AN INTERPRETATION OF PEIRCE'S EXISTENTIAL GRAPHS

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Abstract

The American philosopher Charles Sanders Peirce has come to be regarded as one of the founding fathers of modern day logic. One of his most original contributions is the logical system called existential graphs. Through the existential graphs, Peirce sought a means of evaluating the workings of necessary inference. Peirce develops the existential graphs in three sections, the alpha graphs, beta graphs, and gamma graphs. The purpose of this study is to offer a coherent interpretation, complete with examples, of Peirce's existential graphs. A careful evaluation of Peirce's notes on the existential graphs shows the alpha graphs to be a version of propositional logic and the beta graphs to be a version of quantificational logic. The gamma graphs, however, are not as fully developed as modern day modal logic.

INTRODUCTION

Biography

Charles Sanders Peirce was born in Cambridge, Massachusetts in 1839. His father, Benjamin Peirce, was a well-known mathematician and astronomer who taught at Harvard. Through the influence of his father and his uncle, Charles Henry Peirce, a physician and chemist, young Charles was introduced to the rigors of science and math at an early age (Eschbach, p. xii). He attended Harvard sporadically from 1855 until 1863, receiving a B.A. and an M.A. in chemistry in 1859 and 1862 respectively. Upon leaving Harvard, Peirce accepted a position as a computing aide with the U.S. Coast and Geodetic Survey. Peirce worked for the Coast Survey for the next thirty years doing geographical measurements and meteorological research. During his work with the Survey Peirce devised a method of measuring the gravitational constant g using a set of pendulums. While working for the Coast Survey Peirce continued researching and writing in his spare time on a variety of topics. From 1879 until 1884 he taught logic at Johns Hopkins University. Low enrollment in his classes and personal disputes with several of his colleagues led to his dismissal. From 1900 until his death in 1914 Peirce and his wife lived in poverty in Pennsylvania.

Upon Peirce's death his wife sold his letters and papers to Harvard University for $500. Not until the 1930's when Charles Hartshorne and Paul Weiss published a six volume set of Peirce's papers did many realize the extent of Peirce's work. Two additional volumes of Peirce's papers were added to the Hartshorne and Weiss Collected Papers by Arthur Burks in 1958. Only recently has the academic community come to honor Charles Sanders Peirce as one of America's greatest thinkers for his original work in chemistry, geology, logic, mathematics, history, ethics, and philosophy. The University of Indiana has published three volumes of a planned twenty volume set of Peirce's papers. This project as well as various specialized works and a forthcoming biography of Peirce will greatly increase the information available to the public.

Peirce's Contributions to Logic

Peirce is reported to have regarded logic as his favorite field of study. To this field he made many contributions, but his work did not form a well-defined system (Feibleman, p. 81). Peirce's work spans several areas of logic, including work on the fundamental basis of logic. Logic, according to Peirce, is based upon the facts of experience (Feibleman, p. 85). This statement is fundamental to Peirce's conception of pragmatism, a movement in American philosophy which he helped to found. Peirce's work on pragmatism, or as he termed it, pragmaticism, centers on the problem of defining concepts (Moore, p. 19). Through his work on a method of defining concepts, Peirce developed for the first time the field of semiotics, the study of signs and their meanings (Moore, p. 3).

In addition to his work on the fundamental basis of logic, Peirce is responsible for the introduction of the logical connective called material implication. He also worked to expand the work of Venn and Euler. While the accomplishments listed here are those for which Peirce is well-known, he is also responsible for a little-known system of logic called the "existential graphs. " Peirce developed the existential graphs in some two hundred pages of loosely organized notes today contained in Volume IV of the Peirce Collected Papers, edited by Hartshorne and Weiss. The Collected Papers are abbreviated CP when referenced in the remainder of the text. An explication of Peirce's existential graphs, together with examples, is the topic of this paper.

EXISTENTIAL GRAPHS

Of particular interest among Peirce's accomplishments in logic is his work on existential graphs. While most logical systems use algebraic methods of representing verbal propositions (cf. Russell-Whitehead, Boole), Peirce sought to express propositions pictorially. His system of pictorial representations of propositions and rules by which the pictures may be manipulated are known as the existential graphs. Under the method of the existential graphs, a proposition is diagrammed using a notation of circles and various types of connectives in such a way that the resulting diagram exactly represents the written verbal proposition. The resulting diagram may then be manipulated according to rules set forth by Peirce so that all resulting graphs correspond to propositions that follow validly from the given propositions. This approach is used to evaluate arguments, for manipulations of the graphs of the premises will yield a graph of the conclusion if the argument is valid.

Peirce's ultimate goal in working with the system of existential graphs was to display the workings of necessary inference (CP, p. 346). By breaking up the transformation from premises to conclusion in an argument into as many simple steps as possible, Peirce outlined the workings of reasoning. The graphs and the rules of transformation by which the graphs are manipulated are said by Peirce to represent a generalized diagram of the reasoning powers of the human mind (CP, p. 469).

Peirce developed the system of existential graphs in three parts, which he termed the "alpha", "beta", and "gamma" graphs, respectively. Each type of graph encodes a different type of proposition and has a separate set of symbols and rules of transformation.

An overview of Peirce's text together with interpretations and examples follows. For each representation of a proposition using an existential graph, the corresponding representation has been given in the more commonly used algebraic notation of Russell and Whitehead. The numbering shown here of the conventions and rules for drawing, interpreting, and transforming graphs is designed to provide the maximum clarity. The numbering does not necessarily follow any of the various numbering schemes outlined by Peirce in his development of the existential graphs.

ALPHA GRAPHS - PROPOSITIONAL LOGIC

Conventions

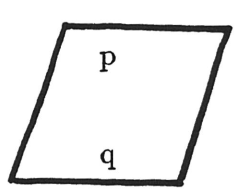
The alpha graphs have three conventions for graphing propositions.

Convention 1

All graphing of propositions will be done upon a sheet of paper. This sheet of paper is called the sheet of assertion (CP, pp. 331-332, 337, 348).

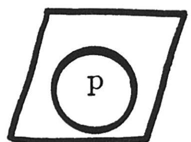
Convention 2

All graphs written upon the sheet of assertion are asserted to be true in the particular problem under discussion. Hence the logical operator of conjunction consists in Peirce's system in writing the conjoined propositions on the same sheet of paper (CP, pp. 332, 349).

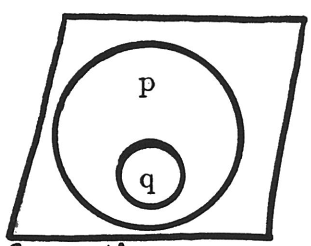
Example	E.G.	R-W	Translation
		$p \cdot q$	p and q

Convention 3

A circle enclosing a graph negates what is enclosed. Peirce calls the circle itself a cut or a sep. The inner space which the circle encloses is not asserted and hence is not on the sheet of assertion. The cut together with the area it encloses is on the sheet of assertion (CP, pp. 332, 337, 350-351).

Example	E.G.	R-W	Translation
		$\sim p$	not p

A conditional proposition stating that if one proposition is true then another proposition is true is expressed as a pair of circles, one within the other, with the antecedent written in the space between the circles and the consequent written in the inner circle. Peirce calls this set of two circles a double cut or a scroll. The structure of the conditional proposition graph is derived from Convention 3 (CP, pp. 333, 351-353).

Example	E.G.	R-W	Translation
		$p \supset q$ or $\sim (p \cdot \sim q)$	If p then q

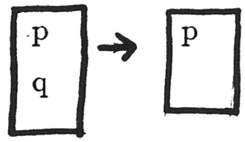
Rules of Transformation

The alpha graphs have seven rules of transformation governing manipulations of graphs on the sheet of assertion.

Rule 1

Any graph on the sheet of assertion that has no enclosures or is enclosed by an even number of circles may be erased (CP, pp. 322, 338, 385).

Example E.G. R-W



$p \cdot q$

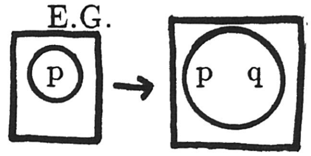
$\therefore p$

Translation: p and q are simplified to p

Rule 2

Any graph may be inserted within an odd number of circles (CP, pp. 323, 385).

Example E.G. R-W



$\sim p$

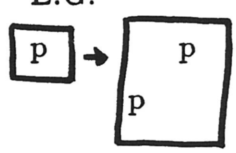
$\therefore \sim (p \cdot q)$

Translation: not p is transformed by addition to not both p and q

Rule 3

Any graph already on the sheet of assertion may be reiterated on any other part of the sheet (CP, pp. 324, 338, 385).

Example E.G. R-W



p

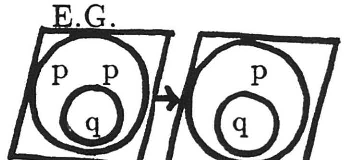
$\therefore p \cdot p$

Translation: p is transformed by repitition to p and p

Rule 4

A graph may be erased if it is located at the same level within a set of cuts as a graph identical to itself.

Example E.G. R-W



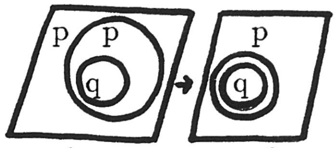
$\sim [(p \cdot p) \cdot \sim q]$

$\therefore \sim (p \cdot \sim q)$

Translation: not(p and p and not q) becomes not both p and not q

Also, a reiterated graph located within a greater number of cuts (counting from the outside) than the graph(s) identical to it may be erased.

Example E.G. R–W



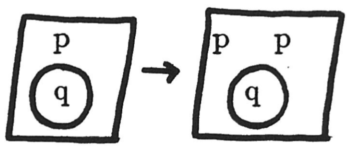
modus ponens

modus tollens

Translation: p and if p then q becomes p and not q

An alternative way of expressing this is that the opposite of anything permissable on the sheet of assertion is permissable within a cut (CP, pp. 325, 338, 385).

Example E.G. R–W

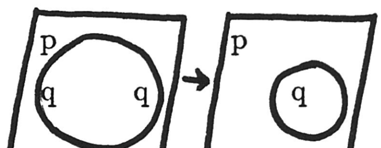


$p \cdot \sim q$

$\therefore p \cdot p \cdot \sim p$

Translation: p and not q becomes p and p and not q

Example E.G. R–W



$p \cdot \sim (q \cdot q)$

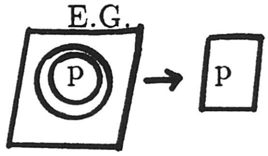
$\therefore p \cdot \sim q$

Translation: p and not both q and q becomes p and not q

Rule 5

Any graph enclosed by two circles may be written on the sheet of assertion without the circles (CP, pp. 324, 338, 385).

Example E.G. R–W



$\sim\sim$ p

$\therefore$p

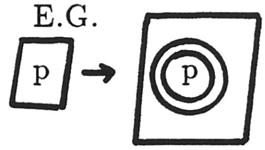
Translation: not not p is p

Rule 6

This rule allows the reversal of Rule 5. A double cut may be written on the sheet of assertion and any graph already on the sheet of assertion may be placed in the inner area of the double cut (CP, p. 338).

Example

E.G.



R–W

p

$\therefore \sim\sim p$

Translation: p is not not p

Rule 7

When any graph one wishes is able to be graphed upon the sheet of assertion, the problem under consideration is absurd (CP, p. 338).

The conventions and rules of transformation associated with the alpha graphs form the basis of what is today called propositional logic. The alpha graphs are capable of evaluating any argument the analysis of the validity of which propositional logic is sufficient. Two examples of logical proofs using the alpha graphs are given below together with the corresponding proof in the algebraic notation of Russell and Whitehead.

EXAMPLES OF ALPHA GRAPHS

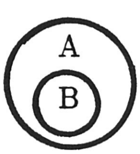
Proof of modus ponens

In Russell–Whitehead notation this argument has the following form.

$$A \supset B$$

$$A / \ B$$

In Peirce's existential graphs this argument is proved in the following way.



A

given

A — by alpha rule 4

B A — by alpha rule 5

B — by alpha rule 1

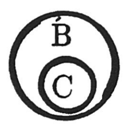
Proof of hypothetical syllogism

In the notation of Russell–Whitehead this argument has the following form.

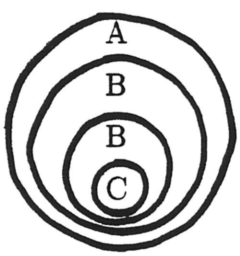
$$A \supset B$$

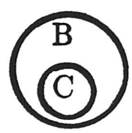
$$B \supset C / \quad A \supset C$$

Using Peirce's existential graphs the argument looks like this:

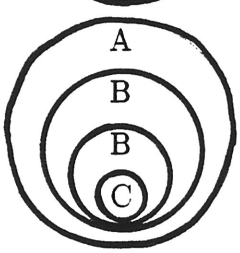


given

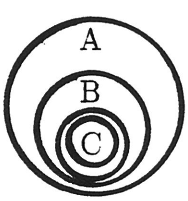




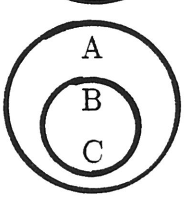
by alpha rule 3



by alpha rule 4



by alpha rule 4



by alpha rule 5

by alpha rule 1

BETA GRAPHS - QUANTIFICATIONAL LOGIC

We now turn from the alpha graphs to a discussion of the beta graphs. The beta graphs use all of the conventions and rules of transformation included in the alpha graphs with the addition of seven new conventions defining the use of new graphing methods and the addition of four new rules of transformation governing manipulations of graphs using the new conventions. As we shall see the added rules of the beta graphs are designed to broaden the use of the existential graphs to cover statements having quantified subject terms. This branch of logic is today called quantificational logic.

To show that the beta graphs are an expansion of the alpha graphs the numbering of the conventions and the rules resumes where the alpha numbering ended.

Conventions

Seven additional conventions form the beta graphs.

Convention 4

When a dash is inserted in place of a noun in a proposition, the result is termed a rhema (plural: rhemata). A rhema is not a proposition. Examples of rhemata, as opposed to propositions, follow (CP, pp. 353-357).

Examples	Proposition	Rhema
	Chocolate is good.	——— is good.
	Joe likes chocolate.	———likes chocolate. Joe likes ———. ——— likes ———.

The dashes are interpreted as "some particular thing which is not currently specified." Dashes do not mean "anything" or "everything."

Peirce defines a spot as "the unanalyzed expression of a rhema" (CP, p. 357). Graphically a spot is an area on the sheet of assertion that is sufficiently defined such that it is separate and distinct from all other areas on the sheet of assertion (CP, p. 339). Thus a spot on the sheet of assertion asserts existence.

Each spot has a hook located at a distinct place on its periphery corresponding to each dash in the rhema.

A dot attached to a hook shows that the particular dash to which the hook corresponds is filled with an indefinite individual (CP, pp. 334, 339).

In the notation of Russell and Whitehead a spot is called a propositional function. The quality expressed in the predicate of the propositional function is symbolized by a capital letter, usually the first letter of the quality. An unspecified individual is represented by a lower case letter usually x,y, or z.

Example	E.G.	R–W
	——— is human.	$(\exists x)(Hx)$

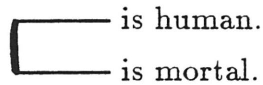
Translation: Some unspecified thing exists which is human.

Convention 5

To express that two "somethings" refer to the same thing the respective hooks are connected by a heavy line called a line of identity. The line of identity must come in contact with no other signs except those at its extremities (CP, pp. 334, 339, 358).

Russell-Whitehead notation simply uses the same lower-case letter to indicate that the referenced individual is the same.

Example E.G. R–W



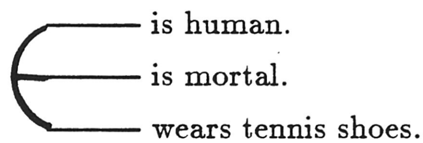
$(\exists x)(Hx \cdot Mx)$

Translation: There exists some unspecified thing that is both a man and is a mortal.

Convention 6

A point where three lines of identity meet is a point of teridentity. That is, all three unspecified individuals filling the dashes (hooks) in the rhema refer to the same individual (CP, pp. 334, 339, 358-359).

Example E.G. R–W



$(\exists x)(Hx \cdot Mx \cdot Tx)$

Translation: There exists at least one thing that has the three qualities of being human, being mortal and wearing tennis shoes.

Convention 7

A ligature is a term referring to the entire collection of connected lines of identity in a given graph (CP, pp. 335, 389).

Thus, in the Example of Convention 6 the three branching lines alone are a ligature.

Convention 8

When a line of identity crosses a sep, the portion of the line lying on the sep itself is not a graph, for a graph must lie either inside or outside a sep. Therefore, points of a line of identity on a sep are defined to be outside the sep for interpretational purposes. The portion of a line of identity inside a sep asserts the hypothetical conditional identity of the term on the inside of the sep and the term on the outside of the sep located at the extremities of the line of identity (CP, pp.363, 456-457).

Example — E.G. — R–W

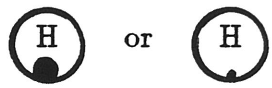
$(x)(Hx \supset Mx)$

Translation: All humans are mortal.

Convention 9

A pseudograph expresses a proposition implying that every proposition is true. The pseudograph is drawn as a black spot entirely filling the circle in which it is located. The size of the spot makes no difference (CP, p. 366).

Example — E.G.



Translation: If H is true, then anything is true.

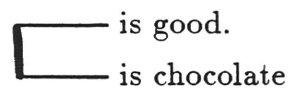
Convention 10

A capital letter, called a selective, may be used to signify an indeterminate individual. The recurrence of the same letter in different portions of the graph refers to the same unspecified individual. A selective may be used to fill any hook (i.e. any blank in the rhema). The use of selectives is entirely optional, for lines of identity may be used for the same purpose. In general Peirce uses selectives only to avoid ambiguity when lines of identity would cross unavoidably (CP, pp. 335, 368-370).

Example

using selectives
X is good.
X is chocolate.

using lines of identity



The above set of conventions has several applications when used to interpret graphs.

Peirce calls these interpretational corollaries. Each one is listed here without extended explanations (CP, pp. 366-367).

Corollary 1

A double cut having the pseudograph in the inner cut is equivalent to the negation of the graph(s) in the outer cut (CP, p. 366).

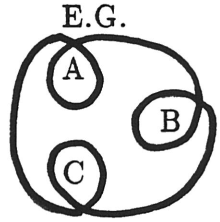
Example

Corollary 2

An inclusive-or proposition may be expressed by placing each alternative in a small loop, with each small loop attached to one cut (CP, p. 366).

Example

E.G.



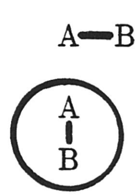
R–W

$A \vee B \vee C$

Corollary 3

Spots in the same compartment may be interpreted as joined by conjunction when enclosed by no loops or by an even number of loops. Spots in the same compartment enclosed by an odd number of loops are combined disjunctively.

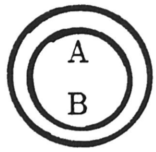
A line of identity whose outermost part is evenly enclosed refers to 'something'; a line of identity whose outermost part is oddly enclosed refers to 'anything' (CP, pp. 366-367).



$(\exists x)(Ax \cdot Bx)$

$\sim (\exists x)(Ax \cdot Bx)$

$\therefore (x)(\sim Ax \vee \sim Bx)$



$\sim\sim(\exists x)(Ax \cdot Bx)$

$\therefore (\exists x)(Ax \cdot Bx)$

Corollary 4

When a line of identity crosses an empty cut the graphs at each end of the line of identity do not refer to the same individual (CP, pp. 367-368).

Example	E.G.	R–W
	 is a man. is a man.	$(\exists x)(\exists y)(Mx \cdot My)$

Translation: There are at least two men.

From these conventions a complete system of quantificational logic emerges. Highlights of the system are outlined below with representations from the logical systems of both Peirce and Russell-Whitehead. A capital letter is used to represent the predicate of an expression with an unspecified subject. Examples of the type of expressions represented include the aforementioned examples ——— is good, or, ——— likes chocolate.

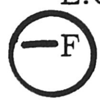
1. As stated previously, writing a spot upon the sheet of assertion asserts existence.

Example	E.G.	R–W
	—F	$(\exists x)Fx$

Translation: Something is F.

2. To negate the entire expression a cut is placed around it as specified in the alpha graph conventions.

Example E.G. R–W



$\sim (\exists x) Fx$

$\therefore (x) \sim Fx$

Translation: Everything is not F; nothing is F.

3. To assert that not all things have the quality F, a cut is drawn enclosing the predicate and part of the line of identity. By Convention 8 extension of the line of identity across the cut or only to the edge of the cut has the same meaning, for the portion of the line touching the cut is interpreted as outside the cut.

Example E.G. R–W

or

$(\exists x) \sim Fx$

$\therefore \sim (x) Fx$

Translation: There exists a non-F.

4. To negate # 3 a cut is drawn around the entire graph. The result expresses universal quantification.

Example E.G. R–W

$\sim (\exists x) \sim Fx$

$\therefore (x) Fx$

Translation: Everything is F.

The preceding four examples utilized a single predicate. Compound predicate forms such as ——— is good and ——— likes chocolate are also expressable in Peirce's system.

5. To express that two properties are possessed by the same thing, the properties are written joined by a line of identity. This is Aristotle's I propositional form.

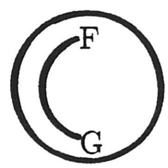
Example | E.G. | R–W

$(\exists x)(Fx \cdot Gx)$

Translation: An unspecified thing exists having both the properties F and G.

6. Negating the entire expression in #5 by drawing a cut around it results in Aristotle's E propositional form.

Example | E.G. | R–W



$\sim (\exists x)(Fx \cdot Gx)$

$\therefore (x)(Fx \supset \sim Gx)$

$\therefore (x)(Gx \supset \sim Fx)$

Translation: No F is G.

7. Negating only one property results in Aristotle's O form.

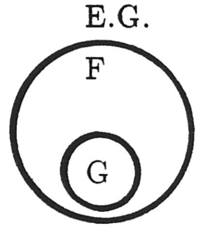
Example | E.G. | R–W

$(\exists x)(Fx \cdot \sim Gx)$

Translation: Something is an F and not a G.

8. To negate #7 a circle is drawn around it. The A form results.

Example | E.G. | R–W



$\sim (\exists x)(Fx \cdot \sim Gx)$

$\therefore (x)(Fx \supset Gx)$

Translation: All F's are G's.

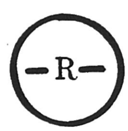
Statements expressing a relation between two or more different things are also representable by the beta graphs. In the examples that follow a capital R with two lines attached is used to express a relation between two unspecified things. An example of this type of expression is ——— likes ——— .

1. So, the basic form from which we are working is shown below. For the Russell-Whitehead representation the x represents the line drawn to the left of the R and the y represents the line drawn to the right of the R.

Example	E.G.	R–W
	— R —	$(\exists x)(\exists y)Rxy$

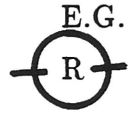
Translation: something is R to something

2. The basic form given in #1 can be negated by drawing a cut around the entire graph.

Example	E.G.	R–W
		$\sim (\exists x)(\exists y)Rxy$
		$\therefore (x)(y) \sim Rxy$

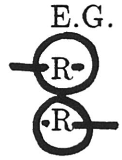
Translation: in no case is anything R to anything

3. When the lines of identity extend across the cut the translation is as follows.

Example	E.G.	R–W
		$(\exists x)(\exists y) \sim Rxy$

Translation: something is not R to something

4. The final alteration of the two subject examples involves one line of identity extending across the cut and the other line of identity remaining inside the cut.

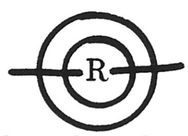
Example	E.G.	R–W
		$(\exists x)(y) \sim Rxy$
		$(\exists y)(x) \sim Rxy$

Translation: something does not have the relation R to anything

From these examples the use of Corollary 3 is evident. Lines of identity ending in an odd number of cuts express 'anything'; lines of identity ending in an unenclosed area express 'something.'

The final four examples in this demonstration of the use of beta graphs as a system of quantificational logic are shown below. Each involves a dyadic relation with double cuts.

5. A relation with both lines of identity crossing two cuts is equivalent to an unenclosed relation.

Example	E.G.	R–W
		$(\exists x)(\exists y) \sim\sim Rxy$ $\therefore (\exists x)(\exists y) Rxy$

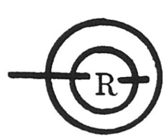
Translation: something has the relation R to something

6. Negating #3 by drawing a cut around the entire graph results in the following interpretation. Note that the ends of the lines are oddly enclosed, so they translate as 'anything' or 'everything.'

Example	E.G.	R–W
		$(x)(y)Rxy$

Translation: everything is R to everything

7. Mixed quantifiers are expressed by lines ending at different levels of cuts. Existential followed by universal quantification is shown below.

Example	E.G.	R–W
		$(\exists x)(y)Rxy$

Translation: something has the relation R to everything

8. Negating #4 results in universal followed by existential quantification.

Example	E.G.	R–W
		$(x)(\exists y)Rxy$

Translation: everything has the relation R to something

Other permutations of the two subject relational graphs are possible, but will not be shown here. Representations of relations involving more than two individuals are expressed by adding additional lines of identity.

The structure of this section owes much to Kenneth L. Ketner's article "Peirce's 'Most Lucid and Interesting Paper': An Introduction to Cenopythagoreanism." A full reference may be found in the bibliography.

Rules of Transformation

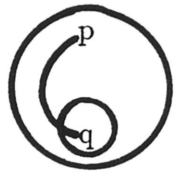
All of the rules of transformation applying to the alpha graphs also apply to the beta graphs. More specifically this means that a line of identity may be erased where it is unenclosed or enclosed by an even number of cuts. Also a line of identity may be extended (reiterated) to any unoccupied portion of the sheet of assertion. Double cuts may still be added or deleted if the lines of identity pass through both cuts and there is nothing else in the area between the two cuts.

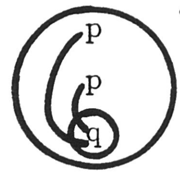
The beta graphs have the following additional rules of transformation.

Rule 8

Any graph may be iterated within the same or additional enclosures provided connections to other graphs are also iterated exactly (CP, p. 397).

Example original change

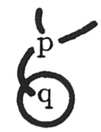




Rule 9

Any number of unattached lines of identity may be placed on the sheet of assertion. These may be connected as desired by spots of teridentity (CP, pp. 339-340).

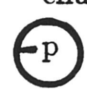
Example original change



Rule 10

A line of identity extending to a cut from the outside may be retracted (CP, pp. 326, 340, 391-392).

Example original change



A line of identity extending to a cut from within the cut may not be retracted (CP, pp. 340, 391-392).

Example original change

A line of identity outside a cut may be extended to the cut (CP, pp. 340, 391-392).

Example original change

A line of identity may be inserted within a cut (CP, pp. 340, 391-392)

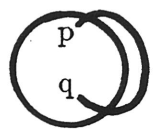
Example original change

Rule 11

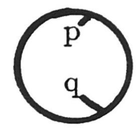
If two spots are within a cut and are not joined within the cut by a line of identity, then any lines joining them outside the cut are of no effect and so may be made or broken (CP, pp. 340, 458).

Example

original



change



GAMMA GRAPHS - MODAL LOGIC

The gamma graphs are the least developed portion of Peirce's system of existential graphs. While the alpha graphs correspond to what is today called propositional logic and the beta graphs correspond to quantificational logic, the gamma graphs are primarily Peirce's attempt to include modal logic in his system of existential graphs.

The modern development of modal logic is largely attributable to the logician C. I. Lewis. Modal logic, rather than dealing with the truth or falsity of propositions, distinguishes between propositions as necessary, possible, or not possible (i.e. impossible). In Lewis' notation a square ($\Box$) is used to symbolize necessity and a diamond is used to symbolize possibility ($\diamond$). Using these three operators several systems of modal logic have been developed. This section will focus only on modal logic as conceived by Peirce in his gamma graphs.

The gamma graphs, as an extension of the alpha and beta graphs, use all of the signs thus far developed. These signs include the sheet of assertion, the cut, the spots, and the lines of identity. In the modal graphs, Peirce adds the broken cut to the list of permissible signs. The broken cut expresses 'possibly not.' From this definition, each of the three fundamental modal operators may be derived (CP, pp. 398-399, 401-403).

Example — E.G. — Lewis

$\diamond \sim x$

Translation: possibly not x

Example — E.G. — Lewis

$\sim \diamond \sim x$

$\therefore \Box\, x$

Translation: not possibly not x; necessarily x

Example — E.G. — Lewis

$\diamond \sim\sim x$

$\therefore \diamond x$

Translation: possibly not not x; possibly x

Example — E.G. — Lewis

$\sim \diamond \sim\sim x$

$\therefore \sim \diamond x$

Translation: not possibly not not x; not possibly x

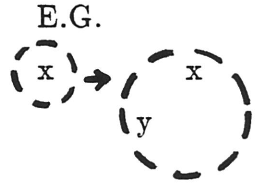
Rules of Transformation

Having defined the broken cut, Peirce outlines the following rules of transformation for the new symbol (CP, pp. 402-403).

Rule 1

Any graph may be inserted in a broken cut already on the sheet of assertion (CP, p. 402).

Example — E.G. — Lewis



$\diamond \sim x$

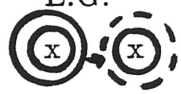
$\therefore \diamond \sim (x \cdot y)$

Rule 2

An evenly enclosed solid cut may be half erased such that a broken cut results (CP, p. 402). This rule expresses the Latin argument form known as *ad esse ad posse.*

Example

E.G.



Lewis

$\sim\sim x \rightarrow \Diamond \sim\sim x$

$\therefore x \rightarrow \Diamond x$

Translation: If x is true, x is possible.

Rule 3

An oddly enclosed broken cut may be filled in to form a solid cut (CP, pp. 402-403). This rule is known in Latin as *ad necessitate ad esse.*

Example

E.G.

Lewis

$\sim \Diamond \sim x \rightarrow \sim\sim x$

$\therefore \Box x \rightarrow x$

Translation: If x is necessary, x is true.

These rules of transformation define what is today called the S1 system of modal logic. It is the basis from which all other modal systems are derived.

In addition to the broken cut, Peirce introduces a second symbol specific to the gamma graphs. He calls this symbol a cross mark (CP, p. 404). The cross mark, in its various interpretations, is Peirce's attempt to deal with knowledge specific to a person, a place, or a time. After some general demonstrations of the use of the cross mark, each of these three possibilities for interpretation will be discussed.

Example

E.G.

Translation: At a specified time (or to a specific person, or within a particular possible world), x may not be the case.

Example E.G.

Translation: At no time (or to no knower, or within no possible world), is x possibly the case.

Example E.G.

Translation: For the specified time (or for this knower, or for this possible world), x is necessarily the case.

The first function of the cross mark is as a restriction of modality to a particular case. The cross mark makes possible the transformation from a true proposition to a necessarily true proposition (CP, pp. 404-405). This aspect of the cross mark is explained quite well in Don D. Robert's book on Peirce's existential graphs. Roberts, following Peirce, postulates a flow of events much like the following (Roberts, p. 58).

Step 1

Suppose we have an information state S1, such that both $\diamond p$ and $\diamond \sim p$ are consistent with the information in S1. In gamma graphs, then, we have the following information.

Step 2

Then, through some means (such as deduction using other information in S1), we learn that p is true. This yields a new state of information, call it S2, for which we have graphs showing the following.

p

Step 3

Given the addition of the information that p is true, it is no longer the case that p is possibly not true. So we have a new graph expressing this.

p

But, the graph on the left above expresses necessarily p.

Hence, for a particular case, a proposition known to be true is necessarily true in that case. The use of the cross mark prevents, however, the equivalence of truth and necessary truth in universal cases (Roberts, p. 84).

A second function of the cross mark in Peirce's system is that of a time keeping mechanism. By using varying numbers of crossmarks in a series of graphs it is possible to distinguish between earlier and later states of information. Peirce expresses this as "a definite order of succession" (CP, p. 404). Hence to show that necessarily p is an information state coming after knowledge that p is true in a particular case, one uses one then two crossmarks respectively (CP, p. 404). A graph with three crossmarks would succeed graphs with two crossmarks, etc.

Example E.G.

Along the same line, Peirce briefly mentions another symbol to show that one proposition follows another. This symbol is shown below, where the graph on the left is defined as preceding the graph on the right.

Example E.G.

Varying the focus of his changing states of information theme from changes occuring in time to changes occuring in one's knowledge, Peirce introduces an element of epistemic logic to his existential graphs. The cross mark, then, may be interpreted as signifying what information a knower knows (CP, pp. 405-406). A knower ignorant of any knowledge of p can form either of the two propositions possibly p or possibly not p. That is,

Should the knower procede to a second state of information in which it is known that p is true, the knower can then procede to a third state of information in which the knower knows that p is necessarily true.

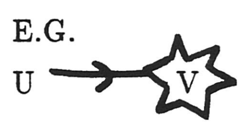
Modal propositions such as these are, according to Peirce, propositions about the information one knows, or is ignorant of, rather than about the universe of things (CP, p. 405).

A third function of the use of the crossmark in Peirce's writing is as a means of distinguishing between different 'possible worlds'. Rather than information specific to a particular time or to a specific knower, the crossmark may symbolize information available in a particular possible world. Graphs containing the same number of crossmarks all belong to the same possible world, rather than to the same time or to the same state of information known to a knower.

Peirce outlines an additional means of distinguishing possible worlds in his use of the book metaphor. Under this view one has a stack of sheets of assertion, each one representing a particular possible world. One of the sheets coincides with the existing world. Whether Peirce conceives of these sheets as including all possible future courses of action or all conceivable states of affairs in the past, present, and future, is not clear (CP, p. 401).

Aside from the development of the gamma graphs as a modal logic system, Peirce defines several signs by which the gamma graphs may be used as a meta-language. The meta-language characteristic provides a means of reasoning about the existential graphs using existential graphs. Towards this end, Peirce defines several new signs. Of these new signs, the saw tooth circle is the central innovation. When placed around a graph, the saw tooth cut indicates that the graph within it is on the sheet of assertion.

Example



Translation: The graph V is on the sheet of assertion U.

The meta-language aspect of the gamma graphs is an interesting sideline of Peirce's system of existential graphs. It is not, however, a central portion of the graphs and so will not be discussed in any greater depth here (CP, pp. 408-410).

In his later work Peirce made what appears to be a major addition to the gamma graphs. He calls this addition the tinctures. Briefly, the tinctures are a system of background patterns on the sheet of assertion on which the graphs are drawn. From the background pattern one can express the type of information the graph represents, namely possibility, intention, or actuality. Unfortunately, the existing notes by Peirce on the tinctures are not sufficient to provide an in-depth interpretation here. While remaining an interesting and possibly fruitful idea, an expansion upon Peirce's work on the tinctures remains beyond the scope of this paper (CP, pp. 439-443).

CONCLUSION

A study of Peirce's system of existential graphs has shown the alpha graphs to correspond to what is today called propositional logic and the beta graphs to correspond to quantificational logic. The gamma graphs present the basis of what is today called modal logic, but are not as well-developed as the alpha and beta portions of the existential graphs. Fragments of epistemic logic and a metalanguage capability are also contained in Peirce's notes on the gamma graphs. An expansion on Peirce's work on the gamma portion of the existential graphs is a topic worthy of future study.

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