ESTIMATION OF THE SPATIALLY VARYING PERMEABILITY OF A POROUS SOLID

by

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ABSTRACT

Proper function of industrial society mandates large supplies of oil both now and in the foreseeable future. In order to meet the great demand for oil, the oil industry uses several enhanced oil recovery techniques. One of the most important of these techniques is water injection.

A water injection system uses water, injected into an oil reservoir, to push the immiscible oil in the reservoir out of an oil producing well. Design of a water injection system would be facilitated by knowledge of the ease with which the oil and water could flow through the reservoir.

Ease of flow through a porous medium is defined as permeability (denoted by the symbol, K). If the permeability varies with distance through a reservoir, this K is known as spatially varying permeability.

The research objective was the estimation of spatially varying permeability. Transient pressure drop data (where the pressure drop is measured between a water injection and an oil producing well) composed the information from which K was estimated. The estimation was conducted with a computer, and the Fortran language was used.

The investigator found spatially varying permeability was estimable. However, problems in the program prevented the code's proper function for all the problems tested. Hence, the objective was attained, but the program requires more testing.

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1. Introduction

Modern industrial societies rely heavily upon oil for use both as a fuel and as a raw material for conversion into finished chemical products. Not only does society depend on oil, but this dependence will probably continue well into the 21st century. Hence, it is of vital importance to maintain the highest possible level of oil production from all oil wells. In order to recover the maximum amount of oil from a given well, one of several enhanced recovery techniques may be employed.

One of the most popular enhanced recovery techniques is that one known as water injection or water flooding. In water injection, water is pumped into an oil reservoir to push the oil out of the reservoir through a producing well.

Design of a water injection system would be facilitated by the knowledge of the ease with which water and oil could flow through the reservoir. This "ease of flow" is known as permeability and is denoted by the symbol K. If the permeability varies with distance through the reservoir, it is known as spatially varying permeability.

In this investigation, the main objective was to design a Fortran computer code which would estimate the spatially varying permeability for two phase flow through a porous solid (e.g., an oil- and water-containing rock reservoir). This K would be estimated from several pieces of data related to the reservoir (See Appendix 1).

Note: This report generally follows the form of those found in the Society of Petroleum Engineers Journal.

A secondary problem (the first one solved, however) was to modify an existing computer program which calculated information about the reservoir. Originally the program computed this information from a constant K. Modification took the form of changing this program so that it would calculate the same information from a spatially varying permeability.

2. Review of Literature

In the study of reservoir permeability, some information about K may be obtained from a literature search. For an incompressible fluid in horizontal linear flow through a porous solid, permeability, K, is defined as:[1]

$$K = \frac{q\mu}{A'(\Delta P/L)} \text{ (Darcies)}$$

Where: A' = Cross-sectional area of sample of porous material

L = Length of sample in direction of flow

q = Fluid flow rate (volume/unit time)

ΔP = Applied pressure difference

μ = Viscosity of fluid

$$\text{Where: } 1 \text{ Darcy} = \frac{1\ (\text{cm.}^3/\text{sec.})\ 1\ (\text{cp.})}{1\ (\text{cm.}^2)\ 1\ (\text{atm./cm.})}$$

This permeability can be experimentally measured. A typical value of permeability for a sandstone might range from 5 x 10^{-4} to 3.0 Darcies.[2] Also, the relative permability of the reservoir to the flow of oil (K_{RO}) and water (K_{RW}) may be measured from an experiment involving simultaneous flow of oil and water (two-phase flow).[3] Spatially varying permeability may also be determined from a two-phase flow experiment.

Finally, spatially varying K can be estimated (previously not done) from a method to be described in the main body of this report.

3. Record of Study

3.1. Scope:

At the start of this project, the investigator was supplied with a computer program which computed the transient pressure drop ($\Delta P(t)$) for two phase (oil and water) fluid flow through a porous medium. The program used several reservoir data, including a K which was constant throughout the reservoir.

The first problem solved was the modification of the program to calculate $\Delta P(t)$ from a spatially varying, not constant, K. The second problem, the point of the research, was to estimate the spatially varying permeability from observations or measurements of the transient drop, $\Delta P(t)$.

3.2 Description of a Water-Injection System:

Prior to describing the computer program and its changes, it would help to describe a water injection system. Figure 1 shows a cross-section of a water injected reservoir. As may be seen, the intent is to use the water from the injection well to push the water-immiscible oil out of the producing well. In a typical water injection system, there are several injection wells and several producing wells.

Figure 2 describes the behavior in the reservoir of both water saturation and water flow within a water-flooded reservoir. As water is injected into the reservoir, a "flood front" of water builds up and traverses the distance between the injection and the producing wells. As seen in Figure 2, the saturation of water behind the flood front follows a smooth curve until the saturation decreases to that at the flood front. At the location of the flood front, there is a

Fig. 1. Water injected reservoir.

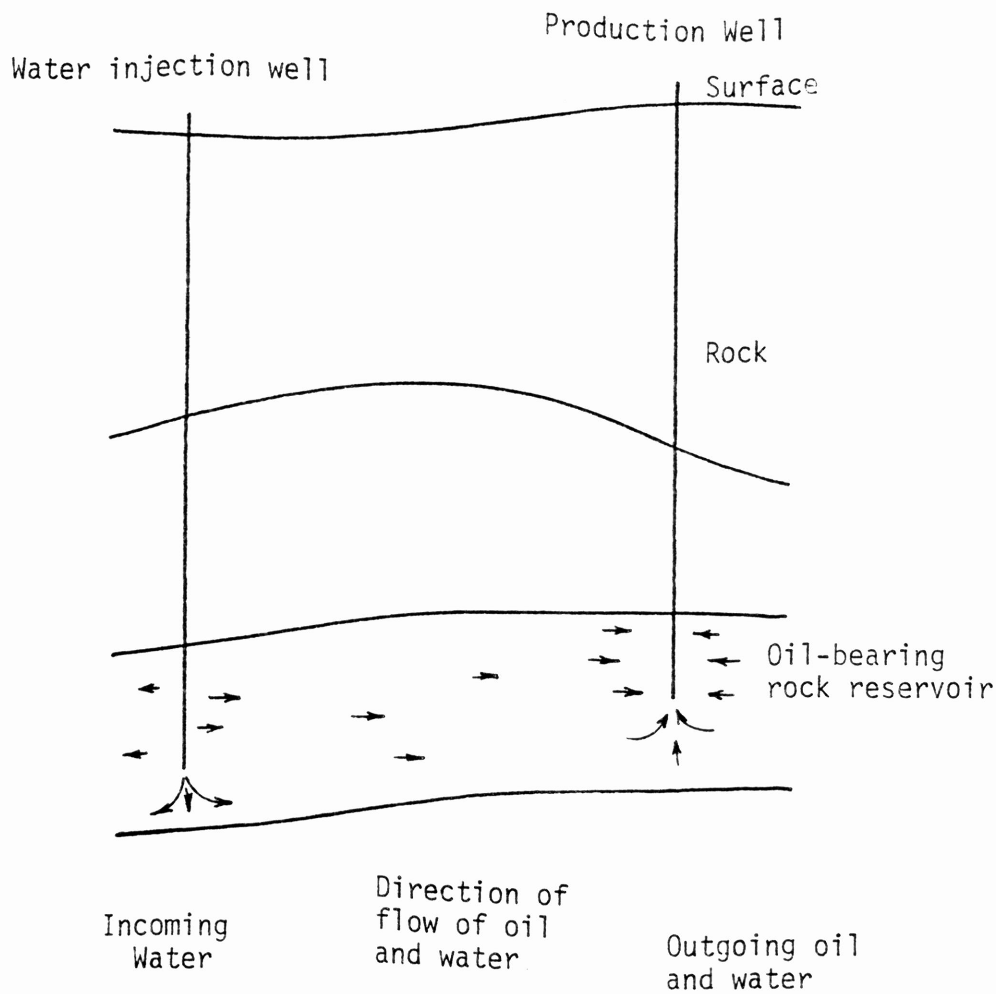

Figure 1 schematically describes a reservoir undergoing water injection. The distances are not to scale.

Fig. 2. Water saturation vs. distance through reservoir.

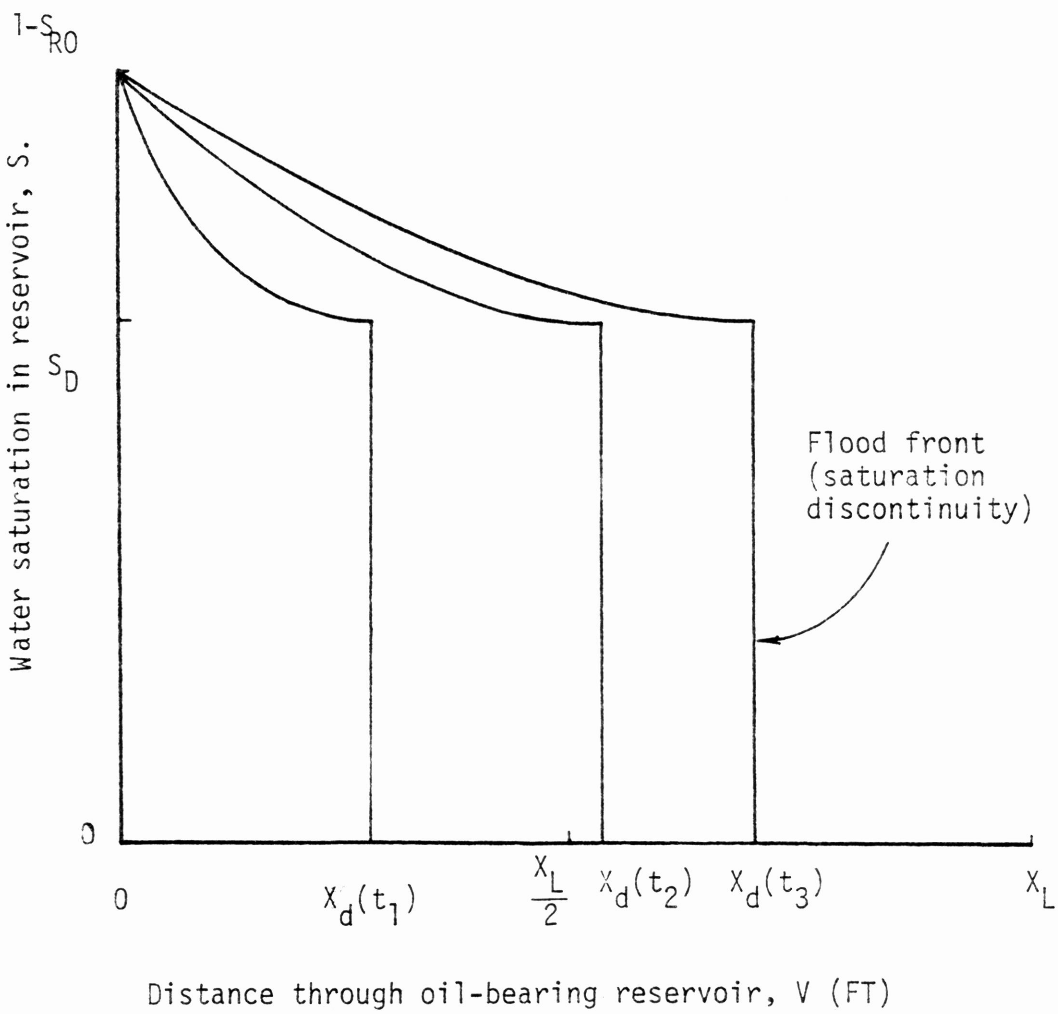

Fig. 2 describes fractional water saturation, S, versus distance, S. The fractional saturation of water may be as high as 1-S_{RO}, where S_{RO} is the residual oil saturation. S_D denotes the saturation at the discontinuity. $X_d(t_1)$ represents the location of the discontinuity at any time, t_1.

discontinuity in the curve. For this reason, the flood front is also known as the saturation discontinuity. In front of this flood front is oil and connate water (water in the reservoir prior to water injection). Behind the flood front is water (mostly injected water, the rest being connate water) and oil. When the flood front arrives at the producing well, "breakthrough" is said to occur. Oil is produced both before and after breakthrough.

The reservoir is injected with water until no more oil can be economically produced. The remaining oil is known as residual oil and has a saturation known as the residual oil saturation (S_{ro}). An infinite amount of water must be injected to recover all of the oil in the reservoir.

3.3 The First Problem--Variable K

At the start of the investigation, the program computed the pressure drop for flow through the reservoir using a code based on these equations:

Pre-breakthrough[4]:

$$\Delta P(t) = V_t \left(\mu_w \int_0^{x_d} f_w/(K * K_{rw})\, dx + u_o/K_{ro}(Sc) \int_{x_d}^{X_L} (1/K)\, dx\right) \qquad 1$$

Post-breakthrough[5]:

$$\Delta P(t) = V_t \mu_w \int_0^{X_L} f_w/(K * K_{rw}) dx \qquad 2$$

Where: f_w = flow fraction of water in oil-water flow

K = Absolute constant permeability*

K_{rw} = Relative permeability to water

$K_{ro}(Sc)$ = Relative permeability to oil at the connate water saturation (Sc)

* K is referred to as "absolute constant permeability" or simply "permeability".

μ_o = Viscosity of oil

μ_w = Viscosity of water

V_t = Velocity through reservoir, constant

x_d = Distance through the reservoir to the discontinuity

x_L = Total distance between injection and producing wells

Equation 1 is a special pre-breakthrough case of equation 2. In the computer program, equation 1 was used prior to breakthrough time and equation 2 after breakthrough time.

The pressure drop was time varying because the location, X, of each specific value of saturation was transient;

where[6]: $x = (V_t\ t/\phi)f'(s)$

t = elapsed from time of injection

ϕ = porosity of reservoir

$f'(s)$ = first derivative of flow fraction for a given S

The actual equations used in the computer were slightly modified as described in Appendix 2.

Though the computer performed many calculations, only those involved in the pressure drop calculations are discussed here. First, the program determined the saturation of water at the discontinuity. Next, the program determined the breakthrough time and the initial pressure drop at time equal to zero. A pressure drop equation (either equation 1 or equation 2 depending on whether the time was before or after breakthrough) was then integrated over distances at a given time (the time was specified and incremented by the computer). The elapsed time of injection was then incremented and the integration was performed again. The values of $\Delta P/\Delta P_o$ were tabulated for three different permeability functions of x in Appendix 3.

The reason for converting from a constant to a spatially varying K in the first problem was to enable the investigator to check the results of the second problem. The time varying pressure drops from the first problem could be entered as observed pressure drop data into the computer program which was the result of the second problem. Hence, the estimated K from the second computer program could be checked with the K which was specified in the modified program from the first problem.

3.4. The Second Problem--Estimating K

The second portion of this research involved estimating the spatially varying permeability, given a specified pressure drop. The equations to be solved for K as a function of X are developed here from the post-breakthrough pressure drop equation, (2):

$$\Delta P(t) = V_t \mu_w \int_0^{X_L} f_w/(K * K_{rw})dx \qquad 2$$

The spatially varying permeability, K, may also be written as K(x), since K varies with distance through the reservoir. Allowing $g = f_w/K_{rw}$ and $P = \Delta P(t)$ we obtain:

$$P = V_t \mu_w \int_0^{X_L} (1/K(x))\ g\ dx \qquad 4$$

However, we may substitute for K with the expression:

$$1/K(x) = \sum_{i=1}^{N} c_i\ \Phi_i(x) \qquad 5$$

Equation (5) describes a series of discrete constant permeabilities within a reservoir. If N were equal to two, there would be two constant permeabilities in a reservoir, as described in Figure 3.

Fig.3 Discrete values of K.

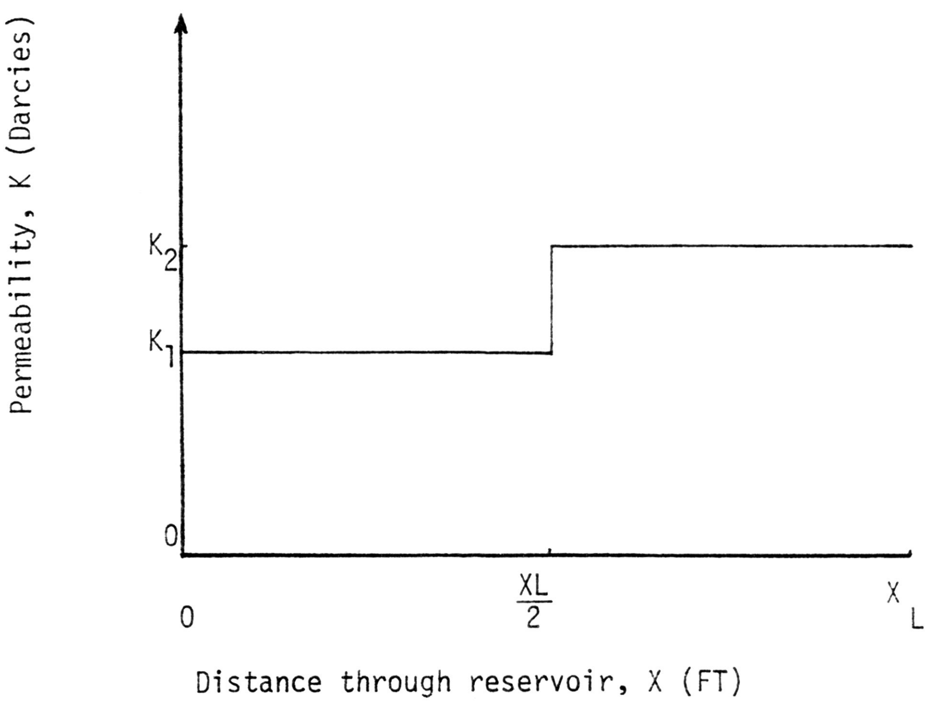

Fig. 3 describes permeability versus distance throu h reservoir. In this case, there are two discrete values of K_i, K_1 and K_2

The quantity $\Phi_i(x)$ is a weighting function. It is specified prior to performing the calculations.

Hence, each value of $K_i(x)$ is specified when each value of c_i is estimated. The unknowns of interest, now, are each value of c_i. Thus, equation (4) may now be written,

$$P = V_t \mu_w \int_0^{X_L} \sum_{i=1}^{N} (c_i \Phi_i) \; g \; dx \qquad 6$$

In order to estimate K(x) accurately, any given pressure drop calculation should be nearly equal to an observed pressure drop measured under the same circumstances. Thus, the quantity, E, should be minimized in this equation:

$$E = P^{\text{observed}} - P^{\text{calculated}}$$

or

$$E = P^{\text{obs.}} - V_t \mu_w \int_0^{X_L} \sum_{i=1}^{N} (c_i \; \Phi_i) g \; dx \qquad 7$$

This equation describes only one comparison; for comparisons at M different times, equation 7 becomes

$$E = \sum_{j=1}^{M} [P_j - V_t \; \mu_w \int_0^{X_L} \sum_{i=1}^{N} (c_i \; \Phi i) \, g_j \; dx] \qquad 8$$

However, some of the difference in equation 8 might be positive, and others might be negative. A summation of these differences might yield a small E, even though the errors were large. Thus, the sum of the squares of the errors should be minimized:

$$E = \sum_{j=1}^{M} [P_j - V_t \mu_w \int_0^{X_L} \sum_{i=1}^{N} (c_i \Phi_i) g_j \; dx]^2 \qquad 9$$

To minimize any function such as E, the first derivatives with respect to the variables of interest (c_i) should be taken, and the result should be equated with zero.

Thus,

$$\frac{\partial E}{\partial c_\ell} = 2[\sum_{i=1}^{M} P_j - V_t\mu_w\int_0^{X_L}\sum_{i=1}^{N}(c_i\Phi_i)g_j\,dx - V_t\,u_w\int_0^{X_L}\Phi_1 g_j\,dx] \qquad 10$$

where $\ell = 1, \ldots\ldots N$.

Hence, there are N derivative equations, each one a summation of calculations at M measurement times, and each of these containing a summation to N of the quantity, $(c_i\Phi_i)$. Rearranged, equation 10 is:

$$2\sum_{j=1}^{M}[-P_jV_t\mu_w\int_0^{X_L}\Phi_L g_j\,dx + (V_t\mu_w)^2\int_0^{X_L}\sum_{i=1}^{N}(c_i\Phi_i)g_j\,dx\int_0^{X_L}\Phi_1 g_j dx] = 0, \qquad 11$$

where $\ell = 1, \ldots N$.

By dividing both sides of (11) by two and rearranging, we obtain

$$\sum_{j=1}^{M}[P_j\,V_t\,\mu_w\int_0^{X_L}\Phi_1 g_j dx] = \sum_{j=1}^{M}[(V_t\mu_w)^2\int_0^{X_L}\sum_{i=1}^{N}(c_i\Phi i)g_j dx\int_0^{X_L}\Phi_1\,g_j\,dx] \qquad 12$$

Where $\ell = 1, \ldots N$.

Since c is not a function of X, it may be moved to the outside of the integral sign. Also, equation 12 may be divided by $V_t\mu_w$ on both sides to obtain,

$$\sum_{J=1}^{M}[P_j\int_0^{X_L}\Phi_1\,g_j\,dx] = \sum_{j=1}^{M}[V_t\,u_w\sum_{i=1}^{N}c_i\int_0^{X_L}\Phi_i\,g_j\,dx\int_0^{X_L}\Phi_1\,g_j\,dx] \qquad 13$$

Where $\ell = 1, \ldots N$.

Rearranging again, we obtain,

$$\sum_{j=1}^{M} P_j \int_0^{X_L} \Phi_\ell \, g_j \, dx = \sum_{i=1}^{N} [c_i \cdot \sum_{j=1}^{M} (V_t \mu_w \int_0^{X_L} \Phi_i \, g_j \, dx \int_0^{X_L} \Phi_\ell \, g_j \, dx) \qquad 14$$

Where $\ell = 1, \ldots N$.

The N equations represented by equation 14 can be put in the following matrix form:

$$\vec{\vec{A}}\vec{c} = \vec{b} \qquad 15$$

In equation 15, $\vec{b}_\ell = P_j \int_0^{X_L} \Phi_\ell g_j dx$. Thus, $\vec{b}$ is a vector of size N x 1. For example, if M = 2 and N = 2,

$$[b] = \begin{matrix} P_1 \int_0^{X_L} \Phi_1 \, g_1 \, dx + P_2 \int_0^{X_L} \Phi_1 \, g_2 dx \\ P_1 \int_0^{X_L} \Phi_2 \, g_2 dx + P_2 \int_0^{X_L} \Phi_2 g \, dx \end{matrix} \qquad 16$$

Likewise, $\vec{c}$ is a vector of size N x 1. If N = 2,

$$[c] = \begin{matrix} c_1 \\ c_2 \end{matrix} \qquad 17$$

Finally, $(\vec{\vec{A}})$ is a matrix of size NxN where $\vec{\vec{A}}_{ij} = V_t \mu_w (\int_0^{X_L} \Phi_i g_j dx \int_0^{X_L} \Phi_\ell g_j \, dx)$

If M = 2 and N = 2,

$$[A] = \begin{matrix} v_t u_w \int_0^{X_L} \Phi_1 \, g_1 \, dx \int_0^{X_L} \Phi_1 \, g_1 \, dx + & v_t u_w \int_0^{X_L} \Phi_1 \, g_1 \, dx \int_0^{X_L} \Phi_2 g_1 \, dx + \\ v_t u_w \int_0^{X_L} \Phi_1 \, g_2 dx \int_0^{X_L} \Phi_1 \, g_2 dx & v_t u_w \int_0^{X_L} \Phi_1 \, g_2 dx \int_0^{X_L} \Phi_2 g_2 dx \\ v_t u_w \int_0^{X_L} \Phi_2 \, g_1 dx \int_0^{X_L} \Phi_1 \, g_1 dx + & v_t u_w \int_0^{X_L} \Phi_2 \, g_1 dx \int_0^{X_L} \Phi_2 g_1 dz \\ v_t u_w \int_0^{X_L} \Phi_2 g_2 dx \int_0^{X_L} \Phi_1 \, g_2 dx & + \, v_t u_w \int_0^{X_L} \Phi_2 g_2 dx \int_0^{X_L} \Phi_2 g_2 dx \end{matrix} \qquad 18$$

Since each c_i was the quantity of interest, the equation $\vec{A}\vec{c} = \vec{b}$ was solved by inverting $\vec{A}$ to obtain $\vec{A}^{-1}$. Then, $c = A^{-1}b$. Each C_i was then inserted into equation 5, to obtain each K_i. Thus, in theory, the problem was solved.

The equations actually used by the computer were modified slightly as described in Appendix 2.

The programs solving both problems of the research were combined to allow a check on the answers for K(x). In actual use, the program for the estimation of K would receive as part of its data physically observed pressure drops for the quanity P_j. However, in testing the solution, the program which estimated K was simply added to the end of that program used to calculate the pressure drops. This action allowed comparison between the estimated values of K(x) and the actual values of K(x) used to produce the pressure drops used in the estimation of K(x). Thus, the estimates could be checked against knowns.

3.5 Results

Once the program was made to solve the estimation problem, several parameters in the program were varied, in order to test the program. The number of data points in the matrices (M) were varied, as was the size of the solution vector c (variation of N). Also varied was the value of the weighting function Φ, and the value of K entered for the calculation of the pressure drops used to test the K estimation routine.

Four sets of values of M and N were tested in the program. Both M and N were set equal to 2, 5, 8, and 10. The results were plotted in table 1.

Table 1

Estimated values of K for various vector sizes, with input pressure drop calculated from constant K.

M, N	Value of K supplied to a pressure drop calculation	Estimated value of K from that pressure drop
2, 2	.2737	.2737007
	.2737	.2736996
5, 5	.2737	.2737064
	.2737	.2736892
	.2737	.2737036
	.2737	.2736996
	.2737	.2736996
	.2737	.2737041
8, 8	.2737	*
10, 10	.2737	*

As may be seen with the case of M = 2, N = 2, the program predicted a constant K correctly. The same held true for M = 5, N = 5. However, the program did not have enough time to compute values of K when M and N were set equal to 8 and 10. (While M and N were equal to each other in these cases, such was not necessary. The quantity M simply must be greater than or equal to N.)

In addition to variation of sample size, the weighting function, Φ, was varied in several ways. When other parameters were varied (for instance, M or N), $\Phi_i(x)$ was defined as $\Phi_i = 1$ if $X_{i=1} \leq X < X_i$ and $\Phi_i = 0$ otherwise, for the $i = 1, \ldots, N$. When this definition of Φ was used, and other parameters were varied, the program predicted K properly.

However, when Φ was specified as one of these functions of distance through the sample, X_i:

*Inadequate computer time did not allow computation.

$\Phi = 1$ (for all x)

$= \cos(\pi Xi)$

$= \cos(2\pi Xi)$

$= \sin(\pi Xi)$

$= \sin(2\pi Xi)$,

the [A] matrix in equation 15 became singular. When the [A] matrix became singular, it could not be inverted and the program could not solve the problem. The reason for this singularity is unknown.

In addition to varying the $\Phi(x)$ function, the spatially varying permeability employed in the pressure drop calculation was also varied. Hence, while the program estimated a constant K, it also needed to estimate variable K. This second ability was tested. As may be seen in Table 2, the program was able to estimate single step functions of K with accuracy.

Table 2

Estimated values of K for constant vector sizes, step function input K.

M, N	Value of K supplied to a pressure drop calculation	Estimated value of K from that pressure drop
2, 2	.48	.4876263
	.1	.09975868
2, 2	.1	.09967554
	.48	.4856497

An input of K in the form of several step functions was also attempted, but too few data points were used to obtain meaningful results.

4. Conclusions

1. K(x) can be estimated using the present computer program.
2. The present Fortran code needs more testing with more difficult problems to insure its reliability.
3. The reason for matrix singularity when Φ_i is equal to unity or a cos sin function needs to be determined.

NOMENCLATURE

<u>English Symbols</u>

1.	A'	cross-sectional area of porous rock sample
2.	$\bar{\bar{A}}$	N*N matrix used to solve for K
3.	$\bar{b}$	N*1 vector used to solve for K
4.	$\bar{C}$	N*1 solution vector for discrete values of k(x)
5.	c_i	The ith member of c
6.	f_w	flow fraction of water in two-phase (oil and water) flow
7.	g	f_w/K_{rw}
8.	K	permeability (describes ease of fluid flow) of a porous solid
9.	K_{ro}	relative permeability of sample to oil flow
10.	K_{ro} (Sc)	relative permeability to oil at connate water saturation
11.	K_{rw}	relative permeability to water
12.	L	length of reservoir or sample
13.	M	number of measurement times
14.	N	size of solution vector; number of samples for solution
13.	ΔPs	Fluid pressure drop for flow through a porour solid. In this investigation, the pressure drop was measured between the water injection and producing wells.
14.	$\Delta P(t)$	transient pressure drop
15.	q	fluid flow rate in a porous solid
16.	S_D	fractional saturation of water at the saturation discontinuity or flood front
17.	S_{RO}	fractional residual oil saturation
18.	V_t	linear velocity of flow through reservoir
19.	X_d	distance location of saturation discontinuity in reservoir
20.	$X_d(t_i)$	transient distance location

21. X_L total distance between injection and production well, or distance through sample.

Greek Letters

22. E error function describing the difference between observed and calculated pressure drops

23. $\Phi_i(x)$ distance-varying specified forcing function

24. μ fluid viscosity

25. μ_o viscosity of oil

26. μ_w viscosity of water

27. ϕ porosity of reservoir or sample

Subscripts

1. D
2. d

implies that this value is measured at the discontinuity

3. i the ith value measured

4. L length of reservoir or sample

5. O oil

6. RO
7. ro

residual oil

8. rw residual water

9. w water

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APPENDIX

7.1 Appendix 1

Reservoir Data for Pressure Drop Calculation

The program which calculated the transient pressure drops required several pieces of reservoir data. The datum under study was the spatially varying permeability. However, the program also required:

S_c fractional connate water (water in reservoir prior to water flooding) saturation

S_{RO} fractional saturation of oil remaining after water flooding

X_T total distance between injection well (X=0) and production well ($X = X_T$) (FT)

V_t linear flooding velocity through reservoir (FT/HR)

μ_o viscosity of oil in reservoir (cp)

μ_w voscosity of water in reservoir (cp)

ϕ porosity of reservoir

Other pieces of information were also fed to the program. Some of these parameters determined the methods by which the calculations were performed. Other information set tolerances for iteration error.

7.2 Appendix 2

Derivation of the Change from Integration Over Distance to Integration Over Saturation

While the listed pressure drop equations were integrated over distance, X, the program performed some integrations, over S, for convenience.

To change from integration over X to S, one followed this procedure:

$$X = \frac{vt}{\phi} \quad f'(s) \; (V_t, \phi \text{ constant}) \qquad 7.2.1$$

$$dx = \frac{V_t t}{\phi} f''(s) \; ds \qquad 7.2.2$$

Equation 7.2.2 could be substituted wherever dx occurred.

The limits of integration also required change. The location, X=0, corresponds to the saturation, $1-S_{RO}$. Likewise, X_D corresponds to S_D.

For example

$$\int_0^{Xd} \Phi_i \frac{f}{Krw} \, dx = \int_{1-S_{ro}}^{Sd} \Phi_i \; \frac{f}{Krw} \frac{V_t \; t \; f''(s)}{\phi} \, ds \qquad 7.2.3$$

7.3 Appendix 3

Permeability Functions and Transient Pressure Drops for the First Problem

Table 7.3.1

Permeability Functions of Distance for Three Test Cases

Case	Value of K	Location
1	0.2737	For all X
2	0.1	$0 \leq X < .5X_L$
	0.48	$.5X \leq X \leq X_L$
3	0.48	$0 \leq X < .5X_L$
	0.1	$.5X_L \leq X \leq X_L$

Fig. 7.3.1

Graphical representation of permeability functions.

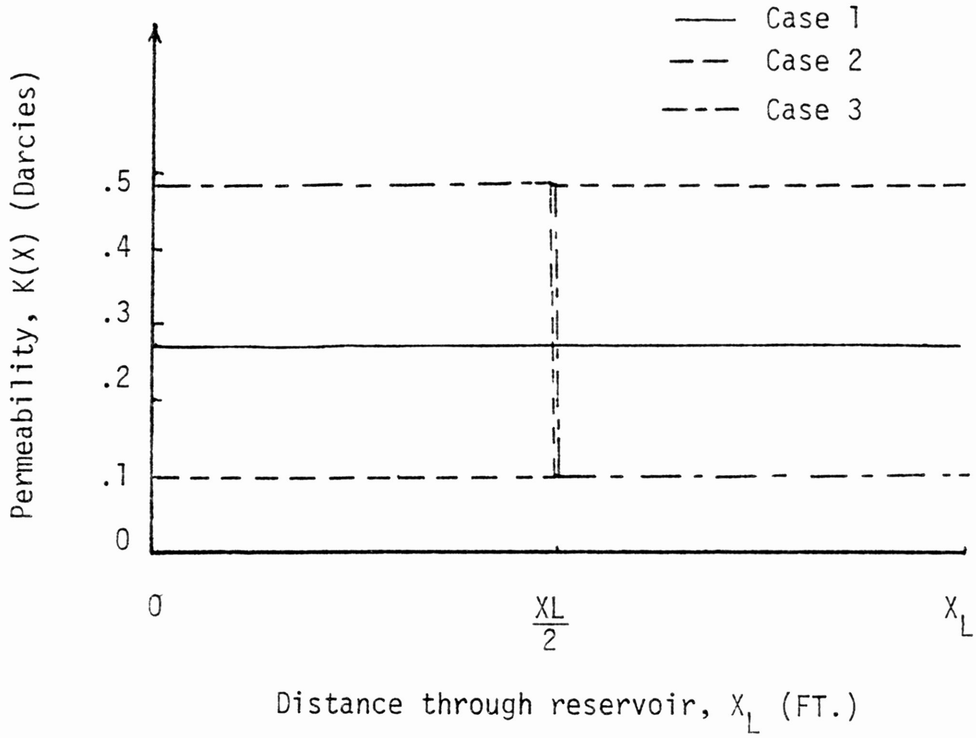

Figure 7.3.1 describes permeability versus distance through reservoir. As may be seen, case 1 used a constant permeability while cases 2 and 3 used a step function permeability.

Table 7.3.2

Transient Pressure Drops for Three Cases

Case	T_i, Elapsed time of water flooding, expressed as a fraction of break-through time*	Quotient of pressure drop at T_i and pressure drop at T_0
1		
(Pressure drop, ΔP_0, at t_0 = 4.81 (ATM))	0.000	1.000000
	0.100	1.136333
	0.200	1.272668
	0.300	1.409002
	0.400	1.545337
	0.500	1.681671
	0.600	1.818006
	0.700	1.954340
	0.800	2.090674
	0.900	2.227010
	1.000	2.363344
	1.100	2.304798
	1.200	2.253578
	1.300	2.208229
	1.400	2.167730
	1.500	2.131263
	1.600	2.098225
	1.700	2.068062
2		
(ΔPo = 7.94 (ATM))	0.000	1.000000
	0.100	1.219707
	0.200	1.451619
	0.300	1.675685
	0.400	1.899752
	0.500	2.123600
	0.600	2.090547
	0.700	2.089628
	0.800	2.090902
	0.900	2.106197
	1.000	2.100324
	1.100	2.046752
	1.200	2.042711
	1.300	2.004212
	1.400	1.971023
	1.500	1.941995
	1.600	1.916334
	1.700	1.857138

Table 7.3.2, continued

Transient Pressure Drops for Three Cases

Case	T_i, elapsed time of water flooding, expressed as a fraction of breakthrough time*	Quotient of pressure drop at T_i and pressure drop at T_0
3		
(ΔP_0 = 7.94 (ATM)	0.000	1.000000
	0.100	1.052585
	0.200	1.092911
	0.300	1.141109
	0.400	1.189308
	0.500	1.237725
	0.600	1.544266
	0.700	1.818462
	0.800	2.090457
	0.900	2.348370
	1.00	2.627547
	1.100	2.564002
	1.200	2.465398
	1.300	2.413183
	1.400	2.365342
	1.500	2.321402
	1.600	2.280962
	1.700	2.279955

*Breakthrough time: 1602 days