CHAOS IN A ROTOR SYSTEM SUPPORTED BY BALL BEARINGS

A Senior Honors Thesis

By

JAMES ROBERT FISHER

Submitted to the Office of Honors Programs
& Academic Scholarships
Texas A&M University
In partial fulfillment of the requirements of

UNIVERSITY UNDERGRADUATE
RESEARCH FELLOWS

April 2001

Group: Engineering

CHAOS IN A ROTOR SYSTEM SUPPORTED BY BALL BEARINGS

A Senior Honors Thesis

By

JAMES ROBERT FISHER

Submitted to the Office of Honors Programs
& Academic Scholarships
Texas A&M University
In partial fulfillment of the requirements of

UNIVERSITY UNDERGRADUATE
RESEARCH FELLOWS

Approved as to style and content by:

Sherif Noah

Edward A. Funkhouser

April 2001

Group: Engineering

ABSTRACT

Chaos in a Rotor System Supported by Ball Bearings.

(April 2001)

James Robert Fisher
Department of Mechanical Engineering
Texas A&M University

Fellows Advisor: Dr. Sherif Noah
Department of Mechanical Engineering

Nonlinearities in a system are often thought of as unimportant and negligible. It is only recently that many of the impacts of nonlinearities in a system are being fully realized. One of these impacts is chaos. Chaos is a bounded steady state behavior that appears to be random, yet has some sort of order associated with it. Much work has been carried out in observing chaos analytically, however there is little experimental work in existence. In addition, there has been little work applied to observing chaos in rotor systems. Further, there has been little work carried out in the area of chaos control, of which only a small portion has been applied to rotor systems.

Originally, the goal of the research described in this paper was to control the chaotic response observed previously in a rotor system by Ortiz (Ortiz 2000). However, it soon became clear that the chaotic response was not as strong as previously believed, and may have been attributed to extraneous conditions. The goal then became to exploit other nonlinearities to observe a chaotic response.

This work was successful. The same rotor system used by Ortiz in his research was used again for this research. Chaos was observed by examining the case where the bearings supporting the rotor vibrate in their mounts. By loading the rotor and examining its response at various speeds, it was discovered that a chaotic response was observed. As the rotor speed increases, the response alternates between "chatter" and chaotic behavior. A control was also attempted using an attempt of Occasional Proportional Feedback as described by Barr, Myneni, Corron, and Pethel, but work was unsuccessful.

ACKNOWLEDGEMENTS

To begin with, I would like to thank my advisor, Dr. Noah. Thank you for pushing me to work hard and to work hard to achieve my goals.

Additionally, I would like to thank Zane Rhodes. Without him, I would have never been able to get the research off the ground.

Thanks also to Dr. Suh, who patiently helped me look at data and helped me work through my problems with finding chaos. Thank you for unselfishly taking your small amount of time to help me.

Also, thanks goes to Baozhong Yang who graciously took time to analyze my data using his wavelet technique.

Finally, thanks goes to Bethany Johanson and Joel Brandt, who graciously took time to help me with experiments and listen to my frustrations. I could never have accomplished anything without your support.

TABLE OF CONTENTS

Page

ABSTRACT iii

ACKNOWLEDGEMENTS v

TABLE OF CONTENTS vi

LIST OF FIGURES vii

INTRODUCTION 1

OBJECTIVE 3

EXPERIMENTAL SETUP 4

TESTING FOR CHAOS 9

RESULTS 12

CONTROLLING CHAOS 25

CONCLUSION 30

REFERENCES 31

VITA 32

LIST OF FIGURES

Figure Page

1 Rotor Setup for Experiment 4

2 Simulink Model for Data Acquisition 6

3 Signal and Frequency Spectrum at 6792 rpm 13

4 Response and Fourier Spectra at 5053 rpm 16

5 Response and Fourier Spectra at 7213 rpm 17

6 Phase portrait at 6792 rpm (non chaotic) 18

7 Phase portrait at 8145 rpm (chaotic response) ... 19

8 Response and Frequency Spectra at 7496 rpm 20

9 Response and Fourier Spectrum at 8145 rpm 21

10 Time and frequency response at 8474 rpm 22

11 Response and frequency spectrum at 8900 rpm 23

12 Time and frequency response at 10827 rpm 24

13 Schematic of Control Method 26

14 Simulink Model for Controller 26

INTRODUCTION

Chaotic behavior is usually thought of as something that is far-fetched and rarely observed in any real or useful system. It is only recently that Engineering Science is beginning to take the effects of nonlinearities and chaos seriously. The fact remains that chaotic behavior is something that can be observed in many of the systems that we consider very stable and linear. Sometimes this behavior is fortunately negligible, but other times, this behavior can result in a crisis in the system in which ultimately failure and injury can ensue. Specifically, this is the case in rotordynamics. A simple ball-bearing supported shaft in rotation can experience chaotic behavior due to many factors that are often neglected or considered unimportant. Two major factors which contribute to chaotic behavior during normal operation of the rotor are the internal clearances in the ball bearings and the ball contact stresses. The ball bearings have small clearances between their contact with the shaft and the outer casing.

This thesis follows the style and format of the *Journal of Vibration and Control.*

This means that not every ball is in contact with the shaft at once. This causes the balls to vibrate in the bearing and in turn perturb the system. These perturbations can lead to bifurcations and chaos. Additionally, the contact stresses on the balls (Hertz) are extremely nonlinear. This in turn can result in an exhibition of chaotic behavior in the rotor. In this paper, another cause for nonlinearity is examined. Many times in operation, the outer casing of a bearing will deform due to vibration and wear. As time goes on, the entire bearing begins to vibrate when the rotor is in operation. This vibration is very nonlinear because there are many contact points. This vibration of the bearing is expected to result in chaotic behavior in the rotor at various speeds.

OBJECTIVE

There were two main objectives to this research in the beginning. The first objective was to reproduce and prove results found previously in this system by Ortiz (Ortiz 2000). The second major objective of this research is to further implement a fast chaos control method as described by Barr, Myneni, Corron, and Pethel (Bar 1999). However, the results previously obtained were unable to be clarified as being as chaotic as was originally believed. In addition, this meant that the control method would be useless. Therefore, the new objective of the research became creating a more definitive chaotic response in the system.

EXPERIMENTAL SETUP

The system previously used by Steven Ortiz (Ortiz 2000) was again used in an attempt to build off of his results. The rotor assembly used to do the testing is a Bently Nevada system as shown below in Figure 1.

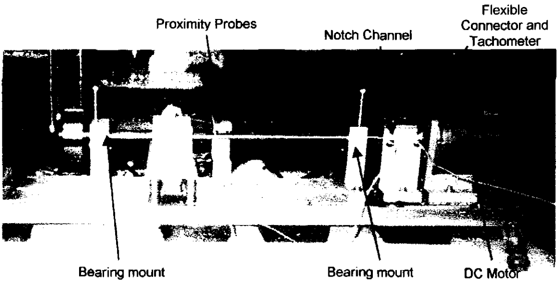


Figure 1: Rotor Setup for Experiment

This consists of a shaft mounted between two ball bearings as shown below. The shaft is turned using a DC motor. The shaft is mounted flexibly to the motor. This means that the effects seen by the proximity probes (which will be explained later) are due only to the shaft's vibration in the bearings and not due to any type of mounting problem between the shaft and the motor. It also means that the motor has only a very small longitudinal force transfer to the shaft and that the only reaction transferred is the torque reaction between the rotating motor and the shaft.

As mentioned previously, proximity probes are used to measure the displacement of the shaft in the ball bearings. These probes operate on an eddy current principle. These probes are attached to a Proximitor® which demodulates the signal. The Proximitor® produces a negative output voltage that is proportional to the distance of an object from the probe. As the object gets closer to the probe, the voltage tends toward zero, and as the object moves away from the probe, the voltage goes toward infinity. The positioning of the probes can be seen in Figure 1.

These voltages were read digitally using a computer equipped with an Analog/Digital Card. MATLAB Simulink was used to interface with the data acquisition card and obtain the data. A Simulink model was compiled containing the data acquisition cards and the sample rates. Additionally, this model was used to make Poincaré maps and a controller to control the ensuing chaos as well. The controller will be discussed in more detail later in the report. The Simulink model is shown below:

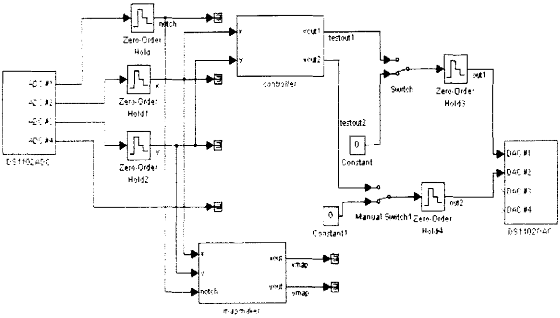


Figure 2: Simulink Model for Data Acquisition

This model was then compiled using MATLAB's Real Time Workshop. This creates and executes a program to interface with the card and collect data that runs exterior to MATLAB. To be able to collect this data and save it for later use, a program called dSpace is used. This program links into the C-code being executed in the system and compiles a trace file of the data, or a recording of the voltages from the analog input channels of the A/D card. Each trace of the data can be saved in a separate file for processing. For this experiment, three voltages were recorded. The displacement of the rotor in the x-direction, the displacement of the rotor in the y-direction, and a "notch" channel. The notch channel is used to determine the velocity of the rotor. The rotor setup has a closed loop velocity control, however, the rotor does not give any external information about its velocity. A knob on the speed control is used to select a velocity and this velocity is matched using the control system. However, the velocity cannot be known exactly just by looking at the knob setting. To know the velocity of the rotor, the "notch" channel is used. On the motor, there is a disc with a notch cut into it. A probe is placed in position to measure the distance from the disk to itself. When the notch is encountered, the probe has a voltage difference across it of well under -10 V. Since the A/D card used only measures -10/10 V, this shows up as -10 V. The number of times the notch channel is passed can be divided by the time it takes for the revolutions to get the rotational speed of the rotor. This is how the speed of the rotor is determined. This means that the sample rate needs to be fairly high to be able to make sure that the occurrence of a notch in the "notch" channel is recorded. For this experiment, the sample rate used was 4 kHz. This means that even at the rotor's maximum speed

(12000 rpm or 200 Hz), 20 samples per revolution can be recorded.

TESTING FOR CHAOS

At the onset of the research there were two main goals. The first was to reproduce previous results and the second was to control ensuing chaos. This section of the report describes the methodology employed in testing for chaos. Testing a signal for chaos is by no means an exact science. There are many factors that go into understand whether or not a signal is chaotic and there is no fool-proof method for knowing exactly whether a signal is chaotic or not. The main characteristic of chaos to look for is a broadband frequency spectrum. A chaotic signal is made up of many different unstable orbits of many different frequencies superimposed onto one another. This means that the signal will exhibit a broadband frequency characteristic when the signal is examined in the frequency domain. To look at this characteristic, the Fourier Transform is used. The formulation for a Fourier Transform is made up of a summation of the frequency terms found in the signal. This summation is given below:

$$X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$

Usually a Radix-2 algorithm is employed to make the process of finding each of these terms very quick. To know which frequencies are dominant, we simply examine the magnitude of the $X(\omega)$ term at each frequency. A broadband frequency has many values of $X(\omega)$ which are high in comparison to the dominant frequencies. Examples of this can be found in the results section.

In addition, to determine whether a signal is chaotic or not, it is important to look at the signal itself and examine its attributes. A chaotic signal will start with one orbit, and then suddenly change into another one. After "jumping" from orbit to orbit, it will later come back and nearly repeat itself. One way to confirm that a signal is chaotic is to look for places in a candidate signal where the signal nearly repeats. In other words, you are testing the ergocity of the signal.

Poincaré maps are also used to look for chaotic attractor. The data is plotted at once per cycle for the slowest revolution present in the system. If the system is behaving periodically, there will be one point on the map. If the system is behaving quasiperiodically, there will be some sort of structure, such as a ring. Finally, if chaos is present, there will be structure present, but there will be no repetition. This can demonstrate the structure of a chaotic attractor.

Phase portraits are also used. In this system, x versus y is plotted. For an equilibrium point solution, the phase portrait will show one point. For a periodic solution, there will be a ring. For a quasiperiodic solution, there will be some sort of complex structure that results in a closed loop of some sort. For chaos, there will be a bounded result that never repeats exactly. You will end up with an infinite amount of lines that seem to follow some sort of quasiperiodic pattern.

Finally, another method employed is a method developed by Yang, Suh, and Chan, which is in the process of being published. This method employs the use of wavelets to approximate a signal. The energy of each waveform is then found using the wavelet approximation. By examining the energy path and following the energy for each cycle, another means of detecting chaos is found.

RESULTS

The original intent of this paper was to reproduce the previous results found by Ortiz (Ortiz 2000). This however proved to be a major complication. The tests were performed in the same manner as those done previously on the same rotor system, and data was collected at the same parameter values for the system. Chaos was previously reported as being obtained at a rotor speed of 6710 rpm. The system was tested at similar rotational speeds and similar results to those previously found were recorded. However, further testing and examination of this data shows that it is very doubtful that the data was in fact chaotic. Figure 3 below shows the signal value versus time as well as a frequency spectrum of the signal obtained by computing a Fast Fourier Transform.

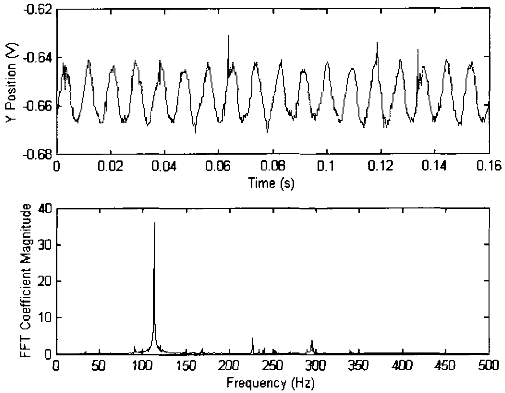


Figure 3: Signal and Frequency Spectrum at 6792 rpm

Additionally, it was found that a large amount of the signal characteristics were in place because of outside influences. In other words, some of the signal transitions that were shown in the previously mentioned paper were not in fact due to the rotor system, but due to other things in the experimental area vibrating with the system. To alleviate this, the large masses were placed on the rotor assembly and the rotor was mounted on rubber blocks to keep vibrations from the outer elements from affecting the rotor.

The discovery that the previous response was not in fact as chaotic as previously thought meant that the rotor had to be tested in other ways to find a chaotic response. Different nonlinearities were examined, including putting mass on the rotor and an imbalance on the rotor. Adding imbalance to the rotor would lead to chaotic behavior, however, with the equipment and its sensitivity, a speed to where this was possible could not be reached without damaging the system. Finally, the nonlinearity associated with the bearings rattling was examined. When a fast enough speed is reached, the bearing casing begins to vibrate inside the bearing mount. This vibration is strongly nonlinear, and under the right conditions, can be chaotic. For an unloaded rotor, this takes place at very high speeds, which are close to the maximum velocity of the rotor, so it is difficult to study this behavior completely. Because of this, the rotor was loaded with a mass. This caused the bearings to vibrate freely after only about 4000 rpm as opposed to 11000.

With the bearings vibrating there were two main types of responses observed, chaos and chatter. Chatter occurs when the system has several dominant frequencies at which it is vibrating. During this behavior, there are several definite single frequency noises that can be heard while the rotor is turning. In addition, a look at the frequency spectrum reveals that this is the case as well. Finally, observation of the response reveals that there is a definite periodic nature to the signal that is repeated throughout it.

In this system, chatter and chaos are intermittent. When the bearing first begins to rattle free at about 4000 rpm, the signal is chatter. Then, as speed increases to about 7500 rpm, the signal turns chaotic. Additionally, at about 8500 rpm, the signal goes back to chatter, and again as speed is increased, back to chaos. Eventually, the ball bearing vibration ceases as speeds of over about 11000 rpm are reached. Shown here are some of the results obtained from 4000 to 7500 rpm.

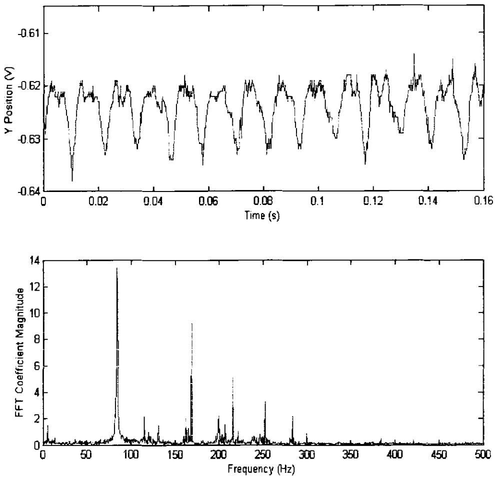


Figure 4: Response and Fourier Spectra at 5053 rpm

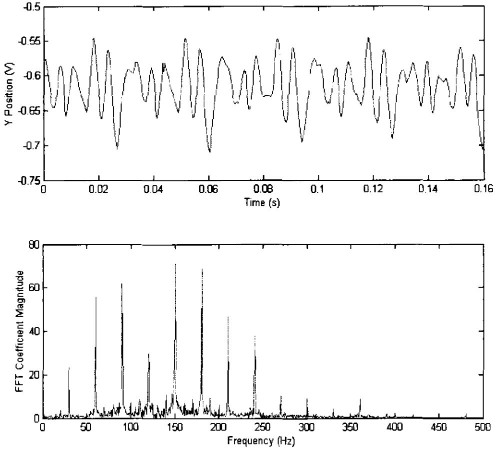


Figure 5: Response and Fourier Spectra at 7213 rpm

As demonstrated in these plots, there are several large frequency spikes in the frequency spectrum. This is a strong indicator that these responses are in fact chatter. This is especially true with the response at 7213 rpm. As in the paper by Ortiz, Poincaré maps were indeterminate. The maps revealed no real attractors or characteristics of the signal. Additionally, the phase space plots are indeterminate as well. These plots revealed the periodic or quasiperiodic nature of the solutions, but no real structure that could shed light onto whether or not the signal was chaotic. The probes used in this experiment are fairly noisy, so it is difficult to use the phase portraits to determine any specific orbits. This also created problems in attempting to control any chaotic response, which will be discussed in more detail later. Some example phase portraits are shown below:

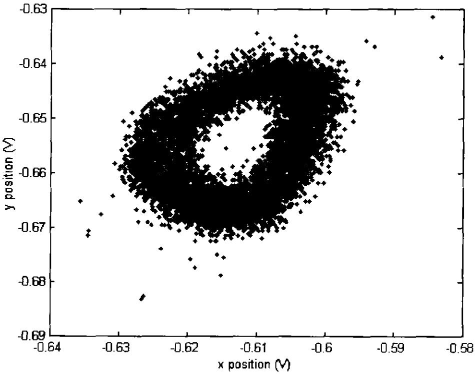


Figure 6: Phase portrait at 6792 rpm (non chaotic)

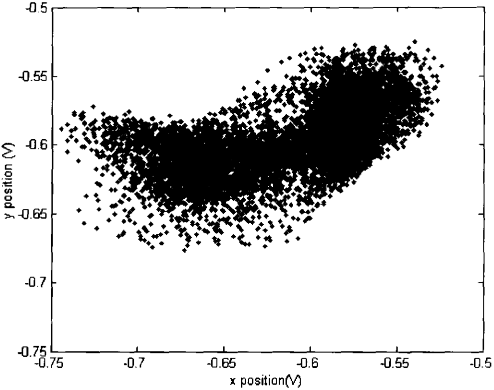
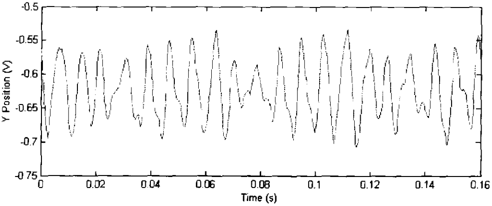


Figure 7: Phase portrait at 8145 rpm (chaotic response)

The other type of behavior observed in this range is the chaotic response. This response differs from the chatter response in that while there are a large number of frequencies present in the signal, only one or so dominate. The result is a broad band of low magnitude Fourier coefficients with maybe one or two peaks. Additionally, upon examining the signal, it is clear that the signal repeats in some stages, but that there is no clear periodic behavior. In other words, the signal exhibits an ergotic response. The chaotic response is observed for a range from about 7500 rpm to 8500 rpm. The responses and Fourier Spectra are shown below:



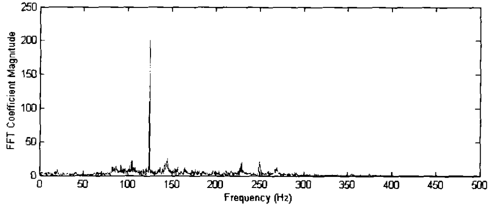


Figure 8: Response and Frequency Spectra at 7496 rpm

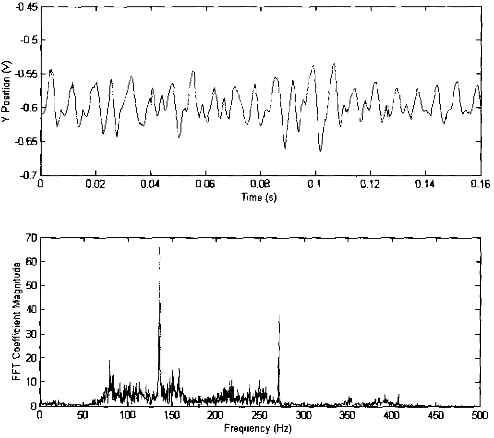


Figure 9: Response and Fourier Spectrum at 8145 rpm

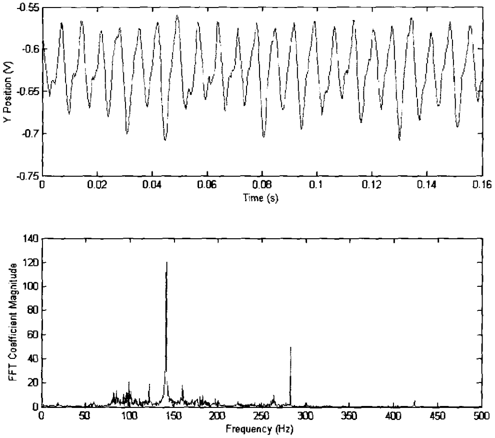


Figure 10: Time and frequency response at 8474 rpm

After 8500 rpm, the response changed from chaotic to chatter again. This can be observed by looking at emerging peaks in the following Fourier Spectra.

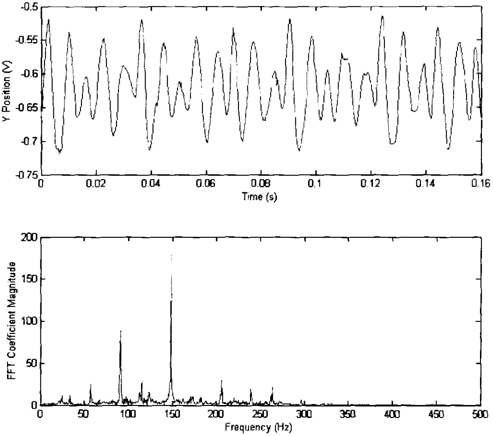


Figure 11: Response and frequency spectrum at 8900 rpm

Finally, at about 10000 rpm, the response changed back to chaotic. This chaotic response, however, is very weak, and eventually as speed is increased, settles into a periodic response. The response is given below:

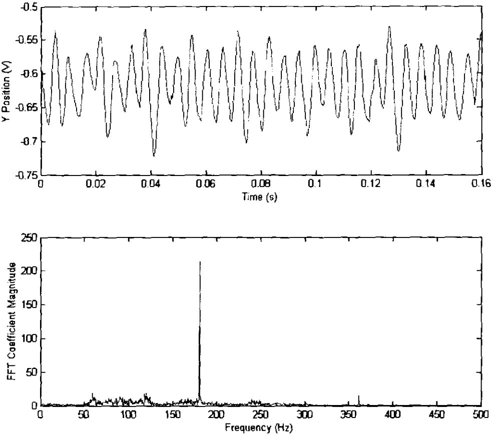


Figure 12: Time and frequency response at 10827 rpm

CONTROLLING CHAOS

As the original intent of this paper was in fact to control the chaos resulting from a previous experiment, a great deal of time and investigation was spent on examining the responses found in this paper and on using a method to control the resulting chaotic behavior. The methodology employed to attempt this control is a form of Occasional Proportional Feedback, or OPF, as described by Barr, Myneni, Corron, and Pethel (Barr 1999). This method is an adaptation of OGY that can be used on very fast chaotic systems. The method requires much less calculation and CPU time, and is also very simple to implement. The method can even be implemented using an analog controller. This makes it very attractive for use in many systems.

The method itself is very simple. Using a phase portrait, a window is placed at a certain location that determines if the controller is on or off. If the controller is on, the system is perturbed with some constant perturbation, γ; otherwise there is no perturbation. The controller seeks to perturb the system into a stable orbit. This was performed successfully by

Barr, Myneni, Corron, and Pethel (Barr 1999). A schematic of how this can work is shown below.

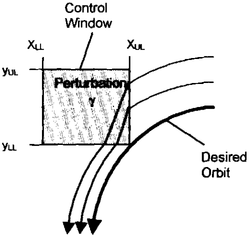


Figure 13: Schematic of Control Method

As shown above, when the signal enters the window it is perturbed so that it is forced into the orbit just below the window. This orbit can be of any period. The MATLAB Simulink implementation of this controller used for experimentation is shown below:

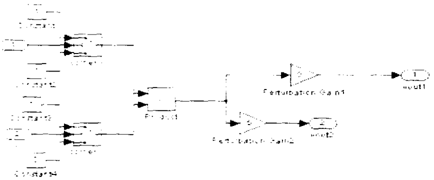


Figure 14: Simulink Model for Controller

There were several difficulties encountered while implementing this controller. The first major problem involved the dependence on the controller to state space. As shown previously, in Figure 7, there is a large amount of noise present in the x-y plot of the response. For the controller to work, it would have to be given very accurate results, else it would perturb when it was not supposed to, ruining the intended effect of the controller. Conventional noise reduction techniques such as filters cannot be employed because they remove many components that contribute to the chaotic response of the signal. There are many methods that have been proposed for noise reduction in chaotic time series data, however these methods mostly involve processing the data after all of it has been collected. Few to none of the methods can be implemented on a real time system. So, even if chaotic noise reduction was employed, it could not be done in order to feed values to the controller, so it would be of no use.

In addition, the phase portraits used examined the x-y plane in search of a chaotic phase portrait. However, many papers that demonstrate analytically the chaotic behavior in ball bearings demonstrate a chaotic phase portrait by examining the displacement of the shaft versus its speed in the displacement direction. The speed in x-y plane of a shaft, however, is not something that can be easily measured. This means that the most realistic phase portrait that can be used to control chaos, the x-y plane, is most likely not one that is useful.

Finally, the method of perturbation might be problematic as well. For this experiment, the system was perturbed physically using a piezoelectric actuator that perturbed the shaft when the controller was on. In most chaos control schemes, the control is used to modify the value of some parameter on which the chaos of the system depends. For this system, that would be speed of the rotor. However, the rotor speed is not controllable for the system studied, and so it was thought that perturbing the system by physically displacing the rotor in some method might be the best possible solution.

Naturally the question of is it possible for this type of control to work for this system arises. The answer to this, despite the problems observed, is yes. The only finding determined by this paper is the fact that the work that has been done on the system thus far has not been successful. It is clear, however, that in order for the control to be successful, some of the problem areas described above must be solved.

CONCLUSION

The previous results obtained on the Bently Nevada rotor system when examined more thoroughly shed doubt on whether the system behaved chaotically in its normal behavior. Therefore, other nonlinearities in the system were studied more closely to draw out a stronger chaotic behavior in the system. Chaos was found that by examining the case where the bearings begin to vibrate in their mounts for a loaded rotor. Fast Fourier Transforms yielded broadband frequency behavior for the system proving the chaotic behavior in the system. It was found that periodic behavior is exhibited for speeds up to about 4000 rpm. Chatter was observed from 4000 rpm to 7500 rpm. Chaos was observed from 7500 rpm to 8500 rpm. After 8500 rpm is reached, the behavior turns to chatter until about 10000 rpm, where it returns to chaos. After about 11000 rpm, the behavior once again turns periodic in nature.

REFERENCES

Barr, T. A., Myneni, K., Corron, N. J. and Pethel, S. D., 1999, "New Method for the Control of Fast Chaotic Oscillations," *Physical Review Letters* **83**(11), 2175-2178.

Choi, S. and Noah, S. T., 1992, "Mode-Locking and Chaos in a Modified Jeffcott Rotor With a Bearing Clearance," *Nonlinear Vibrations* **144**, 21-28.

Fyrillas, M. M. and Szeri, A. J., 1998, "Control of Ultra- and Subharmonic Resonances," *Journal of Nonlinear Science* **8**, 131-159.

Kostelich, E. J. and Schreiber, T., 1993, "Noise Reduction in Chaotic Time-Series Data: A Survey of Common Methods," *Physical Review E* **48**(3), 1752-1763.

Nayfeh, A. H. and Balachandran, B., 1995, John Wiley & Sons, New York.

Sankaravelu, A., Noah, S. T., and Burger, C. P., 1994 "Bifurcation and Chaos in Ball Bearings," *Nonlinear and Stochastic Dynamics* **192**, 313-325.

VITA

James Robert Fisher

Permanent Address:
1800 Clayton
Baytown, TX 77520

Education:
Bachelor of Science, Mechanical Engineering
Expected Graduation Date: May 2001

Honors:
Phi Eta Sigma National Honor Society
Jesse H. Jones Houston Endowment Scholarship
Chevron REACH Scholarship
Lyondell-Citgo Scholarship
Dean's List (1997-2000)
University Honors Graduate
Graduate Merit Fellowship - Texas A&M University
Phi Kappa Phi Honor Society