THE EFFECTS OF CONSTRUCTIVIST TEACHING APPROACHES

ON MIDDLE SCHOOL STUDENTS' ALGEBRAIC UNDERSTANDING

A Dissertation

by

AMANDA ANN ROSS

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Curriculum and Instruction

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ABSTRACT

The Effects of Constructivist Teaching Approaches

on Middle School Students' Algebraic Understanding. (August 2006)

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The goal in mathematics has shifted towards an emphasis on both procedural knowledge and conceptual understanding. The importance of gaining procedural knowledge and conceptual understanding is aligned with *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), which encourages fluency, reasoning skills, and ability to justify decisions. Possession of only procedural skills will not prove useful to students in many situations other than on tests (Boaler, 2000). Teachers and researchers can benefit from this study, which examined the effects of representations, constructivist approaches, and engagement on middle school students' algebraic understanding.

Data from an algebra pretest and posttest, as well as 16 algebra video lessons from an NSF-IERI funded project, were examined to determine occurrences of indicators of representations, constructivist approaches, and engagement, as well as student understanding. A mixed methods design was utilized by implementing multilevel structural equation modeling and constant comparison within the analysis. Calculation of descriptive statistics and creation of bar graphs provided more detail to add to the findings from the components of the statistical test and qualitative comparison method.

The results of the final structural equation model revealed a model that fit the data, with a non-significant model, $p > .01$. The new collectively named latent factor of constructivist approaches with the six indicators of enactive representations, encouragement of student independent thinking, creation of problem-centered lessons, facilitation of shared meanings, justification of ideas, and receiving feedback from the teacher was shown to be a significant predictor of procedural knowledge ($p < .05$) and conceptual understanding ($p < .10$). The indicators of the original latent factor of constructivist approaches were combined with one indicator for representations and two indicators for engagement. Constant comparison revealed similar findings concerning correlations among the indicators, as well as effects on student engagement and understanding. Constructivist approaches were found to have a positive effect on both types of student learning in middle school mathematics.

DEDICATION

To my wonderful parents who gave me so much love and assurance, my loving husband who supports me each and every day, my amazing grandmother, and all of my family, friends, and mentors.

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I would like to thank my wonderful parents, Fred and Ann McAlexander, for all of the love and words of wisdom given to me during our time together. I have been so blessed to have the best parents in the entire world, whose love I will take with me for the rest of my life. Mama and Daddy, thank you for teaching me about the important things in life. Most of all, thank you for loving me and spending your time with me. I miss you with all of my heart and wish I could share all of this with you. I know you would be here each step of the way no matter what. Mama and Daddy, I love you so much!

I would like to thank my loving husband, Kevin, for all of the love and support given to me during our five years together. Kevin, you have given me so much happiness. You have given me another chance to have a family and know the sounds of life, love, and laughter. Whenever I need you, you are always there to hold my hand and guide me through whatever comes my way. You have given me so much joy since you came into my life. I only hope I can make you half as happy as you have made me. I love you!

I would also like to thank my grandmother, Estelle Burgay, for stepping in and watching over me after Mama and Daddy passed away. Granny, you have been like a second mother to me. I am so thankful to have a grandmother who calls almost every day to see how things are going. I am so lucky to be your granddaughter. I hope you always know how much I love and admire you. To my other family members, thank you for your love and support during the years.

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CHAPTER I

INTRODUCTION

Statement of the Problem

In middle school mathematics, the focus in learning now includes conceptual understanding, in addition to procedural knowledge. The promotion of procedural knowledge and conceptual understanding are in alignment with *Principles and Standards* for *School Mathematics* (National Council of Teachers of Mathematics, 2000), which encourages higher-level thinking and reasoning skills, as well as communicative justifications. Van de Walle (2001) defined procedural knowledge by stating, "Procedural knowledge of mathematics is knowledge of the rules and procedures that one uses in carrying out routine and mathematical tasks and also the symbolism that is used to represent mathematics" (p. 31). In addition, Van de Walle (2001) defined conceptual understanding by stating, "Conceptual knowledge of mathematics consists of logical relationships constructed internally and existing in the mind as part of a network of ideas" (p. 31). In order to be mathematically literate, students must possess procedural knowledge, reasoning skills, and conceptual understanding (Wilkins, 2000). Students can rarely use procedural knowledge alone in situations other than on tests (Boaler, 1999). With the acquisition of both procedural knowledge and conceptual understanding, students are more apt to develop skills related to applicability and obtaining an understanding of connected ideas in mathematics (Hiebert & Carpenter, 1992).

This dissertation follows the style of *Journal for Research in Mathematics Education.*

Effective pedagogical strategies for teaching middle school students both procedural knowledge and conceptual ideas related to the algebra strand need to be determined. Students must understand the ways in which algebra topics relate to other mathematics topics, thereby encouraging a broader picture of understanding of mathematics. Woodbury (2000) stated, “The point of teaching and learning about any algebraic topic is how the topic connects with the larger conceptual arenas of number systems and number theory and with symbolic representation and the theory of equations” (p. 230).

Existing literature on the need for both procedural knowledge and conceptual understanding consists primarily of research studies examining learning in areas other than algebra (Chappell & Killpatrick, 2003; Porter & Masingila, 2000; Rittle-Johnson, Siegler, & Alibali, 2001; Sierra-Fernandez & Perales-Palacios, 2003), as well as suggested activities (Ducolon, 2000; Friel, 1998). Literature on the acquisition of both procedural knowledge and conceptual understanding in the area of algebra is quite scarce and consists of suggested activities (Davis, 2005), with few research studies (Hickey, Moore, & Pellegrino, 2001; Vlassis, 2002).

By examining the predictors of both procedural knowledge and conceptual understanding of middle school algebra students, researchers and teachers can determine best practices for promoting such knowledge. Researchers can examine effects on procedural knowledge and conceptual understanding in other contexts, as well. Teachers can plan effective lessons that build both kinds of learning acquisition.

Purpose of This Dissertation

This study examined a hypothesized model whereby representations (Bruner, 1966) and constructivist teaching approaches (Piaget, 1954; Piaget, 1970; Piaget, 1973; Vygotsky, 1978) produce engagement (Boaler, 2000; Dewey, 1900; Lave & Wenger, 1991; Vygotsky, 1978), which in turn impacts middle school students' procedural knowledge and conceptual understanding of algebra. Additionally, the study explored relationships among the types of representations, constructivist approaches, and engagement realized in the lessons, as well as concurrent descriptions of the occurrences. The hypothesized model relating these variables is shown in Figure 1.

Research Questions

This study considered the following research questions:

1. To what extent do representations, constructivist teaching approaches, and engagement predict middle school students' procedural knowledge and conceptual understanding?

2. To what extent do representations and constructivist teaching approaches predict middle school students' engagement?

3. How do types of representations overlay types of constructivist teaching approaches according to student engagement?

4. In what ways do the teachers' presentations and students' actions differ for the various algebra lessons?

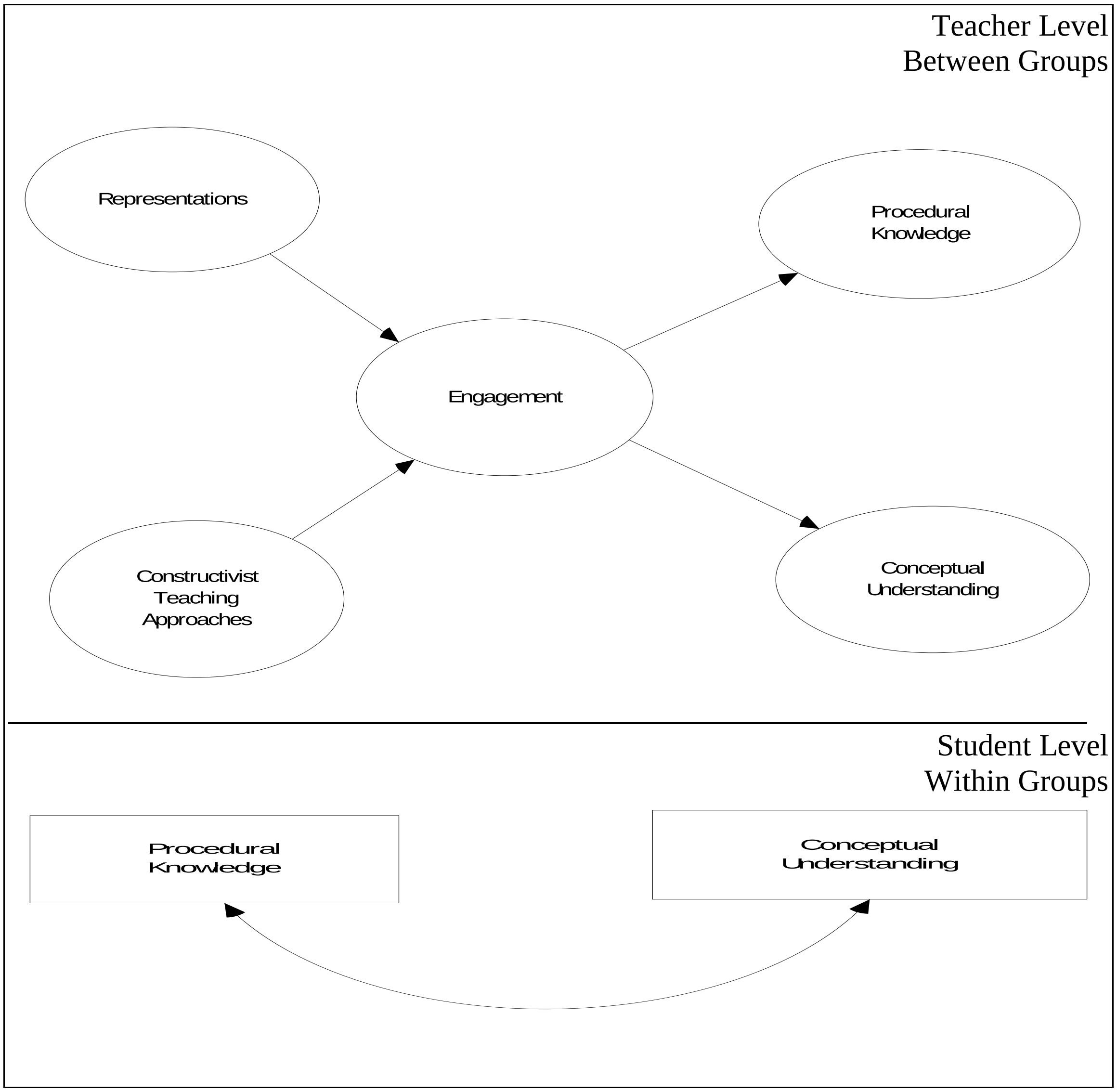

Figure 1. Theoretical model for the study.

Limitations

In this study, the lessons were assumed to represent different teacher engagement. There may be dependency for some lessons due to the same teacher teaching more than one lesson, which is not acknowledged. In addition, the teachers teach either 7th or 8th grade. It should be noted that the algebra posttest was not a high-stakes test, so some students may not have performed at their best.

Key Terms

Procedural knowledge: Knowledge of rules and procedures involving routines for mathematical tasks and use of symbolism (Van de Walle, 2001).

Conceptual understanding: Knowledge of logical relationships and connectedness in mathematics, which involves internal construction (Van de Walle, 2001).

Representation: A learning tool that represents something other than itself (Goldin & Shteingold, 2001). Three components of representations of learning, as described by Bruner (1966) are enactive (hands-on), iconic (pictures and numbers), and symbolic (numbers, symbols; words/discussion).

Constructivist teaching approaches: Teacher encouragement of independent thinking, creation of problem-centered lessons, and facilitation of shared meanings (Piaget, 1954; Piaget, 1970; Piaget, 1973; Vygotsky, 1978).

Engagement: Students' expression of ideas; clarification, justification, and representation of ideas; and receiving feedback from teachers about ideas (AAAS, 2001).

CHAPTER II

BACKGROUND LITERATURE

Representations, constructivist teaching approaches, and engagement are important contributors to students' learning. These variables rest upon the ideas of Jerome Bruner, Jean Piaget, Lev Vygotsky, Ernst von Glasersfeld, John Dewey, Jean Lave, Etienne Wenger, Jo Boaler, and James Greeno. The ideas involved in the theoretical framework set the stage for application of the variables to the field of mathematics education.

Representations

Representations are components of learning that can be used to help students move from concrete thought to more abstract thought. A representation, in fact, represents something other than itself (Goldin & Shteingold, 2001). Representations are therefore needed to enhance students' understanding and ability to make connections in mathematics (National Council of Teachers of Mathematics, 2000). According to Bruner (1966), the components of representations of learning are enactive (hands-on), iconic (pictures), and symbolic (symbols, numbers; words/discussion). For example, children can understand the concept of a balance beam through actually sitting on a seesaw (enactive), drawing a picture of the balance beam with rings (iconic), and finally writing Newton's Law of Moments (symbolic). When learning about the commutative and distributive properties of equations, students can first work with enactive materials, such as algebra tiles, proceed to iconic representations, and finally convert the idea to symbols. This progression of representations provides the students with prior imagery to relate to in the case that symbolic representations do not transfer to certain problem-solving situations. Bruner (1966) described the importance of concrete images by stating, “When they searched for a way to deal with new problems, the task was usually carried out not simply by abstract means but also by ‘matching up’ images” (p. 65). Therefore, it is important for students to be able to abstract meaning through symbolic representations, but also to have a repertoire of visual imagery to recall upon and compare new ideas to in mathematics (Bruner, 1966).

Enactive representations, or hands-on materials, such as manipulatives can be used in helping students to learn through actions. Bruner (1966) stated, “We know many things for which we have no imagery and no words, and they are very hard to teach to anybody by the use of either words or diagrams and pictures” (p. 10). Play with enactive representations, such as building blocks, can promote the beginning of intrinsic learning. Concrete representations enable students to develop a conceptual understanding of the concepts, as well as develop an understanding of future theorems due to exposure to intuitive situations (Bruner, 1966).

Several types of enactive representations, such as manipulatives and technological tools can be used to provide a solid foundation for knowledge and understanding of underlying ideas in mathematics. The use of algebra tiles can facilitate understanding of the variable concept (Chappell, 2001). Based on extensive data collection, consisting of interviews, students’ work, and fieldwork notes, Cedillo (2001) reported that graphing calculators can help middle school students better understand algebraic concepts in a conceptual manner. Attribute blocks can also enable students to acquire deeper levels of conceptual understanding (Bird, 2000), as well as develop an ability to build upon prior knowledge and experiences and make much needed connections through engagement while discovering new concepts (Quinn, 1997). In a study examining only classroom percentages on items correct, Moch (2001) found that cubes, tangrams, candy, paper-folding, and cards increased middle school students' performance on a test assessing all mathematical concepts by 10 percent. Similarly, Kennedy (2000) revealed a significantly higher performance for students using such manipulative models as integer chips, pattern blocks, and fraction strips on an exam measuring algebraic and proportional reasoning.

Manipulatives can also provide students with opportunities to connect ideas to previous ideas through concrete images, thus creating a basis for long-lasting understanding of meaning. Algebra tiles can increase students' mental imagery, resulting in an easier acquisition of learning, as reported by Sharp (1995) in a study examining a relatively small sample size. In addition, in a study involving two eighth grade classes in Belgium, Vlassis (2002) examined students of low ability levels from areas of low socioeconomic status ($n = 40$) in order to determine the effects of the implementation of a balance model in the study of solving equations. A convenient sample of teachers and students was utilized in the study with observations of 16 sessions using the materials given by the researcher. After interviewing students following an eight month lapse in usage of a balance model, results revealed retained learning as indicated by higher performance, mental images, and long-term memory and understanding. Students could more easily understand the solving of an equation with two unknown members, provide clarification as to the role of the equal sign, and explain each step of the process involved in solving an equation.

Iconic representations relate to imagery in the form of pictures and provide students with the tools needed to move beyond the concrete stage. With the use of iconic representations, students are able to represent concrete materials in pictorial form, which indicates a transition in understanding of the true meaning of the mathematical concept. Albeit, students are not making abstractions yet, but they are at an integral part of the process where they can next become successful at applying a symbol, or a type of language to represent something else. Bruner (1966) stated, “Images develop an autonomous status; they become great summarizers of action” (p. 13). Such representations pave the path for complete abstract thought. Students no longer require the actual concrete, physical object, and may then begin to delve deeper in thought and make much needed connections (Bruner, 1966).

Students’ understanding of the number strand and computation has increased through the use of imagery. In a study examining one class of fourth graders with only procedural knowledge revealed prior to any intervention, Saenz-Ludlow (1995) studied the effects of numerical diagrams on students’ numerical understanding. With close collaboration between the researcher and teacher, specifically with the inclusion of team teaching, many opportunities for extensive data collection were afforded. Thus, videotaping of class lessons, including student presentations, and weekly interviews of small groups of students were examined in the data analysis, along with transcribed notes of classroom occurrences and examples of students’ work. The results revealed an increase in students' abilities towards the number strand, increased numerical reasoning, and concurrent engagement when using numerical diagrams to add two numbers. In a study examining a group of fifth grade students of mixed ability (n = 48), Ainsworth, Bibby, and Wood (2002) used a two-factor mixed design, including systems of representations and time to determine the effects of different representations on students' success with computational estimation. Posttests, in the form of paper and pen, were examined and compared to pretest results after the intervention of a computer-based learning environment (CENTS) had taken place. This intervention utilized various representations, while promoting prediction and explanations and providing subsequent, immediate feedback. The results revealed that the use of pictorial representations resulted in a statistically significant improvement in students' abilities to succeed in the area of computational estimation, $p < .001$.

Iconic representations have also been used to increase students' understanding of the number strand in conjunction with the learning of fractions. In a study examining a non-diverse group of students, Brinker (1997) reported that pictorial models, such as a ratio table, can help students apply informal knowledge of fractions. Fifth graders of mixed ability in two different schools (n = 23) participated in another study examining the abilities of students' to use various venues for solving division of fractions. These students did not hold any prior knowledge concerning the division of fractions. The use of well-triangulated data collection, including examination of pretest and posttest results, students' solutions, field notes, and videotaped classroom discussions, revealed that pictorial representations help students develop a conceptual understanding of the division of fractions (Sharp & Adams, 2002).

Symbolic representations, such as numbers, symbols, or words can promote students' abilities to relate ideas and explain their reasoning. When using symbolic representations, students are no longer dependent upon the physical actions and imagery. Additionally, symbolic representations help students condense information into a form that fits into a given attention span. The use of such representations helps students internalize ideas, while reaching a new level of understanding, one including abstract thought (Bruner, 1966).

Numbers or other symbols can help students think in new ways about mathematics topics. Ainsworth, Bibby, and Wood (2002) reported that the use of numerical or symbolic representations improved students' abilities to perform computational estimations, $p < .001$. The development of symbolic representations through constructive procedures can help students develop an understanding of the conceptual underpinnings related to the division of fractions (Sharp & Adams, 2002).

Symbolic representations, in the form of classroom discussions using words as abstraction, can also deepen students' understanding of mathematics. In a study comprised of 21 sixth grade female students enrolled in a Canadian private school, Nason and Woodruff (2003) examined the effects of discourse on students' abilities related to the number strand, specifically order. A plethora of data sources, including observation notes on lessons, student interviews, observation notes on student models, and notes on a computer model were used to examine the benefits of a learning environment supported by computers. The Knowledge Forum, a computer supported learning environment, was used to provide opportunities for immediate responses, and thus plenty of abstraction of numerical ideas. The results revealed engagement through discussions increased students' achievement related to numeracy and order. Additionally, discussions in mathematics classrooms can increase students' performance on items related to the algebra strand. After extensive data collection over a nine week period, Pugalee (2001) found student engagement in conversations about graphs of linear equations, using graphing calculators to develop increased conceptualizations. Examples of student work and other writing, as well as anecdotal notes were used to ascertain such achievement. The high school algebra students involved in the study (n = 16) were characterized as low math performers and gained much success in abilities to explore, explain ideas, and make connections related to slope and y-intercept after use of discussions.

Constructivist Teaching Approaches

Constructivist teaching approaches play an important role in developing students' conceptual understanding and ability to communicate learned ideas. These approaches include teacher encouragement of student independent thinking, creation of problem-centered lessons, and facilitation of shared meanings. The theory of constructivism is the basis for such teaching approaches.

Constructivism is aligned with active learning and encourages comparison of new ideas to prior knowledge (Piaget, 1954; Piaget, 1970; Piaget, 1973; von Glasersfeld, 1997; Vygotsky, 1978). Constructivism was defined by Slavin (2000) as, "[The] view of cognitive development that emphasizes the active role of learners in building their own understanding of reality" (p. 32). In this manner, students are not analogous to sponges through absorption of new information. Instead, students must participate in classrooms, which promote constant reflection of one's own ideas, as well as the connection to others' ideas (Van de Walle, 2001). The use of physical actions, one component of constructivism, can prevent students from simply memorizing information, and therefore, promote use of senses to obtain underlying meaning (Vygotsky, 1978). This action promotes students' control of their own learning situation, or independent thinking. von Glasersfeld (1996) stated, "For whatever things we know, we know only insofar as we have constructed them as relatively viable permanent entities in our conceptual world" (p. 19). Designing classroom activities promoting communication and justification of ideas is important in helping students develop problem-solving skills (Piaget, 1973). There is great importance in the facilitation of correct mathematical language, justification of ideas, and sharing ideas with others (Ball & Bass, 2000).

The process of assimilation and accommodation as used in the learning of new ideas can promote students' abilities to make connections in ways that will foster the retention of knowledge. When the student uses the external environment to make accommodations to a previously learned set of ideas, assimilation is taking place (Piaget, 1954). Such learning approaches are present when students actively engage in the learning process while maintaining a freedom to explore ideas that are of interest. Piaget (1973) stated, "…He will have acquired a methodology that can serve him for the rest of his life, which will stimulate his curiosity without the risk of exhausting it" (p. 93).

Therefore, in order to foster internalization, students need to participate in active exploration and discussion, so they can integrate these actions into similar ideas existing in the mind (Piaget, 1973).

Mathematics education relies closely on constructivism and its exploratory and inquisitive strategies. Students can construct meaning in mathematics from each other or from use of individual objects, both of which are part of experiences (von Glasersfeld, 1997). Students at all levels can benefit from such hands-on approaches. Students who typically perform lower in mathematics foresee ideas in an entirely new light when using concrete learning experiences. Thus, all students can learn mathematics when given appropriate instruction that is tailored to their needs. This type of instruction fosters the linkage between concrete and abstract thought in a meaningful and coherent manner. Piaget (1973) explained the importance of use of hands-on, active types of play in learning by stating, “Once these mechanisms are accomplished, it becomes possible to introduce the numerical data which take on a totally new significance from what they would have had if presented at the beginning” (p. 101). In fact, without such approaches, students often approach mathematics in a haphazard fashion through trying already known procedures, which are detrimental to the ability to use reasoning skills (Piaget, 1973).

The constructivist teaching approach encourages the use of invented algorithms, solving of one’s own problems, and question-asking in order to foster independent thinking in students. Invented ideas in a constructivist classroom will encourage problem-solving through engagement (Warrington & Kamii, 1998). In one study, Kamii, Pritchett, and Nelson (1996) examined data collection from fourth grade observations and student interviews to determine the effects of invented methods for finding the average of a set of numbers on students' understanding and reasoning capabilities. With no prior knowledge of finding the average of a set of numbers, students eagerly invented algorithms and explained their reasoning, as well as provided a definition for the meaning of an average. The results revealed the presence of depth of thinking skills that abounded as students invented their own procedures for finding the average. Specifically, students revealed intuitive understanding, as evidenced by adequate qualitative, spatial, and numerical reasoning. Additionally, students moved beyond the need for paper and pencil in their computations. Inventing and creating procedures can also enhance students' understanding of equivalent fractions (Curcio & Schwartz, 1998). Furthermore, in mathematics, the act of solving one's own problems (Wood, Cobb, and Yackel, 2000), as well as the process of asking questions with other students concerning various strategies applicable to the mathematics topic (Carpenter, et al, 1994) can increase students' mathematical abilities.

The constructivist teaching approach of creating problem-centered lessons includes realistic situations and the posing of problems. Teachers need to present cumulative problems to increase students' abilities to make connections in mathematics (Carpenter, et al, 1994; Cunningham, 2004; Wood & Sellers, 1997). In a study examining the effects of problem-centered lessons on developmental math students' problem-solving skills, Verhovsek and Striplin (2003) reported higher engagement and a statistically significant increase in performance on a post-test exam. Activities that incorporate real-life situations into problems posed in mathematics lessons promote the understanding and realization of applicability of such skills (Tepper, 1999). In a study examining a four-way analysis of variance, Hickey, Moore, and Pellegrino (2001) reported higher student performance on standardized tests and deeper conceptual understanding when learning in a constructivist classroom that implemented video usage and real-world situations into lessons involving the posing of problems. Two pairs of schools, including 19 fifth grade classrooms, were matched according to students' socioeconomic status for inclusion in the study. A significant difference in achievement on a subtest resulted for students taught from realistic contexts provided through problem situations, specifically in the areas of problem solving and data interpretation. Sharp and Adams (2002) reported similar results concerning the manner in which realistic situations can help students develop a foundational understanding of division of fractions without relying on the invert-and-multiply procedure.

The constructivist approach of facilitation of shared meanings involves the creation of small group activities and use of negotiated meanings. Wood, Cobb, and Yackel (2000) explained the importance of negotiated meanings and encouragement of small group collaboration in socially communicative situations with students as they provide their explanations. Group interaction develops deeper-level thinking as students explain ideas (Wood & Sellers, 1997).

Engagement

Engagement plays an important role in students' understanding and application of ideas. The development of a classroom community where engagement in communicative situations can flourish will promote students' abilities to apply knowledge learned. An engaging classroom community can be defined as one that encourages choice and application of mathematical ideas in a discursive environment (Boaler, 2000). Thus, students interact with other students while discussing ideas related to the mathematics lesson, as well as interact with the teacher concerning the sharing of meanings.

In mathematics, the use of discourse helps the students formulate ideas and develop appropriate reasons for such thoughts. Such activity allows the students to make connections, build upon others' ideas, and realize their current level of understanding concerning a mathematics topic. The use of speech and communication (Vygotsky, 1978) in the learning process, as well as social engagement (Dewey, 1900) can help students internalize the concept at hand. Dewey (1938), in opposition to Piaget (1973) and von Glasersfeld (1997), believed that learning cannot take place solely within the individual child. Children are believed to learn best through social situations due to the fact that all experiences are mostly social (Dewey, 1938). Thus, the creation of a social learning environment is of much importance in fostering students' understanding and internalization of ideas.

Communities of practice, involving engagement through discussion and active group collaboration is at the forefront of mathematics education. Engagement in communicative environments is derived from a situated perspective in which students are involved in work on applications of mathematics in groups (Lave & Wenger, 1991). The discussion of mathematical ideas through everyday activities, realized in a practical setting, can help students make connections between learned knowledge and ways to apply such knowledge (Lave, 1988). Engagement in activities involving discourse promotes individuals' abilities to make sense of experiences and develop identities as learners (Greeno & The Middle-School Mathematics through Applications Project Group, 1997). Engagement consists of students' expression of their ideas; clarification, justification, and representation of their ideas; and receiving feedback about their ideas (AAAS, 2001).

Engagement is a product of both the use of different types of representations and constructivist teaching approaches in the mathematics classroom. Goldsmith and Mark (1999) explained that mathematical ways of thinking results from engagement in lessons. Inquiry-based and exploratory activities, such as problem-centered contexts, as well as use of various types of representations, including enactive, iconic, and symbolic, promote engagement, which in turn promotes both procedural and conceptual understanding. Boaler (2000) explained that the use of choice in methods, i.e. independent thinking, and negotiating meanings, i.e. facilitating shared meanings, are constructivist approaches that affect engagement and subsequently affect learning. Goldsmith and Mark (1999) stated, "The emphasis on engaging students in doing mathematics is intended to help students understand the why as well as the how of the mathematics they study" (p. 43). Therefore, engagement is a mediator between constructivist teaching approaches and use of representations and resulting learning.

Engagement in mathematics lessons and activities can increase students' overall knowledge, understanding, and reasoning abilities. Student engagement in mathematics lessons is promoted by use of representations (Nason & Woodruff, 2003; Pugalee, 2001; Quinn, 1997; Saenz-Ludlow, 1995) and constructivist teaching approaches (Cunningham, 2004; Verhovsek & Striplin, 2003; Warrington & Kamii, 1998). High engagement and academic competence are closely related, as revealed by higher academic achievement (Rodriguez, 2004), indicating a high-quality classroom (Weiss & Pasley, 2004). In a study utilizing constructivist teaching approaches involving problem-posing, Cunningham (2004) found students to become more engaged in the lesson when discussing ideas in small groups of three to four students. With the examination of numerous examples of student responses, the results revealed that mathematics students gained higher reasoning skills and deeper understanding of mathematics, as well as more reflection from the concurrent engagement.

Engagement includes components related to classroom discussion involving expression of ideas, justification of ideas, and receiving feedback concerning ideas. Expression of ideas and the opportunity to receive feedback can indeed promote internalization of mathematics ideas (Artzt, 1996). The action of explaining reasons behind ideas in mathematics promotes students' abilities to further their understanding (Burns, 2004). Justification, another component of engagement, has been found to increase students' development and understanding of number sense through examination of qualitative data taken from a large sample size (Schneider & Thompson, 2000). This study examined 26 second grade students and their invented methods, as consequential justifications for performing computations and working with arithmetic equations. The invented equations were deeply perused to determine students' success in understanding the material. Specifically, students developed an appropriate understanding of the idea of positive and negative whole numbers, creation of new numbers, and meaning of number properties.

Procedural Knowledge

Procedural knowledge refers to a level of understanding that involves mostly an attainment of facts and algorithms that does not require knowledge of underlying ideas. Therefore, the teaching strategies needed for promotion of such knowledge typically involves a direct approach that encourages teacher lectures and non-discursive listening on the parts of the learners. In this manner, the teacher acts as the deliverer of knowledge, whilst the students readily absorb the knowledge. There is not much room for inquisitions, discussion, or much active engagement in the lesson process. Boaler (2000) described such classroom teaching as fostering much individualism, while delivering closed types of learning that are not applicable to other settings outside of the classroom. Boaler (2000) stated, “The students believed that adopting classroom practices in the real world was inappropriate, so they did not attempt to draw upon school mathematics” (p. 114).

It is hypothesized in this dissertation, however, that the learning of procedural knowledge can increase with the use of constructivist teaching approaches. With the use of teaching approaches that promote conceptual understanding, students will be equipped with foundational knowledge, which can then promote students’ abilities and achievement related to procedural knowledge. In other words, a student can develop other formulas or theorems through a thorough understanding of underlying concepts. In this manner, such approaches can open doors to students in the realm of procedural knowledge. Students, can therefore perform higher on exams, thus exhibiting procedural knowledge, when learning from constructivist teaching approaches, representations, and engagement in the classroom (Ainsworth, Bibby, & Wood, 2002; Hickey, Moore, & Pellegrino, 2001; Kennedy, 2000; Moch, 2001; Verhovsek & Striplin, 2003; Vlassis, 2002, e.g.).

Conceptual Understanding

Conceptual understanding ascribes to a thorough understanding of underlying, foundational concepts behind the algorithms performed in mathematics. With proper conceptual understanding, students possess the necessary skills to recreate formulas and proofs without any memorization or rote process. The base of understanding is present, and thus allows the students to build upon prior knowledge, as realized by constructivist practitioners. With the assimilation and accommodation used when learning new ideas, students are able to make connections and develop deeper meanings through the comparison of new ideas to previously learned ideas (Piaget, 1954). Conceptual understanding thus involves teaching approaches that allow students to make choices and apply their understanding through active engagement in problem-based situations (Boaler, 2000). These approaches are in alignment with constructivist teaching, use of various representations, and engagement. Such approaches promote deeper levels of conceptual understanding and applicability in mathematics students (Cedillo, 2001; Cunningham, 2004; Curcio & Schwartz, 1998; Hickey, Moore, & Pellegrino, 2001; Vlassis, 2002, e.g.).

Relationship between Procedural and Conceptual Knowledge

Procedural knowledge and conceptual understanding are both needed in order to promote students' overall successful learning base in mathematics. Constructivist teaching approaches need to include methods that involve the attainment of both types of knowledge, namely skills and higher-level understanding (Goldsmith & Mark, 1999). Therefore, the desire to promote conceptual understanding does not eliminate the desire to promote procedural knowledge. Goldsmith and Mark (1999) stated, "Nowhere do the Standards contend that computation is unimportant or that students can get by without knowing basic number facts and operations. They do, however, recommend diminishing the amount of class time dedicated to skills practice…" (p. 41).

Students can make much needed connections in mathematics from the cyclical knowledge gained from both types of learning. For example, students can develop conceptual understanding from prior knowledge and skills by comparing the new ideas to old ideas (Piaget, 1954). Previously attained knowledge and procedures can help students connect such ideas to the big conceptual ideas in mathematics (Woodbury, 2000). Furthermore, conceptual understanding can promote new procedural knowledge by promoting mathematical ways of thinking that allow students to use generalizations to discover new theorems or proofs based upon the already present conceptual base (Goldsmith & Mark, 1999).

Gap in Data Analyses

The analyses used thus far in examining the effects of representations, constructivist teaching approaches, and engagement on students' learning involve diverse qualitative methods and some statistical testing. The qualitative methods have mostly consisted of observation notes, examples of student notes, and interview data. The quantitative statistical tests appearing in the literature have involved descriptive statistics and analysis of variance. Therefore, there is a need to examine these variables using a more rigorous statistical test, such as that of multi-level structural equation modeling. Additionally, constant comparison of descriptions of teachers' presentations and students' actions for many different teacher lessons need to be examined for the variables of representations, constructivist teaching approaches, and engagement.

CHAPTER III

METHODOLOGY

A mixed methods approach was used in order to observe the effects of interventions that had already taken place through use of representations, constructivist teaching approaches, and engagement. Mixed methods research provides information from both quantitative and qualitative methodologies, thereby providing generalizable and contextual data, which reveal more completely the aspects related to the research questions (Creswell, 2002). In this study, qualitative data is taken from observations of algebra lesson videos, whereas quantitative data is pulled from algebra pre and post-test results.

Participants

The sample for this study was 16 lessons of seven 7th and 8th grade teachers and their students (n = 971) enrolled in public schools in a rural area of Texas. It should be noted that n = 436 was the number of separate students involved in the analysis. Due to the inclusion of more than one lesson taught by the same teacher, students were duplicated or tripled when performing both descriptive statistics and structural equation modeling. The unit of analysis for statistical tests was the teacher lesson. The teachers participating in this study were of diverse ethnicities and varied in terms of years of experience in teaching. Additionally, the teachers entered the program without any prior professional development experience concerning the use of their textbooks. The teachers utilized various teaching approaches, representations, and strategies. The population of students consisted of diverse ethnicities, also. The ethnic distribution in the year 2004 for

12 year-olds in this region included 48.3% White, 20.8% African American, 27% Hispanic, and 3.9% for other ethnicities. One-third of the students were categorized as low socioeconomic status.

Instruments

The 2003-2004 algebra test taken from the Middle School Mathematics Project, an NSF-IERI funded project, was used to examine middle school students' procedural knowledge and conceptual understanding of the algebra strand. The multiple choice responses and written responses from pre and post-tests were used to ascertain such knowledge and understanding. Eight questions, consisting of three multiple choice, four short responses, and one extended response were used to assess procedural knowledge. The questions assessing procedural knowledge involved relation of algorithmic and rote knowledge of mathematical ideas. Twelve questions, consisting of four multiple choice, three short responses, and five extended responses were used to assess conceptual understanding. The questions assessing conceptual understanding involved much depth and opportunity for students to make connections and applications using a thorough understanding of underlying mathematical concepts. For example, students were asked to match a real-world situation to a mathematical graph of the function. In this situation, students must possess knowledge above and beyond simple rote knowledge. Refer to Appendix A for more information. The information obtained from the 16 videos of teacher lessons was used to determine the pedagogical tools and strategies used. The videos were obtained from the Middle School Mathematics Project, as well.

The middle school algebra strand requires the use of various modeling techniques by students in order to understand, represent, and analyze algebraic ideas. According to *Principles and Standards for School Mathematics*, “[Students are expected to] understand patterns, relationships, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts” (National Council of Teachers of Mathematics, 2000, p. 395). The algebra test from the Middle School Mathematics Project contained questions relating to each of these objectives. Such algebraic content expectations include both proficiency and conceptual understanding that are evidenced through activities utilizing the translation of types of representations such as symbolic to other verbal or graphical representations. Additionally, students should use mathematical models in order to represent and analyze real world situations, whereby they are actively testing mathematical conjectures (National Council of Teachers of Mathematics, 2000).

Three observation sheets were implemented when viewing the videos to determine percentage of time representations and constructivist teaching approaches were used, as well as percentage of time students were engaged. The sheets were also used to determine percentages of time for use of different types of representations, constructivist teaching approaches, and engagement. Refer to Appendix B, C, and D for more information. The three types of representation are enactive, iconic, and symbolic (Bruner, 1966). The criteria for constructivist teaching approaches include encouragement of independent student thinking, creation of problem-centered lessons, and facilitation of shared meanings. These criteria are derived from the components of constructivism (Piaget, 1954; Piaget, 1970; Piaget, 1973; Vygotsky, 1978). The American Association for the Advancement of Science (AAAS, 2001) criteria was used as a reference for types of engagement. The criteria include students' expression of their own ideas relevant to the learning goals; clarification, justification, interpretation, and representation of ideas; and receiving specific feedback from the teacher. Descriptions and examples were documented and categorized using constant comparison methods. With constant comparison techniques, indicators, codes, and categories are compared with one another in an attempt to eliminate needless repetition. In this manner, overall categories emerge from the recorded data (Creswell, 2002). A space for extra observations or pertinent information was also included in the instrument.

The time coding for the indicators of representations, constructivist teaching approaches, and engagement was undertaken using specific operational definitions. Refer to Table 1. With this approach, more accuracy in determining actual occurrences across the 16 video lessons was ensured. Some of the specifics involved in the definitions were more obvious than others, as with the case of viewing use of enactive materials, or manipulatives. Therefore, each code was documented for each portion of a lesson by examining the operational definition and determining its fit. In several cases, time coding for various indicators overlapped due to concurrent use of more than one representation in the same part of the lesson.

Table 1
Description of Indicators of Coded Variables

Indicator	Description
Enactive Representations	The lesson involves the use of physical objects, i.e. blocks, graphing calculators.
Iconic Representations	The lesson involves the use of pictures, i.e. graphs, diagrams.
Symbolic Representations	The lesson involves the use of written numerals or symbols, as well as spoken or written words. Abstraction of meaning is the key in this case.
Encourage Independent Student Thinking	The lesson promotes the discovery of ideas, including invented processes, and question-asking.
Create Problem-Centered Lessons	The lesson includes cumulative math problems whereby connections are made, i.e. students use manipulatives to discover patterns and later place the data into t-tables and then graph the ordered pairs, while discussing the connections between the representations and the meaning involved.
Facilitate Shared Meanings	The lesson promotes participation in discussions and negotiated meanings for mathematical ideas.
Students' Expression of Ideas	Students provide higher-level, descriptive comments during the lesson. Simple, short phrases are not included. Explanations are not included.
Students' Justification of Ideas	Students provide explanations for ideas provided in the lesson. Simple, short phrases are not included.
Receiving Feedback from Teacher	Students receive meaningful feedback from the teacher, i.e. the teacher makes connections, expands on ideas offered students, models, and provides probing questions.

Representations were the easiest indicators to code, due to their apparent presence in the lesson. The use of enactive representations was coded whenever physical objects (manipulatives) were utilized in teaching the algebraic concept, or procedure. For example, the use of wooden, colored blocks for teaching the concept of patterns would be considered an enactive representation. Likewise, the use of graphing calculators to teach the concept of function would also be considered an enactive representation. Iconic representations were coded for the use of pictorial representations, such as an illustration of a diagram, table of values, or graph. Symbolic representations, which involve abstraction of mathematical ideas for students, were coded whenever symbols, numbers, or words/discussions were used. Such symbols allow students to derive deeper meaning while making connections from concrete to abstract thought.

The coding of constructivist teaching approaches required much more deliberation concerning the appropriate fit of the activity or occurrence with the actual operational definition. The first indicator of constructivist teaching approaches was the encouragement of student independent thinking. Independent thinking included the invention of algorithms and independent solving of problems, as well as inquisitive comments by students during the lesson. Questions asked during the lesson that dealt with possibilities for connections to other ideas in algebra, or other topics in mathematics, were seen as acts of independent thinking. The independent solving of problems could involve group collaboration if it was not based upon explicit directions from the teacher. The second indicator, creation of problem-centered lessons, was coded whenever cumulative mathematics problems were posed, whether it was in a realistic situation or simulated one. The critical part of such coding for this indicator included the posing of a problem that combined several mathematical objectives into one large problem, instead of several small problems. An example of a problem-centered lesson would be the use of manipulatives to help students develop and understand patterns, whereby they could then develop tables of values and create graphs representing the relationships. Students might also discuss ideas with peers concerning similarities and differences between graphs and reasons for the various ideas. These activities could be tied to realistic ideas, but were not required to do so in order to be coded as problem-centered. However, whenever realistic activities were utilized, they were coded as problem-based lessons. The third indicator of constructivist teaching approaches, facilitation of shared meanings, involved small group collaboration and discussion, as well as whole-class discussions and negotiation of ideas. Discussions pertained to the lesson and/or connections to other topics in algebra. Such approaches involved both the teacher and students in discourse concerning the mathematics topic and not simply rote performance.

The coding of engagement involved students' expression of ideas; justification, clarification, and interpretation of ideas; and receiving feedback from teachers. When deciding upon the appropriateness of coding occurrences as expression of ideas, comments made by students indicating higher-level, or descriptive ideas related to the lesson were coded. For example, yes or no responses, as well as short phrase responses to the teachers' question were not coded as expression of ideas. Comments offered during discussions, however, were coded as expression of ideas. The coding of justification of ideas involved students' offering of explanations for steps, ideas, or solutions to the algebraic problems. An example of justification of ideas could involve an explanation involving reasons for identifying a function as linear from a table of values. The coding of receiving feedback from teachers involved a high level of deliberation in determining such labeling. It was determined that simple recognition of correct or incorrect responses, as well as restatements of students' answers in a similar or exact form were not forms of feedback indicative of an engaged classroom. However, the use of probing questions, modeling, facilitation of connections to other ideas, and expansion of ideas to other topics were coded as receiving feedback from the teacher.

An experienced mathematics educator coded eleven representative segments of two of the videos to verify and provide a reliability estimate of the coding variables. Inter-rater reliability was calculated in order to provide information concerning the similarity of results concerning the coding of the three variables. Eleven time segments were viewed by a fellow mathematics education doctoral student. These time segments were representative of the various indicators examined in the lessons. The percentage of agreement was 91%. Creswell (2002) stated, "This method has the advantage of obtaining observational scores from two or more individuals, thus negating any bias that might be brought on by one of the individuals" (p. 182). Triangulation was used, therefore, consisting of the coding and descriptions of the videos using the observation sheets completed by the researcher, with a follow-up completion by a colleague. In addition, for the teachers who did follow the textbook, confirmation of lesson presentation and materials used were described from the textbook. It should be noted that in Lesson 7, the use of iconic representations were not coded, due to the inability to discern their inclusion on the worksheet. However, when the actual lesson materials were later viewed, it was determined that pictorial drawings of pattern blocks were used in the lesson. Lastly, observation sheets filled out by those who videotaped the lessons were used to determine teachers' strategies and techniques in the classroom.

Data Analysis

The 16 videos of algebra lessons were selected based upon the opportunity to relate the effects of representations and constructivist teaching approaches to student achievement. The multiple choice responses and written responses from the 2003-2004 algebra test were examined to determine procedural knowledge and understanding. Both the pre-test data and post-test data were used to ascertain the gain in achievement. Responses to multiple choice questions were coded as 1 for correct and 0 for incorrect. Written responses to open-ended questions were coded according to level of correctness using 0 for incorrect, 1 for partially correct, and 2 for correct. The level of correctness for open-ended questions was recorded using a rubric developed by researchers working on the Middle School Mathematics Project. Refer to Appendix E. The data for determining the growth in procedural knowledge and conceptual knowledge were collected from pre-test to post-test gain.

Quantitative data on teaching included the time and percentage of use of representations, constructivist teaching approaches, and engagement. Each time segment of a lesson was recorded for each of the indicators of the three variables. Time segments were divided by overall instructional time to obtain percentages for each type of representation, constructivist teaching approach, and engagement. Minutes and remaining seconds for each occurrence of an indicator were converted to total seconds, which were then converted to minutes that were rounded to the nearest hundredth. A total for each indicator was calculated by adding the time segments together and rounding to the nearest percentage. Software that records specific times from start to finish for each category was used to ensure accurate measurements of time.

Qualitative data included summary descriptions of teacher and student actions for each lesson. Through the use of constant comparison, types of action and clustering of engagement around certain representations were categorized and described. Descriptions of lesson components, student responses, and teacher feedback were recorded to reveal the qualitative aspects related to the situations surrounding teachers' and students' actions. Percentage of representations and constructivist teaching approaches were examined through the creation of two bar graphs. The first bar graph examined the percentages of the indicators of representations and constructivist teaching approaches for the first lesson taught by each teacher. The second bar graph examined the average percentage of the indicators of representations and constructivist teaching approaches for each teacher. The means and ranges associated with each teacher and the concurrent indicators were provided to demonstrate size of differences between lessons for each teacher. The pictorial representation was used to reveal the amount of commonality between representations and constructivist approaches.

Descriptive statistics were calculated on the resulting information from the nine indicators of types of representations, types of constructivist teaching approaches, and types of engagement, as well as for the test data for both types of questions. In addition, calculations were performed using information from the 16 lessons overall, as well as from information gathered from each of the 16 lessons. Thus, mean percentages and standard deviations for each indicator across the 16 lessons were computed. Means and standard deviations for the pre-test and post-test data for both procedural and conceptual types of questions were calculated. Procedural and conceptual gains across the 16 lessons were also computed. These overall descriptive statistics across the lessons were computed in order to obtain a baseline of how students performed for each question item. Also, the overall means and standard deviations for the indicators provided information concerning the amount of occurrences that prevailed when examining several different lessons. The information from each of the 16 lessons provided information concerning percentages of indicators across the board for each unit of analysis. Therefore, it could be discerned which lessons had strong or weak attributes of use of various representations, constructivist teaching approaches, and engagement in the lesson.

Student achievement was estimated by calculating procedural and conceptual gains and standard deviations for each of the lessons in order to observe the differences and similarities between performances on procedural type questions as compared to conceptual type questions. In addition, the standard deviations revealed the distances that scores were from the mean. Also, the mean gains revealed high or low scores which could be examined according to the types of teaching approaches utilized in the lessons. In this manner, an idea of how students perform related to various approaches could be analyzed.

A correlation matrix for the nine indicators was calculated using SPSS. Each pair of indicators was examined to determine slight, limited, or good predictions, as well as need for combination of indicators due to likely measurement of the same item. Thus, r^2 was calculated for each pair of indicators in order to determine the amount of variability accounted for, or predicted by one indicator from another (Creswell, 2002). Of course, r, or the coefficient, was first examined to reveal the degree to which the items were correlated, in addition to any statistically significant correlations at either the .01 or .05 level.

Structural equation modeling was used to determine the relationship between representations, constructivist teaching approaches, engagement, procedural knowledge, and conceptual understanding. A multi-level structural equation model with a student level (within groups) and teacher level (between groups) was examined using MPLUS. The two-level structural equation model prevented the loss of variation at the teacher level because the student variation contributed to the variation at the teacher level. Muthen and Muthen (2006) stated, “Random effects representing across-cluster variation in intercepts and slopes or individual differences in growth can be combined with factors measured by multiple indicators on both the individual and cluster levels” (p. 6). The use of a two-level structural equation model prevents the loss of information, due to the fact that independence is not assumed over all scores, but only those dealing with between-level scores. Otherwise, a single-level structural equation model would ignore certain teacher level (between-level) predictors that do indeed reveal that scores for students in certain clusters are in fact more related than students randomly pulled from participation in different teacher lessons (Kline, 2005).

The study determined whether the paths from representations, constructivist teaching approaches, and engagement indicated significant predictions for procedural knowledge, or conceptual understanding. The study also determined if the path from representations and constructivist teaching approaches indicated a significant prediction for engagement. An overall model fit for the data was reported, including χ^2 and degrees of freedom, as well as the p-values for the variables predicting classroom means for procedural knowledge and conceptual understanding. The Comparative Fit Index (CFI) and Root Mean Square Error of Approximation (RMSEA) were reported to provide further information concerning the model fit. Paths were examined for possible removal, or addition, depending on the significance of the path (non-significant paths removed). The model was determined to fit the data when a non-significant model resulted, $p > .05$. The structural equation model that relates these variables is shown in Figure 2.

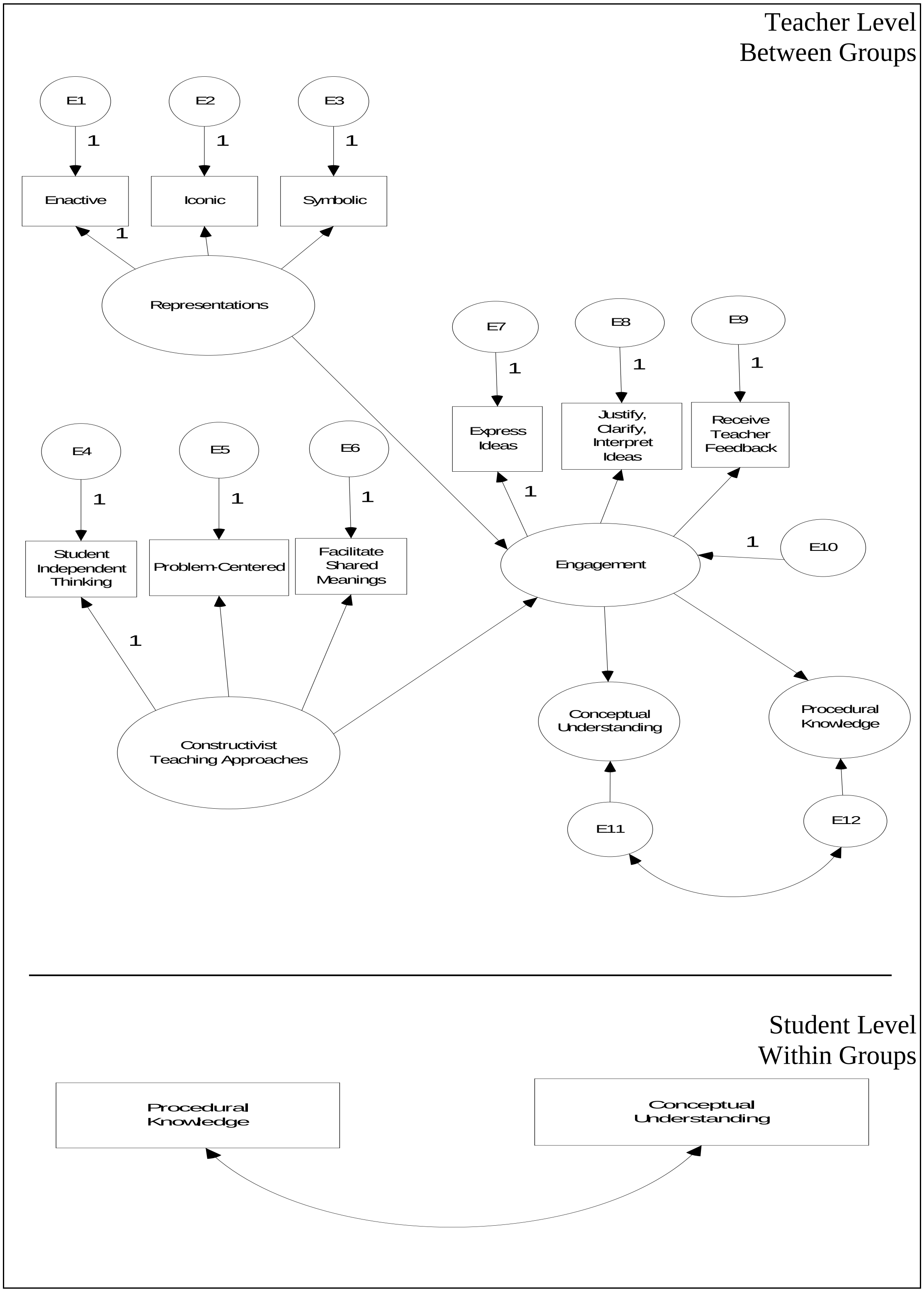

Figure 2. Structural equation model with measures of the latent variables.

CHAPTER IV

ANALYSIS

Examination of Each Lesson

Each lesson was examined through detailed recording of descriptions of teacher and student actions. Thus, examples of occurrences of each indicator were listed, along with any distinct materials utilized within these categories. For example, when looking for iconic representations, examples of such representations, such as a graph were drawn to provide detail as to the kinds of representations used. Likewise, with types of constructivist approaches, examples revealing ways students were working independently, on problem-centered lessons, or using shared meanings were documented. In addition, examples depicting how students appeared to be engaged in the lesson were also provided. The teacher actions during the lesson were also documented, including the types of feedback each one normally provided to the students, the learning environment provided, and the promotion or prohibition of independent thought.

Lesson 1 Taught by Teacher 1

Lesson 1 followed the intended lesson from the textbook, *MathThematics*, with the use of an exploration activity involving the graphing of equations. This activity was observed in the video lesson. However, the teacher did not include the graphing calculator activity provided at the end of the lesson in the textbook. This activity involved entering the equation into the y= screen, creating the appropriate settings for the window, graphing the equation, and using either the trace feature or table feature to find specific values on the graph.

Throughout Lesson 1, presentations of symbolic and iconic representations were used, without any use of enactive representations. Students created tables of values, discovered closed form rules, created graphs, and discussed ideas. They also wrote ordered pairs for the x and y-values of the table of values. Students discussed with the teacher appropriate scales to use when labeling the graphs. Additionally, the table of value was examined through transition of the pictorial form of the numbers into words. For example, one student commented, “For every 10 miles per hour, there’s 15 additional feet in distance.” Students worked with labeling the independent and dependent axes of the graph, as well. They realized such labeling by deciding which variable depends on the other variable. One student stated, “Distance depends on speed.” The students and teacher also discussed the idea of proportionality as it related to the graph. When the teacher asked why a relationship was not proportional, a student responded, “…because it doesn’t go through (0, 0).” Students also learned how to determine whether or not a point was a solution to the equation. One student responded, “If the point is on the graph, the ordered pair is a solution.” Throughout the lesson, it was observed that the use of symbolic representations, and discourse coded from constructivist approaches and engagement were shared with the teacher instead of amongst the students themselves.

Lesson 1 involved limited use of constructivist teaching approaches with no facilitation of shared meaning involved. Students did not solve their own problems, but were instead directly guided throughout the learning process by the teacher. The amount of time coded as student independent thinking for this lesson involved only the process of asking questions during the lesson. The time spent on such tasks was very low. For the creation of problem-centered lessons, the teacher did not present a cumulative problem to solve that involved the students in discovering ideas throughout the learning steps. Instead, the connections between equation, table, and graph were taught using direct teaching. In addition, guess and check was used primarily to find closed form rules instead of well thought processes. The portion of the lesson coded as problem-centered involved use of illustrations of real world examples, such as correlating highway driving to the understanding of functions. During the lesson, the teacher did not foster any negotiation of meaning, or create small groups to foster collaboration.

There was a small amount of engagement in the lesson with no connections or probing offered by the teacher in the form of feedback. It was noted that the teacher lectured the entire time. Thus, there were not many openings for students' expression of ideas and especially justification of these ideas. Again, comments were coded as such, whenever students provided a comment or question pertaining to an ideas related to the lesson or connection to other topic in mathematics. Short phrase responses or simple statements were not coded as comments. Students provided very few justifications. One student did explain how to estimate the number of car lengths that should be in front of a car in order to avoid a wreck. In addition, feedback consisted of reiteration of responses, with no connections or probing questions offered.

Lesson 2 Taught by Teacher 1

Lesson 2 followed the intended lesson from the textbook, *MathThematics*, with an exploration activity involving finding the slope of a line. Students also read a story that fostered discussions involving rate of change. Additionally, they discussed ways to determine a person's height by his/her footprint. Thus, they were learning about independent and dependent variables. These activities were observed in the video lessons.

There was predominant use of symbolic representations, with some iconic representations, and no enactive representations. The students primarily discussed ideas related to slope after examining the story. They also worked with the formula for finding the slope of a line. Interestingly enough, the teacher used algorithmic rules to explain ways to subtract integers. There weren't any conceptual reasons provided for such approaches. During the remainder of the lesson, the teacher used a graph to teach the concept of slope. Students were asked to provide the slope of the line by counting intervals up and over in order to get back on the line. This sort of direct instruction teaching was apparent throughout the lesson. Instead of allowing students the time needed to explore ways to find the slope of the line, instructions were explicitly provided.

This lesson involved a small amount of overall constructivist teaching approaches, with little or no time devoted to encouragement of student independent thinking or creation of problem-centered lessons. The teacher did, however, spent about a third of her time providing opportunities for small group activities. Students worked with partners to explore the slope of a line. In this lesson, some discussion of ideas concerning the story, as well as the creation of graphs and finding slopes ensued. The teacher delineated the meanings, formulas, and definitions of mathematical ideas to the students during the lesson. The small group activities did not, however, provide students to explore ideas on their own. Instead, they provided more practice time to collaborate with peers concerning previously learned ideas. The teacher more or less described the steps needed to take in using the coordinates to find the slope of the line. Additionally, the lesson involved using a graph and its coordinates to find the slope of the line, but did not tie these ideas together with a cumulative problem. Therefore, this lesson did not exude much constructivist approaches as described by the operational definitions.

There was a rather limited amount of engagement observed in the lesson. For example, students provided mostly short responses to questions asked. They did not elaborate, or provide higher-level comments during the lesson. One student expressed an idea in the lesson by stating, “If they’re tall, they have a long foot.” This student was making a conjecture based upon previous data. This sort of expression was an anomaly in this lesson, however. In addition, students did not spend much time making justifications for reasons beyond short responses. One student did explain her reasoning by stating, “A higher slope is not reasonable since the line is flatter.” Slightly more percentage of time was devoted to providing students with meaningful feedback. It should be noted that this percentage is still considerably low. The teachers’ meaningful feedback consisted of providing counterexamples, expounding on ideas, making connections to other ideas, and proving students concerning reasonableness.

Lesson 3 Taught by Teacher 2

Lesson 3 involved material at the beginning of the lesson, other than that provided in the textbook, *Mathematics: Applications and Connections*. The sheet the students were working on wasn’t the same as the sheet included in the observation packet. Towards the end of the lesson, however, students began working on writing expressions from word problems that came from the lesson provided in the textbook. Therefore, the activities provided in the lesson packet did not completely match the material watched on the video lesson.

The predominant type of representation presented in this lesson was symbolic representations. A small amount of time was devoted to iconic representations, with no enactive representations used in the lesson. Most of the lesson consisted of the teacher solving equations on the board. Each step was explicitly shown, without any room for discovery. Likewise, the teacher wrote the operations of addition, subtraction, multiplication, and division on the board along with synonyms used in word problems. This use of symbolic representations did involve more student interaction, as they provided words for the teacher to write down on the board. Additionally, students worked on writing expressions from word problems, both as whole class participation and individual class work. The use of iconic representations consisted of a diagram drawn to illustrate the order of operations. The teacher also used a number line to illustrate the process of addition and subtraction of integers.

The use of constructivist teaching approaches was minimal or non-existent in this lesson. There was not any creation of problem-centered lessons, or facilitation of shared meanings through either small group activities or negotiation of meanings. Students practiced on their work, or in other words, spent time on task. Much time was spent providing students exact algorithms to use when performing mathematical tasks. For example, once during the lesson, the teacher asked, “A positive and negative, you do what?” Rules were therefore encouraged and enforced without providing transitions from underlying conceptual ideas. During the lesson, students did ask some questions related to the writing of expressions, which was labeled as independent thinking. They also provided the teacher with synonyms for the four basic operations without any assistance from the teacher.

This lesson consisted of virtually no engagement as defined by the operational definitions. There were not any cases of expression of ideas, as constituted by higher-level comments instead of simple short responses. Likewise, students did not provide any justifications that would be considered as explanatory. A small amount of time, albeit less than one minute, was devoted to providing meaningful feedback to the students. The feedback provided consisted of teacher probes to help students understand the difference between equations and expressions.

Lesson 4 Taught by Teacher 2

Lesson 4 followed the intended lesson from the textbook, *Mathematics: Applications and Connections*, with the use of t-tables, graphs, ordered pairs, equations, and writing sentences to correspond with the situation described from the function. The lesson began with examining the average height of males and drawing a graph to demonstrate the increase in height. Students learned that height depends on age, or is a function of age. The information from the lesson packet matches the observations from the video lesson.

This lesson involved a high use of symbolic and much higher use of iconic than that used in the previous lesson taught by this teacher. Enactive representations were not used in this lesson either. The lesson was taught in a direct instruction manner with symbolic representation use consisting of copying the definition of a function into their journals, filling in the table of values with numerals and symbols, writing ordered pairs and equations to relate the function. These processes were learned through a step-by-step process lead by the teacher. It should be noted students did discuss the function as it appeared on the graph, while comparing it to the table and graph. The use of iconic representations consisted of creation of t-tables, or tables of values, as well as graphs, which were presented in a non-discovery manner.

There was not any noted use of constructivist teaching approaches in this lesson. For each of the three indicators of constructivist approaches, there was not any time in minutes devoted to them. Students did work on sheets at their desks, which represented time-on-task. However, they were not using invented algorithms, or solving problems that had not been explicitly explained to them beforehand. Question-asking was also not a part of the learning process revealed in this lesson. As for creation of problem-centered lessons, students did work on tying together representations of functions in written, tabular, and graphical form. However, they did not work on one cumulative problem that was given to them to work on as they constructed ideas for themselves. Instead, they worked with worksheets that contained several problems.

Engagement also played a small role in the execution of this lesson, as well. For example, there were not any examples of students' expression of ideas that resembled higher-level comments. In fact, the teacher left little room for these sorts of comments through constant reiteration of steps and facts. At one point, the teacher commented, “What did I say earlier? This is always x, and this is always y.” This comment left no room for students to discover the reasoning behind such labeling. One student did provide an explanation for the particular labeling of the axes. Thus, this occurrence constituted the small amount of time devoted to justifications. A limited amount of time was spent providing meaningful feedback to students in this lesson, as well. Most comments provided by the teacher consisted of, “Why?” The feedback provided that was helpful for students consisted of use of probing questions to help the students make meaningful connections. However, the use of such feedback was quite scarce.

Lesson 5 Taught by Teacher 2

Lesson 5 did not include an observation packet from which alignment with a textbook could be determined. As viewed from the video, the lesson included learning more about functions, whereby students substituted values for x into an expression in order to obtain the solution, or y. Next, ordered pairs were written and used to create a graph.

This lesson included much use of symbolic and iconic representations, due to the inclusion of using numerals and symbols to write expressions, as well as the creation of a table of values and graphs to represent the functions. The students also reviewed the concept of a function. There were not any enactive representations, or manipulatives used in this lesson.

There were virtually no examples of constructivist teaching approaches utilized in this lesson. There was one example whereby a student asked a question concerning how to set up a table of values. Independent thinking, as realized by invented processes and solving of one's own problems, was not presented here. There was absolutely no creation of problem-centered lessons, or facilitation of shared meanings. The components of the lesson were not presented as a coherent whole. Additionally, the students were not provided opportunities to work on the cumulative problem while discovering ideas. Instead, every step was delineated by the teacher, therefore, hindering any opportunity for negotiated meanings and discussion.

The amount of time spent students spent engaged in the lesson was minimal. Students provided mostly short responses when answering the teacher, and comments made on one's own accord were rarely seen. In one case, a student described that a linear function forms a straight line. Justifications were not apparent in the lesson, either. Additionally, the teacher did not provide meaningful feedback to students. Instead, simple, short responses were given concerning the correctness of the response. Other instances of feedback consisted of the teacher following a response with a short question meant to evoke a correct response. These follow-up questions were not probing in the sense of leading students to the correct answer through the use of connections. Instead, they offered much assistance that more or less gave the answer away. Thus, this feedback was not coded.

Lesson 6 Taught by Teacher 3

Lesson 6 did not follow the intended lesson from the textbook, *Mathematics: Applications and Connections*, which consisted of the writing expressions and equations using textbook examples. Instead, the students participated in a Jet Ski activity, whereby they worked in groups exploring proportional relationships. Students also reviewed two Short Stack activities in which the stacking of cups was used to demonstrate non-proportional relationships. Therefore, the lesson that was provided was much more constructivist-based, than the intended lesson. Students did work with expressions and equations, but in a more exploratory manner. The material watched from the video lesson coincided with the observation notes for the lesson.

This lesson involved a variety of each of the three types of representations examined. Symbolic representations were primarily used, and enactive and iconic representations were relatively equal in the percentage of time used during the lesson. Students worked with symbolic representations in a variety of ways including finding the nth term in a sequence, writing formulas for geometric figures, writing expressions and equations, and writing the constant of proportionality. Students worked with cups in order to understand the reasons behind the non-proportional relationship demonstrated with cup stacking. As for iconic representations, students examined graphs to determine proportionality by deciding whether or not the line passed through the origin. The teacher also drew pictures of geometric figures on the overhead at the beginning of the lesson in order to provide students with opportunities to see connections between the drawing of the figure and the formulas for finding the perimeter, area, and circumference of the figure.

Constructivist teaching approaches were highly integrated into the lesson, especially via creation of problem-centered lessons and facilitation of shared meanings. The indicator of encouragement of student independent thinking was present in approximately one-third of the lesson. Students were provided opportunities to construct their own meaning concerning proportionality. It could be discerned that students were struggling with this concept during the lesson, but after it was over, they seemed to have a firm understanding of ways to determine proportionality and non-proportionality in a variety of formats. Throughout the lesson, a definition of proportionality was never offered, but instead students were encouraged to compare different equations and determine the differences between them. As for the creation of problem-centered lessons, students participated in an activity that related representations of proportional relationships via tables, graphs, and equations. Each of these elements was tied together in a constructivist manner, whereby students made connections and discovered ideas through inquiry and collaboration with others. Lastly, students worked in small groups and thus engaged in negotiated meanings with each other and with the teacher. Students discussed ways to determine proportionality, and the whole class decided upon these facets. Students also determined reasons for translation of a word problem into a graphical representation, whereby the line did pass through the origin. One student responded, “If you don’t go skiing, it won’t cost you anything, so it starts at 0.”

A relatively high level of engagement was exhibited during the lesson with mostly feedback from the teacher and expression of ideas. During the lesson, students made several comments concerning the lesson, especially while working in the small group setting. Students were providing higher-level, unique comments concerning their progress in understanding proportionality. Furthermore, when the teacher asked questions, students provided justifications for their responses, which surpassed simple, short phrase answers. Additionally, the teacher provided much probing and connections to previously learned ideas related to expressions, equations, and proportionality.

Lesson 7 Taught by Teacher 4

Lesson 7 followed the intended lesson from the *TAKS Mathematics Preparation Booklet*, with the use of an exploratory patterns activity. Students worked with patterns to determine the numerical value of the nth term, as well as proportionality and non-proportionality. The activity required students to fill in missing cells concerning the visual form of the pattern, written description, process column (or formula), and value for the term. This activity was not in alignment with the activity viewed from the video lesson. In the video, the use of iconic representations could not be discerned. However, it was obvious that enactive representations in the form of blocks were being used to help the students build the necessary patterns as explained by the formulas. Therefore, the coding from the video did not reveal the use of iconic representations.

Symbolic representations and enactive representations were predominantly used in the lesson. Iconic representations were not coded, due to the inability to view the sheet used in the lesson. It was believed that the patterns built from the blocks were being created solely from the use of a written description. The use of enactive representations included the use of blocks to help students understand the progression of the patterns. Additionally, such use was integrated to help students determine a formula for the nth term. Students used symbolic representations when writing formulas for the functions, writing numerical values for the nth term, and using words to describe the situation.

Constructivist teaching approaches were quite apparent throughout the entire lesson. For example, encouragement of student independent thinking and facilitation of shared meanings were present during almost the entire instructional lesson time. Likewise, creation of problem-centered lessons was also predominant, with integration revealed during almost three-fourths of the lesson. During the lesson, the teacher did not explicitly state any algorithms or rules. Instead, she guided students in an endeavor to invent procedures and solutions on their own. This sort of guidance encouraged students to ask questions concerning their learning, and thus inquire into underlying meanings concerning proportionality. The problem-centered lessons invited students to make connections between various representations of functions. Students then made further connections to the idea of proportionality and developed an ability to discern a proportional relationship based upon characteristics in the various representation formats. This activity was developed around small group participation, whereby the students and teacher negotiated meanings concerning appropriate representations and meanings.

Engagement was relatively high and included both whole class discussion and small group interaction. Students specifically discussed pattern-building and ideas underlying proportional relationships. Justifications were provided, along with comments, concerning reasons behind these ideas. One student explained that the relationship shown was proportional because the n was multiplied by 5. During these learning processes, the teacher provided much probing and connections to the various representations.

Lesson 8 Taught by Teacher 4

Lesson 8 followed the intended lesson from the *TAKS Mathematics Preparation Booklet*, which included determining patterns for finding the numerical value for the number of faces to be painted on adjacent cubes. The lesson included filling in the nth term, pictorially representing the cube train, describing the situation in words, writing a formula, and filling in the numerical value for the number of faces to paint. This activity aligned with the observations made from the video lesson.

Each type of representation was used highly during this cube face painting lesson. For example, students worked with actual cubes in order to provide a better visual basis concerning which faces will be showing. Many students have an easier time visualizing special relationships if they can manipulate the objects first. In addition, student drew the cube trains on paper, therefore working with iconic representations. Lastly, students used written words and algebraic expressions to describe the relationships. Additionally, students examined the process column, which contained formulas such as $f(n) = 3n + 2$, which was written as $3(1) + 2$, $3(2) + 2$, $3(3) + 2$, … on the students' chart. Students examined the expression to determine if it represented a proportional relationship, $y = kx$.

Constructivist teaching approaches were paramount throughout this lesson, as well. Students participated in solving cumulative problems, which involved modeling, drawing, writing formulas, finding the value for the nth term, and finding ways to determine proportionality. These ideas were discussed in a coherent manner, so that students did not walk away with an understanding of disconnected topics, but instead with an understanding of the whole picture. Additionally, students were encouraged to figure out ideas for themselves. The teacher provided only assistance in guiding the students towards an understanding. Each of the objectives of the lesson was delivered through interaction, both at the small group level and whole class level. Lecture was not the means used to teach these algebraic concepts.

Engagement in this lesson was quite high, due to the increased use of discussion during a large portion of the lesson. Not only did students discuss ideas in groups, but they also shared discoveries with classmates at the close of the lesson. These ideas were elaborated on by the teacher through probing questions, connections, and expansion to other ideas. The students shared revelations concerning ways to determine proportionality, thus exhibiting a high level of engagement in the lesson. Students were able to speak from understanding, which was prompted from engagement and not from memorization provided from a text.

Lesson 9 Taught by Teacher 4

Lesson 9 followed the intended lesson from the textbook, *MathThematics*, which involved a lesson on patterns and sequences. Students were given word problems and prompted to create a table of values and then write an expression to reveal the value of the nth term, or the closed form rule. Students who had not finished the face painting of the cubes from the previous lesson were provided time to do so, also. The lesson materials matched the observations from the video lesson.

A variety of each of the three representations was used in this lesson, with an especially high use of symbolic and iconic representations. During the lesson, students who had not finished the face painting activity did so with the use of enactive representations, or blocks. The other students worked on writing expressions to represent the closed form rule of the function. Thus, students were creating tables of values that were considered as both iconic, due to the pictorial format, and symbolic due to the use of numbers and symbols. Additionally, students used symbolic representations when writing expressions, reading aloud the word problems, and discussing ideas.

Constructivist approaches were used to help students develop a thorough understanding of the connection between word problems and algebraic expressions relating them. Students worked in small groups with peers in order to discover correct ways to set up t-tables, or tables of values, and determine a closed form rule. Each of the representations was tied together with the emphasis being on the connectedness of the representations. Additionally, the students working on the cube face painting were working on developing connections between representations of the function.

Engagement was definitely a prominent facet of this lesson, as evidenced from discussions concerning functions, closed form rules, proportionality, and constants and variables. Students engaged in discussions with each other, as well as the teacher and provided reasons to support their ideas. The teacher provided meaningful feedback in the form of making connections and probing students to develop a coherent understanding.

Lesson 10 Taught by Teacher 5

Lesson 10 followed the intended lesson from the textbook, *Mathematics: Applications and Connections*, which included solving equations. The sheet including exploratory activities using cups and counters to illustrate the steps involved in solving equations was not used in the lesson. Instead, only symbols were used to reveal the steps taken to proceed through the solution process. The other lesson materials asked students to solve equations and check their answers through substitution. These materials were in alignment with the materials observed in the video lesson.

The only representations utilized in this lesson were symbolic representations. Enactive representations and iconic representations were not used in this lesson. Interestingly enough, the lesson materials called for use of cups and counters, but such objects were not integrated into the lesson. Instead, students were required to use only rote steps while working with symbols in order to progress through the stages to obtain the solution. Likewise, iconic drawings of these manipulatives were not used either. Therefore, reasons underlying the steps were not brought to the students' attention during the lesson.

Constructivist approaches were not used in this lesson by the teacher to facilitate learning. Instead, the teacher relied on a more direct instruction approach, in which lecture was the main mode of deliverance of knowledge. Students were explicitly told each step they should take in the learning process. There wasn't any room for inquiry, or student independent thinking. There was one documented case of a student asking a question about the placement of the word “mean” in a table filled with synonyms for the four basic operations. Otherwise, question-asking was not a component of the lesson. There were many problems modeled for students, as well as many problems given to students for individual seatwork. In this light, it is easy to determine that one cumulative problem with several connected parts was not used here. Also aligned with the lecture method was an avoidance of small group work and discussion of any kind. Students did not participate in developing negotiated meanings with other students, or with the teacher.

Engagement was certainly not a component in this lesson, due to little opportunity for discussion. Students did not spend any percentage of time providing higher-level, thoughtful comments, or justifications of ideas during the lesson. Additionally, the teacher provided feedback in the form of acknowledgement of correct and incorrect responses. Connections or probing questions were not used to guide students in the right direction concerning the learning of algebraic concepts. Instead, the teacher reiterated the correct answer, or restated the steps taken to arrive at the solution.

Lesson 11 Taught by Teacher 5

Lesson 11 did not follow the intended lesson from the textbook, *Mathematics: Applications and Connections*, which included materials on writing expressions and equations. The lesson included materials from the textbook on functions and graphs, which was included in the following lesson. In fact, students drew graphs using information provided in t-tables while learning about labeling of axes and appropriate scaling. The lesson materials, therefore, did not match the materials viewed in the video lesson.

Symbolic and iconic representations were approximately evenly split in their integration into the lesson. Enactive representations, however, were not present in the lesson. This lesson consisted of creating graphs to represent the functions delineated in the t-tables. Students used symbolic representations to label the axes, fill in table values, and write short statements describing the functions. Iconic representations were used via the use of tables and graphs.

Constructivist approaches were not utilized at all during this lesson, due to a heavy dependence upon lecture. The only questions asked were by students who had started seatwork and needed to know how to set up the graphs. It should be noted that these questions were not inquisitive, but rather consisted of learning how to algorithmically solve the problem. The teacher tells them exactly what they should write on their paper. For example, the teacher stated, “What comes first in a table goes on the bottom, and what comes next goes along the side of the graph.” The reasons behind this statement were never explained. Additionally, several mini problems were given to students, as opposed to a cumulative problem encompassing many objectives. Discussion was not integrated into the lesson with the lesson involving short response answers instead.

There was little to no participation in classroom engagement found in the lesson. There were not any forms of student expression of ideas, or justification of ideas presented here. One occurrence of meaningful feedback was provided to a student by the teacher through probing questions used to promote understanding concerning the choice of appropriate intervals on a graph. In other words, this lesson consisted of mostly lecture and independent seatwork without opportunity for sharing with others.

Lesson 12 Taught by Teacher 5

This lesson did not follow the intended lesson from the textbook, *Mathematics: Applications and Connections*, which included worksheets on functions and graphs. The lesson did concentrate on the algebraic objectives, however. Instead of the proposed lesson materials, the teacher used other handouts that asked students to examine processes, ordered pairs, and graphs as they related to functions. Therefore, the lesson examined from the video did not match the lesson materials provided.

Symbolic and iconic representations were the representations of choice for this lesson, as well. Enactive representations, or physical objects, were not integrated into the lesson. Students used tables of value to determine input and output values, rules, and ordered pairs. Then, the students transferred this data into graphs, in which the relationship could be examined to determine the type of function represented. Students learned about linear functions, as well as equations that represent such functions. The teacher asked, "What equation would we use to graph this line?"

In this lesson, constructivist approaches were barely integrated into the learning process. There was not any evidence of problem-centered lesson creation, with only one occurrence of invented algorithms, supporting independent thinking. Two occurrences of facilitation of shared meanings were documented, with both dealing with negotiation of meaning and finding new ways to tackle the problem.

The presence of engagement was also scarce and included only one account of student expression of ideas and two accounts of receiving meaningful teacher feedback. In one situation, a student invented an algorithm for finding the value for the number of riders after 10 hours. Also, the teacher used probing in one situation to help a student discover ways to find the speed of the rollercoaster. At another time, the teacher expanded on a students' idea by revealing the appropriateness of the procedure in solving the problem. The teacher commented, "I had not thought of solving it that way, but that is definitely a way we could do it."

Lesson 13 Taught by Teacher 6

Lesson 13 followed the intended lesson from the textbook, *Texteams*, which included examining the height of cups in order to determine proportional relationships. Students converted the information to graphical form after filling in the process column and determining a closed form rule. Students worked with a Short Stack activity in class and were given another one for homework. These materials matched the materials and activities viewed from the video lesson.

During the lesson, students worked with a variety of representations in order to obtain a thorough understanding of proportional relationships. In this lesson, students worked predominantly with symbolic representations, but also worked with enactive and iconic representations. Students used the graphing calculator in order to graph equations and determine if they passed through the origin, or were indeed proportional. In addition, students drew and labeled graphs to represent the functions. Additionally, students created t-tables, both on paper and using the table function of the graphing calculator. In this manner, they could trace, so they could determine how many cups it would take to make a stack of nth height. Lastly, students set up proportions using equations, wrote equations for the line, wrote the constant of proportionality, and worked with symbols on both the graphs and tables.

Constructivist teaching approaches were a consistent part of this lesson. Each of the components of constructivist approaches was used during approximately half of the instructional part of the lesson. The creation of a problem-centered lesson encompassed the most amount of time, with facilitation of shared meanings and encouragement of independent thinking closely following. The students worked on one problem which tied together a realistic situation in the form of physical modeling, creation of t-tables, creation of graphs, and discussion of meanings associated with the connected ideas. Students determined their own procedures for analyzing and representing data. For example, students were asked to design their own graphs and explain reasons for the set up, in addition to describing whether or not it represented a proportional relationship. Each group presented their results to the class. Students also worked in small groups during the lesson, while sharing knowledge with members of their group, whole class, and teacher.

Student engagement played an important role in the structure of the lesson. Students were given ample opportunity to discuss ideas with others, provide justifications, and receive feedback from the teacher. Students spent the highest percentage of time providing higher-level comments, such as describing whether or not the dots on a graph should be connected. Additionally, students provided justifications concerning reasons for proportionality. Lastly, the teacher provided meaningful feedback mostly in the form of making connections to previously learned knowledge in mathematics. She encouraged them to examine other representations, make comparisons, and then determine the resulting ideas.

Lesson 14 Taught by Teacher 6

Lesson 14 did not follow the intended lesson from the textbook, *Mathematics: Applications and Connections*, which included graphing linear equations. Instead, students participated in a more exploratory activity involving the graphing of linear equations with a Cell Phone activity. Students worked with t-tables and graphs to determine proportional relationships. They also wrote equations found total costs, decided whether or not to connect the dots, and examined what happens at zero months. The Cell Phone activity matched the lesson materials and activities viewed from the video lesson.

Symbolic and iconic representations were the only representations used in the lesson. Students used symbols when working with both graphs and tables, as well as writing equations to represent the relationship. The lesson also involved the use of words to describe situations and reasons behind choices. In addition, students had to use symbols to reveal the process taken to arrive at the value for the nth term. Students worked heavily with making connections between the iconic and symbolic representations. In addition, much discussion concerning reasons behind responses was realized in the lesson.

Constructivist approaches were a very large part of this lesson with over 70% of time devoted to such approaches for each of the three indicators. Students were asked to determine their own strategies, representations, and solutions, as well as provide detailed justifications for their ideas. Such an inquiry-based activity involved much question-asking during the lesson. The lesson involved one large problem, which was a perfect example of a cumulative problem in mathematics. Students were given representations of data, asked to provide other representations, determine meanings for the representations, provide justifications for such meanings, and make connections to other ideas in mathematics. Likewise, the class was divided into groups whereby the sharing of ideas played a major role in the learning process. Also, whole class discussion concerning ideas related to proportionality was used.

Engagement was highly used in the lesson with predominant participation in expression of ideas. Students engaged in in-depth conversations with their group members, other classmates, and the teacher. These discussions were related to the ability to make connections and develop conceptual meaning for ideas, not for simply learning facts or procedures. Additionally, students spent much time providing justification for their responses, due largely to the requirement to explain reasoning during the activity. Lastly, teacher feedback consisted of probing and offering assistance to help students make connections to other ideas that might help them better understand the concept at hand.

Lesson 15 Taught by Teacher 6

Lesson 15 did not follow the intended lesson from the textbook, *Mathematics: Applications and Connections*, which included worksheets on graphing quadratic functions. Instead, the teacher continued the activity from the previous lesson, using the Cell Phone activity. Students worked more with using t-tables, graphs, and equations to determine proportional relationships and describe in words the occurrences realized through the graph. These lesson materials were aligned with the activity viewed from the video lesson.

Enactive, iconic, and symbolic representations were each used in the lesson in a coherent manner. Students used the graphing calculator to model situations, both in tabular and graphical form. Additionally, students used symbols to fill in tables, work with graphs, write equations, and describe relationships as evidenced by the data. Iconic representations in the form of graphs and tables were included in the Cell Phone activity. Each of these representations was incorporated into the lesson in a manner which promoted the ability to make connections in algebra.

Constructivist teaching approaches were once again a large part of the lesson, due to the inclusion of a cumulative mathematical problem with many components. Students were encouraged to find solutions on their own in a setting where mathematics was seen as a coherent whole, not as a source with many disconnected topics. Students were encouraged to develop their own ideas in a classroom environment where direct instruction was avoided. Furthermore, students participated in both small group and whole class discussions concerning their decisions and findings.

Engagement was also a large part of the classroom environment, with expression of ideas spanning over one-third of the classroom instructional time. One student summarized the previous day's findings from the lesson and revealed great understanding in doing so. Justifications were also provided by students during both small group and whole class discussion. One student explained, "You already paid it, so it's already there." He was explaining why you must keep the original amount paid when determining the closed form rule. Teacher expansion of ideas was also prominent in this lesson, as evidenced by her promotion of understanding the meaning of intersecting lines on a graph.

Lesson 16 Taught by Teacher 7

Lesson 16 matched the intended lesson from the textbook, *Mathematics: Applications and Connections*, which included using tables to graph functions. Students completed function tables, including filling in a process column, and then graphed the functions. The lesson was limited to use of the textbook. No other activities were involved in the lesson. These materials from the textbook correlate with the materials viewed in the video lesson.

Symbolic and iconic representations were the only representations used in the lesson. The lesson materials involved students in filling out function tables and creating corresponding graphs. Symbolic representations were utilized during both activities. Likewise, the use of such pictorial drawings indicates the use of iconic representations. Enactive materials were not used in this lesson.

This lesson did not include many constructivist teaching approaches, but instead relied on a more direct instruction teaching approach. The teacher provided information through primarily lecture and did not include a cumulative problem in the lesson. Instead, students were asked to work on several mini-problems that required simple rote procedures and knowledge. They were asked to copy the tables, finish filling them out, and create a graph to represent the data. The only portions of the lesson that were constructivist dealt with the need for students to create graphs without first being shown how to do so. Students were asked to discover ways to represent the information in graphical form. They also were given the opportunity to work with a partner in this endeavor.

Engagement was at a minimum during this lesson with all occurrences accounting for less than 2% of the instructional time. Students did not have many opportunities to provide comments or justifications due to the direct instruction format. Students did work with a partner, but little discourse was observed between the students. Most teacher feedback consisted of acknowledgement of correct or incorrect answers, along with a restatement of correct procedures.

Findings from Constant Comparison

When examining the 16 lessons, several commonalities and differences were found between the lessons. To begin with, three of the teachers, accounting for seven of the lessons, used primarily a constructivist approach to teaching. The other four teachers, accounting for the remaining nine lessons, used little constructivist approaches, with mainly an emphasis on direct teaching. In other words, four of the teachers relied primarily on lecture as the mode to deliver knowledge to the students. It should also be noted that teachers were consistent with their teaching approach across each of the lessons they taught. In other words, if a teacher primarily used constructivist approaches, then these approaches were evident in each of the lessons. There was not much variability in the coding of such approaches for the same teacher across the lessons taught by the teacher.

The use of constructivist teaching approaches seemed to have an effect on students' level of engagement during the lesson. For example, high levels of use of constructivist approaches revealed higher levels of engagement than lessons utilizing a more direct instruction approach. Higher levels of engagement were found for each of the three indicators of expression of ideas, justification of ideas, and receiving meaningful feedback from the teacher. Likewise, lessons with little constructivist approaches apparent revealed low levels of engagement. Thus, constructivist approaches seemed to be related to engagement.

The use of symbolic and iconic representations did not seem to have much effect on whether or not the lesson was constructivist-based or encompassed high levels of engagement. However, the use of enactive representations, or manipulatives, did seem to be related to higher levels of constructivist approaches and engagement. Whenever lessons involved a variety of representations, including enactive representations, the lessons seemed to be more problem-centered. In fact, students were engaged with various representations, which promoted independent thinking and shared meanings through expression and justification of ideas. In addition, the teacher was more involved in the learning process with the students, and thus, provided much more meaningful comments to the students. Thus, there seemed to be a correlation between use of enactive representations and constructivist activities and engagement in the lesson.

High levels of engagement occurring in a lesson revealed much discussion, inquiry, and connection-making in the learning process. As described by the operational definitions, engagement was based upon expression of ideas, justification and interpretation of ideas, and receiving meaningful feedback from the teacher. During lessons with high levels of engagement, students appeared to be participating in a community of practice (Boaler, 2000). Such participation revealed higher levels of understanding, as compared to the understanding of students evidenced by low engagement in the lesson. By this statement, it is meant that students seemed to possess a more complete understanding of the mathematics, as opposed to simply knowledge of algorithms and rote procedures.

All of the lessons involving more constructivist approaches involved specific roles for both teachers and students. When constructivist approaches were applied, the teacher played the role of facilitator of learning, whereby the responsibility was seen as the need to guide students in their endeavor to discover appropriate strategies, develop reasoning skills, and make connections. During these lessons, students were actively involved in the lesson, while creating strategies, testing ideas, discussing solutions and reasons with fellow students, and asking questions. Therefore, in these video lessons, students and teachers were participants in a community of learning. Everyone played an integral role in the learning process, and many shared ideas were available.

The lessons involving less of a constructivist approach and more of a direct instruction, or teacher-directed approach, also involved specific roles for teachers and students. For example, during these lessons, the teacher based lessons on lectures and played the role of “holder of knowledge” while covering the various algebraic topics. Students quietly sat at their desks, rarely working with other students, either taking notes or routinely filling in worksheets. In these lessons, students did not ask many questions, other than those concerning ways to set up a problem, or draw a graph. Instead of being inquiry questions, questions dealt with ways to set up problems and get started on the activity.

Representations and Constructivist Comparisons

In the analysis, it was desired to determine the amount of similarity between use of indicators of representations and indicators of constructivist teaching approaches. To begin with, a bar graph was created that revealed the percentage of occurrence for each indicator through a study of the first lesson taught by each teacher. Refer to Figure 3. Percentage of time for representations and constructivist approaches for each teacher revealed high use of constructivist approaches were present with high use of enactive representations. However, high use of iconic or symbolic representations did not seem to have a correlation with high use of constructivist approaches.

Level of engagement was shown to have an effect on the overlay of representations on constructivist teaching approaches. Total level of engagement for the three indicators of expression of ideas, justification of ideas, and receiving feedback were calculated for the first lesson of each teacher. The percentages of engagement for the first lesson ranged from small to rather large (9%-Teacher 1; 4%-Teacher 2; 55%-Teacher 3; 60%-Teacher 4; 1%-Teacher 5; 16%-Teacher 6; 4%-Teacher 7). High levels of engagement resulted in high levels of enactive representations and high levels of each of the three indicators of constructivist approaches (independent thinking, problem-centered lessons, and shared meanings). An engagement level of 60% resulted in 65% use of enactive representations, 98% independent thinking, 65% problem-centered lessons, and 98% shared meanings. On the contrary, an engagement level of 1% resulted in 0% enactive representations, 2% independent thinking, 0% problem-centered lessons, and 0% shared meanings. High or low levels of engagement did not seem to have an effect on iconic or symbolic representations, as evidenced by 60% engagement and 0% iconic representations realized in the same lesson. In another lesson, 55% engagement resulted in 54% iconic representations. Symbolic representations were high across the board, regardless of level of engagement. Engagement resulted in higher enactive representations and constructivist approaches. Also, enactive representations were the only representations shown to relate to constructivist approaches.

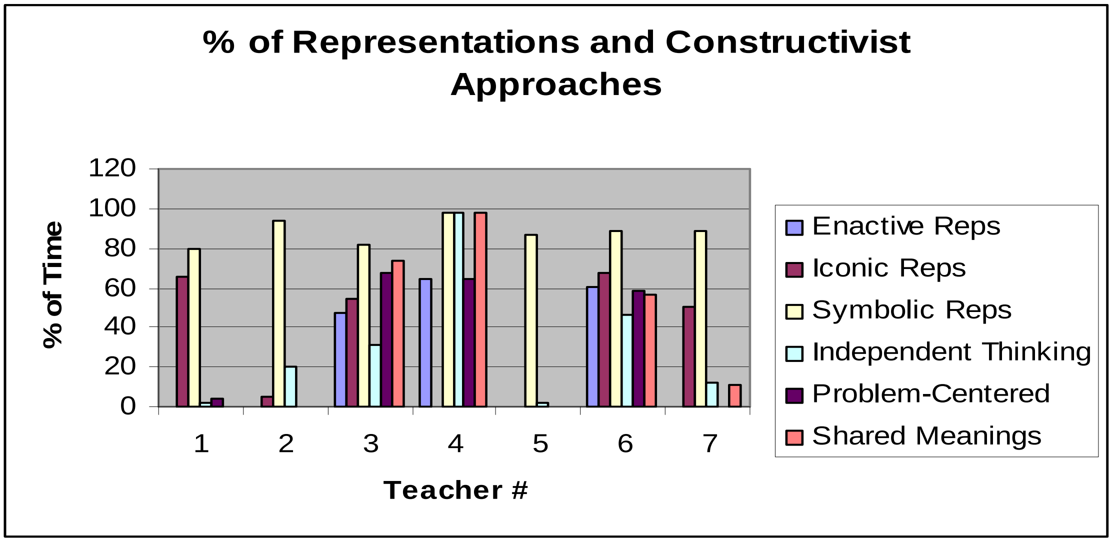

Figure 3. Percentages for first lesson of each teacher.

A bar graph was also created that revealed the percentage of occurrence for each indicator through a study of the average of the indicators for all lessons taught by each teacher. Refer to Figure 4. The average percentage for representations and constructivist approaches for each teacher revealed increased constructivist approaches with the use of enactive representations. Iconic and symbolic representations did not seem to have an effect on use of constructivist approaches. In fact, the use of iconic or symbolic representations alone revealed low levels of constructivist approaches. Also, higher use of iconic representations revealed high levels of constructivist approaches than use of symbolic representations.

Average level of engagement for each teacher was also shown to correlate with higher use of enactive representations and constructivist approaches. The average of total engagement for the three indicators of expression of ideas, justification of ideas, and receiving feedback for each teacher was calculated. The average percentages for overall engagement once again ranged from small to rather large (13%-Teacher 1; 5%-Teacher 2; 55%-Teacher 3; 76%-Teacher 4; 2%-Teacher 5; 58%-Teacher 6; 4%-Teacher 7). The results revealed that 76% engagement resulted in 52% use of enactive representations, 88% independent thinking, 75% problem-centered lessons, and 88% shared meanings. On the contrary, 2% engagement resulted in 0% use of enactive representations, 1% independent thinking, 0% problem-centered, and 1% shared meanings. Again, level of engagement did not have an effect on iconic representations, which fluctuated regardless of engagement, or on symbolic representations, which was consistently high for each of the averaged indicators.

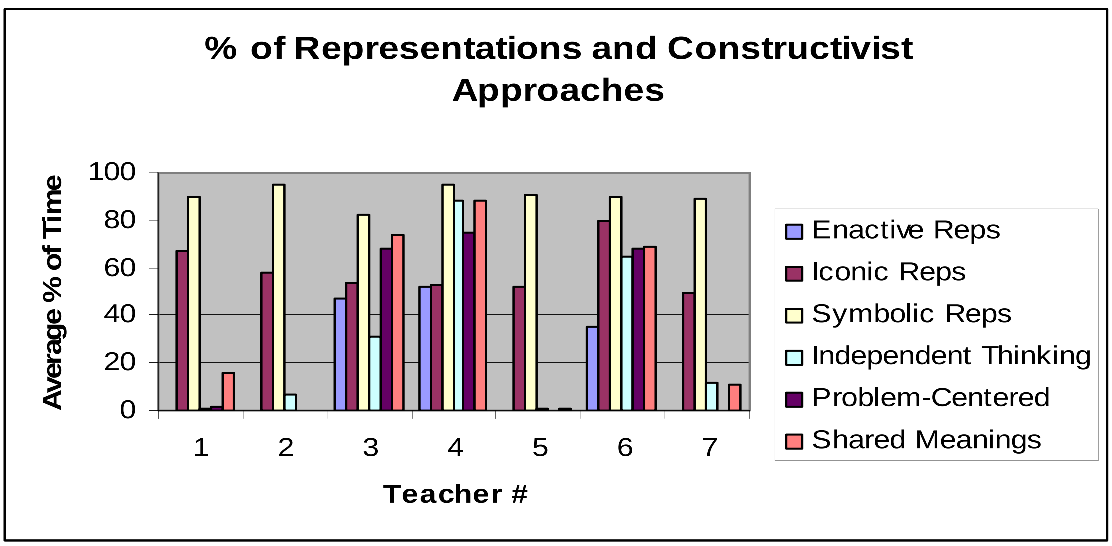

Figure 4. Average percentages for each teacher.

Ranges and Means Examined for Indicators for Each Teacher

Higher means for enactive representations were indicative of higher levels of constructivist approaches. Refer to Table 2. In addition, the teacher with the highest mean for enactive representations had the highest means for independent thinking, creation of problem-centered lessons, and facilitation of shared meanings, as well. Likewise, one teacher with a 0% average for enactive representations had the lowest averages for the three indicators of constructivist approaches, with 1% being the highest average. The ranges revealed whether or not the approach of examining average percentages for each teacher was appropriate. It appeared that the ranges were not large across the board. The ranges for enactive and iconic representations were large in a few cases, due to the extremes of no use of these representations to high usage. Otherwise, the ranges were fairly consistent across the lessons for each teacher.

Table 2
Means and Ranges for Each Teacher for Indicators of Representations and Constructivist Teaching Approaches

	T1		**T2**		**T3**		**T4**		**T5**		**T6**		**T7**	
	M	R	M	R	M	R	M	R	M	R	M	R	M	R
E	0	0	0	0	47	0	52	68	0	0	35	61	0	0
I	67	2	58	80	54	0	53	8	52	87	80	26	50	0
S	90	19	95	2	82	0	95	11	91	12	90	12	89	0
IT	1	2	7	20	31	0	88	25	1	2	65	34	12	0
PC	2	4	0	0	68	0	75	19	0	0	68	18	0	0
SM	16	32	0	0	74	0	88	16	1	2	69	25	11	0

Note. E = Enactive; I = Iconic; S = Symbolic; IT = Independent Thinking; PC = Problem-Centered Lessons; SM = Shared Meanings; **T** = Teacher #

Descriptive Data Analysis

After the data collection from both the algebra pre and post-test results and video observations were finished, descriptive statistics were calculated on the information. To begin with, an overall mean and standard deviation for the percentages of each indicator across the 16 lessons were calculated. Refer to Table 3. By calculating these descriptive statistics, the indicators with the highest and lowest means overall could be discerned, in addition to the distance scores were from the mean.

After examining the results, it was obvious that symbolic representations had the highest mean, whereas justification had the lowest mean. In fact, such results would be easily hypothesized, due to the abundance of use of symbols, numbers, and words in classrooms and less appearance of justifications for reasoning. Although justifications were apparent throughout the viewing of the video lessons, the amount of time spent providing explanations for ideas delivered in discussions seemed to be lowest amount the indicators observed. Most of the discussions were in the form of comments about the task at hand and intriguing ideas they were discovering. Mostly, students only provided explanations if the teacher directly asked for such. In fact, expression of ideas and receiving feedback from teachers also ranked lowest among the means for all of the nine indicators. It should be noted that this does not mean that there was not much discussion between teachers and students and group interaction during the lessons. It does, however, show that use of representations and participation in problem-centered and independent lead lessons with subsequent group work and negotiated meanings will most likely receive higher percentages of time than items including engagement related to discussion. In other words, students may be in the setting for indicators of discussion, but will spend less time on each individual occurrence of discussion than the whole activity of constructivist approaches which encompasses the discussion.

Other indicators seemed to have means that revealed an understandable degree of use, as well. For example, enactive representations had a lower mean than the other types of representation, due to the fact that many of the lessons did not include any use of manipulatives. Additionally, iconic representations had a lower mean than symbolic representations, but a higher mean than enactive representations. This can be evidenced by the high use of pictorial representations in the form of tables, graphs, diagrams, and more that was used in addition to symbolic forms. In some rarer cases, enactive representations were used to help students work with iconic representations and lastly, symbolic representations. Therefore, the means for the indicators seems to fall in expected bounds.

The high standard deviations, however, indicated a large deviation from the means on most of the indicators. Use of symbolic representations and justification of ideas had the lowest standard deviations, which can be accounted for by the consistency of use of these types of representations across lessons and similar time coding for justification across lessons. The other lessons showed more variability in scores' distance from the mean, which reveal the sometimes large differences in time coding across the lessons.

Table 3
Percentages of Instructional Time for Indicators of Latent Variables across 16 Lessons (N = 971)

Indicator	M	SD
Enactive Representations	22.67	29.15
Iconic Representations	58.58	31.54
Symbolic Representations	91.43	6.36
Independent Thinking	35.66	37.22
Problem-Centered Lessons	35.33	36.02
Shared Meanings	41.02	39.34
Express Ideas	13.23	16.41
Justify Ideas	5.47	7.12
Receive Feedback	16.54	17.76

In addition, an overall mean and standard deviation for responses to procedural types of questions were calculated. Refer to Table 4. On both the pre-test and post-test, students from all 16 lessons scored the highest on question 8b, which assessed students' abilities to fill in a table of values for number of apple trees and number of pine trees for each term. On both the pre-test and post-test, students from all 16 lessons scored the lowest on question 10, which assessed students' abilities to find a different pair of values that would still make the equation, or statement, true. Students also scored low on question 12, which examined students' understanding of finding the n that corresponded to the value for the nth term. These results revealed that students did not have much difficulty with filling in a table of values with finding the value of the nth term. However, when asked to think a little more abstractly and find other values than the ones provided that would make a statement true, students had more difficulty. Also, when asked to think in a backwards manner and find the n that relates to the nth term, students scored quite low. Therefore, students had more difficulty applying concepts to problems that were not presented in a certain, straightforward manner. Across the 16 lessons, the two largest gains for procedural questions from pre-test to post-test were on questions 8b and 16, which both dealt with finding values of y. The standard deviations for scores on procedural questions were low (all less than 1), with those for questions 8b and 16 being the highest.

Table 4
Correct Responses to Procedural Type Questions across 16 Lessons (N = 971)

	M		SD	
Question	Pre-test	Post-test	Pre-test	Post-test
#1	.59	.65	.49	.48
#5	.60	.69	.49	.46
#7	.61	.60	.49	.49
#8a	.59	.70	.49	.46
#8b	.68	.97	.91	.94
#10	.32	.42	.47	.49
#12	.39	.44	.49	.50
#16	.56	.77	.81	.88

Descriptive statistics were also calculated for conceptual understanding types of questions across all 16 lessons. Refer to Table 5. On both the pre-test and post-test, students from all 16 lessons scored the highest on question 4, which assessed their understanding of the commutative property. On both the pre-test and post-test, students from all 16 lessons scored the lowest, as well as the same, on questions 8d and 13. Question 8d assessed students' understanding of examining growth of patterns through the need to describe which type of tree increases more quickly, either apple or pine. Question 13 assessed students' abilities to recognize a linear relationship via a constant rate of change through a table of values. With the results from this set of questions, it appears that students did well with having a firm conceptual understanding of properties.

However, the ability to look at patterns and examine changes, as with constant or non-constant rates of change, was more difficult for the students. Across the 16 lessons, the two largest gains for conceptual questions from pre-test to post-test were on questions 8c and 14. Question 8c was quite a high level question dealing with the need to find a value of n that would provide the same number of apple trees as pine trees. Students not only had to find that value of n, but also had to explain how they found the answer. Question 14 asked students to determine if the closed form rule was correct and support the answer with an accurate explanation. The standard deviations for scores on conceptual questions were low (most less than .50), with those for questions 8d and 13 being the lowest.

Table 5

Correct Responses to Conceptual Type Questions across 16 Lessons (N = 971)

	M		SD	
Question	Pre-test	Post-test	Pre-test	Post-test
#2	.33	.42	.47	.49
#3	.31	.35	.46	.48
#4	.59	.65	.49	.48
#6	.32	.30	.47	.46
#8c	.08	.18	.35	.53
#8d	.06	.10	.29	.35
#9	.24	.32	.43	.47
#11	.13	.20	.42	.50
#13	.06	.10	.30	.35
#14	.12	.23	.38	.35
#15a	.20	.24	.40	.43
#15b	.45	.47	.50	.50

Descriptive statistics were also computed on procedural and conceptual gains across the 16 lessons. Refer to Table 6. The mean gain for procedural type questions was higher than the mean gain for conceptual type questions. The means were not very far apart from one another, with a difference of .22. In addition, the standard deviations for both mean gains are low, with both being less than three standard deviations. Therefore, when looking at overall mean gains for all 16 lessons, it can be discerned that students increased gains from pre-test to post-test highest for those types of questions asking for more factual, rote types of knowledge.

Table 6

Procedural and Conceptual Gains across 16 Lessons (N =971)

Question Type	Mean Gain	SD
Procedural Questions	.87	2.76
Conceptual Questions	.65	2.41

Percentages of each indicator for each of the 16 lessons were computed in order to obtain an understanding of the prominence of certain occurrences in the various lessons. Refer to Table 7. After examining the data, use of symbolic representations was the only indicator that did not result in 0%. In fact, the percentage of time spent with these representations was high among each of the 16 lessons. Also, use of enactive representations had the highest number of lessons recorded with 0%. Lessons that did not use manipulatives could not be coded above 0%, whereas several of the other indicators receiving lower percentages, i.e. justification of ideas and facilitation of shared meanings, often included at least one occurrence related to the indicator.

Each lesson was examined according to high or low levels of percentages of time for indicators across the board. The data was analyzed in order to determine lessons that seemed to have high occurrences for all nine indicators. The results revealed that lessons 6, 8, 9, 13, and 15 all had reasonably high scores across each of the nine indicators, when comparing these scores to those from other lessons. Therefore, of the 16 lessons, five of the lessons revealed high levels of constructivist activities, whereas the other 11 illustrated various parts of such activities. It can also be noted that lessons 7 and 14 also had high percentages for the indicators, with the exclusion of either iconic or enactive representations. For the constructivist teaching approaches of fostering student independent thinking, creation of problem-centered lessons, and facilitation of shared meanings, lessons 6-9 and 13-15 had the highest percentages. Likewise, for the components of engagement, which included expression of ideas, justification of ideas, and receiving feedback from the teacher, these lessons also showed the highest percentages. Therefore, a consistency for each lesson across the indicators seemed to be normal. Additionally, it should be noted that the teachers who included enactive representations (manipulatives) in the lesson typically yielded higher percentages for the indicators related to both constructivist teaching approaches and engagement. This was not the case for one lesson whereby the teacher used iconic and symbolic representations in such a problem-centered manner that percentages across the board were very high. It simply appears that use of enactive representations coincided with high percentages for the other indicators that involve use of other representations, constructivist approaches, and engagement in the lesson. Finally, the highest percentages for each of the nine indicators were all above 50%, except for the highest percentage for justification of ideas, which appeared in lesson 8 at 26%. These results solidify the ideas that students are not spending as much time explaining reasons behind their ideas as they are in performing other tasks, even in constructivist settings.

Table 7
Indicator Scores for Lessons 1-16

		Percentages of Lessons Attributed to each Indicator								
Lessons	N	E	I	S	IT	PC	SM	EX	J	RF
1	43	0	66	80	2	4	0	7	2	0
2	43	0	68	99	0	0	32	3	4	10
3	68	0	5	94	20	0	0	0	0	4
4	68	0	84	95	0	0	0	0	0	8
5	68	0	85	96	0	0	0	2	1	1
6	117	47	54	82	31	68	74	20	8	27
7	79	65	0	98	98	65	98	13	5	42
8	79	79	84	87	94	84	84	35	26	58
9	79	11	76	99	73	76	82	19	7	24
10	58	0	0	87	2	0	0	0	0	1
11	58	0	87	88	0	0	0	0	0	3
12	58	0	68	99	1	0	2	1	0	1
13	41	61	68	89	46	58	56	8	5	3
14	41	0	78	85	80	76	70	67	14	22
15	41	43	94	97	71	71	81	37	6	11
16	30	0	50	89	12	0	11	1	1	2

Note. E = Enactive; I = Iconic; S = Symbolic; IT = Independent Thinking; PC = Problem-Centered Lessons; SM = Shared Meanings; EX = Express Ideas; J = Justify Ideas; RF = Receive Feedback

Means and standard deviations for procedural gains and conceptual gains for each lesson were calculated. Refer to Table 8. In order to obtain an understanding of student performance on both types of questions, a lesson-by-lesson analysis of the data was conducted. Using descriptive data results only, it was determined that lessons 1 and 2 had the highest mean gain from pre-test to post-test on procedural type questions. However, lesson 6 had the highest mean gain on conceptual type questions. Therefore, the students who had the highest mean gain on procedural type questions did not also have the highest mean gain on conceptual type questions. Likewise, the students with the second highest mean gain on procedural type questions did not have the highest gain on conceptual type questions. Also, lessons 13-15 had relatively high mean gains on procedural type questions ($M = 1.22$), but had low mean gains on conceptual type questions ($M = .17$). Oddly enough, lesson 16, which had a negative mean gain on procedural type questions ($M = -.43$), had a relatively high mean gain on conceptual type questions ($M = 1.17$). In addition, the lesson involved very little constructivist teaching approaches and engagement and resulted in a higher gain for conceptual type questions than some of the lessons that involved much constructivist approaches and engagement, i.e. lessons 7-9. In fact, this lesson seems to be an anomaly in that it is one of only two lessons whereby students had a higher mean gain on conceptual type questions than procedural type questions. However, the other lesson that had a higher mean gain for students on conceptual type questions than procedural type questions (lesson 6), did involve very high levels of constructivist approaches and concurrent engagement. The standard deviation for each lesson for both procedural and conceptual types of questions was low, with all standard deviations being less than 3.50. Thus, the mean gain for each lesson was not very far from the overall mean of these gains across all 16 lessons.

Table 8
Procedural and Conceptual Gains for Lessons 1-16

		Procedural		Conceptual	
Lesson	N	Mean Gain	SD	Mean Gain	SD
1	43	1.70	2.88	.88	2.24
2	43	1.70	2.88	.88	2.24
3	68	1.25	2.45	.71	1.69
4	68	1.25	2.45	.71	1.69
5	68	1.25	2.45	.71	1.69
6	117	1.09	3.03	1.52	2.62
7	79	1.37	2.29	1.03	2.30
8	79	1.37	2.29	1.03	2.30
9	79	1.37	2.29	1.03	2.30
10	58	- .79	2.61	- .40	2.20
11	58	- .79	2.61	- .40	2.20
12	58	- .79	2.61	- .40	2.20
13	41	1.22	2.87	.17	3.14
14	41	1.22	2.87	.17	3.14
15	41	1.22	2.87	.17	3.14
16	30	- .43	3.49	1.17	2.88

A correlation matrix of the nine indicators of the latent variables was also examined. Refer to Table 9. According to Creswell (2002), the coefficient, r, provides information regarding the degree of the correlation between two variables. The coefficient of determination, or r^2, can be used to provide information concerning the strength of the relationship between the variables. In other words, one can determine the amount of variance accounted for in one variable by another variable. After examining the coefficients in Table 9, it appeared that iconic representations had a low correlation with symbolic representations; iconic representations had a low correlation with facilitation of shared meanings; iconic representations had a low correlation with receiving feedback from the teacher; and symbolic representations had a low correlation with facilitation of shared meanings. The other correlations were pretty high and represented a statistically significant correlation.

After squaring the coefficients and obtaining r^2, the proportion of variance was examined for each pair of indicators. As described by Creswell (2002), coefficients of determination can provide slight predictions (.20-.35), limited predictions (.35-.65), good predictions (.66-.85), or correlations so high that the items should be combined (.86+). After examining all of the data, only seven pairs of indicators represented good correlations, or predictions, of the variance in one variable by that of the other variable. These pairs included use of enactive representations and receiving feedback from the teacher; fostering independent thinking and creating problem-centered lessons; fostering independent thinking and facilitation of shared meanings; fostering of independent thinking and receiving feedback from the teacher; creation of problem-centered lessons and receiving feedback from the teacher; facilitation of shared meanings and receiving feedback from the teacher; and students' justification of ideas and receiving feedback from the teachers. Only one pair of indicators had a coefficient of determination higher than .86, which was that of creation of problem-centered lessons and facilitation of shared meanings. This high r^2 indicates the possible need to combine these indicators since they seem to be measuring the same item.

Table 9

Correlation Matrix of the Indicators

	E	I	S	IT	PC	SM	EX	J	RF
E	1.00	-.07*	-.20**	.74**	.78**	.80**	.44**	.70**	.82**
I		1.00	-.02	-.07*	.11**	-.01	.28**	.28**	-.03
S			1.00	.09**	-.17**	-.03	-.28**	-.30**	-.11**
IT				1.00	.89**	.90**	.72**	.72**	.84**
PC					1.00	.96**	.79**	.76**	.81**
SM						1.00	.70**	.68**	.83**
EX							1.00	.76**	.60**
J								1.00	.85**
RF									1.00

Note. E = Enactive; I = Iconic; S = Symbolic; IT = Independent Thinking; PC = Problem-Centered Lessons; SM = Shared Meanings; EX = Express Ideas; J = Justify Ideas; RF = Receive Feedback

* $p < .05$. ** $p < .01$.

Multi-level Structural Equation Modeling Analysis

Structural equation modeling analyses were chosen for this study in order to provide pertinent information concerning the overall model fit and significance of paths represented in the model. For example, an overall χ^2 and corresponding fit indices were reported for the model as a whole, as well as path coefficients between the latent factors and manifest variables. Multi-level structural equation modeling was used due to the need to examine both the student level and teacher level of data. Individual test score gains were analyzed, as well as classroom test score gains. Due to the fact that the videos revealed classroom occurrences, the student data was nested within the teacher-level (classroom) data. Therefore, it was determined that the relationships of the variables should be examined at both levels. In addition, such modeling would prevent the loss of variation at the teacher level.

The first part of the structural equation modeling process involved writing a program for MPLUS that included the within level for student individual gain scores, in addition to the between level for classroom gain scores on procedural and conceptual knowledge. The within level included only a path between procedural and conceptual knowledge, ascertained by individual student gains from pre-test to post-test. The between level examined the effects of three different latent factors and their indicators on classroom averages of gains from pre-test to post-test for both types of learning. The indicators for the three separate factors were entered at the between level for representations, constructivist approaches, and engagement. Factor 1 (representations) included use of enactive, iconic, and symbolic representations. Factor 2 (constructivist teaching approaches) included fostering independent student thinking, creation of problem-centered lessons, and facilitation of shared meanings. Factor 3 (engagement) included students' expression of ideas, justification of ideas, and receiving feedback from the teacher. In addition, the program was written to examine the paths between representations and engagement, as well as constructivist approaches and engagement, whereby engagement was portrayed as a mediator. The path between constructivist teaching approaches and use of representations was also examined. The paths from engagement to both types of learning, procedural and conceptual, were included in the model. Additionally, the path between procedural and conceptual knowledge at the between level was examined.

The analysis of this original model, as portrayed in Figure 2, resulted in a model of bad fit. Both χ^2 and other fit indices were examined, namely Comparative Fit Index

(CFI), Tucker-Lewis Index (TLI), and Root Mean Square Error of Approximation (RMSEA). The results revealed a significant model, which indicated a model that did not fit the data, $\chi^2(40, N = 971) = 96.38$, $p < .001$. The alpha level for the significance of the model was set at .01. Due to the fact that a large n can provide a significant p-value, the other fit indices must be examined. CFI, TLI, and RMSEA provide a more complete picture of the model fit from the data. The results of fit indices included CFI = 0.84, TLI = 0.78, and RMSEA = 0.04. Both CFI and TLI were lower than the desired 0.90 for each of these. RMSEA was lower than 0.06 and thus was good. Additionally, the model results revealed some abnormal standardized coefficients for factor 1.

The next steps in the analysis procedure involved examining the model estimates to determine possible additions or deletions to the model. At the within level, the path between individual student procedural and conceptual knowledge was significant, $p < .01$. Factor 1 (representations) revealed negative unstandardized coefficients and abnormal output for standardized coefficients. The paths from y4 (independent thinking), y5 (problem-centered lessons), and y6 (shared meanings) to factor 2 (constructivist teaching approaches) were significant, $p < .01$. The paths from y7 (expression of ideas), y8 (justification of ideas), and y9 (receiving feedback) to factor 3 (engagement) were significant, $p < .01$. The path from representations to engagement was not significant, $p > .05$. However, the path from constructivist teaching approaches to engagement was significant, $p < .01$. Factor 3 (engagement) significantly predicted procedural knowledge, $p < .05$, as well as conceptual knowledge, $p < .10$ when using one-tailed distributions. Due to a priori beliefs in direction for procedural and conceptual gains, a one-tailed test was used on these two parts of the model throughout each stage of the analysis process. Also, the correlation between factor 1 (representations) and factor 2 (constructivist teaching approaches) was significant, $p < .01$. Lastly, the path between procedural knowledge and conceptual understanding at the between level was significant, $p < .05$. It should be noted that all other portions of this model other than the gains from the predicting factor to procedural or conceptual knowledge were analyzed using a two-tailed test.

These model results were very interesting and important to report, but a change in the model needed to occur resulting from the overall model fit results, abnormal standardized coefficients for factor 1, and negative variance for factor 1. Therefore, the next step involved removing y2 (iconic representations) and y3 (symbolic representations) from the model. The path from factor 1 to iconic representations was not significant, $p = .309$. Likewise, the path from factor 1 to symbolic representations was not significant, $p = .480$.

An exploratory factor analysis (EFA) was also performed on the data due to the abnormal standardized coefficients for the first factor, negative variance for the first factor, and lack of model fit. Using Principal Component Analysis with Promax as the rotation method, a pattern matrix was produced that revealed all indicators loading onto one factor, except for y2 (iconic representations) and y3 (symbolic representations), which loaded onto two separate factors. These factors were already thrown out of the analysis due to their non-significance in the model estimates. Therefore, the remaining indicators were y1 (enactive representations), y4 (independent thinking), y5 (problem-centered lessons), y6 (shared meaning), y7 (expression of ideas), y8 (justification of ideas), and y9 (receiving feedback from the teacher). These indicators all loaded onto one factor.

The next part of the structural equation modeling process involving writing another program for MPLUS that included this second model, altered from the original. The within level included the path between individual student scores for both procedural knowledge and conceptual understanding. The between level was altered to contain only the seven remaining indicators, which were loaded onto factor 1. Classroom level procedural knowledge and conceptual understanding were predicted from this factor. Refer to Figure 5.

The model fit results of the second model indicated a much better fit than the original. The model was still significant, however, and thus indicated a model that did not fit the data, $\chi^2(26, N = 971) = 55.37$, $p = .001$. The fit indices were much improved with values for CFI and TLI approaching high values (CFI = 0.90, TLI = 0.86). RMSEA was still less than 0.06, indicating no problems (RMSEA = 0.03). The model estimates from the second model revealed information concerning the significance of paths represented, as well as a problem with one of the parameters. Each of the paths represented in the model, both at the within level and the between level were significant. It should be noted that a two-tailed test was used for determining significance of all paths, except for those from factor 1 to either procedural knowledge or conceptual understanding, whereby a one-tailed test was used. Refer to Table 10. The program output notified a problem with the parameter between y7 and factor 1.

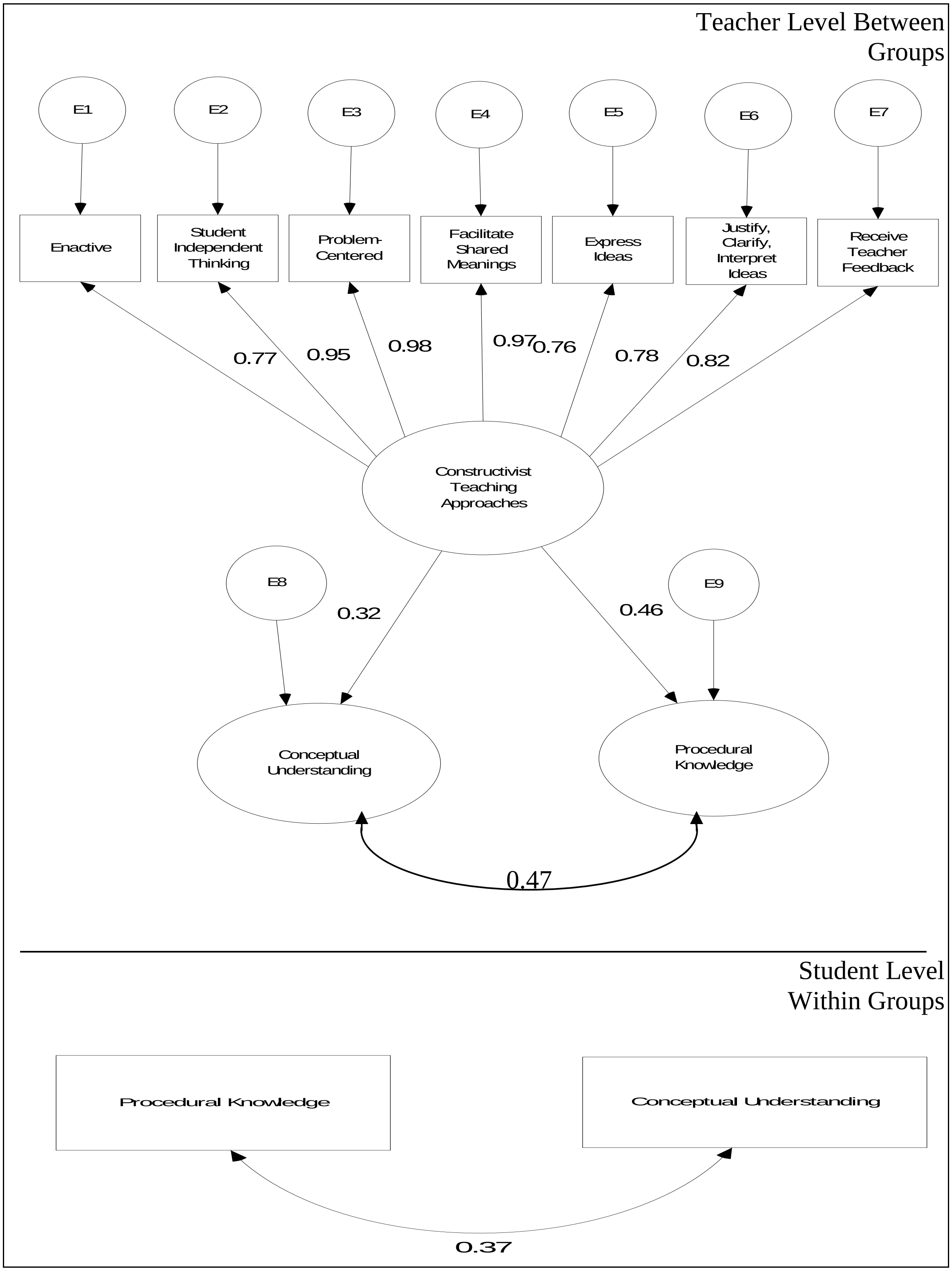

Figure 5. Structural equation model for second model.

Table 10
Path Coefficients from Model II

Path	Unstandardized	Standard Error	Critical Ratio	P	Standardized
Within					
PK and CU	2.46	0.39	6.31	.000**	0.37
Between					
Factor from E	0.62	0.16	3.95	.000**	0.77
Factor from IT	1.60	0.41	3.95	.000**	0.95
Factor from PC	1.61	0.43	3.72	.000**	0.98
Factor from SM	1.70	0.43	3.98	.000**	0.97
Factor from EX	0.65	0.28	2.35	.010*	0.76
Factor from J	0.24	0.08	3.13	.001**	0.78
Factor from RF	0.62	0.13	4.79	.000**	0.82
PK from Factor	0.02	0.01	2.03	.022††	0.46
CU from Factor	0.01	0.01	1.49	.068†	0.32
PK and CU	0.25	0.11	2.25	.013*	0.47

Note. PK = Procedural Knowledge; CU = Conceptual Understanding; E = Enactive; IT = Independent Thinking; PC = Problem-Centered Lessons; SM = Shared Meanings; EX = Express Ideas; J = Justify Ideas; RF = Receive Feedback.
*p < .05, two-tailed. **p < .01, two-tailed. † p < .10, one-tailed. †† p < .05, one-tailed.

The continued steps in the analysis process included excluding y7 from the third and final model. The only change to the third model consisted of including six indicators for factor 1, instead of seven. These indicators were y1 (enactive), y4 (independent thinking), y5 (problem-centered lessons), y6 (shared meanings), y8 (justification of ideas), and y9 (receiving feedback). Refer to Figure 6. Thus, the latent factor 1 was examined in accordance with these indicators to determine the idea represented by the six indicators. It was determined that the remaining six indicators reveal the crux, or main components of constructivist teaching approaches. The use of hands-on materials, independent thinking, cumulative problems, discussion, justification, and receiving meaningful feedback all feed into the process of helping students construct meaning for themselves.

The model fits results from the third and final model were indicative of a model that fit the data. The model was not significant, $\chi^2(19, N = 971) = 30.60$, $p = .045$. The alpha level used to determine significance of the model was set at .01. The other fit indices revealed good values with CFI = 0.96, TLI = 0.93, RMSEA = 0.03. Each of the paths represented in the model were significant. Refer to Table 11. Again, it should be noted that a two-tailed test was used to determine significance in all cases, except for the two paths from factor 1 to procedural knowledge and conceptual understanding. Due to the a priori belief in a gain, a one-tailed test was used. Variances, means, and intercepts were recorded for the final model. Refer to Appendix F. It should be noted the variance of x1 (individual student procedural knowledge) and x2 (individual student conceptual knowledge) at the within level, as well as y1 (enactive representations), y9 (receiving feedback), x3 (classroom procedural knowledge), and x4 (classroom conceptual understanding) at the between level revealed a significant difference from zero. Therefore, the scores for these variables were more widely distributed.

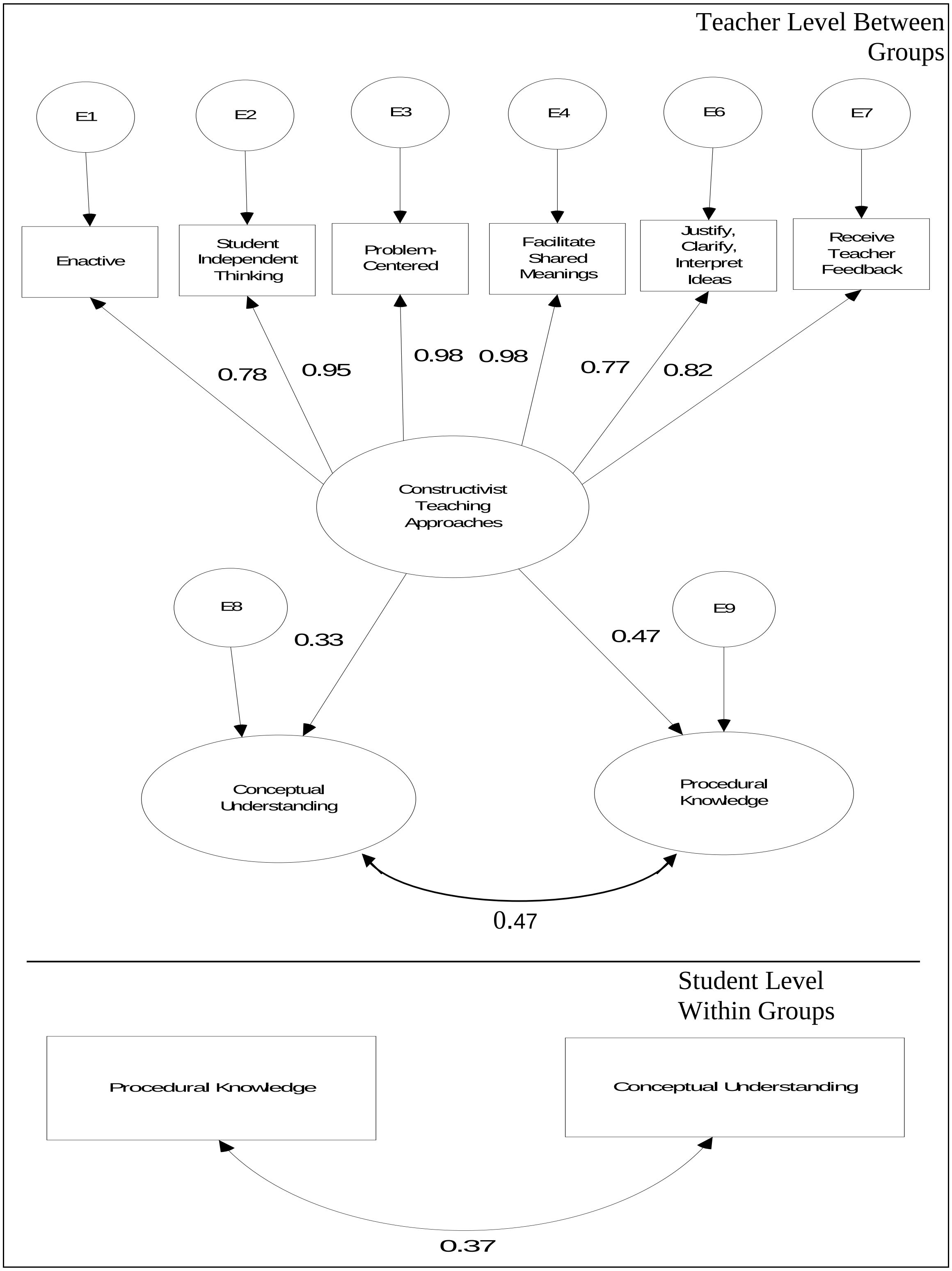

Figure 6. Structural equation model for final model.

Table 11
Path Coefficients from Final Model

Path	Unstandardized	Standard Error	Critical Ratio	P	Standardized
Within					
PK and CU	2.46	0.39	6.31	.000**	0.37
Between					
Factor from E	0.63	0.15	4.14	.000**	0.78
Factor from IT	1.58	0.38	4.14	.000**	0.95
Factor from PC	1.58	0.41	3.87	.000**	0.98
Factor from SM	1.69	0.41	4.08	.000**	0.98
Factor from J	0.23	0.07	3.14	.001**	0.77
Factor from RF	0.62	0.13	4.95	.000**	0.82
PK from Factor	0.02	0.01	2.05	.020††	0.47
CU from Factor	0.01	0.01	1.55	.061†	0.33
PK and CU	0.24	0.11	2.22	.013*	0.47

Note. PK = Procedural Knowledge; CU = Conceptual Understanding; E = Enactive; IT = Independent Thinking; PC = Problem-Centered Lessons; SM = Shared Meanings; EX = Express Ideas; J = Justify Ideas; RF = Receive Feedback.
*$p < .05$, two-tailed. **$p < .01$, two-tailed. † $p < .10$, one-tailed. †† $p < .05$, one-tailed.

After the final model was run using MPLUS, β and R^2 were also examined. All of the beta weights were significant, thus indicating an increase in the dependent variable by a specific number of standard deviations. For example, enactive representations resulted in an increase in factor 1 of .78 standard deviations. Factor 1 resulted in an increase in procedural knowledge at the between level of .47 standard deviations. Factor 1 also resulted in an increase in conceptual knowledge at the between level of .33 standard deviations. Refer to Table 11. R^2 values were reported for each of the observed variables. The observed variables included y1 (enactive, $R^2 = 0.61$), y4 (independent thinking, $R^2 = 0.90$), y5 (problem-centered lessons, $R^2 = 0.95$), y6 (shared meaning, $R^2 = 0.95$), y8 (justification of ideas, $R^2 = 0.60$), y9 (receiving feedback, $R^2 = 0.68$), x3 (classroom procedural knowledge, $R^2 = 0.22$), and x4 (classroom conceptual understanding, $R^2 = 0.11$). These results reveal the largest proportion of variance accounted for are represented by independent thinking, problem-centered lessons, and shared meaning.

The final model thus revealed that of the nine original indicators placed onto three separate latent factors, six actually fit the data. These six indicators loaded onto a single factor, thus collapsing the model of three factors to one factor. The results indicate that there is a significant correlation between students' gains on procedural knowledge and conceptual understanding at the within level. Additionally, there is a significant correlation between classroom level gains on the two types of learning at the between level. Enactive representations, fostering independent thinking, creating problem-centered lessons, facilitation of shared meanings, justification of ideas, and receiving feedback were significant predictors of factor 1, or in other words, constructivist teaching approaches. It should be noted that originally the indicators of fostering independent thinking, creating problem-centered lessons, and facilitation of shared meanings were labeled constructivist approaches. However, as the analysis revealed, other indicators loaded with these onto one factor. Therefore, the factor with the six remaining indicators has been renamed as constructivist teaching approaches. Lastly, constructivist teaching approaches are a significant predictor of both procedural knowledge and conceptual understanding of middle school algebra.

Summary

The original model for the study involved nine indicators predicting three latent factors, which were entered as predicting the two types of learning. The latent factor of representations included enactive, iconic, and symbolic representations. Additionally, the latent factor of constructivist teaching approaches included encouragement of student independent thinking, creation of problem-centered lessons, and facilitation of shared meanings. Lastly, the latent factor of engagement included expression of ideas, justification of ideas (involving clarification and interpretation), and receiving feedback from the teacher. Engagement was included as a mediator in the model, and therefore predicted from representations and constructivist approaches and predicting procedural knowledge and conceptual understanding.

The original model did not fit the data, revealing a significant model with bad fit indices. The low CFI and TLI and negative variance for factor 1 revealed a need to run an exploratory factor analysis (EFA) on the variables to determine which indicators loaded onto certain factors. After doing so, it was determined that each of the indicators loaded onto factor 1, except for y2 (iconic representations) and y3 (symbolic representations), which loaded onto two separate factors. Due to the fact that the paths between y2 and y3 and factor 1 were found to be non-significant, these indicators were taken out of the model.

The second model involved seven indicators loading onto factor 1 and predicting the two types of learning. The model thus included the indicators of y1 (enactive representations), y4 (student independent thinking), y5 (problem-centered lessons), y6 (shared meanings), y7 (expression of ideas), y8 (justification of ideas), and y9 (receiving feedback) as loading onto factor 1. The model revealed better fit indices, but still provided a significant model, which did not fit the data. The results did reveal a problem with the parameter involving y7 (expression of ideas). Therefore, this indicator was taken out of the model.

The third and final model included only six indicators loading onto factor 1 and predicting procedural knowledge and conceptual understanding. The six indicators of factor 1 were y1 (enactive representations), y4 (student independent thinking), y5 (problem-centered lessons), y6 (shared meanings), y8 (justification of ideas), and y9 (receiving feedback). The indicators were examined to determine the name of the latent factor containing such parts. It was decided that enactive representations, independent thinking, problem-centered lessons, shared meanings, justification of ideas, and receiving meaningful feedback from the teacher represented the crux, or main components of constructivist teaching approaches. Therefore, factor 1 was renamed as constructivist teaching approaches. Active, hands-on learning during cumulative problems involving shared ideas is learning that occurs in constructivist-based classrooms. This model was not significant, $p > .01$, and revealed good fit for both CFI (0.96) and TLI (0.93). Additionally, RMSEA was at a good level well below 0.06 at 0.03. Each of the paths in the model was significant, also.

The results from the final structural equation model revealed a significant correlation at the within level for the variables of student procedural knowledge and student conceptual understanding. At the between level, each of the indicators had significant paths from the latent factor of constructivist teaching approaches. The latent factor of constructivist teaching approaches significantly predicted both procedural knowledge and conceptual understanding. Lastly, the correlation between both types of learning at the between level (classroom level) was significant. The results thus revealed that the latent factor of constructivist teaching approaches, with six indicators, had significant effects on both types of student learning. The R^2 results for the observed variables at the between-level revealed y4 (student independent thinking), y5 (problem-centered lessons), and y6 (shared meanings) accounted for the most variance. These variables were the original indicators of constructivist approaches. However, y1 (enactive representations), y8 (justification of ideas), and y9 (receiving feedback) also accounted for a good proportion of the variance. The endogenous variables, x3 (procedural knowledge) and x4 (conceptual understanding) also accounted for a good proportion of the variance. Of course, these were lower than the other R^2 values, with $R^2 = 0.22$ for x3 (procedural knowledge) and $R^2 = 0.11$ for x4 (conceptual understanding), as compared to a range of 0.60-0.95 for R^2 for the indicators. However, the proportion of overlap revealed by the two types of learning was large.

Constant comparison of details from the sixteen lessons revealed similar findings to those realized from the structural equation model. For example, it was determined that the teachers with higher percentages of time with constructivist approaches also had higher levels of engagement in the classroom. Also, use of enactive representations seemed to have an effect on students' level of participation in constructivist activities, as well as engagement in the lesson. Lessons involving use of enactive representations had higher levels of constructivist approaches and student engagement. Furthermore, engagement in the lesson revealed more discussion, inquiry, and connection-making during the lesson.

The examination of the structural equation model and constant comparison revealed similar findings. For example, the structural equation model revealed a connection between enactive representations, the original three indicators of constructivist approaches, and two of the indicators of engagement. These results were closely aligned with the findings from constant comparison. The main difference lay within the correlation between use of enactive representations and higher levels of expression of ideas. In the structural equation model, expression of ideas did not fit into the final model along with enactive representations. However, justification of ideas and receiving feedback, the two other forms of discussion, did fit in the model. It was discerned that such hands-on activities promoted overall discussion. The structural equation model revealed the paths that indicated a model of good fit and thus revealed a closer match with justification of ideas and receiving feedback than with expression of ideas.

The examination of the commonalities between representations and constructivist teaching approaches also revealed similar findings to both the structural equation model and constant comparison. The two bar graphs revealed the presence of higher levels of the three indicators for constructivist approaches of encouragement of student independent thinking, creation of problem-centered lessons, and facilitation of shared meanings with the presence of enactive representations. The presence of iconic and symbolic representations did not reveal high levels of constructivist approaches. On the contrary, presence of these representations alone revealed low levels of constructivist approaches. Therefore, the only representation with high correlation with constructivist teaching approaches was enactive representations. This result was also provided in the final model of the structural equation model, as well as from the constant comparison of the details from the 16 lessons.

The descriptive statistics revealed important information concerning overall presence of the indicators across the lessons examined. For example, symbolic representations had the highest mean for percentage of indicators of representations across the 16 lessons. Facilitation of shared meanings had the highest mean for percentage of indicators of constructivist teaching approaches across the lessons. Receiving feedback had the highest mean for percentage of indicators of engagement across the lessons.

The descriptive statistics also revealed interesting information pertaining to the mean gains for the two types of learning across the 16 lessons. For example, students had a higher mean gain for procedural type questions ($M = .87$) than for conceptual type questions ($M = .65$). Thus, students had a higher increase in learning concerning procedural type questions from pretest to posttest. However, the mean gains were not very far apart. In relation to the structural equation model, the use of constructivist approaches had a significant effect on these main gains for procedural knowledge and conceptual understanding.

In conclusion, this study revealed the indicators that were significant predictors of constructivist teaching approaches. Furthermore, this study revealed that constructivist teaching approaches did, in fact, significantly predict students' procedural knowledge and conceptual understanding. Enactive representations, encouragement of student independent thinking, creation of problem-centered lessons, facilitation of shared meanings, justification of ideas, and receiving meaningful feedback from the teacher promoted higher levels of learning, both procedurally and conceptually. In addition, students were more engaged with constructivist approaches. This engagement revealed increased understanding, as evidenced through increased higher-level discourse. Therefore, classrooms rich in activities that promote construction of one's own knowledge, active engagement in the lesson, and discourse with others was shown to be connected to increased learning. The results from the video data and algebra test data corroborate these findings.

CHAPTER V

CONCLUSIONS

The quantitative and qualitative findings from this study revealed interesting aspects concerning the indicators that align with constructivist teaching approaches. Additionally, the findings revealed the effects of such approaches on students' procedural knowledge and conceptual understanding. The structural equation modeling tests and constant comparison technique provided similar results concerning correlation of indicators and their effects on learning.

Factors Predicting Procedural and Conceptual Knowledge

The original model with the three latent factors of representations, constructivist approaches, and engagement predicting procedural knowledge and conceptual understanding did not fit the data in this study. The final model, which resulted in good fit, involved one latent factor (renamed as constructivist approaches) with the six indicators of enactive representations, encouraging student independent thinking, creating problem-centered lessons, facilitation of shared meanings, student justification of ideas, and receiving feedback from the teacher. The newly renamed, constructivist teaching approaches, significantly predicted procedural knowledge ($p = .02$) and conceptual understanding ($p = .06$), with alpha level set at .10.

There are various possible reasons for the collapsing of the three latent factors into one factor. Classroom approaches that would most closely align with constructivist ideals are those that are hands-on, promote inquiry, and involve reasoning and discussion. The six indicators provided the crux of constructivist approaches, based upon the theory of constructivism. Such indicators of learning provide opportunities for independent learning, construction of knowledge, and communication (Piaget, 1973; von Glasersfeld, 1997; Vygotsky, 1978). Therefore, the findings do align with the theoretical framework.

Some of the variables were most likely excluded from the model, due to either teacher technique when using the approach, or less rigorous requirements from the students. For example, iconic and symbolic representations did not fit into the model of good fit. Their exclusion provides counter findings to previous research that ascribes to their important role in understanding. The manner, in which these representations were used, however, played an important role in the classroom environment, whether it was constructivist, or more teacher-lead instruction. Thus, often the use of these variables in a non-constructivist fashion would not lead to increased learning and understanding. Expression of ideas was most likely excluded, due to the fact that expression of ideas does not require as high a level of understanding as justification of ideas, which did fit the model. Although expression of ideas was coded whenever higher-level comments were made, justification of these comments was an even harder task. Thus, the ability to justify claims in mathematics would certainly be indicative of both procedural and conceptual understanding. Expression of ideas might certainly relate their procedural knowledge, but may not indicate their underlying conceptual knowledge.

Representations and Constructivist Approaches Predicting Engagement

From the original model results, representations were not found to be significant predictors of engagement ($p = .42$), but constructivist approaches were shown to be significant predictors of engagement ($p = .000$). This counters the literature that explains the importance of use of representations, including iconic and symbolic representations on engagement (Saenz-Ludlow, 1995). It can be assumed that representations were not significant predictors of engagement, due to the fact that once again, the manner in which the representations were used affected student engagement. In a classroom based primarily on lecture, students might be using each of the three representations with little or no engagement. If the teacher is modeling each and every step, the students are not engaged, but instead are spending time on task. Constructivist approaches involve inquiry, investigative work on cumulative problems, and sharing ideas with others. Thus, it is easy to understand why constructivist approaches would indeed promote student engagement in the lesson.

The final model included one representation (not all three), three of the original constructivist approaches, and two of the indicators of engagement. These findings support the literature base explaining the correlation of these indicators, with enactive representations promoting engagement (Quinn, 1997), as well as constructivist approaches promoting engagement (Boaler, 2000; Cunningham, 2004; Verhovsek & Striplin, 2003; Warrington & Kamii, 1998). In this study, enactive representations and the original three indicators of constructivist approaches resulted in a model of good fit, along with the indicators of justification of ideas and receiving feedback from the teacher. The reason for the inclusion of enactive representations relates to the typically higher use of these representations in constructivist classrooms than use of iconic or symbolic representations. Although enactive representations can be used in a more direct instruction manner, they are more often than not, integrated as an exploratory activity.

Representations and Constructivist Approaches Overlay

The original three indicators of representations and three indicators of constructivist approaches were qualitatively examined in order to determine similarities and differences with the multi-level structural equation model. An examination of representations and constructivist approaches for both percentages of occurrence during the first lesson for each teacher and the average percentage across a teacher's lessons revealed similar findings. It was determined that use of iconic or symbolic representations produced a low overlap with each of the three original indicators of constructivist approaches. Enactive representations, however, did reveal high levels of constructivist approaches. These variables were also examined according to total engagement for the first lesson, as well as average total engagement across teachers' lessons. In examining the overlap according to engagement, it was determined that high levels of engagement were linked to high levels of use of enactive representations and constructivist approaches. Low levels of engagement were linked to low levels of enactive representations, and concurrently, low levels of constructivist approaches.

The findings of the relationship between enactive representations, constructivist approaches, and engagement are supported by the literature. Again, iconic and symbolic representations are not included as being supported by engagement or supporting constructivist approaches in this study. Thus, the literature that explains their benefits in assisting students' understanding is not evidenced in this study. Similar reasons can be stated for the way in which representations overlap constructivist approaches according to student engagement. When students are engaged, they could easily be working with hands-on materials, while portraying independent thinking in collaboration with others on investigative, cumulative mathematics problems. When students are engaged, they are actively constructing their own meaning (Piaget, 1954).

Teachers' Presentations and Students' Actions

Teachers' presentations and students' actions differed greatly across the 16 algebra lessons. Constant comparison revealed interesting findings concerning the actual occurrences revealed during a teacher lesson. Three of the teachers used mostly constructivist approaches, while the other four utilized a more direct instruction format. The choice in teaching approach promoted vast differences in the behaviors of both teacher and students.

The teachers' presentations and students' actions differed for the lessons that included more constructivist approaches. For example, teachers played the role of facilitator in lessons, whereby students were guided during their inquiry of mathematical ideas. During these lessons, students were actively working on solutions and engaging in discussions with peers. On the contrary, the teachers who used more direct instruction approaches depended on lecture as the main foray of knowledge dissemination. In these lessons, students quietly sat at their desks filling out worksheets. The tasks required very little reasoning skills on the part of the students. Instead, they busily worked on ideas that had already been explicitly lectured to them.

The constant comparison of qualitative recordings revealed similar findings to both the structural equation model and descriptive statistics results. Students learning from constructivist approaches were more engaged in the lesson. When examining the percentages, it was realized that constructivist approaches impacted all three indicators of engagement. The actions of teachers and students in the videos also revealed the connection of enactive representations to higher levels of constructivist approaches and engagement. These findings are supported by the descriptive statistics and structural equation model, as well. Finally, the lessons involving higher levels of engagement showed more student discussion, inquiry, and connection-making.

The actions of the teacher, in the form of design of the lesson and feedback and support given to students, as well as the active participation of students in the lesson revealed deeper levels of understanding. Students were able to justify their ideas when learning in a constructivist setting, which revealed higher levels of procedural knowledge and conceptual understanding. Once again, these variables are included in the structural equation model as supporting both types of understanding.

Final Thoughts

The descriptive statistics in this study revealed that students had a higher procedural knowledge gain than conceptual knowledge gain. Additionally, they scored highest on a question asking them to fill in a table of values and lowest on questions asking them to find two more values that would make the statement true and finding the n that corresponded to the nth term, when using procedural knowledge. They scored the highest on a question asking them to choose the description of the commutative property and lowest on questions pertaining to understanding growth of patterns and recognizing linear relationships, when using conceptual understanding. Therefore, it is obvious that students are in much need of promoting their conceptual understanding and procedural knowledge by thinking in a more abstract manner. The two questions dealing with procedural knowledge that were the most difficult for the students involved more abstraction than simple rote solving of an algebraic equation.

These findings reveal the need for appropriate strategies that boost both types of learning in mathematics. The results of the analyses completed during this study reveal the need for more enactive representations, independent student thinking, problem-centered lessons, shared meanings, justification of ideas, and receiving meaningful feedback from the teacher. The benefits of use of these strategies are supported through both qualitative and quantitative analyses. In fact, the multi-level structural equation model revealed significant increases in procedural and conceptual knowledge with their use. Additionally, there is a correlation among these types of learning at both the individual level and classroom level. Therefore, we have more evidence that procedural and conceptual knowledge are indeed related. To conclude, teachers need to foster mathematical literacy by tapping into both types of knowledge via constructivist approaches that allow hands-on, inquiry-based activities in discursive environments.

Implications for Future Study

This study provides evidence for need of constructivist approaches in the endeavor to help students become more mathematically literate. Thus, the findings related to these six indicators reveal the ability to acquire both procedural and conceptual types of knowledge. This study was conducted examining middle school students and their understanding of the entire algebra strand. Other studies which examine effects of constructivist teaching approaches with different age groups of students on their understanding of the algebra strand at that level need to be completed. In addition, studies need to be completed that examine the effects of constructivist approaches at various grade levels and different areas of mathematics. Rigorous testing, as with structural equation modeling, can provide invaluable information concerning indicators which load onto certain factors that affect these types of learning. Qualitative data completes the picture by providing more information concerning the actual actions of students and teachers. Constant comparison, as well as diagrams can reveal occurrences that support statistical results. In this study, the use of both methods provided evidence and support for one another. Therefore, it would be very interesting to see these analyses completed on other data at other levels in algebra, as well as other levels in other topic areas in mathematics.

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APPENDIX A

ALGEBRA TEST QUESTIONS

Procedural Questions

1. What is the value of □ in this equation?

43 = □ – 28

A. 15

B. 25

C. 61

D. 71

5.

A	B
12	3
16	4
24	6
40	10

What is the rule used in the table to get the numbers in column B from the numbers in column A?

A. Add 9 to the number in column A.

B. Subtract 9 from the number in column A.

C. Multiply the number in column A by 4.

D. Divide the number in column A by 4.

7. The table shows values for the equation y = 2x + 5

X	Y
1	7
2	9
3	11
4	13

Which sentence describes the change in the y values compared to the change in the x values?

A. The y values increase by 6 as the x values increase by 1.

B. The y values increase by 7 as the x values increase by 1.

C. The y values increase by 2 as the x values increase by 1.

D. The y values increase by 5 as the x values increase by 2.

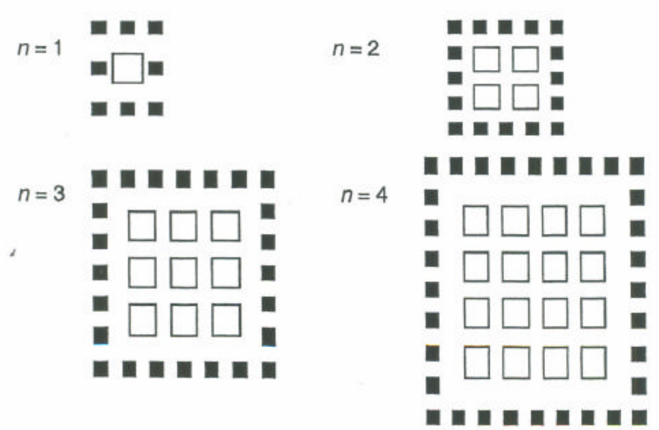
8. A farmer plants his orchard so that pine trees are all around the border and apple trees are in the center in a grid.

Here you see a diagram of this situation where you can see the pattern of apple trees and pine trees for any number of apple trees:

■ = pine tree

□ = apple tree

n = number of rows of apple trees



A) How many pine trees are in an orchard with 2 rows of apple trees?

____

B) Complete the table. (n = number of rows of apple trees)

N	Number of apple trees	Number of pine trees
1	1	8
2	4	
3		
4		
5		

10. a = b – 2 is a true statement when a = 3 and b = 5.

Find a different pair of values for a and b that also make this a true statement.

a = _________________

b = __________________

12. The table represents a relationship between A and B.

A	B
8	3
12	5
20	9
32	15
?	23

Based upon this relationship, what is the missing number in column A? ___________

16. Find the value(s) of y that make the equation true. Show how you got your answer.

19 = 3 + 4y

Conceptual Understanding Questions

2. Mary has some trading cards. Julie has 3 times as many trading cards as Mary. They have 36 trading cards in all. Which of these equations represents their trading card collection?

A. $3x = 36$

B. $x + 3 = 36$

C. $x + 3x = 36$

D. $3x + 36 = x$

3. There are n Girl Scouts marching in a parade. There are 6 girls in each row. Which expression could you use to find out how many rows of Girl Scouts are marching in the parade?

A. $n – 6$

B. $6/n$

C. $6n$

D. $n/6$

4. Jacob writes the following rule: If a and b represent any two numbers, $a + b = b + a$. Which of the following describes Jacob's rule in words?

A. Equals added to equals are equal.

B. Order doesn't matter when adding two numbers.

C. The sum of two whole numbers is a whole number.

D. When adding three numbers, it doesn't matter how the numbers are grouped.

5. Which of the following statements is NOT TRUE about the equation y = 2t, if t is a positive number?

A. It shows how y changes for different values of t.

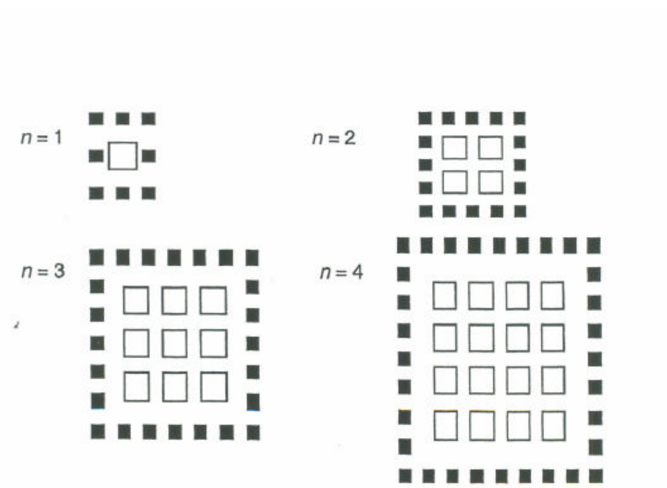
B. It shows a linear relationship between y and t.

C. It shows that the value of y is independent of the value of t.

D. It shows that as t increases, y also increases.

8. A farmer plants his orchard so that pine trees are all around the border and apple trees are in the center in a grid. Here you see a diagram of this situation where you can see the pattern of apple trees and pine trees for any number of apple trees:

■ = pine tree; □ = apple tree; n = number of rows of apple trees



C) Look at the table. You might notice that the number of apple trees can be found by using the formula n x n. The number of pine trees can be found by using the formula 8 x n. Remember, n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of pine trees. Find that value of n. ___________________

Explain how you found that answer.

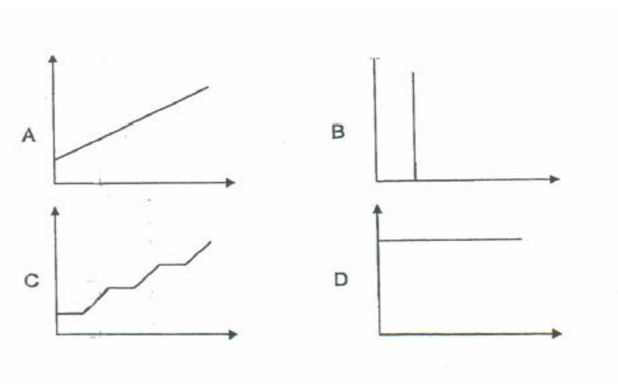
D) Suppose the farmer wants to make a **much larger** orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly, the number of **apple trees** or the number of **pine trees**?

Explain how you found your answer.

9. Tachi is exactly one year older than Bill. Let T stand for Tachi's age and B stand for Bill's age. Write an equation to compare Tachi's age to Bill's age.

____

11. A small boy was raising a flag up a flagpole.



Write the letter of the graph that best represents the height of the flag above the ground as the small boy raises the flag. ________ Explain why you chose this graph.

____

____

13.

Age of car (in years)	Value of car
0	$20,000.00
1	10,000.00
2	5,000.00
3	2,500.00

Circle the correct choice (is or is not) BELOW and complete the statement.

The relationship between the age of the car and the value of the car **is/is not** linear because ____

____

14. Stella has a phone plan. She pays $10.00 each month plus $0.10 each minute for long distance calls.

One month she made 100 minutes of long distance calls and her bill was $20.00.

The next month she made 300 minutes of long distance calls and her bill was $40.00.

Stella said, “If I talk three times as long it only costs me two times as much!”

Will Stella’s rule **always** work?

Show or explain why or why not.

15. Maria sells k donuts. Jinko sells five times as many donuts as Maria.

They sell the donuts for 25 cents each.

The number of donuts Maria sells is a variable.

A. Name another variable in the problem. ________________________

B. Name something in the problem that is NOT a variable.

____

APPENDIX B

REPRESENTATIONS

Enactive (Manipulatives)	**Iconic** (Pictures)	**Symbolic** (Symbols, numbers; words/discussion)
Time in Minutes: ______________ **% of Time in** **Minutes:** ______________	**Time in Minutes:** ________________ **% of Time in** **Minutes:** _________________	**Time in Minutes:** _______________ **% of Time in** **Minutes:** _______________
Description:	**Description:**	**Description:**

APPENDIX C

CONSTRUCTIVIST TEACHING APPROACHES

Encourage Student Independent Thinking (Use of invented algorithms; solving of one's own problems; question-asking)	**Create Problem-centered Lessons** (Realistic situations; posing of problems)	**Facilitate Shared Meanings** (Creation of small group activities; use of negotiated meanings)
Time in Minutes: ______________ **% of Time in Minutes:** ______________	**Time in Minutes:** ______________ **% of Time in Minutes:** ______________	**Time in Minutes:** ______________ **% of Time in Minutes:** ______________
Description:	**Description:**	**Description:**

APPENDIX D

ENGAGEMENT

Students express their own ideas (Provide comments about the lesson/not simple yes-no or short phrase responses)	**Students justify, clarify, and interpret ideas** (Provide explanations, reasons, or background for ideas)	**Students receive feedback from teachers** (Receive feedback other than simple acknowledgement of correct or incorrect answers)
Time in Minutes: ______________ **% of Time in Minutes:** ______________	**Time in Minutes:** ______________ **% of Time in Minutes:** ______________	**Time in Minutes:** ______________ **% of Time in Minutes:** ______________
Description:	**Description:**	**Description:**

APPENDIX E

ALGEBRA RUBRIC

8. **Apple Trees/Pine Trees and Stones/Bricks**

Cell	Code	Description
Part A		
16a1	1	16 or 16 pine trees
16a2	0	Any other response
16a3	0	Blank
Part B		
16b1	2	All entries correct
16b2	1	One incorrect entry in table
16b3	0	Many mistakes
16b4	0	Blank
Part C		
16c1	2	N = 8, because 8 x 8 = 64, n x n = 8 x n and $8^2 = 64$ OR explains something roughly equivalent to this
16c2	2	OR shows algebraically $n^2 = 8n$ $n^2 - 8n = 0$ so n = 8
16c3	2	OR continued pattern in table
16c4	1	N = 8, with fuzzy or incomplete explanation (e.g., 8 x 8 = 64 and 8 x 8 = 64 but does not distinguish n x n or 8 x n) OR incorrect answer (e.g. 64) but correct explanation
16c5	0	Correct response, no explanation
16c6	0	Correct response, incorrect explanation
16c7	0	Incorrect response
16c8	0	Blank
Part D		
16d1	2	[Apple trees/stones] are squared so they increase faster than 8n for [pine trees/bricks] when n > 8

16d2	2	Show graphs of n^2 & 8n and notes that [apple trees/stones] increase faster when n > 8 [apple tees/stones] are quadratic, [pine trees/bricks] are linear so
16d3	2	[apple trees/stones] increase faster when n > 8 n x n and 8n both have a factor of n, but n x n increases faster when n > 8
16d4	2	Extends table and states [apple trees/stones] increase faster when n > 8
16d5	2	[apple trees/stones] for apple trees, you add 1, 3, 5, 9, .. trees for each row
16d6	2	But for pine trees you always add 8 so eventually (> 8) apple trees grow faster
16d7	1	Any of the strategies in 2a-2e but without mentioning n > 8 OR [apple trees/stones] are squared (for example), but does not compare growth of [apple trees/stones] to growth of [pine trees/bricks] OR [apple trees/stones] are filling the inside v. [pine trees/bricks] on the perimeter OR [apple trees/stones] increase faster when n > 8 but offers no explanation
16d8	0	Incorrect [pine trees/bricks] OR [apple trees/stones] with incorrect explanation
16d9	0	Blank

9. **Tachi and Bill**

Cell	Code	Description
8a	1	T = B + 1 or the equivalent (T – B = 1; T – 1 = B)
8b	0	Transposes T and B
8c	0	Any other answer
8d	0	Blank

10. **a = b – 2**

Cell	Code	Description
9a	1	Any (a, b) for which a = b – 2, except a = 3 and b = 5
9b	0	Any other response
9c	0	Blank

11. **Small boy raises a flag**

Cell	Code	Description
10a	2	Shows evidence of understanding that the graph shows height of the flag over time. If A is given, states that height is steadily increasing over time. If C is given, states that height is the same during some time intervals (i.e., there is some pausing/stopping in raising flag)
10b	1	Shows evidence of understanding that the graph shows height of the flag over time but lacks complete explanation (as in 2 above)
10c	0	Correct answer but misunderstood graphical representation; incorrect explanation (e.g., selects A because “you raise a flag sideways”)
10d	0	Correct answer but no explanation
10e	0	B
10f	0	D
10g	0	Blank

12. **Missing number in table**

Cell	Code	Description
11a	1	48
11b	0	Any other response
11c	0	Blank

13. **Value of car not linear**

Cell	Code	Description
12a	2	Is not linear and explains that linear means constant change or rate (may or may not use these words but gets at notion of constant difference)
12b	1	Sees the constant difference is not here, but doesn't articulate it clearly OR tries to draw graph, then concludes it's linear
12c	0	Thinks that it is a regular pattern, even if not a constant difference means linear OR other incorrect OR is not linear but no explanation
12d	0	Blank

14. **Stella's phone plan**

Cell	Code	Description
13a	2	Gives a counterexample (e.g., m = 200, cost = 30; m = 600, cost = 70; \$70 ≠ \$60 OR shows m = 900, cost = \$100 not \$80 **NOTE: must show comparison of costs for the different minutes**
13b	2	States that this is not a case of direct variation (y intercept is not 0)
13c	1	Correct answer but incomplete demonstration of 2a or 2b (e.g., minor errors with counterexample—doesn't add the \$10 or multiplies the minutes by 2 instead of 3) AND/OR no comparison of costs for the different minutes
13d	0	Correct answer, incorrect explanation
13e	0	Correct answer, no explanation
13f	0	Incorrect
13g	0	Blank

15. **Maria and Jinko's donut sales**

Cell	Code	Description
Part A		
14a1	1	Number of donuts Jinko sells OR 5K OR "Jinko sells five times as many donuts as Maria" (no credit for "five times as many/much") OR total profits OR total number of donuts Maria and Jinko sell together
14a2	0	Any other response
14a3	0	Blank
Part B		
14b1	1	Price of donuts OR other irrelevant information (e.g., "they both sell", "five")
14b2	0	Any other response
14b3	0	Blank

16. **19 = 3 + 4y**

Cell	Code	Description
15a	2	16 = 4y, 4 = y
15b	2	Guess and check (substitutes 4 for y in the equation)
15c	2	Other (e.g., running equation—4 x 4 = 16 + 3 = 19)
15d	1	Y = 4 but no explanation OR made other errors (e.g., correct guess and check but reached wrong conclusion) OR may show 3 + 4 x 4 = 19 but does not conclude that y = 4.
15e	0	Completely incorrect (19 = 7y, 2.7 = y) OR correct answer but explanation doesn't support answer
15f	0	Blank

APPENDIX F

VARIANCE TABLE FOR FINAL MODEL

	Variance	Residual Variance	M	Intercept
Within Level				
X1	7.63		0.87	
X2	5.78		0.65	
Between Level				
Y1		304.57		19.13
Y4		138.39		33.13
Y5		63.06		31.38
Y6		70.42		36.88
Y8		17.69		4.94
Y9		86.98		13.53
X3		0.62		0.83
X4		0.30		0.56
Factor 1	477.54			

VITA

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EDUCATIONAL EXPERIENCE

Ph.D., **Texas A&M University**, Curriculum and Instruction with emphasis in Mathematics Education and Educational Research, 2006.
M.Ed., **Stephen F. Austin State University**, Elementary Education with emphasis in Mathematics Education, 2004.
B. S., **Stephen F. Austin State University**, Interdisciplinary Studies with concentration in Mathematics, 2001.

TEACHING EXPERIENCE

Texas A&M University	College Station, TX
Co-Instructor of graduate mathematics education course	2005
Texas A&M University	College Station, TX
Instructor of online mathematics professional development courses	2004-2005
Timber Creek Elementary	Livingston, TX
Math Teacher-4th grade	2002-2003

PUBLICATIONS

Ross, A. (2006). Investigating the effects of manipulative use on middle school students' understanding of equations. *The Lamar Electronic Journal of Student Research, 3*, http://dept.lamar.edu/lustudentjnl/current%20edition.htm.

Ross, A. (2006). A quasi-experimental study examining the effects of access to virtual manipulatives and use of kinesthetic manipulatives on middle school students' understanding of equations. *The Lamar Electronic Journal of Student Research, 3*, http://dept.lamar.edu/lustudentjnl/current%20edition.htm.

SELECTED PRESENTATIONS

Cassidy, S., Wiburg, K., Benedicto, R., Toshima, J., Saldivar, R., Ross, A., et al. (2006). *MathStar project: A collaboration and collection of "electronic" resources for teachers and students*. Talk presented at the Annual Conference of the National Council of Supervisors of Mathematics, St. Louis, MO.

Ross, A., Jolly, D., Cassidy, S., Sims, A., & Saldivar, R. (2006). *Providing online support for middle school teachers: Three studies of success*. Poster presented at the 2nd International Forum for Women in E-Learning Conference, Galveston, TX.