DARK WORLD AND THE STANDARD MODEL

A Dissertation

by

GANG ZHAO

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

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Approved by:	
Chair of Committee,	Dimitri Nanopoulos
Committee Members,	Richard Arnowitt
	Christopher Pope
	Che-Ming Ko
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ABSTRACT

Dark World and the Standard Model.

(August 2006)

Gang Zhao, B.S., Nankai University

Chair of Advisory Committee: Dr. Dimitri Nanopolous

The most popular way to achieve accelerated expansion of the universe is by introducing a scalar field in which motion of state varies with time. The accelerated expanded universe was first observed by Type Ia supernovae and future confirmed by the latest of CMB (Cosmic Microwave Background). The reason for the accelerated universe is the existence of dark energy. In this dissertation, we discuss the relationship between dark matter, dark energy, reheating and the standard model, and we find that it is possible for us to unify dark energy, dark matter and a reheating field into one scalar field. There is a very important stage called inflationary, and we find that the residue of the inflationary field, which is also described by a scalar field, can form bubbles in our universe due to the gravity force. We discuss that these bubbles are stable since they are trapped in their potential wells, and the bubbles can be a candidate for dark matter. We also discuss the scalar singlet filed, with the simplest interaction with the Higgs field, and we find that a static, classical droplet can be formed. The physics picture of the droplet is natural, and it is almost the same as the formation of an oil droplet in water. We show that the droplet is absolutely stable. Due to the very weak interaction with the Standard Model particles, the droplet becomes a very promising candidate for dark matter.

DEDICATION

To my mother and father

ACKNOWLEDGMENTS

I would like to thank my committee chair, Dr. Dimitri Nanopulos, and my committee members, Dr. Richard Arnowitt, Dr. Christopher Pope, Dr. Che-Ming Ko, and Dr. Stephen Fulling, for my guidance and support throughout my research.

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CHAPTER I

INTRODUCTION

Observations of Type Ia high red shift supernova [1] have revealed that the expansion of the universe is speeding up rather than slowing down. The accelerated expansion of the universe which was further confirmed by the observation of Cosmic Microwave Background (CMB), indicates that there is an energy component in the universe which is dark and has negative pressure. The recent data suggest that the universe compose of dark energy (73%) dark matter (23%) and matter (4%) [2]. The simplest way to achieve accelerated expansion of the universe is by introducing a cosmological constant, which has the effective equation of state $p=-\rho$, i.e. $w=p/\rho=-1$, where p is pressure and ρ is energy density. But it is well known that the unevolving cosmological constant faced a serious "fine tuning" problem. A more general way to obtain expansion of the universe is by introducing a scalar field φ ("quintessence") which rolls down a potential $V(\varphi)$ leading to an accelerated expansion at the current epoch. In such models, the dark energy component $\Omega\varphi=\rho\varphi/\rho c$ varies with time. One of these models is AS model [3] in which the parameters involved are roughly of order of one in Plank unites (Mpl=2.44*10^18GeV).

At the stage of inflation, according to the inflationary scenario [4], the universe is void of particles and its energy density is dominated by the potential energy of a scalar

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This dissertation follows the style of Physical Review D.

field φ.

At the end of inflation, the inflation field oscillated quasiperiodicly near the minimum of its effective potential with slowly decreasing amplitude. Quasiperiodic motion of the inflation field leads to the creation of almost all particles in the universe, and after thermalization of particles due to collisions and decays, the universe comes to a state of thermal equilibrium at some temperature Tr, which was called the reheating temperature.

The quintessence field makes the universe undergo in an accelerated expansion today, and the inflation scalar field produces particles at the stage of reheating. Both of them can be described by scalar field. One objective of my thesis is to try to unify these two different fields into one field but at different stages of universe.

In reheating theory, the simplest form of potential is

$$V(\phi)=\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4\,. \tag{1}$$

The equation of motion for field ϕ can be written as

$$\ddot{\phi}-\triangle\phi+m^2\phi+\lambda\phi^3=0\,. \tag{2}$$

And its energy density is

$$\rho=\frac{1}{2}\dot{\phi}^2+\frac{1}{2}m^2\,\phi^2+\frac{1}{4}\lambda\phi^4+\frac{1}{2}\nabla^*\phi\nabla\phi\,. \tag{3}$$

Due to gravity, a static bubble of the reheating field Φ could be formed and becomes a candidate of the dark matter. This is the second objective of my thesis.

Standard Model of particle physics is successful in both theoretical and experiment physics. And in standard model, there is very important scalar which provides the breaking of symmetry and gives particles mass in standard models, is call Higgs field. And the form of this Higgs field can be written as

$$V = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \tag{4}$$

where ϕ is doublet,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

In Standard Model, m^2 is negative and λ is positive number. Generally to say, since ϕ is doublet and has two components ϕ^+ and ϕ^0, m^2 should be a mass matrix. If there is a scalar field which has the same form as (4), but with m^2 as mass matrix, we discussed the possibility that dark world, dark energy and dark matter be described with this scalar field. And we found it cannot be since no droplet of scalar field ϕ^+ can formed in scalar field ϕ^0. Then we moved to a little more complicated case which with potential

$$\begin{aligned} V(\phi_1, \phi_2) = &\frac{1}{2} m_1^2 \phi_1^2 + \frac{1}{4} \lambda_1 \phi_1^4 \\ &+ \frac{1}{2} g \phi_1^2 \phi_2^2 \\ &- \frac{1}{2} m_2^2 \phi_2^2 + \frac{1}{4} \lambda_2 \phi_2^4 \end{aligned} \tag{5}$$

And we found that with this kind of potential, the scalar field ϕ_1 can form a droplet in the scalar field ϕ_2 due to the interaction term. We discussed this model and require the parameter of potential in (5) to satisfy the constraint that the universe is accelerated expansion today and the universe needed a slow expansion in the period of nucleosynthesis. We also discuss some other models which can unify the dark energy and dark matter. Droplet is very important since it is totally classical and it is a solution. We also discussed the shape of droplet, the surface energy, and some problems which we cannot be solved.

In this paper, we also talked some previous work which is useful to our model. We talked Sidney Coleman's Q balls theory. Q balls, although dealing with different questions with our droplet, there is some resemblance, and some techniques in the paper can be used directly to our droplet model.

CHAPTER II

COSMOLOGY, DARK ENERGY AND REHEATING

Cosmology is the study of the universe as whole, that is, how the universe began, how it developed to be like what it looks today, and how it will end in the future. Because of the discovery of accelerated expansion of universe, cosmology becomes one of hottest topics in physics, and many high energy physicists try to explain this phenomenon with superstring and supersymmetry physics. To cosmologist, Big Bang theory is one of the most useful methods to deal with universe problems. Big Bang theory， which is a very successful theory to study universe, presents a clear and natural picture how the evolution of universe from the beginning to our current age, around 13.6 billion years later and with 3k CMB, and to the future. This theory is a theoretical work based on general relativity which is put forward by Albert Einstein and Alexander A. Friedmann in the 1920s, and there are some observational facts to support the Big Bang Theory: First, the expansion of the universe, discovered by Edwin P.Hubble in the 1930s, and accelerated expansion, discovered by the observation of supernova. Second, the relative abundance of light elements, explained by George Gamow in the 1940s, mainly that of helium, deuterium and lithium, which were cooked from the nuclear was a few times hotter than the core of the sun. Third, the cosmic microwave background (CMB), the afterglow of the Big Bang, discovered in 1965 by Arno A. Penzias and Robert W.Wilson as a very isotropic blackbody radiation at a temperature of about 3 degrees Kelvin, emitted when the universe was cold enough to form neutral atoms, and photons decoupled form matter (the photons cannot decays into electrons), approximately 380,000 years after the Big Bang. Today, these observations are confirmed by experiments to within a few of percent accuracy, and have helped establish the hot Big Bang as the preferred model of the universe.

Cosmology begun as a quantitative science with the advent of Einstein's general relativity and the geometry of space-time, and thus the general attraction of matter, is determined by the energy momentum content of content of the universe

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The third term of the equation is cosmology constant term which is important for the accelerated expansion universe. These non-linear equations are very difficult to solve, but with some kinds of symmetry of universe, it can be solved, and sometimes, we can solve it numerically.

Although at small scales the universe looks very inhomogeneous and anisotropic, such as the stars and clusters, the deepest galaxy like 2dF GRS and SDSS suggest that the universe on large scales is very homogeneous and isotropic. And, the cosmic microwave background (CMB), which contains information about the early universe when the photons decoupled from matters, indicates that the deviations from homogeneity and isotropy were just a few parts per million at the time of photon decoupling. Therefore, we can impose those metric satisfying homogeneity and isotropy is the Fridmann-Poberson-Walker(FRW) metric, written here in terms of the invariant geodesic distance $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ in four dimensions μ=0,1,2,3. And by this way, we can simplify the Einstein Equation.

We consider homogenous, isotropic spatially flat cosmology described by the line element

$$ds^2 = dt^2 - a^2(t)dx^2 . \tag{6}$$

The scalar field action is

$$S = \int dx^4 \sqrt{-g}[\frac{1}{2} g^{\mu\nu}\partial_\mu \partial_\nu \phi - V(\phi) - \frac{R}{2}] \tag{7}$$

where R is curvature scalar.

The most general matter fluid consistent with the assumption of homogeneity and isotropy is a perfect fluid, one is which an observer commoving with the fluid would see the universe around it as isotropic. The energy momentum tensor associated with such a fluid can be written as

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)U_\mu U_\nu \tag{8}$$

Where p (t) and $\rho(t)$ are the pressure and energy density of the fluid in the expansion, measured by this commoving observer, and U_μ is the commoving four-velocity, satisfying $U^\mu U_\mu = -1$. For such a commoving observer, the matter content looks isotropic,

$$T^\mu_\nu = diag(-\rho(t), p(t), p(t), p(t)) \tag{9}$$

The conservation of energy ($T^{\mu\nu}_{;\nu} = 0$), a direct consequence of the general covariance of the theory ($G^{\mu\nu}_{;\nu} = 0$), can be written in terms of the FRW metric and the perfect fluid tensor (9) as

$$\dot{\rho}+3\frac{\dot{a}}{a}(p+\rho)=0 \tag{10}$$

In order to find explicit solutions, one has to supplement the conservation equation with an equation of state relating the pressure and the density of the fluid. The most relevant fluids are barotropic, i.e. fluids whose pressure is linearly proportional to the density, $p = w\rho$ and therefore the speed of sound is constant in those fluids. Generally to say, there are three main types of fluids with different value of w:

1) Radiation, w=1/3, with equation of state $p_R = \rho_R / 3$, like photons, associated with relativistic degrees of freedom (i.e. particles with temperatures much greater than their mass). In this case, the energy of matter decays as $\rho_R \sim a^{-4}$ with the expansion of the universe.
2) Matter, w=0, the equation of state $p_M \simeq 0$, associated with nonrelativistic degrees of freedom (i.e. particles with temperatures much smaller than their mass). In this case, the energy density of matter decays as $\rho_M \sim a^{-3}$ with the expansion of the universe. Usually, we treat the dark matter in this group.
3) Vacuum energy, w=-1, with equation $p_v = -\rho_v$, associated with quantum vacuum fluctuations. In this case, the vacuum energy density remains constant with the expansion of the universe. (For dark energy, w can be any value smaller than 0. Recent data indicates that w of dark matter should smaller than -0.7)

Reheating

There is a very important stage of universe called reheating. Reheating theory was introduced by L.Fofman, A.Linde, et al. and I will review their work here since we will use this theory in our model. According to the theory of inflationary scenario, the universe in the past expands exponentially with time while the energy density is dominated by the potential energy of a scalar singlet field φ, called inflation. Right after inflation the universe is empty of particles. Quasiperiodic evolution of the inflation field leads to creation of Standard Model particles and dark matter, after thermalization of which due to collision and decays the universe becomes "hot". There are two kinds of procedure of particles production: Born approximation and Parametric Resonance.

Consider the inflation scenario based on the scalar field dynamics. The Lagrangian of the model is

$$L = \frac{1}{2}(\nabla\varphi)^2 - V(\varphi) - \frac{M_P^2}{16\pi}R$$

With the inflation field potential

$$V(\varphi) = \lambda\mu^{4-q}\,|\,\varphi\,|^q$$

The equation of motion can be written as

$$H^2 = \frac{8\pi}{3M_P^2}[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)]$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

The solution is

$$\dot{\varphi} = a^{-3}(\mathrm{const} - \int V'(\varphi)a^3 dt)$$

As the scalar field rolls towards its smaller values, the equation of motion can be rewritten as

$$H^2 = \frac{8\pi}{3M_P^2}\rho$$

$$\dot{\rho} = -3H\dot{\varphi}^2$$

where

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

is the scalar field energy density. Introducing the positive value $\varphi_0(t)$ by the relation

$$V(\varphi_0(t)) = \rho(t)$$

We can present the evolution of the scalar field in the form

$$\varphi(t) = \varphi_0(t)\cos\int W(t)dt$$

where W (t) is can be written as the form

$$W = \sqrt{\frac{2[\rho - V(\varphi)]}{\varphi_0^2 - \varphi^2}}\left(1 \pm \frac{6H\varphi}{q\sqrt{2\rho}}\sqrt{1 - \frac{V(\varphi)}{\rho}}\right)$$

If $\varphi_0^2 \ll \frac{q}{48\pi}\left(\frac{q+2}{2}\right)^{(q+2)/q} M_P^2$

Then approximately W can be

$$W \approx \sqrt{\frac{2[\rho - V(\varphi)]}{\varphi_0^2 - \varphi^2}}$$

and $\dot{\rho} = -6H[\rho - V(\varphi)]$

taking the average of both sides, we get

$$\frac{\Delta\rho(T)}{T} = -\frac{6}{T}\int_0^T H[\rho - V(\varphi)]dt$$

where $\Delta\rho(T)$ is the change of ρ over the time period T. And approximately we can write

$$\dot{\rho} \approx -\frac{6q}{q+2} H \rho$$

$$\dot{\varphi}_0 \approx -\frac{6}{q+2} H \varphi_0$$

Particle Production Due To Elementary Theory

We take the interaction to be

$$L_{\text{int}} = -f \varphi \bar{\psi} \psi - (\sigma\varphi + h\varphi^2)\chi^2$$

We need to develop the quasiperiodic evolution of the scalar field φ into harmonics

$$\varphi(t) = \sum_{n=1}^{\infty} \varphi_n \cos(n\omega t)$$

$$\varphi^2(t) = \bar{\varphi^2} + \sum_{n=1}^{\infty} \zeta_n \cos(2n\omega t)$$

where the value $\bar{\varphi}^2 \approx \varphi_0^2 / 2$ which is slowly varying with time is φ^2 averaged over the rapid oscillations of the scalar field φ. φ_n and ζ_n are amplitudes which are slowly varying with time, and

$$\omega = \frac{2\pi}{T} = c\frac{\sqrt{\rho}}{\varphi_0}$$

is the leading frequency, also slowly varying with time, related to eh period T of the oscillations of the field φ. Assuming that the oscillation period of φ is small compared to the Hubble time,

$$H \ll \omega$$

We will also assume that particle masses and the coupling constant h are sufficiently small because we haven't found this field in our world. The rates of particle production, that is, the total number of pairs produced per unit and unit time are then given in first-order perturbation theory [12]

$$\Gamma(\varphi \to \chi\chi) = \frac{g^4\sigma^2}{8\pi m_\varphi}$$

$$\Gamma(\varphi \to \psi\psi) = \frac{h^2 m_\varphi}{8\pi}$$

Reheating completes when the rate of expansion of the Universe given by the Hubble constant $H = \sqrt{8\pi\rho / 3M_P^2} \sim t^{-1}$ becomes smaller than the total decay rate $\Gamma = \Gamma(\varphi \to \chi\chi) + \Gamma(\varphi \to \psi\psi)$. The reheating temperature can be estimated by $T_r \simeq 0.1\sqrt{\Gamma M_P}$.

Particle Production Due to Parametric Resonance

Parametric resonance occurs when the frequency ω_k of the quantum field mode is equal to half-integer multiples of the inflation frequency, $\omega_k^2 \approx [(n/2)\omega]^2$. This results in exponentially enhanced production of particles in narrow resonance bands.

The evolution of a particular mode χ_k of the quantum scalar field χ in the presence of the quickly oscillating classical scalar field φ is described by the following equation:

$$\ddot{\chi}_k + 3H\,\dot{\chi}_k + (\underline{k}^2 + m_\chi^2 + 2\sigma\varphi + 2h\varphi^2)\chi_k = 0$$

where $\underline{k} = k/a$ is the physical wave number of the mode under consideration and $k = \sqrt{\vec{k}^2}$ is commoving number.

With the transformation

$$\chi_k = \frac{Y_k}{a^{3/2}}$$

one can get the equation of Y_k as

$$\ddot{Y}_k + [\omega_k^2(t) + g(\omega t)]Y_k = 0$$

where

$$\omega_k^2(t) = \underline{k}^2 + m_\chi^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H} + 2h\bar{\varphi}^2$$

$$g(\omega t) = 2\sigma\varphi + 2h(\varphi^2 - \bar{\varphi}^2)$$

From the general theory of parametric resonance, one knows that parametric resonance occurs for certain values of the frequencies ω_k. Namely, the resonance in the lowest frequency resonance band occurs for those values of ω_k for which

$$\omega_k^2 - (\frac{n}{2}\omega)^2 \equiv \Delta_n < |g_n|$$

where n is an integer and g_n is the amplitude of the nth Fourier harmonic of the function $g(\omega t)$.

Denote by ω_{res} the resonance frequency $n\omega/2$ and by $\Delta\omega_k$ denote the difference $\omega_k - \omega_{res}$. Both ω_k and ω_{res} change with time. In the small time δt the shift between these frequencies will be $\delta\omega = |d\Delta\omega_k / dt|_{\Delta\omega_k=0}\,\delta t$. Hence a new region in phase space region will be given by N_k and each particle will have energy ω_{res}. Then taking the sum over all parametric resonance bands we can write down the equation for the energy density ρ_χ of the scalar particles produced in the following form:

$$\dot{\rho}_\chi\,(\text{Resonance production}) \quad = \sum_{all.resonance.bands} \frac{1}{2\pi^2} N_{res}\omega_{res}^3 \left|\frac{d\Delta\omega_k}{dt}\right|_{\Delta\omega_k=0}$$

where N_{res} is the maximal value of . N_k for the current resonance value of k, achieved after the corresponding mode has passed through the resonance band and has been amplified.

Under the condition

$$m_\psi^2, m_\chi^2 + h\overline{\varphi^2} \ll \omega^2$$

Particles produced will be ultrarelativistic and hence their contribution to the energy density will decrease due to the cosmological expansion. Taking this process also into account we are able to write down the following complete equation for the evolution of the energy density ρ_P of the particles produced

$$\dot{\rho}_P = -4H\rho_P + (\Gamma_\varphi + \Gamma_\chi^{(res)})\rho$$

where

$$\Gamma_\chi^{(res)} = \frac{1}{2\pi^2\rho} \sum_{all.resonance.bands} \frac{1}{2\pi^2} N_{res}\omega_{res}^3 \left| \frac{d\Delta\omega_k}{dt} \right|_{\Delta\omega_k = 0}$$

The simplest case is scalar field potential like

$$V(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \qquad (11)$$

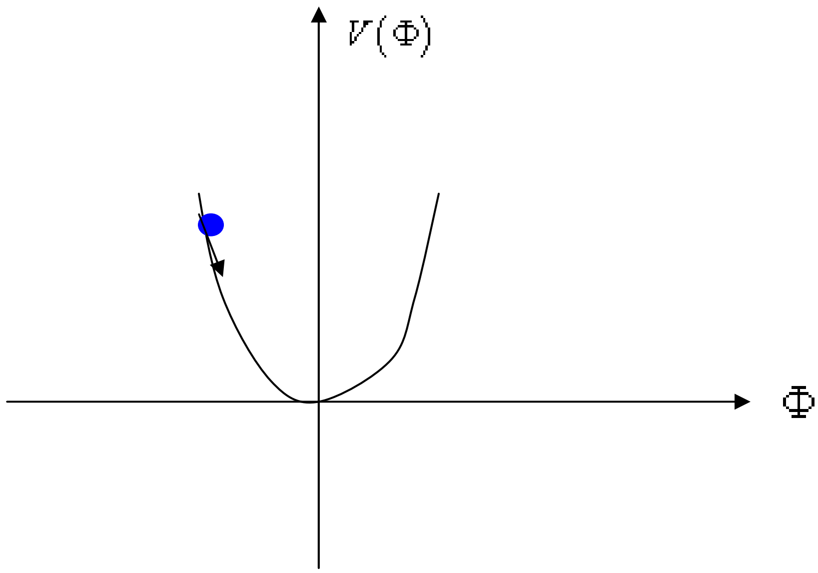

Fig. 1 The simplest scalar field for reheating

Fig.1 shows the potential of the simplest scalar field for reheating.

In this case the effective potential of the field ϕ soon after the end of inflation at $\phi \sim M_P$ is dominated by the term $\frac{1}{4}\lambda\phi^4$. Oscillations of the field ϕ in this theory are not sinusoidal; they are given by elliptic functions, but with good accuracy one can write

$$\phi(t) \sim \Phi \sin(c\sqrt{\lambda}\int \Phi dt)$$

where

$$c = \Gamma^2(3/4)/\sqrt{\pi} \approx 0.85$$

The Universe at that time expands as at the radiation-dominated stage: $\Phi(t) \propto a^{-1}(t)$, so that $a\Phi$ =const. Using a conformal time η, the exact equation for quantum fluctuations $\delta\phi$ of the field ϕ can be reduced to the Lame equation. The results remain essentially the same if we use an approximate equation

$$\frac{d^2(\delta\phi_k)}{d\eta^2} + [k^2 + 3\lambda a^2\Phi^2 \sin^2(c\sqrt{\lambda}a\Phi\eta)]\delta\phi_k = 0$$

$$\eta = \int \frac{dt}{a(t)} = \frac{2t}{a(t)}$$

which leads to the Mathieu equation with

$$A = k^2 / c^2\lambda a^2\Phi^2 + 3/2c^2 \approx k^2 / c^2\lambda a^2\Phi^2 + 2.08$$

and q=1.04. We can see that the resonance occurs in the second band, for $k^2 \sim 3\lambda a^2\Phi^2$.

The maximal value of the coefficient in this band for q~1 is approximately equal to 0.07. As long as the background of created particles is small, expansion of the Universe does not shift fluctuations away from the resonance band, and the number of produced particles grows as $\exp(2c\mu_k\sqrt{\lambda}a\Phi\eta) \sim \exp(\sqrt{\lambda}\Phi_t/5)$.

Reheating stops when the presence of nonzero mass m_ϕ $\sqrt{3\lambda < \phi^2 >}$ though still small as compared to appears enough for the expansion of the Universe to drive a mode away from the narrow resonance. It happens when the amplitude Φ drops up to a value $\sim m_\phi / \sqrt{\lambda}$.

Dark Matter

From the observation of the cosmic microwave background (CMB), the deuterium abundance in the Universe and supernovae, it suggests that $\Omega_{baryon} \simeq 0.04$ [5] if the current Hubble expansion rate is $h = H_0 / 100 km/\sec/Mpc = 0.7$. Although Ω_{baryon} is much larger than the observed mass in stars, $\Omega_{stars} \simeq 0.005$, it is obviously much smaller than the total energy density in the universe inferred from the observed anisotropy in the CMB.

$$\Omega_{total} \equiv \frac{8\pi G\rho_{total}}{3H^2} = 1.02 \pm 0.02 \tag{12}$$

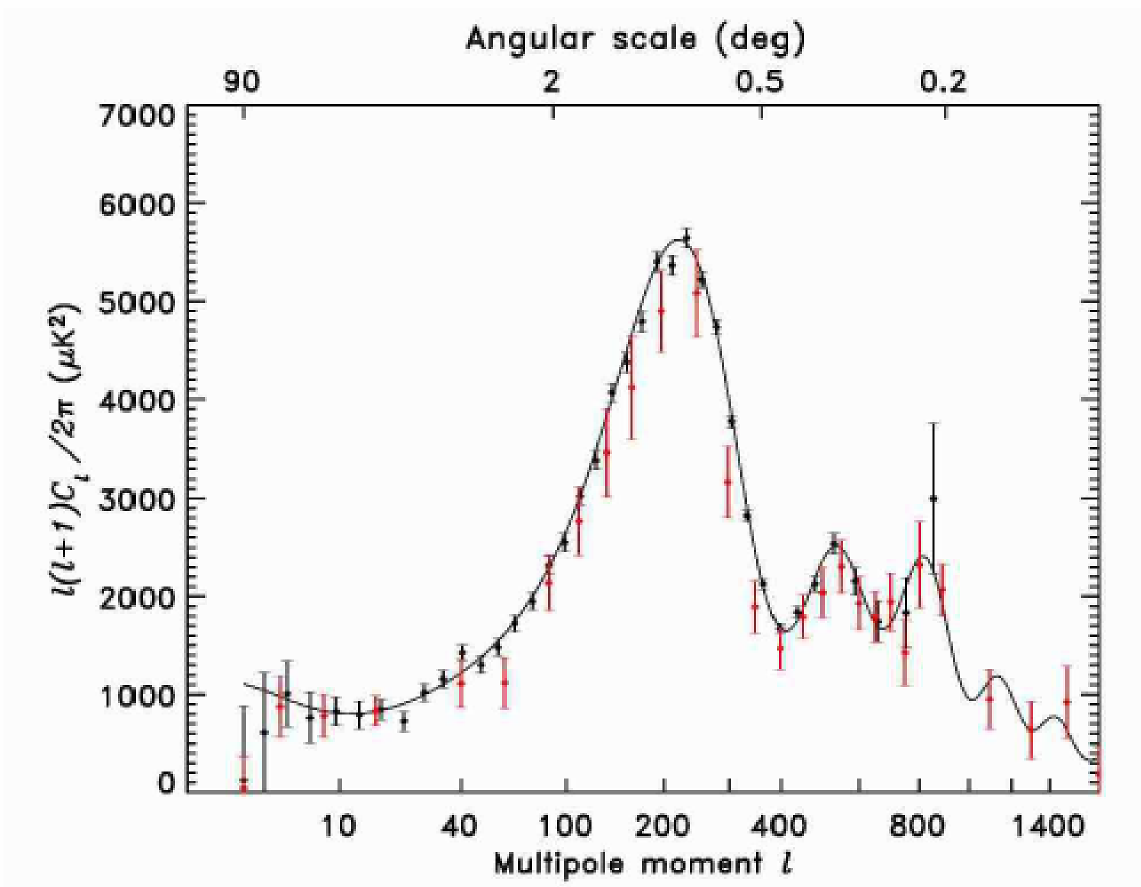

Fig.2 Spectrum of fluctuations in the CMB

Figure 2 compares measurements of CMB fluctuations made before WMAP with the WMAP data themselves.[2] The position of the first acoustic peak represents to flat Universe with $\Omega_{total} \simeq 1$, and two more acoustic peaks gives us a determination of $\Omega_b h^2 = 0.0224 \pm 0.0009$, where h~ 0.7 is the present Hubble expansion rate H, measured in units of 100km/s/Mpc.

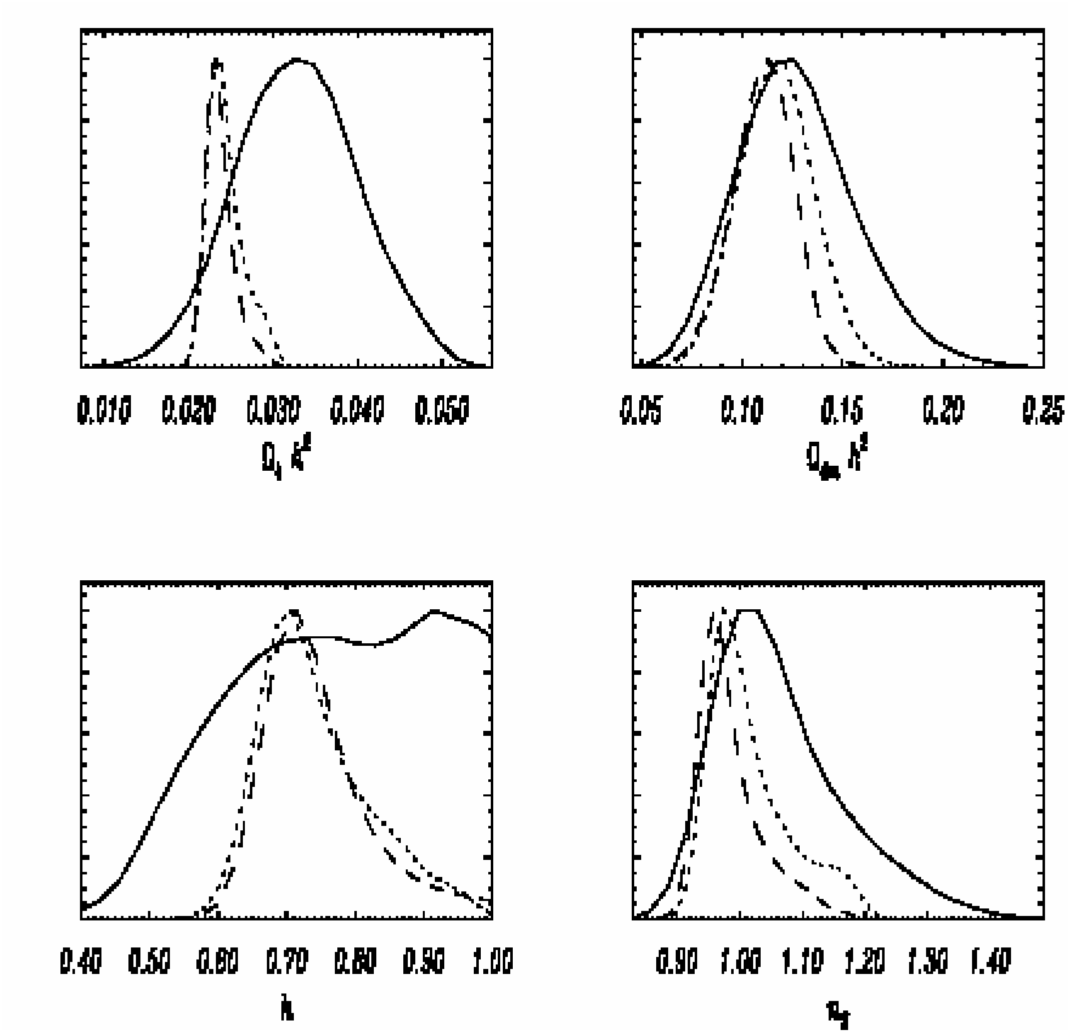

Fig.3 Cosmological Parameters obtained from the WMAP data

Figure 3 shows the functions for various cosmological parameters obtained form the WMAP data analysis. The panels show the baryon density $\Omega_b h^2$, the matter density $\Omega_m h^2$, the Hubble expansion rate h, the strength A, the optical depth τ, the spectral index n_s and its rate of change d n_s /lnk, respectively.[6]

In addition, there is strong evidence to show that

$$\Omega_m \simeq 1/3, \Omega_{DE} \simeq 2/3$$

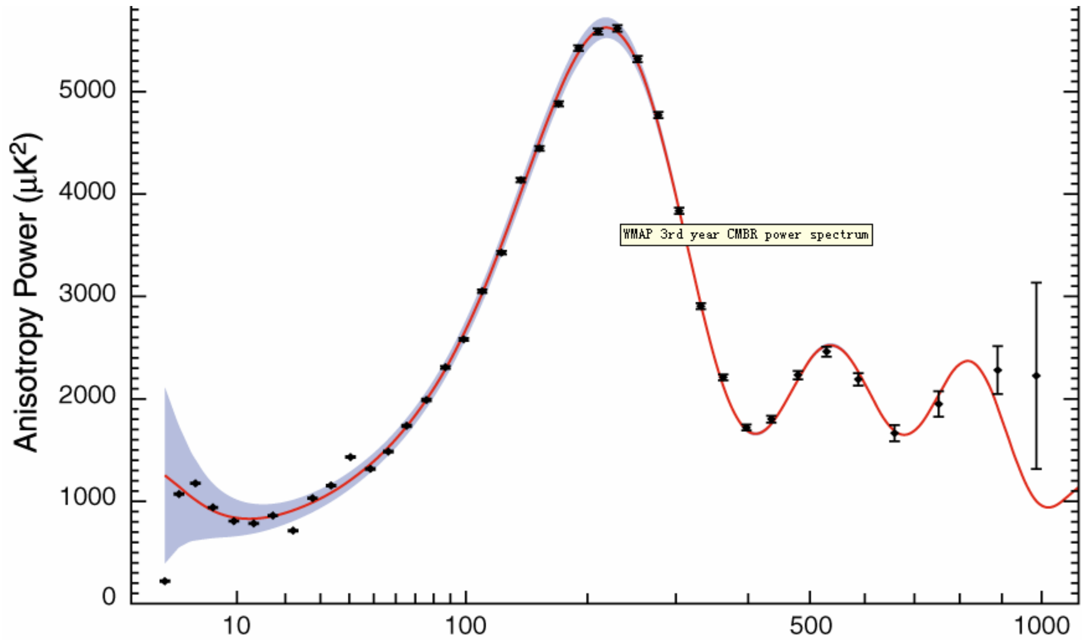

Fig.4 The density of matter Ω_m and dark energy Ω_Λ

As seen in Fig 4, the combination of CMB data with those on high-redshift Type Ia supernovae and on large-scale structure shows a flat Universe with about 30% of matter (dark matter and matter) and 70% of vacuum (dark) energy.

Both dark matter and dark energy are considered essential missing pieces in the cosmic puzzle

$$\Omega_{total} - \Omega_{baryons} = \text{dark world}$$

Though the observational evidence favoring a flat Universe with $\Omega_{total} \simeq 1$ is fairly recent, the nature of the 'unseen' component of the universe (which dominates its mass density), is a long-standing issue in cosmology. Indeed, the need for dark matter was originally pointed out by Zwichy(1933) who realized that the velocity of individual galaxies located within the Coma cluster were too large due to baryons mass, and that this cluster would be gravitationally bound only if its total mass exceeded the sum of the masses of its component galaxies or there is some mass unseeing. For clusters which have relaxed to dynamical equilibrium the mean kinetic and potential energies are related by the theorem

$$K + \frac{U}{2} = 0 \tag{13}$$

where $U \simeq -GM^2 / R$ is the potential energy of a cluster of radius R, $K \simeq 3M \langle v_r^2 \rangle^{\frac{1}{2}}$ is the kinetic energy and $\langle v_r^2 \rangle^{\frac{1}{2}}$ is the dispersion in the line-of-sight velocity of cluster galaxies. (Cluster in the Abell catalogue typically has $R \simeq 1.5h^{-1} Mpc$.) This relation allows us to infer the mean gravitational potential energy if the kinetic energy is accurately known. The mass-to-light ratio in clusters can be as large as $M / L \simeq 300 M_\odot / L_\odot$. However since most of the mass in clusters is in the form of baryons, x-ray emitting intracluster gas, the extent of dark matter in these objects is estimated to be $M / M_{lum} \simeq 20$, where M_{lum} is the total mass in luminous matter including stars and gas.

The Kepler Law says

$$\frac{v^2}{r} = G_N \frac{M(r)}{r^2} \tag{14}$$

The speed v should vary with r liking

$$v = \frac{[G_N M(r)]^{1/2}}{r^{1/2}} \tag{15}$$

Observations of galaxies taken at distances large enough for there to be no luminous galactic component indicate that, instead of declining at the expected rate $v \propto r^{-1/2}$ true if M~ constant, the velocity curves flattened out to v~ constant implying $M(r) \propto r$. This observation suggests that the mass of galaxies continued to grow even when there is no compiled for over 1000 spiral galaxies usually by measuring the 21 cm emission line from neutral hydrogen (HI). The results indicate that $M/L = (10-20)M_{\odot}/L_{\odot}$ in spiral galaxies and in elliptical, while this ratio can increase to $M/L = (200-600)M_{\odot}/L_{\odot}$ in low surface brightness galaxies (LSB's).

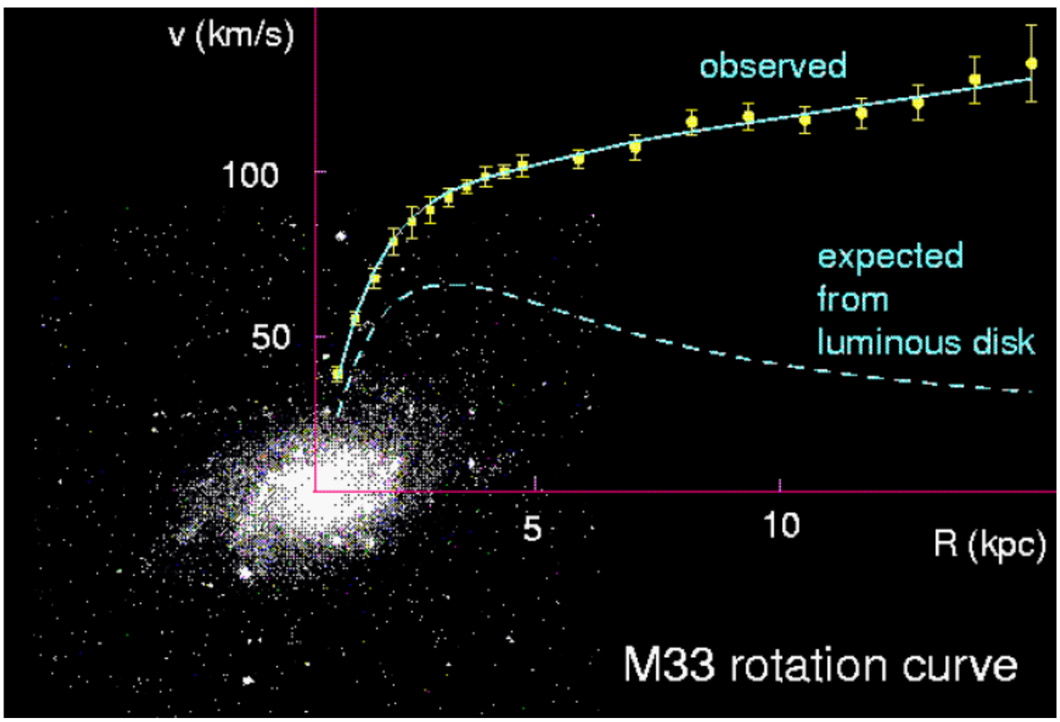

Fig.5 The observed rotation curve of the dwarf spiral galaxy M33

As shown in figure 5, there must be some matter which is non luminous, and this non luminous matter is called dark matter.

A difference between the distribution of dark matter in galaxies and clusters needs to be talked carefully: in galaxies, dark matter appears to increase with distance, while in clusters, the dark matter distribution actually decreases with distance. For example, the rotation curve of dwarfs(such as DD0154) has been measured to almost 15 optical length scales indicating that the dark matter surrounding this object is extremely spread out. On the other hand, acts as a gravitational leans which focuses the light from background objects such as galaxies and QSO's thereby allowing us to determine the depth of the cluster potential well. Observations of strongly concentrated in central regions with a projected mass of $10^{13}-10^{14}M_{\odot}$ being contained within 0.2-0.3 Mpc of the central region

We have discussed that only about 4% of the cosmic density is baryonic, and this means that the dark matter which we care observing must be non-baryonic in origin.(at most the weak interaction is allowed) The need for non-baryonic forms of dark matter also has the indirect support from small initial conditions and hence to reconcile the existence of a well developed comic web of filaments and clusters at the present epoch with the exceedingly small amplitude of density perturbations($\delta\rho/\rho \sim 10^{-5}$ at z~ 1,100) inferred from COBE measurements and more recent CMB experiments. Indeed, it is well known that, linearized density perturbations in a spatially flat matter dominated universe grows at the rate $\delta \propto t^{2/3} \propto (1+z)^{-1}$, where $(1+z)=a_0/a(t)$ is the cosmological red shift. In a baryonic universe, the large radiation pressure ensures that density perturbations in the baryonic component can begin growing only after hydrogen recombines at z~1,100 at which point of time baryons and photons decouple. Requiring $\delta > 1$ today implies

$\delta > 10^{-3}$ at recombination, which contradicts Cosmic Microwave Background observations by over an order of magnitude. In non-baryonic models on the other hand, the absence of any significant coupling between dark matter and radiation allows structure to grow much earlier, significantly before hydrogen n the universe has recombined. After recombination baryons simply fall into the potential wells created for them by the dominant non-baryonic component. As a result, a universe with a substantial non-baryonic dark matter can give rise to the structure which we see today form smaller initial fluctuations.

Here we discuss the two candidate of dark matter.

Neutrinos

Massive neutrinos can be good candidate for hot dark matter.

We have the following direct experimental upper limits on neutrino massed. From measurements of the end-point in Tritium β decay, we know that

$$m_{\nu e} \leq 2.5 eV$$

And there are prospects to improve this limit down to about 0.5eV with the proposed KATRIN experiment. From the measurements of $\pi \to \mu\nu$ decay, we know that

$$m_{\nu_\mu} < 190 KeV$$

And there are prospects to improve this limit by a factor ~ 20. From measurements of $\tau \to n\pi\nu$ decay, we know that

$$m_{\nu_\tau} < 18.2 MeV$$

However, the most stringent laboratory limit on neutrino masses may come from searches for neutrinoless double-β decay, which constrain the sum of the neutrinos' majorana masses weighted by their couplings to electrons

$$\langle m_{\nu} \rangle_{e} \equiv \left| \sum_{\nu i} m_{\nu i} U_{ei}^{2} \right| \leq 0.35 eV \tag{16}$$

The pioneering Super-Kamiokande and other experiments have shown that atmospheric neutrinos oscillate, with the following difference in squared masses and mixing

$$\delta m^{2} \simeq 2.4 \times 10^{-3} eV^{2}, \sin^{2} 2\theta_{23} \simeq 1.0 \tag{17}$$

which is very consistent with the K2K reactor neutrino experiment, as shown the left panel of Figure 6. A flurry of recent solar neutrino experiments, most notably SNO, have established beyond any doubt that they also oscillate, with

$$\delta m^{2} \simeq 6 \times 10^{-5} eV^{2}, \tan^{2} \theta_{31} \simeq 0.5 \tag{18}$$

Most recently, the KamLAND experiment has reported a defict of electron antineutrinos from nuclear power reactors, leading to a very similar set of preferred parameters, as seen in the upper panel of Figure 6.

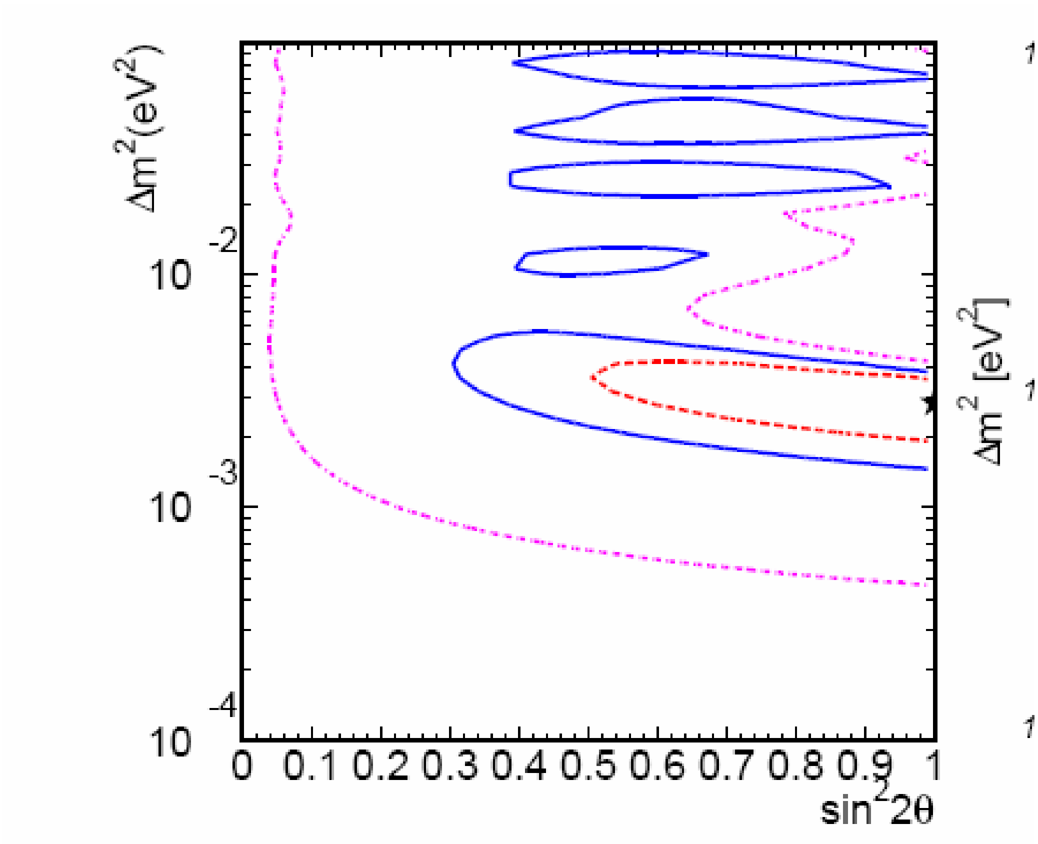

Fig.6 Parameter region for massive neutrino

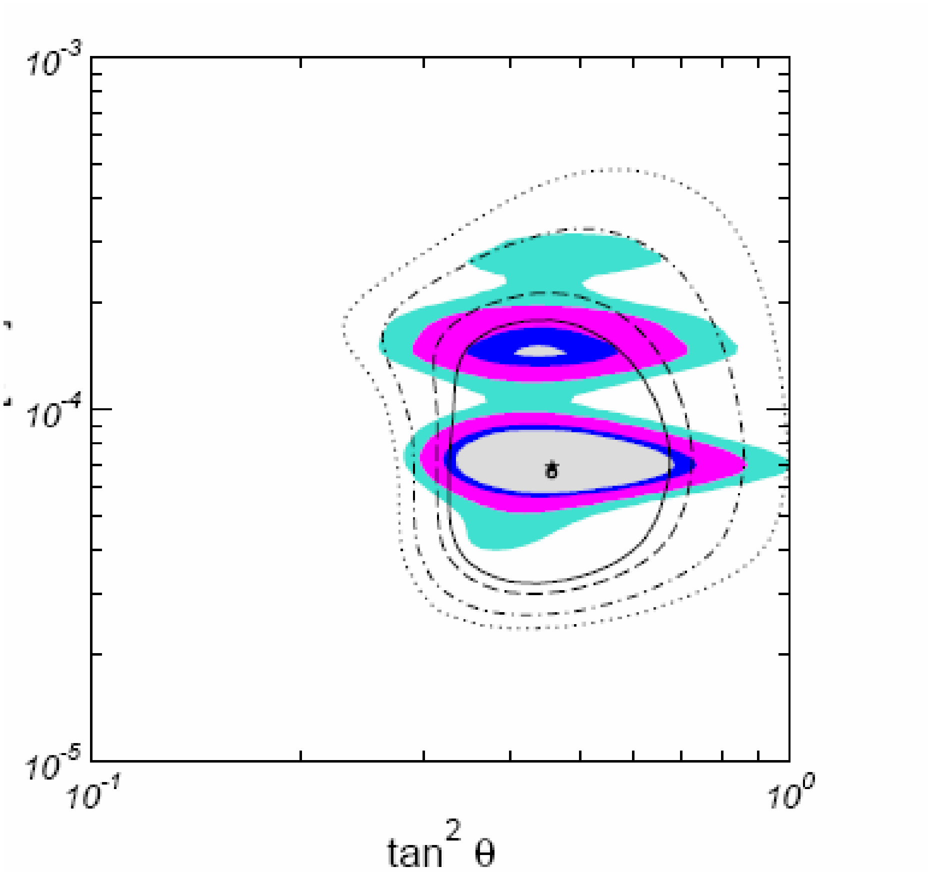

Fig.6 Continued

This figure shows the parameter region for massive neutrino, the right panel inferred from the K2K reactor experiment, and the left panel inferred from the solar-neutrino experiment.

Using the range of θ_{12} allowed by the solar and KamLAND data, one can establish a correlation between the relic neutrino density $\Omega_\nu h^2$ and the neutrinoless double-β decay observable $\langle m_\nu \rangle_e$, as seen in Fig 7. Pre-WMAP, the experimental limit on $\langle m_\nu \rangle_e$ could be used to set the bound

$$10^{-3} \le \Omega_\nu h^2 \le 10^{-1} \qquad (19)$$

Supersymmetric Particle as Dark Matter

When considering the experimental, cosmological and theoretical constraints on the MSSM, it is common to assume that all the unseen spin-0 supersymmetric particles have some universal mass m_0 at some GUT input scale, and similarly for the unseen fermion masses $m_{1/2}$. There are two parameters of this constrained MSSM are restricted by the absences of supersymmetric particles at LEP: $m_{\chi^{\pm}} \ge 103GeV$, $m_{\bar{e}} \ge 99GeV$, and at the Fermilab Tevatron collider.

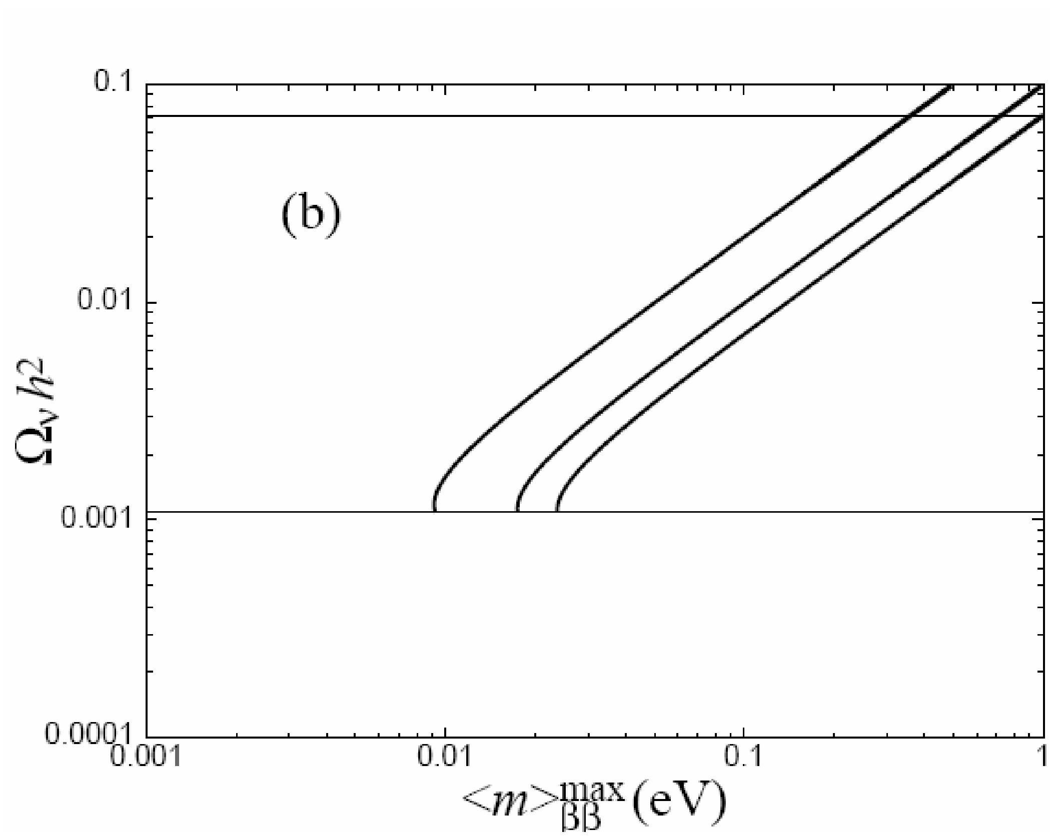

Fig.7 Correlation between $\Omega_\nu h$ and $\langle m_\nu \rangle_e$

They are also restricted indirectly by the absence of a Higgs boson at LEP:

$m_h > 114.4 GeV$, and by the fact that $b \to s\gamma$ decay is consistent with the Standard Model, and potentially by the BNL measurement of $g_\mu - 2$, as seen in Fig 7.（Ellis et al.,2003a; Lahanas & Nanopoulos 2003）

As shown there, the MSSM parameter space is also restricted by cosmological bounds on the amount of cold dark matter, $\Omega_{CDM} h^2$. R-parity indicates that the supersymmetric particle cannot decay into Standard Model particles, so the lightest supersymmetric particle, which cannot decay into Standard Model particles and other supersymmetric particles since it is the lightest one, will be the candidate of Cold Dark Matter. One of these candidates is neotrilino. Since neotrilino can interact with other supersymmetric particles by weak interaction, and its mass is around 100 GeV, it becomes most promising candidate of Cold Dark Matter. Figure 8 shows if neotrilino is the Cold Dark Matter, how it is constraint the parameter space – the netrilino has to be the lightest supersymmetric particle.

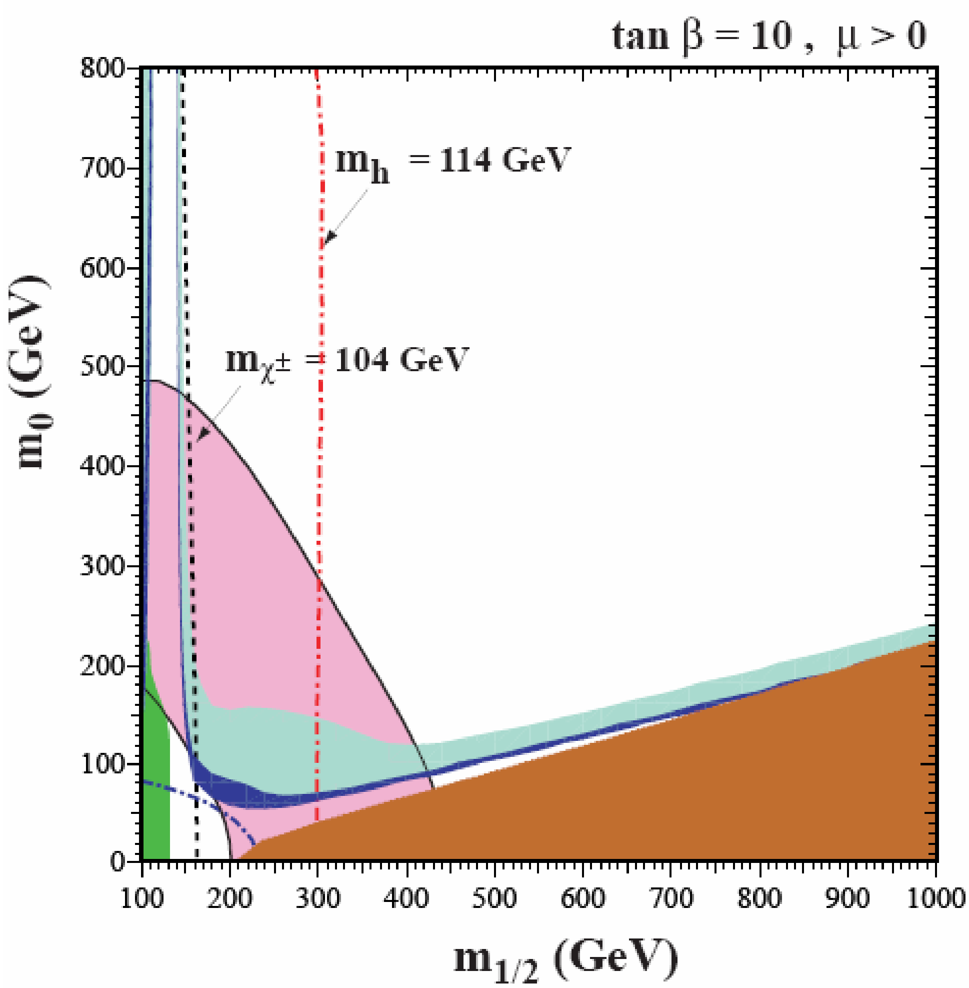

Fig.8 The ($m_{1/2}, m_0$) planes for the mass of Higgs

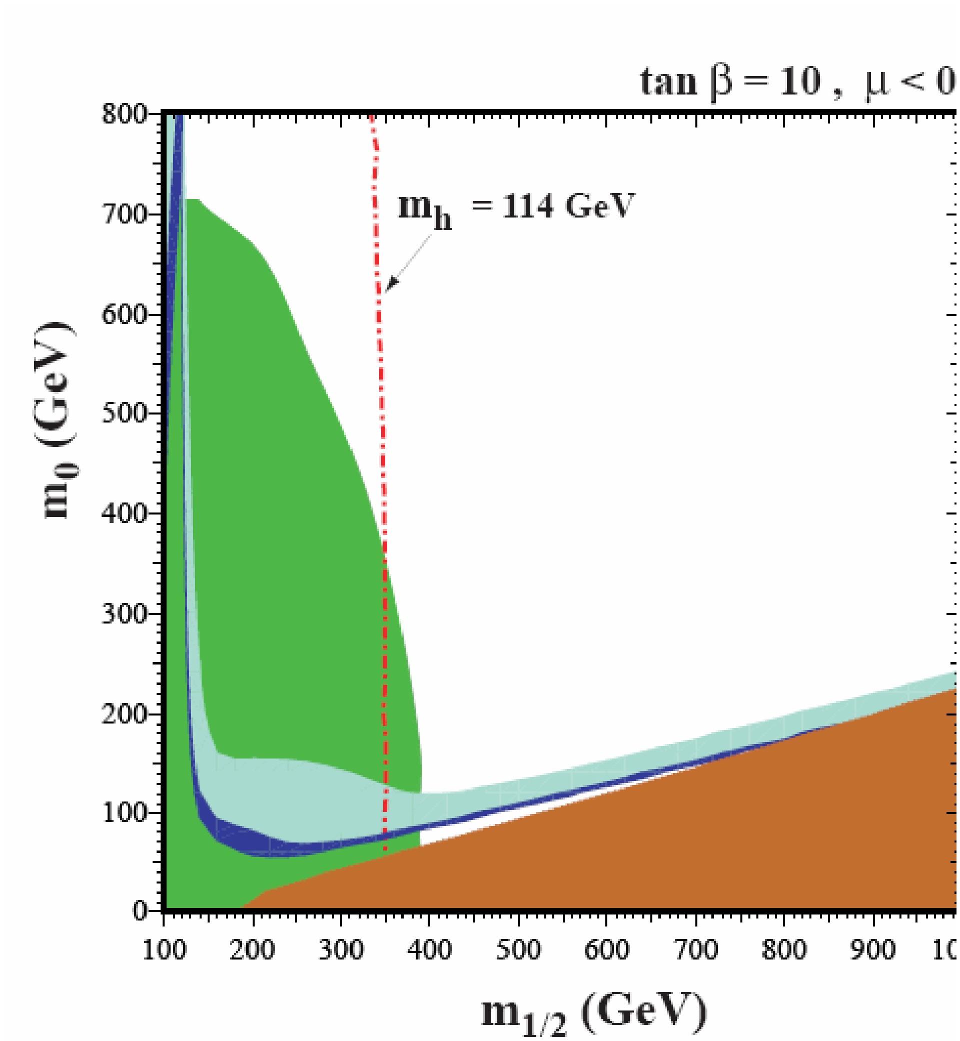

Fig.8 Continued

Figure 8 The ($m_{1/2}, m_0$) planes show (left panel) $\tan\beta = 10, \mu > 0$, and (right panel) $\tan\beta = 10, \mu < 0$. This figure shows allowed region for the mass of Higgs from the constraint of cosmology, g-2 and $b \to s\gamma$.

Dark Energy

Type Ia supernovae, when treated as standardized candles, suggest that the expansion of the universe is speeding up rather than slowing down. The can for an accelerating universe also receives independent support form CMB and large scale structures. The simplest way to explain this accelerating expansion is by introducing a cosmological constant since we are free to input a constant term in Einstein equation. However, there is key problem that we have to explain. The value of energy density stored in the cosmological constant today, this value has to be of order the critical density, namely $\rho_\Lambda \sim 10^{-3} eV^4$ [7].Unfortunately, no sensible explanation exists as to why a true cosmological constant should be at this scale, it should naturally be much larger. Such as in Standard Model, the vacuum value of Higgs field should be around 100 GeV. Typically, since it is conventionally associated with the energy of the vacuum in quantum theory we expect it to have a size of order the typical scale of early Universe phase transitions. Even at the QCD scale it would imply a value $\rho_\Lambda \sim 10^{-3} GeV^4$.

There are some alternative ways to have an accelerating expansion universe. This includes: Quintessence models which invoke an evolving canonical scalar field with a potential and make use of the scaling properties and tracker nature of such scalar fields evolving in the presence of other background matter fields; scalar field models where the small mass of the quintessence field is protected by an approximate global symmetry by making the field a pseddo-Nambu-Goldstone boson; Chameleon fields in which the scalar field couples to the baryon energy density and is homogeneous being allowed to vary across space from solar system to cosmological scales; a scalar field with a non-canonical kinetic tern, know as K-essence based on earlier work of K-inflation; modified gravity arising out of both string motivated or more generally General Relativity modified actions which both have the effect of introducing large length scale corrections and modifying the late time evolution of the universe; the feedback of non-linearities into the evolution equations which can significantly change the background evolution and lead to acceleration at late times without introducing any new matter; Chaplygin gases which attempt unify dark energy and dark matter under one field by allowing for a fluid with an equation of state which evolves between two; tachyons arising in string theory; the same scalar field responsible for both inflation in the early Universe and again today, known as Quintessential inflation; the possibility of a network of frustrated topological defects forcing the universe into a period of accelerated expansion today; Phantom Dark Energy and Ghost Condensates in string theory; the String Landscape arising from the multiple numbers of vacua that exist when the string moduli are make stable as non-abelian fluxes are turned on; Cyclic universe; collision of two D-bran.[8]

Observational Evidence for Dark Energy

In 1998 the accelerated expansion of the universe was pointed out by two groups from the observations of Type Ia supernovae. We use a redshift to describe the evolution of the universe. This is related to the fact that light emitted by a stellar object becomes red-shifted due to the expansion of the universe. The wavelength λ increases proportionally to the scale factor a, whose effect can be quantified by the redshift z, as

$$1+z=\frac{\lambda_0}{\lambda}=\frac{a_0}{a} \tag{20}$$

where the subscript zero denotes the quantities given at the present epoch.(at today a=1).

In Minkowski space time the absolute luminosity Ls of the source and the energy flux F at a distance d is related through

$$F=Ls/(4\pi d^2) \tag{21}$$

By generalizing this to an expanding universe, the luminosity distance, d_L, is defined by

$$d_L{}^2\equiv\frac{Ls}{4\pi F} \tag{22}$$

The Huble parameter takes the convenient form

$$H^2=H_0{}^2\sum_i\Omega_i^{(0)}(1+z)^{3(1+w_i)} \tag{23}$$

where $\Omega_i^{(0)}\equiv 8\pi G\rho_i^{(0)}/(3H_0^2)=\rho_i^{(0)}/\rho_c^{(0)}$ is the density parameter for an individual component at the present epoch. Hence the luminosity distance in a flat geometry is given by

$$d_L=\frac{1+z}{H_0}\int_0^z\frac{dz'}{\sqrt{\sum_i\Omega_i^{(0)}(1+z')^{3(1+w_i)}}} \tag{24}$$

In the figure below, we plot the luminosity distance for a two component flat universe satisfying $\Omega_m^0+\Omega_\Lambda^0=1$. Notice that $d_L\simeq z/H_0$ for small values of z. The luminosity distance becomes larger when the cosmological constant present.

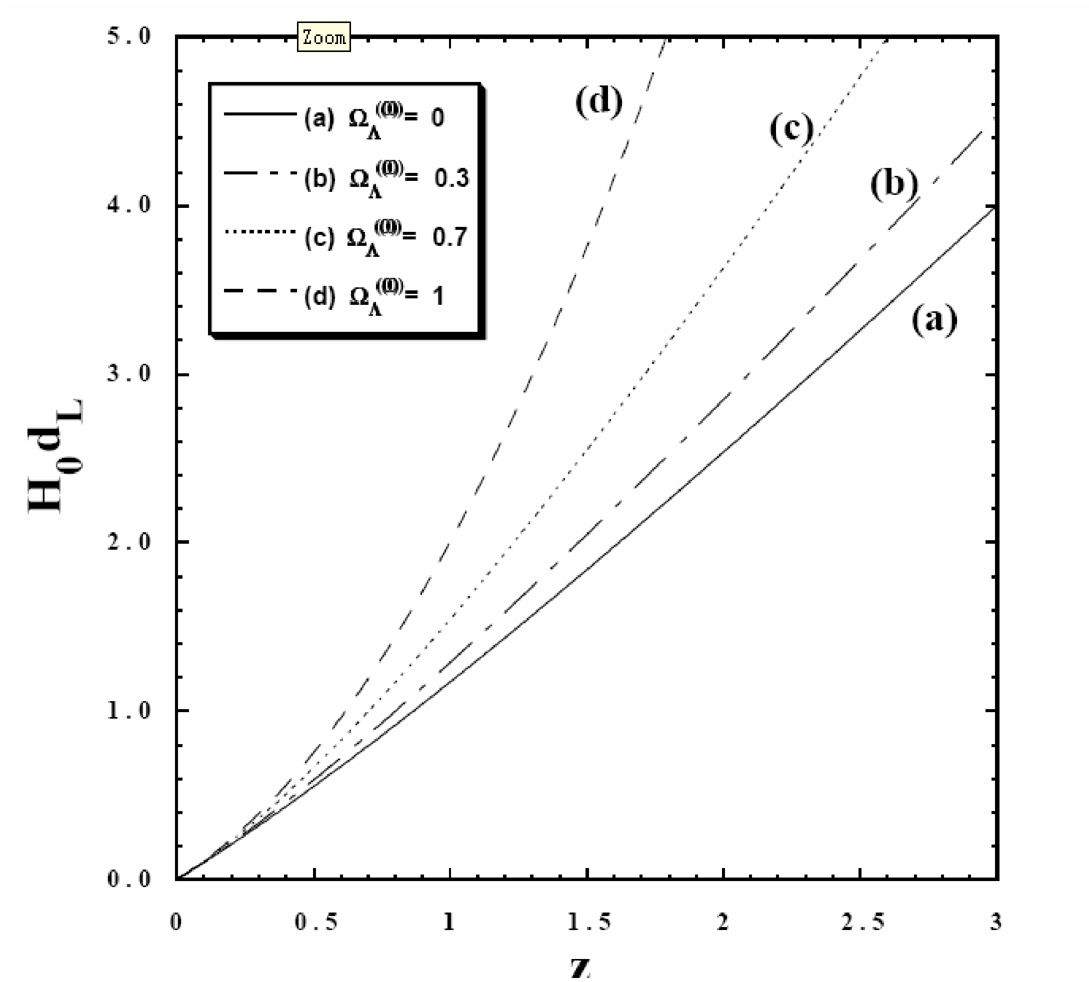

Fig.9 Luminosity distance d_L

Figure 9 shows luminosity distance d_L in the units of H_0^{-1} for a two component flat universe with a non relativistic fluid ($w_m = 0$) and a cosmological constant ($w_\Lambda = -1$).

The direct evidence for the current acceleration of the universe is related to the observation of luminosity distances of high redshift supernovae. And the accelerated expansion of universe can be described by the dark energy. The observations related to the CMB and large scale structure independently support the ideas of a dark energy dominated universe. The CMB anisotropies observed by COBE in 1992 and by WMAP in 2003 exhibited a nearly scale-invariant spectra of primordial perturbations, which agree very well with the prediction of inflationary cosmology.

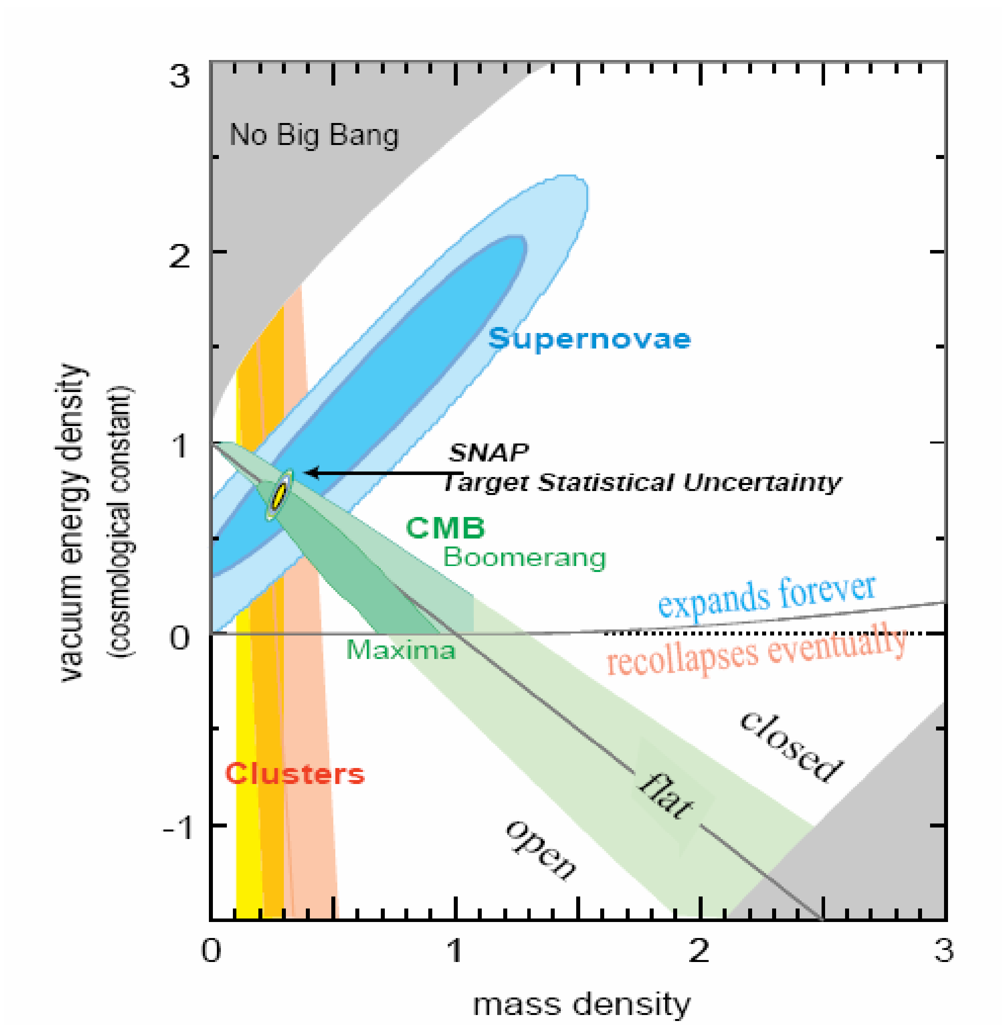

Fig.10 Confidence regions constrained form CMB and large-scale galaxy clustering

In Figure 10, we plot the confidence regions coming from SN Ia, CMB and large-scale galaxy clustering. Clearly the flat universe without a cosmological constant is ruled out. The compilation of three different cosmological data sets strongly reinforces the need for a dark energy dominated universe with $\Omega_{\Lambda}^{0} \simeq 0.7$ and $\Omega_{m}^{0} \simeq 0.3$. Amongst the matter content of the universe, baryonic matter amounts to only 4%. The rest of the matter (23%) is believed to be non-baryonic which is called dark matter. Dark energy is about 73% with its equation of state (w<-1/3).

Cosmological Constant

The Einstein tensor $G^{\mu\nu}$ and the energy momentum tensor $T^{\mu\nu}$ satisfy the Bianchi identities $\nabla_{\nu} G^{\mu\nu} = 0$ and energy momentum conservation $\nabla_{\nu} T^{\mu\nu} = 0$. But since the metric $g^{\mu\nu}$ is constant with respect to covariant derivatives ($\nabla_{\nu} g^{\mu\nu} = 0$), we have the freedom to add a term $\Lambda g^{\mu\nu}$ in Einstein equations where Λ is constant. Then the Einstein equation becomes

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{25}$$

Or we can write it as

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) \tag{26}$$

In the FRW background given by FRW metric

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{27}$$

the modified Einstein equation give

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \tag{28}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{29}$$

If the cosmological constant originates form a vacuum energy density, then this suffers form a severe fine-tuning problem. Observationally we know that Λ is of order the present value of the Hubble parameter H_0, that is

$$\Lambda \approx H_0^{\ 2} = (2.13h \times 10^{-42} GeV)^2 \tag{30}$$

This corresponds to a critical density ρ_Λ,

$$\rho_\Lambda = \frac{\Lambda m_{pl}^2}{8\pi} \approx 10^{-47} GeV^4 \tag{31}$$

Meanwhile the vacuum energy density evaluated by the sum of zero-point energies of quantum fields with mass m is given by

$$\rho_{VAC} = \frac{1}{2}\int_0^\infty \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + m^2} \tag{32}$$

This exhibits an ultraviolet divergence: $\rho_{VAC} \propto k^4$. However we expect that quantum field theory is valid up to some cut-off scale $k_{\max}$, so

$$\rho_{VAC} \approx \frac{k_{\max}^4}{16\pi^2} \tag{33}$$

For the extreme case of General Relativity we expect it to be valid to just below the Plank scale: $m_{pl} = 1.22 \times 10^{19} GeV$. Hence if we treat $k_{\max} = m_{pl}$, we can see that the vacuum energy density will be

$$\rho_{VAC} \approx 10^{74} GeV^4$$

which is about 10^{121} orders of magnitude larger than the observed value. If we take an energy scale of QCD for $k_{\max}$, we get $\rho_{VAC} \approx 10^{-3} GeV^4$ which is also much larger than ρ_Λ.

The cosmological constant problem has led to try a different way to the dark energy issue. Many have investigated the possibility that the dark energy is caused by the dynamics of a light scalar field, or a few scalar field.

Dark Energy and Reheating

The cosmological constant corresponds to a field with a constant equation of state w= -1. Now, the observations which constrain the value of w today to be close to that of the cosmological constant, but it is doesn’t say something about how w vary with time. Scalar field which evolve with time, it’s equation of state will change with time, that is w will change with time.

Quintessence is described by ordinary scalar field ϕ. The action for Quintessence is given by

$$S = \int d^4x\sqrt{-g}[-\frac{1}{2}(\nabla\phi)^2 - V(\phi)] \tag{34}$$

$$(\nabla\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \tag{35}$$

and $V(\phi)$ is the potential of the field. In a flat FRW spacetime the variation of the action with respect to ϕ gives

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{36}$$

The energy momentum tensor of the field is derived by varying the action in terms of $g^{\mu\nu}$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \tag{37}$$

Taking note that $\delta\sqrt{-g} = -(1/2)\sqrt{-g}\,g_{\mu\nu}\delta g^{\mu\nu}$, we can get that

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)] \tag{38}$$

In the flat Friedmann background we obtain the energy density and pressure density of the scalar field:

$$\begin{aligned} T_{00} &= \rho_\phi \\ T_{ij} &= a^2 P_\phi \delta_{ij} \end{aligned} \tag{39}$$

where

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ P_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \tag{40}$$

The equation of motion for the scalar field is then

$$H^2 = \frac{8\pi G}{3}[\frac{1}{2}\dot{\phi}^2 + V(\phi)] \tag{41}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}[\dot{\phi}^2 - V(\phi)] \tag{42}$$

Here we study cosmological dynamics of a scalar field ϕ in the presence of a barotropic fluid; then the equations become:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{43}$$

with
$$H^2 = \frac{1}{3}(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_n)$$
$$\dot{\rho}_n + nH\rho_n = 0$$

where ρ n is the energy density in radiation (n=4) or non relativistic matter (n=3), H=å/a is the Hubble expansion rate of the universe, dots are derivative with respect to time, primes are derivative with respect to the field Φ.

Now consider potential of the form

$$V(\phi) = V_0 e^{-\lambda\phi} \qquad (44)$$

Models with this kind of potential have been studied in [9]. The special features of quintessence field ϕ with this potential is the field with approach a "scaling " solution, independent of initial conditions. Consider $\Omega_\phi = \rho_\phi / \rho_c$, after an initial transient, the quintessence field will take a fixed value $\Omega_\phi = n/\lambda^2$ only depend on λ and n. We can see this special feature of models with potential (44) in figure 1, where a is scale factor of universe. [a(today)=1].

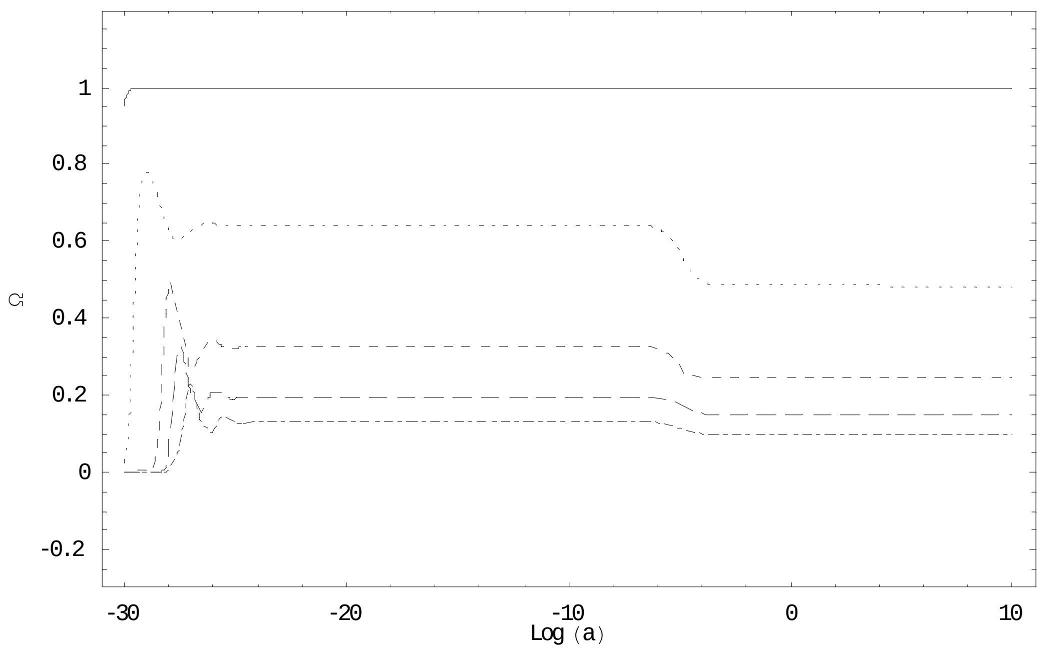

Fig.11 $\Omega_Q(a)$ for different λ in the $V(\phi) = V_0 e^{-\lambda\phi}$

Figure 11 shows the solution for the potential $V(\phi) = V_0 e^{-\lambda\phi}$.

Models with potential (7) can achieve the condition $\Omega_\phi \sim \Omega_{other} = (\rho_c - \rho_\phi)/\rho_c$ naturally through the scaling behavior, but there is a problem that within these models that no choice of λ can fit all constraint. If we choose λ small that can make universe accelerated expansion today, then $\Omega_\phi = n/\lambda^2$ is too large to satisfy the constraint of nucleosynthesis[10].

The tightest constraint on the energy density of dark energy during a radiation dominated ear comes from primarily nucleosynthesis. The introduction of an extra degree of freedom like a light scalar field affects the abundance of light elements in the radiation dominated epoch. The presence of a quintessence scalar field changes the expansion rate of the universe at a given temperature. This effect becomes crucial at the nucleosynthesis epoch with temperature around 1 MeV when the weak interactions (which keep neutrons and protons in equilibrium) freeze-out.

The observationally allowed range of the expansion rate at this temperature leads to a bound on the energy density of the scalar field

The observationally allowed range of the expansion rate at this temperature leads to a bound on the energy density of the scalar field

$$\Omega(\phi)(T \sim 1MeV) < \frac{7\Delta N_{eff}/4}{10.75 + 7\Delta N_{eff}/4} \tag{45}$$

where 10.75 is the effective number of standard model degrees of freedom and ΔN_{eff} is the additional relativistic degrees of freedom used in the literature is $\Delta N_{eff} \simeq 1.5$, whereas a typical one is given by $\Delta N_{eff} \simeq 0.9$. Taking a conservative one, we obtain the following bound

$$\Omega_\phi(T \sim 1MeV) < 0.2 \tag{46}$$

Any quintessence models need to satisfy this constraint at the epoch of nucleosythesis.

The exponential potential $V(\phi) = V_0 e^{-\lambda\phi}$ possesses the following two attractor solutions in the presence of a background fluid:

(1) $\lambda^2 > 3\gamma$: the scalar field mimics the evolution of the barotropic fluid with $\gamma_\phi = \gamma$, and the relation $\Omega_\phi = 3\gamma / \lambda^2$ holds.

(2) $\lambda^2 < 3\gamma$: the late time attractors is the scalar field dominated solution ($\Omega_\phi = 1$) with $\gamma_\phi = \lambda^2 / 3$.

The case (1) corresponds to a scaling solution in which the field density mimics that of the background during radiation or matter dominated era, thus alleviating the problem of a cosmological constant. If this scaling solution exists by the epoch of nucleosynthesis ($\gamma = 4/3$), the constrain gives

$$\Omega_\phi = \frac{4}{\lambda^2} < 0.2 \rightarrow \lambda^2 > 20 \tag{47}$$

In this case, however, one can not have an accelerated expansion at late times.

To solve this problem, Andreas Albrecht and Contantinos Skordis Constructed AS model[3]. They considered the potential of the form

$$V(\phi) = V_p(\phi) e^{-\lambda\phi} \tag{48}$$

$V_P(\phi)$ is a polynomial. In AS model, they choose $V_P(\phi)$ as a simple form

$$V_p(\phi) = (\phi - B)^\alpha + A \tag{49}$$

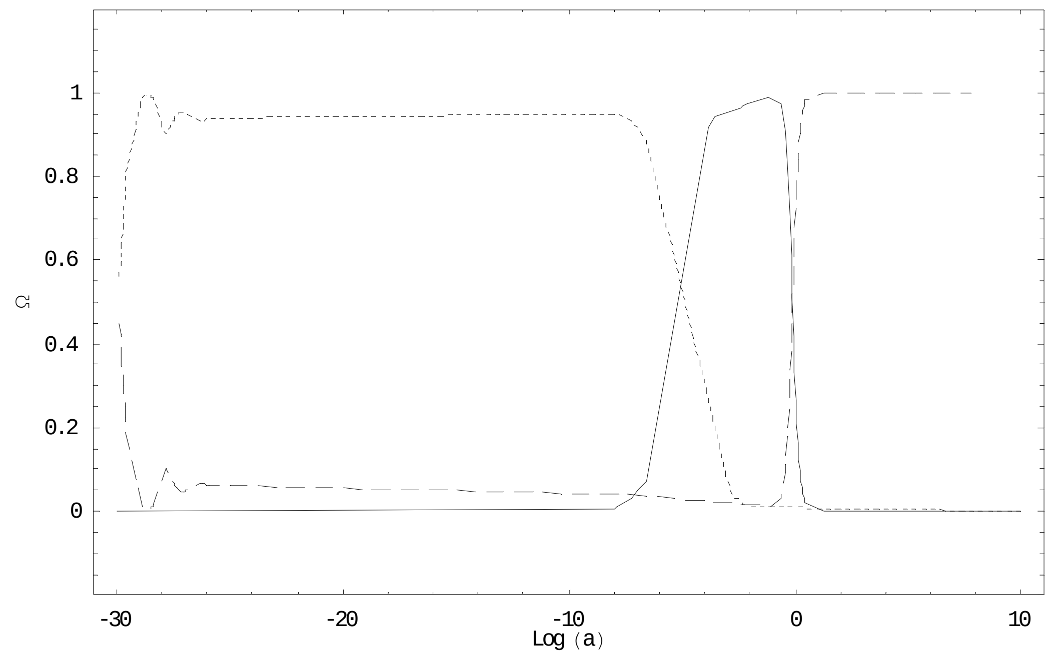

Fig.12 A solution for potential $V(\phi) = V_p(\phi)e^{-\lambda\phi}$, part (a)

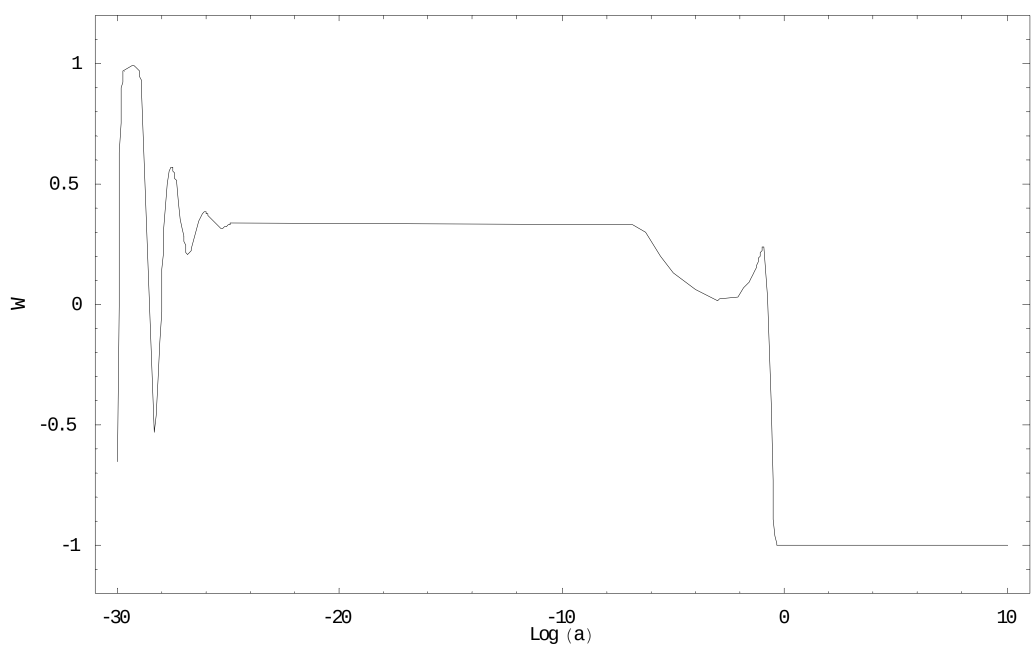

Fig.13 A solution for potential $V(\phi) = V_p(\phi)e^{-\lambda\phi}$, part (b)

Figure 12 and figure 13 show a solution with B=34.8, α =2, A=0.01 and λ =8. In this solution, Ω_ϕ is below the nucleosynthesis bound and the university is accelerating today.

AS model keeps the scaling behavior which can achieve $\Omega_\phi \sim \Omega_{other}$ naturally, accelerates universe today and is consistent with nuclesynthesis bound, but it cannot include the very important stage of universe-"reheating". We find if we choose the polynomial function $V_p(\phi)$ in another way, in the new model, can have AS model's feature and include the reheating stage.

M-theory predicts field with potential of the form

$$V(\phi) = V_p(\phi)e^{-\lambda\phi} \tag{50}$$

Here, instead of taking $V_p(\phi) = (\phi - B)^\alpha + A$, we choose

$$V_p(\phi) = ((\phi - B_1)^2 + A_1)((\phi - B_2)^2 + A_2) \tag{51}$$

Different from AS model which has one minimum, in this model with potential (51), we have two minimum at Φ~B1 and Φ~B2, as shown in figure 3, and reheating can happen at the first minimum Φ ~B1 if quintessence field can interact with particle with small coupling constant.[11]

According to the theory of rehearing, when a scalar field ϕ oscillated near the minimum of its' effective potential and interacts with particle field, parts of its field energy will decay into massive bosoms and fermions due to elementary theory and parametric resonance.

The first minimum of (51) with $V_P(\phi)$ as (5) is $\phi \sim B_1$, and we expand V_ϕ around

$\phi \sim B_1$ and only take the first two terms, we get

$$V_\phi = V_0 + \frac{1}{2} m^2 (\phi - B_1) = V_0 + \frac{1}{2} m^2 \varphi^2 \tag{52}$$

$$L_{\text{int}} = -f \varphi \bar{\psi} \psi - (\sigma\varphi + h\varphi^2)\chi^2 \tag{53}$$

Where $\bar{\psi}$ and ψ describe spinor particles and χ describes scalar particles with corresponding mass m_ψ and m_χ, f and h are dimensionless coupling constant, and σ is a coupling constant of dimension of mass. To potential like (1) and (2), the production of particles by the oscillating scalar field ϕ due to elementary theory and parametric resonance has been given [12]. But we cannot get exactly how much scalar field becomes particle. In fact, because of special feature of "scaling" of exponential potential, Ω_ϕ will take a fixed number $\Omega_\phi = n/\lambda^2$ which is independent of the initial condition.

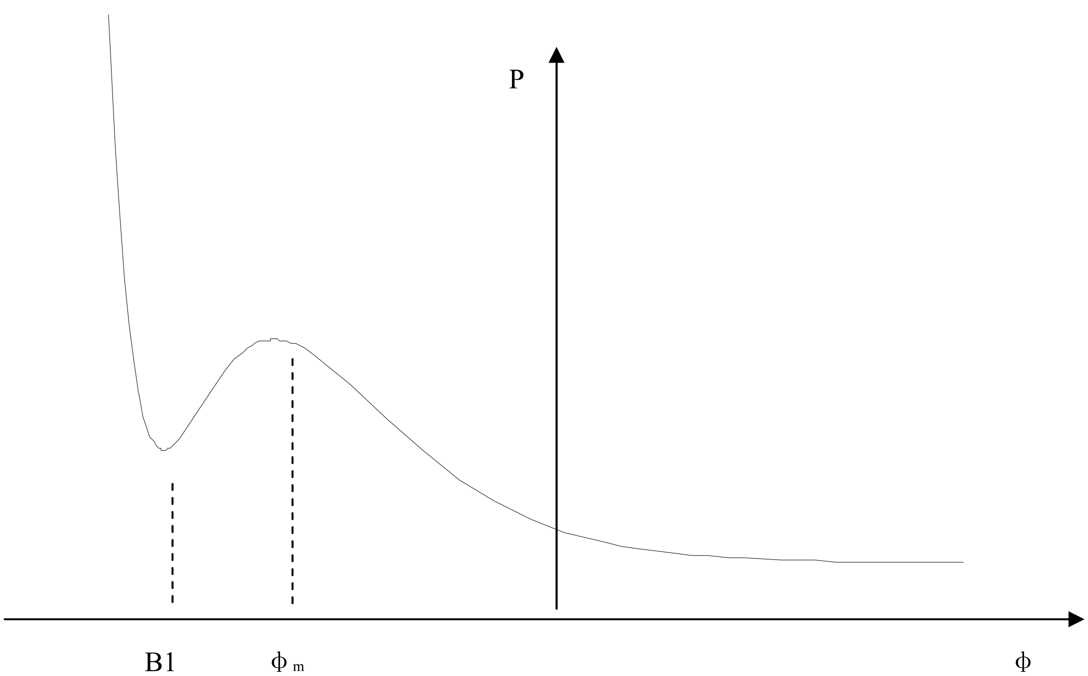

Fig.14 Potential $V(\phi) = V_p(\phi)e^{-\lambda\phi}$ with $V_p(\phi) = ((\phi - B_1)^2 + A_1)((\phi - B_2)^2 + A_2)$

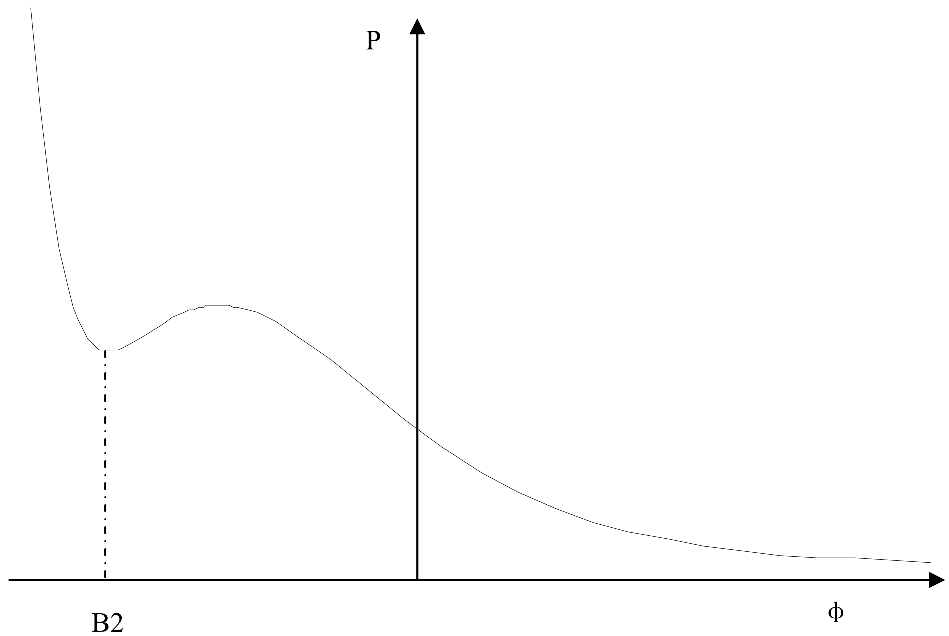

Fig.14 Continued

Figure 14 shows the two minimum in our model.

After reheating, a few percent of matter produced, the ϕ field will penetrate the potential barrier as shown in figure 3 at point ϕ_m due to quantum tunneling [13]. The tunneling rate Γ per volume element V in the semi classical approximation is

$$\Gamma / V = Ae^{-B} \tag{54}$$

where $B = S_E(\phi_{cl}) - S_E(\phi_{cl+})$ with S_E being the action

$$S = \int dx^4 \sqrt{-g}[\frac{1}{2} g^{\mu\nu}\partial_\mu \partial_\nu \phi - V(\phi) - \frac{R}{2}] \tag{55}$$

To AS model with B=34.8, α =2, $\lambda = 8$ and $\delta = 0.01$ in (48) (the reason that AS choose this number is to achieve the accelerated expansion of universe today, we can change one of the numbers above, but once we did that, we have to change the others to achieve that 70% of total energy is dark energy which is consistent with observations), the minimum is as $\phi \sim 34.8$, B in (54) is a bout 10^{123}, so the probability for a tunneling event per unit time, per unit volume is $\Gamma / V \approx \exp(-10^{123})$ which is negligible[14]. But to the potential (50) with V_P as (51), if we choose $B_1 \leq 1$, then B~0(1) in plank unit and the tunneling rate is significant. For a variety of values for A_1, A_2, B_1, B_2 and λ, we can get solutions like the one shown in figure 4 in which reheating happened at $a \sim 10^{-30}$, Ω_ϕ is well below the nucleosythesis bound, the universe is accelerating today. In upper panel of figure 4, we show $\Omega_\phi(a)$ (dashed line), $\Omega_{matter}(a)$ (solid line), $\Omega_{radiation}(a)$ (dotted line), and in lower panel, we show $w_\phi(a)$. At a~0, w equals to (-1) which is needed to archive acceleration universe today.

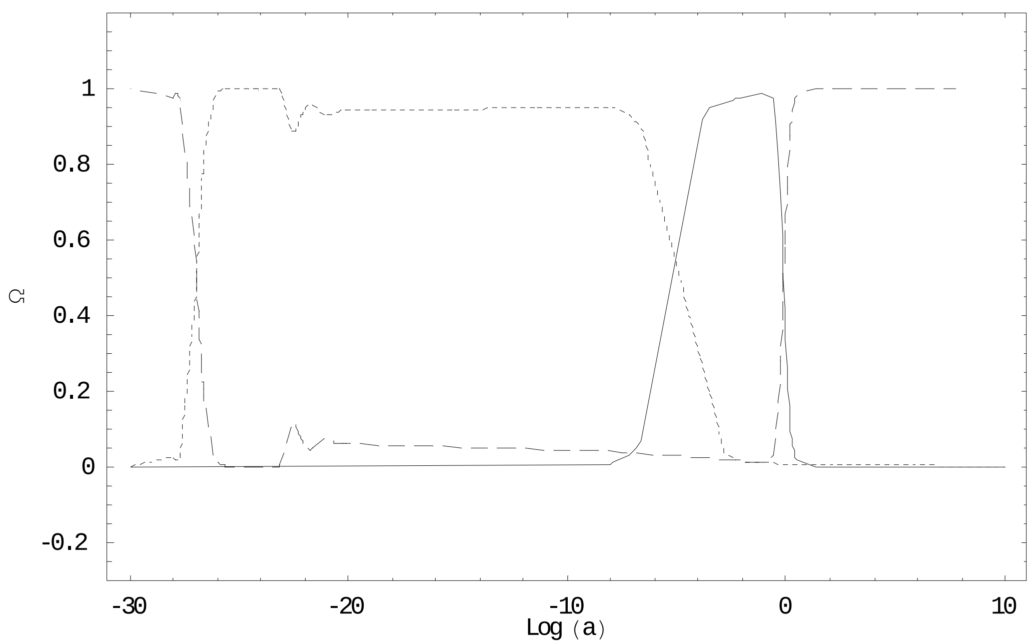

Fig.15 A solution for the potential of Figure 14

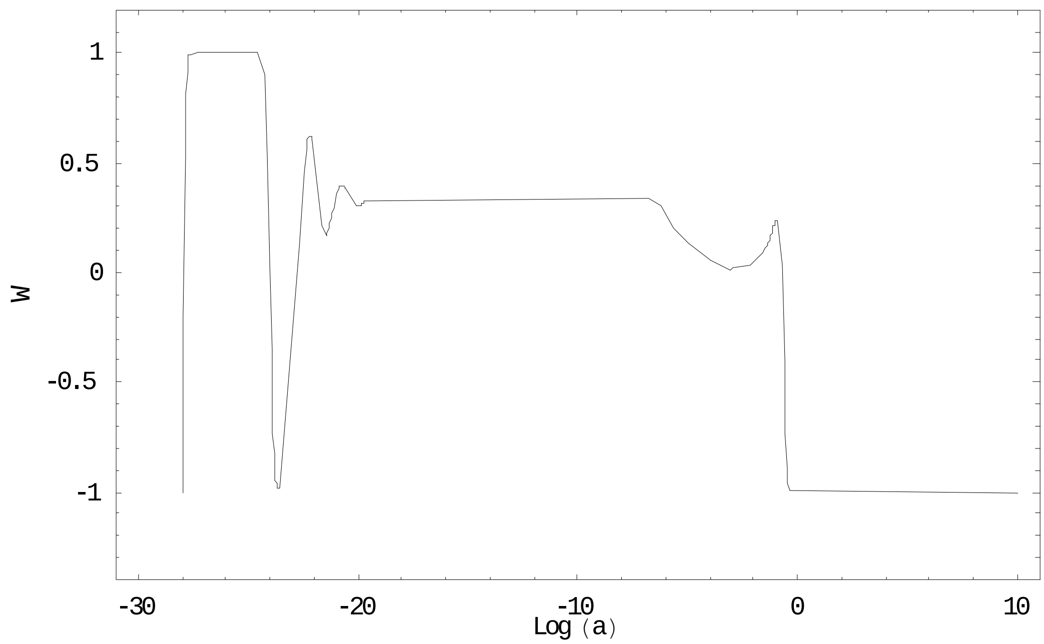

Fig.15 Continued

This figure shows the solution of (50) and (51) with potential A1=0.5, B1=0.005,A2=33.1,B2=0.01, and λ =8. Reheating happened at a~10^-(30), and at a~10^(-28) after the decay of a few percent of scalar field into particles, field penetrated the barrier and rolled down to the second minimum. a~10^(-10) at nucleosynthesis and the radiation-matter transition is evident at a~10^(-5). Today a=1 and w= -1 from the figure.

We have discussed it is possible quintessence field which accelerate universe today could oscillate near minimum of its potential and produce particles at the stage of reheating. Here we only talked the special potential with the form (50)(51), to other potential with minimum, it is also possible to unify quintessence field of reheating into one single field.

Before we move to next chapter, I need to mention that AS model is not the only model of expedient which can satisfy the nucleosynthesis. Nucleosynthesis needs that, when the universe at around loga~-10, the expansion of universe cannot be too fast. If it did expand very fast, then no nuclear can be formed at that stage, then our universe will have no stars and galaxies. Nucleosynthesis is very strong constraint which requires less than 20% of the total energy is dark energy, and more than 80% of the total energy is matter like, either Standard Model particles or dark matter. But at today, we know that about 70% of the total energy is dark energy, and 30% of energy is matter like, so, the varying of dark energy and matter with time should be difference, or, possibly some matter will decay into dark energy. The extremely heavy dark matter decays into dark energy has been talked by many authors.[] In such kind of models, the authors assume the existence of very heavy particle, which is not in Standard Model and with mass in TeV and the motivation of this type of dark matter is the observation of Ultra High Cosmic Ray. This dark matter will decay into scalar field or dark energy. But the problem of this kind of model is that we haven't enough data to support the existence of this super heavy dark matter, and no theory predicts this kind of particles. In fact, since there is some kind of resemblance between dark energy and dark matter, both of them are dark, there are many models trying to unify this two unknown things. The existence of dark matter and dark energy has been proved, but it is very hard to see what it is.

CHAPTER III

STATIC BUBBLE OF REHEATING FIELD DUE TO GRAVITY

Dark energy can be described by quintessence field which has the motion of equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \tag{56}$$

where H is Hubble parameter. With the $3H\dot{\phi}$ term, the amplitude of ϕ will decrease with time. But there is another possibility, that is there is no $3H\dot{\phi}$ term, in other words, the field ϕ is localized in some place without expansion.

Lets look back the equation of state for the quintessence field ϕ.

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ P_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \tag{57}$$

and

$$w = p_\phi / \rho_\phi$$

When $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$, $w \sim -1$, so it looks like dark energy. But if $\frac{1}{2}\dot{\phi}^2 \sim V(\phi)$, then w ~ 0, in this case the scalar field will looks like matter.

First, we assume there is static bubble of scalar field, and then we prove that the bubble is static.

In normal reheating theory, the simplest potential is

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 \tag{58}$$

If there is a static bubble of scalar field ϕ, the equation of scalar field Φ can be written as

$$\ddot{\phi} - \triangle\phi + m^2\phi + \lambda\phi^3 = 0 \tag{59}$$

in this equation, there is no $3H\dot{\phi}$ term since the bubble is static without expansion.

Energy density of the scalar field ϕ becomes

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}\nabla^*\phi\nabla\phi \tag{60}$$

If there is static bubble of this classical scalar field Φ, and assume

When r<R, $\phi = ce^{\mathrm{int}}$, where c is constant

r=R~R+ ε , $\phi = f(r)e^{\mathrm{int}}$, where $f(r) = \{ \begin{matrix} c, r = R \\ 0, r = R + \varepsilon \end{matrix}$

r>R+ ε , Φ =0.

At r<R, since c=constant, $\nabla^2\phi = 0$, the equation of scalar field (59) becomes

$$\ddot{\phi} + m^2\phi + \lambda\phi^3 = 0 \tag{61}$$

And the solution is $\phi = ce^{\mathrm{int}}$ with $n^2 = m^2 + \lambda c^2$.

If we neglect the effect of surface r=R~R+ε which we will show is very small, then at r<R,

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 = m^2c^2 + \frac{3}{4}\lambda c^4 \tag{62}$$

And $\mathrm{p} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 = \frac{1}{4}\lambda c^4 > 0$ with λ>0.

So $w = \frac{p}{\rho} > 0$, it's matter like.

The classical coherently oscillating inflation field φ decays into massive bosons due to elementary theory and parametric resonance. When the amplitude φ drops to a valued ~ $m_\phi / \sqrt{\lambda}$, the parametric resonance process stops, and particles production process is dominant by elementary theory which can be written as

$$\Gamma(\phi \to \chi\chi) = \frac{g^4\sigma^2}{8\pi m_\phi}$$
$$\Gamma(\phi \to \psi\psi) = \frac{h^2 m_\phi}{8\pi} \tag{63}$$

If g and h is very small, the rate of particle production by elementary theory is very weak, so inside the bubble r<R, the total amount of Φ could be conserved.

$$A \equiv \int dv\phi^2 = \frac{4}{3}\pi R^3 \cdot c^2 = cons\tan t \tag{64}$$

The total potential energy of Φ field is

$$\begin{aligned} V_{\phi-potential} &= \int dv(\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4) \\ &= \frac{1}{2}m^2 A + \frac{1}{4}\lambda A\phi^2 \\ &= \frac{1}{2}m^2 A + \frac{1}{4}\lambda A^2 \frac{1}{\frac{4}{3}\pi R^3} \end{aligned} \tag{65}$$

And gravity potential can be written as

$$\begin{aligned} V_{gravity} &= -\int_0^R \frac{4}{3}\pi r^3 \rho G \frac{1}{r} 4\pi r^2 dr \rho \\ &= -\frac{3}{5}G \times \frac{E^2}{R} \end{aligned} \tag{66}$$

where E is total energy of the bubble

$$E = \frac{4}{3}\pi R^3 (m^2c^2 + \frac{3}{4}\lambda c^4)$$

$$= m^2 A + \frac{3}{4}\lambda A^2 \frac{1}{\frac{4}{3}\pi R^3} \approx m^2 A \tag{67}$$

So the gravity potential becomes

$$V_{gravity} \approx -\frac{3}{5} G \times \frac{m^4 A^2}{R} \tag{68}$$

The total potential energy of Φ field and gravity is then

$$V_{total} \approx \frac{1}{2} m^2 A + \frac{1}{4}\lambda A^2 \frac{1}{\frac{4}{3}\pi R^3} - \frac{3}{5} G \times \frac{m^4 A^2}{R} \tag{69}$$

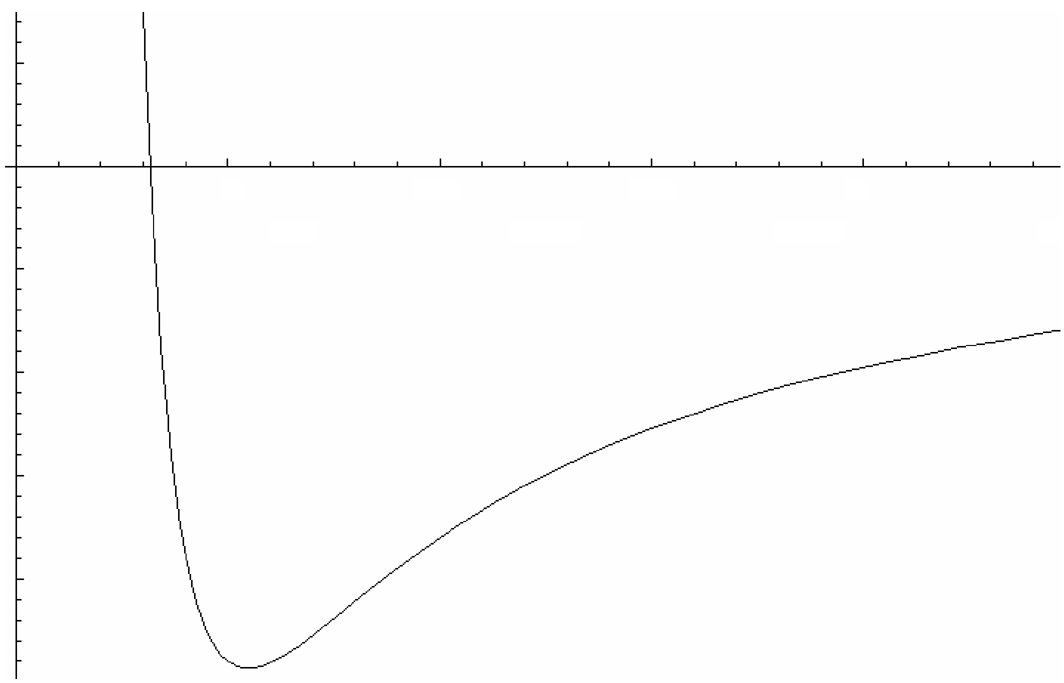

Fig.16 There is a minimum of potential (69)

We can see from figure 16 that there is a minimum in the potential of (69).

The total potential has minimum when $V^{'}=0$,

$$V^{'}=0=-\frac{3\times3}{14}\lambda A^{2}\frac{1}{\pi R^{3}}+\frac{3}{5}G\frac{m^{4}A^{2}}{R^{2}}$$

or
$$R^{2}=\frac{15}{14}\lambda\frac{1}{\pi m^{4}G}$$

if we choose λ =1, m=100GeV, then

$$R\approx0.7\times10^{15}GeV^{-1}\approx0.1m \tag{70}$$

This is the radius of static bubble.

With computer simulation, $\varepsilon<20GeV^{-1}$ while $R\approx0.7\times10^{15}GeV^{-1}$, and we can get $\varepsilon/R\sim10^{-14}$, so the surface effect can be neglected.

We have talked that when we choose λ =1, m=100GeV, we have a static bubble with radius around 0.1 meter. And since the bubble stays in the only minimum, the bubble cannot decay. So the bubble is absolute stable.

The bubble is stable because of the gravity. So, if the bubble is not with intense density, we cannot have such kind of bubble since in this case, the gravity can be neglected and we can never have minimum in the potential (69).

If there is such kind of bubble of reheating field, how can we find it? This is relying on the interaction between reheating field and Standard Model particles. Since there must be some channels through which reheating field can decay into Standard Model particles. And this kind of channels would be used to test this bubble model.

The decay rate of ϕ into Standard Model particles is

$$\Gamma(\phi \to \chi\chi) = \frac{g^4\sigma^2}{8\pi m_\phi}$$

$$\Gamma(\phi \to \psi\psi) = \frac{h^2 m_\phi}{8\pi}$$

And we have neglected the contribution from the parameter resonance, where g and h are interaction constant. We can see that the decay rate is proportional to g and h. Since the bubble is dark and we haven't seen it in experiment, the interaction should be very weak. But it should be greater than gravity, and it is possible to test and find the bubble in experiment.

Here, we only calculate that there is static bubble with radius r~ 0.1m. But this calculation is based on the assumption of λ =1, m=100GeV. We have shown that

$$R^2 = \frac{15}{14}\lambda\frac{1}{\pi m^4 G}$$

So, if the m is very small, the stable radius will becomes very big, for example, with m~1eV and $\lambda = 0.1$ the radius will be ~10^{44} m, and it cannot be treated seriously.

Another thing we haven't talked here is the shape of the bubble. It seems all shapes have degeneracy energy. This is because we neglect the contribution from surface. Since surface energy usually is proportional to its surface area, to have minimum surface energy, the surface should take minimum number, and we know that with same volume, sphere has minimum area. With this consideration, we should get the conclusion that the bubble is sphere.

CHAPTER IV

STANDARD MODEL AND DARK WORLD

Standard Model of Particle Interactions, the basic building blocks of matter are six leptons and six quarks that interact by means of force-carrying particles called bosons. Every phenomenon observed in nature can be understood as the interplay of the fundamental particles and forces of the Standard Model. —FERMILAB [14]

Figure 17 and figure 18 show the fundamental particles and forces in SM. Standard Model can unify the two forces into one-electroweak interaction, and also, it includes the strong force in this model.

But physicists know that the Standard Model does not tell the whole story, and they are searching for physics beyond the Standard Model that will lead to a larger, more elegant "theory of everything." Such of this kind of theory is superstring.

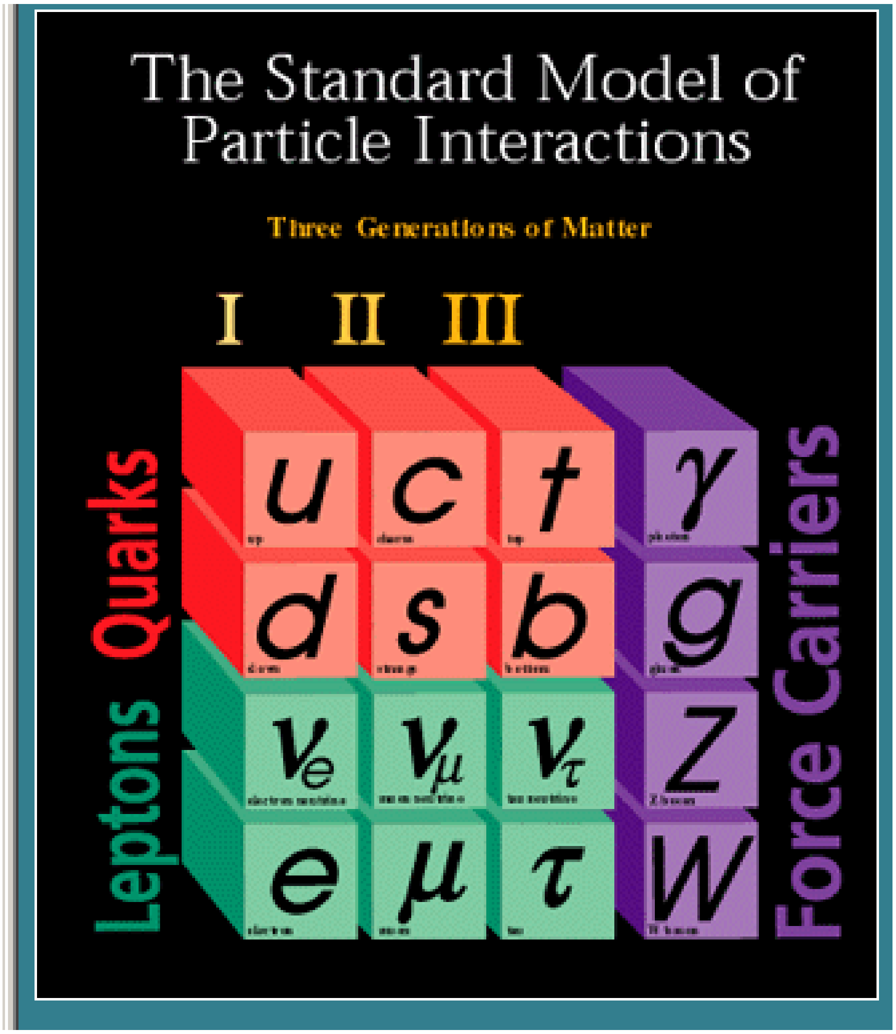

Fig.17 Particles of Standard Model

Forces and symmetries			
Force	**Gauge bosons**	**Gauge group**	**Details**
Electromagnetism	1 Photon	The unbroken U (1) combination of SU(2)xU (1) symmetry	Photon is massless and neutral, couples to electric charge, force is infinite range, theory is called Quantum Electrodynamics, or QED for short.
Weak nuclear force	W^+,W^- ,Z	The broken combination of SU(2)xU (1) symmetry	Gauge symmetry is hidden by interaction with scalar particle called **Higgs**, W and Z are massive, have weak and electric charge, interaction is short range
Strong nuclear force	8 Gluons	SU(3)	Gluon is massless but self-interacting. Charge is called quark color, theory is called Quantum Chromodynamics, or QCD for short.

Fig.18 Forces and symmetry of Standard Model

Standard Model is extremely successful in High Energy Physics, but it is failed when it is used to explain some cosmology phenomena, such as dark energy and dark matter, which has been discovered recently.

Spontaneous Symmetry Breaking

First let us analyze the simple example of a scalar self-interacting real field with Lagrangian

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \tag{71}$$

with

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

In the theory of the phase transition of a ferromagnet, the Gibbs free energy density is analogous to $V(\phi)$ and ϕ playing the role of the average spontaneous magnetization M.

The vacuum can be obtained from the Hamiltonian

$$H = \frac{1}{2}[(\partial_0\phi)^2 + (\nabla\phi)^2] + V(\phi) \tag{72}$$

To guarantee that H is lower bounded, λ must be positive. But there is no constraint that m^2 must be positive. If m^2>0, then the potential $V(\phi)$ looks like

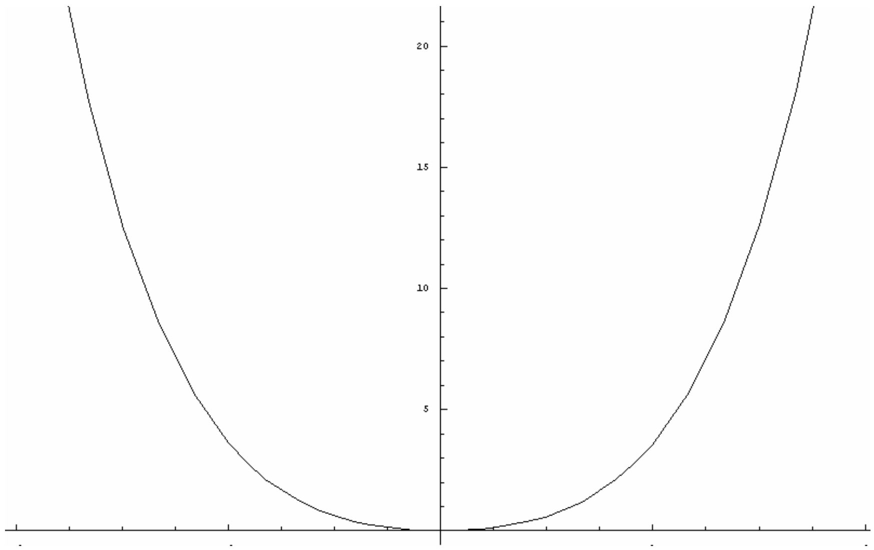

Fig.19 $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4$ with $m^2>0$

Figure 19 shows the form of $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4$ with $m^2>0$.

While if $m^2<0$, the potential $V(\phi)$ looks like figure 20.

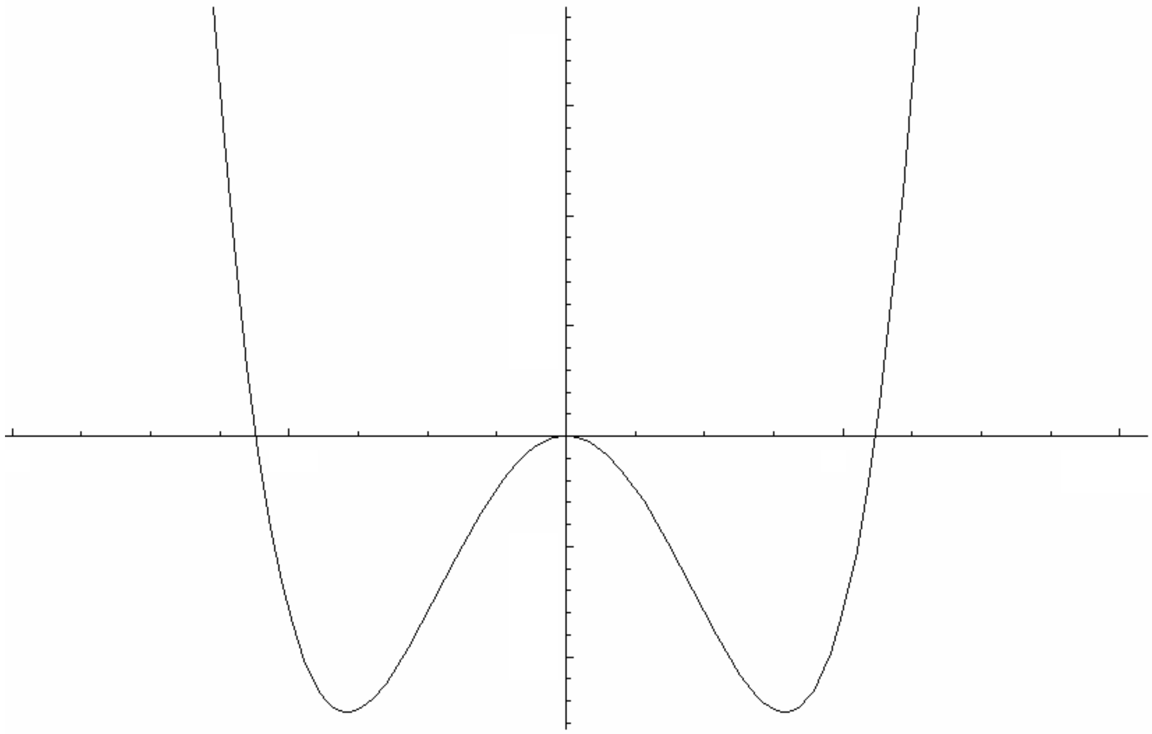

Fig.20 $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4$ with $m^2<0$

For $m^2>0$, the minimum of the potential is at zero, but when $m^2<0$, figure 20 shows there are two minimum of the potential is at $v=\sqrt{-\frac{m^2}{\lambda}}$ and $v=-\sqrt{-\frac{m^2}{\lambda}}$.

Defining $\Phi=\phi+v$, then the vacuum of the new field is at $\Phi_0=0$, and the lagrangian becomes

$$L=\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}(\sqrt{-2m^2})^2-\lambda v\Phi^3-\frac{1}{4}\lambda\Phi^4 \qquad (73)$$

This Lagrangian describes the scalar field Φ with real and positive mass, $M_\Phi=\sqrt{-2m^2}$, but it lost its symmetry due to the Φ^3 term.

A more complicated case is that a continuous symmetry is spontaneously broken. For a charged self-interacting scalar field

$$L = \partial\phi^{*}\partial\phi - V(\phi^{*}\phi)$$

where

$$V(\phi) = \frac{1}{2}m^{2}\phi^{*}\phi + \frac{1}{4}\lambda(\phi^{*}\phi)^{2} \tag{74}$$

The lagrangian is invariant under the global phase transformation

$$\phi \rightarrow \exp(-i\theta)\phi$$

We define the complex field in terms of two real fields by

$$\phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}} \tag{75}$$

The lagrangian becomes

$$L = \frac{1}{2}(\partial_{\mu}\phi_1\partial^{\mu}\phi_1 + \partial_{\mu}\phi_2\partial^{\mu}\phi_2) - V(\phi_1, \phi_2)$$

which is invariant under SO(2) rotations

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \rightarrow \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \tag{76}$$

The vacuum of this potential is at $\phi_1 = \phi_2 = 0$ with m^2 >0, but in the case of m^2 <0, we have a continuum of vacua located at

$$\left\langle |\phi|^{2}\right\rangle = \frac{(\langle\phi_1\rangle^{2} + \langle\phi_2\rangle^{2})}{2} = \frac{-m^{2}}{2\lambda} \equiv \frac{v^{2}}{2} \tag{77}$$

as shown in the figure below.

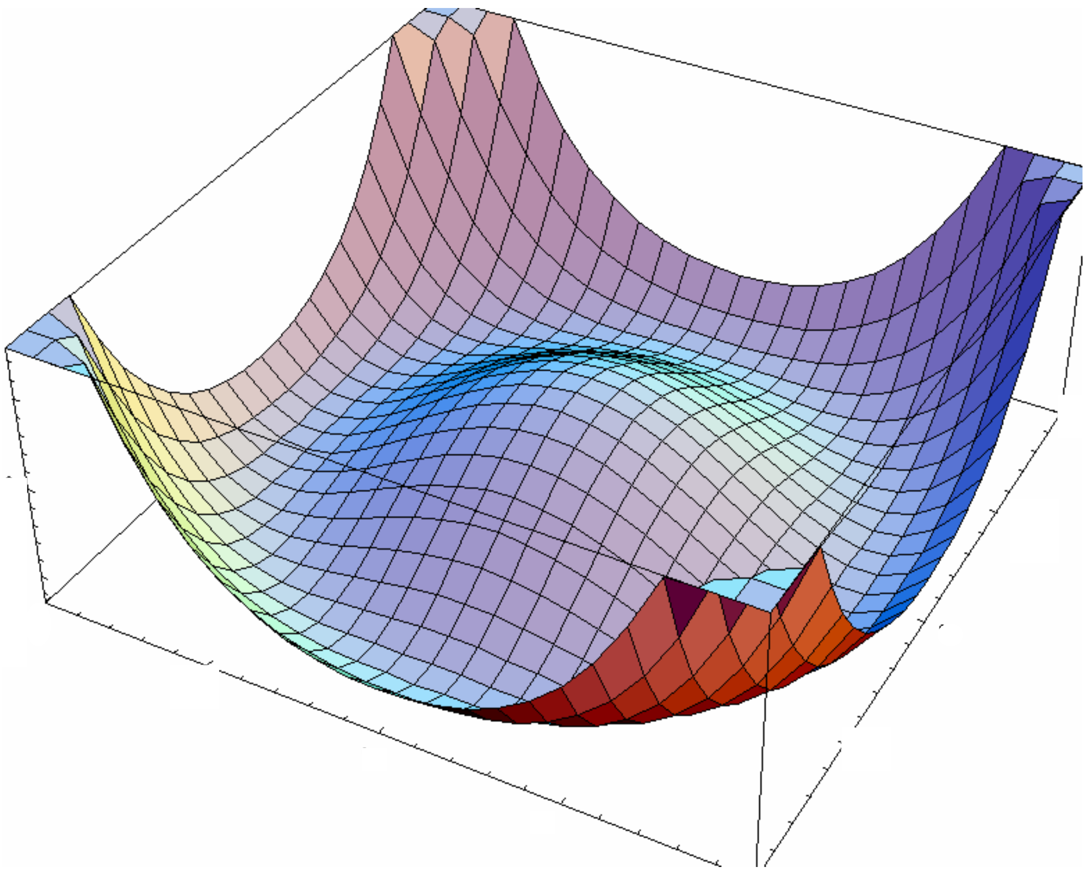

Fig.21 The potential of t with $\phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}}$

Figure 21 shows the form of potential (74).

The SO(2) symmetry of this Lagrangian will broken if we choose a particular vacuum. Let us choose, for example,

$$\phi_1 = v$$
$$\phi_2 = 0$$

We can defined the new fields by

$$\Phi_1 = \phi_1 - v$$
$$\Phi_2 = \phi_2$$

Then the new Lagrangian becomes,

$$L = \frac{1}{2}\partial_\mu \Phi_1 \partial^\mu \Phi_1 - \frac{1}{2}(-2m^2)\Phi_1{}^2$$
$$+\frac{1}{2}\partial_\mu \Phi_2 \partial^\mu \Phi_2$$
$$-\frac{1}{4}\lambda(\Phi_1{}^4 + 2\Phi_1{}^2\Phi_2{}^2 + \Phi_2{}^4) - \lambda v (\Phi_1{}^3 + \Phi_1\Phi_2{}^2)$$
$$+ (-\frac{m^2}{4}v^2) \quad (78)$$

We can see that the scalar field Φ_1 with real and positive mass and field Φ_2 is massless which is called Goldstone particle.

Higgs Mechanism

In experiments, we didn't find the massless Goldstone particle which was predicted by Goldstone theorem with spontaneous symmetry breakdown. The Higgs mechanism can solve this problem by making the massless Goldstone particle massive. This is accomplished by requiring that the Lagrangian that exhibits the spontaneous symmetry breakdown is also invariant under local, rather than global, gauge transformations.

Let us require that the Lagrangian is invariant under the local phase transformation

$$\phi \to \exp[iq\alpha(x)]\phi$$

We introduce a gauge boson(A_μ) and the covariant derivative(D_μ) so that the Lagrangian becomes invariant, following the principles like

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$$
$$A_\mu \to A_\mu' = A_\mu - \partial_\mu \alpha(x) \quad (79)$$

The spontaneous symmetry breaking occurs for m^2 <0, with the vacuum $\langle |\phi|^2 \rangle = \frac{v^2}{2}$.

A convenient way to define the new field by

$$\phi = \exp(i\frac{\phi_2^{'}}{v})\frac{(\phi_1^{'}+v)}{\sqrt{2}} \simeq \frac{1}{\sqrt{2}}(\phi_1^{'}+v+i\phi_2^{'}) = \phi^{'} + \frac{v}{\sqrt{2}} \quad (80)$$

with

$$\phi^{'} = \frac{\phi_1^{'}+i\phi_2^{'}}{\sqrt{2}}$$

The lagrangian becomes,

$$\begin{aligned} L &= \frac{1}{2}\partial_\mu\phi_1^{'}\partial^\mu\phi_1^{'} - \frac{1}{2}(-2m^2)\phi_1^{'2} + \frac{1}{2}\partial_\mu\phi_2^{'}\partial^\mu\phi_2^{'} \\ &+ \mathrm{int}\,eract \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu + qvA_\mu\partial^\mu\phi_2^{'} \end{aligned} \quad (81)$$

In this Lagrangian, it includes a scalar field $\phi_1^{'}$ and a massless scalar field $\phi_2^{'}$(goldstone boson) and a massive vector boson A_μ.

However the presence of the last term in the Lagrangian which is proportional to $A_\mu\partial^\mu\phi_2^{'}$ is quite inconvenient since it mixes the propagators of A_μ and $\phi_2^{'}$ particles. To eliminate this term, we choose

$$\alpha(x) = -\frac{1}{qv}\phi_2^{'}(x) \quad (82)$$

Then the field ϕ becomes,

$$\phi = \frac{1}{\sqrt{2}}(\phi_1^{'}+v) \quad (83)$$

With this choice of gauge, we can acquire

$$L = \frac{1}{2}\partial_\mu \phi_1^{'} \partial^\mu \phi_1^{'} - \frac{1}{2}(-2m^2)\phi_1^{'2}$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad (84)$$
$$+\frac{q^2}{2}(\phi_1^{'} + 2v)A_\mu^{'}A^{\mu'} - \frac{\lambda}{4}\phi_1^{'3}(\phi_1^{'} + 4v)$$

So we have a massive scalar field $\phi_1^{'}$ with freedom 1 and a massive vector field $A_\mu^{'}$ with freedom 3.

Scalar Field in Dark World and Droplet

In above, we have discussed the scalar field in standard model, which is especially important to give mass to fermions and bosons. But there is problem about this scalar field – till now on, we cannot find the massive scalar field, or so called Higgs particle. In experiment, we have searched the energy up to 114 GeV, but there is still no hint for the existence of this particle.

Because of difficulties of Hierarchy Problem, which indicates that the mass of Higgs particle will become infinity due to the second loop correction. To cure this difficulties, super symmetry was introduced, and so the super gravity. But to supersymmetry, itself has some problems, like even in the MSSM, we need about 100 new parameters, and still there is no evidence for the existence of super symmetry particles.

As we discussed in the part of Dark Energy, quintessence is described by ordinary scalar field ϕ, and the action for Quintessence is given by

$$S = \int d^4x\sqrt{-g}[-\frac{1}{2}(\nabla\phi)^2 - V(\phi)]$$

The merit of quintessence is that the equation of state varies with time which can solve the problem of fine tunneling problem.

Is there any relationship between the scalar field in Standard Model and the scalar field quintessence?

Let us introduce a classical scalar field which has the same form as in Standard Model

$$L = -\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \tag{85}$$

with

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

But different with the Standard Model where m^2 <0 and is a constant, here we treat m^2 as mass matrix like

$$m^2 = \begin{pmatrix} m_1^2 & 0 \\ 0 & -m_2^2 \end{pmatrix} \tag{86}$$

The potential $V(\phi)$ is then like

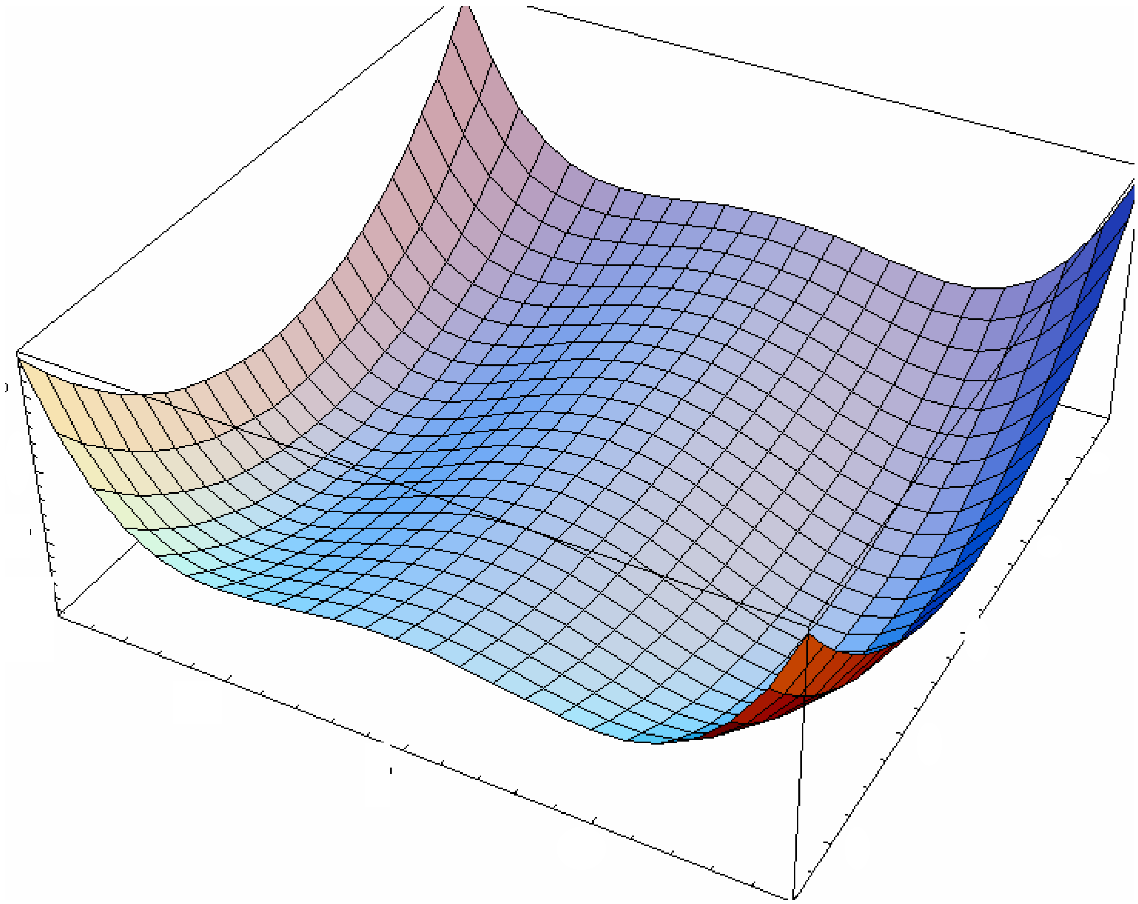

Fig.22 $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4$ with $m^2=\begin{pmatrix} m_1^2 & 0 \\ 0 & -m_2^2 \end{pmatrix}$

In figure 22, we can see there are two minimums at

$$\phi=\begin{pmatrix} 0 \\ \pm\sqrt{\frac{m_2^2}{\lambda}} \end{pmatrix} \tag{87}$$

And the lagrangian haven't SO(2) symmetry invariant.

Let us choose the expectation value is

$$\phi=\begin{pmatrix} 0 \\ \sqrt{\frac{m_2^2}{\lambda}} \end{pmatrix} \tag{88}$$

The new fields, suitable for small perturbations, can be defined as

$$\phi_2^{'} = \phi_2 - \sqrt{\frac{m_2^2}{\lambda}} \tag{89}$$

$$\phi_1^{'} = \phi_1 \tag{90}$$

In terms of these new fields, the potential of $\phi_1^{'}$ and $\phi_2^{'}$ becomes,

$$\begin{aligned} V(\phi) &= \frac{1}{2} m_1^2 \phi_1^{'2} + \frac{1}{4}\lambda \phi_1^{'4} \\ &+ \frac{1}{2}\lambda \phi_1^{'2} (\phi_2^{'2} + \sqrt{\frac{m_2^2}{\lambda}}) \\ &+ \frac{1}{2}(2m_2^2)\phi_2^{'2} + \lambda \sqrt{\frac{m_2^2}{\lambda}} \phi_2^{'3} + \frac{1}{4}\lambda \phi_2^{'4} \\ &- \frac{1}{4}\frac{m_2^4}{\lambda} \end{aligned} \tag{91}$$

or we can write it as

$$\begin{aligned} V(\phi) &= \frac{1}{2} m_1^2 \phi_1^{'2} + \frac{1}{4}\lambda \phi_1^{'4} \\ &+ \frac{1}{2}\lambda \phi_1^{'2} \phi_2^{2} \\ &+ \frac{1}{2}(2m_2^2)(\phi_2 - \sqrt{\frac{m_2^2}{\lambda}})^2 + \lambda \sqrt{\frac{m_2^2}{\lambda}} (\phi_2 - \sqrt{\frac{m_2^2}{\lambda}})^3 + \frac{1}{4}\lambda (\phi_2 - \sqrt{\frac{m_2^2}{\lambda}})^4 \\ &- \frac{1}{4}\frac{m_2^4}{\lambda} \end{aligned}$$

(92)

In normal reheating theory, the simplest reheating field is taken, i.e.

$$V(\phi_1) = \frac{1}{2} m_1^{2} \phi_1^{2} + \frac{1}{4}\lambda \phi_1^{4} \tag{93}$$

which is similar to the first line of (92).

While there is dark energy field, or quintessence, and usually we believe that quintessence field is trapped at a minimum, so it just looks like cosmology constant today. We can write quintessence field potential as

$$V(\phi_2) = \frac{1}{2} m_2{}^2 (\phi_2 - A)^2 + V_0 \tag{94}$$

where V0 is constant, and for convenient, we write it as

$$V(\phi_2) = \frac{1}{2} m_2{}^2 (\phi_2 - A)^2 \tag{95}$$

which looks like the third line of potential (92)

Neglect the higher order term of ϕ_2 in potential (92) and constant term, and $\phi_1^{'} = \phi_1$,the (92) can be written as

$$\begin{aligned} V(\phi) &= \frac{1}{2} m_1^2 \phi_1{}^2 + \frac{1}{4} \lambda \phi_1{}^4 \\ &+ \frac{1}{2} \lambda \phi_1{}^2 \phi_2{}^2 \\ &+ \frac{1}{2} (2m_2^2)(\phi_2 - \sqrt{\frac{m_2^2}{\lambda}})^2 \end{aligned} \tag{96}$$

The first line can be treated as the scalar field for reheating, the third line can be treated as quintessence field, and the interaction term can produce droplet in the quintessence field, which is a candidate of dark matter.

Then a static, matter like droplet of reheating field can be formed which is a candidate for dark matter.

Static droplet of binary mixtures is a very popular topic in solid state.

Here we there is droplet of ϕ_1 in the background ϕ_2, and the formation of the droplet is due to the interaction between ϕ_1 and ϕ_2 with the interaction term $\frac{1}{2}\lambda\phi_1^2\phi_2^2$

Assume the radius of droplet is R, and inside the droplet Φ1 and Φ2 are constant.

Inside the droplet, to ϕ_2 , $\ddot{\phi}$ 2=0 and $\nabla^2\phi$ 2=0, so

$$V'_{\phi_2} = 0 = (2m_2{}^2)(\phi_2 - A) + \lambda\phi_1^2\phi_2 \tag{97}$$

$$\phi_2 = \frac{2m_2^2 A}{2m_2^2 + \lambda\phi_1^2} \tag{98}$$

where

$$A = \sqrt{\frac{m_2^2}{\lambda}}$$

And to ϕ_1, inside the droplet r<R

$$\ddot{\phi}_1 - \nabla^2\phi_1 + m_1^2\phi_1 + \lambda\phi_2^2\phi_1 + \lambda\phi_1^3 = 0 \tag{99}$$

$$\nabla^2\phi_1 = 0$$

$$\phi_1 = \alpha e^{\text{int}}$$

$$n^2 = m_1^2 + \lambda\alpha^2 + \lambda\phi_2^2$$

We can treat $\int dv\phi_1^2 = cons = c = \frac{4}{3}\pi R^3\phi_1^2$ as we talked in second objective.

So inside the bubble, the potential is

$$V(\phi) = \int dv\{\frac{1}{2}m_1^2\phi_1{}^2 + \frac{1}{4}\lambda\phi_1{}^4 + \frac{1}{2}\lambda\phi_1{}^2\phi_2{}^2 + \frac{1}{2}(2m_2^2)(\phi_2 - A)^2\}$$

$$= \int dv\{\frac{1}{2}m_1^2\phi_1{}^2 + \frac{1}{4}\lambda\phi_1{}^4 + \frac{1}{2}\lambda\phi_1^2\frac{2m_2^2A^2}{2m_2^2 + \lambda\phi_1^2}\}$$

$$= \frac{1}{2}m_1^2c + \frac{1}{4}\lambda c\phi_1^2 + \frac{1}{2}\lambda cA^2\frac{2m_2^2}{2m_2^2 + \lambda\phi_1^2} \tag{100}$$

V can take minimum value when

$$\phi_1^2 = \sqrt{\frac{2\times 2m_2^2}{\lambda}}A - \frac{2m_2^2}{\lambda}$$

$$\phi_2 = \frac{2m_2^2 A}{2m_2^2 + \lambda\phi_1^2} = \frac{m_2}{\sqrt{\lambda}}$$

However, when we calculate the ϕ_1^2 with $A = \sqrt{\frac{m_2^2}{\lambda}}$, we found that $\phi_1^2 = 0$. That means there is no static droplet with the potential (92)

The simplest form of scalar field is

$$V(\phi) = \pm\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

where both m^2 and λ are positive.

Now let us consider scalar field ϕ_1 and ϕ_2 with simplest potential form as shown above , and with the simplest interaction term $\frac{1}{2}g\phi_1^2\phi_2^2$

$$\begin{aligned} V(\phi_1,\phi_2) = {} & \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4 \\ & + \frac{1}{2}g\phi_1^2\phi_2^2 \\ & - \frac{1}{2}m_2^2\phi_2^2 + \frac{1}{4}\lambda_2\phi_2^4 \end{aligned} \tag{101}$$

We should notice that the interaction term is symmetric, but the total form of this potential hasn't the symmetry like $\phi_1 \to \phi_2$.

The potential of $V(\phi_2)$ has the form

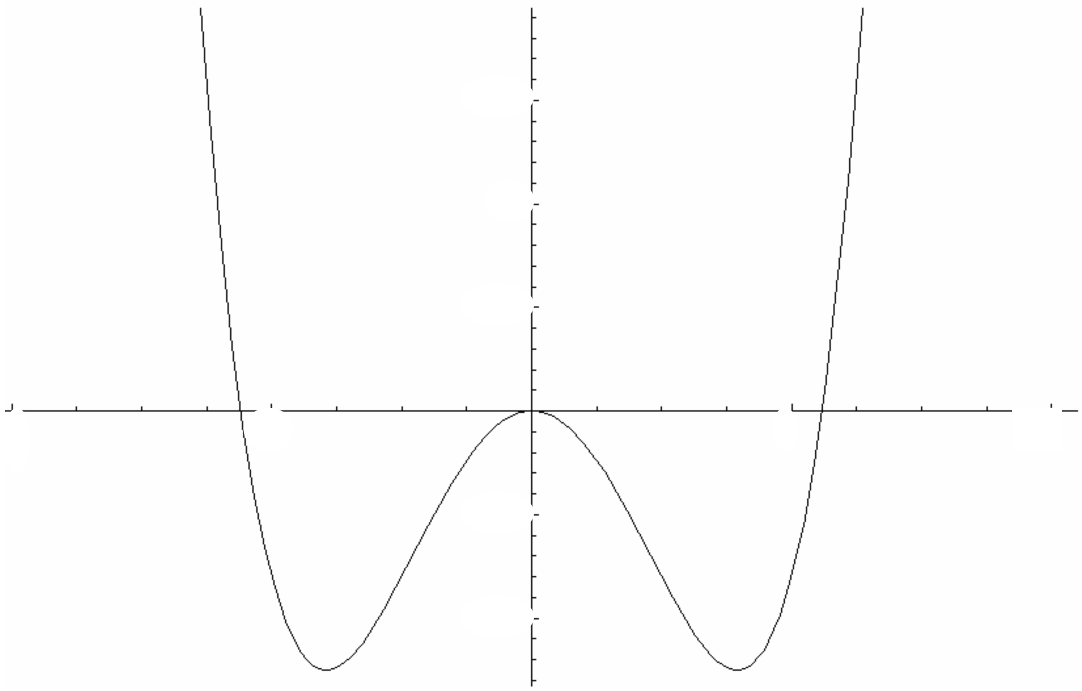

Fig.23 Two minimums in potential (101)

Figure 23 shows there are two minimums in potential (101), which allows symmetry broken in this model.

So it will break the $\phi_2 \to -\phi_2$ with spontaneous broken. The minimum of this potential is

$$\phi_2 = \pm\sqrt{\frac{m_2^2}{\lambda_2}}$$

And we choose the minimum $\phi_2 = v = \sqrt{\frac{m_2^2}{\lambda_2}}$ as the vacuum of universe, and we define replace

$$\phi_2 \to \phi_2 + v$$

then the potential of $V(\phi_1,\phi_2)$ becomes

$$\begin{aligned} V(\phi) &= \frac{1}{2}m_1^2\phi_1^{\ 2}+\frac{1}{4}\lambda_1\phi_1^{\ 4} \\ &+\frac{1}{2}g\phi_1^{\ 2}(\phi_2+v)^2 \\ &+\frac{1}{2}(2m_2^2)\phi_2^{\ 2}+\lambda_2\sqrt{\frac{m_2^2}{\lambda_2}}\phi_2^{\ 3}+\frac{1}{4}\lambda_2\phi_2^{\ 4} \\ &-\frac{1}{4}\frac{m_2^4}{\lambda_2} \end{aligned} \tag{102}$$

Neglect the higher order of ϕ_2, then this potential becomes,

$$\begin{aligned} V(\phi_1,\phi_2) &= \frac{1}{2}m_1^2\phi_1^{\ 2}+\frac{1}{4}\lambda_1\phi_1^{\ 4} \\ &+\frac{1}{2}g\phi_1^{\ 2}\phi_2^{\ 2} \\ &+\frac{1}{2}(2m_2^2)(\phi_2-v)^2 \end{aligned} \tag{103}$$

and define

$$M^2 = 2m_2^2$$

for convenience.

First we assume there is static droplet of ϕ_1 in the field ϕ_2 with interaction $\frac{1}{2}g\phi_1^{\ 2}\phi_2^{\ 2}$, and then we can see whether this droplet is static.

Assume the radius of droplet is R, and inside the droplet Φ1 and Φ2 are constant.

Inside the droplet, to ϕ_2, $\ddot{\phi}_2 = 0$ and $\nabla^2\phi_2 = 0$, so

$$V_{\phi_2}^{'} = 0 = M^2(\phi_2 - v) + g\phi_1^2\phi_2 \tag{104}$$

$$\phi_2 = \frac{M^2 A}{M^2 + g\phi_1^2} \tag{105}$$

And to ϕ_1, inside the droplet $r<R$

$$\ddot{\phi}_1 - \nabla^2\phi_1 + m_1^2\phi_1 + g\phi_2^2\phi_1 + \lambda\phi_1^3 = 0 \tag{106}$$

$$\nabla^2\phi_1 = 0$$

$$\phi_1 = \alpha e^{\text{int}}$$

$$n^2 = m_1^2 + \lambda\alpha^2 + g\phi_2^2$$

We can treat $\int dv\phi_1^2 = cons = c = \frac{4}{3}\pi R^3\phi_1^2$ as we talked in second objective.

So inside the bubble, the potential is

$$V = \int dv(\frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}g\phi_1^2\phi_1^2 + \frac{1}{4}\lambda\phi_1^4 + \frac{1}{2}M^2(\phi_2 - v)^2)$$

$$= \frac{1}{2}m_1^2 c + \frac{1}{4}\lambda c\phi_1^2 + \frac{1}{2}gcv^2\frac{M^2}{M^2 + g\phi_1^2} \tag{107}$$

V can take minimum value when

$$\phi_1^2 = \sqrt{\frac{2}{\lambda_1}}vM - \frac{M^2}{g}$$

$$\phi_2 = \frac{M^2 v}{M^2 + g\phi_1^2} = \sqrt{\frac{\lambda_1}{2}}\frac{M}{g} \tag{108}$$

If $\phi_1^2 = \sqrt{\frac{2}{\lambda_1}}vM - \frac{M^2}{g} > 0$. In this case the potential $V(\phi_1, \phi_2)$ looks like

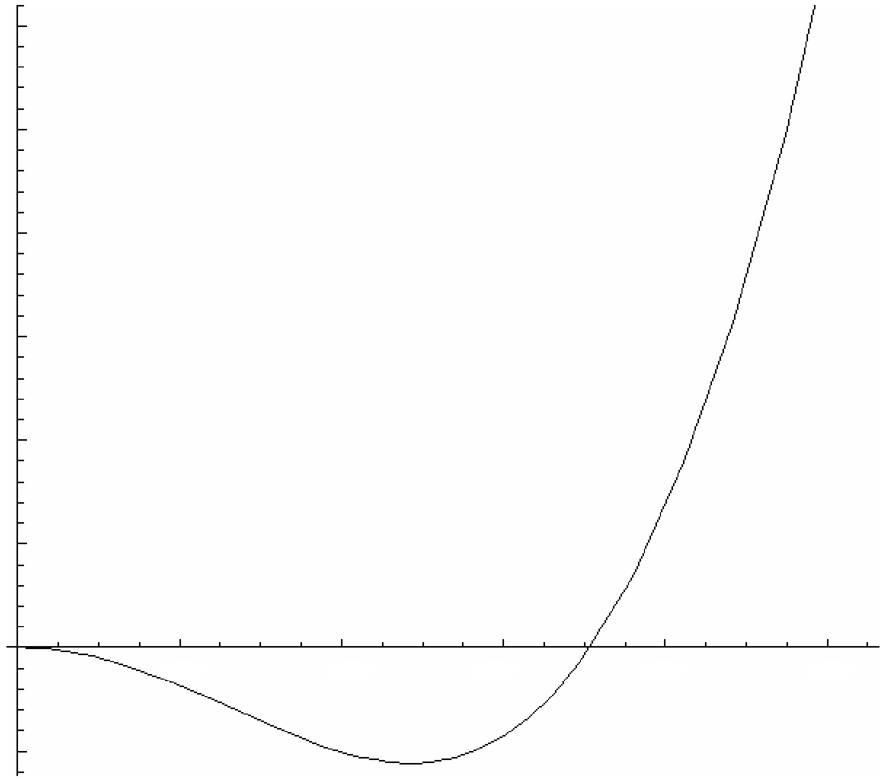

Fig.24 There is a minimum in potential (107)

Figure 24 shows the minimum in potential (107).

The total energy E inside the droplet r<R is then

$$E = \int dv\{(\frac{1}{2}\dot{\phi}_1)^2 + (\frac{1}{2}\dot{\phi}_2)^2 + V(\phi_{1,}\phi_2)\}$$
$$= \frac{4}{3}\pi R^3\{\frac{1}{2}(m_1^2 + \lambda\phi_1^2 + g\phi_2^2)\phi_1^2 + V(\phi_{1,}\phi_2)\}$$
$$= c\cdot(m_1^2 + 2vM\sqrt{\frac{\lambda_1}{2}} - \frac{1}{2}\lambda_1\frac{M^2}{g}) \qquad (109)$$

On the other hand, if ϕ_1 is in free state, that means the scalar field spreads out all of the universe and $\dot{\phi}_1 = \ddot{\phi}_1 = 0$, then total energy $\tilde{E}$ with $\int dv\phi_1^2 =$ constant is

$$\tilde{E} = \int dv\{(\frac{1}{2}\dot{\phi}_1)^2 + (\frac{1}{2}\dot{\phi}_2)^2 + V(\phi_{1,}\phi_2)\} \qquad (110)$$

$$=\frac{1}{2}m_1^2\cdot\frac{4}{3}\pi R^3\phi_1^2+\frac{4}{3}\pi R^3\cdot\frac{1}{2}\phi_1^2 v^2$$
$$=c\cdot(\frac{1}{2}m_1^2+\frac{1}{2}gv^2) \quad (111)$$

With $\lambda << 1, gA^2 > m_1^2$, then $E < \tilde{E}$, on other word, the droplet is a bound state of ϕ_1 !**.**

In this potential, there are five free parameters that we can adjust to satisfy the constraint.

So we can say that the droplet is stable.

$$p=\frac{4}{3}\pi R^3\cdot\frac{1}{2}(m_1^2+\lambda\phi_1^2+g\phi_2^2)\phi_1^2-V=0 \quad (112)$$

w=p/ρ =0, so it is matter like.

Outside the droplet, r>R+ε, $\phi_1=0,\phi_2=v$, and at surface r=R ~ R+ε, the equation becomes,

$$\begin{aligned}&\nabla^2\phi_1+g\beta^2\phi_1+\lambda_1\alpha^2\phi_1-g\phi_2^2\phi_1-\lambda_1\phi_1^3=0\\&\nabla^2\phi_2-g\phi_1^2\phi_2-M^2(\phi_2-v)=0\end{aligned} \quad (113)$$

With

$$\begin{aligned}&\alpha^2=\sqrt{\frac{2}{\lambda}}vM-\frac{M^2}{g}\\&\beta^2=\frac{\lambda_1}{2}\frac{M^2}{g^2}\end{aligned} \quad (114)$$

The equations cannot be solved by hand, but could be solved by numerator (In fact, even numerator, this is still very hard to solve). Here, instead of solving the equations, we simply assume the surface is thin, in other words, we assume that $\frac{\varepsilon}{R}\ll 1$. In this case, we can neglect the effect contributed by the surface.

From the observation, we know that the 73% of total energy is dark energy and 23% is dark matter, and 4% of the total energy is normal matter, as shown in the figure below.

Now, since we treat the vacuum of potential of scalar field ϕ_2 as dark energy, and the droplet of scalar field ϕ_1 as dark matter, this two scalar field must satisfy the constraint from experiment observation.

First, two scalars must satisfy the constraint that 73% of energy is dark energy and 23% is dark matter in today's universe which has temperature of 3K.

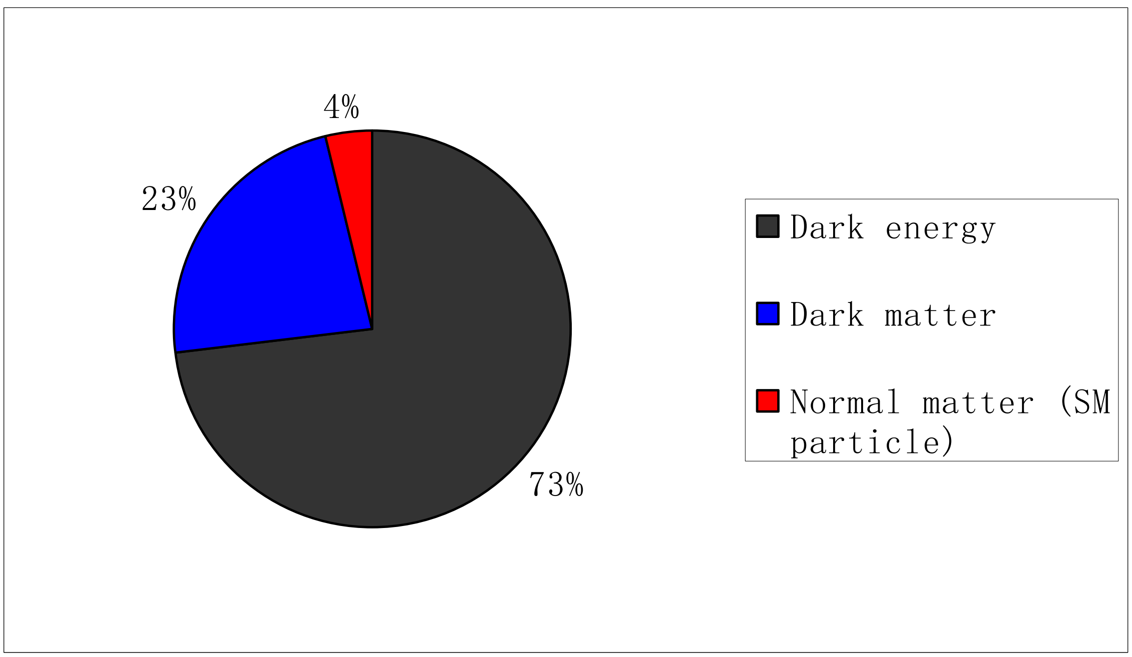

Fig.25 Energy distribution in the universe

Figure 25 shows the energy distribution in our universe.

We discuss one extremely case, that is all the dark energy is contributed by the scalar field ϕ_2, and all the dark matter is droplet of scalar field ϕ_1. Due to the interaction term

$$L_{\text{int}} = \frac{1}{2} g \phi_1^2 \phi_2^2$$

The dark matter droplet of scalar field ϕ_1 very easily decays into the dark energy, so the proportion of dark energy and dark matter should satisfy Boltzman distribution. Let's assume that the droplet can have a series of radius, and the minimum radius is R_1 (I am not sure whether this assumption is correct since I cannot solve the equation at surface) with

$$C_1 = \int dv\phi_1^2 = \frac{4}{3}\pi R_1^3 \phi_1^2$$

Then the energy of this dark matter droplet is

$$E_1 \approx C_1 m_1^2$$

If this droplet decays into scalar field ϕ_2, then the energy is

$$E_{DE} \approx C_1 m_2^2$$

So we can have

$$\frac{\rho_{DM}}{\rho_{DE}} = \frac{23}{73} = 0.315 \approx \exp(-\frac{C_1(m_1^2 - m_2^2)}{3\times 10^{-4} eV}) \tag{115}$$

Define that

$$z \equiv C_1(m_1^2 - m_2^2) \approx 3.47\times 10^{-4} eV$$

So, if the temperature of universe is T when T is ~ 3 K, then

$$\frac{\rho_{DM}(T)}{\rho_{DE}(T)} \approx \exp(-\frac{3.47\times 10^{-4} eV}{T}) \tag{116}$$

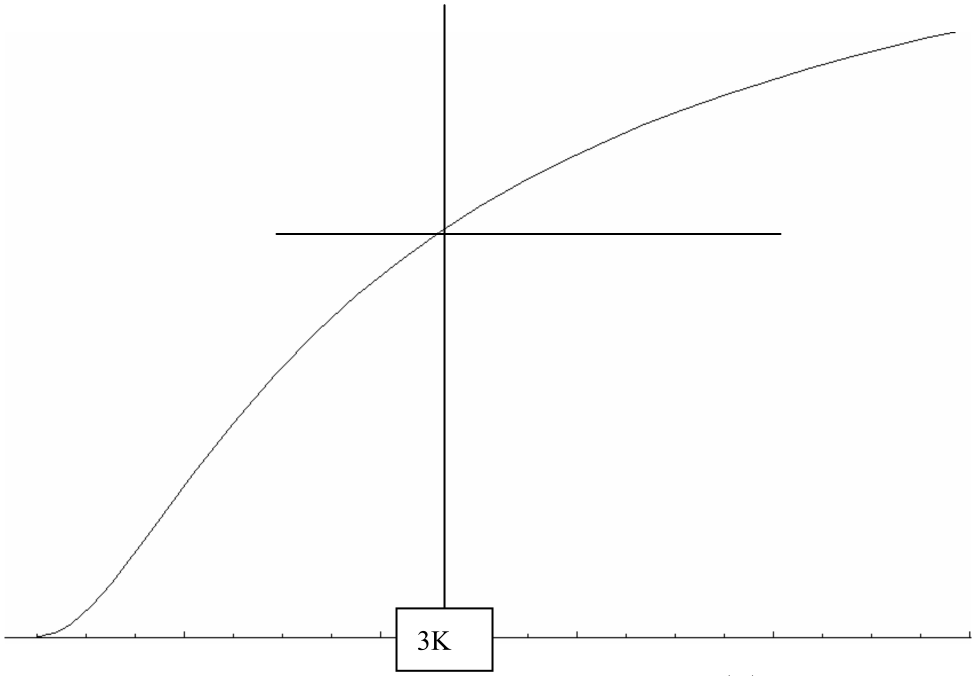

Fig.26 Ratio of dark matter to dark energy $\frac{\rho_{DM}(T)}{\rho_{DE}(T)}$ varies with temperature

Figure 26 shows when T ~ 3K, the ratio of dark matter to dark energy $\frac{\rho_{DM}(T)}{\rho_{DE}(T)}$ varies with time. From the observation of Supernovae, we can testify this model.

When the temperature of the universe much bigger than 3K, we cannot only consider the smallest droplet, but all of the droplet with different radius. For example, when T ~ 1Mev. At this temperature, we have the cosmological constraint for nucleosynthesis, which required that

$$\Omega_{\phi}(T \sim 1MeV) < 0.2$$

Or we can see that

$$\frac{\rho_{DM}(1MeV)}{\rho_{DE}(1Mev)} > 4 \tag{117}$$

It's reasonable the assume that the energy of droplet is much smaller than 1Mev, then we need at least 4 different state of droplet to satisfy the constraint of nucleosynthesis. If there are 6 states, then

$$\Omega_{\phi}(T \sim 1MeV) \sim 0.14$$

We have talked scalar field with potential

$$\begin{aligned} V(\phi_1,\phi_2) &= \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4 \\ &+\frac{1}{2}g\phi_1^2\phi_2^2 \\ &-\frac{1}{2}m_2^2\phi_2^2 + \frac{1}{4}\lambda_2\phi_2^4 \end{aligned} \tag{118}$$

And we have shown that it is possible that the scalar field can act as dark energy and dark matter. In fact, for all kinds of scalar fields which with potential and interaction like

$$\begin{aligned} V(\phi_1) &= \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4 \\ L_{\text{int}} &= +\frac{1}{2}g\phi_1^2\phi_2^2 \\ V(\phi_2) &\approx \frac{1}{2}M(\phi_2 - A)^2 \end{aligned} \tag{119}$$

A droplet of scalar field ϕ_1 can formed in the field ϕ_2.

Another interesting case is scalar field with potential

$$\begin{aligned} V(\Phi_1,\Phi_2) = \frac{1}{2}m_1^2\Phi_1^2 &+ \frac{1}{4}\lambda_1\Phi_1^4 \\ &+\frac{1}{2}g\Phi_1^2\Phi_2^2 \\ &+\frac{1}{2}m_2^2\Phi_2^2 + \frac{1}{4}\lambda_2\Phi_2^4 \end{aligned} \tag{120}$$

Where Φ_1 is a singlet scalar field while Φ_2 is doublet,

$$\Phi_2 = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

And $\lambda_{1,}\lambda_2, g, m_1^2$ are positive number, m_2^2 is negative number.

We can recognize that the potential of Φ_2 just looks like the scalar field in Higgs Mechanism. After spontaneous broken, the potential of (120) will looks like (119) which can act like dark energy and dark matter.

One of the advantage of this model is it is related to Standard Model, and makes it is possible to unify a dark world into standard model.

In 1985, Sidney Coleman published his paper "Q BALLS" [15], and since there is some kind of resemblance between "Q Balls" and Droplet, we would like to talk briefly about Q ball here.

Q Balls

In Sidney Coleman model, he discussed the Lagrange density which is the SO(2)-invariant theory of two real scalar fields with non derivative interactions.

$$L = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - U(\phi) \qquad (121)$$

where $\phi = \sqrt{\phi_1^2 + \phi_2^2}$. The SO(2) symmetry is

$$\begin{aligned} \phi_1 &\to \phi_1 \cos\alpha - \phi_2 \sin\alpha \\ \phi_2 &\to \phi_2 \cos\alpha + \phi_1 \sin\alpha \end{aligned} \qquad (122)$$

The associated conserved current is

$$j_\mu = \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 \qquad (123)$$

And the conserved charge is

$$Q = \int d^3x j_0 \qquad (124)$$

By convention,U(0)=0. If this is the absolute minimum of U (the center of interest),

$\phi = 0$ is the ground state of the theory and the SO(2) symmetry is unbroken. The perturbative particle spectrum consists of spinless mesons with $Q = \pm 1$ and mass μ, where

$$\mu^2 = U''(0) = [2U/\phi^2]_{\phi=0} \quad (125)$$

Sidney Coleman showed in his paper that

New particles appear in the spectrum of the theory if U is such that the minimum of U/ϕ^2 is at some points of $\phi_0 \neq 0$. In equations,

$$\min[2U/\phi^2] \equiv 2U_0/\phi_0^2 < \mu^2 \quad (126)$$

In this case, for sufficiently large Q, there exist nondissipative solutions of the classical field equations that are absolute minima of the energy for fixed Q. Thus they are absolutely stable.

For appropriate choice of the origin of space-time, these solutions are of the form

$$\begin{aligned} \phi_1 &= \phi(r)\cos\omega t \\ \phi_2 &= \phi(r)\sin\omega t \end{aligned} \quad (127)$$

where $\phi(r)$ is a monotonically decreasing function of distance form the origin, going to zero at infinity, and ω is a constant. In other words, these objects rotate with constant angular velocity in internal space and are spherically symmetric in position space.

As Q goes to infinity, ω approaches

$$\omega_0 = \sqrt{2U_0/\phi_0^2} \quad (128)$$

In this same limit, ϕ resembles a smoothed-out step function. Fro r less than a certain radius, R, $\phi = \phi_0$; outside this radius, $\phi = 0$; these two regions are connected by a transition zone with thickness on the order of μ^{-1}. R can easily be computed

$$Q = \frac{4\pi}{3} R^3 \omega_0 \phi_0^2 \qquad (129)$$

This is very much like the description of a ball of ordinary matter, some substance that has a thermodynamic limit; the values of local quantities inside a sample are independent of sample size for sufficiently large samples. The stability of ordinary matter depends on the conservation of particle number, and the radius of a ball of ordinary matter depends on the number of particles in it. Here, the role of particle number is played by Q.

A sphere of ordinary matter has a rich spectrum of small vibrations about its equilibrium state. Some of these have minimum frequencies that go to zero as the radius of the sphere goes to infinity. These lead in quantum theory to extremely low-lying excited states.

Q balls occur whenever we are near a first-order symmetry-breaking phase transition. Figure 27 is a sketch of U for such a situation. U_+, the value of U at the local minimum, ϕ_+, is positive, so the symmetry is unbroken; however, if U_+ is sufficiently small, $2U_+/\phi_+^2$ is less than μ^2, so Q ball exist.

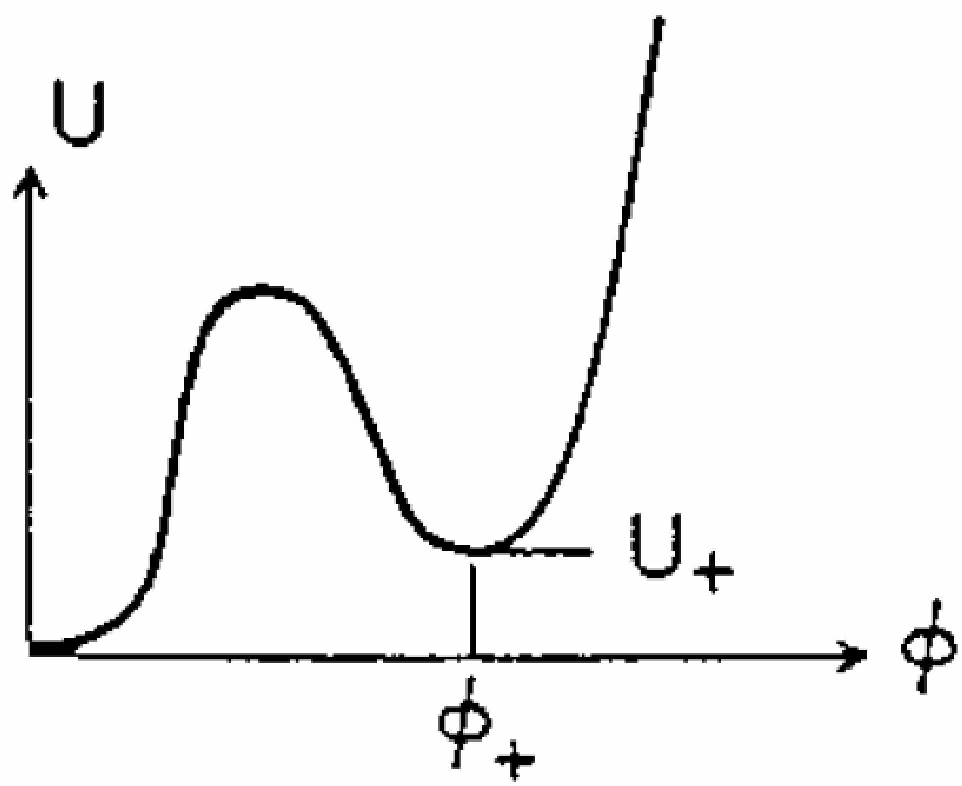

Fig.27 Symmetry-breaking phase transition

Sidney Coleman assumed in his paper that there exists a solution to the equations of motion of the general form: Within some sphere of volume V, ϕ is a constant; outside the sphere, it is zero. Furthermore, ϕ is in stead rotation in internal space, with some frequency ω. And he attempted to find the relations among these quantities by minimizing the energy at fixed Q. (Attention that, in our droplet model, we attempt to find relations by minimizing the potential at fixed amount of the scalar field).

Same as our droplet model, in Sidney Coleman's paper, he also neglected the contributions to both E and Q coming from the transition zone at the surface of the sphere.

The exact expression for the energy is

$$E = \int d^3x[\frac{1}{2}\dot{\phi}_1^2 + \frac{1}{2}\dot{\phi}_2^2 + \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}(\nabla\phi_2)^2 + U] \quad (130)$$

With approximation, this becomes

$$E = \frac{1}{2}\omega^2\phi^2 V + UV \quad (131)$$

And

$$Q = \omega\phi^2 V \quad (132)$$

We wish to minimize E with fixed Q,

$$E = \frac{1}{2}\frac{Q^2}{\phi^2 V} + UV \quad (133)$$

As a function of V, this has its minimum at

$$V = Q / \sqrt{2\phi^2 U} \quad (134)$$

Here,

$$E = Q\sqrt{2U/\phi^2} \quad (135)$$

The last step is to minimize this as a function of ϕ. This gives the definition of ϕ_0.

All shapes of the same volume are degenerate in energy. This is because we have neglected the contributed to the integral form the transition zone connecting the interior of the Q ball to the vacuum outside. We would expect this to make a positive contribution to the Q-ball energy proportional to its surface area. This lifts the degeneracy and selects among all shapes of the same volume the one of minimum area, to wit, the sphere.

The most obvious way for a Q ball to decay is by emitting charged mesons. The energy per unit charge, E/Q, is $\sqrt{2U_0/\phi_0^2}$. Thus, a Q ball is stable under meson emission if this number is less than the meson mass μ .

Another obvious decay mode is quantum tunneling. Q matter is much like a false vacuum, in that it is a homogeneous state of nonzero ϕ . We know a false vacuum decay by quantum tunneling; quantum fluctuations produce a bubble of true vacuum similarly appear inside the false vacuum, which grows classically. Quantum tunneling conserves both Q and E, and, by the arguments, the only such state is a spherical Q ball, not a Q ball with a cavity.

With sufficiently large Q, it is easy to show that the Q ball is a solution of the equations of motion.

Solving the equation of motion, we find

$$\frac{d^2\phi}{dr^2} = -\frac{2}{r}\frac{d\phi}{dr} - \omega^2\phi + U'(\phi) \qquad (136)$$

This is essentially identical to an equation that occurs in the theory of vacuum decay, and can be treated by methods used there. [16]

If we interpret as a particle motion for a particle of unit mass subject to viscous damping(with a coefficient inversely proportional to the time) and moving in the potential $\frac{1}{2}\omega^2\phi^2 - U$. This potential is sketched in Fig.28.

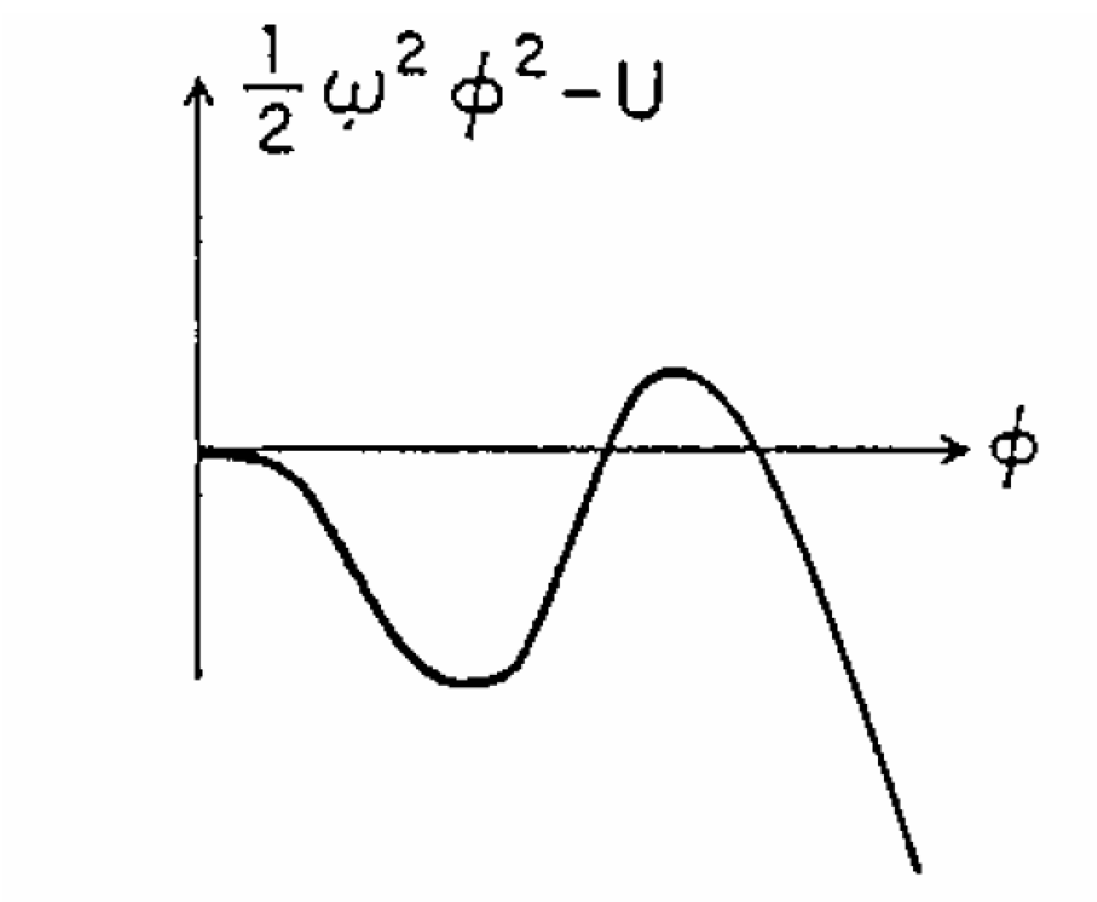

Fig.28 Potential of $\frac{1}{2}\omega^2\phi^2 - U$

The sketch is drawn for $\omega_0^2 < \omega^2 < \mu^2$; this is the range in which we shall find solutions. The curve is qualitatively different outside this range: if ω^2 is greater than μ^2, the hill at the origin becomes a valley; if ω^2 is less than ω_0^2, the hill on the right is lower than the hill at the origin. We are searching the particle starts out at time zero at some position, $\phi(0)$, at rest, $\frac{d\phi}{dr} = 0$, and comes to rest at infinite time at $\phi = 0$.

We can now get a precise description of the surface of a large Q ball, and we can compute the surface tension, the surface-area-dependent term in the energy. For a large Q ball, in the neighborhood of the surface, we can neglect the damping term in the equation of motion; also, we can approximate ω^2 by ω_0^2. Thus, near the surface we must solve

$$\frac{d^2\phi}{dr^2} = \frac{d}{d\phi}[-\frac{1}{2}\omega_0^2\phi^2 + U] \equiv \frac{d}{d\phi}\hat{U}(\phi) \qquad (137)$$

A first integral of this is

$$\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 - \hat{U} = 0 \qquad (138)$$

(This integral must vanish because ϕ goes to zero as r goes to infinity.)

Thus

$$R - r = \int_{\bar{\phi}}^{\phi} \sqrt{2\hat{U}}\, d\phi \qquad (139)$$

Here R is the radius of the Q ball, the place where $\phi = \bar{\phi}$. Of course, we are free to define this radius to be anywhere we want inside the somewhat fuzzy surface, free to choose $\bar{\phi}$ to be anywhere between 0 and ϕ_0.

For purposes, it will convenient to define $\bar{\phi}$ by demanding that

$$\int d^3\underline{x}\phi^2 = \frac{4\pi}{3} R^3 \phi_0^2 \qquad (140)$$

It is easy to shown that this indeed defines a choice of $\bar{\phi}$ independent of R, for large R. The equation about can be rewritten as

$$\int d^3\underline{x}[\phi^2 - \phi_0^2\theta(R-r)] = 0 \qquad (141)$$

If f(r) is some function that is concentrated at r near R, then for large R we can make the approximation

$$\int d^3\underline{x} f = 4\pi R^2 \int_{-\infty}^{\infty} f(r) dr \qquad (142)$$

In this approximation, we have

$$\int_{-\infty}^{\infty} dr[\phi^2 - \phi_0^2\theta(R-r)] = 0 \qquad (143)$$

If we shift the integration variable from r to r-R, we see that this condition is independent of R.

We can now compute Q and E as function of R.

$$Q = \frac{4\pi}{3} R^3 \phi_0^2 \omega_0 \qquad (144)$$

E can be written as $E_{surface}$ and E_{volume}, where

$$E_{surface} = \int d^3\underline{x}[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \hat{U}] \qquad (145)$$

And

$$E_{volume} = \int d^3\underline{x}\omega_0^2\phi^2 \qquad (146)$$

And it can be written as

$$E_{volume} = \frac{4\pi}{3} R^3 \phi_0^2 \omega_0^2 = \frac{8\pi}{3} R^3 U_0 \qquad (147)$$

We can use the approximation

$$\int d^3\underline{x} f = 4\pi R^2 \int_{-\infty}^{\infty} f(r) dr \qquad (148)$$

to calculate the surface energy

$$E_{surface} = 4\pi R^2 \int dr[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \hat{U}]$$

$$= 4\pi R^2 \left[\int_0^{\phi_0} d\phi \sqrt{2\hat{U}}\right] \qquad (149)$$

The quantity in square brackets is the surface-tension coefficient.

For sufficiently large Q, there is existence of Q balls and it is stable.

The first theorem has to do with initial-value data, the fields and their time derivatives at fixed time. In his paper, Sidney Coleman defined a set of initial-value data to be of Q-ball type if $\phi_1 = \phi(r), \phi_2 = 0, \dot{\phi}_1 = 0$ and $\dot{\phi}_2 = \omega\phi(r)$, where ω is a constant and $\phi(r)$ is a positive function monotone decreasing to zero as r goes to infinity.

Theorem 1: For any theory of type（121）, with U>0, given some set of initial-value data. With some Q and E, there is a set of initial-value data of Q-ball type with the same Q and lesser or equal E.

Definition: An interaction is "acceptable" if

(a) U(0)=0 and U is positive everywhere else. U is twice continuously differentiable. $U^{'}(0) = 0, U^{''}(0) = \mu^2$.

(b) The minimum of U/ϕ^2 is attained at some $\phi_0 \neq 0$.

(c) There exist three positive numbers, a, b, and c, with c>2, such that

$$\frac{1}{2}\mu^2\phi^2 - U \leq \min(a, b\phi^c)$$

Theorem 2: If U is acceptable, there exists $Q_{\min} \leq 0$, such that for any $Q > Q_{\min}$, there is initial value of Q-ball type that minimizes E for that value of Q. Furthermore, this is the initial-value data for a Q-ball solution of the equations of motion.

This theorem guarantees both the existence and the absolute stability of Q balls.

There are small vibrations of Q-balls whose frequencies go to zero as R goes to infinity. Upon quantization, these become the excitation levels of lowest energy for large R. In the limit of infinite R, any such family of vibrations must come down to a vibration of zero frequency. By identifying these zero modes we know where to look for the desired vibrations.

If we consider infinite space filled with Q matter, there is an obvious zero mode generated by infinitesimal Q rotations. As we shall see, this is the zero wave-vector limit of a sound wave, which, for small wave-vector, $\underline{k}$, has frequency proportional to $|\underline{k}|$. For a finite Q ball, $|\underline{k}|$ takes discrete values proportional to 1/R. Thus we have a spectrum of excitations (phonons) with energies proportional to 1/R.

Another way of going to the infinite-R limit is to sit, not at the center of Q ball, but at its surface. There is an obvious zero mode, associated with translations of the interface normal to itself. This is the zero wave-vector limit of a surface wave, which, for small $|\underline{k}|$, has frequency proportional to $|\underline{k}^{3/2}|$. Thus we have a spectrum of excitations with energies proportional to $1/R^{3/2}$. In the case of interest, R is much larger than the natural length scales of the theory. Thus, the surface excitations have much lower energies than the phonons.

It is convenient to study small perturbations about Q matter in internally corotating coordinates. Defining δ_1 and δ_2 as,

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\omega_0 t & -\sin\omega_0 t \\ \sin\omega_0 t & \cos\omega_0 t \end{pmatrix} \begin{pmatrix} \phi_0 + \delta_1 \\ \delta_2 \end{pmatrix}$$

If we insert this into the equations of motion, and work only to first order in the δ's, we find

$$\begin{aligned} &\ddot{\delta}_1 + 2\omega_0\dot{\delta}_2 - \nabla^2\delta_1 + U_0''\delta_1 - \omega_0^2\delta_1 = 0 \\ &\ddot{\delta}_2 - 2\omega_0\dot{\delta}_1 - \nabla^2\delta_2 + U_0'\delta_2/\phi_0 - \omega_0^2\delta_2 = 0 \end{aligned} \tag{150}$$

where a zero subscript denotes a quantity evaluated at $\phi = \phi_0$. Because ϕ_0 is a stationary point of U/ϕ.

The equation (150) is invariant under space-time translations. Thus the normal modes can be chosen to be proportional to exp (ikx), where k is a zero of the determinant

$$\begin{vmatrix} -(k^0)^2+|\underline{k}|^2+U_0^{"}-\omega_0^2 & 2i\omega_0 k^0 \\ -2i\omega_0 k^0 & -(k^0)^2+|\underline{k}|^2 \end{vmatrix} \tag{151}$$

Retaining only the leading terms for small k^0 and $|\underline{k}|$, we have

$$(U_0^{"}-\omega_0^2)|\underline{k}|^2=(U_0^{"}+3\omega_0^2)(k^0)^2 \tag{152}$$

This is an acoustic dispersion equation, with the velocity of sound given by

$$v_s^2=\frac{U_0^{"}-\omega_0^2}{U_0^{"}+3\omega_0^2} \tag{153}$$

We need the contribution of the Q-ball surface to the energy-momentum tensor to calculate the vibration mode of surface. Because we are studying waves of very long wavelength, it is reasonable to approximate the surface as being infinitely thin. Thus, for example, for a surface occupying the plane $x^3=0$, the energy density is approximated as

$$T_{surface}^{00}=\alpha\delta(x^3) \tag{154}$$

where α is the surface-tension coefficient

$$\alpha=\int_0^{\phi_0} d\phi[2U-\omega_0^2\phi^2]^{1/2} \tag{155}$$

It is straightforward to use

$$T^{\mu\nu}=\partial\phi^{\mu}\partial\phi^{\nu}-g^{\mu\nu}L \tag{156}$$

to compute the other components of energy momentum tensor. The only nonzero components are

$$T_{surface}^{11}=T_{surface}^{22}=-T_{surface}^{00} \tag{157}$$

And this can be written as

$$T^{\mu\nu}_{surface} = \alpha[g^{\mu\nu} + n^{\mu}n^{\nu}]\delta_s(x) \tag{158}$$

Here n^{μ} is the unit space like normal vector to the surface, and $\delta_s(x)$ is the delta-function concentrated on the surface.

It will be convenient to introduce the characteristic function of the Q ball, $\chi(x)$, defined by $\chi = 1$ inside the Q ball and $\chi = 0$ outside. χ is related to n^{μ} and $\delta_s(x)$ by

$$\partial_{\mu}\chi = n_{\mu}\delta_s \tag{159}$$

In terms of these,

$$T^{\mu\nu} = [(e+p)u^{\mu}u^{\nu} - g^{\mu\nu}p]\chi + \alpha[g^{\mu\nu} + n^{\mu}n^{\nu}]\delta_s \tag{160}$$

Also

$$j^{\mu} = nu^{\mu}\chi \tag{161}$$

Thus, the terms that appear on the right-band side of the conservation laws fall into two sides. From equation (160) and equation (161), we can find that

$$u^{\mu}n_{\mu}\delta_s = 0 \tag{162}$$

$$(-pn^{\nu} + \alpha\partial_{\mu}n^{\mu}n^{\nu} + \alpha n^{\mu}\partial_{\mu}n^{\nu})\delta_s = 0 \tag{163}$$

If we dot n_{ν} into this and use $n_{\nu}n^{\nu} = -1$， we find

$$(-p + \alpha\partial_{\mu}n^{\mu})\delta_s = 0 \tag{164}$$

Let us consider a Q ball which occupies all points obeying

$$x^3 \le \eta(x^0, x^1, x^2)$$

To first order in η

$$\begin{aligned} \chi &= \theta(\eta - x^3) \\ n_{\mu} &= \partial_{\mu}(\eta - x^3) \end{aligned} \tag{165}$$

And the fundamental surface equations becomes

$$\dot{n} = u^3 \qquad \text{(at } x^3 = 0\text{)}$$
$$\alpha \partial_\mu \partial^\mu \eta = \delta p$$

By translational invariances of the problem, we can always choose our normal modes to be of the form

$$\delta p = f(x^3)\exp(ik^0x^0 - ik_\perp x_\perp) \tag{166}$$

From the wave equation,

$$[v_s^2\partial_3^2 + (k^0)^2 - v_s^2 |k_\perp|] f = 0 \tag{167}$$

If $v_s^2 |k_\perp|^2$ is greater than $(k^0)^2$, we can have damped behavior

$$f = e^{kx^3}$$

with

$$v_s^2 k^2 = v_s^2 |k_\perp|^2 - (k^0)^2 \tag{168}$$

These are the surface waves. Eliminating k, we find

$$v_s^2 e_0^2 (k^0)^4 = \alpha^2 [v_s^2 |k_\perp|^2 - (k^0)^2][(k^0)^2 - |k_\perp|^2]^2 \tag{169}$$

For small $|k_\perp|$, we may neglect k^0 on the right. Thus,

$$(k^0)^2 = \frac{\alpha}{e_0} |k_\perp|^3 \tag{170}$$

Before we move on our droplet model, we need to talk some thing about the scalar singlet, and we found that the scalar singlet is very natural in nature.

Scalar Singlet and Droplet

We have talked that we need a scalar singlet to give the inflationary universe at the beginning of universe, and many models of physics beyond the Standard Model suggest the existence of new scalar gauge singlet, e.g., in the so called next-to-minimal super symmetric standard model. Since the only scalar field in Standard Model is Higgs field, which cannot be a candidate of dark energy and dark matter, we need to modification of the Standard Model, and the simplest modification is introducing a single spinless species of new particle field, S, to those of the Standard Model, using only renormalizabel interactions. To keep the new particle from interacting too strongly with ordinary matter, it is taken to be completely neutral under the Standard Model gauge group. The model was first introduced by Veltman and Yndurain [17] in a different context. Its cosmology was later studied by Silverira and Zee [18], and (with a complex scalar) by McDonald [19]. It is the absolute minimal modification of the Standard Model which can explain the dark matter. A good review of this model has been done by C.P.Bergess.[20]. The lagrangian which describes this model has the following form:

$$L = L_{SM} + \frac{1}{2}\partial_{\mu}S\partial^{\mu}S - \frac{m_0^2}{2}S^2 - \frac{\lambda_s}{4}S^4 - \lambda S^2 H^{+}H \tag{171}$$

Where H and L_{SM} respectively denote the Standard Model Higgs doublet and lagrangian, and S is a real scalar field which does not transform under the Standard Model gauge group. We assume S to be the only new degree of freedom relevant at the electroweak scale, permitting the neglect of nonrenormalizable coupling, which contains all possible renormalizable interactions consistent with the field content and symmetry $S \to -S$.

Another simple example [21] for the realization of the idea proposed in [22] of a self-interacting, non-dissipative cold dark matter candidate that is based on anextra gauge singlet, ϕ, coupled to the Standard Model Higgs boson, h, with a Lagrangian density given by:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi \phi^2 - \frac{g}{4}\phi^4 + g^{'} v \phi^2 h$$

where g is the field ϕ self-coupling constant, m_ϕ is its mass, v=246GeV is the Higgs vacuum expectation value and $g^{'}$ is the coupling between the singlet and h. We assume that the mass does not arise from spontaneous symmetry breaking since tight constraints from non-Newtonian forces eliminates this possibility due to the fact that, in this case, there is a relation among coupling constant, mass and vacuum expectation value that results in a tiny scalar self-coupling constant.

So, we have three reasons to have scalar singlet:

1. We need dark matter which is cannot be in Standard Model.

The first candidate of dark matter which has been discussed many years is massive neutrino. The massive neutrino comes from the discovery so called solar neutrino missing. There is standard model for sun which is very successful and predicts the amount of neutrinos which can be received at earth emitting from sun. But surprising, we only find 65% of neutrinos of predicted. One way to explain this phenomenon is add a little mass to neutrino. By this way, electrical neutrino will oscillate into other types of neutrino. But unfortunately, we found that neutrino can only be a candidate of hot dark matter, and we still need the cold dark matter.

2. Dark energy. Dark energy is used to explain the accelerated expansion of universe.

The simplest way to achieve this is by introducing the so called cosmology constant. But cosmology constant has serious fine tunneling problem, and it seems, at least by cosmology constant itself, we cannot explain the expansion of universe. A more general way to solve this problem is by introducing a scalar field so called quintessence, which varies with time. But this scalar field cannot be any thing in Standard Model since the only scalar field in SM is Higgs field.

3. Reheating. As we have talked, there is very important stage of universe called inflationary. At the end of inflationary, the inflationary field begins to oscillate at its minimum and produce almost all particles in this world. Inflationary field usually described as a scalar field, but it cannot be in Standard Model.

4. Scalar singlet is possibly the simplest correction to Standard Model.

We need to mention here that our droplet model, which is classical ball, has the exactly same lagrangian as (171). I don't want to show some other models which also can give such kinds of lagrangian since there are too many such models, you can break higher symmetry to give both Standard Model and scalar singlet, such as 3-3-3 models. [23]

So, it is very natural to have a scalar singlet beyond Standard Model, and our model with potential

$$
\begin{aligned}
V(\phi,\Phi) &= \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda_1\phi^4 \\
&+\frac{1}{2}g\phi^2\Phi^2 \\
&-\frac{1}{2}M^2\Phi^2 + \frac{1}{4}\lambda_2\Phi^4 \\
\Phi &= \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}
\end{aligned}
\tag{172}
$$

becomes a promising model.

I have talked that with potential like (172), where ϕ is scalar singlet which does not transform under the Standard Model gauge group, and Φ is the Standard Model Higgs doublet, then a stable droplet will be formed due to the interaction term $\frac{1}{2}g_z\phi^2\Phi^2$, where g_z is positive interaction constant. But so far, we haven't talked about the surface. It's obvious that Q-balls is totally difference model with our droplet model, they deal with different problems and with different potential. But there is some kind of resemblance. They both treat the field as classical field, and have a sphere structure. Because of this we will deal with the contribution of surface same as Q-balls.

At surface, we have shown that the equations of motion are

$$
\begin{aligned}
&\nabla^2\phi_1 + g\beta^2\phi_1 + \lambda_1\alpha^2\phi_1 - g\phi_2^2\phi_1 - \lambda_1\phi_1^3 = 0 \\
&\nabla^2\phi_2 - g\phi_1^2\phi_2 - M^2(\phi_2 - v) = 0
\end{aligned}
\tag{173}
$$

with

$$
\begin{aligned}
\alpha^2 &= \sqrt{\frac{2}{\lambda}}vM - \frac{M^2}{g} \\
\beta^2 &= \frac{\lambda_1}{2}\frac{M^2}{g^2}
\end{aligned}
\tag{174}
$$

We replace

$$\phi_2 \to \phi_2 - v$$

To reason that we do this transformation is to make the field go to zero as r go to infinity. Then the equation of (173) becomes

$$\begin{aligned} &\nabla^2\phi_1 + g\beta^2\phi_1 + \lambda_1\alpha^2\phi_1 - g(\phi_2+v)^2\phi_1 - \lambda_1\phi_1^3 = 0 \\ &\nabla^2\phi_2 - g\phi_1^2(\phi_2+v) - M^2\phi_2 = 0 \end{aligned} \tag{175}$$

As we have talked, that the equations (175) cannot be solved by hand because of its complicated. But we can get the energy of surface when the droplet becomes truly big, then approximately the equation of (175) can be written as

$$\begin{aligned} &\frac{d^2}{dr^2}\phi_1 + \frac{2}{r}\frac{d}{dr}\phi_1 = \frac{d}{d\phi_1}[-\frac{1}{2}g\beta^2\phi_1^2 - \frac{1}{2}\lambda_1\alpha^2\phi_1^2 + \frac{1}{2}g(\phi_2+v)^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4] \\ &\frac{d^2}{dr^2}\phi_2 + \frac{2}{r}\frac{d}{dr}\phi_2 = \frac{d}{d\phi_2}[g\phi_1^2(\frac{1}{2}\phi_2+v)\phi_2 + \frac{1}{2}M^2\phi_2^2] \end{aligned} \tag{176}$$

With big r, we can neglect the damping terms in equation (176)

$$\begin{aligned} &\frac{d^2}{dr^2}\phi_1 = \frac{d}{d\phi_1}[-\frac{1}{2}g\beta^2\phi_1^2 - \frac{1}{2}\lambda_1\alpha^2\phi_1^2 + \frac{1}{2}g(\phi_2+v)^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4] \\ &\frac{d^2}{dr^2}\phi_2 = \frac{d}{d\phi_2}[g\phi_1^2(\frac{1}{2}\phi_2+v)\phi_2 + \frac{1}{2}M^2\phi_2^2] \end{aligned} \tag{177}$$

A first integral of this is

$$\begin{aligned} &\frac{1}{2}\left(\frac{d\phi_1}{dr}\right)^2 - [-\frac{1}{2}g\beta^2\phi_1^2 - \frac{1}{2}\lambda_1\alpha^2\phi_1^2 + \frac{1}{2}g(\phi_2+v)^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4] = 0 \\ &\frac{1}{2}\left(\frac{d\phi_2}{dr}\right)^2 - [g\phi_1^2(\frac{1}{2}\phi_2+v)\phi_2 + \frac{1}{2}M^2\phi_2^2] = 0 \end{aligned} \tag{178}$$

We have transformed ϕ_2 to make both of ϕ_1 and ϕ_2 go to zero as r go to zero.

It will be convenient to define the radius of droplet R as

$$\int d^3x\phi_1^2 = \frac{4\pi}{3}R^3\alpha^2 \tag{179}$$

If f(r) is some function that is concentrated at r near R, then for large R we can make the approximation

$$\int d^3xf = 4\pi R^2\int_{-\infty}^{\infty} f(r)dr \tag{180}$$

The energy of surface can be written as

$$E_{surface} = \int d^3x[\frac{1}{2}\dot{\phi}_1^2 + \frac{1}{2}\dot{\phi}_2^2 + \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}(\nabla\phi_2)^2 + U]$$

We have calculated that

$$\begin{aligned}\dot{\phi}_1 &= in\phi_1\\ \dot{\phi}_2 &= 0\end{aligned} \tag{181}$$

And with big r, the equations of motion are

$$\begin{aligned}\frac{d^2\phi_1}{dr^2} &= \frac{\partial}{\partial\phi_1}[-\frac{1}{2}n^2\phi_1^2 + U]\\ \frac{d^2\phi_2}{dr^2} &= \frac{\partial}{\partial\phi_2}U\end{aligned} \tag{182}$$

where

$$n^2 = m_1^2 + \lambda\alpha^2 + g\phi_2^2$$

And

$$\begin{aligned}U(\phi_1,\phi_2) &= \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{4}\lambda_1\phi_1^4\\ &+\frac{1}{2}g_z\phi_1^2(\phi_2 + v)^2\\ &+\frac{1}{2}(2m_2^2)\phi_2^2\end{aligned} \tag{183}$$

A first integral of equation (282) is

$$\frac{1}{2}\left(\frac{d\phi_1}{dr}\right)^2 = -\frac{1}{2}n^2\phi_1^2 + U$$
$$\frac{1}{2}\left(\frac{d\phi_2}{dr}\right)^2 = U \quad (184)$$

So the equation (280) becomes

$$E_{surface} = \int d^3x[\frac{1}{2}n^2\phi_1^2 + \frac{1}{2}(\frac{d}{dr}\phi_1)^2 + \frac{1}{2}(\frac{d}{dr}\phi_2)^2 + U]$$
$$= \int d^3x[\frac{1}{2}(\frac{d}{dr}\phi_2)^2 + 2U]$$

With the approximation of (279), we can have

$$E_{surface} = 4\pi R^2 \int dr[\frac{1}{2}(\frac{d}{dr}\phi_2)^2 + 2U]$$
$$= 4\pi R^2 \left[\frac{3}{2}\int_{\beta-v}^{0} d\phi_2 \sqrt{2U}\right] \quad (185)$$

The quantity in square brackets is the surface-tension coefficient.

I have to mention that the integral in equation (185) is still hard to deal with since U is a function of both ϕ_1 and ϕ_2. But at least we can calculate the upper limit of the integral

$$E_{surface} < 4\pi R^2 \left[\frac{3}{2}\int_{\beta-v}^{0} d\phi_2 \sqrt{2U(\alpha,\phi_2)}\right] \quad (186)$$

I have calculated the upper limit of the energy at surface. But all of our calculation is based on the approximation of large Droplet, if it is not, our calculation will be failed. The only way to solve this problem is with help of computer. But, since the equations of (173) are different equations, even with numerical method, we will lose a lot of information. In fact, if we can solve the equations of (173), possibly we can find some solutions with discrete radius, such as when we solve the equation in Quantum Mechanics, we will find discrete energy levels. Such kinds of think are expected in our droplet model, but unfortunately, we cannot get such beautiful things. With very wild assumption, we assume that the droplet can only exist with a defined radius, R_z. That means, when the radius of droplet less than R_z or bigger than R_z, the droplet is unstable, it can decay into background field, the Higgs field ϕ_2, by the interaction term

$$L=\frac{1}{2}g_z\phi_1^2\phi_2^2$$

As we have talked, the droplet can be a good candidate of dark matter, since it is matter like, and except the interaction with ϕ_2, there is no future interactions with Standard Model particles.

But there is another question. That is the dark matter is particle or Droplet? In most of models, they treat the dark matter as particles. So how can we tell the difference of particle and classical droplet? We cannot solve this big problem here.

In our model, we have a lagrangian which include three parts, the lagrangian of Standard Model, Higgs doublet Φ, kinetic term for scalar singlet ϕ, and the interaction term between Higgs doublet and the scalar singlet

$$\begin{aligned} L &= L_{SM} + \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}M^2\Phi^2 - \frac{1}{4}\lambda_1\Phi^4 \\ &+ \frac{1}{2}g_z\phi^2\Phi^2 \\ &+ \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda_2\phi^4 \end{aligned} \tag{187}$$

The last term in (187) is important to keep the droplet from shrinking.

In this scenario, at the beginning of universe, Higgs doublet field Φ oscillates at its' minimum, and much of the energy of the field decays into field ϕ by the interaction term in equation (187), and then, it is spontaneous symmetry breaking. After that, some parts of field Φ decays into Standard Model particles, and the residue of field fail into its' minimum and make the universe accelerated expand today. The scalar singlet field ϕ will eventually cool down form droplet, which is the dark matter. This scenario has been shown in figure 29.

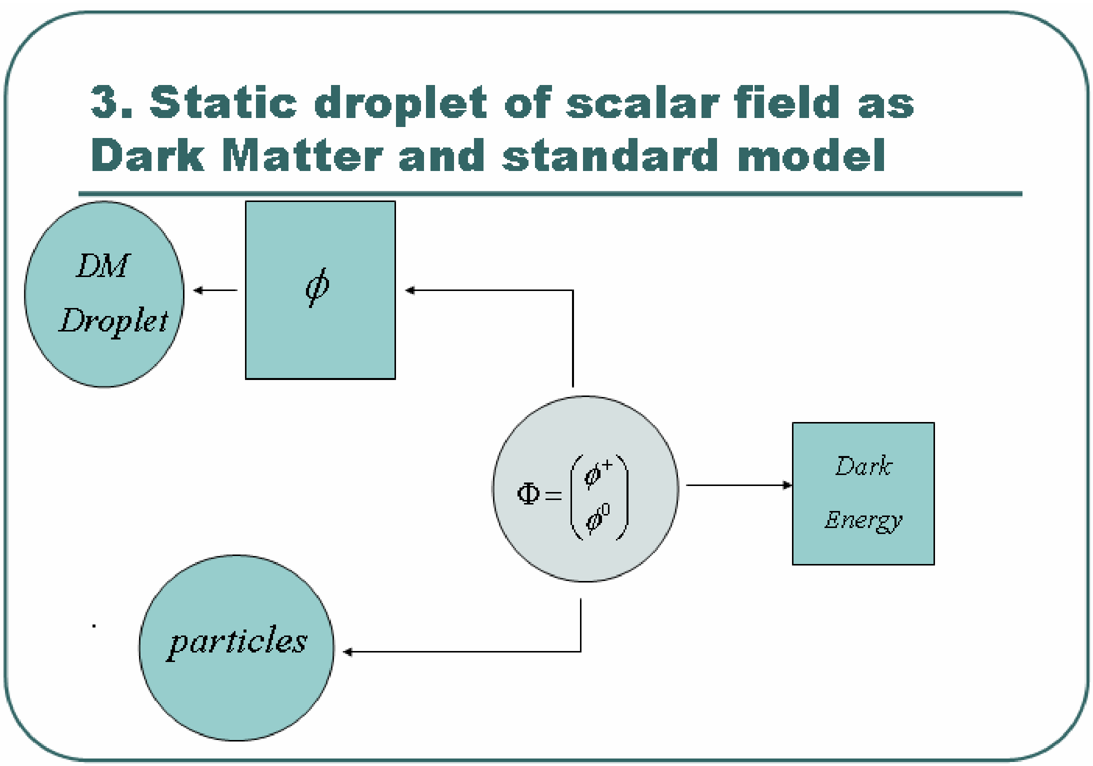

Fig.29 Dark world and the Standard Model

CHAPTER V

SUMMARY AND CONCLUSIONS

Recent years is called cosmology ear because the surprising discovery from the observation of Type Ia Supernovae, which indicated that the expansion is accelerated, and this has been future conformed by the measurement of CMB. We have discussed different aspects for the accelerated universe, dark energy, dark matter and some constraints. Since both reheating field and dark energy can be described by scalar field, at the first parts, we discussed the possibility that we can unify dark energy, dark matter and reheating field into one scalar field. This can be achieved by introduce a scalar potential like (120), and we have showed the solution for this model. At the second parts, we discussed that the reheating field can form a bubble with radius ~ 0.1 m due to the gravity. At third parts, we mainly discussed that a droplet of scalar field can be formed in the background of dark energy which can be a candidate of dark matter. To make things more clear, we have also talked about Q balls since the Q balls have the same structure of our droplet. Both of Q balls and our Droplet are sphere, and are stable. But to our droplet, we need to pay attention that the droplet is absolute stable. Just because of this extremely stable character, it makes the droplet become the candidate of dark matter. It will not break under deforming, or under some kinds of perturbation. The most advantage of this model is that we introduced a way to unify the dark world into standard model.

We leave some problems. First, we cannot solve the equations of (173),

$$\nabla^2\phi_1 + g\beta^2\phi_1 + \lambda_1\alpha^2\phi_1 - g(\phi_2 + v)\phi_1 - \lambda_1\phi_1^3 = 0$$
$$\nabla^2\phi_2 - g\phi_1^2(\phi_2 + v) - M^2\phi_2 = 0$$

at surface since it is too complicated. And because of this difficulty, we can only deal with big droplet, in which we can neglect the damping term and simplify the equation. This problem can be possibly solved with numerical, or by some mathematics expert.

The second is that we didn’t talk about the vibrations in droplet. Vibration is exiting state of droplet, and it is important because, when at the beginning of universe, the temperature is extremely high, and most of droplets are in exciting states. If the ground state is dark energy, then since there are many exciting states of droplet, at the universe stage of nucleosynthesis, when the temperature is about 1MeV degree, most of energy will not stay in ground state, the dark energy state, but most of energy will stay in dark matter states since there are so many states of dark matter. It can let droplet model satisfy the constraint of nucleosymthesis which required that more than 80% of energy should be in matter state. If more than 20% of energy is in the state of dark energy state, then the universe will expand too fast to form nuclear. In fact, this is very strong constraint which makes a lot of models failed. Since there is many excited states of Droplet, most of energy will stay in excited states of droplet. If there are more than 4 excited states of droplet, then 80% of energy stays in dark matter if the energy gap from the dark energy ground to the top excited state much smaller than 1 MeV. Today, our temperature is only 3k degree, so most of energy will stay in dark energy ground. If the 23% energy stays in the lowest state of droplet, then we can predict the varying of energy in dark energy and dark matter in future.

The third question has been talked by C.P.Burgess. Since there is interaction term between Higgs field and scalar singlet, there is window for the decay of Higgs Boson into droplet. And this could be possibly used to find droplet from high energy experiment if we can truly find Higgs Boson in 2007. But if we cannot, we have to worry about our droplet model since we need Higgs field to interact with scalar singlet to form Droplet.

The important thing about the droplet is that it is classical. It is a solution. And possibly it will open a door for another way of physics.

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