AN ASSESSMENT OF MIDDLE GRADES PRESERVICE TEACHERS' MATHEMATICS KNOWLEDGE FOR TEACHING

A Dissertation

by

MARGARET JOAN MOHR

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Curriculum and Instruction

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Approved by:

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ABSTRACT

An Assessment of Middle Grades Preservice Teachers' Mathematics Knowledge for Teaching. (August 2006)

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The overall purpose of this concurrent mixed methods study was to develop an online performance assessment using content questions taken from a reputable seventh and eighth grade standardized assessment that effectively evaluated and allowed preservice middle grades mathematics teachers to demonstrate their mathematics knowledge for teaching in the four main content strands of algebra, probability and statistics, geometry, and number and operations. In addition, this study examined differences in mathematics knowledge for teaching in enrollment characteristics, in courses taken and currently taking, and in three different cohorts, each at different stages in the program, of 122 preservice middle grades mathematics teachers at a large public university in central Texas.

Constant comparative analysis and descriptive statistics revealed average scores on the seventh and eighth grade content questions. The middle grades preservice teachers' content understanding and pedagogical understanding responses indicated several misunderstandings and misinterpretations in the middle grades mathematics they were tested on. Content knowledge, content understanding, and pedagogical understanding together made up a preservice teacher's mathematics knowledge for teaching. The study revealed that although preservice middle grades teachers could answer a content question correctly; they did not necessarily understand the process they used to arrive at their answer. In addition, their lack of explanation and knowledge of how to complete the problem correctly was transferred to their pedagogical understanding of the same problem.

There was a general indication of increasing mathematics knowledge for teaching for each content strand across enrollment characteristics (freshmen, sophomore, etc.) and cohorts. However, there was a noticeable decrease in average mathematics knowledge for teaching scores during middle grades preservice teachers' junior year. Special integrated mathematics and pedagogy courses (MASC) and the middle grades methods course had the greatest affect on preservice teachers' mathematics knowledge for teaching each content strand scores. Recommendations are also included in the study which may be used to help shape reform initiatives in teacher education programs throughout the United States.

DEDICATION

To my grandparents—without their support, dedication, weekly phone calls, and yearly trips to College Station I would have lost my sanity. To Father Fred Barnett—without his numerous prayers I would have lost my patience. Finally, to Cooper and Trinity, my former and current canine companions—without all their energy, kisses, cuddling, demand for walks, and ever-loving support I would not be as lovable as I am today.

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Dr. Kulm, I am not even sure where to begin! Thank you very much for your constant guidance and support over the last two years. Without the opportunities you presented me with and the support you gave during the first year I would not been able to finish my degree as an Aggie. Dr. Goldsby, you have been a source of energy and fresh light. Thank you very much for always listening, calling to check on me, and allowing me extra opportunities to work closely with you. You are my inspiration as a professor! Dr. Smith, I am not sure where to begin either! Thank you very much for believing in me, encouraging me, and pushing me through to the very end. The weekly chats were always a source of renewal and the opportunities you presented me with helped shape me into the person I am today. Dr. Willson, thank you very much for your constant guidance and patience. You helped me gain confidence as an educational researcher in the kindest way. Dr. Allen, thank you for supporting me through the development and the administration of the assessment. To all the instructors who helped me solicit participants for this study, thank you! Without you, this study would not have been possible at all.

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CHAPTER I

INTRODUCTION

A recent survey of select teacher programs in the United States revealed few changes in program characteristics in relation to current education reform recommendations (Graham, Li, & Buck, 2000). In addition, currently at the middle grades level, 68.5% of teachers have no major certification in mathematics and 21.9% do not have a minor in mathematics (Seastrom, Gruber, Henke, McGrath, & Cohen, 2005).

> To deliver the kind of mathematics content in ways that respects middle grades students as learners demands a well prepared and motivated teacher. Few existing teacher preparation programs meet this need, and certification requirements do not support adequate content and pedagogical preparation. (NRC, 2000, p. 15)

The improvement of middle grades mathematics teacher preparation must be grounded in research that provides a theoretical understanding of what prospective mathematics teachers for middle grades need to learn, how they learn it, and how their learning can be assessed (Kulm, Li, Allen, Goldsby, & Willson, 2005). Therefore, the researcher investigated the mathematics knowledge for teaching of middle grades preservice teachers at a large university in central Texas in order to gain a better understanding of the growth of this knowledge for teaching mathematics during their math/science specialist certification and bachelor degree program.

This dissertation follows the style of *Journal for Research in Mathematics Education.*

Statement of the Problem

Given the importance placed on the combination of mathematical content knowledge and pedagogical content knowledge in preparing preservice teachers, it is essential to know the nature of mathematics knowledge for teaching, especially in relation to the four main content strands of algebra, probability and statistics, geometry, and number and operations, in order to help improve middle grades mathematics teacher preparation programs. Not only must teachers understand the material they are teaching, but also teachers must be able to communicate the curriculum to the students as well. Therefore, this study utilized a standards- and literature-based assessment composed of randomly selected problems from a database of questions concerning the following content strands: algebra, probability and statistics, geometry, and number and operations. Each question had a total of three parts in extended-response format, in order to study approximately 500 preservice middle grades mathematics teachers' mathematics knowledge for teaching at a large university in central Texas.

Purpose of the Study

The overall purpose of this concurrent mixed methods study was to develop an online performance assessment that effectively evaluated and allowed preservice middle grades mathematics teachers to demonstrate their mathematics knowledge for teaching in the four content strands of algebra, geometry, probability and statistics, and number and operation. In addition, this study examined differences in mathematics knowledge for teaching in three different cohorts of preservice middle grades mathematics teachers at a large, public university in Central Texas. The ultimate goal of this study was to provide an effective online performance assessment instrument, provide data on preservice middle grades teachers' mathematics knowledge for teaching, and provide recommendations which may be used to help shape reform initiatives in teacher education programs throughout the United States.

Research Questions

This concurrent mixed methods study initiated an online performance assessment instrument which helped determine the nature of the knowledge, and the level of knowledge of preservice middle grades mathematics teachers' mathematics knowledge for teaching four content strands specified in national mathematics standards. Specifically, the following questions were investigated:

1. What is preservice middle grades teachers' mathematics knowledge for teaching number and operations?
2. What is preservice middle grades teachers' mathematics knowledge for teaching algebra?
3. What is preservice middle grades teachers' mathematics knowledge for teaching geometry?
4. What is preservice middle grades teachers' mathematics knowledge for teaching probability and statistics?

Ancillary Questions

1. What is the effect of various sequencing of mathematics courses?
2. What developmental differences are there among cohorts as preservice teachers progress through the courses?

3. Do some types of courses (e.g. algebra, geometry, numerical, statistical or applied, theoretical) have more impact than others upon development of a teacher's mathematical knowledge for teaching (MKT)?
4. Does development happen at greater rates at certain stages of the program than others?

Significance of the Study

Previous research has provided insights into specific areas or parts of teacher knowledge (e.g., Carter, 2005), but no studies have been undertaken to collectively evaluate mathematics knowledge for teaching at the preservice middle grades level and across different cohorts. The design of this study allowed for a snapshot of the development of mathematics knowledge for teaching middle grades during teacher preparation. The current middle grades mathematics certification bachelor degree program at this large, public university in Central Texas includes strong mathematics preparation in courses designed for teachers, specialized courses designed to integrate mathematical knowledge with pedagogy, and school-based methods and practicum. The model used in this specific middle grades program represents a model for teacher preparation recommended by professional organizations (cf. CBMS, 2001). Research on this model has provided the university and the education world with results which are intellectually and scientifically sound; contributed to teacher preparation; and will have significant implications for current and future teacher preparation programs, especially at the middle grades level.

Theoretical Base for the Study

The RAND Mathematics Science Panel (2003) report found a compelling relationship between what teachers could do with their students and their own level of mathematics competence. The obstacle is that "either teachers do not have enough content knowledge, or what they do know is not the 'right' content knowledge" (Sherin, 2002, p. 123). The NCTM (2000) emphasized teachers need different kinds of knowledge, such as knowledge of specific content, curricular goals, the challenges students face in learning these ideas, assessment, and pedagogical knowledge of effective teaching strategies. In addition, "there is a positive connection between subject matter preparation (in both content and specific teaching methods) and teacher performance; however, for some subjects, like mathematics, current subject matter preparation (including an academic subject major) may need to be reformed to increase reasoning skills and conceptual knowledge" (ASCD, 2003, p. 1).

Shulman (1986) introduced the notion of "pedagogical content knowledge" in which there is a conceived complementary relationship between the pedagogical knowledge and the content knowledge of the subject area. Hill, Schilling, and Ball (2004) such specific measures were not yet in place in mathematics education. So they set out, beginning at the elementary level, to map out what inservice elementary teachers knew regarding pedagogical content knowledge. What Hill, Schilling, and Ball found through their multiple-choice assessment was teachers' "mathematics knowledge for teaching" (the specific pedagogical content knowledge of mathematics teachers) elementary grades was partly domain specific rather than relating to their teaching or mathematical ability. In their 2005 article, Hill, Rowan, and Ball formally defined mathematics knowledge for teaching,

> By "mathematical knowledge for teaching," we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this "work of teaching" include explaining the terms and concepts to students, interpreting

> students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and effects of teachers' mathematical knowledge on student achievement providing students with examples of mathematical concepts, algorithms, or proofs. (Hill, Rowan, & Ball, 2005, p. 373)

The following diagram (Figure 1) is Hill, Rowan, & Ball's (Ball, 2006) visual description of mathematics knowledge for teaching and the specific parts this definition implies. Common Content Knowledge (CCK) refers to the mathematical knowledge shared

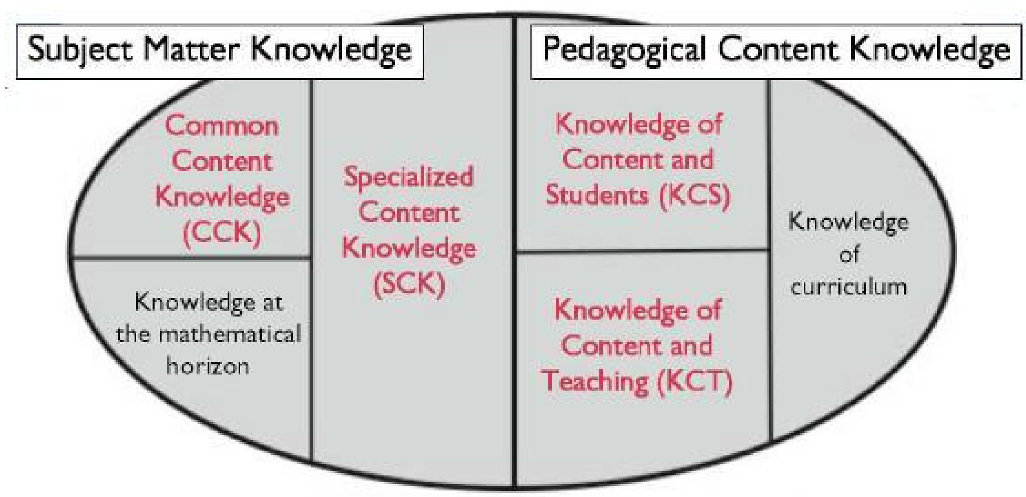

Figure 1. Mathematical Knowledge for Teaching (Ball, 2006).

by most educated adults, such as knowledge of the curriculum. Specialized Content Knowledge (SCK) refers the mathematical knowledge of teachers that goes beyond the knowledge of the curriculum. An example of this specialized content knowledge is providing explanations. Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) are complementary domains concerning the knowledge about mathematics, the knowledge about teaching, and the knowledge about students (Hill, Schilling, & Ball, 2004). Knowledge of curriculum and knowledge at the mathematical horizon are two newer areas of their model they are currently investigating (Ball, 2006). The last two domains (KCS and KCT) are closely related to "pedagogical content knowledge" (Shulman, 1986). Utilizing the above figure as a framework for a specific theoretical model for this study, the following table (Table 1) was developed.

Table 1
Mathematical Knowledge for Teaching the Middle Grades of Preservice Teachers

Preservice Teachers' Mathematical Knowledge for Teaching the Middle Grades			
Content Strand	Content Knowledge		Pedagogical Knowledge for the Teaching of Middle Grades Mathematics
	Correct Answer (Yes/No) **(Common Content Knowledge)**	Explanation of solution provided (Accuracy/Clear/Higher Order Thinking Present, etc.) **(Specialized Content Knowledge)**	Explanation of implementation into middle grades classroom (Accuracy/Clear/Various Methods/Type of Presentation, etc.) **(Integration of Knowledge of Content and Students, and Knowledge of Content and Teaching)**
Algebra			
Geometry			
Probability and Statistics			
Number and Operations			

Since there is a lack of research concerning middle grades teachers' mathematics knowledge for teaching, this model was developed to help explore *what* exactly their mathematics knowledge for teaching was and what it entailed, based off of the research of mathematics knowledge for teaching of elementary school teachers presented by Hill, Schilling, and Ball (2004), and Hill, Rowan, and Ball (2005). The four broad content strands chosen are recommended content areas middle grades teachers need to be studying in their teacher preparation programs (CBMS, 2001). In addition, these content strands closely resembled that of the state standards where the study was held, the state standards where the items were chosen, and the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics* (2000).

In addition to formalizing the definition of mathematics knowledge for teaching, their study of first and third grade teachers found that teachers' mathematical knowledge for teaching was significantly related to student achievement gains in both grade levels. The implementation of the definition of the mathematics knowledge for teaching may very well be the beginnings of the reform ASCD (2003) discusses.

Definition of Terms

The following definitions are provided for terms having special application in this study.

Assessment: the process of collecting, interpreting, and synthesizing information to aid in decision making (Airasian, 1991). The purpose of assessment is to "find out what each student is able to do, with knowledge, in context (Wiggins, 1996/1997, p. 19).

Culturally responsive: using the cultural knowledge, prior experiences, and performance styles of diverse students to make learning more appropriate and effective for them (Gay, 2000).

Mathematical achievement: the attainment and success of a student in mathematics as indicated by scores that go beyond the number of correct responses. One example of mathematical achievement is a score on a text-embedded chapter test.

Mathematics knowledge for teaching: the mathematical knowledge "used to carry out the *work of teaching* mathematics" (Hill, Rowan, & Ball, 2005, p. 373).

Middle grades: fifth to eighth grades.

Middle grades certification: Math/science specialist program: middle grades certification program leading to the Bachelor of Science degree (B.S.) with a major in Interdisciplinary Studies (INST) through the Middle Grades Certification Program in a curriculum and instruction department at a large, public university in Central Texas. It is a field-based program with students spending extensive time in area middle schools. Credit hours required for graduation in the Mathematics/Science strand total 133-134 credit hours.

NCTM: the National Council of Teachers of Mathematics, an organization composed of classroom teachers, supervisors, educational researchers, teacher educators, university mathematicians, and administrators involved in the mathematics education of students.

Pedagogical content knowledge: the knowledge of content, knowledge of teaching, and knowledge of learners' cognition.

Performance assessment: a form of testing requiring students to demonstrate their achievement of understandings and skills by completing a demanding task(s) in which they are asked to respond to the task(s) orally, in writing, or by constructing a product (Gronlund, 2003; Nitko, 2001; Popham, 2005).

Preservice teacher: a college student enrolled and accepted into the College of Education for the Mathematics/Science Specialist degree program who has the intention of teaching in a middle school upon graduation.

Assumptions

1. The participants provided accurate information.
2. The participants did not receive outside help (i.e., internet, other persons, textbooks, etc.) when completing the online assessment.

Delimitations

It is not possible to test *all* preservice middle school mathematics teachers. Thus, the study is limited to the number of preservice teachers and topics feasibly available. The study did not consider gender or ethnicity because of feasibility for the study.

Organization of Study

Chapter I has presented an introduction to the assessment of mathematics knowledge for teaching of middle school preservice teachers while justifying the need for this study. Specialized terms, research questions, and statistical techniques have been described and identified. Chapter II presents a review of relevant literature. Chapter III describes the methodology of the study. Chapter IV reports the results of the analysis of the data obtained in the study. Chapter V discusses the conclusions and implications that can be drawn from the study.

CHAPTER II

BACKGROUND LITERATURE

This chapter review the literature relevant to the learning theory on which performance assessment and mathematics knowledge for teaching is based, constructivism, the research regarding mathematics knowledge for teaching, the research regarding assessment, especially in regards to performance assessment and the assessment of teachers' knowledge, and the research regarding the preparation of teachers, especially in regards to middle grades mathematics.

Currently, there is no literature concerning preservice teachers' mathematics knowledge for teaching. In addition, the only published research on mathematics knowledge for teaching has been at the elementary levels. This lack of literature indicates a dire need to investigate preservice teachers' mathematics knowledge for teaching at the middle grades level.

The National Council of Teachers of Mathematics (NCTM) has had a major impact on the teaching and learning of mathematics. The emphasis the NCTM has placed on student involvement, alternative assessment techniques, and the creation of culturally responsive mathematical environments shows its ideas are more consistent with constructivism than other various learning theories. Previous research studies (e.g., Hill, Rowan, & Ball, 2005) have relied heavily on the use of multiple-choice assessments to evaluate inservice teachers' mathematics knowledge for teaching. However, their model of mathematics knowledge for teaching (see Figure 1) suggests many of components have an underlying constructivism theory.

Constructivism

Borrowed from cognitive psychology, constructivism currently dominates mathematics and science education. Constructivism began as a theory of learning, but has since been described as a philosophy, an epistemology, a cognitive position, or a pedagogical orientation (Matthews, 2000; Noddings, 1998). "Constructivism has become education's version of the 'grand unified theory'" (Matthews, 2000, p. 1). Constructivism can be generally described the constructing of one's own knowledge with the premise that knowledge is not the result of passive reception (Noddings, 1998).

Directly applied to mathematics education, constructivism has the following tenets:

1. Knowledge is actively created or invented by the child, not passively received from the environment.
2. Children create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.
3. No one reality truly exists, only individual interpretations of the world. These interpretations are shaped by experience and social interactions.
4. Learning is a social process in which children grow into the intellectual life of those around them (Bruner, 1986, cited in Clements & Battista, 1990). Mathematical ideas and truths, both in use and in meaning, are cooperatively established by the members of a culture.
5. When a teacher demands students use set mathematical methods, the sense-making activity of students is seriously curtailed. Their beliefs about the nature of mathematics change from viewing mathematics as sense making to

viewing it as learning set procedures that make little sense. (Clements & Battista, 1990)

Constructivism in education can be traced back to Jean Piaget who sought to "identify the structures of the mind underlying cognitive behaviors characteristic at each stage of mental development" (Noddings, 1998, p. 115). Piaget's work was attractive to educators because of the active involvement of the child in his or her own learning. Active engagement of students in the classroom was encouraged while lecturing and telling was de-emphasized. Although Piaget and his learning stages are still popular today, critics felt he neglected the social aspects of learning. This eventually led to a split into two more specific areas of constructivism, radical (also has been called personal) constructivism and social constructivism. Radical constructivism has its origin in Piaget, but is most clearly defined through the works of Ernst von Glasersfeld (Matthews, 2000). Social constructivism, our topic of interest, has its origins with Lev Vygotsky, a Soviet contemporary of Piaget. More recent enunciated works of social constructivism in mathematics education can be found in the writings of Paul Ernest.

Vygotsky's interest in the arts and aesthetics as a young man later influenced his development of psychological theories on consciousness and culture. Many of his colleagues regarded him as an outsider in the field of psychology. He received his share of criticism regarding his points of view and his use of terminology, especially when he "never ceased to uphold the principle of reconstructing psychological phenomena from data seemingly belonging to other disciplines" (Vygotsky, 1986, p. xvi). His work has been labeled *metatheoretical, metapsychological, metapragmatic,* and a *metasematic* – in other words he would be considered a *metatheoretican*. He worked in varied areas of psychology, but concentrated his scientific efforts on the development of human consciousness. According to Davydov and Zinchenko (1993), "Vygotsky created a fundamental theory of human development that is still of considerable practical significance for upbringing and education" (p. 93).

Vygotsky's theories differed from the traditional Russian psychological theory. For example, his views are now seen "as an alternative to behaviorism and to Piaget. It is important to understand that Vygotsky did not try to exclude behaviorism from his overall psychology-philosophy; however it was firmly positioned within the lower mental processes, which should not be confused with the higher mental processes" (Robbins, 2001, p. 7).

The emphasis on social learning was a foundational shift to the current pedagogical approaches of the 1920s and 1930s. Vygotsky "recognized that what was necessary was a theory which focused on how to get the child from his present state of development or learning to a point in the future" (Evans, 1993, p. 32), which became the *Zone of Proximal Development* (ZPD). Succinctly put, the 'zone' is a stage in his development where "a child can resolve a certain range of problems only under the guidance of adults and in collaboration with more intelligent comrades, but cannot do so independently" (Evans, 1993, p. 33; Davydov & Zinchenko, 1993, p. 102.) Vygotsky (1986) noted that "experience has shown that the child with the larger zone of proximal development will do much better in school" (p.187). In the ZPD it appears one only moves towards the highest levels of the higher mental processes, yet later Vygotsky recognized the need to understand education is not linear, but a *spiral* and it might even include *regression*.

Vygotsky's work has permeated the mathematics education world with the most recent writings coming from Paul Ernest. Ernest has written several books concerning the philosophy of mathematics and social constructivism including, *Social Constructivism as a Philosophy of Mathematics* (1998) and *The Philosophy of Mathematics Education* (1991). He believes "mathematics is a social construction, a cultural product, fallible like any other branch of knowledge. This view entails two claims. First of all, the origins of mathematics are social or cultural. …secondly, the justification of mathematical knowledge rests on its quasi-empirical basis" (Ernest, no date, p. 3). Ernest even borrows the two principles of radical constructivism to help construct a social constructivist epistemology.

With the publication of two of von Glasersfeld's works in the *Journal for Research in Mathematics Education* (Richards & von Glasersfeld, 1980; von Glasersfeld, 1981), constructivism was thrust into the mathematics education spotlight, moving it from an "almost hidden, still dualistic phenomenon in the 1960s and 1970s, to a more defined, evolving, and seemingly individually oriented but seriously challenged system for mathematical knowing in the mid 1980s, to an interactionist but nonrepresentationist view of mathematical knowing and teaching today" (Steffe & Kieren, 1994, p. 728). Since the introduction of constructivism, the way people learn mathematics and how mathematics should be taught has been criticized, immortalized, and debated. Constructivism, and more specifically social constructivism, has played a key role in the current mathematics education reform. Standards-based textbooks, which boast of conceptual and active learning, are being implemented in states all across the country. Even the NCTM 1989 *Standards* and the 2000 *PSSM*, reveal constructivist ideas within their documents.

Mathematics Knowledge for Teaching

Concerning teachers' knowledge of mathematics, two things can be repeatedly seen: US teachers' mathematical knowledge continues to be weak, and there is an inherent difference between the mathematical knowledge needed to be an effective teacher and needed by a mathematician (RAND Mathematics Study Panel, 2003). The RAND Mathematics Study Panel (2003) report found a compelling relationship between what teachers can do with their students and their own level of mathematics competence. The obstacle, however, is that “either teachers do not have enough content knowledge, or what they do know is not the ‘right’ content knowledge” (Sherin, 2002, p. 123). The NCTM (2000) emphasized teachers need different kinds of knowledge of specific content, curricular goals, the challenges students face in learning these ideas, assessment, and pedagogical knowledge of effective teaching strategies. In addition, “there is a positive connection between subject matter preparation (in both content and specific teaching methods) and teacher performance; however, for some subjects, like mathematics, current subject matter preparation (including an academic subject major) may need to be reformed to increase reasoning skills and conceptual knowledge” (ASCD, 2003, p.1).

Shulman (1986) introduced the notion of “pedagogical content knowledge” in which there is a conceived complementary relationship between the pedagogical knowledge and the content knowledge of the subject area. Shulman (1987) formally defines pedagogical content knowledge as the ability of the teacher to transform the content knowledge into forms which are “pedagogically powerful and yet adaptive to the variations in ability and background presented by the students” (p. 15). This pedagogical content knowledge links content, students, and pedagogy, revealing a special kind of teacher knowledge (RAND Mathematics Study Panel, 2003). “To be a teacher requires extensive and highly organized bodies of knowledge” (Shulman, 1985, p. 47). Teacher pedagogical content knowledge is closely connected with the subject matter being taught, how this subject matter achieves the transformation through the learning process, in the way which one knows how the students think, and with teachers' beliefs (Fennema & Franke, 1992).

Fennema and Franke (1992) used Shulman's model as a base in discussing the five components of their model of teachers' knowledge: the knowledge of the content of mathematics, knowledge of pedagogy, knowledge of students' cognitions, context specific knowledge, and teachers' beliefs. The content of mathematics includes teachers' knowledge of the concepts, procedures, and problem-solving processes within the domain in which they teach. Pedagogical knowledge is teachers' knowledge of teaching procedures. Learners' cognitions include knowledge of how students think and learn. This model further illustrates the complex and dynamic nature of teachers' knowledge. Ma's study (1999), *Knowing and Teaching Elementary Mathematics* presented a generative form of pedagogical content knowledge by describing "knowledge packages" and the idea of "profound understanding of fundamental mathematics." An important aspect of this profound understanding is linked to the knowledge of teachers who have the ability to explain mathematics effectively to students.

Taking all the aforementioned research into consideration, Hill, Schilling, and Ball (2004) argued specific measures frequently mentioned in relation to pedagogical content knowledge and mathematical content knowledge were not yet in place in mathematics education. They set out, at the elementary level, to map out what elementary teachers knew regarding pedagogical content knowledge. What they found through their multiple-choice assessment was teachers' mathematics knowledge for teaching (their phrase used in place of pedagogical content knowledge) the elementary grades was partly domain specific rather than relating to their teaching or mathematical ability. In their 2005 article, Hill, Rowan, and Ball formally defined mathematics knowledge for teaching (see Figure 1):

> By "mathematical knowledge for teaching," we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this "work of teaching" include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and effects of teachers' mathematical knowledge on student achievement providing students with examples of mathematics concepts, algorithms, or proofs. (Hill, Rowan, & Ball, 2005, p. 373)

Hill, Rowan, and Ball's (2005) study of mathematical knowledge for teaching in the elementary grades presented remarkable and groundbreaking research for the mathematics education community. In addition to finding that teachers' mathematical knowledge for teaching positively mathematics achievement during the first and third grades, their results suggested measures of teacher knowledge should be at least content specific and better yet, specific to the teaching of grade level. In addition, their results confirmed studying Shulman's (1986) pedagogical content knowledge as a subject-specific behavior is critical. The implementation of the definition of the mathematics knowledge for teaching may very well be the beginnings of the reform the ASCD (2003) discusses.

Assessment

With the introduction of *No Child Left Behind* (NCLB) into education in the United States (US), high-stakes testing and assessment in general has been thrust into the spotlight. Currently, many US schools are using high-stakes testing to measure student performance. These tests not only provide valuable feedback to teachers regarding individual student performance, curriculum, and teaching methods, but they also help to hold schools and school districts accountable for students' performances (American Psychological Association [APA], 2001). Although tests are among one of the best methods to assess student learning, high-stakes testing is often perceived as a single "snapshot" of a student's academic achievement (APA, 2001). Schools give the students one chance to successfully pass a paper-and-pencil test. In addition, the tests often "have unintended and potentially negative consequences for individual students, groups of students, or the educational system more broadly" (APA, 2001, ¶ 5).

Airasian (1991) describes assessment as the process of collecting, interpreting, and synthesizing information to aid in decision making. The purpose of assessment is to "find out what each student is able to do, with knowledge, in context" (Wiggins, 1996/1997, p. 19). Traditionally, effective assessment has been defined by what it is not (i.e., standardized tests). However, effective assessment is "built on current theories of learning and cognition and grounded in the views of what skills and capacities students will need for future success" (Herman, 1992, p. 75). Webb (1992) described assessment as having multiple cameras at all different views taking pictures at all different angles all at the same time. However, the most common forms of assessment in classrooms today do not give different angles to a students' knowledge. Instead, the assessments give a single snapshot, usually a snapshot indicating whether or not they know the content material (Bransford, Brown, & Cocking, 2000). These assessments are commonly in multiple choice formats and are graded as right or wrong. Summative assessments can be effective forms of assessment, but monies, grade advancement, certification, and other labels are often associated with these assessments.

Although assessment has taken a more negative connotation in the past decade, assessment does have a necessary and useful function in classrooms today. Assessments allow students to demonstrate proficiency on specified topics and content areas in order to help teachers and others determine whether students are moving satisfactorily toward learning goals and outcomes specified by the teacher (Popham, 2005).

In general, there are two types of assessment: formative and summative. Formative assessments are evaluations intended to improve unsuccessful yet still modifiable instruction. Summative assessments, most commonly utilized and talked about, refer to tests whose purpose is to make a final success/failure decision about a relatively unmodifiable set of instructional activities (Popham, 2005).

Gradually more schools, school districts, and states are experimenting with alternative forms of assessment. Most are not convinced the traditional standardized tests measure many of the important aspects of learning. In addition, traditional standardized tests fail to support many of the useful teaching strategies preservice teachers are currently being taught in their college courses (Darling-Hammond, Ancess, & Falk, 1995). Researchers (cf. Dorr-Bremme & Herman, 1983; Herman & Golan, 1991; Kellaghan & Madaus, 1991; Shepard, 1991; Smith & Rottenberg, 1991) have also concluded "the time focused on test content has narrowed the curriculum by overemphasizing basic-skill subjects and neglecting higher-order thinking skills" (Herman, 1992, p. 74). Although some test scores have improved through the use of the "teaching to the test" method, the standardized test scores represent only the content and formats included on the tests and in the end, teaching to the test does not result in meaningful learning for the students (Herman, 1992).

Performance Assessment

Alternative assessment practices are being developed to look directly at "students' work and their performance in ways that can evaluate the performances of students, classes, and whole schools" (Darling-Hammond et al., 1995, p. 10). The alternative practices are known as "authentic" assessments or "performance" assessments because students are engaged in "real world" tasks rather than multiple choice exercises. A performance assessment is a form of testing in which a student is given a task, usually a demanding one, then asked to respond to the task orally, in writing, or by constructing a product (Popham, 2005). The students are then evaluated according to criteria (typically a rubric of some sort is used) important for the actual performance in the particular field (Wiggins, 1989). Basically, in performance assessment, a student (the examinee) is asked to demonstrate and apply the skills and knowledge they have learned (Stiggins, 1987). "Performance tasks give the teacher and students feedback as to the current level of achievement and suggest ways for teacher and students to improve future outcomes" (Huetinck & Munshin, 2004, p. 373).

Performance assessments are broader in scope than traditional pencil and paper assessments. They are more authentic and are more likely to elicit a student's full repertoire of skills and reflect a range of goals the teacher wants the students to meet (Campbell, Melenyzer, Nettles, & Wyman, 2000). There are various forms of performance assessment including, oral presentations along with student written work, experiments, debates, teacher observation and inventories, inquiries, and portfolios (Archbald & Newman, 1988; Meisels, 1996/1997; O'Neil, 1992). These various forms of performance assessments are also a valuable tool in special education where paper-and-pencil tests are unable to be utilized or fail to test important skills (Stiggins, 1987). Although there are various forms of performance assessment, all forms have the same three features in common: multiple evaluative criteria, prespecified quality standards, and judgmental appraisal (Popham, 2005).

Typically, performance assessments are integrated within the curriculum, not considered separate, as some tests are. Performance assessments can become an integral part of a classroom because the students are no longer stressed with comparing scores against each other. Instead, they are challenged to meet at least minimum proficiency on specified tasks (Huetnick & Munshin, 2004). In addition, performance assessments can be used in group or individual settings, increasing its flexibility for use in the classroom.

Using performance assessments at the middle grades level has many advantages for students (both classroom and college students), teachers, and even parents and administrators. Performance assessments allow students the opportunity to display more than just speed and accuracy. In addition, students have more opportunities to do their own organizing and thinking as they solve more creative problems, allowing them to focus more on the process rather than just memorizing the rules. In performance task assessment—an assessment activity requiring a student to produce a written or spoken response, to engage in an activity or to create a product (Nitko, 2001)—students are involved in real tasks which engage and motivate. For teachers, assessment is an integral part of instruction and student learning. By utilizing performance assessment in their classrooms, teachers increase the quality of information they can obtain about student understanding and ability to do mathematics and they are able to gain more comprehensive information for instructional decisions about student misconceptions or errors. For administrators and parents, the use of performance assessment in classrooms provides direct evidence of students learning to think and use mathematics in new situations. A connection between schoolwork and real life is demonstrated.

Although performance assessment appears to be unbeatable in the classroom, there are some limitations which must be considered when implementing such an assessment system. The development of performance assessment requires considerable time and effort on the teacher's part. In addition rubrics have to be created or adapted to each assessment situation. The scoring of the performance assessments themselves can be burdensome and typically has low reliability in the end, much of it due to the subjectiveness of performance assessments.

With NCLB and the implications of high-stakes testing, the push for an alternative form of assessment is greater than ever. Although a nation-wide effort and federal funding would be the best scenario for the implementation of an alternative assessment program such as performance assessment, the schools, ultimately, will be the deciding factor. The amount of time and effort needed for the proper development, design, and actual implementation of performance assessments will be the largest barrier schools will have to get over (Mohr, 2006).

Assessment of Teachers' Knowledge

Until recently, typical approaches to providing measures of teacher knowledge in mathematics included using mathematics courses taken in their later high school years and in college. However, these proximate measures have been found to be poor indicators of what teachers actually know and how they use that knowledge in teaching mathematics (Hill, Rowan, & Ball, 2005; RAND Mathematics Study Panel, 2003). Specifically at the middle grades level these proximate measures likely not exist due to the lack of specific middle grades programs and lack of content-specific certification in several states (Seastrom et al., 2005).

Although assessing procedures and facts is important, assessments, especially in relation to teacher knowledge and understanding, should focus also on understanding since the goal is not just to learn, but to learn with understanding (Bransford et al., 2000). Assessing understanding can be tricky though since one is not able to assess it directly and it usually cannot be inferred from a single response on a single task. Instead, a “variety of tasks are needed to generate a profile of behavioral evidence” (Hiebert & Carpenter, 1992, p. 89).

In the late 1980s, the National Center for Research on Teacher Education took a step in the direction of the assessment reform and developed a better method of assessing content knowledge by posing questions in the context of teaching. Their results revealed teachers’ thin understanding of mathematics and mathematics pedagogy. Moreover, the results also revealed a lack of understanding of the concepts behind the answers the elementary and secondary teachers gave when they were asked to explain their reasoning (RAND Mathematics Study Panel, 2003). Further research has indicated similar results (e.g., Eisenhardt, Borko, & Underhill, 1993; Ma, 1999)—teachers got the “right answers” but they lacked understanding of why they were doing certain computation procedures or they were unable to explain why they arrived at the solution they did. In a study of preservice secondary teachers over topics common to preservice middle grades and elementary teachers, Bryan (1999) found that about 35% of the time, the group of nine participants offered no explanation or offered a flawed explanation in response to direct interview questions. Only 22% offered a sound explanation in the same situation.

A lack of understanding and failure to successfully justify answers is not just a problem in preservice and inservice teachers. In the analysis of the results of the National Assessment of Education Progress (NAEP) for rational numbers, Wearne and Kouba (2000) found students at all grade levels had difficulty justifying their responses and explaining how they arrived at these responses. One might speculate the lack of teacher and preservice teacher understanding has an affect on student understanding. However, in order to fully understand and target this idea, teacher misconceptions and knowledge must be dealt with first. "We have to first deal with teachers' misconceptions before we can expect them to be competent at helping their students to overcome misconceptions" (Shaughnessy, 1992, p. 484). In order to deal with the misconceptions of teachers, one must first understand the nature of the misconceptions.

> The lack of sophisticated, robust, valid, and reliable measures of teachers' knowledge has limited what we can learn empirically about what teachers need to know about mathematics and mathematics pedagogy. The lack of measures also limits our understanding about how such knowledge affects the learning opportunities of particular students and their development of mathematical proficiency over time. …A range of tools is needed, including survey measures, performance tasks, and written and interactive problems. (RAND Mathematics Study Panel, 2003, p. 26)

Mathematical content knowledge, pedagogical content knowledge, and mathematics knowledge for teaching are critical and central components of teacher preparation. However, there is a lack of research concerning what mathematics middle grades teachers and preservice teachers actually do know and understand (Post, Harel, Behr, & Lesh, 1991).

Teacher Preparation for Middle Grades Mathematics

Reforms seem to be happening all over the US. Ongoing education reform efforts have been escalated in line with new accountability systems. Mathematics education continues its "math wars" and is now struggling to put and keep teachers in the classrooms. Another reform effort is the new teacher education. The new teacher education grows out of changing notions of accountability and more specifically, the recent educational reform movements in the US (Cochran-Smith, 2005). The quality of classroom instruction had not been a major concern of policy makers and educators until it was compared to other educational systems (e.g., Silver, 1998). Now the ever-static American classroom (NCMST, 2000) is gradually seeing changes thanks to numerous professional development programs, nationally-funded research projects in cooperation with major universities, and reform initiatives in teacher preparation programs.

For almost all of the last century, teacher preparation has been located within higher education institutions. This is not the case anymore. Almost every state in the US has some sort of alternative certification program—some are attached to universities while others bypass them altogether (Cochran-Smith, 2005). This has caused colleges and universities to look hard at the way their teachers are prepared. Some programs have been disbanded all together while others work hard to adapt and compete with the new teacher education. Mathematics education programs have been thrust into the spotlight, especially at the middle grades levels, because of the recent comparison studies revealing poor student performance and static teacher instruction in middle grades classrooms (Hiebert et al., 2003; Stigler & Hiebert, 1997, 1999, 2004). However, it is not the poor student performance or the methods of instruction that is of major concern. It is of the mathematical and pedagogical content knowledge of teachers (Hill, Rowan, & Ball, 2005).

National agencies and professional organizations in various reports have expressed concern about teachers' mathematical preparation.

> There is evidence of a vicious cycle in which too many prospective teachers enter college with insufficient understanding of school mathematics, have little college instruction focused on the mathematics they will teach and then enter their classrooms inadequately prepared to teach mathematics to the following generation of students. (CBMS, 2001, p. 5)

As teachers take more mathematical courses at the college level, they often move away from the curriculum they will teach, resulting in better preparation for graduate school than for teaching in the classroom (Usiskin, 2001). Mathematics courses are needed to help teachers develop a strong and deep understanding of the mathematics they will teach. Since the introduction of the *Professional Standards for Teaching Mathematics* (NCTM, 1991), other national organizations have followed suite, many of them providing specific recommendations for the creation of effective mathematics teacher education programs (e.g., CBMS, 2001; INTASC, 1995; RAND Mathematics Study Panel, 2003).

A common theme among the recommendations is the creation or reformation of teacher preparation program targeted for middle grades (grades 5-8) mathematics teachers. If no middle grades program is available, the teacher is prepared in either elementary school mathematics or secondary school mathematics. If the teacher is prepared in elementary school mathematics they often lack the broader background needed to teach the more advanced mathematics of the middle grades (CBMS, 2001). If the teacher is prepared in secondary school mathematics, they often lack the pedagogical knowledge needed to be successful at the middle grades levels. Existing middle grades teacher preparation programs tend to differ substantially depending on whether they gaining generalist (all content areas, similar to that of an elementary teacher preparation program) or specialist (i.e., mathematics and science) certification partly because of the organization of schools, universities, and state accreditation and certification programs (NCES, 1993, 1995; Nelson, Weiss, & Conaway, 1992).

The Conference Board of the Mathematical Sciences (CBMS, 2001) made the following recommendation:

> *Recommendation 11. Mathematics in middle grades (grades 5 – 8) should be taught by mathematics specialists.* This recommendation mirrors similar recommendations by a number of other groups seeking to improve U.S. school mathematics instruction. Middle grades mathematics teachers must know the high school mathematics curriculum well and understand the foundation that is being laid for it in their instruction. As concepts like fractions and decimals enter the curriculum, teaching mathematics well requires subject matter expertise that non-specialists cannot be expected to master. Having mathematical specialists, beginning in middle grades, both reduces the education burden for those teaching mathematics in these grades and provides opportunities for prospective teachers of these grades who like mathematics to specialize in it. (CBMS, 2001, p. 11)

The CBMS (2001) recommends at least 21 semester hours of mathematics that includes four broad content areas (i.e., number and operations, algebra and functions, geometry and measurement, and data analysis, statistics, and probability). The courses must be designed to such a way that prospective teachers develop a deep understanding of the mathematics they will be teaching. In addition, courses are needed that will strengthen prospective teachers' "own knowledge of mathematics and broaden their understanding of mathematical connections between one educational level and the next, connections between elementary and middle grades as well as between middle grades and high school" (pp. 25-26). Further recommendations include: more focus on transitioning prospective teachers from a world of college courses into a world in which they are the teacher of record (Bransford et al., 2000), more effective communication between education and mathematics departments, especially amongst instructors (Bransford et al., 2000), and development of a prospective teachers' mathematics knowledge for teaching in which both content knowledge, pedagogy, and pedagogical content knowledge for mathematics are focused upon (CBMS, 2001).

Future teachers of mathematics at any level will be expected to teach and follow more challenging mathematics (INTASC, 1995) in order to successfully fulfill the ultimate goal of school: helping students transfer what they have learned in school to the ever-increasingly complexities of students' everyday settings (Bransford et al., 2000). Therefore teacher preparation programs, especially at the middle grades level, must focus on developing teachers who have a thorough understanding of mathematics, a deeper understanding of how this knowledge is developed throughout each grade band (elementary, middle grades, secondary), and who believe in and can provide challenging learning opportunities for all students when it is their turn to go into the classroom (CBMS, 2001; Cochran-Smith, 2005; INTASC, 1995).

Conclusions

"In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed" (NCTM, 2000, p. 5). There is limited research available concerning competency of prospective teachers at the middle grades level. In addition there is little or no research concerning the levels of competency middle grades preservice teachers can achieve through specific program studies such as the one found in this research study. Thus it is imperative to undertake a comprehensive study of middle grades preservice teachers' mathematics knowledge for teaching.

CHAPTER III

METHODOLOGY

The intent of this concurrent mixed methods study was to assess the nature of mathematics knowledge for teaching in middle grades mathematics teachers. This study utilized a mixed-model design (cf. Johnson & Onwuegbuzie, 2004) where the data were qualitatively analyzed initially followed by a quantitative analysis. The data were qualitatively analyzed in order to understand the nature and structure of the participants' responses. The data were quantitatively analyzed in order to investigate the overall implications. The results from the qualitative analysis were used to compliment and provide depth to the findings from the quantitative analysis.

This chapter will discuss the research design, population, instrumentation, research questions, and statistical design. A description of the scoring procedures for the tests is also provided.

Mixed Methodology Research Design

Mixed methods research has its origins in psychology and was first used when Campbell and Fiske were researching the validity of psychological traits in 1959. Campbell and Fiske (1959) created a multitrait-multimethod matrix that helped researchers examine "multiple approaches to data collection in a study" and encouraged other researchers to use the matrix as well (Creswell, 2003, p. 15). In the 1970s triangulation of data sources emerged as a system for uniting qualitative and quantitative methods. Currently, several authors (e.g., Collins, Onwuegbuzie, & Sutton, 2006; Johnson & Onwuegbuzie, 2004; Tashakkori & Teddlie, 1998) are discussing the ever-expanding motives for conducting a mixed methods study, and researchers from all over the world have begun developing procedures for mixing quantitative and qualitative methods.

Mixed methods research can be defined as a unique integration of qualitative and quantitative research methodologies in order to conduct a single study or several studies in an agenda of investigation. Mixed methods researchers realize limiting oneself to including only quantitative or qualitative methods "falls short of the major approaches used today in social and human sciences" (Creswell, 2003, p. 4). Therefore researchers employ mixed methods for several reasons. Mixed methods designs have been used to expand understanding from one method to another, or to substantiate results from a different data source. Researchers have used mixed methods study so that one form of investigation informs another.

Although mixed method designs have been seen as a well-rounded method for conducting research, there are several challenges mixed methods researchers face. Researchers who choose to mix methods must be familiar with both quantitative and qualitative methods of conducting research. Also, mixed methods research is very time consuming in the fact there is a need for extensive data collection and analyzation. These challenges come as a small price to pay for the powerful data that can be gleaned from a mixed methods study.

This study will employ a concurrent triangulation mixed methods strategy, the most familiar of the six major mixed methods models, in order to answer the aforementioned research questions and ancillary questions. The concurrent triangulation strategy uses separate quantitative and qualitative methods in order to give strength to one method in areas where the other method is inherently weak. In this strategy it is ideal for quantitative and qualitative approaches to be given equal treatment with the integration of the results of the two methods happening at the interpretation phase (Creswell, 2003).

The researcher felt it necessary to include a qualitative measure, in addition to a quantitative measure, in this study to gain a deeper understanding of preservice middle grades teachers' mathematics knowledge for teaching. Although connecting test scores of mathematics knowledge for teaching the middle grades to various course sequencing, cohorts, class, etc. could easily be done with quantitative measures, the researcher felt the richness of the study could be found in more qualitative aspects such as the open-ended responses on the Mathematics Knowledge for Teaching Middle Grades Online Assessment. Figure 2 below gives a more detailed picture of the concurrent triangulation strategy (Creswell, 2003) being used in this study.

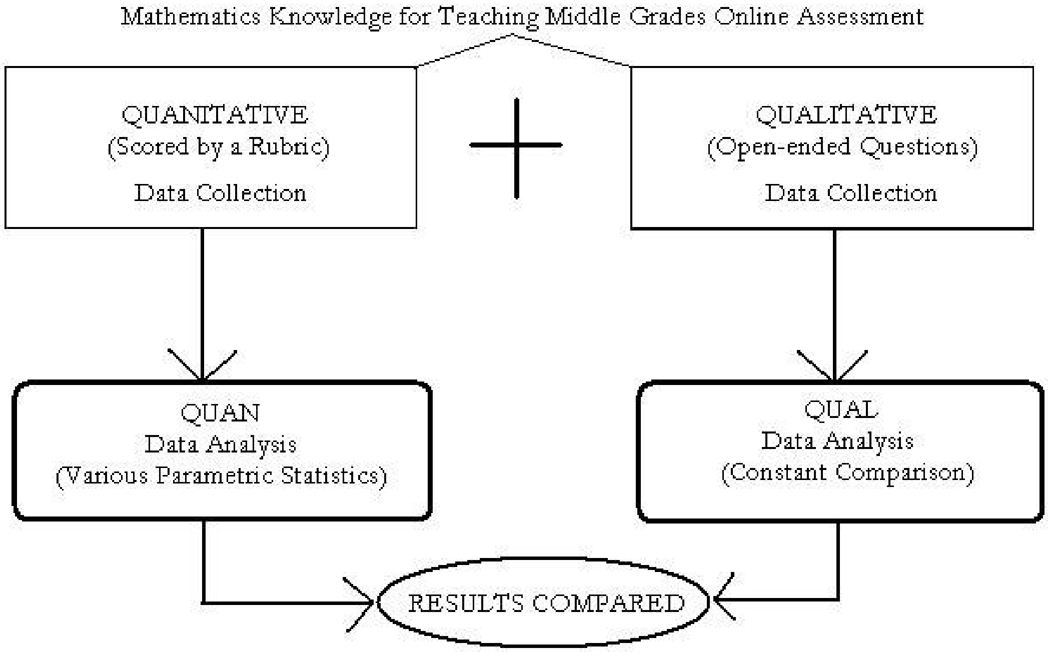

Figure 2. Mixed Methods Design for the Study (adapted from Creswell, 2003; Tashakkori & Teddlie, 1998).

Population

This study occurred at a large, southern, public university. The population for the study was preservice teachers pursuing a Mathematics/Science Specialist degree, who intend to teach in a middle school upon graduation, and who are currently enrolled in the following courses: MATH 365 Structure of Math I, MATH 366 Structure of Math II, MATH 367 Basic Concepts of Geometry, MATH 368 Introduction to Abstract Math, MATH 403 Math and Technology, MASC 351 Problem Solving, MASC 450 Integrated Mathematics, MEFB 460 Methods of Teaching Middle Grades Mathematics, and MEFB 497 Teaching Middle Grades (see Appendix F for course descriptions). MATH 365 Structure of Math I, MATH 366 Structure of Math II, MATH 367 Basic Concepts of

Geometry, MATH 368 Introduction to Abstract Math, and MATH 403 Math and Technology were taught through the Department of Mathematics; MASC 351 Problem Solving, MASC 450 Integrated Mathematics, MEFB 460 Methods of Teaching Middle Grades Mathematics, and MEFB 497 Teaching Middle Grades were taught through the Department of Teaching, Learning and Culture. All instructors for these courses were asked to encourage their students to participate. One of the instructors did not respond. However, confirmation of enrollment in the courses revealed 90% of these students were concurrently enrolled in one of the other courses and were therefore accessed through those courses. The final population total for the study was 122.

Of the 122 participants in the sample, 109 (89.3%) were female. Of the participants, 2 (1.6%) were freshmen, 17 (13.9%) were sophomores, 45 (36.9%) were juniors, and 58 (47.5%) were seniors. of the participants, 3 (2.5%) were Asian or Pacific Islander, 2 (1.6%) were African American, 11 (9.0%) were Hispanic/Latino, 106 (86.9%) were White (non-Hispanic), and none were American Indian or Alaskan Native. None chose not to respond.

Instrumentation

The primary data collection tool was the Mathematics Knowledge for Teaching Middle Grades Online Assessment given to the targeted population. The assessment instrument was comprised of four different assessments targeting the following content strands: Algebra, Number and Operation, Geometry, and Probability and Statistics. Each of the four strands consisted of seven questions with three parts a piece. The four assessments were a compilation of items from the 2005 seventh-grade and eighth-grade New York State Assessment published items.

The assessment was subject to a two-stage pilot test. In the first stage, the assessment was administered online to three informed practitioners and content experts. Two grammar revisions, on the error of the researcher, were made. In the second stage, the assessment was administered online to a sample of 41 undergraduate preservice elementary (PK-4 certification) students in their final semester of coursework. These 41 undergraduate preservice elementary students will be student teaching in the fall semester in PK-4 settings. The data were analyzed and no revisions were made.

A validation study was not needed since these items were published state assessment items created by a major publishing company. The test is considered content valid as it was designed to test the specific state standards for the State of New York. Content validity is of prime importance in the type of measure to be used. The test publisher also claims the tests are content valid as they are created specifically to test for material for the specific topics addressed in the New York State Standards for the grade levels of sixth thru eighth. In addition, specific procedures were followed during the development in pilot testing of the New York State Assessments for eliminating bias and minimizing differential item functioning. Although these procedures were believed to improve the quality of the state assessment, there is evidence to suggest that expertise in the area is no substitute for data (Jensen, 1980). Therefore the researcher opted to not include any items that were reported to be biased and that were flagged for their differential item functioning. This did not affect the content quality and variation on any of the four assessments. The reliability coefficient is the correlation coefficient between scores on parallel tests and is an index of how well scores on one parallel test predict scores from another parallel test. The Feldt-Raju index reported for the 2005 Eighth Grade New York State Assessment Test was 0.94 which is a number comparable to that of the 2004 eighth grade state assessment (CTB/McGraw-Hill, 2005).

The assessment instrument contained seven algebra items, seven probability and statistic items, seven geometry items, seven number and operation items, and eight demographic items. Each of the items under the four content strands contained three parts. Six versions of the overall assessment instrument were created. Each version contained all eight demographic items, found at the beginning of the assessment, and two of the content strands, seven questions each, for a total of 22 items for each assessment. Content questions were kept together (i.e., all seven algebra items followed each other), however, there was no demarcation as to when a new content strand began. In addition, the participants were not aware of which content strands they were answering; however, it could easily be deduced by the participant as they were answering each item. The versions of the tests were content-ordered as follows: 1) Algebra and Geometry, 2) Number and Operation and Probability and Statistics, 3) Algebra and Number and Operation, 4) Probability and Algebra, 5) Geometry and Number and Operation, and 6) Geometry and Probability and Statistics. Within each content strand, the items were randomly ordered using a random number generator for the numbers one thru seven so no version has the same order of items within the content strand. Versions one and two of this assessment can be found in Appendices A and B, respectively.

The standards for grades sixth thru eighth in the State of New York closely resemble the Texas Essential Knowledge and Skills (TEKS), respectively. In addition, both sets of standards closely resemble NCTM'S *Principles and Standards for School Mathematics* (2000) for their respective grade levels. The content part of each question was not changed at all in the development of this assessment. However, there was a removal of the multiple choice answers. Each of the items in the assessment contained the same three open-ended responses. The first part simply provided a space for the participant to type the answer to the question. All of the items, with the exception of one, were number responses. The one item that was not a number response was a one word answer. If there were characters needed to answer the question properly, a “Note:” was provided above the answer space to help participants with the correct characters and to help keep the answers looking relatively similar. There was no limitation to the number of characters in this response. The second part of the item asked for the participant’s explanation of his or her answer. A box was provided for the student to type their response. There was no limitation in characters for this response. These first two items were aimed at gathering the participant’s content knowledge of the strand addressed. The third part of the item addressed the participant’s pedagogical content knowledge of the content strand addressed. The participant was asked to respond to this question: “How would you explain, model, and/or demonstrate this item to someone who did not understand?” In addition to content knowledge and pedagogical content knowledge, the communication and vocabulary were also of interest to the researcher.

The items comprising the algebra assessment contained items addressing the Algebra Strand of the Grade 8 Standards for the State of New York. Items 2, 3, and 5 addressed translating verbal sentences into algebraic equations. Item 1 addressed adding and subtracting polynomials and integer coefficients. Item 4 addressed multiplying a binomial by a monomial or binomial with integer coefficients. Item 6 addressed factoring a trinomial in the form $ax^2 + bx + c$; $a = 1$ and c having no more than three sets of factors. Item 7 addressed applying algebra to determine the measure of angles formed by or contained in parallel lines cut by a transversal and by intersecting lines.

The items comprising the geometry assessment contained items addressing the Geometry Strand of the Grade 8 Standards for the State of New York. Items 1 and 2 addressed calculating the missing angle in a supplementary or complementary pair. Item 3 addressed identifying pairs of supplementary and complementary angles. Item 4 addressed determining angle relationships when given two parallel lines cut by a transversal. Items 5 and 6 addressed identifying pairs of vertical angles as congruent. Item 7 addressed calculating the missing angle measurements when given two parallel lines cut by a transversal.

The items comprising the number and operation assessment contained items addressing the Number Sense and Operations Strand of the Grade 8 and Grade 7 for the State of New York. Item 1 addressed developing and applying the laws of exponents for multiplication and division. Items 2 and 4 addressed estimating a percent of quantity, given an application. Items 3 and 6 addressed applying percents to: tax, percent increase/decrease, commission, interest rates, simple interest, gratuities, and sale price situations. Item 5 addressed evaluating expressions with integral exponents. Item 7 addressed determining multiples and least common multiples of two or more numbers.

The items comprising the probability and statistics assessment contained items addressing the Grade 7 and Grade 6 Statistics and Probability Strand of the State of New York Standards in Mathematics. Item 1 addressed calculating the range for a given set of data. Items 2 and 6 addressed predicting the outcome of an experiment. Item 3 addressed reading and interpreting data represented graphically through a pictograph. Item 4 addressed determining the number of possible outcomes for a compound event by using the fundamental counting principle and use this to determine the probabilities of events when the outcomes have equal probability. Item 5 addressed determining the probability of dependent events. Item 7 addressed interpreting data to provide the basis for predictions and to establish experimental probabilities.

Collection of the Data

The assessment instrument was created for distribution using Form Management System (Strader, 2006), which allowed for online data collection. From the time the instruments were posted until they were removed, participants and instructors had 24-hour access. Therefore, the assessment instruments could be completed at the convenience of the participants. The participants were instructed to go to the home page of the assessment instruments. The home page gave a brief summary of what the goal of the research study was investigating as well as the contact information of the researcher. The students then clicked on a link on the bottom of the page that took them to the participant consent form (Appendix C). The form could be printed out if needed, however, it was not necessary nor did the participants have to turn in the signed form. At the bottom of the consent form was a link to the test. The link was coded so that the participant would get an assessment that was randomly selected using a random number code. The participant then chose to agree or not agree to the research study and proceeded with the assessment. No data was collected if the participant did not agree to the consent form. The term "questionnaire" was used instead of assessment in order to reduce or avoid any test anxiety a participant might have. This term also helped to reinforce to the participants that the results would not be counted against them in their coursework.

Instructors agreed to ask their students to access the assessment online between April 4th and April 30th, 2006. Form Management System (Strader, 2006) randomly assigned one of the six versions of the assessment to each student upon access to the website after the consent form was provided. Each student took an assessment containing eight demographic questions and two content strands. Each content strand (algebra, number and operation, geometry, and probability and statistics) contained seven questions. Therefore, each student took an assessment containing a total of 22 items. Students were allowed to use calculators but no other aids on the assessment.

Two of the instructors required all students in their course to take the assessment as a participation/completion grade. Four of the instructors provided no incentives for the students to complete the online assessment. The rest of the instructors offered minimal extra credit. The instructors of the MATH courses handed out slips of paper to the students in class containing the title of the questionnaire and the link to the assessment. In addition several of the MATH instructors emailed reminders to the students. Two of the instructors (both MASC) relied on classroom time to take the assessment. The other instructors relied on emailed reminders from the instructor and the researcher. Using this procedure, approximately 475 students were asked to respond to the assessment. The computer logged 309 responses. Eleven of the responses were removed from the data due to the first stage of the pilot study. Thirty-nine responses were removed from the data due to the second stage of the pilot study. Four responses were removed from the data due to not agreeing to the consent form. This left an initial sample size of 255. However, MATH 365 Structure of Math I and MATH 366 Structure of Math II are courses for elementary education majors in addition to the middles grades specialist degree program. Therefore, 133 responses were removed because the individual had indicated they were not a middle grades math/science specialist degree student. Response rate was approximately 68% (see Appendix G for more details). The 122 participants left in the sample were seeking certification in 4-8 math/science and matched the population desired for this study. Therefore 122 middle grades preservice teachers comprise the sample investigated in this study.

Scoring

All of the data collected on the assessment, with the exception of the demographic information, were in open-ended question format. Therefore in order to conduct the quantitative analysis for this study, the assessments had to be scored. Each item on the assessment contained three identical parts. The first part, answering the content question, was scored right or wrong, indicated by a 0-wrong or 1-right. The second part, explaining the solution to the problem, was scored on a 7-point holistic rubric (described below). The third part, explaining the item to someone who did not understand, was scored on a similar 7-point holistic rubric (described below).

The weights of the scores in the rubric were determined using Ball's (2006) definition of mathematics knowledge for teaching (see Figure 3). The figure suggests common content knowledge is just a small part of the subject matter knowledge, therefore, part one of the assessment (answering the content item) was scored as 0 or 1. Parts two and three of the assessment appear to be equally weighted in the diagram, therefore, they were scored on a 7-point scale described below (for rubric see Appendix E).

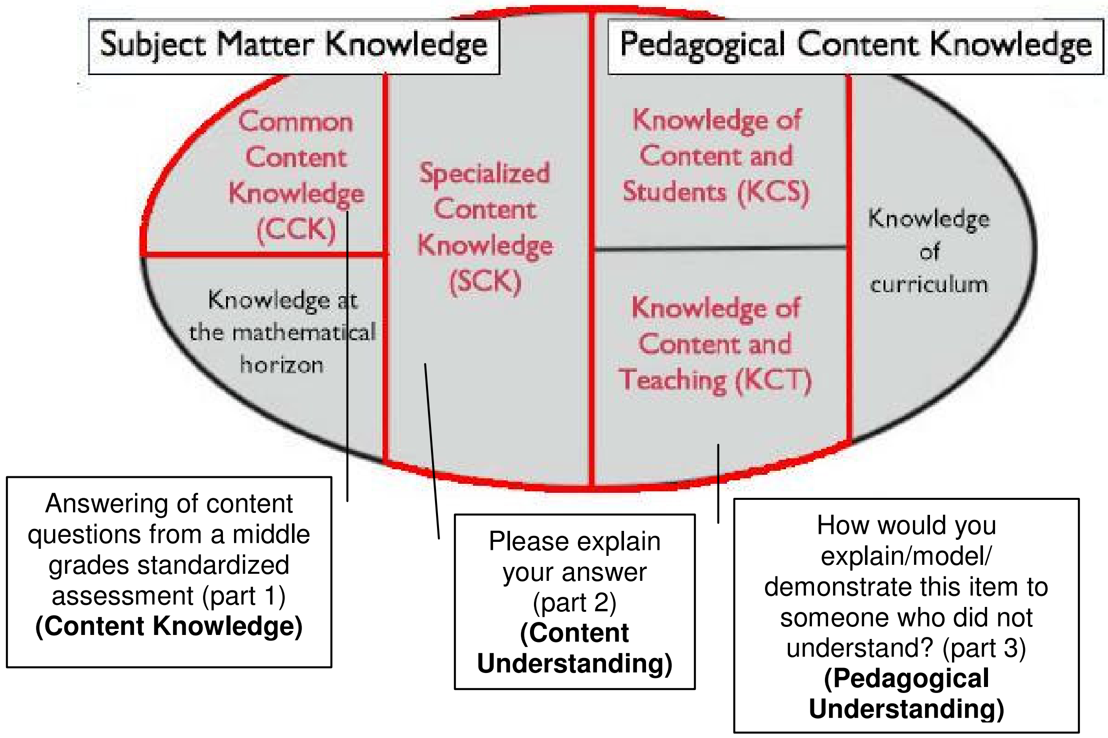

Figure 3. The Assessment of Preservice Middle Grades Teachers' Mathematics Knowledge for Teaching.

The researcher and an outside consultant graded the assessments according to the rubrics. The outside consultant is a graduate student at the university where the study was conducted and has had numerous elementary and middle grades experiences. The researcher and consultant graded seven assessments, with each content strand being represented at least twice, together in order to become familiar with the rubric and to help establish consistency in the coding. Consistency of the coding is essential to the usefulness of the results, so intra-rater and inter-rater reliability studies were conducted (Huck, 2004). To measure intra-rater reliability, each grader re-analyzed a random sample of four different assessments making sure each content strand was represented twice. Intra-rater reliability for grader 1 was 97.7%. Intra-rater reliability for grader 2 was 96.7%. To establish inter-rater reliability of the rubrics, the graders analyzed the same random sample of four different assessments making sure each content strand was represented twice. Inter-rater reliability was 99.3%.

A 7-point holistic rubric (Appendix E) was used to code the open-ended responses from the online assessment. This rubric was adapted from a 4-point rubric (a standard 4-level rubric—exceeds standard, meets standard, partially meets standard, does not meet the standard) published by the New York State Testing Program (2005) and used to grade the 2005 state assessment where these questions were pulled. A two point scaling for three of the parts was adapted so as to account for variations in answers. For example, say a student's answer provided an answer worthy of a score of 4. However, there was one minor error which did not seem to affect the overall answer, but according to the rubric, did not meet the criterion for a level 4 answer. This student would earn a score of 3 then. A scale of this sort helps to minimize unnecessary level penalizations of students' answers.

Two 7-point holistic rubrics were adapted. One for the coding of part 2, the explanation of the solution, and one for the coding of part 3, the explanation to someone who did not understand, of the online assessment. For the coding of part 2, the explanation of the solution, the following indicators were focused on in the rubric:

- Demonstration of a thorough understanding of the mathematics concepts and/or procedures embodied in the task;
- Indication of a correct and complete task, using mathematically sound procedures; and
- Contains clear, complete explanations.

For the coding of part 3, the explanation, modeling, and/or demonstration of the item to someone who did not understand, the following indicators were focused on in the 7-point holistic rubric:

- Demonstration of a thorough understanding of the mathematics pedagogy and/or pedagogy embodied in the task;
- Indication of a complete and correct task, using mathematically and pedagogically sound procedures;
- Contains clear, complete explanations; and
- Method of instruction/explanation is culturally responsive and fosters cultural understanding, safety, emotional well being and is conducive to learning for diverse learners.

Analysis of the Data

The research design was a concurrent mixed methods strategy with equal focus being given to quantitative and qualitative methods. Initially, skewness and kurtosis were computed and reported. In addition normality was assessed because it is an underlying assumption that needs to be met when using parametric analysis.

Research Questions

What is preservice middle grades teachers' mathematics knowledge for teaching number and operations, algebra, geometry, and probability and statistics?

These four separate research questions will be answered using descriptive statistics and representative participant responses from data analysis based on content and pedagogical content knowledge literature. The categories that emerged from the analysis of the written explanations were identified and unifying commonalities were grouped in meta-categories (Denzin & Lincoln, 2000). Each content strand will be presented separately with specific descriptive statistics, meta-categories, and representative participant responses for each part, content knowledge, explanation of solution (content understanding), and presenting the item to someone who did not understand (pedagogical understanding). In addition, item analysis for the content part of each content strand was conducted in order to evaluate each test item to determine each item's discrimination and difficulty level. Distractors are usually identified in the process of an item analysis. However, since this was not a multiple-choice exam, there was no distractor analysis to conduct.

Ancillary Questions

What is the effect of various sequencing of mathematics courses on middle grades mathematics teachers?

This question was explored using a multivariate analysis of covariance (MANCOVA) followed by polynomial trend contrasts with grade point average of early college mathematics courses as the covariate to "level the playing field." Since the participants follow a specific degree plan for their program, enrollment characteristics (freshman, sophomore, junior, senior) were investigated. The comparisons of interest were the covariate, mathematics grade point average (GPA), enrollment characteristics (class), and the interaction of the covariate, GPA, with the class. Each content strand was analyzed and reported separately.

What cohort development differences are there among students as they progress through the courses identified in the middle grades mathematics and science program?

This question utilized a cohort design and was also investigated using a MANCOVA followed by polynomial trend contrasts with GPA as the covariate. The comparison of interest was GPA, cohort, and the interaction of the covariate with the cohort. The participants were placed into the following cohorts according to their current course enrollments. In order to account for overlapping students, two criteria were established: 1) If a student is concurrently enrolled in any MATH course and MASC course, they were placed into the MASC cohort; and 2) If a student was concurrently enrolled in MASC 450 and MEFB 460, they were placed into the last cohort with the rest of the MEFB 460 participants. The reasoning for being placed into the MASC cohort over the MATH cohort was because the MASC courses are junior-level courses designed to help bridge the gap between mathematics and mathematics pedagogy. Therefore students enrolled in the MASC courses are theoretically more advanced than those taking the mathematics courses. The reasoning for placing the MASC 450 students in the MEFB 460 follows similarly. MEFB 460 is in the methods block which is the semester before the students go and student-teach in the schools. The methods block contains a field component where the students are in the real classroom three days a week and on campus two days a week. Therefore students enrolled in the MASC 450 and MEFB 460 are theoretically more advanced in pedagogy than those not concurrently enrolled. These two criteria also helped to level out the cohorts. The cohort design with their respective numbers can be found in Table 2.

Table 2
Cohort Design for Data Analysis

Cohort	**Courses and Instructors**	**Participants**
1 Mathematics Courses	**MATH 365:** Structure of Math I	**40**
	MATH 366: Structure of Math II	
	MATH 367: Basic Concepts of Geometry	
	MATH 368: Introduction to Abstract Math	
	MATH 403: Math and Technology	
2 Mathematics Education Courses	**MASC 351:** Problem Solving	**48**
	MASC 450: Integrated Mathematics	
3 Methods Course/Student Teaching	**MEFB 460:** Methods of Teaching Middle Grades Mathematics	**34**
	MEFB 497: Teaching Middle Grades	
TOTAL NUMBERS		**122**

Do some types of courses (e.g. algebra, geometry, numerical, statistical or applied, theoretical) have more impact than others upon the development of a teachers' mathematics knowledge for teaching?

This question was initially explored using MANCOVA with courses *taken* as a fixed factor, and then again with courses *currently enrolled in* as the fixed factor. The dependent variables were the three parts of the assessment along with the total scores of each content assessment. The covariate was GPA in both cases. However, both analyses failed the homogeneity of variances (*$p < .05$*) for each content strand. Therefore a univariate analysis of covariance (ANCOVA) followed by polynomial trend contrasts was run for each content strand. Since the several of the different parts (content knowledge, content understanding, pedagogical understanding) of each content strand failed the homogeneity of variance, total scores for mathematics knowledge for teaching was used as the dependent variable for each content strand. The covariate in each case was again, mathematics grade point average.

The comparisons of interest for current courses were: each course separately; MASC 351 and MASC 450; MATH 368, MATH 403, and MASC 351; and MATH 368, MATH 403, MASC 351, and MASC 450. The reason for MASC 351 and MASC 450 together was because these are two courses that were developed by the mathematics education program at the site of this study specifically aimed at bridging mathematics content and mathematics pedagogy together. These two courses were developed off of models recommended by various national organizations (e.g., CBMS, 2001; INTASC, 1995; RAND Mathematics Study Panel, 2003). The reason for MATH 368, MATH 403, and MASC 351 together was because these three courses are typically taken together in the same semester by second semester sophomores or first semester juniors. The reason for MATH 368, MATH 403, MASC 351, and MASC 450 was because MASC 450 was being pulled out of the methods block the participants take the semester before they student teach. The researcher was interested in any possible new interactions the changing of the position of this course might make on preservice teachers' mathematics knowledge for teaching.

The comparisons of interest for taken courses were: each course separately; MASC 351 and MASC 450; MATH 368, MATH 403, MASC 351, and MASC 450; MATH 365, MATH 366, MATH 367, MATH 368, and MATH 403; and all courses together. The reason for MASC 351 and MASC 450 was again because these courses were developed to bridge mathematics content and mathematics pedagogy together. The reason for MATH 368, MATH 403, MASC 351, and MASC 450 was because of the new recommended sequence of courses where MASC 450 was pulled out of the methods block. The reason for MATH 365, MATH 366, MATH 367, MATH 368, and MATH 403 was because these are the mathematics courses specific to middle grades teachers. The reason for all the courses together was to see if the overall program contributed to preservice teachers' mathematics knowledge for teaching.

Does development happen at greater rates in certain stages of the program than others?

Predictor variables for mathematics knowledge for teaching were saved after running MANCOVAs for each of the content strands as described above and used in the analyses of this ancillary question. In order to determine if the predictors were good predictors, a correlation was run between the predictors and their corresponding variables.

Initially, the predictor variables for mathematics knowledge for teaching each content strand were graphed in order to determine their rates (slopes). The rates for each course, currently taking and then have taken, were determined by graphing using the course as the independent variable and the predictor variable for mathematics knowledge for teaching for each content strand for the dependent variable. The rates for each respective course were recorded and rates were compared to those of the predictor variables.

Next, the question initially investigated the results the predicted test scores as the dependent variables and enrollment characteristics (class) and then cohort as the independent variables in MANCOVAs and ANCOVAs followed by difference contrasts and then again by Helmert contrasts for each content strand. Difference contrasts compare the mean of each level (except the first) to the mean of previous levels. Helmert contrasts are just the reverse. Helmert contrasts compare the means of each level of the factor (except the last) to the mean of subsequent levels. However the assumption of the homogeneity of variances was not met.

Nonlinear regression was used instead to determine a model of fit for enrollment characteristics onto the mathematics knowledge for teaching predictor variables for each content strand. After an initial analysis of the all the possible fits, it was determined that the three possible fits for all of the data were linear, quadratic, and cubic. These three models were used for the rest of the analyses for each content strand. The same process was repeated for the cohorts onto the mathematics knowledge for teaching predictor variables for each content strand.

Summary of Research Procedures

The primary source of data collection was from the online assessment administered mid-semester of Spring 2006 to 122 preservice middle grades teachers. The assessment consisted of eight demographic questions and two content strands (randomly assigned: algebra, geometry, probability and statistics, and number and operation) containing seven questions each. Each content question contained the same three parts. The first part had the student answer the question and was scored as right or wrong. The second part had the student explain their solution to the item and was scored on a 7-point scale using a holistic rubric specific for this part. The third part had the student write how they would explain, model, and/or demonstrate this item to someone who did not understand and was scored on a 7-point scale using a holistic rubric for this part. The online assessment contained 22 total questions. The data were analyzed qualitatively and quantitatively. The open-ended responses were analyzed qualitatively using constant comparative analysis (Denzin & Lincoln, 2000). The data were quantitatively analyzed using univariate and multivariate statistics as well as nonlinear regression.

CHAPTER IV

RESULTS

Analysis of the mathematics knowledge for teaching algebra, probability and statistics, number and operations, and geometry of preservice middle grades teachers involved in this study are presented in this chapter. The data were restricted to the 122 participants who were enrolled in MATH 365, MATH 366, MATH 367, MATH 368 MATH 403, MASC 351, MASC 450, MEFB 460, and MEFB 490 and who indicated they were pursuing certification in Grades 4-8 Mathematics and Science.

Each participant took an online assessment consisting of eight demographic questions and two of the possible four content strands (algebra, number and operations, probability and statistics, and geometry). Figure 4 gives the frequencies of participants for each content test. All variables were then coded with a maximum score of one on the first part of the item, six on the second part of the item, and six on the third part of the item.

This assessment was specific to middle grades (grades 5-8) material. There were three parts to each item on each content assessment. The parts were identical across all contents. The three parts—content solution, explanation of solution, and explanation/modeling/demonstrating to someone who did not understand the item—together make up mathematics knowledge for teaching. These three parts are based on Hill, Rowan, and Ball's (2005) definition of mathematics knowledge for teaching. Throughout the analysis the three parts to each item will be referred to as content knowledge (part 1), content understanding (part 2), and pedagogical understanding (part 3).

Skewness and kurtosis were computed for the total assessment scores first by item part, then by the total (referred to as the mathematics knowledge for teaching) for each content strand, number and operations (N&O), algebra (Alg), geometry (Geom), and probability and statistics (P&S), which was computed by summing the respective rubric scores (see Table 3). A lack of extreme skewness and kurtosis was noted.

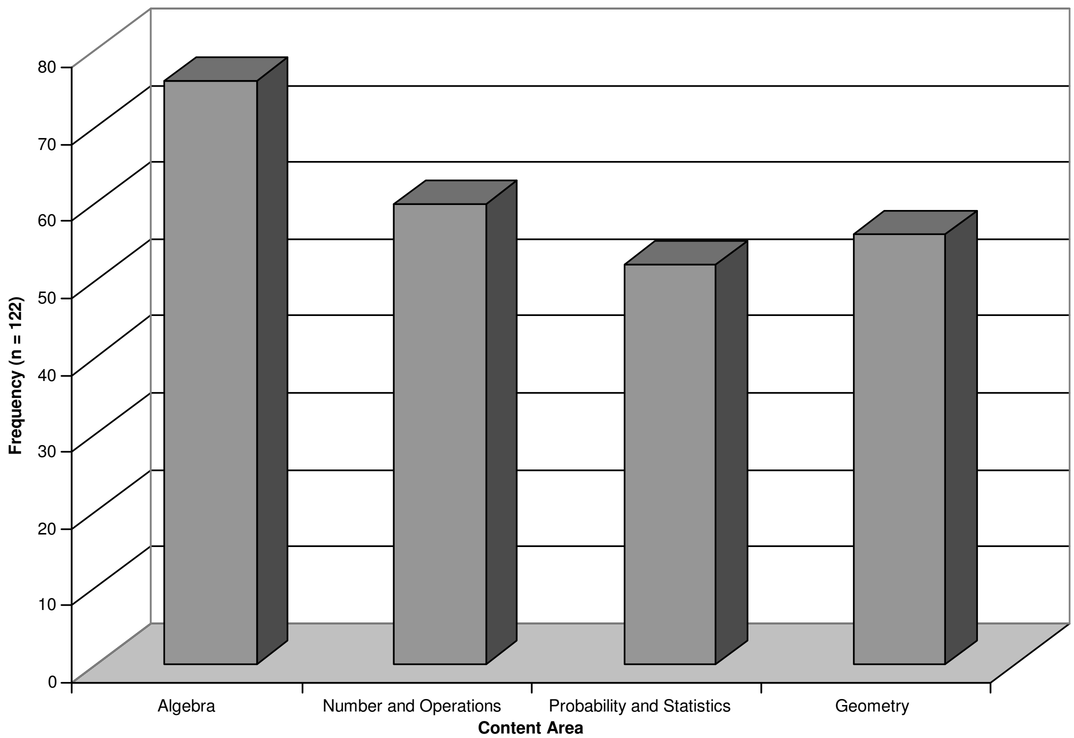

Figure 4. Frequency of Participants for Each Content Strand.

The fit of the total content scores to a normal distribution through a Kolmogorov-Smirnov Test for each content strand, number and operation (z = 1.051 with $N = 60$, p = .219), algebra (z = .770 with $N = 76$, p = .593), geometry (z = .855 with $N = 56$, p = .457), and probability and statistics (z = .563 with $N = 52$, p = .909) revealed no statistical significance. Normality was assessed because it is an underlying assumption that needs to be met when using parametric analysis.

Table 3
Skewness and Kurtosis

	Skewness	Ratio of Skewness to Standard Error of Skewness	Kurtosis	Ratio of Kurtosis to Standard Error of Kurtosis
N&O—Answer of item (content question)	.09	.30	-.33	-.54
N&O—Explanation of solution	-.88	-2.83	.70	1.15
N&O—Explain/model/demonstrate to someone who did not understand	-.40	-1.30	-.58	-.96
N&O—Mathematics knowledge for teaching algebra	-.61	-1.97	-.59	-.97
Alg—Answer of item (content question)	-.91	-3.30	.69	1.27
Alg—Explanation of solution	-.61	-2.25	.81	1.49
Alg—Explain/model/demonstrate to someone who did not understand	-.45	-1.62	.21	.39
Alg—Mathematics knowledge for teaching algebra	-.47	-1.70	.09	.17
Geom—Answer of item (content question)	-1.50	-4.69	1.87	2.97
Geom—Explanation of solution	-.37	-1.15	-.80	-1.28
Geom—Explain/model/demonstrate to someone who did not understand	-.36	-1.13	-.99	-1.58
Geom—Mathematics knowledge for teaching algebra	-.50	-1.58	-.82	-1.31
P&S—Answer of item (content question)	-1.26	-3.81	1.41	2.16
P&S—Explanation of solution	-.19	-.57	.09	.14
P&S—Explain/model/demonstrate to someone who did not understand	-.51	-1.54	-.03	-.04
P&S—Mathematics knowledge for teaching algebra	-.51	-1.55	.24	.37

Research Question 1

What is preservice middle grades teachers' mathematics knowledge for teaching number and operations?

The analysis of this question was broken into four parts: analysis of the content knowledge, analysis of the content understanding, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching number and operations. The three parts—content knowledge, content understanding, and pedagogical understanding—together make up mathematics knowledge for teaching number and operations in the middle grades. Beginning with the analysis of the content knowledge part of the number and operations exam, the Spearman-Brown prediction formula was used to calculate the reliability of the assessment. The coefficient alpha for the Number and Operations assessment was .924.

The next step was to conduct an item analysis for the content knowledge part of the number and operations assessment. An item analysis evaluates each test item to determine its discrimination and difficulty level. Item discrimination refers to the ability of an item to differentiate among students on the basis of how well they know the material being tested. An item's difficulty index is expressed as the proportion of students who responded correctly to an item to the total number who responded. Distractors are usually identified in the process of an item analysis. However, since this was not a multiple-choice exam, there was no distractor analysis to conduct. The item difficulty and discrimination is again associated with the content part (part 1) of the items. The item discrimination for the content question items in Number and Operations can be found in Table 4. Item difficulty can be found in Table 5. The average item difficulty was 56% (*p* = .56) which is considered moderately difficult on the item difficulty scale.

Items for which less than half of the participants answered correctly (1, 2, and 4) were investigated for commonalities. The first item on the number and operations assessment addressed developing and applying the laws of exponents for multiplication. There were two common errors found in student answers to this item. The first error was after the student correctly multiplied the solution, the solution was left in incorrect scientific notation. The coefficient was larger than ten (i.e., 29.5). The second error was after the student multiplied the solution correctly, the coefficient was changed so as to reflect a number less than ten. However, moving of the decimal was not reflected as the exponent

Table 4
Item Discriminations for Content Question (Part 1) in Number and Operations

Item	% Item Discrimination	Level*
1—Developing and applying laws of exponents for multiplication	20%	Usually Unacceptable
2—Estimating percent of quantity, given an application	30%	Good
3—Applying percents to commission	20%	Usually Unacceptable
4—Estimating percent of quantity, given an application	27%	Good
5—Evaluating expressions with integral exponents	20%	Usually Unacceptable
6—Applying percents to sale price situations	37%	Good
7—Determining multiples and least common multiples of two or more numbers	57%	Excellent

*Negative = unacceptable; 0% - 24% = Usually unacceptable, might be approved; 25% - 39% = Good Item; 40% - 100% = Excellent Item

Table 5
Item Difficulty for Content Question (Part 1) in Number and Operations

Item	% Item Difficulty	Level*
1—Developing and applying laws of exponents for multiplication	47%	Moderately Difficult
2—Estimating percent of quantity, given an application	28%	Moderately Difficult
3—Applying percents to commission	87%	Easy
4—Estimating percent of quantity, given an application	23%	Difficult
5—Evaluating expressions with integral exponents	83%	Easy
6—Applying percents to sale price situations	68%	Moderately Difficult
7—Determining multiples and least common multiples of two or more numbers	58%	Moderately Difficult
Average Item Difficulty for Number and Operations	**56%**	**Moderately Difficult**

*<20% = difficult; 20% - 80% = moderately difficult; >80% = easy

remained unchanged on the power of 10. Items two and four on the number and operations assessment addressed estimating a percent of quantity, given an application. The most common error on both of these items was simply not estimating at all. A majority of the students who missed this item reported the exact number instead of the estimate (i.e., $562.50 instead of $550 or $600; or 38% instead of 40%).

Next, the open-ended parts, content understanding and pedagogical understanding (parts 2 and 3), of the online number and operations assessment of preservice middle grades teachers were explored using constant comparative analysis (Denzin & Lincoln, 2000). For consistency of data interpretation, all items were scored on an ordinal scale from zero to six according to the holistic rubric (see Appendix G). Although the data were not strictly interval in nature, they approximate an interval scale. Therefore, statistics requiring interval scaling, such as mean and standard deviation, can be reasonably interpreted. Descriptive statistics including mean, median, mode, standard deviation, and quartiles are reported for each item on the number and operations assessment and are reported in Table 6. Frequencies for each rubric score on explanation (part 2) and understanding (part 3) were computed for each item and are reported in Table 7.

Table 6
Descriptive Statistics of Item Responses (Parts 2 and 3) on Number and Operations

	Mean	Median	Mode	Standard Deviation	Percentiles 25	50	75
Item 1—Explanation	2.15	2	2	1.60	1	2	3
Item 1—Understanding	2.01	2	0	1.65	0	2	3
Item 2—Explanation	2.22	2	2	1.28	2	2	3
Item 2—Understanding	2.05	2	2	1.38	1	2	3
Item 3—Explanation	2.87	3	3	1.21	3	3	3
Item 3—Understanding	2.32	3	3	1.43	2	3	4
Item 4—Explanation	1.73	2	0	1.55	0	2	3
Item 4—Understanding	1.45	1	0	1.50	0	1	3
Item 5—Explanation	2.78	3	3	1.30	2	3	3
Item 5—Understanding	2.72	3	4	1.73	2	3	4
Item 6—Explanation	2.85	3	3	1.77	2	3	4
Item 6—Understanding	2.57	3	3	1.69	1	3	4
Item 7—Explanation	2.75	3	0	2.10	1	3	5
Item 7—Understanding	2.15	2	0	1.86	0	2	4

In terms of explanations of solutions (content understanding—part 2) to items in the number and operations assessment, a majority of the students scored either a two or three according to the rubric for this item. A score of three on the explanation of the solution generally meant the student just explained their exact algorithmic or mental mathematic procedure for the given item. The procedures were mathematically correct and

Table 7
Frequencies for Explanation and Understanding (Parts 2 and 3) of Items in Number and Operations

	Score						
	0	1	2	3	4	5	6
	Reported as: Frequency/Percent (n = 60)						
Item 1—Content Understanding	15/25.0%	3/5.0%	16/26.7%	16/26.7%	6/10.0%	2/3.3%	2/3.3%
Item 1—Pedagogical Understanding	19/31.7%	3/5.0%	12/20.0%	14/23.3%	8/13.3%	4/6.7%	
Item 2—Content Understanding	8/13.3%	2/3.36%	31/51.7%	11/18.3%	5/8.3%	2/3.3%	1/1.7%
Item 2— Pedagogical Understanding	11/18.3%	8/13.3%	19/31.7%	14/23.3%	5/8.3%	3/5.0%	
Item 3—Content Understanding	6/10.0%	1/1.7%	3/5.0%	40/66.7%	6/10.0%	3/5.0%	1/1.7%
Item 3— Pedagogical Understanding	10/16.7%	2/3.3%	7/11.7%	26/43.3%	12/20.0%	3/5.0%	
Item 4—Content Understanding	18/30.0%	8/13.3%	17/28.3%	12/20.0%	1/1.7%	2/3.3%	2/3.3%
Item 4— Pedagogical Understanding	23/38.3%	11/18.3%	11/18.3%	9/15.0%	3/5.0%	3/5.0%	
Item 5—Content Understanding	6/10.0%	1/1.7%	8/13.3%	40/66.7%	1/1.7%		4/6.7%
Item 5— Pedagogical Understanding	12/20.0%	2/3.3%	9/15.0%	14/23.3%	17/28.3%	3/5.0%	3/5.0%
Item 6—Content Understanding	7/11.7%	5/8.3%	13/21.7%	20/33.3%	3/5.0%	4/6.7%	8/13.3%
Item 6— Pedagogical Understanding	10/16.7%	10/16.7%	5/8.3%	14/23.3%	13/21.7%	8/13.3%	
Item 7—Content Understanding	12/20.0%	10/16.7%	6/10.0%	9/15.0%	7/11.7%	8/13.3%	8/13.3%
Item 7—Pedagogical Understanding	16/26.7%	11/18.3%	8/13.3%	8/13.3%	9/15.0%	6/10.0%	2/3.3%

their solution was correct, however, there was no more explanation provided other than their algorithmic procedure. A common example of a score of three:

- Multiplied 5 by 5.9, and got 29.5 Added the exponents together to get 10^15. Added one more to the exponents when I moved the decimal point over one space to get 2.95 x 10^16.

Another common example of an explanation of a solution scoring a three according to the rubric:

- Found out what 20% of 37 was, then found what 15% of 29.6 was. Then subtracted the two from the original price.

A score of two on the content understanding for number and operations items generally meant the student had a mathematically sound procedure, however, their answer did not match what the problem was wanting. The following statement is mathematically correct, however, the question was not asking for an exact answer, it was looking for an estimate.

- Subtracted 511 from 705. Divided that number by 511, then multiplied it by one hundred.

Two other items on the number and operations assessment are noteworthy due to their common themes across the explanation of solutions, especially in the misuse of mathematical vocabulary. The first concerns item 6 which addressed applying percents to sale price situations. A common error among the incorrect content answers was the adding together of two percentages given in an original price and then a sale price situation. A common answer for the explanation of the solution was:

- Add the total percent discount and then multiply to 37 then subtract that from 37.

A more detailed explanation of the same item:

- If you add the percentages off together, you get 35 percent off. So then I broke it down into 30 percent and 5 percent. 30 percent is 10 percent 3 times and 10 percent is 3.70 so I multiplied 3.70 by 3. 5 percent is half of 3.70 so I added this all together and subtracted from 37 to get the new price.

The next item of noteworthy concern is item seven which addressed determining least common multiples of two or more numbers. An overwhelming majority of the students who missed this item mistook the least common multiple for the greatest common factor. Examples of common explanations include (each of these students reported 3 as their final answer):

- 3 is the smallest number that evenly "goes into" 3, 6, and 27
- Knowing what the multiples of all those numbers were because they were small helped me to determine what number was the smallest multiple of all three
- It is the lowest value they all have in common

On this same item, misuse of mathematical vocabulary was evident in the explanation of their solution although the student received credit for a correct answer on the content (part 1). Common explanations include:

- Factor than multiply by all the factors.
- To have 27 as a factor, the number had to be bigger than 27, and to have 6 as a factor it had to be even. The best way to get a number that fits those criteria is to multiply 27 by 2.
- 27 times 2 because 27/6 is not a whole number.

For the pedagogical understanding of number and operations, being able to communicate the material to someone who did understand, many of the same themes and trends found in the explanation of the solution emerged from the understanding data. If the student did not receive a correct answer on the original content problem or received a low score on the explanation of their solution, they were likely to not receive a high score on the pedagogical understanding part. A majority of the participants scored a two, three, or four on the pedagogical understanding part of the number and operation assessment. A score of two on the pedagogical understanding part of the items on the number and operation assessment generally meant the procedure was somewhat correct, but contained minor mathematical errors. A score of three generally indicated the student could successfully explain their own procedure to a person who did not understand. Scores of two and three on the pedagogical understanding part tended to be very algorithmic in nature and the general method of instruction could assume to be direct since there was no mention of any other method. Examples of scores of two and three on the understanding part of the number and operations assessment:

- I subtracted 511 from 705 to get 194. I then divided 194 by 511 to get .37964 and then multiplied by 100 to get the percent of about 38.
- I'd explain the definition of exponents and how 4^3 is short hand for 4*4*4. And then work the multiplication.
- I could write out all the multiples for all 3 numbers until I found a number that appeared as a multiple for all 3.

A score of four on the pedagogical understanding part indicated the student could do more than just explain their algorithmic procedure. They could relate their procedure back to different parts of the problem and could often explain why they got a certain number or conducted a certain procedure. It is also at this level that the researcher first saw any signs of cultural responsiveness on the student's part: mention of various instructional techniques, application to real world situations, and use of hands-on learning. Examples of a score of four on the pedagogical understanding part of the number and operations assessment:

- When we solve percent problems, we set up a proportion in words first. The words are PART over WHOLE equals PERCENT over 100: part/whole= /100. Look at the first part. The shirt cost 37. is that the whole cost or part of the cost of the shirt? The whole. And we want to know what 20% is… so we have ?/37=20/100. Divide 100 by something to get 37. Do long division and figure out that something is 7.4. So the 20% sale gives him $7.40 off the price. Subtract that from the cost and now the shirt costs $29.60. Then we set it up the same way and try to find what 15% of the sale price is: it's $4.44. Subtract that from the new cost to get the final cost of: $25.16.
- List out about five multiples of each number. That means 3 times 1, 3 times 2, 3 times 3, etc. 3: 3, 6, 9, 12, 15 6: 6, 12, 18, 24, 30 27: 27, 54, 81, 108 OK, well we need to keep going with 3 and 6 until we get much higher. We are looking for the FIRST number that is in all three lists. Let's try just with 6 (because it is bigger, and then see if the number also works with 3) 6: 6, 12, 18, 24, 30, 36, 42, 48, 54 54!!! So 54 is the LCM of 6 and 27, is it a multiple of 3 also? 3x18=54. YES. So the LCM of 3, 6, and 27 is 54.
- Since he gets 5% of the sale and the sale was $180 we would discuss why we would multiply the two together. They would then multiply 180 by .05, which is 5%, to get $9.00, which is what he made off of the sale.

- You could use the colored rod method. Build three trains of 3, 6, and 27 in length. Keep building each train using trains of the same length until all three trains are equal. The length of the equal trains is the least common multiple.

All three of the parts (content knowledge, content understanding, and pedagogical understanding) mentioned above in the analysis together make up the component called mathematics knowledge for teaching. Total scores for this assessment ranged from 3 to 59, a range of 56. Total possible points for the entire number and operations assessment was 91 points. Descriptive statistics for the total of each part (content knowledge, content understanding, and pedagogical understanding) and the total mathematics knowledge for teaching number and operations can be found in Table 8.

Table 8
Descriptive Statistics for Mathematics Knowledge for Teaching Number and Operations

	Mean	Median	Mode	Standard Deviation	Percentiles		
					25	50	75
Total Content (7 pts possible)	3.95	4	4	1.40	3	4	5
Total Explanation (42 pts possible)	17.32	18	17	6.65	15	18	22
Total Understanding (42 pts possible)	15.57	16.50	20	8.10	9	17	21
Mathematics Knowledge for Teaching Number and Operation (91 pts possible)	36.83	40	40	14.63	25	40	49

Research Question 2

What is preservice middle grades teachers' mathematics knowledge for teaching algebra?

The analysis of this question was again broken down into four parts: analysis of the content, analysis of the explanation, analysis of the understanding, and analysis of the mathematics knowledge for teaching. The three parts— content knowledge, content understanding, and pedagogical understanding—together make up mathematics knowledge for teaching algebra in the middle grades. Beginning with the analysis of the content knowledge part of the algebra exam, the Spearman-Brown prediction formula was used to calculate the reliability of the assessment. The coefficient alpha for the Algebra assessment was .935.

The next step was to conduct an item analysis for the content knowledge part of the algebra assessment. An item analysis evaluates each test item to determine its discrimination and difficulty level. Item discrimination refers to the ability of an item to differentiate among students on the basis of how well they know the material being tested. An item's difficulty index is expressed as the proportion of students who responded correctly to an item to the total number who responded. Distractor analysis was not conducted because this assessment did not contain any distractors. The item difficulty and discrimination is again associated with the content part (part 1) of the items. The item discrimination for the content question items in the algebra assessment can be found in Table 9. Item difficulty can be found in Table 10. The average item difficulty was 79% (p = .79) which is considered moderately difficult on the item difficulty scale.

Item two, the only item in which less than half of the participants answered correctly, was investigated for commonalities. This item addressed translating verbal sentences into algebraic equations. There were two common errors found in students answers to this item. The first error was simply not following the directions of the item. The item asked for the student to set up an inequality for the information given. About half of the students who missed this item solved the inequality and reported an answer instead. The second error was the misinterpretation of the information given in the item. The item stated, "The web site fee is $30." The inequality was to be used to determine a minimum profit. The other half of the students who missed this problem mistook the website fee for a profit and added 30 to the inequality instead of subtracting the fee.

Table 9

Item Discriminations for Content Question (Part 1) in Algebra

Item	% Item Discrimination	Level*
1—Adding and subtracting polynomials and integer coefficients	32%	Good
2—Translating verbal sentences into algebraic equations	37%	Good
3—Translating verbal sentences into algebraic equations	26%	Good
4—Multiplying a binomial by a monomial or binomial with integer coefficients	39%	Good
5—Translating verbal sentences into algebraic equations	18%	Usually Unacceptable
6—Factoring a trinomial in the form $ax^2 + bx + c$; $a = 1$ and c having no more than 3 sets of factors	16%	Usually Unacceptable
7—Applying algebra to determine the measure of angles formed by or contained in parallel lines cut by a transversal and by intersecting lines	16%	Usually Unacceptable

*Negative = Unacceptable; 0% - 24% = Usually unacceptable, might be approved; 25% - 39% = Good Item; 40% - 100% = Excellent Item

Table 10
Item Difficulty for Content Question (Part 1) in Algebra

Item	% Item Difficulty	Level*
1—Adding and subtracting polynomials and integer coefficients	82%	Easy
2—Translating verbal sentences into algebraic equations	42%	Moderately Difficult
3—Translating verbal sentences into algebraic equations	79%	Moderately Difficult
4—Multiplying a binomial by a monomial or binomial with integer coefficients	78%	Moderately Difficult
5—Translating verbal sentences into algebraic equations	88%	Easy
6—Factoring a trinomial in the form $ax^2 + bx + c$; $a = 1$ and c having no more than 3 sets of factors	92%	Easy
7—Applying algebra to determine the measure of angles formed by or contained in parallel lines cut by a transversal and by intersecting lines	89%	Easy
Average Item Difficulty for Algebra	**79%**	**Moderately Difficult**

*<20% = difficult; 20% - 80% = moderately difficult; >80% = easy

Two other items are noteworthy concerning incorrect answers on the content knowledge part of the algebra online assessment. Item five addressed translating verbal sentences into algebraic equations. Of the three items on this assessment addressing the idea, this item was the only one not in a real world context. The one major error common to a majority who answered this problem incorrectly was the incorrect placement of the "eight less than" in the equation. Instead of subtracting eight from "twice a number" (i.e., $2x - 8$), students subtracted the twice a number from the eight (i.e., $8 - 2x$). Item six address factoring a trinomial in the form of $ax^2 + bx + c$. Of the students who missed this item, with the exception of those who did not complete the item, all simply mixed up their signs when reporting their factored answer (i.e., $(y + 3)(y - 6)$ instead of $(y + 6)(y - 3)$).

Next the open-ended parts, content understanding and pedagogical understanding (parts 2 and 3), of the online algebra assessment of preservice middle grades teachers were explored using constant comparative analysis. For consistency of data interpretation, all items were scored on an ordinal scale from zero to six according to the holistic rubric designated for each part (see Appendix G). Although the data were not strictly interval in nature, they approximate an interval scale. Therefore, statistics requiring interval scaling, such as mean and standard deviation, can be reasonably interpreted. Descriptive statistics including mean, median, mode, standard deviation, and quartiles are reported for each item on the algebra assessment and are reported in Table 11. Frequencies for each rubric score on content understanding (part 2) and pedagogical understanding (part 3) were computed for each item and are reported in Table 12.

In terms of content understanding (part 2) of items in the algebra assessment, a majority of the students scored either a two or three according to the rubric for this item. A score of three on the explanation of the solution generally indicated the student just explained or algorithmically showed their exact mathematic procedure for the given item. The procedures were mathematically correct and their solution was correct, however, there was no more explanation provided other than their algorithmic procedure. Common examples of a score of three:

- I distributed the minus sign throughout the problem in the second set of parenthesis. Then I took away the parenthesis and added the problem.
- Subtract the second parenthesis from the first. Look for like terms. Subtract like terms to finish the problem.

Table 11
Descriptive Statistics of Item Responses (Parts 2 and 3) on Algebra

	Mean	Median	Mode	Standard Deviation	Percentiles 25	50	75
Item 1—Explanation	3.26	3	3	1.60	2	3	5
Item 1—Understanding	3.14	3	3	1.77	2	3	5
Item 2—Explanation	2.82	2	2	1.85	2	2	4
Item 2—Understanding	2.57	3	2	1.42	2	3	4
Item 3—Explanation	3.34	3	3	1.79	2	3	5
Item 3—Understanding	2.79	3	3	1.62	2	3	4
Item 4—Explanation	3.29	3	3	1.45	3	3	4
Item 4—Understanding	3.11	3	3	1.37	3	3	4
Item 5—Explanation	3.24	3	3	1.60	2	3	4
Item 5—Understanding	3.03	3	4	1.41	2	3	4
Item 6—Explanation	3.34	3	3	1.60	2	3	5
Item 6—Understanding	2.80	3	3	1.44	2	3	4
Item 7—Explanation	3.37	3	3	1.63	3	3	5
Item 7—Understanding	3.07	3	4	1.74	2	3	4

A score of two on the explanation of the solution (content understanding) for algebra items generally meant the student had a mathematically sound procedure; however, their answer did not match what the problem was wanting. The following example of an explanation is of a student who got a correct answer for the item, but they misused mathematical vocabulary and had a limited explanation and therefore scored a two:

- I multiplied each factor using FOIL.

Five of the seven items on the content understanding part of the algebra assessment are noteworthy. Items two, three, and five addressed translating verbal sentences into algebraic equations. Although a majority of the students got the items correct in terms of content, they had trouble explaining how they came up with their solutions. A common

Table 12
Frequencies for Explanation and Understanding (Parts 2 and 3) of Items in Algebra

	Score						
	0	1	2	3	4	5	6
	Reported as: Frequency/Percent (n = 76)						
Item 1—Content Understanding	7/9.2%	1/1.3%	12/15.8%	27/35.5%	9/11.8%	14/18.4%	6/7.9%
Item 1—Pedagogical Understanding	11/14.5%	2/2.6%	10/13.2%	21/27.6%	11/14.5%	16/21.1%	5/6.6%
Item 2— Content Understanding	10/13.2%	3/3.9%	28/36.8%	13/17.1%	6/7.9%	4/5.3%	12/15.8%
Item 2— Pedagogical Understanding	9/11.8%	5/6.6%	23/30.3%	18/23.7%	16/21.1%	4/5.3%	1/1.3%
Item 3— Content Understanding	7/9.2%	4/5.3%	12/15.8%	20/26.3%	10/13.2%	12/15.8%	11/14.5%
Item 3— Pedagogical Understanding	12/15.8%	2/2.6%	16/21.1%	19/25.0%	15/19.7%	11/14.5%	1/1.3%
Item 4— Content Understanding	4/5.3%	3/3.9%	8/10.5%	36/47.4%	9/11.8%	9/11.8%	7/9.2%
Item 4— Pedagogical Understanding	7/9.2%	3/3.9%	4/5.3%	33/43.4%	21/27.6%	6/7.9%	2/2.6%
Item 5— Content Understanding	6/7.9%	2/2.6%	15/19.7%	22/28.9%	15/19.7%	8/10.5%	8/10.5%
Item 5— Pedagogical Understanding	5/6.6%	5/6.6%	15/19.7%	20/26.3%	23/30.3%	5/6.6%	3/3.9%
Item 6— Content Understanding	4/5.3%	3/3.9%	17/22.4%	19/25.0%	14/18.4%	10/13.2%	9/11.8%
Item 6— Pedagogical Understanding	9/11.8%	3/3.9%	16/21.1%	22/28.9%	18/23.7%	8/10.5%	
Item 7— Content Understanding	7/9.2%	3/3.9%	6/7.9%	26/34.2%	14/18.4%	13/17.1%	7/9.2%
Item 7— Pedagogical Understanding	10/13.2%	5/6.6%	11/14.5%	14/18.4%	21/27.6%	10/13.2%	5/6.6%

theme across all explanations for these items was "just follow the words…they tell you what to do." Common examples of explanations for these items include:

- Well you really just have to be familiar with the terminology of a word problem or if you know how to solve the problem on your own you could do that. But the word problem practically tells you the equations; you just have to be able to put it all together.

- All I did in this problem is read the statement and translate what it was saying into numbers and operations.
- It is quite simple. All it is is that the number of cars making $4 per car minus the fee must be at least $50 for him to make a profit.

The other two items of interest are items four and six. These two items complement each other in that item four addressed the multiplication of binomials and item six addressed factoring a trinomial. In item four, the common explanation was "I just used FOIL to multiply it out and then I added like terms." Another example of a common solution was:

- I came to this answer by foiling or multiplying the two parentheses together. By doing this I get 6x^2-24x-10x+40. Then combining like terms I get 6x^2-34x+40, which is my answer.

Another method mentioned, but not as frequently, was the "box method" or "tic-tac-toe method." Students who mentioned this method in their explanation of their solution tended to score lower because they were unable to clearly communicate this procedure:

- I did this problem using a tic tac toe method. You draw what looks like a tic tac toe board and then put the first binomial across the top and second binomial down the side. I will try to demonstrate what this looks like on paper but without the lines: 2x -8 3x 6x^2 -24x -5 -10x 40 There would be lines separating the lines and columns. Next you combine like terms. -24x plus -10x can be added together and nothing else can. This gives you our final answer: 6x^2 -34x +40

A lack of the phrases "distributive property" and "multiplication of each term" amongst the majority of the explanations was noted. Although a majority of the students answered item six correctly, there was a theme of the misuse of mathematical vocabulary and procedures in their explanations of their solutions. For example, several students explained they used "FOIL" to factor their trinomial:

- The solution is (y – 3)(y + 6) because when you FOIL these two factors together it gives you y^2+3y-18. FOIL is just a way of multiplying the two factors together without leaving out a part.
- y is squared so it needs to be in both parentheses. Then use FOIL.

Across item six explanations, the most common method of arriving at a solution was to work the FOIL method backwards:

- I did the FOIL method backwards in a way. I know y^2 is y times y so that will be the first portion of each binomial. Then I thought about the factors of 18 and picked on that when subtracted would equal 3.
- I got this by working backwards from my original polynomial using the FOIL method. I knew that I needed each of my binomials to have y as the first term so that they would give me y^2 as my first term in the polynomial. I then looked at what two numbers multiplied together would give me 18, that would also give me a -3 when added together. I came up with 6 and 3, and I knew that to get a +3, the 6 had to be the binomial with the + sign and the 3 with the – sign. Thus giving me (y+6)(y-3).

Although the explanations above considered the y^2 term, this was not common to most answers. Most of the explanations just concentrated on explaining how the factors of 18 were combined to find the middle term:

- I just did this problem in my head by breaking it down into what factors of 18 could be subtracted to give me a difference of 3.
- I arrived at this solution by deciding the factors of 18 and then once I did that I knew one would be positive and the other negative because the 18 was negative. All I had to do then was figure which two values subtracted from each other was +3. The solution to this is +6 and -3 which when added equal +3 and when multiplied equal -18 which we are looking for.

For the pedagogical understanding of algebra, again, many of the same themes and trends found in the explanation of the solution emerged. If the student did not receive a correct answer on the original content problem or received a low score on the explanation of their solution, they were likely to not receive a high score on the pedagogical understanding part. A majority of the participants scored a two or three on the pedagogical understanding part of the items on the algebra assessment. A score of two on the pedagogical understanding part of the items on the number and operation assessment generally meant the procedure was somewhat correct, but contained minor mathematical errors or had missing parts to the item missing. A score of three generally indicated the student could successfully explain their own procedure to a person who did not understand. Scores of two and three tended to be very algorithmic and procedure-oriented in nature. The general method of instruction could be assumed to be direct since there was often no mention of any other method. Examples of scores of two on the understanding part of the algebra assessment include:

- They would need to know a little bit of geometry and understand the meaning of transversal and what that means in regards to parallel lines.

- I would show them the FOIL method and how I went about solving this particular problem using it.
- I would list out the possible combinations of factors of -18, and then use trial and error to find which pair satisfied the second requirement of a sum of three.

Examples of scores of three include:

- After reading "eight less than twice a number is forty-two" first we know that the equation = 42 and we know that twice a number is 2xN or 2N. Eight less of 2N is 2N-8 which equals 42.
- Use the method First—multiply the first set of numbers in the parentheses and carry the values, outside—multiply the outside sets of numbers, inside—multiply the inside set of numbers, last—multiply the last set of numbers. Then add or subtract the 2nd answer and 3rd answer you got for the outside part and inside part. FOIL
- I would have him or her set up variables of x = how much tank will hold (which would equal 12), and g = number of gallons already in tank. In order to find out how much it is empty, you would subtract g from 12. You would then multiply the price (p) to see how much the total would be.

Across all the pedagogical understanding responses, there was a greater indication of cultural responsiveness than was found on the number and operations items. Exemplar pedagogical understanding explanations include:

- I would write 3x(2x-8) + -5(2x-8) and draw arrows from the coefficients outside the parentheses to the two characters inside of each.
- I would start plugging in real numbers instead of using a variable right away.

- I would bust out a protractor to show that the angles are indeed equal, and proceed in a similar matter as my explanation of my solution. If someone did not understand the algebra, I would do a mini-lesson on doing like-operations on both sides of an equal equation in order to solve it.
- I would bring in a 1 gallon jug and demonstrate the equation using the info from the problem. We could say the capacity of the jug is 12 gallons, but we would only fill the container partially. Then, calculate how much it would cost to fill the rest of the jug.
- Just work it out step by step with them maybe using different colors or underlining to show common terms and such.

It should also be noted that there were a few examples of multiplying binomials where the participants stated they would use Base-10 blocks to model this procedure. Only one participant correctly incorporated the use of a hands-on material in the multiplication of binomials. The participant suggested algebra tiles be used to model this item.

Other themes noteworthy for pedagogical understanding in algebra were the teaching of the concepts of the multiplication of binomials and the factoring of trinomials. Again, FOIL, forwards and backwards, was a common theme in the responses. A major concentration on just finding the correct combination of factors in factoring trinomials was noted. Common examples for the multiplication of binomials include:

- Ok, I want to teach you FOIL. Front, Outside, Inside, Last. SO when we see a problem like this one we know what to do. Front: 3x*2x=6x^2 Outside: 3x*-8=-24x Inside: -5*2x=-10x Last:-5*-8=40. We have 6x^2-24x-10x+40. Now we collect like terms and end with 6x^2-34x+40.

- Explain the FOIL method means first, outer, inner, last. So we must first multiply the first variables in each expression. Then we multiply the outer numbers, then the inner numbers, and then the last two variables in each expression. After we've found the solutions we combine our like terms, (usually found with the inner and outer answers) and add all of them together. I would make sure to tell the students to watch out for negative signs!

Common examples for the factoring of trinomials include:

- I'd tell the kids to automatically put (y)(y), because we know y x y is y^2. Then I'd show them that since the second sign is -, one sign must be + and one -. Then I'd tell them to factor 18, find the numbers that will equal 3 when subtracted, since it's positive three, you know the larger number goes with the plus sign and thus we get our answer.
- I would have them find factors of 18 and then those factors must somehow (either by adding or subtracting) equal 3.
- Review the FOIL process with models and arrows then help them work it backwards.

The box/tic-tac-toe method was again mentioned. As with the explanation of the solution, there was no clear explanation of understanding of how the method was to be used:

- There are many types of ways to model factoring. I was taught using a box method, cut into 4 quadrants and is filled in accordingly, with the variables that the rows and columns have in common on the outside of the box. I would then make sure they knew to go back and check their work to make sure they did it correctly!

It was again noted that there was a lack of the use of the term "distributive property" and "multiplication of each term."

All three of the parts (content knowledge, content understanding, and pedagogical understanding) mentioned above in the analysis together make up the component called mathematics knowledge for teaching algebra in the middle grades. Total scores for this assessment ranged from 4 to 80, a range of 76. Total possible points for this algebra assessment were 91. Descriptive statistics for the total of each part (content, explanation, and understanding) and the total mathematics knowledge for teaching algebra can be found in Table 13.

Table 13
Descriptive Statistics for Mathematics Knowledge for Teaching Algebra

	Mean	Median	Mode	Standard Deviation	Percentiles		
					25	50	75
Total Content (7 pts possible)	5.50	6	6	1.17	5	6	6
Total Explanation (42 pts possible)	22.66	23	29	7.61	18	23	29
Total Understanding (42 pts possible)	20.50	21	20	7.92	16	21	26
Mathematics Knowledge for Teaching Algebra (91 pts possible)	48.66	51	46	14.98	39	51	59

Research Question 3

What is preservice middle grades teachers' mathematics knowledge for teaching geometry?

The analysis of this question was again broken into four parts: analysis of the content knowledge, analysis of the content understanding, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching geometry. The three parts— content knowledge, content understanding, and pedagogical understanding— together make up mathematics knowledge for teaching geometry in the middle grades. Beginning with the analysis of the content knowledge part of the geometry assessment, the Spearman-Brown prediction formula was used to calculate the reliability of the assessment. The coefficient alpha for the geometry assessment is .973.

The next step was to conduct an item analysis for the content knowledge part of the geometry assessment. An item analysis evaluates each test item to determine the discrimination and difficulty level. Item discrimination refers to the ability of an item to differentiate among students on the basis of how well they know the material being tested. An item's difficulty index is expressed as the proportion of students who responded correctly to an item to the total number who responded. Distractors are usually identified in the process of an item analysis. However, a distractor analysis was not conducted because this assessment did not contain any distractors. The item discrimination for the content question items in geometry can be found in Table 14. Item difficulty can be found in Table 15. The average item difficulty was 76% (p = .76) which is considered moderately difficult on the item difficulty scale.

Table 14

Item Discriminations for Content Question (Part 1) in Geometry

Item	% Item Discrimination	Level*
1—Calculating the missing angle in a supplementary or complementary pair	39%	Good
2— Calculating the missing angle in a supplementary or complementary pair	43%	Excellent
3—Identifying pairs of supplementary and complementary angles	18%	Usually Unacceptable
4—Determining angle relationships when given two parallel lines cut by a transversal	54%	Excellent
5—Identifying pairs of vertical angles as congruent	29%	Good
6— Identifying pairs of vertical angles as congruent	29%	Good
7—Calculating the missing angle measurements when given two parallel lines cut by a transversal	21%	Usually Unacceptable

*Negative = unacceptable; 0% - 24% = Usually unacceptable, might be approved; 25% - 39% = Good Item; 40% - 100% = Excellent Item

Table 15

Item Difficulty for Content Question (Part 1) in Geometry

Item	% Item Difficulty	Level*
1—Calculating the missing angle in a supplementary or complementary pair	77%	Moderately Difficult
2— Calculating the missing angle in a supplementary or complementary pair	64%	Moderately Difficult
3—Identifying pairs of supplementary and complementary angles	70%	Moderately Difficult
4—Determining angle relationships when given two parallel lines cut by a transversal	66%	Moderately Difficult
5—Identifying pairs of vertical angles as congruent	86%	Easy
6— Identifying pairs of vertical angles as congruent	86%	Easy
7—Calculating the missing angle measurements when given two parallel lines cut by a transversal	86%	Easy
Average Item Difficulty for Geometry	**76%**	**Moderately Difficult**

*<20% = difficult; 20% - 80% = moderately difficult; >80% = easy

There were no items for which less than half of the participants answered correctly, therefore the three items with the smallest percentage (items 2, 3, and 4) of the participants answering correctly were investigated for commonalities across each item. The first item, item two, addressed calculating the missing angle in a supplementary pair. There was one major error type identified. Participants who incorrectly answer this item most often identified their answer as 58 degrees instead of 60 degrees. This is a minor "plugging in" error. The item requires one to solve an equation for x. However, the item is looking the measure of an angle, given an expression $x + 2$. The second item, item three, addressed identifying pairs of complementary angles. In this item, the majority of those who answered this question incorrectly reported angle Q as the complementary angle to the given angle X. Angle Q is actually vertical to angle X. The third item, item four, addressed determining angle relationships when given two parallel lines cut by a transversal. A majority of the participants who answered this item incorrectly described the relationship between the two angles as equaling 180 degrees. This is partially correct, but the researcher was looking for the specific term, supplementary. It was noted that a few of the responses to this item reported a complementary relationship instead of a supplementary relationship.

Next, the open-ended parts, content understanding and pedagogical understanding (parts 2 and 3), of the online algebra assessment of preservice middle grades teachers were explored using constant comparative analysis. For consistency of data interpretation, all items were scored on an ordinal scale from zero to six according to a holistic rubric specific to each part (see Appendix G). Although data were not strictly interval in nature, they approximate an interval scale. Therefore, statistics requiring interval scaling, such as mean and standard deviation, can be reasonably interpreted. Descriptive statistics including mean, median, mode, standard deviation, and quartiles for each item on the algebra assessment are reported in Table 16. Frequencies for each rubric score on explanation (part 2) and understanding (part 3) were computed for each item and are reported in Table 17.

Table 16
Descriptive Statistics of Item Responses (Parts 2 and 3) on Geometry

	Mean	Median	Mode	Standard Deviation	Percentiles		
					25	50	75
Item 1—Explanation	3.93	4	6	2.07	2	4	6
Item 1—Understanding	3.20	3	3	1.79	2	3	5
Item 2—Explanation	3.61	4	3	1.88	2	4	5
Item 2—Understanding	3.00	3	3	1.72	2	3	4
Item 3—Explanation	3.04	3	0	2.22	0	3	5
Item 3—Understanding	2.55	3	0	1.94	0	3	4
Item 4—Explanation	3.05	3	0	2.22	1	3	5
Item 4—Understanding	2.46	3	0	1.89	0	3	4
Item 5—Explanation	2.64	3	0	2.20	0	3	5
Item 5—Understanding	2.38	3	0	2.09	0	3	4
Item 6—Explanation	2.71	3	3	2.05	1	3	4
Item 6—Understanding	2.46	3	3	1.82	0	3	4
Item 7—Explanation	3.34	3	3	2.00	2	3	5
Item 7—Understanding	2.50	3	0	1.79	1	3	4

In terms of explanations of solutions (part 2) to items in the geometry assessment, a majority of the students scored three according to the rubric for this item. A score of three on the explanation of the solution generally meant the student just explained their exact algorithmic or mental mathematic procedure with no additional explanations given for the given item. The procedures were mathematically correct and their solution was correct. Common examples of a score of three on the geometry explanation part include:

- In order to find angle STU, you must do 180 – 2x

- I combined both A and B expressions and set them equal to 180. I solved for the missing variable, plugged that back into the expression for angle A and got the answer.
- Vertical angles are basically opposite angles. So since 2 is opposite of 4, they are vertical and since 1 is opposite of 3, they are also vertical.
- Congruent angles mean they are the same degree measure. So since t and s are vertical angles, they are congruent and the same goes for r and u.

Overall, the explanations of the solutions for the geometry assessment revealed incorrect mathematical vocabulary. When dealing with parallel lines and a transversal such as with items four and seven, a majority of the students mixed up terminology for corresponding angles, and alternate interior and exterior angles. In addition it was noted in item three that several students thought complementary and congruency were analogous terms:

- By looking at the picture you can see that m<R=90 by opposite angles of a line. You can see that m<P=m<S by opposite exterior angles. So then we can see that Q is equal to X. (Item 3—the angles are actually supplementary and the item asked for complementary, not congruency)
- I just looked for the angle that was directly opposite of the angle X (Item 3)
- Corresponding angles are angle 1 and angle 2 (Item 4—the angles are actually supplementary)
- Opposite exterior angles are congruent. (Item 4)

Vertical angle terminology, especially in item five, revealed that students often associated vertical angles with longitudinal directions instead of intersecting lines and their subsequent

Table 17

Frequencies for Explanation and Understanding (Parts 2 and 3) of Items in Geometry

	Score						
	0	1	2	3	4	5	6
	Reported as: Frequency/Percent (n = 56)						
Item 1—Content Understanding	7/12.5%		7/12.5%	9/16.1%	6/10.1%	7/12.5%	20/35.7
Item 1—Pedagogical Understanding	9/16.1%	2/3.6%	3/5.4%	15/26.8%	12/21.4%	12/21.4%	3/5.4%
Item 2— Content Understanding	5/8.9%	3/5.4%	7/12.5%	12/21.4%	8/14.3%	9/16.1%	12/21.4%
Item 2— Pedagogical Understanding	9/16.1%	1/1.8%	8/14.3%	14/25.0%	14/25.0%	7/12.5%	3/5.4%
Item 3— Content Understanding	15/26.8%	1/1.8%	6/10.7%	7/12.5%	7/12.5%	12/21.4%	8/14.3%
Item 3— Pedagogical Understanding	16/28.6%	3/5.4%	5/8.9%	9/16.1%	13/23.2%	9/16.1%	1/1.8%
Item 4— Content Understanding	13/23.2%	2/3.6%	7/12.5%	12/21.4%	3/5.4%	7/12.5%	12/21.4%
Item 4— Pedagogical Understanding	16/28.6%	1/1.8%	10/17.9%	9/16.1%	12/21.4%	6/10.7%	2/3.6%
Item 5— Content Understanding	18/32.1%	3/5.4%	3/5.4%	11/19.6%	4/7.1%	12/21.4%	5/8.9%
Item 5— Pedagogical Understanding	22/39.3%		3/5.4%	9/16.1%	12/21.4%	8/14.3%	2/3.6%
Item 6— Content Understanding	13/23.2%	5/8.9%	5/8.9%	15/26.8%	6/10.7%	4/7.1%	8/14.3%
Item 6— Pedagogical Understanding	15/26.8%	3/5.4%	4/7.1%	18/32.1%	9/16.1%	5/8.9%	2/3.6%
Item 7— Content Understanding	8/14.3%	2/3.6%	8/14.3%	13/23.2%	7/12.5%	6/10.7%	12/21.4%
Item 7— Pedagogical Understanding	14/25.0%	2/3.6%	9/16.1%	13/23.2%	10/17.9%	7/12.5%	1/1.8%

angle relationships. Other common terms used instead of vertical angles were opposite and diagonal:

- a is diagonal from 51, so I assumed it was also 52, and then subtracted from 180 to get 129 (Item 7)
- Vertical means up and down. Horizontal means side-to-side, so the angles that are facing up and down are 2, 4. (Item 5)
- Vertical angles would be the ones that are going up and down…like north and south (Item 5)

- I first knew that angles that were diagonal from each other are equal and I also used that when two angles are next to each other and create a line they equal 180, since the lines are parallel and have a transversal they will all have one of the two measurements just depending on where they are in the diagram. (Item 7)
- They are opposite angles of each other so they are equal which implies they are congruent. (Item 6)

The last interesting and noteworthy trend across the explanation of solutions in the geometry assessment was the use of the term linear pairs often instead supplementary:

- The given angle and angle a are vertical angles and vertical angles are congruent. The given angle and angle d are a linear pair and linear pairs are supplementary
- Angle 1 makes a linear pair with, call it angle x, and angle x and angle 2 are corresponding angles. Since they are corresponding angles, angle x and angle 2 are congruent.
- Angle 1 and angle 2 make a linear pair.
- Angle A and Angle B are a linear pair which means the sum of the two angles = 180 degrees. We set up the equation and solve for x. We then know what x equals and can figure out the exact measurement of angle A.

For the pedagogical understanding of geometry, many of the same themes and trends found in the explanations (content understanding) emerged again. If the student did not receive a correct answer on the original content problem or received a low score on the explanation of their solution, they were likely to not receive a high score on the understanding part. A majority of the participants scored a three or a four on the pedagogical understanding part of the geometry assessment. A score of three generally indicated that the student could successfully explain their own procedure to a person who did not understand. A score of four indicated the student could go just beyond the explanation of their own procedure, many times adding terms and definitions associated with the problem or providing a basic definition of why they did what they did. Examples of a score of three include:

- First we would review the meaning of supplementary angles. Then I would work out the problem vertically step by step, writing next to each step what I was doing if necessary.
- If they knew what vertical angles were then I would remind them that vertical angles are congruent and therefore, S&T and R&U are congruent because they are vertical angles.

Examples of a score of four include:

- I would teach supplementary angles, then parallel lines and corresponding angles. I would show the relationship between all these and then explain like that to the solution described above.
- Complementary means two angles sums are equal to 90 degrees. Since 90 degrees is a right angle, and we know that there is an angle that is complementary to X, we need to find the angle that completes the right angle with X. P is the only choice. R is wrong because it is supplementary to X.

Although directional association with vertical angles was prevalent in the explanation of solutions, there was only one directional association in the understanding part of the problem:

- Show a map or some other object with north and south orientation and then demonstrate how the picture is similar.

Of the students who mentioned directional associations in the explanation (content understanding) part, a majority just mentioned describing the definition of vertical angles to someone who did not understand. There was no additional explanation given as to what this definition might be.

There was again an evident misconception that congruency and complementary are analogous terms. All of the following understanding explanations refer to item three, identifying the complementary angle to *X*:

- Find the angles of each with numbers and show that they are equal
- I could demonstrate this very problem to my class and help them understand why the two angles are equal.
- I would again show the basic way degrees on a line works and how the angle opposite diagonally from the given angle is the same.

Across all pedagogical understanding parts of the items on the geometry assessment there were several instances of exploration activities. Most of these activities involved providing actual measurements and protractors to students so they could measure the angles and then come up with their own idea of the relationships between angles (i.e., supplementary, complementary, corresponding, etc.). In addition a majority of the responses contained some kind of definition or discussion of giving and/or explaining definitions necessary for students to understand the problems. Common examples include:

- Have the students explore angles and what it means to be supplementary. Have them measure the angles formed on both sides of an intersecting line. This should result in an understanding that no matter what the measure of the angle (x) is on one side of a intersecting line, the measure of the angle on the other side of the line is 180-x. Now have the students use this knowledge and armed with the definition of supplementary to tackle this problem. If the two angles are supplementary, what does this mean? (Their measure adds up to 180) How would you write this using the above angles? m<PQR + m<STU = 180. If the measure of <PQR is 2x, what is the measure of <STU? 2x + <STU = 180 which means 180 - 2x = <STU

- **This example contains another example of complementary = congruency** Have the students review the concepts of supplementary and corresponding angles. After the students have physically explored angles, then measured pictorial representations, have the students review the relationship between the angles formed by two intersecting lines. Have the students list the relationships between <a, <d, <b, and angle having measure of 51. What angles are complementary? <a and <d; <a and <b; <d and angle with measure 51; <b and angle with measure of 51. What does it mean for an angle to be complementary? m<a + m<d = 180; m<d + 51 = 180, etc. With this knowledge, what is the measure of angles <a, <b, <d? m<d = m<b = 180-51 = 129; m<a = 51 (either because they form vertical angles with the first or by being supplementary with <b and <d). Now that we know what the measure of these angles are, how are these angles related to the angles formed by the intersection of the same line with a parallel line? (forms angles with same measure) Which angle corresponds to <a? <z What are the relationships between angles <z, <y, <w, and <x? <z is supplementary with <y and <w; With this knowledge, what is the measure of <z, <y, <w, and <x? m<z + m<y = 180; m<z +

m<w = 180; m<z = m<a m<z = 51; 51 + m<y = 180; m<y = 180-51; m<y = 129; m<y = m<w = 129; m<z = m<x = 51

- After students have investigated real world examples of angles, then pictorial representations of angles have students measure a lot of angles and come up with the relationships between them. They should come to the realization that the measure of the angle on one side of an intersecting line is the same as that of the opposite angle (the vertical angle). Define the term "congruent angles". Stress that congruence "- angles having equal measure" relates the measurement of the angles, not the size of the representation. Ask the students which angles have the same measure. What does having the same measure mean? So...which angles are congruent?

All three parts (content knowledge, content understanding, and pedagogical understanding) mentioned in the above analysis together make up the component called mathematics knowledge for teaching geometry in the middle grades. Total scores for this assessment ranged from 0 to 81, a range of 80. The total possible points for the geometry assessment was 91 points. Descriptive statistics for the total of each part (content, explanation, and understanding) and the total mathematics knowledge for teaching geometry can be found in Table 18.

Table 18
Descriptive Statistics for Mathematics Knowledge for Teaching Geometry

	Mean	Median	Mode	Standard Deviation	Percentiles 25	50	75
Total Content Knowledge (7 pts possible)	5.34	6	6	1.77	5	5	7
Total Content Understanding (42 pts possible)	22.32	25	6	11.33	15	25	32
Total Pedagogical Understanding (42 pts possible)	18.55	20.50	0	10.66	9	21	27
Mathematics Knowledge for Teaching Geometry (91 pts possible)	46.21	50.50	59	22.29	31	51	65

Research Question 4

What is preservice middle grades teachers' mathematics knowledge for teaching probability and statistics?

The analysis of this question was again broken into four parts: analysis of the content knowledge, analysis of the content understanding, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching probability and statistics. The three parts— content knowledge, content understanding, and pedagogical understanding—together make up mathematics knowledge for teaching probability and statistics in the middle grades. Beginning with the analysis of the content knowledge part of the probability and statistics assessment, the Spearman-Brown prediction formula was used to calculate the reliability of the assessment. The coefficient alpha for the probability and statistics assessment was .907.

The next step was to conduct an item analysis for the content knowledge part of the probability and statistics assessment. An item analysis evaluates each test item to determine its discrimination and difficulty level. Item discrimination refers to the ability of an item to differentiate among students on the basis of how well they know the material being tested. An item's difficulty index is expressed as the proportion of students who responded correctly to an item to the total number who responded. Distractors are usually identified in the process of item analysis. However, a distractor analysis was not conducted because this assessment did not contain any distractors. The item discrimination for the content question items on the probability and statistics assessment can be found in Table 19. Item difficulty can be found in Table 20. The average item difficulty was 76% ($p = .76$) which is considered moderately difficult on the item difficulty scale.

Table 19
Item Discriminations for Content Question (Part 1) in Probability and Statistics

Item	% Item Discrimination	Level*
1—Calculating the range for a given set of data	23%	Usually Unacceptable
2—Predicting the outcome of an experiment	19%	Usually Unacceptable
3—Reading and interpreting data represented graphically through a pictograph	0%	Usually Unacceptable
4—Determining the number of possible outcomes for a compound event by using the fundamental counting principle and use this to determine the probabilities of events when the outcomes have equal probabilities	31%	Good
5—Determining the probability of dependent events	38%	Good
6— Predicting the outcome of an experiment	35%	Good
7—Interpreting data to provide the basis for predictions and to establish experimental probabilities	15%	Usually Unacceptable

*Negative = unacceptable; 0% - 24% = Usually unacceptable, might be approved; 25% - 39% = Good Item; 40% - 100% = Excellent Item

Table 20
Item Difficulty for Content Question (Part 1) in Probability and Statistics

Item	% Item Difficulty	Level*
1—Calculating the range for a given set of data	65%	Moderately Difficult
2—Predicting the outcome of an experiment	90%	Easy
3—Reading and interpreting data represented graphically through a pictograph	100%	Easy
4—Determining the number of possible outcomes for a compound event by using the fundamental counting principle and use this to determine the probabilities of events when the outcomes have equal probabilities	81%	Easy
5—Determining the probability of dependent events	46%	Moderately Difficult
6— Predicting the outcome of an experiment	63%	Moderately Difficult
7—Interpreting data to provide the basis for predictions and to establish experimental probabilities	88%	Easy
Average Item Difficulty for Probability and Statistics	**76%**	**Moderately Difficult**

*<20% = difficult; 20% - 80% = moderately difficult; >80% = easy

One item, item five, had less than half of the participants answer it correctly. This item was investigated for commonalities in the errors of the answers given. Item five addressed determining the probability of dependent events. There were three common errors. The majority of the participants who did not answer this question correctly simply forgot to reduce the fraction for their final answer. There were two types of answers to be reduced. When the two original probabilities were multiplied together, the answer needed to be reduced. The other type was a result from multiplying the first probability, which was initially reduced, by the second probability, which could not be initially reduced. The result was another probability which needed to be reduced, and a majority of the time was not. The other two types of errors, minimal but still evident in student answers, were the adding of the two probabilities and the treatment of the item as “with replacement” even though the problem directly states “without replacing the first cookie.”

Next, the open-ended parts, content understanding and pedagogical understanding (parts 2 and 3), of the online probability and statistics assessment of preservice middle grades teachers were explored using constant comparative analysis. For consistency of data interpretation, all items were scored on an ordinal scale from zero to six according to the holistic rubric for each part (see Appendix G). Although the data were not strictly interval in nature, they approximate an interval scale. Therefore, statistics requiring interval scaling, such as mean and standard deviation, can be reasonably interpreted. Descriptive statistics including mean, median, mode, standard deviation, and quartiles are reported for each item on the probability and statistics assessment and are reported in Table 21. Frequencies for each rubric score on explanation (part 2) and understanding (part 3) were computed for each item and are reported in Table 22.

Table 21
Descriptive Statistics of Item Responses (Parts 2 and 3) on Probability and Statistics

	Mean	Median	Mode	Standard Deviation	Percentiles 25	50	75
Item 1—Content Understanding	2.71	3	3	1.18	2	3	3
Item 1—Pedagogical Understanding	3.21	3	3	1.53	2	3	5
Item 2— Content Understanding	3.87	4	3	1.21	3	4	5
Item 2— Pedagogical Understanding	3.33	3	3	1.42	3	3	4
Item 3— Content Understanding	3.46	3	3	1.35	3	3	4
Item 3— Pedagogical Understanding	3.65	4	4	1.30	3	4	5
Item 4— Content Understanding	2.79	3	3	1.26	3	3	3
Item 4— Pedagogical Understanding	3.12	3.50	4	1.75	2	4	4
Item 5— Content Understanding	2.88	3	2	1.65	2	3	4
Item 5— Pedagogical Understanding	2.87	3	4	1.63	2	3	4
Item 6— Content Understanding	3.33	3	3	1.45	2	3	4
Item 6— Pedagogical Understanding	3.21	3	3	1.39	2	3	4
Item 7— Content Understanding	3.38	3	3	1.44	3	3	4
Item 7— Pedagogical Understanding	3.10	3	3	1.55	2	3	4

Table 22

Frequencies for Explanation and Understanding (Parts 2 and 3) of Items in Probability and Statistics

	Score						
	0	1	2	3	4	5	6
	Reported as: Frequency/Percent (n = 52)						
Item 1— Content Understanding	3/5.8%	3/5.8%	12/23.1%	27/51.9%	3/5.8%	3/5.8%	1/1.9%
Item 1— Pedagogical Understanding	3/5.8%	4/7.7%	9/17.3%	14/26.9%	8/15.4%	13/25.0%	1/1.9%
Item 2— Content Understanding	--	--	5/9.6%	18/34.6%	16/30.8%	5/9.6%	8/15.4%
Item 2— Pedagogical Understanding	4/7.7%	--	6/11.5%	19/36.5%	15/28.8%	4/7.7%	4/7.7%
Item 3— Content Understanding	1/1.9%	--	7/13.5%	29/55.5%	4/7.7%	3/5.8%	8/15.4%
Item 3— Pedagogical Understanding	1/1.9%	2/3.8%	6/11.5%	12/23.1%	18/34.6%	10/19.2%	3/5.8%
Item 4— Content Understanding	6/11.5%	2/3.8%	--	37/71.2%	4/7.7%	2/3.8%	1/1.9%
Item 4— Pedagogical Understanding	9/17.3%	1/1.9%	4/7.7%	12/23.1%	14/26.9%	11/21.2%	1/1.9%
Item 5— Content Understanding	5/9.6%	2/3.8%	16/30.8%	15/28.8%	5/9.6%	3/5.8%	6/11.5%
Item 5— Pedagogical Understanding	6/11.5%	4/7.7%	12/23.1%	9/17.3%	13/25.0%	6/11.5%	2/3.8%
Item 6— Content Understanding	1/1.9%	2/3.8%	13/25.0%	15/28.8%	12/23.1%	2/3.8%	7/13.5%
Item 6— Pedagogical Understanding	3/5.8%	1/1.9%	10/19.2%	17/32.7%	13/25.0%	5/9.6%	3/5.8%
Item 7—Content Understanding	3/5.8%	1/1.9%	4/7.7%	25/48.1%	9/17.3%	4/7.7%	6/11.5%
Item 7— Pedagogical Understanding	5/9.6%	3/5.8%	5/9.6%	20/38.5%	10/19.2%	6/11.5%	3/5.8%

In terms of explanations of solutions (content understanding—part 2) to items on the probability and statistics assessment, a majority of the participants scored a three according to the rubric for this part of the assessment. A score of three on the explanation of the solution generally meant the student just explained their exact algorithmic or mental mathematic procedure for the given item. The procedures were mathematically correct and their solution was correct, however, there was no further explanation provided other than their algorithmic procedure. Common examples of a score of three include:

- Since there are 21 possible options for the first letter, 5 for the second, and 21 for the third, you just do 21*5*21 and that gives you 2,205 possible tag codes.
- Just looking at the pictograph I could see that the yellow lens had sold three times the amount of the brown lens.
- I subtracted 73 from 97.

There are a few items with minor errors in explanations of noteworthiness. The first is item one which addressed calculating the range for a given set of data. Of the students who incorrectly answered this problem, a majority of them indicated the range of data as an actual range (i.e., 73 – 97) instead of calculating the range by subtracting the highest and lowest scores. Another common error revealed in the explanation was the ordering of the numbers. The numbers are presented in non-numerical order and instead of reordering them, the student took the last score minus the first score.

Item three addressed reading and interpreting data represented graphically through a pictograph. This item is the only item received 100% correct responses out of all four assessments. However, in the comparison of the explanations, it was revealed that although the participants correctly chose yellow, their explanations were not correct. The pictograph displayed three times greater sales for yellow than brown. However, participants misinterpreted three times as three more. Green has three less or two times fewer sales than yellow. Common examples of this misinterpretation include:

- Yellow has three more sunglasses in the pictograph than green, so that is how I got my answer.
- You would look at the graph and start with the smallest number of sunglasses, which is brown. The pictography has only two sunglasses (20,000) for that. The next

one up, green, has three sunglasses (30,000). You can see that that is not the answer, because you are looking for something that is three times greater, which means it needs to have three more sunglasses in the pictograph. If you compare all of them, you find out that yellow has three more (30,000) than green. So, yellow is the answer.

- Yellow had sales three times greater than green because there are three more sunglasses pictures for the yellow than there are for the green.

Item six addressed predicting the outcome of an experiment. Of the students who incorrectly answered this problem, a misinterpretation of the graphic representation given was the most common. The representation contained ten cards numbered and with the color written on them. The black cards were darkened a deep gray while there was little room for distinction between the white and gray cards. Therefore participants reported probabilities of 7/10 instead of 4/10 in order to predict the outcome of the experiment:

- He has a 7 out of 10 chance to pick a white card every time he draws a card and since he replaces the card each time he always has the same probability of picking a white card. If he does this 100 times then you would expect him to pick a white card about 70 out of the 100 times.
- Because every single time he has 10 cards to pick from and every single time there is a 70 chance (7 out of 10) that he will pick a white card. Conducted 100 times, the theoretical number of times that he should pick a white card is 70. That said, it will probably not be the actual number.

- There is a probability that Derek will pick up a white card 7 out of 10 times. You multiply this 100 times. You don't have to change the number because he replaces the card each time.

For the pedagogical understanding part of probability and statistics, a majority of the participants scored either a three or a four. A score of three generally indicated the student could successfully explain their own procedure to a person who did not understand. These explanations tended to be very algorithmic in nature and the general method of instruction could be assumed to be direct since there was no mention of any other method. Common examples of a score of three on various items within the probability and statistics assessment include:

- I divided 240 by 3 since 1 through 5 contains 3 odd numbers.
- Show that if there is no tampering with the odds of certain cards, each card has an equal chance of getting picked. So if there are 100 tries than they need to divide the total number of tries by the number of cards. Then multiply the number of certain cards times that answer they got.
- See if they could find the probability of grabbing a red in one try. They have that same probability every time they grab from the bag, so they just need to multiply the probability times the total number of tries.

A score of four on the pedagogical understanding part indicated that the student could do more than just explain their own procedure. They could relate their procedure back to different parts of the problem and could often explain why they got a certain number or conducted a certain procedure. Responses at this level tended to be more culturally responsive than a score of three. Common examples of a score of four on various items within the probability and statistics assessment include:

- I honestly don't know any other method to show this other than drawing out a line to show how you read the information given in the problem and how to interpret it in order. I wrote three lines to show three letters on the tag. Each letter had a specific requirement. The first and third had only twenty-one choices each time and the middle had five. I multiply across to find out how many possibilities.
- I would start with the word problem and ask them what they are looking for and to write the word problem out in symbols. So they are looking for X glasses that are 3 times greater than Y, so Y = 3X and then I would have them figure out the number of sunglasses and find which ones fit the equation.
- Show them on a spinner how the spinner is 3/5ths (60%) odd and thus there is a 60% chance each time that the spinner will land on an odd number, so 60% times 240 is 144.

Analysis across pedagogical understanding of all items on the probability and statistics assessment revealed several trends of interest. The first had to do with demonstrating an item on a smaller scale first before doing the actual problem. Common responses provided include:

- I would demonstrate the problem with a smaller number and teach the process of the problem and then let them do it on their own with the bigger numbers.
- Scale the problem down to smaller numbers. Ex: 3shoes 2 pants 2 shirts: how many combos are there?

Another theme was the use of hands-on material in order to conduct experiments. It was noted that few of these responses addressed theoretical probability versus experimental probability. Common examples include:

- I would use hands on objects such as the miniature blocks that you can also put on the overhead. A lot of teachers use this to teach multiplication. Easily you could show the relationship in smaller number of sales and show what "three times greater" means. By putting each row into block groups you could add three more groups of "sunglass" blocks and see which one = which original groups sales. Then you would know that Yellow=Brown*x*3
- I would actually lay out all of the cookies or something to represent them. I would then have the person choose a cookie and go through the steps in the question. I would then talk to them about how to work the problem.
- I would simply model this by using a deck of cards and discussing black and red cards. What is the probability of drawing a red card? Then you could make it more difficult and really test their understanding by discussing hearts, spades, diamonds, and other things like numbers. This would give the children an understanding of how probability works but also a real life example that they might use one day. Also it really helps them to understand a concept, but have fun with it.

If the pedagogical understanding response did not include the use of hands-on material, it generally contained some sort of another representation of the item being addressed. The majority of representations presented were pictorial. Common examples of the use of pictorial representations on the understanding part of the probability and statistics assessment include:

- I would explain this by drawing out in a picture format the probability of each cookie draw. Since we are only drawing out the cookie jar twice in this situation I would only need to draw to that point. The first draw is whatever the amount of cookies of that type are out of the total number of cookies. I then would draw three more branches and here I would notice that we no longer have 24 cookies; we took one away so now we have 23. Well that changes the oatmeal's probability because we drew an oatmeal. So it would be 7/23 for example. Now the second draw on a chocolate chip doesn't change because we didn't draw a chocolate chip. So the probability for that is 12/23. Now we multiply across because these two probabilities multiplied together form the total probability of those two draws out of the cookie jar.
- I would draw a blank for the three spots on the tag. Then I would say how many possible letters could be the first letter and that would be 21. Then I would say that 5 possible letters could be the second letter and 21 could be the last letter. Then I might begin to make a tree diagram. I would not create the whole tree but I would show them that say the first letter was B then the second letter could be A, E, I , O, or U so for all 21 consonants there would be 5 different combinations of vowels just for that first consonant. So each first consonant is taken 5 times and that gives you 21 times 5 but then there is a third consonant so now those 105 combinations are taken 21 times which gives you 21*5*21 and that equals 2205.

All three of the parts (content knowledge, content understanding, and pedagogical understanding) mentioned above in the analysis make up mathematics knowledge for teaching probability and statistics in the middle grades. Total scores for this assessment ranged from 18 to 73, a range of 55. The probability and statistics assessment had a total of 91 possible points on it. Descriptive statistics for the total of each part (content, explanation, and understanding) and the total mathematics knowledge for teaching can be found in Table 23.

Table 23
Descriptive Statistics of Mathematics Knowledge for Teaching Probability and Statistics

	Mean	Median	Mode	Standard Deviation	Percentiles 25	50	75
Total Content Knowledge (7 pts possible)	5.34	6	6	1.17	5	6	6
Total Content Understanding (42 pts possible)	22.42	23.50	24	5.41	19	24	25
Total Pedagogical Understanding (42 pts possible)	22.48	22	22	7.69	18	22	27
Mathematics Knowledge for Teaching Probability and Statistics (91 pts possible)	50.25	51.50	44	12.63	43	52	60

Ancillary Questions

The statistical design for the first three ancillary questions used was the multivariate analysis of covariance (MANCOVA) and the univariate analysis of covariance (ANCOVA). Since the assessment was only administered once, the MANCOVA and the ANCOVA could be used to adjust for preexisting differences among the individuals. The MANCOVA and ANCOVA, which combine regression analysis and analysis of variance, control for the effect of extraneous variables. The covariate for the study was an overall grade point average (GPA) derived from self-reported grades on the following courses or their equivalents: MATH 142 (Business Mathematics II), MATH 131 (Mathematical Concepts—Calculus), MATH 166 (Topics in Contemporary Mathematics II), and STAT 303 (Statistical Methods). Thirteen participants did not self-report grades. NORM software (Schafer, 1999) was used to conduct multiple imputations, a simulation-based approach to the statistical analysis of incomplete data, in order to compute the missing data (Schafer, 1997).

Each of the following questions are comprised of four parts: mathematics knowledge for teaching number and operations, mathematics knowledge for teaching algebra, mathematics knowledge for teaching geometry, and mathematics knowledge for teaching probability and statistics.

Ancillary Question 1

What is the effect of various sequencing of mathematics courses on middle grades mathematics teachers?

The middle grades mathematics/science degree program has a specific degree plan in which students are required to take courses in certain semesters with little room and tolerance for variance from this plan. In order to closely account for a proper sequence of courses, enrollment classification (freshman, sophomore, 1st semester junior, 2nd semester junior, 1st semester senior, 2nd semester and beyond senior, other) was collected. Initially the data were analyzed using a MANCOVA with grade point average as the covariate.

However, numerous parts rejected the null hypothesis that the error variance of the dependent variables are equal across groups (Levene's Test of Equality of Variances, $p <$ .05). Therefore, enrollment classification data were recoded. Juniors were grouped together and seniors were grouped together. Descriptive statistics for enrollment characteristics can be found in Table 24.

Mathematics Knowledge for Teaching Number and Operations

Using a multivariate regression with the variable adjusted enrollment classification in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching number and operation as well as the composite variable of mathematics knowledge for teaching number and operation, the overall model yielded $R^2=0.274$ for content knowledge, $R^2=.267$ for explanation of solution, $R^2=.240$ for pedagogical understanding, and $R^2=.294$ for mathematics knowledge for teaching number and operations for summary data (see Table 25). Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Enrollment classification (adjusted) had a statistically significant ($p < .05$) contribution to content

Table 24

Descriptive Statistics of Adjusted Enrollment Characteristics for Mathematics Knowledge for Teaching

	Enrollment Classification	Mean	Standard Deviation	N
Number and Operation Content Knowledge	Freshman	--	--	--
	Sophomore	4.86	1.21	7
	Junior	3.42	1.33	26
	Senior	4.22	1.34	27
	Total	3.95	1.40	60
Number and Operation Explanation of Solution	Freshman	--	--	--
	Sophomore	19.14	9.26	7
	Junior	14.73	6.72	26
	Senior	19.33	5.03	27
	Total	17.32	6.65	60
Number and Operation Pedagogical Understanding	Freshman	--	--	--
	Sophomore	18.57	5.22	7
	Junior	11.62	7.63	26
	Senior	18.59	7.68	27
	Total	15.57	8.10	60
Mathematics Knowledge for Teaching Number and Operation	Freshman	--	--	--
	Sophomore	42.57	14.93	7
	Junior	29.77	14.21	26
	Senior	42.15	12.32	27
	Total	36.83	14.63	60
Algebra Content Knowledge	Freshman	5.50	.71	2
	Sophomore	5.13	.64	8
	Junior	5.17	1.40	24
	Senior	5.57	1.19	42
	Total	5.50	1.23	76
Algebra Explanation of Solution	Freshman	18.50	4.95	2
	Sophomore	18.50	9.46	8
	Junior	20.25	8.17	24
	Senior	25.02	6.28	42
	Total	22.66	7.61	76
Algebra Pedagogical Understanding	Freshman	15.50	4.95	2
	Sophomore	21.13	7.92	8
	Junior	17.13	6.71	24
	Senior	22.55	8.13	42
	Total	20.50	7.93	76
Mathematics Knowledge for Teaching Algebra	Freshman	39.50	9.19	2
	Sophomore	45.75	15.93	8
	Junior	42.54	14.72	24
	Senior	53.14	13.97	42
	Total	48.66	14.98	76

Table 24
Continued

	Enrollment Classification	Mean	Standard Deviation	N
Geometry Content Knowledge **Removed from Analysis**	Freshman	4.50	3.54	2
	Sophomore	5.55	1.57	11
	Junior	5.75	1.16	20
	Senior	4.96	2.14	23
	Total	5.34	1.77	56
Geometry Explanation of Solution	Freshman	11.00	12.73	2
	Sophomore	37.82	9.99	11
	Junior	20.10	11.77	20
	Senior	22.61	10.82	23
	Total	22.32	11.33	56
Geometry Pedagogical Understanding	Freshman	11.50	16.26	2
	Sophomore	20.00	11.21	11
	Junior	17.10	10.42	20
	Senior	19.74	10.60	23
	Total	18.55	10.66	56
Mathematics Knowledge for Teaching Geometry	Freshman	27.00	32.53	2
	Sophomore	53.63	20.93	11
	Junior	42.95	21.77	20
	Senior	47.30	22.71	23
	Total	46.21	22.29	56
Probability and Statistics Content Knowledge	Freshman	--	--	--
	Sophomore	5.25	1.39	8
	Junior	5.25	1.21	20
	Senior	5.46	1.10	24
	Total	5.35	1.17	52
Probability and Statistics Explanation of Solution	Freshman	--	--	--
	Sophomore	22.00	5.13	8
	Junior	22.00	6.08	20
	Senior	22.92	5.09	24
	Total	22.42	5.41	52
Probability and Statistics Pedagogical Understanding	Freshman	--	--	--
	Sophomore	20.50	8.90	8
	Junior	19.95	8.10	20
	Senior	25.25	6.15	24
	Total	22.48	7.69	52
Mathematics Knowledge for Teaching Probability and Statistics	Freshman	--	--	--
	Sophomore	47.75	14.36	8
	Junior	47.20	13.96	20
	Senior	53.63	10.38	24
	Total	50.25	12.63	52

Table 25

Multivariate Regression on Adjusted Enrollment Classification and GPA (Covariate) for Number and Operations

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	31[a]	5	6	4.07	.003	.274
	Content Understanding	695[b]	5	139	3.93	.004	.267
	Pedagogical Understanding	928[c]	5	86	3.41	.010	.240
	Mathematics Knowledge for Teaching N&O	3710[d]	5	742	4.49	.002	.294
Intercept	Content Knowledge	936	1	936	606.02	p < .001	.918
	Content Understanding	17992	1	17992	508.16	p < .001	.904
	Pedagogical Understanding	14539	1	14539	266.95	p < .001	.832
	Mathematics Knowledge for Teaching N&O	81402	1	81402	492.78	p < .001	.901
GPA	Content Knowledge	16	1	16	10.44	.002**	.162
	Content Understanding	271	1	271	7.65	.008*	.124
	Pedagogical Understanding	238	1	238	4.38	.041*	.075
	Mathematics Knowledge for Teaching N&O	1290	1	1290	7.81	.007**	.126
Enrollment Classification	Content Knowledge	12	2	6	4.09	.022*	.132
	Content Understanding	242	2	121	3.41	.040*	.112
	Pedagogical Understanding	625	2	313	5.74	.005**	.175
	Mathematics Knowledge for Teaching N&O	1911	2	956	5.79	.005*	.176
Enrollment Classification * GPA	Content Knowledge	23	2	1	.86	.429	.031
	Content Understanding	183	2	91	2.58	.085	.087
	Pedagogical Understanding	64	2	32	59	.559	.021
	Mathematics Knowledge for Teaching N&O	508	2	254	1.539	.224	.054
Error	Content Knowledge	83	54	2			
	Content Understanding	1911	54	35			
	Pedagogical Understanding	2941	54	54			
	Mathematics Knowledge for Teaching N&O	8920	54	165			
Total	Content Knowledge	1051	60				
	Content Understanding	20599	60				
	Pedagogical Understanding	18408	60				
	Mathematics Knowledge for Teaching N&O	94032	60				
Corrected Total	Content Knowledge	115	59				
	Content Understanding	2607	59				
	Pedagogical Understanding	3869	59				
	Mathematics Knowledge for Teaching N&O	12630	59				

* *p < .05. **p < .01.*

[a]R Squared = .274 (Adjusted R Squared = .206)

[b]R Squared = .267 (Adjusted R Squared = .199)

[c]R Squared = .240 (Adjusted R Squared = .169)

[d]R Squared = .294 (Adjusted R Squared = .228)

knowledge, explanation of solution (content understanding), pedagogical understanding, and mathematics knowledge for teaching number and operations. The covariate, GPA, had a statistically significant contribution on number and operations content knowledge, content understanding, pedagogical understanding and mathematics knowledge for teaching number and operations ($p < .05$, $p < .05$, $p < .01$, and $p < .01$, respectively). Enrollment classification and GPA together did not contribute significantly to any of the dependent variables. Effect sizes were small to medium (Huck, 2004). Enrollment classification had a larger impact on pedagogical understanding and mathematics knowledge for teaching number and operations than on content knowledge and explanation of the solutions. GPA had a larger effect on content knowledge than on explanation of solution and mathematics knowledge for teaching number and operations. The largest effect was enrollment classification on mathematics knowledge for teaching number and operations.

Mathematics Knowledge for Teaching Algebra

Using a multivariate regression with the variable adjusted enrollment classification in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching algebra as well as the composite variable of mathematics knowledge for teaching algebra, the overall model yielded $R^2=0.162$ for content knowledge, $R^2=.229$ for explanation of solution, $R^2=.144$ for pedagogical understanding, and R^2=*.202* for mathematics knowledge for teaching algebra for summary data (see Table 26). Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Enrollment classification (adjusted) did not contribute significantly to content knowledge, but did have a statistically significant ($p < .05$) contribution to explanation of solution, pedagogical understanding, and mathematics knowledge for teaching algebra. The covariate, GPA, did not contribute significantly to any of the dependent variables. Enrollment classification and GPA together did not contribute significantly to pedagogical understanding or mathematics knowledge for teaching algebra, but did have a statistically significant ($p < .05$) contribution to content knowledge and to explanation of solutions. Effect sizes were small. Enrollment classification had approximately equal effects on explanation of solution, pedagogical understanding, and mathematics knowledge for teaching algebra. The interaction of enrollment classification and GPA had approximately equal effects on content knowledge and explanation of solutions.

Table 26

Multivariate Regression on Adjusted Enrollment Classification and GPA (Covariate) for Algebra

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	18[a]	6	3	2.22	.051	.162
	Content Understanding	998[b]	6	166	3.42	.005	.229
	Pedagogical Understanding	678[c]	6	113	1.93	.088	.144
	Mathematics Knowledge for Teaching Algebra	3400[d]	6	567	2.91	.014	.202
Intercept	Content Knowledge	2299	1	2299	1675.14	p < .001	.960
	Content Understanding	39017	1	39016	803.30	p < .001	.921
	Pedagogical Understanding	31939	1	31939	545.95	p < .001	.888
	Mathematics Knowledge for Teaching Algebra	179937	1	179937	925.26	p < .001	.931
GPA	Content Knowledge	4	1	4	2.75	.102	.038
	Content Understanding	106	1	106	2.18	.144	.031
	Pedagogical Understanding	94	1	94	1.60	.210	.023
	Mathematics Knowledge for Teaching Algebra	480	1	480	2.47	.121	.035
Enrollment Classification	Content Knowledge	5	3	2	1.24	.39	.051
	Content Understanding	563	3	188	3.87	.013*	.144
	Pedagogical Understanding	496	3	165	2.83	.045*	.109
	Mathematics Knowledge for Teaching Algebra	1985	3	662	3.40	.022*	.129
Enrollment Classification * GPA	Content Knowledge	9	2	5	3.43	.038*	.090
	Content Understanding	328	2	164	3.38	.040*	.089
	Pedagogical Understanding	89	2	44	.76	.471	.022
	Mathematics Knowledge for Teaching Algebra	935	2	468	2.40	.098	.065
Error	Content Knowledge	95	69	1			
	Content Understanding	3351	69	49			
	Pedagogical Understanding	4037	69	59			
	Mathematics Knowledge for Teaching Algebra	13419	69	194			
Total	Content Knowledge	2412	76				
	Content Understanding	43366	76				
	Pedagogical Understanding	36654	76				
	Mathematics Knowledge for Teaching Algebra	196756	76				
Corrected Total	Content Knowledge	113	75				
	Content Understanding	4349	75				
	Pedagogical Understanding	4715	75				
	Mathematics Knowledge for Teaching Algebra	16819	75				

* $p < .05$. **$p < .01$.

[a]R Squared = .162 (Adjusted R Squared = .089)

[b]R Squared = .229 (Adjusted R Squared = .162)

[c]R Squared = .144 (Adjusted R Squared = .069)

[d]R Squared = .202 (Adjusted R Squared = .133)

Mathematics Knowledge for Teaching Geometry

Using a multivariate regression with the variable adjusted enrollment classification in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching geometry as well as the composite variable of mathematics knowledge for teaching geometry, the overall model yielded R^2=*.168* for explanation of solution, R^2=*.088* for pedagogical understanding, and R^2=*.117* for mathematics knowledge for teaching geometry for summary data (see Table 27). The dependent variable content knowledge for geometry had to be removed from the analysis because it rejected the null hypothesis that the error variance of the dependent variable is equal across groups. Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Enrollment classification, GPA, and its interaction did not contribute significantly to any of the dependent variables. Effect sizes were small.

Table 27

Multivariate Regression on Adjusted Enrollment Classification and GPA (Covariate) for Geometry

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Understanding	1182[a]	6	197	1.64	.155	.168
	Pedagogical Understanding	552[b]	6	92	.79	.581	.088
	Mathematics Knowledge for Teaching Geometry	3208[c]	6	535	1.09	.384	.117
Intercept	Content Understanding	27902	1	27902	232.77	$p < .01$	.826
	Pedagogical Understanding	19277	1	19277	165.72	$p < .01$	.772
	Mathematics Knowledge for Teaching Geometry	119603	1	119603	242.88	$p < .01$	.832
GPA	Content Understanding	216	1	216	1.80	.186	.035
	Pedagogical Understanding	219	1	219	1.88	.176	.037
	Mathematics Knowledge for Teaching Geometry	884	1	884	1.80	.187	.035
Enrollment Classification	Content Understanding	757	3	252	2.11	.112	.114
	Pedagogical Understanding	259	3	86	.74	.532	.043
	Mathematics Knowledge for Teaching Geometry	1840	3	613	1.25	.304	.071
Enrollment Classification * GPA	Content Understanding	210	2	105	.87	.424	.034
	Pedagogical Understanding	74	2	37	.32	.729	.013
	Mathematics Knowledge for Teaching Geometry	485	2	242	.49	.614	.020
Error	Content Understanding	5874	49	120			
	Pedagogical Understanding	5700	49	116			
	Mathematics Knowledge for Teaching Geometry	24130	49	492			
Total	Content Understanding	34958	56				
	Pedagogical Understanding	25529	56				
	Mathematics Knowledge for Teaching Geometry	146940	56				
Corrected Total	Content Understanding	7056	55				
	Pedagogical Understanding	6252	55				
	Mathematics Knowledge for Teaching Geometry	27337	55				

[a]R Squared = .168 (Adjusted R Squared = .066)
[b]R Squared = .088 (Adjusted R Squared = -.023)
[c]R Squared = .117 (Adjusted R Squared = .009)

Mathematics Knowledge for Teaching Probability and Statistics

Using a multivariate regression with the variable adjusted enrollment classification in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching probability and statistics as well as the composite variable of mathematics knowledge for teaching probability and statistics, the overall model yielded $R^2=0.061$ for content knowledge, $R^2=.033$ for explanation of solution, $R^2=.163$ for pedagogical understanding, and $R^2=.104$ for mathematics knowledge for teaching probability and statistics for summary data (see Table 28). Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Enrollment classification, GPA, and its interaction did not contribute significantly to any of the dependent variables. Effect sizes were small.

Ancillary Question 2

What cohort developmental differences are there among students as they progress through the courses identified in the middle grades mathematics and science program?

As mentioned above, the middle grades mathematics/science degree program has a specific degree plan with highly suggested sequence of courses. Many of the mathematics and mathematics education courses taken for the program occur simultaneously. These courses can be grouped into three major categories: mathematics courses, specialized middle grades mathematics education courses, and methods and student teaching experiences/course work. Please refer to Table 2 for the cohort design and number of individuals. Descriptive statistics for the cohorts for all assessments can be found in Table 29.

Table 28

Multivariate Regression on Adjusted Enrollment Classification and GPA (Covariate) for Probability and Statistics

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	4[a]	5	1	.60	.699	.061
	Content Understanding	49[b]	5	10	.31	.902	.033
	Pedagogical Understanding	492[c]	5	98	1.80	.133	.163
	Mathematics Knowledge for Teaching P&S	845[d]	5	169	1.07	.392	.104
Intercept	Content Knowledge	1486	1	1486	1043.97	p < .01	.958
	Content Understanding	26145	1	26145	833.20	p < .01	.948
	Pedagogical Understanding	26280	1	26280	479.20	p < .01	.912
	Mathematics Knowledge for Teaching P&S	131303	1	131303	828.39	p < .01	.947
GPA	Content Knowledge	1	1	1	.986	.327	.021
	Content Understanding	11	1	11	.350	.557	.008
	Pedagogical Understanding	101	1	101	1.85	.181	.039
	Mathematics Knowledge for Teaching P&S	212	1	212	1.34	.254	.028
Enrollment Classification	Content Knowledge	1	2	1	.15	.859	.007
	Content Understanding	9	2	5	.14	.866	.006
	Pedagogical Understanding	306	2	153	2.79	.072	.108
	Mathematics Knowledge for Teaching P&S	445	2	222	1.40	.256	.058
Enrollment Classification * GPA	Content Knowledge	2	2	10	.86	.430	.036
	Content Understanding	29	2	15	.47	.631	.020
	Pedagogical Understanding	85	2	43	.78	.467	.033
	Mathematics Knowledge for Teaching P&S	188	2	94	.59	.557	.025
Error	Content Knowledge	65	46	1			
	Content Understanding	1443	46	31			
	Pedagogical Understanding	2523	46	55			
	Mathematics Knowledge for Teaching P&S	7291	46	159			
Total	Content Knowledge	1556	52				
	Content Understanding	27638	52				
	Pedagogical Understanding	29295	52				
	Mathematics Knowledge for Teaching P&S	139439	52				
Corrected Total	Content Knowledge	70	51				
	Content Understanding	1493	51				
	Pedagogical Understanding	3015	51				
	Mathematics Knowledge for Teaching P&S	8136	51				

[a]R Squared = .061 (Adjusted R Squared = -.041)

[b]R Squared = .033 (Adjusted R Squared = -.072)

[c]R Squared = .163 (Adjusted R Squared = .072)

[d]R Squared = .104 (Adjusted R Squared = .006)

Table 29
Descriptive Statistics for Cohorts for Mathematics Knowledge for Teaching

	Cohort	Mean	Standard Deviation	N
Number and Operation Content Knowledge	1	3.86	1.56	21
	2	4.00	1.24	23
	3	4.00	1.46	16
	Total	3.95	1.40	60
Number and Operation Explanation of Solution	1	15.62	7.74	21
	2	18.26	5.96	23
	3	18.19	5.95	16
	Total	17.32	6.65	60
Number and Operation Pedagogical Understanding **Removed from Analysis**	1	11.62	8.58	21
	2	18.70	5.45	23
	3	16.25	8.89	16
	Total	15.57	8.09	60
Mathematics Knowledge for Teaching Number and Operation	1	31.10	16.69	21
	2	40.96	11.20	23
	3	38.44	14.56	16
	Total	36.83	14.63	60
Algebra Content Knowledge	1	5.35	1.43	23
	2	5.54	1.03	26
	3	5.59	1.25	27
	Total	5.50	1.23	76
Algebra Explanation of Solution	1	18.96	7.55	23
	2	23.54	8.02	26
	3	24.96	6.23	27
	Total	22.66	7.61	76
Algebra Pedagogical Understanding	1	16.70	7.91	23
	2	21.04	7.12	26
	3	23.22	7.67	27
	Total	20.50	7.93	76
Mathematics Knowledge for Teaching Algebra	1	41.00	15.34	23
	2	50.12	14.34	26
	3	53.78	12.99	27
	Total	48.66	14.98	76
Geometry Content Knowledge	1	5.41	1.56	22
	2	5.42	1.80	19
	3	5.13	2.10	15
	Total	5.34	1.77	56
Geometry Explanation of Solution	1	18.50	11.72	22
	2	26.63	10.23	19
	3	22.47	70.74	15
	Total	22.32	11.33	56
Geometry Pedagogical Understanding	1	14.00	11.11	22
	2	22.63	7.48	19
	3	20.07	11.49	15
	Total	18.55	10.66	56

Table 29
Continued

	Cohort	Mean	Standard Deviation	N
Mathematics Knowledge for Teaching Geometry	1	37.91	22.81	22
	2	54.68	17.98	19
	3	47.67	23.42	15
	Total	46.21	22.29	56
Probability and Statistics Content Knowledge	1	5.21	1.37	14
	2	5.36	1.22	28
	3	5.50	.71	10
	Total	5.35	1.17	52
Probability and Statistics Explanation of Solution	1	20.64	5.31	14
	2	23.11	5.86	28
	3	23.00	3.92	10
	Total	22.42	5.41	52
Probability and Statistics Pedagogical Understanding	1	18.21	8.30	14
	2	23.68	7.34	28
	3	25.10	5.78	10
	Total	22.48	7.69	52
Mathematics Knowledge for Teaching Probability and Statistics	1	44.07	13.96	14
	2	52.14	12.66	28
	3	53.60	7.63	10
	Total	50.25	12.63	52

Mathematics Knowledge for Teaching Number and Operations

Using a multivariate regression with the variable cohort in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching number and operations as well as the composite variable of mathematics knowledge for teaching number and operations, the overall model yielded R^2=*.170* for explanation of solution, R^2=*.144* for content knowledge, and R^2=*.223* for mathematics knowledge for teaching number and operations for summary data (see Table 30). The dependent variable pedagogical understanding for number and operations had to be removed from the analysis because it rejected the null hypothesis that the error variance of the dependent variable is equal across groups. Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Cohort and the interaction of GPA and cohort did not contribute significantly to content knowledge, explanation of solutions, or mathematics knowledge for teaching number and operations. However, GPA had a statistically significant *($p < .05$)* contribution on number and operations content knowledge, explanation of solutions, and mathematics knowledge for teaching number and operations. Effect sizes were small. GPA had approximately equal effects on content knowledge, explanation of solutions, and mathematics knowledge for teaching number and operations.

Mathematics Knowledge for Teaching Algebra

Using a multivariate regression with the variable cohort in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching algebra as well as the composite variable of mathematics knowledge for teaching algebra, the overall model yielded $R^2=.139$ for content knowledge, $R^2=.183$ for explanation of solution, $R^2=.138$ for pedagogical understanding, and $R^2=.183$ for mathematics knowledge for teaching algebra for summary data (see Table 31). Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Cohort did not contribute significantly to content knowledge, but did have a statistically significant *($p < .05$)* contribution to explanation of solution, pedagogical understanding, and mathematics knowledge for teaching algebra. The covariate, GPA, and Cohort did not contribute significantly to any of the dependent variables. Cohort and GPA

Table 30

Multivariate Regression on Cohort and GPA (Covariate) for Number and Operations

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	17[a]	5	3	1.82	.125	.144
	Content Understanding	444[b]	5	89	2.22	.066	.170
	Mathematics Knowledge for Teaching N&O	2817[c]	5	563	3.10	.016	.223
Intercept	Content Knowledge	936	1	936	514	p < .001	.905
	Content Understanding	17992	1	17992	449	p < .001	.893
	Mathematics Knowledge for Teaching N&O	81402	1	81402	448	p < .001	.892
GPA	Content Knowledge	16	1	16	9	.004**	.141
	Content Understanding	271	1	271	7	.012*	.111
	Mathematics Knowledge for Teaching N&O	1290	1	1290	7	.010**	.116
Cohort	Content Knowledge	.093	2	.046	.025	.975	.001
	Content Understanding	50	2	25	.628	.538	.023
	Mathematics Knowledge for Teaching N&O	823	2	412	2	.114	.077
Cohort * GPA	Content Knowledge	.310	2	.155	.085	.918	.003
	Content Understanding	123	2	61	2	.225	.054
	Mathematics Knowledge for Teaching N&O	703	2	352	2	.154	.067
Error	Content Knowledge	98	54	2			
	Content Understanding	2163	54	40			
	Mathematics Knowledge for Teaching N&O	9814	54	182			
Total	Content Knowledge	1051	60				
	Content Understanding	20599	60				
	Mathematics Knowledge for Teaching N&O	94032	60				
Corrected Total	Content Knowledge	115	59				
	Content Understanding	2607	59				
	Mathematics Knowledge for Teaching N&O	12630	59				

* *p* < *.05.* ***p* < *.01.*

[a]R Squared = .144 (Adjusted R Squared = .065)

[b]R Squared = .170 (Adjusted R Squared = .094)

[c]R Squared = .223 (Adjusted R Squared = .151)

Table 31

Multivariate Regression on Cohort and GPA (Covariate) for Algebra

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	16[a]	5	3	2.27	.057	.139
	Content Understanding	794[b]	5	159	3.13	.013	.183
	Pedagogical Understanding	651[c]	5	130	2.24	.060	.138
	Mathematics Knowledge for Teaching Algebra	3075[d]	5	615	3.13	.013	.183
Intercept	Content Knowledge	2299	1	2299	1654.62	< .001	.959
	Content Understanding	39017	1	39017	768.20	< .001	.916
	Pedagogical Understanding	31939	1	31939	550.02	< .001	.887
	Mathematics Knowledge for Teaching Algebra	179937	1	179937	916.44	< .001	.929
GPA	Content Knowledge	4	1	4	2.71	.104	.037
	Content Understanding	106	1	106	2.09	.153	.029
	Pedagogical Understanding	94	1	94	1.61	.208	.023
	Mathematics Knowledge for Teaching Algebra	480	1	480	2.45	.122	.034
Cohort	Content Knowledge	1	2	1	.20	.820	.006
	Content Understanding	441	2	221	4.34	.017*	.110
	Pedagogical Understanding	508	2	254	4.38	.016*	.111
	Mathematics Knowledge for Teaching Algebra	1955	2	978	4.98	.010**	.125
Cohort * GPA	Content Knowledge	11	2	6	4.11	.021*	.105
	Content Understanding	247	2	123	2.43	.096	.065
	Pedagogical Understanding	48	2	24	.41	.663	.012
	Mathematics Knowledge for Teaching Algebra	639	2	320	1.63	.204	.044
Error	Content Knowledge	97	70	1			
	Content Understanding	3555	70	51			
	Pedagogical Understanding	4065	70	58			
	Mathematics Knowledge for Teaching Algebra	13744	70	196			
Total	Content Knowledge	2412	76				
	Content Understanding	43366	76				
	Pedagogical Understanding	63354	76				
	Mathematics Knowledge for Teaching Algebra	196756	76				
Corrected Total	Content Knowledge	113	75				
	Content Understanding	4349	75				
	Pedagogical Understanding	4715	75				
	Mathematics Knowledge for Teaching Algebra	16819	75				

* $p < .05$. **$p < .01$.

[a]R Squared = .139 (Adjusted R Squared = .078)

[b]R Squared = .183 (Adjusted R Squared = .124)

[c]R Squared = .138 (Adjusted R Squared = .076)

[d]R Squared = .183 (Adjusted R Squared = .124)

together did not contribute significantly to explanation of solutions, pedagogical understanding, or mathematics knowledge for teaching algebra, but did have a statistically significant ($p < .05$) contribution to content knowledge. Effect sizes were small.

Mathematics Knowledge for Teaching Geometry

Using a multivariate regression with the variable cohort in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching geometry as well as the composite variable of mathematics knowledge for teaching geometry, the overall model yielded $R^2=.022$ for content knowledge, $R^2=.133$ for explanation of solution (content understanding), $R^2=.189$ for pedagogical understanding, and $R^2=.148$ for mathematics knowledge for teaching geometry for summary data (see Table 32). Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. GPA and the interaction between GPA and Cohort did not contribute significantly to any of the dependent variables. Cohort did not contribute significantly to content knowledge, content understanding, or mathematics knowledge for teaching geometry, but did have a statistically significant ($p < .05$) contribution to pedagogical understanding. Effect sizes were small.

Table 32

Multivariate Regression on Cohort and GPA (Covariate) for Geometry

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	4[a]	5	1	.23	.948	.022
	Content Understanding	937[b]	5	187	1.53	.197	.133
	Pedagogical Understanding	1184[c]	5	237	2.34	.055	.189
	Mathematics Knowledge for Teaching Geometry	4045[d]	5	809	1.74	.143	.148
Intercept	Content Knowledge	1597	1	1596	473.23	<.01	.904
	Content Understanding	27902	1	27902	228.00	<.01	.820
	Pedagogical Understanding	19278	1	19277	190.20	<.01	.792
	Mathematics Knowledge for Teaching Geometry	119603	1	119603	256.74	<.01	.837
GPA	Content Knowledge	1	1	1	.02	.898	.000
	Content Understanding	216	1	216	1.76	.190	.034
	Pedagogical Understanding	219	1	219	2.16	.148	.041
	Mathematics Knowledge for Teaching Geometry	884	1	884	1.90	.175	.037
Cohort	Content Knowledge	1	2	1	.13	.883	.005
	Content Understanding	618	2	309	2.52	.090	.092
	Pedagogical Understanding	758	2	379	3.74	.031*	.130
	Mathematics Knowledge for Teaching Geometry	2695	2	1347	2.89	.065	.104
Cohort * GPA	Content Knowledge	3	2	1	.44	.645	.017
	Content Understanding	104	2	52	.43	.656	.017
	Pedagogical Understanding	207	2	104	1.02	.367	.039
	Mathematics Knowledge for Teaching Geometry	466	2	233	.50	.609	.020
Error	Content Knowledge	169	50	3			
	Content Understanding	6119	50	122			
	Pedagogical Understanding	5068	50	101			
	Mathematics Knowledge for Teaching Geometry	23292	50	466			
Total	Content Knowledge	1769	56				
	Content Understanding	34958	56				
	Pedagogical Understanding	25529	56				
	Mathematics Knowledge for Teaching Geometry	146940	56				
Corrected Total	Content Knowledge	173	55				
	Content Understanding	7056	55				
	Pedagogical Understanding	6252	55				
	Mathematics Knowledge for Teaching Geometry	27337	55				

* $p < .05$.

[a] R Squared = .022 (Adjusted R Squared = -.075)

[b] R Squared = .133 (Adjusted R Squared = .046)

[c] R Squared = .189 (Adjusted R Squared = .108)

[d] R Squared = .148 (Adjusted R Squared = .063)

Mathematics Knowledge for Teaching Probability and Statistics

Using a multivariate regression with the variable cohort in addition to the covariate, GPA, regressed on the variables for the different components of mathematics knowledge for teaching probability and statistics as well as the composite variable of mathematics knowledge for teaching probability and statistics, the overall model yielded $R^2=0.061$ for content knowledge, $R^2=.033$ for explanation of solution, $R^2=.163$ for pedagogical understanding, and $R^2=.104$ for mathematics knowledge for teaching probability and statistics for summary data (see Table 33). Polynomial trend contrasts were conducted on the covariate, the variable, and the interaction of the two. Cohort, GPA, and its interaction did not contribute significantly to any of the dependent variables. Effect sizes were small.

Ancillary Question 3

Do some types of courses (e.g., algebra, geometry, numerical, statistical or applied, theoretical, education) have more impact than others upon development of a teachers' mathematics knowledge for teaching?

As mentioned above, the middle grades mathematics/science degree program has a specific degree plan with highly suggested sequence of courses. Many of the mathematics and mathematics education courses taken for the program occur simultaneously. These courses can be grouped into three major categories: mathematics courses, specialized middle grades mathematics education courses, and methods and student teaching experiences/course work.

Table 33
Multivariate Regression on Cohort and GPA (Covariate) for Probability and Statistics

Source	Dependent Variable	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	Content Knowledge	4[a]	5	1	.611	.692	.062
	Content Understanding	110[b]	5	22	.734	.601	.074
	Pedagogical Understanding	559[c]	5	112	2.093	.083	.185
	Mathematics Knowledge for Teaching P&S	1182[d]	5	236	1.564	.189	.145
Intercept	Content Knowledge	1486	1	1486	1044.945	<.01	.958
	Content Understanding	26145	1	26145	870.032	<.01	.950
	Pedagogical Understanding	26280	1	26280	492.189	<.01	.915
	Mathematics Knowledge for Teaching P&S	131303	1	131303	868.566	<.01	.950
GPA	Content Knowledge	1	1	1	.983	.327	.021
	Content Understanding	11	1	11	.365	.549	.008
	Pedagogical Understanding	101	1	101	1.895	.175	.040
	Mathematics Knowledge for Teaching P&S	212	1	212	1.401	.243	.030
Cohort	Content Knowledge	1	2	1	.250	.780	.011
	Content Understanding	76	2	38	1.258	.294	.052
	Pedagogical Understanding	446	2	223	4.180	.021*	.154
	Mathematics Knowledge for Teaching P&S	932	2	466	3.082	.055	.118
Cohort * GPA	Content Knowledge	2	2	1	.786	.462	.033
	Content Understanding	24	2	12	.395	.676	.017
	Pedagogical Understanding	11	2	6	.106	.899	.005
	Mathematics Knowledge for Teaching P&S	38	2	19	.127	.881	.005
Error	Content Knowledge	65	46	1			
	Content Understanding	1382	46	30			
	Pedagogical Understanding	2456	46	53			
	Mathematics Knowledge for Teaching P&S	6954	46	151			
Total	Content Knowledge	1556	52				
	Content Understanding	27638	52				
	Pedagogical Understanding	29295	52				
	Mathematics Knowledge for Teaching P&S	139439	52				
Corrected Total	Content Knowledge	70	51				
	Content Understanding	1493	51				
	Pedagogical Understanding	3014	51				
	Mathematics Knowledge for Teaching P&S	8136	51				

* $p < .05$.
[a] R Squared = .062 (Adjusted R Squared = -.040)
[b] R Squared = .074 (Adjusted R Squared = -.027)
[c] R Squared = .185 (Adjusted R Squared = .097)
[d] R Squared = .145 (Adjusted R Squared = .052)

Univariate Analysis of Covariance (ANCOVA) was run first on the current mathematics courses with the mathematics knowledge for teaching as the dependent variable and the GPA as the covariate. In addition polynomial trend contrasts were run.

Mathematics Knowledge for Teaching Number and Operations

Using a Univariate regression with the variables Current MATH 365, Current MATH 366, Current MATH 367, Current MATH 368, Current MATH 403, Current MASC 351, Current MASC450, Current MEFB 460, Current MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching number and operations, the overall model yielded $R^2=.406$ for mathematics knowledge for teaching number and operations for summary data (see Table 34). Polynomial trend contrasts were conducted for each current course separately, the covariate GPA, and then the following interactions: Current MASC 351* Current MASC 450, Current MATH 368* Current MATH 403* Current MASC 351, Current MATH 368* Current MATH 403* Current MASC 351* Current MASC450. Current enrollment in MASC 450 had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching number and operations. The covariate GPA also had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching number and operations. Effect sizes were small. The largest effect size was current enrollment in MASC 450 on mathematics knowledge for teaching number and operations.

Table 34

Univariate Regression on Current Courses with GPA as Covariate for Number and Operations

Dependent Variable: Mathematics Knowledge for Teaching Number and Operations

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	5131[a]	18	285	1.56	.119	.406
Intercept	81402	1	81402	445.01	<.001	.916
GPA	1290	1	1290	7.05	.011*	.147
Current MATH365	66	1	66	.36	.552	.019
Current MATH366	168	1	168	.92	.344	.022
Current MATH367	2	1	2	.01	.918	<.001
Current MATH368	159	1	159	.87	.356	.021
Current MATH403	130	1	130	.71	.403	.017
Current MASC351	309	1	309	1.69	.201	.040
Current MASC450	1162	1	1162	6.36	.016*	.134
Current MEFB460	39	1	39	.21	.646	.005
Current MEFB497	2	1	2	.01	.915	<.001
Current MASC351 * Current MASC450	64	1	64	.35	.556	.009
Current MATH368 * Current MATH403 * Current MASC351	191	4	48	.26	.901	.025
Current MATH368 * Current MATH403 * Current MASC351 * Current MASC450	1547	3	516	2.82	.051	.171
Error	7500	41	183			
Total	94032	60				
Corrected Total	12630	59				

* $p < .05$

[a] R Squared = .406 (Adjusted R Squared = .146)

Using a Univariate regression with the variables Taken MATH 365, Taken MATH 366, Taken MATH 367, Taken MATH 368, Taken MATH 403, Taken MASC 351, Taken MASC450, Taken MEFB 460, Taken MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching number and operations, the overall model yielded R^2=*.396* for mathematics knowledge for teaching number and operations for summary data (see Table 35). Polynomial trend contrasts were conducted for each taken course separately, the covariate GPA, and then the following interactions: Taken MASC 351* Taken MASC 450; Taken MATH 368* Taken MATH 403* Taken MASC 351* Taken MASC 450; Taken MATH 365*Taken MATH 366*Taken MATH 367* Taken MATH 368*Taken MATH 403; Taken MATH 365*Taken MATH 366*Taken MATH 367*Taken MATH 368*Taken MATH 403*Taken MASC 351*Taken MASC450*Taken MEFB 460*Taken MEFB 497 (all taken courses together). The covariate GPA had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching number and operations. Effect sizes were small.

Mathematics Knowledge for Teaching Algebra

Using a Univariate regression with the variables Current MATH 365, Current MATH 366, Current MATH 367, Current MATH 368, Current MATH 403, Current MASC 351, Current MASC450, Current MEFB 460, Current MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching algebra, the overall model yielded $R^2 = .368$ for mathematics knowledge for teaching algebra for summary data (see Table 36). Polynomial trend contrasts were conducted for each current course separately, the covariate GPA, and then the following interactions: Current MASC 351* Current MASC 450, Current MATH 368* Current MATH 403* Current MASC 351, Current MATH 368* Current MATH 403* Current MASC 351* Current MASC450. Current enrollment in MATH 365, MASC 450, and MEFB 497 all had statistically significant ($p < .01$, $p < .05$, $p < .05$, respectively) contributions to mathematics knowledge for teaching algebra. Effect sizes were small. The largest effect size was current enrollment in MATH 365 on mathematics knowledge for teaching algebra.

Table 35

Univariate Regression on Taken Courses with GPA as Covariate for Number and Operations

Dependent variable: Mathematics Knowledge for Teaching Number and Operations

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	5001[a]	18	277	1.49	.143	.396
Intercept	81402	1	81402	437.46	<.01	.914
GPA	1290	1	1290	6.93	.012*	.145
Taken MATH365	424	1	424	2.28	.139	.053
Taken MATH366	195	1	195	1.05	.312	.025
Taken MATH367	163	1	163	.87	.355	.021
Taken MATH368	98	1	98	.53	.473	.013
Taken MATH403	261	1	261	1.40	.243	.033
Taken MASC351	15	1	15	.08	.778	.002
Taken MASC450	34	1	34	.18	.670	.004
Taken MEFB460	112	1	112	.60	.443	.014
Taken MEFB497	146	1	146	.79	.380	.019
Taken MASC351 * Taken MASC450	0	0	.	.	.	.000
Taken MATH368 * Taken MATH403 * Taken MASC351 * Taken MASC450	928	4	232	1.25	.306	.108
Taken MATH365 * Taken MATH366 * Taken MATH367 * Taken MATH368 * Taken MATH403	1215	3	405	2.18	.105	.137
Taken MATH365 * Taken MATH366 * Taken MATH367 * Taken MATH368 * Taken MATH403 * Taken MASC351 * Taken MASC450 * Taken MEFB460 * Taken MEFB497	121	1	121	.65	.424	.016
Error	7629	41	186			
Total	94032	60				
Corrected Total	12630	59				

* $p < .05$.

[a] R Squared = .396 (Adjusted R Squared = .131)

Table 36
Univariate Regression on Current Courses with GPA as Covariate for Algebra
Dependent Variable: Mathematics Knowledge for Teaching Algebra

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	6184[a]	16	386	2.14	.018	.368
Intercept	179937	1	179937	998.20	<.001	.944
GPA	480	1	480	2.67	.108	.041
Current MATH 365	1310	1	1310	7.27	.009**	.123
Current MATH 366	337	1	337	1.97	.177	.024
Current MATH 367	132	1	132	.73	.396	.008
Current MATH 368	234	1	234	1.30	.259	.025
Current MATH 403	1	1	1	.001	.977	<.001
Current MASC 351	239	1	239	1.33	.254	.013
Current MASC 450	1123	1	1123	6.23	.015*	.102
Current MEFB 460	47	1	47	.26	.613	.003
Current MEFB 497	923	1	923	5.12	.027*	.076
Current MASC 351 * Current MASC 450	80	1	80	.44	.509	.007
Current MATH 368 * Current MATH 403 * Current MASC 351	356	3	119	.66	.581	.032
Current MATH 368 * Current MATH 403 * Current MASC 351 * Current MASC 450	921	2	461	2.56	.086	.080
Error	10635	59	180			
Total	196756	76				
Corrected Total	16819	75				

* $p < .05$. ** $p < .01$.
[a]R Squared = .368 (Adjusted R Squared = .196)

Using a Univariate regression with the variables Taken MATH 365, Taken MATH 366, Taken MATH 367, Taken MATH 368, Taken MATH 403, Taken MASC 351, Taken MASC450, Taken MEFB 460, Taken MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching algebra, the overall model yielded R^2=*.354* for mathematics knowledge for teaching algebra for summary data (see Table 37). Polynomial trend contrasts were conducted for each taken course separately, the covariate GPA, and then the following interactions: Taken MASC 351* Taken MASC 450; Taken MATH 368* Taken MATH 403* Taken MASC 351* Taken MASC 450; Taken MATH 365*Taken MATH 366*Taken MATH 367* Taken MATH 368*Taken MATH 403; Taken MATH 365*Taken MATH 366*Taken MATH 367*Taken MATH 368*Taken MATH 403*Taken MASC 351*Taken MASC450*Taken MEFB 460*Taken MEFB 497 (all taken courses together). MATH 403, which had already been taken, had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching algebra. Effect sizes were small.

Mathematics Knowledge for Teaching Geometry

Using a Univariate regression with the variables Current MATH 365, Current MATH 366, Current MATH 367, Current MATH 368, Current MATH 403, Current MASC 351, Current MASC450, Current MEFB 460, Current MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching geometry, the overall model yielded $R^2=.381$ for mathematics knowledge for teaching geometry for summary data (see Table 38). Polynomial trend contrasts were conducted for each current course separately, the covariate GPA, and then the following interactions: Current MASC 351* Current MASC 450, Current MATH 368* Current MATH 403* Current MASC 351, Current MATH 368* Current MATH 403* Current MASC 351* Current MASC450. Current enrollment in MATH 365, and MASC 450 both had statistically significant ($p < .01$, $p <.05$, respectively) contributions to mathematics knowledge for teaching geometry. Effect sizes were small to medium. The largest effect size was current enrollment in MATH 365 on mathematics knowledge for teaching geometry.

Table 37
Univariate Regression on Taken Courses with GPA as Covariate for Algebra
Dependent Variable: Mathematics Knowledge for Teaching Algebra

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	5947[a]	19	313	1.61	.085	.354
Intercept	179937	1	179937	926.85	<.001	.943
GPA	480	1	480	2.83	.098	.048
Taken MATH365	279	1	279	1.21	.276	.021
Taken MATH366	745	1	745	3.83	.055	.064
Taken MATH367	300	1	300	1.07	.306	.019
Taken MATH368	28	1	28	.15	.697	.003
Taken MATH403	1071	1	1071	4.89	.031*	.080
Taken MASC351	177	1	177	1.41	.241	.025
Taken MASC450	170	1	170	.96	.333	.017
Taken MEFB460	40	1	40	.11	.742	.002
Taken MEFB497	602	1	602	3.59	.063	.060
Taken MASC351 * Taken MASC 450	0	0	.	.	.	.000
Taken MATH368 * Taken MATH403 * Taken MASC351 * Taken MASC450	948	4	237	1.22	.312	.080
Taken MATH365 * Taken MATH366 * Taken MATH367 * Taken MATH368 * Taken MATH403	407	3	136	.70	.557	.036
Taken MATH365 * Taken MATH366 * Taken MATH367 * Taken MATH368 * Taken MATH403 * Taken MASC351 * Taken MASC450 * Taken MEFB460 * Taken MEFB497	701	2	351	1.81	.174	.061
Error	10872	56	194			
Total	196756	76				
Corrected Total	16819	75				

* $p < .05$.
[a]R Squared = .354 (Adjusted R Squared = .134)

Table 38

Univariate Regression on Current Courses with GPA as Covariate for Geometry

Dependent Variable: Mathematics Knowledge for Teaching Geometry

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	10406[a]	15	694	1.64	.106	.381
Intercept	119603	1	119603	282.57	<.001	.876
GPA	884	1	884	2.09	.156	.050
Current MATH365	4968	1	4968	11.74	.642	.227
Current MATH366	93	1	93	.22	.300	.005
Current MATH367	467	1	467	1.10	.707	.027
Current MATH368	61	1	61	.14	.877	.004
Current MATH403	10	1	10	.02	.343	.001
Current MASC351	390	1	390	.92	.034*	.022
Current MASC450	2051	1	2051	4.85	.720	.108
Current MEFB460	55	1	55	.13	.346	.003
Current MEFB497	385	1	385	.91	.156	.022
Current MASC351 * Current MASC450	0	0	.	.	.	.000
Current MATH368 * Current MATH403 * Current MASC351	808	3	269	.64	.596	.046
Current MATH368 * Current MATH403 * Current MASC351 * Current MASC450	235	2	117	.28	.759	.014
Error	16931	40	423			
Total	146940	56				
Corrected Total	27337	55				

*p < .05. **p < .01.*

[a]R Squared = .381 (Adjusted R Squared = .148)

A Univariate regression with the variables Current MATH 365, Current MATH 366, Current MATH 367, Current MATH 368, Current MATH 403, Current MASC 351, Current MASC450, Current MEFB 460, Current MEFB 497 in addition to the covariate, GPA, was run with mathematics knowledge for teaching geometry as the dependent variable. However, it was rejected that the error variance of the dependent variable was equal across groups. Further analyses (ANCOVA) were run with current courses and the components of mathematics knowledge for teaching geometry; the tests all failed Levene's Test of Equality of Error Variances.

Mathematics Knowledge for Teaching Probability and Statistics

Using a Univariate regression with the variables Current MATH 365, Current MATH 366, Current MATH 367, Current MATH 368, Current MATH 403, Current MASC 351, Current MASC450, Current MEFB 460, Current MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching probability and statistics, the overall model yielded $R^2=.426$ for mathematics knowledge for teaching probability and statistics for summary data (see Table 39). Polynomial trend contrasts were conducted for each current course separately, the covariate GPA, and then the following interactions: Current MASC 351* Current MASC 450, Current MATH 368* Current MATH 403* Current MASC 351, Current MATH 368* Current MATH 403* Current MASC 351* Current MASC450. The interaction of Current MATH 368* Current MATH 403* Current MASC 351* Current MASC450 together had a statistically significant ($p <.05$) contribution to mathematics knowledge for teaching probability and statistics. Effect sizes were small.

Table 39

Univariate Regression on Current Courses with GPA as Covariate for Probability and Statistics

Dependent Variable: Mathematics Knowledge for Teaching Probability and Statistics

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	3467[a]	15	231	1.78	.078	.426
Intercept	131303	1	131303	1012.44	< .001	.966
GPA	212	1	212	1.43	.240	.038
Current MATH365	185	1	185	.010	.938	<.001
Current MATH366	1	1	1	1.88	.179	.050
Current MATH367	244	1	244	1.95	.172	.051
Current MATH368	252	1	252	.91	.648	.025
Current MATH403	117	1	117	2.22	.145	.058
Current MASC351	288	1	288	1.36	.252	.036
Current MASC450	176	1	176	1.35	.253	.036
Current MEFB460	175	1	175	.33	.570	.009
Current MEFB497	43	1	43	1.63	.209	.043
Current MASC351 * Current MASC450	8	1	8	.06	.807	.002
Current MATH368 * Current MATH403 * Current MASC351	984	3	328	2.53	.073	.174
Current MATH368 * Current MATH403 * Current MASC351 * Current MASC450	782	1	782	6.03	.019*	.143
Error	4669	36	130			
Total	139439	52				
Corrected Total	8136	51				

* $p < .05$.

[a] R Squared = .426 (Adjusted R Squared = .187)

Using a Univariate regression with the variables Taken MATH 365, Taken MATH 366, Taken MATH 367, Taken MATH 368, Taken MATH 403, Taken MASC 351, Taken MASC450, Taken MEFB 460, Taken MEFB 497 in addition to the covariate, GPA, regressed on the variable mathematics knowledge for teaching probability and statistics, the overall model yielded $R^2=.459$ for mathematics knowledge for teaching probability and statistics for summary data (see Table 40). Polynomial trend contrasts were conducted for each taken course separately, the covariate GPA, and then the following interactions: Taken MASC 351* Taken MASC 450; Taken MATH 368* Taken MATH 403* Taken MASC 351* Taken MASC 450; Taken MATH 365*Taken MATH 366*Taken MATH 367* Taken MATH 368*Taken MATH 403; Taken MATH 365*Taken MATH 366*Taken MATH 367*Taken MATH 368*Taken MATH 403*Taken MASC 351*Taken MASC450*Taken MEFB 460*Taken MEFB 497 (all taken courses together). The interaction Taken MASC 351* Taken MASC 450 together had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching probability and statistics. In addition, the interaction of all courses together (Taken MATH 365*Taken MATH 366*Taken MATH 367*Taken MATH 368*Taken MATH 403*Taken MASC 351*Taken MASC450*Taken MEFB 460*Taken MEFB 497) had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching probability and statistics. Effect sizes were small to medium. The largest effect size was the interaction of the courses, taken previously, together on the mathematics knowledge for teaching probability and statistics.

Ancillary Question 4

Does development happen at greater rates in certain stages of the program than others?

Predictors for each component of mathematics knowledge for teaching across each content strand were saved during the MANCOVA to be used in the analysis of this question. A Pearson's *r* correlation was run between the predictor and its corresponding component to be sure it was a "good" predictor. Table 40 reveals all of the correlations were significant at $p < .01$.

Table 40

Univariate Regression on Taken Courses Data with GPA as Covariate for Probability and Statistics

Dependent variable: Mathematics Knowledge for Teaching Probability and Statistics

Source	Type I Sum of Squares	df	Mean Square	F	*p*	Partial Eta Squared
Corrected Model	3736[a]	18	208	1.56	.132	.459
Intercept	131303	1	131303	984.77	<.001	.968
GPA	212	1	212	1.59	.216	.046
Taken MATH 365	93	1	93	.70	.410	.021
Taken MATH 366	49	1	49	.37	.548	.011
Taken MATH 367	95	1	95	.71	.405	.021
Taken MATH 368	222	1	222	1.66	.206	.048
Taken MATH 403	50	1	50	.37	.545	.011
Taken MASC 351	91	1	91	.68	.415	.020
Taken MASC 450	2	1	2	.02	.898	.001
Taken MEFB 460	237	1	237	1.78	.192	.051
Taken MEFB 497	25	1	25	.19	.668	.006
Taken MASC 351*MASC450	658	1	658	4.93	.033*	.130
Taken MATH 368 *MATH 403 *MASC 351* MASC 450	329	3	110	.82	.490	.070
Taken MATH 365* MATH 366*MATH 367*MATH 368 *MATH 403	19	1	19	.14	.711	.004
Taken MATH 365* MATH 366*MATH 367*MATH 368 *MATH 403*MASC 351*MASC 450 *MEFB 460*MEFB 497	1655	3	552	4.14	.014*	.273
Error	4400	33	133			
Total	139439	52				
Corrected Total	8136	51				

* $p < .05$

[a] R Squared = .459 (Adjusted R Squared = .164)

Table 41
Correlations of Predictors with Real Scores

	Correlation	Pearson Correlation
Number and Operations	Content Knowledge*Predicted Content Knowledge	.556**
	Explanation*Predicted Explanation	.552**
	Understanding* Predicted Understanding	.518**
	Mathematics Knowledge for Teaching N&O* Predicted Mathematics Knowledge for Teaching N&O	.560**
Algebra	Content Knowledge*Predicted Content Knowledge	.448**
	Explanation*Predicted Explanation	.501**
	Understanding* Predicted Understanding	.488**
	Mathematics Knowledge for Teaching Algebra* Predicted Mathematics Knowledge for Teaching Algebra	.510**
Geometry	Content Knowledge*Predicted Content Knowledge	.393**
	Explanation*Predicted Explanation	.441**
	Understanding* Predicted Understanding	.420**
	Mathematics Knowledge for Teaching Geometry* Predicted Mathematics Knowledge for Teaching Geometry	.405**
Probability and Statistics	Content Knowledge*Predicted Content Knowledge	.384**
	Explanation*Predicted Explanation	.343**
	Understanding* Predicted Understanding	.415**
	Mathematics Knowledge for Teaching P&S* Predicted Mathematics Knowledge for Teaching P&S	.364**

** Correlation is significant at the 0.01 level (2-tailed)

The predictors for mathematics knowledge for teaching algebra, number and operations, probability and statistics, and geometry were used for the rest of the analyses of this question.

The next step was to find the rate of the predictors of mathematics knowledge for teaching across each content area. Finding the trendline of the data allowed one to obtain the rate (see Table 42). Since course information was dichotomous, each course was

Table 42
Rates for Predictors of Mathematics Knowledge for Teaching

	Rate
Number and Operations	.0037
Algebra	-.1738
Geometry	.2169
Probability and Statistics	-.1208

graphed as the independent variable against the predictor variable of mathematics knowledge for teaching (dependent variable) and the trendline was used to determine the slope, or rate, for each course in its corresponding content area. Table 42 contains the values described above. From Table 43, the course having the most impact, according to their rates, across all content areas was MEFB 460 Teaching mathematics in the middle grades. This course is designed for preservice middle grades mathematics teachers focusing on pedagogy of mathematics in the classroom. It is taken simultaneously with field experiences which occur three times a week. The following semester, the preservice teachers student teach.

Table 43
Rates for Predictors of Mathematics Knowledge for Teaching vs. Course

Course	N&O Currently Enrolled	N&O Already Taken	Algebra Currently Enrolled	Algebra Already Taken	Geom. Currently Enrolled	Geom. Already Taken	P&S Currently Enrolled	P&S Already Taken
MATH 365	-1.86	-2.43	-8.54	4.10*	-13.68	17.92*@	-4.11	.64*
MATH 366	2.41*	-1.55	-4.96	6.87*	9.91*@	4.83*	-1.19	1.17*
MATH 367	-.74	-1.45	-1.83	6.66*	4.12*	1.97*	-3.99	3.53*
MATH 368	3.15*	3.15*	-2.76	6.19*	-.96	1.80*	-.33	4.44*
MATH 403	5.15*	5.15*	-1.63	5.42*	.03*	3.03*	.19*	6.00*@
MASC 351	-.91	2.05	-3.58	8.40*@	3.34*	3.03*	-2.22	3.52*
MASC 450	4.23*	4.23*	8.36*	4.61*	1.20*	.03*	4.84*	3.82*
MEFB 460	7.13*@	4.62*@	8.37*@	7.18*	.27*	.03*	5.22*@	2.81*
MEFB 497	5.61*	--	6.39*	--	.03	--	3.56*	--
Rate of Predictor	.0037		-.1738		-.2169		-.1208	

* Indicates a positive relationship
@ Highest rate for characteristic (taken/current)

The next thing to investigate was enrollment characteristics against the predictor variables. Nonlinear regression was used to determine a model of fit for enrollment characteristics onto the predictor variables for mathematics knowledge for teaching. Enrollment characteristics are coded as: 1—Freshman, 2—Sophomore, 3—1st Semester Junior, 4—2nd Semester Junior, 5—1st Semester Senior, and 6—2nd Semester Senior and Beyond. Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching number and operations was found to be quadratic (Figure 5, R^2=*.495)*.

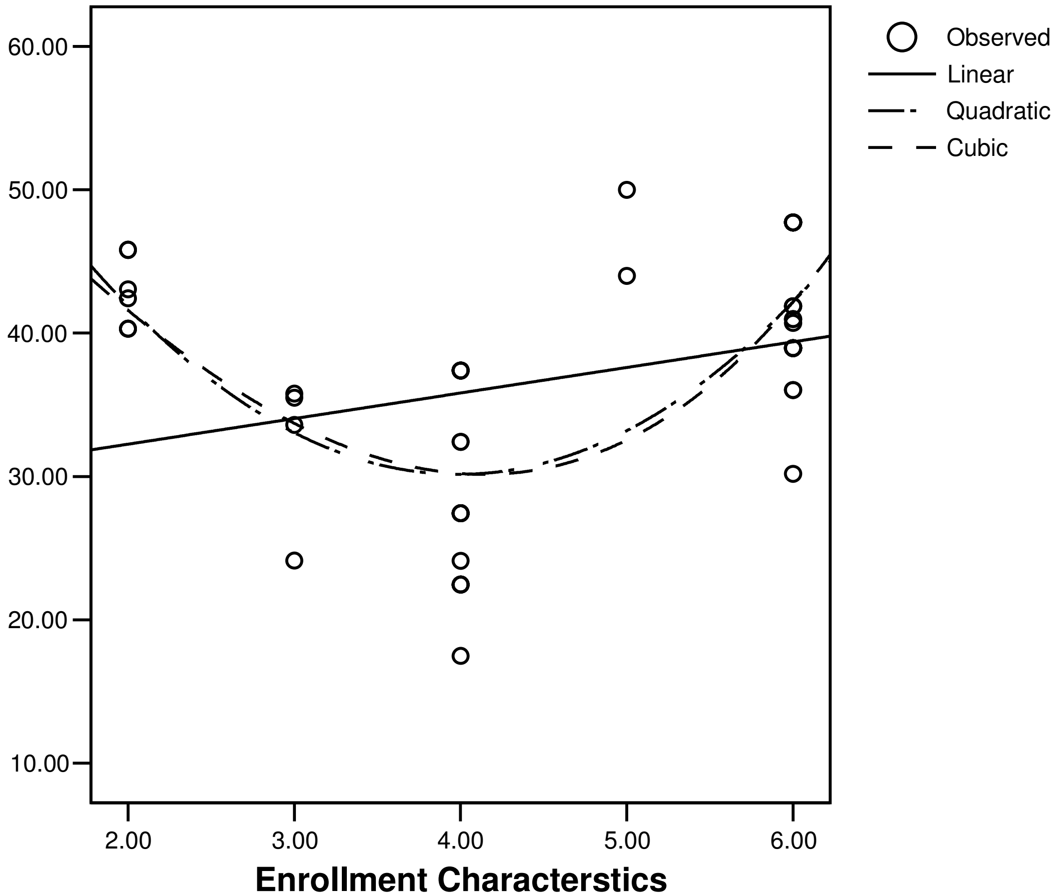

Figure 5. Curve Fit of Enrollment Characteristics Onto the Predictor Variable for Mathematics Knowledge for Teaching Number and Operations.

Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching algebra was found to be cubic (Figure 6, R^2=*.550)*.

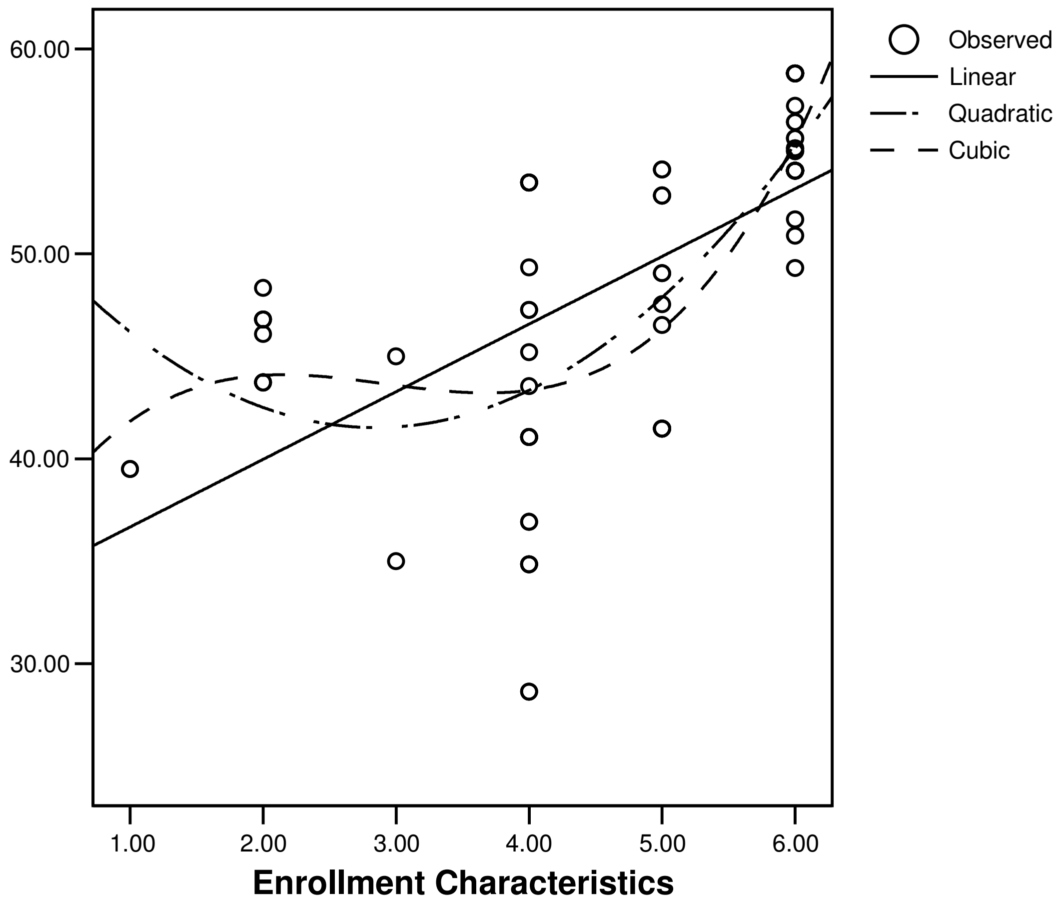

Figure 6. Curve Fit of Enrollment Characteristics Onto the Predictor Variable for Mathematics Knowledge for Teaching Algebra.

Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching geometry was found to be cubic (Figure 7, R^2=*.040)*.

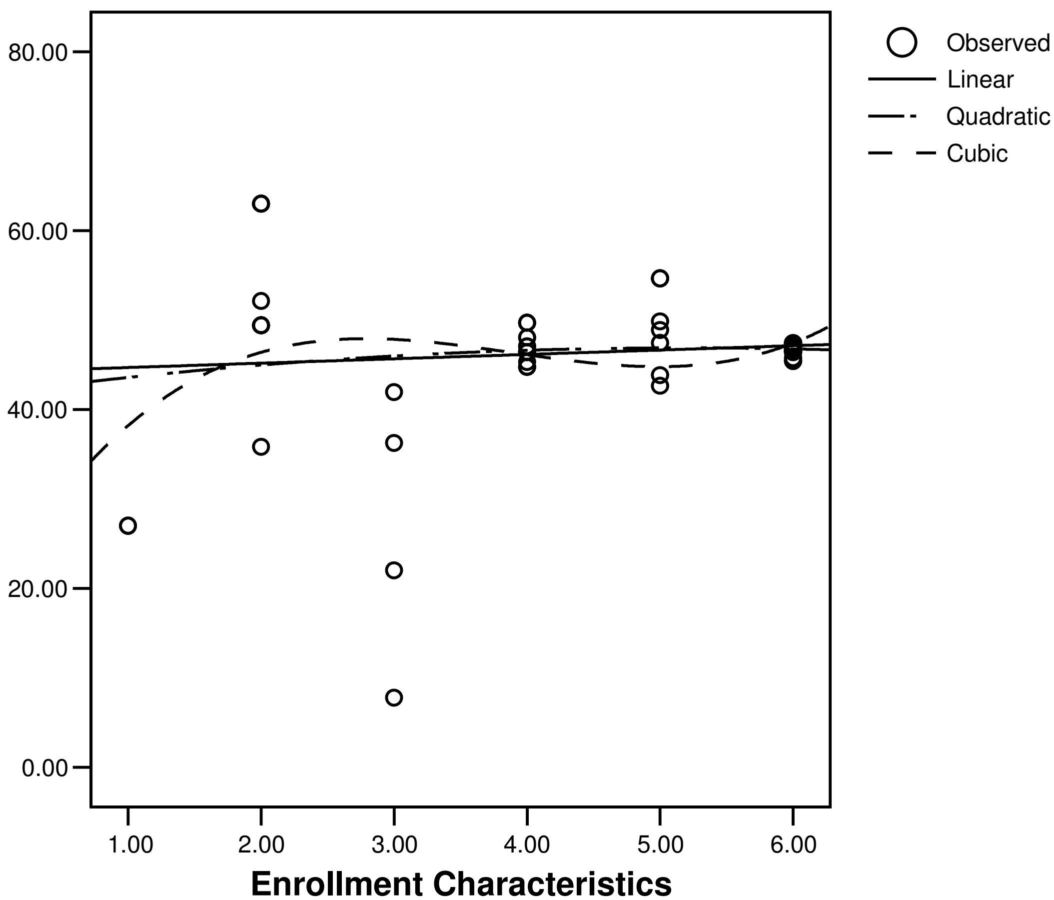

Figure 7. Curve Fit of Enrollment Characteristics Onto the Predictor Variable for Mathematics Knowledge for Teaching Geometry.

Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching probability and statistics was found to be quadratic (Figure 8, R^2=*.583)*.

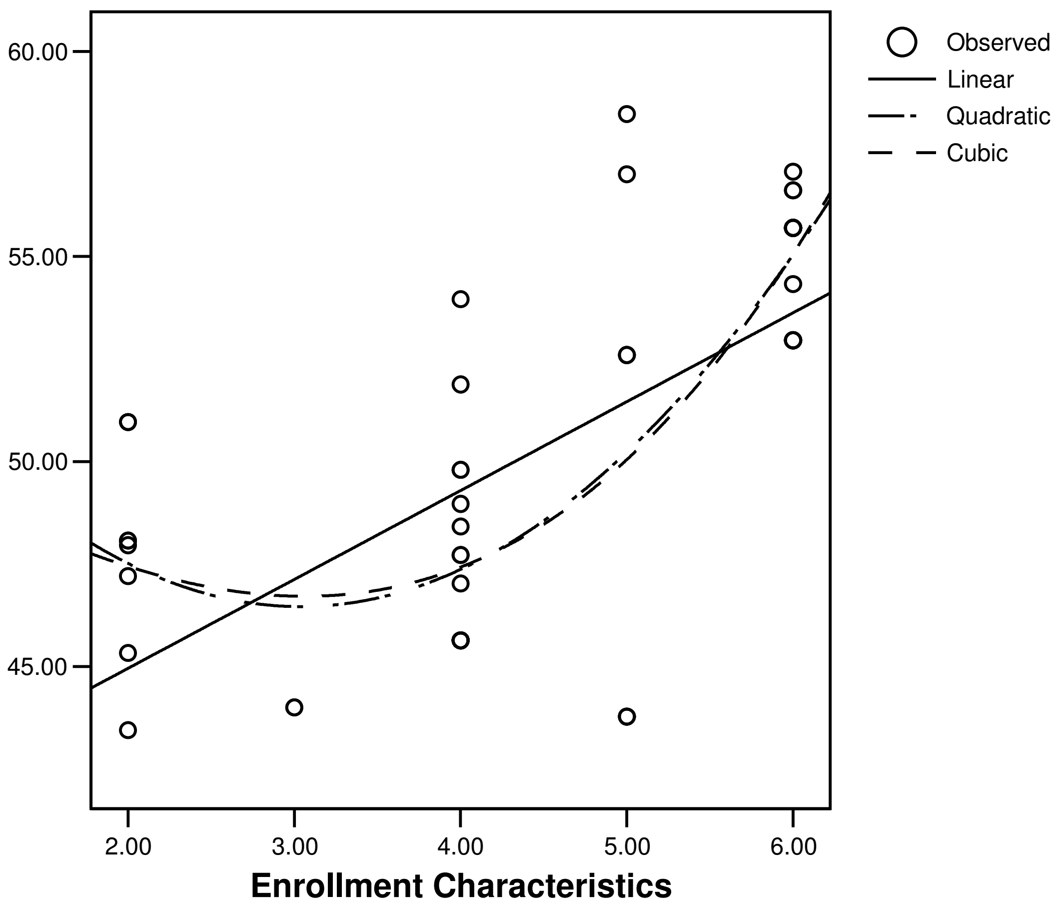

Figure 8. Curve Fit of Enrollment Characteristics Onto the Predictor Variable for Mathematics Knowledge for Teaching Probability and Statistics.

Finally, cohorts were investigated through nonlinear regression to determine a model of fit for cohorts onto the predictor variables for mathematics knowledge for teaching. Cohorts were coded as: 1—Mathematics courses; 2—Integrated Mathematics and Science (MASC) courses; and 3—Methods block and student teaching. Cohorts onto the predictor variable for mathematics knowledge for teaching number and operations was found to be quadratic (Figure 9, R^2=*.191)*.

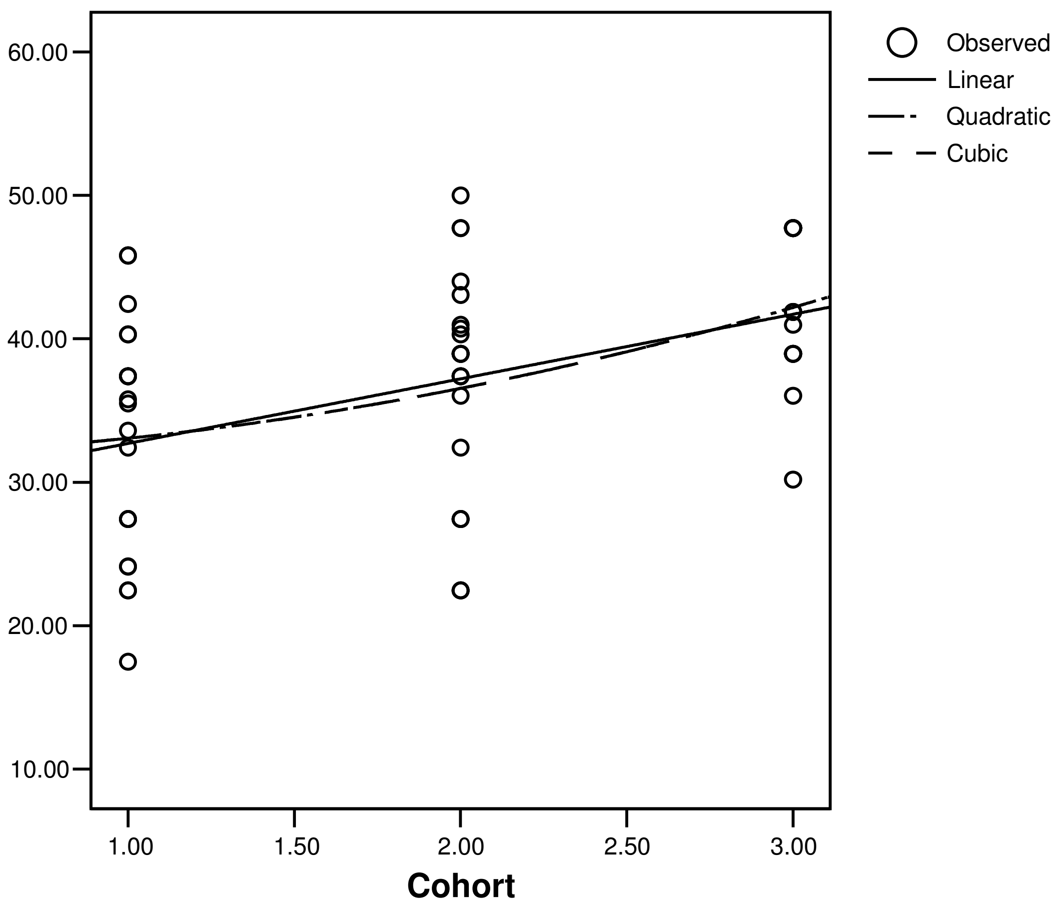

Figure 9. Curve Fit of Cohorts Onto the Predictor Variable for Mathematics Knowledge for Teaching Number and Operations.

Cohorts onto the predictor variable for mathematics knowledge for teaching algebra was found to be linear (Figure 10, R^2=*.411)*.

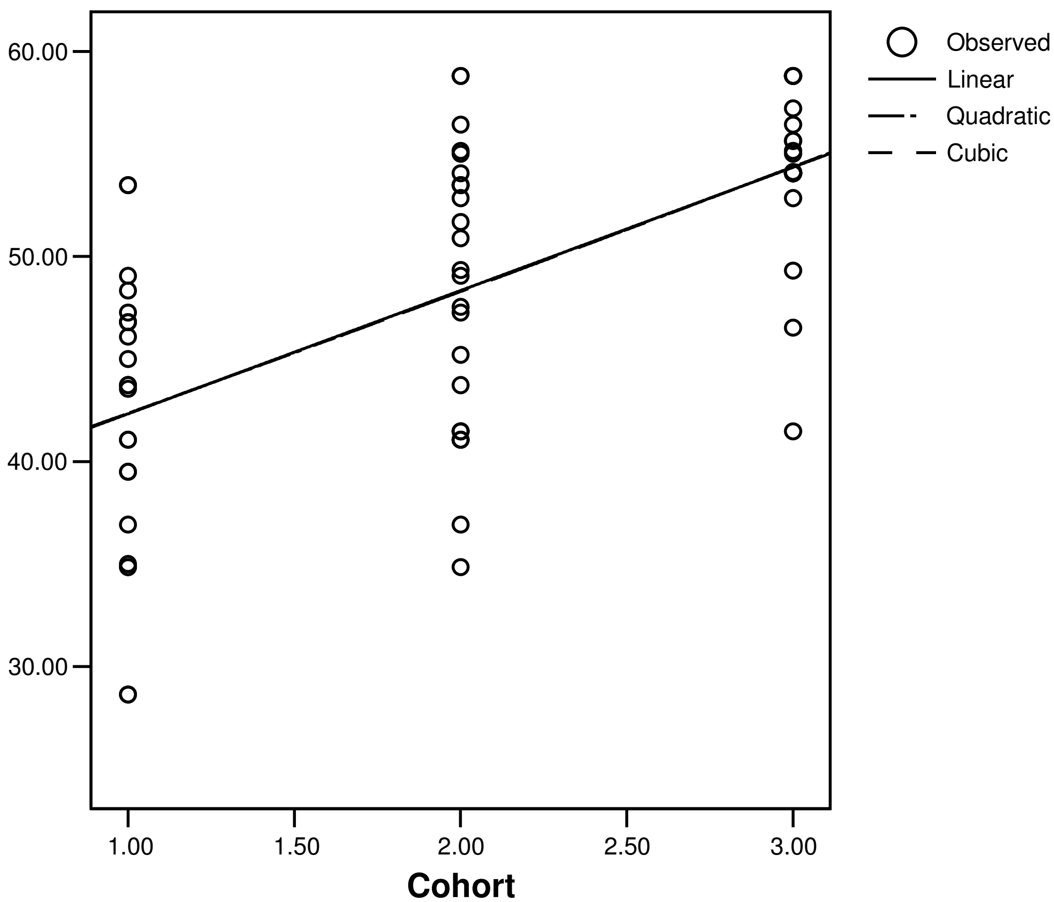

Figure 10. Curve Fit of Cohorts Onto the Predictor Variable for Mathematics Knowledge for Teaching Algebra.

Cohorts onto the predictor variable for mathematics knowledge for teaching geometry was found to be quadratic (Figure 11, R^2=*.034)*.

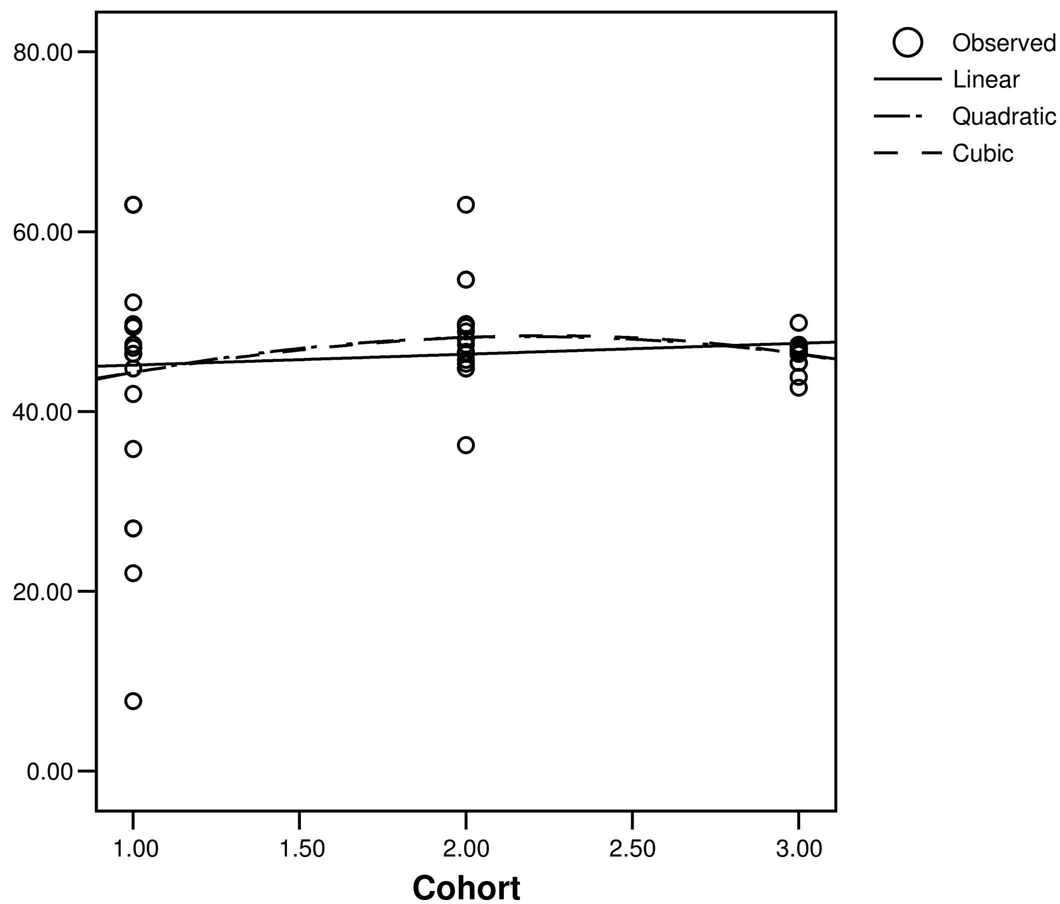

Figure 11. Curve Fit of Cohorts Onto the Predictor Variable for Mathematics Knowledge for Teaching Geometry.

Cohorts onto the predictor variable for mathematics knowledge for teaching probability and statistics was found to be quadratic (Figure 12, R^2=*.244)*.

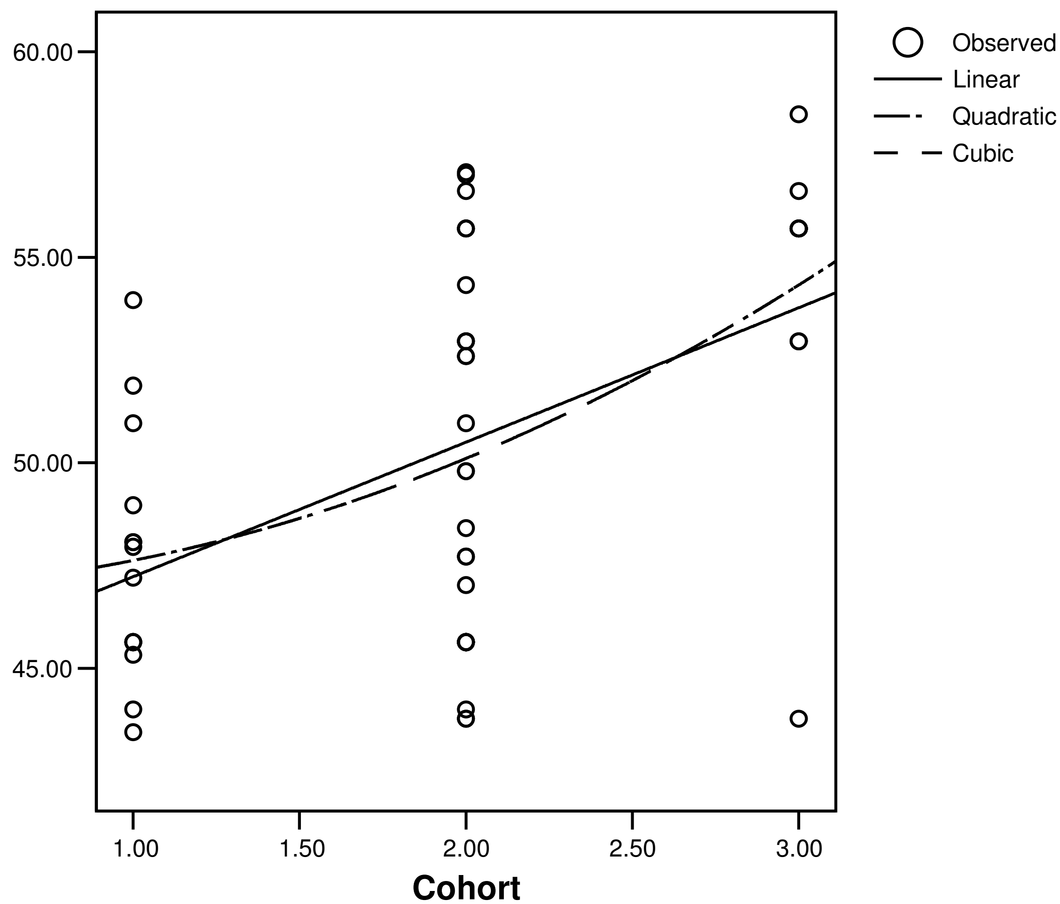

Figure 12. Curve Fit of Cohorts Onto the Predictor Variable for Mathematics Knowledge for Teaching Probability and Statistics.

Summary of Results

Analysis of the mathematics knowledge for teaching algebra, probability and statistics, number and operations, and geometry of preservice middle grades teachers involved in this study were presented in this chapter. Skewness and kurtosis were first computed for each item and then the total (mathematics knowledge for teaching) for each content strand (algebra, geometry, number and operations, and probability and statistics). A lack of extreme kurtosis and skewness was noted. Normality was assessed because it was an underlying assumption that needed to be met when using parametric analyses. A Kolmogorov-Smirnov Test for each content strand revealed no statistical significance, therefore it was concluded that the scores were normally distributed.

Research Question 1

The analysis of question one was broken into four parts: analysis of the content, analysis of the explanation, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching number and operations. The reliability of the number and operations assessment was found to be .924. An item analysis revealed an average item difficulty of 56% ($p = .56$) and the item discrimination revealed three usually unacceptable items, three good items, and one excellent item.

Items for which less than half of the participants answered correctly were investigated for commonalities. Common errors on the item addressing developing and applying laws of exponents for multiplication included leaving the coefficient a number greater than or equal to ten, and not changing the exponent when moving a decimal left or write. The two other items addressed estimating a percent of quantity, given an application. The most common error on both of these items was simply not estimating at all. In analyzing the content understanding (part 2) of the number and operations assessment, a majority of the participants scored either a two or three according to the rubric for this item. Misuse of vocabulary was noted on two other items. A common error on one item was adding together two percents given in an original price and then a sale price situation. A common error on another item was the mixing up of greatest common factor and the least common multiple.

For the pedagogical understanding of number and operations, a majority of the participants scored a two, three, or four. Scores of two and three on the pedagogical understanding part tended to be very algorithmic in nature and the general method of instruction could be assumed to be direct since there was no mention of any other method. With a score of four there was often a culturally responsive component included.

The mathematics knowledge for teaching number and operations scores ranged from 3 to 59, a range of 56, out of a total possible 91 points.

Research Question 2

The analysis of this question was again broken into four parts: analysis of the content knowledge, analysis of the content understanding, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching algebra. The reliability of the algebra assessment was found to be .935. An items analysis revealed an average item difficulty of 79% ($p = .76$) and the item discrimination revealed three usually unacceptable items and four good items.

Only one item, which less than half of the participants answered correctly, was investigated for commonalities. The item addressed translating verbal sentences into algebraic equations. Two common errors were found in student answers. The first error was simply not following the directions of the item. The second error was the misinterpretation of the information given in the item. Two other items were noteworthy because of the commonalities in the incorrectness of their answers. One error was the misplacement of the "eight less than" in the equation they were supposed to translate from verbal sentences. Another item concerning the factoring trinomials revealed that participants often mixed up the signs in the parentheses when they had an incorrect answer.

A majority of the participants scored either a two or three according to the rubric on content understanding for algebra. Other areas of noteworthiness for the content understanding first concerned translating verbal sentences into algebraic equations. A common them across all explanations for these items was "just follow the words…they tell you what to do." The other two items of interest complemented each other in that one addressed multiplying binomials together and the other addressed factoring trinomials. The common explanation for content understanding in the item address the multiplication of binomials was "I just used FOIL to multiply it out and then I added like terms." "Box method" or "tic-tac-toe method" was also mentioned, but not as frequently. Although a majority of the students answered the factoring item correctly, there was a theme of misuse of mathematical vocabulary and procedures in the explanations of their solutions.

For the pedagogical understanding of algebra, a majority of the participants scored a two or a three according to the rubric. Scores of two and three tended to be very algorithmic and procedure-oriented in nature. The general method of instruction could be assumed to be direct since there was often no mention of any other method. Across all the pedagogical understanding responses, there was a greater indication of cultural responsiveness than was found on the number and operations items. Themes noteworthy for the pedagogical understanding of algebra dealt with the teaching of the concepts of the multiplication of binomials and the factoring of trinomials. Again, FOIL, forwards and backwards, was a common them in the responses. A major concentration on just find the correct combination of factors in factoring trinomials was noted.

The mathematics knowledge of teaching algebra total scores ranged from 4 to 80, a range of 76, out of a possible 91 points.

Research Question 3

The analysis of this question was again broken into four parts: analysis of the content knowledge, analysis of the content understanding, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching geometry. The reliability for geometry assessment was found to be .973. An item analysis of the content of each item revealed an average item difficulty of 76% ($p = .76$) and item discrimination revealed two usually unacceptable items, three good items, and two excellent items. There were no items for which less than half of the participants answered correctly. Therefore, the three items with the smallest percentage of the participants answering correctly were investigated for commonalities. The first item addressed calculating the missing angle in a supplementary pair. The common error found was participants' lack of "plugging in" the x value in order to find the measure of the specified angle. In the item addressing complementary angles, the majority of those who answered this question incorrectly reported angle Q as the complementary angle to the given angle X. Angle Q is actually vertical to angle X. The third item, item four, addressed determining angle relationships when given two parallel lines cut by a transversal. A majority of the participants who answered this item incorrectly described the relationship between the two angles as equaling

180 degrees. This is partially correct, but the researcher was looking for the specific term, supplementary.

On the content understanding part, a majority of the participants scored a three. Overall, the explanations of the solutions for the geometry assessment revealed incorrect mathematical vocabulary. When dealing with parallel lines and a transversal, a majority of the students mixed up terminology for corresponding angles, and alternate interior and exterior angles. In addition it was noted in one item that several students thought complementary and congruency were analogous terms. Vertical angle terminology revealed that students often associated vertical angles with longitudinal directions. Other common terms used instead of vertical angles were opposite and diagonal. The last interesting and noteworthy trend across the content understanding in the geometry assessment was the use of the term linear pairs, often instead supplementary.

For the pedagogical understanding of geometry, a majority of the participants scored a three or a four. Of the students who mentioned directional associations in the explanation part, a majority just mentioned describing the definition of vertical angles to someone who did not understand. There was no additional explanation given as to what this definition might be. There was again an evident misconception that congruency and complementary are analogous terms. Across all understanding parts of the items on the geometry assessment there were several instances of exploration activities. In addition a majority of the responses contained some kind of definition or discussion of giving and/or explaining definitions necessary for students to understand the problems.

For the mathematics knowledge for teaching geometry, total scores ranged from 0 to 81, a range of 80, out of a possible 91 points.

Research Question 4

The analysis of this question was again broken into four parts: analysis of the content knowledge, analysis of the content understanding, analysis of the pedagogical understanding, and analysis of the mathematics knowledge for teaching probability and statistics. The reliability for the probability and statistics assessment was found to be .907. An item analysis revealed an average item difficulty of 76% ($p = .76$) and item discrimination revealed four usually unacceptable items and three good items. One item had less than half of the participants answer it correctly. This item was investigated for commonalities in the errors of the answers given. There were three common errors found in the item addressing determining the probability of dependent events. The majority of the participants who did not answer this question correctly simply forgot to reduce the fraction for their final answer. The other two types of errors, minimal but still evident in student answers, were the adding of the two probabilities and the treatment of the item as "with replacement" even though the problem directly states "without replacing the first cookie."

On the content understanding part of the probability and statistics assessment, a majority of the participants scored a three according to the rubric for this part of the assessment. There are a few items with minor errors in explanations of noteworthiness. The first is an item which addressed calculating the range for a given set of data. Of the students who incorrectly answered this problem, a majority of them indicated the range of data as an actual range (i.e., 73 – 97). Another common error revealed in the explanation was the ordering of the numbers. The numbers are presented in non-numerical order and instead of reordering them, the student took the last score minus the first score. In an item addressing reading and interpreting data represented graphically through a pictograph, although every participant who took the probability and statistics assessment got this answer correct, participants often misinterpreted three times as three more. On an item addressing predicting the outcome of an experiment a majority of the participants who missed this item misinterpreted the graphic representation given.

For the pedagogical understanding part of probability and statistics, a majority of the participants scored either a three or a four. These explanations tended to be very algorithmic in nature and the general method of instruction could be assumed to be direct since there was no mention of any other method. Responses at the score level of four tended to be more culturally responsive than a score of three. Analysis across pedagogical understanding of all items on the probability and statistics assessment revealed several trends of interest. The first had to do with demonstrating an item on a smaller scale first before doing the actual problem. Another theme was the use of hands-on material in order to conduct experiments. It was noted that few of these responses addressed theoretical probability versus experimental probability. If the understanding response did not include the use of hands-on material, it generally contained some sort of another representation of the item being addressed. The majority of representations presented were pictorial.

Mathematics knowledge for teaching probability and statistics scores ranged from 18 to 73, a range of 55, out of a possible 91 points.

Ancillary Question 1

Enrollment classification (adjusted) had a statistically significant ($p < .05$) contribution to number and operations content knowledge, content understanding, pedagogical understanding, and mathematics knowledge for teaching number and operations. The covariate, GPA, did not contribute significantly to pedagogical understanding of number and operations, but did have a statistically significant ($p < .05$) contribution on number and operations content knowledge, content understanding, and mathematics knowledge for teaching number and operations. Effect sizes were small to medium (Huck, 2004). Enrollment classification had a larger impact on pedagogical understanding and mathematics knowledge for teaching number and operations than on content knowledge and content understanding. GPA had a larger effect on content knowledge than on content understanding and mathematics knowledge for teaching number and operations. The largest effect was enrollment classification on mathematics knowledge for teaching number and operations.

Enrollment classification (adjusted) did not contribute significantly to algebra content knowledge, but did have a statistically significant ($p < .05$) contribution to content understanding, pedagogical understanding, and mathematics knowledge for teaching algebra. Enrollment classification and GPA together did not contribute significantly to pedagogical understanding or mathematics knowledge for teaching algebra, but did have a statistically significant ($p < .05$) contribution to content knowledge and to content understanding. Effect sizes were small. Enrollment classification had approximately equal effects on content understanding, pedagogical understanding, and mathematics knowledge for teaching algebra. The interaction of enrollment classification and GPA had approximately equal effects on content knowledge and content understanding.

The dependent variable content knowledge for geometry had to be removed from the analysis because it rejected the null hypothesis that the error variance of the dependent variable is equal across groups. Enrollment classification, GPA, and its interaction did not contribute significantly to any of the dependent variables concerning the mathematics knowledge for teaching geometry. Effect sizes were small.

Enrollment classification, GPA, and its interaction did not contribute significantly to any of the dependent variables concerning the mathematics knowledge for teaching probability and statistics. Effect sizes were small.

Ancillary Question 2

Cohort and the interaction of GPA and cohort did not contribute significantly to number and operations content knowledge, content understanding, or mathematics knowledge for teaching number and operations. However, GPA had a statistically significant ($p < .05$) contribution on number and operations content knowledge, content understanding, and mathematics knowledge for teaching number and operations. Effect sizes were small. GPA had approximately equal effects on content knowledge, content understanding, and mathematics knowledge for teaching number and operations.

Cohort did not contribute significantly to algebra content knowledge, content understanding, pedagogical understanding, or mathematics knowledge for teaching algebra. Cohort and GPA together did not contribute significantly to content understanding, pedagogical understanding, or mathematics knowledge for teaching algebra, but did have a statistically significant ($p < .05$) contribution to algebra content knowledge. Effect sizes were small. Cohort had a larger effect on mathematics knowledge for teaching algebra than on pedagogical understanding or content understanding.

GPA and the interaction between GPA and Cohort did not contribute significantly to any of the dependent variables concerning mathematics knowledge for teaching geometry. Cohort did not contribute significantly to geometry content knowledge, content understanding, or mathematics knowledge for teaching geometry, but did have a statistically significant (*$p < .05$)* contribution to pedagogical understanding. Effect sizes were small.

Cohort, GPA, and its interaction did not contribute significantly to any of the dependent variables concerning mathematics knowledge for teaching probability and statistics. Effect sizes were small.

Ancillary Question 3

Current enrollment in MASC 450 had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching number and operations. The covariate GPA also had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching number and operations. Effect sizes were small. The largest effect size was current enrollment in MASC 450 on mathematics knowledge for teaching number and operations. Taken courses revealed no statistical significance with regards to mathematics knowledge for teaching number and operations. The covariate GPA had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching number and operations. Effect sizes were small.

Current enrollment in MATH 365, MASC 450, and MEFB 497 all had statistically significant ($p < .01, p <.05, p<.05$, respectively) contributions to mathematics knowledge for teaching algebra. Effect sizes were small. The largest effect size was current enrollment in MATH 365 on mathematics knowledge for teaching algebra. MATH 403, which had already been taken, had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching algebra. Effect sizes were small.

Current enrollment in MATH 365 and MASC 450 both had statistically significant ($p < .01, p <.05$, respectively) contributions to mathematics knowledge for teaching geometry. Effect sizes were small to medium. The largest effect size was current enrollment in MATH 365 on mathematics knowledge for teaching geometry. Analyses on taken courses for mathematics knowledge for teaching geometry failed Levene's Test of Equality of Error Variances.

The interaction of Current MATH 368 x Current MATH 403 x Current MASC 351 x Current MASC450 together had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching probability and statistics. Effect sizes were small. The interaction Taken MASC 351 x Taken MASC 450 together had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching probability and statistics. In addition, the interaction of all courses together (Taken MATH 365 x Taken MATH 366 x Taken MATH 367 x Taken MATH 368 x Taken MATH 403 x Taken MASC 351 x Taken MASC450 x Taken MEFB 460 x Taken MEFB 497) had a statistically significant ($p < .05$) contribution to mathematics knowledge for teaching probability and statistics. Effect sizes were small to medium. The largest effect size was the interaction of the courses, taken previously, together on the mathematics knowledge for teaching probability and statistics.

Ancillary Question 4

Predictors for each component of mathematics knowledge for teaching across each content strand were saved during the MANCOVA to be used in the analysis of this question. A Pearson's r correlation run between the predictor and its corresponding component to be sure it was a "good" predictor revealed all of the correlations were significant at $p < .01$.

Since course information was dichotomous, each course was graphed as the independent variable against the predictor variable of mathematics knowledge for teaching (dependent variable) and the trendline was used to determine the slope, or rate, for each course in its corresponding content area. The rates were then compared to the rates of the mathematics knowledge for teaching each content strand. The course having the most impact, according to their rates, across all content areas was MEFB 460 Teaching mathematics in the middle grades.

Nonlinear regression was used to determine a model of fit for enrollment characteristics onto the predictor variables for mathematics knowledge for teaching. Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching number and operations was found to be quadratic (R^2=*.495)*. Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching algebra was found to be cubic (R^2=*.550)*. Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching geometry was found to be cubic (R^2=*.040)*. Enrollment characteristics onto the predictor variable for mathematics knowledge for teaching probability and statistics was found to be quadratic (R^2=*.583)*.

Finally, cohorts were investigated through nonlinear regression to determine a model of fit for cohorts onto the predictor variables for mathematics knowledge for teaching. Cohorts onto the predictor variable for mathematics knowledge for teaching number and operations was found to be quadratic (R^2=*.191)*. Cohorts onto the predictor variable for mathematics knowledge for teaching algebra was found to be linear (R^2=*.411)*. Cohorts onto the predictor variable for mathematics knowledge for teaching geometry was found to be quadratic (R^2=*.034)*. Cohorts onto the predictor variable for mathematics knowledge for teaching probability and statistics was found to be quadratic (R^2=*.244)*.

CHAPTER V

DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

Discussion

Initially, normality was assessed for each content strand (algebra, number and operations, probability and statistics, and geometry) because it was an underlying assumption that needed to be met when using parametric analyses. It was concluded that the scores were normally distributed. Therefore, the researcher was able to conduct parametric analyses on the data.

Mathematics Knowledge for Teaching Number and Operations

The reliability of the assessment was found to be comparable to other standardized assessments. This was to be expected because the content questions were taken from a reputable standardized test and the content was not changed. The item analysis revealed an item difficulty of moderately difficult and the item discrimination revealed three usually unacceptable items, three good items, and one excellent items. This is good news and bad news. The bad news is that there were three unacceptable items on the test and therefore these three items should be looked at and modified or thrown out all together. However, item analysis is based on the number of correct responses. The questions given were taken from a middle grades standardized assessment; therefore, the content questions *should* have been easy for the participants to answer.

Common errors in answering number and operations content questions included leaving the coefficient of a number in scientific notation greater than or equal to ten, and not changing the exponent when moving a decimal left or right when dealing with scientific notation. Another common reason for incorrect answers was failure to estimate, specifically when the directions directly asked for estimation. These responses could indicate a lack of understanding of the properties of scientific notation; however, it could also just be they did not look over their answer carefully before recording it. The failure to estimate could have been a failure to read the instructions carefully or could indicate a lack of understanding when it comes to proper estimation strategies. Several of the students in the content understanding part indicated they estimated the answer *after* they performed the actual operation. This indicates that the failure to estimate and to estimate properly could definitely be a lack of understanding. The purpose of estimation is get an idea of what your answer should be without having to do any *burdening* mathematics.

A majority of participants scored a two or a three on the content understanding part. A score of two generally meant the participant had a mathematically sound procedure; however, their answer did not match what the problem was wanting. A score of three generally meant the participant just explained their exact algorithm or mental mathematic procedure for the given item. A score of three indicated the participant's did nothing above and beyond what was asked of them. It could argued, however, that explaining is *not* simply writing out an algorithmic procedure. However, if the solution was correct and was mathematically sound, the student had to receive a minimum score of three according to the rubric. This could be the fault of the rubric.

Misuse of vocabulary was noted on two other items in the number and operations assessment. A common error on one item was adding together two given percents in an original price and then a sale price situation. This is a common error often found in sale price situations. The students think they can shorten the amount of work they do by simply adding the two percents together. They do not realize one discount applies to the original price and the other discount applies to the sale price. One way to overcome this error is to give an example involving percentages that add up to 100. Then, when the participants would have added the percents together, they would have seen that the shirt would have "cost" them $0. This cannot be, so they would have to go through and think about what the problem was asking them to do. The other common error was the mixing up of greatest common factor and least common multiple. Even when the participants got the answer right, several used the word factor instead of multiple. These two terms, factor and multiple, are especially important in instruction in the middle grades. The greatest common factor is found when adding and subtracting fractions in number and operations as well as algebra settings. Least common multiple is a term introduced around the fifth grade and used throughout the rest of the participant's schooling. A way to show the proper relationship between factors and the least common multiple is to find the prime factorization of all the numbers given and then find the least common multiple based on the number of times a factor occurs in each number. Only two participants on the number and operations assessment explained their solution in this way.

For the pedagogical understanding of number and operations, a majority of the participants scored a two, three, or four. A score of two generally meant the procedure was somewhat correct, but contained minor mathematical errors. A score of three generally indicated the student could successfully explain their own procedure to a person who did not understand. A score of four generally indicated the student could do more than just explain their own procedure. They could relate their procedure back to different parts of the problem and could often explain why they got a certain number or conducted a certain procedure. Scores of two and three on the pedagogical understanding part tended to be very algorithmic in nature and the general method of instruction could be assumed to be direct since there was no mention of any other method. With a score of four, there was often an indication of some culturally responsiveness in the explanation. Scores of two and three are a little discouraging because participants *should* be able to explain their procedures a little more thoroughly, especially knowing they are going to have to teach the concept or item to someone who did not understand. It is encouraging however, that they can at least explain their own procedure correctly to a student. Scores of four on this part were encouraging since this indicated a little bit of cultural responsiveness in the explanation as well as going beyond just explaining their own algorithmic procedure.

Although the scores reported for the mathematics knowledge for teaching number and operations could initially be discouraging to the reader, it is important to not associate letter grades with these percentages. The mean was used to calculate the averages and it is important to remember the students are all at various places in their program. Since this study is the first of its kind, there can be no comparisons made. Instead, these numbers are setting the bar for future studies.

Mathematics Knowledge for Teaching Algebra

The reliability of the assessment was found to be comparable to other standardized assessments. This was to be expected because the content questions were taken from a reputable standardized test and the content was not changed. The item analysis revealed an item difficulty of moderately difficult and the item discrimination revealed three usually unacceptable items, and four good items. This was good news and bad news again. The bad news is that there were three unacceptable items on the test and therefore these three items should be looked at and modified or thrown out all together. However, item analysis is based on the number of correct responses. The questions given were taken from a middle grades standardized assessment; therefore, the content questions *should* have been easy for the participants to answer.

There was only one item on the algebra assessment for which less than half of the participants answered correctly. The reason for the higher scores in the area of algebra could be from the concentration on algebra in often found in high school and early college courses in mathematics. Participants had trouble translating verbal sentences into algebraic equations. Common errors included simply not following the directions of the item and misinterpretation of the given information. In the item, several participants solved an inequality when they were just asked to set it up. Many of their answers were correct; however, it was not the information the item was looking for. A majority of students who missed this item though misinterpreted the given information regarding cost and profit. The students added a fee to a profit instead of subtracting it out. This could be from a lack of problem solving skills of the participants. Reading items carefully and carefully extracting all given information properly are essential to effective teaching in the middle grades. Misinterpretation of given information showed up again in another item, although over fifty percent of the participants answered the question correctly. Another noteworthy item dealt with the factoring of trinomials. Of the students who missed this item, a majority switched their signs in the parentheses. Using the distributive property, participants could have checked their answer. Checking your answer is an essential part of the problem solving process and is often the most skipped over part. Several of the participants stated they multiplied their answers back out to check their answer. Interestingly, a majority of the participants who had switched their signs and used the distributive property to check their answer got the original problem.

A majority of the participants scored either a two or a three on content understanding for algebra. A score of two generally meant the student had a mathematically sound procedure; however, their answer did not match what the problem was looking for. A score of three generally indicated the student just explained or algorithmically showed their exact mathematic procedure for the given item. These majority scores indicate students know *how* to do the algorithmic procedures needed to be successful in algebra. However, they may not understand why they are doing these procedures or even understand *what* they are doing when they complete an algorithmic sequence.

Interestingly, the noteworthy area for the algebra assessment for content understanding was the translating of verbal sentences into algebraic equations again. A common theme across all explanations for these items was "just follow the words…they tell you what to do." Again, this could be tied back to problem solving processes. Problem solving is an essential component of middle grades mathematics curriculum and is a strand in the NCTM (2000) standards. Participants take a course in problem solving (MASC 351) during the sophomore or junior year, so this class should help with problems and explanations such as these.

The other two items of interest complemented each other in that one addressed multiplying binomials together and the other address factoring trinomials. The common explanation for content understanding in the item addressing the multiplication of binomials was "I just used FOIL to multiply it out and then I added like terms." FOIL is a process which is supposed to aid students in using the distributive property to multiply two binomials together. However, from this example, one can see it is evident the participants memorized the procedure and may not understand what they doing. It was noted there was a lack of the term distributive property in a majority of the answers. FOIL, or other methods such as the box method, is taught as early as sixth grade and engrained into students' minds all through high school and even on into college. There are alternative procedures to teaching the multiplication of binomials that the participants are taught during the middle grades program at the site of the study, including algebra tiles. When it came to factoring the trinomials, the FOIL method backwards was the most commonly chosen method. Several students even went so far as to call factoring "FOILing." The most notable thing about their content understanding explanations was the fact the participants concentrated on explaining how the factors of the last term were combined to find the middle term. There was little or no concentration on where the first term came from.

For the pedagogical understanding of algebra, a majority of the participants scored a two or a three. Scores of two and three tended to be very algorithmic and procedure-oriented in nature. A score of two generally meant the procedure was somewhat correct, but contained minor mathematical errors or had missing parts to the item. A score of three generally indicated the student could successfully explain their own procedure to a person who did not understand. However, their explanation did not go beyond their own algorithmic procedure. The general method of instruction could be assumed to be direct since there was no mention of any other method. Across all the pedagogical understanding responses, there was a great indication of cultural responsiveness than was found on the number and operations items.

Themes noteworthy for the pedagogical understanding of algebra dealt with the teaching of the concepts associated with the multiplication of binomials and the factoring of trinomials. Again, FOIL, forwards and backwards, was a common theme in the responses. It was again noted there was an extreme concentration on finding the two second terms from the last term of the trinomial by factoring it and finding combinations of numbers to give one the middle term. There was little or no concentration on how and why the parentheses were split and what happened with the y^2 term. This is most likely because this was the only way the participants were taught and they are not aware of anything else. However, in the upper level courses in education, there is a focus on different methods of instruction. An example of this is the use of algebra tiles to demonstrate multiplication of binomials and then factoring of trinomials. Only one participant suggested algebra tiles. A few responses indicated the use of Base-10 blocks to model these procedures. However, Base-10 blocks are not effective because they all fit together. Algebra tiles have an odd length for the x -bar and the unit squares do not fit evenly across the bar. This is to help represent x as an unknown. Base-10 blocks cannot do this.

Mathematics knowledge for teaching algebra scores overall could again be discouraging. However, it is important to not assign letter grades with these percentages. The mean was used to calculate the averages and it is important to remember the students are all at various places in their program. Since this study is the first of its kind, there can be no comparisons made. Instead, these numbers are setting the bar for future studies.

Mathematics Knowledge for Teaching Geometry

The reliability of the assessment was found to be comparable to other standardized assessments. This was to be expected because the content questions were taken from a reputable standardized test and the content was not changed. The item analysis revealed an item difficulty of moderately difficult and the item discrimination revealed two usually unacceptable items, three good items, and two excellent items. This is again good news and bad news. The bad news is that there were two unacceptable items on the test and therefore these two items should be looked at and modified or thrown out all together. However, item analysis is based on the number of correct responses. The questions given were taken from a middle grades standardized assessment; therefore, the content questions *should* have been easy for the participants to answer.

There were no test items for which less than half of the participants answered correctly. There were some interesting commonalities on items that were missed though. The first common error found was participants' lack of "plugging in" the *x*-value in order to find the measure of a specified angle after using algebra to find the missing value. This error often happens when there is more work to be done *after* an equation has been solved. Participants need to be sure to read the problem carefully, apply problem solving techniques every time, and be sure they are answering what the problem is asking for. On an item identifying complementary angles, a majority of the students who incorrectly answered the problem identified vertical angles instead of complementary angles. Further analysis, discussed later, revealed that these participants are thinking complementary and congruent are analogous terms. On another item, participants who incorrectly answered the item described a relationship between two angles as equaling 180 degrees instead of supplementary.

On the content understanding part of geometry, a majority of the participants scored a three. A score of three generally indicated the student just explained their exact algorithmic or mental mathematic procedure with no additional explanations given for the item. The procedures were mathematically correct and their solution was correct. Overall, the explanations of the solutions for the geometry assessment revealed incorrect vocabulary. When dealing with parallel lines and a transversal a majority of the students who incorrectly answered these items mixed up the terminology for corresponding angles, alternate interior angles, and alternate exterior angles. There were also several students who thought complementary angles were congruent angles. Vertical angle terminology revealed that students often associate vertical angles with longitudinal directions. Other common terms used instead of vertical angles were opposite and diagonal. The researcher is not sure why there was so much confusion with the vocabulary associated with the geometry items. Vertical is a directional term in everyday life and the participants may have not seen geometry material since their early years of high school. So the participants may have forgotten much of the terminology and relationships associated with geometry. This however is not good because these terms are very important to middle grades geometry. With the way the high school mathematics courses are sequenced, most middle grades students will not see geometry again until their sophomore or junior years of high school.

The last and interesting noteworthy trend across the content understanding in geometry was the use of the term linear pairs, often instead of supplementary. A linear pair is a term the participants learn in one of their mathematics courses introducing abstract mathematics and different methods of proving problems. A linear pair is a conjecture in geometry. A linear pair is formed when two lines intersect. Two angles are said to be linear if they are adjacent angles formed by two intersecting lines. The measure of a straight angle is 180 degrees, so a linear pair of angles must add up to 180 degrees. Two angles are called supplementary angles if the sum of their degree measurements equals 180 degrees. So, comparing the two words, linear pair is a specific instance where adjacent angles add up to 180 degrees whereas supplementary angles describe the relationship between two angles whose sum is 180 degrees. The angles do not necessarily have to be adjacent.

For the pedagogical understanding of geometry, a majority of the participants scored a three or a four. A score of three indicated the student could successfully explain their own procedure to a person who did not understand. However, they did no more than explain their exact procedure. A score of four indicated the student could go just beyond the explanation of their own procedure, many times adding terms and definitions associated with the problem or providing a basic definition of why they did what they did. Of the students who mentioned directional associations in the content understanding part, a majority just mentioned describing the definition of vertical angles to someone who did not understand. There was no additional explanation as to what this definition might be. Although it cannot be stated for sure, but it can be deduced those students who incorrectly associated vertical angles with directions would pass on this information to their students. A couple of participants even went so far as to say "horizontal angles" or even to bring a map in to help students out with directions. Of the participants who incorrectly associated complementary and congruent, their pedagogical understanding revealed similar explanations to someone who did not understand.

Across all pedagogical understanding of the items on the geometry assessment there were several instances of exploration activities. Most of the activities involved providing actual measurements and protractors to students so they could measure the angles and the come up with their own idea of the relationships between angles (i.e., supplementary, complementary, corresponding, etc.). The use of these kinds of activities promotes cultural responsiveness in a classroom, allows for student-centered learning, and can aid in practicing problem solving processes.

Mathematics knowledge for teaching geometry scores overall could again be discouraging. However, it is important to not assign letter grades with these percentages. The mean was used to calculate the averages and it is important to remember the students are all at various places in their program. Since this study is the first of its kind, there can be no comparisons made. Instead, these numbers are setting the bar for future studies.

Mathematics Knowledge for Teaching Probability and Statistics

The reliability of the assessment was found to be comparable to other standardized assessments. This was to be expected because the content questions were taken from a reputable standardized test and the content was not changed. The item analysis revealed an item difficulty of moderately difficult and the item discrimination revealed four usually unacceptable items and three good items. This is good news and bad news. The bad news is that there were four unacceptable items on the test and therefore these four items should be looked at and modified or thrown out all together. However, item analysis is based on the number of correct responses. The questions given were taken from a middle grades standardized assessment; therefore, the content questions *should* have been easy for the participants to answer.

One item on the probability and statistics assessment has less than half of the participants answer it correctly. This item was investigated for commonalities in the errors of the answers given. The majority of the participants who did not answer the question correctly simply forgot to reduce the fraction for their final answer. This error can again be associated with problem solving practices. It is very important to double check the answer to make sure it is accurate and makes sense. Checking an answer at the end can catch minor errors such as reducing fractions. The other two types of errors, minimal but still evident in student answers, were the adding of the two probabilities and the treatment of an item concerning with or without replacement. Errors such as these reveal a misunderstanding in the concepts of independent and dependent events.

On the content understanding part of the probability and statistics assessment, a majority of the participants scored a three. A score of three generally indicated the student just explained their exact algorithmic or mental mathematic procedure for the given item. The procedures were mathematically correct and their solution was correct; however, there was no further explanation provided other than the algorithmic procedure. There were a few minor errors noteworthy on the probability and statistics assessment. The first was an incorrect form of stating the range of a data set. Of the students who incorrectly answered this problem, a majority of them indicated the range of the data as an actual range of numbers (i.e., 73 – 97) instead of calculating the range by subtracting the highest and lowest scores. This could come from real life experiences. In real life, the range is usually reported, "the test scores *ranged* from 73 to 93." Participants could have confused this with the definition of range in probability and statistics. Another common error on this item was the ordering of the numbers. Several participants subtracted the first number from the last number. It could perhaps be assumed that the participants just *forgot* to reorder the items, but it could also be assumed that the participants think the range is the difference between the first and last scores.

Probably one of the most interesting items out of all four assessments was the item addressing reading and interpreting data represented graphically through a pictograph. This item is the only item out of all four assessments that received 100% correct responses. However, in comparison with the explanations, it was revealed that although the participants had given the correct answer, their explanations were not correct. The most common error was interpreting "three times greater" as "three more." This could be from misunderstanding of the common vocabulary associated with multiplication and addition.

Another common error found was the misinterpretation of a graphic on the assessment. The participants went by the number of colors of the cards instead of the words that were written on the cards. This again falls back the problem solving process reading the problem carefully and going back to check one's answer to make sure it makes sense.

For the pedagogical understanding of the probability and statistics assessment, a majority of the students scored either a three or a four. A score of three generally indicated the student could successfully explain their own procedure to a person who did not understand. These explanations tended to be very algorithmic in nature and the general method of instruction could be assumed to be direct since there was no mention of any other method. A score of four on the understanding part indicated the student could do more than just explain their own procedure. They could relate their procedure back to different parts of the problem and could often explain why they got a certain number or conducted a certain procedure. Responses at this level tended to be more culturally responsive than a score of three. Again, a reason for similar procedures being explained to their own could be because they do not understand any other way or method to do the problem. It also could have to do with how they were taught probability and statistics items when they were in school.

Analysis across pedagogical understanding of all items on the probability and statistics assessment revealed several trends of interest. The first had to do with demonstrating an item on a smaller scale. This is often a recommended strategy for developmentally teaching a concept. Another theme was the use of hands-on material in order to conduct experiments. The participant responses wanted their students to conduct the experiments in order to have a better understanding of what the problem was asking for. This use of hands-on material and various methods of instruction are encouraging because it helps to promote cultural responsiveness in the classroom. However, the researcher noticed there was little or no connection of experimental probability to theoretical probability. This is an important concept to convey to students because the experiments conducted in the classroom may or may not be close to the theoretical probability of that experiment. If the use of hands-on material was not mentioned, the pedagogical response generally contained some sort of other representation of the item being addressed. The most common representation of these was pictorial. Although this method is not as culturally responsive as the hands-on method, it at least gives students another way to look at a problem.

Mathematics knowledge for teaching probability and statistics scores overall could again be discouraging. However, it is important to not assign letter grades with these percentages. The mean was used to calculate the averages and it is important to remember the students are all at various places in their program. Since this study is the first of its kind, there can be no comparisons made. Instead, these numbers are setting the bar for future studies.

Effect of Enrollment Classification on Mathematics Knowledge for Teaching

Enrollment classification (freshman, sophomore, junior, senior) had a statistically significant contribution to number and operations content knowledge, content understanding, pedagogical understanding, and mathematics knowledge for teaching number and operations. Although enrollment classification did not contribute significantly to algebra content knowledge, it did have a statistically significant contribution to algebra content understanding, pedagogical understanding, and mathematics knowledge for teaching algebra. There was no interactions or statistical significance found in the areas of geometry and probability and statistics. It makes sense that enrollment classification would contribute to mathematics knowledge for teaching since mathematics knowledge for teaching should theoretically increase as one advances in courses and classification. A reason for no contribution for content knowledge could be because the student came into college with the same algebra content knowledge being taught in the courses, therefore there was nothing new to learn. Probably the main reason for no significant contributions in geometry was because the content knowledge failed the homogeneity of variance test. Even though after content knowledge was removed from the analysis, there was a significant decrease from R^2 to adjusted R^2 for each part of the geometry indicating a loss of power. In the case of pedagogical understanding, everything and more was lost. The case was similar for the probability and statistics strand. A larger sample size could perhaps help get more power to run the statistical analyses. Effect sizes were small across each analyses, especially among probability and statistics and geometry. However, there is no interpretation for the effect sizes because there are no studies to compare this one to.

Cohort Development Differences on Mathematics Knowledge for Teaching

Cohort type did not contribute significantly to mathematics knowledge for teaching number and operations or any of its parts. However, in this analysis, GPA, the covariate, had a statistically significant contribution on number and operations content knowledge, content understanding, and mathematics knowledge for teaching number and operations. This could be because much of the content addressed in number and operations is learned by the participants in the middle grades and then again in high school. There is very little specific focus on this strand in college mathematics courses. Therefore, students will have already come in knowing what they need to know to perform well on the number and operations assessment. Cohort did not contribute significantly to algebra content knowledge, pedagogical understanding, and mathematics knowledge for teaching algebra. Cohort did not have a statistically significant contribution to geometry content knowledge, content understanding, or mathematics knowledge for teaching geometry, but did have a statistically significant contribution to pedagogical understanding. This makes sense because as one advances through the cohorts, the more education-related classes (i.e., MASC 351) are integrated into the coursework. By the end of the last cohort, one should be an expert in middle grades mathematics, especially since they will be going out and teaching the middle grades. As cohort level increases, the more pedagogy is integrated. In fact in the last cohort, participants were either in their methods block and spending three days a week in the classroom, or student teaching. Again, effect sizes were small, but there was nothing to compare them to. Cohort did not contribute anything to mathematics knowledge for teaching probability and statistics or any of its parts. Effect sizes were small. On the mathematics knowledge for teaching probability and statistics and mathematics knowledge for teaching geometry there was a loss of predictive power to where the adjusted R-squared was negative. This indicates the analysis essentially lost what it had to begin with and lost more in the process. Again, an increase in sample size could help counteract the loss of power.

Impact of Various Types of Courses

Current enrollment in MASC 450 had a statistically significant contribution to mathematics knowledge for teaching number and operations, mathematics knowledge for teaching algebra, and to mathematics knowledge for teaching geometry. MASC 450 is a special middle grades mathematics course developed to help students bridge their mathematics knowledge to mathematics pedagogy. It is very encouraging to see this course contribute to mathematics knowledge for teaching for three of the four content areas. Current enrollment in MATH 365 had a statistically significant contribution to mathematics knowledge for teaching algebra and to mathematics knowledge for teaching geometry. MATH 365 is a course designed for elementary and middle grades teachers which concentrates on advanced number and operations and algebra concepts. It was interesting that current enrollment in MATH 365 contributed to mathematics knowledge for teaching geometry. Perhaps this course built a good foundation for the information asked of the participants in the geometry assessment. Current enrollment in MEFB 497 had a statistically significant contribution to mathematics knowledge for teaching algebra. Students enrolled in MEFB 497 the semester of the study were student teaching. A reason for this significant contribution could be the topics the student teacher experienced during their student teaching experience; perhaps there was more of a concentration on algebraic concepts during the second semester. The interaction of current enrollment in MATH 368 x MATH 403 x MASC 351 x MASC 450 had a statistically significant contribution to mathematics knowledge for teaching probability and statistics. These courses were chosen together because a junior student is likely to take all of them in one semester or within a semester of each other. These courses demand higher levels of thinking than previous courses and force the students to use their problem solving abilities in order to be successful in the courses. Probability and statistics takes often takes a lot of higher order thinking skills in order to make sure one can correctly predict the experiment or actually conduct the experiment. Effect sizes were small for all of the above mentioned interactions.

Math 403, which had already been taken, had a statistically significant contribution to mathematics knowledge for teaching algebra. MATH 403 is a mathematics and technology course geared to middle grades teachers. Many of the technologies used in this course use algebra and geometry concepts for exemplars, so it is encouraging to see this course contribute to the mathematics knowledge for teaching algebra. The interaction of MASC 351 x MASC 450, which have both already been taken, had a statistically significant contribution to mathematics knowledge for teaching probability and statistics. These two courses were often taken together this semester and last semester. It is likely to happen more in the future because the placement of MASC 450 is changing. Both courses focus on integrating mathematics knowledge with mathematics pedagogy, and promote higher order thinking skills. Higher order thinking skills are essential in probability and statistics; therefore, it is encouraging to see the interaction of these two courses, which have already been taken at the time of the assessment, contribute to participants' mathematics knowledge for teaching probability and statistics. The interaction of all courses together, which have already been taken, had a statistically significant contribution for mathematics knowledge for teaching probability and statistics. Essentially, this is telling us that the specified middle grades content courses, once taken, can significantly contribute to mathematics knowledge for teaching, specifically probability and statistics. This is definitely encouraging since this is the goal of the current middle grades program.

Development in Certain Stages of the Middle Grades Program

Predictors for mathematics knowledge for teaching each content strand were found to be statistically significantly correlated with its corresponding component, mathematics knowledge for teaching each content strand. Comparing the rates of each course, taken and currently enrolled, against the rates for the predictor for mathematics knowledge for teaching each content strand revealed MEFB 460 having the most impact across all content strands. MEFB 460 is the methods course middle grades teachers take the semester before they student teach. In addition, the preservice teachers are in middle grades schools three days a week for observations of classroom teachers. It is encouraging to see this course making an impact on mathematics knowledge for teaching since it is the last "real" course the participants take before going to student teach. It is also a course where mathematics pedagogy is more of a concentration than mathematics content.

A model of fit for enrollment characteristics for the predictor variables for mathematics knowledge for teaching in each content area was determined. Mathematics knowledge for teaching number and operations was found to be quadratic. There was a dip in scores at the second semester juniors. This could be because of the type of courses the participants take their junior year. The courses include an abstract mathematics course, a

Euclidean Geometry course, and both MASC courses. These are courses requiring higher level thinking and the students are also taking science courses with their labs in order to fulfill their science certification requirements. These courses also do not have much concentration, if any, on number and operations concepts. Mathematics knowledge for teaching algebra was found to be cubic. There was an evening of the curve around the junior year and a steep increase during the senior year. The steep increase was encouraging and should theoretically happen because this is when the participants are taking their methods courses and going out into the schools. Mathematics knowledge for teaching geometry was found to be cubic. Although the model was cubic, the picture itself revealed very little change in the scores across each semester. However, there was a slight increase in the last semester of the participants' second semester senior year. This is again encouraging since this shows that participant's mathematics knowledge for teaching geometry increased as the participants progressed through their degree programs. Mathematics knowledge for teaching probability and statistics was found to be quadratic. There was again a slight decrease in scores around the junior year and then a steep increase in scores during their senior year. Again the increase in scores during their senior year fits the theoretical model of increasing mathematics knowledge for teaching as participants progress through their degree programs.

A model of fit for cohort for the predictor variables for mathematics knowledge for teaching in each content area was determined. Mathematics knowledge for teaching number and operations was found to be quadratic. The picture appears to be linear; however there was a slight dip in scores on the second cohort, which were the MASC courses. The trend otherwise was increasing. Again, the theoretical model would assume increasing mathematics knowledge for teaching with each cohort. According to this model, there seems to be a slight decrease in scores during the MASC courses indicating a negative effect on mathematics knowledge for teaching number and operations on cohort two. Mathematics knowledge for teaching algebra was found to be linear. The scores steadily increased with each cohort, which follows the theoretical model of increasing mathematics knowledge for teaching with each cohort. Mathematics knowledge for teaching geometry was found to be quadratic. However, instead of a slight decrease in scores during cohort 2, there was a slight increase in scores. This indicates the MASC courses had a more positive impact on geometry scores than the other two cohorts did. Mathematics knowledge for teaching probability and statistics was found to be quadratic. There was a gradual increase in scores across each cohort with a steeper grade in cohort 3.

Conclusions

Although average mathematics knowledge for teaching scores were low among individuals and there were several indications of student misunderstandings and misinterpretations, there was a general indication of increasing mathematics knowledge for teaching across cohorts and across enrollment characteristics. Again the purpose of this study was diagnostic and to investigate growth, not to establish benchmarks or norms. It was noted, however that there was a noticeable dip in mathematics knowledge for teaching scores during the participants' junior semesters of courses. This could be from a number of factors including type of courses and course loads during the junior year. Hill, Rowan, and Ball (2005) found teachers' mathematics preparation positively predicted student gains in the third grade. Although their results were not statistically significant ($p < .06$), there was enough evidence to note this positive prediction. MASC 450 and MEFB 460 were two courses found to contribute the most to mathematics knowledge for teaching algebra, geometry, number and operations, and probability and statistics.

Several other studies have found content understanding and pedagogical understanding to be weak among teachers. In the late 1980s, the National Center for Research on Teacher Education found elementary and secondary teachers were unable to explain their reasoning or why the algorithms they used worked (RAND, 2003). Instead, the exhibited a rule-bound sense of understanding. This rule-bound sense of understanding reflects the nature of teaching and curriculum teachers experienced in elementary and secondary schools (RAND, 2003). Although many of the studies (e.g., Eisenhardt, Borko, & Underhill, 1993; Even, 1990; Graeber & Tirosh, 1991; Ma, 1999; Simon, 1993; Wheeler & Feghali, 1993) revealed "right answers" by their participants, the participants lacked an understanding of the meanings behind their procedures or their solutions.

Although this study was conducted at one site focusing on one middle grades mathematics specialist program, these results should be able to be generalized to other programs that follow such a model.

Recommendations (Issues for Further Investigation)

The need for further research concerning mathematics knowledge for teaching is more essential than ever. Preliminary research has been conducted and published at the elementary grade level; however, there is a lack of literature and research concerning mathematics knowledge for teaching at the middle grade level. This study was only a preliminary study that highlighted the beginnings and ends of preservice middle grades teachers' mathematics knowledge for teaching. Research on this mathematics knowledge for teaching of preservice middle grades teachers serves as a foundation for future studies.

Effect sizes found in this study ranged from small to medium and should be comparable across future studies.

It is recommended that research continue on this very important topic by first conducting a longitudinal study involving the same assessments and format. Results from the longitudinal study should be compared to the initial results found in this study. Analyses or future studies varying the weights of the items on the rubric should be conducted in order to see if the weights on the parts and criteria for mathematics knowledge for teaching significantly change. Including more items, randomly assigned, for a bigger sample population would allow for more depth in studying preservice teachers' mathematics knowledge for teaching, and encourage more volunteer student participation by decreasing the number of items given to one student. Including inservice teachers in a study with the preservice teachers would be essential to forming growth models and to study the increases in mathematics knowledge for teaching, especially during the first two years teaching. Additional items including the two other components (see Figure 1) of Ball's (2006) mathematics knowledge for teaching model should be included in the assessment in order to better compare results to that of Ball and her staff at the University of Michigan. Finally, continued research on the middle grades mathematics specialist program should continue in order to produce more sound results that would enable the study site to continue to improve their program so as to produce middle grades teachers whose mathematics knowledge for teaching any content or process strand is exemplary.

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APPENDIX A

MATHEMATICS KNOWLEDGE FOR TEACHING THE MIDDLE GRADES: A PRESERVICE TEACHER ASSESSMENT—VERSION 1

Mathematics Knowledge for Teaching the Middle Grades of Preservice Teachers Questionnaire

Although this questionnaire is a required portion of this course, if you DO NOT wish to have your scored included in the study, please indicate so below. No adverse actions will be taken against you or your grades if you choose this option. You will still participate in all the same tests, assignments, and other classroom activities as the rest of the class.

- I agree to participate in this study
- I do **NOT** agree to participate in this study

By clicking to agree, you are agreeing to the consent form from the previous page. (If you would like to read it again, please use the back button and then click on take test and start again. You can also print the consent form from the previous screen).

I have read and understand the explanation provided to me. I have had all my questions answered to my satisfaction, and I voluntarily agree to participate in this study. I have been given a copy of the consent form (you may print off the consent form from the previous page).

Any questions can be directed to Margaret Mohr (runmohr@tamu.edu).

A calculator is not necessary, but may be helpful. You are welcome to use a calculator on this questionnaire, but please do not use any other resources (including other people and/or websites)—the results of this questionnaire will NOT average into your class grade. Instead, you will receive credit for completing the questionnaire as designated by your teacher. We just want to know what you remember off the top of your head.

Your answers will not be recorded until you submit the questionnaire. If you choose not to be a part of this study, your answers will be discarded, but you will still receive credit for completing the questionnaire for the purposes of a completion grade for your course. Please note, you MUST submit the questionnaire in order to receive completion credit.

Thank you for your time and effort! We really appreciate your help!

Name: ______________________

Course: [ ]

Instructor: [ ]

Background Information

What is your gender?
- Male
- Female

What is your current grade level?
- Freshman
- Sophomore
- First Semester Junior
- Second Semester Junior
- First Semester Senior
- Second (or more) Semester Senior
- Other

Which of the following best describes your major?
- Education—Early Childhood
- Education—Gr. 4-8 Language Arts/Social Studies
- Education—Gr. 4-8 Math/Science
- Other

What is your ethnic background?
- American Indian or Alaskan Native
- Asian or Pacific Islander
- African American
- Hispanic/Latino
- White (Non-Hispanic)
- Other or prefer not to answer

From the following list of courses, please indicate which ones you have taken:
- ❑ MATH 365
- ❑ MATH 366
- ❑ MATH 367
- ❑ MATH 368
- ❑ MATH 403
- ❑ MASC 351
- ❑ MASC 450
- ❑ MEFB 460
- ❑ MEFB 497

From the following list of courses, please indicate which ones you are **currently** enrolled in **this semester**:

- ❑ MATH 365
- ❑ MATH 366
- ❑ MATH 367
- ❑ MATH 368
- ❑ MATH 403
- ❑ MASC 351
- ❑ MASC 450
- ❑ MEFB 460
- ❑ MEFB 497

What is the highest level of mathematics you took in high school and what grade did you receive? If you do not remember your grade in this course, please report the course and just omit the grade.

Course: [ ] Grade: A, B, C, D, F, --

Please indicate the letter grade you received in each of the following courses. If you did not take a course listed, please just omit it from the list

A, B, C, D, F, Did Not Take	MATH 142
A, B, C, D, F, Did Not Take	MATH 131
A, B, C, D, F, Did Not Take	MATH 166
A, B, C, D, F, Did Not Take	STAT 303

Content Questions

1. Simplify the expression below.

$$\left(3x^2y - 5xy + 12xy^2\right) - \left(5xy^2 + 4xy\right)$$

NOTE: Please use ^ to indicate powers where necessary.

Answer: [ ]

Please explain your answer:

[ ]

How would you explain, model, and/or demonstrate this item to someone who did not understand?

2. Hank sells toy cars on a web site. The web site fee is $30. Hank sells each toy car for $4. Set up an inequality for Hank to use to determine how many toy cars, *c*, he must sell to make a profit of **at least** $50.

   Note: Please use <= to indicate "less than or equal to" and >= to indicate "greater than or
   equal to".

   Answer:

   Please explain your answer:

   How would you explain, model, and/or demonstrate this item to someone who did not understand?

3. Linda must calculate the cost of filling her car's 12-gallon gas tank. She calculates the difference between how much gasoline her gas tank will hold and the number of gallons of gas, *g*, already in the tank. Then she multiplies the difference by the price, *p*, of one gallon of gas. Set up an expression for Linda to use to calculate the cost to fill her gas tank.

   Answer:

   Please explain your answer:

   How would you explain, model, and/or demonstrate this item to someone who did not understand?

4. Multiply the expression below.

$(3x - 5)(2x - 8)$

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

5. Write an equation that represents "eight less than twice a number is forty-two."

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

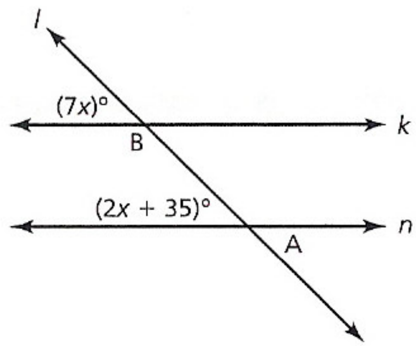
6. Factor $y^2 + 3y - 18$ into two binomials.

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

7. In the figure below, lines *k* and *n* are parallel. Line *l* is a transversal.



[not drawn to scale]

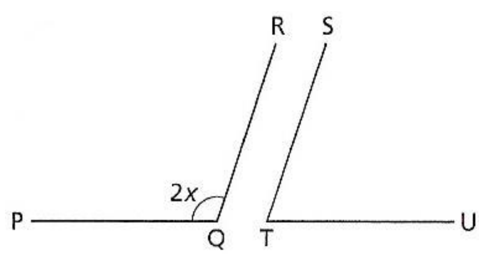
What is the value of *x*?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

8. The angles shown below are supplementary. The measure of $\angle PQR$ is $2x$.



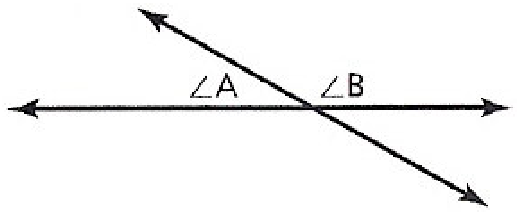
Please write an expression that represents the measure of $\angle STU$.

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

9. $\angle A = x + 2$ *and* $\angle B = 2x + 4$



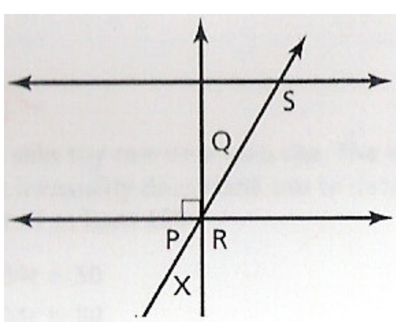
What is the measurement of $\angle A$?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

10. Michael drew the diagram below.



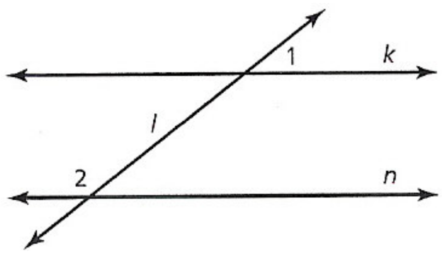
Which angle is complementary to $\angle X$?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

11. In the diagram below, line *k* and line *n* are parallel. Line *l* is a transversal.



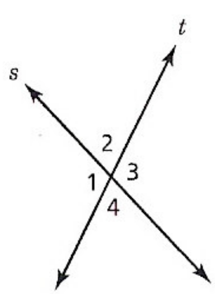
What is the relationship between $\angle 1$ *and* $\angle 2$?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

12. Line *s* and line *t* intersect, as shown below.



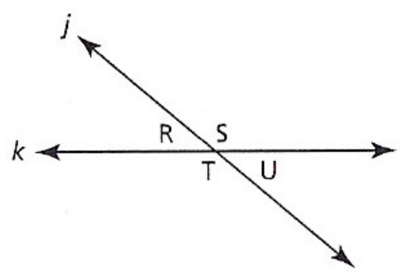
Which angles are vertical?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

13. Line *j* and line *k* intersect, as shown below.



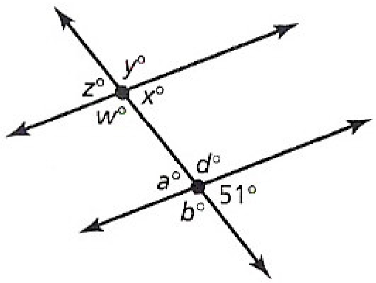
Which **two** pairs are congruent angles? Please use a semi-colon to separate your answers.

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

14. The figure below shows parallel lines cut by a transversal.



[not drawn to scale]

Based on the information, what is the measure of $a^o, d^o, x^o, and\ z^o$? Please use semi-colons to separate each of your answers.

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

Please feel free to comment and give feedback on this assessment: the questions it contains, how long it took you, concerns, wording, etc. Thank you very much for your help!!!

APPENDIX B

MATHEMATICS KNOWLEDGE FOR TEACHING THE MIDDLE GRADES: A PRESERVICE TEACHER ASSESSMENT—VERSION 2

Mathematics Knowledge for Teaching the Middle Grades of Preservice Teachers Questionnaire

Although this questionnaire is a required portion of this course, if you DO NOT wish to have your scored included in the study, please indicate so below. No adverse actions will be taken against you or your grades if you choose this option. You will still participate in all the same tests, assignments, and other classroom activities as the rest of the class.

- I agree to participate in this study
- I do **NOT** agree to participate in this study

By clicking to agree, you are agreeing to the consent form from the previous page. (If you would like to read it again, please use the back button and then click on take test and start again. You can also print the consent form from the previous screen).

I have read and understand the explanation provided to me. I have had all my questions answered to my satisfaction, and I voluntarily agree to participate in this study. I have been given a copy of the consent form (you may print off the consent form from the previous page).

Any questions can be directed to Margaret Mohr (runmohr@tamu.edu).

A calculator is not necessary, but may be helpful. You are welcome to use a calculator on this questionnaire, but please do not use any other resources (including other people and/or websites)—the results of this questionnaire will NOT average into your class grade. Instead, you will receive credit for completing the questionnaire as designated by your teacher. We just want to know what you remember off the top of your head.

Your answers will not be recorded until you submit the questionnaire. If you choose not to be a part of this study, your answers will be discarded, but you will still receive credit for completing the questionnaire for the purposes of a completion grade for your course. Please note, you MUST submit the questionnaire in order to receive completion credit.

Thank you for your time and effort! We really appreciate your help!

Name: ____________________

Course: [ ]

Instructor: [ ]

Background Information

What is your gender?
- Male
- Female

What is your current grade level?
- Freshman
- Sophomore
- First Semester Junior
- Second Semester Junior
- First Semester Senior
- Second (or more) Semester Senior
- Other

Which of the following best describes your major?
- Education—Early Childhood
- Education—Gr. 4-8 Language Arts/Social Studies
- Education—Gr. 4-8 Math/Science
- Other

What is your ethnic background?
- American Indian or Alaskan Native
- Asian or Pacific Islander
- African American
- Hispanic/Latino
- White (Non-Hispanic)
- Other or prefer not to answer

From the following list of courses, please indicate which ones you have taken:
- ❑ MATH 365
- ❑ MATH 366
- ❑ MATH 367
- ❑ MATH 368
- ❑ MATH 403
- ❑ MASC 351
- ❑ MASC 450
- ❑ MEFB 460
- ❑ MEFB 497

From the following list of courses, please indicate which ones you are **currently** enrolled in **this semester**:

- ❑ MATH 365
- ❑ MATH 366
- ❑ MATH 367
- ❑ MATH 368
- ❑ MATH 403
- ❑ MASC 351
- ❑ MASC 450
- ❑ MEFB 460
- ❑ MEFB 497

What is the highest level of mathematics you took in high school and what grade did you receive? If you do not remember your grade in this course, please report the course and just omit the grade.

Course: [ ] Grade: A, B, C, D, F, --

Please indicate the letter grade you received in each of the following courses. If you did not take a course listed, please just omit it from the list

A, B, C, D, F, Did Not Take	MATH 142
A, B, C, D, F, Did Not Take	MATH 131
A, B, C, D, F, Did Not Take	MATH 166
A, B, C, D, F, Did Not Take	STAT 303

Content Questions

1. The Horseshoe Nebula is about 5.0×10^{3} light years away from Earth. One light year is equal to approximately 5.9×10^{12} miles. What is the approximate distance, in miles, between Earth and the Horseshoe Nebula?

   Note: Please use the ^ to indicate a power

   Answer: [ ]

   Please explain your answer:

   [ ]

How would you explain, model, and/or demonstrate this item to someone who did not understand?

2. During the summer, Breanna works at a coffee shop. She saves 75% of her earnings to buy new school clothes. If Breanna earns $750, what is the **best** estimate for the amount of money she saves to buy clothes?

   Answer:

   Please explain your answer:

   How would you explain, model, and/or demonstrate this item to someone who did not understand?

3. Thomas earns a 5% commission for each cellular phone he sells. On Tuesday, he sells a cellular phone for $180. How much commission does Thomas earn on this sale?

   Answer:

   Please explain your answer:

   How would you explain, model, and/or demonstrate this item to someone who did not understand?

4. The table below shows the number of students who attended Walters Middle School each year during a five-year period.

Walters Middle School

Year	Number of Students
2000	511
2001	548
2002	587
2003	664
2004	705

What is the **approximate** percent increase in the number of students from 2000 to 2004?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

5. Simplify the expression below.

   4^3

   Answer:

   Please explain your answer:

   How would you explain, model, and/or demonstrate this item to someone who did not understand?

6. Xavier bought a shirt that was on sale for 20% off the original price. He also used a coupon that gave him an additional 15% off the sale price of the shirt. The original price of the shirt was $37. What is the new price of the shirt before tax?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

7. What is the least common multiple of 3, 6, and 27?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

8. Jacob received the following scores on his last five science tests.

81, 73, 80, 94, 97

What is the range of Jacob's scores for these five science tests?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

9. A spinner is divided into five equal sections numbered 1 through 5. Predict how many times out of 240 spins the number is most likely to stop on an odd number.

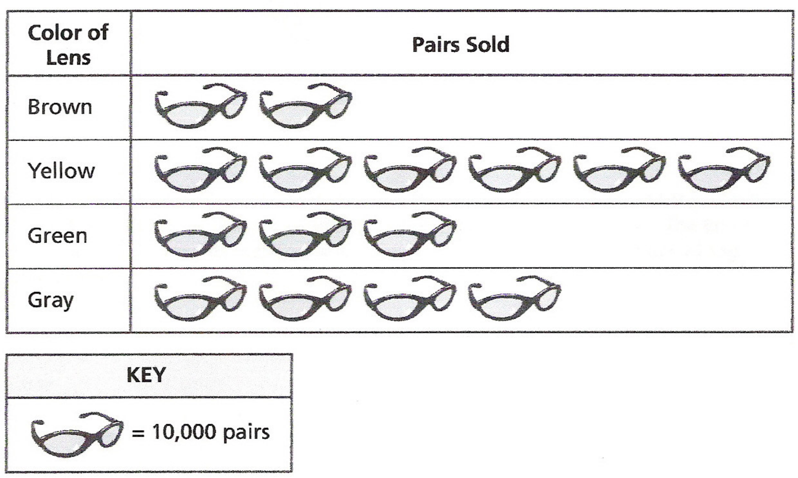
Answer: [ ]

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

10. The pictograph below records Vista Sunglass sales for 2004.

VISTA SUNGLASS SALES FOR 2004



Which color of lens had sales three times greater than one of the other color of lens?

Answer: [ ]

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

11. A shipping company uses baggage tags with 3-letter city codes. The first and third letters of each code are always consonants and the middle letter is always a vowel. The English language uses 21 consonants and 5 vowels. How many different combinations of tag codes are possible?

    Answer: [ ]

    Please explain your answer:

    [ ]

    How would you explain, model, and/or demonstrate this item to someone who did not understand?

    [ ]

12. Eric's mother wants to help him with his math homework. She puts 24 cookies in a cookie jar. Twelve of the cookies are chocolate chip, 8 are oatmeal, and 4 are peanut butter. She then has Eric select a cookie from the jar without looking. Next, without replacing the first cookie, Eric picks a second cookie without looking in the jar. What is the probability Eric will pick an oatmeal cookie first and a chocolate chip cookie second?

    NOTE: Please use / to indicate a fraction and/or ratio.

    Answer: [ ]

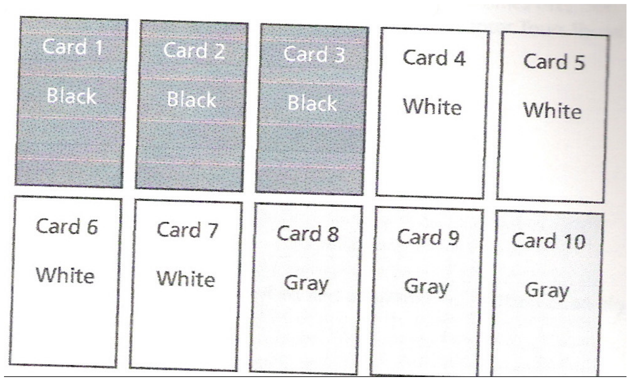
    Please explain your answer:

    [ ]

    How would you explain, model, and/or demonstrate this item to someone who did not understand?

    [ ]

13. Derek conducts a probability experiment for his mathematics class. He uses the ten cards shown below.



Derek randomly picks one of the ten cards from a container, looks at the color, and replaces the card. He repeats this 100 times. How many times would you expect Derek to pick a white card?

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

14. Dylan has a bag containing 15 marbles. The table below shows the number of marbles of each color in the bag. As part of a probability experiment for his science class, Dylan randomly picks a marble from the bag and then replaces it. He repeats this 300 times.

DYLAN'S BAG OF MARBLES

Marble Color	Number of Marbles
White	3
Red	8
Blue	3
Black	1

Predict the number of times out of 300 Dylan will pick a red marble.

Answer:

Please explain your answer:

How would you explain, model, and/or demonstrate this item to someone who did not understand?

Please feel free to comment and give feedback on this assessment: the questions it contains, how long it took you, concerns, wording, etc. Thank you very much for your help!!!

APPENDIX C

PARTICIPANT CONSENT FORM FOR INFORMATION RELEASE

Mathematics Knowledge for Teaching of Preservice Middle Grades Teachers

The purpose of the study:

I understand the purpose of this study is to understand more about what future middle grades teachers know about teaching middle grades mathematics. Since many students who take this class are preparing to teach, I have been asked to participate regardless of my major. This is not an experiment. The researcher will not attempt to change the manner in which this class is taught.

I agree to the following during Spring 2006:

1. My instructor may provide information to the researcher including my grades from this class, samples of my work from this class, my age, gender, major, and classification (Freshman, Sophomore, Junior, or Senior).

2. The researcher may request additional background information (such as previous course grades, MATH scores, and/or ethnicity). I can accept or decline to provide this information without repercussions and still participate in other parts of the study.

I understand that:

1. Participation is strictly voluntary. I can refuse to include my questionnaire.

2. The information gathered will not affect grades or any other evaluations made by the teacher of this course.

3. The information gathered will be confidential. Student and teacher names or any other identifying factors will be removed from any report or publications of the data or results.

4. I may opt out of the project at any time and for any reason I deem necessary with no repercussions if I give written notice to the researcher.

5. Approximately 600 students per semester in MATH 365, 366, 367, 368, & 403; MASC 351 & 450; MEFB 460 & 497 have been asked to participate.

6. Participation in this study will not directly provide any benefits to me. Declining participation in this study will not cause adverse actions to be taken against me or my grades.

I understand that this research study has been reviewed and approved by the Institutional Review Board—Human Subjects in Research, Texas A&M University. For research-related problems or questions regarding subjects' rights, I can contact the Institutional Review Board through Ms. Angelia M. Raines, Director of Research Compliance, Office of Vice President for Research at (979) 458-4067 (araines@vprmail.tamu.edu).

I have read and understand the explanation provided to me. I have had all my questions answered to my satisfaction, and I voluntarily agree to have the results of my questionnaire included in this study. I have been given a copy of this consent form.

Students name PRINTED____

Student's Signature ____Date _____________

Researcher's Signature ______________________________________ Date _____________

If I do NOT wish to participate I will not return this form. No adverse actions will be taken against me or my grades if I choose this option. I will still participate in all the same tests, assignments, and other classroom activities as the rest of the class.

If you have any questions or concerns, please contact:

Researcher: Margaret J. Mohr

TLAC Ph.D. student, Texas A&M University, MS 4232, College Station, TX 77843-4232, (979) 458-4174

Student of: Dr. Gerald Kulm, Curtis D. Robert Professor

TLAC Department, Texas A&M University, MS 4232, College Station, TX 77843-4232, (979) 862-4407

APPENDIX D

ANSWER KEY TO ASSESSMENT

Mathematics Knowledge for Teaching the Middle Grades: A Preservice Teacher Assessment

NUMBER AND OPERATIONS CONTENT QUESTIONS

1. The Horseshoe Nebula is about 5.0×10^3 light years away from Earth. One light year is equal to approximately 5.9×10^{12} miles. What is the approximate distance, in miles, between Earth and the Horseshoe Nebula?

   Note: Please use the ^ to indicate a power

   Answer: 2.95 x 10^16

2. During the summer, Breanna works at a coffee shop. She saves 75% of her earnings to buy new school clothes. If Breanna earns $750, what is the **best** estimate for the amount of money she saves to buy clothes?

   Answer: $550.00

3. Thomas earns a 5% commission for each cellular phone he sells. On Tuesday, he sells a cellular phone for $180. How much commission does Thomas earn on this sale?

   Answer: $9.00

4. The table below shows the number of students who attended Walters Middle School each year during a five-year period.

   Walters Middle School

Year	Number of Students
2000	511
2001	548
2002	587
2003	664
2004	705

   What is the **approximate** percent increase in the number of students from 2000 to 2004?

   Answer: 40%

5. Simplify the expression below.

   4^3

   Answer: 64

6. Xavier bought a shirt that was on sale for 20% off the original price. He also used a coupon that gave him an additional 15% off the sale price of the shirt. The original price of the shirt was $37. What is the new price of the shirt before tax?

   Answer: $25.16

7. What is the least common multiple of 3, 6, and 27?

   Answer: 54

PROBABILITY AND STATISTICS CONTENT QUESTIONS

1. Jacob received the following scores on his last five science tests.

   81, 73, 80, 94, 97

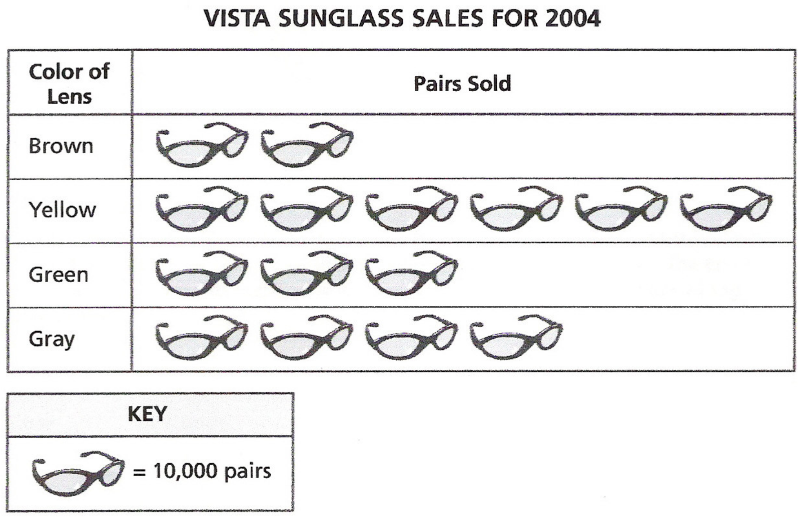
   What is the range of Jacob's scores for these five science tests?

   Answer: 24

2. A spinner is divided into five equal sections numbered 1 through 5. Predict how many times out of 240 spins the number is most likely to stop on an odd number.

   Answer: 144

3. The pictograph below records Vista Sunglass sales for 2004.



Which color of lens had sales three times greater than one of the other color of lens?

Answer: Yellow

4. A shipping company uses baggage tags with 3-letter city codes. The first and third letters of each code are always consonants and the middle letter is always a vowel. The English language uses 21 consonants and 5 vowels. How many different combinations of tag codes are possible?

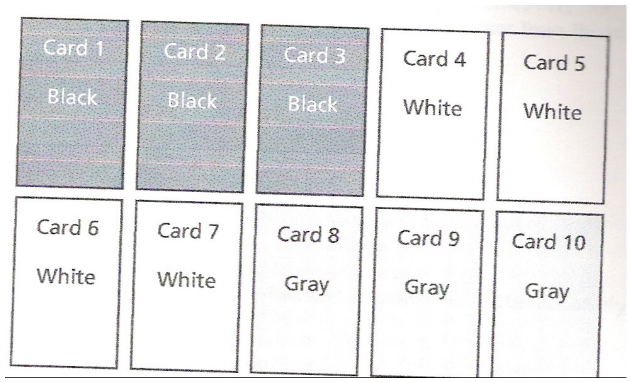
   Answer: 2,205

5. Eric's mother wants to help him with his math homework. She puts 24 cookies in a cookie jar. Twelve of the cookies are chocolate chip, 8 are oatmeal, and 4 are peanut butter. She then has Eric select a cookie from the jar without looking. Next, without replacing the first cookie, Eric picks a second cookie without looking in the jar. What is the probability Eric will pick an oatmeal cookie first and a chocolate chip cookie second?

   NOTE: Please use / to indicate a fraction and/or ratio.

   Answer: 4/23

6. Derek conducts a probability experiment for his mathematics class. He uses the ten cards shown below.



Derek randomly picks one of the ten cards from a container, looks at the color, and replaces the card. He repeats this 100 times. How many times would you expect Derek to pick a white card?

Answer: 40 times

7. Dylan has a bag containing 15 marbles. The table below shows the number of marbles of each color in the bag. As part of a probability experiment for his science class, Dylan randomly picks a marble from the bag and then replaces it. He repeats this 300 times.

DYLAN'S BAG OF MARBLES

Marble Color	Number of Marbles
White	3
Red	8
Blue	3
Black	1

Predict the number of times out of 300 Dylan will pick a red marble.

Answer: 160 times

ALGEBRA CONTENT QUESTIONS

1. Simplify the expression below.

$$\left(3x^2y - 5xy + 12xy^2\right) - \left(5xy^2 + 4xy\right)$$

NOTE: Please use ^ to indicate powers where necessary.

Answer: 3x^2y – 9xy + 7xy^2

2. Hank sells toy cars on a web site. The web site fee is \$30. Hank sells each toy car for \$4. Set up an inequality for Hank to use to determine how many toy cars, *c*, he must sell to make a profit of **at least** \$50.

   Note: Please use <= to indicate "less than or equal to" and >= to indicate "greater than or equal to".

   Answer: 4c – 30 >= 50

3. Linda must calculate the cost of filling her car's 12-gallon gas tank. She calculates the difference between how much gasoline her gas tank will hold and the number of gallons of gas, *g*, already in the tank. Then she multiplies the difference by the price, *p*, of one gallon of gas. Set up an expression for Linda to use to calculate the cost to fill her gas tank.

   Answer: (12-g)p or p(12-g)

4. Multiply the expression below.

   $$(3x-5)(2x-8)$$

   Answer: 6x^2 – 34x + 40

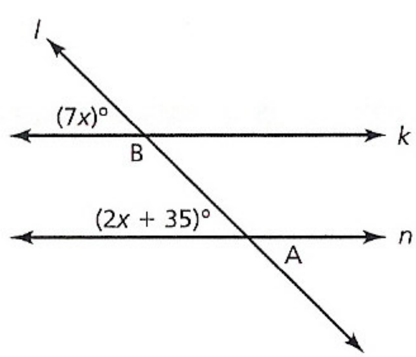
5. Write an equation that represents "eight less than twice a number is forty-two."

   Answer: 2n – 8 = 42

6. Factor $y^2 + 3y - 18$ into two binomials.

   Answer: (y + 6) (y – 3) or (y – 3) (y + 6)

7. In the figure below, lines *k* and *n* are parallel. Line *l* is a transversal.



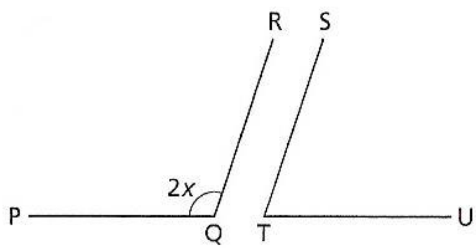
[not drawn to scale]

What is the value of *x*?

Answer: x = 7

GEOMETRY CONTENT QUESTIONS

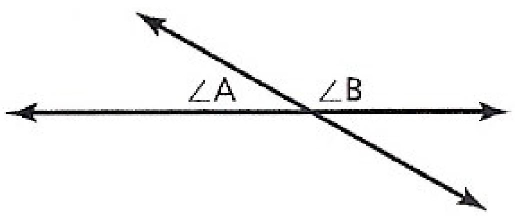
1. The angles shown below are supplementary. The measure of $\angle PQR$ is $2x$.



Please write an expression that represents the measure of $\angle STU$.

Answer: 180 – 2x

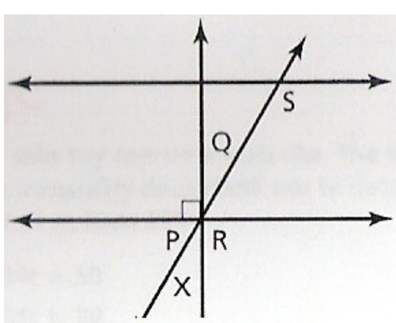
2. $\angle A = x + 2 \;\; and \;\; \angle B = 2x + 4$



What is the measurement of $\angle A$?

Answer: 60 degrees

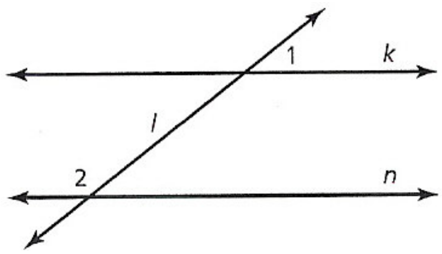
3. Michael drew the diagram below.



Which angle is complementary to $\angle X$?

Answer: < P or angle P

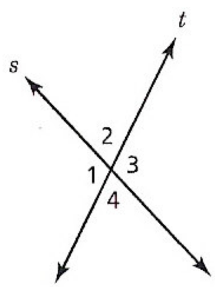
4. In the diagram below, line *k* and line *n* are parallel. Line *l* is a transversal.



What is the relationship between $\angle 1$ *and* $\angle 2$?

Answer: Supplementary

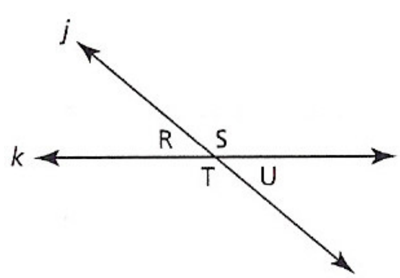
5. Line *s* and line *t* intersect, as shown below.



Which angles are vertical?

Answer: < 3 and < 1 OR < 2 and < 4 (can have both)

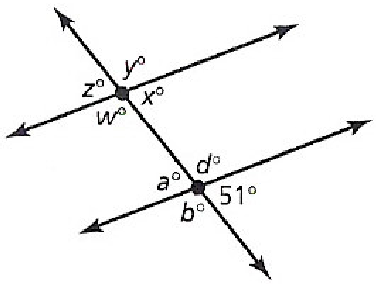
6. Line *j* and line *k* intersect, as shown below.



Which **two** pairs are congruent angles? Please use a semi-colon to separate your answers.

Answer: < T & < S; < U & < R (HAVE TO HAVE BOTH!!!!)

7. The figure below shows parallel lines cut by a transversal.



[not drawn to scale]

Based on the information, what is the measure of $a^o, d^o, x^o, and\ z^o$? Please use semi-colons to separate each of your answers.

Answer: a = 51; d = 129; x = 51; z = 51 (all of these are degrees, but DON'T have to put degrees)

APPENDIX D

RUBRIC FOR ASSESSMENT

Holistic Rubric for Open-Ended Questions	
Content Knowledge	
0 point	Did NOT correctly answered the given problem
1 point	Correctly answered the given problem
5-6 points	A five- to six-point response is complete and correct. This response: ▪ Demonstrates a thorough understanding of the mathematics concepts and/or procedures embodied in the task ▪ Indicates the student has completed the task correctly, using mathematically sound procedures ▪ Contains clear, complete explanations and/or adequate work when required
3-4 points	A three- to four-point response is partially correct. This response: ▪ Demonstrates partial understanding of the mathematical concepts and/or procedures embodied in the task ▪ Addresses most aspects of the task, using mathematically sound procedures ▪ May contain an incorrect solution but applies a mathematically appropriate process with valid reasoning and/or explanation ▪ May contain a correct solution but provide faulty or incomplete procedures, reasoning, and/or explanations ▪ May contain a correct solution but lacks work when required ▪ May reflect some misunderstanding of the underlying mathematical concepts and/or procedures
1-2 points	A one- to two-point response is incomplete and exhibits many flaws but is not completely incorrect. This response: ▪ Demonstrates only a limited understanding of the mathematical concepts and/or procedures embodied in the task ▪ May have addressed some elements of the task correctly but reached an inadequate solution and/or provided reasoning that was faulty or incomplete ▪ Exhibits multiple flaws related to a misunderstanding of important aspects of the task, misuse of mathematical procedures, or faulty mathematical reasoning ▪ Reflects a lack of essential understanding of the underlying mathematical concepts
0 points	A zero-point response is completely incorrect, irrelevant, or incoherent.
/7	**TOTAL POINTS AWARD FOR CONTENT SECTION**

*Adapted from the New York State Testing Program (2005) *Grade 8 Test Sampler Draft*

Pedagogical Knowledge	
5-6 points	A five- to six-point response is complete and correct. This response: ▪ Demonstrates a thorough understanding of the mathematics pedagogy and/or pedagogy embodied in the task ▪ Indicates the student has completed the task correctly, using mathematically and pedagogically sound procedures ▪ Contains clear, complete explanations and/or adequate work when required ▪ Exemplary method of instruction/explanation is culturally responsive and fosters cultural understanding, safety, emotional well being and is conducive to learning for diverse learners
3-4 points	A three- to four-point response is partially correct. This response: ▪ Demonstrates partial understanding of the mathematics pedagogy and/or pedagogy embodied in the task ▪ Addresses most aspects of the task, using mathematically and pedagogically sound procedures ▪ May contain an incorrect solution but applies a mathematically or pedagogically appropriate process with valid reasoning and/or explanation ▪ May contain a correct explanation but provides faulty or incomplete procedures, reasoning, and/or explanations ▪ May contain a correct solution but lacks work when required ▪ May reflect some misunderstanding of the underlying mathematics pedagogy and/or pedagogy ▪ Competent method of instruction/explanation fosters cultural understanding, safety, emotional well being and is conducive to learning for diverse learners
1-2 points	A one- to two-point response is incomplete and exhibits many flaws but is not completely incorrect. This response: ▪ Demonstrates only a limited understanding of the mathematics pedagogy and/or pedagogy embodied in the task ▪ May have addressed some elements of the task correctly but reached an inadequate solution and/or provided reasoning that was faulty or incomplete ▪ Exhibits multiple flaws related to a misunderstanding of important aspects of the task, misuse of mathematical procedures or concepts, or faulty mathematical reasoning ▪ Reflects a lack of essential understanding of the underlying mathematics pedagogy and/or pedagogy ▪ Insufficient method of instruction/explanation regarding the creation of learning environments that foster cultural understanding, safety, emotional well being, and are conducive to learning for diverse learners
0 points	A zero-point response is completely incorrect, irrelevant, or incoherent. No evidence; undocumented demonstration of competence.
/6	**TOTAL POINTS AWARDED FOR PEDAGOGY**
/13	**TOTAL POINTS AWARDED FOR MATHEMATICS KNOWLEDGE FOR TEACHING**

APPENDIX E

COURSE DESCRIPTIONS

MATH 365 Structure of Mathematics I
This course primarily deals with the topics of: informal logic, sets, relations, functions, whole numbers, numeration systems, binary operations, integers, elementary number theory, modular systems, rational numbers and the system of real numbers. This course is designed primarily for elementary teacher certification.

MATH 366 Structure of Mathematics II
This course primarily deals with the topics of: geometry, measurement and coordinate geometry. This course is designed primarily for elementary teacher certification.

MATH 367 Basic Concepts of Geometry
This course primarily deals with the formal development of geometry: finite {Euclidean and non-Euclidean}. This course is designed primarily for elementary teacher certification.

MATH 368 Introduction to Abstract Mathematical Structures
This course primarily deals with the topics of: mathematical proofs, sets, relations, functions, infinite cardinal numbers, algebraic structures, and structure of the real line. This course is designed primarily for elementary teacher certification.

MATH 403 Mathematics and Technology
This course primarily deals with mathematics problem-solving and communication through the use of various technologies (both hardware and software). This course is intended primarily, but not limited to, students working toward teacher certification.

MASC 351 Problem Solving in Mathematics
This course primarily deals with topics including: problem solving strategies in math and science; evaluate conjectures and arguments; writing and collaborating on problem situations; posing problems and conjectures; constructing knowledge from data; developing relationships from empirical evidence; and connecting mathematics concepts; readings, discussions, and analyses will model and illustrate mathematics problems solving and proofs.

MASC 450 Integrated Mathematics
This course primarily deals with topics including: integration and connections among topics and ideas in mathematics and other disciplines; connections between algebra and geometry and statistics and probability; and focus for integration with authentic problems requiring various branches of mathematics.

MEFB 460 Math Methods in the Middle Grades
This course examines theories, provides practice in teaching methods essential to successful mathematics learning; focuses on content and criteria central to teaching mathematics for understanding, skill development, and problem solving; readings, discussions, analyses; and modeling and practicing mathematics teaching and learning.

MEFB 497 Supervised Student Teaching
This course includes: observation and participation in an accredited public school classroom; and techniques of teaching student's teaching fields, and appropriate instructional strategies for assigned student population in fulfillment of endorsement requirements.

APPENDIX F

SURVEY/ASSESSMENT PARTICIPATION

Course	Number of students enrolled in Course*	Number who took Assessment*	Percentage of Class Participating	Number in Analysis*
MATH 365	123	83	67%	13
MATH 366	140	58	41%	18
MATH 367	18	12	67%	12
MATH 368	47	32	68%	32
MATH 403	57	27	47%	27
MASC 351	28	25	89%	25
MASC 450	58	52	90%	52
MEFB 460	36	31	86%	29
MEFB 497	50	8	16%	8

*Some students were enrolled in more than one of these courses during Spring 2006, so they would be counted twice in this chart.

VITA

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EDUCATIONAL EXPERIENCE

Ph.D., **Texas A&M University**, Curriculum and Instruction with emphases in Mathematics Education and Educational Research, 2006.

M.S., **Pittsburg State University**, Mathematics, 2004.

B.S.Ed., **Pittsburg State University**, Mathematics, 2002.

PROFESSIONAL EXPERIENCE

2005-2006	Middle School Mathematics Project (MSMP) Project Supervisor**, Texas A&M University.**
	Learning Assistant for Student Athlete Center, **Texas A&M University.**
	Co-Chair of Educational Research Exchange/Faculty Research Symposium, **Texas A&M University.**
August 2005	Co-Team Leader for PEICs (PreK thru 16 Educational Improvement Consortia), **Texas A&M University.**
2004-2005	Mathematics Education Instructor, **Texas A&M University.**
2003-2004	Adjunct Mathematics Instructor, **Labette Community College.**
2003-2004	Mathematics Teacher, **St. Mary's-Colgan Junior High School.**
2002-2004	Mathematics Instructor, **Pittsburg State University.**

SELECTED PUBLICATIONS

Lee, Y., Mohr, M., & Lowry, K. J. (in press)**.** Re-imaging accountability: Looking for educational quality beyond test scores. In K. Sloan (Ed.), *Holding schools accountable: A handbook for educators and parents* (pp. TBA). Westport, CT: Greenwood Publishing Group.

Mohr, M. J. (2006). Performance assessment at the high school level. In D. L. Smith and L. J. Smith (Eds.), *Restructuring high schools: Searching for solutions.* College Station, TX: Mid America Training and Development. Available from http://tlac.tamu.edu

Smith, D. L., Ezrailson, C. M., Binks, E., Delzer, L., Mohr, M. J., Warren, C. (2006). '*Connected Teacher' professional development module II: Can reading strategies help improve 8th grade students' TAKS mathematics performance?* Austin, TX: Texas Education Agency.

Mohr, M. (2003). Reading teachers – Option D: Analyze scores. *American Careers: Teaching Guide, 10,* 17, 36. *Invited publication.*

SELECTED PRESENTATIONS

Mohr, M. J., Binks, E., Shaw, B., & Smith, D. L. (2006, February). *Using research-based literacy strategies to improve mathematics achievement in the middle grades.* Paper presented at the annual meeting of the Southwest Educational Research Association, Austin, TX.

Mohr, M. J. (2005, November). *Teaching problem solving to preservice middle school mathematics and science teachers.* Paper presented at the annual meeting of the School Science and Mathematics Association, Fort Worth, TX.