NUMERICAL MODELING OF HAILSTONE GROWTH AND TRAJECTORY

by

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ABSTRACT

A numerical model of a convective storm is used to generate conditions in which the study of the growth and trajectories of hailstones is made. The model consists of a one-dimensional, time-dependent system in which adiabatic conditions are assumed to exist. It was designed for conditions existing in the Texas panhandle-Oklahoma region with a maximum vertical velocity of 30 m sec^{-1}.

The hailstone growth equations used in the computations account for wet gtowth, dry growth, and melting of the hailstone. Four sizes of hail embryos consisting of frozen droplets are considered starting at six different heights resulting in twentyfour hailstones. The embryos are produced by the cloud.

Results indicate that hailstones would reach the ground with a diameter of 1.00 cm within fifteen minutes of precipitation formation in the cloud and have an internal ring structure characteristic of naturally occuring hailstones.

ACKNOWLEDGMENTS

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1. Introduction

The processes involved in hail formation have long been a subject of study. In the 17th century, René Descartes, noting that hailstones often have snow in the middle, speculated that large hailstones are formed because the winds drove numerous snowflakes together, which then partly thawed and again froze. Later in history, Alessandro Volta proposed that hailstones bounce up and down above the top of the clouds by electric repulsion until they reach such a size that the electrical forces can no longer suspend them and they fall to the ground. Today, progressing much beyond these early speculations, we attempt to describe with basic physical laws what appears to be occuring in hailstorms. In order to achieve quantitative results, we use numerical modeling which simulates the hail growth process.

The purpose of this study was to describe the growth and trajectories of hailstones in conditions generated by a numerical model of a convective storm. The numerical model initiated by Dr. P. Das, was developed by Smith (1973) at Texas A&M University and consists of a one-dimensional, time-dependent system in which adiabatic conditions are assumed to exist. The model, originally considering only size classes of hailstones as a group, was expanded to give the life history of certain individual hailstones.

2. Review of the Literature

The earliest model of hailstone growth, offered by many workers, was the multiple incursion model which had its basis in the internal ring pattern of most hailstones. Hailstones are observed to consist of alternate rings of clear and opaque ice with three or four layers being the most common number. It was believed that during its

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growth period, the hailstone oscillated around the 0°C level, the opaque ice being deposited while it was above the freezing level and the clear ice while it was below.

Schumann (1938) provided a careful, analytical decription of the accretion of liquid water by a hailstone as it fell through a cloud. He described the growth process as being "dry" or "wet" in which the nature of the ice deposited, whether opaque or clear, was determined by a heat balance at the surface of the hailstone. Ludlam (1958), following these ideas, found that one simple up-and-down trajectory was sufficient to produce a hailstone with an internal ring pattern. To allow for the growth of large hailstones, Browning and Ludlam (1962) proposed a model in which wind shear and the resulting tilted updraft allowed for the recycling of hailstones more than once in an updraft twisting upward around the downdraft.

Another attempt to account for large hailstone growth in a short amount of time was that of Hitschfield and Douglas (1963) who postulated the accumulation of rain water leading to high water contents in the cloud. Sulakvelidze et al. (1967) also included the accumulation zone in their model, trapped in the upper part of the cloud above the level of maximum vertical velocities. Gokhale (1975) recently has shown that multiple fluctuations on the updraft and the resulting changes in the ambient temperature of the hailstone, are capable of explaining the multi- layered structure of hailstones and large variations in the liquid-water content are not required.

3. The Cloud Model

General Conditions. Cloud formation is simulated numerically in an environment which is constant with respect to time and whose parameters, for example, temperature, water-vapor content, and pressure, vary only with altitude. The cloud on the other hand, varies with both altitude and time. This dependence on time produces a more realistic situation in which the hailstones grow. A time step of ten seconds was used between each series of computations.

The model was designed for conditions existing in the Texas panhandle-Oklahoma region on days of extensive hailstorms. The cloud base was assumed to be at 1500 meters and 17°C. The maximum vertical velocity of 30 meters per second was modelled as located initially at a height of 9500 meters with the cloud top set at 13,500 meters. Initially there is no precipitation in the cloud. Precipitation formation is initiated at time zero and grows according to processes recognized by cloud physicists.

Vertical Velocity. A time-height cross section of vertical velocity is shown in Fig. 1 in which vertical velocity is shown in meters per second with positive values indicating updraft and negative values downdraft. In the first five minutes, the vertical velocity profile was relatively constant, however, the next five minutes showed a general weakening of the updraft through all levels with the depth of the downdraft region increasing. At a time of fifteen minutes, the downdraft is most pronounced reaching a maximum depth of 6,000 meters and a maximum strength of 15.9 m sec^{-1} at the surface. By the twentieth minute, the downdraft had decreased only to reoccur somewhat weaker around the twentyfifth minute producing a pulsating character in the vertical velocity profile.

Liquid Water Content. Fig. 2 shows a time-height cross section of liquid precipitation content in units of grams of water per kilogram of air (gm kg^{-1}), but this can be used to approximate the liquid water content of the cloud since once the precipitation is formed, it dominates over the other forms of cloud water. A buildup of water is evident by the fifth minute centered around 1200 meters. This increase continues and spreads to lower levels reaching a maximum value of over 20 gm kg^{-1}. At eight minutes, precipitation began reaching the surface with the hardest shower occuring around the eighteenth minute. This corresponds well with the occurrence of the maximum downdraft. Afterwards, there appears to be a lull in the shower with the liquid water content decreasing at all levels.

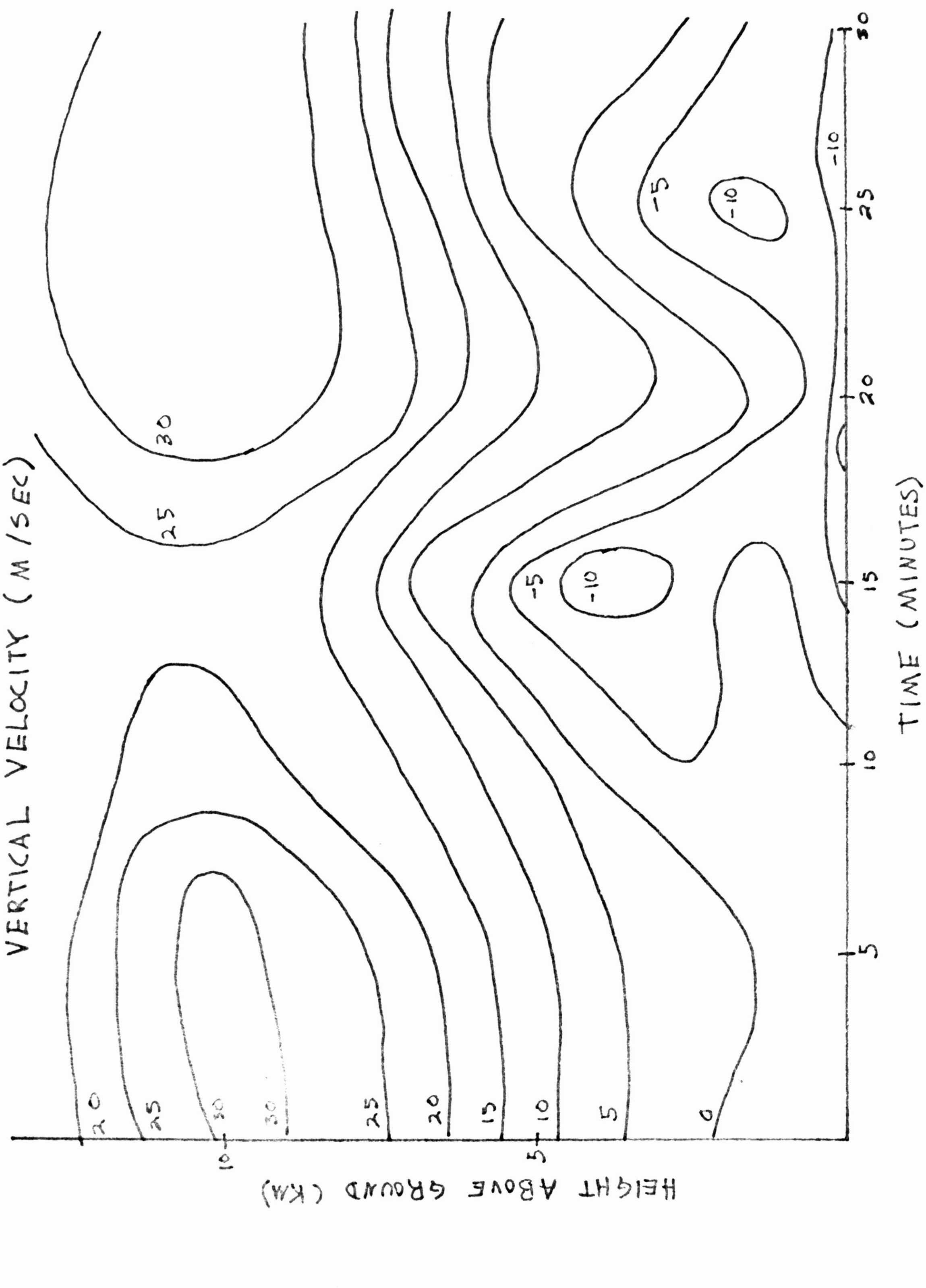

Fig. 1 Time-height cross section of vertical velocities in the cloud.

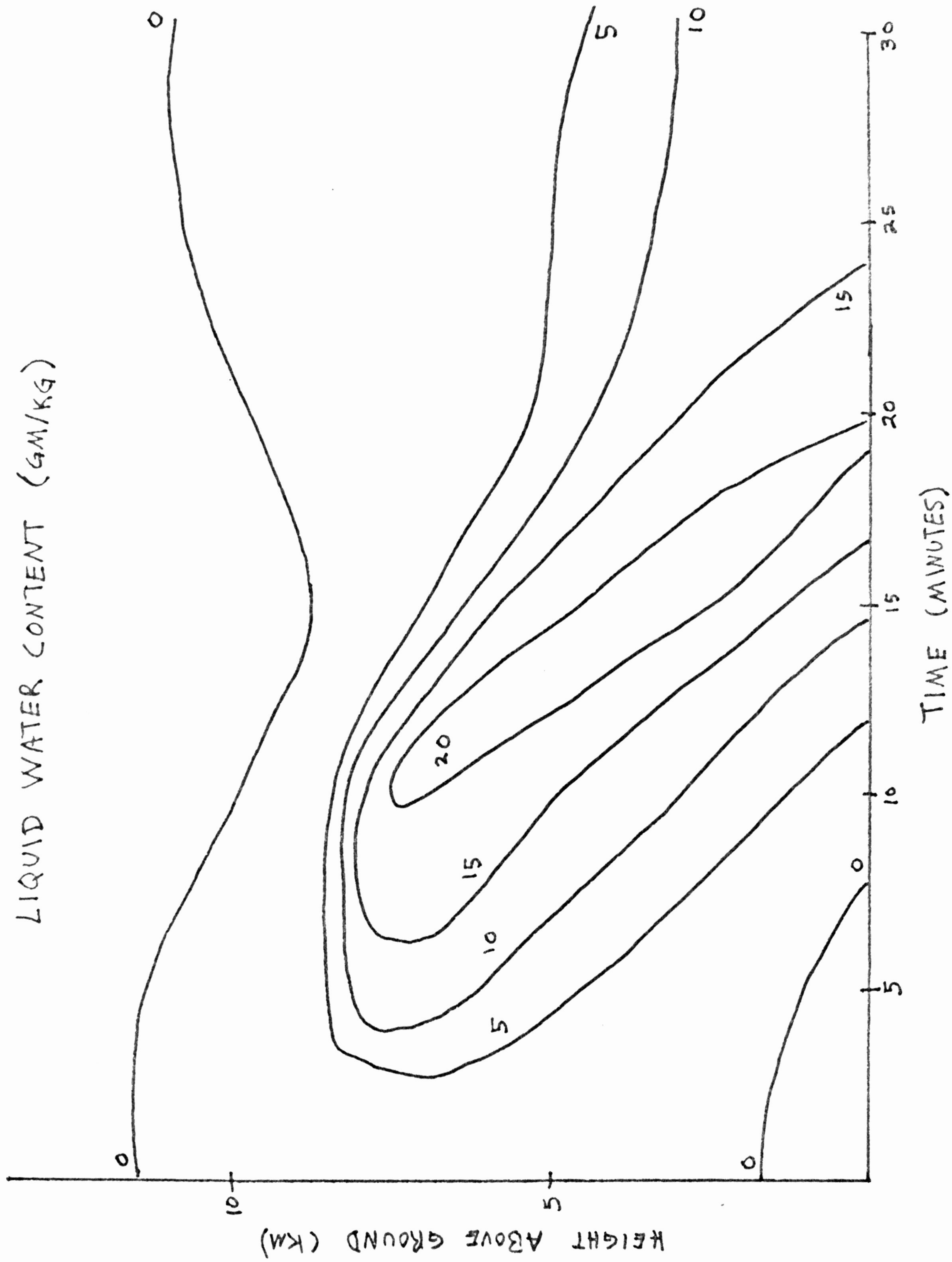

Fig. 2 Time-height cross section of liquid precipitation content of the cloud.

4. The Physics of Hail Growth

Basic Concepts. Hailstones grow primarily in the updrafts of convective storms. An important requirement for the velocity of the updraft, and by implication, the severity of the storm, is the updraft velocity must be great enough to hold the hailstones aloft long enough for them to grow to the observed sizes.

A hailstone embryo, the beginning of a hailstone, may be either a frozen drop or a graupel partical, a snowflake cluster which has obtained a coating of ice by collisions with supercooled droplets. Only embryos resulting from the freezing of drops were considered in this study.

A hailstone is heavier than the water drops and droplets surrounding it and so falls faster than the latter do relative to the updraft, and in the process, collects or sweeps up some of the drops and droplets that are in its path. Most of the drops and droplets are in the supercooled state at temperatures below freezing.

The growth of hailstones by the accretion of droplets is controlled by a heat balance between the hailstone and its environment. If the hailstone is able to freeze quickly all the water it collects, it is in a phase known as dry growth. Ice deposited in this phase is often opaque since many air bubbles are trapped in the process and is called rime ice. The temperature of the hailstone surface stays at a temperature between 0°C and that of the environment. However, if the heat exchange is not sufficient to freeze all the deposited water immediately, it has time to spread over the surface of the stone before freezing. The hailstone is said to be undergoing wet growth and any deposited ice is transparent since it freezes relatively slowly and fewer air bubbles are trapped. This glaze ice surface remains at a temperature of 0°C. Another possibility is that the part of the water that freezes forms a framework which aids in the retention of the rest of the liquid water. The resulting ice-water mixture is called spongy ice.

Hailstone Growth Equations. The equations of growth of a hailstone are obtained by considering the heat transfer to and from the hailstone. Such transfer consists of conduction, evaporation-sublimation, and accretion. In the following derivation, heat transfers from the hailstone are treated as positive.

The conduction term accounts for the transfer of sensible heat to and from the hailstone depending on whether it is colder or warmer than its environment. Following a development by Byers (1965), the sensible heat conducted from the stone is given by,

$$dQ_s/dt = -4\pi RaKT, \tag{1}$$

where R is the radius of the hailstone, a the ventilation factor, K the conductivity of the air, and T is the temperature in °C.

The latent heat transfer due to evaporation is given by,

$$dQ_l/dt = 4\pi RaLD\Delta\rho_w, \tag{2}$$

The combined heat transfer is,

$$dQ_v/dt = 4\pi Ra(-KT+LD\Delta\rho_w). \tag{3}$$

The transfer of heat by accretion involves two processes. The first is the transfer of latent heat of fusion to the hailstone as collected supercooled droplets freeze to the stone. Second, the sensible heat of the collected droplets is also transferred to the hailstone. The transfer of latent heat of fusion is given by,

$$dQ_f/dt = \pi R^2 U_T \chi L_f \;, \tag{4}$$

where U_T is the hailstone terminal velocity, χ the liquid water content of the air, and L_f the latent heat of fusion.

The transfer of sensible heat due to accretion is given by,

$$dQ_s/dt = \pi R^2 U_T \chi T. \tag{5}$$

The combined effect of accretion would be,

$$dQ_c/dt = \pi R^2 U_T \chi (L_f+T). \tag{6}$$

Dry growth would result when $dQ_v/dt > dQ_c/dt$ and the growth rate would be given by,

$$dR/dt = U_T \chi/(4\delta_r) \ , \qquad (7)$$

where δ_r is the density of the coating of rime.

Wet growth would occur when $dQ_c/dt > dQ_v/dt$, and would be expressed by the heat balance,

$$dQ_f/dt + dQ_s/dt - dQ_v/dt = 0.$$

Substitution and manipulation of this equation results in,

$$dR/dt = -U_T \chi T/(4\delta_e L_f) + a(-KT+LD\Delta\rho_w)/(R\delta_e L_f) \ , \qquad (8)$$

where δ_e is the density of the ice.

Equation (8) also was used to calculate the melting rate of hailstones.

Three different schemes were used to determine terminal velocities of the hailstones depending on their size, and therefore, the range of their Reynolds number. A development of these schemes can be found in the Appendix.

Assumptions Made. For lack of information on the exact physical conditions of a growing hailstone, the following assumptions were made:

(1) Water Shells. It was assumed that whenever a hailstone was undergoing wet growth, excess water that was collected and could not be frozen, was shed in the wake of the hailstone. Chung and Chen (1974) found that water may exist as a shell on the surface of hailstones with a radius of less than 0.45 cm but that thick water shells on larger stones were unstable. In fact, the larger the hailstone, the smaller the water shell that could exist in equilibrium.

(2) Liquid Water Content. In a similar fashion to the first assumption, we assumed that the hailstone could not undergo spongy growth, or stated differently, that there was no internal liquid water in the hailstone. Basis for this assumption comes from a study of nine hailstorms by Gitlin, et al. (1968) in which 90% of the hailstones sampled had less than 4% liquid water content.

(3) Collection Efficiencies. The collection efficiency of the hailstones was assumed to be unity for drops and droplets, and zero

for ice crystals. According to Wisner et al. (1972), the fact that no cloud crystals were collected would cause the wet growth rate to be underestimated by a small amount since their collection would have reduced the rate at which heat must be lost to the environment to balance the heat gained due to accretion. However, the assumption is likely to have little effect on the dry growth rate since the collection of ice crystals by dry, cold hailstones is very low anyway.

(4) Density of the Hailstones. For wet growth, an ice denstiy of 0.9 gm cm^{-3} was used, a value agreed upon by most authors. The density resulting from dry growth has a much wider range of reported values but a value of 0.7 gm cm^{-3} was used in accordance with earlier work done by Das (1962).

5. Results

Sample Trajectories. Four sizes of embryos were started at six different heights resulting in twentyfour hailstone life histories. Instead of presenting the results on all of them, we present a small sample by way of illustrating the main conclusions.

Fig. 3 shows the trajectories of three hailstones which started at a height of 8,000 meters but differed in the size of the embryos on which they formed. In the figure, the solid line indicates dry growth, the dashed line shows wet growth, the dotted line shows when the hailstone was melting, and the dot-dashed line refers to when the hailstone remained the same size. Due to the preliminary nature of this study, the model was run for a simulated cloud life of thirty minutes only.

Curve 1 is the trajectory of a hailstone resulting from an embryo of 0.18 cm diameter and mass 0.003 gm - the smallest embryo considered. By comparing it to Fig. 1, we see that it quickly rose above the updraft maximum into a part of the cloud with low water content mostly composed of ice crystals where it grew very slowly. In fact, part of the time it had zero growth as illustrated by the dot-dashed line. During this time, it was in a region of the cloud

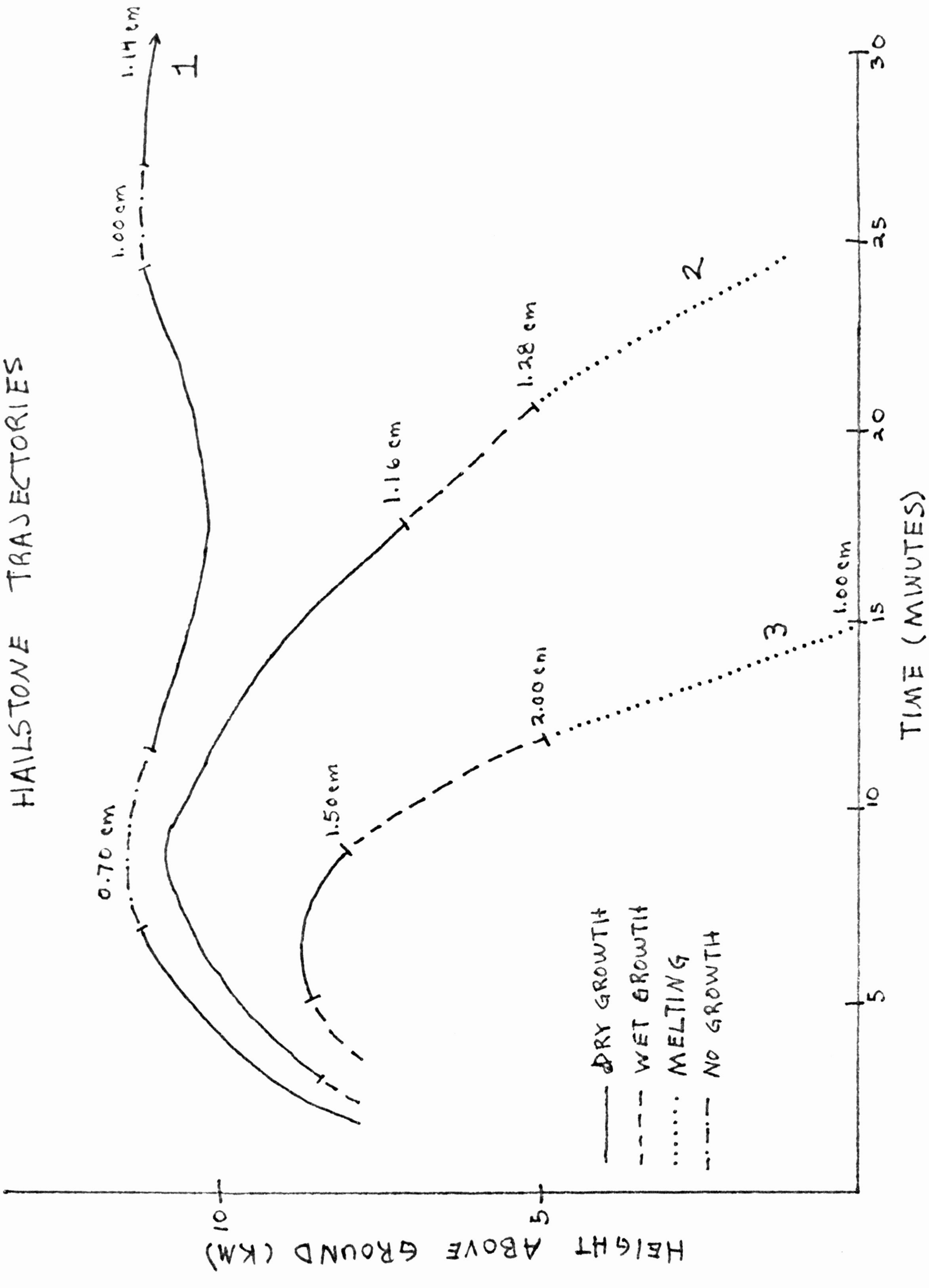

Fig. 3 Trajectories of hailstones starting in the initial updraft.

with a temperature below -40°C and all the cloud substance was in the form of ice crystals. Its very slow growth enabled it to remain above the updraft maximum for the entire thirty minutes at the end of which, it had a diameter of 1.14 cm and 0.56-gm mass.

Curve 2 is the trajectory of a hailstone starting from an embryo of diameter 0.26 cm and mass 0.0088 gm . The hailstone was initially blown upwards in the strong updraft, but as it grew the updraft also weakened causing it to fall downward. However, the stone melted before reaching the ground. A maximum diameter of 1.26-cm and 0.82-gm mass was reached before melting began.

The trajectory shown by curve 3 resulted from an embryo of 0.52-cm diameter, the largest considered, with a mass of 0.07 gm . It also was initially blown upwards but eventually turned back downward. The maximum diameter it achieved was 2.00 cm and it reached the ground 1.00 cm in diameter with a mass of 0.16 gm .

Comparing curves 1, 2, and 3 reveals a trend of increasing initial wet growth in the hailstone life history. This probably has two causes: first, the size of the embryos considered were larger for each succeeding hailstone. The larger embryos were able to collect more liquid water and thus had an increase in its tendency toward wet growth. Second, by referring to Fig. 2 , we see that there was an increase in the liquid-water content at the height where the embryos initially were. This would have the effect of increasing the collection of water, and thereby making the hailstone grow in the wet mode.

Fig. 4 shows the trajectories of three hailstones which began during the downdraft. The same line scheme designated for Fig. 3 was used. All the hailstones started at a height of 7,500 meters and ranged in size from 0.26 cm diameter and 0.0088 gm mass, to 0.52 cm diameter and 0.07 gm mass. None of the hailstones experienced an upward trajectory and all immediately began to fall. Therefore, their growth period was very short and they did not survive the melting which took place. A diameter of 0.70 cm and a mass of 0.17 gm was the maximum any of the hailstones attained.

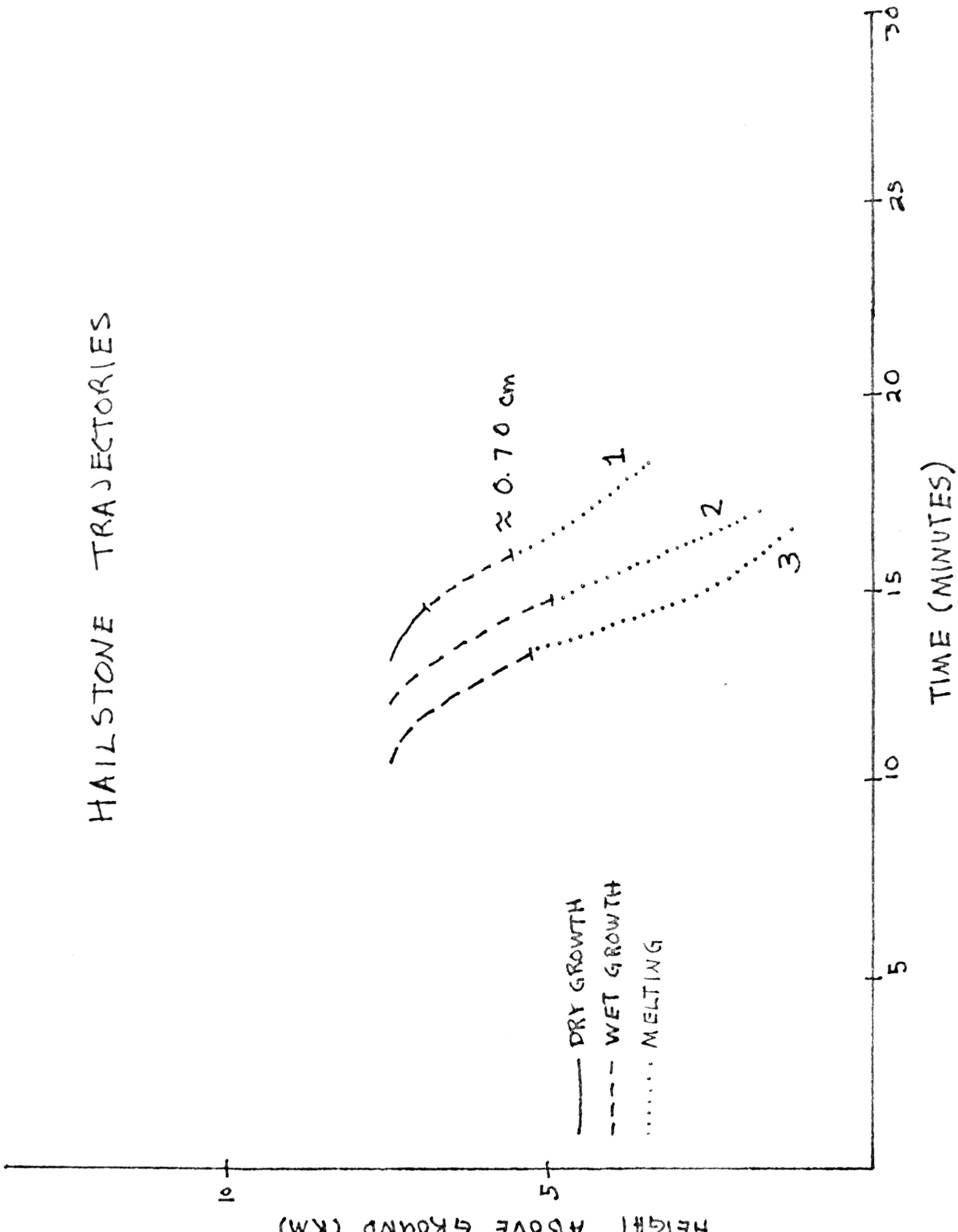

Fig. 4 Trajectories of hailstones starting in the downdraft.

Fig. 5 shows the trajectories of three hailstones which began approximately ten minutes later in the simulated life of the cloud. They all began during the final stages of the downdraft in the cloud and the resurge of the updraft. Curve 1 is the trajectory of the small 0.18 cm, 0.003 gm embryo. Initially it fell for approximately three minutes, but as the downdraft began to weaken and an updraft began to take its place, the hailstone was small enough to be blown upwards. It continued to grow in its path upward until reaching the upper cloud, composed almost entirely of ice crystals, where it remained the same size. At the end of the thirty minute period it had a diameter of only 0.52 cm and a mass of 0.053 gm.

Curves 2 and 3 are embryos of the same size as that in curve 1 but which started lower in the cloud. They also initially fell but were carried upward in the resurge of the updraft. However, they remained in a part of the cloud where growth was possible, and at the end of the thirty minuts, they had begun to fall again with diameters of 1.1 cm and 0.82 cm and masses of 0.58 gm and 0.41 gm respectively.

Development of Internal Structure. The hailstones produced by the model had internal structures consisting of alternate rings of clear and opaque ice, characteristic of those found in nature. These rings correspond to the periods in the life of the hailstone when it was experiencing wet or dry growth.

Fig. 6 shows the development of the internal structure of two hailstones. The hailstone reaching the ground corresponds to the hailstone whose trajectory is shown as curve 3 in Fig. 3. Both stones had a maximum of three rings before melting began. The one reaching the ground had retained its two inner rings.

Fig. 7 shows the internal structure of the hailstone whose trajectory is curve 2 in Fig. 5. At the end of the thirty minutes, it consisted of four rings and was still growing. It is unfortunate that our limited objective did not permit following this hailstone for a longer period.

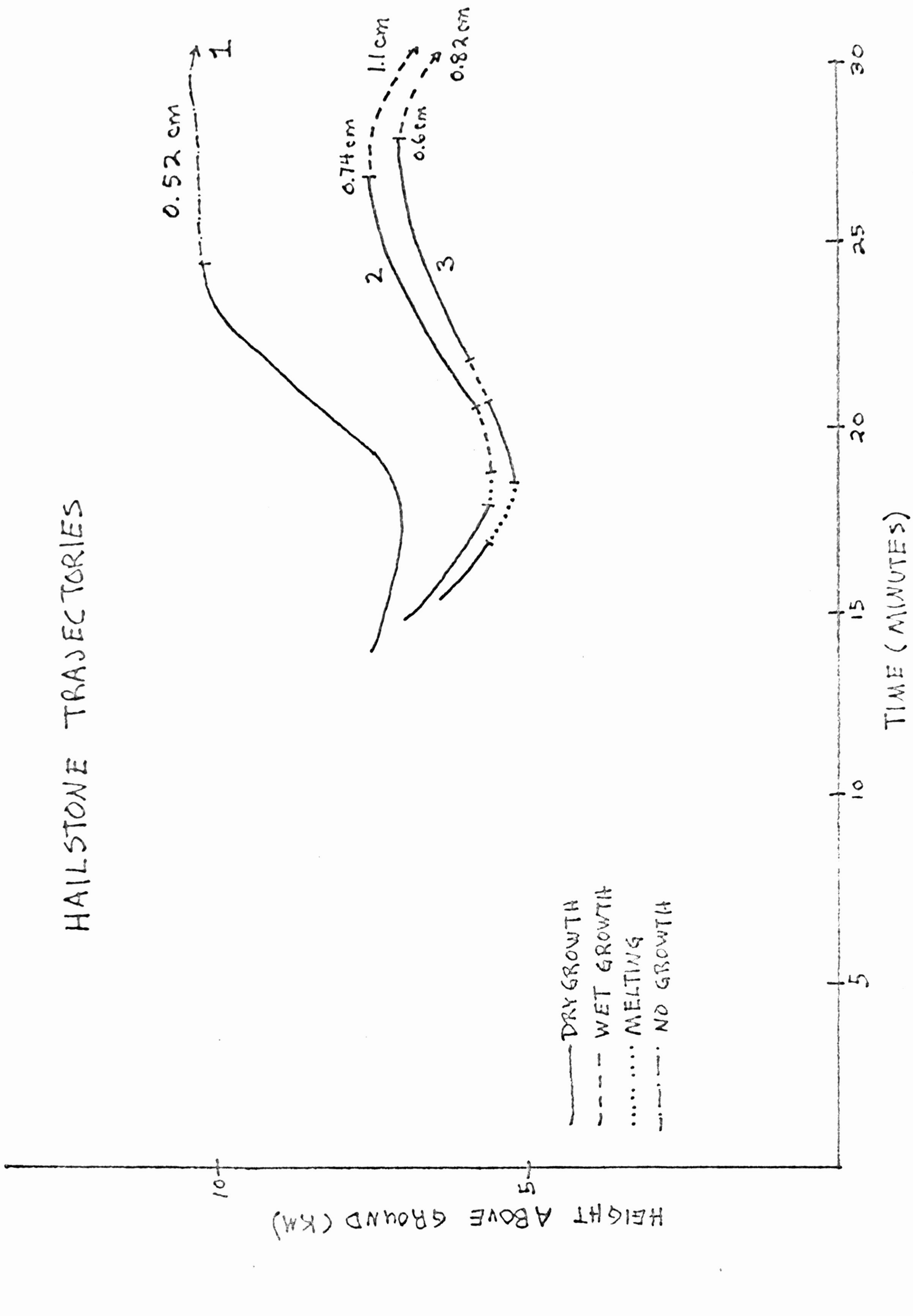

Fig. 5 Trajectories of hailstones caught in the resurge of the updraft.

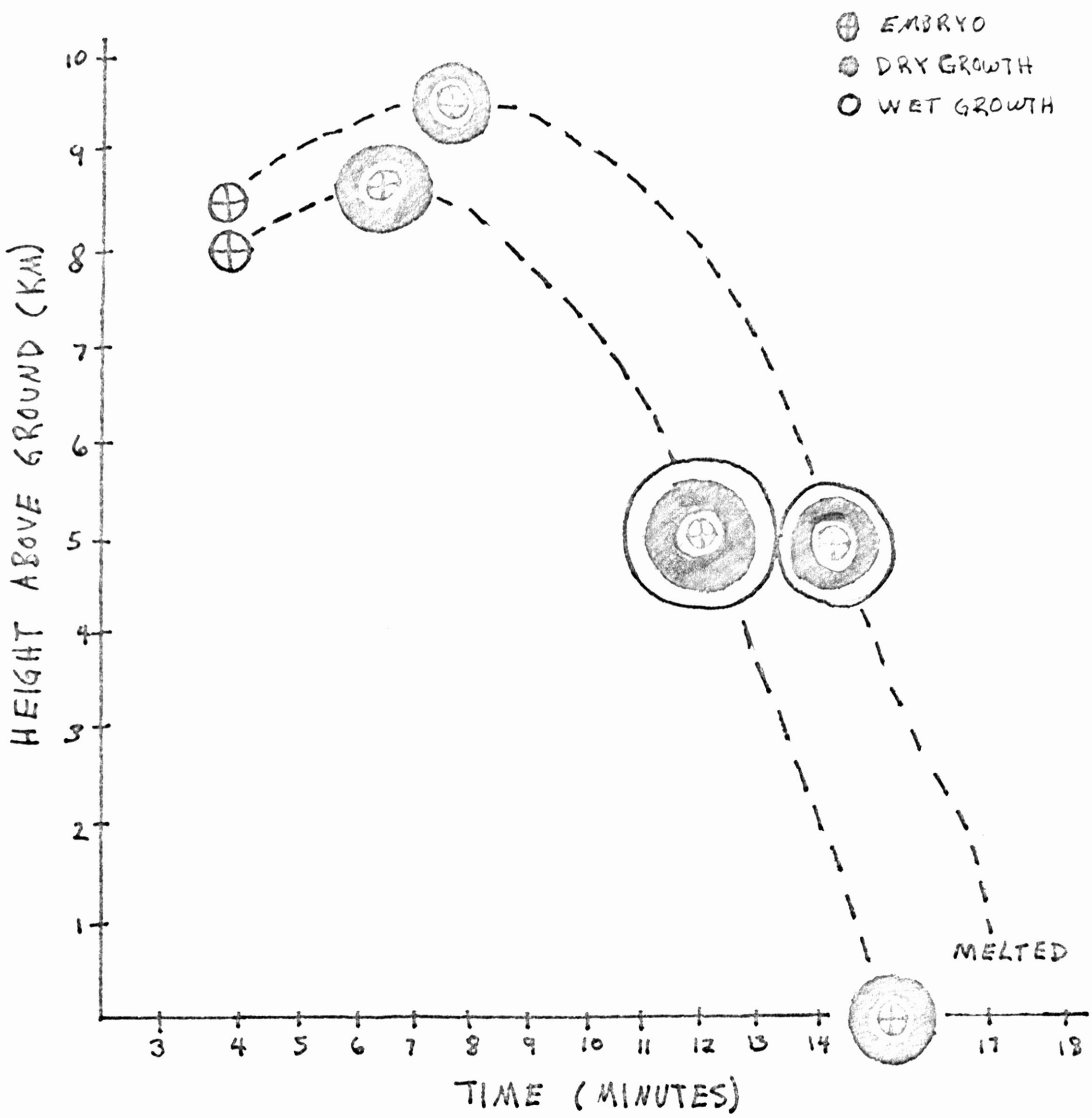

Fig. 6 Development of the internal ring structure of hailstones following a simple up-and-down trajectory.

6. Conclusions

The results of the study indicate that hailstones would readily be produced by the conditions simulated in the numerical model. The resulting hailstones grew, in most cases, during their traverse along a simple up-and-down trajectory and had an internal ring structure characteristic of naturally occurring hailstones. However,

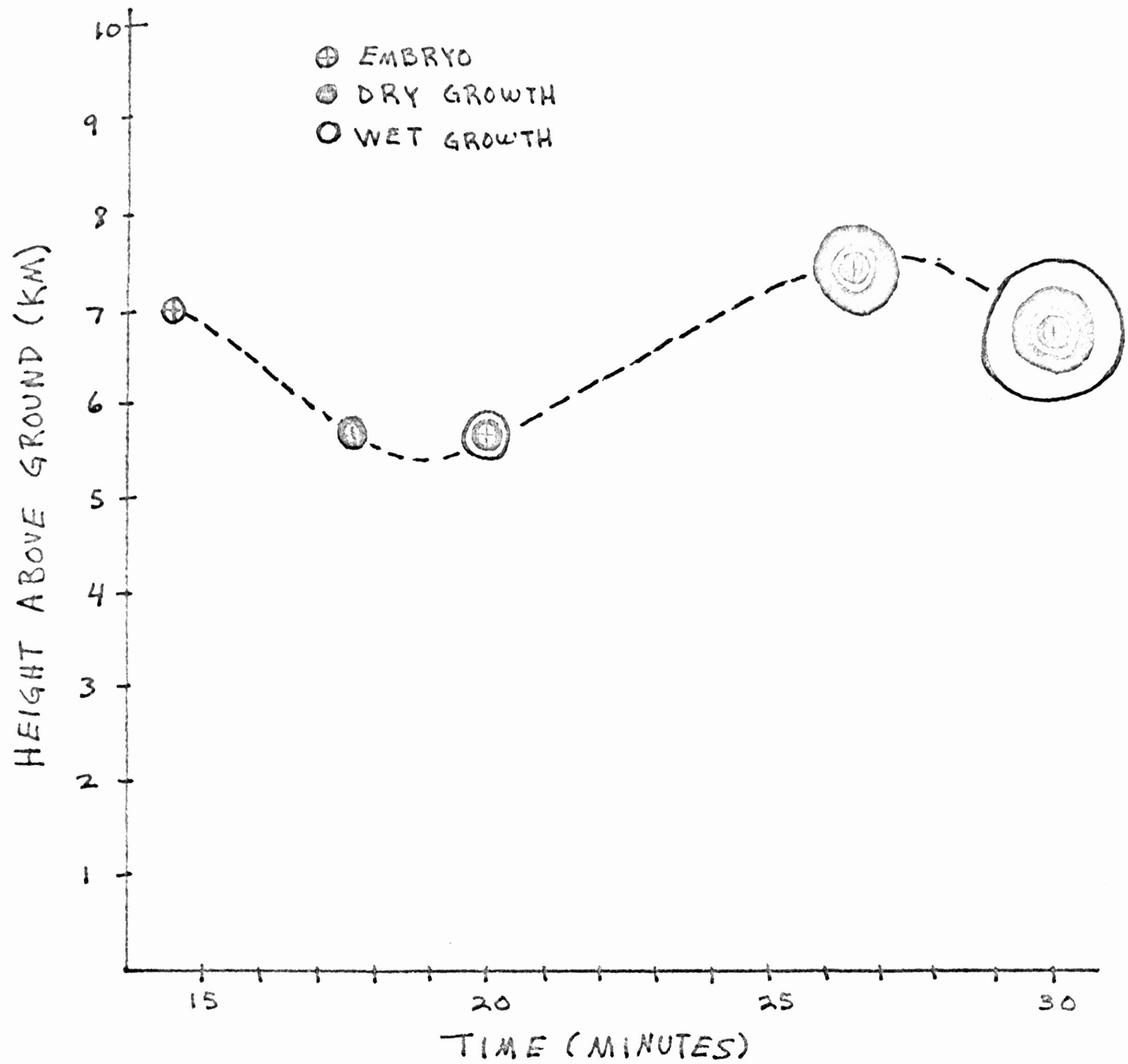

Fig. 7 Development of the internal ring structure of hailstone caught in the resurge of the updraft.

hailstones trapped above the level of maximum vertical velocity grew very slowly or even not at all. In three-dimensional situations, represented by a real cloud, horizontal motion would eject these hailstones out of the regions near the top of the cloud. Most hailstones produced, melted before reaching the ground, especially those which began to grow during the existence of the downdraft.

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APPENDIX

DETERMINATION OF HAILSTONE TERMINAL VELOCITY

Three different schemes were used to determine terminal velocities of the hailstones depending on their size, and therefore, the range of their Reynolds number. Following Berry and Pranger (1974), the methods involve the computation of $C_D Re^2$ where C_D is the drag coefficient of the hailstone, and Re is the Reynolds number. This quantity is independent of fall velocity and can be determined by the equation,

$$C_D Re^2 = (8/\pi)(mg\rho/\eta^2),$$

where m is the mass of the hailstone, g the acceleration of gravity, ρ is the air density, and η the dynamic viscosity. The first two methods of determining terminal velocity, for hailstones of radius less than 1.5 cm but differing in their range of $C_D Re^2$, involved a least-squares curve fit to empirical data relating $C_D Re^2$ as a function of Re. It was necessary to make a logarithmic curve fit for the larger values of $C_D Re^2$ within this range. The coefficients for the resulting polynomials are given by Davies (1945).

For hailstones of radius greater than 1.5 cm, the drag coefficient was assumed to have a constant value of 0.55, a value recommended for spherical hailstones by Macklin and Ludlam (1961).

Once the Reynolds number is determined, the fall velocity is given by,

$$V = Re\eta/(2r\rho),$$

where r is the radius of a spherical hailstone.