VALUATION OF GOVERNMENTAL GUARANTEE IN BOT PROJECT FINANCE WITH REAL OPTION ANALYSIS

A Dissertation

by

JAE BUM JUN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2008

Major Subject: Urban and Regional Sciences

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Approved by:

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ABSTRACT

Valuation of Governmental Guarantee in BOT Project Finance with Real Option Analysis. (December 2008)

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Chair of Advisory Committee: Dr. M. Atef Sharkawy

The limitation of public funds available for infrastructure projects has induced governments to attract private entities to participate in long-term contracts for financing, constructing, and operating huge infrastructure projects through Public Private Partnerships (PPPs) to reduce debt, constrain taxation, and share financial risks and rewards between the public and private sectors. Because these projects have such complicated risk evolutions, diverse contractual forms for project members to hedge their risks are necessary. Hence, the Build-Operate-Transfer (BOT) model has been considered as a very popular type to accomplish PPPs with the characteristic of a shared-ownership. For the government to attract private sector's participation, they have used incentive systems such as debt payment guarantee, Minimum Revenue Guarantee (MRG), or direct cash support. These incentive systems have been important critical success factors in BOT projects yet they have remained unfavorable in bidding process by failure of the traditional capital budgeting theory, Net Present Value (NPV) analysis, in evaluating the guarantee values. This is because NPV analysis can not reflect the guarantee agreements' contingent characteristic. For this reason, "Real Option Concept" imported from "Option Pricing Theory" in finance has been used as an effective way in estimating the guarantee value during the construction and operation of the project.

However, there are still open issues in identifying, formulating, and calculating the guarantee agreements' contingency due to the complexity of option pricing theory and in considering the uncertainty of the underlying asset. Furthermore, in recent real option-related research that evaluate BOT investment projects, the volatility of rate of

return in underlying asset (project value) is assumed to be just given or too simplified in its calculating process despite its significant impact on the guarantee value.

The purpose of this research is to develop the binomial real option model to better evaluate the MRG value by complementing existing real option models without violating the option pricing theory. To do so, the developed model in this research is to formulate the MRG agreement as a put option, consider the uncertainty of the underlying asset, and use the more detailed level of volatility with a Monte Carlo simulation approach.

To verify the applicability of the developed model, the model is applied to three different BOT project case studies, then, the results are compared with those by NPV analysis, Cheah and Liu (2006)'s real option model, and option pricing theory derived from Black-Scholes model.

Finally, based upon the results and analyses, the developed real option model appears to provide a practical and theoretical framework to quantitatively evaluate the MRG agreement under the BOT scheme and help the government establish better BOT policies and help the developer make appropriate bidding strategies in its investment.

DEDICATION

To my parents with my greatest appreciation and gratitude for all they have given me throughout my life

And, to my beloved brother and sister

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This research would not have been possible without the help and encouragement of many people. I am happy to dedicate this section to those who have helped me a great deal.

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Finally, I am glad that I had the unique and valuable opportunity to come to College Station, study, and finish my work. Now, I am preparing for another exciting world.

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1. INTRODUCTION

1.1 Background

In many countries, limitations upon the public funds available for infrastructure have led governments to attract private sector entities to participate in long-term contractual agreements for the finance, construction and operation of huge infrastructure projects. The public sector, therefore, insists on methods to ensure that value-for-money has been achieved. In contrast to the public sector's intent in value to society, private developers seek value for shareholders. Often, shareholders gain on projects that require little equity and rely on direct revenues to cover operating and financing costs. In order for the large and complex projects to match public and private intents, substantial study of project financing is needed.

In these projects, risk evolution is so complicated that it is difficult to conduct proper risk analysis, from the different perspectives of both the public and private sectors. These projects take many forms and may incorporate diverse features. In general, Public Private Partnerships (PPPs) arrangements include any collaboration between government and the construction development industry. Among the various ways to accomplish PPP projects, Build-Operate-Transfer (BOT) is the most frequently used type that includes a shared-ownership between the public and private sectors.

By definition, in the BOT model, private entities receive incentives to finance, build, and operate the project for a fixed period of time, after which ownership will be transferred to the government. Here, ownership reversion is planned to occur only after the private-sector entity has received a target return on the capital invested in the project. In return for the ownership reversion, the government might be asked to furnish some limited credit support.

Basically, the BOT type is implemented following risk and return negotiations among governments, project companies, and lenders. Through the bidding stage, these members will negotiate with one another to develop a mutually satisfactory project fina-

This dissertation follows the style and format of *Real Estate Economics*.

ncing structure. It is no wonder that the success of a risk and return sharing scheme depends on the financial soundness of the BOT proposal. Essentially, better financial planning provides a higher probability that the BOT project will succeed.

The BOT projects have very complicated and tightly structured contracts for each project member as a mean of hedging risks. Due to its unique characteristics in risks, managerial flexibilities, and heavily structured arrangements, there are large asymmetric payoffs for some parties to consider and respond appropriately.

The risk characteristics, diverse managerial flexibilities, and the financially structured arrangements, which stem from the uncertainties of project size, long concession periods and contractual complexities, make BOT project complicated assets that cannot be easily assessed by traditional evaluation methods such as Net Present Value (NPV) analysis. Modern financial theory suggests that option pricing models can be applied in the valuation to consider many complicated asset features such as financing schemes and managerial options. Analogies in the process between the BOT financial feasibility evaluation and option pricing can help evaluate the asymmetric payoff condition. However, because it is rather complicated to evaluate the financial feasibility identifying and considering these diverse factors in the BOT project, studies on the valuation of infrastructure projects based on managerial options have been limited. The various management flexibilities in the BOT project can include: a subsidy, a Minimum Revenue Guarantee (MRG), a debt payment guarantee, or a direct capital contribution (Huang and Chou, 2006; Klein, 1997). Any of these mechanisms can be used to address the concerns of the private sector and attract investor participation in financing the project. This is because an infrastructure project under the BOT arrangement is risky due to its huge size, long term concessionaire and complicated contractual arrangements. However, some governments do not explicitly account for the contingent liabilities and implications associated with these support packages (Mody and Patro, 1995).

Despite these guarantees often found in BOT projects, the cost to government of these financial incentives and their value to the private sector are not well understood (Mason and Baldwin, 1988). As well, it is difficult and sometimes controversial to estimate the exact guarantee value that the government has to support in order for the developers to decide to undertake the project. Furthermore, unsolicited proposals of the private sector's participation in projects are not uncommon (Hodges, 2003) and these factors can cause government to be exploited by a developer proposing a lopsided deal. This situation can become worse in situations that involve weak host country regulatory frameworks.

The importance of balancing risk and value has been highlighted by many financial theorists, and, in public infrastructure, Cheah (2004). All concession agreements cover risk, hence, it is important to fully understand all components of value enhancement associated with the agreement. Governmental support such as subsidies and guarantees add direct value to the transaction. In addition, value can be created through operating options and flexibilities during the design process and project execution. Without proper evaluation of these options, the matching of risk and expected return cannot proceed in a guided manner. When the expected return of the concessions granted is properly accounted for, the level of risk tolerance of the concessionaire who has received an expected return from these incentives may be higher. Although it is difficult to derive the exact expected return and value of these incentives by using traditional NPV method, efforts are needed to pursue this direction, since they ultimately lead to a more equitable alignment between risk and value. Hence, the ideas of the expected return and risk form the core of deal-making and are closely related to the bidding negotiation (Lewis and Mody, 1998).

Most importantly, all parties need to know how much they can benefit from its expected return and tolerate in risk. For the private sector, the expected return of these incentives is an asset to let the private sector consider the project worthy. On the other hand, for the public sector, because the expected return from these incentive systems is the cost and liability to government, the government can save time in a bidding process to quit lopsided deal against the private sector through understanding the contingencies that happen.

The negotiation process is largely facilitated by various valuation techniques that can include the gains from managerial flexibility. As the MRG and debt payment guarantee is a liability to the government while being an asset to the BOT firm, it is crucial for project participants to evaluate the guarantee value especially during the bidding process. Developers failing to consider the value of guarantee will underestimate the investment value, and, if the guarantee value is too large, the government over-subsidizes the BOT firm (Baldwin et al., 1983). Unfortunately, the NPV method cannot price the value of those guarantees that create an asymmetric payoff; instead, an option pricing framework is needed.

1.2 Significance and Contribution of Research

This study will contribute to providing a practical and theoretical real option framework to evaluate the real assets related to BOT project finance under the agreements of the MRG between the government and the private sector. This MRG concession agreement allows for detail on the capital structure of the developers and highlights the asymmetric payoff inherent in the MRG. By doing so, the developed real option model is expected to be easily used in practical applications and help both governments and developers establish better project finance bidding policies and decision-making in their investments.

1.3 Organization of Dissertation

Section 1 introduces research background and brief problems, then, describes the significance and contributions of this research. Section 2 summarizes the related literature regarding traditional capital budgeting analysis, real option valuation analysis, real option analyses that have been applied in infrastructure projects, and the valuation of governmental guarantees using a real option analysis in infrastructure projects. Section 3 includes the description of the related theories; traditional capital budgeting theories and real option theory. Section 4 investigates more detailed problems, which have been observed based on the literature review and the existing studies, in evaluating the impact of guarantee agreements in BOT infrastructure projects. Section 5 describes the research question, conceptual framework, research hypotheses, and validation process necessary to verify the applicability of the developed real option model. Section 6 explains the methodology used to evaluate the MRG agreement. This Section falls into two parts. The first describes existing project evaluation methodologies which have been used to evaluate the MRG in infrastructure projects. This part includes the NPV analysis and Cheah and Liu (2006) real option model. The second focuses on the process of developing a new real option model to evaluate the MRG agreement. Section 7 describes three BOT project case studies and provides analysis with the existing valuation models; NPV analysis and Cheah and Liu's real option model (2006), and developed real option model. This Section also explains the analyses and verification process to test the applicability of the developed real option model with the results obtained from the three BOT case studies. The goal is to show whether the results can satisfy the research hypotheses or not. Finally, Section 8 gives the conclusions and the limitations of this research and recommendations for further study.

2. LITERATURE REVIEW

2.1 Traditional Capital Budgeting Analysis

The purpose of capital budgeting process is to select financially feasible long-term investment projects. Under the certain outcomes followed by stable cash flows, capital budgeting is simple and clear. However, these projects are just few, most often, there is great focus on how to consider uncertainty and risk in the project evaluation (Aggarwal, 1993). The following are descriptions of traditional capital budgeting analyses.

2.1.1 Net Present Value (NPV) Analysis

The representative of traditional Discounted Cash Flow (DCF) analysis, NPV analysis, works well when the risks of an asset remain stable over time. This traditional valuation method is adequate for investment decisions regarding assets-in-place if operations guarantee relatively stable cash flows (Luehrman, 1998; Myers, 1984). NPV analysis also works well for typical engineering investments such as equipment replacement if the main benefit is cost reduction. However, sometimes projects create contingency possibilities such as delaying, abandoning or expanding the projects by the management decision changes. And, future cash flows change as development proceeds or new information is received. In this case, NPV analysis has either underestimated or ignored the value of this management's flexibility (Amram and Kulatilaka, 1999; Trigeorgis, 1999; Dixit and Pindyck, 1994; Myers, 1984). Once risk comes into play in investment, the NPV analysis can accommodate risk attitude by using a risk-adjusted discount rate to discount the expected cash flows. In the real world, generally, many firms classify different risk categories of projects and assign each category different rates to reflect the risk involved (Trigeorgis, 1999) or use different discount rates in different periods to reflect the change of nominal rates of interest (Aggarwal, 1993). Even as the NPV analysis has been widely used in almost every industry as an effective project valuation method, there are also some criticisms that can be leveled against it.

First, the NPV analysis assumes that the cash outflow is certain and occurs at the beginning of a project. Even when there are cash outflows in different time periods other than time 0, they are usually assumed to have the same risk characteristics as the cash inflows. That is, the NPV analysis just uses the same discount rate as the cash inflows when discounting the future cash outflows when addressing the uncertainty. But, in real projects (such as large construction projects) even if the future cash inflows are assumed to be certain based on the contracts, the uncertainty mainly comes from the cash outflows. This causes two important problematic issues. One is how to decide on the right discount rate for the certain cash inflows. Choosing the right risk adjusted discount rate is important in cash flow analysis, because it adjusts the project analysis for risk (Butler and Schachter, 1989) requiring the knowledge of economic indicators and market attitudes. The other is, if the same discount rate as cash inflows is used in cash outflows, it would underestimate the present value of the cost/cash out flows and, in turn, overestimate the NPV of the project.

Secondly, when NPV is applied to construction projects, it cannot appropriately evaluate managerial flexibility (Baldwin, 1982; Copeland and Antikarov, 2001; Dixit and Pindyck, 1995; Trigeorgis, 1999). The NPV analysis ignores the management’s flexibility to adapt or revise later decision when, as uncertainty is resolved, future events turn out differently from what management expected at the beginning of the project (Trigeorgis, 1999). When a project is associated with high uncertainty, if an investment requires sequential decision-making and if early investment can reveal information about the future profitability of the project, it deserves to invest even when NPV is negative (Roberts and Weitzman, 1981). The cumulative error of ignoring all the operating/managerial options embedded in a project can cause a significant underestimation of its value (Mason and Merton, 1985). For the evaluation of long-horizon projects in which future profitability can only be imprecisely estimated, it is critical to consider the associated managerial or strategic options.

For the reasons above, the real options analysis is suggested by many researchers as the most appropriate methodology due to its ability to incorporate these management flexibilities. In fact, infrastructure development often proceeds in a series of stages that aim to better define the project scope and discover unknown information. Moreover, flexibility is often incorporated as an intuitive managerial approach to effectively deal with uncertainty. Preliminary planning and feasibility studies, such as environmental impact studies, geotechnical surveys and traffic volume analyses, can reveal information that may alter future investment and development decisions. Flexible design also permits infrastructure projects to more readily adapt to changeable conditions, such as an increase or decrease in expected demand for the project's output. Staged infrastructure projects can give management the opportunity to obtain more information as market conditions become more certain. That is to say, flexibility can effectively reduce life-cycle costs by allowing a more timely and cheaper response to a risky environment. Flexibility adds value, but it comes at a cost in terms of money, time, and complexity. This added value should be weighed against its cost, but the NPV analysis cannot appropriately support such analyses.

2.1.2 Internal Rate of Return (IRR) Analysis

Along with NPV analysis, Internal Rate of Return (IRR) analysis is another popular capital budgeting approach due to its intuitiveness as to the rate of return. Generally, NPV decreases as the discount rate increases. At some value of the discount rate, the NPV of a specific cash flow stream is zero. This discount rate that makes NPV equal to zero is called the IRR. That is, the IRR is the rate of return on project investment reflected in its set of future cash inflows (Aggarwal, 1993; Herbst, 1982). If the IRR is higher than a required rate of return, then the project is considered acceptable. In general, the required rate of return is obtained based on the cost of borrowing for a similar project plus several percentage points higher than the cost of borrowing to compensate for the risk, time, and trouble associated with the investment (Butler and Schachter, 1989). Unlike NPV analysis, this approach may help remove the problem of choosing the proper discount rate but it still produces another question in how to decide the reasonable hurdle rate that will be used as a standard to estimate if the IRR is high enough to approve to the project.

Like the NPV analysis, the IRR approach also has some controversial issues. The first is the fact that the calculation of IRR assumes that all project cash inflows are reinvested in other projects at the same rate of return as the IRR. Due to this assumption, when a project's IRR is higher than a firm's normal rate of return, it will be difficult to justify how the reinvestment could have the same high rate of return. Finally, the IRR could lead to a conflicting recommendation when projects are mutually exclusive (Weston and Brigham, 1993). In a long-term project, this will be even more serious. On the other hand, NPV analysis assumes that the reinvestment's rate of return is its opportunity cost, NPV's discount rate. The assumption of reinvestment return underlying the NPV method can be justified by business practice. Second, this approach can not appropriately handle unconventional cash flows such as negative cash flows coming in later years (Butler and Schachter, 1989). If the signs of the net cash flows change in successive periods, it is possible for the calculations to produce multiple IRRs (Copeland and Weston, 1988; Brigham and Gapenski, 1997). While multiple rates are theoretically possible, only one rate of return is significant in an 'accept' or 'reject' decision.

2.1.3 Decision Tree Analysis (DTA)

Unlike the NPV and IRR analyses, which are based upon the assumption that investment decision is now or never, the DTA approach considers the uncertainty and later decisions made by management to capture the value of managerial flexibility contingent on a project (Copeland and Antikarov, 2001). This approach provides an effective structure where all feasible alternative decisions and the implications of taking those decisions are laid down and evaluated. Finally, this approach helps analyze complicated sequential/staged investment decisions when the uncertainty is resolved at discrete time (Trigeorgis, 1999). In general, since the DTA trees are structured with all the possible alternatives available to management identified at the different nodes of the tree and the management is faced with the decision of choosing the best alterative consistent with the maximization of the project's net present value, this approach forms an accurate and balanced picture of the risks and rewards that can result from a particular choice.

In spite of its effectiveness in project valuation, the DTA also has practical limitations. As an example, when being applied to a real-world problem where the number of different paths through the tree expands geometrically it will become an unmanageable black-box in the decision making process.

The presence of operating options in future decision nodes which changes the payoff structure and the risk characteristics of an asset can cause confusion and difficulties in selecting the proper discount rates (Trigeorgis, 1999).

2.1.4 Other Traditional Analyses

2.1.4.1 Accounting Rate of Return (ARR) Approach

The Accounting Rate of Return (ARR), which is also called as Return on Investment (ROI), is an accounting ratio that represents the profit of an organization before interest and taxation as a percentage of the capital employed. The rate of return of a project is compared with a predetermined and targeted rate by management to be the minimum required rate of return. In the ARR, the project is acceptable if the ratio is higher than the targeted rate.

The inherent weakness in the ARR approach is that it does not consider the time value of money (Herbst, 1982). And, because the proposed project is measured in percentage terms, it can not take into account the financial size of a project when the management favors small projects over large profitable ones and alternatives are compared (Weston and Brigham, 1993). Therefore, it is likely that the target rate of return is an ad hoc risk adjustment (Butler and Schachter, 1989).

2.1.4.2 Monte Carlo Simulation Approach

The Monte Carlo Simulation Approach, as one of the extensions of the NPV analysis, is to account for the uncertainty (Razgaitis, 2003; Trigeorgis, 1999) and was originally suggested by Hertz (1964). Simulation procedure is used to identify the key variables that determine the cash flows of a project and then simulate these variables to obtain the distribution of the resulting cash flows or NPVs. This is an analytical approach that depends on repeated random sampling from the probability distributions of the main variables underlying the cash flows of the project to arrive at risk profiles, which represents the distribution of the cash flows or NPV (Evan and Olsen, 1998). This simulation reflects real world decision settings by using a mathematical model to capture the important characteristics of the project as it evolves through time and encounters random events. As a common type of simulation approach, the Monte Carlo simulation generally follows the following processes (Evans and Olsen, 1998; Trigeorgis, 1999). The first is to identify all the uncertain variables in a project's cash flow setup, noting the interdependencies between different variables and any serial dependency. In the second process, for each uncertain variable, it defines the possible values with a probability distribution. Here, the type of distribution selected is based on the nature of uncertainty surrounding that particular variable. The third, a random value is then selected for each uncertain variable based on its probability distribution to calculate the net cash flow for each period and subsequently the NPV. Finally, this process will be repeated for a number of iterations, which results in a probability distribution for the project's NPV.

Even if this approach has an advantage in dealing with the complicated/uncertain decision problems and the interaction of the variables with one another and across time (Trigeorgis, 1999), it also has its limitations. First, if NPV is calculated with a risk adjusted discount rate, any further adjustment for risk is double-counting. If a risk-free rate is used instead, then one obtains a distribution of what the project's value would be tomorrow if all uncertainty about the project's cash flows were resolved between today and tomorrow. But as uncertainty is not resolved in this way, the interpretation of the distribution is not clear (Myers, 1976). The second limitation is in the estimation of the interdependencies and the serial dependencies. This is a difficult and time-consuming process, which often leads management to delegate this task to experts (Trigeorgis, 1999) and thus management does not fully understand the simulation results which consequently affect their decision making. Third, it is possible that the interpretation of the probability distribution of NPV given by the simulation process is questionable, because it is unclear as to how to value the risk-return trade-off (Evans and Olsen, 1998). Fourth, like other traditional capital budgeting approaches, this approach can not deal with asymmetric payoff conditions caused by management's flexibility to maximize the project's value when the actual cash flows differ from what they were supposed to be as uncertainty gets resolved over time (Trigeorgis, 1999).

2.1.4.3 Pay-Back Period (PBP) Approach

The Pay-Back Period (PBP) analysis, which is referred to as the capital recovery period, is defined as the length of time required for the cash inflows produced by an investment to equal to the initial investment (Weston and Brigham, 1993). If an investment is expected to produce constant cash inflows from year to year, the PBP will be determined as follows (Butler and Schachter, 1989; Weston and Brigham, 1993).

PBP = Initial cost of Investment / Annual Net cash inflow

On the other hand, when the expected cash inflows vary from year to year, the PBP is determined by separately adding up the expected cash flows in successive years until the total is equal to the initial investment cost. In this approach, the project will be accepted when the pay back of the initial investment cost is obtained within a predetermined time. So, projects with shorter payback periods rank higher than those projects with longer payback periods. One major limitation is that this approach does not consider the time value of money (Herbst, 1982) and ignores any benefits that occur after the payback period (Weston and Brigham, 1993). Further, even if the cash flows in many projects are slow to start for some reason (Butler and Schachter, 1989), the PBP approach favors of projects with higher cash flows at early stages.

2.2 Real Option Valuation Analysis

2.2.1 Real Option Analysis

The option pricing theory, which is devised in financial assets by Black and Scholes (1973) and Merton (1973), is the building block of the “Real Option Theory.” The option pricing concept finds out the interactions among option holder’s profit maximizing behaviors, an asset’s uncertainty, and market disciplines. This financial concept is imported to evaluate the real assets of physical investments, which are then called “real options.” So, the real option theory is the application of financial option pricing theory as a management tool to evaluate the investments of real assets (Amram and Kulatilaka, 1999). The real option analysis, in many ways, is considered as an effective tool to evaluate the options and flexibilities embedded in projects such as oil and gas, pharmaceutical, manufacturing, airlines, mining, real estate, and other industries. In financial assets like stocks, bonds, currency and so on, an option is a right, but not an obligation, to exercise a certain action in the face of the uncertainty. Likewise, in a real asset, which is tangible, such as a project or business relative to the financial assets (Copeland and Antikarov, 2001), a real option is the right, but not the obligation, to take some specific action. These are referred to as “management flexibilities” such as deferring, abandoning, or expanding at a specific cost for a specific time (Reiss, 1998).

Real option theory provides a framework to analyze strategic capital investments by identifying the management flexibilities to be dealt with as valuable opportunities (Dixit and Pindyck, 1995) so that the decision-makers can keep investment options open when facing uncertainty and undertake them after resolving uncertainty with time or more information (Trigeorgis, 1993a). When the present value of the expense of making decision changes during construction or operation is greater than the additional cost of designing flexibility into the investment opportunity at the outset (Lander and Pinches, 1998), the real option is available.

The main difference between real option analysis and NPV analysis stems from the different impact that some variables have on the models. For example, the higher uncertainty, greater interest rates, or more time before undertaking a project do not necessarily make an investment opportunity less valuable in the real options analysis, as is the case with the NPV analysis (Trigerogis, 1999). That is to say, although each of these factors has a negative effect on the NPV of an investment opportunity, these factors can enhance the option value created by managerial flexibility.

In general, real option models fall into two categories: the continuous-time model and the discrete-time model, which referred to as “Black-Scholes Model” and “Binomial Model” respectively. Depending on the particular context of the application, the two approaches incorporate different assumptions, which ultimately determine the suitability of the selected model.

2.2.2 “Black-Scholes Model” – Continuous Time Approach

The Black, Scholes (1973), and Merton (1973)’s option pricing model, which is referred to as the continuous-time model, represents one or more variables as stochastic processes. In general, most continuous time models are restricted to just one or two variables due to computational complexity and detailed mechanics of these stochastic processes being examined (Dixit and Pindyck, 1994, Hull, 1997; McDonald, 2002; Wilmott, 1998). The continuous-time model often requires the derivation of a Partial Differential Equation (PDE). The procedure then shifts from an intuitive consideration of strategic issues into mathematical manipulation where the PDE is solved subject to a set of boundary conditions related to the features of the option. The Black-Scholes Equation used to evaluate a European option is a good example (Hull, 1997). Pindyck (1991) provides another example of an analytical solution for the simplest version of an investment timing problem. Brennan and Schwartz (1978) and Hull and White (1990) have developed and published algorithms that simplify and improve the valuation procedures using finite difference methods.

Although the Black-Scholes model is widely used for financial option valuation, when being applied to real assets, it has been criticized due to practical limitations. First, this model assumes that the option can only be exercised at maturity as a type of a European option because it only calculates the option value at maturity. But, there are many options other than European-type options, that have to be exercised at any time prior to maturity as American-type options. For example, in infrastructure projects, it is expected that management would exercise any of the real options available to the project at any time within the maturity period when market conditions are suitable. This requires that the option be evaluated as an American-type option. However, this is not possible in the Black-Scholes model because it calculates the value of the option at a point in time, the maturity date. Second, this model excludes the possibility of evaluating a rainbow option and a compound option as it considers a single source of the uncertainty and a single underlying asset. But most investment decisions have compound options embedded in projects because they occur in phases and there are usually several correlated sources of uncertainty. Third, if the options are considered in combination, this model can not be used. With the investment having more than one option, the Black-Scholes model is not suitable because this model is applied to evaluate only one option at a time, which means that the valuation of multiple options is ruled out. The final limitation of Black-Scholes model is that because this model is derived from a variety of advanced mathematical techniques and knowledge, it requires a sophisticated mathematical background to understand the derivation and intuition of the model. However, this mathematical background and these techniques are not common to managers and practitioners (Graham and Harvey, 2000). These criticisms mentioned above are mainly related to the basic assumption underlying the derivation of the model. A relaxation of some of the underlying assumptions can significantly affect the results of the real option analysis. These are the limitations which make it very difficult to use for analyzing most real options with Black-Scholes model.

2.2.3 "Binomial Model" - Discrete-Time Approach

As another common form of the real option model, the discrete-time model involves the binomial model, trinomial model, and lattice model. Many of these models have been developed to provide an approximation to continuous-time models 'in the limit' and, in general, when a discrete-time model is described, it is a binomial model. Cox et al. (1979) recommended values for the up and down movements of the underlying variable within a binomial model so that its volatility matches that of the stochastic process given in a limiting situation. Then, this approach was expanded by Boyle (1988) for a trinomial and a lattice model with recommendations of appropriate up/down movements of an underlying asset and risk neutral probabilities. With a discrete time model, the problems of payoff structure, risk characteristics and non-constant discount rates that are primarily due to the flexibility and asymmetry embedded in the decision-making process can be overcome (Trigeorgis, 1999). This is achieved by converting the real situation into a risk-neutral one. More importantly, due to its resemblance to the DTA approach, managerial flexibility can be easily formulated in the tree structure. Consequently, modeling real options in this way is more intuitive. This approach is especially valuable when simply imposing stochastic processes on underlying variables cannot represent a real option scenario.

There are some advantages of the binomial model that are not possible in the continuous time approach. The first is that, unlike the Black-Scholes model, the binomial model can be used to accurately price American-type options (Copeland and Antikarov, 2001; Trigeorgis, 1999) because this model makes it possible to check at every point in an option's life the possibility of early exercise. Second, the binomial model is not dependent on the probability of certain outcomes, which means that this model can be beneficial to investors who have different subjective probabilities about an upward and/or downward movement in the value of the underlying asset (Cox et al., 1979). Third, the binomial model is easier to formulate and use because it does not require sophisticated mathematical background and skill compared to the Black-Scholes (Copeland and Antikarov, 2001). Finally, the model can formulate the effects of interactions among different options that may be present in real projects. With binomial trees, the valuation of multiple options can be taken into account as well. On the other hand, the disadvantage of this model is that, while modeling and valuing management flexibilities, the trees will quickly grow large as the number of time periods and the intervals within those time periods increase (Copeland and Antikarov, 2001; Hull, 2002).

2.3 Real Option Valuation Analysis in Infrastructure Projects

The option pricing concept has been considered in evaluating infrastructure projects. Wey (1993) found that crude oil pricing follows a mean-reverting process for a long time horizon and that a real option evaluation contingent on oil prices can be modeled with this concept. Leviäkangas and Lähesmaa (2002) suggested the option approach as one of the useful evaluation methods for intelligent transport system investments. Garvin and Cheah (2004) used a simple binomial model to evaluate a deferment option in a toll road project. Ford et al. (2002) also used a similar approach to quantify the value of design flexibility in an engineering project. Wooldridge et al. (2002) evaluated the flexibility in private toll road development with real option analysis based on the case of Dulles Greenway. Other works include Ho and Liu (2002) who adopted the real option approach to the equity value in an infrastructure project and Ng et al. (2004) who set up a model to value a price cap and to determine an optimal exercise policy in construction material procurement. Here, Ho and Liu adopted a discrete-time approximation to model the stochastic processes of two log-normally distributed variables such as project value and construction cost to subsequently solve for equity value using a lattice model. By broadly categorizing models into continuous-time and discrete-time, Garvin and Cheah (2004) commented on the merits and challenges of applying each of them to the context of infrastructure projects. Borison (2003) also categorized the different real option approach based on their assumptions and the mechanics involved. As for BOT type project evaluation as related to option pricing theory, Ho and Liu (2002) developed a real-option pricing model to evaluate government debt guarantees and negotiation options in BOT projects. Ford et al. (2002)

show that the real option approach can be applied in traditional project planning by using a binomial option pricing approach to quantify the value of design flexibility. Tien (2002) analyzed time-to-build options in sequential construction and Garvin and Cheah in 2004 developed a model to evaluate strategic project deferment.

2.4 Evaluation of Governmental Guarantee with Real Option Analysis in BOT Projects

As for the related recent research on evaluation of governmental guarantees with real option analysis in a BOT infrastructure project, few studies have been published. The research by Ho and Liu (2002) and Cheah and Liu (2006) are representative. Many subsidies and guarantees represent a form of options and all options have their own values but the valuation method of a minimum revenue guarantee (MRG) or debt payment guarantee is still an open issue (Mason and Baldwin, 1988).

Ho and Liu (2002) examined the financial viability of privatized infrastructure projects based upon the option pricing model. Ho and Liu developed the quantitative and theoretical binomial pyramid model that considered the BOT characteristics such as uncertainties in project value and construction cost and asymmetric payoffs due to limited liability of equity that can cause bankruptcy and terminal conditions. They then evaluated the debt payment guarantee value from the perspectives of the developers and the government. Finally, through this model, they evaluated the impact of the debt payment guarantee and the developer negotiation option on the equity value of the project. In its analysis, the authors used the binomial pyramid model that was developed to consider the uncertainties and dynamics of two risk variables of project value and construction cost simultaneously with an illustrative example of a case study. As a result, based upon traditional NPV analysis, this research showed that because the equity value is less than the equity investment, the equity investor will realize an investment loss. But, under the condition of a debt payment guarantee, the equity value is greater than the equity investment so that the BOT type project is financially viable from an equity investor's point of view. Finally, the debt payment guarantee value is significant when compared to equity value. A contribution of Ho and Liu's research is that it provided a theoretical and quantitative framework for evaluating the financial viability of BOT type projects from the perspectives of both the project developer and the government and provided an important policy implication to be considered in the bidding process.

In the research by Cheah and Liu (2006), they investigated the option value of a minimum revenue guarantee, repayment and expansion in infrastructure projects with an asymmetric payoff concept using a Monte Carlo simulation. This approach is in principle very similar to the case of Moel and Tufano (1998). Moel and Tufano simulated the prices of copper and zinc following specific diffusion processes. Then they evaluated the option which was related to copper and zinc prices in a discounted cash flow model. In Cheah and Liu's research, they assumed traffic volume growth rate and initial traffic volume as risky variables. They devised two cash flow models; one is the expected cash flow model used for the concept of a put option and the other is the actual cash flow model where risky variables are simulated with specific mean and standard deviation values in a Monte Carlo simulation. Finally, by calculating the difference between the expected cash flow and actual cash flow models and then discounting that difference back at a risk-free rate, they calculated the value of the minimum revenue guarantee. This research also shows that the minimum revenue guarantee and repayment option have significant value compared to the project value, and, subject to changes in the volatilities of initial traffic volume and traffic volume growth rate, the guarantee value is more sensitive to the volatility of the initial traffic volume. Cheah and Liu have shown a clear and relatively simple view of how the real option concept can be applied to evaluate management's flexibility in the bidding process and have given a policy implication that a governmental guarantee can be appropriately designed to balance the concepts of risk and return.

3. THEORY

3.1 Traditional Capital Budgeting Analyses

3.1.1 Net Present Value (NPV) Analysis

The NPV analysis has been widely used to evaluate project values in industries with the concept of discounting the future cash flows at a required rate of return (Brigham and Houston, 2004):

$$\begin{aligned} NPV &= -I_0 + \sum_{i=1}^{t} \frac{FCF_i}{(1+r)^i} \\ &= -I_0 + \sum_{i=1}^{t} \frac{FCF_i}{(1 + WACC)^i} \end{aligned} \tag{3.1}$$

where, I_0 is the initial investment, FCF_i is the future net cash flow after tax at time i, r is the required rate of return that will be used to discount the future cash flow FCF_i and i is the time increment. In general, the required rate of return "*r*" is WACC (Weighted Average Cost of Capital) of the firm or project as defined by Equation (3.2).

$$WACC = R_e \cdot \frac{E}{A} + R_d \cdot \frac{D}{A} \cdot (1 - T) \tag{3.2}$$

where, E is the equity, R_e is the cost of equity, A is total invested capital, R_d is the cost of debt, D is the debt, and T is the corporate tax (Modigliani and Miller, 1963). Sometimes higher interest rates than WACC are required. If a firm wants to increase its growth rate, then it will want to fund projects that have a higher level of return. Some firms impose a high interest rate as a hedge against risk, requiring high rates of return for high-risk projects. This higher interest rate is, in general, called the Hurdle Rate (Meredith and Mantel, 2003). In the case of infrastructure projects, generally, the

WACC to discount the future cash flows will be determined by using Equation (3.2). And, this WACC will be used as in Equation (3.1) to find NPV of the project. Typically, WACC represents a company's weighted average cost of capital which includes the cost of debt and cost of equity, and it is employed to evaluate projects that match a firm's existing operating assets and associated risks. Thus, determining R_d , T, D, E and A is not that difficult, and the last variable, cost of equity, R_e, is often estimated by the well-known Capital Asset Pricing Model (CAPM). The cost of equity, R_e, is a measure of the appropriate required return that equity investors should expect on equity investments, given the level of risk of such investments. Equation below is used to estimate the cost of equity, R_e, is based on the CAPM developed by Sharpe (1964), which is expressed as follows.

$$R_e = R_f + \beta_e (R_m - R_f) \tag{3.3}$$

In general, with huge infrastructure projects, since some risk premiums should be added to the cost of equity or cost of debt to reflect the risks involved in the projects such as specific country or sector risk, actual risk-adjusted discount rate can be greater than R_e.

3.1.1.1 Determination of Risk Premium

In NPV analysis, identifying and quantifying the risk involved in a project is important in project evaluation. When risk appears in a project the investment problem becomes complicated, and, more information related to individual risk attitudes should be considered in the analysis. Every project has a specific amount of risk involved and, based on the basic assumption that investors are risk averse, the project demands more returns which includes the higher risk premium as uncertainty gets resolved in project (Megginson, 1997). In general, the investment projects have two main types of risks (Trigeorgis, 1999). Theses include systematic risk and unsystematic risk. Systematic risk is defined as the risk which impacts on the project returns and is caused by its correlation with the market returns. On the contrary, unsystematic risk is the risk that is specific to the investment project. This risk can be eliminated through investor's diversifying their investment portfolios by holding more than one investment project in a portfolio at one time (Trigeorgis, 1999). But, because in investments such as in infrastructure projects, where it is often impossible to diversify the projects, both systematic and non-systematic risks have to be taken into account in the process of investment analysis. Followings are two methods used to adjust the discount rate for risk in an investment.

1) Capital Asset Pricing Model (CAPM)

The CAPM provides a way to simultaneously relate the required return of a project/security to its relevant systematic risks (Sharpe, 1964). That is, the CAPM describes how the market reaches a price for risk through the formulation of a market efficient portfolio, M , and a risk-free security, r , and how individual securities are priced through their covariant relationship with M (Levy and Sarnet, 1990). In derivation of the CAPM, investors are assumed to avoid the non-systematic risk through portfolio diversification. That is, a portfolio's expected return solely depends on its systematic risk, which is called "β" (Brigham and Gapenski, 1997). Therefore, the CAPM measures a portfolio's risk and the return for taking that risk. Followings are some basic assumptions for the CAPM (Sharpe, 1964; Trigeorgis, 1999).

- ✓ *All assets are perfectly divisible and liquid.*
- ✓ *Investors are rational, which means that their objectives are to maximize their expected utility of wealth.*
- ✓ *Investors are risk averse, which means that they diversify their portfolio efficiently based upon the mean and variance of portfolio return.*
- ✓ *A risk free rate exists and investors can borrow/lend any amount of money at this rate.*

- ✓ *The market is competitive, which means that the investors are price takers.*
- ✓ *The expectations about asset returns are homogeneous, that is, identical estimates if the expected values, variances, and covariances of returns for risky assets.*
- ✓ *No taxes, no transaction costs, cost of bankruptcy is negligible and information is freely available to investors.*

Based upon the assumptions mentioned above, the CAPM determines the expected returns for investors to be compensated for the corresponding level of systematic risk related to an asset or portfolio in equilibrium as in Equation (3.4) below (Sharpe, 1964; Trigeorgis, 1999). Here, when an asset has no systematic risk, β will be "0" and, according to the Equation, the expected rate of return for the asset is the same as the risk-free rate. As shown earlier, because the expected rate of returns given by the CAPM provides no compensation for risk that is unique to the investment, the CAPM is useful for investors holding diversified portfolios. However, it can not give adequate risk measures for investments where investors find it difficult to diversify.

$$E(R_j) = R_f + \beta_j \, [E(R_M) - R_f] \tag{3.4}$$

where

$E(R_j)$ is the expected rate of returns on asset j

R_f is the risk free rate

$E(R_M)$ is the expected market rate of return

$\beta_j = Cov(R_j, R_f)/Var(R_M)$ is the asset's beta or a measure of the level of the asset's systematic risk

2) Capital Market Line (CML)

The Capital Market Line (CML) is the set of portfolios formed by combinations of risk-free assets and risky portfolios (Damodaran, 1997). These are made by combining assumptions regarding individuals and the market to derive the most efficient portfolios (Brigham and Gapenski, 1997). These portfolios have the highest returns for a given level of risk, which is measured by the standard deviation of returns. The CML provides a linear relationship between the risk and return for efficient portfolios of assets (Brigham and Gapenski, 1997). Because the CML considers both systematic risk and unsystematic risk of a project, unlike the CAPM, it is a more appropriate risk measure to use in determination of its discount rate for non-diversifiable investment projects. When a project is a large part of an investor's portfolio, the CML can be used to determine the discount rate for the project (Damodaran, 1997). Here, the discount rate consists of a risk-free rate and a risk premium which is equal to $[E(R_m) - R_f]/\sigma_m$, multiplied by the investment's standard deviation, σ_p. Based upon Copeland and Weston (1983) and Brigham and Gapenski (1997), the following is the capital market line Equation:

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m} \cdot \sigma_p \tag{3.5}$$

where

$E(R_p)$ is the individual expected asset rate of return

R_f is the risk free rate

$E(R_m)$ is the expected market rate of returns

σ_m is the market risk

σ_p is the risk associated with the asset

3.1.2 Internal Rate of Return (IRR) Analysis

Along with the NPV analysis, the IRR, which is the value of the discount rate at which the NPV is equal to zero, is regarded as the most fundamental financial decision criteria for project evaluation. But, as shown in below Equation, unlike the NPV approach the IRR is the discount rate that makes the net present value of future cash in/outflows equal to zero, or more simply put, is the long-term rate of return for project. Of course, this approach can be an indicator to determine whether the project is financially feasible or not by comparing the obtained IRR with the required rate of return or hurdle rate. If the IRR is greater than or equal to the required rate of return or hurdle rate, it means that the project will be financially feasible. Following is the Equation to calculate the IRR.

$$0 = \sum_{i=1}^{t} \frac{FV_i}{(1 + IRR)^i} - I_0 \tag{3.6}$$

3.1.3 Decision Tree Analysis (DTA)

The DTA approach illustrates all the possible options from which management can choose. The implications of choosing each option are then described as an additional option as shown in Figure 3.1. Here, the square boxes depict decision nodes where a decision is made and the circles indicate outcome nodes where management has no control pf the outcome due to natural occurrences. To try to briefly understand the value of managerial flexibility which the DTA approach considers, the project value with the NPV analysis will be calculated with the decision tree approach.

Figure 3.1 A General Structure of a Decision Tree (Trigeorgis, 1999)

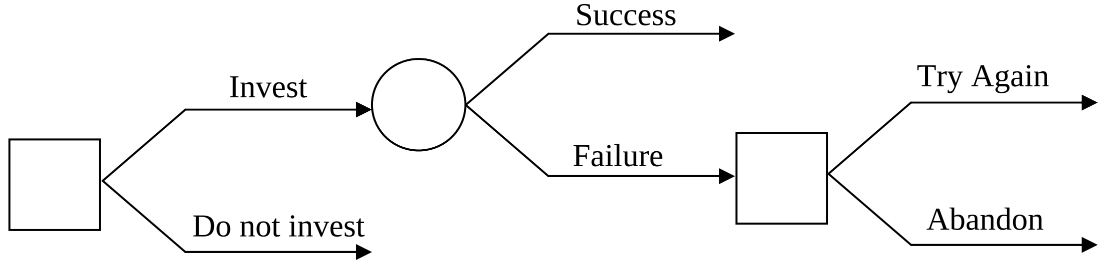
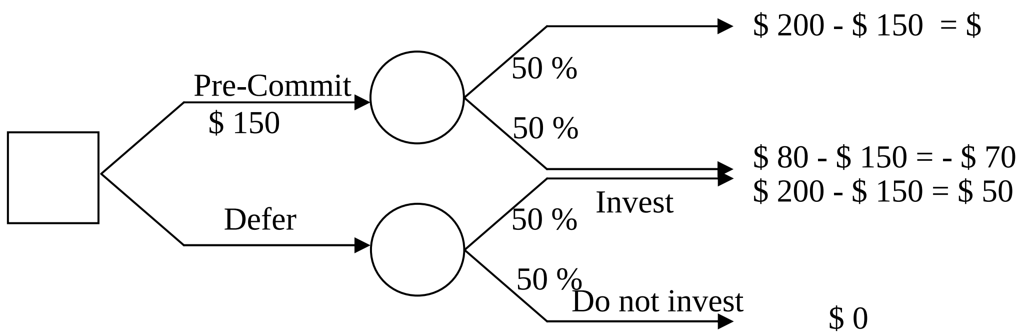


Figure 3.2 Various Alternative Investment Decision in a Project (Copeland and Antikarov, 2001)



First, as shown in Figure 3.2, if we assume pre-committing to undertake the investment with the initial cost of $150, we can consider two situations based on the future economic condition. When the market is good at year 1, the outcome will be $200 and management has $50 as cash flow at year 2. On the other hand, when the market is bad at year 1, the cash flow is only $80 at year 2 so it will result in a negative NPV of $70. In the case of pre-committing, management has no control over future outcomes. Then, the expected net payoff of this project is calculated as follows:

$$0.5(\$\,200 - \$150) + 0.5(\$80 - \$150) = -\,\$10 \tag{3.7}$$

If we assume that the risk-adjusted discount rate is 15 %, the net present value of this payoff without the right to defer will be obtained by discounting the expected cash flows at the risk-adjusted discount rate as:

$$NPV = \frac{-\,\$\,10}{(1+0.15)^1} = -\,\$\,8.7 \tag{3.8}$$

Second, unlike the traditional NPV analysis mentioned above, the decision tree allows management to evaluate the alternative of deferring the investment until the end of the period and decide on whether to invest \$150 depending on the state of nature. When considering deferral option for the project, if the economy looks good at year 1, management will decide to invest \$150 and have \$50 as cash flow at yare 2. But, if the economy does not look positive, the cash flow is only \$80 at year 2, which means that the net present value will be negative \$70. Then, management would choose not to invest in the project on this outcome and the net payoff would be \$0. So, the expected net payoff of this project will be obtained as follows:

$$0.5(\$\ 200 - \$150) + 0.5(\$0) = \$25 \tag{3.9}$$

And, the net present value of the decision with the right to defer is calculated at the risk-adjusted discount rate as follows:

$$NPV = \frac{\$25}{(1+0.15)^1} = \$\ 21.74 \tag{3.10}$$

With the decision tree approach, the project value has increased to \$21.74 with the deferral option as opposed to the \$-8.7 given the NPV without flexibility. As we can see in this example, the flexibility of the deferral option in the DTA approach can have a significantly higher value as compared to the project value given the traditional NPV. The value of deferral option from this approach is as follows:

$$\$\,21.74 - (\$ - 8.7) = \$\,30.4 \tag{3.11}$$

When the variables and the outcomes increase the discount rate for the analysis, despite its considerations of the management's flexibility, this approach is likely to seen as a very complex black box (Trigeorgis, 1999; Copeland and Antikarov, 2001). Basically, the level of uncertainty in an investment project varies due to the changes in payouts at the various points on the decision tree. In turn, this will result in modification of the risk-adjusted discount rate to reflect the related risks associated with varying conditions. Even if this means that the discount rate should be appropriately adjusted when the decision to defer and possibility to abandon the project was considered, this approach allows the use of a constant risk-adjusted discount rate throughout the project. To correctly use the decision tree approach, we need to adjust the discount rate whenever the management's flexibilities happen (Trigeorgis, 1999; Copeland and Antikarov, 2001; Neely and De Neufville, 2001). So, the use of the real option analysis has been considered to correct the deficiencies of the traditional NPV and the DTA approach (Trigeorgis, 1999; Copeland and Antikarov, 2001).

3.2 Real Option Valuation Analysis

As indicated earlier, the option pricing theory, which was developed by Black, Scholes (1973), and Merton (1973), for financial assets is the foundation of the real option theory. This theory was subsequently 'exported' to value options on real assets. The option pricing theory is based on the assumption that stock price follows a diffusion process of a geometric Brownian motion. The geometric Brownian motion process, which follows a lognormal distribution, has been proven to be appropriate for modeling the price of a limited liability security (Luenberger, 1998) and the values of the underlying assets such as an oil reserve or a start-up venture (Brennan and Schwartz, 1984; Dixit and Pindyck, 1994; Leland, 1994; Schwartz and Moon, 2000).

3.2.1 Call and Put Options in a Financial Asset

An option is a security that gives the option holder the right, but not the obligation, to buy or sell a stock at a fixed price on or before a specified date (Trigeorgis, 1999; Hull, 2002). The fixed price in option terminology is called the exercise price and the given date the maturity date. The main characteristic of the option is that the option holder has the right but not the obligation to satisfy the option contract. If the option has a negative intrinsic value when exercised, the option is said to be out of the money. If the option is out of the money, the option holder can let the option expire and only lose the money that they paid as the premium to buy the option (Cox et al., 1979). Generally, the option can be categorized based on the kinds of options, positions to the option contracts, and time to be exercised.

A call option gives the option holder the right to buy the stock at a fixed price on or before the expiration date of the option. At expiration, if the stock price is less than the exercise price, because the option holder can not take any profit from buying the stock the option is not exercised and it expires as being worthless. But, if the stock price is greater than the exercise price, the option is exercised. A put option gives the option holder the right to sell the stock at an exercise price on or before the expiration date of the option. If the stock price is greater than the exercise price, the option will not be exercised and will expire as being worthless. However, if the stock price is less than the exercise price, the put option holder will exercise the option.

There are two sides to every option contract. They are the long and short positions. A long position refers to the holding/purchasing position of the option while a short position refers to the selling/writing of the option. The option issuer receives cash up-front but has to endure the potential liabilities that can happen later. The profit loss of the issuer is the reverse of that for the option purchaser. And, based on the kinds of options and the option contract sides mentioned above, the option position can be categorized as a long position in a call option, a long position in a put option, a short position in a call option, and a short position in a put option (Hull, 2002). Under the assumptions that X is the exercise price, S_0 is the initial stock price, and S is the current stock price, when S is greater than X, in a call option, the option will be exercised. On the other hand, if X is greater than S, in a put option, the option will be exercised. So, to describe the option values according to the four option positions, they are $MAX\ [S - X, 0]$ in a long call position, $MAX\ [X - S, 0]$ in long put position, $-MAX\ [S - X, 0]$ in short call position, and $-MAX\ [X - S, 0]$ in short put position. Third, according to their time to be exercised, the financial options also fall into two different categories. If the option can be exercised at any time up to the expiration date it is called an American option and if the option can only be exercised at the expiration date it is referred as a European option. In Section 3.2.2, the similarity and the limitation in analogy between financial and real option are described.

3.2.2 Similarities and Limitations of Analogy between Financial and Real Options

The real options analysis is based on importing the concept of financial options into the real assets. An opportunity to invest in a real asset is similar to an option to invest in a financial asset. Both assets involve the right, but not the obligation, to acquire an asset by paying a certain amount of money at a specific time. As a call option gives the buyers the right, but not the obligation, to buy a security at a specific price in the future, some types of capital investments today give the investor the right in the future, but not the obligation, to make an investment. Of course, as main determinants of financial option values are also important in determining the value of real options, a close analogy can be made between the variables that determine the value of a financial option and a real option on a project investment. Even if most of all capital investments have some specific characteristics which are in common and important between both financial and real options for the analogy (Dixit and Pindyck, 1994), there are also problematic aspects to prevent the analogy between financial and real options (Kester, 1993; Trigeorgis, 1999).

3.2.2.1 Similarities between Financial and Real Options

1) Uncertainty

The most important similarity between a financial option and a firm's option to take a specific action in a specific project is that the option value is obtained under the assumption that the future value of the asset is uncertain. Here, we can see two uncertainties. One is economic uncertainty and the other is technical uncertainty. Economic uncertainty, which is caused by variables such as interest rate, inflation, industry prices, cost movements and so on, is present in the real market and can not be influenced by management decision changes, it simply follows the general trend of the economy. In order for the management to actively respond to the changes in economic variables in order to maximize the company's wealth, it will appropriately revise its decisions related to the investments and operating over time. However, we know that the management can not affect economic movements. On the other hand, technical uncertainty is limited to the project and does not follow the general trends of the economy. It is affected by management decisions as it is only possible to reduce the uncertainty in the outcomes of a R&D project with an actual staged investment strategy in which each step provides valuable information, which reduces the uncertainty.

2) Irreversibility

In general, the initial investment cost may be completely or partially irreversible in both financial and real assets and we call this investment cost a "sunk cost" (Dixit and Pindyck, 1994). An investment on a financial call option is close to a completely irreversible investment opportunity because once the investor exercises the option it will be irreversible and the investor cannot retrieve the option or the money that was paid to exercise the option. On the other hand, if an investor invests in a project that shows itself to be unfavorable, the investor will end the project and try to recover as much as possible but may not be able to recover all the initial investment cost put into the project at the outset. This is an example of partial irreversibility in a real asset.

3) Managerial Flexibilities

Another similarity between financial and capital investment opportunities is the managerial flexibility embedded in the projects. The higher the degree the of the managerial flexibility, the better and higher value of the investment opportunity, which means that embedded options can only add to the value of an investment (Sharp, 1991). In general, the managerial flexibility can fall into two types. They are internal and external flexibility. Internal flexibility is the managers' flexibility in the project itself, which is the power to modify the project as the future conditions change, involving expanding, altering, shrinking or abandoning the projects. External flexibility is the growth option, which renders it possible to be able to invest in another project that may not have been possible originally (Flatto, 1996).

4) Dividends

A final similarity in financial and real options is the way dividend payments are taken into account in the process of valuing the options. In the case of financial options, when the stock pays dividends, the option value has to be adjusted. This is because the dividend reduces the underlying asset value, stock price, and, in turn, affects the option value which is contingent on it. Likewise, in real asset if the underlying asset of a real option pays dividends, the asset value decreases by the amount of the dividends paid at the ex-dividend date (Amram and Kulatilaka, 1999). In real assets, there are some kinds of value leakages, which can be expressed as dividend payments. These will cause the reduction of option value contingent on the underlying asset value and the timing of the optimal investment decision. Adjustment for these value leakages should be made while valuing a real option of a dividend-paying asset in order to consider the impact of the dividend payments on the option value.

3.2.2.2 Limitations of Analogy between Financial and Real Options

Despite the similarities mentioned above that can help apply the concept of a financial option into a real option in valuing the option, there still exist some questionable issues that make the analogy limited. These follow three potential factors which render real options different from financial options (Kester, 1993; Trigeorgis, 1999).

1) Simple or Compound

The financial option is called “simple” because the option value is derived only from the underlying asset which depends on the value of the underlying asset at the maturity date (Trigeorgis, 1999). Likewise, some real options are “simple” because their values are determined based on the value of the underlying asset of the project (Kester, 1984). However, when a real option gives the holder the right not only to receive the underlying asset but also to receive further investment opportunities in the future, the real option will be more complicated. We call this "compound option" or "the options on options," whose payoff is another option (Trigeorgis, 1999; Copeland and Antikarov, 2001). As theses options can not be seen as individual options and as such can not be added together, the fact that the calculation becomes more complicated and cumbersome rather than in a simple option is the main problem with these options (Trigeorgis, 1993b). The investments in a lease on an undeveloped tract with potential oil reserves or R&D projects can be the examples of these types of options.

2) Proprietary or Shared

A financial option is proprietary because it gives its owner the exclusive right of when and whether to exercise it. In general, the real option holder will do so when a company has a unique know-how or a patent right which is the proprietary characteristic since the company has the exclusive right to exercise these options. However, the limitation of the real option in this analogy is the fact that some investment opportunities are not proprietary if 1) they are jointly held by more than a single competitor (Smit and Ankum, 1993) or 2) if the choice to introduce a new product is not protected from a possible introduction of close substitutes by a competitor or 3) there is the chance to enter a new market with no barriers to competitive entry. These kinds of real options are called "shared real options" since they could be exercised by any of the participants (Trigeorgis, 1999). As a result, all things being equal, shared real options are less attractive than proprietary ones because competitors can make counter investments that can erode the profitability of a company (Kester, 1984; Smit and Ankum, 1993).

3) Non- tradeability

Financial options which are traded in a security market allow easy determination of their parameters while real options like most investment projects do not because real options are generally not tradable. In a financial market, a security price is observable and, in turn, the variance of its rate of return can be easily obtained from historical data or the implied variance of similar options on the same security (Copeland and Antikarov, 2001). On the other hand, with shared real options, as they are a public good for the whole industry they may not be tradable on the market (Smit and Ankum, 1993), except for some proprietary real options, such as investment opportunities related to licensing agreements and patents, which may be traded but at a expensive cost in imperfect markets (Trigeorgis, 1999). Furthermore, some projects are very unique and do not have any historical data available. These characteristics make the real option difficult to estimate with respect to the variance of the rate of return of the underlying asset (Copeland and Antikarov, 2001). Therefore, sometimes management uses the subjective estimates to solve this problem. Table 3.1 is the comparison of variables used to value financial and real options (Trigeorgis, 1999).

Table 3.1 Comparison of Variables Used to Value Financial and Real Options (Trigeorgis, 1999)

Financial Options	*Real Options*
Current Value of Stock (S)	*Present Value of Expected Cash Flows* (V)
Exercise Price (X)	*Investment Cost* (I)
Time to Maturity (T)	*Time until Opportunities Disappear* (T)
Stock Value Uncertainty (σ)	*Project Value Uncertainty* (σ)
Risk-Free Interest Rate (r)	*Risk-Free Interest Rate* (r)
Dividend (d)	*Free Cash Flows* Paid Out by the Underlying Asset* (q)

* Defined as the per-period cash inflows less its corresponding cash outflows, excluding the initial cost of the investments that leave the business.

In next Section, there will be descriptions of the basic assumptions for two major real option theories; the Black-Scholes model and the binomial models, to support them and of the derivations of those models with underlying concepts.

3.2.3 Basic Assumptions for Black-Scholes and Binomial Models

The Black-Scholes and binomial models are two major elements in the real option valuation method, which are most widely used to price the options in option pricing theories. This Section describes the basic assumptions for the applications of the Black-Scholes and binomial models. Even if many assumptions look restrictive, the Black-Scholes and binomial models have proven to be two rigorous and solid methods that produce correct prices for financial options. We can easily find out that such assumptions as that non-arbitrage opportunities and stock price behavior can be applied well in financial assets, for which the historical and observable market data are readily available.

- Stock price and option price depends on the same underlying uncertainties. Moreover, stocks and options are traded in a perfect market which has characteristics as follows.

- ✓ *Risk-free asset exists in the market and this market is perfectly competitive*
- ✓ *It operates in equilibrium and there should be infinitely divisible securities*
- ✓ *Individuals have equal access to the capital market*
- ✓ *There are no transaction costs or taxes and short-selling is allowed*

- There should not be any arbitrage opportunities which benefits from price differences in different markets. If an asset is bought in one market and sold immediately at a higher price in another market, the investor can make a risk-free profit without investing anything. So, the assumption is that in a competitive and well-developed market, if arbitrage opportunities exist, the law of supply and demand will immediately force the two asset prices to be equal (Baxter and Rennie, 1996). Therefore, a portfolio of the stock and the option with a same risk profile can be set up so that the payoffs of the option exactly replicate the payoffs of the stock, hence the stock and the option should be traded at the same price. As there is no risk involved in creating this kind of portfolio, the investor's risk attitude does not have to be considered in pricing the option. We call this the "Non-arbitrage" pricing concept which helps use a risk-free rate to discount future cash flows and ignore the investor's risk attitude.

- The stock price is continuous. Even if it looks unrealistic since the trading can not be continuous, the Black-Scholes model, which is called as a continuous-time model, performs well in the real world where stocks trade only intermittently with price jumps.

- Based on Samuelson (1965), even though there may be a known seasonal pattern in the current price of a commodity the future price will fluctuate randomly, stock prices fluctuate randomly in a complete and efficient market. Therefore, in a short period of time, stock price jump is formulated by a lognormal distribution, which means that the logarithmic value of the rate of return follows a normal distribution. Changes in the magnitude of the jump are described by a geometric Brownian motion process, where the logarithm value of the underlying asset follows a generalized Wiener process (Hull, 2002).

- In the binomial model, there are some assumptions for the asset price evolution as follows.

- ✓ *Each state leads to two other states over a time step*
- ✓ *The paths to a state are independent of each other*
- ✓ *The intermediate branches are all recombining*
- ✓ *The price evolution is stationary over time*

3.2.4 Black, Scholes, and Merton's Option Pricing Model

3.2.4.1 Stochastic Processes

The Black-Scholes' option pricing theory, based on the assumption that stock prices follow diffusion processes, models the option prices as continuous-time stochastic processes by applying *Itô*'s lemma, which is considered as a fundamental theorem of stochastic calculus and used to determine the differential of a function of a diffusion process when being used in dynamic models in finance (Pennacchi, 1997). A pure Brownian motion or a Wiener process, is the basic stochastic building block for the general continuous-time stochastic processes and can be called diffusion processes including an arithmetic Brownian motion and a geometric Brownian motion.

1) Pure Brownian Motion Process

If we consider the stochastic process of $z(t)$, the change in $z(t)$ over Δt is,

$$z(t+\Delta t) - z(t) \equiv \sqrt{\Delta t} \cdot \tilde{\varepsilon} \tag{3.12}$$

where $E(\tilde{\varepsilon})=0$ and $Var[\tilde{\varepsilon}]=1$. Under the assumption that T is composed of N intervals multiplied by the length Δt,

$$z(t) - z(0) = \sum_{i=1}^{N} \Delta z_i \equiv \sum_{i=1}^{N} \sqrt{\Delta t} \cdot \tilde{\varepsilon}_i \tag{3.13}$$

Here,

$$E[z(t) - z(0)] = \sqrt{\Delta t} \sum_{i=1}^{N} E[\tilde{\varepsilon}_i] = 0 \tag{3.14}$$

$$Var[z(T) - z(0)] = (\sqrt{\Delta t})^2 \sum_{i=1}^{N} Var[\tilde{\varepsilon}_i] = \Delta t \cdot N \cdot 1 = T \tag{3.15}$$

So, the mean of $z(t) - z(0)$ is 0 and the variance is T. Based on the Central Limit Theorem, $z(t) - z(0)$ follows a normal distribution with mean 0 and variance T as N increases to the unlimited. Here, we can assume that each $\tilde{\varepsilon}_i$ has a standard normal distribution and define

$$dz(t) \equiv \lim_{\Delta t \to 0} \Delta z = \lim_{\Delta t \to 0} \sqrt{\Delta t}\, \tilde{\varepsilon} \tag{3.16}$$

Here, dz is called as a "pure Brownian motion" or a "Wiener process" which follows normal distribution with a mean of 0 and a variance of dt. Then, the result of $z(t) - z(0)$ is obtained as a distribution from the stochastic integral as follows,

$$z(t) - z(0) = \int_0^T dz(t) \sim N(0, T) \tag{3.17}$$

2) Diffusion Processes

A pure Brownian motion process such as an arithmetic Brownian motion or a geometric Brownian motion process can help generalize the diffusion processes. At first, in the arithmetic Brownian motion process with a non-zero drift and any desired volatility. If we consider a new process $x(t)$, which is distributed as a normal distribution with mean 0 and variance of $\sigma^2 T$, the distribution is obtained as follows,

$$dx(t) = \sigma \cdot dz(t) \tag{3.18}$$

$$\int_0^T dx = x(T) - x(0) = \int_0^T \sigma \cdot dz(t) = \sigma \int_0^T dz(t) \sim N(0, \sigma^2 T) \qquad (3.19)$$

Here, arithmetic Brownian motion process $x(t)$ is defined as

$$dx = \mu\, dt + \sigma\, dz \qquad (3.20)$$

The above Equation, where μ is deterministic change per unit time to the $x(t)$ process, follows that the arithmetic Brownian motion process is distributed as follows:

$$\int_0^T dx = x(T) - x(0) = \int_0^T \mu \cdot dt + \int_0^T \sigma \cdot dz(t) = \mu T + \sigma \int_0^T dz(t) \sim N(\mu T, \sigma^2 T)$$

(3.21)

As μ and σ are functions of time or $x(t)$, then the arithmetic Brownian motion's Stochastic Differential Equation (SED) will be as follows:

$$dx(t) = \mu[x(t), t]dt + \sigma[x(t), t]dz \qquad (3.22)$$

In the above Equation, $dx(t)$ is described as being instantaneously normally distributed with mean $\mu[x(t), t]dt$ and variance $\sigma[x(t), t]dz$. Second, the geometric Brownian motion process is defined as:

$$dx = \mu\, x\, dt + \sigma\, x\, dz \qquad (3.23)$$

or

$$\frac{dx}{x} = \mu\, dt + \sigma\, dz \qquad (3.24)$$

where μ and σ are constants. The geometric Brownian motion process has been proven to follow a lognormal distribution, which is appropriate for modeling the price of a limited-liability security such as a common stock.

3) Itô 's lemma

Itô 's lemma has been thought to give us a clear rule to find the differential of a function of one or more variables, some of which follow a diffusion process. If we assume that the function of $F(x(t),t)$ is governed by the variable of $x(t)$, which follows a diffusion process, because $x(t)$ follows $dx = \mu[x, t]dt + \sigma[x, t]dz$, the differential of $F(x(t),t)$ will be as follow:

$$dF(x(t), t) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2 \tag{3.25}$$

As $dx = \mu[x, t]dt + \sigma[x, t]dz$ and $(dx)^2 = \sigma^2 dt$, if we substitute these for dx and $(dx)^2$, Equation (3.25) will be as follow:

$$dF(x(t), t) = \frac{\partial F}{\partial x}\left(\mu[x,t]dt + \sigma[x,t]dz\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\sigma[x,t]^2 dt$$

$$= \left(\frac{\partial F}{\partial x}\mu[x,t] + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\sigma[x,t]^2\right)dt + \frac{\partial F}{\partial x}\sigma[x,t]\,dz \tag{3.26}$$

In general, because the function of $F(x_1, x_2, x_3, ...x_m, t)$ is at least a twice differentiable, based on the *Itô* 's lemma, the generalized form of the differential Equation will be as follow:

$$dF(x_1,\ x_2,\ x_3,.....,x_m,\ t) = \sum_{i=1}^{m} \frac{\partial F}{\partial x_i} dx_i + \frac{\partial F}{\partial t} dt + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 F}{\partial x_i \partial x_j} dx_i\, dx_j \tag{3.27}$$

Here, because of $dx_i\, dx_j = \sigma_i\, \sigma_j\, dt$, Equation (3.27) can be rewritten as

$$dF(x_1,\ x_2,\ x_3,...,x_m,t) = \left[\sum_{i=1}^{m} \frac{\partial F}{\partial x_i} \mu_i[x,t] + \frac{1}{2} \frac{\partial^2 F}{\partial x_i{}^2} \sigma_i[x,t]^2 + \frac{\partial F}{\partial t} + \sum_{i=1}^{m} \sum_{j>1}^{m} \frac{\partial^2 F}{\partial x_i \partial x_j} \right] dt$$

$$+ \sum_{i=1}^{m} \frac{\partial F}{\partial x_i} \sigma_i[x,t] dz_i \tag{3.28}$$

In the real world, $F(x,t)$ can be a project valuation function or a profit function and can be analyzed using $It\hat{o}$'s lemma. As the non-zero drift, μ, depicts the trend of the growing demand and σ of the volatility, the uncertainty of the risks, in a geometric Brownian motion, during the concession period in an infrastructure project, the project's demand trend and uncertainty can be modeled with this diffusion process as shown in Equation (3.24).

3.2.4.2 Black, Scholes, and Merton's Option Pricing Model

The option pricing theory (Black and Scholes 1973; Merton, 1973), which recognizes the interactions among an asset's uncertainty, the option holder's behaviors to maximize the profit, and market disciplines, is the foundations of the modern dynamic asset pricing theories in light of overcoming difficulties in discounting approach and more realistically computing the value of an investment. The most remarkable feature of the option pricing theory is the fact that the price can be solved independently regardless of investor's risk attitude.

This Section includes the description of the derivation of the Black-Scholes model concerning a European call option based on the geometric Brownian motion process. When we consider that the stock price follows a geometric Brownian motion process, we see that:

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz \tag{3.29}$$

where S is the stock price, μ is the instantaneous rate of return, and σ^2 is the instantaneous variance of rate of return. If we assume that the value of a European call option is $dF(S,t)$ on this stock, which is governed by the variables of $S(t)$ and t. According to $It\hat{o}$'s lemma,

$$dF(S,t) = \left[\frac{\partial F}{\partial S}\mu S + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2\right]dt + \frac{\partial F}{\partial S}\sigma S\, dz \tag{3.30}$$

As long as the market is ideal that the short-term interest rate is known and constant, there are no transaction costs, the market is frictionless, and the stock pays no dividends or other distributions, the return on the portfolio becomes certain by forming a hedging portfolio and continuously adjusting the portfolio (Black and Scholes, 1973). Therefore, the return of this portfolio is instantaneously risk-less and the rate of return must be risk-free under the non-arbitrage condition. When we consider the value of the hedge portfolio, $W(t)$, at time t with one option short and $\frac{\partial F}{\partial S}$ shares of stock long,

$$W(t) = -F(S,t) + \left(\frac{\partial F}{\partial S}\right)S \tag{3.31}$$

Then, the instantaneous change of this hedge portfolio is

$$dW(t) = -dF(S,t) + \left(\frac{\partial F}{\partial S}\right) dS$$

$$= -\left[\frac{\partial F}{\partial S}\mu S + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2\right] dt - \frac{\partial F}{\partial S}\sigma S\, dz + \left[\left(\frac{\partial F}{\partial S}\right)\mu S\, dt + \left(\frac{\partial F}{\partial S}\right)\sigma S\, dz\right]$$

$$= -\left[\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2\right] dt \tag{3.32}$$

In Equation (3.32), $dW(t)$ is certain with the uncertain term being dropped out and because the risk-less return under continuous hedging is equal to the risk-free return (Black and Scholes, 1973), we will obtain the following Equation.

$$dW(t) = r\, W(t)\, dt = r\left[-F + \left(\frac{\partial F}{\partial S}\right) S\right] dt \tag{3.33}$$

where r is the risk-free rate which is assumed to be constant.

To solve the price that is consistent with capital market in Equation (3.33), we takes into account non-arbitrage opportunity that, when the asset is mis-priced, the arbitrage transactions will adjust the prices until the market reaches the equilibrium and there are non-arbitrage opportunities. So, with Equation (3.32) and (3.33), we will find following relationship.

$$-\left[\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2\right] dt = r\left[-F + \left(\frac{\partial F}{\partial S}\right) S\right] dt \tag{3.34}$$

Through this process, we have the Black and Scholes partial differential Equation (PDE) by making Equation (3.34) look simple.

$$\frac{\partial F}{\partial t}+\frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2+rS\left(\frac{\partial F}{\partial S}\right)-rF = 0 \tag{3.35}$$

or

$$F_t+\frac{1}{2}\sigma^2 S^2 \; F_{SS} + r\,S\,F_S - r\,F = 0 \tag{3.36}$$

Finally, as for the European call option, the option value will be calculated in Equation (3.35) or (3.36) with three terminal and boundary conditions. First, the terminal condition based on the option value when at maturity date T is as follows.

$$F\,(\,S(T),\; T\,) = S\,(T) - X\,, \qquad S(T) \geq X \tag{3.37}$$

$$F\,(\,S(T),\; T\,) = 0, \qquad\qquad S(T) < X \tag{3.38}$$

where X is the exercise price. This condition can also be combined as

$$F\,(\,S(T),\; T\,) = MAX\;[\;0,\; S(T) - X\;] \tag{3.39}$$

Second, two boundary conditions are

$$F\,(\,0\;,\,t\,) = 0 \tag{3.40}$$

$$F\,(\,S\;,\,t\,) = S \qquad\qquad S \to \infty \tag{3.41}$$

By calculating the Black-Scholes' PDE with these terminal and boundary conditions, we finally obtain the exact and unique solution which is also expressed as the option valuation formula as follow:

$$F\,(\,S(t),\,t\,) = S(t)\,N(d_1) - \,X\,e^{-r(T-t)}N(d_2) \tag{3.42}$$

Where

$$d_1 = \frac{\ln\left(\dfrac{S(t)}{X}\right) + \left(r + \dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\,\sqrt{T-t}} \tag{3.43}$$

and

$$d_2 = d_1 - \sigma\,\sqrt{T-t} \tag{3.44}$$

$N(d_1)$, which is the hedging ratio of shares of stock to options, $\dfrac{\partial F}{\partial S}$, is the cumulative normal density function and $N(d_2)$ is the probability that the option will finish in-the-money (Copeland and Weston, 1988).

3.2.4.3 "Non-Arbitrage" Principle and Risk-Neutral Valuation Solutions

$$\frac{1}{2}\sigma^2(S,t)\;F_{SS} + (rS - b(S,t))F_S - r\,F + F_t = 0 \tag{3.45}$$

Equation (3.45) is a generalized form of the option price by Cox and Ross (1976). Here, $b(S,t)$ is the continuous pay out stream of the underlying asset of S. And, S is governed by

$$dS = \mu\,(S,t)\,dt + \sigma\,(S,t)\;dz \tag{3.46}$$

In Equation (3.45), when $\mu(S,t) = \mu S$, $\sigma(S,t) = \sigma S$ and $b(S,t) = 0$, the Equation (3.45) becomes Equation (3.35) or (3.36). In Equation (3.35) or (3.36) and Equation (3.45), if we use the risk-free interest rate, r , as the underlying asset's drift instead of μ , that is, if the drift term μ in Equation (3.46) is replaced by r , the solution will not be affected. Then, it becomes possible to pretend that the world is risk neutral through the assumption that S evolves with drift r not with μ , in turn, calculate the contingent claim solution consistent with the real world.

We call this technique "risk neutral valuation." According to this technique, the fact that the option price is obtained by creating a risk-free hedge portfolio help the contingent claim not rely on the investor's risk attitudes (Cox and Ross, 1976). Based on this, the investor's risk preferences/attitudes become risk neutral and finally we have the evidence to use the risk-free returns as the expected returns on both the underlying assets and the contingent claims. This risk-neutral valuation concept, which has been considered to be powerful in numerical computation methods, has an interpretation that, by assuming the world is risk neutral and taking the expectation of the contingent claim's payoff and discounting at risk-free rate, we can compute the value of a contingent claim.

3.2.5 Binomial Model

The binomial option pricing model, which is derived by Cox, Ross, and Rubinstein (1979), has been regarded as the simplest option pricing approach (Elton and Gruber, 1995). They use probability theory to develop a binomial lattice approach to option pricing that applies discrete mathematics. The binomial option valuation model is based on a simple representation of the up and down movements of the value of the underlying asset and is possible to price both European and American options. This model disassembles the life time of option into a large number of time steps then the binomial tree of the underlying asset is produced working forward from the present to expiration. At every time step the value of the underlying asset is assumed to move up or down by an amount calculated using the volatility and the time interval of the binomial. Of course, the option pricing theory, which assumes that the stock price follows a geometric Brownian motion process (Black and Scholes, 1973; Merton, 1973), is the building block of the binomial model as following formula.

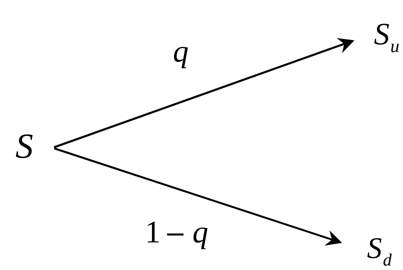
$$\frac{dS}{S} = \mu \; dt + \sigma \; dz \tag{3.47}$$

where S is the stock price, μ is the instantaneous rate of return, σ is the instantaneous standard deviation of the rate of return, and dz is a random increment to a standard Wiener process. The following section is the description of the detailed derivation of the binomial model based on Cox, Ross, and Rubinstein (1979).

3.2.5.1 Derivation of the Single-period Binomial Model

To drive the binomial option pricing theory, it is necessary to assume that the value of the underlying asset, stock price, follows a multiplicative binomial process over times (Cox, Ross, and Rubinstein, 1979). And, the change of the underlying asset's value over time steps has two possibilities of going up or down with specific probabilities.

Figure 3.3 Single-period Stock Price Movement (Cox et al., 1979)



First of all, as shown in Figure 3.3 we need to assume that the current asset price, S, can either go up to S_u or down to S_d at next time step and the possibilities of up and down movements are q and $(1-q)$ respectively. Then, under the assumptions that the interest rate is constant, individuals may borrow or lend as much as they wish at this rate, and there is no taxes, no transaction costs, or margin requirements, individuals can sell short any security and receive full use of the proceeds, we can think of the value of a call option, C, on the underlying asset, where C_u and C_d are the values of the call option when the stock price is S_u and S_d respectively, and X is the exercise price in Figure 3.4.

Figure 3.4 Single-period Movement of Call and Put Option Values (Cox et al., 1979)

$$\begin{tikzcd} & C_u = Max[S_u - X, 0] \\ C \arrow[ur, "q"] \arrow[dr, "1-q"'] & \\ & C_d = Max[S_d - X, 0] \end{tikzcd}$$

$$\begin{tikzcd} & P_u = Max[X - S_u, 0] \\ P \arrow[ur, "q"] \arrow[dr, "1-q"'] & \\ & P_d = Max[X - S_d, 0] \end{tikzcd}$$

Here, we assume creating an equivalent portfolio, which is risk hedging, by buying a particular number of shares of the underlying asset and borrowing against them an appropriate amount at the risk free rate that would exactly replicate the future returns of the options in any state of nature. If we assume that this risk hedge portfolio is formed with Δ shares of stocks and the dollar amount B in risk-free bonds, the cost of this portfolio is $\Delta S + B$. And, at next time step the values of this portfolio will be as follows for the up and down statuses of value movements respectively.

$$\Delta S_u + (1+r)\ B \tag{3.48}$$

$$\Delta S_d + (1+r)\ B \tag{3.49}$$

In this risk hedging portfolio, whether the stock price moves up or down, to avoid risk free arbitrage profit opportunities, the option and the portfolio will provide the same future returns. This means that they must sell for the same current price.

$$\Delta S + B = C \tag{3.50}$$

$$\Delta S_u + (1+r)\ B = C_u \tag{3.51}$$

$$\Delta S_d + (1+r)\ B = C_d \tag{3.52}$$

In Equation (3.51) and (3.52), if we find the value of Δ and B with the condition of $C_u = C_d$, the results is the following.

$$\Delta = \frac{C_u - C_d}{S\,(u-d)} \tag{3.53}$$

$$B = \frac{u\,C_d - d\,C_u}{(u-d)\,(1+r)} \tag{3.54}$$

Finally, the number of the stock Δ and the loan B obtained from above processes help calculate the value of the option at time t. If these are to be no risk free arbitrage opportunities, the current value of the call option, C, can not be less than the current value of the replicating portfolio, $\Delta S + B$. Therefore, $C = \Delta S + B$ is true if there are to be no risk free arbitrage opportunities. So, by substituting the values Δ and B obtained from above processes into $C = \Delta S + B$, we can find the value of the call option as follows.

$$
\begin{aligned}
C &= \frac{C_u - C_d}{S(u-d)} \cdot S + \frac{u\,C_d - d\,C_u}{(u-d)(1+r)} \\
&= \frac{C_u - C_d}{(u-d)} + \frac{u\,C_d - d\,C_u}{(u-d)(1+r)} \qquad (3.55) \\
&= \frac{\left[\left(\dfrac{(1+r)-d}{u-d}\right)C_u + \left(\dfrac{u-(1+r)}{u-d}\right)C_d\right]}{(1+r)}
\end{aligned}
$$

To make the above Equation simple, we assume the values of q and *1-q* which are the risk neutral probabilities and dependent on the magnitude of the up/down movements and risk-free rate.

$$q = \frac{(1+r)-d}{u-d} \qquad (3.56)$$

$$1-q = \frac{u-(1+r)}{u-d} \qquad (3.57)$$

So, the Equation (3.55) becomes the following Equation for the value of a call option over one period:

$$Call\ \ Option\ \ Value,\ \ C = \frac{q\,C_u + (1-q)\,C_d}{(1+r)} \tag{3.58}$$

And, with the same process as a call option, the put option value over one period can be obtained as follows.

$$Put\ \ Option\ \ Value,\ \ P\ =\ \frac{q\,P_u + (1-q)\,P_d}{(1+r)} \tag{3.59}$$

Here, we need to keep in mind that Equation (3.55) is a discrete approximation and can become a continuous approximation when the $(1+r)$ is replaced with the exponential of the risk free rate $e^{r\Delta t}$. As we know, Δt is the time interval and n is the number of intervals per year. Here, if n is 1 and Δt is 1, Δt times n becomes 1. From now on, all the Equations written in this research will be expressed as continuous approximation form. In Equation (3.58) and (3.59), we can find out that the option value at each node in the binomial tree is the same as the expected option value which is discounted at the risk free rate in the next time step. Here, there are some remarkable points that we have to think about. The one is that the actual probability disappear in that Equation because the actual probability distribution is incorporated into the stock price and, therefore, already in that option Equation. Practically, this fact helps the model be independent on investors who have different subjective probabilities about up and down movements in the stock. The other is that we do not have to take into account investor's risk attitudes in valuing the call or put option based on the assumption that the individuals prefers more wealth to less and therefore has the incentive to take advantage of profitable risk-free arbitrage opportunities. As a result, it is not dependent on the random prices of other securities or portfolios then what we have to consider is just the stock price which is the random variable on which the call or put option value depends (Cox et al., 1979).

3.2.5.2 Binomial Model and Risk-Neutral Valuation

The Equation (3.58) and (3.59) have a simple interpretation consistent with the risk-neutral valuation introduced in Section 3.2.3 and Section 3.2.4.3 by pretending the world is risk neutral, the option price can be computed by taking the expectation of the contingent payoff and discounting the expectation at a risk-free rate. This option price will be valid for the world of risk averse. To see how Equation (3.58) and (3.59) pretends the world is risk neutral, if we use the risk neutral probability q to compute the expected price of the stock after a time increment, Δt, where q is obtained by no-arbitrage argument. The expected stock price under probability q would be

$$\begin{aligned} E(S_{t+\Delta t}) &= q\,S_u + (1-q)\,S_d \\ &= \left(\frac{R-d}{u-d}\right) u\,S + \left(\frac{u-R}{u-d}\right) d\,S \\ &= \frac{R\,S\,(u-d)}{u-d} = e^{r\,\Delta t}\,S \end{aligned} \tag{3.60}$$

Equation (3.60) shows that by using q, the expected rate of return of the stock is a risk-free rate; that is, the stock behaves as if it were in the risk-neutral world. In other worlds, using q for stock dynamics is equivalent to pretending the world is risk neutral. This is why q is also named as a risk-neutral probability.

3.2.5.3 Risk-Neutral Stock Dynamics and Values of u and d

Under the risk-neutral valuation framework, u and d can be obtained to compute $q = R - d / u - d$. Suitable values for u, d, and q could be found by matching both the mean and variance of the logarithm of a price change (Luenberger, 1998). Assuming that $S_0 = 1$, the matching gives us:

$$E(S_{t+\Delta t}) \;=\; qS_u \;+\; (1-q)\,S_d \tag{3.61}$$

$$\longrightarrow \quad E(\ln S_{t+\Delta t}) \;=\; q\ln u + (1-q)\ln\, d \tag{3.62}$$

$$\begin{aligned}\operatorname{var}(\ln S_{t+\Delta t}) \;&=\; q(\ln u)^2 + (1-q)(\ln\, d)^2 - (q\ln u + (1-q)\ln d)^2 \\ &=\; q(1-q)(\ln u - \ln d)^2\end{aligned} \tag{3.63}$$

According to risk-neutral valuation, it is assumed that the stock dynamics is $\frac{dS}{S} = rdt + \sigma\, dz$. Applying introduced *Itô* 's lemma in Section 3.2.3 to the stock dynamics, Equation (3.64) can be obtained

$$d\ln S = \left(r - \frac{1}{2}\sigma^2\right)dt + \sigma\, dz \tag{3.64}$$

Equation (3.64) implies that changes of S are normally distributed with mean $\left(r - \frac{1}{2}\sigma^2\right)dt$, and variance, $\sigma^2\, dt$. Converting the mean and variance to their discrete-time forms, Equations (3.61), (3.62), and (3.63) can be written as follows:

$$qU + (1-q)\,D = \left(r - \frac{1}{2}\sigma^2\right)\Delta t \tag{3.65}$$

$$q\,(1-q)(U-D)^2 = \sigma^2\,\Delta t \tag{3.66}$$

where $U = \ln u$ and $D = \ln d$. By imposing another condition for convenience, $u = 1/d$, that is, $U = -D$, we can solve the Equation (3.65) and (3.66). In Equation

(3.67) and (3.68), higher order terms than Δt are ignored and the results are as follows (Cox et al. 1979):

$$u = e^{\sigma\sqrt{\Delta t}} \tag{3.67}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{3.68}$$

However, in this research, we follow Hull (1997) and obtain u and d after imposing the probability value $q = 0.5$ in Equation (3.65) and (3.66). In this case, u and d can be as follows:

$$u = e^{\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}} \tag{3.69}$$

$$d = e^{\left(r - \frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}} \tag{3.70}$$

3.2.5.4 Multi-Period Binomial Model

The Equation for pricing the call and put options in the multi-period of time to maturity can be simply obtained by extending the single-period of time binomial option pricing model derived above. If we replace the initial underlying asset S_0 in financial market with the initial underlying asset V_I in real assets, Figure 3.5 shows the three-step binomial tree for the project value, V. Figure 3.6 is the description of the three-step binomial tree for the call and put option values for three periods. The tree represents a forward calculation of all the possible values that the asset could take in the future during the life of the option.

Figure 3.5 Three-step Binomial Tree for Project Value

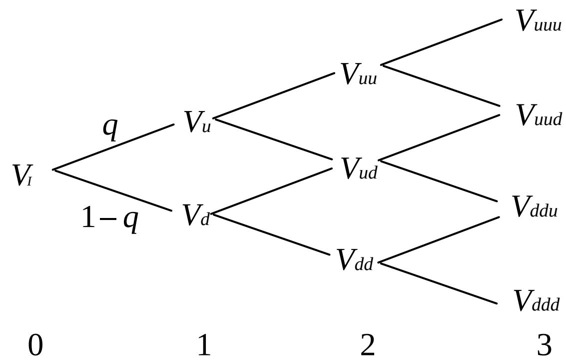
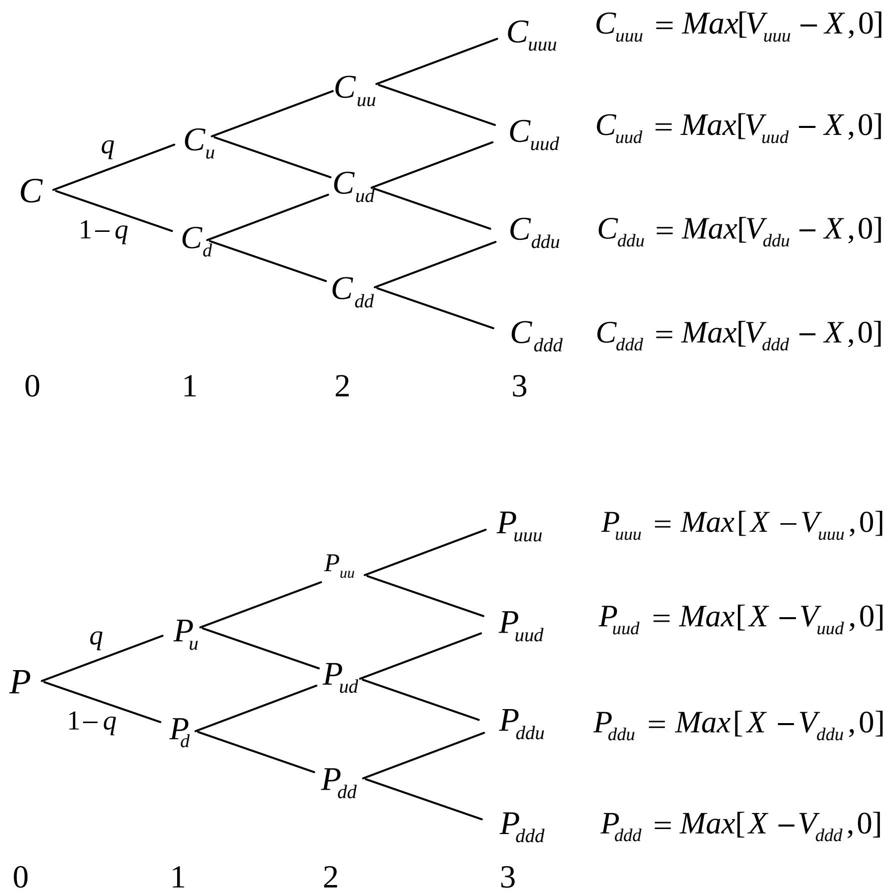


Figure 3.6 Three-step Binomial Tree for Call and Put Option Values



In Figure 3.5, starting at the first node at $t = 0$, the initial value of the project is V_I. At the next period, the value could move up to V_u which is calculated as V times the up movement ratio, u, or down to V_d with a value calculated as V times the down movement ratio, d. The initial value of the underlying asset, V, will move either up by u with risk neutral probability q to V_u or down by d with a complementary probability $1-q$ to V_d at the end of the first step. Here u and d are multiplicative factors, which are called “up” and “down” movements, to express the value change of underlying asset at next step. By following the same process, the value of the project in the next stage will be V_{uu} (V times u squared), V_{ud} (V times u times d), and V_{dd} (V times d squared). This forward calculation of the project value will be repeated for every node in every time step until the project values at the end nodes reach V_{uuu}, V_{uud}, V_{udd} and V_{ddd}. Here, as mentioned earlier, the values of u and d are Equation (3.71) and (3.72).

$$u = e^{\sigma\sqrt{\Delta t}} \tag{3.71}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \tag{3.72}$$

where σ is the standard deviation of the rate of return in the project value and Δt is the time interval of the binomial tree. In the binomial model, by assuming the future values of the underlying asset follow a multiplicative binomial distribution over discrete periods, the underlying asset’s values at the branches of the tree have the range from “0” at the bottom to infinity as the number of time step grows (Copeland and Antikarov, 2001). And, as the number of time step increases the probability distribution of the underlying asset value follows the lognormal distribution. Then, this binomial tree help take into account the uncertainty in the project value by showing all the possibilities that the underlying asset value could be during the life of the option over time. At the end of the binomial tree, as all the final option values for each of the final possible values are known, the option values at each time step will be calculated backward recursively from at the end of the binomial tree to the starting point. Where, every option value at each time step will be used to calculate the option value at time "0." When all the possible values of the project have been determined in the binomial tree, the option values on the project will be calculated backward recursively from the end nodes of the tree to the starting point. The calculation procedure is to start at the end of the tree at $t = 3$ and calculate the value of the option at the previous time period $t = 2$ and, in turn, $t = 1$. Finally, it end up with a final value of the call option at $t = 0$. In Figure 3.6, the values of a call and put options in the uppermost node (V_{uuu}) at the end of the binomial tree ($t = 3$) are as follows respectively:

$$C_{uuu} = Max\,[V_{uuu} - X,\, 0] \tag{3.73}$$

$$P_{uuu} = Max\,[\,X - V_{uuu},\; 0\,] \tag{3.74}$$

where X is the exercise price. In a European option, the process is repeated until all the option values are determined at the end nodes while in an American option whether to exercise the option can be considered at every point at every time step. As shown in Figure 3.6, in case of the call option, it will be exercised if the project value is greater than the exercise price. As opposed to the call option, put option will be exercised if the exercise price is greater than the value of the project.

The next step is to move back into the previous period ($t = 2$) to determine the decision of if management will take at that node. The value of call option C_{uu} at this node is calculated as follows:

$$C_{uu} = Max\,[\ \frac{qC_{uuu} + (1-q)\,C_{uud}}{e^{r\Delta t}}\ ,\ V_{uu} - X\ \ ]$$

$$= Max\left[\frac{qMax\,[V_{uuu} - X\,,\,0] + (1-q)\,Max\,[V_{uud} - X\,,\,0]}{e^{r\Delta t}}\ ,\ V_{uu} - X\ \right]$$

(3.75)

In Equation (3.75), $qC_{uuu} + (1-q)\,C_{uud}\,/\,e^{r\Delta t}$ is the present value of the option if it is kept alive. But the second term $V_{uu} - X$ is the option value if exercised. Therefore, this Equation can be interpreted that the management would choose between the option value if exercised or its present value if kept alive at that node, whichever is larger. As a result, if the present value of the option if being kept alive is larger that the option value if exercised, the option will be kept alive and will not be exercised at this node. This shows that the option will be exercised only when its exercised value is greater than its value if kept alive. This calculation process will be continued backward recursively at every rest of the nodes from the end of the binomial tree to the starting point and the final option value C at time "0" will be the result that we want to have. The followings are the other call option values at second time step ($t = 2$).

$$C_{ud} = Max\left[\frac{qC_{udu} + (1-q)\,C_{udd}}{e^{r\Delta t}}\ ,\ V_{ud} - X\right]$$

$$= Max\left[\frac{qMax\,[V_{udu} - X\,,\,0] + (1-q)\,Max\,[V_{udd} - X\,,\,0]}{e^{r\Delta t}}\ ,\ V_{ud} - X\ \right]$$

(3.76)

$$C_{dd} = Max\left[\frac{qC_{ddu} + (1-q)\,C_{ddd}}{e^{r\Delta t}}\ ,\ V_{dd} - X\right]$$

$$= Max\left[\frac{qMax\,[V_{ddu} - X\,,\,0] + (1-q)\,Max\,[V_{ddd} - X\,,\,0]}{e^{r\Delta t}}\ ,\ V_{dd} - X\right]$$

(3.77)

Based on the Equation (3.55), as the call option values of C_u and C_d depend on the value of C_{uu}, C_{ud}, and C_{dd} at second time step of $t = 2$, the values of C_u and C_d are given as follows.

$$C_u = Max\left[\frac{qC_{uu} + (1-q)\,C_{ud}}{e^{r\Delta t}}\ ,\ V_u - X\right] \tag{3.78}$$

$$C_d = Max\left[\frac{qC_{du} + (1-q)\,C_{dd}}{e^{r\Delta t}}\ ,\ V_d - X\right] \tag{3.79}$$

Then, finally, the value of the call option at time "0" can be obtained by substituting Equation (3.78) and (3.79) into Equation (3.58) as Equation (3.80).

$$C = \frac{q\,C_u + (1-q)C_d}{e^{r\Delta t}}$$

$$= \frac{q\,Max\left[\frac{qC_{uu} + (1-q)\,C_{ud}}{e^{r\Delta t}}\ ,\ V_u - X\right] + (1-q)Max\left[\frac{qC_{du} + (1-q)\,C_{dd}}{e^{r\Delta t}}\ ,\ V_d - X\right]}{e^{r\Delta t}}$$

$$= \frac{\left(qMax\left[\frac{\left(qMax\left[\frac{qMax[V_{uuu}-X,0]+(1-q)Max[V_{uud}-X,0]}{e^{r\Delta t}}, V_{uu}-X\right]+ (1-q)Max\left[\frac{qMax[V_{udu}-X,0]+(1-q)Max[V_{udd}-X,0]}{e^{r\Delta t}}, V_{ud}-X\right]\right)}{e^{r\Delta t}}, V_u - X\right] + (1-q)Max\left[\frac{\left(qMax\left[\frac{qMax[V_{udu}-X,0]+(1-q)Max[V_{udd}-X,0]}{e^{r\Delta t}}, V_{ud}-X\right]+ (1-q)Max\left[\frac{qMax[V_{ddu}-X,0]+(1-q)Max[V_{ddd}-X,0]}{e^{r\Delta t}}, V_{dd}-X\right]\right)}{e^{r\Delta t}}, V_d - X\right]\right)}{e^{r\Delta t}}$$

(3.80)

Likewise, with the same processes, we can find the following Equation for the value of the put option at time “0.”

$$P_{uu} = Max\left[\frac{qP_{uuu}+(1-q)P_{uud}}{e^{r\Delta t}}, \ X - V_{uu}\right]$$

$$= Max\left[\frac{qMax\,[X - V_{uuu}, 0] + (1-q)Max\,[X - V_{uud}, 0]}{e^{r\Delta t}}, \ X - V_{uu}\right]$$

(3.81)

$$P_{ud} = Max\left[\frac{qP_{udu} + (1-q)\,P_{udd}}{e^{r\Delta t}}\ ,\ X - V_{ud}\right]$$

$$= Max\left[\frac{qMax\,[\,X - V_{udu},\, 0] + (1-q)\,Max\,[X - V_{udd},\, 0]}{e^{r\Delta t}}\ ,\ X - V_{ud}\right] \tag{3.82}$$

$$P_{dd} = Max\left[\frac{qP_{ddu} + (1-q)\,P_{ddd}}{e^{r\Delta t}}\ ,\ X - V_{dd}\right]$$

$$= Max\left[\frac{qMax\,[X - V_{ddu},\, 0] + (1-q)\,Max\,[X - V_{ddd},\, 0]}{e^{r\Delta t}}\ ,\ X - V_{dd}\right] \tag{3.83}$$

$$P_u = Max\left[\frac{qP_{uu} + (1-q)\,P_{ud}}{e^{r\Delta t}}\ ,\ X - V_u\right] \tag{3.84}$$

$$P_d = Max\left[\frac{qP_{du} + (1-q)\,P_{dd}}{e^{r\Delta t}}\ ,\ X - V_d\right] \tag{3.85}$$

Finally, the value of the put option is obtained as follows.

$$P = \frac{q\,P_u + (1-q)\,P_d}{e^{r\Delta t}}$$

$$= \frac{q\,Max\left[\frac{qP_{uu} + (1-q)\,P_{ud}}{e^{r\Delta t}}\ ,\ X - V_u\right] + (1-q)Max\left[\frac{qP_{du} + (1-q)\,P_{dd}}{e^{r\Delta t}}\ , X - V_d\right]}{e^{r\Delta t}}$$

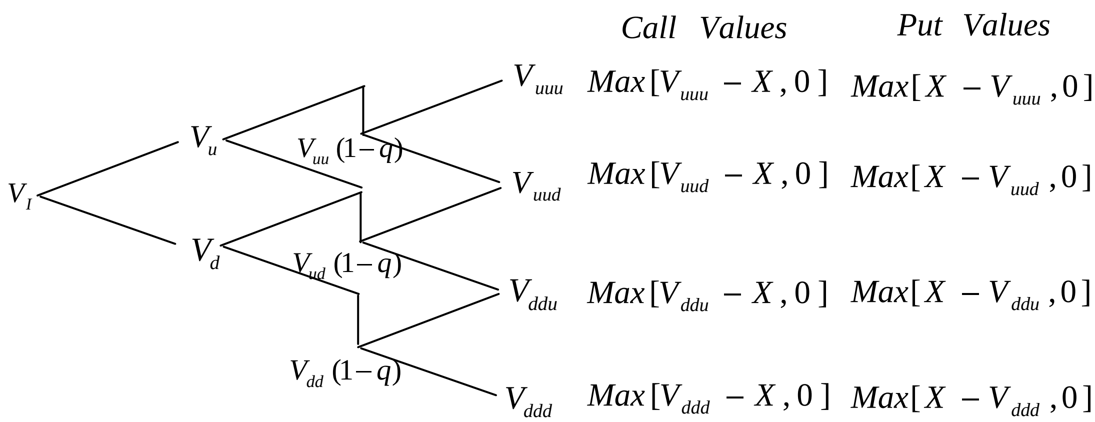
$$
= \frac{\left(\begin{array}{l} qMax\left[\dfrac{\left(\begin{array}{l} qMax\left[\dfrac{qMax\,[X - V_{uuu}, 0] + (1-q)\,Max\,[X - V_{uud}, 0]}{e^{r\Delta t}}, X - V_{uu} \right] + \\ (1-q)Max\left[\dfrac{qMax\,[X - V_{udu}, 0] + (1-q)Max\,[X - V_{udd}, 0]}{e^{r\Delta t}}, X - V_{ud} \right] \end{array} \right)}{e^{r\Delta t}}, X - V_u \right] \\ + (1-q)Max\left[\dfrac{\left(\begin{array}{l} qMax\left[\dfrac{qMax\,[X - V_{udu}, 0] + (1-q)Max\,[X - V_{udd}, 0]}{e^{r\Delta t}}, X - V_{ud} \right] + \\ (1-q)Max\left[\dfrac{qMax[X - V_{ddu}, 0] + (1-q)\,Max[X - V_{ddd}, 0]}{e^{r\Delta t}}, X - V_{dd} \right] \end{array} \right)}{e^{r\Delta t}}, X - V_d \right] \end{array} \right)}{e^{r\Delta t}} \tag{3.86}
$$

So far, we have seen the extending the single-step binomial model to three-step binomial model. The further extending of the time step will be achieved with the same process mentioned above no matter how the time steps are large. As we can see, the derivation of the extended binomial model into three steps from single-step binomial model eventually makes the calculations so large that we usually use the spreadsheet program such as Microsoft Excel to consider the multi-step binomial model to calculate the option value more easily. Of course, as the project should be, in general, considered for a long-term period of time, the multi-step binomial model will take huge spaces in Microsoft Excel program and, as a way to solve this problem, we can use the computer language such as Visual Basic which is built in Microsoft Excel program.

3.2.5.5 Multi-Period Binomial Model with Dividends Adjustments

In above Section, the risk neutral probability approach of the binomial model is extended from the single-period case to a multi-period aspect. The concepts applied in the single-period example are the same as for the multi-period case. This section is going to show how the binomial model can be adjusted to evaluate dividend-paying assets with the example that considers a three-period time step. Among some different ways to consider the dividend adjustment in evaluating the binomial model when valuing real options, the most generally used approach is that it is assuming the underlying asset pays a discrete known dividend yield at a specific time period (Kolb, 2000; Hull, 2002). In this approach, as the dividend yield, q, is a proportion of the underlying asset value whenever the dividend is paid, the underlying asset value will be affected by the dividend rate in the binomial model. As a result, the reduction of the underlying asset value will affect the option value as well. When dividend is accounted for in this manner, the holder of an American call or put option can decide to exercise the option early. In the case of an American call option, the holder may exercise the option immediately before the dividend payment (ex-dividend date) while the owner of an American put option may exercise the option immediately after the ex-dividend date. This is because the owner of the call may profit from high asset value by exercising before the dividend payment while the put owner may benefit from the low asset value as result of the dividend payment. The process of valuing real option using the binomial model in a multi-period stage is discussed below. Figure 3.7 shows the binomial tree where the dividend is paid.

Figure 3.7 Underlying Asset with Discrete Dividend Adjustment



Suppose that the stock pays annual dividend yield, q, continuously, the stock's risk-neutral dynamics can be modeled as

$$\frac{dS}{S} = (r - q)dt + \sigma\, dz \tag{3.87}$$

As a result, the formula of u and d will become:

$$u = e^{\left(r - \frac{1}{2}\sigma^2 - q\right)\Delta t + \sigma\sqrt{\Delta t}} \tag{3.88}$$

$$d = e^{\left(r - \frac{1}{2}\sigma^2 - q\right)\Delta t - \sigma\sqrt{\Delta t}} \tag{3.89}$$

Other computations remain the same as in the non-dividend paying stock's options. Note that this dividend-paying feature is crucial in the real options analysis.

3.2.6 Determinants and Impacts of the Input Variables on Real Option Values

To implement the real option valuation, we need to know the necessary input variables. The option value is dependent on six main variables relating to the underlying asset (Hull, 2002). They are the initial price of the underlying asset, the exercise price, time to maturity, risk free rate, volatility which is defined as the standard deviation of rate of return in the underlying asset, and dividends expected during the life of the option. Table 3.2 describes these factors and their predicted effects on call and put option values in real option values. The effect indicates a change in one factor holding all other factors constant.

Table 3.2 Determinants and Their Effects on Option Value (Hull, 1997 and 2002)

	American		*European*	
Increase in Factor	*Call Value*	*Put Value*	*Call Value*	*Put Value*
Stock Price	+	-	+	-
Exercise Price	-	+	-	+
Time to Maturity	+	+	?	?
Risk-Free Rate	+	-	+	-
Volatility	+	+	+	+
Dividend	-	+	-	+

3.2.6.1 Initial Underlying Asset Value "V_I"

As options are assets that derive their value from the underlying asset, the changes of the underlying asset value such as a stock price in the financial market, affect the value of the options on the asset (Hull, 1997). Since a call option provides the right to buy the underlying asset at a fixed price, an increase in the underlying asset value will increase the call option value. On the other hand, a put option becomes less valuable as the underlying asset value increases. In real assets, the initial underlying asset value is the present value of the expected cash flow to be received from the project. This value does not account for the initial capital requirements for the project and can be obtained from a discounted cash flow calculation without consideration of flexibility. The expected cash flow to estimate this value will be discounted to the present at a risk-adjusted discount rate for the project. The expected net cash flow consists of all the revenues/expenditures generated from the investment except for the initial capital cost. If we know that the project will generate a specific revenue per year for the next n years starting from year one, and the total variable and fixed costs per year for the project, we can easily find the annual expected cash flow FCF_i by deducting the annual variable and fixed cost from the total annual revenue. Assuming r is a risk-adjusted discount rate and FCF_i is the future free cash flow, the initial underlying asset value, V_I , is estimated by Equation (3.90). And, Equation (3.91) shows the change of the call and put option values as the initial underlying asset value V_I increases (Hull, 1997).

$$V_I = \sum_{i=1}^{n} \frac{FCF_i}{(1 + r)^i} \tag{3.90}$$

$$\frac{\partial C}{\partial V_I} < 0 \quad \text{and} \quad \frac{\partial P}{\partial V_I} < 0 \tag{3.91}$$

3.2.6.2 Exercise Price “ X ”

In a real option model, the exercise price is the condition for any contingent action to occur. This value can be given by the management or be calculated based on the risks or characteristics in which the project is involved (Copeland and Antikarov, 2002). For example, when it comes to the expansion option, the value of the investment outlay is equivalent to the exercise price. The initial capital cost is the present value of the start-up cost, or the lump sum cost, incurred at the beginning of a project. The entire cost may be incurred at the beginning of the project or could be incurred over a certain time period. Where the initial investment cost is incurred over a period of time, future costs have to be discounted to the present. If we assume that the project will cost $I_0 + I_1$ over two years, for example I_0 in year 0 and I_1 in year 1, for the chance to receive future free cash flow, the initial cost of the investment can be estimated in Equation (3.92):

$$X = I_0 + \frac{I_1}{(1+r)^1} \tag{3.92}$$

Equation (3.93) shows the change of the call and put option values as the exercise price increases (Hull, 1997).

$$\frac{\partial C}{\partial X} > 0 \quad \text{and} \quad \frac{\partial P}{\partial X} > 0 \tag{3.93}$$

3.2.6.3 Time to Maturity "T"

The time to maturity or expiration in real projects is the time left until the right of the option has expired. Unlike the financial option, as the time to maturity is usually not pre-defined in real asset projects, sometimes it is vaguely specified and should be subjectively defined by the management as the time it takes for competitors to exploit the same opportunity. However, in some cases the time to maturity is fixed in advance. In relation to the option values, both American call and put options become more valuable as the time to expiration increases because the longer time to maturity gives more opportunities for the underlying asset value to move, increasing the value of both types of options. On the other hand, European call and put options do not necessarily become more valuable as the time to expiration increases because the owner of a European option can use his/her right to exercise the option only at the maturity of the option (Hull, 1997). Equation (3.94) shows the change of the call and put option values as the time to maturity increases (Hull, 1997).

$$\frac{\partial C}{\partial T} > 0 \quad \text{and} \quad \frac{\partial P}{\partial T} > 0 \tag{3.94}$$

3.2.6.4 Volatility of the Rate of Return in Project Value "σ"

Risk is involved in virtually all types of projects. Even if planning cannot overcome all forms of risk, good planning can deal with risk in an objective way. In terms of the financial outcome of projects, risk can be defined as the probability that a project will not return the desired outcome. The standard deviation or volatility of the rate of return, σ, has been used as a measure of financial risk. It is defined as the square root of the variance, where the variance is defined by the following Equation.

$$\sigma^2 = \sum p(s)[r(s) - E(r)]^2 \tag{3.95}$$

Here, σ^2 is the variance, *p(s)* is the probability of occurrence, *r(s)* is the actual return and *E(r)* is the expected rate of return. In a real option analysis, rather than directly adjusting the required rate of return for the level of risk, the risk-free rate of return is used in conjunction with a separate volatility parameter. It is this inclusion of volatility that mathematically differentiates the real option analysis from the NPV analysis. This option analysis recognizes that different types of projects will have different levels of volatility. The volatility may be the most difficult and important of all of the variables to forecast and measure. And, it is a key parameter because the option value depends highly on the volatility estimate. While in financial markets the volatility of a security traded is relatively easy to estimate based on the historical volatility of the returns or the implied volatility of the option, for a specific project it is more complicated to determine the correct volatility. However, the volatility of projects usually have been estimated by some representative methods such as the logarithmic cash flow returns approach, twin security which correlates with the project (Trigeorgis, 1996), and management estimate (Sharp, 1991; Copeland and Antikarov, 2001).

Generally in measuring the value of the volatility σ, an easy method is to use the Logarithmic Cash Flow Returns Approach (Copeland and Antikarov, 2001; Mun, 2002). The Logarithmic Cash Flow Returns Approach has been used with historic or future estimates of cash flows, along with their logarithmic returns (Lewis, Enke, and

Spurlock, 2004). This approach is considered valid and widely used in estimating the volatility of financial assets. The logarithmic return of the cash flow is defined by following formula. Here, CF_i is a cash flow at time *i* and r_i is a rate of return of cash flow, CF_i, at time *i*;

$$r_i = \ln CF_i - \ln CF_{i-1} \tag{3.96}$$

$$\bar{r} = \sum_{i=1}^{n} \frac{r_i}{n} \tag{3.97}$$

Equation (3.96) is an approximation of the percentage of change. To find the value of a logarithmic return of cash flow, forecast cash flows of a project will be used. Then, the standard deviation of the cash flow return "σ" will be calculated through Equation (3.98).

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (r_i - \bar{r})^2} \tag{3.98}$$

Here, $\bar{r}$ is the average log of the returns gained by Equation (3.97) and r_i is the log of return for the cash flow at each time step in Equation (3.96).

A higher volatility always causes an increase of call and put option values as shown in Equation (3.99) because the higher volatility makes the distribution of the value of future assets in the binomial tree widen helps to induce the possibility of increase in the option's intrinsic value when exercised. For instance, the call option value depends on the underlying asset value exceeding the exercise price. As the distribution of the future asset widens due to an increase in the uncertainty of the project value, the expected payoff is conditional on the option expiring in the money also increases (Hull, 1997).

$$\frac{\partial C}{\partial \sigma} > 0 \text{ and } \frac{\partial P}{\partial \sigma} > 0 \tag{3.99}$$

3.2.6.5 Risk-Free Interest Rate "*r*"

The variable is the risk-free interest rate for a risk-free bond with the same expiration date as the option being evaluated. It is usually ideal to consider this variable after the time to maturity variable is known since it is normally derived from government bonds or Treasury bills with the same maturity as the option (Perlitz et al., 1999). As shown in Equation (3.100), an increase in the risk-free interest rate will cause the increase of a call option value since the exercise price will only have to be paid at the maturity date, making it possible to invest money elsewhere. However, a put option value has an opposite effect against the increase of the risk-free interest rate since the proceeds from the sale of the underlying assets are received in the future. Equation (3.100) illustrates the change of the call and put option values as the stock price increases (Hull, 1997).

$$\frac{\partial P}{\partial r} < 0 \text{ and } \frac{\partial C}{\partial r} < 0 \tag{3.100}$$

3.2.6.6 Dividend "*q*"

The dividends are paid out by the underlying asset and the dividend payments can be seen as leakage in value arising from cash flow because it reduces the value of the underlying asset (Amram and Kulatilaka, 1999). Many real assets experience diverse leakages such as dividends, royalty and licensing fees, loss of value through competition, and loss from perishable damage in their values, which changes the value of the underlying asset, in turn, affecting the option value and the timing of the optimal decision. If the underlying asset pays dividends, its value will be reduced and it may become optimal to slowly or quickly exercise the option in a call or a put respectively before or after the dividend is paid. In this case, the call option holder will wait until the time is close to maturity before deciding whether or not to exercise if there is no dividend (Trigeorgis, 1991; Hull, 1997). So, the value of the underlying asset will decrease if dividend payments are made on the asset during the life of the option. Finally, the call option value decreases as the dividend increases, and the put option value increase as dividend payments increase (Hull, 1997).

$$\frac{\partial C}{\partial q} < 0 \quad \text{and} \quad \frac{\partial P}{\partial q} > 0 \tag{3.101}$$

4. PROBLEM STATEMENT

As previously mentioned, the NPV analysis has rejected the projects that have a negative NPV, although it is likely that the future profit by management flexibilities would be written in during that year and added to the present value, because it cannot take into account the value of management flexibilities (Amram and Kulatilaka, 1999; Trigeorgis, 1999; Dixit and Pindyck, 1994; Myers, 1984). When it comes to BOT projects related to infrastructure construction, due to asymmetric payoff conditions by its complicated contractual agreement to allocate risks to each project finance member, the need of real option analysis is growing. For this reason, real option analysis has been considered as an important tool to help evaluate governmental guarantees that have been proven to be an important success factor in BOT project finance (Zhang, 2005). Still, there have been some problematic issues in evaluation techniques of previous representative real option approaches. If we briefly mention the major problems of the NPV analysis and evaluation techniques in previous real option methods used for estimating debt payment guarantees and minimum revenue guarantee values in a BOT project, they are as follows.

First, the NPV approach treats the choice of whether or not to take a specific action in a management decision as mutually exclusive alternatives at the beginning of the project, and then chooses the only one with the higher NPV (Copeland and Antikarov, 2001). On the other hand, real option analysis works back in time from the end points of the decision tree, making the value-maximizing decision tree at each node, when the choice is actually available, contingent on the underlying risky variable. And, finally, the result is gained as a single present value that is used to decide whether to start the project today. The deferral decision is a good example to illustrate the difference between NPV and real option analyses. If we use an NPV analysis in evaluating a project, deferrals must be treated as a large set of mutually exclusive deferral dates - defer for one year, defer for two years, and so forth. However, the real option approach tells us whether or not to start the project today and provides a value for the right to defer without saying when. It then gives a value-maximizing rules of thumb for deciding when to defer.

Second, in real option approaches to evaluate guaranteed value, Ho and Liu's research (2002) did not consider debt payment guarantees as an option. Instead, upon assuming that the project value and construction costs follow a geometric Brownian motion process, by formulating the limited liability of equity as an option, they evaluated the project value on equity. And, through the notion that the difference of the present value between a risky loan and a risk-free loan will be the amount that the lender will require as a debt payment guarantee from the government when an adverse situation occurs, Ho and Liu found the loan guarantee value. This means that the option formulation of the guarantee is not used in their research. In light of this fact, strictly speaking, although the method used in their research to find the loan guarantee value can look like a real option analysis, it cannot be regarded as an actual real option analysis. Furthermore, in this case, another issue is that because the interest rate that the bank applies when they lend money for the project can be changed - according to the bank's policies and situation- the method of findingthe loan guarantee value from the difference of interest rates can look like a very subjective and indirect method compared to directly formulating the guarantee as an option. Besides, the developed real option model in their research uses a long and complicated process to find the guarantee option value, which cannot be easily applied to practical fields.

Third, in Cheah and Liu's real option approach (2006), there are some points that the author overlooked. First, their research uses the basic concept of a real option analysis based upon the asymmetric payoff condition. But, unlike in a basic real option analysis that assumes that the underlying risky asset follows a random path, this approach just constructs a cash flow model for the expected cash flow that considers a base case and actual cash flow model that reflects initial traffic volume and traffic volume growth rate as key risky variables. And then, by discounting the difference between the two cash flow models with a risk-free rate, the authors evaluate the present value of the minimum revenue guarantee option. Actually, the methodology used in this case has a gap compared to the original real option analysis in light of not taking into account the dynamics and uncertainty of underlying risky assets that usual real option theory follows. Basically, as one of the most important characteristics of real option analysis, which can be distinguished from other evaluation techniques, because the assumption that the risk variables follow a random walk such as the geometric Brownian motion process has been proved to be able to reasonably model the change in the value of risk variables, this stochastic process of risk variable or underlying asset should be considered. Second, in analyzing the final results, upon applying the real option concept, although they use the median value of the minimum revenue guarantee in its probability distribution gained from a Monte Carlo simulation, because the probability distribution of governmental guarantee value is excessively skewed to the right and has an extremely long tail in its right side, it is not easy to regard the mean or median value of guarantee options as reasonable guaranteed values.

The final issue that we need to consider is that previous real option approaches have ignored that the volatility of the project return is not the same as the volatility of the input variables that compose the return of project (Copeland and Antikarov, 2001). For example, when the price per unit is given a volatility of 10%, the volatility of the project returns can be different from 10%. The point is that the volatility of a project can be higher or lower than that of the input variables. In turn, because this volatility value is the most important factor that can significantly affect the option value in a real option approach, this can make the government and developer overestimate or underestimate the guarantee value. This biased information can lead the government and developer to fail in the process of investment decision-making and building bidding strategies. For this reason, it is necessary to take into account the decomposed input variables to find a more detailed level of volatility instead of just directly considering the rate of return in the cash flow.

Based upon the facts mentioned in the problem statement, the purpose of this research is to numerically evaluate the minimum revenue guarantee (MRG) with the developed option pricing model. It is intended that this thoroughly follow option pricing theory be able to be easily used in practice while better reflecting the dynamics and uncertainties of the cash flow components by using decomposed input variables. To find the guaranteed value with more detailed considerations based upon real option analysis, the minimum revenue guarantee in BOT projects will be treated as a form of option and analyzed to find the appropriate value to help the government and the developer make appropriate decisions in their bidding process. Finally, in terms of significance, this research will contribute to providing a more detailed numerical framework to evaluate the minimum revenue guarantee in BOT projects based on a binomial tree model in real option analysis. It will also provide some useful empirical evidence using case studies to further validate the applicability of the developed method.

5. CONCEPTUAL FRAMEWORK AND RESEARCH HYPOTHESES

5.1 Research Question

Based upon the purpose and significance of the research in Section 1.2, this study will consider the following research question:

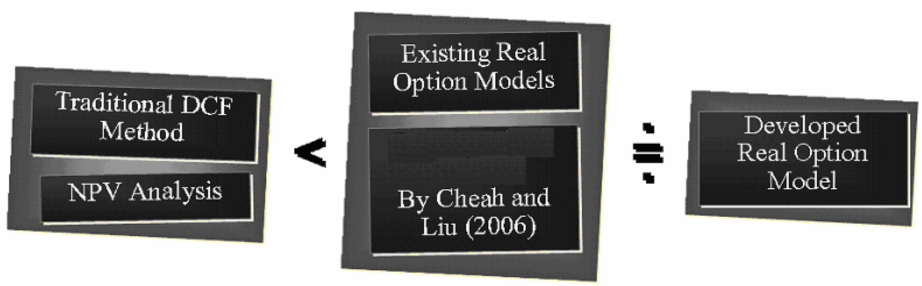
- ✓ *The newly developed real option model, which thoroughly follows real option theories and considers a more detailed level of the volatility of the rate of return in a project, will provide us with a reasonable Minimum Revenue Guarantee (MRG) option value.*

To verify the applicability of the developed real option model, it is necessary to test the model with some appropriate research hypotheses which reflect the financial characteristics related to the real option theories through three BOT project case studies. The verification will look into how well the model can consistently give the expected results, which are pre-assumed in research hypotheses, and how well the numbers and results are interpreted from the case studies. The case studies comprise three different toll road systems such as an expressway, a bridge, and a beltway/ring road, which are contracted with MRG agreements under a BOT project finance scheme between the government and the BOT developer during operation. These three case studies are used to generalize the results and render a reasonable conclusion through theoretical investigations, and the government BOT policies on these cases will be discussed as well.

5.2 Conceptual Framework and Research Hypotheses

In this research, for erecting reasonable research hypotheses to test if the developed real option method works well while evaluating the value of the MRG agreement, we will consider three project valuation methods; NPV analysis, Cheah and Liu's real option model (2006), and developed real option model, which will help show if the results from three different project valuation methods are reasonable and if the relationships between the major determinants on the MRG option value and the MRG option value follow the proven real option theory.

Figure 5.1 Conceptual Approach: MRG Agreement Value Comparison



First, three BOT project valuation methods are the traditional NPV analysis of method 1, method 2 (Cheah and Liu's model, 2006) and method 3 which is a developed real option model considering the dynamics of underlying asset "project value" through a more detailed level of volatility considering decomposed risky variables. In method 3, the Monte Carlo simulation is taken into account to randomize the cash flow with detailed cash flow components and this method is intended to improve the practical real option modeling technique that can be easily used in real BOT projects under MRG agreement which traditional valuation methods have overlooked. Method 3 may be expected to give us a more reasonable MRG value through considering more detailed levels of dynamics of risky variables and to help for both the government and developer build strategies and policies for decision making during the bidding process. The following are the three different project valuation methods used in this research:

- ✓ ***Method 1:*** *Traditional DCF model – NPV Analysis*
- ✓ ***Method 2:*** *Cheah and Liu (2006)'s Real Option Model*
- ✓ ***Method 3:*** *Developed Real Option Model*

In erecting research hypotheses with three project valuation methods, we will calculate the values of the three BOT projects with given case study data sets using method 1, a traditional NPV analysis. Here, the project values gained from this process will be called the passive NPV because they do not consider the option of MRG agreement. Next, with the same data set used in the traditional NPV analysis, the evaluation of the MRG agreement value with the option pricing model developed by method 2 will be conducted. In this case, as proven in their research, the value of the MRG agreement will be reflected in the project's present value. Next, we will calculate the value of the MRG agreement through method 3 by formulating the MRG agreement as a put option. Here, we will call the project values gained from methods 2 and 3 an expanded NPV because they take into account MRG agreement that could not be considered by the traditional NPV analysis.

Second, we will consider the relationships between major determinants of MRG option value and the MRG option values as already mentioned in Section 3.2.6. According to Hull (2002), it is already proved that the value of an option is determined by a number of input variables such as initial underlying asset value, exercise price, time to maturity, risk free interest rate, volatility, and the dividends expected during the life of the option. Based upon the option pricing theory, the predicted impacts of these major determinants on call and put option values in the developed binomial real option model have to be consistent with those proved by the option pricing theory based on the Black-Scholes model to justify the validity of the developed real option model. The impact indicates a change in one factor holding all other factors constant and as each factor increases the call and put option value change (Hull, 2002).

Through the conceptual approach mentioned above and Figure 5.1, we can come up with the following reasonable research hypotheses to verify the applicability of the developed real option model. Here, to test the validity and the reliability of the developed real option model, we will conduct the calculation of the MRG option values for three BOT project case studies through the same process. This is due to the fact that the result of only one case study is not enough to verify whether or not the developed model is working well and providing consistent results. The followings are research hypotheses constructed to test the applicability of the developed real option model.

Hypothesis One **-** *The project value using the two option pricing method 2 and 3 under the MRG agreement will show significant value rather than the project value under NPV analysis.*

Hypothesis Two **-** *Based upon the option pricing theory, the predicted effects of major determinants (current price of the underlying asset V_I, exercise price X, time to maturity T, volatility σ, and risk free interest rate r) on an MRG option value in the developed real option model, method 3, have to follow those of option pricing theory based on the Black-Scholes model.*

Hypothesis Three **-** *The MRG agreement value gained from the developed real option model, method 3, will be consistent with that of method 2. (The MRG agreement value gained from method 3 will be located within a range of $\pm 2 \cdot \sigma$ from the median and mean in probability distribution of the MRG value by method 2).*

5.3 Validation Test

As mentioned above, to test the validity and reliability of the developed real option model, we are going to use three different case studies on toll road projects of the BOT type. The results of each case study through the developed method must satisfy the research hypotheses to show that the model has a reasonable level of validation.

6. METHODOLOGY

6.1 Introduction

This research presents an acceptable binomial real option valuation model considering the MRG agreement that the NPV analysis and the related previous real option methods developed by Ho and Liu (2002) and Cheah and Liu (2006) could not consider. This developed model is consistent with option pricing theories. And, this model will be applied to three BOT project finance case studies to verify the applicability of the developed model. As BOT project valuation methods, we will consider three evaluation techniques; NPV Analysis, Cheah and Liu (2006)'s real option model, and a developed real option model. Before the description of the project valuation methods, basic assumptions in a BOT investment valuation and the data collection process will be explained.

6.2 Basic Assumptions in a BOT Project Valuation

To evaluate the BOT project with theoretically developed real option model, we need to examine the conditions and problems related to the BOT project in advance. These conditions and problems are the basis of the fundamental framework to erect the appropriate and reasonable assumptions to facilitate developing the real option model.

6.2.1 Assumptions in a BOT Investment Environments and Conditions

There are various kinds of project finance types and the main differences between BOT and other types of project finance schemes mainly stem from the concession period and the project ownership.

In implementing the BOT project, there should be some necessary conditions or environments as shown in Table 6.1. However, to facilitate the derivation of the BOT valuation model in this research, it is assumed that some conditions are excluded in the process of the model derivation. In general, the following are the necessary conditions and environments for the BOT project implementation (Augenblic and Custer, 1990).

The risks caused by the political, legal, economic environment, and host country credit rating, are excluded in this research to focus on more sensitive issues. However, it is impossible to rule out all these conditions completely.

Table 6.1 Conditions and Environments for BOT Project Implementation

Conditions & Environments	*Description*
Political Condition	*The political stability and continuity of the host country is satisfactory.*
Legal Condition	*The host government's legal system is ready to support the contractual issues of the infrastructure privatization and fairly mature.*
Host Country Credit Rating	*The BOT investment requires at least an intermediate credit rating.*
Economic Environment	*The host country has a developed banking system/organized capital market which are mature enough to provide equity investors/lenders with enough financial tools to make investments or hedge risks.*

6.2.2 Assumptions regarding a BOT Investment Risks

In general, risks are present in every BOT project and can be categorized as construction risk, completion risk, operating risk, financial risks, political risk, technical risks, and country risks (political and regulatory risks) etc. There is no specific definition for risk categories and from time to time the same category name can refer to different risks depending on the researchers. Below are brief descriptions of the risk categories according to the various researchers:

- ✓ *Augenblick and Custer (1990): completion risk, performance and operating risk, cash flow risk, inflation and foreign exchange risk, insurable risks, uninsurable risks (force majeure), and political risk.*

- ✓ *Walker and Smith (1995): financial risks, political risks, and technical risks. They also grouped risks by different stages of a project.*

- ✓ *Dias and Ioannou (1995c): country risks (political and regulatory risks), force majeure risks, development risks, financial risks, revenue risks, promotion risks,* *procurement risks, development risks, construction risks, and operating risks.*

Table 6.2 Risks related to BOT Project Investment

BOT Investment Risks	Description
Construction/ Completion Risks	*Many insurable risks such as plant or equipment casualties and physical loss or damage and workmen's compensation are in this category. Mainly due to the construction phase including completion delays/cost overruns/technical difficulties.*
Operating/ Economic Risks	*Primarily related to the economic situations/changes during the operation/construction phase, and include price, demand/supply changes, management, business cycle, and liability. Uninsurable force majeure risks may be included in construction/completion and economic/operating risks categories.*
Financial Risks	*Due to interest rates, foreign exchange rates or inflation rates.* *Can be hedged by certain financial tools in a developed financial market or by contractual arrangement.* *Include a firm's default risk that is related to the firm's financial structure arrangement, such as debt service, repayment, and equity investment.*
Political/ Environmental Risks	*The hardest to predict/control among all categories. These risks include political support risks, forced buy-out risks, cancellation of concessions, and changes in environmental regulations.*

Based on the related research, we can summarize the risks in a BOT investment as shown in Table 6.2. In this research, it is assumed that quantifiable risks such as operating risks and economic risks are significant in project valuation. The risks that can be hedged by using financial tools or contractual arrangements or that can be insured by commercial insurance are not considered.

6.2.3 Assumptions of Financial Feasibility Analysis in BOT Project Valuation

It is obvious that measuring the financial feasibility is important in a BOT project where different participants have different perspectives. In this research, the financial feasibility of the BOT project is taken into account from the BOT developer and the government's point of views.

First, in a BOT project, the BOT developer aims to maximize the NPV on equity, "equity value," against its equity investment (Ho and Liu, 2002). This is why the BOT developer holds the complete or partial fraction of the equity of the BOT project and receives dividends. Since the main reward for the BOT developer in the BOT project is this equity value, which is the sum of the BOT project's discounted free cash flow on equity, the equity value can be assumed to be the important component in measuring the BOT developer's and equity investor's financial feasibility. Second, from the perspective of the government, it can be considered that once the BOT project goes into a bankruptcy condition where the debt value is greater than the project asset value, which means that the project equity value is below 0, the government may face the political burden for failure to provide for the public interest. Therefore, the equity value of the BOT project can be assumed to be an important component for the government to consider in measuring the BOT project's financial feasibility (Ho and Liu, 2002).

For these reasons, this research assumes that the BOT project's equity value is a measure in evaluating the BOT project's financial feasibility.

6.3 Data Collection

In this research, the financial cash flow model used in method 1 is the basic framework to conduct the analyses of methods 2 and 3. Once the data set is collected to conduct the project valuation by method 1, this data set and some parameters calculated by method 1 are again used as key input variables for the project valuations of methods 2 and 3.

For the analysis, three different BOT case studies associated with a toll road system are considered in this research. The following basic data necessary to this research is extracted from different sources with logical assumptions.

- ✓ *Initial traffic volume*
- ✓ *Traffic volume growth rate*
- ✓ *Average toll rate and toll rates for different vehicle classes*
- ✓ *Cost of the project*
- ✓ *Debt and equity ratio*

- ✓ *Loan term and interest rate*
- ✓ *Concession period*
- ✓ *Operating and maintenance costs*
- ✓ *Corporate tax rate, and so on.*

Generally, since the above data can be obtained from the BOT developer's expected cash flow model, the BOT firms, construction companies, or equity investors can provide BOT project related data sets which will be supplemented with market information captured from reliable newspapers, reports, or the Internet. In this research, Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. and Macquarie Shinhan Infrastructure Asset Management (MSIAM) Co., Ltd. support related data sets for three BOT project case studies; Ma-Chang Bridge (MCB) Project, Kwangju Ring Road Section 3-1 (KRRC) project, and Cheonan-Nonsan Expressway (CNE) project.

As for market data, this research uses the stock and bond market data publicly issued by reliable sources and organizations. The risk free rate (Treasury bill rate) and overall market rate of return are based on the Bond Information Service (BIS) of the Korea Securities Dealers Association (KSDA) (Source: http://www.ksdabond.or.kr). The values of β of the equity investment institutions or construction companies, which join as equity investors in three BOT project cases, are referred from the publicly reliable financial information website (Source: http://kr.stock.yahoo.com).

6.4 Method 1 - Valuation of the BOT Project with NPV Analysis

In general, NPV of the project is calculated by discounting the cash flow at a risk-adjusted discount rate considering the project's risk premium as shown in Equation (6.1).

$$NPV = -I_0 + \sum_{i=1}^{n} \frac{FCF_i}{(1+WACC)^i} \tag{6.1}$$

However, since this research focuses on the BOT developers and the governments' points of view, the project value on equity investment is considered rather than the project value on the whole investment. So, Equation (6.1) is appropriately adjusted as Equation (6.2) (Damodaran 1996).

$$NPV_e = -I_e + \sum_{i=1}^{n} \frac{FCFe_i}{(1 + R_e)^i} \tag{6.2}$$

Where,

I_e is the initial equity investment

$FCFe_i$ is the free cash flow on equity at year i

R_e is the cost of the equity.

Here, $FCFe_i$ can be obtained by deducting the annual debt service from the annual free cash flow FCF_i. NPV_e is the net present value of $FCFe_i$ considering the initial equity investment I_e in the project. As the risk-adjusted interest rate, unlike the case of the WACC, which reflects both the cost of debt and the cost of equity, since $FCFe_i$ is the free cash flow that already excludes the effect of the debt payment, it is reasonable to only reflect the portion of the equity by R_e. Finally, through discounting the future cash flow on equity $FCFe_i$ at R_e to the present, the net present value on equity, NPV_e, can be estimated.

Basically, the cost of equity is a measure of the required return that the BOT developers or the equity investors expect on equity investments. However, in infrastructure projects, since these projects have been implemented under different risks levels, which stem from the characteristics of the different countries or sectors, it needs to reflect these risk premiums within the cost of equity, R_e. Equation (6.3) shows diverse risk premiums such as country risk and sector risk. It seems that the cost of equity estimated by Equation (6.3) will be greater than the general cost of equity by Equation (3.3) because of the additional risk premium terms of *CRP* and *SRRP*.

$$R_e = R_f + \beta\ (R_m - R_f) + CRP + SRRP \qquad (6.3)$$

where, R_f is *the* risk-free rate, $R_m - R_f$ is market risk premium, β is sector beta, which is the measure of risk for a certain industry and calculated by averaging the betas of comparable firms. *CRP* is the country risk premium and *SRRP* is the sector and regulatory risk premium. Each of these parameters in Equation (6.3) corresponds to a level of return necessary to compensate for some specific risks.

The risk-free rate is the minimum return that can be earned on a risk-free investment. It is measured as the average interest rate on the each country's Treasury bill (i.e., U.S. Treasury bill in U.S.) over a BOT concession period.

The market risk premium $R_m - R_f$ is the additional return that must be earned on equity investments over risk-free investments to compensate for their additional non-diversifiable risk. It is generally measured as the average excess return on the each country's stock market (i.e., return on the S&P 500 in U.S.) above the risk-free rate over a BOT concession period.

β is a measure of the non-diversifiable risk of stock market investments in a specific industry. It can be easily estimated by specialist firms or publicly reliable financial website (i.e., http://kr.stock.yahoo.com in South Korea or http://finance.yahoo.com in U.S.) based on the many financial, operational, and strategic characteristics of each industry. The market risk premium $R_m - R_f$ is multiplied by β, because investors are compensated only for risks that cannot be diversified by an appropriate portfolio management.

The country risk premium, *CRP*, is a measure of the extra risk taken when investing in a specific country. It is generally measured on the basis of the country's Moody or other credit rating compared to the U.S. rating, or by comparing the average spread on bonds of that country with equivalent spreads in U. S. over a long historical period.

The sector and regulatory risk premium, *SRRP*, is a measure of the risk of government noncompliance with agreed-upon regulatory terms or of unilateral changes by government on the regulatory framework. This can be measured by an index capturing the historical volatility of regulatory changes and noncompliance, and by the degree of independence of the regulatory agency. Often, it is also measured by surveying existing and potential operators. In general, this can fall in the range of 2 to 6 % (Guasch, 2004). If the overall return earned by project shareholders on their investment is lower than the cost of equity measured in this way, they would have been better off investing their money in alternative investments given that they earned too little compared to the risk they took.

6.5 Method 2 - Valuation of BOT Project with Cheah and Liu (2006)'s Real Option Model

6.5.1 Introduction

The purpose of Cheah and Liu's research is to quantitatively evaluate the MRG paid out from the government to the BOT developer and the repayment from the BOT developer to the government as a form of the options. When it comes to the aspects of the real option valuation concept, an MRG agreement is considered as a put option and the repayment agreement is formulated as a call option. Then, the values of these options are evaluated quantitatively in order for the BOT project members to use in the bidding process as effective information to determine the balance between risk and benefit.

In their BOT project valuation model, Cheah and Liu (2006) show a clear and simple framework as to how the real option concept can be applied to quantitatively evaluate the values of the MRG and repayment agreements, and provides policy implications that the BOT project members have to consider when joining the bidding processes. Their model demonstrates that the option value of an MRG agreement is substantial compared to the project value based on equity from the NPV analysis and, to match the value that has been conferred, a repayment scheme can be designed to place a cap on the private sector's return.

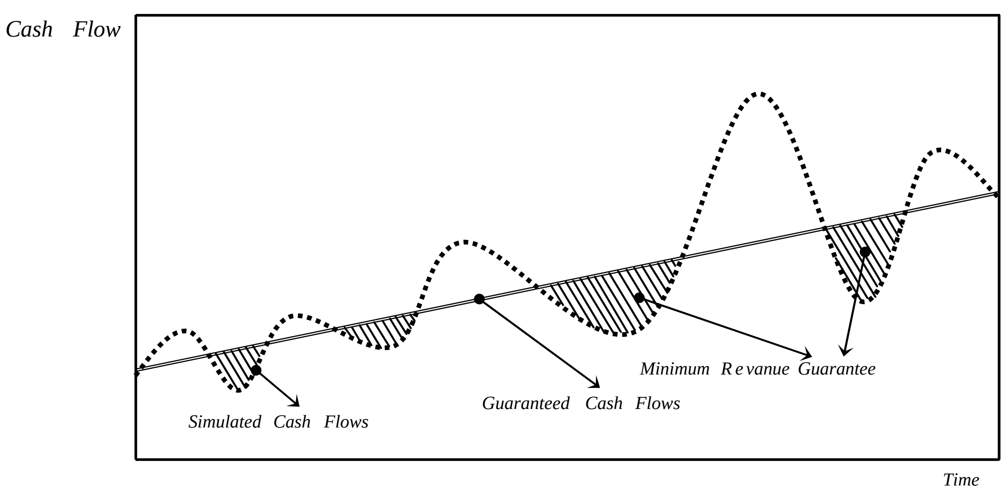
Despite clear results in terms of the option values of the MRG and repayment agreement, this model has some issues that are not consistent with real option methodology. In this research, Cheah and Liu used the basic concept of real option valuation based upon the asymmetric payoff condition. But, unlike traditional real option valuation methods, the Black-Scholes model and the binomial tree model, this approach only constructed a cash flow model for the guaranteed cash flow, which is based on the BOT developer's expected cash flow model and a simulated cash flow model. This reflects the uncertainties of the two most important cash flow components, initial traffic volume and the growth rate of traffic volume, that are likely to happen in a real world. Here, Cheah and Liu use a Monte Carlo simulation to consider the cash flows that are likely to happen in a real world. Then, based on the asymmetric payoff condition with a guaranteed cash flow model and simulated cash flow model, by discounting the cash flow difference between these two cash flow models at each time step with a risk-free rate, they evaluate the present values of the MRG and repayment options separately.

Unlike Cheah and Liu's model, which takes into account the repayment option, this research focuses on evaluating the value of the MRG agreement option. The repayment agreement will be ignored in the analysis process.

6.5.2 Conceptual Approach

The MRG that the government provides to the BOT developer for the shortfall of the toll revenue can be effectively formulated as a put option. Basically, the idea of the MRG agreement stems from the fact that, if the projected (simulated or realized) cash flow in each year i satisfies the guaranteed cash flow level, which is signed in the BOT contract based on the expected cash flow model estimated by the BOT developer, the government does not have to pay any MRG to the developer. Here, the projected cash flow represents the cash flow which is likely to happen in a real world. Otherwise, the government should make up for the shortfall in revenue by paying the BOT developer. For the purpose of evaluating the MRG agreement, the government's obligation to pay in each year, SF_i, would depend on the relative value between guaranteed cash flow at year i, CF_{ig}, and simulated cash flow at year i, CF_{ip}, as shown in Figure 6.1 and Equation (6.4). In Figure 6.1, when the guaranteed cash flow is greater than the simulated cash flow, the MRG, the government pays the difference between the guaranteed cash flow and simulated cash flow to the BOT developer as the MRG.

Figure 6.1 Asymmetric Payoff Condition of MRG Agreement in a BOT Project



Therefore, the government's MRG payment to the BOT developer at year i, SF_i, can be estimated by using the following asymmetric condition.

$$SF_i = Max[Guaranteed\ FCF_e\ at\ Year\ i - Projected(Simulated\ or\ Realized)FCF_e\ at\ Year\ i\ , 0]$$

$$= Max[CF_{ig} - CF_{ip}\ ,\ 0\ ] \tag{6.4}$$

Where, FCF_e is the free cash flow on equity at year i and SF_i is the free cash flow difference on equity at year i between the guaranteed cash flow and simulated cash flow in the MRG agreement. Finally, based on the above asymmetric payoff condition, we can find the MRG value in the following Equation.

$$MRG = \sum_{i=1}^{n} \frac{SF_i}{(1 + r)^i} \tag{6.5}$$

Where, MRG is the total minimum revenue guarantee value during concession period at time "0", r is the risk-free rate, and n is the years of the BOT concession period. As we can see in Equation (6.5), by discounting the annual cash flow differences between guaranteed cash flow CF_{ig} and simulated cash flow CF_{ip} at risk free rate and, then, summing them, MRG option value based on Cheah and Liu's real option model (2006) can be obtained. In a real BOT contract, the guaranteed cash flow CF_{ig} can be estimated on the basis of the BOT developer or private consortium's cash flow model as a pre-determined condition. The guaranteed cash flow CF_{ig}, in BOT contract/agreements, to exercise the MRG option can be described as the percentage of the BOT developer's expected cash flow. For instance, the BOT contract can indicate that the MRG would be paid from the government to the BOT developer when the projected or simulated cash flows are lower than 80 % of the expected cash flows. On the contrary, when the projected or simulated cash flows are higher than 110 % of the expected cash flows, the condition to exercise the repayment option is met. Finally, the guaranteed cash flow CF_{ig}, which is a specific percentage of the expected cash flow, is used as a standard whether or not the MRG option is being exercised in evaluating the MRG agreement. In contracts of real BOT projects, it is usual that the MRG from the government to the BOT developer is annually paid at the end of the year when CF_{ip} is lower than CF_{ig} as agreed upon.

Cheah and Liu (2006), when it comes to the cash flows CF_{ig} and CF_{ip} to calculate the MRG value, use the expected cash flow model estimated by the BOT developer or the private consortium as standard of the guaranteed cash flows and the simulated cash flow model to consider the uncertain toll revenues by generating the likely cash flows in the future with a Monte Carlo simulation.

There are many ways in modeling the real option valuation depending on the types of options, characteristics of the project contracts, capital structures, risk characteristics, the methods of acquiring volatility, and the selection of the underlying assets. However, although Cheah and Liu (2006) apply the asymmetric payoff concept, which is one of the most important characteristics in real option valuation methods, the approach used in this research is closer to the Monte Carlo simulation than the real option analysis in light of considering uncertainties of underlying assets through the simulation method.

6.5.3 Data

The BOT project case study data used for method 1 of NPV analysis is used as a basic data set to apply to Cheah and Liu's real option concept (2006) in method 2 since it is the data estimated by the BOT developer to construct the expected cash flow model. As mentioned, this expected cash flow model is the standard of the guaranteed cash flow based on the BOT contract/agreement.

As for the simulated cash flow model, because it is the result of the simulation of the revenue projection, the appropriate assumptions to implement the Monte Carlo simulation are made for two major key input variables; initial traffic volume and traffic volume growth rate, which define the revenue (Cheah and Liu, 2006). In method 2, these two major revenue components, initial traffic volume and traffic volume growth rate, are assumed to follow lognormal distribution and normal distribution respectively, based on the assumption of Cheah and Liu's research (2006). And, this enables simulation of these variables in reflecting the uncertainties embedded in a real BOT project. As for the initial traffic volume, since logarithmic value do not go below zero and the combination of the logarithmic values is also a logarithmic value, which makes it easy to calculate, it is assumed to follow a lognormal distribution. With regard to the traffic volume growth rate, this variable is modeled to follow a normal distribution in their research. When it comes to the data related to initial traffic volume and the traffic volume growth rate, we can refer to the historical cash flow models of similar past BOT projects or expected cash flow models estimated by the BOT developer or private consortium. So, the mean and standard deviation of initial traffic volume and traffic volume growth rate, which are important input variables to simulate the likely cash flows and, in turn, revenue in method 2, can also be obtained from the data from the expected cash flow model in method 1. Finally, this simulated cash flow is expressed as a form of the probability distribution since it is the result of a Monte Carlo simulation.

6.6 Method 3 - Valuation of BOT Project with Developed Real Option Model

6.6.1 Introduction

The real option model that is developed in this research is to try to support several problematic issues that the NPV analysis and previous representative real option valuation methods by Ho and Liu (2002) and Cheah and Liu (2006) did not consider in evaluating the MRG agreement.

If briefly describing the points that the previous approaches missed, first, in Ho and Liu (2002)'s research, the governmental guarantee is not formulated as an option. Instead, they consider the characteristics of the limited liability of equity as options that can fall into two asymmetric payoff conditions; a terminal condition and a bankruptcy condition. Then, the project value and construction costs are assumed to be underlying assets which are following a geometric Brownian motion process to reflect the uncertainties of the underlying assets. Afterward, by deducting the present value of debt payment discounted at the bank loan interest rate from the present value of the debt payment discounted at a risk-free rate, they calculate the governmental guarantee value. In this case, because the bank interest rate can differ greatly between banks according to their policies, the guaranteed value can be arbitrary and cannot be objective. Moreover, despite the theoretically rigorous framework of the model, this model is hard to apply to the practical world due to the complexity of understanding and formulating the model.

Second, regarding Cheah and Liu's model (2006), the way of considering the uncertainties of risky variables such as initial traffic volume and the traffic volume growth rate are implemented by simulating those two variables with a Monte Carlo approach, not by assuming they follow a geometric Brownian motion process according to the real option theory. Even if this model uses the basic concept of a real option analysis based upon the asymmetric payoff condition, it is not exactly following the real option theory in light of not considering an underlying asset's random walk. Basically, in implementing a real option analysis, because the assumption that the risky variables follow a geometric Brownian motion process has been proven to reasonably be able to model the dynamics and uncertainties of the underlying asset, this random walk has to be considered.

Third, the research of Ho and Liu (2002) and Cheah and Liu (2006) do not consider the uncertainties of a more detailed level of input variables (initial traffic volume and traffic volume growth rate) which directly affect the project value. Actually, the volatility of the underlying assets are not the same as the volatility of any of the input variables that consist of that underlying asset (Copeland and Antikarov, 2001). For this reason, professionals may decide to model cash flow components at a more detailed level, so the value of the underlying asset is further decomposed into variables such as price, costs, volume and quantity. Furthermore, even if the volatility of the project return has to be determined based on the level of the project value rather than the level of the cash flow, because the volatility has been decided by the change of the cash flow returns, it is necessary to convert the volatility of the rate of return in cash flows into the volatility of the rate of return in project value.

For these reasons, the purpose of this research is to support the controversial issues mentioned above. Therefore, the process of developing the model focuses on applying the governmental guarantee agreement as an option, considering the uncertainty of the underlying assets, and calculating the volatility through more detailed level of input variables. Furthermore, this real option model will thoroughly follow the option pricing theory without violation it and be easily applied in practice.

6.6.2 Conceptual Approach

What this research considers in developing a real option model are the MRG agreement as a put option, the random walk of the underlying asset by option pricing theory, and the more detailed level of volatility of the project returns, which is gained by taking into account more detailed input variables. To develop the real option valuation model, we will use the following processes.

6.6.3 The Developing Process of Binomial Real Option Model

6.6.3.1 Selection of the Underlying Risky Asset and Determination of Its Dynamics

In developing the real option model, the first step is to choose the underlying risky asset and determine its dynamics. The changes of the underlying asset value, project value, are important to the value of the real options because the real options are assets that derive their value from underlying assets.

The expected cash flow is a key to any financing scheme. When it come to the project's finances, as the lenders look especially to the forecasted cash flow rather than to project assets as collateral for the loan, the forecasted cash flow is the main credit support of the capital needed (Beidleman et al., 1990). Therefore, it is reasonable to determine the value of the BOT project based on its forecasted cash flow instead of physical asset value. The major source of the future cash flow is the operating profit from the project. And, other minor additional sources of cash flow, which may come from other project related business activities, that are granted to the BOT developer or firm can be considered. The fluctuation of the operating profit is the main risk during the operation in a BOT project because in some projects that are exposed to market competition the operating/economic risks may be large (Finnerty, 1996). Here, the BOT project value is defined as the expected cash flow discounted at an appropriated risk-adjusted discount rate during the operation. As a result, the project value is based on the future cash flow of the entire concession period (Majd and Pindyck, 1987). In this research, the BOT valuation problems will focuson the perspectives of the BOT developer and the government while joining the bidding process. Therefore, the major issue will concern the stochastic nature of the project value on equity investment, which is subject to change or fluctuations due to various market conditions during the operation period.

To model the dynamics of the project value during the operation period in this research, the project value V will be defined as an underlying risky asset and assumed to follow a geometric Brownian motion process (Majd and Pindyck, 1987; Dixit and Pindyck, 1994; Schwartz and Moon, 2000). The dynamics of project value V are given as follows:

$$\frac{dV}{V} = \mu \, dt + \sigma \, dz \tag{6.6}$$

Where, V represents the market value of a completed project, μ is the market required rate of return from the project, σ represents the volatility of the rate of return of the project value, and dz is an increment to a standard Wiener process. Through this step, we will assume a structure for the dynamics and uncertainties of the underlying risky asset "project value."

6.6.3.2 Finding the Initial Project Value "V_I"

In real option analysis the initial underlying asset value is the present value of the expected cash flows, which consist of all the revenues and expenditures generated from the investment, excluding the initial investment cost in the project. Then, by discounting these future cash flows at a proper discount rate to the present, the initial project value can be estimated. In a general real option model, the annual expected free cash flows FCF_i can be easily calculated by deducting the annual variable and fixed cost from the total annual revenue if we know the annual revenues and the total annual variable and fixed costs for the project during the operation period. Then, by assuming r is a risk-adjusted discount rate and FCF_i is the future free cash flows at year i, the initial underlying asset value, V_I, can be estimated as Equation (3.90) in Section 3.2.6.1. However, since this research focuses on the BOT developers and the governments' points of view, the dynamics of the project value on equity investment should be dealt with and the initial project value used in this research is initial project value on equity investment. So, Equation (3.90) can be adjusted as following.

$$V_I = \sum_{i=1}^{n} \frac{FCFe_i}{(1 + R_e)^i} \tag{6.7}$$

Where, $FCFe_i$ is the free cash flow on equity at year i and R_e is the cost of equity. $FCFe_i$ can be obtained by deducting the annual debt service from the annual free cash flows FCF_i.

6.6.3.3 Selection of Volatility "σ"

In usual real option analysis, we have easily found the volatility "σ" from the "Logarithmic Cash Flow Returns Approach," which is mentioned in Section 3.2.6.4. If volatility is calculated based only on the historic or future estimates of cash flow returns this method seems easy and valid and, for this reason, this method has been widely used in estimating the volatility of real assets in many industries. However, the volatility of the underlying asset, which is obtained by this approach, is not the same as the volatility which is gained from considering more detailed cash flow components (Copeland and Antikarov, 2001). This is why to calculate a more detailed level of volatility it is necessary to consider and model more decomposed cash flow components.

In this research, the Monte Carlo simulation approach is adopted to provide a more detailed level of volatility gained from considering the dynamics of detailed cash flow components that will significantly affect the dynamics of the underlying asset by thoroughly following the real option theory (Copeland and Antikarov, 2001).

1) Monte Carlo Simulation with a More Detailed Level of Cash Flow Components

The Monte Carlo simulation approach used in this research is able to combine many uncertainties into one uncertainty by running them through a spreadsheet. Having identified the uncertain variables, which are cash flow components; price, cost, quantity, etc, in the valuation model, the best stochastic process; mean reversion, geometric Brownian motion, etc. that fits the historical data is then used to forecast the expected future values of the variables. The mean and standard deviation of the cash flow components from the histogram or the standard deviation of the error terms in the cash flow components from the regression is used to define the level of uncertainty for each uncertain variable for the Monte Carlo simulation. When there is more than one uncertain variable, the correlation between the variables can be accounted for by using the correlation coefficient of the error term between the variables estimated from the regression. The stochastic variables are then incorporated into a static valuation model such as DCF analysis to simulate the rate of return of the gross present value of the project. Using any simulation software, the Monte Carlo simulation within the DCF analysis is then run on a number of iterations to calculate the standard deviation of the rate of returns, which is the volatility of the project value.

To calculate a more detailed level of volatility based upon a Monte Carlo simulation, we need to first identify the detailed cash flow components which consist of cash flow returns in the project. For example, in general industries, quantity and unit cost of the product can be the detailed cash flow components because they mainly generate the cash flow of the project by multiplying with each other. These components, which can be obtained from the historical or projected data, have their own pattern of change in their quantity and unit cost over time and, by drawing the histogram with each component, we can obtain the mean and standard deviation of each component. As for the BOT project case data, as shown in Section 6.5.3, key cash flow components, initial traffic volume and traffic volume growth rate, will be considered and these components are assumed to follow lognormal and normal distributions respectively with a specific mean and standard deviation value (Cheah and Liu, 2006).

Next, in a spreadsheet model, there would be a proper assumption for the expected toll rate per vehicle and its annual growth rate. The cash flow model used in method 1 will be used as a framework for the Monte Carlo process in the spreadsheet program and the basic assumptions related to the data aside from the two major cash flow components will follow those of method 1. Here, the mean and standard deviation value of each cash flow component is the same as that obtained from method 2. Then, to convert the uncertainties of the two cash flow components, initial traffic volume and traffic volume growth rate, into uncertainties of the rate of return in the project value, we will pursue the following steps with the professional Monte Carlo simulation program, "Crystal Ball," which is made by ORACLE (formerly Decisioneering, Inc) and working as a built-in program of the Microsoft Excel program.

2) Monte Carlo Simulation Process

Step 1. Define the assumptions of the Input Variables

In randomizing the cash flow components of the two detailed levels, initial traffic volume and traffic volume growth rate, to find the possible probability distribution of the cash flow that can occur in the real world, we need to choose an appropriate pattern for each cash flow component, which can show all the possibilities that each cash flow component can have and can be expressed as a form of the probability distribution. As the probability distribution of each cash flow component directly affects the probability distribution of the cash flow, this process needs to be done carefully.

In understanding the pattern and range of the cash flows, the basic information needed to randomize each cash flow component is mean, standard deviation, and the shape of the probability distribution that the each cash component data shows. With the historical data of projects that have similar characteristics to the BOT case study, or the projected data estimated by the developer of the BOT case study, by drawing the histogram of each cash flow component, we can have the mean, standard deviation, and the pattern of the probability distribution of each component.

Then, to randomize the cash flows, we choose the obtained mean, standard deviation, and pattern of the probability distribution of each cash flow component in the Crystal Ball program. In defining the input variables, there are various probability distribution choices, binomial, uniform, normal, and lognormal, that we can choose in the Monte Carlo program, and the probability distribution type will be selected based on the shape of the probability distribution we have from drawing the histogram of each cash flow component. However, in this research, the shapes of the cash flow components are assumed to follow the research of Cheah and Liu (2006). The initial traffic volume follows a lognormal distribution because it will never go below “0” and the combinations of the lognormal are themselves lognormal. And, it generally increases in subsequent years as in real cases of many other toll road projects. For the traffic volume growth rate, for simplicity, it is assumed to follow a normal distribution, which will be reduced to a specific rate until traffic volume reaches the capacity of the toll road. However, in reality, these two cash flow components would be sought and modified by the transportation modeling experts in establishing the cash flow model. As stated, we will define the mean, standard deviation, and shape of the probability distribution for each cash flow components, initial traffic volume and traffic volume growth rate, for the first year in the Crystal Ball program. Likewise, we then define the traffic volume growth rate and repeat the selection of the distribution for the second year by setting the mean equal to the expected value of the first year and the standard deviation equal to that of the first year. This process will be repeated until the end of the operating year.

Step 2. Define the Forecast Variable

In second step, we define the forecast variable whose distribution will be simulated by the Monte Carlo program. In this research, the forecast variable is the rate of return for the BOT project value. Strictly speaking, the standard deviation of this forecast variable is the volatility of the rate of return for the BOT project value, which is used to calculate up and down movements, u and d, and risk neutral probabilities, q and $1-q$, in a developed real option model. As the volatility needed is the volatility of the project returns not the cash flow returns, there should be a process to convert project values produced by the spreadsheet into rates of return by using the following relationship (Copeland and Antikarov, 2001):

$$V_t = V_0 \ e^{kt} \tag{6.8}$$

$$\ln V_t \ - \ \ln V_0 \ = \ kt \tag{6.9}$$

Where, k is the rate of return for the project, V_0 is the project value at time “0,” which can be computed by discounting the cash flows at a risk-adjusted discount rate, and V_t is the project value at time t. And, when $t=1$, Equation (6.9) becomes a simple transformation to convert between continuous random draws of project value estimates in a Monte Carlo simulation and the standard deviation of the rate of return. Based upon Equation (6.8) and (6.9), the rate of project value change from one time period to the next as shown in the following:

$$z = \ln\left(V_1 + FCF \right) - \ln\left(V_0 \right) \tag{6.10}$$

$$V_1 = \sum_{t=2}^{i} \frac{FCF_t}{(1+r)^{t-1}} \tag{6.11}$$

Where, r is the risk-adjusted discount rate. As shown in Equations (6.10) and (6.11), we can easily understand that "z" is the rate of return in the project, which is obtained from the project value change not from the cash flow change. z can be calculated using the present value at time "0", V_0, as a denominator and the present value at time 1, V_1, plus the free cash flow at time 1, FCF_1. Here, as project value and cash flow are the result of the simulation, the values of V_0, V_1, and FCF_1 have their own probability distributions. Therefore, z can provide the value of the mean and standard deviation of the growth rate in project value and the standard deviation here is the volatility, which we want. However, as this research focuses on the BOT developer and the governments' points of view, the project value and free cash flows are considered on the level of the equity. So, Equation (6.10) and (6.11) are considered in this research as follows:

$$z = \ln\left(V_1 + FCFe_1 \right) - \ln\left(V_0 \right) \tag{6.12}$$

$$V_1 = \sum_{t=2}^{i} \frac{FCFe_t}{(1+R_e)^{t-1}} \tag{6.13}$$

Where, $FCFe_i$ is the free cash flow on equity at year i and R_e is the cost of the equity.

Step 3. Run the Monte Carlo Simulation

Upon finishing these processes, we will run the Monte Carlo simulation program. For this research, the Crystal Ball software built into Microsoft Excel will be used. After iterations using the parameters that we choose, we will get the mean return, the annual standard deviation, and the frequency distribution of the annual rates of return.

6.6.3.4 Up and Down Movements, "*u*" and "*d*" and Risk Neutral Probability, "q" and "$1-q$"

We then calculate the value of the up and down movement *u* and *d* that will be multiplied by the initial project value V_I to reflect the up/down movement of the project value V. Under the binomial tree framework, *u*, *d*, and R are needed in order to compute the risk neutral probabilities $q = (R-d)/(u-d)$ and $1-q = (u-R)/(u-d)$. Here, R equals to $e^{r\Delta t}$. By imposing *u* = 1/*d*, for convenience, the jump amplitudes may be found from (Cox et al., 1979):

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6.14}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{6.15}$$

$$q = \frac{R-d}{u-d} = \frac{e^{r\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} \tag{6.16}$$

$$1-q = \frac{u-R}{u-d} = \frac{e^{\sigma\sqrt{\Delta t}} - e^{r\Delta t}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} \tag{6.17}$$

Equation (6.14) and (6.15) show that the up and down movements are determined by the risky variable's volatility and Δt. As explained in Section 3.2.5.3, in this research, we will alternatively obtain *u* and *d* by imposing a fixed pseudo probability, *q* = 0.5, for the convenience of the calculation. Then, the up and down movements are given by (Hull, 1997):

$$u \;=\; e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}} \tag{6.18}$$

$$d \;=\; e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}} \tag{6.19}$$

$$q \;=\; \frac{R-d}{u-d} = \frac{e^{r\Delta t} - e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}}}{e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}} - e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}}} \;=\; 0.5 \tag{6.20}$$

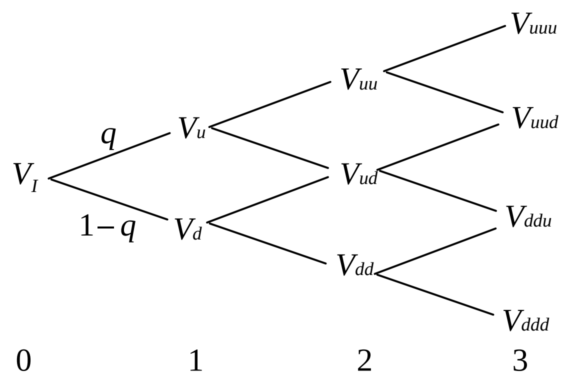
$$1-q \;=\; \frac{u-R}{u-d} = \frac{e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}} - e^{r\Delta t}}{e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}} - e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}}} \;=\; 0.5 \tag{6.21}$$

Although we could get the values of u and d from Equations (6.16) and (6.17), we will use 0.5 for risk neutral probabilities q and $1-q$ since this procedure help the risk neutral probabilities remain at 0.5 regardless of the value of σ or the number of time steps Δt so that we can compute a huge binomial tree which has a lot of time steps (Hull, 1997).

6.6.3.5 Construct a Reverse Binomial Tree with an Underlying Asset "V_I"

The next step is to construct the binomial tree with up and down movements of u and d calculated in the previous step. And here, initial project value V_I will be taken from Section 6.6.3.2. With up and down movements of u and d and an initial project value of V_I, if we construct three step binomial trees, it is shown in Figure 6.2.

Figure 6.2 Binomial Tree of Underlying Asset, V



The binomial tree in Figure 6.2 includes all the possible project values that the initial project value V_I can have considering the uncertainties over time. So, the project values shown in this binomial tree can be considered as "projected (or realized) project values." The change of this project value over time is as follows based on the up movement "u" and down movement "d":

$$\text{At } t = 1 \qquad V_u = u \cdot V_I = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.22}$$

$$V_d = d \cdot V_I = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.23}$$

$$\text{At } t = 2 \qquad V_{uu} = u^2 \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.24}$$

$$V_{ud} = u \cdot d \cdot V_I = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.25}$$

$$V_{dd} = d^2 \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.26}$$

$$\text{At } t = 3 \qquad V_{uuu} = u^3 \cdot V_I = e^{3\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.27}$$

$$V_{uud} = u^2 \cdot d \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.28}$$

$$V_{ddu} = d^2 \cdot u \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.29}$$

$$V_{ddd} = d^3 \cdot V_I = e^{3\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \tag{6.30}$$

6.6.3.6 Formulation of the MRG Agreement as a Put Option

This step is to identify the option that management can exercise. The MRG agreement is formulated as a put option. Cheah and Liu (2006) considered the fact that the amount of an MRG for each time step has to be the same as the difference between the guaranteed cash flow and simulated cash flow. Because if there is any shortfall that can cause dissatisfaction with regard to a BOT developer’s expected revenue, that much should be paid as an MRG. However, this research takes a position that is slightly different. If the projected (realized) project value at each time step is higher than that of guaranteed project value, there is no reason for the government to pay the MRG to developers because the project value is enough. But, if the realized project value is less than the guaranteed project value, there should be an MRG for the developers to be able to quit an adverse condition where they can not obtain the profit that was expected. This is the asymmetric payoff condition, formulated as a put option, which is applied in this research.

Here, unlike Cheah and Liu’s research that considers just the difference between projected, which can be called realized, cash flow and guaranteed cash flow, the idea applied in this model is that as the underlying asset, project value, is assumed to follow a specific random walk, a geometric Brownian motion process, with the level of the project value not with the level of the cash flow, it seems reasonable that the MRG agreement be formulated as the same unit with the project value. So, this research considers the guaranteed project value as an exercise price instead of the guaranteed cash flow, which is applied in Cheah and Liu's model. This transformation process from the guaranteed cash flow to the guaranteed project value is to consider the exercise price reflecting the uncertainty of the underlying asset to thoroughly follow the real option theory without violating it. The MRG agreement formulated as a put option is shown as:

$$MRG_i = Max\,[\,Guaranteed\ \ Project\ \ Value\ \ on\ \ Equity\ \ at\ Year\ \ i$$

$$- Projected\ \ (Realized)\ \ Project\ \ Value\ \ on\ \ Equity\ \ at\ Year\ \ i,\ 0\,] \tag{6.31}$$

As for the guaranteed project value, which is used as the exercise price, the basic assumption of the real option analysis is applied. In real option analysis, based on the assumption that the underlying asset follows a geometric Brownian motion process, we understand that the change of the underlying asset value has two major factors in its value change process: the term of the fixed rate of return and that of the uncertain rate of return, which is randomly selected at every time step over time. And if the initial project value increases at a growth rate of a certain fixed rate of return, which is defined in the geometric Brownian motion process, it can be considered that this value change represents the project value change without considering the uncertainty. For this reason, when the initial project value is assumed to increase at this fixed rate of return, since the initial project value already reflects all cash flows that the project will generate, this increasing project value represents the project value which this project will reach over time without uncertainty. Therefore, this increasing project value is assumed as the exercise price in this research. In the developed model, the guaranteed project value is a certain percentage of this increasing project value according to the BOT agreement (i.e., the guaranteed cash flow is 80 % of the expected cash flow).

Since this project value increases over time, the exercise price, which is the guaranteed project value, increases. Hence, this real option model has a varying exercise price at every time step. The MRG option can be exercised in every time step as long as the conditions are met and the exercise price at every time step is as follows:

$$\text{At } t = 1 \quad X_1 = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t\right)} \cdot V_I \tag{6.32}$$

$$\text{At } t = 2 \quad X_2 = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t\right)} \cdot V_I \tag{6.33}$$

$$\text{At } t = 3 \quad X_3 = e^{3\cdot\left(\left(r-\frac{1}{2}\right)\Delta t\right)} \cdot V_I \tag{6.34}$$

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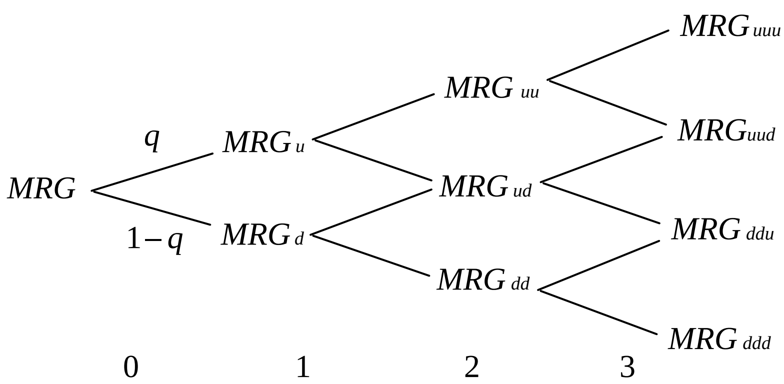
$$\text{At } t = n \quad X_n = e^{n\cdot\left(\left(r-\frac{1}{2}\right)\Delta t\right)} \cdot V_I \tag{6.35}$$

Where, X_n is the exercise price at time n.

6.6.3.7 Asymmetric Payoff Condition at Each Node in the Binomial Tree

Based on the project value, exercise price, and asymmetric condition, we can construct the following asymmetric conditions as shown in Figure 6.3.

Figure 6.3 MRG Option Value and Asymmetric Payoff Condition in Binomial Tree



Where,

At $t = 1$ $$MRG_u = Max\left[\ X_1 - V_u\ ,\ 0\ \right] \tag{6.36}$$

$$MRG_d = Max\left[\ X_1 - V_d\ ,\ 0\ \right] \tag{6.37}$$

At $t = 2$ $$MRG_{uu} = Max\left[\ X_2 - V_{uu}\ ,\ 0\ \right] \tag{6.38}$$

$$MRG_{ud} = Max\left[\ X_2 - V_{ud}\ ,\ 0\ \right] \tag{6.39}$$

$$MRG_{dd} = Max\left[\ X_2 - V_{dd}\ ,\ 0\ \right] \tag{6.40}$$

At $t = 3$ $$MRG_{uuu} = Max\left[\ X_3 - V_{uuu}\ ,\ 0\ \right] \tag{6.41}$$

$$MRG_{uud} = Max\left[\ X_3 - V_{uud}\ ,\ 0\ \right] \tag{6.42}$$

$$MRG_{ddu} = Max\left[\ X_3 - V_{ddu}\ ,\ 0\ \right] \tag{6.43}$$

$$MRG_{ddd} = Max\left[\; X_3 - V_{ddd}\;, 0\;\right] \tag{6.44}$$

6.6.3.8 Implementing the Calculation Backward Recursively

It is time to start calculating the MRG option from the end of the binomial tree backward recursively in Figure 6.3. In this process, the selected option value based on the asymmetric payoff functions at each node from the end of binomial tree to the present will be calculated backwards recursively by using risk neutral probabilities $q = (R-d)/(u-d)$, $1-q = (u-R)/(u-d)$, and discount factor R which is equal to $e^{r\Delta t}$.

For example, in Figure 6.3, if we find the option value at time 2 when the MRG option value is MRG_{uu}, its process will be as follows. First, we will consider two cases. One is when an MRG option is exercised and the other is when a MRG option is not exercised. As mentioned earlier, because the real option analysis will find the maximized value in each time step, we will choose the larger value between the cases when the option is exercised and not exercised. Then, the higher value between the two cases will be the MRG option value at time 2 and this process will be iterated in each time step from the right end to the left end of the real option binomial tree. In detail, if we find the MRG option value when the MRG agreement is not exercised, the calculation to find the option value of MRG_{uu} will be as follows. When the option is not exercised, its value is:

$$MRG_{uu\,(not\;\;exercised\,)} = \frac{\left[\; q\cdot MAX\left(X_3 - V_{uuu}\,,\,0\right) + \left(1-q\right)\cdot MAX\;\left(X_3 - V_{uud}\,,\,0\right)\;\right]}{e^{r\Delta t}} \tag{6.45}$$

And, when being exercised:

$$MRG_{uu\,(exercised\,)} = MAX\;\;\left[\;\; X_2 - V_{uu}\;\;,\;\;0\;\;\right] \tag{6.46}$$

Finally, here, because the larger option value between two values at time 2 has to be chosen, its value will be as follows.

$$MRG_{uu} =$$

$$MAX\left[\frac{q\,MAX\,\left[X_3 - V_{uuu}\ ,\ 0\right] + \left(1-q\right)MAX\,\left[X_3 - V_{uud}\ ,\ 0\right]}{e^{r\Delta t}},\ MAX\,\left[X_2 - V_{uu}\ ,\ 0\right]\right] \tag{6.47}$$

Through iterations of this process at all nodes for every time step backward recursively, at the end, we can find the value of MRG at time “0”. The following shows every asymmetric condition and MRG option value which can be obtained at each node in the binomial tree and process to calculate the final MRG option value at time 0.

At $t = 3$

$$MRG_{uuu} = Max\left[\ X_3 - V_{uuu}\ ,\ 0\ \right] \tag{6.48}$$

$$MRG_{uud} = Max\left[\ X_3 - V_{uud}\ ,\ 0\ \right] \tag{6.49}$$

$$MRG_{ddu} = Max\left[\ X_3 - V_{ddu}\ ,\ 0\ \right] \tag{6.50}$$

$$MRG_{ddd} = Max\left[\ X_3 - V_{ddd}\ ,\ 0\ \right] \tag{6.51}$$

At $t = 2$

$$MRG_{uu} = Max\left[\frac{q\,MRG_{uuu} + (1-q)\,MRG_{uud}}{e^{r\Delta t}}\ ,\ X_2 - V_{uu}\right]$$

$$= Max\left[\frac{qMax\,[X_3 - V_{uuu}\,,\,0] + (1-q)\,Max\,[\,X_3 - V_{uud}\,,\,0]}{e^{r\Delta t}}\ ,\ X_2 - V_{uu}\right]$$

(6.52)

$$MRG_{ud} = Max\left[\frac{q\,MRG_{udu} + (1-q)\,MRG_{udd}}{e^{r\Delta t}}\ ,\ X_2 - V_{ud}\right]$$

$$= Max\left[\frac{qMax\,[\,X_3 - V_{udu}\,,\,0] + (1-q)\,Max\,[X_3 - V_{udd}\,,\,0]}{e^{r\Delta t}}\ ,\ X_2 - V_{ud}\right]$$

(6.53)

$$MRG_{dd} = Max\left[\frac{q\,MRG_{ddu} + (1-q)\,MRG_{ddd}}{e^{r\Delta t}}\ ,\ X_2 - V_{dd}\right]$$

$$= Max\left[\frac{qMax\,[X_3 - V_{ddu}\,,\,0] + (1-q)\,Max\,[X_3 - V_{ddd}\,,\,0]}{e^{r\Delta t}}\ ,\ X_2 - V_{dd}\right]$$

(6.54)

At $t = 1$

$$MRG_{u} = Max\left[\frac{q\,MRG_{uu} + (1-q)\ MRG_{ud}}{e^{r\Delta t}}\ ,\ X_1 - V_u\right]$$

$$= Max\left[\frac{qMax\,[X_2 - V_{uu}\,,\,0] + (1-q)\,Max\,[\,X_2 - V_{ud}\,,\,0]}{e^{r\Delta t}}\ ,\ X_1 - V_u\right]$$

(6.55)

$$MRG_{d} = Max\left[\frac{q\,MRG_{du} + (1-q)\,MRG_{dd}}{e^{r\Delta t}}\ ,\ X_1 - V_d\right]$$

$$= Max\left[\frac{qMax\,[X_2 - V_{du}\,,\,0] + (1-q)\,Max\,[\,X_2 - V_{dd}\,,\,0]}{e^{r\Delta t}}\ ,\ X_1 - V_d\right]$$

(6.56)

At $t = 0$

MRG

$$= \frac{q\,MRG_u + (1-q)\,MRG_d}{e^{r\Delta t}}$$

$$= \frac{qMax\left[\frac{q\,MRG_{uu} + (1-q)\,MRG_{ud}}{e^{r\Delta t}}, X_1 - V_u\right] + (1-q)Max\left[\frac{qMRG_{du} + (1-q)\,MRG_{dd}}{e^{r\Delta t}}, X_1 - V_d\right]}{e^{r\Delta t}}$$

$$= \frac{\left[\begin{array}{l} qMax\left[\frac{qMax\left[\frac{qMRG_{uuu}+(1-q)MRG_{uud}}{e^{r\Delta t}}, X_2 - V_{uu}\right] + (1-q)Max\left[\frac{q\,MRG_{udu}+(1-q)MRG_{udd}}{e^{r\Delta t}}, X_2 - V_{ud}\right]}{e^{r\Delta t}}, X_1 - V_u\right] + \\ (1-q)Max\left[\frac{qMax\left[\frac{qMRG_{udu}+(1-q)MRG_{udd}}{e^{r\Delta t}}, X_2 - V_{ud}\right] + (1-q)Max\left[\frac{qMRG_{ddu}+(1-q)MRG_{ddd}}{e^{r\Delta t}}, X_2 - V_{dd}\right]}{e^{r\Delta t}}, X_1 - V_d\right] \end{array}\right]}{e^{r\Delta t}}$$

$$= \frac{\left[\begin{array}{l} qMax\left[\frac{\left[\begin{array}{l} qMax\left[\frac{qMax[X_3 - V_{uuu}, 0] + (1-q)Max[X_3 - V_{uud}, 0]}{e^{r\Delta t}}, X_2 - V_{uu}\right] + \\ (1-q)Max\left[\frac{qMax[X_3 - V_{udu}, 0] + (1-q)Max[X_3 - V_{udd}, 0]}{e^{r\Delta t}}, X_2 - V_{ud}\right] \end{array}\right]}{e^{r\Delta t}}, X_1 - V_u\right] + \\ (1-q)Max\left[\frac{\left[\begin{array}{l} qMax\left[\frac{qMax[X_3 - V_{uud}, 0] + (1-q)Max[X_3 - V_{ddu}, 0]}{e^{r\Delta t}}, X_2 - V_{ud}\right] + \\ (1-q)Max\left[\frac{qMax[X_3 - V_{ddu}, 0] + (1-q)Max[X_3 - V_{ddd}, 0]}{e^{r\Delta t}}, X_2 - V_{dd}\right] \end{array}\right]}{e^{r\Delta t}}, X_1 - V_d\right] \end{array}\right]}{e^{r\Delta t}}$$

(6.57)

7. CASE STUDY

7.1 The Ma-Chang Bridge (MCB) Project

7.1.1 Background

From Mokpo to Pusan in South Korea the Masan gulf crossing project was planned in order to play a role as an alternative to the second national highway and to solve the expected traffic and logistics demand for local economic development. The Masan gulf crossing project, which is 10.47 km in length, and a total of $ 699 million will be invested connecting between Woosan-Dong in Masan city and Guisan-Dong in Changwon city.* This project has two components. The first is the Ma-Chang Bridge construction project and the second is the road construction project to connect both ends of the Ma-Chang Bridge. This project began in 2004 and completion is expected in 2008.

The Ma-Chang Bridge is the first project in South Korea, to be suggested by the private, 'Hyundai Construction Co., Ltd.', as a BOT/BTO type privatized infrastructure project to the government, Gyeongsangnam Provincial Government. The Gyeongsangnam Provincial Government contracted with Hyundai Construction Co., Ltd. and Bouygues Co., Ltd. (France) which are equity investors in the Masan-Changwon Bridge (MCB) Co., Ltd. in 2002. The Ma-Chang Bridge linking between Gapo-Dong, Happo-Gu in Masan city and Guisan-Dong in Changwon city is cable-stayed girder bridge of 1.7 km in length and consisting of 4 lanes. In 2002, the Gyeongsangnam Provincial Government assigned the MCB Co., Ltd. as a Special Purpose Vehicle (SPV) and the MCB Co., Ltd. is expected to complete the construction, operate this bridge during concession period based upon the concession agreement between MCB Co., Ltd. and Gyeongsangnam Provincial Government and transfer the ownership to Gyeongsang-

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*In this research, three BOT project case studies are planned to finance with Korean currency (KRW). However, for the convenience of understanding, the Korean currency is converted into U.S dollar ($ 1 = KRW 950 as of March in 2008) (Source: http://www.bok.or.kr/index.jsp).

nam Provincial Government upon expiration of the concession period. Here, Hyundai Construction Co., Ltd. and Bouygues Co., Ltd. invested total $ 59 million (50 : 50) on MCB Co., Ltd as equity investors. MCB Co., Ltd. will operate and take the profit from this bridge during 30 years with a toll rate of about $ 2.10 per vehicle.

Besides the bridge construction, the connecting road, which totals 8.77 km in length (Masan direction 5.58 km and Changwon direction 3.19 km), links Woosan-Dong in Masan city and Yanggok-Dong in Changwon city. Both ends of this bridge will be completed with the support of $ 413.90 million from the Gyeongsangnam Provincial Government. Unlike the Ma-Chang Bridge, the construction of this connecting road will be conducted by Masan city, Changwon city, and Gyeongsangnam Provincial Government as a public project. The connecting road in the direction of Masan, which is 3.19 km, will include Gapo Interchange and Gapo Tunnel (1.2 km) and in the direction of Changwon which is 5.58 km, will include the Guisan interchange, Guisan tunnel (0.35 km) and Yanggog tunnel (1.00 km).

Upon the completion of this project, it will contribute to solving the chronic traffic congestion problem that is present on the second national road between Woosan-Dong in Masan city and Guisan-Dong in Changwon city by reducing the physical length of the road from 16.2 km to 9.2 km, and the driving time from 37 minutes to 7 minutes. This will help Masan and Changwon city share and strengthen the urban connection with each other thereby improving the quality of life for the people who live in this area. Finally, the completion of this bridge is supposed to give Geongsangnam Provincial Government comprehensive solutions for the appropriate distribution of existing traffic, for the new explosive traffic demand accompanied by Masan harbor and new city development projects, an improved logistics system, and an attractive tourism opportunity as a beautiful landmark with excellent scenery lighting.

7.1.2 Contractual Structure

The Ma-Chang Bridge, which is the first project suggested by the private companies; Hyundai Construction Co., Ltd. and Bouygues Co., Ltd., in Korea is considered a huge infrastructure project amounting to about $ 316 million in total project size. In 2002, Gyeongsangnam Provincial Government signed the concession agreement with the project equity investors, Hyundai Construction Co., Ltd. and Bouygues Co., Ltd., and selected MCB Co., Ltd, formed by these two equity investors as a Special Purpose Vehicle (SPV).

To support the construction cost, the contractual structure was considered in forms of equity, senior debt, and subordinated debt from various funding sources. In the construction cost of about total $ 316 million, MCB Co., Ltd. (Hyundai Construction Co., Ltd. 50 % and Bouygues Co., Ltd. 50 %) will account for 18.7 % of construction cost, $59 million, and 80 % of the remaining construction cost, $ 206 million, will be provided at a fixed interest rate of 8.11 % during 15 years by leading Korean financial institutions such as Kookmin Bank Co., Ltd., Kyobo Life Insurance Co., Ltd., Korean Life Insurance Co., Ltd., and Korea Credit Guarantee Fund in the form of senior debt. The low interest rates trend, which has been the result of an interaction between the cheap money policy by the Korean government after a financial crisis in 1997, a stable price level, and a strong Korean currency, enables these financial institutions to provide the debt service at a relatively low interest rate. Moreover, the senior debt guarantee agreement by the Gyeongsangnam Provincial Government makes this low interest rate possible.

Finally, the outstanding amount of construction cost, $ 51 million, will be supported by the Korea Road Infrastructure Fund (KRIF) planned from the Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. as a type of subordinated debt with a tenure of 20 years and a fixed rate of 20%. Finally, as the MKIF Co., Ltd. will buy the equity portion of the MCB Co., Ltd. up to 49% during the construction period and up to 100 % when the construction is completed under the agreement, the total amount MKIF Co., Ltd. will invest in this project is about $ 106.3 billion in equity and subordinated debt. When it comes to the debt payment, this financing is planned with the type of Project Finance (PF) loan as non-recourse where there is no guarantee for debt payment by equity investors. Then, during the concession period, Hyundai Construction Co., Ltd.

and Bouygues Co., Ltd. seconded by Gyeongsangnam Provincial Government will have all rights, liabilities and obligations related to Ma-Chang Bridge project under the concession agreement to receive tolls in return for designing, constructing, operating this bridge (approximately 1.7 km). The fixed interest rate loan would be repaid out of following sources:

(1) Tolls will be collected through the 30-year concession to operate the project with its operation starting in 2008.

(2) The Gyeongsangnam Provincial Government will provide MCB Co., Ltd. with the revenue guarantee if the expected revenue that MCB Co., Ltd. estimated is not reached. The guarantee will be considered from 2008 to 2037 for 30 years at the rate of 80% of the expected revenue.

(3) There is a repayment option that the MCB Co., Ltd. will pay to the Gyeongsangnam Provincial Government in case that the actual revenue will surpass far beyond the expected revenue that MCB Co., Ltd. estimated. The repayment option will continue during 30 years from the beginning of operation at the rate of 120 % of the expected revenue.

The Ma-Chang Bridge is the first huge infrastructure project in South Korea, which has been suggested by a private company, and in which the investors joined from the construction process. Moreover, when it comes to the Hyundai Construction Co., Ltd., because it owns 50% of the equity in the MCB Co., Ltd. consortium and is in charge of construction of about 50 % of Ma-Chang Bridge, 30% of the connecting road into the Masan direction, and 40 % into the Changwon direction, it will take the profit related to the construction as well as income through operating Ma-Chang Bridge during the concession period.

7.1.3 Financial Analysis and Cash Flow Model

The following key points have been obtained from data set given by the Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. and Macquarie Shinhan Infrastructure Asset Management (MSIAM) Co., Ltd. with reasonable assumptions.

(1) The total cost of the project and the cost of the equity investment total about $ 316 million and $ 59 million respectively, and will be assumed to be used during the first year of the construction period.

(2) The debt to equity ratio is 81.3:18.7. The loan terms are 15 years and 20 years for senior and subordinated debt respectively. Their interest rates are 8.11% and 20% respectively.

(3) The concession period is 30 years.

(4) Total Capital expenditures are around $ 2.21 million. This amount which is about 7% of the total construction cost will be evenly distributed with $ 3.66 million for every 5 years during concession period and escalated with the same growth rate as inflation rate which is 3%. Operating expenditure totals $ 75.9 million. This amounts to 23.8% of total construction cost and will be evenly distributed with $2.53 million for every year during concession period. It escalates at 3 % annually.

(5) The corporate tax rate is 27.5 %.

(6) Under the MCB Co., Ltd.'s proposal, the initial toll rates are $ 2.10 for cars, $ 2.63 for buses, $ 3.16 for vans, and $ 4.21 for lorries, respectively. The average toll rate is $3.00 and would escalate at inflation rate of 3 % annually. The class of vehicle, its portion against total traffic volume, and initial toll rate are as following Table 7.1.

Table 7.1 Initial Toll Rates for Different Vehicle Classes in the MCB Project

Vehicle Class	Toll Rate ($)	Proportion (%)
Cars	2.10	20
Buses	2.63	20
Vans	3.16	25
Lorries	4.21	25
Taxies	2.10	10

(7) The distribution of the initial traffic volume (in 2008) is assumed to follow a lognormal distribution based on the assumptions mentioned earlier (Cheah and Liu, 2006). The original traffic volume projection, 8.61 million vehicles, estimated by the private consortium, MCB Co., Ltd. will be taken as the mean value of the initial traffic volume variable. In practice, inputs from transportation modeling experts would be sought, and suitable modifications can even be made in the cash flow model. The following Table is the annual traffic volume and traffic volume growth rate estimated by MCB Co., Ltd.

Table 7.2 Annual Traffic Volume and Traffic Volume Growth Rate Estimated by the MCB Co., Ltd. (Traffic Volume: Million, Growth Rate: %)

Year	***2008***	***2009***	***2010***	***2011***	***2012***	***2013***	***2014***	***2015***	***2016***	***2017***	***2018***	***2019***	***2020***	***2021***	***2022***	***2023***
Growth Rate	-	4.10	3.83	3.58	3.86	4.29	3.29	3.18	2.82	3.08	2.74	1.95	1.83	1.88	1.77	1.96
Traffic Volume	8.61	8.97	9.32	9.66	10.04	10.48	10.83	11.18	11.50	11.86	12.19	12.43	12.66	12.90	13.13	13.39

Year	***2024***	***2025***	***2026***	***2027***	***2028***	***2029***	***2030***	***2031***	***2032***	***2033***	***2034***	***2035***	***2036***	***2037***	***2038***
Growth Rate	1.92	1.81	1.78	1.82	1.79	1.35	1.40	1.38	1.36	1.41	1.39	1.37	1.41	1.39	1.37
Traffic Volume	13.65	13.90	14.15	14.41	14.67	14.87	15.08	15.29	15.50	15.72	15.94	16.16	16.39	16.62	16.85

(8) The traffic volume increases in subsequent years and the annual traffic volume growth rate will be obtained by annual traffic volume data estimated by MCB Co., Ltd (Table 7.2). From the first operation year (2008), the traffic volume will gradually grow. And, the traffic volume growth rate that MCB Co., Ltd. used in its cash flow model for

financial feasibility analysis will be used as one of the most important risky variables with initial traffic volume and can be gained from the expected traffic volume estimated by MCB Co., Ltd. during the concession period. Table 7.2 also shows the traffic volume growth rate according to the annual traffic volume data.

(9) With the estimated traffic volume and traffic volume growth rate data, we can obtain the necessary parameters; mean and standard deviation of traffic volume and traffic volume growth rate. Later on, these parameters will be used to reflect the uncertainty of project returns in method 2 and 3. Table 7.3 illustrates the distributions and parameters chosen for two risky variables based on Table 7.2.

Table 7.3 Probability Distribution of Two Input Variables in the MCB Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal	Normal
Mean	8.61	2.30
Standard Deviation	2.38	0.989

(10) Given MCB project data, a cash flow model will be evaluated from the project equity investors' point of view. Therefore, the relevant interest rate which will be used to discount future cash flows is the required rate of return on equity, or cost of equity, which can be estimated using CAPM. To calculate the private consortium's required rate of return based upon CAPM, we will use following formula.

$$R_e(Cost\ of\ \ Equity) = R_f + (MRP \times \beta) \tag{7.1}$$

R_f : *Risk-Free Rate*

$MRP = R_m - R_f$: *Market Risk Premium*

β : *Sector Beta*

R_e : *Cost of Equity*

1. The risk-free rate we will use is the 10-year Korean Treasury bill rate which is 5.3 % in March 2008 based on the Bond Information Service (BIS) of the Korea Securities Dealers Association (KSDA). (Source: http://www.ksdabond.or.kr)

2. Market risk premium, *MRP*, is the difference between the return of the market, Korea Composite Stock Price Index (KOSPI) and the risk-free rate (Treasury bills). The overall market rate of return will be used to measure *MRP*. From 1990 to 2005, the KOSPI averaged yearly returns of 10.40 % according to the Korea Securities Dealers Association (KSDA). So, *MRP* is 10.40 % – 5.3 % = 5.1 %. (Source: http://www.ksdabond.or.kr)

3. Beta, β, is a measure of risk for a certain industry or company. So, it can be calculated with the weighted average beta of construction companies, which join as equity investors, in proportion to each company's equity investment. In this case, since the firms, Hyundai Construction Co., Ltd. and Bouygues Co., Ltd., have betas of 1.45 and 1.22, respectively, the average of two betas is 1.335 (Source: http://kr.stock.yahoo.com). If the company is not listed in stock market, the average beta of construction industry sector, which is 1.20 (in 2007), in market, can be alternatively used (Source: http://kr.stock.yahoo.com).

$$\begin{aligned} R_e(Cost\ \ of\ \ Equity) &= R_f + (MRP \times \beta) \\ &= 5.3\% + (5.1 \times 1.335) \qquad (7.2) \\ &= 12.11\ \% \end{aligned}$$

So, MCB Co., Ltd.'s required rate of return is 12.11 %.

7.1.4 Implementation of the BOT Project Valuation

7.1.4.1 Method 1 - NPV Analysis

1) Financial Model

Based on the data set obtained from the earlier processes, the cash flow model is constructed to capture a comprehensive picture of the financial feasibility of the project and is summarized in Figure 7.1.

Figure 7.1 Cash Flow Model of the MCB Project by Method 1

(M: Million / $: Dollar)

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012
Traffic Volume Growth Rate (%)									
Traffic Volume (M)					8.61	8.97	9.32	9.66	10.04
Toll Rate ($)					3.00	3.09	3.18	3.28	3.38
Gross Revenue (M, $)					26	28	30	32	34
CAPEX (M, $)	59								4.14
OPEX (M, $)					2.53	2.61	2.68	2.76	2.85
EBIT (M, $)	-59				23.30	25.11	26.98	28.90	26.91
Senior Debt Service (M, $)					24.27	24.27	24.27	24.27	24.27
Sub Debt Service (M, $)									
Taxes (M, $)					-0.27	0.23	0.75	1.28	0.73
FCF on Equity (M, $)	-59				-0.70	0.61	1.97	3.36	1.92

Year	2013	2014	2015	2016	2017	2018	2019	2020
Traffic Volume Growth Rate (%)								
Traffic Volume (M)	10.48	10.83	11.18	11.50	11.86	12.19	12.43	12.66
Toll Rate ($)	3.48	3.58	3.69	3.80	3.91	4.03	4.15	4.28
Gross Revenue (M, $)	36	39	41	44	46	49	52	54
CAPEX (M, $)					4.80			
OPEX (M, $)	2.93	3.02	3.11	3.20	3.30	3.40	3.50	3.61
EBIT (M, $)	33.51	35.77	38.14	40.50	38.32	45.75	48.12	50.54
Senior Debt Service (M, $)	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27
Sub Debt Service (M, $)						10.35	10.35	10.35
Taxes (M, $)	2.54	3.16	3.81	4.46	3.87	3.06	3.71	4.38
FCF on Equity (M, $)	6.71	8.34	10.06	11.77	10.19	8.07	9.78	11.54

Figure 7.1 (Continued)

(M: Million / $: Dollar)

Year	2021	2022	2023	2024	2026	2027	2028	2029
Traffic Volume Growth Rate (%)								
Traffic Volume (M)	12.90	13.13	13.39	13.65	14.15	14.41	14.67	14.87
Toll Rate ($)	4.41	4.54	4.67	4.81	5.11	5.26	5.42	5.58
Gross Revenue (M, $)	57	60	63	66	72	76	79	83
CAPEX (M, $)		5.57				6.45		
OPEX (M, $)	3.72	3.83	3.94	4.06	4.31	4.44	4.57	4.71
EBIT (M, $)	53.12	50.19	58.64	61.65	67.96	64.91	74.92	78.28
Senior Debt Service (M, $)	24.27	24.27						
Sub Debt Service (M, $)	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Taxes (M, $)	5.09	4.28	13.28	14.11	15.84	15.00	17.75	18.68
FCF on Equity (M, $)	13.41	11.29	35.01	37.19	41.76	39.56	46.81	49.25

Year	2030	2031	2032	2033	2034	2035	2036	2037
Traffic Volume Growth Rate (%)								
Traffic Volume (M)	15.08	15.29	15.50	15.72	15.94	16.16	16.39	16.62
Toll Rate ($)	5.75	5.92	6.10	6.28	6.47	6.66	6.86	7.07
Gross Revenue (M, $)	87	91	95	99	103	108	112	117
CAPEX (M, $)			7.48					8.67
OPEX (M, $)	4.85	4.99	5.14	5.30	5.46	5.62	5.79	5.96
EBIT (M, $)	81.84	85.54	81.90	93.45	97.67	102.07	106.71	102.86
Senior Debt Service (M, $)								
Sub Debt Service (M, $)	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Taxes (M, $)	19.66	20.67	19.68	22.85	24.01	25.22	26.50	25.44
FCF on Equity (M, $)	51.82	54.51	51.87	60.24	63.31	66.49	69.86	67.07

2) Project Evaluation

In this research the cash flow model will be analyzed from the equity investor and the government's points of view and this means that the project value needs to be observed against the level of the equity value. So, the risk-adjusted discount rate used to discount the free cash flow on equity is the cost of equity, R_e, which is 12.11 %. Then, we can calculate the NPV of this project without considering the MRG agreement value.

$$\text{NPV on Equity} = \frac{-59.00}{(1+0.12.11)^0} + \frac{0}{(1+0.1211)^1} + \frac{0}{(1+0.1211)^2} + ...+ \frac{69.860}{(1+0.1211)^{32}} + \frac{67.070}{(1+0.1211)^{33}}$$

$$= \$\ 6.11\ \mathit{Million} \tag{7.3}$$

As a result, we have the NPV of equity of $6.11 million. This value is called the static/passive NPV on equity since it does not consider the MRG agreement option. Then, the free cash flow on equity shown in Figure 7.1 will be used as guaranteed cash flows to build an asymmetric payoff condition in the cash flow models of method 2 and 3.

7.1.4.2 Method 2 - Cheah and Liu's Real Option Model

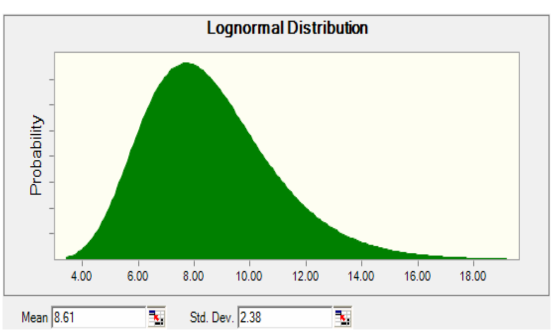
1) Key Variables of the Monte Carlo Simulation

In this BOT project, the uncertainties of the project value are affected by two detailed cash flow components; initial traffic volume and the traffic volume growth rate, which are assumed to follow a lognormal distribution and a normal distribution respectively. The detail of these two cash flow components is compiled in Table 7.4, Figure 7.2, and Section 7.1.3.

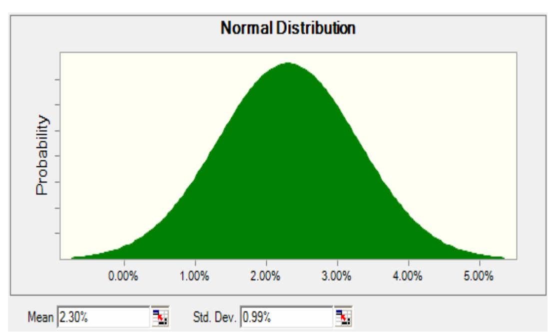
Table 7.4 Probability Distribution of Two Input Variables in Method 2

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal Distribution	Normal Distribution
Mean	8.61	2.3
Standard Deviation	2.38	0.989

Figure 7.2 Defining the Assumption of Two Input Variables in Method 2 at the MCB Project



1) Initial Traffic Volume



2) Traffic Volume Growth Rate

Figure 7.3 Cash Flow Model of the MCB Project by Method 2

(M: Million / $: Dollar)

Year	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Traffic Volume Growth Rate (%)						2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) - Expected					8.61	8.97	9.32	9.66	10.04
Traffic Volume (M) - Simulated					8.61	8.81	9.18	9.53	9.88
Toll Rate ($)					3.00	3.09	3.18	3.28	3.38
Gross Revenue (M, $) - Expected					25.83	27.72	29.66	31.67	33.90
Gross Revenue (M, $) - Simulated					25.83	27.22	29.21	31.26	33.37
CAPEX (M, $)									4.15
OPEX (M, $)					2.53	2.61	2.68	2.76	2.85
EBIT (M, $) - Expected					23.30	25.11	26.98	28.90	26.91
EBIT (M, $) - Simulated					23.30	24.61	26.52	28.49	26.37
Senior Debt Service (M, $)					24.27	24.27	24.27	24.27	24.27
Sub Debt Service (M, $)									
Taxes (M,$) – Expected					-0.27	0.23	0.75	1.28	0.73
Taxes (M,$) – Simulated					-0.27	0.09	0.62	1.16	0.58
Expected FCF on Equity (M, $)					-0.70	0.61	1.97	3.36	1.91
Guaranteed FCF on Equity (M, $)					-0.56	0.49	1.57	2.69	1.53
Simulated FCF on Equity (M, $)					-0.70	0.25	1.64	3.06	1.53
Cash Flow Difference (M, $)					0.48	2.24	0.05	0.00	0.10

Year	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**
Traffic Volume Growth Rate (%)	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) - Expected	10.48	10.83	11.18	11.50	11.86	12.19	12.43	12.66	12.90
Traffic Volume (M) - Simulated	10.27	10.72	11.08	11.44	11.76	12.13	12.47	12.72	12.95
Toll Rate ($)	3.48	3.58	3.69	3.80	3.91	4.03	4.15	4.28	4.41
Gross Revenue (M, $) - Expected	36.45	38.79	41.25	43.70	46.42	49.15	51.62	54.15	56.83
Gross Revenue (M, $) - Simulated	35.72	38.40	40.88	43.46	46.05	48.92	51.79	54.39	57.06
CAPEX (M, $)					4.81				
OPEX (M, $)	2.93	3.02	3.11	3.20	3.30	3.40	3.50	3.61	3.72
EBIT (M, $) - Expected	33.51	35.77	38.14	40.50	38.32	45.75	48.12	50.54	53.12
EBIT (M, $) - Simulated	32.79	35.38	37.77	40.26	37.94	45.52	48.28	50.78	53.34
Senior Debt Service (M, $)	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27
Sub Debt Service (M, $)						10.35	10.35	10.35	10.35
Taxes (M, $) – Expected	2.54	3.16	3.81	4.46	3.86	3.06	3.71	4.38	5.09
Taxes (M, $) - Simulated	2.34	3.06	3.71	4.40	3.76	3.00	3.76	4.44	5.15
Expected FCF on Equity (M, $)	6.71	8.34	10.06	11.77	10.19	8.07	9.78	11.54	13.41
Guaranteed FCF on Equity (M, $)	5.36	6.67	8.05	9.41	8.15	6.45	7.83	9.24	10.73
Simulated FCF on Equity (M, $)	6.18	8.06	9.79	11.60	9.92	7.90	9.91	11.72	13.57
Cash Flow Difference (M, $)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Year	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**
Traffic Volume Growth Rate (%)	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) - Expected	13.13	13.39	13.65	13.90	14.15	14.41	14.67	14.87
Traffic Volume (M) - Simulated	13.20	13.43	13.70	13.96	14.22	14.48	14.74	15.01
Toll Rate ($)	4.54	4.67	4.81	4.96	5.11	5.26	5.42	5.58
Gross Revenue (M, $) - Expected	59.58	62.58	65.71	68.92	72.27	75.80	79.49	82.99
Gross Revenue (M, $) - Simulated	59.88	62.78	65.94	69.24	72.62	76.15	79.87	83.75
CAPEX (M, $)	5.57					6.46		
OPEX (M, $)	3.83	3.94	4.06	4.18	4.31	4.44	4.57	4.71
EBIT (M, $) – Expected	50.18	58.64	61.65	64.74	67.96	64.91	74.92	78.28
EBIT (M, $) – Simulated	50.48	58.84	61.88	65.06	68.32	65.25	75.30	79.05
Senior Debt Service (M, $)	24.27							
Sub Debt Service (M, $)	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Taxes (M, $) – Expected	4.28	13.28	14.11	14.96	15.84	15.00	17.75	18.68
Taxes (M, $) – Simulated	4.36	13.33	14.17	15.04	15.94	15.10	17.86	18.89
Expected FCF on Equity (M, $)	11.28	35.01	37.19	39.43	41.76	39.55	46.81	49.25
Guaranteed FCF on Equity (M, $)	9.03	28.01	29.75	31.54	33.41	31.64	37.45	39.40
Simulated FCF on Equity (M, $)	11.50	35.15	37.36	39.66	42.02	39.80	47.09	49.80
Cash Flow Difference (M, $)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 7.3 (Continued)

(M: Million / $: Dollar)

Year	2030	2031	2032	2033	2034	2035	2036	2037
Traffic Volume Growth Rate (%)	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) - Expected	15.08	15.29	15.50	15.72	15.94	16.16	16.39	16.62
Traffic Volume (M) - Simulated	15.21	15.43	15.64	15.86	16.08	16.31	16.53	16.77
Toll Rate ($)	5.75	5.92	6.10	6.28	6.47	6.66	6.86	7.07
Gross Revenue (M, $) - Expected	86.68	90.53	94.52	98.74	103.13	107.69	112.50	117.50
Gross Revenue (M, $) - Simulated	87.44	91.34	95.39	99.60	104.04	108.67	113.47	118.54
CAPEX (M, $)			7.49					8.68
OPEX (M, $)	4.85	4.99	5.14	5.30	5.46	5.62	5.79	5.96
EBIT (M, $) – Expected	81.84	85.54	81.89	93.45	97.67	102.07	106.71	102.85
EBIT (M, $) – Simulated	82.60	86.35	82.76	94.30	98.59	103.05	107.68	103.89
Senior Debt Service (M, $)								
Sub Debt Service (M, $)	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Taxes (M, $) – Expected	19.66	20.67	19.67	22.85	24.01	25.22	26.50	25.44
Taxes (M, $) – Simulated	19.87	20.90	19.91	23.09	24.26	25.49	26.76	25.72
Expected FCF on Equity (M, $)	51.82	54.51	51.87	60.24	63.31	66.49	69.86	67.06
Guaranteed FCF on Equity (M, $)	41.46	43.60	41.49	48.19	50.64	53.19	55.89	53.65
Simulated FCF on Equity (M, $)	52.37	55.09	52.49	60.86	63.97	67.20	70.56	67.82
Cash Flow Difference (M, $)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

In Figure 7.3 of the cash flow model by method 2, the spreadsheet of the method 1 became a basic framework to construct the asymmetric payoff condition by deducting the simulated free cash flows on equity from guaranteed free cash flows on equity, which is 80 % of the expected free cash flows on equity. And, the cash flow model by method 2 in Figure 7.3 is the step to define the two important cash flow components; initial traffic volume and traffic volume growth rate, by setting up these two cash flow component cells and the free cash flow difference cell as input and forecast variables respectively for the Monte Carlo simulation approach.

When it comes to the “define assumption” of two input variables, the “Crystal Ball” software includes choices such as binomial, uniform, normal, and lognormal distributions and asks to choose an appropriate probability distribution for each cash flow component. In this step, as assumed earlier, we will select lognormal and normal distributions for the initial traffic volume and the traffic volume growth rate respectively. While choosing the probability distributions, we set up the values of the mean and standard deviation for each cash flow component based on the Table 7.4 and Figure 7.2. As the operation period is 30 years, we need to define the traffic volume growth rate variable for every year during the operation period. However, the initial traffic volume will be defined just for the first year of the operation.

2) Formulation of MRG agreement as a Put Option with Guaranteed Cash Flow Model

As mentioned earlier, the guaranteed free cash flow on equity by method 1 is used to compose the asymmetric payoff condition with the simulated free cash flow on equity in method 2. That is, the guaranteed free cash flow on equity plays a role as an exercise price in formulating the MRG agreement as a put option based on Section 6.5.2. In the spreadsheet of Figure 7.3, we define the asymmetric condition for each year that when the guaranteed free cash flow on equity is higher than the simulated free cash flow on equity, the MRG value for each year is the difference between two cash flows. Otherwise, the MRG is "0". To set this asymmetric condition up will be repeated until the end of the operation in year 2037.

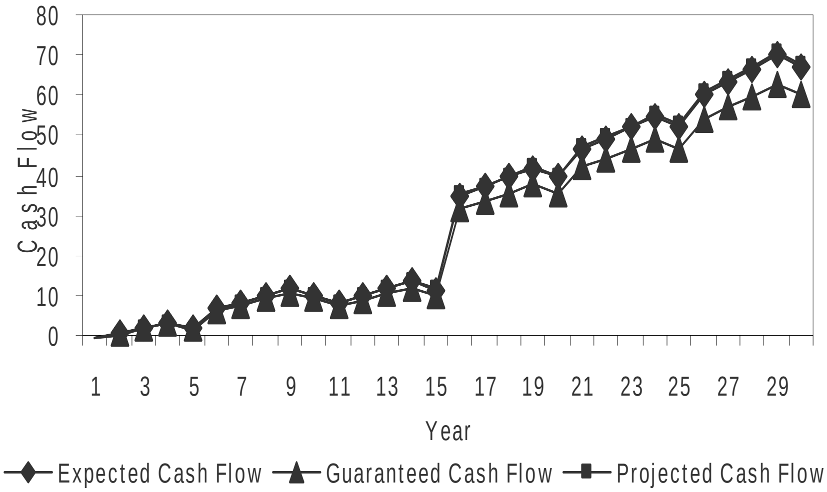
3) Define Forecast Variable

This is the step to define the forecast variable, which is the present value of the cash flow differences between the guaranteed and simulated free cash flows on equity and whose probability distribution will be simulated from the cash flow components of initial traffic volume and traffic volume growth rate by the Monte Carlo simulation process.

4) Run the Monte Carlo Simulation Program with Crystal Ball Software

We will run the Monte Carlo simulation with the Crystal Ball software which is built into the Microsoft Excel program. The iteration will be run 10,000 times and we can then see the value change of simulated free cash flow as the number of iterations increase while running the simulation. The result of the iteration process in this simulation provides the probability distribution of the simulated free cash flows. Figure 7.4 is the comparison of the cash flows of the MCB BOT Project.

Figure 7.4 Comparison of the Cash Flows of the MCB BOT Project



5) MRG Option Value

With the guaranteed free cash flow on equity by method 1 (80 % of the expected cash flow estimated by method 1) and simulated free cash flow on equity by the Monte Carlo simulation in method 2, we have the cash flow differences between these two cash flows for each year, and, by discounting the differences at risk free rate, presently 5.3 %, we finally obtain the following MRG option value in Figure 7.5 and Table 7.5.

Figure 7.5 MRG Value by Method 2 in the MCB Project

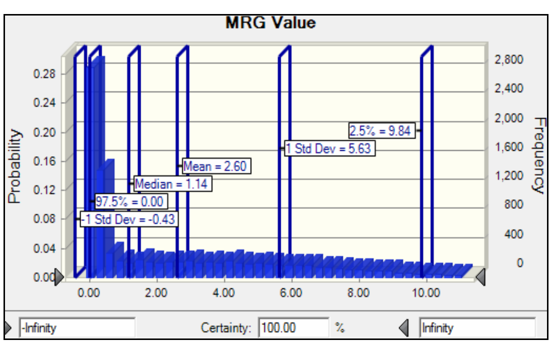


Table 7.5 MRG Value by Method 2 in the MCB Project

	MRG Value (Million, $)
Maximum	14.32
2.5 %	9.84
+ 1 · σ	5.63
Mean	2.60
Median	1.14
-1 · σ	-0.43
97.5 %	0.00
Minimum	0.00

Here, the mean and median values of MRG agreement are \$ 2.60 million and \$ 1.14 million respectively. These values seem to have some impact with respect to \$ 6.11 million which is the project value on equity from method 1 not considering the MRG option and \$ 59 million of initial equity investment.

7.1.4.3 Method 3 - Developed Real Option Model

1) Find the Initial Project Value V_I

As an initial project value that will be used in a binomial tree to reflect the dynamics of project value by following geometric Brownian motion process, the discounted free cash flows on equity are used. This is the present value of the expected free cash flows on equity, which consist of all the revenues and expenditures generated from the investment, excluding the initial investment cost in the project. Based on the cash flow model in Figure 7.1, V_I is estimated as following.

$$V_I = \sum_{i=1}^{n} \frac{FCFe_i}{(1+R_e)^i}$$

$$= \frac{-0.70}{(1+0.1211)^1} + \frac{0.61}{(1+0.1211)^2} + \frac{1.97}{(1+0.1211)^3} + \ldots + \frac{69.86}{(1+0.1211)^{29}} + \frac{67.07}{(1+0.1211)^{30}}$$

$$= \$\ 91.82\ \textit{Million} \tag{7.4}$$

Since the operation period is 30 years and the cost of equity R_e is 12.11 %, then, the initial project value V_I is \$ 91.82 million.

2) Selection of Volatility "σ"

To calculate a more detailed level of volatility, the Monte Carlo simulation approach is adopted by considering the dynamics of detailed cash flow components

(Copeland and Antikarov, 2001). The following is the Monte Carlo simulation process used to randomize the cash flows of the BOT project.

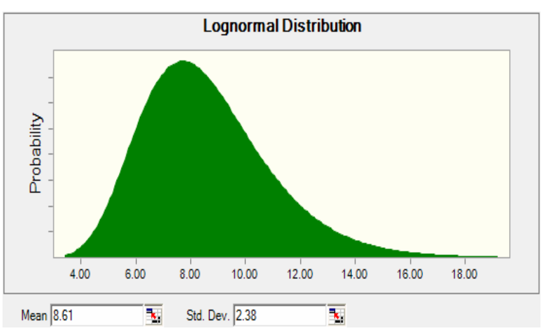
Step 1. Define the assumptions of Input Variables

In randomizing the detailed level of the two cash flow components; initial traffic volume and traffic volume growth rate, we need to have the basic information of the mean, standard deviation, and probability distribution of each cash flow component. And, by drawing the histogram of the expected cash flow component data estimated by the BOT developer, we can have these values. In the simulation process, we use the same mean, standard deviation, and probability distribution of each cash flow component in method 2 since method 2 also applies the Monte Carlo simulation approach to reflect the uncertainties of the cash flow components. Then, as shown in Table 7.6 and Figure 7.6, the process and the concept of defining the input variables follow the way the method 2 did.

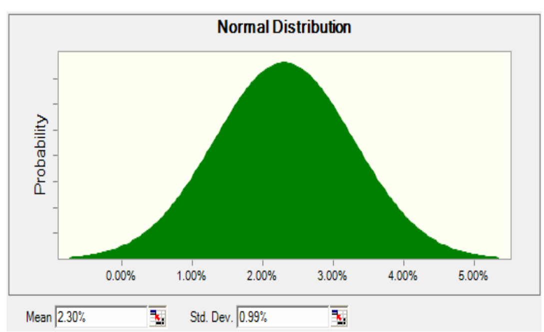
Table 7.6 Probability Distribution of Two Input Variables in Method 3 at the MCB Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal Distribution	Normal Distribution
Mean	8.61	2.3
Standard Deviation	2.38	0.989

Figure 7.6 Defining the Assumption of the Two Input Variables in Method 3 at the MCB Project



1) Initial Traffic Volume



2) Traffic Volume Growth Rate

Step 2. Define Forecast Variable

In the second step, we define the forecast variable cell of “z” in the spreadsheet, which is the rate of return in project value, whose distribution will be simulated by the Monte Carlo program. As the standard deviation of the “ z ” is the volatility of the project returns, which we need, and not from the cash flow returns, there should be a process to convert project values produced by the spreadsheet into rates of return by using the following relationship. Equation (7.5) shows this process. The volatility from one time period to the next is as following (Copeland and Antikarov, 2001).

$$z = \ln(V_1 + FCFe_1) - \ln V_0 \tag{7.5}$$

The process of calculating V_1, $FCFe_1$, V_0, and z from the cash flow model projected by the Monte Carlo simulation is described in following steps.

Step 3. Run the Monte Carlo Simulation

Upon defining the input and forecast variable cells, we will run the Monte Carlo simulation with the Crystal Ball software built within Microsoft Excel. After 10,000 iterations using the parameters that we choose, we will get the mean, standard deviation, and the probability distribution of the annual rates of return of “z.” Figure 7.7 is the cash flow model for the Monte Carlo simulation process in method 3. Since this cash flow model will be the result of the simulation process, the free cash flows on equity are also in a form of a probability distribution.

Figure 7.7 Cash Flow Model for Monte Carlo Simulation in Method 3 at the MCB Project

(M: Million / $: Dollar)

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012
Traffic Volume Growth Rate						2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) - Expected					8.61	8.97	9.32	9.66	10.04
Traffic Volume (M) - Simulated					8.61	8.81	9.18	9.53	9.88
Toll Rate ($)					3.00	3.09	3.18	3.28	3.38
Gross Revenues (M, $)-Expected					25.83	27.72	29.66	31.67	33.90
Gross Revenues (M, $)-Simulated					25.83	27.22	29.21	31.26	33.37
CAPEX (M, $)									4.14
OPEX (M, $)					2.53	2.60	2.68	2.76	2.85
EBIT (M, $) – Simulated					23.30	25.11	26.98	28.90	26.91
Senior Debt Service (M, $)					24.27	24.27	24.27	24.27	24.27
Sub Debt Service (M, $)									
Taxes (M, $) – Simulated					-0.27	0.10	0.62	1.16	0.58
FCF on Equity (M, $)- Simulated					**A**	**B**	**C**	**D**	**E**

Year	2013	2014	2015	2016	2017	2018	2019	2020	2021
Traffic Volume Growth Rate	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) – Expected	10.48	10.83	11.18	11.50	11.86	12.19	12.43	12.66	12.90
Traffic Volume (M) – Simulated	10.27	10.72	11.08	11.44	11.76	12.13	12.47	12.72	12.95
Toll Rate ($)	3.48	3.58	3.69	3.80	3.91	4.03	4.15	4.28	4.41
Gross Revenues (M, $)- Expected	36.45	38.79	41.25	43.70	46.42	49.15	51.62	54.15	56.83
Gross Revenues (M, $)-Simulated	35.72	38.40	40.88	43.46	46.05	48.92	51.79	54.39	57.06
CAPEX (M, $)					4.80				
OPEX (M, $)	2.93	3.02	3.11	3.20	3.30	3.40	3.50	3.61	3.71
EBIT (M, $) – Simulated	33.52	35.78	38.14	40.50	38.32	45.75	48.12	50.54	53.12
Senior Debt Service (M, $)	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27	24.27
Sub Debt Service (M, $)							10.35	10.35	10.35
Taxes (M, $) – Simulated	2.34	3.06	3.71	4.40	3.76	5.84	3.76	4.44	5.15
FCF on Equity (M, $)-Simulated	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**

Year	2022	2023	2024	2025	2026	2027	2028	2029
Traffic Volume Growth Rate	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) - Expected	13.13	13.39	13.65	13.90	14.15	14.41	14.67	14.87
Traffic Volume (M) - Simulated	13.20	13.43	13.70	13.96	14.22	14.48	14.74	15.01
Toll Rate ($)	4.54	4.67	4.81	4.96	5.11	5.26	5.42	5.58
Gross Revenues (M, $)-Expected	59.58	62.58	65.71	68.92	72.27	75.80	79.49	82.99
Gross Revenues (M, $)-Simulated	59.88	62.78	65.94	69.24	72.62	76.15	79.87	83.75
CAPEX (M, $)	5.57					6.45		
OPEX (M, $)	3.83	3.94	4.06	4.18	4.31	4.43	4.57	4.70
EBIT (M, $) – Simulated	50.19	58.64	61.65	64.74	67.96	64.92	74.92	78.28
Senior Debt Service (M, $)	24.27							
Sub Debt Service (M, $)	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Taxes (M, $) – Simulated	4.36	13.33	14.17	15.04	15.94	15.10	17.86	18.89
FCF on Equity (M, $)-Simulated	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**

Year	2030	2031	2032	2033	2034	2035	2036	2037
Traffic Volume Growth Rate	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Traffic Volume (M) – Expected	15.08	15.29	15.50	15.72	15.94	16.16	16.39	16.62
Traffic Volume (M) – Simulated	15.21	15.43	15.64	15.86	16.08	16.31	16.53	16.77
Toll Rate ($)	5.75	5.92	6.10	6.28	6.47	6.66	6.86	7.07
Gross Revenues (M, $)- Expected	86.68	90.53	94.52	98.74	103.13	107.69	112.50	117.50
Gross Revenues (M, $)-Simulated	87.44	91.34	95.39	99.60	104.04	108.67	113.47	118.54
CAPEX (M, $)			7.48					8.67
OPEX (M, $)	4.85	4.99	5.14	5.30	5.45	5.62	5.79	5.96
EBIT (M, $) – Simulated	81.84	85.54	81.90	93.45	97.67	102.07	106.71	102.87
Senior Debt Service (M, $)								
Sub Debt Service (M, $)	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Taxes (M, $) – Simulated	19.87	20.90	19.91	23.09	24.26	25.49	26.77	25.73
FCF on Equity (M, $)- Simulated	**W**	**X**	**Y**	**Z**	**AA**	**AB**	**AC**	**AD**

Upon running the cash flow simulation of Figure 7.7, as a result, we will have the simulated free cash flows on equity (cell A, B, C, ….. , AC, and AD in Figure 7.7) necessary to calculate Equation (6.12) with R_e, which is 12.11 %. Afterward, if we calculate the necessary parameters after simulation process, it is as follows:

$$\begin{aligned} V_0 &= \sum_{t=1}^{30} \frac{FCFe_t}{(1+R_e)^t} \\ &= \frac{A}{(1+0.1211)^1} + \frac{B}{(1+0.1211)^2} + \ldots\ldots + \frac{AC}{(1+0.1211)^{29}} + \frac{AD}{(1+0.1211)^{30}} \end{aligned} \tag{7.6}$$

$$\begin{aligned} V_1 &= \sum_{t=2}^{30} \frac{FCFe_t}{(1+R_e)^{t-1}} \\ &= \frac{B}{(1+0.1211)^1} + \frac{C}{(1+0.1211)^2} + \ldots\ldots + \frac{AC}{(1+0.1211)^{28}} + \frac{AD}{(1+0.1211)^{29}} \end{aligned} \tag{7.7}$$

$$FCFe_1 = A \tag{7.8}$$

Finally, as a result of the simulation, z is obtained as follow.

$$z = \ln\left(\frac{V_1 + FCFe_1}{V_0}\right) = \ln\left(\frac{V_1 + A}{V_0}\right) = 0.1112 \tag{7.9}$$

Here, the rate of return of the project value, z, is expressed as a probability distribution such other parameters as V_0, V_1, and $FCFe_1$. Figure 7.8 and Table 7.7 show the randomized value of "z" and other parameters as results of the simulation process. Here, we finally gain the volatility of 0.0948, then, we are ready to calculate other parameters, u and d needed to implement the real option analysis.

Figure 7.8 Probability Distribution of "z" in the MCB Project

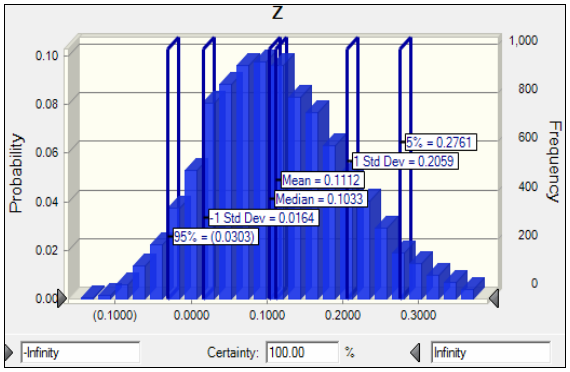


Table 7.7 Volatility of Project Value in the MCB Project

Statistics	*Forecast Values*
Volatility	0.0948
Maximum	0.603
95 %	0.2761
$+1\cdot\sigma$	0.2059
Mean	0.1112
Median	0.1033
$-1\cdot\sigma$	0.0164
5 %	-0.0303
Minimum	-0.1487

3) Up/down movements "u" and "d" and the risk neutral probabilities "q" and "$1-q$"

It is time to calculate the up and down movements u and d that are found from equation (7.10) and (7.11). Up and down movements of the project value, u and d, are necessary to reflect the uncertainties of the project value.

$$u = e^{\left(\left(r-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\sqrt{\Delta t}\right)} = e^{\left(\left(0.053-\frac{1}{2}(0.0948)^2\right)\cdot 1+0.0948\cdot\sqrt{1}\right)} = 1.154 \tag{7.10}$$

$$d = e^{\left(\left(r-\frac{1}{2}\sigma^2\right)\Delta t-\sigma\sqrt{\Delta t}\right)} = e^{\left(\left(0.053-\frac{1}{2}(0.0948)^2\right)\cdot 1-0.0948\cdot\sqrt{1}\right)} = 0.955 \tag{7.11}$$

As for the risk neutral probabilities, 0.5 is assigned for the values of q and $1-q$ in this research (Hull, 1997).

4) Construct a reverse binomial tree with a risk variable

Now, we know the values of V_I, u, d, q, and $1-q$ from the above stages. With these parameters, and based upon Figure 6.3, we construct a reverse binomial tree.

In this binomial tree, we will reflect all possibilities that the project value can have over time by using up/down movements of u and d during the concession period of 30 years. Table 7.8 is the basic information and parameters needed to be put to construct binomial tree. The binomial tree, which is constructed in Microsoft Excel program, follows in Figure 7.9. The varying project values over time in the binomial tree reflect the uncertainty of the project value.

Table 7.8 Calculated Parameters in the MCB Project

Calculated Parameters			
Initial Project Value "V_I"(Million, $)	91.82	*Volatility "σ"*	0.0948
Up Movement "u"	1.154	*Concession Period (Year)*	30
Down Movement "d"	0.955	*Risk Neutral Probability "q"*	0.5
Risk Free Rate "r" (%)	5.3	*Risk Neutral Probability "$1-q$"*	0.5

Figure 7.9 Binomial Tree of the MCB Project Value

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
91.82	106	121	139	160	184	211	243	279	321	368	423	486	559	642	738	848	975	1120	1287	1479	1699	1952	2243	2578	2962	3403	3911	4494	5164	5933
	88	101	116	134	154	177	203	233	268	308	354	406	467	537	617	708	814	935	1075	1235	1419	1631	1874	2153	2474	2843	3267	3753	4313	4956
		85	97	112	128	147	169	195	224	257	295	339	390	448	515	592	680	781	898	1032	1185	1362	1565	1798	2066	2374	2728	3135	3602	4139
			81	93	107	123	142	163	187	215	247	284	326	374	430	494	568	653	750	862	990	1138	1307	1502	1726	1983	2279	2619	3009	3458
				78	90	103	118	136	156	179	206	237	272	313	359	413	474	545	626	720	827	950	1092	1255	1442	1657	1904	2187	2513	2888
					75	86	99	113	130	150	172	198	227	261	300	345	396	455	523	601	691	794	912	1048	1204	1384	1590	1827	2099	2412
						72	82	95	109	125	144	165	190	218	251	288	331	380	437	502	577	663	762	875	1006	1156	1328	1526	1754	2015
							69	79	91	105	120	138	159	182	209	241	276	318	365	419	482	554	636	731	840	965	1109	1275	1465	1683
								66	76	87	100	115	132	152	175	201	231	265	305	350	403	463	531	611	702	806	927	1065	1223	1406
									63	73	84	96	111	127	146	168	193	222	255	293	336	386	444	510	586	674	774	889	1022	1174
										61	70	80	92	106	122	140	161	185	213	244	281	323	371	426	490	563	646	743	854	981
											58	67	77	89	102	117	135	155	178	204	235	270	310	356	409	470	540	620	713	819
												56	64	74	85	98	112	129	148	171	196	225	259	297	342	392	451	518	595	684
													54	62	71	82	94	108	124	142	164	188	216	248	285	328	377	433	497	572
														52	59	68	78	90	104	119	137	157	180	207	238	274	315	362	415	477
															50	57	65	75	86	99	114	131	151	173	199	229	263	302	347	399
																48	55	63	72	83	95	110	126	145	166	191	220	252	290	333
																	46	53	60	69	80	92	105	121	139	160	183	211	242	278
																		44	50	58	67	76	88	101	116	133	153	176	202	232
																			42	48	56	64	73	84	97	111	128	147	169	194
																				40	46	53	61	70	81	93	107	123	141	162
																					39	45	51	59	68	78	89	103	118	135
																						37	43	49	56	65	75	86	98	113
																							36	41	47	54	62	72	82	94
																								34	39	45	52	60	69	79
																									33	38	43	50	57	66
																										32	36	42	48	55
																											30	35	40	46
																												29	33	38
																													28	32
																														27

5) Option Formulation

In this step, the MRG agreement will be formulated as a put option. In building the asymmetric payoff condition, it needs the exercise price, which is defined as a guaranteed project value as shown in Section 6.6.3.6. Here, at time "0", the guaranteed project value is the same as the initial project value multiplied by 0.8 since the concession agreement in MCB BOT project indicates that the 80 % of the expected cash flow will be guaranteed as a minimum revenue for the BOT developer. So, the guaranteed project value for the first year is $0.8 \times 91.82 = 73.50$. And, from the second year, this initial project value will annually increase with the rate of $r - (1/2)\sigma^2 = 0.053 - (1/2) \cdot 0.0948^2 = 0.049$ over time until the end of the concession agreement. If we describe these guaranteed project values that will be used as exercise prices, they will be as follows in Table 7.9. As the BOT project value increases over time, the exercise price grows as well and this developed real option model has a different exercise price for each year during the concession period. Then, the MRG option can be exercised and calculated every year as long as the condition to exercise the option is met. For all nodes at every year in the spreadsheet, we will recognize whether the condition of exercising the MRG option is met or not. Once the guaranteed project value is higher than the projected project value, the option will be exercised. Otherwise, the MRG option value will be "0." Figure 7.10 is the binomial tree of the asymmetric payoff based on the projected project value and guaranteed project value.

Table 7.9 Guaranteed Project Value during Concession Period in the MCB Project (Million, $)

Year	***2007***	***2008***	***2009***	***2010***	***2011***	***2012***	***2013***	***2014***	***2015***	***2016***	***2017***	***2018***	***2019***	***2020***	***2021***
Guaranteed Project Value	73.5	77.1	80.9	85	89.2	93.6	98.3	103.2	108.3	113.7	119.3	125.3	131.5	138	144.9

Year	***2022***	***2023***	***2024***	***2025***	***2026***	***2027***	***2028***	***2029***	***2030***	***2031***	***2032***	***2033***	***2034***	***2035***	***2036***	***2037***
Guaranteed Project Value	152.1	159.6	167.6	175.9	184.7	193.8	203.5	213.6	224.2	235.3	247.0	259.3	272.2	285.8	300.0	314.9

Figure 7.10 Binomial Tree of Asymmetric Payoff in the MCB Project

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				12	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					20	14	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						28	22	15	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							36	31	24	17	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
								44	40	34	27	18	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
									52	49	44	37	30	20	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
										61	58	53	48	41	33	22	9	0	0	0	0	0	0	0	0	0	0	0	0	0
											69	67	63	59	53	46	36	24	10	0	0	0	0	0	0	0	0	0	0	0
												78	76	74	70	65	58	50	40	27	11	0	0	0	0	0	0	0	0	0
													87	86	84	81	77	72	64	55	44	30	12	0	0	0	0	0	0	0
														96	96	95	93	89	85	79	71	61	48	33	13	0	0	0	0	0
															105	106	105	104	102	99	94	87	78	67	53	36	15	0	0	0
																115	116	116	116	115	112	109	103	96	86	74	59	40	0	0
																	125	127	128	128	128	127	124	120	114	106	95	82	0	0
																		135	138	140	141	141	141	140	137	132	125	117	0	0
																			146	149	152	154	155	156	155	154	151	145	0	0
																				157	161	164	167	170	171	172	171	169	0	0
																					168	173	177	181	184	187	189	189	0	0
																						180	185	190	195	199	203	206	0	0
																							192	198	204	210	215	220	0	0
																								205	212	218	225	231	13	0
																									218	225	233	241	24	0
																										231	240	249	33	0
																											246	255	40	0
																												260	47	0
																													52	0
																														0

6) Implementing the Calculation Backward Recursively

The last step is to calculate the MRG option value in the binomial tree. This calculation will be conducted from the end of the binomial tree to present backward recursively considering whether the asymmetric condition is met at each year or not. The selected value of asymmetric payoff from the end of the binomial tree, at year 30, will be the starting point and will be calculated backwards recursively using the risk neutral probabilities 0.5 and a risk-free rate of 5.3 %. Figure 7.11 is the description of the binomial tree for the MRG option value. We have the MRG option value in 2007, which is $ 5.948 million and, by discounting this value to 2004 with a risk-free rate of 5.3%, as a result, we obtain the MRG value of $ 5.075 million.

7.1.4.4 Results of the BOT Project Valuation Methods

In the BOT case study of the MCB project, three valuation methods to evaluate the MRG option value are considered. They are NPV analysis, Cheah and Liu's real option model, and the developed real option model. If we describe the results of the valuations, they will be as follows.

First, through method 1 of NPV analysis, we calculated the NPV on equity of the MCB project. In this case, the project value on equity is $ 6.11 million. And, MRG value is "0" since this method can not take into account the MRG agreement.

Second, in method 2 we calculated the MRG value based upon Cheah and Liu's real option model. The mean and median values of MRG agreement are $ 2.6 million and $ 1.14 million respectively. The mean value of MRG agreement option accounts for 42.55 % of NPV on equity of $ 6.11 million and 4.41 % of initial equity investment of $ 59 million and median accounts for 18.66 % and 1.93 % respectively.

Third, with the developed real option model, method 3, we found the MRG agreement value, which is $ 5.075 million. This value accounts for 83.06 % of project value on equity and 8.60 % of initial equity investment respectively.

Figure 7.11 MRG Value of the MCB Project (MRG Value = $ 5.074 million)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
5.948	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		11	9	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			14	11	9	7	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				18	15	12	9	7	5	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					23	20	16	13	10	7	5	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						30	26	21	17	14	10	7	4	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							37	32	28	23	19	14	10	7	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
								45	40	36	30	25	20	15	10	6	4	2	1	0	0	0	0	0	0	0	0	0	0	0
									53	49	45	39	33	27	21	15	10	6	3	1	0	0	0	0	0	0	0	0	0	0
										62	59	54	49	43	36	29	22	15	10	5	2	0	0	0	0	0	0	0	0	0
											70	68	65	60	54	47	39	31	23	15	9	4	1	0	0	0	0	0	0	0
												79	77	75	71	66	60	51	42	33	23	14	7	2	0	0	0	0	0	0
													88	87	85	82	78	73	66	57	46	35	23	13	4	0	0	0	0	0
														97	97	96	94	91	86	80	72	62	50	37	22	9	0	0	0	0
															106	107	106	105	103	100	95	89	80	69	55	38	20	0	0	0
																116	117	118	117	116	114	110	105	98	88	76	60	41	0	0
																	126	128	129	130	129	128	125	121	115	107	97	83	0	0
																		136	139	141	142	143	142	141	138	134	127	118	0	0
																			147	150	153	155	157	157	157	155	152	147	0	0
																				157	161	165	168	171	173	173	173	171	0	0
																					169	173	178	182	185	188	190	191	0	0
																						180	186	191	196	200	204	207	0	0
																							192	199	205	211	216	221	2	0
																								205	212	219	226	232	15	0
																									218	226	234	241	26	0
																										232	240	249	35	0
																											246	255	42	0
																												261	48	0
																													53	0
																														0

Table 7.10 depicts the comparison of the MRG values by three project valuation methods. As shown in Table 7.10, from the total project value, which is NPV on equity plus MRG option value, we realize that the MRG value has an impact on NPV on equity and equity investment. This relatively small MRG value against NPV on equity and equity investment may be due to the favorable estimation of base cash flow model by the MCB Co., Ltd. However, when it comes to the investment decision of the BOT developer, if the BOT developer hesitates to decide the investment because of slightly negative NPV, this MRG value may give them the implication to cause a major change in their decision-making by converting the negative NPV into positive through being added to the negative NPV.

Table 7.10 MRG Option Value in the MCB Project by Three Valuation Methods

(MRG, NPVe: Net Present Value on Equity, Ie: Equity Investment / Million, $)

		MRG Value	MRG Value/NPVe	MRG Value/Ie	MRG Value + NPVe	(MRG Value + NPVe)/NPVe
Method 1		0.00	**0.00%**	**0.00%**	6.11	100.00%
Method 2	Max	14.32	**234.37%**	**24.27%**	20.43	29.91%
	2.5%	9.84	**161.05%**	**16.68%**	15.95	38.31%
	$+1 \cdot \sigma$	5.63	**92.14%**	**9.54%**	11.74	52.04%
	Mean	2.60	**42.55%**	**4.41%**	8.71	142.55%
	Median	1.14	**18.66%**	**1.93%**	7.25	118.66%
	$-1 \cdot \sigma$	0.43	**7.04%**	**0.73%**	6.54	107.04%
	97.5%	0.00	**0.00%**	**0.00%**	6.11	100.00%
	Min	0.00	**0.00%**	**0.00%**	6.11	100.00%
Method 3		5.075	**83.06%**	**8.60%**	11.185	183.06%

In Table 7.10, we easily find that, aside from the method 1 by NPV analysis, method 2 and 3 show that the MRG option values account for some portion of the project value on equity and initial equity investment.

7.1.4.5 Validation Test of Developed Real Option Model

Although the developed quantitative real option model to evaluate the BOT project by considering MRG agreement depends entirely on real option theories, the applicability of the developed model is still open issue that has to be verified. In light of this fact, the purpose of a validation test is to know if newly developed real option model can provide us with a reasonable degree of the validity and reliability through the reasonable results in evaluating MRG value. To test the validity of the model, we will examine whether the results of the model satisfy the research hypotheses or not. In the validity test, we already have three research hypotheses which seem to be reasonable to show the applicability of the developed model. This validity test will be conducted with three different BOT case studies to give us the satisfactory reliability. And, the MCB project is the first case among three BOT case studies we are going to examine.

Hypothesis One - *"The project value using the two option pricing methods 2 and 3 under the MRG agreement will show significant value rather than the project value of method 1, which is called NPV analysis."*

As we can see in Table 7.10, NPVs on equity by project valuation methods 2 and 3 under MRG agreement turn out to have some impact compared to that of method 1. The NPV on equity by method 2 and 3 are, 42.55 % (mean) and 18.66 % (median), and 83.06 % higher than that of method 1 respectively.

Hypothesis Two - *"Based upon the option pricing theory, the predicted effects of major determinants (current price of the underlying asset V_I, exercise price X, time to maturity T, volatility σ, and risk free interest rate r) on a MRG option value in the developed binomial real option model have to follow those of option pricing theory based on the Black-Scholes model."*

This hypothesis is to verify the applicability of the developed real option model by comparing the MRG option value change against the change of each important input variable; initial project value V_I, exercise price X, time to maturity T, volatility σ, and risk-free rate r, with that of the continuous time model (Black-Scholes model). As the real option model in this research is derived from the discrete approximation from the binomial model, if developed real option model is reasonable and applicable, the change trends of the MRG option value against input variable changes by the developed real option model should follow those by the continuous time model which is called Black-Scholes model as mentioned on the Table 3.2 and Section 3.2.6.

Initial Project Value V_I

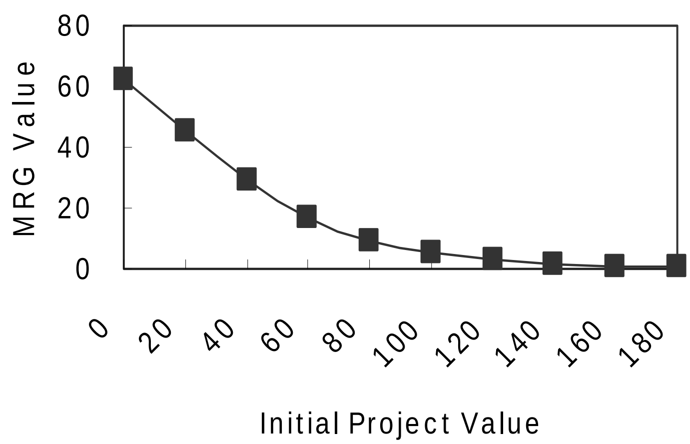
The value of the MRG option will vary with the changes in the input variables. One of the most critical variables is the value of the underlying asset, initial project value V_I which is the present value of the future cash flows of the project. In this research, as the MRG agreement is formulated as a put option, we need to determine if the graph drawn by the change of the initial project value against the change of the MRG value follows the graph drawn by the underlying asset and the put option value by the Black-Scholes model.

$$\frac{\partial P}{\partial V_I} < 0 \tag{7.12}$$

Equation (7.12), $\partial P / \partial V_I$, which is known as the hedge ratio and is a continuous function in the case of a put option based on the Black-Scholes model, can be analogous to the slope of the change of the MRG value on the change of the initial project value since the MRG agreement is formulated as a put option in this research. Therefore, we need to check if the change of the MRG option value against the change of the initial project value in the developed model can produce a trend similar to Equation (7.12) at a given point. Figure 7.12 shows the changes of the MRG option value as V_I increases in the developed model. In this graph, we can easily see that as V_I increases the MRG option value decreases while the initial exercise price is held constant at $ 73.50 million in the first year of the binomial tree, the volatility is held constant at 0.0948, the risk free rate is held constant at 5.3 %, and the time frame is constant at 30 years with 30 time-steps. The maximum value of a MRG option, \$ 62.69 million, will be the present value of sum of all the exercise prices at every time step as the initial project value V_I is “0” and the minimum value is approximately “0” when the V_I is higher than the exercise price of the last year in binomial tree, which is \$ 315 million. Here, the MRG option value will never be less than zero since the option can be allowed to expire without being exercised. As we can see in Figure 7.12, the change of the MRG option value against the change of the initial project value V_I is consistent (but not identical) with the result of the partial differential $\partial P / \partial V_I < 0$ of Equation (7.12) which is based on the Black-Scholes model.

Figure 7.12 MRG Value Change against Initial Project Value V_I Change in the MCB Project

(MRG, V_I : Million, \$)



(*Initial* X =73.50, T =30, σ =0.0948, r =0.053)

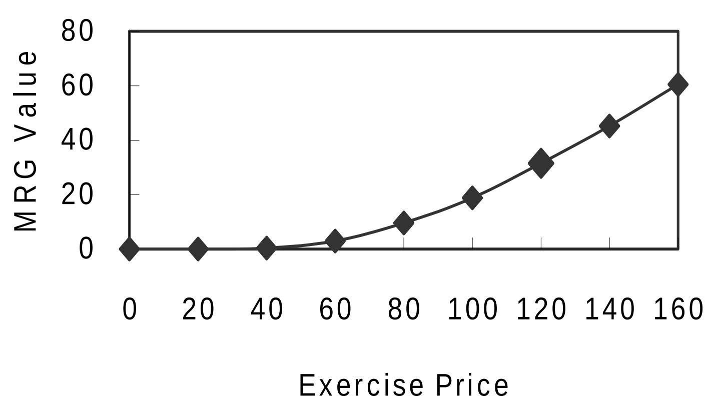
Exercise Price X

The results of the MCB project case with developed real option model shows that the MRG option value is positively dependent on the exercise price X. Figure 7.13 shows the nature of how the MRG option value will change with the change of the exercise price in developed real option model. As mentioned earlier, in the developed real option model the exercise price at every time step is calculated from $e^{\left(r-\left(\frac{1}{2}\right)\sigma^2\right)\Delta t} \cdot V_I$. Here, to understand the sensitivity of the MRG value against the change of the exercise price X, we need to gradually increase the initial project value V_I, since $e^{\left(r-\left(\frac{1}{2}\right)\sigma^2\right)\Delta t}$ is constant. So, while increasing the exercise price X by increasing V_I, we recognize that the MRG option value increases. This is expected because increasing X increases option value at every node in binomial tree as long as other input variables are held constant. Equation (7.13) shows the relationship between exercise price X and put option value P based on the Black-Scholes model (in Section 3.2.6 and Table 3.2).

$$\frac{\partial P}{\partial X} > 0 \tag{7.13}$$

In Figure 7.13, the change of the MRG value on the change of the exercise price seems to follow the Equation (7.13), which is mathematically defined as the partial differential $\partial P / \partial X$ based on the Black-Scholes model. Figure 7.13 shows this relationship between the MRG option value and the exercise price X.

Figure 7.13 MRG Value Change against Exercise Price X Change in the MCB Project (MRG, X: Million, \$)



(V_I = 91.82, T =30, σ =0.0948, r =0.053)

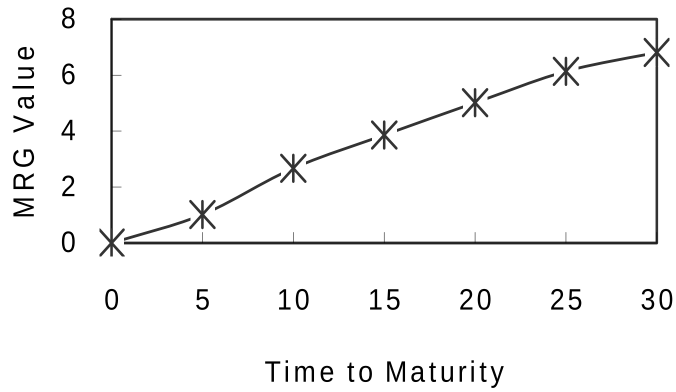
Time to Maturity T

The time variable is the time from the present until the time that the option might be exercised. For example, if a project is being considered for funding sometime in the next 10 years, then the time to maturity is 10 years. The time frame is important for the other factors as well, as it determines the timeline that the option is being valued within. A 10–year time line means that the interest rate must be an annualized rate good for 10 years and the volatility must be based on an annualized standard deviation. The sensitivity function, $\partial P / \partial T$, for time is defined as a positive correlation as shown in Equation (7.14), because as time passes the option becomes less valuable.

$$\frac{\partial P}{\partial T} > 0 \tag{7.14}$$

According to the Equation (7.14) which is based on the Black-Scholes model, the put option value increases with increases in the time that the option is held open. And, it seems because the put option value increases if the chances of ending with a positive value increase with time, while the chances of ending with a negative value do not (the option will never be worth less than zero). Figure 7.14 shows the change of the MRG value against the change of the time to maturity. And, we can see here that the impact of the time to maturity on the MRG value is positive and follows the tendency of Equation (7.14).

Figure 7.14 MRG Value Change against Time to Maturity T Change in the MCB Project (MRG: Million, $; T: Year)



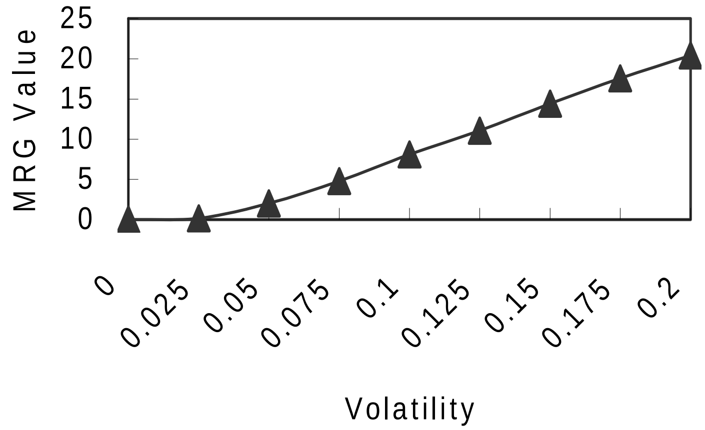
(V_I = 91.82, *Initial* X =73.50, σ =0.0948, r =0.053)

Volatility σ

The volatility is an important variable in estimating the option value and is perhaps one of the most difficult variables to estimate, especially in the BOT project. Figure 7.15 shows the relationship of the change of the MRG option value to the change of the volatility based on the results by developed when the other variables are equal. As shown in Figure 7.15, the MRG option value increases with increases in volatility σ. This is because the probability of the upside potential increases as the variability increases. The probability of the downside potential does not increase since the minimum value of the MRG option value is “0.” Figure 7.15 shows that the increase of σ results in an increased MRG option value and the decrease of σ decreases the MRG option value and we find out from the trend of the MRG option value change against volatility change that the results of the developed real option model is similar to Equation (7.15) by the Black-Scholes model.

$$\frac{\partial P}{\partial \sigma} > 0 \qquad (7.15)$$

Figure 7.15 MRG Value Change against Volatility σ Change in the MCB Project (MRG: Million, $)



(V_I = 91.82, *Initial* X =73.50, T =30, r =0.053)

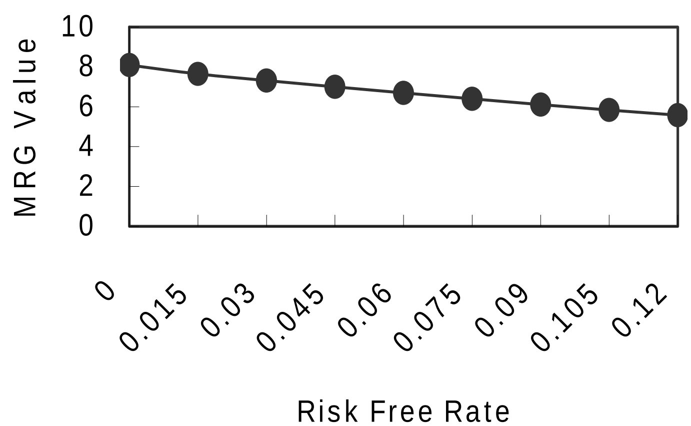
Risk-free Rate r

The interest rate that is used in the option valuation is a risk-free interest rate r. In NPV analysis, the interest rate is often increased to compensate for risk. "Hurdle rates" are often used instead of the WACC to ensure a high return and to hedge against risk. However, risk and interest rates are difficult to correlate with any accuracy. In real options, the risk-free rate is used and the volatility parameter is used to reflect risk. So, the interest rate used in the real option model is a risk-free rate based on the time horizon. If the project has a timeline of 30 years, then the rate for 30-year Treasury bonds is chosen. If the project has an option timeline different from 30 years, a corresponding Treasury rate is used. According to the Section 3.2.6 based on the Black-Scholes model, the sensitivity function of the risk-free rate to the put option value, which is known as $\partial P / \partial r$, is as Equation (7.16).

$$\frac{\partial P}{\partial r} < 0 \qquad (7.16)$$

Based on the results from the developed model, the MRG option value decreases with increasing risk-free interest rate r, as shown in Figure 7.16. The exercise price X is dependent on the risk-free rate in the developed model because the exercise price is the function of the risk-free rate like $e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t}\cdot V_I$. On the other hand, while discounting the MRG option values to present, we use r as discount rate. It seems that these two effects will have an impact on the MRG option value and, since the effect of discounting the MRG option value at every time step to present is more significant than that of increasing the exercise price, the MRG option value decreases as the risk-free rate r increases. So, Figure 7.16 has a similar shape to the continuous function for $\partial P/\partial r$ in Equation (7.16). However, it seems that the sensitivity of the MRG option value to the risk-free interest rate is relatively minor compared to the effects of the other input variables such as initial project value, exercise price, time to maturity, and volatility.

Figure 7.16 MRG Value Change against Risk-free Rate r Change in the MCB Project (MRG: Million, \$)



(V_I = 91.82, *Initial* X =73.50, T =30, σ =0.0948)

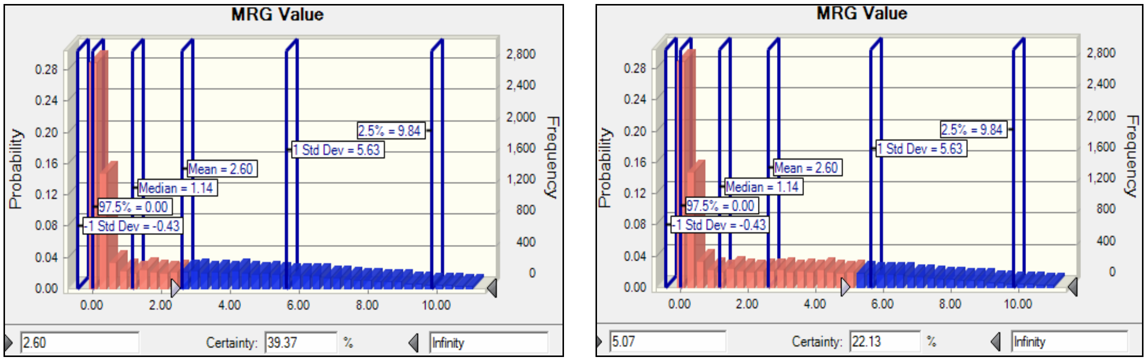
So far, according to the effects of the input variables on the MRG option value with the developed real option model, it seems that the developed real option model with the MCB BOT project case study satisfies hypothesis two that is based upon the Black-Scholes model.

Hypothesis Three **-** *"The MRG agreement value gained from the developed real option method 3 will be consistent with that of method 2. (The MRG agreement value gained from the developed method 3 will be located within the range of* $\pm 2 \cdot \sigma$ *from the median and mean in probability distribution of the MRG value by method 2)."*

Statistically speaking, if the probability distribution of the MRG value by method 2 follows a normal distribution, by investigating how far the MRG value by method 3 is away from the mean with the range of standard deviation, for example $\pm 1 \cdot \sigma$ (68.26 %) or $\pm 2 \cdot \sigma$ (95 %), we can easily find out if the MRG value by method 3 is statistically consistent with that of method 2 or not.

However, as mentioned in Section 4 of problem statement, since the probability distribution of the MRG value by method 2 (Cheah and Liu, 2006) is seriously skewed to the right side and not to follow a normal distribution, it seems to be reasonable to use the cumulative probability in the probability distribution of the MRG value of method 2 in understanding the degree of the consistency between MRG values by method 2 and 3. Moreover, In statistics, as a sample which is located within the range of $\pm 2 \cdot \sigma$ (95 %) from mean value is considered as generally acceptable, this research selects the range of $\pm 2 \cdot \sigma$ to test the consistency of the MRG value obtained by the developed real option model compared to that by method 2.

Figure 7.17 Comparisons of MRG Values by Method 2 (Mean) and 3 in the MCB Project



The MRG value of \$ 5.075 million by method 3 can be shown as cumulative probabilities of 87.87 % in the probability distribution of the MRG value by method 2 while the mean and median values of MRG option by method 2 are \$ 2.60 million and \$ 1.14 million which can be shown as cumulative probabilities of 60.63 % and 50 % respectively. By using these cumulative probabilities, if we are going to find out how far the MRG value by method 3 is away from the mean and median of the MRG value by method 2, it will be shown as in Table 7.11. The MRG value of \$ 5.075 million by method 3 is located within ± 27.24 % from mean and within ± 37.87 % from median in probability distribution of the MRG value by method 2.

From this fact, since the MRG value obtained by the developed real option model "method 3" is located within the range of $\pm 2 \cdot \sigma$ (± 47.5 %), as shown in Figure 7.17 and Table 7.11, from the mean and median in the probability distribution of the MRG value by method 2, it seems that the developed method 3 satisfies the research hypothesis 3.

Table 7.11 MRG Values in Cumulative Probability Distributions by Method 2 and 3 in the MCB Project

	Method 2 (By Cheah and Liu's Real Option Model)	
	Mean \$ 2.6 million (60.63 %)	Median \$ 1.21 million (50 %)
Method 3: \$ 5.075 million (87.87 %)	± 27.24 % (< ± 47.5 %)	± 37.87 % (< ± 47.5 %)

Based on the results of testing research hypotheses 1, 2, and 3, we can recognize that the developed real option model gives a reasonable degree of validation in its applicability with the MCB BOT project case study. In Section 7.2 and 7.3, we will further the test of the research hypotheses 1, 2, and 3 with two other BOT project case studies to see the generalization of the applicability of the developed real option model.

7.1.4.6 Sensitivity Analysis of MRG Value to Standard Deviation of the Initial Traffic Volume and Traffic Volume Growth Rate

It is necessary to investigate as to how sensitive the estimated MRG value is to the various input variables for the purpose of using in a negotiation process. Table 7.12 shows the results of the sensitivity analyses of the MRG value subject to changes in the standard deviations of initial traffic volume and growth rate. This table shows that the value of MRG option is more sensitive to the standard deviation of the initial traffic volume assumed rather than that of the traffic volume growth rate (Cheah and Liu, 2006).

It should be noted that the standard deviations of these two variables are determine the volatility of the project cash flows, which is a key determinant of the value of the MRG option evaluated.

Table 7.12 Sensitivity of MRG Value to Standard Deviations of Initial Traffic Volume and Traffic-Volume Growth Rate in the MCB Project

(MRG Value: Million, $)

		Standard Deviation of Growth Rate (%)			
		0.49	**0.99**	1.48	1.98
Standard Deviation of Traffic Volume (Million)	1.19	1.486	1.490	2.020	1.486
	2.38	5.020	**5.075**	5.020	5.075
	3.57	7.720	7.611	7.774	7.720
	4.76	9.369	9.330	9.369	9.333

7.2 The Kwangju Ring Road Section 3-1 (KRRC) Project

7.2.1 Background

The Kwangju Ring Road Section 3-1, which is 3.5 km length and 6 lanes in both directions and where a total of $ 143.47 million was invested connecting between Hyodeok district and Pungam district in Kwangju Metropolitan city of Korea, was planned in 2001 by the suggestion of Kwangju Metropolitan city. In 2001, Kwangju Metropolitan city made a concession agreement with a SPV, Kwangju Ring Road (KRRC) Co., Ltd. which was sponsored by a single project equity investor, Doosan Engineering and Construction Co., Ltd. The construction began in 2002 and finished in 2004.

This project was suggested by Kwangju Metropolitan city as a BOT/BTO type of privatized infrastructure project, and the Kwangju Metropolitan city entered into a contract with Doosan Engineering and Construction Co., Ltd. which is a single equity investor in the SPV of KRRC Co., Ltd. in 2001. In 2001, the Kwangju Metropolitan city assigned the KRRC Co., Ltd. as a SPV and the KRRC was expected to complete the construction, operate the road during the concession period based on the concession agreement between KRRC Co., Ltd. and Kwangju Metropolitan city and transfer the ownership to Kwangju Metropolitan city after the concession period. Here, Doosan Engineering and Construction Co., Ltd. invested a total of $ 40.46 million (100%) on KRRC Co., Ltd as a single equity investor. The KRRC Co., Ltd. was supposed to operate and take the profit from this road during 30 years at an average toll rate of $ 1.87 per vehicle.

The completion of the KRRC project is expected to contribute to solving the serious traffic problems in front of Kwangju University and reduce the driving time by 30 minutes from 50 minutes to 20 minutes between Hyodeok district and Pungam district. Moreover, this project is also supposed to help alleviate traffic congestion of the main freeway and Kwangju Ring Road Section 1, which is the inner beltway within the Kwangju Metropolitan city, and give easier accessibility from the suburbs of Kwangju

Metropolitan city to the downtown. Because this road can provide an entrance that is more convenient to Mokpo and Whasoon city without penetrating the Kwangju city downtown, it is expected to increase the traffic benefit as well.

7.2.2 Contractual Structure

The KRRC project, which was suggested in 2000 by Kwangju Metropolitan city, amounts to about $ 143.47 million in its total project cost. In 2001, Kwangju Metropolitan city made a concession agreement with a KRRC Co., Ltd.

To afford this huge construction cost, the capital structure was planned with forms of equity and senior debt. The KRRC Co., Ltd. (Doosan Engineering and Construction Co., Ltd., 100 %) covered 28.2 % of the construction cost, which is $ 40.46 million, and 71.8 %, the rest of the construction cost, $ 103 million was supported with a fixed interest rate of 7.25 % during 13 years by Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. as a type of senior debt. As MKIF Co., Ltd. had a plan to buy more equity of the KRRC Co., Ltd. after completion of construction under the take-out agreement, it was possible to support the senior debt with the relatively low interest rate of 7.25 %. During the operation period, KRRC Co., Ltd. has all rights, liabilities and obligations related to KRRC project under the concession agreement to receive tolls in return for designing, constructing, operating and maintaining this beltway (approximately 3.5 km). The debt would be repaid out of following sources:

(1) Tolls will be collected through the 30-year concession to operate the project and its operation will start from 2005.

(2) There is an agreement related to the minimum revenue guarantee, MRG, by Kwangju Metropolitan city to KRRC Co., Ltd. if the expected revenue that KRRC Co., Ltd. estimated isn’t reached. The MRG will be considered from 2005 to 2034 during 30 years at the rate of 90% of the expected revenue. And, there will be a repayment agreement which will be paid from the KRRC Co., Ltd. to Kwangju Metropolitan city if

the projected revenue goes far beyond the guaranteed revenue that KRRC Co., Ltd. estimated. The repayment agreement will be considered during the same period as revenue guarantee at the rate of 110% of the guaranteed revenue.

7.2.3 Financial Analysis and Cash Flow Model

In constructing the cash flow model, the following data are gained from the Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. and Macquarie Shinhan Infrastructure Asset Management (MSIAM) Co., Ltd. with different sources of information out of public/private organizations.

(1) The total cost of the project and the cost of the equity investment are taken as about $ 143.47 million and $ 40.46 million respectively, and assumed to be used during the first year of the construction period.

(2) The debt and equity ratio is 71.8 : 28.2. The loan terms are 13 years for senior debt at a fixed rate of 7.25 %.

(3) The concession period is 30 years.

(4) Capital expenditure is total around $ 15.16 million. This amount, which accounts for about 10.6 % of total construction cost, is evenly distributed with $ 2.61 million for every 5 years during the concession period and will escalate at 3%. Operating expenditures are around $ 75.1 million. This accounts for 52.3 % of total construction cost. Like the capital expenditure, this amount is evenly distributed with $ 2.59 million for every year during concession period and it escalates at 3% annually.

(5) The corporate tax rate is 27.5 %.

(6) Under the KRRC Co., Ltd.'s proposal, the initial toll rates are $ 0.947 for cars, $ 2.42 for buses, $ 2.00 for vans, and $ 2.42 for lorries, respectively. The average toll rate is $ 1.87 and would escalate at 3 % annually. The class of vehicles of total traffic volume and initial toll rates are as follows in Table 7.13.

Table 7.13 Initial Toll Rates for Different Vehicle Classes in the KRRC Project

Vehicle Class	*Toll Rate ($)*	*Proportion (%)*
Cars	0.947	20
Buses	2.42	20
Vans	2.00	25
Lorries	2.42	25
Taxies	0.947	10

(7) The distribution of the initial traffic volume (in 2005) is assumed to follow a lognormal distribution (Cheah and Liu, 2006). The original traffic volume projection, 7.153 million vehicles, estimated by KRRC Co., Ltd. is taken as the mean value of the initial traffic volume variable in 2005. The following table illustrates the annual traffic volume and traffic volume growth rate estimated by KRRC Co., Ltd.

Table 7.14 Annual Traffic Volume and Traffic Volume Growth Rate Estimated by the KRRC Co., Ltd. (Traffic Volume: Million, Growth Rate: %)

Year	***2005***	***2006***	***2007***	***2008***	***2009***	***2010***	***2011***	***2012***	***2013***	***2014***	***2015***	***2016***	***2017***	***2018***	***2019***	***2020***
Growth Rate	-	3.44	2.80	3.87	3.72	3.71	3.80	3.66	1.73	1.59	1.67	1.64	1.62	1.59	1.66	1.64
Traffic Volume	7.15	7.40	7.61	7.91	8.21	8.52	8.85	9.18	9.34	9.49	9.65	9.81	9.97	10.13	10.3	10.47

Year	***2021***	***2022***	***2023***	***2024***	***2025***	***2026***	***2027***	***2028***	***2029***	***2030***	***2031***	***2032***	***2033***	***2034***
Growth Rate	1.61	1.68	2.10	2.15	2.10	2.15	2.10	2.14	2.09	2.13	2.16	2.11	2.14	2.10
Traffic Volume	10.64	10.82	11.05	11.29	11.53	11.78	12.03	12.29	12.55	12.82	13.10	13.38	13.67	13.96

(8) The traffic volume growth rate is assumed to follow a normal distribution (Cheah and Liu, 2006). Based on the Table 7.14, we can calculate the mean and standard deviation of the traffic volume and traffic volume growth rate. The distributions and parameters of the initial traffic volume and traffic volume growth rate are compiled in Table 7.15.

Table 7.15 Probability Distribution of Two Input Variables in the KRRC Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal	Normal
Mean	7.153	2.34
Standard Deviation	1.943	0.794

(9) To calculate the private consortium's required rate of return based upon CAPM, we will use following information.

1. The risk free rate used is 10-year Korean Treasury bill rate which was 5.3 % in March 2008 based on the Bond Information Service (BIS) of the Korea Securities Dealers Association (KSDA). (Source: http://www.ksdabond.or.kr)

2. The market risk premium, *MRP*, is the difference between the market rate of return based on the KOSPI (Korea Composite Stock Price Index) and the risk-free rate (Treasury bills). The overall market rate of return will be used to measure *MRP*. From 1990 to 2005, the KOSPI averaged yearly returns of 10.40 % according to the Korea Securities Dealers Association (KSDA), so, *MRP* is 10.40 % – 5.3 % = 5.1 %. (Source: http://www.ksdabond.or.kr)

3. Beta, β, is a measure of risk for a certain industry or company. So, it is calculated with a weighted average beta of construction companies, which joined as equity

investors, in proportion to each company's equity investment. In this case, since the equity investor of KRRC Co., Ltd. is Doosan Construction and Engineering Co., Ltd alone, the β of this company, 1.48, is used. (Source: http://kr.stock.yahoo.com) If the company is not listed in stock market, the average beta of the construction industry sector, which is 1.20, in the market, can be alternatively used.

$$\begin{aligned} R_e(Cost\ \ of\ \ Equity) &= R_f + (MRP \times \beta) \\ &= 5.3\% + (5.1\% \times 1.48) \\ &= 0.1285 = 12.85\ \% \end{aligned} \tag{7.17}$$

Finally, the private consortium, KRRC Co., Ltd.'s required rate of return, R_e, is 12.85 %.

7.2.4 Implementation of the BOT Project Valuation

7.2.4.1 Method 1 - NPV Analysis

$$\text{NPV on Equity} = \$\ 2.02\ Million \tag{7.18}$$

In the KRRC BOT project, we have the NPV on equity of \$ 2.02 million. Figure 7.18 shows the cash flow model of the project by method 1. The free cash flow on equity shown in Figure 7.18 will be used as the standard of the guaranteed cash flow to build an asymmetric payoff condition in cash flow models of method 2 and 3.

Figure 7.18 Cash Flow Model of the KRRC Project by Method 1

(M: Million / $: Dollar)

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010
Traffic Volume Growth Rate (%)									
Traffic Volume (M)				7.153	7.398	7.614	7.905	8.206	8.520
Toll Rate ($)				1.87	1.93	1.99	2.05	2.11	2.17
Gross Revenue (M, $)				13.40	14.28	15.14	16.18	17.31	18.51
CAPEX (M, $)	40.46							2.85	
OPEX (M, $)				2.50	2.58	2.65	2.73	2.81	2.90
EBIT (M, $)	-40.46			10.90	11.70	12.48	13.45	11.64	15.61
Debt Service (M, $)						12.50	12.50	12.50	12.50
Taxes (M, $)				3.00	3.22	0.00	0.26	-0.24	0.85
FCF on Equity (M, $)	-40.46	0.00	0.00	7.90	8.48	-0.01	0.69	-0.62	2.25

Year	2011	2012	2013	2014	2015	2016	2017	2018
Traffic Volume Growth Rate (%)								
Traffic Volume (M)	8.846	9.184	9.335	9.489	9.646	9.806	9.969	10.130
Toll Rate ($)	2.24	2.30	2.37	2.44	2.52	2.59	2.67	2.75
Gross Revenue (M, $)	19.79	21.16	22.16	23.20	24.29	25.43	26.63	27.87
CAPEX (M, $)				3.30				
OPEX (M, $)	2.99	3.07	3.17	3.26	3.36	3.46	3.56	3.67
EBIT (M, $)	16.81	18.09	18.99	16.64	20.93	21.97	23.07	24.20
Debt Service (M, $)	12.50	12.50	12.50	12.50	12.50	12.50	12.50	12.50
Taxes (M, $)	1.18	1.54	1.78	1.14	2.32	2.60	2.91	3.22
FCF on Equity (M, $)	3.12	4.05	4.70	3.00	6.11	6.87	7.66	8.48

Year	2019	2020	2021	2022	2023	2024	2025	2026
Traffic Volume Growth Rate (%)								
Traffic Volume (M)	10.300	10.470	10.643	10.819	11.051	11.288	11.530	11.778
Toll Rate ($)	2.83	2.92	3.01	3.10	3.19	3.29	3.38	3.49
Gross Revenue (M, $)	29.19	30.56	32.00	33.51	35.25	37.09	39.02	41.05
CAPEX (M, $)	3.83					4.44		
OPEX (M, $)	3.78	3.89	4.01	4.13	4.26	4.38	4.52	4.65
EBIT (M, $)	21.58	26.67	27.99	29.37	30.99	28.27	34.50	36.40
Debt Service (M, $)	12.50							
Taxes (M, $)	2.50	7.33	7.70	8.08	8.52	7.77	9.49	10.01
FCF on Equity (M, $)	6.58	19.33	20.29	21.30	22.47	20.49	25.01	26.39

Year	2027	2028	2029	2030	2031	2032	2033	2034
Traffic Volume Growth Rate (%)								
Traffic Volume (M)	12.032	12.290	12.554	12.824	13.100	13.381	13.668	13.962
Toll Rate ($)	3.59	3.70	3.81	3.92	4.04	4.16	4.29	4.42
Gross Revenue (M, $)	43.20	45.45	47.82	50.31	52.93	55.69	58.59	61.65
CAPEX (M, $)			5.14					5.96
OPEX (M, $)	4.79	4.93	5.08	5.23	5.39	5.55	5.72	5.89
EBIT (M, $)	38.41	40.51	37.59	45.08	47.54	50.14	52.87	49.80
Debt Service (M, $)								
Taxes (M, $)	10.56	11.14	10.34	12.40	13.07	13.79	14.54	13.69
FCF on Equity (M, $)	27.84	29.37	27.25	32.68	34.47	36.35	38.33	36.10

7.2.4.2 Method 2 - Cheah and Liu's Real Option Model

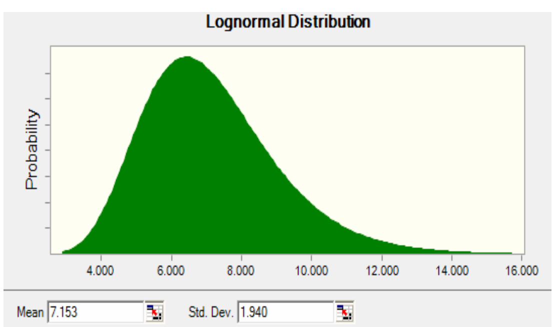
As mentioned in Section 6.5 and 7.1.4.2, we calculate the MRG option value for the KRRC BOT project by method 2. Table 7.16 and Figure 7.19 show the probability distribution of two risky variables; initial traffic volume and traffic volume growth rate, and the process of defining those variables in a Monte Carlo program (Crystal Ball software) respectively. Figure 7.20 shows the simulated cash flow model, which is the result of the simulation (10,000 iterations) by method 2. Figure 7.21 is the description of the comparison between expected, guaranteed, and simulated free cash flows on equity in the KRRC BOT Project.

1) Key Variables of the Monte Carlo Simulation

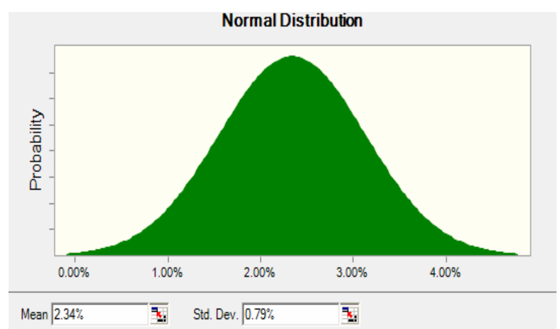
Table 7.16 Probability Distribution of Two Input Variables in Method 2 at the KRRC Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal Distribution	Normal Distribution
Mean	7.153	2.34
Standard Deviation	1.940	0.79

Figure 7.19 Defining the Assumption of the Two Input Variables in Method 2 at the KRRC Project



1) Initial Traffic Volume



2) Traffic Volume Growth Rate

Figure 7.20 Cash Flow Model of the KRRC Project by Method 2

(M: Million / $: Dollar)

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010
Traffic Volume Growth Rate (%)					2.34%	2.34%	2.34%	2.34%	2.34%
Traffic volume (M) - Expected				7.153	7.398	7.614	7.905	8.206	8.520
Traffic volume (M) - Simulated				7.151	7.322	7.573	7.794	8.092	8.400
Toll Rate ($)				1.87	1.93	1.99	2.05	2.11	2.17
Gross Revenue (M, $) - Expected				13.40	14.28	15.14	16.18	17.31	18.51
Gross Revenue (M, $) - Simulated				13.40	14.13	14.56	14.99	15.44	15.90
CAPEX (M, $)								2.85	
OPEX (M, $)				2.50	2.58	2.65	2.73	2.81	2.90
EBIT (M, $) - Expected				10.90	11.70	12.48	13.45	11.64	15.61
EBIT (M, $) - Simulated				10.90	11.56	11.90	12.26	9.78	13.01
Debt Service (M, $)						12.50	12.50	12.50	12.50
Taxes (M, $) - Expected				3.00	3.22	0.00	0.26	0.00	0.85
Taxes (M, $) - Simulated				3.00	3.18	0.00	0.00	0.00	0.14
Expected FCF on Equity (M, $)				7.90	8.48	-0.02	0.69	-0.86	2.25
Guaranteed FCF on Equity (M, $)				7.11	7.64	-0.02	0.62	-0.77	2.03
Simulated FCF on Equity (M, $)				7.90	8.38	-0.60	-0.24	-2.72	0.37
Cash Flow Difference (M, $)				0.00	0.00	0.58	0.86	1.95	1.66

Year	2011	2012	2013	2014	2015	2016	2017	2018
Traffic Volume Growth Rate (%)	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic volume (M) - Expected	8.846	9.184	9.335	9.489	9.646	9.806	9.969	10.130
Traffic volume (M) - Simulated	8.722	9.055	9.401	9.556	9.714	9.874	10.038	10.205
Toll Rate ($)	2.24	2.30	2.37	2.44	2.52	2.59	2.67	2.75
Gross Revenue (M, $) - Expected	19.79	21.16	22.16	23.20	24.29	25.43	26.63	27.87
Gross Revenue (M, $) - Simulated	16.38	16.87	17.38	17.90	18.44	18.99	19.56	20.15
CAPEX (M, $)				3.30				
OPEX (M, $)	2.99	3.07	3.17	3.26	3.36	3.46	3.56	3.67
EBIT (M, $) - Expected	16.81	18.09	18.99	16.64	20.93	21.97	23.07	24.20
EBIT (M, $) - Simulated	13.40	13.80	14.21	11.34	15.08	15.53	16.00	16.48
Debt Service (M, $)	12.50	12.50	12.50	12.50	12.50	12.50	12.50	12.50
Taxes (M, $) - Expected	1.18	1.54	1.78	1.14	2.32	2.60	2.91	3.22
Taxes (M, $) - Simulated	0.25	0.36	0.47	0.00	0.71	0.83	0.96	1.09
Expected FCF on Equity (M, $)	3.12	4.05	4.70	3.00	6.11	6.87	7.66	8.48
Guaranteed FCF on Equity (M, $)	2.81	3.65	4.23	2.70	5.50	6.18	6.89	7.63
Simulated FCF on Equity (M, $)	0.65	0.94	1.24	-1.16	1.87	2.20	2.53	2.88
Cash Flow Difference (M, $)	2.16	2.71	2.99	3.86	3.63	3.98	4.36	4.75

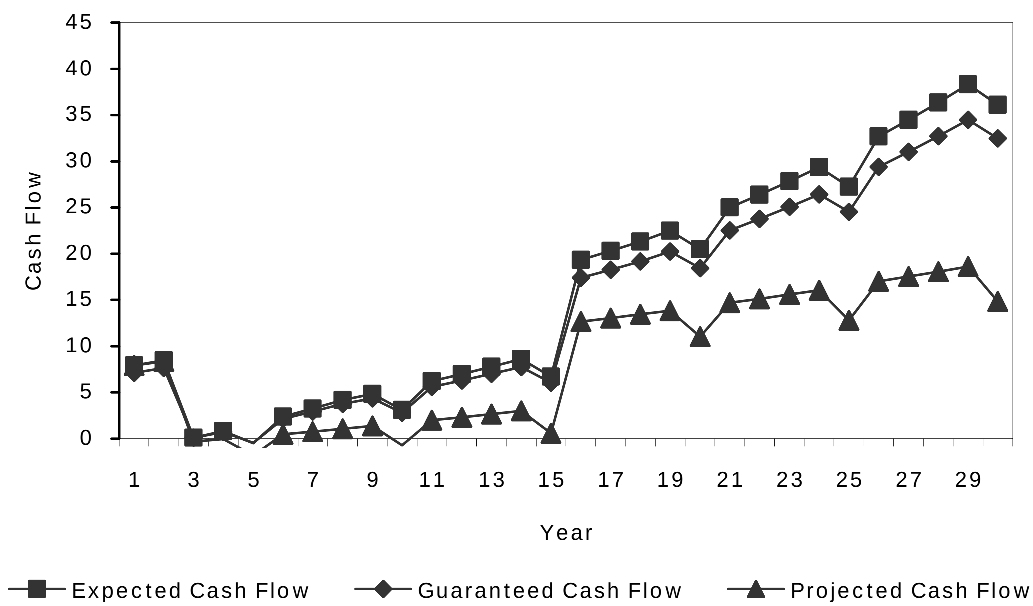
Year	2019	2020	2021	2022	2023	2024	2025	2026
Traffic Volume Growth Rate (%)	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic volume (M) - Expected	10.300	10.470	10.643	10.819	11.051	11.288	11.530	11.778
Traffic volume (M) - Simulated	10.370	10.544	10.718	10.895	11.075	11.313	11.555	11.803
Toll Rate ($)	2.83	2.92	3.01	3.10	3.19	3.29	3.38	3.49
Gross Revenue (M, $) - Expected	29.19	30.56	32.00	33.51	35.25	37.09	39.02	41.05
Gross Revenue (M, $) - Simulated	20.75	21.37	22.02	22.68	23.36	24.06	24.78	25.52
CAPEX (M, $)	3.83					4.44		
OPEX (M, $)	3.78	3.89	4.01	4.13	4.26	4.38	4.52	4.65
EBIT (M, $) - Expected	21.58	26.67	27.99	29.37	30.99	28.27	34.50	36.40
EBIT (M, $) - Simulated	13.14	17.48	18.00	18.54	19.10	15.24	20.26	20.87
Debt Service (M, $)	12.50							
Taxes (M, $) - Expected	2.50	7.33	7.70	8.08	8.52	7.77	9.49	10.01
Taxes (M, $) - Simulated	0.18	4.81	4.95	5.10	5.25	4.19	5.57	5.74
Expected FCF on Equity (M, $)	6.58	19.33	20.29	21.30	22.47	20.49	25.01	26.39
Guaranteed FCF on Equity (M, $)	5.93	17.40	18.26	19.17	20.22	18.44	22.51	23.75
Simulated FCF on Equity (M, $)	0.47	12.67	13.05	13.44	13.85	11.05	14.69	15.13
Cash Flow Difference (M, $)	5.46	4.73	5.21	5.72	6.38	7.40	7.82	8.62

Figure 7.20 (Continued)

(M: Million / $: Dollar)

Year	2027	2028	2029	2030	2031	2032	2033	2034
Traffic Volume Growth Rate (%)	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic volume (M) - Expected	12.032	12.290	12.554	12.824	13.100	13.381	13.668	13.962
Traffic volume (M) - Simulated	12.057	12.317	12.581	12.851	13.128	13.410	13.698	13.992
Toll Rate ($)	3.59	3.70	3.81	3.92	4.04	4.16	4.29	4.42
Gross Revenue (M, $) - Expected	43.20	45.45	47.82	50.31	52.93	55.69	58.59	61.65
Gross Revenue (M, $) - Simulated	26.29	27.08	27.89	28.73	29.59	30.48	31.39	32.33
CAPEX (M, $)			5.14					5.96
OPEX (M, $)	4.79	4.93	5.08	5.23	5.39	5.55	5.72	5.89
EBIT (M, $) - Expected	38.41	40.51	37.59	45.08	47.54	50.14	52.87	49.80
EBIT (M, $) - Simulated	21.50	22.14	17.66	23.49	24.20	24.92	25.67	20.48
Debt Service (M, $)								
Taxes (M, $) - Expected	10.56	11.14	10.34	12.40	13.07	13.79	14.54	13.69
Taxes (M, $) - Simulated	5.91	6.09	4.86	6.46	6.65	6.85	7.06	5.63
Expected FCF on Equity (M, $)	27.84	29.37	27.25	32.68	34.47	36.35	38.33	36.10
Guaranteed FCF on Equity (M, $)	25.06	26.43	24.53	29.41	31.02	32.72	34.50	32.49
Simulated FCF on Equity (M, $)	15.59	16.05	12.81	17.03	17.54	18.07	18.61	14.85
Cash Flow Difference (M, $)	9.47	10.38	11.72	12.38	13.48	14.65	15.89	17.64

Figure 7.21 Comparison of the Cash Flows of the KRRC BOT Project



2) MRG Option Value

With the guaranteed free cash flows on equity by method 1 (90 % of the expected cash flows estimated by BOT developer) and simulated free cash flows on equity by simulation in method 2, we discount the cash flow difference at risk free rate, 5.3%, and have the MRG option value as shown in Figure 7.22 and Table 7.17.

Figure 7.22 MRG Value by Method 2 in the KRRC Project

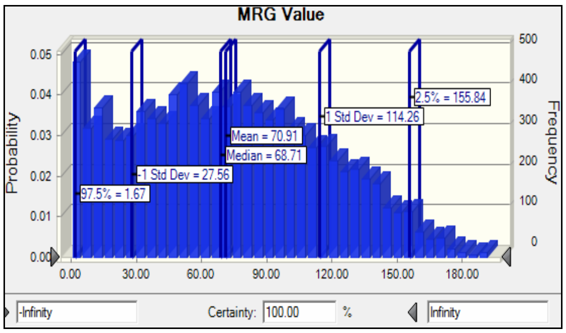


Table 7.17 MRG Value by Method 2 in the KRRC Project

	MRG Value (Million, $)
Maximum	216.89
2.5%	155.84
$+1\cdot\sigma$	114.26
Mean	70.91
Median	68.71
$-1\cdot\sigma$	27.56
97.5 %	1.670
Minimum	0.00

7.2.4.3 Method 3 - Developed Real Option Model

In calculating the MRG option value for the KRRC BOT project based on method 3, Table 7.18 and Figure 7.23 show the probability distribution of two risky variables; initial traffic volume and traffic volume growth rate, and the process of defining those variables in a Monte Carlo program (with Crystal Ball software) respectively.

Figure 7.24 shows the cash flow model, which is the result of the simulation (10,000 iterations), to calculate the more detailed level of volatility in project returns in method 3. And, we have the volatility of 0.082 as shown in Figure 7.25 and Table 7.19. Based on this volatility value we calculate the parameters necessary for the real option analysis in Table 7.20 and, with the exercise price in Table 7.21, construct a binomial tree of the project value in Figure 7.26. Figure 7.27 is the MRG option value obtained from the developed real option model.

1) Find the Initial Project Value

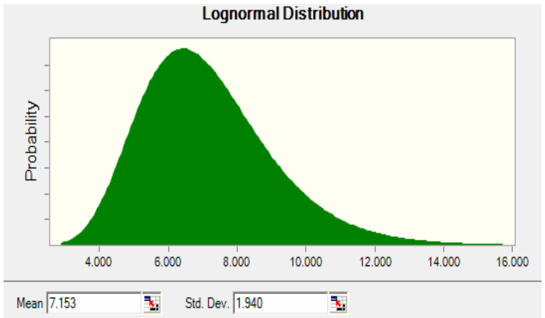
$$V_I = \$\ 54.10\ Million \tag{7.19}$$

2) Selection of Volatility "σ"

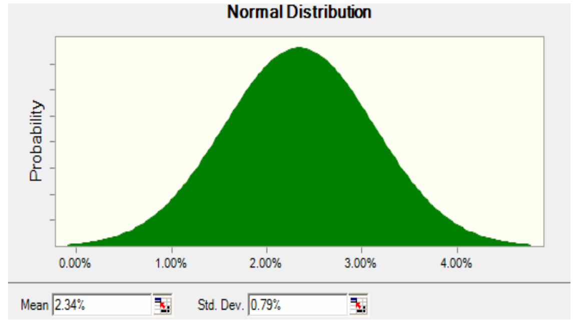
Table 7.18 Probability Distribution of Two Input Variables in Method 3 at the KRRC Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal Distribution	Normal Distribution
Mean	7.153	2.34
Standard Deviation	1.940	0.79

Figure 7.23 Defining the Assumption of the Two Input Variables in Method 3 at the KRRC Project



1) Initial Traffic Volume



2) Traffic Volume Growth Rate

Figure 7.24 Cash Flow Model for Monte Carlo Simulation in Method 3 at the KRRC Project

(M: Million / $: Dollar)

Year	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
Traffic Volume Growth Rate					2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic Volume (M)				7.153	7.398	7.614	7.905	8.206	8.52	8.846
Toll Rate ($)				1.87	1.93	1.99	2.05	2.11	2.17	2.24
Gross Revenue (M, $) - Expected				13.40	14.28	15.14	16.18	17.31	18.51	19.79
Gross Revenues (M, $) – Simulated				13.40	14.13	15.05	15.96	17.07	18.25	19.51
CAPEX (M, $)								2.85		
OPEX (M, $)				2.50	2.58	2.65	2.73	2.81	2.90	2.99
EBIT (M, $)				10.90	11.70	12.48	13.45	11.64	15.61	16.81
Debt Service (M, $)						12.50	12.50	12.50	12.50	12.50
Taxes (M, $) - Expected				3.00	3.22	0.00	0.26	-0.24	0.85	1.18
Taxes (M, $) – Simulated				3.00	3.18	-0.03	0.20	-0.30	0.78	1.11
FCF on Equity (M, $) - Expected				7.90	8.48	-0.01	0.69	-0.62	2.25	3.12
FCF on Equity (M, $) - Simulated				**A**	**B**	**C**	**D**	**E**	**F**	**G**

Year	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
Traffic Volume Growth Rate	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic Volume (M)	9.184	9.335	9.489	9.646	9.806	9.969	10.13	10.3
Toll Rate ($)	2.30	2.37	2.44	2.52	2.59	2.67	2.75	2.83
Gross Revenue (M, $) - Expected	21.16	22.16	23.20	24.29	25.43	26.63	27.87	29.19
Gross Revenues (M, $) – Simulated	20.87	22.31	23.36	24.46	25.61	26.82	28.08	29.39
CAPEX (M, $)			3.30					3.83
OPEX (M, $)	3.07	3.17	3.26	3.36	3.46	3.56	3.67	3.78
EBIT (M, $)	18.09	18.99	16.64	20.93	21.97	23.07	24.20	21.58
Debt Service (M, $)	12.50	12.50	12.50	12.50	12.50	12.50	12.50	12.50
Taxes (M, $) - Expected	1.54	1.78	1.14	2.32	2.60	2.91	3.22	2.50
Taxes (M, $) – Simulated	1.46	1.83	1.18	2.36	2.65	2.96	3.27	2.55
FCF on Equity (M, $) - Expected	4.05	4.70	3.00	6.11	6.87	7.66	8.48	6.58
FCF on Equity (M, $) - Simulated	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**

Figure 7.24 (Continued)

(M: Million / $: Dollar)

Year	2020	2021	2022	2023	2024	2025	2026
Traffic Volume Growth Rate	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic Volume (M)	10.47	10.643	10.819	11.051	11.288	11.53	11.778
Toll Rate ($)	2.92	3.01	3.10	3.19	3.29	3.38	3.49
Gross Revenue (M, $) - Expected	30.56	32.00	33.51	35.25	37.09	39.02	41.05
Gross Revenues (M, $) – Simulated	30.78	32.23	33.74	35.33	37.17	39.10	41.14
CAPEX (M, $)					4.44		
OPEX (M, $)	3.89	4.01	4.13	4.26	4.38	4.52	4.65
EBIT (M, $)	26.67	27.99	29.37	30.99	28.27	34.50	36.40
Debt Service (M, $)							
Taxes (M, $) - Expected	7.33	7.70	8.08	8.52	7.77	9.49	10.01
Taxes (M, $) – Simulated	7.39	7.76	8.14	8.54	7.80	9.51	10.03
FCF on Equity (M, $) - Expected	19.33	20.29	21.30	22.47	20.49	25.01	26.39
FCF on Equity (M, $) - Simulated	**P**	**Q**	**R**	**S**	**T**	**U**	**V**

Year	2027	2028	2029	2030	2031	2032	2033	2034
Traffic Volume Growth Rate	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%	2.34%
Traffic Volume (M) – Simulated	12.032	12.29	12.554	12.824	13.1	13.381	13.668	13.962
Toll Rate ($)	3.59	3.70	3.81	3.92	4.04	4.16	4.29	4.42
Gross Revenue (M, $) – Expected	43.20	45.45	47.82	50.31	52.93	55.69	58.59	61.65
Gross Revenues (M, $) - Simulated	43.29	45.55	47.92	50.42	53.05	55.81	58.72	61.78
CAPEX (M, $)			5.14					5.96
OPEX (M, $)	4.79	4.93	5.08	5.23	5.39	5.55	5.72	5.89
EBIT (M, $)	38.41	40.51	37.59	45.08	47.54	50.14	52.87	49.80
Debt Service (M, $)								
Taxes (M, $) – Expected	10.56	11.14	10.34	12.40	13.07	13.79	14.54	13.69
Taxes (M, $) – Simulated	10.59	11.17	10.37	12.43	13.10	13.82	14.58	13.73
FCF on Equity (M, $) - Expected	27.84	29.37	27.25	32.68	34.47	36.35	38.33	36.10
FCF on Equity (M, $) - Simulated	**W**	**X**	**Y**	**Z**	**AA**	**AB**	**AC**	**AD**

Figure 7.25 Probability Distribution of "z" in the KRRC Project

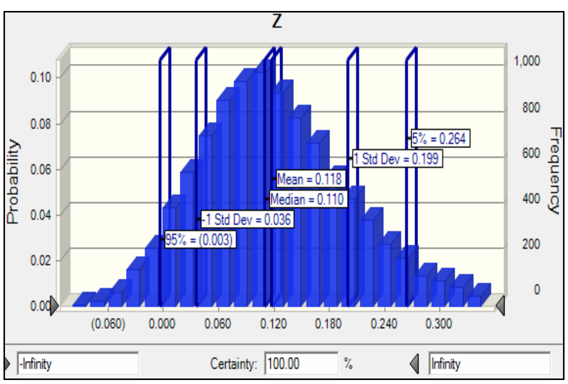


Table 7.19 Volatility of Project Value in the KRRC Project

Statistics	Forecast Values
Volatility	0.082
Maximum	0.495
5 %	0.264
+ 1 ⋅ σ	0.199
Mean	0.118
Median	0.110
-1 ⋅ σ	0.036
95 %	0.003
Minimum	-0.100

Table 7.20 Calculated Parameters in the KRRC Project

Calculated Parameters			
Initial Project Value "V_I"(Million, $)	54.10	*Volatility "σ"*	0.082
Up Movement "u"	1.141	*Concession Period (Year)*	30
Down Movement "d"	0.958	*Risk Neutral Probability "q"*	0.5
Risk Free Rate "r" (%)	5.3	*Risk Neutral Probability "$1-q$"*	0.5

Table 7.21 Guaranteed Project Value during Concession Period in the KRRC Project (Million, $)

Year	*2004*	*2005*	*2006*	*2007*	*2008*	*2009*	*2010*	*2011*	*2012*	*2013*	*2014*	*2015*	*2016*	*2017*	*2018*	*2019*
Guaranteed Project Value	48.69	51.17	53.77	56.51	59.38	62.41	65.58	68.92	72.43	76.11	79.99	84.06	88.33	92.83	97.55	102.52

Year	*2020*	*2021*	*2022*	*2023*	*2024*	*2025*	*2026*	*2027*	*2028*	*2029*	*2030*	*2031*	*2032*	*2033*	*2034*
Guaranteed Project Value	107.74	113.22	118.98	125.04	131.40	138.09	145.11	152.50	160.26	168.41	176.98	185.99	195.46	205.40	215.86

3) Implementing Calculation Backward Recursively

Figure 7.26 and 7.27 are the binomial tree of the KRRC BOT project and asymmetric payoff. Figure 7.28 is the description of the binomial tree for the MRG option value. We have the MRG option value at time "0" (at year 2004), which is $ 4.99 million and, by discounting this value to 2002 with a risk-free rate of 5.3%, we obtain the MRG value of $ 4.49 million.

Figure 7.26 Binomial Tree of the KRRC Project Value

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
54.10	62	70	80	92	104	119	136	155	177	202	230	263	300	342	390	445	507	578	660	753	859	979	1117	1274	1454	1658	1891	2157	2461	2807
	52	60	68	78	89	101	115	132	150	171	195	223	254	290	331	377	430	491	560	639	729	831	948	1082	1234	1407	1605	1831	2089	2383
		51	58	66	75	86	98	112	127	145	166	189	216	246	281	320	365	417	475	542	618	705	805	918	1047	1194	1362	1554	1773	2022
			49	56	64	73	83	95	108	123	141	161	183	209	238	272	310	354	403	460	525	599	683	779	889	1014	1156	1319	1505	1716
				48	54	62	71	80	92	105	119	136	155	177	202	231	263	300	342	391	446	508	580	661	754	860	981	1120	1277	1457
					46	52	60	68	78	89	101	116	132	150	172	196	223	255	291	331	378	431	492	561	640	730	833	950	1084	1236
						45	51	58	66	75	86	98	112	128	146	166	190	216	247	281	321	366	418	476	543	620	707	806	920	1049
							43	49	56	64	73	83	95	108	124	141	161	184	209	239	272	311	354	404	461	526	600	685	781	891
								42	48	54	62	71	81	92	105	120	137	156	178	203	231	264	301	343	391	446	509	581	663	756
									40	46	53	60	68	78	89	102	116	132	151	172	196	224	255	291	332	379	432	493	562	642
										39	45	51	58	66	76	86	98	112	128	146	167	190	217	247	282	322	367	419	477	545
											38	43	49	56	64	73	83	95	109	124	141	161	184	210	239	273	311	355	405	462
												37	42	48	54	62	71	81	92	105	120	137	156	178	203	232	264	301	344	392
													36	41	46	53	60	69	78	89	102	116	132	151	172	197	224	256	292	333
														34	39	45	51	58	66	76	86	99	112	128	146	167	190	217	248	283
															33	38	43	49	56	64	73	84	95	109	124	142	162	184	210	240
																32	37	42	48	55	62	71	81	92	105	120	137	156	178	204
																	31	36	41	46	53	60	69	78	89	102	116	133	151	173
																		30	34	39	45	51	58	67	76	87	99	113	129	147
																			29	33	38	43	50	56	64	74	84	96	109	124
																				28	32	37	42	48	55	62	71	81	93	106
																					27	31	36	41	46	53	60	69	79	90
																						27	30	35	39	45	51	58	67	76
																							26	29	33	38	44	50	57	65
																								25	28	32	37	42	48	55
																									24	27	31	36	41	47
																										23	27	30	35	39
																											23	26	29	34
																												22	25	28
																													21	24
																														20

Figure 7.27 Binomial Tree of Asymmetric Payoff in the KRRC Project

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			8	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				12	8	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					17	13	9	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						21	18	15	10	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							26	24	20	16	11	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
								31	29	26	22	18	12	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
									36	34	32	29	25	20	14	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
										41	40	38	35	32	27	22	15	7	0	0	0	0	0	0	0	0	0	0	0	0
											47	45	44	42	39	35	30	24	17	8	0	0	0	0	0	0	0	0	0	0
												52	51	50	48	46	43	39	33	27	18	9	0	0	0	0	0	0	0	0
													58	57	57	55	54	51	47	43	37	29	20	9	0	0	0	0	0	0
														64	64	63	63	61	59	56	52	47	41	32	22	10	0	0	0	0
															70	70	70	70	69	68	65	62	58	52	45	36	25	11	0	0
																76	77	78	78	77	76	75	72	68	64	57	49	40	27	13
																	82	84	85	86	86	85	84	82	80	76	70	63	55	44
																		89	91	93	94	94	95	94	93	91	88	83	78	70
																			96	98	100	102	104	104	105	104	103	101	97	92
																				103	106	109	111	113	114	115	115	115	114	111
																					111	114	117	120	123	125	126	127	128	127
																						119	123	126	129	133	135	138	139	140
																							127	131	135	139	143	146	149	152
																								136	140	145	149	154	158	162
																									145	150	155	160	165	170
																										154	160	165	171	177
																											164	170	176	183
																												174	181	188
																													184	192
																														195

Figure 7.28 MRG Value of the KRRC Project (MRG Value = $ 4.49 million)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
4.99	4	3	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	5	4	3	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		8	7	6	4	3	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			11	9	8	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				14	12	10	8	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					17	15	13	11	9	7	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						21	19	16	14	11	9	7	5	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							26	23	21	18	15	12	10	7	5	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0
								31	28	26	23	20	16	13	10	7	5	3	1	1	0	0	0	0	0	0	0	0	0	0
									36	34	31	28	25	21	18	14	10	7	5	2	1	0	0	0	0	0	0	0	0	0
										41	39	37	35	31	27	23	19	15	11	7	4	2	1	0	0	0	0	0	0	0
											46	45	44	41	38	35	30	25	20	15	11	7	3	1	0	0	0	0	0	0
												52	51	50	48	46	42	38	33	28	22	16	11	6	3	1	0	0	0	0
													57	57	56	55	53	50	47	42	36	30	23	16	10	5	1	0	0	0
														63	63	63	62	61	59	56	52	47	40	33	25	16	9	3	0	0
															69	70	70	70	69	67	65	61	57	51	44	35	26	16	6	0
																76	76	77	77	77	76	74	71	68	63	57	49	39	27	12
																	82	83	84	85	85	85	84	82	79	75	70	63	54	43
																		89	91	92	93	94	94	94	92	90	87	83	77	69
																			96	98	100	102	103	104	104	103	102	100	96	91
																				103	106	108	110	112	114	115	115	114	113	110
																					111	114	117	120	122	124	126	127	127	126
																						119	122	126	129	132	135	137	139	140
																							127	131	135	139	142	146	149	151
																								135	140	145	149	153	157	161
																									144	150	155	160	165	169
																										154	159	165	171	176
																											163	170	176	182
																												174	180	187
																													184	192
																														195

7.2.4.4 Results of the BOT Project Valuation Methods

Like the MCB BOT project, three valuation methods are considered to evaluate the MRG option value in the KRRC project.

As results, first, we have the NPV on equity of $ 2.02 million by method 1. This value does not consider the MRG agreement option.

Second, we obtained the MRG option value with method 2. The mean and median of MRG value are $ 70.91 million and $ 68.71 million respectively. The mean of MRG value accounts for 3510.4 % of NPV on equity of $ 2.02 million and 177.28 % of initial equity investment of $ 40 million. And, the median accounts for 3401.49 % and 171.78 % respectively. As already mentioned in Section 4, these mean and median values of the MRG by method 2 do not seem to be reasonable since these values are quite higher than the NPV on equity and equity investment. In real world, if these MRG values are likely to happen, the government does not have to attract the private consortium since it is much better for the government to implement this project within their own finance and control. Furthermore, as the probability distribution of the MRG value by method 2 is excessively skewed to right side, it is hard to regard the mean or median value of the MRG as reasonable.

Third, with method 3, we found that the MRG value is $ 4.49 million. This value accounts for about 222.28 % of NPV on equity and 11.23 % of initial equity investment respectively. Table 7.22 shows the comparison of the MRG values by three project valuation methods and here we can recognize that the MRG values by method 2 and 3 have significant impact on NPV on equity and initial equity investment cost.

Table 7.22 MRG Option Values in the KRRC Project by Three Valuation Methods

(MRG, NPVe: Net Present Value on Equity, Ie: Equity Investment / Million, $)

		MRG Value	MRG Value/NPVe	MRG Value/Ie	MRG Value + NPVe	(MRG Value + NPVe)/NPVe
Method 1		0	**0.00%**	**0.00%**	2.02	100.00%
Method 2	Max	216.89	**10737.13%**	**542.23%**	218.91	0.92%
	2.5%	155.84	**7714.85%**	**389.60%**	157.86	1.28%
	$+1 \cdot \sigma$	114.26	**5656.44%**	**285.65%**	116.28	1.74%
	Mean	70.91	**3510.40%**	**177.28%**	72.93	3610.40%
	Median	68.71	**3401.49%**	**171.78%**	70.73	3501.49%
	$-1 \cdot \sigma$	27.56	**1364.36%**	**68.90%**	29.58	1464.36%
	97.5%	1.67	**82.67%**	**4.18%**	3.69	182.67%
	Min	0	**0.00%**	**0.00%**	2.02	100.00%
Method 3		4.49	**222.28%**	**11.23%**	6.51	322.28%

7.2.4.5 Validation Test of Developed Real Option Model

As mentioned above, we will use three different BOT projects to test if the developed real option model can provide reasonable degree of the validation. The KRRC project is the second case among three BOT case studies we are going to examine.

Hypothesis One - *"The project value using the two option pricing methods 2 and 3 under the MRG agreement will show significant value rather than the project value of method 1, which is called NPV analysis."*

As shown in Table 7.22, the NPV on equity by method 2 and 3 are 3510.40 % (mean) and 3401.49 % (median), and 222.28 % respectively higher than that of method 1. So, we can see that the NPV on equity by project valuation methods 2 and 3 under MRG option are significant compared to that of method 1.

Hypothesis Two - *"Based upon the option pricing theory, the predicted effects of major determinants (current price of the underlying asset V_I, exercise price X, time to maturity T, volatility σ, and risk free interest rate r) on a MRG option value in the developed binomial real option model have to follow those of option pricing theory based on the Black-Scholes model."*

As we can see in Figure 7.29, the change of the MRG option value against the change of the initial project value V_I is coincident with the result of the partial differential $\partial P / \partial V_I < 0$ of Equation (3.91) based on the Black-Scholes model.

As shown in Figure 7.30, the change of the MRG value on the change of the exercise price X follows the relationships of Equation (3.93).

Figure 7.31 shows that the impact of the time to maturity on the MRG value is positive and follows the tendency of the Equation (3.94).

Figure 7.32 shows that as σ increases the MRG option value increases. And, the decrease of σ causes the decrease of the MRG value. Here, this shows that the trend of the MRG option value change against volatility change from the results of the developed real option model follows that of Equation (3.99) by the Black-Scholes model.

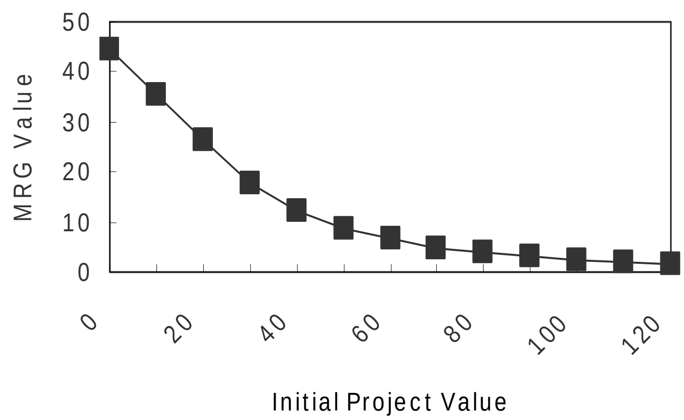
Figure 7.33 has similar shape to the continuous function for $\partial P / \partial r$ in Equation (3.100). Like in MCB case, the sensitivity of the MRG option value to the risk-free interest rate seems to be relatively minor compared to the effects of other input variables.

Finally, like the results of the MCB project case, in KRRC project case we find out that the predicted effects of five input variables on a MRG option value with the developed real option model follow those of option pricing theory based on the Black-Scholes model.

Initial Project Value V_I

Figure 7.29 MRG Value Change against Initial Project Value V_I Change in the KRRC Project

(MRG, V_I : Million, $)

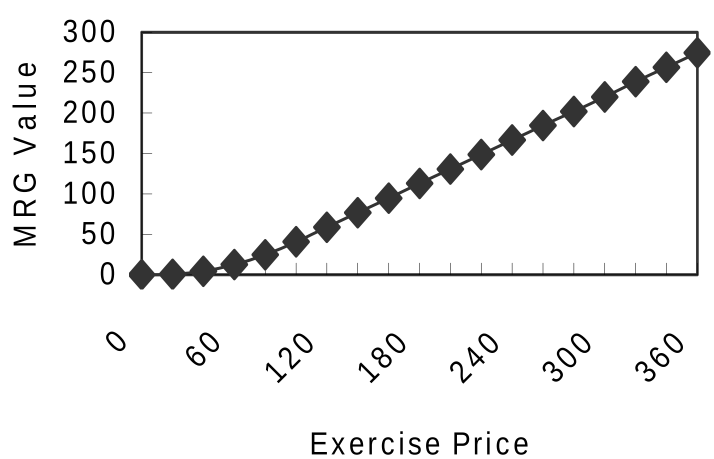


(*Initial* X =48.69, T =30, σ =0.082, r =0.053)

Exercise Price X

Figure 7.30 MRG Value Change against Exercise Price X Change in the KRRC Project

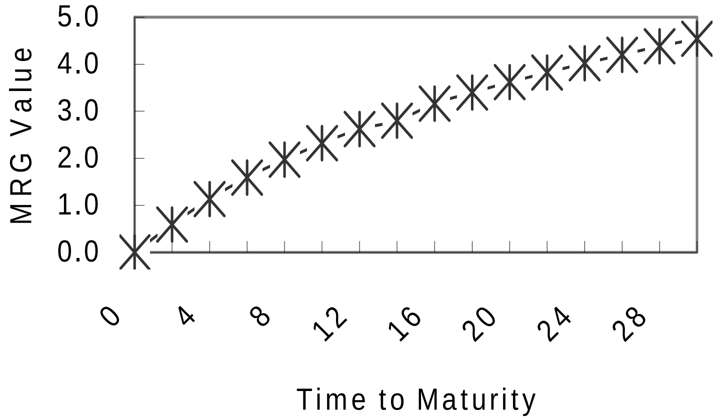
(MRG, X: Million, $)



(V_I = 54.10, T =30, σ =0.082, r =0.053)

Time to Maturity T

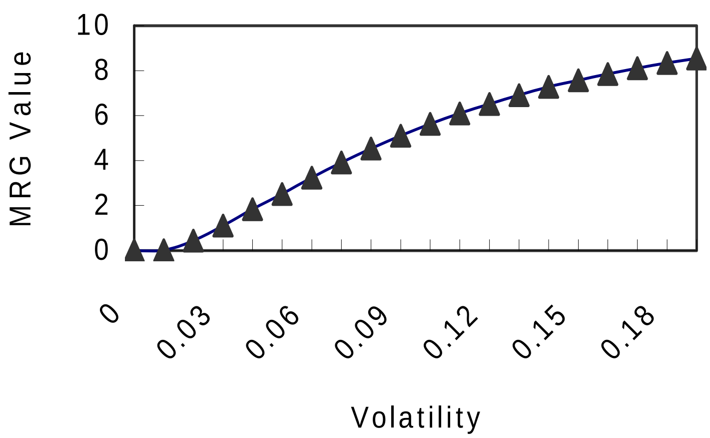
Figure 7.31 MRG Value Change against Time to Maturity T Change in the KRRC Project (MRG: Million, $; T: Year)



(V_I = 54.10, *Initial* X =48.69, σ =0.082, r =0.053)

Volatility σ

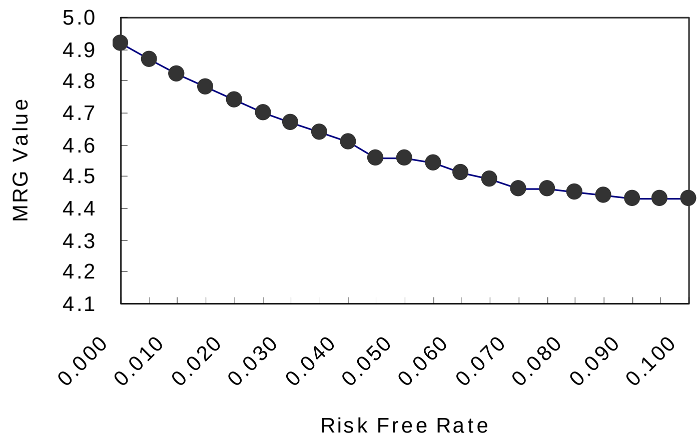
Figure 7.32 MRG Value Change against Volatility σ Change in the KRRC Project (MRG: Million, $)



(V_I = 54.10, *Initial* X =48.69, T =30, r =0.053)

Risk-free Rate r

Figure 7.33 MRG Value Change against Risk-free Rate r Change in the KRRC Project (MRG: Million, $)



(V_I = 54.10, *Initial* X =48.69, T =30, σ =0.082)

Hypothesis Three **-** *"The MRG agreement value gained from the developed real option method 3 will be consistent with that of method 2. (The MRG agreement value gained from the developed method 3 will be located within the range of $\pm 2 \cdot \sigma$ from the median and mean in probability distribution of the MRG value by method 2)".*

By using the cumulative probabilities of the MRG value, if we are going to examine how far the MRG value by method 3 is away from the mean and median of the MRG value by method 2, it is shown in Figure 7.34 and Table 7.23. The MRG value of $ 4.49 million by method 3 is located within $\pm$ 43.80 % from mean and within $\pm$ 45.57 % from median in probability distribution of the MRG value by method 2. Here, since the MRG value by the developed real option model "method 3" is located within the range of $\pm 2 \cdot \sigma$ ($\pm$ 47.5 %) from mean and median in probability distribution of the MRG value by method 2, we understand that the developed method 3 satisfies the research hypothesis 2.

Figure 7.34 Comparison of MRG Values by Method 2 (Mean) and 3 in the KRRC Project

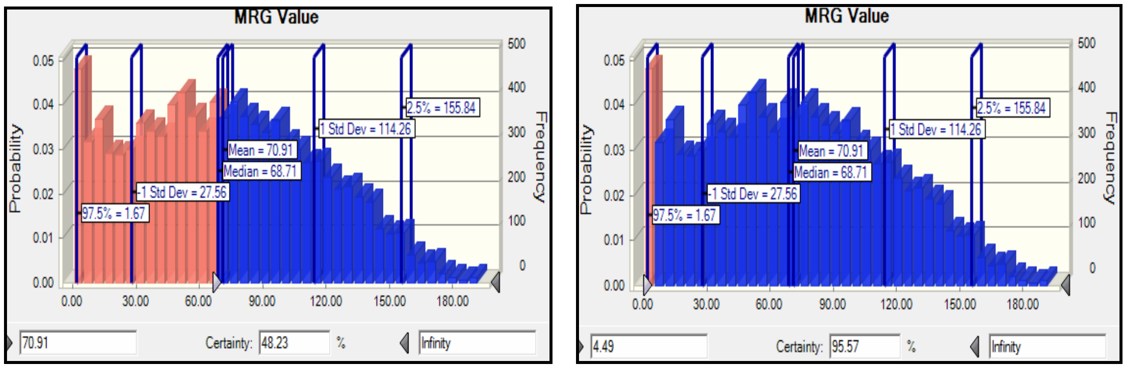


Table 7.23 MRG Values in Cumulative Probability Distributions by Method 2 and 3 in the KRRC Project

	Method 2 (By Cheah and Liu's Real Option Model)	
	Mean \$ 70.91 million (51.77 %)	Median \$ 68.71 million (50 %)
Method 3: \$ 4.49 million (95.57 %)	± 43.80 % ($< \pm 2 \cdot \sigma$)	± 45.57 % ($< \pm 2 \cdot \sigma$)

7.2.4.6 Sensitivity Analysis of MRG Value to Standard Deviation of the Initial Traffic Volume and Traffic Volume Growth Rate

Table 7.24 shows the results of the sensitivity analyses of MRG value subject to changes in the standard deviations of initial traffic volume and growth rate in KRRC project. Like in MCB case, the MRG value seems to be more sensitive to the standard deviation of the initial traffic volume assumed rather than that of traffic volume growth rate.

Table 7.24 Sensitivity of MRG Value to Standard Deviations of Initial Traffic Volume and Traffic-Volume Growth Rate in the KRRC Project

(MRG Value: Million, $)

		Standard Deviation of Growth Rate (%)			
		0.40	**0.79**	1.19	1.58
Standard Deviation of Traffic Volume (Million)	0.97	1.90	1.83	1.83	1.83
	1.94	4.49	**4.49**	4.49	4.54
	2.91	6.18	6.18	6.22	6.18
	3.88	7.39	7.33	7.39	7.33

7.3 The Cheonan-Nonsan Expressway (CNE) Project

7.3.1 Background

The Cheonan-Nonsan expressway (CNE), which was suggested by the Korean Ministry of Construction and Transportation in 1996, was planned to link Mokcheon-Myun, Cheonan city (Southern part of Kyungbu expressway) and Yunmoo-Eup, Nonsan city (Northern part of Honam expressway) with a length of 81 km and 4 lanes within Chungcheongnam Province in South Korea. This expressway was expected to reduce travel by about 30 km in distance and by 20 to 25 minutes in driving time compared to existing route through Kyungbu and the Honam expressway. The CNE project which was expected to cost a total of about $ 1.242 billion in its construction, began in 1998 and was finished in 2002.

The CNE project was suggested by the Korean Ministry of Construction and Transportation as a BOT/BTO type of privatized infrastructure project in 1996 and the Korean Ministry of Construction and Transportation entered into a contract with Daewoo Construction Co., Ltd. which is the leading equity investor of a SPV, Cheoan-Nonsan Expressway (CNE) Co., Ltd. in 1997. As a lead manager in an equity investment of the CNE project, Daewoo Construction Co., Ltd. tried to find some construction companies as equity investors with the condition that the construction work of this project would be divided in proportion to the amount of equity investments. Finally, contributing to 25 % of total equity investment, this lead to the joining of several leading Korean construction companies including LG Construction Co., Ltd.(15.0%), Hyundai Construction Co., Ltd.(12.5%), Kumho Engineering and Construction Co., Ltd.(12.0%), ISU Construction Co, Ltd.(11.0%), Hanwha Construction Co., Ltd.(10.0%), Ssangyong Construction Co., Ltd.(5.0%), Hanil Construction Co., Ltd.(5.0%), Kyoungnam Construction Co., Ltd.(4.5%). With a lead manager of Daewoo Construction Co., Ltd., the eight leading Korean construction companies built CNE Co., Ltd. as a SPV in 1997 and delegated to CNE Co., Ltd. all rights, liabilities and obligations related to the CNE project under the concession agreement to receive tolls in return for designing, constructing, operating and maintaining it. The CNE project linking between Cheonan and Nonsan cities within Chungcheongnam Province is a 4 lane-road which is 81 km long including 2 tunnels (2.95 km), 44 bridges (11.81 km), and 8 interchanges.

In 1997, the Korean Ministry of Construction and Transportation assigned the CNE Co., Ltd. as a SPV to complete the construction, operate this expressway during the concession period based upon the concession agreement between CNE Co., Ltd. and the Korean Ministry of Construction and Transportation. After the concession period ownership would transfer to the Korean Ministry of Construction and Transportation. The 9 construction companies invested a total $ 154 million on CNE Co., Ltd. as equity investors and the CNE Co., Ltd. was planned to operate and take the profit from this expressway during 30 years with a toll rate of about $ 8.42 per vehicle.

7.3.2 Contractual Structure

The CNE project which was suggested by the Korean Ministry of Construction and Transportation in 1996 was a huge privatized infrastructure project costing about $ 1.242 billion in its construction. In 1997, Korean Ministry of Construction and Transportation signed the concession agreement with the lead equity investor, Daewoo Construction Co., Ltd. and selected CNE Co., Ltd. as a SPV which is composed of 9 equity investors.

To find the money sources spent on this huge construction project, the capital structure was tightly planned with the forms of equity, senior debt, and subordinated debt. Of the construction cost of about $ 1.242 billion, CNE Co., Ltd. covered 12.4 % of construction cost, $ 154 million and, the 61.87 % of the construction cost, $ 768 million, was provided with a form of a Project Finance (PF) loan as a senior debt at fixed interest rate of 8.62 % during 25 years by a Korean leading financial institution, the Korea Development Bank. Finally, the outstanding amount of construction cost, $ 320 million, was supported by Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. in a form of subordinated debt with a tenure of 20 years and a fixed rate of 20 %.

The CNE Co., Ltd. has all rights, liabilities and obligations related to Cheonan-Nonsan expressway under the concession agreement to receive tolls in return for designing, constructing, operating and maintaining it (approximately 81 km). The fixed interest rate loan would be repaid out of following sources:

(1) Tolls will be collected through 30-year concession to operate the project and its operation will start from 2003.

(2) The Korean Ministry of Construction and Transportation has a plan to provide CNE Co., Ltd. with a revenue guarantee in case that the expected revenue that they estimated isn't reached. The minimum revenue guarantee was planned to consider from 2003 to 2022 during 20 years at the rate of 82 % of the expected revenue.

(3) On the other hand, there is a repayment agreement that will be paid to the Korean Ministry of Construction and Transportation from CNE Co., Ltd. if the actual revenue goes far beyond the expected revenue that CNE Co., Ltd. estimated. The repayment will be considered during the same period as the guarantee at the rate of 110% of the expected revenue.

7.3.3 Financial Analysis and Cash Flow Model

In order to construct the cash flow model, the following key points are extracted from the sources given by the Macquarie Korea Infrastructure Fund (MKIF) Co., Ltd. and Macquarie Shinhan Infrastructure Asset Management (MSIAM) Co., Ltd. with logical assumptions.

(1) The total cost of the project and the cost of the equity investment are taken as about $ 1.242 billion and $ 154 million respectively, and assumed to be used during the first year of the construction period.

(2) The debt and equity ratio is 87.6:12.4. The loan terms are 25 years and 20 years for senior and subordinated debt and their interest rates are 8.62 % and 20 % respectively.

(3) The concession period is 30 years.

(4) Capital expenditures which include costs for scheduled repairs, pavement resurfacing and structural replacement which are not part of routine maintenance will be assumed to be total around \$ 110.63 million. This amount which is about 9 % of total construction cost will be evenly distributed with \$ 19.10 million for every 5 years during concession period and will escalate at 3 % annually. Operating expenditures, which are primarily expenses such as office administration, utility costs, toll revenue collection and routine maintenance work such as repair, cleaning and winter maintenance, will be total \$ 522.5 million. This accounts for 42 % of the total construction cost. Like the capital expenditure, this will be evenly distributed with \$ 17.42 million for every year during concession period and escalate at 3 % annually.

(5) The corporate tax rate is 27.5 %.

(6) Under the CNE Co., Ltd.'s proposal, the initial toll rates are \$ 8.42 for cars, \$ 8.95 for buses, \$ 8.63 for vans, and \$ 12 for lorries, respectively. The average toll rate is \$ 9.47 and would escalate at inflation rate of 3 % annually. The class of vehicle, its portion against total traffic volume, and initial toll rate are as follow in Table 7.25.

Table 7.25 Initial Toll Rates for Different Vehicle Classes in the CNE Project

Vehicle Class	*Toll Rate ($)*	*Proportion (%)*
Cars	8.42	20
Buses	8.95	20
Vans	8.63	25
Lorries	12.00	25
Taxies	8.42	10

The uncertainties in the project value are assumed to be affected by initial traffic volume in 2003 (the first year of operation) and the traffic volume growth rate.

(7) The distribution of the initial traffic volume (in 2003) is assumed to follow a lognormal distribution based on the assumptions by Cheah and Liu (2006). The original traffic projection, 11.175 million vehicles, estimated by the private consortium, CNE Co., Ltd. will be taken as the mean value of the initial traffic volume variable. The following is the projection of the annual traffic volume and traffic volume growth rate estimated by CNE Co., Ltd.

Table 7.26 Annual Traffic Volume and Traffic Volume Growth Rate Estimated by the CNE Co., Ltd. (Traffic Volume: Million, Growth Rate: %)

Year	*2003*	*2004*	*2005*	*2006*	*2007*	*2008*	*2009*	*2010*	*2011*	*2012*	*2013*	*2014*	*2015*	*2016*	*2017*	*2018*
Growth Rate	-	6.85	5.92	6.07	2.73	3.90	3.88	7.86	3.68	3.17	3.02	2.83	2.80	2.78	2.75	2.67
Traffic Volume	11.18	12.80	13.58	14.43	14.83	15.42	16.03	17.34	17.99	18.57	19.14	19.69	20.25	20.82	21.40	21.98

Year	*2019*	*2020*	*2021*	*2022*	*2023*	*2024*	*2025*	*2026*	*2027*	*2028*	*2029*	*2030*	*2031*	*2032*
Growth Rate	2.34	1.98	1.98	1.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Traffic Volume	22.50	22.95	23.41	23.88	23.88	23.88	23.88	23.88	23.88	23.88	23.88	23.88	23.88	23.88

(8) The traffic volume growth rate that CNE Co., Ltd. used in its cash flow model for financial feasibility analysis will be used as one of the important risky variables with traffic volume and assumed to follow the normal distribution (Cheah and Liu, 2006). According to Table 7.26, we can calculate the parameters chosen for two risky variables as compiled in Table 7.27.

Table 7.27 Probability Distribution of Two input Variables in the CNE Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal	Normal
Mean	11.175	2.385
Standard Deviation	4.05	2.21

(9) To calculate the CNE Co., Ltd's required rate of return, we will use following information.

1. The risk free rate we will use is the Korean Treasury bill rate which is a risk-less asset whose value in March 2008 is 5.3 % for a 10 year bond. (Source: http://www.ksdabond.or.kr)

2. Market risk premium, *MRP*, is the difference between the return of the market, KOSPI (Korea Composite Stock Price Index) and the risk-free rate (T-bills). The overall market rate of return will be used to measure *MRP*. From the 1990s to 2000s, the KOSPI averaged yearly returns of 10.40 %. So, *MRP* is 10.4 – 5.3 = 5.1. (Source: http://www.ksdabond.or.kr)

3. Beta, β, is a measure of risk for a certain industry or company. It can be calculated with weighted average beta of 9 construction companies, which joined as equity investors, in proportion to each company's equity investment. (Source: http://kr.stock.yahoo.com)

Table 7.28 Weighted Average Beta and Betas of 9 Equity Investors

Company	*Equity*	*Beta*	*Weighted Beta*	*Weighted Average Beta*
Daewoo Construction Co., Ltd.	25 %	1.28	0.320	1.247
LG Construction Co., Ltd.	15.0 %	1.22	0.183	
Hyundai Construction Co., Ltd.	12.5 %	1.45	0.181	
Kumho Engineering and Construction Co., Ltd.	12.0 %	1.43	0.172	
ISU Construction Co, Ltd.	11.0 %	1.22	0.134	
Hanwha Construction Co., Ltd.	10.0 %	1.22	0.122	
Ssangyong Construction Co., Ltd.	5.0 %	0.91	0.046	
Hanil Construction Co., Ltd.	5.0 %	0.84	0.042	
Kyoungnam Construction Co., Ltd.	4.5 %	1.06	0.048	

9 equity investor's betas are mentioned in Table 7.28 and the weighted average beta of these 9 companies is 1.247.

$$
\begin{aligned}
R_e &= R_f + (MRP \times \beta) \\
&= 5.3\% + (5.1 \times 1.247) \qquad (7.20) \\
&= 11.66\%
\end{aligned}
$$

Finally, according to the results from the above formula, we can calculate the private consortium, CNE Co., Ltd.'s required rate of return, R_e, which is 11.66 %.

7.3.4 Implementation of the BOT Project Valuation

7.3.4.1 Method 1 - NPV Analysis

$$
\text{NPV on Equity} = \$\ 126.39\ \textit{Million} \qquad (7.21)
$$

As shown in the cash flow model of Figure 7.35, the CNE project has NPV on equity of $ 126.39 million by method 1.

Figure 7.35 Cash Flow Model of the CNE Project by Method 1

(M: Million / $: Dollar)

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Traffic Volume Growth Rate (%)						2.70%	2.70%	2.70%	2.70%
Traffic Volume (M)					11.175	12.804	13.581	14.433	14.831
Toll Rate ($)					9.47	9.76	10.05	10.35	10.66
Gross Revenue (M, $)					105.87	124.94	136.50	149.41	158.14
CAPEX (M, $)	154.02								20.75
OPEX (M, $)					17.42	17.94	18.48	19.03	19.60
EBIT (M, $)	-154.02				88.45	107.00	118.02	130.38	117.78
Senior Debt Service (M, $)							117.75	117.75	117.75
Sub Debt Service (M, $)							65.63	65.63	65.63
Taxes (M, $)					24.32	29.43	0.00	0.00	0.00
FCF on Equity (M, $)	-154.02	0.00	0.00	0.00	64.13	77.58	-65.36	-53.00	-65.60

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Traffic Volume Growth Rate (%)	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic Volume (M)	15.423	16.027	17.343	17.995	18.569	19.139	19.686	20.253	20.820
Toll Rate ($)	10.98	11.31	11.65	12.00	12.36	12.73	13.11	13.51	13.91
Gross Revenue (M, $)	169.38	181.30	202.07	215.96	229.53	243.67	258.16	273.56	289.66
CAPEX (M, $)					24.06				
OPEX (M, $)	20.19	20.80	21.42	22.06	22.73	23.41	24.11	24.83	25.58
EBIT (M, $)	149.19	160.50	180.65	193.89	182.75	220.27	234.05	248.73	264.08
Senior Debt Service (M, $)	117.75	117.75	117.75	117.75	117.75	117.75	117.75		
Sub Debt Service (M, $)	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63
Taxes (M, $)	0.00	0.00	0.00	2.89	0.00	10.14	13.93	50.35	54.57
FCF on Equity (M, $)	-34.19	-22.88	-2.73	7.62	-0.64	26.74	36.73	132.75	143.87

Year	2019	2020	2021	2022	2023	2024	2025	2026
Traffic Volume Growth Rate (%)	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic Volume (M)	21.400	21.975	22.502	22.954	23.413	23.883	23.883	23.883
Toll Rate ($)	14.33	14.76	15.20	15.66	16.13	16.61	17.11	17.62
Gross Revenue (M, $)	306.66	324.34	342.09	359.43	377.61	396.75	408.65	420.91
CAPEX (M, $)	27.89					32.33		
OPEX (M, $)	26.35	27.14	27.95	28.79	29.65	30.54	31.46	32.40
EBIT (M, $)	252.42	297.21	314.14	330.64	347.96	333.87	377.19	388.51
Senior Debt Service (M, $)								
Sub Debt Service (M, $)	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63
Taxes (M, $)	51.37	63.68	68.34	72.88	77.64	73.77	85.68	88.79
FCF on Equity (M, $)	135.42	167.89	180.17	192.13	204.69	194.47	225.88	234.09

Year	2027	2028	2029	2030	2031	2032	2033	2034
Traffic Volume Growth Rate (%)	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic Volume (M)	23.883	23.883	23.883	23.883	23.883	23.883	23.883	23.883
Toll Rate ($)	18.15	18.70	19.26	19.84	20.43	21.04	21.68	22.33
Gross Revenue (M, $)	433.54	446.54	459.94	473.74	487.95	502.59	517.67	533.20
CAPEX (M, $)			37.48					43.46
OPEX (M, $)	33.37	34.38	35.41	36.47	37.56	38.69	39.85	41.05
EBIT (M, $)	400.16	412.17	387.05	437.27	450.39	463.90	477.82	448.70
Senior Debt Service (M, $)								
Sub Debt Service (M, $)								
Taxes (M, $)	110.05	113.35	106.44	120.25	123.86	127.57	131.40	123.39
FCF on Equity (M, $)	290.12	298.82	280.61	317.02	326.53	336.33	346.42	325.30

7.3.4.2 Method 2 - Cheah and Liu's Real Option Model

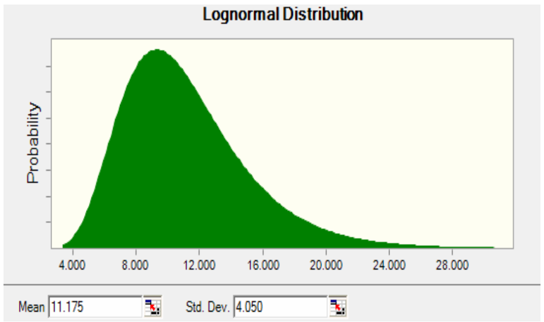
As mentioned in Section 6.5, 7.1.4.2, and 7.1.4.2, we implemented the calculation of the MRG value for the CNE BOT project by method 2. Table 7.29 and Figure 7.36 are the description of the probability distribution of initial traffic volume/traffic volume growth rate and the assumption in defining those variables in a Monte Carlo program respectively. Figure 7.37 shows the simulated cash flow model through the simulation of 10,000 time iterations by method 2. Figure 7.38 is the description of the comparison between expected, guaranteed, and simulated free cash flows on equity in the CNE BOT Project.

1) Key Variables of the Monte Carlo Simulation

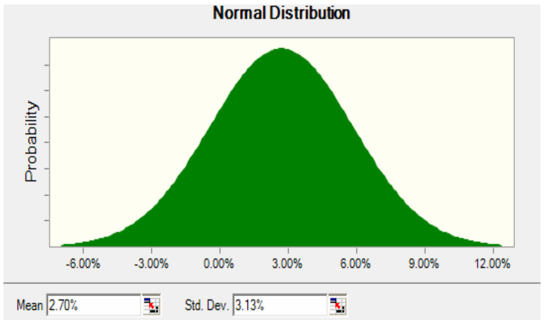
Table 7.29 Probability Distribution of Two Input Variables in Method 2 at the CNE Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal Distribution	Normal Distribution
Mean	11.175	2.70
Standard Deviation	4.050	3.13

Figure 7.36 Defining the Assumption of Two Input Variables in Method 2 at the CNE Project



1) Initial Traffic Volume



2) Traffic Volume Growth Rate

Figure 7.37 Cash Flow Model of the CNE Project by Method 2

(M: Million / $: Dollar)

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Traffic Volume Growth Rate (%)						2.70%	2.70%	2.70%	2.70%
Traffic Volume (M) – Expected					11.18	12.80	13.58	14.43	14.83
Traffic Volume (M) – Projected					11.18	11.48	11.80	12.12	12.45
Toll Rate ($)					9.47	9.76	10.05	10.35	10.66
Gross Revenue (M, $) – Expected					105.87	124.94	136.50	149.41	158.14
Gross Revenue (M, $) – Projected					105.87	112.03	118.55	125.45	132.75
CAPEX (M, $)									20.75
OPEX (M, $)					17.42	17.94	18.48	19.03	19.60
EBIT (M, $) – Expected					88.45	107.00	118.02	130.38	117.78
EBIT (M, $) – Projected					88.45	94.09	100.07	106.41	92.39
Senior Debt Service (M, $)							117.75	117.75	117.75
Sub Debt Service (M, $)							65.63	65.63	65.63
Taxes (M, $) – Expected					24.32	29.43	0.00	0.00	0.00
Taxes (M, $) – Projected					24.32	25.87	0.00	0.00	0.00
Expected FCF on Equity (M, $)					64.13	77.58	-65.36	-53.00	-65.60
Guaranteed FCF on Equity (M, $)					57.71	69.82	-58.83	-47.70	-59.04
Projectedc FCF on equity (M, $)					64.13	68.21	-83.31	-76.97	-90.99
Cash Flow Difference (M, $)					8.73	9.05	37.35	41.15	44.99

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Traffic Volume Growth Rate (%)	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic Volume (M) – Expected	15.42	16.03	17.34	18.00	18.57	19.14	19.69	20.25	20.82
Traffic Volume (M) – Projected	12.79	13.14	13.50	13.87	14.25	14.64	15.04	15.45	15.87
Toll Rate ($)	10.98	11.31	11.65	12.00	12.36	12.73	13.11	13.51	13.91
Gross Revenue (M, $) - Expected	169.38	181.30	202.07	215.96	229.53	243.67	258.16	273.56	289.66
Gross Revenue (M, $) - Projected	140.47	148.64	157.29	166.45	176.13	186.38	197.22	208.70	220.85
CAPEX (M, $)					24.06				
OPEX (M, $)	20.19	20.80	21.42	22.06	22.73	23.41	24.11	24.83	25.58
EBIT (M, $) – Expected	149.19	160.50	180.65	193.89	182.75	220.27	234.05	248.73	264.08
EBIT (M, $) – Projected	120.28	127.85	135.87	144.38	129.34	162.97	173.11	183.87	195.27
Senior Debt Service (M, $)	117.75	117.75	117.75	117.75	117.75	117.75	117.75		
Sub Debt Service (M, $)	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63
Taxes (M, $) – Expected	0.00	0.00	0.00	2.89	0.00	10.14	13.93	50.35	54.57
Taxes (M, $) - Projected	0.00	0.00	0.00	0.00	0.00	0.00	0.00	32.52	35.65
Expected FCF on Equity (M, $)	-34.19	-22.88	-2.73	7.62	-0.64	26.74	36.73	132.75	143.87
Guaranteed FCF on Equity (M, $)	-30.77	-20.59	-2.46	6.86	-0.57	24.07	33.06	119.47	129.49
Projectedc FCF on equity (M, $)	-63.10	-55.54	-47.51	-39.00	-54.04	-20.41	-10.27	85.72	93.99
Cash Flow Difference (M, $)	44.22	46.55	54.34	55.40	63.16	55.88	56.26	37.01	38.87

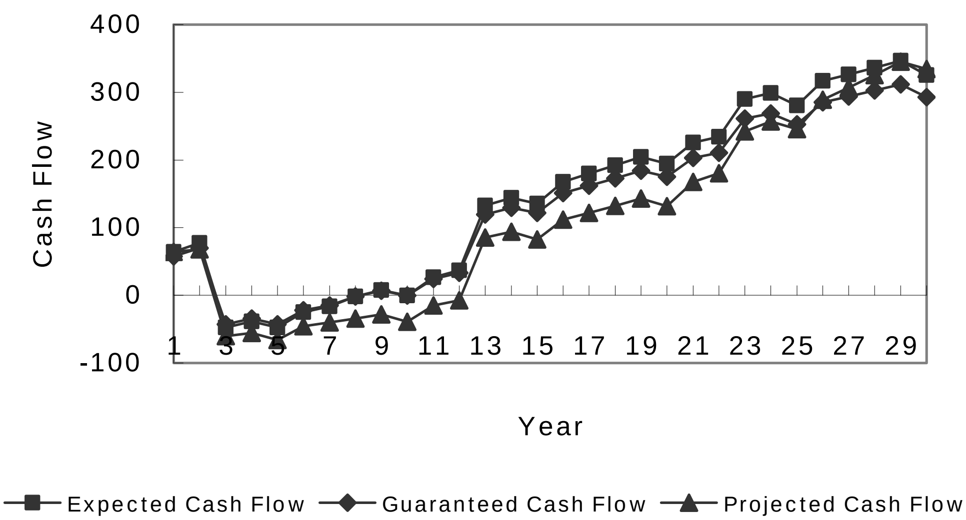
Year	2019	2020	2021	2022	2023	2024	2025	2026
Traffic Volume Growth Rate (%)	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic Volume (M) – Expected	21.40	21.98	22.50	22.95	23.41	23.88	23.88	23.88
Traffic Volume (M) – Projected	16.31	16.75	17.21	17.68	18.17	18.67	19.18	19.70
Toll Rate ($)	14.33	14.76	15.20	15.66	16.13	16.61	17.11	17.62
Gross Revenue (M, $) - Expected	306.66	324.34	342.09	359.43	377.61	396.75	408.65	420.91
Gross Revenue (M, $) - Projected	233.70	247.29	261.68	276.91	293.02	310.08	328.12	347.21
CAPEX (M, $)	27.89					32.33		
OPEX (M, $)	26.35	27.14	27.95	28.79	29.65	30.54	31.46	32.40
EBIT (M, $) – Expected	252.42	297.21	314.14	330.64	347.96	333.87	377.19	388.51
EBIT (M, $) – Projected	179.46	220.16	233.73	248.12	263.37	247.20	296.66	314.81
Senior Debt Service (M, $)								
Sub Debt Service (M, $)	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63
Taxes (M, $) – Expected	51.37	63.68	68.34	72.88	77.64	73.77	85.68	88.79
Taxes (M, $) - Projected	31.30	42.50	46.23	50.19	54.38	49.93	63.53	68.52
Expected FCF on Equity (M, $)	135.42	167.89	180.17	192.13	204.69	194.47	225.88	234.09
Guaranteed FCF on Equity (M, $)	121.88	151.10	162.15	172.92	184.22	175.03	203.29	210.68
Projectedc FCF on equity (M, $)	82.52	112.03	121.87	132.31	143.36	131.64	167.50	180.65
Cash Flow Difference (M, $)	43.61	42.74	44.39	45.49	46.76	50.85	46.06	44.19

Figure 7.37 (Continued)

(M: Million / $: Dollar)

Year	2027	2028	2029	2030	2031	2032	2033	2034
Traffic Volume Growth Rate (%)	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic Volume (M) - Expected	23.88	23.88	23.88	23.88	23.88	23.88	23.88	23.88
Traffic Volume (M) - Projected	20.24	20.79	21.36	21.95	22.55	23.17	23.80	24.45
Toll Rate ($)	18.15	18.70	19.26	19.84	20.43	21.04	21.68	22.33
Gross Revenue (M, $) - Expected	433.54	446.54	459.94	473.74	487.95	502.59	517.67	533.20
Gross Revenue (M, $) - Projected	367.41	388.79	411.42	435.36	460.69	487.50	515.86	545.88
CAPEX (M, $)			37.48					43.46
OPEX (M, $)	33.37	34.38	35.41	36.47	37.56	38.69	39.85	41.05
EBIT (M, $) – Expected	400.16	412.17	387.05	437.27	450.39	463.90	477.82	448.70
EBIT (M, $) – Projected	334.04	354.42	338.53	398.89	423.13	448.81	476.01	461.38
Senior Debt Service (M, $)								
Sub Debt Service (M, $)								
Taxes (M, $) - Expected	110.05	113.35	106.44	120.25	123.86	127.57	131.40	123.39
Taxes (M, $) - Projected	91.86	97.47	93.09	109.69	116.36	123.42	130.90	126.88
Expected FCF on Equity (M, $)	290.12	298.82	280.61	317.02	326.53	336.33	346.42	325.30
Guaranteed FCF on Equity (M, $)	261.11	268.94	252.55	285.32	293.88	302.69	311.78	292.77
Projectedc FCF on equity (M, $)	242.18	256.95	245.43	289.19	306.77	325.39	345.11	334.50
Cash Flow Difference (M, $)	37.42	35.72	36.44	32.29	30.62	29.02	27.61	28.52

Figure 7.38 Comparison of the Cash Flows of the CNE BOT Project



3*) MRG Option Value*

Based on Section 6.5, with the guaranteed free cash flows on equity by method 1 (82 % of the expected cash flows estimated by the BOT developer) and simulated free cash flows on equity by simulation in method 2, we finally obtain the MRG option value by discounting two cash flow differences at a risk-free rate, 5.3%, as shown in Figure 7.39 and Table 7.30.

Figure 7.39 MRG Value by Method 2 in the CNE Project

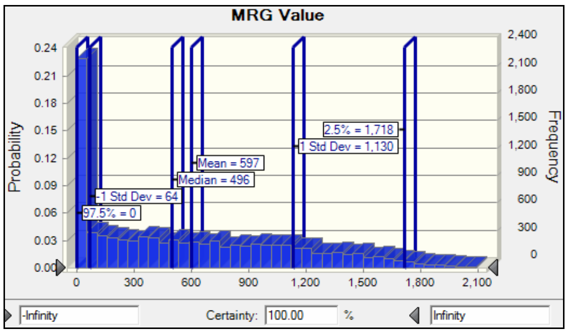


Table 7.30 MRG Value by Method 2 in the CNE Project

	MRG Value (Million, $)
Maximum	2,287.00
2.5%	1,718.00
$+1\cdot\sigma$	1130.00
Mean	597.00
Median	496.00
$-1\cdot\sigma$	64.00
97.5 %	0.00
Minimum	0.00

7.3.4.3 Method 3 - Developed Real Option Model

With the process mentioned in Section 6.6, 7.1.4.3, and 7.2.4.3, we obtained the MRG value for the CNE BOT project by method 3. Table 7.31 and Figure 7.40 show the probability distribution of initial traffic volume/traffic volume growth rate and the assumption of those variables in the Monte Carlo program.

Figure 7.41 is the cash flow model by the simulation (10,000 time iterations) to obtain the more detailed level of the volatility in project value in method 3. As a result, we find the volatility of 1.17 as shown in Figure 7.42 and Table 7.32. With this volatility, we can calculate the necessary parameters (Table 7.33) for the real option analysis. Finally, we can, with the exercise price in Table 7.34, build the binomial tree of project value (Figure 7.43). Figure 7.45 is the description of the MRG value obtained from the developed real option method.

1) Find the Initial Project Value

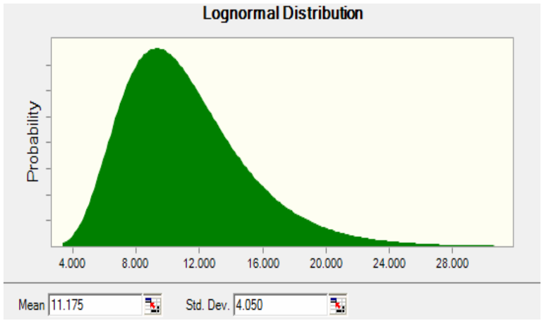
$$V_I = \$\ 390.38\ Million \tag{7.22}$$

2) Selection of Volatility "σ"

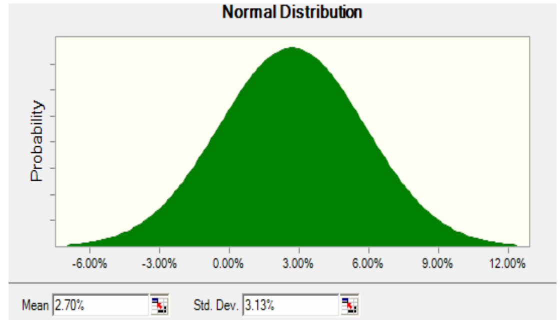
Table 7.31 Probability Distribution of Two Input Variables in Method 3 at the CNE Project

	Initial Traffic Volume (Million)	*Traffic Volume Growth Rate (%)*
Type of Distribution	Lognormal Distribution	Normal Distribution
Mean	11.175	2.70
Standard Deviation	4.050	3.13

Figure 7.40 Defining the Assumption of Two Input Variables in Method 3 at the CNE Project



1) Initial Traffic Volume



2) Traffic Volume Growth Rate

Figure 7.41 Cash Flow Model for Monte Carlo Simulation in Method 3 at the CNE Project

(M: Million / $: Dollar)

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Traffic Volume Growth Rate						2.70%	2.70%	2.70%	2.70%	2.70%
Traffic volume (M)-Expected					11.175	12.804	13.581	14.433	14.831	15.423
Traffic Volume (M) – Simulated					13.09	14.51	15.16	16.32	16.99	16.72
Toll Rate ($)					9.47	9.76	10.05	10.35	10.66	10.98
Gross Revenue (M, $) - Expected					100.42	124.94	136.50	149.41	158.14	169.38
Gross Revenues (M, $) - Simulated					124.03	141.55	152.32	168.92	181.19	183.61
CAPEX (M, $)									20.75	
OPEX (M, $)					17.42	17.94	18.48	19.03	19.60	20.19
EBIT (M, $) - Expected					83.01	107.00	118.02	130.38	117.78	149.19
EBIT (M, $) - Simulated					106.61	123.61	133.84	149.89	140.83	163.42
Senior Debt Service (M, $)							117.75	117.75	117.75	117.75
Sub Debt Service (M, $)							65.63	65.63	65.63	65.63
Taxes (M, $) – Expected					22.83	29.43	0.00	0.00	0.00	0.00
Taxes (M, $) – Simulated					29.32	33.99	0.00	0.00	0.00	0.00
FCF on Equity (M, $)-Expected					60.18	77.58	-65.36	-53.00	-65.60	-34.19
FCF on Equity (M, $)-Simulated					**A**	**B**	**C**	**D**	**E**	**F**

Year	2011	2012	2013	2014	2015	2016	2017	2018
Traffic Volume Growth Rate	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic volume (M)-Expected	16.027	17.343	17.995	18.569	19.139	19.686	20.253	20.820
Traffic Volume (M) – Simulated	17.82	19.08	19.48	19.85	20.67	21.31	22.07	23.54
Toll Rate ($)	11.31	11.65	12.00	12.36	12.73	13.11	13.51	13.91
Gross Revenue (M, $) - Expected	181.30	202.07	215.96	229.53	243.67	258.16	273.56	289.66
Gross Revenues (M, $) - Simulated	201.64	222.32	233.74	245.33	263.15	279.40	298.09	327.43
CAPEX (M, $)				24.06				
OPEX (M, $)	20.80	21.42	22.06	22.73	23.41	24.11	24.83	25.58
EBIT (M, $) - Expected	160.50	180.65	193.89	182.75	220.27	234.05	248.73	264.08
EBIT (M, $) - Simulated	180.84	200.90	211.67	198.54	239.74	255.29	273.26	301.85
Senior Debt Service (M, $)	117.75	117.75	117.75	117.75	117.75	117.75		
Sub Debt Service (M, $)	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63
Taxes (M, $) – Expected	0.00	0.00	2.89	0.00	10.14	13.93	50.35	54.57
Taxes (M, $) – Simulated	0.00	4.82	7.78	4.17	15.50	19.77	57.10	64.96
FCF on Equity (M, $)-Expected	-22.88	-2.73	7.62	-0.64	26.74	36.73	132.75	143.87
FCF on Equity (M, $)-Simulated	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**

Figure 7.41 (Continued)

(M: Million / $: Dollar)

Year	2019	2020	2021	2022	2023	2024	2025	2026
Traffic Volume Growth Rate	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic volume (M)-Expected	21.400	21.975	22.502	22.954	23.413	23.883	23.883	23.883
Traffic Volume (M) – Simulated	24.71	25.31	26.18	27.00	26.91	26.73	27.81	30.48
Toll Rate ($)	14.33	14.76	15.20	15.66	16.13	16.61	17.11	17.62
Gross Revenue (M, $) - Expected	306.66	324.34	342.09	359.43	377.61	396.75	408.65	420.91
Gross Revenues (M, $) - Simulated	354.05	373.54	398.01	422.82	433.96	444.11	475.86	537.13
CAPEX (M, $)	27.89					32.33		
OPEX (M, $)	26.35	27.14	27.95	28.79	29.65	30.54	31.46	32.40
EBIT (M, $) - Expected	252.42	297.21	314.14	330.64	347.96	333.87	377.19	388.51
EBIT (M, $) - Simulated	299.82	346.40	370.06	394.04	404.31	381.23	444.40	504.73
Senior Debt Service (M, $)								
Sub Debt Service (M, $)	65.63	65.63	65.63	65.63	65.63	65.63	65.63	65.63
Taxes (M, $) – Expected	51.37	63.68	68.34	72.88	77.64	73.77	85.68	88.79
Taxes (M, $) – Simulated	64.40	77.21	83.72	90.31	93.14	86.79	104.16	120.75
FCF on Equity (M, $)-Expected	135.42	167.89	180.17	192.13	204.69	194.47	225.88	234.09
FCF on Equity (M, $)-Simulated	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**

Year	2027	2028	2029	2030	2031	2032	2033	2034
Traffic Volume Growth Rate	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%	2.70%
Traffic volume (M)-Expected	23.883	23.883	23.883	23.883	23.883	23.883	23.883	23.883
Traffic Volume (M) – Simulated	30.12	29.60	30.53	31.53	32.36	35.39	35.76	34.81
Toll Rate ($)	18.15	18.70	19.26	19.84	20.43	21.04	21.68	22.33
Gross Revenue (M, $) - Expected	433.54	446.54	459.94	473.74	487.95	502.59	517.67	533.20
Gross Revenues (M, $) - Simulated	546.72	553.45	588.02	625.43	661.09	744.68	775.17	777.09
CAPEX (M, $)			37.48					43.46
OPEX (M, $)	33.37	34.38	35.41	36.47	37.56	38.69	39.85	41.05
EBIT (M, $) - Expected	400.16	412.17	387.05	437.27	450.39	463.90	477.82	448.70
EBIT (M, $) - Simulated	513.35	519.08	515.13	588.96	623.52	705.99	735.32	692.59
Senior Debt Service (M, $)								
Sub Debt Service (M, $)								
Taxes (M, $) – Expected	110.05	113.35	106.44	120.25	123.86	127.57	131.40	123.39
Taxes (M, $) – Simulated	141.17	142.75	141.66	161.96	171.47	194.15	202.21	190.46
FCF on Equity (M, $)-Expected	290.12	298.82	280.61	317.02	326.53	336.33	346.42	325.30
FCF on Equity (M, $)-Simulated	**W**	**X**	**Y**	**Z**	**AA**	**AB**	**AC**	**AD**

Figure 7.42 Probability Distribution of "z" in the CNE Project

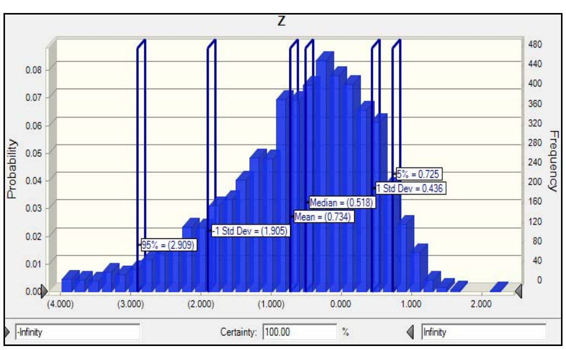


Table 7.32 Volatility of Project Value in the CNE Project

Statistics	*Forecast Values*
Volatility	1.17
Maximum	2.287
5 %	0.725
$+1\cdot\sigma$	0.436
Mean	-0.734
Median	-0.518
$-1\cdot\sigma$	-1.905
95 %	-2.909
Minimum	-8.280

Table 7.33 Calculated Parameters in the CNE Project

Calculated Parameters			
Initial Project Value "V_I"(Million, $)	390.38	*Volatility "σ"*	1.17
Up Movement "u"	1.71	*Concession Period (Year)*	30
Down Movement "d"	0.17	*Risk Neutral Probability "q"*	0.5
Risk Free Rate "r" (%)	5.3	*Risk Neutral Probability "$1-q$"*	0.5

Table 7.34 Guaranteed Project Value during Concession Period in the CNE Project (Million, $)

Year	***2005***	***2006***	***2007***	***2008***	***2009***	***2010***	***2011***	***2012***	***2013***	***2014***	***2015***	***2016***	***2017***	***2018***	***2019***	***2020***
Guaranteed Project Value	320.11	170.24	90.54	48.15	25.61	13.62	7.24	3.85	2.05	1.09	0.58	0.31	0.16	0.09	0.05	0.02

Year	***2021***	***2022***	***2023***	***2024***	***2025***	***2026***	***2027***	***2028***	***2029***	***2030***	***2031***	***2032***	***2033***	***2034***	***2035***
Guaranteed Project Value	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

3) Implementing Calculation Backward Recursively

Figure 7.43 and 7.44 are the binomial tree of project value and asymmetric payoff in the CNE project. Figure 7.45 is the binomial tree for the MRG option value. We have the MRG option value, $ 54 million, in 2004 and, by discounting this value to 2001 with risk-free rate of 5.3%, obtain $ 46 million.

Figure 7.43 Binomial Tree of the CNE Project Value

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
390.4	668.9	1146.2	1964.1	3.E+03	6.E+03	1.E+04	2.E+04	3.E+04	5.E+04	9.E+04	1.E+05	3.E+05	4.E+05	7.E+05	1.E+06	2.E+06	4.E+06	6.E+06	1.E+07	2.E+07	3.E+07	5.E+07	9.E+07	2.E+08	3.E+08	5.E+08	8.E+08	1.E+09	2.E+09	4.E+09
	64.4	110.4	189.2	3.E+02	6.E+02	1.E+03	2.E+03	3.E+03	5.E+03	8.E+03	1.E+04	2.E+04	4.E+04	7.E+04	1.E+05	2.E+05	4.E+05	6.E+05	1.E+06	2.E+06	3.E+06	5.E+06	9.E+06	2.E+07	3.E+07	5.E+07	8.E+07	1.E+08	2.E+08	4.E+08
		10.6	18.2	3.E+01	5.E+01	9.E+01	2.E+02	3.E+02	5.E+02	8.E+02	1.E+03	2.E+03	4.E+03	7.E+03	1.E+04	2.E+04	3.E+04	6.E+04	1.E+05	2.E+05	3.E+05	5.E+05	9.E+05	1.E+06	3.E+06	4.E+06	7.E+06	1.E+07	2.E+07	4.E+07
			1.8	3.E+00	5.E+00	9.E+00	2.E+01	3.E+01	4.E+01	8.E+01	1.E+02	2.E+02	4.E+02	7.E+02	1.E+03	2.E+03	3.E+03	6.E+03	1.E+04	2.E+04	3.E+04	5.E+04	8.E+04	1.E+05	2.E+05	4.E+05	7.E+05	1.E+06	2.E+06	4.E+06
				3.E-01	5.E-01	9.E-01	1.E+00	2.E+00	4.E+00	7.E+00	1.E+01	2.E+01	4.E+01	6.E+01	1.E+02	2.E+02	3.E+02	5.E+02	9.E+02	2.E+03	3.E+03	5.E+03	8.E+03	1.E+04	2.E+04	4.E+04	7.E+04	1.E+05	2.E+05	3.E+05
					5.E-02	8.E-02	1.E-01	2.E-01	4.E-01	7.E-01	1.E+00	2.E+00	4.E+00	6.E+00	1.E+01	2.E+01	3.E+01	5.E+01	9.E+01	2.E+02	3.E+02	5.E+02	8.E+02	1.E+03	2.E+03	4.E+03	7.E+03	1.E+04	2.E+04	3.E+04
						8.E-03	1.E-02	2.E-02	4.E-02	7.E-02	1.E-01	2.E-01	3.E-01	6.E-01	1.E+00	2.E+00	3.E+00	5.E+00	9.E+00	1.E+01	3.E+01	4.E+01	7.E+01	1.E+02	2.E+02	4.E+02	6.E+02	1.E+03	2.E+03	3.E+03
							1.E-03	2.E-03	4.E-03	7.E-03	1.E-02	2.E-02	3.E-02	6.E-02	1.E-01	2.E-01	3.E-01	5.E-01	8.E-01	1.E+00	2.E+00	4.E+00	7.E+00	1.E+01	2.E+01	4.E+01	6.E+01	1.E+02	2.E+02	3.E+02
								2.E-04	4.E-04	6.E-04	1.E-03	2.E-03	3.E-03	5.E-03	9.E-03	2.E-02	3.E-02	5.E-02	8.E-02	1.E-01	2.E-01	4.E-01	7.E-01	1.E+00	2.E+00	3.E+00	6.E+00	1.E+01	2.E+01	3.E+01
									4.E-05	6.E-05	1.E-04	2.E-04	3.E-04	5.E-04	9.E-04	2.E-03	3.E-03	5.E-03	8.E-03	1.E-02	2.E-02	4.E-02	7.E-02	1.E-01	2.E-01	3.E-01	6.E-01	1.E+00	2.E+00	3.E+00
										6.E-06	1.E-05	2.E-05	3.E-05	5.E-05	9.E-05	1.E-04	3.E-04	4.E-04	7.E-04	1.E-03	2.E-03	4.E-03	6.E-03	1.E-02	2.E-02	3.E-02	6.E-02	1.E-01	2.E-01	3.E-01
											1.E-06	2.E-06	3.E-06	5.E-06	8.E-06	1.E-05	2.E-05	4.E-05	7.E-05	1.E-04	2.E-04	4.E-04	6.E-04	1.E-03	2.E-03	3.E-03	5.E-03	9.E-03	2.E-02	3.E-02
												2.E-07	3.E-07	5.E-07	8.E-07	1.E-06	2.E-06	4.E-06	7.E-06	1.E-05	2.E-05	3.E-05	6.E-05	1.E-04	2.E-04	3.E-04	5.E-04	9.E-04	2.E-03	3.E-03
													3.E-08	5.E-08	8.E-08	1.E-07	2.E-07	4.E-07	7.E-07	1.E-06	2.E-06	3.E-06	6.E-06	1.E-05	2.E-05	3.E-05	5.E-05	8.E-05	1.E-04	2.E-04
														4.E-09	7.E-09	1.E-08	2.E-08	4.E-08	6.E-08	1.E-07	2.E-07	3.E-07	6.E-07	9.E-07	2.E-06	3.E-06	5.E-06	8.E-06	1.E-05	2.E-05
															7.E-10	1.E-09	2.E-09	4.E-09	6.E-09	1.E-08	2.E-08	3.E-08	5.E-08	9.E-08	2.E-07	3.E-07	5.E-07	8.E-07	1.E-06	2.E-06
																1.E-10	2.E-10	3.E-10	6.E-10	1.E-09	2.E-09	3.E-09	5.E-09	9.E-09	2.E-08	3.E-08	4.E-08	8.E-08	1.E-07	2.E-07
																	2.E-11	3.E-11	6.E-11	1.E-10	2.E-10	3.E-10	5.E-10	8.E-10	1.E-09	2.E-09	4.E-09	7.E-09	1.E-08	2.E-08
																		3.E-12	6.E-12	9.E-12	2.E-11	3.E-11	5.E-11	8.E-11	1.E-10	2.E-10	4.E-10	7.E-10	1.E-09	2.E-09
																			5.E-13	9.E-13	2.E-12	3.E-12	5.E-12	8.E-12	1.E-11	2.E-11	4.E-11	7.E-11	1.E-10	2.E-10
																				9.E-14	2.E-13	3.E-13	4.E-13	8.E-13	1.E-12	2.E-12	4.E-12	7.E-12	1.E-11	2.E-11
																					1.E-14	2.E-14	4.E-14	7.E-14	1.E-13	2.E-13	4.E-13	6.E-13	1.E-12	2.E-12
																						2.E-15	4.E-15	7.E-15	1.E-14	2.E-14	4.E-14	6.E-14	1.E-13	2.E-13
																							4.E-16	7.E-16	1.E-15	2.E-15	3.E-15	6.E-15	1.E-14	2.E-14
																								7.E-17	1.E-16	2.E-16	3.E-16	6.E-16	1.E-15	2.E-15
																									1.E-17	2.E-17	3.E-17	5.E-17	9.E-17	2.E-16
																										2.E-18	3.E-18	5.E-18	9.E-18	2.E-17
																											3.E-19	5.E-19	9.E-19	1.E-18
																												5.E-20	8.E-20	1.E-19
																													8.E-21	1.E-20
																														1.E-21

Figure 7.44 Binomial Tree of Asymmetric Payoff in the CNE Project

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		80	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			46	23	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				25	13	6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					14	7	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						7	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
								2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
									1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
										1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
															0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
																0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
																	0	0	0	0	0	0	0	0	0	0	0	0	0	0
																		0	0	0	0	0	0	0	0	0	0	0	0	0
																			0	0	0	0	0	0	0	0	0	0	0	0
																				0	0	0	0	0	0	0	0	0	0	0
																					0	0	0	0	0	0	0	0	0	0
																						0	0	0	0	0	0	0	0	0
																							0	0	0	0	0	0	0	0
																								0	0	0	0	0	0	0
																									0	0	0	0	0	0
																										0	0	0	0	0
																											0	0	0	0
																												0	0	0
																													0	0
																														0

Figure 7.45 MRG Value of the CNE Project (MRG Value = $ 46 Million)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
54	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	106	15	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		80	30	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			46	23	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				25	13	6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					14	7	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						7	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
								2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
									1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
										1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
															0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
																0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
																	0	0	0	0	0	0	0	0	0	0	0	0	0	0
																		0	0	0	0	0	0	0	0	0	0	0	0	0
																			0	0	0	0	0	0	0	0	0	0	0	0
																				0	0	0	0	0	0	0	0	0	0	0
																					0	0	0	0	0	0	0	0	0	0
																						0	0	0	0	0	0	0	0	0
																							0	0	0	0	0	0	0	0
																								0	0	0	0	0	0	0
																									0	0	0	0	0	0
																										0	0	0	0	0
																											0	0	0	0
																												0	0	0
																													0	0
																														0

7.3.4.4 Results of the BOT Project Valuation Methods

Like above two BOT cases, we use three valuation methods to evaluate the MRG value in the CNE BOT project.

As results of those three valuation methods, first, we obtain the NPV on equity of $ 126.39 million by method 1.

Second, we have the mean and median of MRG value of $ 597 million and $ 496 million respectively. Here, the mean of MRG value accounts for 472.35 % of NPV on equity of $ 126.39 million and 387.66 % of initial equity investment of $ 126.39 million and the median accounts for 392.44 % and 322.08 % respectively.

Third, with method 3, we found that the MRG option value is $ 46 million, which accounts for about 36.40 % of NPV on equity and 29.87 % of initial equity investment respectively. Table 7.35 is the description of the MRG values by three project valuation methods. It shows that like cases of MCB and KRRC projects the MRG values by method 2 and 3 are significant on NPV on equity and initial equity investment cost.

Table 7.35 MRG Option Value in CNE Project by Three Valuation Methods in the CNE Project

(MRG, NPVe: Net Present Value on Equity, Ie: Equity Investment / Million, $)

		MRG Value	MRG Value/NPVe	MRG Value/Ie	MRG Value + NPVe	(MRG Value + NPVe)/NPVe
Method1		0.00	**0.00%**	**0.00%**	126.39	100.00%
Method2	Max	2287.00	**1809.48%**	**1485.06%**	2413.39	5.24%
	2.5%	1718.00	**1359.28%**	**1115.58%**	1844.39	6.85%
	$+1 \cdot \sigma$	1130.00	**894.06%**	**733.77%**	1256.39	10.06%
	Mean	597.00	**472.35%**	**387.66%**	723.39	572.35%
	Median	496.00	**392.44%**	**322.08%**	622.39	492.44%
	$-1 \cdot \sigma$	64.00	**50.64%**	**41.56%**	190.39	150.64%
	97.5%	0.00	**0.00%**	**0.00%**	126.39	100.00%
	Min	0.00	**0.00%**	**0.00%**	126.39	100.00%
Method3		46.00	**36.40%**	**29.87%**	172.39	136.40%

7.3.4.5 Validation Test of the Developed Real Option Model

The CNE project is the last BOT case that we are going to examine to test if the developed real option model can provide a reasonable degree of validation among three BOT case studies. The following are the results of testing the three research hypotheses.

Hypothesis One **-** *"The project value using the two option pricing methods 2 and 3 under the MRG agreement will show significant value rather than the project value of method 1, which is called NPV analysis."*

The NPV on equity by method 2 and 3 are 472.35 % (mean) and 392.44 % (median), and 36.40 % higher than that of method 1 respectively. Therefore, it seems that the NPV on equity by project valuation methods 2 and 3 under MRG option are significant compared to that of method 1. Table 7.35 shows the NPVs on equity of the CNE Project by three valuation methods.

Hypothesis Two **-** *"Based upon the option pricing theory, the predicted effects of major determinants (current price of the underlying asset V_I, exercise price X, time to maturity T, volatility σ, and risk free interest rate r) on a MRG option value in the developed binomial real option model have to follow those of option pricing theory based on the Black-Scholes model."*

The MRG option value change on the change of five input variables seems to follow the partial differential based on the Black-Scholes model as long as other variables are held constant.

As shown in Figure 7.46, the MRG option value change on the initial project value change is negative. This trend seems to follow the partial differential $\partial P / \partial V_I < 0$ of Equation (3.91) based on the Black-Scholes model.

As we can see in Figure 7.47, as the exercise price X increases the MRG value increases and this looks coincident with Equation (3.93).

Figure 7.48 is the description of the impact of the time to maturity on the MRG value, which is positive and follows the tendency of Equation (3.94).

Figure 7.49 shows that the trend of the MRG option value change against volatility change follows that of Equation (3.99) by the Black-Scholes model.

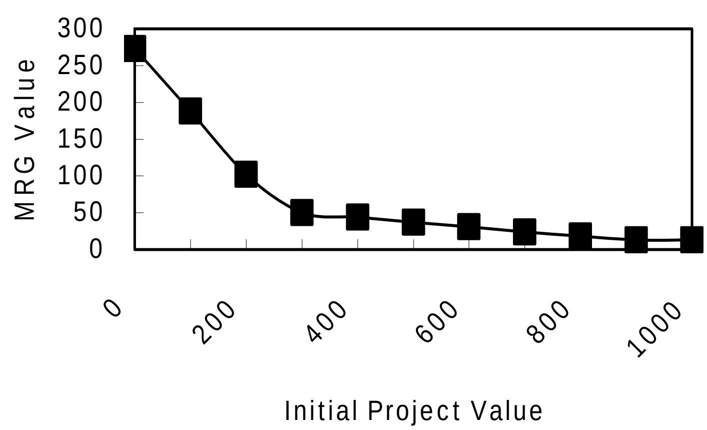
As shown in Figure 7.50, the risk-free interest rate seems to have a negative relationship with the MRG value and this relationship seems to follow Equation (3.100).

As a result, the CNE project clearly shows that like the other two BOT cases the predicted effects of five input variables on a MRG value with the developed real option model are coincident to those by the option pricing theory based on the Black-Scholes model.

Initial Project Value V_I

Figure 7.46 MRG Value Change against Initial Project Value V_I Change in the CNE Project

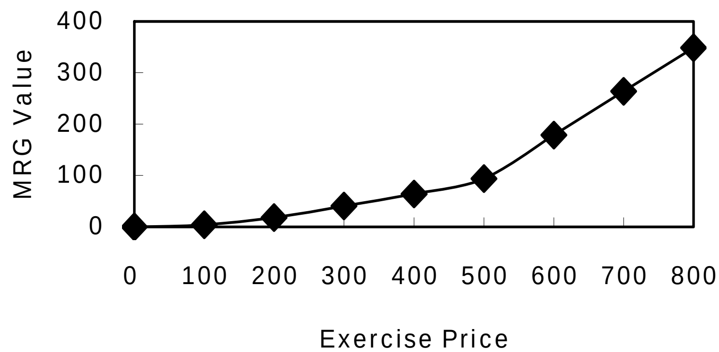
(MRG, V_I : Million, \$)



(*Initial* X =320, T =30, σ =1.17, r =0.053)

***Exercise Price** X*

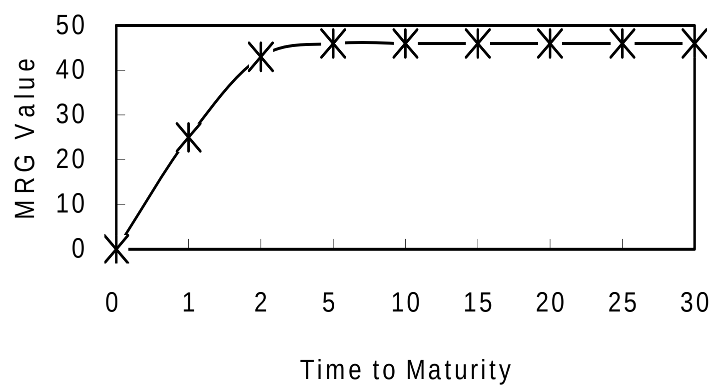
Figure 7.47 MRG Value Change against Exercise Price X Change in the CNE Project (MRG, X: Million, $)



(V_I = 390.38, T =30, σ =1.17, r =0.053)

***Time to Maturity** T*

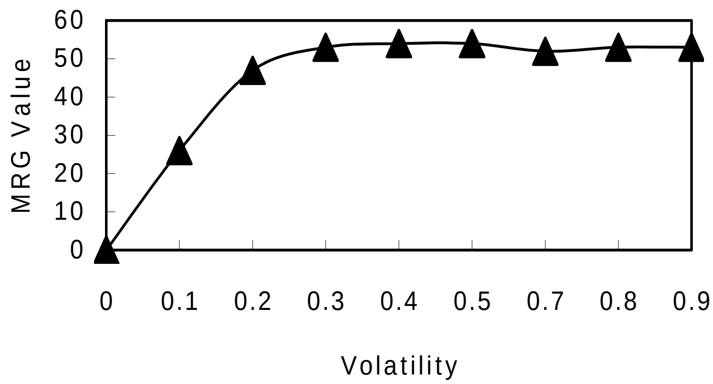
Figure 7.48 MRG Value Change against Time to Maturity T Change in the CNE Project (MRG: Million, $; T: Year)



(V_I = 390.38, *Initial* X =320, σ =1.17, r =0.053)

Volatility σ

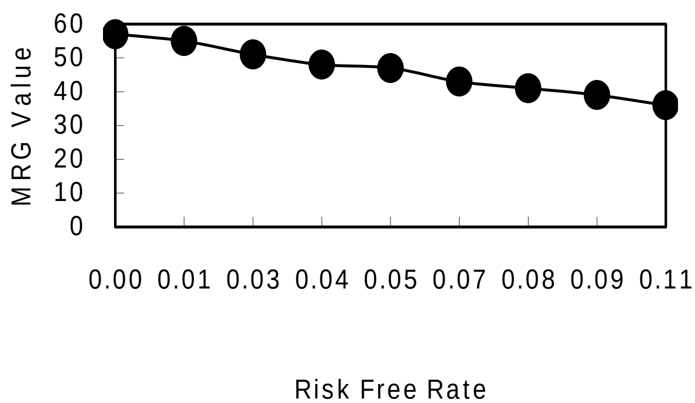
Figure 7.49 MRG Value Change against Volatility Value σ Change in the CNE Project (MRG: Million, $)



(V_I = 390.38, *Initial* X =320, T =30, r =0.053)

Risk-free Rate r

Figure 7.50 MRG Value Change against Risk-free Rate r Change in the CNE Project (MRG: Million, $)



(V_I = 390.38, *Initial* X =320, T =30, σ =1.17)

***Hypothesis Three** - "The MRG agreement value gained from the developed real option method 3 will be consistent with that of method 2. (The MRG agreement value gained from the developed method 3 will be located within the range of $\pm 2 \cdot \sigma$ from the median and mean in probability distribution of the MRG value by method 2)".*

Figure 7.51 and Table 7.36 show that the MRG value of \$ 46 million by method 3 is located within ± 33.18 % from mean and within ± 27.81 % from median in probability distribution of the MRG value by method 2, which are within the boundary of $\pm 2 \cdot \sigma$ (± 47.5 %) from mean and median in probability distribution of the MRG value by method 2. So, it seems that the CNE case satisfies the research hypothesis 2.

Figure 7.51 Comparison of MRG Values by Method 2 (Mean) and 3 in the CNE Project

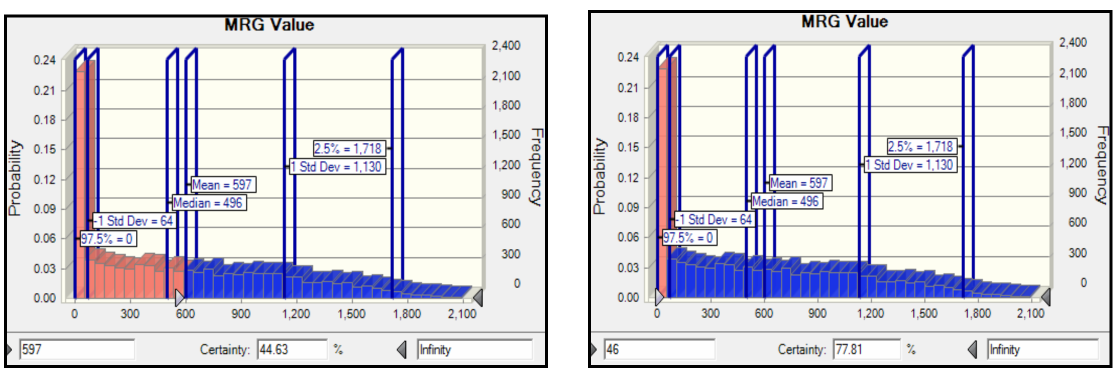


Table 7.36 MRG Values in Cumulative Probability Distributions by Method 2 and 3 in the CNE Project

	Method 2 (By Cheah and Liu's Real Option Model)	
	Mean \$ 597 million (55.37 %)	Median \$ 496 million (50 %)
Method 3: \$ 46 million (22.19 %)	± 33.18 % ($< \pm 2 \cdot \sigma$)	± 27.81 % ($< \pm 2 \cdot \sigma$)

7.3.4.6 Sensitivity Analysis of MRG Value to Standard Deviation of the Initial Traffic Volume and Traffic Volume Growth Rate

Table 7.37 is the description in the sensitivity of the MRG value to the standard deviations of initial traffic volume and growth rate in CNE project. Unlike other two BOT cases, the MRG value is neither sensitive to the standard deviation of the initial traffic volume or traffic volume growth rate.

Table 7.37 Sensitivity of MRG Value to Standard Deviations of Initial Traffic Volume and Traffic-Volume Growth Rate in the CNE Project

(MRG Value: Million, $)

		Standard Deviation of Growth Rate (%)			
		1.57	**3.13**	4.70	6.26
Standard Deviation of Traffic Volume (Million)	2.03	45.00	46.00	46.00	45.00
	4.05	46.00	46.00	46.00	45.00
	6.08	45.00	45.00	45.00	44.00
	8.10	45.00	45.00	44.00	45.00

7.4 Result Summary

7.4.1 Results of three Project Valuation Methods with BOT Project Case Studies

With three different BOT case studies; the MCB bridge, the KRRC ring road, and the CNE expressway projects, three project valuation methods, which are NPV analysis, Cheah and Liu's real option model, and the developed real option model, to evaluate the MRG values are considered in this research. Then, the results of the project valuations are as follows.

First, with method 1 of NPV analysis, we have each NPV on equity where the MRG agreement is not considered as to three BOT case projects. Second, in method 2 which is based on Cheah and Liu's real option model, we calculate the mean and median values of the MRG agreements for three BOT project cases. The mean values of the MRG options in the three BOT project cases account for approximately 42.55 % to 3510.40 % of NPV on equity and 4.41 % to 387.66 % of initial equity investment. And, the median values of the MRG options account for 18.66 % to 3401.49 % of NPV on equity and 1.93 % to 322.08 % of initial equity investment respectively. Third, through the developed real option model, method 3, we found the MRG option values of three BOT projects. These values range from 36.40 % to 222.28 % of NPV on equity and 8.60 % to 29.87 % of initial equity investment.

As a result, we understand that, aside from the method 1 that does not consider the MRG agreement, the results from method 2 and 3 consistently indicate that the MRG values have a significant impact on NPV on equity and initial equity investment cost in the three BOT projects.

7.4.2 Validation Test

In testing the validity of the developed real option model, we investigate whether the results from the developed model consistently satisfy three research hypotheses, which are reasonable to show the applicability of the model, for all three BOT project cases.

For hypothesis one, since NPVs on equity by project valuation methods 2 and 3 under MRG agreement for the three BOT cases are shown to be significant relative to those in method 1, the developed real option model turns out to satisfy hypothesis one.

When it comes to the hypothesis two, for all three BOT project cases, the change of the MRG option value against the change of the initial project value V_I is consistent with the result of the partial differential $\partial P / \partial V_I < 0$ which is based on the Black-Scholes model by showing that as V_I increases the MRG value decreases while holding other variables constant. Furthermore, the maximum value of the MRG option seems to be the present value of the sum of all the exercise prices at every time step when the initial project value V_I is "0" while the minimum value of the MRG option becomes approximately "0" from the point that V_I is higher than the exercise price of the last year in binomial tree. Second, the results of the three BOT project cases with developed real option model show that the MRG value is positively correlated with the exercise price X as long as other input variables are held constant. This result is shown to follow a mathematically defined partial differential, $\partial P / \partial X > 0$, based on the Black-Scholes model. Third, we can see in the three BOT project cases that the impact of the time to maturity on the MRG value is consistently positive and, in turn, also follows that of the Black-Scholes model. Fourth, by finding that the increase of σ results in an increased MRG value while the decrease of σ decreases the MRG value in every BOT project case, we understand that the trend of the MRG value change on volatility change in this research follows that of the Black-Scholes model. Fifth, from the results of the developed real option model, the MRG value decreases with an increasing risk-free interest rate r in the three BOT cases. This result turns out to follow the sensitivity function of the risk-free rate to the put option value $\partial P / \partial r < 0$ based on the Black-Scholes model. However, the sensitivity of the MRG value to risk-free rate seems to be minor relative to the effects of other input variables.

Finally, according to the effects of the five input variables on the MRG value with the developed real option model, the results consistently indicate that the developed real option model with the three BOT project cases satisfies hypothesis two based upon the Black-Scholes model.

In hypothesis three, we realize that the MRG values by method 3 for every BOT case is located within the range of $\pm 2 \cdot \sigma$ from the mean and median in the probability distribution of the MRG value by method 2. For the three BOT project cases, the MRG values by method 3 appears to be located within $\pm$27.24 % to 43.80 % from means and within $\pm$27.81 % to 45.57 % from medians in the probability distribution of the MRG value by method 2. Therefore, since all of the MRG values obtained by method 3 are located within the range of $\pm 2 \cdot \sigma$ ($\pm$47.5 %), we understand that the developed method 3 satisfies hypothesis three. So, the developed model seems to reasonably show the MRG value.

As for the sensitivity analyses of the MRG value subject to changes in the standard deviations of initial traffic volume and traffic volume growth rate, the results in three BOT project cases consistently indicate that the MRG values are relatively more sensitive to the standard deviation of the initial traffic volume than to that of the traffic volume growth rate. This result also shows the consistency with that of sensitivity analysis by Cheah and Liu (2006).

Finally, according to the results of testing research hypotheses one, two, and three with the three BOT project cases, the developed real option model is shown to provide a reasonable degree of the validation in its applicability for the three BOT project case studies.

8. CONCLUSIONS

8.1 Conclusions

Governments facing financial constraints have tried to attract private developers of huge infrastructure projects. Among diverse project finance schemes, the BOT type is one of the most popular means for the government and the BOT developer to coordinate an infrastructure project finance. To decide whether to undertake the BOT project, there should be a more in-depth investigation in terms of the financial feasibility, since BOT related projects generally have complicated and tightly structured contracts for each project. There are some difficulties in evaluating the BOT projects with traditional capital budgeting analyses because management flexibilities are not taken into account. Moreover, practical and quantitative studies to evaluate the BOT project investment have been scarce. This is why this research investigates the characteristics of the BOT project with option pricing theory.

The BOT project's major concern is the minimum revenue guarantee (MRG) agreement which is one of the most important critical success factors from the perspectives of the BOT developer and the government. Through this research, a quantitative binomial real option model is developed and the developed real option model appropriately evaluates the BOT project. Finally, this model provides the BOT project developers, contractors, and governments with a practical methodology to quantitatively evaluate the BOT project by considering the MRG agreement option. The conclusions drawn from this research are:

Conclusion 1: The real option pricing theory is appropriate to evaluate the MRG agreement, which has been considered as one of the most important critical success factors in BOT projects.

The developed binomial real option model involves the process of converting the theoretical framework in the BOT project valuation into a quantitative one. The developed binomial model based on the real option theory formulates one of the most remarkable characteristics in a BOT project, MRG agreement, as an option. Then, the initial project value is drawn from the future free cash flows on equity and, by constructing the binomial tree, the uncertainty of the BOT project value is reflected under the assumptions related to the BOT project investment. Then, the asymmetric payoff condition with the guaranteed project value and projected project value is considered to formulate the MRG agreement as a put option.

Conclusion 2: With the characteristic of MRG agreement in a BOT project, the asymmetric payoff falls into two conditions; when the guaranteed project value is greater than the projected project value and when the guaranteed project value is less than the projected project value. The MRG value calculated based on this asymmetric payoff condition is significantly greater than the static net present value on equity.

From this research, we found out that the MRG option values of three BOT projects range from 36.40 % to 222.28 % of NPV on equity and 8.60 % to 29.87 % of initial equity investment. Therefore, we know that the asymmetric payoff condition has a significant impact on net present value on equity and, hence, the BOT developer does not fully assume the operational risk should things go wrong. So, it seems to be quite important to understand the negotiation process which occurs between the BOT developer and the government since each contract/agreement decided through the negotiation process has a significant impact on the value of the BOT project.

Conclusion 3: The negotiation process associated with the MRG agreement is important because it directly affects the MRG value.

Although we intuitively understand that the MRG agreement determined by the negotiation process between the government and the BOT developer affects the MRG value, it is shown through this research that the effect of the MRG agreement on the BOT project value can not be ignored.

The results from the developed real option model indicate that the MRG value directly relies on five input variables; initial project value, exercise price, time to maturity, volatility, and risk-free rate. However, since most of these input variables are deterministic and, in turn, uncontrollable through the negotiation process aside from the variable of the exercise price, the exercise price seems to be the only factor which the government and the BOT developer can decide through the negotiation process. Furthermore, since, according to the developed real option model, the exercise price, which is decided based on a certain percentage of the expected project value, depends on the agreement associated with the MRG, the negotiation process as to the MRG agreement should be taken into account in the BOT project value.

Conclusion 4: The developed binomial real option model is easier for the management to use in the practical world as opposed to the Black-Scholes model.

Unlike the Black-Scholes real option model, the developed real option model is derived from the numerical framework of a binomial model not from a set of complicated mathematics and is easier for the BOT developer, the government, and the management who are already familiar with the algebra level of the NPV analysis. And, due to the convenience and simplicity of the binomial model in formulating the complex asymmetric payoff conditions, the process of formulating the MRG agreement as a put option may be easily applied to formulate other management flexibilities which can take place in the BOT contracts through proper modifications.

8.2 Limitations and Further Research

In this research, the developed real option model provides a satisfactory way to verify the applicability of its usage and demonstrates that the model can be applied to the BOT project valuation. However, there are still some open issues which mainly arise from the characteristics of the BOT project itself or the real option analyses in the project valuation process. So, it is supposed that further investigations concerning the following issues should be taken into account.

First, one of the main purposes of this research is to better evaluate the BOT project under the MRG agreement. However, in the real world there can be more diverse and complicated management flexibilities, which are likely to be formed as proper contracts/agreements, that cause the asymmetric payoff condition in project cash flows or project value. These should be properly assessed in addition to the MRG agreement. Efforts to identify the possible management flexibilities in BOT project in advance are essential. Then, the appropriate formulation process of those management flexibilities is necessary in estimating the BOT project. Afterward, the additive effect of each management flexibility on project value and its practical implication also need to be interpreted so that the BOT developer and the government can utilize these in building their bidding strategies and policies. To do so, the binomial real option model is an effective tool because it is relatively easy to formulate the complicated management flexibilities without heavy and complicated mathematics.

Second, the problem related to moral hazard can affect the credibility and accuracy of the developed real option model. If the government can reasonably predict the project revenue based on the proper assumptions as to demand and cost, then, if this estimated revenue is consistent with that of the BOT developer, there can be little probability for the moral hazard problem to occur. However, in case that the government has to solely rely on the BOT developer in predicting the project revenue because the government does not have any appropriate system to estimate the project revenue, it is possible for the BOT developer to purposely overestimate project revenue since the MRG value significantly depends on the estimated revenue in light of the fact that overestimated project revenue causes the overestimation of the MRG value. In reality, this moral hazard problem which has occurred in some developing countries has resulted in the consideration for the government to have an independent system to objectively evaluate the project revenue. For this reason, the estimation of the MRG value by developed real option model can be guaranteed as long as the estimation of the project revenue is close to accurate. Moreover, the application of the game theory with the real option consideration between the developer and the government remains as a part that should be taken into consideration in future research.

Finally, even if the real option theory is applied in evaluating the BOT project under the MRG agreement and proved efficient, the application of real option analysis in the construction field has not been well studied prior to this research. Further empirical studies are essential to improve the fundamentals of the real option application and to validate the applicability of the real option model, have been scarce in the BOT project world. For this reason, it will be necessary to focus greater study on parameter calibration for the BOT real option framework because the degree of accuracy in the parameters affects the preciseness of the real option analysis.

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