Application of Specialized Optimization Techniques in Water Quantity and Quality Management with Respect to Planning for the Trinity River Basin

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APPLICATION OF SPECIALIZED OPTIMIZATION TECHNIQUES
FOR WATER QUALITY AND QUANTITY MANAGEMENT WITH RESPECT
TO PLANNING FOR THE TRINITY RIVER BASIN

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PREFAE

This report describes results of research performed under Project B-024-TEX sponsored by the U. S. Department of the Interior, Office of Water Resources Research and the Texas A&M University Water Resources Institute. Research results have been published in the Proceedings Of The American Water Resources Association and the Journal of Mathematical Analysis And Applications. Additional results of research under this project will be published in the future.

The research reported herein represents the first time that a water quality - quantity management model has been developed which incorporates both the consideration of increased water treatment costs due to waste return flows and the trade-off between treatment costs and costs of flow augmentation. Although much remains to be done, this research represents a first step in this important area of water resources management.

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CHAPTER I

INTRODUCTION

Philosophy of Water Quality Management

One of the major problems in the United States today involves controlling and limiting the pollution of the nation's water resources. The quality of water in streams, reservoirs and estuaries is continually being altered by natural and man-made processes. Water development takes place as man seeks to alter the natural occurrence of water and channel it to his use. Because water occurs in nature in places, times, and quantities different from the desires of man, reservoirs and aqueducts are built to smooth and tame the vagaries of nature. This alteration of the natural flow processes changes the stream regimen and affects environmental stream conditions.

In addition, the quality of water is altered as man uses it. Return flows from cities and industries contain waste constituents in various concentrations which affect its suitability for further use. In the broadest sense, pollution can be thought of as any alteration of the quality of water. However, pollution is generally thought of as being some deleterious change in the water quality which renders it unfit for some particular use.

Effective water quality management is the goal of water pollution control activities in this country. Streams serve both as sources of water supply and avenues for the disposal of wastes. Thus, there is a clear interaction between quantity and quality of water. However,
largely due to the agency structure in state and federal government, water quality management and water development activities are usually separated and often delegated to different agencies. As an example, in the State of Texas, the water quality control problems are the major concern of the Texas Water Quality Board, and the water development aspects are handled by the Texas Water Development Board. Although the professional personnel working within these agencies recognize the clear interaction between water quantity and quality, the present institutional structure is not necessarily conducive to the active consideration of this interaction. The State of Texas is not unusual in this regard as many states and the federal government have similar institutional arrangements. Even local governmental units are often formed in the same pattern.

The consideration of water quality management should be an integral part of water planning practice. Water resources planning involves the selection of physical works and operational procedures which will redistribute resources in time and space to satisfy demands for water. Consideration of water quality and quantity in water planning is increasing considerably. Because of past planning practice stemming from institutional structure and legislative directive, this consideration has not been widespread or comprehensive in nature up to the present time. Although there are many reasons for this, two are paramount. Firstly, the past solutions in which quality and quantity were considered separately have been adequate in most cases. As long as the needs for water were small in comparison to the total available
supply or the water use and waste return points were widely separated, interaction of water quality and quantity was a local phenomenon. However, with population increase and a large increase in the demand for water has come the problem of degradation of water quality and more direct interaction of water quality and water quantity. Another reason for the failure to integrate water quality and quantity considerations completely in water planning activities stems from the complexity generated in the planning problems. Numerous methods for water quality management are in use or have been proposed. Some of these include direct waste treatment, in-stream treatment, retention and later release, flow augmentation, and others. Each adds a dimension of added complexity to an already difficult planning problem. This report describes a methodology which can be used in water planning to consider water quality and quantity interactions while remaining computationally tractable.

Purpose and Scope

The purpose of this report is to provide an analytical framework based on the decomposition approach of dynamic programming within which it is possible to develop optimum water quality and quantity management plans. Emphasis will be placed on model development and diversity of applicability rather than highlighting a specific application. The model structure is developed and explained and example problems are formulated and solved.
CHAPTER II

ELEMENTS OF PLANNING FOR WATER QUALITY MANAGEMENT

Basic Concepts

In developing a plan for water quality management, a variety of options is available to the planner. If this plan is to be complete, it is not possible to consider water quality exclusive of water quantity. Even a cursory review of the literature will reveal that most previous research has separated water quality and quantity considerations for separate analysis [13,18,38]. Considering only pollution control, the least costly solution at a waste treatment site would involve treatment of the effluent only to a degree at which the combination of treatment and natural purification just satisfies the constraints specified by regulatory agencies. However, in most cases, the watercourse is also a source of water supply. Clearly, because the watercourse must serve a dual purpose of both a source of water supply and an avenue for treated waste disposal, water development plans should integrate water quality and water quantity considerations.

A discussion of the economic forces and some of the options available to the planner in developing a water quality management plan are enumerated in the remainder of this chapter. Each of the options available to the planner must be considered in the decision-making process associated with the planning. Economic measures or some other suitable measures of utility must be devised and used
to rank the available alternatives. A flexible procedure developed and described in the chapter to follow is then necessary which will permit consideration of the alternatives available in light of the economic goals.

Economic Considerations

In order to formulate goals in a water quality management plan, one must consider the interaction of the users and the consequences of their actions on one another. This discussion will include a brief review of the economics of water quality management and discuss the consequences of shared resources. Some consideration of the trends in control measures will be presented. Finally, the assumptions regarding the economic control of the water quality management system to be used in this report will be presented.

The present practice of neglecting the interaction between users of the water resources and waste dischargers within a river basin results in an economic situation which tends to encourage pollution. Upstream waste dischargers have a direct effect on the quality of the water utilized by downstream users. The downstream user is forced to remove pollutants for which he is not directly responsible. Kneese [15] indicates that the physical character of waste disposal in streams is such that virtually all of the damages and costs resulting from discharge of waste occur external to the waste discharger. Thus, there are sizable spillover effects in a water quality management system. Whenever the upstream user or waste discharger is not forced to consider these spillover effects, quality degradation tends to increase. Kneese [15]
further indicates that any society which allows waste dischargers to neglect external costs will devote too few resources to waste treatment and tend to degrade the quality of their water resources.

Regulatory agencies have devised a variety of methods to safeguard the quality of the water resources under their jurisdiction. However, even with increased interest in pollution control, degradation of water quality still remains a problem of paramount importance. Part of the problem stems from the failure to include economic considerations in a corrective method. One corrective measure which has been tried and will serve as a case in point involves the imposition of damage charges upon a waste discharger when the discharge of the waste results in the degradation of quality in the stream. In this case, when the marginal treatment costs exceed marginal damage costs, it is more economical for the discharger to accept damages rather than increase costs by treating the waste. Furthermore, damage costs are imposed after the damage has occurred. This procedure therefore may cause irreparable damage such as fish kills before corrective measures are taken. Thus, water quality management activities, if they are to be adequate, must consider the high degree of interaction present in river systems and the economic consequences resulting from regulatory actions.

A further complication in water quality management results from the consequences of shared resources. Christy [4] notes that the consequences of the exploitation of natural resources manifest themselves in three distinct ways. Initially, uncontrolled exploitation
usually results in the depletion of the resource. Secondly, unconstrained use leads to an inefficient economic condition. Finally, exploitation usually results in pollution or congestion of the resource. Inherent in this idea is the principle that common property attracts more users than is physically and economically desirable. Although some form of public control is necessary to overcome these consequences, control alone is not the answer. The constraints that society imposes must be considered in light of economic criteria before control can be made equitable.

Other methods often used to limit water quality degradation involve the establishment of stream standards and effluent standards which place minimum and maximum standards on potential pollutants in the waste effluent or in the stream. These standards are usually set based on critical stream conditions. In either case, if the stream and effluent standards have been equitably determined at the outset, waste dischargers will be treating to a higher or lower degree than actually necessary depending on whether the actual stream condition is better or worse than the assumed critical condition on which the standard was based. Furthermore, if the standard states only the minimum or maximum concentration of a quality constituent without considering the total volume of waste discharged, a potentially dangerous situation can occur in which the concentration meets the required standard, but the quantity greatly exceeds the quantity on which the original quality standard was based. In any real system, stream and effluent standards should be viewed as complements of one another in
a total management program. As was mentioned previously, effluent standards often result in corrective measures which are too severe. On the other hand, stream standards in and of themselves are seemingly impossible to meet by a number of independent waste dischargers using a common watercourse. This again points up the difficulty caused by interaction between waste dischargers on a common stream.

The trend toward higher degrees of interaction between waste dischargers and water users coupled with the development of larger and more complex water development systems has forced the trend in water quality management toward consideration of regional management plans. Fox and Herfindahl [9] point out two very highly desirable modifications in the techniques of quality management in river basin systems. Firstly, there should be increased use of effluent charges and prices which are imposed at waste outfalls and depend on the degree of pollution resulting from the discharge of the waste. This type of control has been demonstrated to be effective in the Ruhr area of Germany where penalty prices are adjusted to the amount and kind of effluent present [9]. The second measure proposed by Fox and Herfindahl concerns the creation of a central control unit with the facilities necessary to audit independently the activities within the control region and consider whether the full range of different methods of satisfying water demands which are possible have received adequate consideration. Fox and Herfindahl point out that using this approach, alternative plans can be developed and the cost for providing the services can be calculated. Comparisons could then be made to determine
whether the benefits are justified by the costs incurred. This technique is being used to express important intangible benefits in terms of alternative plans according to Fox. Thus, it is possible to determine the cost of water recreation by comparing the cost of maintaining the recreation to the cost of some alternative plan.

In addition, there has been consideration by Kneese and others [16] of the relative advantages of effluent charges as discussed above versus the imposition of effluent standards as is the common practice in water pollution control agencies today. Effluent charges are considered to be more equitable than effluent standards and to contribute in a more substantial way to the achievement of higher water quality standards. Kneese points out that "charges offer some incentive to take action even to the lowest level of waste discharge while standards provide no incentive to curtail waste discharge beyond the required level even though it is possible to do so quite inexpensively". The charge system exerts continuous pressure to improve waste treatment while standards on effluents provide no additive incentive. In other words, the charge system forces the discharger to keep incremental costs of waste reduction in balance with the charge by improving waste treatment. Furthermore, effluent charges yield revenue. Each waste discharger is charged in proportion to the use he makes of the resource. The waste discharger compares his marginal cost and charges and decides whether it will pay him to reduce his waste load on the stream and to what degree it should be reduced. This amounts to little more than a rental charge for the use of the
natural resource. Charges are leveled on all dischargers which result in external costs. Although under ideal conditions the imposition of effluent standards and effluent charges will yield identical results, the ease of administration and equity of income distribution and cost favor the imposition of effluent charges over the imposition of effluent standards.

In this investigation it will be assumed that a regional authority has the power to regulate and manage the water quality within the region of its authority. The purpose of the authority will be to minimize all costs of water quality management whether they arise due to waste treatment, water treatment, or in-stream treatment by low flow augmentation. The regional authority through the use of the model contained in this report can control the basin in such a way that each discharger removes a percentage of the pollutant which he adds to the water resource while meeting in-stream constraints on water quality and minimizing overall water quality management costs within the basin.

Stream - Quality Relationships

Pollutants which are discharged into surface waterways are classed as either conservative or non-conservative. Conservative pollutants are not changed appreciably in the river system except by dilution, evaporation or other physical transport mechanisms. Non-conservative pollutants, on the other hand, are degradable biologically in addition to those physical transport mechanisms associated with the conservative pollutants. Thus, the description for the stream quality effects
due to non-conservative pollutants is much more complicated mathematically than that for conservative pollutants. In fact, the description for conservative pollutants can be handled as a special case for non-conservative pollutants as the spatial variability considerations can be eliminated. Thus, only the effect of non-conservative pollutants on stream quality will be considered in the remainder of this report.

In general, the major waste characteristics considered in the description for stream quality effects are:

1) Biochemical oxygen demand
2) Nutrients materials other than carbon
3) Surface active agents
4) Indicator organisms
5) Temperature

The BOD (biochemical oxygen demand) is the total oxygen requirement for the oxidation of biodegradable organics contained in a pollutant and is considered to be a measure of the strength of a biodegradable waste. Since the BOD consumes the dissolved oxygen in natural streams, non-conservative pollutants are usually considered as one of the primary sinks of dissolved oxygen. Hence, the DO (dissolved oxygen) is one of the key parameters indicating the natural balance of the aquatic ecosystem, and the BOD-DO relationship has become the principal indicator governing the management and control of stream quality.
There are many factors affecting the BOD-DO relationship in surface streams. These factors can be considered in two categories: geophysical and biochemical. Geophysical characteristics include all of the factors which make up the physical description of the stream and its drainage area. Such factors as quantity of streamflow, geomorphology of the drainage basin, and meteorological conditions can be included in this list. Biochemical considerations on the other hand include the factors relating the various sources of organic pollutants and dissolved oxygen whether natural or of man-made origin.

Each of these many factors interacts to influence the sources and sinks of organic waste and dissolved oxygen to determine the character of the stream. If all of the factors are in balance, the stream will be in equilibrium with natural purification factors operating on waste inputs to maintain an environment which supports aquatic life and is pleasing and not harmful to man. If the waste loading exceeds the stream's natural purification capacity, offensive, septic conditions can result with associated fish kills, sludge banks, unsightly conditions, and unpleasant odors. Thus, analytical methods have been developed to describe the sources and sinks for dissolved oxygen so that a balance can be maintained when wastes are introduced into streams. The complexity of these models depends largely on how many sources and sinks for dissolved oxygen are included in them.

Streeter and Phelps [30] first suggested an analytical description of the BOD-DO relationship including only the BOD oxidation and natural reaeration processes. The analytical models developed by Streeter and
Phelps are given in Equations 2.1 and 2.2

\[ L_t = L_0 e^{-K_1 t} \]  \hspace{1cm} (2.1)

\[ D_t = \frac{K_1 L_0}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) + D_0 e^{-K_2 t} \]  \hspace{1cm} (2.2)

in which

\begin{align*}
 t & \quad = \text{Time of travel (since release of waste) (days)} \\
 D & \quad = \text{Oxygen deficit below saturation concentration (mg/l)} \\
 K_1 & \quad = \text{BOD rate constant (days}^{-1}) \\
 K_2 & \quad = \text{Reaeration rate constant (days}^{-1}) \\
 L & \quad = \text{Oxygen demand of organic matter (mg/l)} \\
 e & \quad = \text{Base of the Naperian logarithm}
\end{align*}

Equation 2.1 describes the BOD remaining at any time \( t \) after release of a waste with BOD concentration \( L_0 \). Equation 2.2 represents the dissolved oxygen deficit below the saturation level, at any time \( t \) which results from the opposing forces of deoxygenation and reaeration in the stream when the initial concentration of BOD and DO are \( L_0 \) and \( D_0 \), respectively.

Camp \cite{Camp1} later modified the equation of Streeter and Phelps by adding terms accounting for the BOD which settles to the bottom of the stream, the BOD addition detached from the bottom sediments, and the DO production due to photosynthesis. These generalizations are accomplished by the addition of constants to Equations 2.1 and 2.2 to form revised
equations for BOD and DO deficit as given in Equations 2.3 and 2.4.

\[ L_t = \left[ L_0 - \frac{P}{(K_1+K_2)} \right] e^{-(K_1+K_3)t} + \frac{P}{(K_1+K_3)} \]  

\[ D_t = \frac{K_1}{K_2-(K_1+K_3)} \left[ L_0 - \frac{P}{(K_1+K_3)} \right] \left( e^{-(K_1+K_3)t} - e^{-K_3t} \right) \]  

\[ + \frac{K_1}{K_2} \left[ \frac{P}{(K_1+K_3)} \right] \left( 1 - e^{-K_2t} \right) + D_0 e^{-K_2t} \]  

(2.4)

where

- \( K_3 \) = Rate of BOD settling (days\(^{-1}\))
- \( P \) = Rate of addition of BOD from bottom sediments

Dobbins [6] proposed another equation describing the BOD-DO relationship which provided for the inclusion of the steady input of BOD within a stream segment, the first-order reaction rate for BOD removal by sedimentation and absorption, the consumption of dissolved oxygen by benthal demand, and the constant rate of addition of DO by photosynthesis. Dobbin's equation for the DO deficit is expressed in Equation 2.5
\[ D_t = \frac{K_1}{K_2 - (K_1 + K_3)} \left[ L_o - \frac{L_a}{(K_1 + K_3)} \right] \left[ e^{-(K_1 + K_3)t} - e^{-K_2t} \right] \]

\[ D_0 e^{-K_2t} + \left[ \frac{D_B}{K_2} + \frac{K_1 L_a}{K_2 (K_1 + K_3)} \right] (1 - e^{-K_2t}) \]

(2.5)

In this equation the variables and constants are as defined previously.

The analytical models presented above are designed to describe steady-state conditions. Other researchers have devised analytical techniques designed to describe the dynamic situation existing in the stream. Models have been proposed by Li [17] and Frankel and Hansen [10] which account for the variation of dissolved oxygen deficit as a function of both time and distance. The technique of Frankel and Hansen was developed for digital computation. As the purpose here is not to present a summary of previous work but only an indication of the types of models in use, the equations of Li and Frankel will not be presented.

O'Connor and DiToro [24] recently proposed a model for the BOD-DO relationship accounting for spatial and time variations. In this model, they have considered both the nitrogenous and the carbonaceous BOD at the same time. Likewise, they have considered a variety of sources and sinks for dissolved oxygen. Algal photosynthesis and respiration and Benthal respiration are examples of additional sources in sinks.
considered. This model also accounts for the variation of the cross-sectional area of the river. The equation of O'Connor and DiToro is presented in Equation 2.6 and is given here to indicate the complexity of some of the available models describing the BOD-DO relationship.

\[
D(x,t) = D_0(t - \frac{x}{u}) e^{-\frac{K_a x}{u}} + \frac{K_{d0}}{K_a - K_d} \left( e^{-\frac{K_n x}{u}} - e^{-\frac{K_a x}{u}} \right) + \frac{K_{n0}}{K_a - K_n} \left( e^{-\frac{K_n x}{u}} - e^{-\frac{K_a x}{u}} \right) + \left( \frac{S}{K_a} + R \right) (1 - e^{-\frac{K_n x}{u}})
\]

\[ - p_m \left\{ \frac{2P}{\pi K_a} \left( 1 - e^{-\frac{K_n x}{u}} \right) \right\} \]

\[
+ \sum_{n=1}^{\infty} \frac{b_n}{\sqrt{K_a^2 + (2\pi n)^2}} \cos \left\{ 2\pi n \left( t - t_s - \frac{P}{2} \right) - \tan^{-1} \left( \frac{2\pi n}{K_a} \right) \right\}
\]

\[- e^{-\frac{K_a x}{u}} \sum_{n=1}^{\infty} \frac{b_n}{\sqrt{K_a^2 + (2\pi n)^2}} \cos \left\{ 2\pi n \left( t - t_s - \frac{P}{2} - \frac{x}{u} \right) - \tan^{-1} \left( \frac{2\pi n}{K_a} \right) \right\} \]

where

- \( x \) = Distance from the point of waste discharge
- \( u \) = Average velocity of the stream
- \( K_a \) = Reaeration coefficient
\( K_d \) = Coefficient of carbonaceous oxidation  
\( K_r \) = Removal rate constant of carbonaceous organics in streams  
\( K_n \) = Coefficient of nitrogenous oxidation  
\( S \) = Benthic respiration sink of DO  
\( R \) = Algal respiration sink of DO  
\( t_s \) = Time at which photosynthesis source begins  
\( N_0 \) = Initial nitrogenous BOD concentration  
\( P_m \) = Maximum rate of photosynthetic oxygen production  
\( P \) = Duration in time of the photosynthetic oxygen source  
\( b_n \) = nth Fourier coefficient of the photosynthetic oxygen source

As presented in the foregoing section, a variety of different BOD-DO relationships have been proposed and tested against limited DO data. Each specific relationship possesses its own merit in terms of practical applicability. There is no relationship which can be applied for all physical systems because many arbitrary simplifications were assumed by each investigator. Although the relationship presented by O'Connor and DiToro has included more parameters than other models, there are still many variables encountered in practical problems which have not been considered.

Mathematical models outlining the stream-quality relations must be included as transformations in any water quality management planning problem. The research described in this report has as its goal the presentation of a generalized approach which can be adapted to include any of the BOD-DO functions now available or developed in the future.
In view of the flexibility needed, a transfer function approach was utilized which correlates and predicts the BOD-DO relationship between upstream discharge points and downstream quality control points. For the purpose of this study, these transfer functions were considered to be simple tables and graphs. The formulation and use of these graphical transformations will be illustrated in the following chapters of this report.

Waste Treatment

Waste treatment quite obviously is one of the most significant components in water quality management within a river system. In treating wastes, a portion of the waste is removed from the water carrying it before the remaining waste-water mixture is returned to the stream. The present design procedure for waste treatment plants includes consideration of characteristics of the waste water inflows, quality requirements for the treated effluent in terms of the receiving stream quality conditions, specific requirements imposed by regulatory agencies, and the reliability of the waste treatment operation. Techniques permitting minimization of the total costs of treatment while meeting overall treatment requirements have been developed previously [27].

Generally in a waste treatment plant design, the factors involving the treatability of the waste water, concentration of the raw waste, regulatory requirements, total quantity of waste flow and receiving stream conditions must be considered. However, for a specific waste treatment facility, the treatability of the waste and the quantity
and quality of the raw waste are basically fixed conditions in the determination of the cost of the waste treatment plant. Thus, the waste treatment cost can be expressed as a unique function of the treated effluent quality and receiving stream conditions for each specific waste treatment facility.

In this research investigation, the total cost of waste treatment will be assumed to be a function of the quality of the treated effluent with the effluent subject to limitations specified by regulatory agencies. The regulatory limitations usually specify minimum dissolved oxygen requirements and/or maximum allowable concentration of BOD.

Water Treatment

Although other studies of the analysis of water quality management within river basins have considered waste treatment effects, there has been almost no consideration or mention of the effect of waste flows on the cost of water treatment at locations downstream of waste return points.

The cost for the water treatment is primarily dependent upon the quality of the raw water, the quantity of the water to be treated, the quality requirements for the treated water, and the treatment process alternatives included for the treatment plant. The quality requirement for the treated water is normally specified by the usage to which the water is to be put. For example, municipal water requires a lesser removal of constituents such as inorganic salts than do some industrial waters. All the purification requirements can be related to
the concentration of specific substances in the water. Some of the factors of greatest concern in municipal water are color, taste, odor, and undesirable organic contents which result from non-conservative pollutant concentration. The chemical consumption in coagulation and sedimentation, filtration efficiencies, backwash requirements for the filter, and chlorination requirements are directly related to the quality of the raw water received by the water treatment plant.

Thus, the cost of water treatment will vary for each different raw water quality. Knowing the specific degree of treatment required for each water treatment plant, a cost function can be developed which depends only on the quality of the raw water at the intake of plant. In the methodology presented in this report, the raw water quality will be assumed to be dependent on the quality of the waste effluent released immediately upstream of the water treatment plant, the quality of the river upstream of that waste return point, the quantity of flow in the stream, and the quantity of that waste return flow.

Low Flow Augmentation

Considerable attention has been given in the last few years to the concept of using streamflow regulation or augmentation as a means for water quality control [32]. In many streams in the State of Texas, most if not all of the flow at low-flow periods is return flow from waste treatment plants. The concept of streamflow regulation is based on the release of a quantity of higher quality water from an upstream reservoir to maintain or improve the quality in the stream by dilution
of return flows. In water short areas such as the State of Texas, releases for quality control usually require curtailing or limiting other uses to which the water can be put. Thus, there is an identifiable cost associated with water released for quality control equal to the capital cost at the reservoir and opportunity cost associated with the water itself. For the purpose of this research, these costs can be related to the quantity of water released and the quality of the release.

Direct Benefits

An important component in water quality - quantity management involves direct and indirect benefits derived from the recreational use, tourism potential and scenic beauty afforded by good water management practices. There has been and, no doubt, will be much discussion of the magnitude and reality of these benefits. These benefits will be a function of the volume of flow and the quality in the stream. If it is decided to include benefits such as these in a planning model, they can be incorporated into the models with little difficulty by adding them to return functions at the appropriate stream reach.

Engineering Alternatives

Previous studies have suggested other methods for quality management which should be considered in a water quality management plan. Several of these procedures will be outlined below.

By-pass piping [11] has been proposed as a means of improving stream quality conditions. Using this method, interceptor lines are
provided to collect waste effluents or surface drainage and transport these to a downstream point where no critical water quality conditions exist. This can lead to a reduced requirement for waste-treatment while meeting water quality requirements. However, the cost of the installation, operation, and maintenance of the piping system may limit the use of this alternative.

Storage of wastes for later release during more favorable stream quality conditions also has been proposed to lessen the treatment requirement and provide greater operational flexibility [19]. During periods of high flow, stream quality is usually significantly better than it is during periods of low flow. If waste effluents can be impounded during periods of low flow and released during periods of high flow, an improved stream quality situation often results. Furthermore, impoundment of waste effluents with post aeration will provide additional stabilization and oxidation of residue organics and improve quality. However, the probabilistic nature of the river flow and the treated effluent quality must be studied cautiously if this method is to be used. The cost of construction of the impoundment and the waste water transfer system must be weighed against the additional treatment cost in arriving at a decision.

Regional collection and treatment of wastes [8] has been proposed as a means of improving water quality by reducing costs of attaining the same degree of treatment through collective regional action and economics of scale associated with larger, better managed waste treatment facilities. In large metropolitan areas, this method appears
to be particularly attractive.

Some consideration has been given to the use of in-stream aeration and distributing the waste effluent return over a section of stream to improve quality [5]. In-stream aerators increase the diffusion of oxygen into surface flows and thereby enhance stream quality conditions. Distributing the waste effluent return over a segment of the stream rather than returning waste effluent at a single point reduces the effect of shock loading on the stream and has been found to be useful in some cases.

Although there are many more alternatives which can be considered, these are representative of the many methods of quality management available to the planner. Any analytical framework for planning water quality management programs should be able to accept these alternatives in the analysis procedure.
CHAPTER III

DEVELOPMENT OF AN APPROACH TO WATER QUALITY MANAGEMENT

Basic Concepts

The purpose of this chapter is to describe the development of an analytical framework for use in planning water quality management systems as a part of an overall water resources planning problem. As was mentioned earlier in this report, water quality management considerations within the water resources planning framework have not been considered in the same detail as water quantity aspects. The highly interactive nature of a water resource system coupled with the fact that water in typical rivers is used and reused numerous times in its transit down the stream illustrates the hazard in following this approach.

The goal in this report is to develop a basic methodology which is of a general nature. The implementation demonstrated in this report is limited mainly because of time and budgetary constraints coupled with a paucity of data. The major limitations in this analytical development relate to the fact that the formulation is deterministic rather than stochastic and that the representation of the component subsystems is not as precise as some planners might desire. A deterministic framework was developed principally because of time and budgetary limitations. The theory and practice of stochastic mathematical programming are developing to the point that the extension of the models proposed to include stochastic elements is not outside the realm of possibility in the very near future. Furthermore, concerning the representation of
the subsystems, use of a technique of imbedding other optimization problems within a total dynamic programming framework later will afford a method of representing subsystems to greater and greater detail. This will permit the planner to analyze a highly complex water development system including water quality management aspects while retaining the advantages of dynamic programming decomposition in overcoming problem complexity.

System Complexity Problems

The death knell of optimization problems in water planning has been the exploding nature of the complexity of the problem. A typical water planning problem contains numerous decision variables and constraints interacting in a system of nonlinear functions. This mathematical form results from the fact that water planning is usually done on a regional scale with the size of the regions determined by the degree of interaction of the water development elements. The competition among water uses for a limited quantity of the available water resource has caused the regional extent of water planning problems to be increased in size and the number of physical facilities and alternative designs under consideration to be increased at the same time. The addition of new water uses and increased competition for water and the increase in the number of alternative means for satisfying water needs have joined together to cause the complexity of water planning problems to increase tremendously in recent years.

Thus, the methodology developed and presented within this report has been formed on the basic premise that one of the great needs in
water planning practice today is for methods permitting decomposition of the overall water planning problem in a region into simpler problems which may be imbedded in the larger problem for total optimization of the system. The techniques presented in this report provide for this decomposition and total optimization and also provide for the sensitivity analysis of the optimum plan that is so necessary for planning.

Use of Dynamic Programming

An examination of the structure of a typical water quality management problem shown in Figure 3.1 exhibits a multistage structure with stages composed of physical facilities such as waste and water treatment plants, reservoirs, stream quality reaches, and alternate engineering works. This structure and the problems associated with problem complexity suggest the applicability and utility of dynamic programming.

The purpose of this section will be to briefly describe the dynamic programming method discussing principally innovations which have computational significance and were devised in this investigation. Dynamic programming is an approach to problem solving which permits the analyst to decompose the original problem into a set of smaller optimization problems - designated stages - for recursive solution. The major effort in solving a dynamic programming problem is associated with problem formulation. In the formulation phase of the solution process, the stages, states, decisions, state transformations, returns, and incidence identities involved in the dynamic programming solution must be identified. Each of the items involved in the formulation
SCHEMATIC REPRESENTATION OF RIVER SYSTEM

STAGED REPRESENTATION OF RIVER SYSTEM

Figure 3.1
will be explained before defining their specific function in the water quality management problem undertaken in this investigation.

A functional diagram of a staged system shown in Figure 3.1 will be used in the definition of terms. Stages analogous to decision points are represented by boxes numbered in reverse order by computational convention. States, illustrates by horizontal arrows, carry information from stage to stage in a dynamic programming problem and represent inputs to and outputs from the stages. In the diagram, input and output states at stage \( i \) are represented by \( s_i \) and \( \tilde{s}_i \), respectively. Returns at stage \( i \), represented by \( r_i \), are measures of utility or cost resulting from the decision, \( d_i \). Decisions are the planning variables which can be controlled and manipulated by the decision-maker. Both the returns and decisions are indicated in Figure 3.1 by vertical arrows. State transformations are functions, tables, or graphs which describe the way in which an input state is transformed into an output state through the making of the decision at a stage. Incidence identities describe the manner in which an output state from one stage becomes an input state to the next succeeding stage.

Dynamic programming has been adequately described in a variety of books \([1,23,40]\) and has been found useful in solving a variety of water resources problems including water quantity and quality considerations \([28,13]\). In describing the problem shown in Figure 3.1, one would class it as a serial multistage optimization problem. It is serial because all of the stages are arranged one following another in a series. If the initial state \( s_N \) is fixed at a constant value
but the final state $s_1$ is free to take on any value, the problem is known as an initial value problem. If the final state is fixed and the initial state is free to assume any value, the problem is classed as a final value problem. When both initial and final states are fixed, the problem is known as a two-point boundary value problem.

The dynamic programming approach to problem solving treats all problems as if they are serial, initial-value problems in the solution process. The solution is obtained by processing the stages sequentially in reverse order in a recursive computational scheme outlined by Bellman [1] in his Principle of Optimality and stated mathematically in the functional equation given in Equation 3.1

$$f_n (s_n) = \max/\min \left[ g_n (s_n, d_n) \right]$$

where

$$g_n (s_n, d_n) = \begin{cases} 
  r_n (s_n, d_n) + f_{n-1} (s_{n-1}), & n \geq 2 \\
  r_1 (s_1, d_1), & n = 1 
\end{cases}$$

and

$$r_n (s_n, d_n) = \text{stage return at stage } n$$
$$s_{n-1} = t_n (s_n, d_n) = \text{known state transformation}$$
$$f_n (s_n) = \text{optimum return for stage } n \text{ and all successive stages.}$$

Equation 3.1 is applied in a stagewise fashion as described in detail elsewhere [23].
Wilde and Beightler [40] suggest using techniques known as decision inversion and state inversion for solving final value problems. Using state inversion, the state transformations are inverted, and the direction of recursion is reversed. Thus, a final value problem is transformed into an equivalent initial value problem and solved using the standard backward recursive procedure. Applying the decision inversion approach involves invoking the state transformation at the final stage as a constraint thereby reducing the number of decisions at the final stage by one. Recall from Equation 3.1 that a state transformation is of the form

\[ \tilde{s}_n = t_n (s_n, d_n) . \]

Using state inversion, this transformation is inverted to become

\[ s_n = \tilde{t}_n (\tilde{s}_n, d_n) \]

and substituted into the return to yield a new return function of the form

\[ r_n = \tilde{r}_n (\tilde{s}_n, d_n) . \]

In decision inversion if the final state, \( \tilde{s}_1 \), is fixed at \( s_0 \), the state transformation

\[ s_0 = t_1 (s_1, d_1) \]

may be solved uniquely for the value of \( d_1 \) (if the final stage is a single decision stage) in terms of the known final state, \( s_0 \), and any
feasible value of the input state, \( s_1 \). The final stage becomes a "decisionless" stage in that the state transformation can be substituted into the return function to yield

\[
    r_1 = r'_1(s_0, s_1)
\]

If more than one decision is present at stage 1, one of the decisions may be eliminated by substitution. These techniques will be referred to in the discussion of computational aspects which follows.

Study of Figure 3.1 will reveal that the system portrayed is not a general water quality management system because no tributary streams are considered. As discussed previously [20], techniques have been developed to permit the solution of branched multistage systems as shown in Figure 3.2 by decomposing the nonserial systems into equivalent serial systems for solution using dynamic programming. The basic procedure for accomplishing the solution of nonserial systems is discussed below.

Examination of the functional diagram given in Figure 3.2 reveals that, if the state link between \( A_l \) and stage \( j \) is cut, two serial staged systems will result similar to those shown in Figure 3.3. In fact, the technique for solving these problems involves introducing a "cut state" for \( s_{A_l} \) to obtain the equivalent serial systems. In general, if there are \( P \) branches present in the problem \( P+1 \) serial problems will result. If the initial states \( s_N \) and \( s_{A_\alpha} \) are fixed, the resulting serial problem 1, the main system, will be an initial value problem, and serial problem 2, the branch, will be a two-point
Figure 3.2
FUNCTIONAL DIAGRAM OF A BRANCHING MULTISTAGE SYSTEM
SERIAL SYSTEMS EQUIVALENT TO BRANCHING SYSTEMS

SERIAL PROBLEM 2

SERIAL PROBLEM 1

Figure 3.3
boundary problem.

In the solution process, the branch is solved as a two-point boundary problem for the fixed value of the input state \( s_{A_{x}} \) and all feasible values of the output state \( s_{A_{\text{f}}} \) to yield the optimum branch return in terms of \( s_{A_{x}} \) and \( s_{A_{\text{f}}} \), i.e., \( f_{x}(s_{A_{x}}, s_{A_{\text{f}}}) \). The methodology for accomplishing this will be discussed in the section on computational aspects in this chapter.

The main system, serial problem \( l \), will be solved as an initial value problem or a two-point boundary problem depending on whether or not the final state, \( s_{i} \), is fixed. In either case, it is not a conventional initial or two-point boundary problem. The reason for this is the cut state that enters at the junction stage, \( j \). It is possible to solve the first \( j-1 \) stages using serial dynamic programming procedures. However, stage \( j \) violates the standard dynamic programming format by receiving state input from more than one stage.

Meier and Beightler [20] outline a procedure for optimizing stage \( j \) as a two decision problem as shown in Equation 3.2

\[
\begin{align*}
    f_{j+1, x}(s_{j+1}) &= \max/\min \left[ r_{j+1}(s_{j+1},d_{j+1}) + f_{x}(s_{A_{x}}, s_{A_{\text{f}}}) + f_{j}(s_{j}) \right] \\
    d_{j+1, s_{A_{\text{f}}}}
\end{align*}
\]  

(3.2)

where

\[
    f_{j}(s_{j}) = \text{Optimum return from } j \text{ stages in terms of input state } s_{j}.
\]
\( f_{\alpha}(s_{A\alpha}, \tilde{s}_{A1}) \) = Optimum return from branch stages in terms of input and output states.

\( f_{j+1+\alpha}(s_{j+1}) \) = Optimum return from the first \( j+1 \) main system stages plus the \( \alpha \) branch stages in terms of the input state \( s_{j+1} \).

\( r_{n+1}(s_{j+1}, d_{j+1}) \) = Return at stage \( j+1 \)

\( \max/\min \) = Optimization operator - either maximum or minimum.

\[ s_j = \phi_1(s_{A1}, \tilde{s}_{j+1}) \]

\[ \tilde{s}_{j+1} = \psi_j(s_{j+1}, d_j). \]

The solution of problems using this approach is described in the literature in technical papers [20,23]. Fundamentally, this approach involves finding the optimum branch return as a function of the connecting state, \( \tilde{s}_{A1} \), at the junction of the two staged systems. The effect of the branch on the main system of stages then has been distilled into a form such that it can be "absorbed" into the functional equation used to optimize the main system stage, \( j+1 \). At stage \( j+1 \), the analyst is free to choose that value of the connecting state, \( \tilde{s}_{A1} \), which along with \( d_{j+1} \), the decision variable, optimizes the terms in brackets in Equation 3.2. The freedom to choose state \( \tilde{s}_{A1} \) arises because no decisions or states at stages \( j+1 \) to \( N \) are directly dependent on the value of \( \tilde{s}_{A1} \). A choice state is in fact a decision
variable since a decision variable is just a variable which the analyst is free to choose.

An improved solution procedure known as the "branch compression" approach was presented by Meier and Beightler [21]. This procedure involves solving the branch as a two-point boundary problem as before. However, in absorbing the effect of the branch into the main system of stages, a pseudo stage is introduced at which the optimum values of the branch output state, \( \tilde{s}_{A1} \), is chosen. This approach can be used to solve an \( M \)-stage serial set of stages and \( \alpha \) stages on one or more branches with no more difficulty than required to solve \( K+\alpha \) serial stages. This result follows from invoking the Principle of Optimality [2] to decompose the functional equation appearing in Equation 3.2 to obtain the two shown in Equation 3.3

\[
f_{j+\alpha}(\tilde{s}_{j+1}) = \max/\min \left[ f_{\alpha}(s_{A\alpha} \tilde{s}_{A1}) + f_j(s_j) \right]
\]

\[
d_{j+1}(\tilde{s}_{j+1}) = \max/\min \left[ f_{j+\alpha}(s_{j+1}, d_{j+1}) + f_{j+\alpha}(\tilde{s}_{j+1}) \right]
\]

(3.3)

where all terms are as defined previously.

In considering the difficulties generated in solving branching problems, it should be noted that, at each branch junction, additional decision variables equal in number to the number of state variables in
the problem result. As an example, two single-state branches joining a main string of stages at the same point would generate two additional decision variables. Using the procedure shown in Equation 3.2, a special three decision stage will result with its associated computational difficulties. However, use of the methodology given in Equation 3.3 will result only in the addition of two single-decision stages. Therefore, the "branch compression" technique capitalizes on the computational efficiencies afforded by dynamic programming while eliminating, in most cases, the need for more than a simple single-decision stage algorithm. The major problem of computational significance relating to the solution of branching problems is that posed in the solution of final value and two-point boundary dynamic programming problems. This will be the subject of discussion in the following section.

Computational Aspects

The computational solution of final value, two-point boundary value, and branching problems has not been studied in detail in previous investigations. Considerable study of the computational aspects of these problems was undertaken as a part of this investigation. One result of this study was the "branch compression" principle discussed in the previous section. Other considerations are discussed below.

One of the major considerations involving computational aspects is that the direction of recursion in dynamic programming problems is quite often the choice of the analyst. Another is that state and decision variables can be treated differently in the optimization process. Finally, it is possible to combine stages in some instances
to reduce computational effort. There was not sufficient time or funds in this research effort to consider these aspects to the detail necessary to permit making more than qualitative judgements at this time. Each was tried in a variety of instances, however. The experience obtained is reported here in the hope that some computation time may be saved for other investigators. Other research is being conducted now to solidify the results alluded to here.

To indicate the importance of the direction of recursion in problem solution, consider the two-point boundary solution of the branch as a function of the input and output states. In most staged representations of spatial systems such as the one discussed in this report, the initial state on the branch, $s_{A\alpha}$ is known constant while the solution, $f_{\alpha}(s_{A\alpha},\tilde{s}_{A1})$, is sought for a feasible set of final states, $\tilde{s}_{A1}$. If the two-point boundary solution is obtained by processing the stages in the order $A1$ to $A\alpha$, a set of solutions will have to be obtained for the branch with one solution for each feasible value of $\tilde{s}_{A1}$. However, if the sequence of solution or recursion is inverted using state inversion initially, the two-point boundary solution of $f_{\alpha}(s_{A\alpha},\tilde{s}_{A1})$ can be obtained for all feasible values of $\tilde{s}_{A1}$ in one pass through the set of branch stages. This results from the fact that, at every stage in the recursive computational procedure, the optimum return for the entire set of feasible input state values is determined. This affords a considerable savings in computation time equal to $K-1$ passes through the branch stages with $K$ equal to the number of feasible values of the final state, $\tilde{s}_{A1}$, considered.
State variables are linking variables which serve as information carriers. Decision variables are under the control of the decision-maker, and, at each stage, the optimum value is sought for each feasible value of the input state. In the dynamic programming solution process, it is possible to use direct search methods to obtain optimum decisions and eliminate some of the computations required. Rather than enumerating returns for each feasible value of the decision variable, search methods can be used to accelerate the selection of an optimum. In this investigation, studies were made using the Fibonacci and "pattern" search procedures [40]. As the number of discrete values of the decision variable used in the conventional discrete dynamic programming procedure is increased, the advantage in reduced computation time of using search methods increases significantly. Because state variables are information carriers, tabular results in terms of enumerated state variables must be included in sufficient detail to obtain a computationally meaningful solution.

Other techniques such as polynomial approximation in the state space [2], and stage combination [40] are useful in improving computational efficiency. When the number of state enumerations was high or the number of state variables was greater than one as in this investigation, polynomial approximation was tested and found to be useful in improving computation speed. Stage combination, first discussed by Wilde [40], was used in the computational solution of the two-point boundary dynamic programming problems occurring in this research effort. Application of this procedure reduces the computer storage
requirements and the computation time. Thus, the overall computational
effort is reduced.

Each of these techniques involves additional computer logic in
the coding process. For problems in which a production code (to be
run repetitively) is desired, it becomes advantageous to implement these
approaches. For programs which will be run only a limited number of times,
these refinements are of limited usefulness. Each individual analyst
faces a strategic decision in the development of his algorithm to
develop one which balances program development cost and computation
costs to obtain efficient solution methods while keeping the specific
application in mind.
Elements of the Method

As described in Chapter 2, the major components attributing economic impacts on the quality and quantity management of a river system are:

1) Waste treatment
2) Water treatment
3) Low flow augmentation
4) Direct benefits.

The relationship between the cost or benefit incurred and various parameters defining the water quality and quantity for each of these components has been discussed in detail. In the remainder of this section, the cost functions are expressed in terms of specific independent variables for each individual component. These elements will form the objective function for the quality - quantity planning model.

Waste Treatment

The wastewater treatment cost at the ith waste treatment plant can be expressed as a unique function of the treated effluent quality and the receiving stream conditions.

\[ W_i = W_i \{E_i < E_{a_i}\} \quad (3.4) \]

As shown in Equation 3.4, the cost of the waste water treatment plant, \( W_i \), can be expressed as a function of the quality of the treated effluent, \( E_i \), while the effluent is subject to a limitation, \( E_{a_i} \), specified by the regulatory agency. Normally, this limitation
specifies the maximum allowable concentration of the key pollutant (for example - BOD) in the effluent. The specific limitation also may be based on the minimum DO concentration in the receiving stream. In order to assess the impact of the treated waste effluent on the receiving stream, the initial water quality of the receiving stream and the quantity of the river flow must be included, as shown in Equation 3.5.

$$\bar{W}_i = \bar{W}_i (E_i, L_{o1}, Q_i)$$  \hspace{1cm} (3.5)

Water Treatment

The water treatment cost will be included in the optimization procedure as a function of the quality of the raw water at the intake of the plant. The quality of the raw water is measured by the concentration of the key pollutant (such as BOD) as expressed in Equation 3.6.

$$P_{ij} = P_{ij} (L_{ij})$$  \hspace{1cm} (3.6)

In Equation 3.6, $L_{ij}$ and $P_{ij}$ are the concentration of BOD in the plant influent and the cost of water treatment at the jth plant in the ith stream reach. The pollutant concentration, $L_{ij}$, is a function of the flow in the ith reach, $Q_i$; the initial pollutant concentration at the head of the reach, $L_{o1}$; and the pollutant concentration in the waste return flow of this ith reach, $E_i$. Thus, Equation 3.6 can be rewritten

$$\bar{P}_{ij} = \bar{P}_{ij} (Q_i, L_{o1}, E_i)$$  \hspace{1cm} (3.7)
Low-Flow Augmentation

The costs for water released for low-flow augmentation is a measure of the cost of providing reservoir storage (capital plus operation and maintenance) and the opportunity costs associated with other uses to which the quality releases might be put. The utility of a release for quality control through streamflow regulation reflected in the cost of water is a function of the river flow at the entrance of reservoir, $Q_i$, concentration, $l_i$, of the pollutant in the release and the quantity, $Q_i$, released from the reservoir in the $i$th stream reach as given in Equation 3.8.

$$ R_i = R_i \{Q_i, l_i\} \quad (3.8) $$

However, the concentration of pollutant in the release, $l_i$, is dependent upon the concentration of pollutant at the entrance of reservoir, $L_{0i}$. Thus, the cost function for low-flow augmentation may be rewritten as

$$ R_i = R_i \{Q_i, Q_i, L_{0i}\} \quad (3.8) $$

Direct Benefits

The economic benefits derived from scenic beauty and recreational potential will be termed direct benefits resulting from quality-quantity practices. The direct benefit associated with the $i$th river reach is a function of the quality and quantity of the flow in the reach.

$$ B_i = B_i \{Q_i, l_i\} $$

The pollutant concentration in the river water of $i$th reach, $l_i$, varies with the concentration and flow quantity at the head of the
ith reach, \( L_{0i} \), and the pollutant concentration, \( E_i \), and quantity, \( q_i \), of the waste return flow. Thus, Equation 3.9 can be expressed as,

\[
\overline{B}_i = \overline{B}_i \{0_i, L_{0i}, =_i E_i\}
\]

(3.9)

Formulation of the Optimization Procedure

In this section, the dynamic programming approach to the water quality - quantity management problem discussed in the foregoing sections of this report will be outlined. The costs for water treatment, waste treatment, and flow augmentation will be combined with the direct benefits and the stream - quality relations to form an overall optimization framework useful in planning. As was mentioned in a previous section of this chapter, the formulation of a dynamic programming problem requires the specification of the states, decisions, stages, returns, state transformations, and incidence identities.

Stages in a dynamic programming problem are decision points. In a system of the type shown in Figure 3.1, the major decisions which will be made are the degree of waste treatment, \( E_i \), required at each waste treatment plant and the release, \( 0_i \), required at each reservoir to maintain acceptable quality levels. It was desired in this formulation to have only one decision variable at each stage. Therefore, the stages will be stream reaches containing no more than one reservoir release or waste return point. Thus, the system shown on the upper part of Figure 3.1 could be represented by six stages. The information carriers and, hence, the state variables in the problem are the quantities, \( Q_i \),
of regulated flow and the concentration, $L_{0i}$, of pollutant.

The general stage return is the total of the costs due to waste treatment, water treatment, and flow augmentation plus the negative of the benefits due to the improved quality of the river reach as shown in Equation 3.10.

$$C_i = \alpha_i \bar{W}_i + \alpha'_i \bar{R}_i + \beta_i \sum_{j=1}^{M_i} \bar{P}_{ij} - \bar{B}_i$$  \hspace{1cm} (3.10)

where $i = 1, 2, \ldots, N$ (N = total number of reaches).

$\alpha_i$ = Indicator of presence or absence of waste treatment plant in ith reach = 0 or 1.

$\alpha'_i$ = Indicator of presence of a reservoir in the ith reach = 1 or 0.

$\beta_i$ = Indicator of presence of one or more water treatment plants in the ith reach = 1 or 0.

$M_i$ = Total number of water treatment facilities in the ith reach.

The values of $\alpha$ and $\alpha'$ are opposite to each other so that either a reservoir or treatment plant but not both will appear at each stage.

The total cost function which represents the problem objective is given as
\[ C_T = \sum_{i=1}^{N} C_i = \sum_{i=1}^{N} \alpha_i \bar{W}_i (E_i, L_{0i}, Q_i) + \alpha_i \bar{R}_i (O_i, Q_i, L_{0i}) \]

(3.11)

\[ M_i \sum_{j=1}^{M_i} \bar{P}_{ij} (E_i, L_{0i}, Q_i) - \bar{E}_i (Q_i, L_{0i}, \alpha_i E_i) \]

The objective of the analysis will be to select \( E_i \) and \( O_i \) which minimize \( C_T \).

The constraints in the general problem concern principally meeting water quality standards specified by regulatory agencies and minimum stream flow rates as specified by contract. In describing pollutant concentrations in the remainder of this report, \( L_{0i} \) and \( \ell_i \) represent respectively the initial concentration of the key pollutant (assumed to be BOD here) at the entrance to the \( i \)th stage and the intermediate BOD concentration at some "within-stage" control point. If the variable \( \ell \) is doubly subscripted (i.e. \( \ell_{ij} \)), it represents the influent concentration of key pollutant for the \( j \)th water treatment plant within the \( i \)th stage. BOD-DO models will be used as quality constraints and state transformations in this development. In the particular formulation illustrated in this section, the O'Connor - DiToro model will be used to illustrate the use of a complex BOD-DO relation within a dynamic programming context. The reader should realize that any other stream - quality models could be used - possibly requiring less effort.
The functional diagram for the dynamic programming formulation of the water quality - quantity methodology is given in Figure 3.4. A diagramatic representation of the inner parts of a stage is shown in Figure 3.5. The variable, \( q \), represents within stage quantities of water demands and return flows. Singly subscripted variables represent return flows (i.e. \( q_i \) represents the waste return flow in the \( i \)th stage) and doubly subscripted variables are used for water demands (i.e. \( q_{ij} \) represents the quantity of water intake at the \( j \)th water user intake point in the \( i \)th stage).

Consider now the restrictions on the dynamic programming problem. The decision variable, \( E_i \), must be made such that the allowable concentrations of BOD and DO in the stream are not violated. The O'Connor - DiToro equation given in Equation 2.6 can be partially differentiated with respect \( x \) and \( t \) to find the point at which the minimum dissolved oxygen would occur. The time, \( t_{mi} \), and distance, \( x_{mi} \), to the occurrence of the point of minimum dissolved oxygen content are obtained solving Equations 3.12 and 3.13 simultaneously.

\[
\frac{D_{oi}}{D_{mi}} = \sum_{n=1}^{\infty} \frac{2\pi n b_{ni}}{N_{k_{ai}}^2 + (2\pi n)^2} \sin \left[ 2\pi n \left( t_{mi} - t_{si} - \frac{P_i}{2} \right) \right] - \tan^{-1} \left( \frac{2\pi n}{k_{ai}} \right)
\]

\[
+ \sum_{n=1}^{\infty} \frac{2\pi n b_{ni}}{N_{k_{ai}}^2 + (2\pi n)^2} \sin \left[ 2\pi n \left( t_{mi} - t_{si} - \frac{P_i}{2} \frac{x_{mi}}{u_i} \right) \right] - \tan^{-1} \left( \frac{2\pi n}{k_{ai}} \right)
\]

(3.12)
Figure 3.4

FUNCTIONAL DIAGRAM OF A WATER QUALITY QUANTITY MANAGEMENT PROBLEM
Figure 3.5
SCHEMATIC REPRESENTATION OF A STAGE
\[ D_{0i} e^{-\frac{k_{ai} x_{mi}}{u_i}} (1 - k_{ai} \frac{x_{mi}}{u_i}) + \psi_i(x_{mi}, t_{mi}) \]

\[ + \frac{k_{di}}{k_{ai} - k_{di}} (k_{ri} e^{-\frac{k_{ri} x_{mi}}{u_i}} - k_{ai} e^{-\frac{k_{ai} x_{mi}}{u_i}}) \]

\[ + \frac{k_{ni} \eta_i}{k_{ai} - k_{ni}} (k_{ni} e^{-\frac{k_{ni} x_{mi}}{u_i}} - k_{ai} e^{-\frac{k_{ai} x_{mi}}{u_i}}) = e^{-\frac{k_{ai} x_{mi}}{u_i}} (s + k_{ai} R) \]

where

\[ \psi_i(x_{mi}, t_{mi}) = p_m \left\{ \frac{2p_i}{\pi} e^{-\frac{k_{ai} x_{mi}}{u_i}} + k_{ai} e^{-\frac{k_{ai} x_{mi}}{u_i}} \sum_{n=1}^{\infty} \frac{b_{ni}}{\sqrt{k_{ai}^2 + (2\pi n)^2}} \right\} \]

\[ \cos [2\pi n (t_{mi} - t_{si} - \frac{P_i}{2} \frac{x_{mi}}{u_i})] - \tan^{-1} \left( \frac{2\pi n}{k_{ai}} \right) \]

\[ + 2\pi e^{-\frac{k_{ai} x_{mi}}{u_i}} \sum_{n=1}^{\infty} \frac{nb_n}{\sqrt{k_{ai}^2 + (2\pi n)^2}} \sin [2\pi n (t_{mi} - t_{si} - \frac{P_i}{2} \frac{x_{mi}}{u_i})] \]

\[ - \tan^{-1} \left( \frac{2\pi n}{k_{ai}} \right) \]
where

\[ i = \text{Subscript denoting the stage number.} \]

\[ D_{mi} = C_{si} - C_{mi} = \text{Maximum dissolved oxygen deficit in ith reach.} \]

\[ C_{si} = \text{Saturation concentration of dissolved oxygen in ith river reach.} \]

\[ C_{mi} = \text{Minimum dissolved oxygen requirement in the ith reach.} \]

Using the O'Connor - DiToro approach, it is necessary to account for the nitrogenous BOD in the stream. The nitrogenous BOD can be treated as an additional sink for DO in determining a value for \( E_{ai} \). However, it need not be carried as an additional state variable in the staged optimization of the problem. Because there exists a ratio between the nitrogenous and carbonacious BOD concentrations in each specific treated-waste effluent, it is possible to define a relation

\[ N_i = \mu_i E_i \] (3.14)

with \( N_i \) representing the nitrogenous BOD concentration in the effluent. Then

\[ n_i = \frac{\mu_i E_i q_i + O_i N_{0i}}{Q_i + q_i} \] (3.15)

where \( n_i = \text{The nitrogenous BOD concentration downstream of the treated-waste return.} \)

and \( N_{0i} = \text{The nitrogenous BOD concentration upstream of the waste return point.} \)
The maximum concentrations of nitrogenous and carbonaceous BOD in the stream below a waste return point may be computed using the equations shown in Equation 3.13.

\[
\ell_{ai} = \frac{E_{ai} q_i + Q_i L_{oi}}{Q_i + q_i}
\]  

(3.16)

\[
n_{ai} = \frac{v_{ai} q_i + Q_i L_{oi}}{Q_i + q_i}
\]

Where \(E_{ai}\), \(\ell_{ai}\), and \(n_{ai}\) represent respectively the computed maximum allowable concentration of carbonaceous BOD in the effluent, the maximum allowable carbonaceous BOD concentration in the stream below the point of waste discharge, and the maximum allowable concentration of nitrogenous BOD in the stream below the point of waste discharge.

Using the values of \(t_{mi}\) and \(x_{mi}\) found in Equations 3.12 and 3.13 along with the two equations (3.16) and the equation below (3.17), the computed maximum allowable concentration of the BOD in the effluent, \(E_{ai}\), can be determined.
\[ D_{mi} = D_{oi} \left( t_{mi} - \frac{x_{mi}}{u_i} \right) e^{-k_{ai} \frac{x_{mi}}{u_i}} + \frac{k_{di} \ell_{ai}}{k_{ai} - k_{di}} \left( e^{\frac{k_{di}}{k_{ai} - k_{di}} - \frac{k_{di}}{k_{ai} - k_{di}}} \right) \]

\[ + \frac{k_{ni} n_{ai}}{k_{ai} - k_{di}} \left( e^{-\frac{k_{ni}}{k_{ai} - k_{di}}} - e^{\frac{k_{ni}}{k_{ai} - k_{di}}} \right) \]

\[ + \left( \frac{S_i}{k_{ai}} + R_i \right) (1 - e^{-\frac{k_{ai}}{k_{ai} - k_{di}}}) - p_{mi} \left\{ \frac{2p_i}{n_k a_i} \left( 1 - e^{-\frac{k_{ai}}{k_{ai} - k_{di}}} \right) \right\} \]

\[ + \sum_{n=1}^{\infty} \frac{b_{ni}}{N_{k_{ai}^2 + (2\pi n)^2}} \left[ \cos \left( 2\pi n (t_{mi} - t_{si} - \frac{p_i}{2}) - \tan^{-1} \left( \frac{2\pi n}{k_{ai}} \right) \right) \right] \]

\[ - e^{-\frac{k_{ai}}{u_i}} \sum_{n=1}^{\infty} \frac{b_{ni}}{N_{k_{ai}^2 + (2\pi n)^2}} \left[ \cos \left( 2\pi n (t_{mi} - t_{si} - \frac{p_i}{2} - \frac{x_{mi}}{n_i}) \right) \right] \]

\[ - \tan^{-1} \left( \frac{2\pi n}{k_{ai}} \right) \]
Sometimes, there may exist additional arbitrary regulation on pollutant concentration in waste return flow, $E_{mx}$, (e.g. BOD concentration in treated waste return flow cannot exceed 20 mg/l in Texas). Then, comparing the $E_{ai}$ value computed from optimums 3.12 and 3.13 with this arbitrary maximum allowable concentration requirement, $E_{mx}$ apply the smaller one as effective constraint.

Thus, in the dynamic programming formulation, the functional equation guiding optimization becomes

$$f_n(Q_n, L_{0n}) = \min \left[f_n(Q_n, L_{0n}, E_n \text{ or } O_n)\right]$$

$$E_n \text{ or } O_n$$

where

$$g_n(Q_n, L_{0n}, E_n \text{ or } O_n) = \begin{cases} 
C_n + f_{n-1}(Q_{n-1}, L_{0n}, E_n \text{ or } O_n) & n \geq 2 \\
C_1, & n = 1 
\end{cases}$$

$C_1$ is as defined in Equation 3.10 and the constraints include:

$$E_i \leq \min \{E_{mi}, E_{ai}\}$$

$$N_i = \mu_i E_i$$

$$n_i = \frac{\mu_i E_i q_i + Q_i N_{bi}}{Q_i + q_i}$$
and

\[ O_{mi} \leq O_i \leq O_{si} \]  \hspace{1cm} (3.21)

where \( O_{si} \) = maximum allowable release from the reservoir in \( i \)th reach based on total storage capacity,

\( O_{mi} \) = minimum required release from the reservoir in \( i \)th reach specified by contracts.

The state transformation for the river quality and quantity are given below

\[ N_{oi} = n(i+1) e^{-K_n(i+1)T(i+1)} \]  \hspace{1cm} (3.22)

\[ L_{oi} = \ell_{i+1} e^{-K_r(i+1)T(i+1)} = L_{oi+1} \]  \hspace{1cm} (3.23)

\[ \ell_{ij} = \ell_{i} e^{-K_{ri}T_{ij}} \]  \hspace{1cm} (3.24)

\[ Q_i = r_{i+1}(Q_{i+1} + q_{i+1}) + r_{i+1}O_{i+1} - B_{i+1} \sum_{j=1}^{M_{i+1}} q_{i+1} - V(i+1) \]

\[ = \tilde{Q}_{i+1} \]  \hspace{1cm} (3.25)
where \( n_{i+1} \) = Nitrogenous BOD carried downstream of the beginning point of \((i + 1)^{th}\) reach

\( \ell_{i+1} \) = Carbonatous BOD carried downstream of the beginning point of \((i + 1)^{th}\) reach

\( T_{(i+1)} \) = Travel time between the beginning points of \(i^{th}\) and \((i + 1)^{th}\) river reaches

\( T_{ij} \) = Travel time between the beginning point of \(i^{th}\) reach and the water intake of \(j^{th}\) water treatment plant

\( Q_i \) = River flow to \(i^{th}\) river reach

\( Q_{i+1} \) = River flow from \((i+1)^{th}\) river reach while \((i+1)^{th}\) river reach consistency of reservoir operation.

\( V_{i+1} \) = Evaporation loss of river water in \((i+1)^{th}\) reach

in which Equations (3.22)(3.23), (3.24) represent the BOD decay which occurs in the natural stream purification process.

As long as there are both reservoir and waste water treatment plants located upstream, this two-state, one decision multistage optimization process must be utilized. But, wherever either reservoir or waste water treatment plant is not located upstream, one of these two state variables can be dropped by treating it as constant. Then, the problem becomes a one-state, one-decision optimization problem.
CHAPTER IV
APPLICATION OF MODEL

In order to present a clear picture of optimization procedure of this river basin water quality management model, two example problems are presented. The first example is concerned with total quantity and quality management of a river basin while the second example is focused upon the quality management only. A computer program was written to utilize the methodology presented in problem solution. A flow diagram for the program is included in this chapter. Finally, a brief discussion regarding the implementation of the model is presented. A diagram describing a general system to which this methodology is applicable is given in Figure 4.1.

Illustrative Example I

In this problem the objective is to find optimal management policies based on minimum costs for waste water treatment, water treatment and reservoir operation in a river system which consists of three water supply-waste complexes and one multipurpose reservoir as shown in Figure 4.2.

The lack of adequate water quality and cost data severely hampered the application of the model developed to a practical water planning problem. Adequate water quality data particularly relating to organic constituents is almost non-existent except for general references [22,35] and programs begun within the last two years [31]. Costs for waste treatment operations are available only on a limited scale [33]. Water
Figure 4.1
GENERALIZED WATER QUALITY-QUANTITY MANAGEMENT SYSTEM
treatment costs are non-existent outside consulting engineering offices. Data describing the quality of waste effluents are available to a limited degree in the files of the Texas Water Quality Board and in a general form in published works [38,39]. Water use information is usually not keyed to specific water treatment plants [36]. Thus, short of an expensive sampling program, it was not possible to obtain field-level data with which to exercise the model. Using the published data as a guide and the authors' experience, the data used in these examples were developed.

In Figure 4.2, the treated waste effluents are designated as $W_1$, $W_2$, and $W_4$, and intakes for water supply are designated as $P_{11}$, $P_{21}$, and $P_{42}$. The reservoir is designated at $R_2$, water intake located downstream of reservoir $R_2$ is designated at $P_{21}$. The cost functions for waste treatment are expressed in terms of treated waste effluent quality, $E_i$, as shown in Figure 4.3. Those for water treatment are described as being dependent upon river quality downstream of the outfall of the treated waste effluent, $E_i$, Figure 4.4. In constructing the cost functions, qualitative and quantitative characteristics of the raw water and the purification requirement must be considered simultaneously. The cost function for reservoir operation is described in terms of the quantity of the flow to be released and the quality of the water to be maintained in Figure 4.5. The costs for reservoir operation were derived from the direct benefits due to recreation and aesthetic values as well as the opportunity cost associated with the specific quality of the water.

The maximum allowable BOD in the treated waste effluent and the minimum allowable DO in the river are assumed to be 30 mg/l and 2 mg/l respectively. Because of assumed quality control upstream, the initial
L_i  BOD UPSTREAM OF i^{th} WASTE OUTFALL
L_{i+} BOD DOWNSTREAM OF i^{th} WASTE OUTFALL
q_i  TREATED WASTE EFFLUENT FLOW RATE IN i^{th} REACH
E_i  BOD CONC. IN TREATED WASTE EFFLUENT
Q_i  RIVER FLOW TO i^{th} REACH
q_{ij}  FLOW RATE TO j^{th} WATER TREATMENT IN i^{th} REACH

Figure 4.2
QUALITY-QUANTITY MANAGEMENT - EXAMPLE SYSTEM 1
$q_1 = 10\text{mgd}$
$q_3 = 15\text{mgd}$
$q_4 = 16\text{mgd}$

**Figure 4.3**
WASTE TREATMENT COST RELATIONS
Example 1
Figure 4.4
WATER TREATMENT COST RELATIONS
Example 1
Figure 4.5
FLOW AUGMENTATION COST RELATIONS
Example 1
BOD loading in this system is assumed not to exceed 12 mg/l. Therefore, the maximum allowable BOD in the effluent will be assumed to be 30 mg/l. This simplification is made to preserve the clarity of the application and promote understanding of the optimization model. However, in actual implementation, the development of the maximum allowable BOD level has to be included in the computation. Assuming that BOD concentration in the treated waste effluent are all 30 mg/l, the maximum BOD concentration upstream of each of the waste outfalls are calculated to be respectively, 12, 10, and 12 mg/l for \( W_4 \), \( W_3 \), and \( W_1 \). The BOD decay functions between the initial points of two consecutive stages were computed using stream data and presented as transfer functions in Figure 4.6. The relationships between \( L_{0i} \), \( \ell_i \), and \( E_i \), for different treated waste-effluents were computed as,

\[
\ell_3 = \frac{1}{115} \left( 100 \, L_{03} + 15E_3 \right)
\]

\[
\ell_4 = \frac{1}{116} \left( 100 \, L_{04} + 16E_4 \right)
\]

The relationship between \( L_{0i} \), \( \ell_i \), and \( Q_i \) for the stages located downstream of reservoir \( R_2 \) are as follows:

\[
\ell_2 = T_2(L_{02})
\]

\[
\ell_1 = \frac{Q_1 \, L_{01} + 8E_1}{Q_1 + 8}
\]

\[
Q_1 = Q_2 - 10
\]

For computational convenience, representative values of the regulated streamflow quantities which can occur were determined from published data [37] and expressed in MGD (million gallons per day) for computational convenience as follows:
Figure 4.6
WATER QUALITY STATE TRANSFORMATIONS
Example 1
1) Regulated flow range in first reach, $Q_1 = 20 - 300$ (MGD)
2) Waste water treatment $W_1$, $q_1 = 8$ (MGD)
3) River flow in second reach, $Q_2 = 30 - 310$ (MGD)
4) Water treatment $q_2 = 10$ (MGD)
5) River flow in third reach, $Q_3 = 100$ (MGD)
6) Waste treatment $W_3$, $q_3 = 15$ (MGD)
7) River flow in fourth reach, $Q_4 = 100$ (MGD)
8) Waste water treatment $W_4$, $q_4 = 16$ (MGD)

As described in Chapter III, the water quality and quantity management for a river basin involves the minimization of the total cost for waste treatment, water treatment, and reservoir will be typical two-state, single-decision, initial value, serial dynamic programming problem. For all the stages downstream of any reservoir there will be two states for each stage. One state variable is the regulated flow quantity in the river. Another is the concentration of BOD in the stream upstream of each waste outfall for stages including waste treatment. For the stages including reservoir release and water treatment, the other state variable will be the BOD concentration in the river flow at the entrance of a reservoir. For those stages located in the most upstream reaches of the river where there is not any reservoir located upstream of them, there will be only one state variable which is the BOD concentration upstream of the waste in the treated-waste effluent.

The general approach of the application of this optimization procedure to Example 1 can be described as follows:
1) Use the BOD concentration upstream of the waste outfall as one state variable and the streamflow quantity as another state variable, the stage return including the cost for waste treatment and water treatment facilities is computed for different values of the decision variable, the concentration of BOD in the treated-waste effluent (varying between 0 and 30 mg/l).

2) Select the minimum cost stage return for different values of initial BOD concentration upstream of the treated-waste outfall and of quantity of the streamflow. Store the optimal returns and the optimal decisions for the waste treatment plant in a two-dimensional arrays.

3) Combine the optimal stage return of the last stage together with the stage return of the next-to-last stage. Using the BOD decay function as the transition function between the stages, the two-stage, three dimensional arrays can be constructed.

4) Select the minimum return values for each row in the combined stage return (cubic matrix) and the optimal two-stage return table can be obtained.

5) For the stage including only reservoir operations, the combined stage return can be computed similar to the others. However, the decision variable for the reservoir stage is the quantity of flow to be released to the downstream reaches.

6) Continue the optimization as shown in steps 2) to 4) for the rest of stages until the stage with the most upstream reservoir in the river system is included.

7) For the reservoir stage located upstream of any other reservoir, the stage variable will be reduced to only one (the BOD concentration in
the river water at the entrance of the reservoir).

8) Optimize the remaining stages as single-state, single-decision, initial-value problems where the state variable is the initial concentration of BOD upstream of the waste outfall and the decision variable is the BOD concentration in the treated-waste effluent.

Following the optimization methodology described in Chapter III, the first stage of the optimal return can be obtained by applying the functional equation as follows:

\[ f_1 (L_{01}, Q_1) = \min_{E_1} \left( \bar{W}_1 (E_1) + \bar{P}_{11} (L_{01}, E_1, Q_1) \right) \]

\[ = \min_{E_1} \left( \bar{W}_1 (E_1) + \bar{P}_{11} (\xi_1) \right) \]

(4.4)

where \( 0 \leq L_{01} \leq 12; \ 20 \leq Q_1 \leq 300 \)

\[ 0 \leq E_1 \leq 30 \]

and \( \xi_1 = \frac{(Q_1 L_{01} + 8E_1)}{(Q_1 + 8)} \)

With the cost values obtained from Figure 4.3 and 4.4, the stage return for the first reach of the river can be summarized as shown in Table 4.1. As expressed in Equation 4.4 above, the stage return for the first reach of the river includes the waste treatment cost for \( \bar{W}_1 \) and the water treatment cost for \( \bar{P}_{11} \). The cost for waste treatment \( \bar{W}_1 \) is obtained from Figure 4.3 based on the BOD concentration, \( E_1 \), in the treated-waste effluent. The cost for water treatment \( \bar{P}_{11} \) depends upon the concentration of BOD downstream of the waste outfall, \( \xi_1 \). As it is defined in Equation 4.4, the concentration of BOD downstream of the waste outfall, \( \xi_1 \), is changing with both the concentration of BOD upstream of
TABLE 4.1
Stage Return of First Reach, C₁

<table>
<thead>
<tr>
<th>L₀₁ (mg/l)</th>
<th>E₁ (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>6850</td>
</tr>
<tr>
<td>10</td>
<td>5920</td>
</tr>
<tr>
<td>8</td>
<td>5200</td>
</tr>
<tr>
<td>6</td>
<td>4480</td>
</tr>
<tr>
<td>4</td>
<td>3710</td>
</tr>
<tr>
<td>2</td>
<td>3385</td>
</tr>
<tr>
<td>0</td>
<td>2925</td>
</tr>
</tbody>
</table>
the waste outfall and the quantity of the river flow in the reach, \( Q_1 \). Thus, the value for the cost of water treatment, \( P_{11} \), must be obtained through the use of \( \ell_1 \) which is expressed in terms of \( L_{01} \), \( E_1 \), and \( Q_1 \).

For instance, when \( Q_1 = 100 \) MGD, \( L_{01} = 12 \) mg/l, and \( E_1 = 30 \) mg/l, \( \ell_1 \) is calculated to be 12.33 mg/l from Equation 4.4. Then, using \( \ell_1 \) equal to 12.3 mg/l, \( P_{11} \) equals $663,000 when read from Figure 4.4. The cost for the waste water treatment plant \( W_1 \) is found from Figure 4.3 based on \( E_1 \) of 30 mg/l. Thus,

\[
C_1 = W_1 + P_{11} = 663,000 + 22,000 = 685,000
\]

With the variation of both the values of BOD concentration upstream of the waste outfall, \( L_{01} \), and the quantity of the flow in the river in the first reach, \( Q_1 \), the stage return for the first reach of the river can be constructed as a three dimensional array. For each combination of the two state variables in the three dimensional table, the decision yielding the minimum cost is selected. The optimal decisions and returns are shown in Tables 4.2 and 4.3.

The stage return for the second reach of the river, \( C_2 \), may be obtained through the use of both Figures 4.4 and 4.5. The relationship shown in Equation 4.5 is applied,

\[
C_2 = \overline{R}_2 (L_{02}, 0_2, \ell_2) + P_{21} (L_{02})
\]

where \( 0 < L_{02} < 12; \)

\[
\ell_2 = L_{02}
\]

and \( Q_2 = Q_1 + 10 \)

As expressed in Equation 4.5, the cost for the second stage is equal to the total cost of both reservoir operation, \( \overline{R}_2 \), and the cost of water
### TABLE 4.3
Optimal Policies, $E_1^*$, for First Reach

<table>
<thead>
<tr>
<th>$L_0$</th>
<th>$E_1^*$ (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mg/l)</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
treatment $F_{21}$. The cost for reservoir operation, $R_{2}$, varies with the quantity of flow to be released, $O_{2}$, and the quality of release, $L_{2}$.

The cost for water treatment, $P_{21}$ varies with the concentration of BOD to be maintained in the reservoir, $L_{2}$. The quality of water released from reservoir, $L_{2}$, is assumed to be the same as that entering the reservoir, $L_{02}$. Then, with the use of Figure 4.6 which contains the BOD decay relationships and serves as a state transformation to link $L_{02}$ and $L_{01}$, the two stage optimal return can then be obtained as shown in Table 4.4 following Equation 4.6.

$$f_{2}(L_{02},Q_{2}) = \min_{02} \left[ C_{2} + f_{1}(t_{2}(L_{02}, Q_{2})) \right]$$

where $30 \leq Q_{2} \leq 310$

$$0 \leq L_{02} \leq 12$$

Using Figure 4.6, the $L_{01}$ value can be defined in terms of the values of $L_{02}$. The minimum cost for the first reach of the river, $f_{1}$, corresponding to each of the $L_{02}$ values can then be determined. Combining this $f_{1}$ value together with the values of $C_{2}$, the two-stage return is then obtained. For instance, when the $L_{02}$ value = 10 mg/l, the $L_{01}$ will be 9.9 mg/l based on Figure 4.6. According to Table 4.2, the optimal return of the first reach of the river, $f_{1}$, is thus found to be $545,400$ for an $L_{01}$ value of 9.9 mg/l. Therefore, the return for stages 1 and 2 is $851,800$ while $Q_{2} = 160$ MGD. The optimum returns and decisions at stage 2 are shown in Table 4.4 by asterisks.

It should be noted that the decision variable for the second reach of the river is the quantity of the river flow to be regulated which is different from the decision variable of the first reach of the river.
TABLE 4.4

Two-Stage Return of Reaches 1 and 2.

\[ C_2 + f_1[t_1(Q_2 \cdot L_{O2})] \]

\[ (\$ \times 100/yr.) \]

<table>
<thead>
<tr>
<th>( L_{O2} ) (mg/l)</th>
<th>( Q_2 ) (MGD)</th>
<th>30</th>
<th>60</th>
<th>110</th>
<th>160</th>
<th>210</th>
<th>260</th>
<th>310</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td>7642*</td>
<td>8163</td>
<td>8423</td>
<td>8518</td>
<td>8557</td>
<td>8595</td>
<td>8610</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>6720*</td>
<td>7353</td>
<td>7240</td>
<td>7313</td>
<td>7350</td>
<td>7378</td>
<td>7492</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>5946*</td>
<td>6239</td>
<td>6334</td>
<td>6379</td>
<td>6399</td>
<td>6397</td>
<td>6440</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>5331*</td>
<td>5482</td>
<td>5523</td>
<td>5569</td>
<td>5532</td>
<td>5514</td>
<td>5626</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>4178</td>
<td>4167</td>
<td>4026*</td>
<td>4079</td>
<td>4136</td>
<td>4172</td>
<td>4203</td>
</tr>
</tbody>
</table>
Also it should be noted that the optimization will become a single-state, single-decision serial problem starting above the second stage. This will be true at any point in the stream above which no further stream-flow regulation is possible.

The remaining stages are processed in the manner described above using the functional equation shown in Equation 4.7 as a guide.

\[
f_n(L_{On}) = \min \left[ C_n + \sum_{i=1}^{n-1} (L_{On}) \right]
\]

The results are shown in Tables 4.5 and 4.6.

Finally, the optimal decision values for each stage in terms of the initial BOD loading upstream of the river reach 4 are summarized together with their optimal annual cost as presented in Table 4.7. It should be noted that, in this example, the least-cost policy for river quality management is not a policy of meeting maximum allowable BOD loading for waste water treatment only. It varies significantly with the tradeoff between the waste water treatment costs, water purification costs, and the maintenance and opportunity costs associated with the reservoir operations involved in a normal river system.

*Illustrative Example II*

The proposed model described in Chapter III may also be applied directly to the river system shown in Figure 4.7. This example does not contain flow augmentation but does contain water treatment operations. Because flow regulation does not appear in the problem, the dynamic programming formulation does not include regulated flow as a state variable. Thus, a one-state, one-decision problem results which will be solved below. All variables are described as in the previous example problem.
TABLE 4.5
Third Stage Return of Reaches 1, 2, and 3.

\[ C_3 + f_2[t_2(L_{03})] \]

\((\$ x 100/yr.)\)

<table>
<thead>
<tr>
<th>(L_{03}) (mg/l)</th>
<th>(E_3) (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>7982*</td>
</tr>
<tr>
<td>10</td>
<td>7982</td>
</tr>
<tr>
<td>8</td>
<td>7982</td>
</tr>
<tr>
<td>6</td>
<td>7521</td>
</tr>
<tr>
<td>4</td>
<td>6670</td>
</tr>
<tr>
<td>2</td>
<td>6128</td>
</tr>
<tr>
<td>0</td>
<td>5490</td>
</tr>
</tbody>
</table>
TABLE 4.6
Fourth Stage Return for Reaches, 4th, 3rd, 2nd,
And 1st, $C_4 + f_3(t_3|L_{04}|)$
($\$ \times 100/yr.$)

<table>
<thead>
<tr>
<th>$L_{04}$ (mg/l)</th>
<th>30</th>
<th>25</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>25902</td>
<td>25622</td>
<td>24560</td>
<td>24620</td>
<td>23590</td>
<td>23370</td>
<td>22800*</td>
</tr>
<tr>
<td>10</td>
<td>25462</td>
<td>23750</td>
<td>23240</td>
<td>23040</td>
<td>22430</td>
<td>23390</td>
<td>21651*</td>
</tr>
<tr>
<td>8</td>
<td>23110</td>
<td>22450</td>
<td>22470</td>
<td>21750</td>
<td>21430</td>
<td>20220</td>
<td>19300*</td>
</tr>
<tr>
<td>6</td>
<td>21510</td>
<td>21451</td>
<td>20780</td>
<td>19458</td>
<td>18270</td>
<td>17230*</td>
<td>23650</td>
</tr>
<tr>
<td>4</td>
<td>17680</td>
<td>18600</td>
<td>17380</td>
<td>16976</td>
<td>16000</td>
<td>15920</td>
<td>15300*</td>
</tr>
<tr>
<td>2</td>
<td>16700</td>
<td>16300</td>
<td>15780</td>
<td>15470</td>
<td>14750</td>
<td>14070</td>
<td>13400*</td>
</tr>
<tr>
<td>0</td>
<td>15870</td>
<td>15210</td>
<td>17270</td>
<td>13660</td>
<td>13449</td>
<td>12970</td>
<td>12515*</td>
</tr>
<tr>
<td>$L_{04}$ (mg/l)</td>
<td>$E_{4}^{*}$ (mg/l)</td>
<td>$E_{3}^{*}$ (mg/l)</td>
<td>$O_{2}^{*}$ MGD</td>
<td>$E_{1}^{*}$ (mg/l)</td>
<td>Total Cost ($$ x 100/yr.$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>5</td>
<td>30</td>
<td>0</td>
<td>22800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>0</td>
<td>21651</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>10/5</td>
<td>30</td>
<td>0</td>
<td>19300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>17230</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>5</td>
<td>46</td>
<td>5</td>
<td>15300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>5</td>
<td>74</td>
<td>10</td>
<td>13400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>5</td>
<td>100</td>
<td>15</td>
<td>12515</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.7
QUALITY MANAGEMENT
Example System 2
In this example, there are three waste treatment outfalls, \( W_3 \), \( W_2 \), and \( W_1 \), together with five water supply intakes, \( P_{31} \), \( P_{32} \), \( P_{21} \), \( P_{11} \), and \( P_{12} \), given in this system. In this notation \( P_{31} \) and \( P_{32} \) denote the first and second water treatment plants with intakes located downstream of the third treatment plant outfall.

Cost functions for waste and water treatment are given in Figures 4.8 and 4.9 respectively. Water treatment costs are given as a function of the intake BOD concentration, \( \ell_{ij} \), and waste treatment costs are conveniently specified for values of effluent concentration, \( E_i \). The water quality state transformation was computed using a BOD decay relation and converted to a graphical interpretation shown in Figure 4.10. Use of Figure 4.10 enables one to quickly determine how the BOD concentration varies between stages.

For this problem, it will be assumed that the maximum allowable BOD in the treated waste effluent and the minimum DO in the river are 30 mg/l and 2 mg/l respectively as specified by the pollution control agency. Furthermore, initial BOD loading is less than or equal to 10 mg/l and average initial DO is 7 mg/l. It is assumed that the allowable BOD concentrations based on Equations 3.12 and 3.13 and minimum DO requirements would give the maximum \( E_{a1} \) values greater than 30 mg/l. Thus, the maximum allowable BOD concentration of 30 mg/l in the effluent of each waste treatment plant becomes the limiting criterion for quality control in this problem.

For example, with the use of Equation 3.17, the requirement of a minimum DO of 2 mg/l in the third river reach will yield a value of \( \ell_{a3} \).
Figure 4.8
WASTE TREATMENT COST FUNCTIONS
Example 2
Figure 4.9
WATER TREATMENT COST FUNCTIONS
Example 2
Figure 4.10
WATER QUALITY STATE TRANSFORMATIONS
Example 2
the allowable in-stream BOD concentration, which when used with the
given maximum value of $L_{03} = 10$ in the following equation

$$E_{a3} = \frac{1}{q_3} \left[ (Q_3 + q_3) \ell_{a3} - 10 Q_3 \right]$$

produces a value of $E_{a3}$ larger than the 30 mg/l specified by the
pollution control agency. Therefore, $E_{a3}$ must satisfy the tighter
constraint of 30 mg/l imposed by the control agency. Assume that the
relationships between $L_{0i}$, $\ell_i$, and $E_i$ for different treated waste
outfall are as follows:

$$\ell_1 = 0.2 \left( 4 \, L_{01} + E_1 \right)$$
$$\ell_2 = 0.2 \left( 3 \, L_{02} + 2 \, E_2 \right)$$
$$\ell_3 = 0.5 \left( L_{03} + E_3 \right)$$

These relations together with Equations 3.12 and 3.15 and the given value of
$L_{03} = 10$ may be used to determine the largest possible values for $E_2$, $E_1$,
$L_{02}$ and $L_{01}$. For this problem it was found that the largest values
of $E_3 = E_2 = E_1 = 30$ mg/l and the highest values of $L_{03}$, $L_{02}$, and $L_{01}$
equal to 10, 10, 12, (mg/l).

As previously stated, the system may be viewed as an initial
value dynamic programming problem. Therefore, the optimization
procedure begins at stage one with the following minimization problem:

$$f_1 (L_{01}) = \min_{E_1} \left[ \bar{W}_1 (E_1) + \sum_{j=1}^{2} p_{1j} (\ell_{1j}) \right]$$

(4.9)
where

\[ 0 \leq L_{01} \leq 12; \ 0 \leq E_1 \leq 30 \]

\[ \ell_1 = 0.2 (4 \ L_{01} + E_1) \]

and \( \ell_{1j} \) is a function of \( \ell_1 \)

It is quite easy to construct a table of returns at the first stage for various values of the decision variable, \( E_1 \), and the state variable, \( \ell_1 \), by using Figures 4.8 and 4.9. Table 4.8 gives the results of these computations with a grid size of two. For example using \( L_{01} = 12 \) and \( E_1 = 30 \), the \( \ell_1 \) value is given by

\[ \ell_1 = 0.2 (4 \ L_{01} + E_1) = 0.2 (48 + 30) = 15.6. \]  

(4.10)

\( E_1 = 30 \) reveals a waste treatment cost of 23.5 from Figure 4.8.

The water treatment cost has been defined in terms of the function of the BOD concentration at the outflow of the waste treatment plant immediately upstream as shown in Figure 4.9. The water treatment costs can be determined from Figure 4.9 to be 12 and 8.3 for \( \bar{P}_{11} \) and \( \bar{P}_{12} \).

The total of water and waste treatment costs for this policy are

\[ C_1 = \bar{P}_{11} + \bar{P}_{12} + \bar{W}_1 = 12 + 8.3 + 23.5 = 43.8 \]

The remaining values of Table I may be calculated in a like manner. The asterisks in Table 4.8 denote the optimal one stage returns for each value of \( L_{01} \).

The second stage calculations may be obtained by solving the following optimization problem:
<table>
<thead>
<tr>
<th>$L_{01}$ (mg/l)</th>
<th>$E_1$ (mg/l)</th>
<th>30</th>
<th>25</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>43.8</td>
<td>43.4*</td>
<td>43.7</td>
<td>44.1</td>
<td>44.5</td>
<td>46.3</td>
<td>53.2</td>
<td></td>
</tr>
<tr>
<td>10</td>
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<td>42.0</td>
<td>42.5</td>
<td>42.6</td>
<td>42.7</td>
<td>44.7</td>
<td>51.8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>40.8*</td>
<td>41</td>
<td>41.2</td>
<td>41.5</td>
<td>41.2</td>
<td>43.4</td>
<td>50.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>39.5</td>
<td>39.5</td>
<td>39.4*</td>
<td>39.7</td>
<td>39.7</td>
<td>41.7</td>
<td>49.8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>37.8*</td>
<td>38</td>
<td>38.1</td>
<td>37.8*</td>
<td>38.2</td>
<td>39.8</td>
<td>47.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36.5</td>
<td>36.4*</td>
<td>36.6</td>
<td>36.6</td>
<td>36.5</td>
<td>38.5</td>
<td>45.5</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>34.8</td>
<td>35.0</td>
<td>35.1</td>
<td>35.8</td>
<td>34.4*</td>
<td>35.5</td>
<td>44.0</td>
<td></td>
</tr>
</tbody>
</table>
$$f_2(L_{02}) = \min_{E_2} \left \{ w_2(E_2) + \bar{p}_{21}(\ell_{21}) + f_1[t_1(L_{02})] \right \}$$  \hspace{1cm} (4.11)

where $0 \leq L_{02} \leq 10$; $0 \leq E_2 \leq 30$

$$\ell_2 = 0.2 \left( 3L_{02} + 2E_2 \right); \; \ell_{21} = \ell_2$$

For various values of $L_{02}$ and $E_2$, one may calculate $\ell_2$ which may be converted to $L_{01}$ by using Figure 4.10. The value of $L_{01}$ determines a unique optimal cost from Table I. The one stage optimal cost is then combined with the cost at stage two for water and waste treatment to yield a total optimal two stage return for the specified values of $L_{02}$ and $E_2$. This procedure is repeated for all values of $L_{02}$ and $E_2$ to obtain Table II.

For instance, for $L_{02} = 10$ and $E_2 = 30$, $\ell_2$ is given by

$$\ell_3 = 0.2 \left( 3L_{02} + 2E_2 \right) = 18.0$$  \hspace{1cm} (4.12)

Using Figure 4.10, one obtains $L_{01} = 10$ for $\ell_2 = 18$ and $f_1(10) = 41.8$ from Table 4.8. The total cost for water and waste treatment for $L_{02} = 10$, $E_2 = 30$ and $\ell_2 = 18$ is 49.5 from Figures 4.8 and 4.9 for stage 2. The optimal two stage return for $\ell_2 = 10$ and $E_2 = 10$ is $(49.5 + 41.8) = 91.3$. The results of these calculations for the full range of $\ell_2$ and $E_2$ are given in Table 4.9.

This same procedure may be applied to the third stage by solving the following problem:
TABLE 4.9
Two-Stage Return for 1st and 2nd Reaches
of River S, $C_2 + f_1 [t_1 (L_{O2})]$
($\times 10,000$/yr.)

<table>
<thead>
<tr>
<th>$L_{O2}$ (mg/l)</th>
<th>$E_2$ (mg/l)</th>
<th>30</th>
<th>25</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<td>92.7</td>
<td>92.4</td>
<td>90.8</td>
<td>89.8</td>
<td>87.9*</td>
<td>89.9</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>90.0</td>
<td>91.3</td>
<td>91.4</td>
<td>88.6</td>
<td>85.9*</td>
<td>36</td>
<td>88.4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>89.3</td>
<td>88.7</td>
<td>88.3</td>
<td>85.0</td>
<td>82.4*</td>
<td>83.9</td>
<td>84.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>87.6</td>
<td>86.4</td>
<td>85.8</td>
<td>80.9</td>
<td>80.4*</td>
<td>82.9</td>
<td>83.6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>86.7</td>
<td>84</td>
<td>83</td>
<td>79.3*</td>
<td>79.9</td>
<td>79.4</td>
<td>82.1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>83.8</td>
<td>81.8</td>
<td>78.4</td>
<td>77.9*</td>
<td>78.4</td>
<td>78.1</td>
<td>80.4</td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{align*}
    f_3(L_{03}) &= \min_{E_3} \left\{ \overline{W}_3(E_3) + \overline{P}_{31}(\ell_{31}) + \overline{P}_{32}(\ell_{32}) + f_2 [t_2(L_{03})] \right\} \\
    \text{where} & \\
    0 & \leq L_{03} \leq 10 \\
    0 & \leq E_3 \leq 30 \\
\end{align*}
\]

and
\[
\ell_3 = 0.5 (L_{03} + E_3) 
\]

The results for various values of \( E_3 \) and \( L_{03} \) are given in Table 4.10. Table 4.10 specifies the optimal three stage returns for values of \( \ell_3 \). Again, the asterisks indicate the optimal policies.

Table 4.11 presents the optimal policies and costs obtained by tracing back through previous calculations. It is of importance to note that the optimal policies for various initial BOD loadings do not correspond to the maximum allowable BOD discharges as allowed by the pollution control agency. Therefore, a concise systems approach to the interaction of water and waste treatment has shown that one must consider this interdependency in order to obtain the optimal water quality management policy. A policy which allowed maximum waste effluent discharges would fail to consider the effect upon water treatment costs and thus result in a higher total system cost.

**Computer Algorithm**

In the solution of any real problem, a computer algorithm must be utilized. In this investigation, a number of programs were developed
### TABLE 4.10
Three-Stage Return for 3rd, 2nd, and 1st Reaches of River $S, C_s + f_2 [t_2 (L_{03})]
($ x 10,000/yr.)

<table>
<thead>
<tr>
<th>$L_{03}$ (mg/l)</th>
<th>$E_3$ (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>140.9</td>
</tr>
<tr>
<td>8</td>
<td>138.5</td>
</tr>
<tr>
<td>6</td>
<td>138.1</td>
</tr>
<tr>
<td>4</td>
<td>137.8</td>
</tr>
<tr>
<td>2</td>
<td>136.2</td>
</tr>
<tr>
<td>0</td>
<td>135.0</td>
</tr>
</tbody>
</table>
TABLE 4.11
Optimal Management Policies and Minimum Annual Costs

<table>
<thead>
<tr>
<th>Initial BOD $L_{D3}$ (mg/l)</th>
<th>BOD in the Treated Waste (mg/l)</th>
<th>Total Annual Cost ($ x 10,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_3^*$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_2^*$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_1^*$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>129.2</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>130.5</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
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<td>20</td>
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<tr>
<td>10</td>
<td>20</td>
<td>136.9</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>30/15</td>
</tr>
</tbody>
</table>
and modified in the course of the research. The model settled on was written in FORTRAN-IV and has been implemented on the IBM 360/65 computer system at Texas A&M University. The program will solve problems of the type presented in this chapter.

The logical flow diagrams for the program are shown in Figures 4.11, 4.12, 4.13, 4.14, and 4.15. In addition to the flow charts shown, routines for one, two, and three dimensional table look-up and interpolation are included. Figure 4.11 depicts the logic diagram for the executive program which consists of little more than subroutine calls.

Figure 4.12 describes the basic dynamic programming subroutine. The dynamic programming routine calls subroutines for evaluating stage returns (Figure 4.13) and evaluating successive stage returns (Figure 4.14) to be combined with the stage returns. The last subroutine (Figure 4.15) for which a flow diagram is shown is the one for tracing the optimum policy after recursive computations are completed.

For simple problems of the sort given in this chapter, the execution time required to solve the problem is approximately 45 seconds. However, considerably more difficult problems including more stages and system complexity may be run without a great increase in computer time. The major problem encountered in using this algorithm involves the preparation of the data tables (equivalent to the curves presented in this chapter) for use in the model.
Figure 4.11
LOGICAL FLOW - EXECUTIVE PROGRAM
DPI
DYNAMIC PROGRAMMING ROUTINE

STAGE LOOP

INITIALIZE DECISION LIMIT
INITIALIZE STATE 1 LIMIT

TWO STATES

YES

INITIALIZE STATE 2 LIMIT

NO

STATE 1 LOOP

STATE 2 LOOP

COMPUTE DISCRETE STATE 2 VALUE

COMPUTE DISCRETE STATE 1 VALUE

SET STATE RETURN COMPARISON = ∞

Figure 4.12
DYNAMIC PROGRAMMING SUBROUTINE
Figure 4.12 Continued
Figure 4.12 CONTINUED
Figure 4.13
RETURN FUNCTION SUBROUTINE
Figure 4.14

Optimal Return Subroutine
POLICY

TRACE OPTIMAL POLICY

STAGE LOOP

INITIALIZE STATE 1 & 2

STATE 1 & 2 LOOP

COMPUTE TRANSITION FOR STREAM QUALITY & QUANTITY

LOOK UP OPTIMAL DECISION AND STAGE RETURN BASED ON STREAM QUALITY & QUANTITY

SAVE OPTIMAL DECISION AND STAGE RETURN

2

A

Figure 4.15
OPTIMAL POLICY SUBROUTINE
Figure 4.15 Continued
Implementation of the Model

In the original research schedule, it was planned to solve a field-level problem using data from the Trinity River in Texas. Available data for the flow and quality in the Trinity River were gathered and found to be inadequate to complete anything other than a reconnaissance level planning effort. If the data necessary to complete the field-level problem solution were gathered, a considerable amount of field work would have been required. The budgeted funds and available sampling equipment were not sufficient to conduct the study. Therefore, it was decided to spend the time developing a methodology which would be applicable and useful in a field-level planning effort when the data could be gathered. The water quality data are now being gathered by the Texas Water Development Board and the U. S. Geological Survey under a new cooperative program. Streamflow data are available. The project staff is now assembling waste treatment and water treatment quality and cost data which will be used to further test and strengthen the methodology presented in this report.

This research represents an important first step in combining water quality and water quantity considerations within a single planning model. In addition, the effect of waste treatment operations on water treatment is treated for the first time. Both of these considerations are believed by the authors to be of paramount importance to water planners.

In order to apply these techniques to a planning problem, a planner must first determine waste and water treatment costs as illustrated in this chapter. This can be accomplished for new plants by estimating
the construction and operation costs for plants with varying efficiencies. Costs for providing regulated flow releases must be determined from reservoir cost estimates and opportunity costs of the stored water. The planner must also select a stream-quality relation to be used to transform the water quality at one stream location to that at another. These data are then used to develop curves, tables, or functions of the form shown in this chapter to be applied within a dynamic programming format to obtain treatment efficiencies and reservoir releases which minimize total system costs.
CHAPTER V

DISCUSSION OF RESULTS

This report has presented a mathematical model for optimal water quality and quantity management in a general water resource system. The technique was applied to determine the minimum total cost policy in two illustrative examples given in the previous chapter. The results of these computations illustrate the important fact that effective water quality management must consider the off-site costs of water treatment which result from upstream waste discharges and the economic trade off available through flow augmentation. Effluents from waste treatment facilities which satify maximum BOD or minimum DO requirements will result in a minimum cost policy for waste treatment processes as independent units. However, since this policy directly affects the resultant cost of water treatment and reservoir releases and the return from the recreational use of the river, the total minimum cost system may not be obtained by using such a management approach.

Consider the results of the examples presented in this paper. The cost for overall water quantity and quality management is determined to be $2,165,100/year for the system in Example I while the initial BOD concentration in the river water is 10 mg/l. If all the waste treatment facilities were operated such that the maximum allowable BOD concentration requirement (20 mg/l in Texas) was barely satisfied, the total cost for the system would be $2,520,500 with \( E_1 = E_2 = E_3 = 20 \) mg/l and \( Q_2 = 150 \) MGD. This is about 20 percent higher than the cost
resulting from optimal policies summarized in Table 4.7. In addition to this economic savings, the river quality will definitely improve because all the optimal waste water treatment policies are providing a higher degree of treatment than the minimum requirement.

Similarly, the total cost of waste and water treatment for the system in Example II would be $1,463,000 if all waste effluent was treated to satisfy only the 30 mg/l requirement on effluent BOD. However, the optimal total cost of $1,369,000 was obtained and waste treatment costs in a systems context using the methodology described in Chapter III. The savings of $94,000 indicates a significant deficiency in the traditional management approach.

Unfortunately, the existence of two separate agencies for water pollution control and water supply does not necessarily enhance the possibilities of effective water quality management. The optimal management of river systems can only be realized by increasing the amount of interaction between such agencies. The consideration of mutual trade off studies between water and waste treatment is essential to good water quality management. The method presented in this report is designed to enable both agencies to study the mutual effects of their decisions in a search for an equitable management policy.

In order to present a simplified example problem, a branching problem was not solved in Chapter IV. Using the branch compression technique described in Chapter III, it is no more difficult to solve a branching system than it is to solve a serial system. The branches are solved first as two-point boundary problems using the stage combination
approach discussed in Chapter III. The optimum returns from the branches in terms of the output states are stored and input to the total system optimization as pseudo-stages at the location of the branch junction. Care should be taken when using this methodology to incorporate the state inversion (discussed in Chapter III under "computational aspects") into the branch solutions to minimize computation time.

The most severe limitation in the methodology presented in this report is its neglect of the stochastic aspect of the problem. Research is now under way at Texas A&M University to incorporate variability measures into the techniques presented herein to improve the incapability of the procedures.

It must be pointed out that more refined and more accurate cost information and performance data are required before this method can be broadly applied with confidence. Most information and data available in current literature sources are not readily adaptable to this method. This is particularly true of the data relating total annual cost as a function of plant size only. It is hoped that more accurate and reliable cost information may be derived in the future from the actual experience gained in treatment plant construction and operation.

One of the most neglected areas in economic evaluation for water quality management is the impact of quality on water treatment cost and direct recreational benefits. Scientists and engineers tend to place their emphasis on waste treatment cost alone. The cost for waste water treatment is purposely kept as low as possible. The present
practice of just meeting quality standards does not consider possible hazardous damage to the environment. The economic burden and sacrifice borne by the water users and the general public is often neglected. This results neither from the lack of concern of control agencies or the intention of pollution control scientists. However, it may be explained by the lack of use of a total integrated systems approach to water quality and quantity management.
LIST OF REFERENCES


