ZERO PRESSURE BALLOON DESIGN

by

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ABSTRACT*

In atmospheric balloon research it is very important to accurately describe the delicate balance of forces applied to a film and to use this analytical description to produce a suitable manufactured balloon shape. This paper investigates several additions to the presently accepted method for free balloon design. This includes a study of the non-uniform stress state created by the inclusion of load tapes in the theoretical model. The effects of balloon material properties and deformations is also considered. These refinements result in a more accurate theoretical model and an improved design for atmospheric research balloons.

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ACKNOWLEDGEMENTS AND DEDICATION

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SYMBOLS

a	distance the zero pressure level is below the bottom of the balloon
A	balloon surface area
b	specific bouyancy of lifting gas
E_x	film modulus in x-direction
K_t	tape stiffness
n	number of load tapes
p	pressure
P	total balloon payload
r	radius
s	distance measured along a gore
t	film thickness
T	force created by film stress
V	balloon volume
w_f	film weight per unit area
w_t	tape weight per unit length
z	height
ε_f	film strain
ε_t	tape strain
θ	angle between balloon film and the vertical axis of symmetry
λ	normalizing length $(P/b)^{1/3}$
σ_c	circumferential film stress (force/length)
σ_m	meridional film stress (force/length)

NON-DIMENSIONAL DEFINITIONS

$\bar{A} = A/\lambda^2$

$\bar{E}_x = E_x t/b\lambda^2$

$k = (2\pi)^{-1/3}$

$\bar{r} = r/\lambda$

$\bar{s} = s/\lambda$

$\bar{V} = V/\lambda^3$

$\bar{z} = z/\lambda$

$\bar{\sigma}_c = \sigma_c/b\lambda^2$

$\bar{\sigma}_m = \sigma_m/b\lambda^2$

$\Sigma_f = w_f/kb\lambda$

$\Sigma_t = nw_t/b\lambda^2$

INTRODUCTION

Atmospheric balloon research is a relatively young and often overlooked area of engineering science. However, the origins of the basic ideas relating to ballooning can be traced through history to before the time of Christ. In 240 B.C. Archimedes stated that a statically floating body must displace its own weight of the fluid in which it floats.[1] The practical application of this prinicple to balloon flight was not to come until many years later when the Montgolfier brothers constructed several hot-air balloons in the late 1700's. In 1783, hydrogen was first used by J. A. C. Charles and M. N. Robert as an improved lifting gas.[1] Prior to this point in history and for approximately 150 years thereafter, the entire range of knowledge in balloon design and construction consisted of only a few simple "rule-of-thumb" concepts. It was not until the late 1930's that the University of Michigan began an extensive study of atmospheric balloons. This program, prompted by the failure of the first Explorer flight, produced several notable achievements: 1) lightweight plastics were considered for use in the construction of the gas barrier and 2) a balloon shape and stress analysis was introduced.

This work done by the University of Michigan marked the beginning of modern atmospheric balloon research. The state-of-the-art in balloon design was advanced further through work done by Ralph H. Upson and later by Justin H. Smalley. In his study the equilibrium balance of forces acting on the balloon film was investigated and used to derive a set of equations defining the shape of a specific balloon.[2] This has become the accepted method of balloon design in recent years. Research continues at several institutions in such areas as material property investigation and design and analysis of various balloon systems.

As is typical in any growing area of engineering and science, increased demands are encountered which can only be satisfied by a continually improved subject knowledge. This is true of the field of atmospheric balloon research as well. More stringent design criteria demand a more accurate theoretical description of reactions in the balloon film and a more precise design procedure. The main purpose of this paper will be to investigate improvements in this area relating to load tape effects and material properties and deformations.

THEORY

The main goal in atmospheric balloon design is to be able to define a specific balloon shape for the manufacturer which will accurately satisfy a particular set of mission requirements. The presently accepted method by which this is done was developed by Justin H. Smalley.[2] This procedure determines the characteristics of various balloon shapes through the solution of a set of differential equations. These equations are developed by considering the forces acting on a differential film element of a symmetrically deployed balloon in the float configuration. The boundaries of this element of balloon film are defined as shown in Figure 1. The film is then considered to be in equilibrium with the forces acting on it as shown in Figure 2. By summing the forces in the meridional direction and perpendicular to the film the equilibrium equations are found to be:

$$T_{m2} \cos\theta_2 - T_{m1} \cos\theta_1 - F_p \sin\theta - F_w = 0$$

$$T_{m2} \sin\theta_2 - T_{m1} \sin\theta_1 - F_p \cos\theta - 2T_c \sin\frac{\Psi}{2} = 0$$

Upon substitution of terms, eliminating higher order differentials, and assuming $\sin \Psi/2 = \Psi/2$ (i.e. small angles) these equations reduce to the following

$$\Delta(r\sigma_m \cos\theta) - rw_f \,\Delta s - pr \sin\theta \,\Delta s = 0$$

$$\Delta(r\sigma_m \sin\theta) - \sigma_c \Delta s + pr \cos\theta \,\Delta s = 0$$

After dividing by Δs and forming the derivative with respect to gore position, s, these equations become:

$$\frac{d}{ds}(r\sigma_m \cos\theta) - rw_f - pr \sin\theta = 0$$

$$\frac{d}{ds}(r\sigma_m \sin\theta) - \sigma_c + pr \cos\theta = 0$$

Applying the chain rule to the derivatives results in the following:

$$\frac{d}{ds}(r\,\sigma_m)\cos\theta - (r\sigma_m)\sin\theta\frac{d\theta}{ds} - rw_f - pr\sin\theta = 0$$

$$\frac{d}{ds}(r\,\sigma_m)\sin\theta + (r\sigma_m)\cos\theta\frac{d\theta}{ds} - \sigma_c + pr\cos\theta = 0$$

These equations are then combined and rearranged with the additional assumption that the pressure differential is proportional to the vertical distance from the zero pressure point, a.

$$(r\sigma_m)\frac{d\theta}{ds} = \sigma_c\cos\theta - rw_f\sin\theta - br(z+a)$$

$$\frac{d}{ds}(r\sigma_m) = \sigma_c\sin\theta + rw_f\cos\theta$$

Four other differential equations can be written from the geometric consideration of the balloon shape as follows:

$$\frac{dr}{ds} = \sin\theta$$

$$\frac{dz}{ds} = \cos\theta$$

$$\frac{dA}{ds} = 2\pi r$$

$$\frac{dV}{ds} = \pi r^2\cos\theta$$

These differential equations may be written in non-dimensional form for greater convenience. This is done by introducing the non-dimensional film weight parameter, Σ_f, along with the parameters λ and k to yield the final set of non-dimensional differential equations

$$(\overline{r\sigma_m})\ \theta' = \bar{\sigma}_c\bar{z}' - k\ \Sigma_f\ \bar{r}\ \bar{r}' - \bar{r}(\bar{z}+\bar{a})$$

$$(\overline{r\sigma_m})' = \bar{\sigma}_c\bar{r}' + k\ \Sigma_f\ \bar{r}\ \bar{z}'$$

$$\bar{r}' = \sin\theta \qquad \bar{A}' = 2\pi\bar{r}$$

$$\bar{z}' = \cos\theta \qquad \bar{V}' = \pi\bar{r}^2\bar{z}'$$

with differentiation with respect to s represented by the prime notation. These equations are solved simultaneously to yield a specific balloon shape for a given set of conditions.

This technique instituted by Smalley using these differential equations is generally used today for the majority of the balloon design work. This formulation assumes that the balloon is constructed of a continuous unbroken shell, and thus maintains a constant load distribution as shown in Figure 3. However, in reality these balloons are constructed of several gore sections. Along each seam between the gores a load tape is attached for reinforcement and load introduction purposes. This then alters the original assumption of a smooth continuous film. It is evident that the total load carried by the balloon will be carried jointly between the film and the load tapes as shown in Figure 4.

The relationship between the tape and film loadings at any meridional location can be defined by

$$P = F_t + F_f$$

where P is the total load carried by the balloon and F_t and F_f represent the tape and film loads respectively.[3] The tape and film force can be expressed in terms of the tape and film strains according to the following equations:

$$F_t = n\ K\ \varepsilon_t$$

$$F_f = \sigma_m\ 2\pi r t_f = E_f\ \varepsilon_f\ 2\pi r t_f$$

This simplified relationship between film stress and strain should be sufficiently accurate to evaluate the magnitude of the load distribution between tape and film.

Combining these equations and assuming tape and film strains to be equal in the meridional direction yields:

$$F_f = P(1 + \frac{nK}{2\pi r E_f t_f})^{-1}$$

If load tapes are not included in the theoretical model this equation would show that the entire load is carried in the balloon film. However, when typical values for the constants are substituted, this equation demonstrates that the film carries only about 40% of the total load.

It is apparent from this that any accurate theoretical model should in some way account for the load tape influence in the overall balloon design. For this reason Smalley's differential equations defining a balloon shape are reexamined including load tape effects. Again a differential element of balloon film and its associated loadings are considered. The forces acting on the circumferential boundaries are now defined allowing for load tape influence as

$$T_{m1} = (r - \frac{\Delta r}{2}) \, \Psi \, (\sigma_m - \frac{\Delta\sigma_m}{2}) + (\varepsilon_t - \frac{\Delta\varepsilon_t}{2})K_t$$

$$T_{m2} = (r + \frac{\Delta r}{2}) \, \Psi \, (\sigma_m + \frac{\Delta\sigma_m}{2}) + (\varepsilon_t + \frac{\Delta\varepsilon_t}{2})K_t$$

and the weight force is rewritten as

$$F_w = w_f \, (r\Psi\Delta s) + w_t \Delta s$$

to include tape weight as well as film weight. These modified weight and force terms are then used to write the equilibrium equations. These equations are manipulated in a manner similiar to the original Smalley derivations to yield the following differential equations:

$$(r\sigma_m + \frac{nK_t}{2\pi}\varepsilon_t)\frac{d\theta}{ds} = \sigma_c \cos\theta - \sin\theta(rw_f - \frac{n}{2\pi} w_t) - br(z+a)$$

$$\frac{d}{ds}(r\sigma_m + \frac{nK_t}{2\pi}\varepsilon_t) = \sigma_c \sin\theta + \cos\theta \, (rw_f - \frac{n}{2\pi} w_t)$$

The non-dimensional forms of these two new equations can then be written using the non-dimensional tape stiffness and weight parameters, $\overline{K_t}$ and Σ_t, as follows:

$$(\bar{r}\bar{\sigma}_m + k^3\bar{K}_t\varepsilon_t)\theta' = \bar{\sigma}_c\,\bar{z}' - \bar{r}(\bar{z} + \bar{a}) + k^3\,\Sigma_t\,\bar{r}' - k\,\Sigma_f\,\bar{r}\,\bar{r}'$$

$$(\bar{r}\bar{\sigma}_m + k^3\bar{K}_t\varepsilon_t)' = \bar{\sigma}_c\,\bar{r}' + (k\Sigma_f\,\bar{r} - k^3\,\Sigma_t)\bar{z}'$$

The four differential equations describing the geometric balloon parameters from the original Smalley derivations remain unchanged. The solution of this set of equations then defines a balloon shape which realistically includes load tape effects.

A second consideration that would improve the existing design procedure involves film deformation and material properties. It would be advantageous to be able to define the meridional and circumferential film stresses in terms of the appropriate material properties and film strains. It is obvious that some material deformation does occur when the balloon fully deploys in the float configuration. By allowing for this material deformation in the design procedure a more accurate estimation could be made of the actual film stress levels. This would also be beneficial in the balloon shape description. For this reason this film stresses are defined in the following non-dimensional equations

$$\bar{\sigma}_m = \bar{E}_m\varepsilon_m + \bar{E}_{mc}\varepsilon_c$$

$$\bar{\sigma}_c = \bar{E}_{mc}\varepsilon_m + \bar{E}_c\varepsilon_c$$

using the non-dimensional material properties $\bar{E}_{mc}$, $\bar{E}_c$, and $\bar{E}_m$ together with the meridional and circumferential film strains. This information concerning material properties and deformation together with the inclusion of load tapes in the theoretical model is added as modifications to the present design method. These improved design equations are then solved numerically by Gill's modification of the Runga-Kutta method.[4]

NUMERICAL EVALUATION

The differential equations derived from an investigation of a balloon surface element can be solved simultaneously using Gill's modified Runga-Kutta method to yield a specific balloon shape. This is done numerically with the aid of any of several computer programs designed specifically for this purpose[4]. Typically these programs use the bottom of the balloon as a starting point for a solution. Assuming an initial angle, θ_0, the set of equations are integrated over incremental positions until the apex of the balloon is reached. An existing balloon design program was modified to incorporate the proposed changes (i.e. inclusion of tape effects, material deformation, material properties) into the design procedure. An iterative technique added to the original program insures that the shape produced is for a flat-topped balloon ($\theta_{final} = \frac{-\pi}{2}$). The solution at each gore position yields both an r and z value which eventually defines the entire balloon shape as shown in Figure 5.

DICUSSION OF RESULTS

For demonstration purposes a typical scientific mission was selected to evaluate the impact of load tapes and material properties (Table I). A payload of 3000 pounds floating at 120,000 feet is quite nominal. Several cases of various tape parameters were used to evaluate the influence of these quantities on the final balloon shape and stress distribution. The results of this study are presented in Figures 6 and 7.

Considering the non-dimensional tape parameters, $\overline{K_t}$ and Σ_t, it is seen that a change in the number of tapes linearly affects each of these constants. Therefore a change in n would essentially be the same as an appropriate change in the tape weight and stiffness. Allowing n to go to zero, the tape influence is completely negated, and the resulting design reverts back to the original method using a theoretical model with a uniform load distribution. This was done and the results agreed to three significant figures with those reported by Smalley[4]. This check serves to verify the validity of the newly modified design procedure.

The results presented in Figures 6 and 7 clearly demonstrate the load tape effects on both the balloon shape and stress levels. By considering the shape changes in Figure 6 it can be seen that the balloon assumes a smaller overall radius and height as the tape influence increases. Of these two dimensional changes the difference between the "no-tape configuration" is more pronounced in the decreased radius. The percentage change in maximum radius is approximately twice that of the difference created in overall height. The maximum radius change of approximately six percent could represent as much as nine feet difference in actual size.

As shown in Figure 7 the balloon film loads are also considerably affected by the inclusion of load tapes in the design procedure. The influence on meridional film stress is especially obvious. The difference in stress that is created by the addition of load tapes appears in two forms: 1) the elimination of extreme stresses at the top and bottom of the balloon and 2) the overall decrease in the magnitude of meridional film stress. The stress magnitude decreases approximately eighty percent along the majority of the gore length.

CONCLUSIONS

The influence of tape weight and stiffness have been included in the design procedure for high altitude scientific balloons. Typical values of these parameters currently used by the industry have been used to evaluate their influence on balloon shape and stress distributions. It has been found that:

a. The maximum radius may be reduced by as much as six percent from the normal design.

b. The gore length may be reduced by as much as four to five percent of its original length.

c. The flim stresses in the meridional direction are reduced approximately eighty percent over the majority of the film surface.

It is concluded that typical scientific balloons could be designed with significant size, weight, and cost reductions without compromising the structural integrity or reliability of the system by including the appropriate tape parameters.

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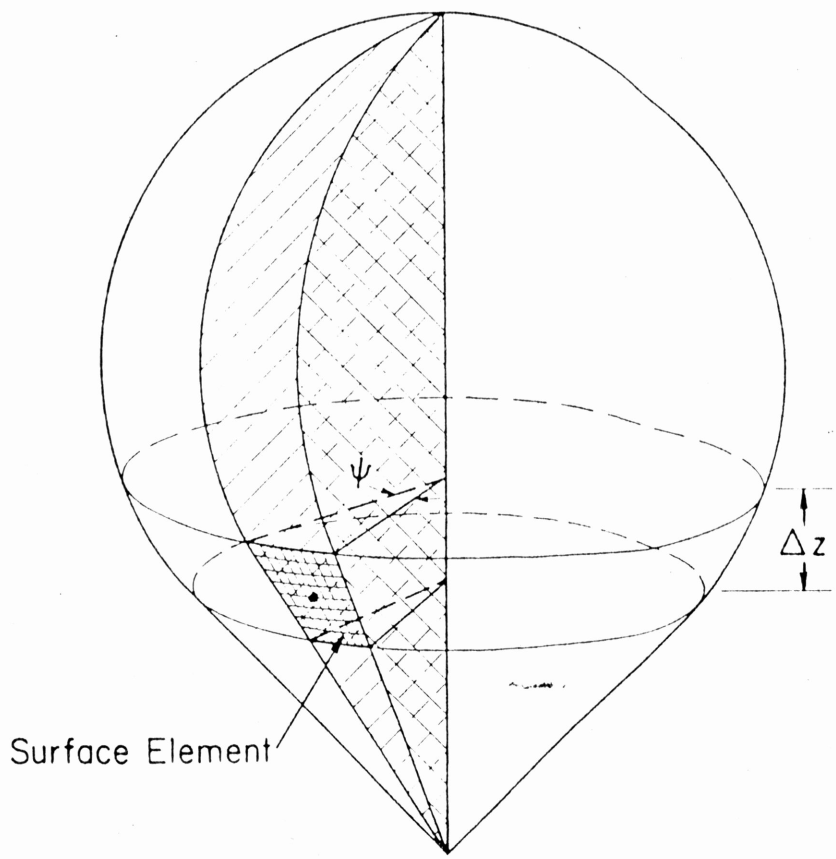

Figure 1. Differential surface element

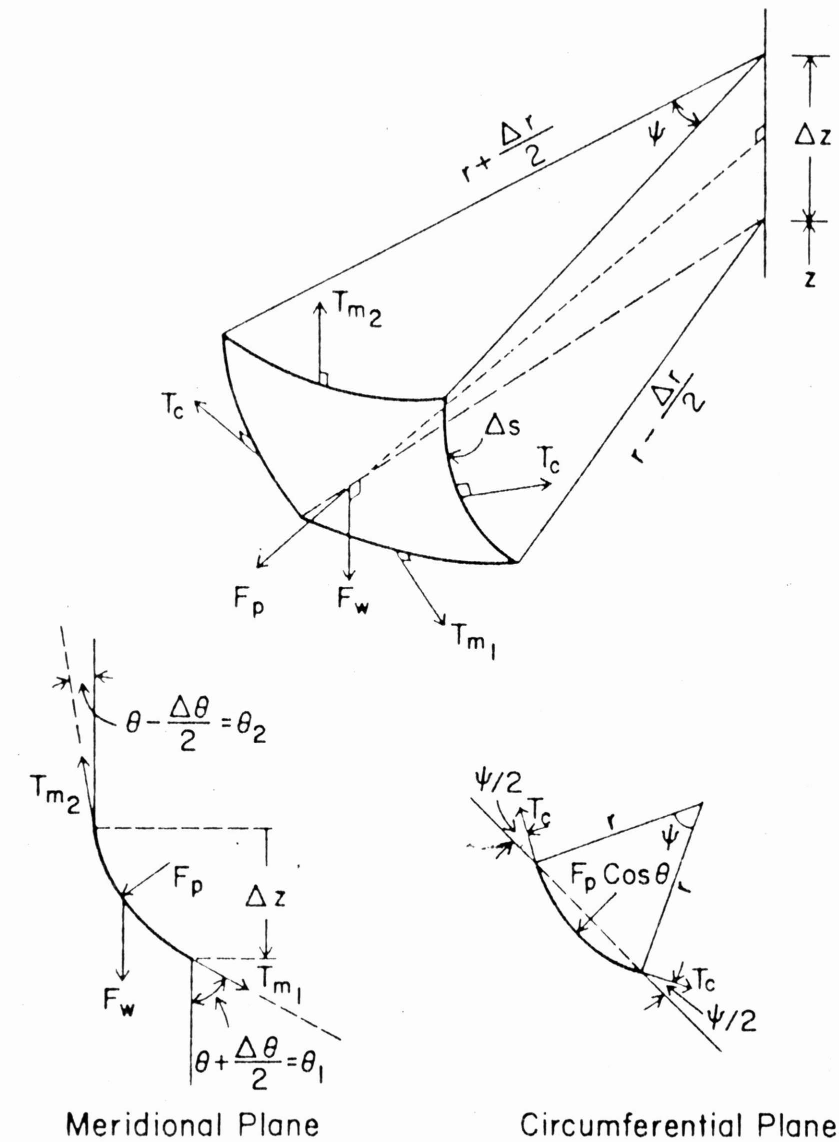

Figure 2. Equilibrium forces acting on the film element

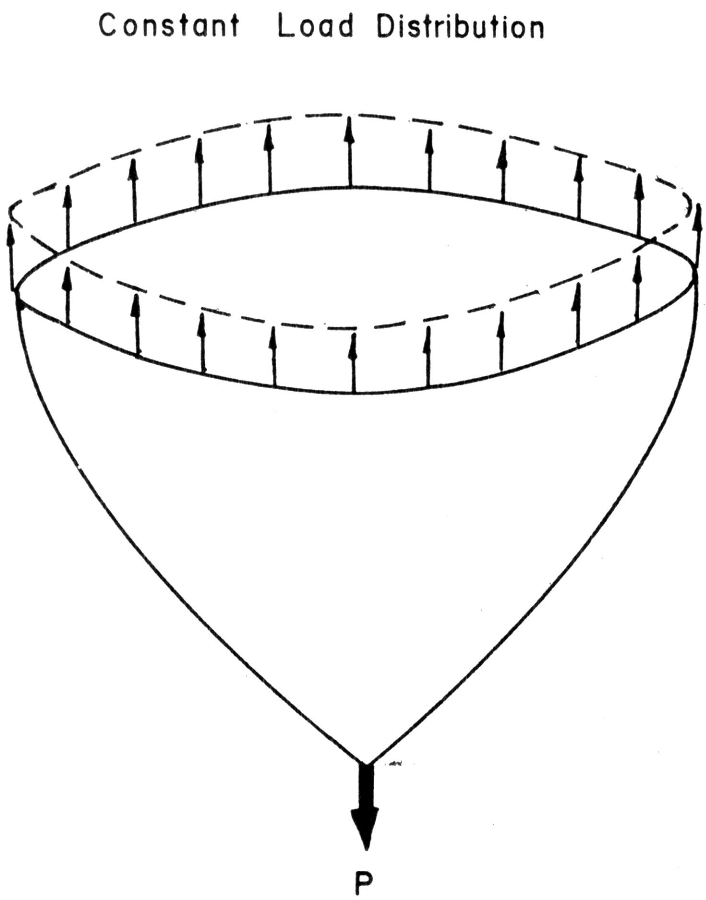

Figure 3. Constant load distribution model

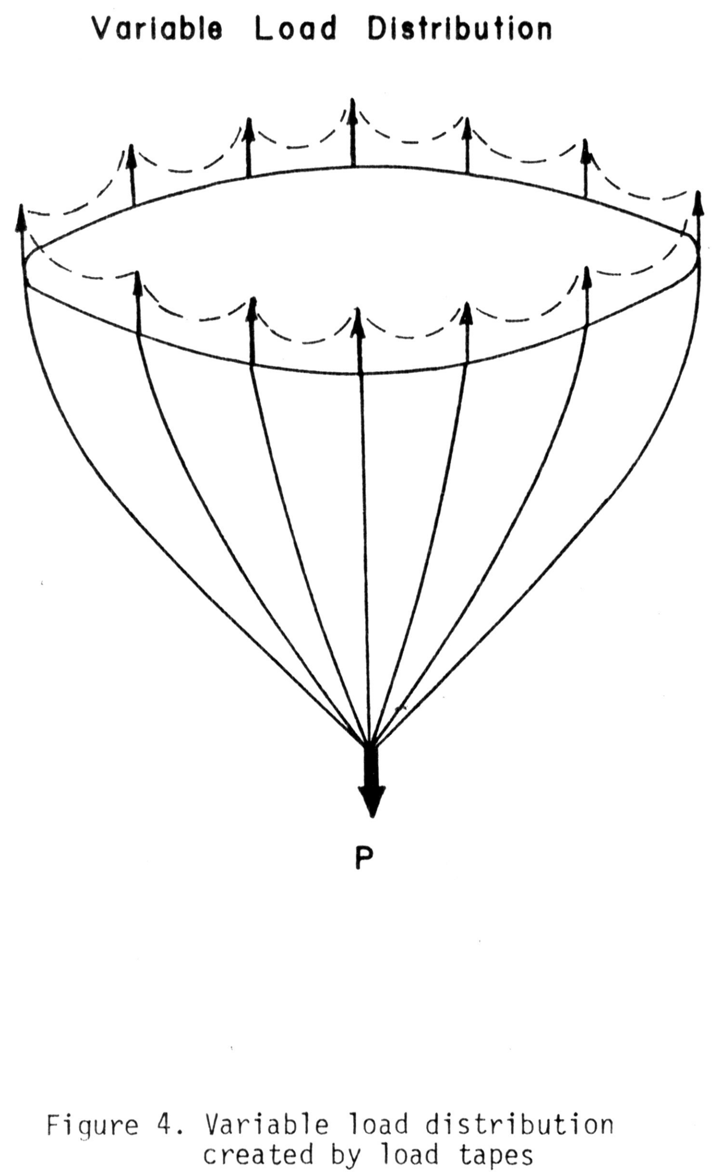

Figure 4. Variable load distribution
created by load tapes

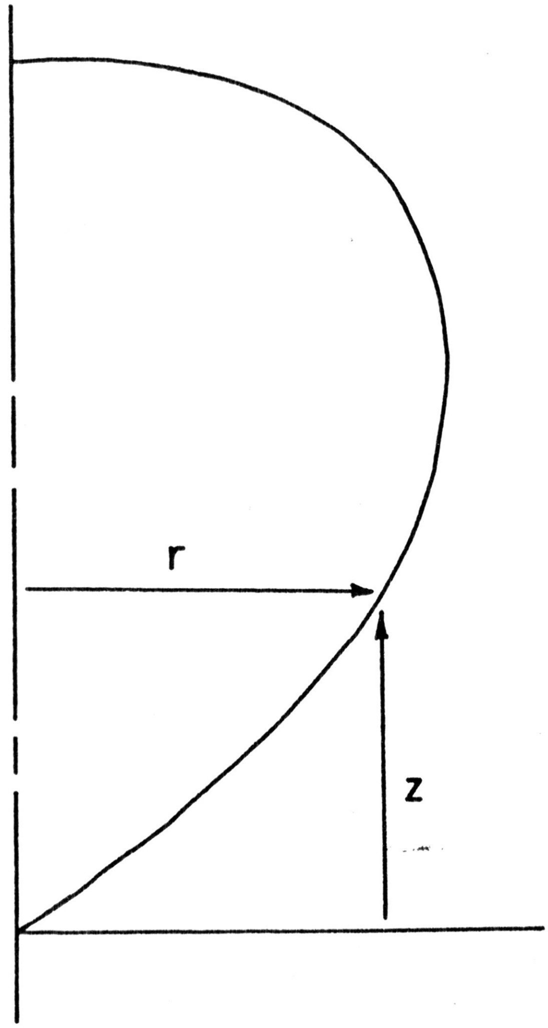

Figure 5. Balloon shape coordinates

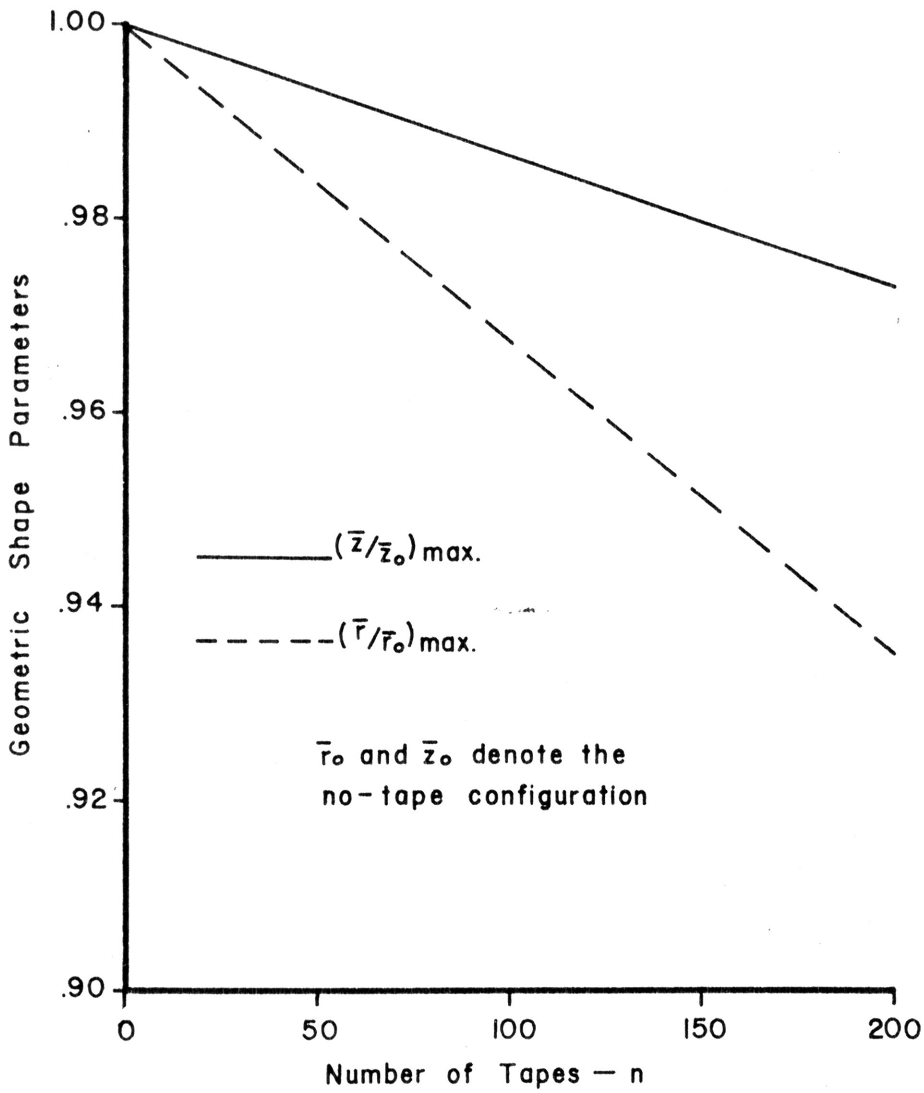

Figure 6. Load tape effects on balloon shape

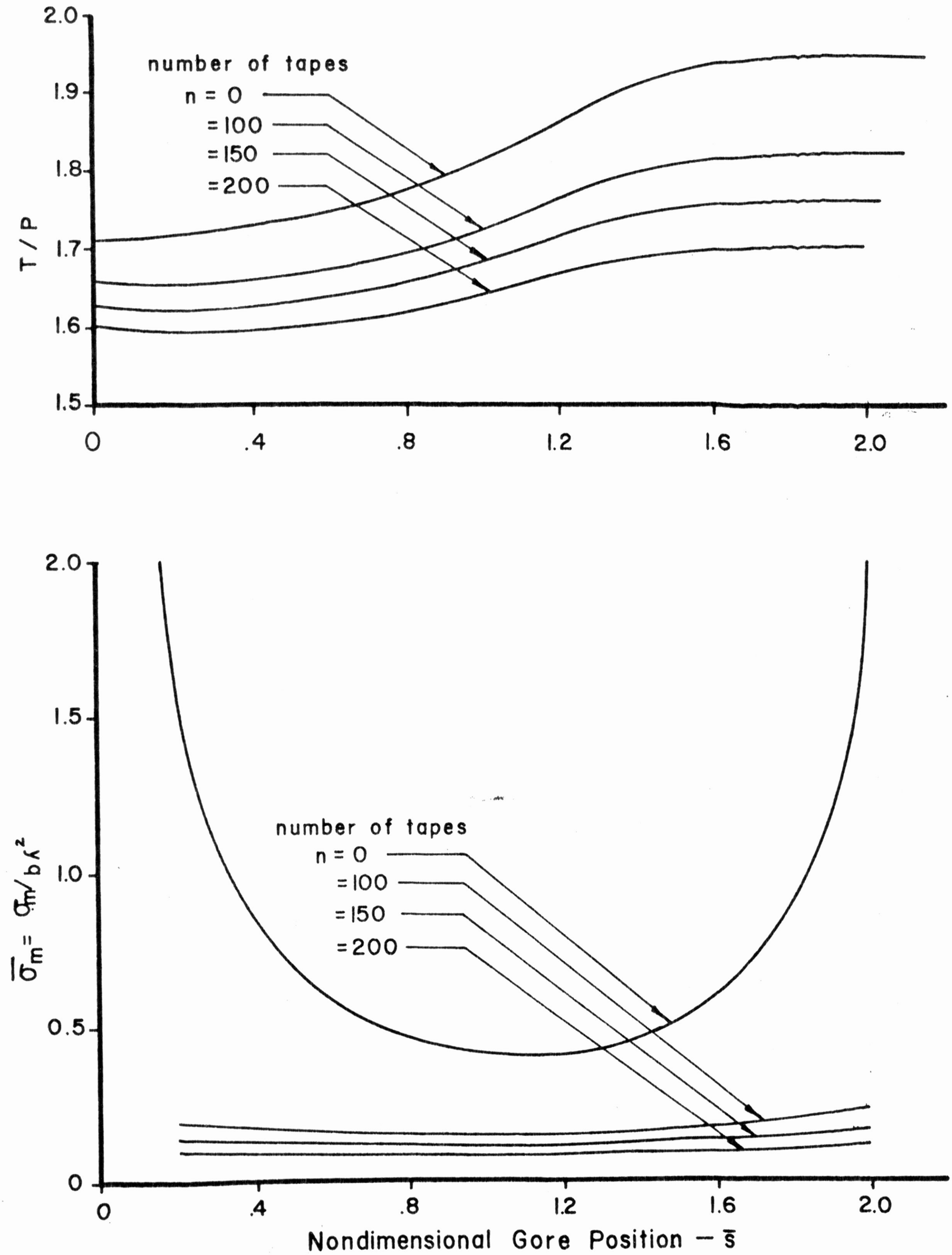

Figure 7. Load tape effects on balloon film loading

TABLE I - SAMPLE CASE PARAMETERS

Design Parameters	
Float Altitude (ft)	120,000
Payload (lbs)	3,000
Specific Bouyancy (lbs/ft^3)	3.455×10^{-4}
Normalizing Length (ft)	205.5
Film Properties	
Thickness (in)	0.0008
Σ_f	0.10
E_f (lbs/in^2)	80×10^3
Poisson's Ratio	0.5
E_m, E_c (lbs/in^2)	131.8
E_{mc} (lbs/in^2)	65.9
$\bar{E}_m, \bar{E}_c$	70.2
$\bar{E}_{mc}$	35.1
Load Tape Properties	
Number of Tapes	150
Type	500 lb. Polyester
Tape Stiffness (lbs)	1.5×10^4
Tape Weight (lbs/ft)	0.00628
Σ_t	0.0646
$\bar{K}_t$	600.3