Measurement Of The Forward Scattered Light From A Small Sphere

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Abstract

The goal of this project is to measure the forward scattered light from a small sphere at angles down to zero degrees. By cancelling the unscattered illumination beam with a Mach-Zehnder interferometer, it was possible to observe the scattered light plus the interference term from the scattered light interfering with the residual unscattered beam. It was found that in order to measure the scattered light alone, it will be necessary to make the residual unscattered beam 100-1000 times darker. This paper describes the experimental progess toward this goal and analyzes the effects of interferences on the light exiting the interferometer.

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I. Introduction

This paper describes our progress in measuring the forward scattered light from a small sphere. The purpose of this section is to acquaint the reader with the significance of the forward scattering, the difficulties involved in measuring it, the methods my advisor and I are using to overcome these difficulties and our progress toward this goal.

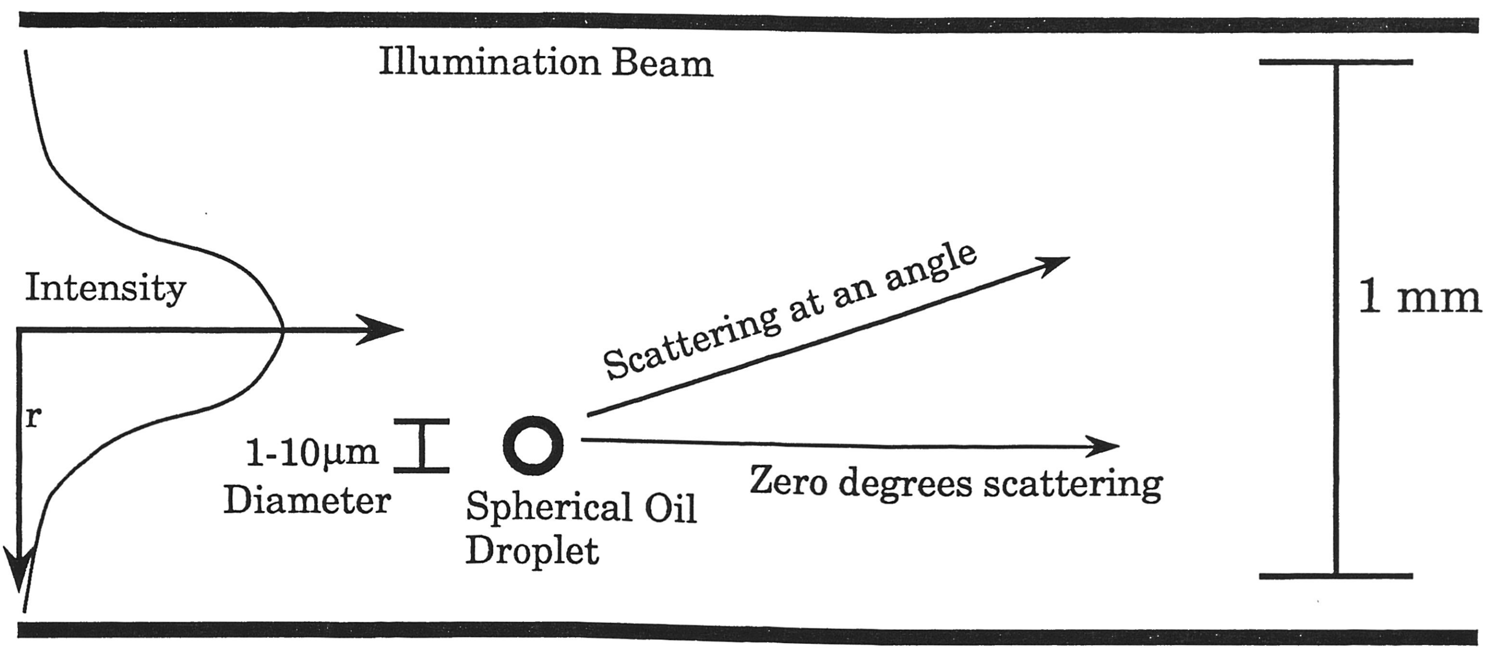
The forward scattering we are concerned with is the light scattered by a small sphere in the same direction as the light that illuminates the sphere. Specifically, we desire to measure the scattered light intensities at angles down to and including zero degrees. (see figure I-1) This scattering of light is described completely by the solution of Maxwell's equations for a homogeneous sphere which was obtained by Gustav Mie, [Van de Hulst, 1957, #1] in 1908. The zero degree scattering has never been measured experimentally.

The region close to zero degrees is the most significant direction to study, because the scattering into this direction is much more intense than in any other region. This peak in the scattering pattern makes zero degrees the most important direction to consider when making theoretical calculations. A practical application of this type of light scattering is the simulation of laser beams passing through the atmosphere, ocean, dust clouds or smoke screens. The same techniques we are using here could be applied to the measurement of the zero degree scattering from cubic scatterers which are much more difficult to treat theoretically then spheres.

It is difficult to measure the forward scattering because as the detector swings in front of the sphere, it has to pass through the unscattered illumination beam. Any detector sensitive enough to measure the scattered light would be saturated by the unscattered beam. Also, in order to compare the zero degrees scattering with the rest of the scattering pattern, our photodetector must be capable of operation with light levels that may vary by several orders of magnitude.

Our plan is to interferometrically cancel out the unscattered beam by using a Mach-Zehnder interferometer with the sphere held in one arm. This provides a way to decrease the amount of unscattered light hitting the photodetector when it scans through zero degrees. By modulating the sphere's position in the gaussian beam, we can modulate the intensity of the scattered light. This allows us to use lock-in amplification to differentiate the scattering from constant stray light and noise. There is a problem with trying to pick out the scattered beam by its modulation, though. The scattered light and the unscattered beam interfere to give an interference term that oscillates with the same frequency as the scattered light. This term is picked up by the lock-in amplifier, since it has the same frequency as the scattered light. This means that the level of residual light must be reduced by a factor of 1000. Some methods that might be used to do this are described in appendix A.

I-1 Forward Scattering



II-1 Experimental Apparatus

The first part of this chapter describes the forward scattering meter. Its individual components are described in the succeeding parts.

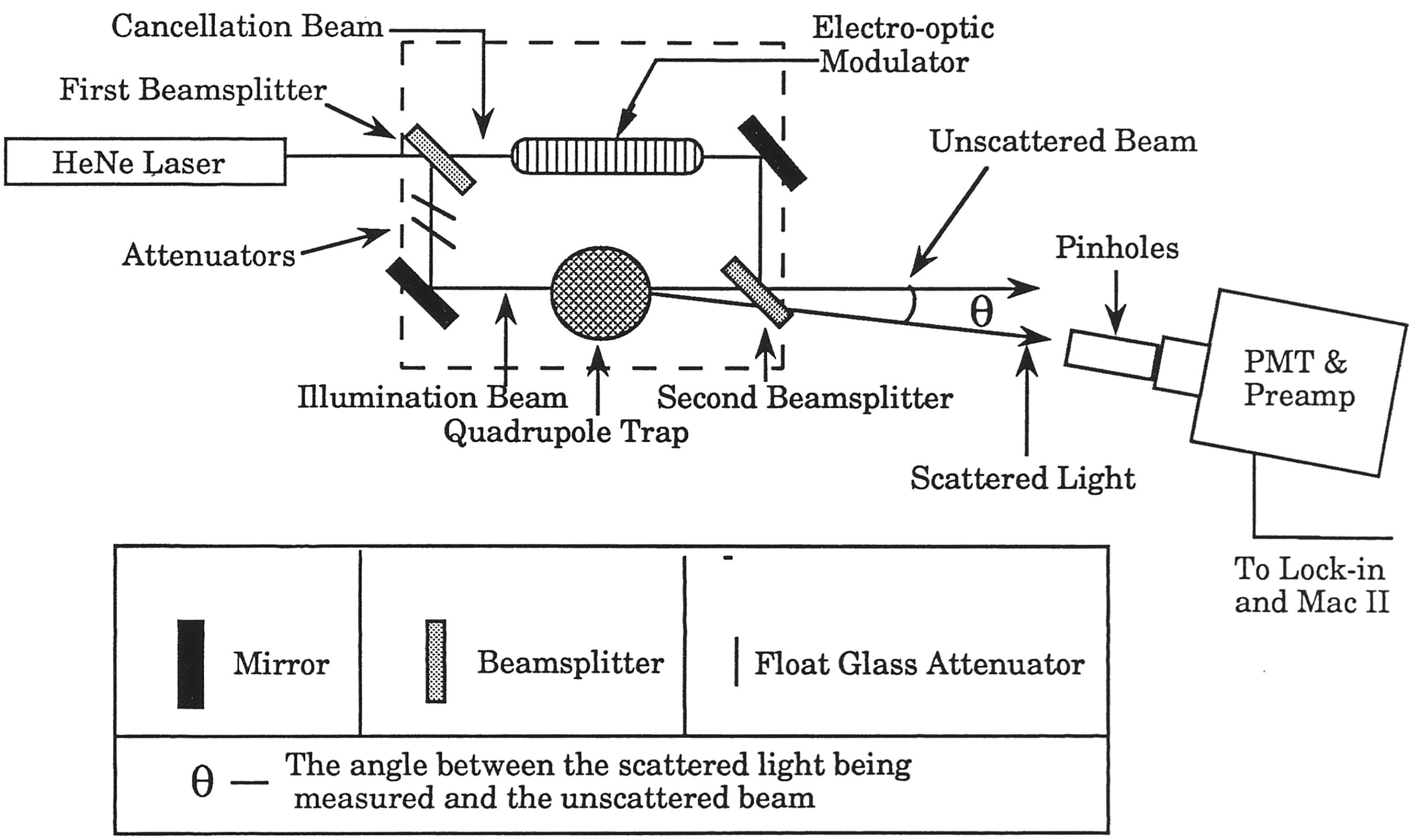
First, the beam from a 4 milliwatt Helium-Neon laser is sent into the Mach-Zehnder interferometer, Figure II-1. It is split into two roughly equal halves by the first beamsplitter. One half, the illumination beam, is sent through the quadrupole trap and illuminates the oil droplet. The other half is sent through an electro-optic modulator which has its optical axis lined up with the S-polarization of the laser beam. This keeps the modulator from rotating the polarization. It only shifts the phase of that beam so that maximum destructive interference can be obtained when the cancellation beam is recombined with the illumination beam at the second beamsplitter.

The forward scattered light and the recombined beams exit the interferometer through the second beamsplitter. The side scattered light exits the quadrupole trap through a curved window on the side. This allows the scanning arm to swing the photomultiplier tube in front for forward scattering measurements or to the side for side scattering. An optical encoder provides the computer with the angular position of the scanning arm.

Because the trap modulates the scattered light by moving the sphere back and forth through the gaussian beam, the lock-in detector could use the trap's oscillation frequency as a reference frequency for detection of the scattered light, except that the intensity term from the interference of the scattered light and the residual unscattered beam is modulated at the same frequency.

The output from the lock-in is then fed to the Macintosh II computer via a GPIB-488 interface. It records and processes the data. After the data is collected, we compare it with Mie theory calculations that are also run on the Macintosh. This allows us to size the sphere if we use the side scattering data or to check the forward scattering calculations.

II-1 Experimental Apparatus



II-2 Quadrupole Trap

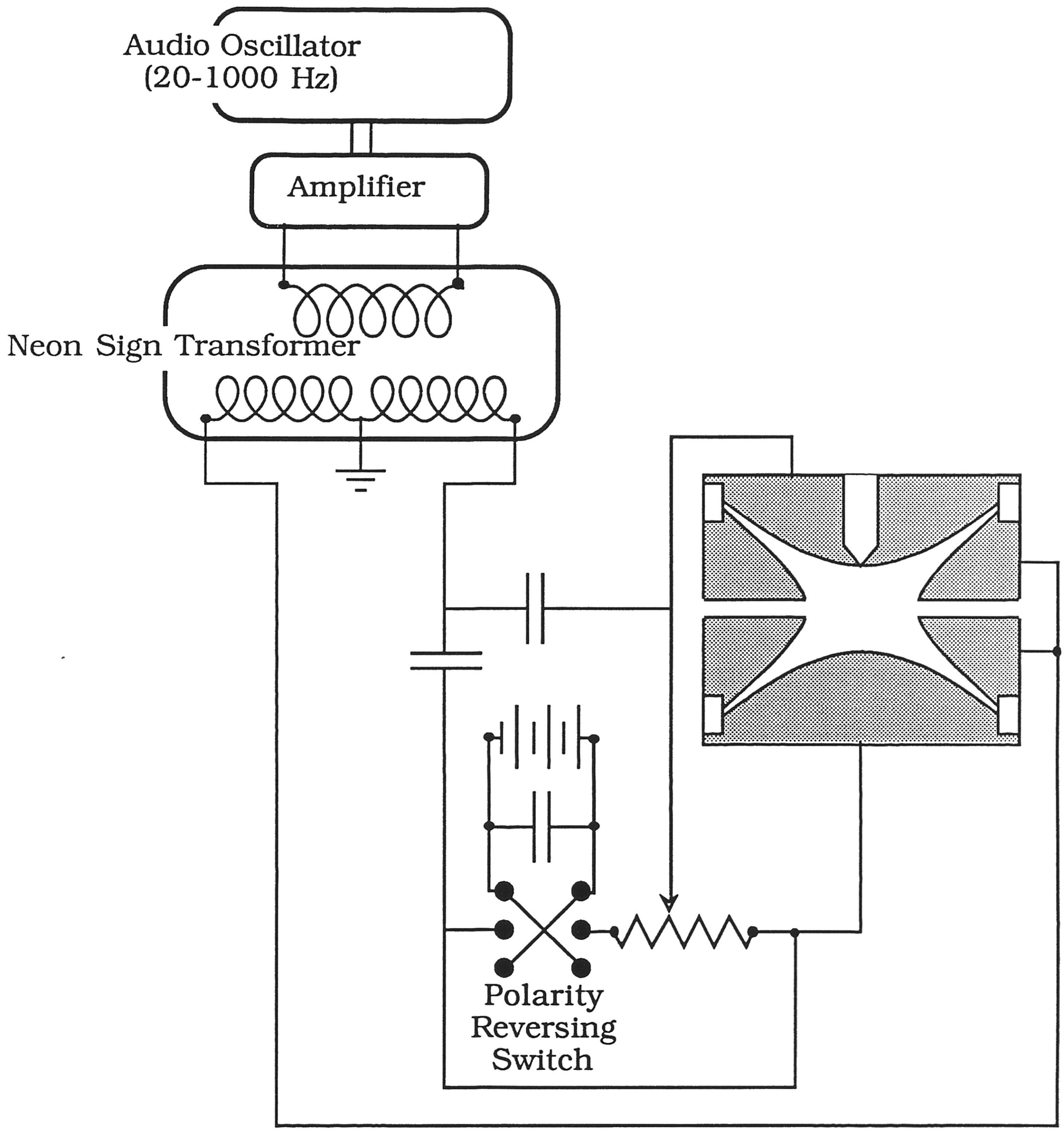
The quadrupole trap was used to suspend the sphere in the laser beam and to modulate its position. The spheres used in this experiment are droplets of microscope objective immersion oil. This material was chosen because the liquid naturally assumes a spherical shape and the low vapor pressure keeps the size from changing due to evaporation. This oil also has had its refractive index determined to three places at the factory.

The trap has a cylindrical shape consisting of a top and bottom electrode with a ring electrode in the middle. (see figure II-2) The three electrodes are insulated from each other by plastic rings. Our trap is similar in design to that used by R. R. Wuerker, [Wuerker, et al., 1950, #2]. It suspends the charged oil droplet because the electric fields from the hyperbolic electrodes alternately provide restoring forces in the z and radial directions. The time average of these forces provides a net restoring force toward the center of the trap.

The oil droplets are charged when they are injected into the trap. This is done by taking a sharp hypodermic needle and coating the end with microscope oil. The needle is then connected to the ring electrode through a 15 megaohm resistor. Since the ring electrode is opposite in polarity to the top electrode, placing the needle inside the injection port in the top electrode produces a low current arc. This arc atomizes the oil into tiny droplets and charges them. This procedure usually generates several droplets with enough charge to be caught by the trap. Since we desire to look at only one sphere, the others must be eliminated. This is done by decreasing the 2000 V rms

400 Hz a.c. potential between the ring and end electrodes and increasing the d.c. potential between the top and bottom electrodes. This condition of reduced stability and increased force pulling the droplets away from the center will deposit the less stable droplets against the bottom of the trap. By using these techniques, single droplets have been selected and then trapped for weeks.

The sphere's position in the beam is naturally modulated by the force of gravity. Since the restoring force in the vertical z direction is only present during each half cycle of the driving voltage, the sphere is free to fall down during the other half cycle. This makes it oscillate with the frequency of the trap driving voltage. The d.c. potential between the top and bottom plates can be applied to mimic the effect of higher gravity for larger oscillations, or it can be used to counteract the effect of gravity to produce smaller oscillations.


Audio Oscillator
(20-1000 Hz)
Amplifier
Neon Sign Transformer
Polarity
Reversing
Switch

II-3 Mach-Zehnder Interferometer

In order for the cancellation beam to destructively interfere with the illumination beam in such a way as to completely cancel it, there are five criteria that must be satisfied. The first of these is that the intensities of the illuminating beam and the cancellation beam must be equal. If one is more intense than the other, there will be residual light when they interfere destructively. The intensity balance is controlled by the attenuators (figure II-1) placed in the more intense beam. These are simply thin glass windows which attenuate the beam by reflecting some of the light out of it. In accordance with the Fresnel equations, the amount of attenuation depends on the angle that the window makes with the beam. The intensity balancing procedure is a simple one. First, the beam going through the attenuators is blocked off. A photodiode detector is placed in front of the second beamsplitter. The fixed intensity beam is then measured. The fixed beam is then blocked, and the variable beam's intensity is adjusted by rotating the windows until its intensity is equal to the fixed beam. Using this procedure it has been possible to balance the beams to within 1%.

Secondly, the polarizations of the two beams must be in the same direction, which has been chosen to be perpendicular to the plane of the interferometer, i.e., S-polarized light. This is necessary since a beam polarized in one direction cannot cancel an electric field perpendicular to it. To adjust the polarization, the cancellation beam is blocked off and a crossed polarizer is placed in front of the detector. The laser is then rotated until the light passing through the crossed polarizer is a minimum. At this point the illumination beam is S-polarized. Then the illumination arm is blocked off and the laser beam is sent through the cancellation arm. The cancellation arm contains the modulator which will rotate the polarization when voltage is applied unless the optical axis is aligned parallel to the polarization. This must be done because the polarization cannot be allowed to change; it must stay S-polarized. In order to align the modulator, we keep the crossed polarizer in front of the detector and block off the illumination beam. The photodiode detects the amount of light getting through the crossed polarizer, and this is visually monitored on an oscilloscope. Voltage is applied to the modulator and the light level is observed. Ideally, the light level should not change when voltage is applied if the modulator is properly oriented. In practice, the modulator is rotated around the beam direction until the polarization change with voltage is a minimum. Using this procedure, the polarization change with voltage has been limited to .5% of the cancellation beam intensity.

Thirdly, the phase of the cancellation beam must be adjustable so that this beam can be phase-shifted π radians from the illumination beam and thus cause destructive interference. This is accomplished by using the modulator to shift the phase. The modulator is a KD*P crystal with the electric field oriented longitudinally. It is powered by a 0-3.6 KV voltage multiplier circuit. The output voltage can by varied by adjusting a variable transformer that supplies the input voltage to the multiplier.

other is rotated so as to keep the two beams superimposed on the screen. The fringes either broaden or become more closely spaced. The adjustment should be made so that the fringes broaden. When the fringes are uniform horizontally, the lines of the fringes will be horizontal. Now the vertical is adjusted in a similar way. However, when the vertical is adjusted, the horizontal adjustment will be slightly changed, so it must be readjusted, and then the vertical has to be readjusted. This continues for a few cycles until the field is nearly uniform. At this point, it is hard to tell if the pattern is narrowing or broadening because the force of the operator's hand on the mount is making the pattern flash bright and dark. We can make use of this by continuing to tweak the thumbscrews until the pattern flashes brightly. In between adjustments, the pattern will be still and the proximity to uniform phase can be judged. When the pattern flashes the brightest, you are probably on or at uniform phase. This is because at uniform phase the whole pattern must change if there is a phase shift. At less than uniform phase, one part of the pattern can brighten and another can darken, so the total light doesn't change much. Steady hands, patience, and experience in looking at the optical cues provided by the interference pattern are the key to achieving uniform phase.

In addition to proper alignment, the beam path must be free of nonuniformities in order to have uniform phase. This is because any irregularities in one part of the beam will cause that portion to be phase shifted which would make it impossible to have that part and the rest of the beam be at the same level of darkness. By paying careful attention to what we put in the beam path, uniform phase can be maintained. For example, the interferometer optics are specified to be flat to $\lambda/10$ over their diameter of 2.54 cm. This would seem to be a grossly inadequate flatness, but it must be remembered that the beam is only a few millimeters in diameter, so the optics are sufficiently flat over this small area. This is convenient since it allows us to use this flatness rating which is relatively cheap. Also, the attenuator and trap windows are made of float glass. This glass is flat to about $\lambda/3$ per centimeter. It is produced by allowing liquid glass to spread out on a surface of molten tin. We simply buy it in small sheets and cut it to size with a carbide tipped scribing pen.

The fifth factor is vibration control. Not only do we require uniform phase in space, we require uniform phase over a period of time. This means that drifts and vibrations must be minimized because a shift of only $\lambda/2$ in the path difference between the arms changes the pattern from fully dark to completely bright. Vibrations were minimized by the use of a three stage system. First, the optics are not mounted on the usual tall floppy steel rods and loose mounts, but are mounted on bolts that go through a 25 kilogram block of lead. The optical mounts sit slightly above the lead block surface so that the adjusting knobs can turn, but this is counteracted by having 2.54 cm diameter washers around the supporting 10-32 bolt. Thus, the washers act as a very wide short post. The lead block provides good positional stability. Because of lead's high internal damping, it also tends to dissipate vibrations well. This can easily be demonstrated by dropping a piece of lead. It hits the ground, does not bounce, and makes a dull noise. A dropped piece of aluminium or steel will bounce and ring for a long time. The second vibration reduction stage is a foam/rubber support system. The lead block is supported by a layer of rubber, polyurethane foam, and another layer of rubber. This acts in two ways. The first is that foam is a poor conductor of mechanical energy because of its low density. Rubber also dissipates vibrations because of its good internal damping. The second way this works is that the foam has a low spring constant and the lead mass is heavy. Together they form a harmonic oscillator with an extremely low resonant frequency. This means that most vibrations will move the block very little. This is important because if the laser beams travel a slightly different path through the interferometer the altered alignment may not provide uniform phase or there may be sudden phase shifts. These layers of foam and rubber rest on a wooden board which supports the interferometer over the optical encoder. This board in turn is supported on each end by two layers. Each of these is composed of 3 cm of rubber and 5 cm of rigid polystyrene foam. Once again the rubber damps out vibrations. The polystyrene foam is a poor mechanical conductor because of its low density. Another important quality is that it is rigid. This keeps the interferometer from shifting around on a tall unstable stack of flexible polyurethane foam layers. The third part of this system is the optical table the entire assembly rests on. The optical table has special internal damping and is supported on the floor by four carefully designed shock absorbers.

By controlling the 5 factors of polarization, intensity, phase setting, uniform phase, and vibration control, we have greatly increased the ratio of scattered light to residual unscattered light hitting the detector at zero degrees.

II-4 Lock-in Amplification

We need to be able to separate the scattered light from the unscattered beam. Lock-in amplification was the method we attempted to use to do this, because it provides a means to selectively measure the rms value of only the signals that have the same frequency as a chosen reference signal.

The droplet oscillates in the gaussian illumination beam at the trap oscillation frequency and by making this frequency our reference signal, we tried to separate the scattered signal light from that of the unscattered beam. The problem is that the interference term has the same frequency as the scattered light.

The lock-in performs a correlation between the incoming signal which may have noise and undesirable signals and the reference signal which is of the same frequency as the signal we wish to measure. [Horowitz and Hill, 1980, #5]

Suppose that you have a signal modulated at a known frequency, ω_0 . Let's make it a sine wave for simplicity and assume that it is contaminated with noise. Write the signal plus noise as:

$$F(T) = A(\omega_0)\sin(\omega_0 T) + \sum_{\omega \neq \omega_0} \left\{ A(\omega)\sin(\omega T) + B(\omega)\cos(\omega T) \right\}$$

Now the lock-in multiplies this function with the reference frequency, G(T).

$$G(T) = \sin(\omega_0 T),$$

to obtain

$$F(T)\ G(T) = \sum_{\omega \neq \omega_0} \Big\{ A(\omega)\sin(\omega T)\ \sin(\omega_0 T) + B(\omega)\cos(\omega T)\ \sin(\omega_0 T) \Big\} .$$

Averaging this for an infinite amount of time gives

$$\langle F(T)\ G(T) \rangle = A(\omega_0)\ /\ 2$$

Actually, the signal is only averaged for a finite time, which will remove all frequencies except for those close to the reference frequency. This bandwidth is specified by $\Delta\omega = 1\ /\ (4T_c)$, where T_c is the time constant of the low pass filter used to average the product of the signal and reference frequencies.

Our particular lock-in is an EG & G PARC model 5210. In addition to basic lock-in amplification it has a variety of electronic filters and also possesses a GPIB-488 interface.

II-5 Photomultiplier assembly

The photomultiplier assembly consists of the photomultiplier, preamplifier, power supply and scanning arm. The purpose of this setup is to convert the light to an electrical signal, amplify it, move the detector through an angular range and determine its angular position.

We chose a photomultiplier as the photodetector for this experiment because it had the high gain that was required to measure the weak side scattering while looking through the small aperture required for high angular resolution. The particular PMT (photomultiplier tube) we used was an RCA 8644. It was connected to a preamplifier that used a 1 volt per microamp current to voltage converter and a 220× voltage amplifier for a total gain of 220V per microamp. The power supply for the PMT was a Kepco 1500 power supply (0 - 1250 volts d.c.).

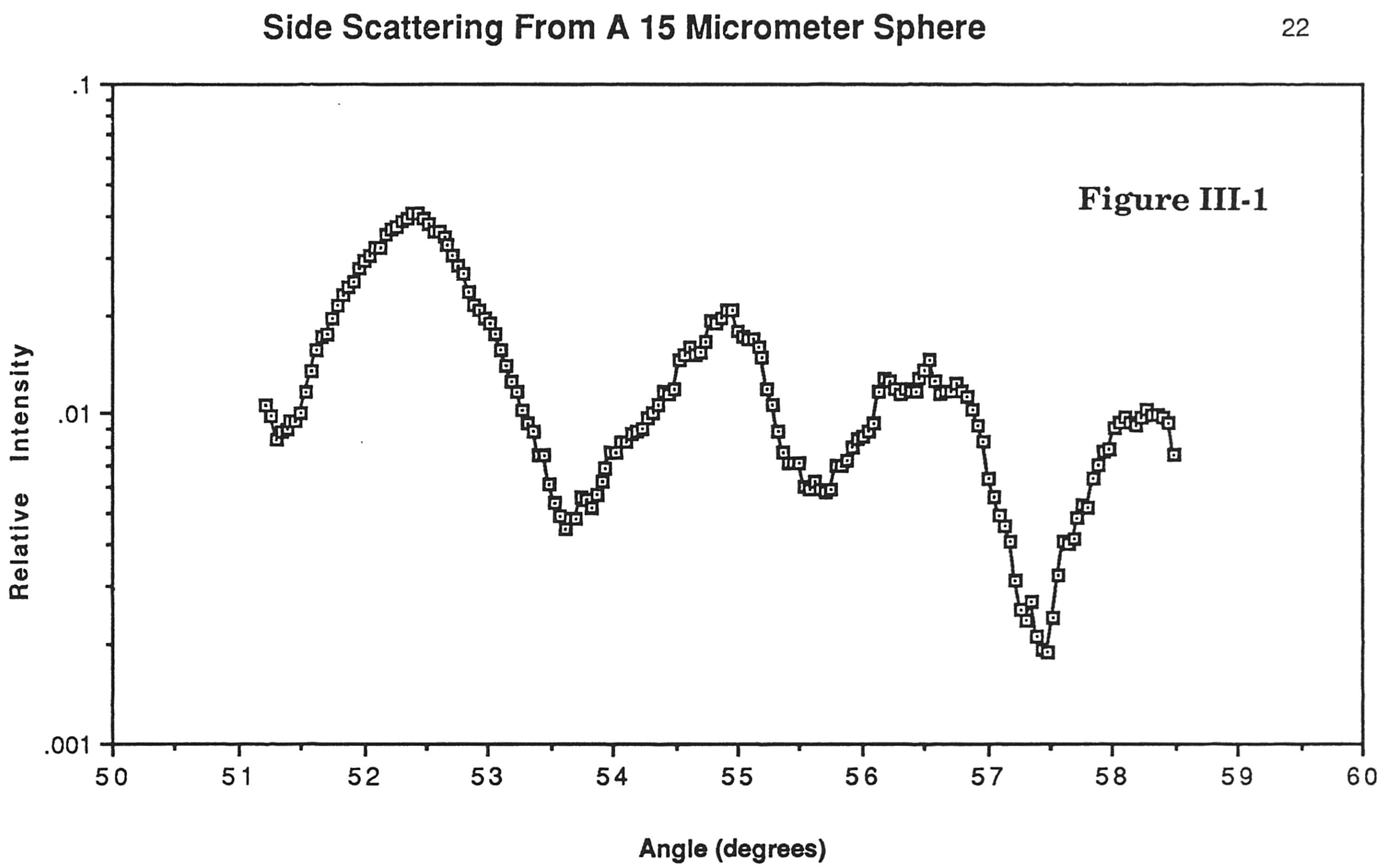
Small spikes were present in the output of this power supply. While these were not visible on an oscilloscope at 5 millivolts/division on ac setting, the PMT and preamp amplified them up to about 20 millivolts. Even with the lock-in amplifier, these spikes would cause the output to jump by 100-200 microvolts with a 1 second time constant. This was a problem when observing signals in the 10's of microvolts. We solved this problem by adding an RC filter with a time constant of 18 seconds to the power supply output.

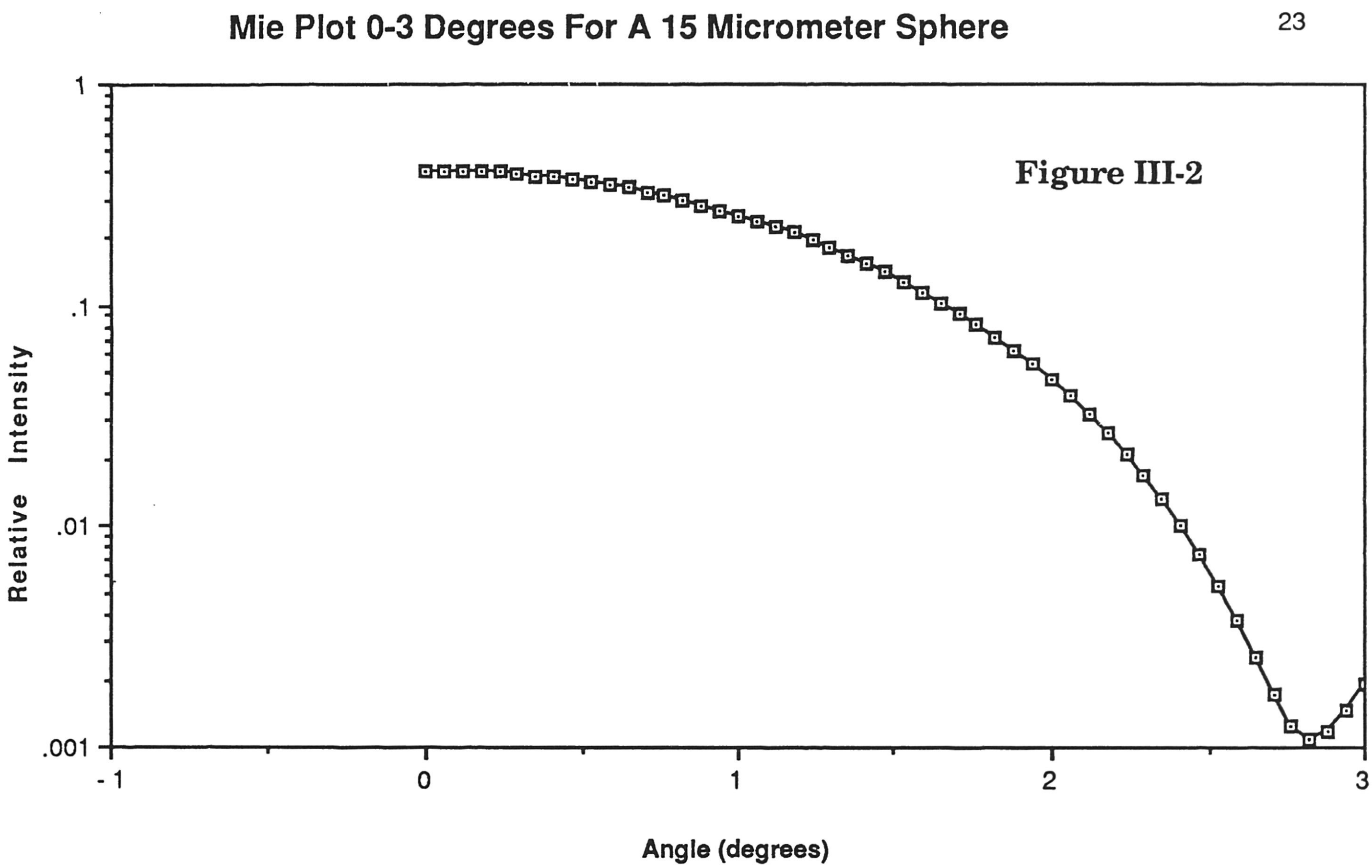
The PMT and preamp were mounted on an optical rail 45 cm long. This scanning arm was free to rotate about a vertical axis through the levitated sphere. Thus, the PMT could be moved from – 5 to + 90 degrees around the sphere. In order to achieve an angular resolution of 1 milliradian, the PMT was equipped with a .4 mm aperture positioned 30 cm from the axle. Alignment of the axis was facilitated by a 4 cm long 3 mm diameter tube. The axis position was adjusted by looking through the pinhole and tube while rotating the arm. When the particle remains centered in the field of view over a 90 degree scan, the axis is aligned to the sphere. The PMT assembly was scanned through the desired angular range by a geared down motor attached to a 5 cm diameter wheel at the end of the scanning arm. By running the motor for a brief period of time, the arm could be precisely moved through angular distances less than a milliradian. The angular position was determined by an optical encoder attached to the scanning arm axis. The particular model we used had a resolution of .0459 degrees. In summary, this assembly provides an easy and accurate means of scanning the light scattering pattern of the sphere.

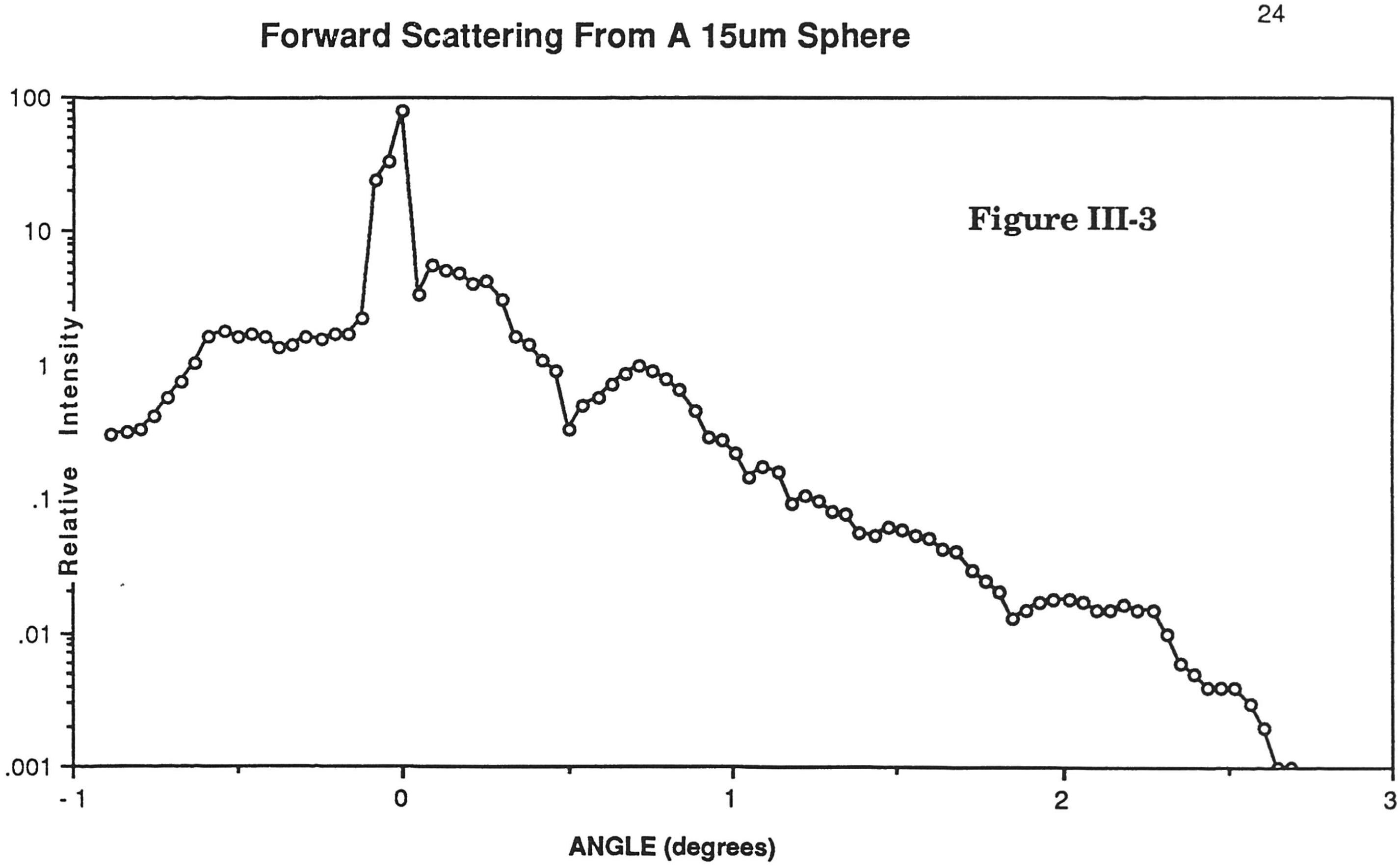
III- PROGRESS REPORT

As of April 1, 1989, we have proved that the basic principle of our experiment is sound. By measuring the side scattering (figure III-1) we were able to determine the size of a sphere and calculate the Mie theory prediction for the forward scattering down to zero degrees. (figure III-2) The forward scattering was measured but without the degree of cancellation necessary to separate the scattered light from the interference term caused by the interaction of the scattering and the unscattered beam. Because of the interference term, the forward scattering (figure III-3) showed an extremely sharp peak compared to the Mie plot as we expected to see with the interference term included in the measurement.

After this, we plan to implement feedback controls to more precisely darken the unscattered beams and to use better beam and polarization adjustment procedures. These will hopefully allow us to measure only the scattered light. We plan to study several sphere sizes and to compare our data with the Mie theory calculations. If there is good agreement, we will try to measure the zero degrees scattering for a cubic scatterer which is much more difficult to calculate than the scattering for a sphere.


Side Scattering From A 15 Micrometer Sphere
Figure III-1
.1
.01
.001
Relative Intensity
50
51
52
53
54
55
56
57
58
59
60
Angle (degrees)


Mie Plot 0-3 Degrees For A 15 Micrometer Sphere
Figure III-2
1
.1
.01
.001
Relative Intensity
- 1
0
1
2
3
Angle (degrees)



Forward Scattering From A 15um Sphere
Figure III-3
100
10
1
.1
.01
.001
Relative Intensity
-1
0
1
2
3
ANGLE (degrees)

IV. Bibliography

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3. William White, Doctoral Thesis, Texas A & M University

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Appendix A:
Calculation of Interference Effects

All electric fields are S - polarized.

A - the amplitude of the cancellation beam modulation

A_i - amplitude of illumination electric field

A'_i - amplitude of cancellation electric field

A_s - amplitude of scattered electric field

C - the average position of the sphere in the beam

d - the phase shift of the scattered field

d' - the phase shift introduced by the electro-optic modulator

$\mathbf{E_i}$ - electric field of illumination beam

$\mathbf{E'_i}$ - electric field of cancellation beam

$\mathbf{E_s}$ - electric field of scattered light

$\hat{\mathbf{z}}$ - unit vector

m - the change in intensity per radial distance in the illumination beam

r - the radial position of the sphere from the axis of the beam

R - the amplitude of the sphere's motion

S - scattering coefficient

w - the frequency of the sphere's motion

w' - the frequency of the cancellation beam modulation

The intensity that the photodetector sees is given by

$$I_{out} = \left\langle (\varepsilon / \mu)^{1/2} \; [\mathbf{E_i} + \mathbf{E_i}' + \mathbf{E_s}] \bullet [\mathbf{E_i} + \mathbf{E_i}' + \mathbf{E_s}]^* \right\rangle$$

$$= \left\langle (\varepsilon / 4\mu)^{1/2} \left\{ \begin{array}{ccccc} \mathbf{E_i \cdot E_i^*} & + & \mathbf{E'_i \cdot E_i^*} & + & \mathbf{E_s \cdot E_i^*} \\ \mathbf{E_i \cdot E'_i{}^*} & + & \mathbf{E'_i \cdot E_i'^*} & + & \mathbf{E_s \cdot E'_i{}^*} \\ \mathbf{E_i \cdot E_s^*} & + & \mathbf{E'_i \cdot E_s^*} & + & \mathbf{E_s \cdot E_s^*} \end{array} \right\} \right\rangle$$

The illumination beam actually has a gaussian intensity profile, but since we are considering the intensity at a fixed detector position, we can approximate the beam at that point as a plane wave.

$$\mathbf{E_i} = A_i \, e^{i(kx - wt)} \, \hat{\mathbf{z}}$$

Similarly, the cancellation beam can be considered to be a plane wave which is phase shifted by the modulator with respect to the illumination beam.

$$\mathbf{E_i}' = A'_i \, e^{i(kx - wt + d')} \, \hat{\mathbf{z}}$$

We assume the sphere is positioned off center in a region where the intensity is most nearly linear with position. The scattered intensity at a particular angle will then be proportional to the sphere's distance from the center of the beam, r, times the spatial slope of the illumination intensity, m, times a scattering coefficient, S.

$$I_s = mrS$$

The sphere moves up and down sinusoidally about some fixed distance from the beam center, so

$$r = C + R\cos(wt).$$

Note that as the particle moves, the intensity it sees changes. We are assuming the gradient in the intensity is sufficiently small that the change in intensity across the sphere is negligible and that the sphere therefore effectively sees an infinite plane wave at any instant.

Taking the scattered electric field to be the square root of the intensity I_s, we get

$$\mathbf{E_s} = [mS(C + R\cos(wt)]^{1/2}\, e^{i(kx - wt + d)}\, \hat{\mathbf{z}}$$

Plugging the electric fields into I_{out}, we get

$$I_{out} = A_i^2 + A_i'^2 + 2A_iA_i'\cos(d') + mS(C + R\cos(wt)) + 2[mS(C + R\cos(wt))]^{1/2}\, [A_i\cos(d) + A_i'\cos(d' - d)]$$

In order to get destructive interference between $\mathbf{E_i}$ and $\mathbf{E'_i}$, d' will be set to π.

$$I_{out} = A_i^2 + A_i'^2 - 2A_iA_i' + Sm(C + R\cos(wt)) + 2[Sm(C + R\cos(wt)]^{1/2}\, [A_i\cos(d) - A_i'\cos(d)]$$

Now add in the modulation of the cancellation beam. Let A_i' become $A_i' + A\cos(w't)$, where A is the modulation amplitude and w' is the modulation frequency.

$$I_{out} = A_i^2 + A_i'^2 + 2A_i'A\cos(w't) + A^2 \cos^2 (w't) - 2A_iA_i' - 2A_iA\cos(w't) + mS(C + R\cos(wt)) + 2[mS(C + R\cos(wt))]^{1/2} [A_i - A_i' - A\cos(w't)] \cos(d)$$

If the beams are balanced, A_i will be almost equal to A_i' . This will leave a residual amount of light from the interference of the illumination and cancellation beams, X1. In the interference term, $A_i - A_i'$ will be some non-zero amplitude, X.

$$I_{out} = X1 + A^2 \cos^2 (w't) - AX\cos(w't) + mS(C + R\cos(wt)) + 2[mS(C + R\cos(wt))]^{1/2} [X - A\cos(w't)] \cos(d)$$

$$I_{out} = X1 + A^2 (1 + \cos(2w't)) / 2 - AX\cos(w't) + mS(C + R\cos(wt)) + 2[mS(C + R\cos(wt))]^{1/2} [X - A\cos(w't)] \cos(d)$$

Considering only the time varying terms, we get

$$i_{out} = A^2 \cos(2w't) / 2 + SmR\cos(wt) - AX\cos(w't) + 2[Sm(C + R\cos(wt))]^{1/2} [X - A\cos(w't)] \cos(d)$$

$$i_{out} = A^2 \cos(2w't) / 2 + SmR\cos(wt) - AX\cos(w't) + 2[SmC(1 + R\cos(wt) / C)]^{1/2} [X - A\cos(w't)] \cos(d).$$

Expanding the square root, we get

$$(1 + R\cos(wt) / C)]^{1/2} \cong 1 - (R/4C)^2 + (R/2C) [1 + (3/32) (R/C)^2] \cos(wt) - (R/4C)^2 \cos(2wt) + (R/4C)^3 \cos(3wt)$$

Unfortunately, the cos(wt) term in this expansion multiplies X and thus will give us an interference term which also has a frequency w. This means that we still have to obtain an X that is 100 to 1000 times smaller than the current value. But we can now use this result to better balance the beams. Specifically, the sphere is moved out of the beam, making S = 0. Then

$$i_{out} = A^2 \cos(2w't) / 2 - AX\cos(w't),$$

and we can use lock-in detection on the AXcos(w't) term and adjust it accurately to zero.

The modulator will be better aligned by putting an a.c. voltage on it and watching the polarization modulated signal on an oscilloscope. The modulator will be rotated until the a.c. change in polarization goes to zero. This continuous adjustment will be much better than the current approximate procedure described earlier. A polarizer will be placed in front of the modulator to eliminate any residual polarization changes. Lastly, the phase must be electronically controlled with a feedback loop, because even small phase changes must be eliminated now.