are approximations of the $\sqrt{3}/1$ ratio. There is no evidence of the use of Golden proportions. The $\sqrt{2}$, $\sqrt{3}$ and the golden proportion are incommensurable ratios, as they cannot be expressed as ratios of whole numbers, and are derived from geometric constructions. Palladio’s preference for commensurable ratios could be an influence of Vitruvius’ view, which suggests that incommensurable ratios should be precluded from architecture. Most Renaissance architects seem to have held to this view. Hence it is probable that Palladio used very little geometry in generating proportions.

Though there is little evidence of Palladio’s conscious use of geometry his approach to form and space suggests certain correlations with geometry in mathematics. His emphasis on proportion deals with dimensions or measurements which were the basis of many Euclid’s theorems. He uses simple Euclidean constructions to demonstrate the method of obtaining the height of the vault from the width of the room. The idea of integrating his building through dimensions could have its roots in a Pythagorean worldview, which believed in the musical harmony of the world. Palladio tried to coordinate heights and widths in plans, sections, and elevations which were basically two-
dimensional projections of the building. Two sections, one along each of the cross axis, were sufficient to understand the interior spaces of Palladio’s buildings. This reflects a simple two-dimensional approach to space as the complexity of a building increases with the number of sections required to explain it. Hence, one could conclude that Palladio focused on the plan and elevations, and not on developing interior spaces. His buildings were perpendicular projections of two-dimensional drawings into the third dimension. Despite Renaissance architects’ knowledge of perspective projection, Palladio adopted plans, sections and elevations to represent his buildings. He used light and shadow rendering to add depth to his drawing. His two-dimensional conception of space places him closer to Euclidean geometry, than perspective.

The square and sometimes the circle were the geometries Palladio used in plan. A finite cuboid was the typical form for most of his villas. Elements like the dome or barrel vault were either added to it or carved inside it. The vaults he used to span the interior ceilings were rarely expressed on the façade. Thus, it was not important that the geometry of the form be an expression of the structure. Both the plan and elevation followed strict orthogonal grids. The grids were not open ended, but were finite, and contained within a square. His designs usually have a well-defined centre which is also the geometric centre of the plan. Such characteristics give a very finite and static quality to space and form, which again connects his works to Euclidean geometry.

Perspective drawing was the first geometric understanding of three-dimensional space. The structure of a perspective was basically a grid that converged at the centre or the vanishing point. The grid Palladio used in his designs had a definite centre from
Fig. 3.24 – Drawing of the entrance to the Palazzo Farnese. An example of a Renaissance perspective drawing. (Branko Mitrović, Learning from Palladio, 2004).
Fig. 3.25 – Alignment of doors in Villa Poiana.
(Branko Mitrovic, Learning from Palladio, 2004).
which the design was generated. The hierarchy present in his designs was very similar to that present in a perspective projection. Most Renaissance architects’ works are related to perspective drawings in the way they structure their interior space. The lines from square floor tiling, the coffers on the vaulted ceiling, and the straight lines of column lintels were structured to converge at a point under the dome. This was similar to the depiction of architectural interiors in paintings. The squares on the floor that receded in size and the coffers of the ceiling were spatial co-ordinates in a perspective picture.  

[Fig.3.24] Palladio’s Villas do not reveal such an approach to interior arrangement. He created a series of openings in the walls along an axis that provides axial vistas. One can see receding horizontal planes and doorways, which makes one’s eye move along the axis till it ended in a window, a niche, or a fire place.[Fig.3.25] These axial vistas connected various parts of the building. Palladio used proportion, symmetry and these vistas to integrate his buildings, which he believed reflected the harmony of the human body and the universe. The Greeks developed geometry as an interpretation of the essence of the universe. For them geometry represented an ideal order, far removed from reality. The Renaissance artists continued their tradition with the belief that the human body and the universe, represented perfection. Palladio’s use of symmetry, axial and hierarchical organizations, proportions, closed orthogonal grids, finite spaces and static form reflect this search for the ideal.

End Notes

1 Michele Furnari, Formal design in renaissance architecture from Brunelleschi to Palladio, (New York, NY: Rizzoli International, 1995), 175.
2 Ibid, 175.
3 Ibid, 185.
6 Ibid, 4.
7 Ibid, 5.
10 Guido Beltramini and Antonio Padoan, 6.
12 Ibid, 67.
13 Ibid 68.
14 James S. Ackerman, 168.
15 Ibid, 169.
16 Andrea Palladio, 57.
18 James S. Ackerman, 162.
19 James S. Ackerman, 162.
21 James S. Ackerman 162.
23 Guido Beltramini and Antonio Padoan, 46.
24 James S. Ackerman, 165.
25 The intercolumniation according to Vitruvius had to be less than 3 time the column diameter. If the intercolumniation was more the stone entablatures broke easily. Renaissance architects tended to ignore this as they used arches over these columns or the columns were pilasters. In both cases the span did not have structural implications. Palladio and some his contemporaries tried to bring it back to what Vitruvius had prescribed in his treatise.
27 Guido Beltramini and Antonio Padoan, 179.
28 Colin Rowe, 11.
29 Rudolf Wittkower, 12.
30 Andrea Palladio, 128.
31 Guido Beltramini and Antonio Padoan, 65.
32 Ibid, 65.
33 James S. Ackerman 169.
34 Ibid, 65.
35 Michele Furnari, 72.
37 Michele Furnari, 172.
38 Camillo Semenzato, 18.
39 James S. Ackerman, 70.
40 Andrea Palladio, 94.
41 Branko Mitrovic, Learning from Palladio, 64.
43 Branko Mitrovic, Learning from Palladio, 39.
Stephen R. Wassell, 184.
Branko Mitrović, Learning from Palladio, 87.
Villa Emo has rooms of dimension of 16’ by 16’, 12’ by 16’, 16’ by 27’ and a hall of dimension 27’ by 27’. The ratio 16: 27 is a major sixth on the Pythagorean scale.
Branko Mitrović, Learning from Palladio, 64.
CHAPTER IV
FRANK LLOYD WRIGHT: 1876-1959

Introduction

The beginnings of Modern architecture are often traced to the Industrial Revolution. The mass production of materials like steel, and the invention of ferroconcrete helped architects build bigger and larger structures. These materials led to new structural systems which in turn led to new expressions in architecture. The whole Modern period, from 1750 to 1950 C.E, saw different styles, schools, and movements in architecture across the world. The main theme unifying these various approaches was a search for a new expression in architecture, which was appropriate to the machine age, whether it was the use of a new material, structural system, functional organization, or a spatial concept. Iron and steel were widely used for construction in the early Modern period (around 1750 to 1900 C.E). Frank Lloyd Wright’s practice began in Chicago around late 1800s. What was remarkable about Wright’s work was that he did not directly continue the iron skeleton of the Chicago School.\(^1\) “He did not carry over the use of new material like steel and glass into his own sphere: housing.”\(^2\) He adapted the Balloon frame\(^3\) construction that was used in the housing industry to the flat surface aesthetic of the Chicago sky scraper. His architecture eventually developed with pure, independent geometric shapes that were fused into larger compositions.\(^4\) Modernism as we know it today is associated with mass production, functionalism, and pure geometric forms also known as the machine aesthetic. Most of these qualities are attributed to the
Fig. 4.1. - The Bauhaus, Dessau. An example of Bauhaus architecture. (Sigfried Giedion, Space Time Architecture, 1941).
Bauhaus School of design or to the works of Le Corbusier and others from the first half of the twentieth century. [Fig.4.1] Wright’s work, which embodies most of these qualities, was a precursor to all them.

Wright began his work at the atelier of Louis Sullivan and Dankmar Adler around 1887 C.E., and worked with them for six years. Wright was more influenced by Sullivan’s philosophy than by his actual approach to form in architecture. Modern architecture was not just about construction and technological advance, it also had deep rooted philosophical tenants by which people acted. One of the main themes of modernism especially of Sullivan was organic architecture. This idea evolved from the extensive research that was being conducted in the biological sciences around the 1850s. Scientists around this time were trying to determine the factors which determine the morphology of living organisms.\(^5\) Sullivan’s approach to form in architecture had its roots in these sciences. In his, A System of Architectural Ornament According with a Philosophy of Man’s Powers, he demonstrated, in morphological terms, with a series of geometric transformations, the way a figurative seed germ exfoliated into complex organic forms.\(^6\) [Fig.4.2] Therefore, seed germs have a latent structure that could consist of simple shapes like the pentagon, triangle and square, which can develop to complex geometric forms. With one’s imagination these simple forms can be given character by recognizing their latent structure.\(^7\) Sullivan’s approach to living organisms was to reduce their morphology to some basic geometric forms. This was most expressed in his use of ornamentation which was essentially composed of simple geometric shapes. [Fig.4.3]

Wright’s approach to geometry went beyond mere ornamentation. According to
Fig. 4.2 - The geometric transformations of a figurative seed germ. As shown in Sullivan’s A System of Architectural Ornament According with a Philosophy of Man’s Powers. (Kenneth Frampton, Studies in tectonic Culture: The Poetics of Construction in Nineteenth and Twentieth Century Architecture, 2001).

Fig. 4.3 - Sullivan’s use of geometry for ornamentation. Left: Roosevelt University foyer. Right: Roosevelt University waiting area (author).
Wright, design was an abstraction of nature in purely geometric terms. He approached form in an analytical rather than an imitative way. Wright was interested in the “practice of the thing not on it, [or the] innate property of all form, [something] not merely looked at but looked into as structure.” This thought process is reflected in the grid and straight line aesthetic typical of Wright’s works. Here manipulation of geometry facilitated the inter-weaving of parts of a building into an organic whole. Wright in this manner came closer to Sullivan’s philosophy of an organic architecture than Sullivan himself had ever had.

Wright’s experience while working with Sullivan resurrected ideas from his childhood. Wright constantly emphasized that he acquired his ability of abstraction through his childhood training with Froebel’s toys. Froebel, a crystallographer turned educator, designed toys, called ‘Gifts’ that were given to a child starting from his first birthday. ‘Froebel’s toys were simple, sturdy and crystalline in form.’ They consisted of basic geometric forms and shapes with which one could play, observe patterns and construct complex objects. According to Lionel March, “The ‘Gifts’ basically dealt with three themes. The first theme was ‘forms of knowledge’, which demonstrated quantitative and arithmetical relationship. The second was ‘forms of beauty’ that showed qualitative geometrical relationships like symmetry, reflection, rotation, translation, balance and rhythm. The third was ‘forms of life’ that express evocative compositions of a figurative representational or pictorial nature.” Froebel hoped this would encourage the child to see that geometric forms underlie all natural objects. He had developed these exercises from his experience as a
Fig. 4.4 - Froebel’s gifts. (Lionel March, Frank Lloyd Wright: The Phoenix Papers, Vol.2, The Natural Pattern of structure, 1995).
Fig.4.5 - Some examples of patterns created with Froebel’s toys. (Robert McCarter ed, Frank Lloyd Wright: primer on architectural principles, 1991).

Fig.4.6 - Two dimensional figurative representation of a praying monk. (Lionel March, Frank Lloyd Wright: The Phoenix Papers, Vol.2, The Natural Pattern of structure, 1995).
crystallographer and made a basic assumption that the underlying principles of crystals and organisms were the same. The consequence was an application of inorganic structure to organic form which led to an idealized concept of nature. Wright’s architecture was composed of pure abstract horizontal and vertical planes, where the horizontal plane was more prominent. Thus, he gave an original expression to modern architecture.

**The Prairie Houses**

From 1893 to 1910 C.E Wright designed dwellings in and around Chicago, which were known as the Prairie houses. In this thesis I cite the Willitts House and the Robie House [Fig.4.7] as examples of Prairie houses. The Willitts House, completed in 1902, is considered to be the first developed Prairie house master piece. The Robie House (1906-1910) marks the terminal point for his career and practice in Chicago.

These buildings were designed for urban blocks, which were themselves rectangular divisions of the city grid. Wright further divided them into asymmetrical rectangular grids. [Fig.4.8] The grid was not meant to be restrictive but was used to integrate the various parts of the building. “A vocabulary of forms was used to translate or express the grid at all points.” In his Prairie houses, balconies, planters and piers were used to evoke the underlying structure of the house. In Palladian Villas, the walls ran along the entire grid line, and a cuboid confined the grid. Contrastingly Wright used the grid as a three-dimensional matrix to locate elements of the house. He used minimum partitions on the inside and organized the rectangular rooms to overlap each other. The space that overlapped became a transition space and minimized the corners present in a
Fig. 4.7 – First floor plan, Robie House, Chicago, Illinois. (Henry-Russell Hitchcock, In The Nature of Materials, 1942).

Fig. 4.8 – Asymmetric grid in the Robie House. Modified by author (Donald Hoffmann, Frank Lloyd Wright’s Robie House, 1984).
room. This eliminated the classical notion of box like separation of spaces, and is often referred to as the ‘breaking of the box’. [Fig.4.9]

Wright reduced the external forms of his Prairie buildings to horizontal and vertical planes. The surfaces near the ground were heavier with fewer openings, which gave way to piers and longer windows as one reached the top floors. The hipped roof projected beyond the vertical confines of the house which gave an outward thrust to the interior space.[Fig.4.10] The prow or the triangular bay window in the Robie house culminates in a vertical line when seen in perspective from the other end of the living room. This eliminates the barrier one experiences when seeing a surface, and further emphasizes the outward thrust of space.[Fig. 4.11] The projecting roof gave a strong visual sense of horizontality, and was balanced by a chimney that emphasized the vertical. [Fig.4.12] Palladio’s Villas are contained within a cubical mass but in Wright’s houses the horizontal plane extends beyond the vertical confines to release the internal space. 23

Another typical characteristic of Wright’s Prairie architecture was the cross axial arrangement of spaces. The plan of the Willitts house is clearly structured on a cross axis with the centre occupied by the chimney. [Fig.4.13] The Robie House has a more asymmetric plan than the other Prairie house. Here the cross axis divides the lot into four equal parts. [Fig.4.14] The living and dining rooms are organized along an axis with the chimney between them. Here the centre is not occupied by the chimney, but by a staircase. The living and dining areas form a single unit, and are symmetrical about both their axes.[Fig.4.15] In most of the house only the overall plan or certain parts are
Fig. 4.9 – Breaking the box. Left: a typical house with box-like rooms. Right: Wright’s first step in breaking the box by minimizing corners. (Allen Brooks, Frank Lloyd Wright and the Destruction of the Box, JSAH, 1979).

Fig. 4.10 – Horizontality in the Robie House. Left: The horizontal planes on the lower floors change to piers and larger opening on the top floors. Right: The hipped roof projects beyond the confines of the Robie House. (author)
Fig. 4.11 - The living room of the Robie House. Left: The ‘prow’, eliminates a surface at the end of a perspective. (author) Right: View of the prow from the opposite end of the living room. (Donald Hoffmann, Frank Lloyd Wright’s Robie House, 1984).

Fig. 4.12 – The horizontal and the vertical elements in the Robie House. The horizontal planes are balanced by the vertical of the chimney. (author)
Fig. 4.13 - The cross axis in the Willitts House. (Henry-Russell Hitchcock, In The Nature of Materials, 1942).

Fig. 4.14 - The cross axis of the Robie House divide the site into four parts. (Donald Hoffmann, Frank Lloyd Wright's Robie House, 1984).
Fig. 4.15 – Axial organization in the Robie House. The living and dining rooms together are symmetrical about both their axes. The longitudinal axis anchors the two horizontal axes. (author)

Fig. 4.16 – Movement inside the Robie House is not along the axis. Modified by author. (Henry-Russell Hitchcock, In The Nature of Materials, 1942). (author)
symmetric, and is not extended to details like the distribution of openings, walls, and other details of the plan. These were different from the cross axis of the Palladian Villas that had a centralized and hierarchical organization, and a well defined central space at the geometric centre of the plan. This is particularly evident in Villa Malcontenta and Villa Rotunda. Contrastingly, in Wright’s Prairie houses the centre is occupied by a chimney mass or a staircase. In Wright’s houses the cross axis did not result in a centralized or hierarchical organization of spaces. The spaces were arranged on a cross axis in plan, but did not follow strict symmetry in every detail like some of the Renaissance architects’ works did.

In a Palladian Villa movement is always along the axis, while in Wright’s house physical movement was almost never along the axis. The entry to a building is never on an axis in any of Wright’s buildings. Within elements organized on a rectangular grid he tried to introduce movement along the diagonal. [Fig.4.16] Wright believed that diagonal movement, both visual and physical, gave more freedom than movement along an axis. Free movement on the inside connected various spaces in a non-hierarchical manner, within rigorously ordered formal elements. Wright further gave character to the spaces of his buildings by varying the scales of the spaces. Entrances and transition spaces were generally lower in height than the dining and living rooms. Wright was influenced by the words of Lao Tzu, “The reality of the building does not consist in the four walls and the roof but in the space within to be lived in.” On the inside, wooden panels run across the ceiling down to the lintels and match the rhythm of the piers. The continuous visual line
thus merges the pier with the lintel and the roof. It also breaks the ceiling into lines, and 
does not read as a surface. [Fig. 4.11] In this manner, Wright reduced the expanse of 
stark surfaces. He tried to merge boundaries between the inside and outside, roof and 
wall to form a continuous interior space.

The Unity Temple

The Unity Temple located in Oak Park, Illinois, is often considered to be Frank Lloyd Wright’s greatest public building. [Fig. 4.17, 4.18] Though it was not a Prairie house, it still shares many characteristics with his Prairie houses that have been 
translated to an institutional scale. Hence, “geometric form was universally applied, but scale and order determine the nature of the space”.26 It was designed in 1905 when Frank Lloyd Wright was thirty eight years old. This temple represented the more liberal views 
of the Unitarian Universalists in Chicago.27 Wright was not concerned with the 
traditional symbolic aspects of geometry; he was concerned with the logic or the rationale behind the shape (geometry) of the form. Wright himself claims that the Unity Temple was one of the first buildings that he “designed consciously with an organic ideal.” “The Unity Temple may be seen as the decisive formal and conceptual synthesis of Wright’s development as an architect, fusing his philosophical search for the [meaning] of geometry.”28 The design for the Unity temple was approached from the stand point of building production.”29 The form was a simple cube that helped expedite construction.30 Concrete was chosen as the material for construction as it was cheaper than traditional masonry construction. Wooden forms were expensive for concrete casting. Hence making a simple building with all four sides identical would increase the
Fig. 4.17 - First floor plan, Unity Temple, Oak Park, Illinois. (Robert McCarter, Phaidon, Frank Lloyd Wright).

Fig. 4.18 - Sanctuary level plan, Unity Temple, Oak Park, Illinois. (Robert McCarter, Phaidon, Frank Lloyd Wright).
repetitive unit module. This in the simplest terms meant a square plan where all the four sides were identical.\footnote{\[Fig.4.19\] That would make the temple a cube, “a noble form”.} For the roof, a reinforced concrete slab proved to be the most economic solution. The slab too belonged to the cube by nature.\footnote{\[Fig.4.20\]}

The conventional form of a place of worship adopted in the nineteenth century usually followed the nave and transept form of the medieval cathedrals, which placed the preacher apart from the gathering and made him the focus. Wright’s use of the square floor plan in the sanctuary of the Unity temple made the audience face each other more than the pulpit. This arrangement gave the auditorium of the “Unity Temple the requirements of a modern congregation where the speaker was placed well out in the auditorium, his audience gathered about him in the fashion of a friendly gathering.” A separate building, the Unity House, was designed in the same lot for the more secular functions of the church. Hence, the floor plan of the Unity Temple is in the shape of a simple square attached to a rectangle with notched sides. There is a common courtyard linking both buildings which also became the main entrance for the building. The plan is symmetric along the main axis, but similar to his other works, circulation is not along this main axis; one enters the building perpendicular to the main axis. Even the auditorium building is not entered on the axis. People are taken to a lower level along the sides of the square and enter the main sanctuary of the auditorium from the rear.

The main theme of the plan is a square intersected with a cross. Lighting fixtures, furniture, stained glass windows, decorative features, and wood trims were all designed
Fig. 4.19 - A view of the Unity Temple showing the identical facades. (author)

Fig. 4.20 - Relationship of the axis and movement in the Unity Temple. Modified by author. (Robert McCarter, Phaidon, Frank Lloyd Wright).
Fig. 4.21 - Theme of the square intercepted with the cross in the Unity Temple. Left: The plan showing the theme. Modified by author. Middle: a sketch by Leonardo da Vinci for Bramante’s St. Peters, taken from folio 310V, Codex Atlanticus. Right: Compared to Wright’s Unity Temple. (Robert McCarter, Phaidon, Frank Lloyd Wright).

Fig. 4.22 - The repletion of the theme of the plan in the Unity temple. Left: light fixtures. Right: window patterns. (author)
and constructed so as to follow the general theme of the building. This theme continued ordering principles of centralized Renaissance churches, especially an analytical sketch of Bramante’s design for St. Peters made by Leonardo da Vinci.\textsuperscript{35} [Fig.4.21, 4.22] “Renaissance architects advocated centrally planned churches that were not satisfactory from many practical points of view”.\textsuperscript{36} A linear space required for the liturgy and seating people was absent in the centralized plans. In the Unity Temple a centralized organization is realized without the use of a circle. By seating people around the four sides of the square Wright puts forth his interpretation of a church. The central square of the sanctuary rises the highest and is top lit, which gives the sanctuary some verticality, though this feature is not very prominent when seen from outside. The four corners of the cube with the staircases seem to be anchored to the base as they are shorter than the central part of the cube.\textsuperscript{37} They are also detached from the cube by narrow windows which accentuate the cruciform. When seen from outside, the low scale of the central space along with the treatment of the corner gives the effect of a folded surface that is hiding the cube. [Fig.4.23]

The corner is where we traditionally look to get a reading of the interior space. In the Unity Temple the stairwell almost dissolves the corner of the interior space. The complex layering of surfaces at the corner suggests mystery and wonder rather than certainty and familiarity.\textsuperscript{38} [Fig.4.24] The crossing of the lamp fixtures, the wooden paneling going around the corners, the recessed mezzanine floor, and the half walls near the staircases add complexity to the corner, despite the fact that they are composed of simple geometries of the square and rectangle. These details add to the idea of folding in
Fig. 4.23 - Treatment of the corner at the Unity Temple. Left: The treatment of the corner gives the effect of a folded surface that is hiding the cube. (author) Right: The corner detail of the sanctuary at the Unity temple. (Robert McCarter, Phaidon, Frank Lloyd Wright).

Fig. 4.24 - The complex layering dissolves the corner in the Unity Temple. This is which one usually gets a reading of the geometry of the space. Left: corner near the pulpit in the Unity Temple. Right: corner opposite the pulpit in the Unity Temple. (author).
the Unity Temple. The wooden paneling running from wall to ceiling is consistent with the idea of folding, as they avoid emphasizing the corners by running around them and not along them.³⁹

The Unity Temple and the Unity House are ordered on two different grids. [Fig.4.25] The central sanctuary determines the dimension of the entire building. Its size was based on the number of people who were to be seated. The central space with the four columns is 33’ square in plan, and the internal height of the central space is 30’, which makes this space close to a cube. Each arm of the cruciform is 33’ by 16’-6”, thus, each of them form two squares of sides 16’-6”. The four corner squares have sides of length 16’-6” in plan; this makes the exterior a square of side 66 feet on each side, which is exactly double (2:1) the interior dimension. The plan of the Unity House is a double square (1:2) making it geometrically related to the auditorium building. [Fig.4.26] In this manner the various parts, inside and outside are all integrated into an organic whole.

**Hanna House**

The Usonian houses were planned for a modest budget, catering to an average American family. Most characteristics of form and space in these houses were continued from the Prairie houses. The grid was standardized for all Usonian house projects. The horizontal module was 2ft by 4ft, which was a subdivision of a 4ft by 8ft plywood sheet. The vertical module was 13”, which was the distance between battens in a balloon frame construction. The grid lines were marked at the outset on the concrete floormat. The plan was usually L-shaped which allowed for easy expansion. In these houses Wright came closest to an ideal of standardization, which was as an expression of the machine age.
Fig. 4.25 – The different grids of the Unity Temple and the Unity House. (author).

Fig. 4.26 – Dimensional relationships in the Unity Temple. (author)
Fig. 4.27 – Plan, Hanna House, Palo Alto, California. (Paul and Jean Hanna, Frank Lloyd Wright’s Hanna House: The Clients’ Report, 1981).
Most of his Usonian houses were built on a rectangular grid. The Hanna House was one of the first Usonian houses that was built on a non-rectangular grid. [Fig.4.27] Though he had envisaged a number of houses in the twenties and the thirties that were based on the hexagon or triangular module, none of them were actually built. Therefore, the Hanna House was a significant development in Wright’s career.

The Hanna House, also known as the Honey Comb House in Stanford, California, was one of the first hexagonal plans used by Frank Lloyd Wright. It was designed in 1936 for Paul and Jean Hanna, two Stanford professors, who were drawn to Wright’s philosophy when they came across some of Wright’s published Princeton lectures. The main requirement, mostly programmatic, was a house that could grow and change in harmony with the family. It was similar to some of Wright’s ideas on modern architecture: “Form is made by function but qualified by use. Therefore, form changes with changing conditions; Principle is the safe precedent; an organic form grows its structure out of conditions as a plant grows out of soil. Both unfold similarly from within.” The house used a hexagon as the module for the generation of the plan. Wright believed this to be a flexible spatial form that would give greater freedom of movement. He also refers to the analogy of a beehive which is made from a hexagonal unit. “The sociological structure within the beehive demands a unit adaptable to the tremendous growth and activity that flows from one generation of bees to another.” These ideas of the integrity of function and form, typical to Wright’s notion of organic architecture lead to the use of the hexagonal geometry.
The interior angles of the hexagon are 120 degrees making the sides form corners at an obtuse angle. This introduces movement along the diagonal more naturally than a square.\textsuperscript{49} Wright associated diagonal movement as being closer to natural human movement and freedom. [Fig.4.28] The idea of free movement which he juxtaposed with rectangular geometry in his earlier works was integrated in this house. The hexagon with its six sides could expand easily in more directions than the square with its four sides. These possibilities of the hexagonal module made Wright believe that it had a rhythm more in tune with human rhythm, than other geometries. He wrote: “I am convinced that the cross-section of the honeycomb has more fertility and flexibility where human movement is concerned than the square. The obtuse angle is more situated to human ‘to and fro’ than the right angle. That flow and movement is, in this design, a characteristic lending itself admirably to life, as life is to be lived in it.”\textsuperscript{50} This is also the geometry that is found in nature, and is more adaptive to growth and change.

The whole plan was laid using a hexagonal module; the basic ‘mat’ of hexagons is still visible in the concrete flooring of the house. The side of the hexagon, which was 26”, was double of the 13” module used on the vertical plane. All walls were laid on this hexagonal pattern where the distance between parallel sides of the hexagon was approximately 45”. The house was built on a three dimensional gridline of 26” by 13”, which was laid on a hexagonal mat.[Fig.4.29] The basic 13” vertical dimension was obtained from 12” solid redwood boards that were offset on ½” thick redwood battens.[Fig.4.30] The redwood boards were interlocked with the redwood battens which in turn were screwed to 7/8” x 8” plywood studs set 2’2” on center. This method of
Fig. 4.28 - The living room of the Hanna House. The obtuse angle creates diagonal movement more naturally than a square. (Paul and Jean Hanna, Frank Lloyd Wright’s Hanna House: The Clients’ Report, 1981).

Fig. 4.29 – Section through the living room of the Hanna House. This shows the 13” by 26” timber frame in elevation. (Paul and Jean Hanna, Frank Lloyd Wright’s Hanna House: The Clients’ Report, 1981).
Fig. 4.30 – Timber walls in the Hanna House. These 12” boards are offset on redwood battens. (Author)

Fig. 4.31 – Interlocking of the redwood battens and boards in the Hanna House. (Paul and Jean Hanna, Frank Lloyd Wright’s Hanna House: The Clients’ Report, 1981).
interlocking the redwood boards prevented the use of any screws and nails, and saved
the wood from splits or cracks.[Fig.4.31] Fenestrations which run from the ceiling to the
floor level also follow this plaited structure. Timber frames ran along the 26” by 13” grid
lines and glass panels are used as in-fills. This produced a plaited structure that was
similar to a textile. Wright could have been influenced by some of the later exercises set
by Froebel which had mat weaving. In such plaited structures, Wright suppressed the
vertical and emphasized the horizontal. While the Usonian houses were standardized for
mass production on a ‘quadratic grid’, the hexagonal module with its obtuse angle was
not suited for standardization. Craft-trained carpenters were employed to assemble the
walls of the Hanna House.

The house almost seems to rise horizontally on a 13” vertical interval. Horizontal
bands run continuously around the house; even brick courses are matched to the 13”
interval. The horizontal pointing of the bricks are recessed while the vertical mortar
joints are flush. The low height of the overhangs, which is almost 6’8” at some points,
further emphasizes the horizontal. The flat roof runs along the edge of the building and
helps resist the outward thrust of the sloping roof. It is anchored in the brick chimney
and its significantly low height near the exterior walls gives the sloped roof an extra lift.
Like all his Usonian houses which have three plans, the Hanna house also has three
plans - the floor plan, the deck plan at door height, and a roof plan (some Usonian
houses had a ceiling plan at clear story level if the roof was flat). The three plans while
determined by the same geometric module, do not necessarily coincide.[Fig.4.32] This
makes the space more fluid and the geometry more subtle.
Fig. 4.32 - The three distinct floor plans of the Hanna House. (author)

Fig. 4.33 - The repetition of hexagonal module for the chimney and the furniture. (Paul and Jean Hanna, Frank Lloyd Wright’s Hanna House: The Clients’ Report, 1981).
In his autobiography Wright states that the square modified by the triangle gave the hexagon. The hexagon appears in various places at various scales in the Hanna House. There are some hexagonal elements that act like columns. Small seats, cushions, and tables take the form of a hexagon. Partial hexagons occur at fireplaces, niches, shelves, and furniture again. [Fig. 4.33] Many external walls are folded along the lines of the hexagonal grid present on the floor. Some of these walls, though 2-3/4” thick act as a folded plate and provide support for the overhang above. [Fig. 4.34] The absence of heavy structural elements adds to the lightness of structure. One would notice folding in the plan of the house where ever support is required. [Fig. 4.35] The overall plan also defines a partial hexagon, with the wings of the house marking thirty degree axial shifts along the sides of the hexagon. In the Hanna House one is only conscious of fluid space and not any geometric three dimensional objects. The hexagonal ‘mat’ embedded on the concrete floor comes as a reminder of the geometry used.

**Conclusion**

“Geometric order was Wright’s primary method of producing structured space; it was this structure that enabled the spaces to be both independent as pure geometric forms and interdependent as part of a woven or continuous whole”\(^5\) He believed, “Realization of form is always geometric. Geometry is the obvious frame-work upon which nature works to keep her scale in designing. She relates things to each other and to the whole.”\(^5\) According to Wright, nature possessed an underlying geometric structure that was a result of function. This integrated idea of nature was Wright’s approach to organic architecture. The grid provided a three dimensional geometric structure within
Fig. 4.34 - The thin folded walls provide support for the roof. (Author)

Fig. 4.35 - The plan showing the folded walls in the Hanna House. These walls act as points of support. (Paul and Jean Hanna, Frank Lloyd Wright's Hanna House: The Clients' Report, 1981).
Fig. 4.36 – The three dimensional grid of the Unity Temple. (author)
which elements could be manipulated. Wright did not use a rectangular grid always; at times a building also had more than one system of grids present. The grid also helped standardization and mass production, a goal that was better realized in his Usonian houses than in his other buildings. Hence, both geometric purity and the grid were not just ideals, but were also used to expedite construction.

Some of Froebel’s tools had a table of grids to guide arrangements of blocks. Some arrangements from these exercises seemed as if mass and void were interwoven. These childhood experiences helped Wright interweave formal elements, which had rigorous geometric ordering, with free meandering movement. The themes of spatial continuity and geometric order constantly appear in Wright’s works. Wright achieved spatial continuity within the grid by ‘breaking the box’. The interiors had few partitions, and corners were always treated in an unconventional way to break the notion of a confined interior space. Wright folded the corner, introduced a corner window, or overlapped spaces to eliminate the corner. While formal elements were structured on a grid, they were interwoven with free movement to give a sense of spatial continuity.

Dimensions in Wright’s works did not represent any ideal proportion that was derived from the universe or from the human being. In Wright’s architecture the human proportions were not idealized at all. Wright constantly related the lintel heights on the elevation to the average height of a human being, or to himself. At various places within the houses, he would have the lintel run across high doorways to relate the space to real human dimensions. Similarly, the grid in the Usonian houses was based on material
dimensions. Thus dimensions arose from functional requirements, and were not idealized.

Wright was often close to Renaissance architects in his approach to principles of ordering formal elements along the vertical and horizontal axis, the cross axis or the grid. He adopted these principles as he believed that they were fundamental to man and nature, and refrained from imitating traditional forms like the classical orders and pediments. New forms and spaces were created by adopting new construction techniques, while giving expression to structure and construction. Thus, while continuing some universal principles, he refrained from mere imitation of his predecessors.

**Correlations**

Wright used geometry as shapes, grids and organizational principles. His early education with Froebel’s toys developed an intuitive understanding of geometry. His preference for axial arrangements, symmetry and his emphasis on the plan suggest his more two-dimensional approach to geometry. The plan was of preeminent importance in Wright’s work. The plan was the solution and the elevation the expression of a building. “Wright’s architecture was designed in precise orthogonal projections, plan, section and elevation.” The three dimensional model or the perspective was more for the client. His two dimensional use of geometry is further emphasized in the window patterns which were composed of lines, squares, circles and triangles.

Lionel March demonstrates that certain sides and heights of triangles in Froebel’s gifts had sides that were proportions of 1, 2, $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ (an approximation of
(1+\sqrt{5})/2). He further relates the proportions of drafting triangles, the 45 degree right triangle and the 30-60 degree right triangle, to the triangles of Froebel’s gifts. The 45 degree right triangle has sides of ratio 1: 1: \sqrt{2}, and the 30-60 degree right triangle has sides of ratio 1: \sqrt{3}:2. Placing two 45 right triangle together along their short sides produces a parallelogram with a long diagonal \sqrt{5} times the length of the short side.

March tries to draw relationships between the proportions of the 45 and 30-60 right triangles and some dimensions in the Windows of the Coonley house. Though March draws such parallels, there is no evidence of any use of geometric proportions like the golden proportion, \sqrt{2}, and \sqrt{3} proportion in Wright’s writings.

Wright was trained in descriptive geometry which was an essential part of all architectural education by the nineteenth century. It taught architects to project three dimensional buildings on two dimensional surfaces. Therefore, despite the fact that his design approach and principles are very two-dimensional, his actual buildings are not mere three-dimensional projections of the plan. Wright used geometry to structure three dimensional spaces. Like Palladio he did not use the grid to determine the dimensions of the form, but used the grid to structure formal elements that define space. Wright’s architecture was a skilful manipulation of two-dimensional geometry to achieve spatial continuity three-dimensionally. Cartesian geometry gave a three-dimensional rectangular grid as a model for the space we live in. Wright’s architecture tends to evoke this model.

Euclidean geometry was not only two-dimensional, but also dealt with form and space in a static and finite manner. Wright constantly tried to break finite and contained volumes in his architecture. Horizontal and vertical planes always extended beyond the
confines of the form. Wright’s cantilevered roofs and the treatment of the corner were important architectural gestures of Wright that released interior space. This was Wright’s approach to ‘breaking the box’ in architecture. He worked with interior spaces three dimensionally by varying their scales. The constant contraction and expansion of spaces along with diagonal movement gave a sense of dynamism for anybody moving through his buildings. The spiral is a geometric form mostly associated with dynamism and movement.

The spiral occurs at various instances in Wright’s career. His Home and Studio in Oak Park, Illinois, has an octagonal shaped library. Starting from floor level to the roof, the octagon is reiterated at various levels. Every time the octagon is repeated it undergoes a rotation, thus creating a spiraling effect. The Guggenheim museum, which was built towards the end of Wright’s career, resurrected the idea of a spiral in a more literal manner. Though Wright’s design principles and design process was very two-dimensional, he used them to create spatial continuity and dynamism. In this sense Wright’s work goes beyond Euclidean geometry.

As already discussed, the plan was very important in Wright’s architecture. Its importance not only lay in organization of spaces, but as a theme it was reduced to a two-dimensional pattern that repeated itself in windows, fixtures and carpets. This idea of self-similarity is visible in some of his Prairie houses, the Unity temple and to a greater extent in the Hanna House. Some scholars associate this idea of self similarity to fractal geometry. Nature was an important source of geometric abstraction for Wright. His early works tended to reduce nature to abstract vertical and horizontal planes. His
later works, especially starting with the Hanna house, tended to use non-rectangular geometry. The fractal approach to form is visible in the way the hexagon repeats itself as furniture, the fireplace, columns and floor patterns. Even the structure of Hanna house uses folded panels of timber frames with glass in fills, instead of flat thick masonry walls, to support the roof. This meandering wall is typical of fractals, like the Koch snowflake, which has an infinite perimeter enclosing finite area. The first idea of a fractal dimension was proposed in 1861, though it was not until 1975 that Mandelbrot suggested that fractals were an essential part of nature. Hence, one could say that it was unlikely that Wright knew about fractal geometry as it might have only been a part of obscure mathematics during his time. Self similarity and scaling is definitely present in Wright’s works but it is questionable if the scaling dimensions are fractal in nature. It is possible that in his attempt to create an organic architecture he subconsciously approached fractal ideas of geometry.

End Notes

1 The founder of the Chicago School was Baron Jenney an engineer by training. His professional practice provided training for young architects and was not a formalized institute. He pioneered the use of the iron skeleton for tall office buildings in Chicago. This type of construction was later used for high-rise hotels and housing.


3 The ancient method of using timber as the structural frame, which was mostly hand crafted, had very complicated joints like the mortised and tenoned joint. With the advent of machine cut wood, it was possible to have thin wooden members called studs that could be joined by iron only. This resulted in a closely spaces structural timber frame structure that was easy to assemble and is still used in the housing industry.


Robert McCarter, 14.

As quoted in Richard MacCormac, 106.


Vincent Scully, 19.


Lionel March, 18.


Robert McCarter, 258.

J. Sergeant, 212.

Scully 21.

J. Sergeant, 213.

J. Sergent , 211.

Vincent Scully, 18.

Ibid, 17.

Robert McCarter, 15.

Ibid.


Kenneth Frampton, 102.

Robert McCarter, Unity temple.

Frank Lloyd Wright, An autobiography: Frank Lloyd Wright, (London, New York : Longman’s, Green and company, 1932), 139.

Kenneth Frampton, 102.

Frank Lloyd Wright, 139.

Robert McCarter, Unity temple.

Ibid.


Robert McCarter, Unity temple.

Ibid.

Ibid.


Ibid.


Richard Joncas, 309.
Wright’s interest in the hexagonal module was triggered when he saw a sketch of a house made by one of his apprentices, Cornelia Brierly.

Richard Joncas, 313.

Wright’s study of some of Wright’s sketches reveal that they were not conceptual but seemed like developed drawings with dimensions on them. That Wright conceived buildings as whole is further justified by the often repeated story: Wright drew the Falling water design from scratch in the two hours it took the client to drive to his office.


Carl Bovill discusses the fractal dimension in the elevations of Wright’s work to demonstrate richness in detail, but he does not address the idea of self similarity. Carl Bovill, Fractal Geometry in Architecture and Design, (Boston : Birkhauser, c1996).
CHAPTER V
FRANK GEHRY: 1929-PRESENT

Introduction

Towards the 1960s the abstract and minimalist aesthetics of modern architecture was criticized by Robert Venturi as being reductive.\(^1\) In 1972, the demolition of Pruitt Igoe, a modernist public housing project in St. Louis, Missouri, was proclaimed by Jenks as the ‘death of modernism’. Due to its depressing environment, the housing project had become a host for crime and vandalism, and it was blown up. Charles Jenks attributed architectural reasons for the failure of the project; especially the aesthetic tenants of modern architecture that failed to consider practical and sociological issues.\(^2\) This was the context in which post-modern architecture developed as a style that renewed ornamentation and historical styles.\(^3\) The post 1960 period saw the development of many other theories in architecture like structuralism, post-structuralism, deconstruction and critical regionalism. Though each of these theories had a different premise, they were all critical of modernism. No single theory dominated this post-modern phase; but in all there was also a serious reexamination of many fundamental principles of architecture. Hence, many scholars refer to the post 1960 theories as the post-modern phase. In this thesis I refer to post-modernism as the general post 1960 phase in architecture, and not to a particular post-modern style.

It is difficult to categorize Frank Gehry’s work into any particular theory or style within the post-modern phase. Since he distorts forms, his works are often considered to
be a part of deconstruction. This is a misnomer, since deconstruction as a theory has its roots in linguistic theory and does not primarily advocate distortion of form. Gehry himself does not categorize his works in any manner. His early works have a lot of modern characteristics from which he slowly departed to design more fragmented forms. These works tended to question modernist principles of architecture. With the introduction of the computer in his office, his works, which have complex forms, have attained sculptural qualities. The computer has had an important role in visualization of projects in the post-modern phase. Gehry’s works stand out among these as he is one of the first architects to use the computer for both design and manufacture of structural units.

Frank Gehry’s work is characterized by unconventional shapes and forms. His buildings have distorted forms that either collide to form a single sculptural object, or are arranged individually in a seemingly random manner within a landscape. Most of his buildings are volumes that seem to have been arrested in a moment of motion. Complex forms and geometries are used to capture motion and dynamism, which are an essential part of our life. In his attempt to free himself from the dictates of the right angle, motion and lightness become central themes for his building. He tries to access different ideas or experiences of motion to free himself from the grid, which was typically used in the Modern movement. Gehry borrows from nature extensively as it provides him with numerous instances of dynamism. Yet he does not view nature and the animal world as a standard, from which one derives measurements, proportions and rules. Zoomorphic forms have undergone metamorphosis and possess qualities of change and dynamism.
embedded in them. Gehry focuses on natural organisms because their forms possess qualities of change and dynamism. Of the various zoomorphic forms possible, the fish seems to be a theme that constantly occurs in his works. He started designing fish, or fish like forms, as the perfection and the variety of their forms fascinated him. Gehry also believed that the “primitive beginnings of architecture come from zoomorphic yearnings and skeletal images, of which the fish is important as it preceded man on earth.” It occurs at various scales and forms; sometimes it is a fish that is supporting a bridge, or at times the fish takes the form of a lamp. [Fig.5.1] Some structures also take the shape of a horse. [Fig.5.2] He does not approach these in an analytical manner, but tries to borrow their shape or texture. “As the poetic quality of metamorphosis endows animals with meaning that escapes rational analysis.” This unconventional approach to form has its sources in Gehry’s interaction with other artists and art movements.

Since his school years Gehry associated himself with artists and art as he admired their sense of personal freedom. As in art, there are no rules and principles that his architecture follows. As he said once: “My approach to architecture is different. I search out the work of artists, and use art as a means of inspiration. I want [architecture] to be open-ended. There are no rules, no right or wrong. I am confused as to what [is] ugly and what [is] pretty.” For Gehry, artists often provided the points of departures he was looking for from the more rigid and rectangular forms of the modernists. One early influences was Claes Oldenburg whose exhibition in 1965, included works that consisted of geometrical and materially independent objects framed within a single field. [Fig.5.3] His works were distorted as if in perspective and were accentuated by grossly textured
Fig. 5.1 – The fish in Gehry’s works. Top: Low White Fish lamp designed by Gehry. Bottom: The fish takes the form of a hotel in Gehry’s design for a urban development project. (Rosemarie Haag Bletter...[et al.], The Architecture of Frank Gehry, 1986).
Fig. 5.2 – The DG Bank project, Berlin. The structure is shaped like the head of a horse. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).
Fig. 5.3 – Works of Claes Oldenburg. Left: a Bedroom Ensemble. Right: a Leopard Chair. (www.artnet.com).

Fig. 5.4 - A sketch for the Jung institute, an unbuilt project. (Francesco Dal Co and Kurt Forster, Frank.O.Gehry: the completed works, 2003).
A decade later Gehry also conceived of buildings in terms of a number of individual objects that float apart like boats on a pond. The Jung institute, LA California, was designed as “individual structures of distinct sculptural forms” with a shallow pool to unify these objects. [Fig.5.4] Though this project was never built, this idea continued to evolve in his other projects. Another artist friend, Ron Davis, became one of Gehry’s early clients. Davis’ work mainly consisted of two-dimensional wall pieces that give the illusion of spatial perspective. [Fig.5.5] Gehry’s studio for Davis took the form of a trapezoid that was essentially perspective forced on a rectangle. [Fig.5.6] The roof was tilted from a height of 30 ft to 10 ft with window openings so that it would appear to be a part of the elevation. Complexity and dynamism are achieved by creating surfaces at different angles that seem to have multiple vanishing points. Gehry further plays with our perception when he conjures up forms that seem to be literal translations of two dimensional representations of objects.

**Early Rectilinear Works**

Most of his early works do not reflect a very naturalistic approach to geometry. Some of his buildings from the 1970s and the 1980s are very rectilinear in nature. The influence of Wright is evident in works like the Steeves House. [Fig.5.7] The plan of the house has rectangular rooms organized in shape of a cruciform. The overhangs on the pool side of the house emphasize the horizontal, which is similar to Wright’s works. An original approach is seen in the Danziger Studio and residence in Hollywood, California. This house is an assemblage of simple stucco cuboids with unadorned surfaces that exemplify the commercial strips of Los Angeles. [Fig.5.8] One of the goals was to
Fig. 5.5 - ‘To and Fro’, by Ron Davis. An example of optical illusion in the works of Ron Davis. (www.irondavis.com).

Fig. 5.6 – Drawing, Ron Davis Studio, Malibu, California. (Rosemarie Haag Bletter et al., The Architecture of Frank Gehry, 1986).
Fig. 5.7 – Drawing, Steeves House, Brentwood, California. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).

Fig. 5.8 – Danzinger Studio, Los Angeles, California. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).
provide an introverted house that sealed the inhabitants from the noise of the street. Hence, unlike the Steeves House where he was trying to blur the barrier between inside and outside, here he tries to make the barrier stronger. Windows are eliminated at the street level and at some points some cubic masses are pulled outwards for clearstory lighting. Hence while the plan is simple, with two offset rectangles, the building in reality looks more like an agglomeration of cuboids.

Gehry’s Frances Howard Goldwyn Regional Library, California, is a public project that has simple rectangular geometry. The Goldwyn Library has a very symmetric plan with rectangular walls that are perpendicularly extruded from the plan with no tilts or skews.[Fig.5.9] One enters the building on the main axis in the manner of a classical arrangement. The torquing away from symmetry is achieved while responding to the sun.\(^\text{21}\) Openings on the façade seem to be asymmetrical as the building faces the east, and Gehry tries to open the northern walls, while closing the south.[Fig.5.10] The rectangular groove lines of the stucco works and the window mullions all align to make the building look like a three dimensional gridded cage. Contrastingly, the inside of the building has beams that are not aligned, which are evidences of breaking away from the rectangular grid. The corners of the reading room have supports and surfaces layered in such a manner that they almost dematerialize the cube.[Fig.5.11] This effect is further enhanced by the treatment of the ceiling. The ceiling drops low for defining certain functions or is projected up to get in light.[Fig.5.12] In this manner Gehry breaks the rectilinear geometry of the exterior on the inside. Gehry’s treatment of
Fig. 5.9 – Ground floor plan, Frances Howard Goldwyn Regional Library, Los Angeles, California. (Rosemarie Haag Bletter et al., The Architecture of Frank Gehry, 1986).

Fig.5.10 - Front view, Frances Howard Goldwyn Regional Library. Opening up of the north walls led to an asymmetrical façade. (Rosemarie Haag Bletter et al., The Architecture of Frank Gehry, 1986).
Fig. 5.11 – Interior views of the Goldwyn Library. Left: Beams are not aligned, which suggest breaking away from the grid. (Henry Cobb, The Architecture of Frank Gehry, 1986). Right: The treatment of the corner in the Goldwyn Library. (author)

Fig. 5.12 - Treatment of the roof in Goldwyn Library. Left: ceiling over the reading area. Right: ceiling near the stacks. (author).
interior space is very similar to some of Wright’s approaches. Though, the exterior surfaces of Gehry’s buildings form well defined volumes when compared with Wright’s works, which could be reduced to vertical or horizontal elements.

**Fragmented Forms in Houses**

More fragmented and dynamic forms appear in his houses of the 1970s and 1980s. His own house serves as an example of such forms. Here he took a small pink bungalow and tried to make it more important by adding a shell to it. The gabled house with a rectangular plan is surrounded by irregular geometric figures. A corrugated metal shell was added around the north and east ends of the house. The north facing kitchen was designed to seem like a falling glass cube that is trying to escape the existing cubic volume. Here Gehry makes a “tangible demonstration of the instability of geometry as a basis for compositional logic.” The north-east corner occupied by the dining room has a second distorted glass cube. This is a more literal way of breaking the corner than Wright’s subtle treatment of the corner. On the inside, these interesting forms bring in varying qualities of light. Gehry uses everyday materials like corrugated metal and chain link panels to cover parts of his house. These materials give a temporary feeling to his house. Here the shapes and dimensions are not idealized to give an effect of something finished and permanent. He believes in exploring the various ways a material can be used, and does not necessarily enhance the structural properties of materials. This gives Gehry more freedom to experiment with materials and forms.

Some of the houses designed by Gehry for his various clients seem like a
Fig. 5.13 – Drawing, Gehry’s house, Santa Monica, California. (Rosemarie Haag Bletter et al., The Architecture of Frank Gehry, 1986).
Fig. 5.14 - The falling cube on the north facing of Gehry's house. (author)

Fig. 5.15 - A distorted glass cube on the north-east end of Gehry's house. (author)
collection of free floating rooms and fractured the very notion of the house as a unifying shell. In this manner he redefines the notion of a single family house. The Tract House project, which reflected these ideas, was divided into nine equally shaped blocks, where each room was an individual house surrounded by “a street”. As the design evolved, the grid patterns broke away to an even more complex organization. [Fig.5.16] The Schnabel House was the final step in the evolution of these ideas. The house was broken into individual units evoking the appearance of a small village set back from the street. [Fig.5.17] There is an axial path leading to the front door of the house that seems to be a contrast to the irregularly shaped main building. In plan, the main part of the house with the living and dining room is a cruciform structure with walls at obtuse and acute angles. Gehry distorts the cross in plan and does not align it to the main axis. [Fig.5.18] The intersection of the cross axis of the living room is accentuated by a cuboid perched precariously on the roof, which also distorts the vertical axis. [Fig.5.19] This completely departs from the structure of a Palladian Villa, which had a well defined linear axis that was intercepted by a cross axis, and a central space that emphasized the vertical axis. In Wright’s houses the axis was used to structure spaces, and free movement distorted the axis. Contrastingly, in Gehry’s buildings, form itself is distorted. The office of the Schnabel House has a dome with pendentives, which are also skylights. This makes the dome appear as if it is floating. The dome was used on the client’s request as it brought back childhood memories of a planetarium. [Fig. 5.20,5.21]
Fig. 5.16 – Distortion of the grid by Gehry. Top: The nine square diagram of the Tract House with streets separating each module; a conceptual sketch. Middle: Developed model of the Tract House, where Gehry breaks away from the rectangular grid. Bottom: The Smith House where the grid is further distorted. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).
Fig.5.17 - Aerial view, Schnabel House, Brentwood, California. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).
Fig. 5.18 – Axial organization in the plan of the Schnabel House. The main building, which is in the shape of a distorted cross, does not align with the main axis. Modified by author. (James Steele, Schnabel House: Frank Gehry, 1993).

Fig. 5.19 – The entrance to the Schnabel House. The cuboid distorts the vertical axis. (James Steele, Schnabel House: Frank Gehry, 1993).
Fig. 5.20 – The dome in the office of the Schnabel House rests on glass pendentives. (James Steele, Schnabel House: Frank Gehry, 1993).

Fig. 5.21 - The office of the Schnabel House. (James Steele, Schnabel House: Frank Gehry, 1993).
The Aerospace Museum

The fragmented aesthetic that Gehry used in his houses was transferred to the Aerospace Museum in Los Angeles, which was one of his early public projects to get noticed. Known for his houses, which were the type of projects he dealt with in his earlier years, this museum was designed as the same time as other public buildings like the Loyola Law School and the Frances Goldwyn Library. The building was an addition to an already existing armory building. An important criterion for his design was that the architecture should evoke the idea of the new frontier provided by limitless space. The exterior of the building looks like a collision of two steel structures: one a relatively plain stucco clad box and the other a metal clad polygon. Both these structures are mediated by a large glazed wall which on the inside forms a Gantry or viewing tower. Here different materials are used to accentuate the different geometries of the building. The structure is a steel frame for both the forms. The dimensions of rectangular metal sheet cladding or the rectangular grooves of the stucco come about from their intrinsic material properties and do not reflect the structural grid inside. A truncated polygonal cone on the west end rests awkwardly on a wedge shape structure and a wall that projects beyond the polygon. On the inside, the wall forming the polygonal cone cuts beyond the glazing and ends abruptly. Hence, Gehry himself would like to call this composition a relative positioning of disparately shaped objects where he tried to express the awkwardness with which they touch. The entrance forms a dynamic geometric sculpture consisting of a truncated conical polygon, a skewed staircase and a ramp leading up to the staircase. From the outside,
Fig. 5.22 - The Aerospace Museum, Los Angeles, California. (James Steele, California Aerospace Museum: Frank Gehry, 1994).

Fig. 5.23 – Plan, The Aerospace Museum. (James Steele, California Aerospace Museum: Frank Gehry, 1994).
Fig. 5.24 - The truncated polygonal cone rests on a wedge shaped projection. (author)

Fig. 5.25 - The wall of the truncated polygonal cone cuts beyond the glazing. (author)
Fig. 5.26 - The entrance to the Aerospace Museum. (author)

Fig. 5.27 – The inside of the Aerospace Museum. (author)
the building appears as discrete forms, but forms a continuous space on the inside with
different models of aircrafts suspended as exhibits.\textsuperscript{30} [Fig.5.27] The initial sketches and
models “show massing composed of separately articulated building objects, surrounded
by a landscape of symbolic imagery [pertaining to flight].”\textsuperscript{31} Ultimately the design was
reduced in scale, which resulted in a more compact sculpture. The roof of the polygonal
cone structure has a diamond shaped skylight and that of the box structure has a cross
shaped skylight. [Fig.5.28,5.29] The interior is continuous with viewing platforms
suspended at different levels which allow the exhibits to be viewed at different angles.
The whole effect evokes an idea of space with the roof studded with sky lights similar to
constellations and space crafts in midair. The various angles of the skylights take the
viewer’s eye beyond the confines of the space to give glimpses of the vast expanse of the
sky.

The Disney Hall: The Post Computer Phase

The project was started when Lillian Disney contributed funds to the Los
Angeles Philharmonic to build a new building. Gehry’s design was chosen over several
other architects like Hans Hollien and Jame Stirling. The Disney Hall was a landmark
project for Frank Gehry as he was able to realize many ideas that were in rudimentary
stages in previous projects. His search for breaking barriers between art and architecture,
going beyond the grid and his experiments with zoomorphic forms all culminated in this
dynamic and ground-breaking sculptural form. He wanted this “building to be one idea,
inside and outside, one aesthetic, unlike traditional concert halls, [it had ] to express the
joy and feeling of music.”\textsuperscript{32} Wright wanted to achieve spatial continuity in his works,
Fig. 5.28 - Roof plan of the Aerospace Museum. (James Steele, California Aerospace Museum: Frank Gehry, 1994).

Fig. 5.29 - The skylights of the Aerospace Museum. Left: The skylight over the truncated polygonal cone. Right: The skylight over the box like space. (author)
whereas Gehry wanted form to appear integrated. He merged differences between roof wall, inside surface and outside surface by creating shapes that twist and curve in more than one direction. Gehry had animated a part of the rectangular grid of Los Angeles downtown, and had also captured the action and dynamism that are a part of Disney animations.[Fig.5.30 and 5.31]

One of the primary requirements of the program was to provide the best possible acoustical environment for the orchestra. The geometry of the interior was predominantly influenced by the acoustical parameters. There were two prevailing architectural strategies for designing such a hall: one was with curves of organic forms similar to Hans Scharoun’s Berlin Concert Hall, and the other was the shoe box type similar to Boston’s symphony hall. Gehry, in association with Minoru Nagata, came up with a third strategy that modified both the other strategies. Gehry himself admits borrowing Hans Scharoun’s form as it worked better. The hall took the form of a shoebox with its corners tilted upward to follow the path of the sound. From outside the auditorium looks like two cubes tilted into one another to from a shallow “v”. Convex surfaces were used on the inside to disperse sound. [Fig.5.32] The inside is therefore symmetrical in plan and one sees four large convex surfaces defining the seating, which is spread on all the four sides of the auditorium, and the musicians are placed at the center. [Fig. 5.34] This arrangement again borrowed from the Berlin Philharmonic, suggests a centralized organization in a loose manner, unlike examples of centralized organizations in Renaissance architecture.

It is quite impossible to attribute any shape to the exterior of the building. The
Fig. 5.30 – Rear View, The Disney Hall, Los Angeles, California. (author).

Fig. 5.31 – Front view, the Disney Hall. (author)
Fig. 5.32 - Model of the auditorium. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).

Fig. 5.33 - Convex forms in the ceiling of the auditorium. (author).
Fig. 5.34 - Convex surfaces defining the seating in the auditorium. The orchestra is surrounded by seating making it the focus. (author)

Fig. 5.35 – Plan, Disney Hall. (Francesco Dal Co and Kurt Forster, Frank O. Gehry: the completed works, 2003).
curved metallic surfaces that seem like walls move apart from the ground to reveal a rectangular grid of glazing. At places these surfaces do not touch the ground at all, and act more like a suspended shelter. In this manner the roof and wall are completely synthesized to form just dynamic surfaces. The metallic surface acts as a skin that is layered around the rectilinear auditorium to define volumes for the lobby and other auxiliary functions of the Disney Hall. In plan, the auditorium is a symmetrical rectangular space with spaces organized in a random manner around it. [Fig.5.35] This arrangement makes movement inside Gehry’s buildings more random than in Wright’s works. The exterior shell tries to depart from the interior geometry completely. The internal rectangular geometry is repeated again in the parking levels and the edges of the site. These have obviously been included for more practical and functional considerations.

The task of the building as Gehry saw it was to provide a passage from something visually stimulating to something that is auditorily suitable. Inside the space is more symmetric and repetitive. This way, from the visually complex outside, the inside, which is made of wood, becomes simpler visually, making it suitable for sound and acoustics. The materials also change, from metal, to plaster and wood in the lobby spaces, and then to wood for the auditorium. The continuous curving skin is similar to varying tones in a musical composition. These undulating forms provide varying qualities of indirect light that yield a soft feeling to the interior. The exterior curves visually soften the rectangular nature of the interior core. The curves are essentially steel frames clad with metal. The steel frame takes the form of a warped grid and its dense,
clumsy detailing is covered by sleek metal sheets. Here the geometry of the skin and the structure are not integrated in any manner. [Fig. 5.36] In fact, the skin is offset from the structure by 10" and there is a separate frame with steel members supporting the skin. This is consistent with his belief that expression of structure and materials does not have any relevance in an age when one can build anything. Therefore, in his practice, materials are not used for what they are but for what they might be used for or become a part of. 

Streamlined shapes on the inside of the auditorium and outside could be related to sails of ships or fish. Some of the forms are very imitative, like the columns on the inside that look like trees, and the form on the rear view of the building that looks like a flower. [Fig. 5.37]

**Gehry’s Design Process**

The unconventional configuration of the Disney hall defied traditional methods of making architectural drawings. CATIA (Computer Aided Three-dimensional Interactive Application) had already been used for a large fish sculpture in Barcelona, but the software was completely adapted for The Disney Hall project. It had been previously used for designing fuselages in aeronautical engineering, and had to be adapted for architectural use. In this context, the Disney Hall is a significant project as the introduction of this software helped Gehry completely break away from the use of conventional forms. The curves from the model of the Disney Hall were difficult to convert to an accurate drawing within a short time and a tight budget for the Venice biennale. This made the office adopt CATIA software, which provided the ability to manipulate and document complicated shapes that the architect first sketched and
Fig. 5.36 – The steel structure behind the metal skin at the Disney Hall. Top: A view of the steel structure behind the metal skin. Bottom: Close up view of the steel structure. (author).
Fig. 5.37 – Interior views of the Disney Hall. Left: columns near the entrance that look like trees. Below: spaces near the auditorium with columns that look like trees. (author).
modeled. Until then, most softwares had been used for visualization purposes, in contrast to CATIA, which helped in manufacturing. The earlier softwares could build a model accurately in the computer, but they could not construct it in reality.\textsuperscript{39} Such softwares were used for the horse head shaped structure in the DG Bank project. Curved surface primitives were introduced and the required shape was attained by manipulating control points. While this serves the purpose, it is not a very congenial way of evolving the design.\textsuperscript{40} While the visualization problem was solved, the construction issue was still not resolved. CATIA is activated by a hand held probe which transfers the complex curves of a scale model to a computer screen.\textsuperscript{41} It is different from other systems as it uses NURBS(Non Uniform Rational B-Spline) as the primary way of representing surfaces. Therefore, instead of locating points in space, as other softwares do, CATIA is capable of defining surfaces as equations.\textsuperscript{42} CATIA has almost eliminated the use of two dimensional drawings in Gehry’s office.

Gehry has always been skeptical about computer modeling software as it does not give him a feel of materials. Hence, he mostly works with models. He feels the model is less abstract than the drawing and he usually makes his models from materials that have similar properties to the materials that are to be used. He focuses on the strength and flexibility of materials while choosing them for the model.\textsuperscript{43} His office is a workshop of forms. The models in his office reflect the gradual way in which he makes decisions using visual and tactile intuition.\textsuperscript{44} The way he selects and refines form is based on a personal aesthetic perception.\textsuperscript{45} His models are usually preceded by sketches. His sketches, which seem like squiggles, reflect the speed at which they were made.
Most of his sketches seem to be a reconciliation of the straight line and the curve. The first sketches for the Disney Hall were a series of squiggles offset by strong diagonal and vertical lines.[Fig.5.38] His next sketch was a series of curved lines controlled by a strong horizontal.[Fig.5.39] This is very similar to his finished work, which also seeks a balance between the right angle and the curve. In Gehry’s work the drawing is more about movement than forms. He calls the drawing process a search for the form in paper – a kind of a two-dimensional sculpture.46

Conclusion

One of the critical points of Gehry’s explorations has been to capture the dynamism of our surroundings. His works can be divided into two distinct phases: the pre-computer phase and the post-computer phase. His early works have more austere forms, which slowly develop to more fragmented and complicated forms. The projects in the post computer phase are dominated by complex curves and plastic forms. In his later works, Gehry makes a deliberate attempt to break away from Cartesian approaches to form. He uses the grid and principles like axis and symmetry in plan, but usually distorts them or breaks them when they translate to three-dimensional form. He constantly tries to juxtapose rectangular geometry with curves or distorted forms. In the Disney Hall, due to functional reasons he had to use rectangular geometry for the auditorium. Wright’s works refer to universal principles but he tries to give them new meaning by giving a different spatial, experiential and formal interpretation. Gehry’s disagrees with any traditional principles of architecture, and expresses his views by three-dimensionally distorting form.
Fig. 5.38 – One of the first sketches for the Disney Hall. It shows a series of squiggles offset by strong diagonal and vertical lines. (Horst Bredekamp et al., Gehry Draws, 2004).

Fig. 5.39 – A later sketch of the Disney Hall. It shows a series of curved lines controlled by a strong horizontal. (Horst Bredekamp et al., Gehry Draws, 2004).
The forms in Gehry’s works are not a result of structural expression. The structure usually uses steel frames with a metal skin wrapped over it. Unlike brick, stone and timber, which need to be constructed within a horizontal and vertical module, this system, with the aid of the computer, can take the form of any curve. The dome in traditional architecture had a spatial significance but in Gehry’s architecture the sphere occurs more as a visual symbol, it has no structural or spatial function. His building forms are sculptural, iconic and often imitate forms in nature. This is particularly true in the post computer phase where it is easy to construct more complex forms. He particularly turns towards zoomorphic forms like the fish. Palladio and Wright looked at nature in an abstract way from which they could derive measurements, proportions and principles of design. Gehry’s approach to geometry is not reductive, abstract, essence-based or rational. He looks at nature in a more imitative way. “The fish and the snake are not employed just to announce the idea that the human being is not a measure for form any more, it is also a way of evoking everything architecture cannot be, like dynamic movement.” While Wright tried to animate the spaces of his building by changing scales and creating free movement, Gehry tries to animate the forms of his building three-dimensionally.

Correlations

The computer has made it possible to construct complicated curves that are characteristic of the later works of Gehry. Such forms have been visualized in architecture before but have been difficult to construct. Some early sketches of the Sydney Opera House by Jorn Utzon show sail like forms that were more dynamic than
the ones actually built. These curved surfaces were slightly modified and standardized for practical reasons, and the resulting form seems more rigid than the original sketch.\footnote{Fig. 5.40, 5.41} CATIA helps with both visualization and manufacturing. Thus, it helps transfer the fluidity of the sketch onto the computer and from the computer to the constructed product. The surfaces of Gehry’s buildings curve in more than one direction. Hence, each piece of the steel structure needs to be custom cut. This is a contrast to the standardization of forms in Wright’s works. The computer not only liberates one from the grid, but also from standardization.

His early projects reflect certain Euclidean concepts like axis and symmetry. The three-dimensional forms define finite volumes in spite of the fact that they seem dynamic. “Gehry does not design his buildings within the confines of abstract space; rather he engages these volumes in intimate relationships with one another.”\footnote{49} By twisting and turning surfaces he attempts to unify inside and outside spaces, but like Wright, he does not achieve unity by dematerializing form. In some of his projects he uses forced perspective to distort forms. In Palladio’s works the hierarchical structure of a perspective was used to organize spaces. Gehry uses ideas of perspective for formal goals. Gehry’s design process is always dominated by models, yet his earlier projects needed to be represented through two-dimensional drawings. Thus the early part of his practice was dominated by Euclidean and descriptive geometry.

During the post computer-phase, many architectural critics claim that Gehry has gone beyond Euclidean approach to space. Concepts from Euclidean geometry like lines, arcs, and other conventional shapes are not sufficient to describe the forms of Gehry’s
Fig. 5.40 – Sketch, the Sydney Opera House, Sydney. (William Mitchell, Roll Over Euclid: How Frank Gehry Designs and Builds, 2001).

Fig. 5.41 – The Sydney Opera House. (Charles Jenks, Modern Movements in Architecture, 1973).
building. Differential geometry is required to generate forms like NURBS, Bezier curves and Coon Patches on the computer. Whether he really transcends the Euclidean notion of space is questionable. Gehry does not use the computer, and directly works on the model. It is evident that Gehry is not thinking of complex geometry when designing. Instead a computer software architect might use some concepts of differential geometry to write codes. Gehry’s sketches are usually of three dimensional forms of the building, and not of plans. They usually are squiggles and cannot easily be differentiated as lines and curves. As we have already seen, he does not approach the geometry of the form in a rational or analytical manner. Compared with Palladio and Wright, Gehry designs in three-dimensions. It is difficult to represent his post-computer buildings as plans, sections, and elevations. The geometry and the forms of his buildings do not reflect any abstraction or essence. Hence, one could assume that his approach to geometry is very intuitive and form oriented.

End Notes

1 Robert Venturi, Complexity and Contradiction in Architecture, with an introd. by Vincent Scully, (New York : Museum of Modern Art ; Boston : Distributed by New York Graphic Society, 1966). Venturi’s book is usually considered to be one of the first post-modern texts that criticize the modern aesthetics. Though, Venturi himself uses examples of Louis Kahn and Le Corbusier to explain his ideas of contradiction and complexity in architecture.
6 Ibid , 7.
8 Francesco Dal Co, Kurt W. Forster, 55.
9 Ibid, 35.
10 Rosemarie Haag Bletter , 76.
11 Francesco Dal Co, Kurt W. Forster, 23.
CHAPTER VI
CONCLUSION

Mario Salvadori described geometry as the bridge between mathematics, an abstract science, and architecture, a concrete art.\footnote{1} Geometry is as important to architecture as letters are to words. Yet, architects only use geometry, and very rarely come up with new concepts in geometry. An architect might be interested in the shape of a triangular object, while a mathematician would be interested in proving that the sum of the internal angles in a triangle is 180 degrees. Such a theoretical approach may be essential for architects to reproduce that shape in a design, but rarely do architects come up with such theoretical concepts. Even Frank Lloyd Wright’s work, which seems to have foreshadowed fractal geometry, does not capture the entire concept of fractal dimension. Hence, one could conclude that “Architects do not produce geometry they only consume it”.\footnote{2}

There is a huge gap in the theoretical aspect of geometry which is more a part of mathematics and the practical aspect of geometry which is more useful for architects.\footnote{3} Medieval masons solved stereotomical problems with a practical knowledge of geometry. They manipulated geometric forms with instruments and tools available to the masons. These were rule of thumb procedures with no mathematical calculations.\footnote{4} Today, with computer programs like CATIA such use of practical geometry seems superfluous. Another use of practical geometry is in architectural representation. Perspective and descriptive geometry were developed as simple tools that helped in
visualization or architectural representation. Both perspective and descriptive geometry had no mathematical calculations, or proofs. One can draw or draft a perspective, or use descriptive geometry to represent objects by standard methods. Such drafting requires architects to draw lines, measure distances, and mark angles. This does not involve theoretical geometry as one is not thinking beyond a line, arc, curve, point, intersection, bisecting etc.

Today, CAD programs use this concept of manual drafting, but the user interface is such that it requires one to think about geometry. To draw a door swing manually in plan, we usually fix the compass on the plan in the position of the door hinge, and draw an arc with radius equal to the width of the opening. To draw the same door swing on the computer we type the command ‘arc’. The computer gives us different options to define the arc: like three points; the start point, the center and an angle; the center, start point and end point etc. Drafting on the computer involves some basic concepts of Euclidean geometry, which are practical in nature. In Contrast, when using 3D modeling softwares like 3ds Max one uses limited concepts from theoretical geometry. While modeling from primitives and using Boolean operation on an object, one is thinking about the shape and ways to model that on the computer. To modify editable meshes and NURBS on the computer, one does not think in terms of geometry. The software uses concepts from differential geometry to generate the model, which is the domain of the software developer who has a more theoretical knowledge of geometry than the user. Hence the divide between practical geometry and theoretical geometry continues to exist today.
From the discussions of the three architects in this thesis, one could conclude that architects use geometry in a subconscious and implicit manner in their designs. There is little evidence of any conscious use of the golden proportions, \( \sqrt{2} \), \( \sqrt{3} \), or other incommensurable ratios that can be expressed geometrically. Architects are not thinking in terms of geometry when they are dealing with space and form. Yet there are correlations between architects’ approach to space and form, and concepts of geometry in mathematics. It is evident from the discussions in this thesis that Euclidean geometry is dominant in architecture, as the three dimensional space we experience is Euclidean in nature. At smaller scales, which are also the scale of architecture, we experience the earth as a flat surface. Spherical geometry, a non-Euclidean geometry, deals with a spherical surface and is used only in larger scales like astronomy and navigation.

Architects like Gehry claim to break away from Euclidean geometry, but they invariably create complex curves and surfaces, which are easy to represent and construct with the aid of computers. NURBS, b-splines and other complex curves that Gehry’s projects have are defined with concepts of differential geometry, yet, they exist in Euclidean space. Cartesian geometry introduced the idea of space as a three dimensional and infinite grid. Many modern architects like Wright adopted the grid, as it increased the efficiency of construction. It is easier to cut a straight line and an arc with a hand saw than to cut a free form curve. It is also easier to align brick and boards at right angles, than along a curve. Hence, the products that were machine made adopted the simple geometry of the straight line and right angle. Drafting instruments are also more congenial to parallel lines, perpendicular lines, angles like the 30 degrees, 60 degrees
and 45 degrees. The right angle, and the grid which are concepts of Cartesian geometry, again belong to Euclidean space. Hence architects associate the grid, and the straight line with Euclidean geometry.

Gehry and his contemporaries were not the first to breakaway from the grid, to deal with complex curved surfaces. Gothic vaulting had ribs with varying curvature that intersected from more than three directions. The surface spanning between these ribs had to accommodate their varying curvature. [Fig.6.1] Most of these ribs were circular arcs, and were also compounded with two or three circles. A particular system of ribs and surfaces were repeated on every bay within a cathedral, and cannot be compared with the warped surfaces of Gehry’s works. Even the modern movement which is usually associated with the grid and rectangular forms has some examples of complex curves. Le Corbusier’s Philips pavilion was a concrete shell structure that was a combination of spiky conoids and hyperbolic parabolic shells. [Fig.6.2] Some of Corbusier’s engineers used ruled surfaces to resolve the form and represent it three dimensionally. 7 This was a complex structure that could not be resolved into circles and arcs. It was with great difficulty that the stresses were calculated for this structure, and the engineers proposed a 5- centimeter concrete shell. 8 Corbusier himself was influenced by Antonio Gaudi who was known for his sculptural approach to form. His most noted building, La Sagrada Familia, had ruled surfaces like hyperbaloids, conoids etc. [Fig.6.3] He resolved the structure with either models or by working on site. He thus escaped from more conventional uses of geometry in his designs, and also projective drawing as a mediator. 9
Fig. 6.1 - Choir vault, Gloucester Cathedral. (www.britannica.com).

Fig. 6.3 - Sagrada Familia, Barcelona. View of interior east facade view. (www.greatbuildings.com).

Fig. 6.4 – Different views of a 3-D projection of a tesseract. (mathworld.wolfram.com).
Gaudi spent forty years on Sagrada Familia. In the present context, the forms of Gehry’s buildings are easier to construct due to the use of the computer for both visualization and construction.

Technology has made it possible to construct almost any form of any shape or geometry. It is easier to distort form today, than it was a couple of decades ago. Simple concepts from Euclidean geometry like line, arc and polygons may not be sufficient to describe these shapes. Yet, they are a part of Euclidean space, and cannot be considered to be non-Euclidean geometry, which is a part of geometry that is yet to be explored in architecture. Cartesian geometry represented reality as a three dimensional space with the three coordinate axes. Albert Einstein developed the model of a spacetime continuum with time as the fourth dimension. This model explains a number of phenomena at astronomical distances. During the early part of the 20th century, Cubism in art tried to incorporate the fourth dimension by juxtaposing different views of an object. Cubism broke away from Renaissance perspective by simultaneously representing the several views of an object. Sigfried Giedion has compared such simultaneous representations in Cubism to the Bauhaus architecture of Walter Gropius. The glass walls of The Bauhaus, which were transparent, simultaneously revealed several views of the building, including the inside and outside. 10 “The eye [could not] sum up this complex at one view; it [was] necessary to go around it on all sides, to see it from above and well as below.” 11 Movement was a new artistic dimension which Giedion described as the fourth dimension in architecture. To get a complete idea of a tesseract (a four dimensional cube), one needs to rotate it. [Fig.6.4] One gets to view different sides, inside, and
outside of the tesseract that are not perceivable from a single view point. It is possible
Giedion and some of the modernists translated this concept of viewing a three-
dimensional projection of a tesseract as movement, the fourth dimension, in architecture.
Non-Euclidean geometry cannot be used in the same instrumental way as Euclidean
geometry, perspective, differential geometry or other concepts of geometry. The
potential of non-Euclidean geometry in architecture is yet to be explored.

**Future Directions**

- I have primarily concentrated on western mathematics and architecture in this
thesis. Eastern cultures had their own systems of mathematics, architecture and
geometry. In India specifically mathematicians focused more on algebra, than
geometry. Given such a context, it would be interesting to study approaches to
geometry in Indian architecture, and the correlations that might exist with
geometry in mathematics.

- In this thesis I have discussed the works of three architects from three different
periods in history. Another possible way of approaching this study would be to
study three different architects from the same period in history. Though the
architects of the modern period had some general tenets by which they
approached their designs, most of them had different architectural expressions.
Within the modern period, for instance, I could study the way Frank Lloyd
Wright, Le Corbusier, and Louis Kahn approached geometry, and the
correlations that might exist between their works and geometry in mathematics.
Nature has been a source of inspiration for many architects. Different perspectives of nature have led to varied formal expressions in architecture. Indirectly this has led to different approaches to geometry in architecture. I could select three architects from three different phases in history, or a particular phase in history, and study the way they have been inspired by nature.

End Notes

3 The distinction between practical and theoretical geometry was introduced in the west by Hugh of St.Victor. “The theoretical is that which investigates spaces and distances of rational dimensions only by speculative reasoning; the practical is that which is done by means of certain instruments, and which makes judgements by proportionally joining together one thing with another.” Lon R. Shelby, Geometric Knowledge of Mediaeval Master Mason, Speculum, Vol.47, No.3. (Jul. 1972), 401.
4 Ibid.
6 Ibid.
7 Robin Evans, 298.
8 Ibid.
9 Ibid, 33.
10 Ibid, 58.
REFERENCES


Websites


http://www.cs.fit.edu/~wds/classes/graphics/History/history/history.html.html. (accessed 5\textsuperscript{th}, February, 2009).


VITA

Name: Urmila Srinivasan

Address: Department of Architecture, 3137 TAMU, College Station, TX, 77843-3137

Email Address: urmilasri@gmail.com

Education: B.Arch, CEPT University, 2002.
M.S., Architecture, Texas A&M University, 2009.