FLUTTER ANALYSIS OF A CASCADE OF ROTOR BLADES*

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Abstract

A classical two-dimensional, bending-torsion flutter analysis of a reference airfoil in a cascade of infinite blades is performed. The unsteady airloads on the reference airfoil are predicted using a numerical lifting surface theory. Several cascade and flow parameters such as interblade spacing, stagger angle, phase angle between blades, Mach number, and frequency are investigated. The bending-torsion flutter speed of the cascaded reference airfoil is studied as a function of the cascade and flow parameters and the results are compared with that of an isolated airfoil.

Introduction

Axial flow compressors usually have several stages consisting of a stator and a rotor. In this study, a classical flutter analysis is performed on a reference blade of a typical set of compressor rotor blades. The set of rotor blades is treated on a two-dimensional basis in a cascade. The reference airfoil is treated as a two-degree-of-freedom system undergoing flapping and pitching oscillation. The usual procedure is to derive Lagrange's equation of motion taking into account the structural geometry and properties with the unsteady airloads acting as external forcing functions. Since the prediction of unsteady airloads play a major role in the flutter analysis, in this study a brief literature survey and the formulation of an unsteady airloads prediction method are given.

Several investigators have developed methods for prediction of unsteady airloads on a blade of a cascade. Steady flow assumptions were not valid for a cascade of blades since each blade row operates in the unsteady wake of the preceding row. Kemp and Sears[1] in one early investigation studied the problem of the unsteady lift generated by a reference airfoil using incompressible flow assumptions. They included the steady interaction between blades in the cascade, but neglected the unsteady interaction, and therefore, the effects of cascade spacing. This method expressed the unsteady lift as a function of the design parameters, for this lift is proportional to the ratio of the airfoil chord and the disturbance wavelength. This enabled the designer to optimize the performance of the turbomachine design instead of analyzing one particular blade setup. This method was extended by Harlock[2] to include the normal and parallel velocities to the airfoil, thereby considering the stagger angle and blade camber in the design analysis.

For the incompressible airloads prediction on airfoils with oscillating flaps, Jones and Moore[3] developed a unique numerical lifting surface technique which differs from previous methods in that it used the velocity potential instead of the acceleration potential doublet distributions. One advantage of this method is the resulting simpler set of linear simultaneous equation to be solved. Rao and Jones[4] applied this method to the oscillatory incompressible flow in a cascade and were in good agreement with results submitted by Schorr and Reddy[5] using approximate integral equation solutions.

Compressibility effects were added by Kaji and Olazaki[6] to the cascade solution. They employed the acceleration potential doublets using linearized flow and reduced the solution to a set of six singular implicit integral equations for each set of cascade parameters. This method is able to account for staging as well as cascading effects on the unsteady airloads.

Jones and Moore[7,8] extended the velocity potential formulation to oscillating two-dimensional airfoils in compressible flow. They were able to replace the slowly converging Hankel function series with a fast converging exponential series in the compressible flow equation. Rao and Jones[9] applied the numerical lifting surface method for the prediction of airloads on a reference blade of a cascade. This lifting surface theory is the one used to compute the airloads for the single airfoil and cascade flutter predictions and will be formulated in the next section.

The unsteady airloads predicted by the lifting surface method are combined with the given blade structural properties, and a two-dimensional bending-torsion flutter analysis using the Lagrange's equation of motion will provide the flutter speed and frequency.[10] The unsteady airloads predicted by the numerical lifting surface method can be added as forcing functions to the two Lagrangean equations of motion representing the bending and torsional degrees-of-freedom of a two-dimensional airfoil. By making use of an iterative procedure which permits frequency variation, the flutter frequency and speed of the reference airfoil can be obtained. Comparison of the flutter speed for the single airfoil and the reference airfoil of a cascade provides the information necessary for predicting the effects of cascade parameters; the interblade spacing, phase lag, and stagger angle, on the flutter speed.

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Theory

Any dynamic system is assumed to flutter at a speed at which the unsteady aerodynamic forces make the system neutrally stable. In other words, flutter occurs at the speed where no net damping forces are acting on the system. Also, there exists a frequency corresponding to the flutter speed and it is called the flutter frequency. In this paper, a unit span of a reference airfoil in a cascade is assumed to have two-degrees-of-freedom, bending and torsion, and the unsteady airloads are predicted using a two-dimensional lifting surface theory. The cascade parameters associated with a staggered row of thin oscillating blades are presented in Figure 1. In a free-stream velocity of U and Mach number of M, the blades have an oscillation frequency of p rad/sec. The airfoils are of a chord length 2ℓ and are inclined to the vertical axis at a stagger angle of λ. The airfoil is assumed to be undergoing flapping and pitching periodic motion, $\ell z(=\ell z' e^{ipt})$ and $\alpha(=\alpha' e^{ipt})$, respectively. This motion is represented in Figure 2 for the reference airfoil, and the expression for the kinetic energy per unit span for a mass element dm undergoing this motion, that is at a distance r from the rotation point is

$$dT = \tfrac{1}{2}(\ell \dot{z} + r\dot{\alpha})^2 \, dm \qquad (1)$$

where z and α are nondimensional coordinates. The kinetic energy per unit span of the entire airfoil is given by

$$T = \tfrac{1}{2}\int_{\ell}^{-\ell} (\ell \dot{z} + r\dot{\alpha})^2 \, dm = \tfrac{1}{2} M_w \ell^2 \dot{z}^2 + S\ell \dot{z}\dot{\alpha} + \tfrac{1}{2} I \dot{\alpha}^2 \qquad (2)$$

where M_w = Mass per unit span of blade

$S = \int r \, dm$ = Static moment about elastic airfoil axis.

$I = \int r^2 \, dm$ = Moment of inertia about elastic airfoil axis.

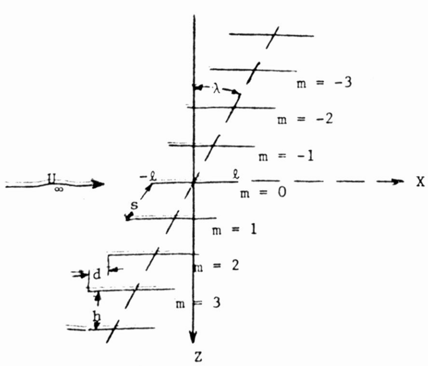

Figure 1 Cascade of Airfoils

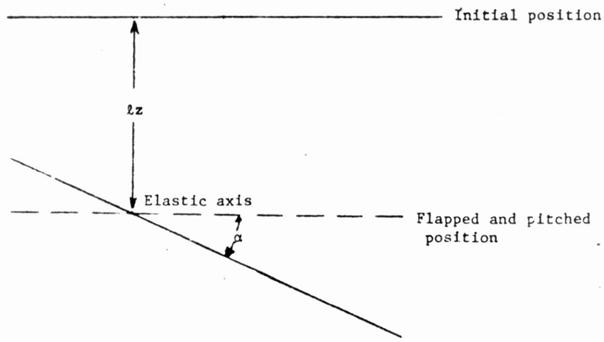

Figure 2 Flutter Coodinates

The potential energy expression for the blade is expressed as

$$U = \tfrac{1}{2} k_z \ell^2 z^2 + \tfrac{1}{2} k_\alpha \alpha^2 \qquad (3)$$

where k_z and k_α are the stiffness in bending and torsion, respectively. These are related to the bending and torsional natural frequencies, ω_z and ω_α as $k_z = M_w \omega_z^2$ and $k_\alpha = I\omega_\alpha^2$. Flutter is a self-exciting aeroelastic phenomenon involving the interaction between the inertial, elastic, and aerodynamic forces. The structural damping of the blade can be neglected compared to the aerodynamic damping terms associated with the forcing function. Neglecting the structural damping, Lagrange's equations of motion[10] can be expressed as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} + \frac{\partial U}{\partial q_k} = F_k \qquad (4)$$

where F_k and q_k are generalized forces and coordinates. The aerodynamic forcing functions are obtained from a two-dimensional unsteady airloads prediction method derived later in this section and are given in the form

$$F_z = -\rho U_\infty^2 \ell \, (C_{\ell_z} z + C_{\ell_\alpha} \alpha)$$
$$F_\alpha = \rho U_\infty^2 \ell \, (C_{m_z} z + C_{m_\alpha} \alpha) \qquad (5)$$

where C_{ℓ_z}, C_{ℓ_α}, C_{m_z}, C_{m_α} are the aerodynamic derivatives referred to the elastic axis. Equations (2), (3), and (5) are combined with (4) to obtain

$$M_w \ell \ddot{z} + S\ddot{\alpha} + k_z \ell z = -\rho U_\infty^2 \ell \, (C_{\ell_z} z + C_{\ell_\alpha} \alpha)$$
$$S\ell \ddot{z} + I\ddot{\alpha} + k_\alpha \alpha = \rho U_\infty^2 \ell \, (C_{m_z} z + C_{m_\alpha} \alpha) \qquad (6)$$

The solutions of equations (6) are assumed to have periodic motions

$$z = z' e^{ipt} = z' e^{i\omega T}$$
$$\alpha = \alpha' e^{ipt} = \alpha' e^{i\omega T} \qquad (7)$$

where $\omega = p\ell/U_\infty$ is the reduced frequency and $T = U_\infty t/\ell$ is a nondimensional time. Equations (6) and (7) are combined to yield

$$(\ell k_z - M_w \ell p^2 + \rho U^2 \ell C_{\ell_z})z' + (\rho U_\infty^2 \ell C_{\ell_\alpha} - Sp^2)\alpha' = 0 \quad (8)$$

$$(C_{m_z} \rho U_\infty^2 \ell^2 + S\ell\, p^2)z' + (k_\alpha - \rho U_\infty^2 \ell^2 C_{m_\alpha} - Ip^2)\alpha' = 0$$

Equations (8) are a set of two simultaneous, linear, homogenous algebraic equations in z' and α'. For this system to have a nontrivial solution, the coefficient determinant of z' and α' must be equal to zero. This determinant is referred to as the flutter determinant and the solution gives the flutter frequency. However, a direct solution cannot be obtained since the aerodynamic derivatives are functions of $\omega(=p\ell/U_\infty)$. Also, aerodynamic derivatives are complex quantities and hence the flutter determinant equation can be expressed in two parts, the real and the imaginary. Noting that the subscripts, R and I, correspond to the real and the imaginary parts of the aerodynamic derivatives, and defining $p' = 1/p^2$, the determinant expansion can be expressed as

Real part

$$[\ell k_z k_\alpha]p'^2 - [\ell k_z (I + \frac{\rho\ell^4}{\omega^2} C_{m_{\alpha R}}) + k_\alpha (M_w \ell - \frac{\rho\ell^3}{\omega^2} C_{\ell_{zR}})]p' + [(M_w \ell I - S^2\ell) - \frac{\rho\ell^3}{\omega^2}(IC_{\ell_{zR}} - M_w\ell^2 C_{m_{\alpha R}} + S\ell C_{m_{zR}}) - \frac{\rho^2\ell^7}{\omega^4}(C_{\ell_{zR}}C_{m_{\alpha R}} - C_{\ell_{zI}}C_{m_{\alpha I}} + C_{\ell_{\alpha I}}C_{m_{zI}} - C_{\ell_{\alpha R}}C_{m_{zR}})] \quad (9)$$

Imaginary part

$$[k_\alpha C_{\ell_{zI}} - \ell^2 k_z C_{m_{\alpha I}}]p' + [S\ell C_{\ell_{\alpha I}} - IC_{\ell_{zI}} + M_w \ell^2 C_{m_{\alpha I}} - S\ell C_{m_{zI}} + \frac{\rho\ell^4}{\omega^2}(C_{\ell_{\alpha R}}C_{m_{zI}} + C_{\ell_{\alpha I}}C_{m_{\alpha R}} - C_{\ell_{zR}}C_{m_{\alpha I}} - C_{\ell_{zI}}C_{m_{\alpha R}})]$$

For an assumed value of ω, if one of the solutions, p_1 or p_2, of the real part is equal to the solution p_3 of the imaginary part, then this ω corresponds to the flutter frequency. If this condition does not exist, then the procedure is repeated for several values of ω until the flutter frequency is obtained.

Prediction of the aerodynamic coefficients for the cascaded reference airfoil is necessary to determine the forcing functions for Lagrange's equations of motion. This is accomplished using a numerical lifting surface method for unsteady airloads prediction developed by Rao and Jones.[9] The cascade parameters as outlined in Figure 1 are stagger angle λ, interblade phase lag σ, and interblade spacing s. Vertical and horizontal separation between blades, h and d, is such that $\tan\lambda = d/h$. To provide for a more general solution the system is nondimensionalized in the following fashion

$$X = x/\ell,\ Z = \beta z/\ell,\ T = Ut/\ell,$$

$$D = d/\ell,\ H = \beta h/\ell,\ S = (D^2+H^2)^{1/2}$$

where $\beta = (1 - M^2)^{1/2}$. If the velocity potential ϕ is replaced by Φ according to

$$\phi = U\ell\Phi e^{i(\varepsilon X + \omega T)} \quad (10)$$

where

$$\varepsilon = \frac{M^2\omega}{\beta^2},\ \kappa = \frac{M\omega}{\beta^2}$$

Then Φ will satisfy the wave equation

$$\frac{\partial^2\Phi}{\partial X^2} + \frac{\partial^2}{\partial Z^2} + \kappa\,\Phi = 0 \quad (11)$$

The downwash $w(=w'e^{i\omega T})$ can be expressed in terms of Φ by using equation (10)

$$w = \frac{\partial\Phi}{\partial z} = \beta U e^{i(\varepsilon X + \omega T)}\frac{\partial\Phi}{\partial Z} \quad (12)$$

Downwash can now be nondimensionalized

$$W = \frac{\partial\Phi}{\partial Z} = \frac{w'e^{-i\varepsilon X}}{\beta U}$$

For oscillating blades with flapping and pitching motion about midchord

$$w_i' = U[i\omega z' + (1 + i\omega X)\alpha'] \quad (13)$$

where z' and α' are amplitudes in flapping and pitching, respectively. Utilizing Euler's equation of motion, the chordwise lift distribution can be given by

$$\tilde{\ell}(X) = \rho U_\infty^2 (iK\nu + \frac{\partial K}{\partial X}) e^{i(\varepsilon X + \omega T)} \quad (14)$$

where $K = \Phi_u - \Phi_\ell$ and $\nu = \omega/\beta^2$. One boundary condition can be obtained from equation (14); since no lift is generated by the wake then

$$iK\nu + \frac{\partial K}{\partial X} = 0 \quad \text{for } x \geq 1 \quad (15)$$

The relation between the downwash at any point (X,Z) on the thin reference blade in subsonic, compressible flow and doublet distribution K(X) is given by

$$2\pi W(X_p) = \int_{-1}^{\infty} K(X)\frac{\partial^2}{\partial Z_p^2} S_o(X_p - X, Z_p, D, H, \sigma)dX \quad (16)$$

where

$$S_o = \frac{\pi i}{2}\sum_{m=-\infty}^{\infty} e^{im\sigma} H_o^{(2)}\{\kappa[(X_p - X + mD)^2 + (mH - Z_p)^2]^{1/2}\}$$

The blades of the cascade are numbered m, with m=0 the reference airfoil. Equation (16) may be rewritten as

$$2\pi W(X_p) = \int_{-1}^{\infty} K(X)\left(\frac{\partial S_1}{\partial X} + \kappa^2 S_o\right) dX \quad (17)$$

where

$$S_1 = \frac{\partial S_o}{\partial X} = \frac{\pi i\kappa}{2}\sum_{m=-\infty}^{\infty} e^{im\sigma} \frac{(X_p - X + mD)\, H_1^{(2)}\{\kappa[(X_p - X + mD)^2 + m^2H^2]^{1/2}\}}{[(X_p - X + mD)^2 + m^2H^2]^{1/2}}$$

It has been shown the convergence of this Hankel series is slow and has been transformed into exponential series to improve the convergence. The transformed relations are

$$S_o = -\frac{1}{2} \sum_{m=-\infty}^{\infty} \frac{e^{-2\pi a(m)|X_p - X|/S}}{[(\delta - m)^2 - \mu^2]^{\frac{1}{2}}} \quad (18a)$$

and

$$S_1 = \pm \frac{\pi}{S} \sum_{m=-\infty}^{\infty} \frac{a(m)\ e^{-2\pi a(m)|X_p - X|/S}}{[(\delta - m)^2 - \mu^2]^{\frac{1}{2}}} \quad \text{for } X_p \neq X \quad (18b)$$

where

$$a(m) = [(\delta - m)^2 - \mu^2]^{\frac{1}{2}} \frac{H}{S} \mp i(\delta - m)\frac{D}{S} \text{ for } X_p \neq X$$

$$\delta = (\sigma + \varepsilon D)/2\pi, \quad \mu = \kappa S/2\pi.$$

Once the doublet distribution K(X) is known, the local lift at a point can be given by $\tilde{\ell}(X)$. The local lift can then be integrated to determine the two-dimensional lift coefficient as well as the moment coefficient. These coefficients are complex and are given in the form $C_\ell = C'_\ell e^{ipT}$ and $C_m = C'_m e^{ipT}$ which can be established in terms of $\bar{K}$ which has been substituted for $K(X)e^{i\varepsilon X}$.

$$C'_\ell = \bar{K}_t + i\omega \int_{-1}^{1} \bar{K}\, dX = C_{\ell_z} z' + C_{\ell_\alpha} \alpha'$$

$$C'_m = -\bar{K}_t + \int_{-1}^{1} \bar{K}\, dX - i\omega \int_{-1}^{1} X\bar{K}\, dX = C_{m_z} z' + C_{m_\alpha} \alpha' \quad (19)$$

where $\bar{K}_t$ is the doublet distribution at the trailing edge and is determined from equation (15).

Numerical Procedure

Classical flutter is an iterative procedure since it involves several variables: flutter frequency, p; flutter speed, U; and reduced frequency, $\omega(=p\ell/U_\infty)$. The primary objective is to find the flutter speed; however, the aerodynamic derivatives are functions of Mach number, M, and reduced frequency. The flutter problem can only be solved after the aerodynamics derivatives are evaluated, so it is necessary to assume a Mach number and a reduced frequency and test whether flutter occurs at these values according to equation (9). If the test results are negative, then iterate on reduced frequency until a flutter speed is obtained for the assumed Mach number. If the two speeds do not match, then iterate on Mach number until the flutter Mach number is equal to the assumed Mach number.

Several FORTRAN computer programs have been written in which an iterative procedure is set up to predict the flutter speed for incompressible and compressible isolated airfoil cases as well as cascaded airfoil cases. In these programs, the flutter program is combined with unsteady airload prediction programs that have been developed at Texas A&M University. In these unsteady airload programs, a unique numerical lifting surface method is used. This lifting surface technique has been applied to a variety of lifting surface problems such as helicopter rotor blades and delta wings. In this numerical technique, the reference airfoil is replaced by an appropriate number of boxes in which the doublet distribution, K(X), is assumed to be constant. With this assumption, it was possible to reduce the complex governing flow equation into a set of linear, simultaneous, homogenous equations which contain the K(X) values at various boxes along the chord of the airfoil as unknowns.

For flapping and pitching motions, the appropriate boundary conditions are given in equation (13). For the known boundary conditions at various boxes, the K(X) distribution can be solved. Once the K(X) distribution is known, the aerodynamic derivatives are evaluated from equation (19) for any chosen value of M and ω. The airload prediction method evaluates the aerodynamic derivatives with the reference axis at the midchord position. For the flutter program, the aerodynamic derivatives are obtained with respect to the elastic axis by a coordinate transformation.

The computational procedure is illustrated in the Appendix by applying it to a two-dimensional airfoil with cascade parameters of: S = 6ℓ, λ = 45°, and σ = 90°. The iteration results are given in the Appendix as well as the geometric and elastic properties.

Results and Discussion

A two-dimensional flutter analysis is performed for the numerical example described in the Appendix. The flutter Mach numbers for the isolated airfoil case are determined to be 0.593 and 0.571 for incompressible and compressible flows, respectively. The flutter Mach numbers for the reference airfoil of a cascade are tabulated in Table 1.

TABLE 1 FLUTTER MACH NUMBER

σ	λ	Flutter Mach Number		
		S = 2	S = 6	S = 10
0°	0°	0.779	0.604	0.599
0°	30°	*	0.621	0.610
0°	45°	*	0.623	0.612
0°	50°	*	0.620	0.611
0°	55°	*	0.615	0.609
0°	60°	*	0.608	0.606
45°	0°	0.782	0.615	0.602
45°	30°	*	0.700	0.618
45°	45°	*	0.741	0.616
45°	50°	*	0.737	0.615
45°	55°	*	0.707	0.639
45°	60°	*	0.674	0.674
90°	0°	–	0.685	0.615
90°	30°	–	–	0.705
90°	45°	–	0.649	–
90°	50°	–	0.659	–
90°	55°	–	0.673	–
90°	60°	–	0.690	0.597
135°	45°	–	–	–
180°	45°	–	0.592	–
225°	45°	–	0.620	–
270°	45°	–	0.613	–
315°	45°	–	0.611	–
360°	45°	–	0.623	–

* - No Flutter Mach Number Exists.

– - No Data At This Case.

Since the objective of the study is to predict the effects of various cascade parameters on the flutter speed, several cases have been studied. In all these cases, the compressible unsteady airload program developed by B. M. Rao at Texas A&M University was used. It has been noted that at small values of inter-blade spacing the aerodynamic derivatives vary rapidly with frequency, while the aerodynamic damping terms, i.e., the imaginary parts of the aerodynamic derivatives remain positive. When the flutter program was run for small values of inter-blade spacing (s = 2) no subsonic Mach number convergence was achieved for most cases. These cases where it was concluded flutter was not possible are marked with '*' in the table; however, this particular conclusion was based upon the assumption that the unsteady airloads program does yield valid aerodynamic derivatives. Additional study is being performed on the effect of truncation of equation (18) for small values of s.

In all the cases studied, the flutter Mach number for the reference airfoil is always greater than the compressible single airfoil case. Also, Figure 3 illustrates that flutter Mach number increases as s decreases; for constant values of inter-blade phase lag, σ, and stagger angle, λ. This can be correlated by observing that the single airfoil can be treated as the limiting case of a cascaded airfoil with an infinite inter-blade spacing. Hence, as s increases in a cascade, the flutter Mach number decreases to the limiting value of the single airfoil.

Variation of flutter speed with stagger angle differs from the variation with inter-blade spacing because the single airfoil is not a limiting value for the cascade in terms of stagger angle. An optimum value is observed for λ in Figure 3 when a constant inter-blade spacing curve peaks at the highest flutter speed possible. For phase lag angles of 0^o and 45^o, a stagger angle of 45^o seems to be the optimum stagger angle value, regardless of the spacing value. Variation between flutter Mach numbers for different inter-blade spacing appears to be the greatest at the optimum stagger angle value and the differences between Mach numbers seems to be the least at the extreme stagger angles. In fact, for a phase lag angle of 45^o, at a stagger angle of 60^o, there is no difference between the flutter Mach numbers for s = 6 and s = 10.

Variation of flutter Mach number for various inter-blade phase lag angles is illustrated in Figure 4 for a constant stagger angle of $\lambda = 45^o$ and inter-blade spacing of 6ℓ. There is extreme variation of flutter Mach values for $\sigma = 0^o$ to 180^o that is indicative of a possible critical phase lag value. This value could probably be determined by using smaller phase lag angle increments. In addition, the flutter Mach number values are cyclic with phase lag, since identical values are obtained for $\sigma = 0^o$ as well as for $\sigma = 360^o$.

Iteration for the flutter Mach number requires careful selection of initial values as well as the use of iterated values. The iteration of the flutter frequency for an assumed Mach number yields three frequencies, p_1, p_2, and p_3, which are the solution for the real and imaginary parts of equation (9). These three frequencies occasionally undergo rapid variations with reduced frequency requiring a long iterative procedure to obtain a solution.

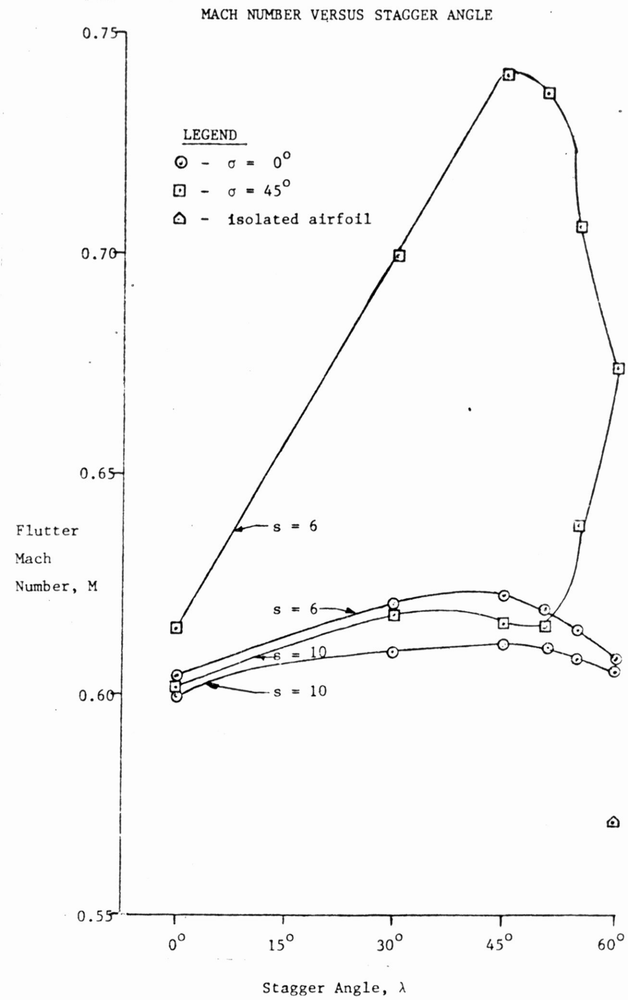

Figure 3

Conclusions and Recommendations

A numerical computational procedure for the prediction of two-dimensional flutter in a cascaded airfoil has been developed. In addition, the effect of cascade parameters on flutter speed has been investigated. The success and reliability of this computational procedure was observed to depend upon the reliability of the unsteady airload prediction method. This study has been only a first attempt to obtain the solution for a complex problem and a more thorough investigation and several other numerical examples need to be studied before one can completely understand the effect of various cascade parameters on flutter Mach number and frequency.

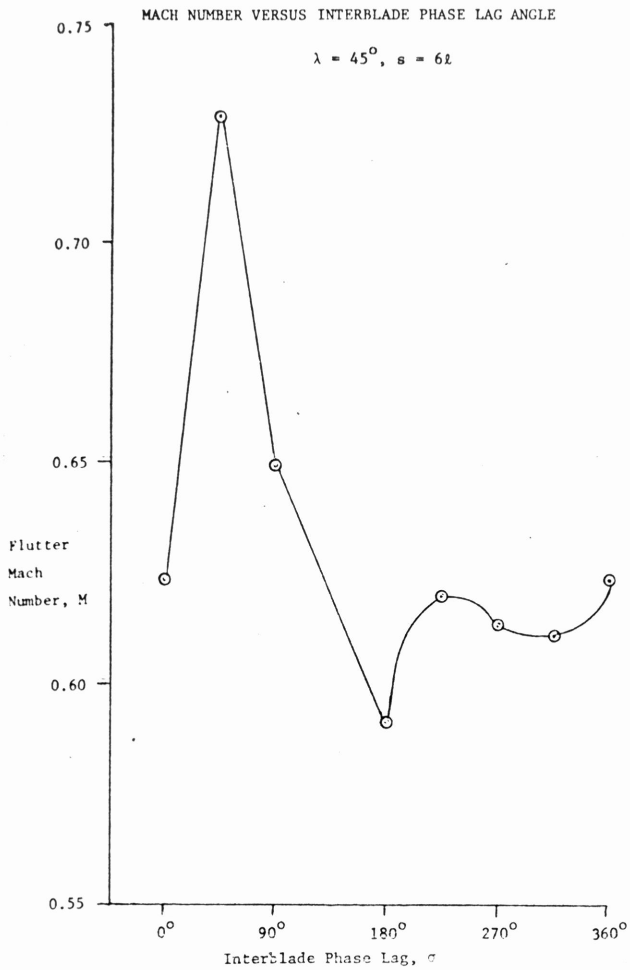

Figure 4

Appendix

A numerical test case for cascaded airfoil flutter was computed using a classical flutter analysis FORTRAN program interfaced with an unsteady airload prediction program evaluating the aerodynamic derivatives. The geometrical and structural properties of the reference airfoil used in this analysis are listed below.

Static moment about the elastic axis = S = 0.456 ft-lb

Moment of inertia about elastic axis = I = 3.375 ft^2-lb

Mass per unit span of blade = M_w = 0.6516 lb

Bending natural frequency = ω_z = 62.2 rad/sec

Torsional natural frequency = ω_α = 100.6 rad/sec

The cascade parameters used in this test case are

Interblade Spacing = S = 6ℓ

Interblade Phase Angle = σ = 90°

Stagger Angle = λ = 45°

To obtain the flutter speed and frequency, the program was executed for the given airfoil. As initial values, the program assumes a Mach number of 0.6 and a reduced frequency, ω, of 0.4. After obtaining the roots of the stability determinant, p_1, p_2, and p_3, it assumes a ω = 0.5. With the roots obtained from this run, it interpolates another ω so that either p_1 or p_2 is equal to p_3. Experience has shown that for many cases if this value is used, divergence may result since the determinant roots are not well behaved. Therefore, a limit of 0.04 has been set for the maximum change in ω that is allowed. Table 1 shows the history of the iteration in which convergence is achieved, but at a flutter Mach number of 0.6587. The new assumed Mach number is 0.7 for the next run and according to Table 2, it converges to a flutter Mach number of 0.6235. Using these values for flutter Mach number, an extrapolated Mach number is obtained so that the assumed Mach number is equal to the flutter Mach number. This value is 0.643 and according to Table 3 converges to a flutter Mach number of 0.649, which is within accuracy tolerances. The associated reduced flutter frequency for this flutter Mach number is 0.3612.

TABLE 1 FIRST ASSUMED MACH NUMBER

Assumed Mach Number	ω	p_1	p_2	p_3	Flutter Mach Number
0.6	0.4	65.26	84.31	80.2	
	0.5	63.90	92.00	79.88	
	0.36	67.80	78.83	79.36	
	0.365	67.32	79.67	79.49	
	0.363	67.48	79.38	79.45	
	0.3634	67.44	79.46	79.46	.6587

TABLE 2 SECOND ASSUMED MACH NUMBER

Assumed Mach Number	ω	p_1	p_2	p_3	Flutter Mach Number
0.7	0.4	69.47	86.69	74.02	
	0.5	64.48	106.13	68.68	
	0.36	72.58	76.87	75.07	
	0.37	71.63	78.76	75.02	
	0.367	72.14	77.90	75.04	
	0.3633	73.70	75.81	75.07	
	0.363	74.03	75.42	75.07	
	0.36274	74.27	75.03	75.07	
	0.36269	74.40	75.03	75.07	
	0.36271	74.36	75.07	75.07	.6235

TABLE 3 THIRD ASSUMED MACH NUMBER

Assumed Mach Number	ω	p_1	p_2	p_3	Flutter Mach Number
0.643	0.4	66.40	83.83	78.33	
	0.5	65.90	95.92	75.20	
	0.364	68.88	78.49	77.90	
	0.360	69.45	77.53	77.81	
	0.3613	69.24	77.86	77.84	
	0.36122	69.25	77.84	77.84	.649

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