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Prediction of Dry-Friction Whirl and Whip Between a Rotor and a Stator

This paper addresses recent test results for dry-friction whip and whirl. Authors of these publications suggest that predictions from Black's 1968 paper (J. Mech. Eng. Sci., 10(1), pp. 1–12) are deficient in predicting their observed transition speeds from whirl to whip and the associated precession frequencies of whirl and whip motion. Predictions from Black's simple Jeffcott-rotor/point-mass stator are cited. This model is extended here to a multimode rotor and stator model with an arbitrary axial location for rotor-stator rubbing. Predictions obtained from this new model are quite close to experimental observations in terms of the transition from whip to whirl and observed precession frequencies. Paradoxically, nonlinear numerical simulations using Black's model fail to produce the whirl and whip solutions. The Coulomb friction force in Black's model has a fixed direction, and Bartha showed in 2000 ("Dry Friction Backward Whirl of Rotors," Dissertation, THE No. 13817, ETH Zurich) that by making the friction-force direction depend on the relative sliding velocity, nonlinear simulations would produce the predicted whirl solutions. He also showed that Black's proposed whip solution at the upper precessionfrequency transition from whirl to whip was unstable. The multimode extension of Black's model predicts a complicated range of whirl and whip possibilities; however, nonlinear time-transient simulations (including the sgn function definition for the Coulomb force) only produce the initial whirl precession range, initial whirl-whip transition, and initial whip frequency. Simulation results for these values agree well with predictions. However, none of the predicted higher-frequency whirl results are obtained. Also, the initial whip frequency persists to quite high running speeds and does not (as predicted) transition to higher frequencies. Hence, despite its deficiencies, correct and very useful predictions are obtained from a reasonable extension of Black's model. [DOI: 10.1115/1.2731412]

Keywords: rotordynamics, whirl, whip, rub, Coulomb friction

Introduction

Black [1,2] analyzed a possible range of motion arising from the rubbing interaction of a rotor and stator with Coulomb friction at the rubbing interface. His model consisted of a flexible rotor and an elastically supported stator with a clearance annulus between the rotor and stator. Bearings with isotropic stiffness connect the rotor to ground, and isotropic springs and dampers connect the stator to ground. Black considered the following classes of motion:

- a. Synchronous rubbing with the rotor precessing at the running speed in the direction of the shaft rotation.
- b. Dry-friction whirl with the rotor precessing in a direction against rotation at a frequency determined by the radiusto-clearance ratio of the annulus between the rotor and stator. This motion results from a rolling-without-slipping condition at the contact point.
- c. Dry-friction whip with continuous slipping between the rotor and the stator, and the rotor precessing in a direction opposed to shaft motion at approximately the combined natural frequency of the connected rotor and stator system.

Black's results are presented in terms of complex receptances for the rotor and stator. His results for synchronous response due to imbalance involve multiple modes for the rotor and stator, but his examples for dry-friction whip and whirl are simpler, using two-degree-of-freedom models for the rotor and stator of Fig. 1. Using this type of model, Black investigated the range of precession frequencies for which dry-friction whip and whirl are possible. Black's analysis predicts that a precession frequency range exists for which dry-friction whirl occurs, and that dry-friction whip develops at the upper limit of this precession frequency range and persists at higher precession frequencies. Crandall [3] used the model of Fig. 1 to obtain very similar results.

Crandall and Lingener's [3–5] test results supported Black's findings. However, test results from Bartha [6,7], Yu et al. [8] and Bently et al. [9] seem to contradict Black. Specifically, their results showed the onset of dry-friction whip at precession frequencies that are well below Black's predictions for the model of Fig. 1. Bartha suggested that the stator model was deficient, requiring nested layers to simulate experimental observations.

Yu et al. [8] and Bently et al. [9] performed analysis on a rotor-only model. However, their formulation cannot be obtained starting with Black's rotor-stator model. During the rub-induced reverse whip region, Williams [10] observed large normal loads leading to large Coulomb friction torques that rapidly decelerated the rotor. Insufficient data were provided in [10] to support an analysis of their test rig and results [11].

The present work revisits Black's original approaches, including the possibility of multiple rotor modes to demonstrate that Black's conclusions remain generally valid. Including multiple rotor modes and accounting for rub possibilities at other than centered locations is sufficient to explain most of the recent test observations. In the balance of this paper, we will review Black's

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Fig. 1 Jeffcott rotor model interacting with a point-mass stator across a clearance from [11]

analysis and results and demonstrate its application to a multiplemode rotor model using a simple distributed-parameter Eulerbeam model.

Simple Rotor Interaction Model and Black's Results

Figure 2 provides the free-body diagram for the model of Fig. 1 with contact. The equations of motion are

$$\begin{bmatrix} m_{r} & 0 & 0 & 0 \\ 0 & m_{r} & 0 & 0 \\ 0 & 0 & m_{s} & 0 \\ 0 & 0 & 0 & m_{s} \end{bmatrix} \begin{cases} \ddot{x}_{r} \\ \ddot{y}_{r} \\ \ddot{x}_{s} \\ \ddot{y}_{s} \end{cases} + \begin{bmatrix} c_{r} & 0 & 0 & 0 \\ 0 & c_{r} & 0 & 0 \\ 0 & 0 & c_{s} & 0 \\ 0 & 0 & 0 & m_{s} \end{bmatrix} \begin{bmatrix} \dot{x}_{r} \\ \dot{y}_{r} \\ \dot{x}_{s} \\ \dot{y}_{s} \end{cases} + \begin{bmatrix} k_{r} & 0 & 0 & 0 \\ 0 & k_{r} & 0 & 0 \\ 0 & 0 & k_{s} & 0 \\ 0 & 0 & 0 & k_{s} \end{bmatrix} \begin{bmatrix} x_{r} \\ y_{r} \\ x_{s} \\ y_{s} \end{cases} + \begin{bmatrix} -N\cos\gamma + f_{f}\sin\gamma \\ -N\sin\gamma - f_{f}\cos\gamma \\ N\cos\gamma - f_{f}\sin\gamma \\ N\sin\gamma + f_{f}\cos\gamma \end{bmatrix} .$$
(1)

Rewriting them using the complex variables

$$\mathbf{Z}_r = \mathbf{x}_r + \mathbf{j}\mathbf{y}_r \quad \mathbf{Z}_s = \mathbf{x}_s + \mathbf{j}\mathbf{y}_s \tag{2}$$

gives

$$m_r \mathbf{\hat{Z}}_r + c_r \mathbf{\hat{Z}}_r + k_r \mathbf{Z}_r = m_r a \omega^2 e^{j\omega t} - (N + \mathbf{j}f_f) e^{j\gamma}$$
$$m_s \mathbf{\hat{Z}}_s + c_s \mathbf{\hat{Z}}_s + k_s \mathbf{Z}_s = (N + \mathbf{j}f_f) e^{j\gamma}$$

For synchronous response due to imbalance, an assumed solution of the form



Fig. 2 Rotor-stator interaction motion

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$$Z_r = z_r e^{j\omega t}$$
 $Z_s = z_s e^{j\omega t}$ and $\gamma = \omega t - \xi$ (4)

gives,

 $C_r e^{\cdot}$

$$(-m_r\omega^2 + jc_r\omega + k_r)z_r = m_r a\omega^2 - (N + jf_f)e^{-j\xi}$$
$$(-m_s\omega^2 + jc_s\omega + k_s)z_s = (N + jf_f)e^{-j\xi}$$
(5)

The clearance is related to the rotor and stator displacements as

$$I^{\gamma} = \mathbf{Z}_{r} - \mathbf{Z}_{s} = (z_{r} - z_{s})e^{j\omega t} \Rightarrow C_{r}e^{-j\xi} = z_{r} - z_{s} = \boldsymbol{\alpha}_{11}(\omega)m_{r}a\omega^{2}$$
$$- [\boldsymbol{\alpha}_{11}(\omega) + \boldsymbol{\beta}_{11}(\omega)](N + jf_{f})e^{-j\xi} \Rightarrow C_{r} + [\boldsymbol{\alpha}_{11}(\omega) + \boldsymbol{\beta}_{11}(\omega)]$$
$$\times (N + jf_{f}) = \boldsymbol{\alpha}_{11}(\omega)m_{r}a\omega^{2}e^{j\xi}$$
(6)

where the rotor and stator receptances are simply

$$\boldsymbol{\alpha}_{11}(\omega) = \frac{1}{-m_r \omega^2 + jc_r \omega + k_r}$$
$$\boldsymbol{\beta}_{11}(\omega) = \frac{1}{-m_s \omega^2 + jc_s \omega + k_s}$$
(7)

This equation can be solved for the complex reaction force $P = N + jf_f$. Black's solution demonstrates the range of running speeds for which synchronous rubbing contact can exist.

Black's solution (Eq. (6)) can be extended to study the backward precession phenomenon. For reverse precession at frequency Ω , solutions are assumed of the form

$$\mathbf{Z}_r = \mathbf{z}_r e^{-j\Omega t}$$
 $\mathbf{Z}_s = \mathbf{z}_s e^{-j\Omega t}$ and $\gamma = -\Omega t + \gamma_0$. (8)

Ignoring the contribution due to imbalance, Eq. (3) becomes

$$m_{r}\ddot{\boldsymbol{Z}}_{r} + c_{r}\dot{\boldsymbol{Z}}_{r} + k_{r}\boldsymbol{Z}_{r} = -(N + jf_{f})e^{j\gamma}$$
$$m_{s}\ddot{\boldsymbol{Z}}_{s} + c_{s}\dot{\boldsymbol{Z}}_{s} + k_{s}\boldsymbol{Z}_{s} = (N + jf_{f})e^{j\gamma}$$
(9)

which becomes

$$(-m_r\Omega^2 - jc_r\Omega + k_r)z_r = -(N + jf_f)e^{j\gamma_0}$$
$$(-m_s\Omega^2 - jc_s\Omega + k_s)z_s = (N + jf_f)e^{j\gamma_0}$$
(10)

 $-i\Omega t$

The imbalance was neglected because test results show that dryfriction whip and whirl motion—once initiated—are largely independent of imbalance [7,8]. However, imbalance can be the initial cause of whip initiation [8].

The clearance is related to the rotor and stator displacements as

$$C_r e^{j\gamma} = Z_r - Z_s = (z_r - z_r) e^{j\gamma_0}$$

$$\Rightarrow C_r e^{j\gamma_0} = z_r - z_r = -[\alpha_{11}(-\Omega) + \beta_{11}(-\Omega)](N + jf_f) e^{j\gamma_0}$$

$$\Rightarrow C_r + [\alpha_{11}(-\Omega) + \beta_{11}(-\Omega)](N + jf_f) = 0$$
(11)

Note that the same definition of α_{11} and β_{11} for the synchronous response (as given in Eq. (7)) is used here, except ω is replaced with $-\Omega$.

The friction force f_f in Eq. (11) is an unknown, and this result would apply for dry-friction whirl, a rolling-without-slipping condition. However, Black [1,2] introduced $f_f = \mu N$ to eliminate the friction force as the basis for synchronous whirl with continuous slipping, which is clearly appropriate and dry-friction whirl where it would appear to be questionable. Introducing $f_f = \mu N$ into Eq. (11), the following two conditions for the required friction factor are found:

Real part:

$$\operatorname{Re}[\boldsymbol{\alpha}_{11}(-\Omega) + \boldsymbol{\beta}_{11}(-\Omega)] = -\frac{N}{C_r} |\boldsymbol{\alpha}_{11}(-\Omega) + \boldsymbol{\beta}_{11}(-\Omega)|^2 < 0$$

Imaginary part:

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(3)

$$\frac{f_f}{N} = \mu = -\frac{\operatorname{Im}[\boldsymbol{\alpha}_{11}(-\Omega) + \boldsymbol{\beta}_{11}(-\Omega)]}{\operatorname{Re}[\boldsymbol{\alpha}_{11}(-\Omega) + \boldsymbol{\beta}_{11}(-\Omega)]}$$
(12)

Black's analysis gives the second result of Eq. (12) but not the first. The solution for μ defines the required Coulomb friction to maintain dry-friction whirl for a range of precession frequencies. Hence, despite introducing $f_f = \mu N$ that implies slipping, Black was solving for the required steady-state friction force (without slipping) to maintain whirl. Dry-friction whirl can exist for finite range of Ω values within the U-shape of μ satisfying Eq. (12). The running speed ω corresponding to a dry-fiction whirl precession frequency is $\omega = -\Omega/r/C_r$. The upper precession frequency corresponds to the natural frequency of the system with the rotor and stator pinned together and is identified as the onset frequency for dry-friction whip. Increasing the running speed above the corresponding limiting running speed will not increase the dry-friction-whip precession frequency.

Comparing Yu et al. [8] Equations of Motion With Black's Equation of Motion

The Yu et al. [8] equations of motion do not have the stator degree of freedom. The contact is modeled as an additional stiffness k_s acting on the rotor system. This same model was used by Chen et al. [12], and Jiang and Ulbrich [13,14]. Comparing their equations to Black's equation shows that the two models are not equivalent and would lead to two different results. Here, an attempt was made to reach the Yu et al. equations from Black's equations. Starting from Black's Eq. (3) and ignoring stator mass and damping gives

$$m_{r} \ddot{\boldsymbol{Z}}_{r} + c_{r} \dot{\boldsymbol{Z}}_{r} + k_{r} \boldsymbol{Z}_{r} = m_{r} a \omega^{2} e^{j \omega t} - (N + j f_{f}) e^{j \gamma}$$

$$k_{s} \boldsymbol{Z}_{s} = (N + j f_{f}) e^{j \gamma}.$$
(13)

Adding the two equations of Eq. (13) removes the normal and frictional force completely and gives

$$m_r \ddot{\mathbf{Z}}_r + c_r \dot{\mathbf{Z}}_r + k_r \mathbf{Z}_r + k_s \mathbf{Z}_s = m_r a \omega^2 e^{j\omega t}$$
(14)

Expressing the stator displacements in terms of rotor displacements and the clearance vector

$$m_r \ddot{\mathbf{Z}}_r + c_r \dot{\mathbf{Z}}_r + k_r \mathbf{Z}_r + k_s (\mathbf{Z}_r - C_r e^{j\gamma}) = m_r a \omega^2 e^{j\omega t}$$
(15)

where

$$\tan \gamma = \frac{y_r - y_s}{x_r - x_s} \tag{16}$$

These equations of motion differ from Yu et al.'s [8], which are

$$m_{r}\ddot{\boldsymbol{Z}}_{r}+c_{r}\dot{\boldsymbol{Z}}_{r}+k_{r}\boldsymbol{Z}_{r}+k_{s}\boldsymbol{Z}_{r}\left(1-\frac{C_{r}}{|\boldsymbol{Z}_{r}|}\right)(1+\boldsymbol{j}\boldsymbol{\mu})=m_{r}a\omega^{2}e^{j\omega t}$$
(17)

They assume that stator displacements are zero, and the normal force is proportional to the rotor intrusion into the stator.

Analyzing a Multimode Rotor Model in Reverse Whirl Using Black's Formulation

As noted earlier, Black's [1,2] formulation holds for general rotor and stator models that can include multiple modes. We are interested in the changes Eq. (7) that arise for a general, multimode rotor model that includes the possibility of rubbing at arbitrary (noncentered) locations. For simplicity, the following uniform, Euler-beam partial differential equation model is used

$$\rho A \frac{\partial^2 u}{\partial t^2} + E I \frac{\partial^4 u}{\partial z^4} = 0 \tag{18}$$

The beam has length L, and pinned-end boundary conditions apply. An arbitrary time-varying force f(t) acts at z=b. Following

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Meirovitch [15], a solution is assumed of the form

$$u(z,t) = \sum_{i=1}^{\infty} \Phi_i(z) \,\boldsymbol{\eta}_i(t) \tag{19}$$

The functions $\Phi_i(z)$ are the orthonormal eigenvectors corresponding to pinned-end eigenvalues ω_{ni} and are defined by

$$\Phi_i(z) = \sqrt{\frac{2}{\rho A L}} \sin \frac{i \pi z}{L}; \quad \omega_{ni} = \sqrt{\frac{EI}{\rho A}} \left(\frac{i \pi}{L}\right)^2; \quad i = 1, 2, \dots, \infty$$
(20)

The corresponding modal differential equations are

$$\ddot{\eta}_i(t) + \omega_{ni}\eta_i(t) = f(t)\Phi_i(b); \quad i = 1, 2, \dots, \infty$$
(21)

We are interested in the frequency-domain transfer function between the motion at z=b and the force f(t). For zero modal initial conditions, the Laplace domain solution to Eq. (21) can be stated

$$s^{2}N_{i}(s) + \omega_{ni}^{2}N_{i}(s) = F(s)\Phi_{i}(b) \Longrightarrow N_{i}(s) = \frac{\Phi_{i}(b)}{s^{2} + \omega_{ni}^{2}}F(s);$$

$$i = 1, 2, \dots, \infty$$
(22)

In general, at any axial location z, we have

$$\frac{U(z,s)}{F(s)} = \sum_{i=1}^{\infty} \frac{\frac{2}{\rho AL} \left(\sin \frac{i\pi b}{L} \right) \left(\sin \frac{i\pi z}{L} \right)}{s^2 + \omega_{n_i}^2}$$
(23)

The desired transfer function at location z=b is

$$\frac{U(b,s)}{F(s)} = \sum_{i=1}^{\infty} \frac{\frac{2}{\rho AL} \left(\sin^2 \frac{i\pi b}{L} \right)}{s^2 + \omega_{n_i}^2}$$
(24)

Now, including proportional modal damping values (ζ_i) , we have

$$\frac{U(b,s)}{F(s)} = \sum_{i=1}^{\infty} \frac{\frac{2}{\rho AL} \left(\sin^2 \frac{i\pi b}{L} \right)}{s^2 + 2\zeta_i \omega_{ni} s + \omega_{ni}^2}$$
(25)

Rewriting the Eq. (25) as the rotor receptance term $\alpha_{11}(b,\omega)$ and substituting $s=j\omega$ gives the desired receptance functions. The stator receptance remains the same as Eq. (7)

$$\boldsymbol{\alpha}_{11}(b,\omega) = \sum_{i=1}^{\infty} \frac{\frac{2}{\rho AL} \left(\sin^2 \frac{i\pi b}{L} \right)}{-\omega^2 + j\omega^2 \zeta_i \omega_{ni} + \omega_{ni}^2}$$
(26)

Note that the rotor receptance depends on the contact location as well as on the higher bending modes of the rotor. For finite number of modes (n), we get

$$\boldsymbol{\alpha}_{11}(b,\omega) = \sum_{i=1}^{n} \frac{\frac{2}{\rho AL} \left(\sin^2 \frac{i\pi b}{L} \right)}{-\omega^2 + j\omega^2 \zeta_i \omega_{ni} + \omega_{ni}^2}$$
$$\boldsymbol{\beta}_{11}(\omega) = \frac{1}{(-m_s \omega^2 + jc_s \omega + k_s)}$$
(27)

Previous researchers have used only one mode (n=1) when they quote Black's model for predicting the backward whirl range. If required, this procedure could easily be extended to a multimode stator receptance $\beta_{11}(b, \omega)$. A general Timoschenko beam model could be used in the present analysis and would make a modest reduction in the eigenfrequencies but would have no impact on the nature of the predicted solutions.

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Fig. 3 Simple rotor with mid and quarter span contact setup (not to scale)

Results and Comparisons

Simple Shaft Model Analysis. The model used for this analysis is a rotor of length L=12.04 m and diameter of 0.1778 m, made of steel. The stator has length 0.3048 m, outer diameter of 0.381 m, and inner diameter of 0.2667 m, also made of steel. The density and modulus of elasticity of steel are $\rho = 7833 \text{ kg/m}^3$, E = 2.068×10^{11} Pa. This model is appropriate for an oil-well drillpipe. To model it as a single mass rotor connected by a single spring, the effective mass is half of its actual weight (m_r) =2342/2=1171 kg), and the rotor's effective stiffness is k_r $=2.832 \cdot 10^5$ N/m. The housing was very stiff in comparison to the rotor. The housing mass could be estimated reasonably, the stiffness could not. The natural frequency of the foundation (\approx 24.75 Hz) was set to be ten times higher than the first natural frequency of the rotor (\approx 2.475 Hz). The rest of the required parameters are $m_s = 139$ kg, $k_s = 3.361 \times 10^6$ N/m, $c_r = 182$ N s/m, and $c_s = 216 \text{ N s/m}$. The results are largely independent of the specific value for k_s . The radial clearance is the difference between the inner radius of the stator and the outer radius of the rotor and is denoted by $C_r = 0.13335 - 0.0889 = 0.04445$ m. Figure 3 shows two different contact locations b=L/2 and b=L/4. The required friction factor (Eqs. (12) and (27)) for backward whirl for these models are plotted in Figs. 4 and 5, respectively. Figures 4 and 5 show a reduction in the whirl range for additional modes, and a significantly greater reduction when contact occurs at offcenter locations. Black's point-mass rotor model results correspond exactly to the one-mode predictions shown in Fig. 4.

The rotor radius-to-radial-clearance ratios r/C_r is not used in the prediction of the backward whirl and whip precession frequen-



Fig. 4 Prediction for simple shaft model at midspan of the shaft as shown in Fig. 3 $\,$



Fig. 5 Prediction for simple shaft model at quarter span of the shaft as shown in Fig. 3

cies. It is required to find the corresponding rotor speeds at which whirl and whip occur. The clearance-to-radius ratio C_r/r is used in turbomachinery applications and the radius-to-clearance ratio r/C_r is used by the researchers working on dry-friction whirl. The simple shaft model example has a low r/C_r value of 2, the Yu et al. [8] model has an intermediate value of 13.3, and Bartha's [6,7] model has a high value of 500. The last value is representative of turbomachinery applications.

Comparison With Yu et al. [8] and Bartha's [6,7] experimental models. This prediction procedure (Eqs. (12) and (27)) was applied to the Yu et al. and Bartha [6,7] experimental rotorstator results. The radius-to-clearance ratios for the two cases are r/C_r =13.3 (Yu et al. [8]) and r/C_r =500 (Bartha [6,7]). Yu et al. [8] cite Black's 1968 work [2] in regard to synchronous whirl but apparently were not aware of his results for dry-friction whip and whirl, since they failed to use it in interpreting their results. They use a uniform, solid rotor with a diameter of 0.01 m and length L=0.56 m. Two disks of mass 0.8 kg are attached at the midspan, and contact occurs at b=0.366 m, as shown in Fig. 6. Porous metal bearings with impregnated lubricants support the rotor.

Figure 7 illustrates a run-up and rundown spectrum from a test, showing that whip is initiated at approximately ω =1500 rpm due to large response at the rotor's critical speed with a supersynchronous response at 38 Hz. This response persists during rundown until speeds lower than 800 rpm (13.33 Hz) when there is gradual reduction in the whip frequency. The reduction continues until 120 rpm (2 Hz), and the corresponding whip frequency is 32 Hz. Using the data provided by Yu et al. [8] and the extended Black model developed here (Fig. 8) shows three predicted whirl-frequency ranges (horizontal arrows) for three different models. One range corresponds to a one-mode model with the rub location at the center of the shaft (n=1, b=L/2). The second range corresponds to a one-mode model with the actual contact location (n = 1, b=0.366 m). Results are also shown for a seven-mode model with the correct off-center contact location (n=7, b=0.366 m).



Fig. 6 The Yu et al. rotor-stator model with noncentered contact location (based on figures in [8,9] and not to scale)

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Fig. 7 Experimental plots from Yu et al. [8] and Bently et al. [9]

The predicted dry-friction whirl frequency Ω range is about 25.56–38.23 Hz with the corresponding ω range of 1.92–2.87 Hz (115-172 rpm). During rundown, this whip phenomenon is predicted to continue to a low rotor speed of 172 rpm and then transition into backward whirl until ω reaches 115 rpm. Reducing the speed further would disengage the rotor and stator.

During the run-up of Fig. 7, contacts occurs at the rotor natural frequency of 1500 rpm, which is much higher than the transition speed to dry friction whip of 172 rpm; hence, whip phenomenon (38 Hz) occurs as predicted. The drop in whip frequency as ω drops from \sim 800 to 172 rpm is not directly predicted by the extended Black model. The dropout in backward whirl in Fig. 7 at \sim 120 rpm is reasonably close to the predicted drop-out frequency for whirl motion of 115 rpm.

The Yu et al. [8] results are quite reasonably predicted by extending Black's analysis. Black's work [1,2] defines the range of possible whirl precession frequencies and their corresponding running speed ranges from the clearance-to-radius ratio. The limiting precession frequency for whirl and its related speed define the onset speed of whip instability and the whip precession frequency. Crandall and Lingener's work [4] showed whirl induced by an outside impulse and then transitioning to whip as the speed was increased. Yu et al. [8] induce whip due to imbalance; however, the agent that causes contact has no bearing on Black's predictions that whip can occur above a limiting speed.

Bartha [6] demonstrated that the solution developed from Black's model for whip at the transition from whip to whirl is unstable. Changing the Coulomb friction force by using a sgn



Fig. 9 Simple layout of Bartha's rotor stator model (modeled based on figures in [6] and not to scale)

function, which depends on the relative velocity at contact, Bartha simulated Crandall and Lingener's [4] whirl-whip transition results. However, Black's predictions (with a less than adequate contact friction model) continue to correctly explain Crandall and Lingener's [4] experimental results.

Figure 9 shows an approximate layout of Bartha's rotor-stator model. The rotor has length of 1.0 m, approximately, with a disk at the center. Contact location was nearly at the middle (b $\approx L/2$) with the disk making contact with the stator mass. Stator mass was 15 kg, and the stator natural frequency was 2260 Hz. The rotor and stator damping ratios were 3.1% and 5.0%, respectively. The rotor contact is off the rotor center.

Extending Black's analysis to Bartha's model, Fig. 10 shows the predicted range of backward whirl as 50-653 Hz. The corresponding rotor speeds are 6–78 rpm (0.1–1.3 Hz). For ω >78 rpm, the predicted whip frequency is 653 Hz. Black's onemode, simple model predicts a whirl range of around 50-1600 Hz with a running speed range of 6-192 rpm (0.1–3.2 Hz) and a whip frequency of 1600 Hz. The major difference between the two predictions concerns the whip precession frequency, 653 Hz versus 1600 Hz, not the onset speed for stable whip motion, 78 rpm versus 192 rpm, since both of these limiting running speeds are quite low.

For a stiffly mounted stator, Bartha observed whip frequencies at ~600 Hz for ω =2400 rpm, declining to ~500 Hz for ω =10,000 rpm. To improve his whip predictions, Bartha developed a nested stator model (retaining a Jeffcott rotor model) with a predicted whip frequency of 754 Hz. As shown in Fig. 10, a significantly better prediction is obtained by properly applying the extended Black modeling approach.

Figure 10 shows other peaks that correspond to the higher rotor bending modes. Five rotor bending modes have lower natural frequencies than the stator natural frequency; ignoring these modes



Fig. 8 Prediction for the Yu et al. model [8]

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Table 1 Whip frequency prediction and comparison with FEM simulation

CASES	Analytical Prediction		Numerical Simulation
	n=1 - Ω (Hz)	n=7 $-\Omega$ (Hz)	$-\Omega$ (Hz)
Simple rotor (midpoint contact)	8.29	7.765	7.765
Simple rotor (quarterpoint contact)	6.26	4.76	4.76
Yu et al. case	40	38	38
	(b=0.366 m) 42	(<i>b</i> =0.366 m)	
	(if $b = L/2$)		
Bartha's case with constant	1600	653	653
speed			(very large k_L) 439
Bartha's case with	1600	653	(specified k_L) 492
decelerating friction torque and suitable k_{NL}			(<i>ω</i> =900 rpm)
			553
			$(\omega = 1500 \text{ rpm})$ 573
			$(\omega = 2400 \text{ rpm})$ 614
			(ω=8200 rpm)

could lead to the wrong prediction of whip frequency. Even though several U-shaped curves are observed in Fig. 10, whip frequency has to satisfy two conditions. First, it has to be a point of intersection between required friction factor and specified friction factor. Second, it has to be the first location where the required friction factor changes sign from positive infinity to negative infinity. Beyond the first whip frequency, no other whirl ranges are observed experimentally by Yu et al. [8] and Bartha [6,7] (see also [8,9]). Numerical simulation on their model also shows that only the initial whip frequency is observed at the higher speeds.

Comparison to Numerical Simulation. Comparisons are also made to numerical simulation of the rotor-stator model described above using FEM and component synthesis methods on Timoshenko beams. The sgn function was used in defining the Coulomb friction force in terms of the relative slip velocity and was required to produce whirl or whip solutions.

Black's model uses a kinematic constraint to enforce coupled rotor-stator motion. However, following Bartha, the nonlinear contact model of Hunt and Crossley [16] was used here to represent a compliant interface between the rotor and stator at contact. Hunt and Crossley [16] relate the normal contact force N to the normal deformation δ and velocity $\dot{\delta}$ as $N = k_{NL} \delta^n + c_{NL} \delta^n \dot{\delta}$. For the linear case, $N = k_L \delta + c_L \dot{\delta}$.

For very large values of k_L and k_{NL} , the clearance between the rotor and stator remains close to C_r , and the analytical predictions match the numerical simulations. Lower values of k_L (and k_{NL}) produce lower effective stator stiffness $k_{s_{\text{eff}}} = k_s k_L / (k_s + k_L)$. For these lower values of contact stiffnesses, the clearance between the rotor and stator is larger than the specified clearance (> C_r), increases with rotor speed, and has different magnitudes for linear and nonlinear cases. The whirl and whip frequencies depend on the ratio of rotor radius to the actual clearance (including deformation).

Contact Coulomb friction factor was not varied with speed in the numerical simulations. Bartha [6,7] observed the contact surfaces melting at higher speeds, indicating a lower friction factor. The whirl region in the numerical simulation matches the predictions with seven modes (Eqs. (12) and (27)) as shown in Table 1. In the simulations, for increasing running speed with contact

maintained, the precession frequency for whipping motion was constant, staying at the upper limit of the backward whirl frequency. Also, if contact occurs between the rotor and stator at a rotor speed beyond the upper limit for sustained backward whirl, the numerical simulations produced whipping motion with this same precession frequency. Higher housing accelerations, higher normal forces, and higher torque requirements were observed for dry-friction whip than for dry-friction whirl. Larger torque values during whip would rapidly decelerate the rotor.

By extending Black's work, the current analysis was able to correctly predict the correct whip and whirl frequencies. However, near the transition point from whirl to whip or vice versa, there was a reduction in the backward whirl frequency. This reduction is not predicted analytically but was obtained by numerical simulations. Figure 11 shows an example of this reduction in the whirl frequency near the transition. The experimentally observed results



Fig. 11 Reduction in whirl and whip frequencies not predicted using Black's [1,2] analysis

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match very well with the numerical simulations at this transition region. The transition region predictions depend on system parameters and differ from model to model.

Conclusions and Discussion

The main results and conclusions from this paper are as follows

- Predictions of dry-friction whirl and whip frequencies depend strongly on the rotor and stator model.
- For the test results that are available, the number of rotor modes and the location of the contact point have a major impact on prediction accuracy.
- A multiple-rotor model with a correct accounting for the rubbing-contact location gives much better predictions than either Black's simple model [1,2] or Bartha's nested-stator model [6,7].
- Black's model [1,2] does not account for the sign of the sliding velocity in defining the contact friction force. In accordance with Bartha' results [6,7], simulations that include this term reproduce Black's solution. Simulations without this term fail to produce the whirl or whip solutions.
- Though several U-shaped curves are produced with a multimode model (yielding multiple possible whirl ranges), the simulations conducted here only produced the first whirl range. Beyond the first whip frequency, no other whirl solution could be found.
- Similarly, a multimode rotor model predicts several possible whip frequencies, but the present simulations only produced the initial whip frequency.
- Numerical simulations with complete rotor and stator model—including multiple modes for the rotor and stator, a correct location for contact, proper accounting for the friction force direction, and proper contact modeling—gave good result when compared to experimental data. Even the observed drop in precession frequency during transition from whirl to whip was produced.
- For the test models considered here, rub occurs at a designated intentional location. For real machines, rubbing can occur at multiple locations, generally at seals. The fact that the rubbing-contact location has a major impact on the outcome argues that separate whirl-whip analysis would be appropriate for all possible rub locations.

The results provided here leave open a larger question. Specifically, potentially destructive dry-friction whip is readily produced in test rigs. However, rubbing contact occurs regularly in turbomachinery yet it rarely leads to dry-friction whip. The question is simple: Why not?

Nomenclature

- A = area of cross section of the rotor (m²)
- C_r = radial clearance between rotor and stator (m)
- \dot{E} = modulus of elasticity for the rotor (Pa)
- F(s) = arbitrary forcing function in Laplace domain (N)
 - I = area moment of inertia of the rotor (m⁴)
- N = normal force acting at contact location (N)
- $N_i(s) = i$ th modal coordinates in Laplace domain (-)
- P = complex reaction force (N)
- U(z,s) = lateral displacement of rotor in Laplace domain (m)
 - V_t = tangential relative contact velocity (m/s)
- $Z_r, Z_s =$ complex rotor and stator displacement vector (m)
 - a = rotor imbalance distance (m)
 - b = axial location on rotor where contact occurs (m)
 - c_L = linear contact damping (N s/m)

 c_{NL} = nonlinear contact damping (N s/m^{m+1})

- c_r, c_s = rotor and stator damping to ground (N s/m)
 - e = base of natural logarithm $(e^{\ln(x)}=x)$ (-)
 - f_f = frictional force acting at contact location (-)
- f(t) = arbitrary forcing function (N)
 - $\dot{J} = \sqrt{-1}$, the imaginary unit for complex numbers (-)
- k_L = linear contact stiffness (N/m)
- k_{NL} = nonlinear contact stiffness (N/m^m)
- $k_r, k_s =$ rotor and stator stiffness to ground (N/m)
- m = nonlinear power, an odd number or fraction (-) m_r, m_s = totor and stator mass (kg)
- $u_r, m_s = \text{totol and statol mass (kg)}$
 - n = number of rotor modes to keep for analysis (-) r = radius of the rotor at contact location (m)
 - s = Laplace domain frequency variable (rad/s)
 - t = time (s)
- u(z,t) = lateral displacement of the rotor (m)
- x_r, x_s = rotor and stator displacements in x-axis (m)
- $y_r, y_s =$ rotor and stator displacements in y-axis (m)
- z = axial location along the rotor length (m)
- $z_r, z_s =$ complex rotor and stator displacements at t=0 (m)
- α_{11} = rotor receptance (measured response at point 1 over the force applied at point 1) (m/N)
- β_{11} = stator receptance (measured response at point 1 over the force applied at point 1) (m/N)
- $\delta, \dot{\delta}$ = normal contact displacement and velocity (m m/s)
- $\Phi_i = i$ th orthonormal eigenvector for pinned-end rotor (-)
- γ = angle between clearance vector and x-axis (rad)
- $\eta_i = i$ th modal coordinates (m)
- μ = Coulomb friction factor (= f_f/N) (-)
- μ^* = required friction factor (satisfying Eq. (12))
- ρ = density of the rotor (kg/m³)
- Ω = rotor whirling speed (rad/s)
- ω = rotor running speed (rad/s)
- $\omega_{ni} = i$ th orthonormal eigenvalue for pinned-end rotor (rad/s)
- ξ = angle between clearance vector and imbalance vector (rad)
- $\zeta_i = i$ th modal damping factor for pinned-end rotor (-)

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