The Economic Pipe Diameter for Non-Newtonian Fluids

by

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Chemical Engineering

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Approved by:

Ron Darby

Dr. Ron Darby

April 21, 1981

ABSRACT

Much work has been done in the past for calculating the economic pipe diameter for Newtonian fluids, but there has been very little done to date for the case of non-Newtonian fluids; i.e.,a fluid whose viscosity is not constant with increasing shear rate. Since so many fluids that occur industrially do not behave as a Newtonian fluid, there is a great need for a way to determine the optimum economic pipe diameter for these non-ideal fluids. This paper presents a method of finding the economic pipe diameter for Bingham plastics and pseudoplastics knowing all the flow parameters except D. Figures V and VI on pages 17 and 20 are dimensionless plots which show the economic Reynold's number as a function of two variables which are independent of D. Ψ incorporates all the economic variables and was calculated in this report to be equal to 0.11507 $ft^{(p-1)}s^3/lb_m$, but,if desired, a new Ψ can be calculated from equation (13) and still be used with the charts.

A preliminary part of the work involve deriving a simple empirical equation for the friction factor of Bingham plastics. The equation that was derived is presented on page 13, and the comparison with a friction factor chart show in figure III on page 6.

A special thanks and acknowledgement goes to Dr. Ron Darby of the Chemical Engineering Department of Texas A&M, who encouraged me to become a part of the University Undergraduate Fellows Program in the beginning, and then advised me and supported my work throughout the project.

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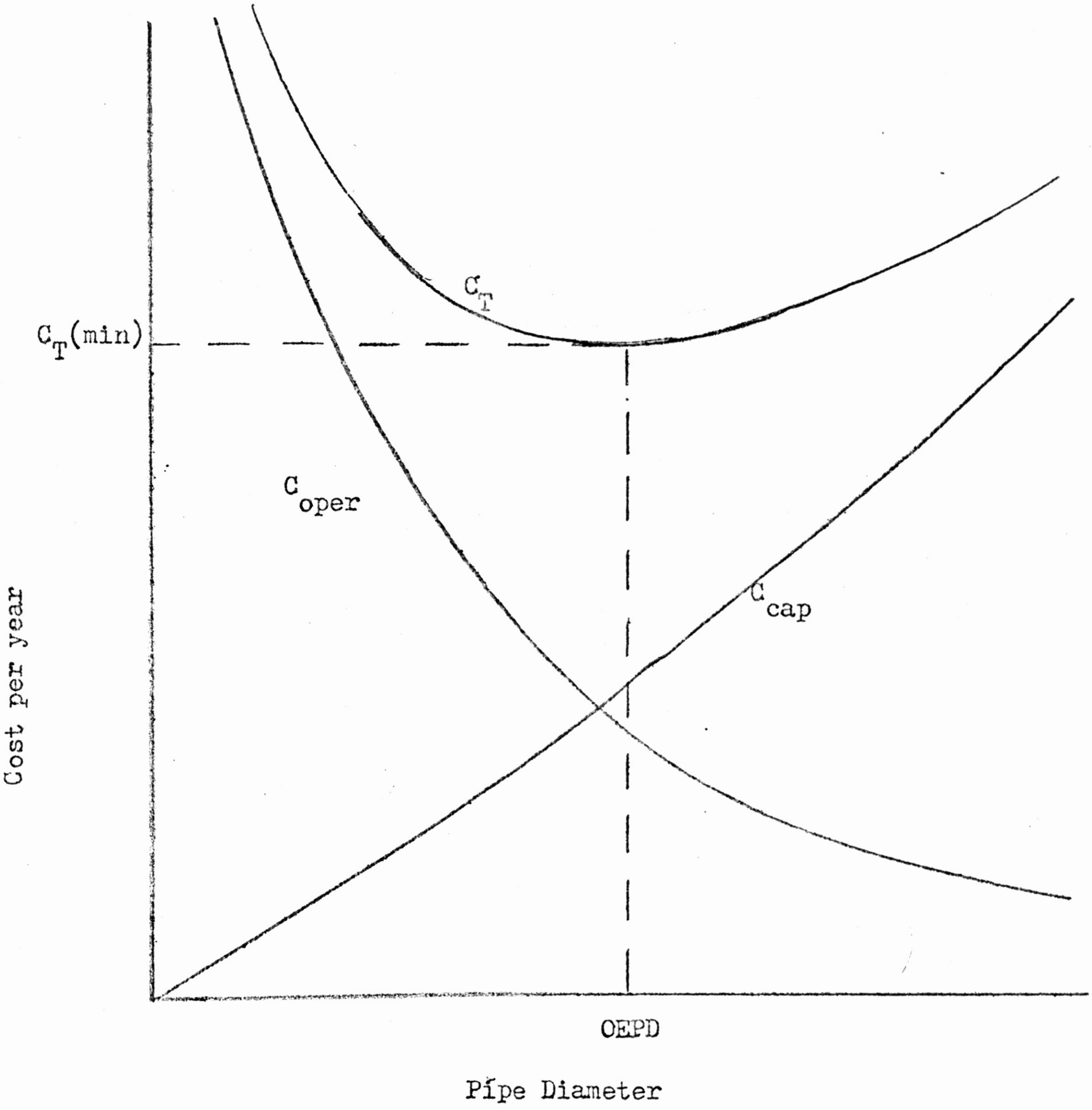
INTRODUCTION

With the costs of constructing and operating a pipeline constantly increasing, it has become extremely important for an engineer to build the pipeline at the most economic diameter. The costs associated with pumping a fluid are directly related to the diameter of the pipe. As the inside pipe diameter increases, the operating costs decrease due to less friction loss. At the same time, however, the capital costs increase due to more material. Especially when the pipeline is to transport a fluid over a great distance, it is an engineer's design strategy to determine the pipe diameter at which the total yearly costs are minimized; i.e., the optimum economic pipe diameter (OEPD) (see figure I,page 2).

While much has been done in the area of Newtonian fluids, very little has been done in determining the OEPD for non-Newtonian fluids. A non-Newtonian fluid may be simply described as one whose viscosity does not remain constant with changing shear rate. In this study Bingham plastics and pseudoplastics were investigated.

The objective of this research was to present a simple method of determining the OEPD to use in pumping a Bingham plastic or pseudoplastic. The final results are presented in the form of dimensionless plots.

Figure I : Variation of operating costs, capital cost, and total costs with pipe diameter.



LITERATURE REVIEW

Much work has been done in developing methods of finding the OEPD for Newtonian fluids. Peters and Timmerhaus present empirical equations to calculate the OEPD (see reference (6), pp. 380 - 381), while Perry provides a nomograph for this purpose (see reference (5), p. 5-31). These are only two examples of numerous efforts in this area. Not all fluids used industrially, though, are Newtonian. In fact, most fluids are probably non-Newtonian. Easy to use methods are not readily available to engineers for this broader case of fluids.

The first fluid to be considered is Bingham plastics. A Bingham plastic is a fluid that will not flow until a certain critical shear stress, τ_y, is applied (see figure IIb, page 4). Examples of Bingham plastics are certain water suspensions of rock, sand, and grain; drilling muds; sewage sludge; and greases. The Reynold's number for Bingham plastics is defined in terms of the plastic viscosity, η, rather than the normal viscosity, μ.

$$N_{Re} = \frac{DV\rho}{\eta} = \frac{\dot{m}}{D\eta}\frac{4}{\pi} \tag{1}$$

Another dimensionless variable used in describing the characteristic of a Bingham plastic is the Hedstrom number, which is dependent on the critical shear stress.

$$N_{He} = \frac{g_c \tau_y D^2 \rho}{\eta^2} \tag{2}$$

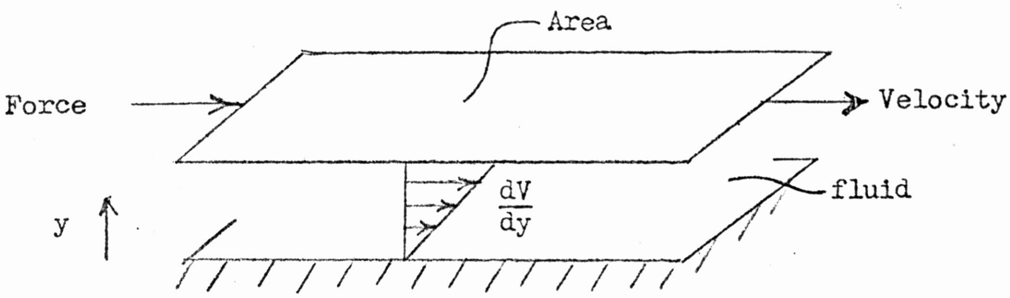


Figure IIa : The sliding plate experiment.

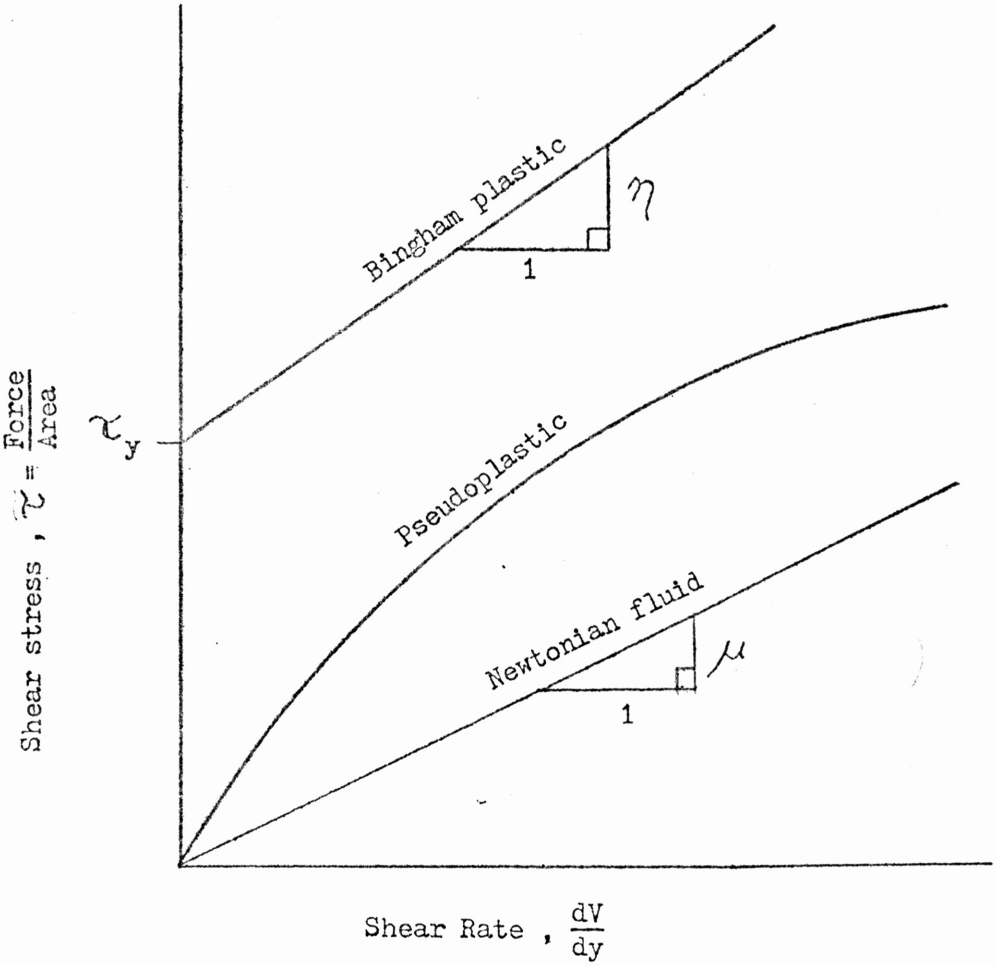


Figure IIb : Outcome of sliding plate experiment.

The friction factor for Bingham plastics is a function of N_{Re} and N_{He}, as shown in figure III, page 6 . The laminar region is described by the implicit equation

$$f_L = \frac{16}{N_{Re}} \left[1 + \frac{1}{6} \frac{N_{He}}{N_{Re}} - \frac{1}{3f_L^3} \frac{N_{He}^4}{N_{Re}^7} \right] \tag{3}$$

The transition and turbulent region of the chart was determined by a semi-empirical equation including a complicated integral (see reference (3)). One objective of this research was to create a simple empirical equation which would relate the friction factor to N_{Re} and N_{He}. (Note: There is evidence to support the assumption that the pipe roughness does not affect the friction factor significantly for non-Newtonian fluids.)

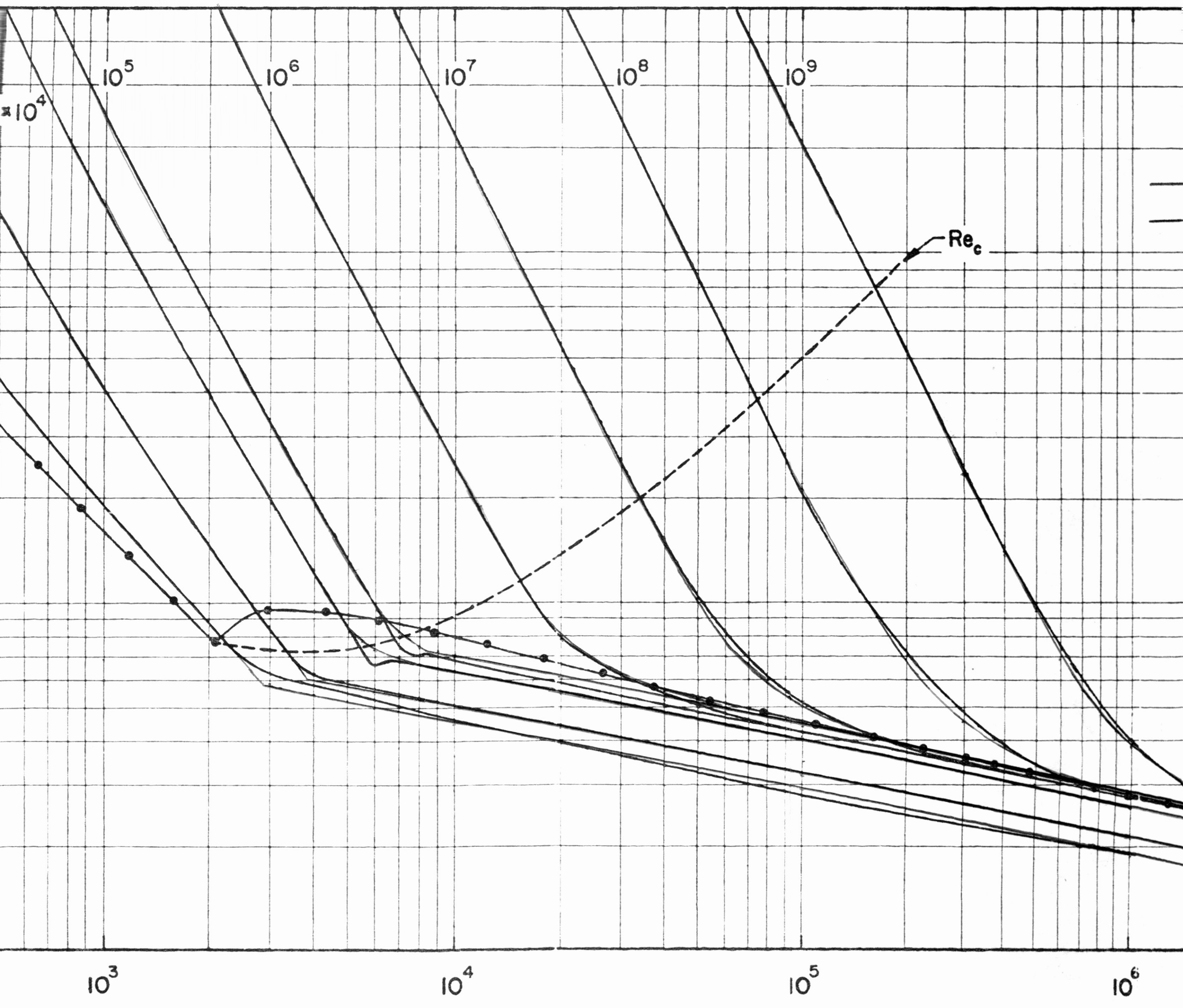
For a pseudoplastic, the relationship between the shear stress and shear rate is not linear, but generally can be related by the power law equation.

$$\tau = K \left(\frac{dV}{dy}\right)^n \tag{4}$$

where $n \leq 1.0$ for a pseudoplastic. K and n are not easily determined experimentally, but similar pair of rhealogical parameters, K' and n', can be readily found from a series of capillary viscometer experiments. The sets of variables are related by the fact that for most cases

$$n' = n \tag{5}$$

$$K' = K \left(\frac{3n + 1}{4n}\right)^n \tag{6}$$



: Friction factor for Bingham plastics.

(from Hanks and Dadia, see referenc

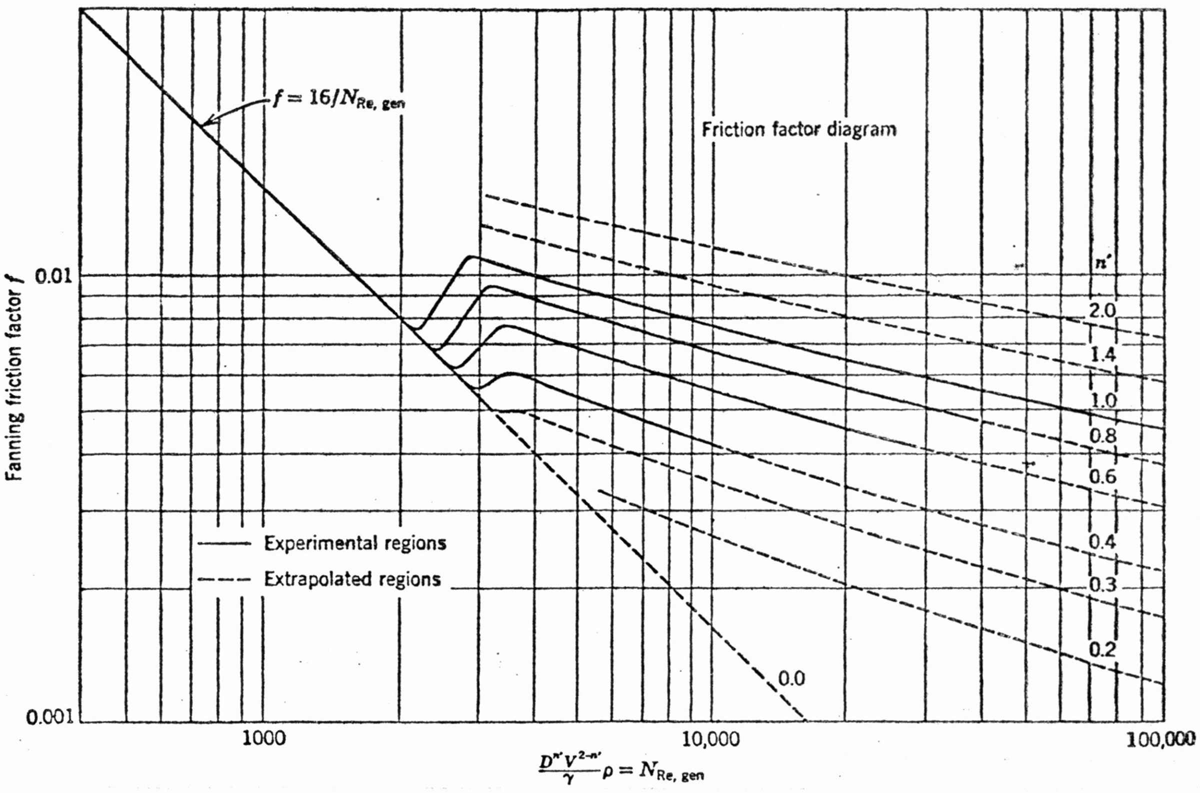


Figure IV : Friction factor for pseudoplastics.

(from Skelland, *Non-Newtonian Flow and Heat Transfer*, p. 194)

Examples of pseudoplastics are many slurries, oils, and polymers.

The friction factor for pseudoplastics is generally determined in terms of K' and n'. The generalized Reynold's number for pseudoplastics is defined as

$$N_{Re} = \frac{D^{n'} V^{(2-n')} \rho}{8^{(n'-1)} K'} = \frac{\dot{m}^{(2-n')} \rho^{(n'-1)}}{D^{(4-3n')} 8^{(n'-1)} K'} \left(\frac{4}{\pi}\right)^{(2-n')} \qquad (7)$$

The most accepted correlation for the friction factor is from the work by Dodge and Metzner. Figure IV, page 7, shows f as a function of N_{Re} and n'. The equation for the laminar region is well established.

$$f_L = \frac{16}{N_{Re}} \qquad (8)$$

Dodge and Metzner's equation for turbulent flow is

$$\sqrt{\frac{1}{f_T}} = \frac{4.0}{n'^{.75}} \log\left[N_{Re}\, f_T^{(1-n'/2)}\right] - \frac{0.4}{n'^{1.2}} \qquad (9)$$

These relationships were accepted and used in this investigation.

It is interesting to note that Newtonian fluids are a special class of both Bingham plastics and power law fluids. If $\tau_y=0$, then $N_{He} = 0$ and $\eta = \mu$, and equations (1) and (3) reduce to the case for Newtonian fluids. The same is true for the case of pseudoplastics where n' = 1.0.

The method used to calculate the OEPD is developed fully by Skelland (see reference(7), pp. 240 - 265). Basically, the operating costs per year per foot diameter are directly proportional to the amount of work required to overcome friction losses.

$$C_{oper} = \frac{2\ PC\ f\ \dot{m}^3\ h\ (4/\pi)^2}{g_c\ D^5 \rho^2\ E} \qquad (10)$$

(Note : C_{oper} also includes other terms related to kinetic and potential energy, but these terms are not a function of pipe diameter and eventually cancel out.)

The yearly capital costs can be expressed as a fraction of the total initial investment cost for the pipeline.

$$C_{cap} = CC \cdot X \cdot D^p \qquad (11)$$

where CC includes the fraction of the initial investment that must be allotted yearly to depreciation, maintenance, etc.; X includes all purchased and installed costs per foot; and p will vary from 1.0 to 1.5 depending on the pipe material and wall thickness.

The total yearly cost is the sum of equations (10) and (11). OEPD is the diameter where the total cost is a minimum; i.e., where $(\partial C_T/\partial D) = 0$.

$$D = \left[\Psi\ f\ \frac{\dot{m}^3}{\rho^2}\ (4/\pi)^2 \right]^{\frac{1}{5+p}} \qquad (12)$$

where Ψ incorporates all the economic variables.

$$\Psi = \frac{10\ \ PC}{g_c \cdot X \cdot p \cdot E \cdot CC} \qquad (13)$$

Equation (12) is implicit in that f is a function of D through the

Reynold's number, and the Hedstrom number for Bignham plastics. Though the equation does converge after only a few iterations, it is not easy to use for most situations.

RESULTS

Economic Variables

Data was obtained from Earl and Wright Consulting Engineers of Houston, Texas in order to determine values for the economic variables in equation (13). To calculate the variables in equation (11), the inside pipe diameter was plotted against the cost of pipe per mile on a log-log graph (see figure VII, page 26). The following results were obtained.

$$X = 130{,}600\ \$/(\text{mi ft}^{p}) = 24.728\ \$/\text{ft}^{(p+1)}$$

$$p = 1.193$$

These values are for 400# ANSI pipe and include all reasonable purchase installation costs.

Likewise, the pump station operating costs were plotted versus horsepower, and the points were fitted to a liear equation (see figure VIII, page 27).

$$C_{oper} = 172{,}860 + 450.818\ \text{hp}$$

The constant term would be eliminated when the cost equation was differantiated; therefore, PC could simply be expressed as

$$PC = 450.818\ \$/(\text{hp yr})$$

Values of CC and E may be approximated as

$$CC = 0.15\ \text{yr}^{-1}$$

$$E = 0.50$$

By plugging the above values into equation (13), Ψ was calculated.

$$\Psi = 0.11507 \ \mathrm{ft}^{(p-1)} \ \mathrm{s}^{3} / \mathrm{lb}_{m}$$

Bingham Plastics

The first task in dealing with this problem was to derive an empirical equation for the friction factor. To account for all regions of flow, including the transition region, the equation was expressed in a form similar to the Churchill equation.

$$f = \left(f_L^{\gamma} + f_T^{\gamma} \right)^{1/\gamma} \tag{14}$$

Equation (3) was used for laminar flow.

$$f_L = \frac{16}{N_{Re}} \left[1 + \frac{1}{6} \frac{N_{He}}{N_{Re}} - \frac{1}{3 f_L^3} \frac{N_{He}^4}{N_{Re}^7} \right] \tag{3}$$

An equation for f_T was derived using figure III, page 6. The N_{He} lines turbulent flow were all assumed to be parallel, with the placement on the chart determined as a function of N_{He}.

$$f_T = \frac{10^A}{N_{Re}^{0.193}} \tag{15}$$

$$A = -1.378 \left[1 + 0.146 \exp(-2.9 \times 10^{-5} N_{He}) \right]$$

Equation (15) is only for N_{He} greater than 1000 since there is a great deal of uncertainty as to f_T for N_{He} between 0 and 10^3.

To complete equation (14), γ was determined as a function of N_{Re}.

$$\gamma = 1.7 + \frac{4 \times 10^4}{N_{Re}} \tag{16}$$

The results of equation (14) are plotted in red on figure III, page 6.

In the problem of determining the OEPD, it was assumed that all other fluid properties would be known, including the mass flow rate. Therefore, the optimum diameter would also correspond to the optimum Reynold's number. The objective of a dimensionless plot would be to present the optimum N_{Re} from dimensionless variables which do not include diameter. These other variables were determined by combining the dimensionless numbers f, N_{Re}, and N_{He} in a couple of forms. Equation (12) was rearranged into

$$f = \frac{D^{(5+p)} \rho^2}{\Psi \; \dot{m}^3} \left(\frac{\pi}{4}\right)^2 \tag{17}$$

This was combined with the Reynold's number, equation (1), in such a way that the D's cancelled.

$$f \; N_{Re}^{(5+p)} = \frac{\dot{m}^{(2+p)} \rho^2}{\Psi \; \eta^{(5+p)}} \left(\frac{4}{\pi}\right)^{(3+p)} \tag{18}$$

Likewise, the Hedstrom number, equation (2), was combined with N_{Re} to form

$$N_{He} \; N_{Re}^{2} = \frac{g_c \; \tau_y \; \dot{m}^2 \rho}{\eta^4} \left(\frac{4}{\pi}\right)^2 \tag{19}$$

There was found to be a definite relationship between N_{Re}, N_{He} N_{Re}^{2}, and f $N_{Re}^{(5+p)}$. Knowing the relationship between N_{Re}, N_{He}, and f, the chart in figure V, page 17, was generated on the computer (see the program on pages 29 - 31). Knowing Ψ and all the fluid parameters of a system except D, equations (18) and (19) can be used to calculate f $N_{Re}^{(5+p)}$ and $N_{He}\, N_{Re}^{2}$. From the chart N_{Re} can be easily found, which specifies the OEPD to use.

All lines of $N_{He}\, N_{Re}^{2}$ have the same asymptote. It was interesting to note that the asymptote corresponded to turbulent flow, and laminar flow occurs where the $N_{He}\, N_{Re}^{2}$ line branches off.

<u>Example 1.</u>

Consider the case where the OEPD is wanted to pump a Bingham plastic at a rate of 200,000 lb_m/hr. The following fluid properties are known:

$\rho = 100\ lb_m/ft^3$

$\eta = 0.1\ lb_m/(ft\ sec)$

$\tau_y = 0.23\ lb_f/ft^2$

Using equation (18) and (19)

$$f\, N_{Re}^{(5+p)} = 7.816 \times 10^{16} \Rightarrow \sqrt[6]{f\, N_{Re}^{(5+p)}} = 653.9$$

$$N_{He}\, N_{Re}^{2} = 2.78 \times 10^{10}$$

On figure V, 654 was found on the abscissa and followed up to the point where $N_{He}\, N_{Re}^{2}$ would be approximately 2.8×10^{10}. This point corresponded to

$$\sqrt{N_{Re}} = 25 \qquad \Rightarrow \qquad N_{Re} = 625$$

From the definition of N_{Re}, equation (1), D was calculated.

$D = 1.132$ ft

To confirm this result, the answer was checked with equation (12). Equation (2) was used to calculate N_{He}.

$$N_{He} = 7.117 \times 10^4$$

The friction factor was found from N_{Re} and N_{He} using either figure III or equation (14).

$$f = 0.38$$

Plugging this value into equation (12) gave

$$D = 1.132 \text{ ft}$$

This shows that figure V agrees with equation (12).

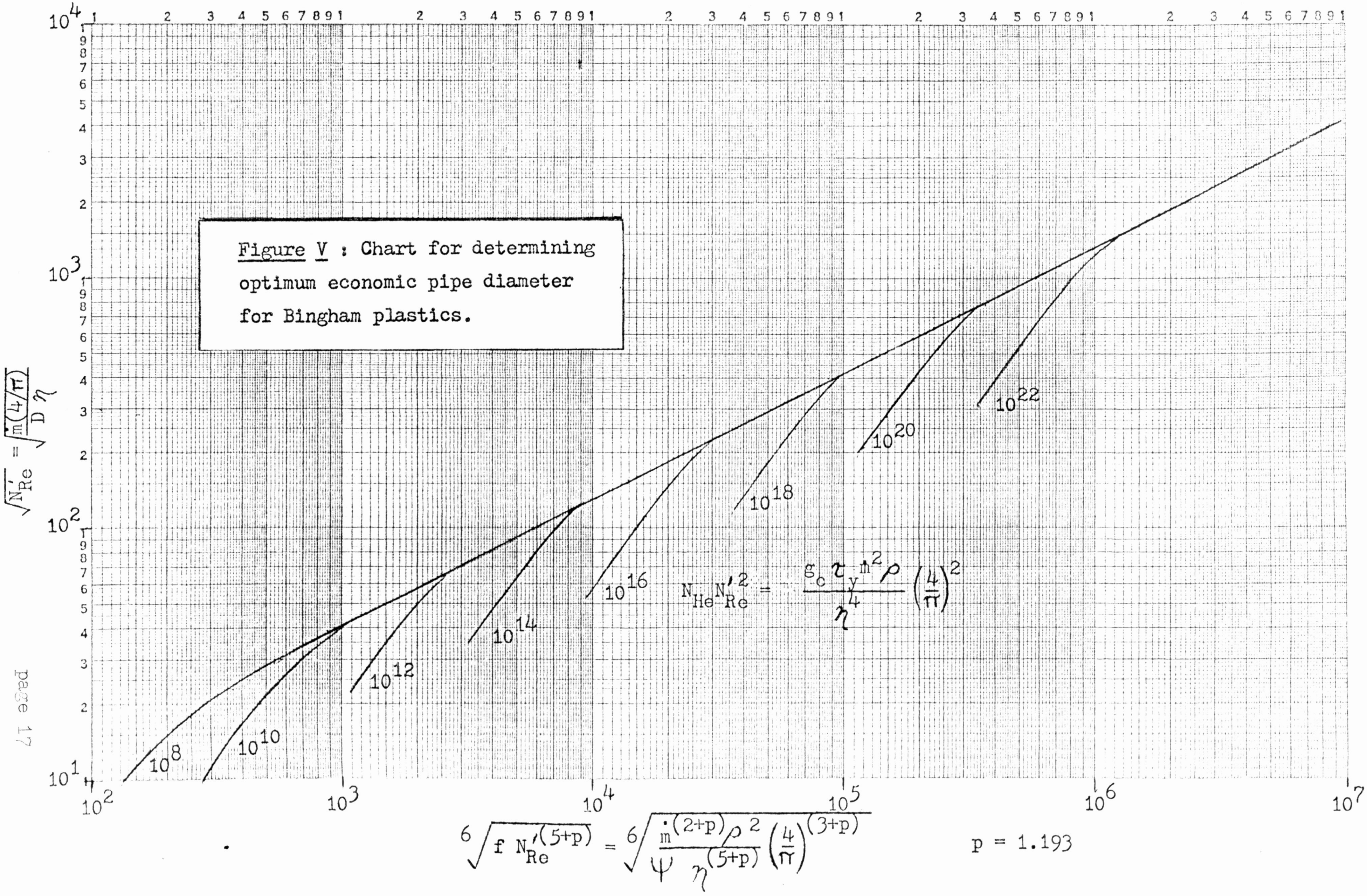


Figure V : Chart for determining optimum economic pipe diameter for Bingham plastics.

Pseudoplastics

The case for pseudoplastics was handled in much the same way as Bingham plastics. However, it was not necessary to derive an equation for the friction factor since correlations already existed. Equations (8) and (9) were used.

Again, two dimensionless variables independent of D were used to determine N_{Re}. One of these variables was simply n'. The other variable was evaluated by combining equation (17) with the generalized Reynold's number, equation (7).

$$f^{(4-3n')} N_{Re}^{(5+p)} = \frac{\rho^{[3-p+(p-1)n']} \dot{m}^{[2p-2+(4-p)n']}}{\psi^{(4-3n')} K'^{(5+p)} 8^{(5+p)(n'-1)}} \left(\frac{4}{\pi}\right)^{[2+2p+(1-p)n']} \qquad (20)$$

The relationship between N_{Re}, n', and $f^{(4-3n')} N_{Re}^{(5+p)}$ is shown in figure VI, page 20. This chart was generated by the computer program on pages 32 - 36. As with the previous case, equation (20) can be evaluated without the diameter of the pipeline. With this and n', figure VI can be used to find N_{Re}, and from this D can be calaulated.

Example 2.

It was necessary to find the OEPD with which to pump 200,000 lb_m/hr of a pseudoplastic. The fluid properties are shown below.

$\rho = 100\ lb_m/ft^3$

$K' = 0.5\ lb_m\ sec^{(n'-2)}\ ft^{-1}$

$n' = 0.6$

Using equation (20),

$$f^{(4-3n')} N_{Re}^{(5+p)} = 1.183 \times 10^{14}$$

$$\sqrt[6]{f^{(4-3n')} N_{Re}^{(5+p)}} = 221.6$$

On figure VI, 221.6 was found on the abscissa and followed up to the line for n'=0.6. This point corresponded to N_{Re} on the ordinate.

$$\sqrt{N_{Re}} = 26.5 \implies N_{Re} = 702.25$$

The diameter was then found using equation (7).

$$D = 0.6616 \text{ ft}$$

Again, this result was compared with equation (12). Since N_{Re} was so low, equation (8) was used to calculate the laminar friction factor.

$$f = 0.0228$$

When this value was plugged into equation (12), the result was

$$D = 0.6552 \text{ ft}$$

This represented a difference of less than 1%, proving the method.

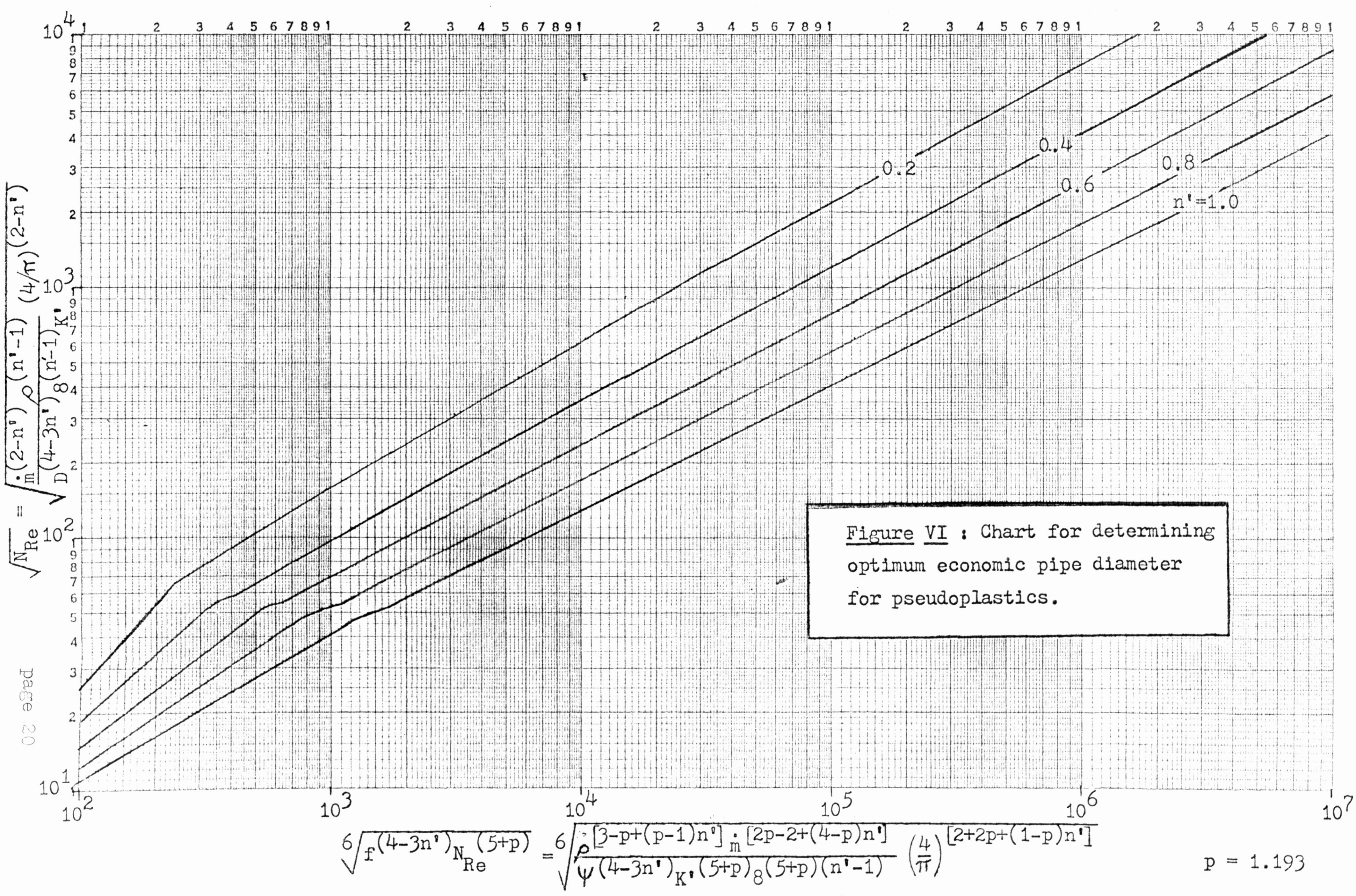


Figure VI : Chart for determining optimum economic pipe diameter for pseudoplastics.

CONCLUSION

Even though equation (14) is not valid for cases where both N_{He} is less than 1000 and N_{Re} is greater than 2100, the asymptotic line in figure V is identical to the line in figure VI for Newtonian fluids; i.e., n'=1.0. This implies that figure V is even good for Hedstrom numbers down to zero.

Figures V and VI were actually generated without using the economic variables. Therefore, the plots will be accurate regardless of the value for Ψ. If the value for Ψ presented here is unacceptable, a new value for Ψ can be calculated from equation (13) and figures V and VI can still be used. A change in the value for p will affect the charts some, but not much since p generally only varies from 1.0 to 1.5.

It would be impossible to prove the results of this report industrially; i.e., to construct pipelines of several diameters and see which one operates most economically. However, each part of the method has been proven by previous investigators and are accepted by the engineering community. It was reasonably assumed that if each step was correct, the combination of the steps would also be correct.

With figures V and VI, the engineer now has a simple method with which to calculate the optimum economic pipe diameter for a non-Newtonian fluid. These results are shown in this form for the first time and are very useful in a world where most fluids are not ideally Newtonian.

APPENDIX A : Data for Determining Economic Variables

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June 2, 1980

Doctor Ron Darby
Texas A & M University
College Station, Texas 77843

Subject: Summaries of Pipeline Installation and Pump Station Costs

Dear Ron:

Enclosed are summaries of pipeline installation and pump station costs that I promised you sometimes ago. Please excuse the long delay; my excuse is that I have been traveling out of the country quite a bit lately and just could not get around to finishing the work. The notes to these studies are self-explanatory. These costs are fairly representative of the cost trends that exist at the present time and on the conservative side, however they should be suitable for feasibility studies. Please let me know if you require any more information or further clarification and details of these costs.

Yours very truly,

EARL AND WRIGHT

Jesse Mason

Jesse E. Mason
Project Manager

JEM:sn
Enclosures

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NORMAL PIPELINE INSTALLATION COSTS
($/MILE-1980 $)

Nom. Diam.	ANSI 300# (720 PSIG)	ANSI 400# (960 PSIG)	ANSI 600# (1440 PSIG)	ANSI 900# (2160 PSIG)	ANSI 1500# (3600 PSIG)
4"	38,500	38,500	46,600	59,500	77,100
6"	55,100	55,100	62,300	74,900	103,100
8"	70,400	70,400	87,400	108,100	150,700
10"	94,000	95,900	113,400	152,500	212,900
12"	118,600	122,200	149,300	190,600	281,900
14"	141,800	144,700	174,000	209,900	310,100
16"	161,600	166,700	203,000	256,000	392,200
18"	184,800	192,400	241,700	311,800	478,700
20"	207,700	216,100	294,600	367,500	542,100
22"	229,800	239,500	344,500	425,800	683,500
24"	248,800	261,900	391,000	501,600	776,500
26"	289,100	322,700	428,100	593,900	865,800
30"	351,400	391,600	? 517,000	715,900	-
32"	372,800	416,500	573,6 513,600	826,900	-
34"	417,200	463,300	648,300	859,600	-
36"	456,200	511,200	683,300	-	-

Notes

(1) Does not include cost of major river crossings but does include small stream crossings.

(2) Pipe required includes a 20-mil corrosion and erosion allowance.

(3) Installed Cost Assumptions

(a) Length of pipeline at least 125 miles.
(b) Flat to rolling terrain with some rock.
(c) Normal valve spacing.
(d) On-stream pigging facilities provided.
(e) Population centers avoided to provide for type I construction except road, highway and RR crossings.

(4) Impresssed current cathodic protection system included.

(5) Plasticized coal tar with reinforced felt inner wrap and reinforced kraft paper external wrap.

(6) Costs do not include a communications and "Scada" system.

(7) Costs include hydrostatic testing and dewatering.

(8) All wall thicknesses for pipe are standard to 1/64".

NORMAL PUMPING STATION COSTS
(1980 $)

Station HP	$/HP Low (1)	High (2)	Median	
0-100	700	1000	850	85,000
500	650	940	800	400,000
1000	600	840	720	720,000
1500	500	750	625	937,500
2000	410	650	530	1,060,000
2500	400	640	520	1,300,000
3000	390	630	510	1,530,000
3500	380	620	500	1750,000
4000	365	615	490	1,960,000
5000	365	610	480	2,400,000
5000 & Over	350	610	480	2,4

Notes

(1) Electric motor driven reciprocating units includes all electrical switch gear and controls but does not include power sub-station.

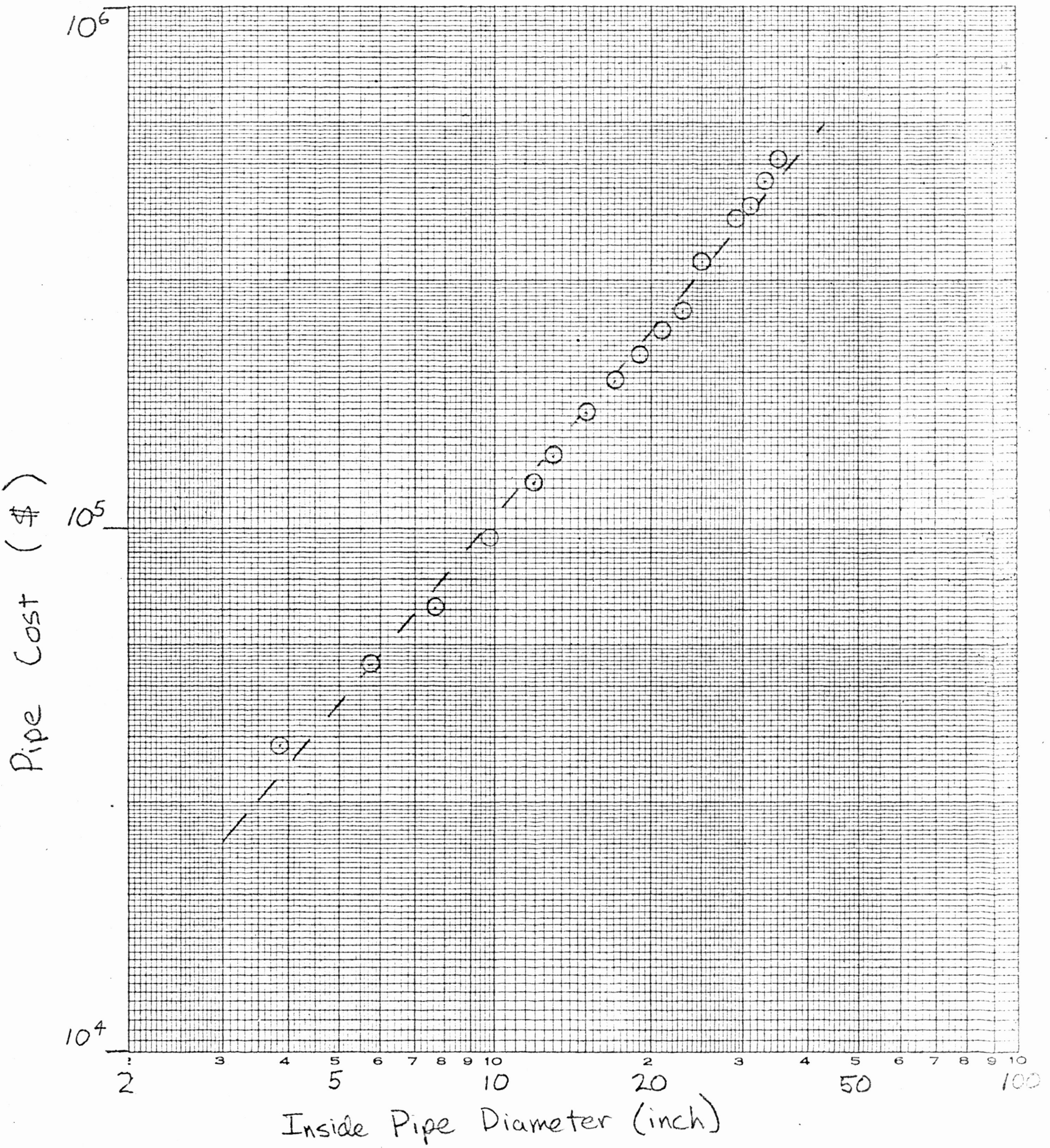
(2) Diesel engine driven reciprocating units includes fuel fuel system and storage tanks and shutdown and control panels.

(3) Includes pump, driver, pulsation dampensers, recirculation system, check valves, suction and discharge valves, duplex suction strainer, pump and driver skids, reduction system and couping, foundations, instrumentation etc.

(4) Although these costs are for reciprocating pump installations, the same costs would be applicable to centrifugal installations with only minor cost differences.

Figure VII:

Cost of 400# ANSI pipe



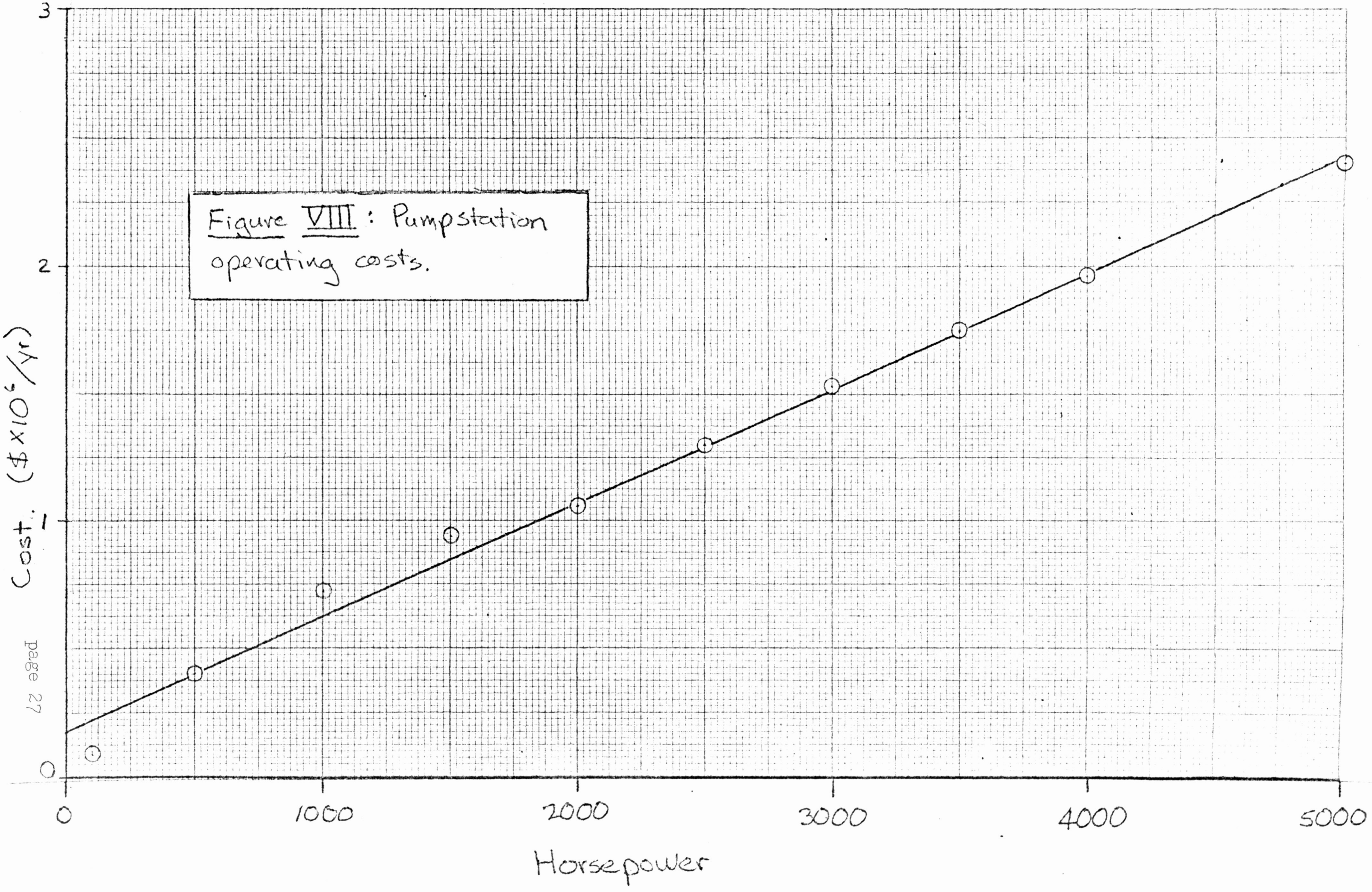


Figure VIII: Pumpstation operating costs.

APPENDIX B : Computer Programs

```
//BINGHAM JOB  (X007,301E,S02,002,JM),'MELSON                    '
//*MAIN          USER=U255$JM,ORG=ZCC
//*TAMU          HOLDOUT
//*PASSWORD*****************************************************
//*WATFIV
      REAL NREY,NHED,NR
      WRITE (6,90)
   90 FORMAT('1','NHED*NREY**2   NREY',7X,'NHED',8X,'FF',4X,
     X'FF*NREY**P ',1X,'SQRT(NREY)',2X,'(FNREP)**1/6')
      P=6.192735
      FF=0.
      VAR =1.E24
      DO 10 I=1,11
      VAR=VAR/100.
      NREY=SQRT(VAR/250.)
      DO 9 J=1,20
      NREY=NREY/2
      NHED=VAR/NREY**2
      CALL FRICT(NREY,NHED,FF)
      FNREP=FF*NREY**P
      ROOT=FNREP**(1./6.)
      NR=SQRT(NREY)
      WRITE(6,100)VAR,NREY,NHED,FF,FNREP,NR,ROOT
  100 FORMAT(' ',3E10.3,F7.4,E10.3,2E10.3)
      IF(FF.GT.15.)GOTO 10
    9 CONTINUE
   10 CONTINUE
      WRITE(6,120)
  120 FORMAT('1')
      STOP
      END

      SUBROUTINE FRICT(NREY,NHED,FF)
      REAL MSFLW,NREY,NHED
      PI=3.141592
      FL1=0.1
    2 FL=16./NREY*(1.+NHED/(NREY*6.)-NHED**4/(NREY**7*3*FL1**3))
      IF(ABS((FL-FL1)/FL).LT.1.E-5)GOTO 6
      FL1=FL
      GOTO 2
    6 IF(NHED.GT.4.E5)GOTO 8
      X=10.**(-.2012*EXP(-2.9E-5*NHED))
      GOTO 9
    8 X=1
   9  FT=.04188*X/NREY**.193
      GAMMA=1.7+4.E4/NREY
      IF(GAMMA.GT.15.)GAMMA=15.
      FF=(FL**GAMMA+FT**GAMMA)**(1./GAMMA)
      RETURN
      END

//$DATA
```

```
NHED*NREY**2   NREY         NHED          FF      FF*NREY**P  SQRT(NREY)   (FNREP)**1/6
 0.100E 23  0.316E 10  0.100E 04  0.0004  0.265E 56  0.562E 05  0.173E 10
 0.100E 23  0.158E 10  0.400E 04  0.0005  0.431E 54  0.398E 05  0.869E 09
 0.100E 23  0.791E 09  0.160E 05  0.0006  0.760E 52  0.281E 05  0.443E 09
 0.100E 23  0.395E 09  0.640E 05  0.0009  0.148E 51  0.199E 05  0.230E 09
 0.100E 23  0.198E 09  0.256E 06  0.0010  0.248E 49  0.141E 05  0.116E 09
 0.100E 23  0.988E 08  0.102E 07  0.0012  0.388E 47  0.994E 04  0.582E 08
 0.100E 23  0.494E 08  0.410E 07  0.0014  0.607E 45  0.703E 04  0.291E 08
 0.100E 23  0.247E 08  0.164E 08  0.0016  0.948E 43  0.497E 04  0.145E 08
 0.100E 23  0.124E 08  0.655E 08  0.0018  0.148E 42  0.351E 04  0.727E 07
 0.100E 23  0.618E 07  0.262E 09  0.0020  0.232E 40  0.249E 04  0.364E 07
 0.100E 23  0.309E 07  0.105E 10  0.0024  0.366E 38  0.176E 04  0.182E 07
 0.100E 23  0.154E 07  0.419E 10  0.0048  0.101E 37  0.124E 04  0.100E 07
 0.100E 23  0.772E 06  0.168E 11  0.0573  0.165E 36  0.879E 03  0.741E 06
 0.100E 23  0.386E 06  0.671E 11  0.9055  0.358E 35  0.621E 03  0.574E 06
 0.100E 23  0.193E 06  0.268E 1214.4510  0.780E 34  0.439E 03  0.445E 06
 0.100E 23  0.965E 05  0.107E 13*******  0.171E 34  0.311E 03  0.346E 06
 0.100E 21  0.316E 09  0.100E 04  0.0006  0.266E 50  0.178E 05  0.173E 09
 0.100E 21  0.158E 09  0.400E 04  0.0007  0.431E 48  0.126E 05  0.869E 08
 0.100E 21  0.791E 08  0.160E 05  0.0009  0.760E 46  0.889E 04  0.443E 08
 0.100E 21  0.395E 08  0.640E 05  0.0013  0.148E 45  0.629E 04  0.230E 08
 0.100E 21  0.198E 08  0.256E 06  0.0016  0.248E 43  0.445E 04  0.116E 08
 0.100E 21  0.988E 07  0.102E 07  0.0019  0.388E 41  0.314E 04  0.582E 07
 0.100E 21  0.494E 07  0.410E 07  0.0021  0.607E 39  0.222E 04  0.291E 07
 0.100E 21  0.247E 07  0.164E 08  0.0024  0.949E 37  0.157E 04  0.145E 07
 0.100E 21  0.124E 07  0.655E 08  0.0028  0.149E 36  0.111E 04  0.728E 06
 0.100E 21  0.618E 06  0.262E 09  0.0037  0.265E 34  0.786E 03  0.372E 06
 0.100E 21  0.309E 06  0.105E 10  0.0232  0.230E 33  0.556E 03  0.248E 06
 0.100E 21  0.154E 06  0.419E 10  0.3562  0.483E 32  0.393E 03  0.191E 06
 0.100E 21  0.772E 05  0.168E 11  5.6568  0.105E 32  0.278E 03  0.148E 06
 0.100E 21  0.386E 05  0.671E 1190.3139  0.229E 31  0.196E 03  0.115E 06
 0.100E 19  0.316E 08  0.100E 04  0.0010  0.266E 44  0.562E 04  0.173E 08
 0.100E 19  0.158E 08  0.400E 04  0.0011  0.431E 42  0.398E 04  0.869E 07
 0.100E 19  0.791E 07  0.160E 05  0.0015  0.761E 40  0.281E 04  0.443E 07
 0.100E 19  0.395E 07  0.640E 05  0.0021  0.148E 39  0.199E 04  0.230E 07
 0.100E 19  0.198E 07  0.256E 06  0.0026  0.249E 37  0.141E 04  0.116E 07
 0.100E 19  0.988E 06  0.102E 07  0.0029  0.389E 35  0.994E 03  0.582E 06
 0.100E 19  0.494E 06  0.410E 07  0.0033  0.608E 33  0.703E 03  0.291E 06
 0.100E 19  0.247E 06  0.164E 08  0.0039  0.971E 31  0.497E 03  0.146E 06
 0.100E 19  0.124E 06  0.655E 08  0.0103  0.351E 30  0.351E 03  0.840E 05
 0.100E 19  0.618E 05  0.262E 09  0.1418  0.660E 29  0.249E 03  0.636E 05
 0.100E 19  0.309E 05  0.105E 10  2.2235  0.141E 29  0.176E 03  0.492E 05
 0.100E 19  0.154E 05  0.419E 1035.3407  0.307E 28  0.124E 03  0.381E 05
 0.100E 17  0.316E 07  0.100E 04  0.0015  0.266E 38  0.178E 04  0.173E 07
 0.100E 17  0.158E 07  0.400E 04  0.0018  0.432E 36  0.126E 04  0.869E 06
 0.100E 17  0.791E 06  0.160E 05  0.0023  0.761E 34  0.889E 03  0.444E 06
 0.100E 17  0.395E 06  0.640E 05  0.0032  0.148E 33  0.629E 03  0.230E 06
 0.100E 17  0.198E 06  0.256E 06  0.0040  0.249E 31  0.445E 03  0.116E 06
 0.100E 17  0.988E 05  0.102E 07  0.0046  0.390E 29  0.314E 03  0.582E 05
 0.100E 17  0.494E 05  0.410E 07  0.0063  0.733E 27  0.222E 03  0.300E 05
 0.100E 17  0.247E 05  0.164E 08  0.0581  0.929E 26  0.157E 03  0.213E 05
 0.100E 17  0.124E 05  0.655E 08  0.8832  0.193E 26  0.111E 03  0.164E 05
 0.100E 17  0.618E 04  0.262E 0913.8815  0.414E 25  0.786E 02  0.127E 05
 0.100E 17  0.309E 04  0.105E 10*******  0.901E 24  0.556E 02  0.983E 04
 0.100E 15  0.316E 06  0.100E 04  0.0023  0.266E 32  0.562E 03  0.173E 06
 0.100E 15  0.158E 06  0.400E 04  0.0028  0.432E 30  0.398E 03  0.870E 05
 0.100E 15  0.791E 05  0.160E 05  0.0036  0.762E 28  0.281E 03  0.444E 05
 0.100E 15  0.395E 05  0.640E 05  0.0051  0.148E 27  0.199E 03  0.230E 05
 0.100E 15  0.198E 05  0.256E 06  0.0063  0.251E 25  0.141E 03  0.117E 05
 0.100E 15  0.988E 04  0.102E 07  0.0259  0.142E 24  0.994E 02  0.722E 04
```

```
0.100E 15 0.494E 04 0.410E 07 0.3603 0.270E 23 0.703E 02 0.548E 04
0.100E 15 0.247E 04 0.164E 08 5.5038 0.564E 22 0.497E 02 0.422E 04
0.100E 15 0.124E 04 0.655E 0886.6721 0.121E 22 0.351E 02 0.327E 04
0.100E 13 0.316E 05 0.100E 04 0.0036 0.267E 26 0.178E 03 0.173E 05
0.100E 13 0.158E 05 0.400E 04 0.0043 0.432E 24 0.126E 03 0.870E 04
0.100E 13 0.791E 04 0.160E 05 0.0055 0.763E 22 0.889E 02 0.444E 04
0.100E 13 0.395E 04 0.640E 05 0.0145 0.273E 21 0.629E 02 0.255E 04
0.100E 13 0.198E 04 0.256E 06 0.1578 0.406E 20 0.445E 02 0.185E 04
0.100E 13 0.988E 03 0.102E 07 2.2345 0.786E 19 0.314E 02 0.141E 04
0.100E 13 0.494E 03 0.410E 0734.3087 0.165E 19 0.222E 02 0.109E 04
0.100E 11 0.316E 04 0.100E 04 0.0058 0.273E 20 0.562E 02 0.174E 04
0.100E 11 0.158E 04 0.400E 04 0.0144 0.929E 18 0.398E 02 0.988E 03
0.100E 11 0.791E 03 0.160E 05 0.0847 0.749E 17 0.281E 02 0.649E 03
0.100E 11 0.395E 03 0.640E 05 0.9664 0.117E 17 0.199E 02 0.476E 03
0.100E 11 0.198E 03 0.256E 0613.8708 0.229E 16 0.141E 02 0.363E 03
0.100E 11 0.988E 02 0.102E 07******* 0.483E 15 0.994E 01 0.280E 03
0.100E 09 0.316E 03 0.100E 04 0.0771 0.234E 15 0.178E 02 0.248E 03
0.100E 09 0.158E 03 0.400E 04 0.4999 0.207E 14 0.126E 02 0.166E 03
0.100E 09 0.791E 02 0.160E 05 5.9313 0.336E 13 0.889E 01 0.122E 03
0.100E 09 0.395E 02 0.640E 0586.1684 0.668E 12 0.629E 01 0.935E 02
0.100E 07 0.316E 02 0.100E 04 2.9688 0.578E 10 0.562E 01 0.424E 02
0.100E 07 0.158E 02 0.400E 0436.4777 0.970E 09 0.398E 01 0.315E 02
0.100E 05 0.316E 01 0.100E 04******* 0.281E 06 0.178E 01 0.809E 01
0.100E 03 0.316E 00 0.100E 04******* 0.166E 02 0.562E 00 0.160E 01
```

```
//PSEUDO JOB (U255,301E,S02,002,JM),'MELSON'
//*MAIN         USER=U255$JM,ORG=ZCC
//*PASSWORD************************************************************
//*WATFIV
      REAL N,NREY
      P=6.192735
      N=1.1
      WRITE (6,100)
  100 FORMAT('1','    NREY',9X,'FF',8X,'FNRE')
      DO 10 I=1,9
      N=N-.1
      WRITE (6,110)N
  110 FORMAT('          N = ',F3.1)
      NREY=2.E8
      DO 20 J=1,24
      NREY=NREY/2.
      CALL FRICT(N,NREY,FF)
      FNRE=FF**(4.-3.*N)*NREY**P
      WRITE(6,120)NREY,FF,FNRE
  120 FORMAT(' ',E11.4,3X,F6.4,E14.4)
   20 CONTINUE
   10 CONTINUE
      STOP
      END

       SUBROUTINE FRICT(N,NREY,FF)
      REAL N,NREY
      FL=16./NREY
      IF(NREY.LT.2000)GOTO 10
      FT=.01
    1 T=4./N**.75*ALOG10(NREY*FT**(1.-N/2.))-.4/N**1.2
      FT1=(1./T)**2
      IF(ABS((FT1-FT)/FT1).LT.1.E-5)GOTO 2
      FT=FT1
      GOTO 1
    2 IF(NREY.GT.4000)GOTO 12
      FTRANS=NREY**1.329*10.**(.6*N-7.14)
      IF(FL.GT.FTRANS)GOTO 10
      IF(FT1.LT.FTRANS)GOTO 12
      FF=FTRANS
      RETURN
   10 FF=FL
      RETURN
   12 FF=FT1
      RETURN
      END

//$DATA
```

```
  NREY              FF            FNRE
        N  = 1.0
0.1000E 09    0.0015        0.5174E 47
0.5000E 08    0.0016        0.7730E 45
0.2500E 08    0.0018        0.1159E 44
0.1250E 08    0.0020        0.1746E 42   7.48   6
0.6250E 07    0.0022        0.2642E 40
0.3125E 07    0.0024        0.4017E 38
0.1563E 07    0.0027        0.6142E 36
0.7813E 06    0.0030        0.9448E 34
0.3906E 06    0.0034        0.1463E 33
0.1953E 06    0.0039        0.2283E 31
0.9766E 05    0.0045        0.3592E 29
0.4883E 05    0.0053        0.5704E 27
0.2441E 05    0.0062        0.9157E 25
0.1221E 05    0.0073        0.1488E 24
0.6104E 04    0.0088        0.2452E 22
0.3052E 04    0.0108        0.4108E 20
0.1526E 04    0.0105        0.5436E 18
0.7629E 03    0.0210        0.1486E 17
0.3815E 03    0.0419        0.4064E 15
0.1907E 03    0.0839        0.1111E 14
0.9537E 02    0.1678        0.3038E 12
0.4768E 02    0.3355        0.8307E 10
0.2384E 02    0.6711        0.2271E 09
0.1192E 02    1.3422        0.6210E 07
        N  = 0.9
0.1000E 09    0.0013        0.6403E 46
0.5000E 08    0.0015        0.9846E 44
0.2500E 08    0.0016        0.1522E 43
0.1250E 08    0.0018        0.2366E 41
0.6250E 07    0.0020        0.3700E 39
0.3125E 07    0.0022        0.5826E 37
0.1563E 07    0.0025        0.9240E 35
0.7813E 06    0.0028        0.1478E 34
0.3906E 06    0.0032        0.2384E 32
0.1953E 06    0.0036        0.3885E 30
0.9766E 05    0.0042        0.6404E 28
0.4883E 05    0.0049        0.1069E 27
0.2441E 05    0.0057        0.1810E 25
0.1221E 05    0.0068        0.3116E 23
0.6104E 04    0.0083        0.5466E 21
0.3052E 04    0.0102        0.9800E 19
0.1526E 04    0.0105        0.1385E 18
0.7629E 03    0.0210        0.4662E 16
0.3815E 03    0.0419        0.1569E 15
0.1907E 03    0.0839        0.5283E 13
0.9537E 02    0.1678        0.1778E 12
0.4768E 02    0.3355        0.5987E 10
0.2384E 02    0.6711        0.2015E 09
0.1192E 02    1.3422        0.6784E 07
        N  = 0.8
0.1000E 09    0.0012        0.7275E 45
0.5000E 08    0.0013        0.1153E 44
0.2500E 08    0.0014        0.1839E 42
0.1250E 08    0.0016        0.2955E 40
0.6250E 07    0.0018        0.4786E 38
0.3125E 07    0.0020        0.7816E 36
0.1563E 07    0.0022        0.1288E 35
0.7813E 06    0.0025        0.2146E 33
```

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0.3906E 06    0.0029      0.3616E 31
0.1953E 06    0.0033      0.6172E 29
0.9766E 05    0.0038      0.1069E 28
0.4883E 05    0.0045      0.1880E 26
0.2441E 05    0.0053      0.3370E 24
0.1221E 05    0.0063      0.6166E 22
0.6104E 04    0.0077      0.1156E 21
0.3052E 04    0.0094      0.2150E 19
0.1526E 04    0.0105      0.3529E 17
0.7629E 03    0.0210      0.1463E 16
0.3815E 03    0.0419      0.6061E 14
0.1907E 03    0.0839      0.2512E 13
0.9537E 02    0.1678      0.1041E 12
0.4768E 02    0.3355      0.4314E 10
0.2384E 02    0.6711      0.1788E 09
0.1192E 02    1.3422      0.7410E 07
         N = 0.7
0.1000E 09    0.0010      0.7438E 44
0.5000E 08    0.0011      0.1217E 43
0.2500E 08    0.0013      0.2007E 41
0.1250E 08    0.0014      0.3339E 39
0.6250E 07    0.0016      0.5610E 37
0.3125E 07    0.0017      0.9524E 35
0.1563E 07    0.0020      0.1636E 34
0.7813E 06    0.0022      0.2845E 32
0.3906E 06    0.0026      0.5021E 30
0.1953E 06    0.0029      0.9000E 28
0.9766E 05    0.0034      0.1642E 27
0.4883E 05    0.0040      0.3057E 25
0.2441E 05    0.0048      0.5822E 23
0.1221E 05    0.0058      0.1138E 22
0.6104E 04    0.0071      0.2291E 20
0.3052E 04    0.0081      0.4071E 18
0.1526E 04    0.0105      0.8992E 16
0.7629E 03    0.0210      0.4588E 15
0.3815E 03    0.0419      0.2341E 14
0.1907E 03    0.0839      0.1194E 13
0.9537E 02    0.1678      0.6093E 11
0.4768E 02    0.3355      0.3109E 10
0.2384E 02    0.6711      0.1586E 09
0.1192E 02    1.3422      0.8094E 07
         N = 0.6
0.1000E 09    0.0009      0.6649E 43
0.5000E 08    0.0010      0.1125E 42
0.2500E 08    0.0011      0.1923E 40
0.1250E 08    0.0012      0.3321E 38
0.6250E 07    0.0013      0.5802E 36
0.3125E 07    0.0015      0.1027E 35
0.1563E 07    0.0017      0.1843E 33
0.7813E 06    0.0019      0.3358E 31
0.3906E 06    0.0022      0.6227E 29
0.1953E 06    0.0026      0.1177E 28
0.9766E 05    0.0030      0.2273E 26
0.4883E 05    0.0036      0.4499E 24
0.2441E 05    0.0043      0.9151E 22
0.1221E 05    0.0052      0.1921E 21
0.6104E 04    0.0064      0.4179E 19
0.3052E 04    0.0071      0.7096E 17
0.1526E 04    0.0105      0.2291E 16
0.7629E 03    0.0210      0.1439E 15
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0.3815E 03    0.0419      0.9040E 13
0.1907E 03    0.0839      0.5678E 12
0.9537E 02    0.1678      0.3567E 11
0.4768E 02    0.3355      0.2241E 10
0.2384E 02    0.6711      0.1407E 09
0.1192E 02    1.3422      0.8841E 07
       N  =  0.5
0.1000E 09    0.0007      0.4977E 42
0.5000E 08    0.0008      0.8735E 40
0.2500E 08    0.0009      0.1551E 39
0.1250E 08    0.0010      0.2789E 37
0.6250E 07    0.0011      0.5085E 35
0.3125E 07    0.0013      0.9413E 33
0.1563E 07    0.0014      0.1772E 32
0.7813E 06    0.0016      0.3398E 30
0.3906E 06    0.0019      0.6654E 28
0.1953E 06    0.0022      0.1333E 27
0.9766E 05    0.0026      0.2741E 25
0.4883E 05    0.0031      0.5802E 23
0.2441E 05    0.0037      0.1270E 22
0.1221E 05    0.0046      0.2885E 20
0.6104E 04    0.0057      0.6844E 18
0.3052E 04    0.0062      0.1138E 17
0.1526E 04    0.0105      0.5837E 15
0.7629E 03    0.0210      0.4514E 14
0.3815E 03    0.0419      0.3491E 13
0.1907E 03    0.0839      0.2700E 12
0.9537E 02    0.1678      0.2088E 11
0.4768E 02    0.3355      0.1615E 10
0.2384E 02    0.6711      0.1249E 09
0.1192E 02    1.3422      0.9657E 07
       N  =  0.4
0.1000E 09    0.0006      0.2909E 41
0.5000E 08    0.0006      0.5313E 39
0.2500E 08    0.0007      0.9841E 37
0.1250E 08    0.0008      0.1850E 36
0.6250E 07    0.0009      0.3537E 34
0.3125E 07    0.0010      0.6884E 32
0.1563E 07    0.0012      0.1367E 31
0.7813E 06    0.0013      0.2776E 29
0.3906E 06    0.0016      0.5778E 27
0.1953E 06    0.0018      0.1236E 26
0.9766E 05    0.0022      0.2728E 24
0.4883E 05    0.0026      0.6235E 22
0.2441E 05    0.0032      0.1483E 21
0.1221E 05    0.0039      0.3688E 19
0.6104E 04    0.0049      0.9662E 17
0.3052E 04    0.0054      0.1681E 16
0.1526E 04    0.0105      0.1487E 15
0.7629E 03    0.0210      0.1416E 14
0.3815E 03    0.0419      0.1348E 13
0.1907E 03    0.0839      0.1284E 12
0.9537E 02    0.1678      0.1222E 11
0.4768E 02    0.3355      0.1164E 10
0.2384E 02    0.6711      0.1108E 09
0.1192E 02    1.3422      0.1055E 08
       N  =  0.3
0.1000E 09    0.0004      0.1169E 40
0.5000E 08    0.0005      0.2235E 38
0.2500E 08    0.0005      0.4345E 36
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0.1250E 08    0.0006      0.8599E 34
0.6250E 07    0.0007      0.1736E 33
0.3125E 07    0.0008      0.3581E 31
0.1563E 07    0.0009      0.7569E 29
0.7813E 06    0.0010      0.1643E 28
0.3906E 06    0.0012      0.3674E 26
0.1953E 06    0.0014      0.8494E 24
0.9766E 05    0.0017      0.2039E 23
0.4883E 05    0.0021      0.5105E 21
0.2441E 05    0.0025      0.1341E 20
0.1221E 05    0.0032      0.3719E 18
0.6104E 04    0.0041      0.1098E 17
0.3052E 04    0.0052      0.3233E 15
0.1526E 04    0.0105      0.3789E 14
0.7629E 03    0.0210      0.4442E 13
0.3815E 03    0.0419      0.5206E 12
0.1907E 03    0.0839      0.6103E 11
0.9537E 02    0.1678      0.7154E 10
0.4768E 02    0.3355      0.8385E 09
0.2384E 02    0.6711      0.9829E 08
0.1192E 02    1.3422      0.1152E 08
         N = 0.2
0.1000E 09    0.0003      0.2460E 38
0.5000E 08    0.0003      0.4975E 36
0.2500E 08    0.0003      0.1026E 35
0.1250E 08    0.0004      0.2164E 33
0.6250E 07    0.0004      0.4676E 31
0.3125E 07    0.0005      0.1038E 30
0.1563E 07    0.0006      0.2372E 28
0.7813E 06    0.0007      0.5605E 26
0.3906E 06    0.0008      0.1374E 25
0.1953E 06    0.0010      0.3510E 23
0.9766E 05    0.0012      0.9394E 21
0.4883E 05    0.0015      0.2649E 20
0.2441E 05    0.0019      0.7930E 18
0.1221E 05    0.0024      0.2540E 17
0.6104E 04    0.0032      0.8781E 15
0.3052E 04    0.0052      0.6690E 14
0.1526E 04    0.0105      0.9655E 13
0.7629E 03    0.0210      0.1393E 13
0.3815E 03    0.0419      0.2011E 12
0.1907E 03    0.0839      0.2902E 11
0.9537E 02    0.1678      0.4187E 10
0.4768E 02    0.3355      0.6043E 09
0.2384E 02    0.6711      0.8721E 08
0.1192E 02    1.3422      0.1259E 08

CORE USAGE        OBJECT CODE=     1776 BYTES,ARRAY AREA=        0 BYTES,TOTAL AREA

DIAGNOSTICS          NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBE

COMPILE TIME=     0.05 SEC,EXECUTION TIME=     0.35 SEC,     15.21.50     TUESDAY
```

APPENDIX C : References

(1) Chontos, "Find Economic Pipe Diameter via Improved Formula," Chemical Engineering, June 16, 1980, pp. 139 - 142.

(2) DeNevers, Fluid Mechanics, Addison-Wesley Publishing Co., 1970, pp. 196 - 201.

(3) Hanks & Dadia, "Theoretical Analysis of the Turbulent Flow of Non-Newtonian Slurries in Pipes," A.I.Ch.E. Journal, Vol.17, No. 3, May 1971, pp. 554 - 557.

(4) Kemblowski & Kolodziejski, "Flow Resistances of Non-Newtonian Fluids in Transitional and Turbulent Flow," International Chemical Engineering, Vol. 13, No. 2, April 1973, pp. 265 - 279.

(5) Perry & Chilton, Chemical Engineers' Handbook, 5th ed., McGraw-Hill Book Company, 1973, pp. 5-31, 5-32, 5-38 - 5-40.

(6) Peters & Timmerhaus, Plant Design and Economics for Chemical Engineers, 3rd ed., McGraw-Hill Book Company, 1980, pp. 377 - 383.

(7) Skelland, Non-Newtonian Flow and Heat Transfer, John Wiley & Sons Inc., 1967, pp. 1 - 23, 27 - 47, 162 - 175, 180 - 205, 240 - 265

(8) Weisman, "Minimum Power Requirements for Slurry Transport," A.I.Ch.E. Journal, Vol. 9 , No. 1, January 1963, pp. 134 - 138.

APPENDIX D : Nomenclature

CC - fraction of initial capital investment that must be yearly alloted for depreciation, maintenance, etc., see equation (11) (yr^{-1})

C_{cap} - yearly capital costs (\$/(yr ft))

C_{oper} - yearly operating costs (\$/(yr ft))

C_T - total yearly costs,($= C_{cap} + C_{oper}$) (\$/(yr ft))

D - inside pipe diameter (ft)

dV/dy - shear rate (sec^{-1})

E - pump efficiency (dimensionless)

f - friction factor (dimensionless)

f_L - friction factor for laminar flow (dimensionless)

f_T - friction factor for turbulent flow (dimensionless)

g_c - gravitational constant ($=32.2\ lb_m\ ft/(lb_f\ sec^2)$)

h - length of yearly operation (hr)

K - fluid consistency index ($lb_m\ sec^{(n-2)}\ ft^{-1}$)

K' - modified fluid consistency index ($lb_m\ sec^{(n-2)}\ ft^{-1}$)

$\dot{m}$ - mass flow rate (lb_m/hr)

n - flow behavior index (dimensionless)

n' - modified flow behavior index (dimensionless)

N_{He} - Hedstrom number for Bingham plastics, defined by equation(2) (dimensionless)

N_{Re} - Reynold's number, defined for Bingham plastics by equation (1), defined for pseudoplastics by equation (7), (dimensionless)

OEPD - optimum economic pipe diameter

p - exponent in pipe cost equation (11) (dimensionless)

PC - power cost (\$/(hp yr))

V - velocity (ft/sec)

X - purchased and installed cost of pipe for D=1 ft ($\$/ft^{(p+1)}$)

γ - exponent in friction factor equation (14) (dimensionless)

η - plastic viscosity ($lb_m/(ft\ sec)$)

μ - viscosity for Newtonian fluid ($lb_m/(ft\ sec)$)

ρ - fluid density (lb_m/ft^3)

τ - shear stress (lb_f/ft^2)

τ_y - critical shear stress of Bingham plastic (lb_f/ft^2)

Ψ - economic variable, defined in equation (13) ($ft^{(p-1)}\ sec^3/lb_m$)