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# Investigation of a Linear Model to Describe Hydrologic Phenomenon of Drainage Basins

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# INVESTIGATION OF A LINEAR MODEL TO DESCRIBE HYDROLOGIC PHENOMENON OF DRAINAGE BASINS

Principal Investigator

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ABSTRACT

This investigation is concerned with the applicability of the linear convolution relationship for approximating the rainfall-runoff phenomenon for small drainage basins. A solution for the transfer function of the convolution relationship is obtained by employing discrete mathematics similar to the Wiener-Hopf equation. The solution is obtained, based on the restraints of the physical system by linear programming.

In this investigation, the hydrologic system is analyzed as a truly linear system. Recorded rainfall intensity is the *input* of the system, and recorded runoff *output*. A major concern of the study involves the effects of antecedent moisture conditions on the transfer function.

Two basins are used to test the model -- an urban basin located within the city limits of Bryan, Texas and a rural basin approximately three miles east of Bryan, Texas. Results are presented which substantiate the use of the proposed linear model as an approximation to the hydrologic system. Generalized transfer functions are developed for each basin and tested with independent events. Antecedent moisture conditions are shown to have a definite predictable effect on the transfer function, and rainfall events are classified with an antecedent moisture condition criteria in order to select the proper transfer function for the event.

Keywords -- rainfall-runoff relationships*/ hydrograph analysis/ surface runoff/ mathematical models/ hydrology/ watersheds/ unit hydrograph.

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# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# CHAPTER I

## INTRODUCTION

The concept of comprehensive management of our water resources has only had active attention during the past thirty years. During the past decade the public has come to realize that water resources, which are vital to man, must have comprehensive management. Senate Document No. 97 (1962) charges the federal agencies with the responsibility for planning the use of water and related land resources (52). Therefore, there has been increased attention and research on problems of water resource management during the past decade.

During this same decade, there also has been other research, seemingly unrelated to water resources, which has developed the technology for solving complex problems. This technology was developed under defense, space, and other research efforts. The opportunity to apply this new technology to water resources problems is great. The Office of Water Resources Research recognized this opportunity in the publication, "Areas of Defense and Space Technology Applicable to Water Resources Research" (31).

The influx of new, powerful and versatile technology, especially adaptable to computer use, prompted the Surface Water

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The citations on the following pages follow the style of the *Transactions of the American Society of Agricultural Engineers*.

Committee of the Hydrology Section of the American Geophysical Union to hold a symposium on analytical methods in hydrology in 1967. An insight to the reasoning behind this symposium can be found in the following quote from the introductory remarks in the published proceedings (51):

> Such rapid influx of new techniques into a discipline based in large degree on empiricism and self-perpetuating and limited methodology is inherently disturbing in that a gap may develop between research leaders and a large segment of practicing hydrologists. Of particular concern to the Committee were those hydrologists not located at research centers and, therefore, not in position to learn new methods through direct contact. It is the feeling of the Committee that the usual references to methodologies in the literature do not provide explanatory detail sufficient to enable most readers to gain a working competence. The concept of a symposium on analytical methods based on a principle of education and retraining evolved from all of these factors.

The need for research on the application of new techniques and mathematical models to water resources also was emphasized in the Office of Water Resources Research publication entitled, "Recommendations for Watershed Research Programs" (39). The panel which compiled this report stated:

> The panel considers that mathematical models are an effective method for analysis of watershed behavior. Perhaps up to one-half of the total watershed research efforts under P.L. 88-379 should be directed toward the development and improvement of general methods of analysis, and research in mathematical models is a good example of this type of activity. Research in digital computer models will be restricted mainly by the shortage of personnel and facilities for this type of study.

> The development of mathematical models is appropriate for both university and federal agency research. High hydrologic competence and the programming and statistical skills are needed in model development. The more elaborate mathematical models require medium to large-scale computers that use the most advanced programming languages. The principal research workers on these projects should combine technical knowledge of all the processes to be included in the mathematical models with a knowledge of computer programming and operation.

The hydrologic cycle can be described in qualitative terms. The principal components of the cycle are reasonably easy to identify and most of the interactions between the major components are known. The extension of this qualitative knowledge about the hydrologic cycle to obtain quantitative results is much more difficult. Few basic quantitative concepts exist in hydrology compared to other fields. The science of hydrology has not developed to the point of being a mathematically precise science.

A good portion of the hydrologic research has been directed toward the runoff-prediction problem. A brief look at hydrologic literature illustrates the tenacity with which hydrologists have attacked this problem. But, the development of hydrology to date does not, in general, permit adequate estimates of runoff response for a given area.

Basically, a method is needed whereby the runoff hydrograph can be predicted with an acceptable degree of reliability. Such a method must be sufficiently general and simple to allow its use under diverse watershed conditions. Only with such a reliable tool for prediction of runoff can water resource projects be designed on a rational basis; viz., to produce optimum conditions for a minimum cost.

## Study Objectives

The general objective of this research was to develop a simple method which is universally applicable that can be used in the prediction of storm runoff for small, natural drainage basins. The specific objectives were:

1. To determine the applicability of available linear mathematical models to the rainfall-runoff hydrologic phenomenon in small natural drainage basins;

2. To evaluate the ability of a specific lumped linear time-invariant system, the convolution relation, to represent adequately the rainfall-runoff hydrologic phenomenon of small, natural drainage basins; and

3. To investigate possible relationships between the unit-impulse response of the convolution relation and drainage basin characteristics provided the validity of the use of the convolution relation is substantiated.

# CHAPTER II

# LITERATURE REVIEW

The development of a method for the accurate prediction of runoff from small drainage basins has had the attention of many researchers in water resource development during the last century. Without a firm estimate of the expected runoff of an area, it is impossible to approach rationally the design of surface water drainage or control structures.

A complete review of watershed hydrology research would be voluminous. Thus, only a brief review of past research in watershed hydrology will be made. For a more thorough historical treatment of the subject, reference is made to Singh (46) and Kulandaiswamy (26).

Before 1925 the main objective of rainfall-runoff research on small drainage basins was the prediction of the peak discharge due to a given or design rainfall. Due to a very limited knowledge of rainfall-runoff processes and a very limited amount of reliable data, proposed techniques were based mostly on empiricism.

## Rational Method

According to Dooge (11), the principles of the "rational method" were established by T. J. Mulvany in 1851. Mulvany's work was based on rainfall-runoff data collected on arterial drainage basins in Ireland. In 1889 the peak-discharge formula now known as the "rational formula" was proposed by Kuichling (25). Kuichling's work was based on urban areas and was proposed for estimating peak discharge in sewer systems.

The rational method of predicting a design, peak runoff rate is expressed by the equation

$$Q = CiA \quad , \qquad 2.1$$

where

$Q$ = design peak runoff rate in cubic feet per second,

$C$ = runoff coefficient,

$i$ = rainfall intensity in inches per hour for the design recurrence interval and for a duration equal to the "time of concentration" of the watershed, and

$A$ = watershed area in acres (43).

The "time of concentration" is defined as the time required for water to flow from the most remote point of the watershed to the outlet. The runoff coefficient, C, is defined as the ratio of the peak runoff rate per unit area to the rainfall intensity. Empirical procedures are available for estimating these quantities (43).

The rational method is still used extensively for determining the design peak runoff for watersheds when the designed structures are relatively inexpensive and the consequences of failure are limited. Numerous methods of predicting peak discharge have been developed and are in use today (43); however, they will not be discussed.

## Time-Area Methods

During the latter part of the 1920's design engineers realized that the time distribution of the flow was important. This led to the development of time-area and routing methods for determination of basin flow. Development of the time-area diagram involves the division of the basin into zones through the use of isochrones of the travel time from selected points to the basin outlet. Use of this method is described by Linsley et al. (29), and continues to be used widely today.

## Unit-Hydrograph Methods

In 1932, Sherman (45) introduced the concept of the unit graph. This approach, known today as the unit-hydrograph approach, has been the basis for almost all methods used in the prediction of stream flow. Today it is one of the most powerful tools in applied hydrology.

As conceived by Sherman (45), in its empirical nature, the unit-hydrograph approach was based on the following assumptions:

1. For a given watershed, runoff producing storms of equal duration will produce surface-runoff hydrographs with an equal time base, regardless of the intensity of the rainfall.

2. For a given watershed, the magnitude of the ordinates representing the instantaneous discharge from an area will be proportional to the volume of surface runoff produced by storms of equal duration.

3. In a given watershed, the time distribution of runoff from a given storm period is independent of precipitation from antecedent or subsequent storm periods.

The unit-hydrograph is the hydrograph of direct surface runoff from a given basin due to a unit rainfall excess (i.e., one inch) distributed uniformly over the entire basin for a duration which is less than the time of concentration. If the duration of the unit rainfall excess is t hours, the unit-hydrograph is termed the unit-hydrograph of t-hours duration. The principle of linearity or superposition is an inherent part of the unit-hydrograph concept. This principle leads to the development of direct surface-runoff hydrographs for storms producing a rainfall excess of variable intensity and duration. Sherman stressed the advisability of deriving the unit-hydrograph from an observed hydrograph which was produced by a rain of high intensity and short duration in order to minimize any errors resulting from an incorrect estimation of base flow. Although he followed the customary method of expressing the amount of direct surface runoff as a percentage of rainfall, he recognized that this percentage increased with an increase in the rate and duration of precipitation and with the occurrence of antecedent precipitation. He further noted that the relative amount of direct surface runoff was affected by vegetal cover, season, and temperature. Whereas application of the rational formula is limited generally to small areas, the unit-hydrograph method does not suffer from such a restriction. No equation was proposed by Sherman for the unit-hydrograph in terms of significant watershed parameters.

When sufficient rainfall-runoff data are available, an average unit-hydrograph can be derived. The average unit-hydrograph is drawn with its peak equal to the average of the peaks of the derived unit-hydrographs, with the time to its peak equal to the average of the times to the derived peaks, and conforming to the general shape of the derived unit-hydrographs. The S-curve method can be used for converting a unit-hydrograph of a given duration to one of a different duration (29).

Generalizing, all unit-hydrograph procedures (hydrologic input-output systems) are based on two fundamental criteria (12, 24):

1. Invariance--The hydrographs of surface runoff from a given watershed due to a given temporal pattern of rainfall excess are time invariant. The first assumption of Sherman's, which implies invariance, also implies the added restriction of equal time base for storms of equal duration.

2. Superposition--The hydrograph resulting from any temporal pattern of rainfall excess can be determined by superimposing hydrographs computed from rainfall excess occurring in unit periods of shorter duration. This restricts us to linear systems (i.e., the ordinates of the hydrograph are proportional to the volume of rainfall excess for a given duration).

One of the first procedures developed for synthetic construction of a unit-hydrograph was presented by Snyder (49) in 1938. Snyder worked on rainfall-runoff data for streams in the Appalachian Highlands and correlated the basin characteristics with peak flow, basin lag, and total time base of the unit-hydrograph.

While correlations were being actively established between pertinent basin characteristics and unit-hydrograph parameters, some investigators developed an interest in finding rational equations to explain the physics of the rainfall-runoff phenomenon. They created conceptual models to simulate basin action in transforming rainfall or rainfall excess to direct surface runoff.

In 1945, Clark (8) suggested that the unit-hydrograph for instantaneous rainfall excess, i.e., the instantaneous unit hydrograph (unit-impulse), could be derived by routing the time-area-concentration curve through a linear storage reservoir. An instantaneous unit-hydrograph is a hypothetical unit-hydrograph whose duration of rainfall excess approaches zero as a limit, while rainfall excess remains one inch.

Given the instantaneous unit-hydrograph, the unit-hydrograph of any desired duration can be obtained easily, as discussed in Chapter III. The instantaneous unit-hydrograph is more basic to the system and was certainly an improvement in the understanding of rainfall-runoff phenomenon.

Clark (8), Dooge (12), and Singh (46) utilized the concept of pure translation and prepared unit, time-area curves, though the base time of the diagram and the method of its determination were not the same. The time-area curves were then routed through linear channels and/or linear reservoirs.

Nash's (26, 37, 46) approach to obtaining an instantaneous unit-hydrograph aroused considerable interest and discussion. By routing the unit-impulse input through a series of n equal linear reservoirs, Nash developed an expression for the instantaneous unit-hydrograph. The Nash equation is very similar to that proposed by Edson (16) in 1951. These developments utilized physical reasoning to obtain mathematical results. Consequently, the assumptions inherent to the development were somewhat obscured. Kulandaiswamy (26) presented a systematic mathematical development of Nash's equation with the assumptions fully stated. The assumptions required to obtain the Nash expression obviously demand liberties on the physical system that are not feasible (26). This observation is not made as a criticism to Nash but as an example of a hydrologic mathematical model that cannot theoretically be applied to the system. Nevertheless, the procedure used is valid in light of the fact that the results obtained from the model are quite satisfactory for many applications (26, 37, 46).

Barnes (4) described a numerical procedure for unit-hydrograph derivation using the "principle of progressive addition". Progressive addition refers to certain variations in the form of the discrete convolution equations. The method is a trial-and-error solution which, according to Barnes, should be attempted only by experienced hydrologists.

Snyder (49) presented a discrete solution for the instantaneous unit-hydrograph using a least-squares technique. Snyder's method requires prior arbitrary assumptions of the number of unit-hydrograph points and their distribution.

O'Donnell (38) suggested the application of harmonic analysis in deriving an instantaneous unit-hydrograph from an effective hyetograph and a direct surface-runoff hydrograph. This method is equivalent to the least-squares method, but perhaps easier to use, because an instantaneous unit-hydrograph is obtained directly.

According to Minshall (35), both the peak of the unit-hydrograph and the time to the peak for small drainage areas (less than 500 acres) are dependent on rainfall intensity and storm pattern. As the intensity of rainfall increases, the time to peak decreases while the magnitude of the peak increases; however, the effect is less for large watersheds than for small ones in the areal range specified. Amorocho (2), in a discussion of Minshall's paper, showed that the parameters in the Nash equation for derivation of the instantaneous unit-hydrograph can be correlated with the intensity of rainfall excess.

Gray (19) correlated the parameters in the Nash equation for the instantaneous unit-hydrograph with the length and slope of the main stream for small drainage areas. He found the period of rise of a hydrograph to be a significant parameter in correlating the salient features of rainfall and runoff. The results were applied to uniformly-distributed, short-duration, and high-intensity storms over small basins.

Levi and Valdes (28) have presented a technique for deriving the instantaneous unit-hydrograph using Fourier transforms. The mathematical procedures are similar to those of Guillemin (20). The resulting instantaneous unit-hydrograph is in the form of an "impulse train" and exhibits negative as well as positive ordinates.

Singh (46, 47) presented a rigorous least-squares analysis (discrete solution) to obtain the instantaneous unit-hydrograph. The solution is obtained by the inversion of the symmetric matrix:

$$\begin{bmatrix} \overline{Q}_1 \\ \overline{Q}_2 \\ \cdot\ \cdot \\ \overline{Q}_n \\ \cdot\ \cdot \\ \overline{Q}_{m-n+1} \end{bmatrix} = \begin{bmatrix} C_1 & C_2 & \cdots & C_n & 0 & \cdots & 0 & \cdots & 0 \\ C_2 & C_1 & \cdots & C_{n-1} & C_n & \cdots & 0 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdots & \\ C_n & C_{n-1} & \cdots & C_1 & C_2 & \cdots & C_n & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdots & \\ 0 & 0 & \cdots & \multicolumn{4}{c}{\text{starting with } C_n} & \cdots & C \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \cdot\ \cdot \\ H_n \\ \cdot\ \cdot \\ H_{m-n+1} \end{bmatrix} \qquad 2.2$$

where (assuming the discrete time interval, $\Delta t = 1$)

$$\overline{Q}(k) = \sum_{j=1}^{n} f(j)\, g(j+k-1), \quad k = 1, 2, 3, \cdots (m-n+1) \ ,$$

$$\overline{C}(p) = \sum_{j=1}^{n} f(j)\, f(j-p+1), \quad p = 1, 2, 3, \cdots n, \ (j-p) \leq 0,$$

$H(k)$ = unit-impulse response (instantaneous unit-hydrograph),

$f(j)$ = input time series,

$g(j)$ = output time series,

$n$ = number of points in input time series, and

$m$ = number of points in output time series.

This matrix is only valid for $(m-n+1) > n$, where $(m-n+1)$ is the number of equations and the number of unknowns.

# CHAPTER III

## RESEARCH METHODOLOGY

A system may be defined as an organized set of connected parts that determines a relation between a specified cause (input) and a specified effect (output), Figure 3.1. The input and output under consideration are functions of one or more variables (i.e., time, location, etc.). A model of a system is a mathematical formulation of the transformation of input into output, or an arbitrarily close approximation to it.

The hydrologic cycle and certainly the transformation of rainfall (input) into runoff (output) can be readily included in the above definition of a system. The model which will adequately describe the rainfall-runoff process should be sufficiently simple in use and manipulation to be readily adaptive to computer use.

Hydrologic systems are, in general, time-lag systems. That is, the output of these systems, at each instant, depends on the input during some time before the instant. Wooding (55, 56, 57, 58) has recently demonstrated theoretically that the processes governing the rainfall-runoff relationships are nonlinear. He also has been quite convincing in exemplifying the additional mathematical restraints necessary to manipulate a nonlinear model.

The need for a simple and adequate model of the rainfall-runoff process has prompted the investigation of some linear

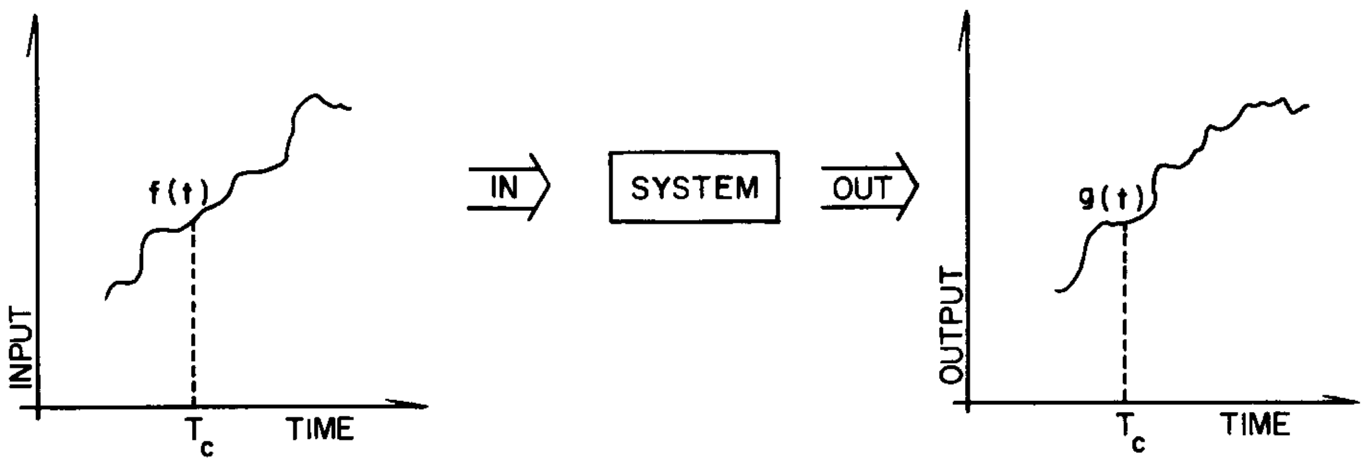


Figure 3.1. Illustration of input-output system.

mathematical models in the past decade. The most common linear model applied to hydrologic phenomenon is that of a lumped-linear, time-invariant system.

According to linear theory, one means of defining this type of input-output relationship is the convolution integral. The convolution integral can be written (13, 27):

$$g(\tau) = \int_{-\infty}^{\infty} f(t)\ h(\tau-t)\ dt \quad , \qquad 3.1$$

where

$g(\tau)$ = output,

$f(t)$ = input, and

$h(\tau-t)$ = characterizing or transfer function of the system.

This integral representation of a linear system is well known. However, for completeness the concepts and assumptions on which the integral are based will be briefly reviewed (27).

A linear system can be characterized by its response to a unit-impulse excitation. When a system is characterized by a certain function, the output of the linear system, for an input function of a general class, is expressible in terms of the characteristic function and the input. In general the input and output are functions of time. One convenient characterizing function of a linear system is the system's response to a unit-impulse function. A unit-impulse is defined as the limiting form of a rectangular pulse of a given height and base such that the area of the pulse is always unity and as the height tends to infinity, the base becomes dt. The unit-impulse, u(t), is therefore defined as an impulse with an infinitesimal duration dt. During the entire duration (not just at an instantaneous point), the amplitude of the impulse tends to infinity. It is an integrable function, since the area is unity in this extremely long and narrow rectangle. The unit-impulse function is located at the origin of the time axis such that t = 0 at the midpoint of the pulse duration. In other words, u(t) will be an even function.

We will now denote by h(t) the response of the linear system to the unit-impulse. Since u(t) is applied at t = 0, it is physically impossible for it to have any effect on the output for t less than zero. With the system initially at rest, it is therefore necessary that h(t) = 0 for t less than zero. For the purpose of illustration, examine Figure 3.2 and suppose that the system unit-impulse response h(t) is as shown.

We now wish to demonstrate that h(t) characterizes the system behavior. We will define an important property of the system and confine our further discussion to time-invariant systems. The invariance in the input-output relation of a system under a translation in time means that if the system were represented by a differential equation involving the input and output,

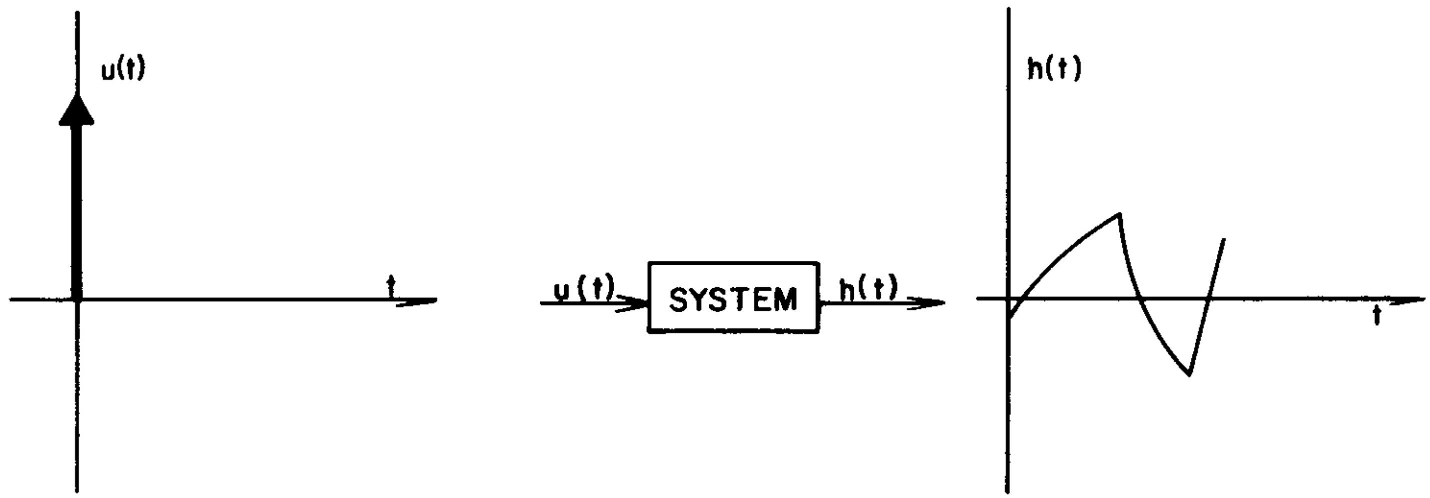


Figure 3.2. Illustrative system response to a unit-impulse.

the coefficients of the equation would be constant. Then, the first important property of the system is that, if the application of the unit-impulse is delayed by time $t = \tau$, the response h(t) also will be delayed by the same length of time, i.e., for the unit-impulse $u(t-\tau)$ the unit response is $h(t-\tau)$, as shown in Figure 3.3.

Let us consider now a general class of functions consisting of periodic, aperiodic and random functions and let f(t) be representative of this class acting as the system input. For purposes of illustration, let f(t) be represented as in Figure 3.4. Let f(t) start at an arbitrary time $t = -a$. We have divided f(t) into elements of infinitesimal widths, dt, as indicated by the fine lines in Figure 3.4. At $t = \tau$, for example, the element has the width $d\tau$ and the height $f(\tau)$. If this particular element were a unit-impulse, we know that the output due to this element alone would be $h(t-\tau)$. However, this element is a very small impulse, in the sense that it has the area $f(\tau)\ d\tau\ h(t-\tau)$. Hence, the differential of the output is

$$dg(t) = f(\tau)\ d\tau\ h(t-\tau) \quad , \qquad 3.2$$

as is shown in Figure 3.4. In arriving at this output element, we have made the assumption that the system response to an impulse is directly proportional to the area of the impulse. Since the widths of the unit-impulse and the infinitesimal impulse at $t = \tau$ are the

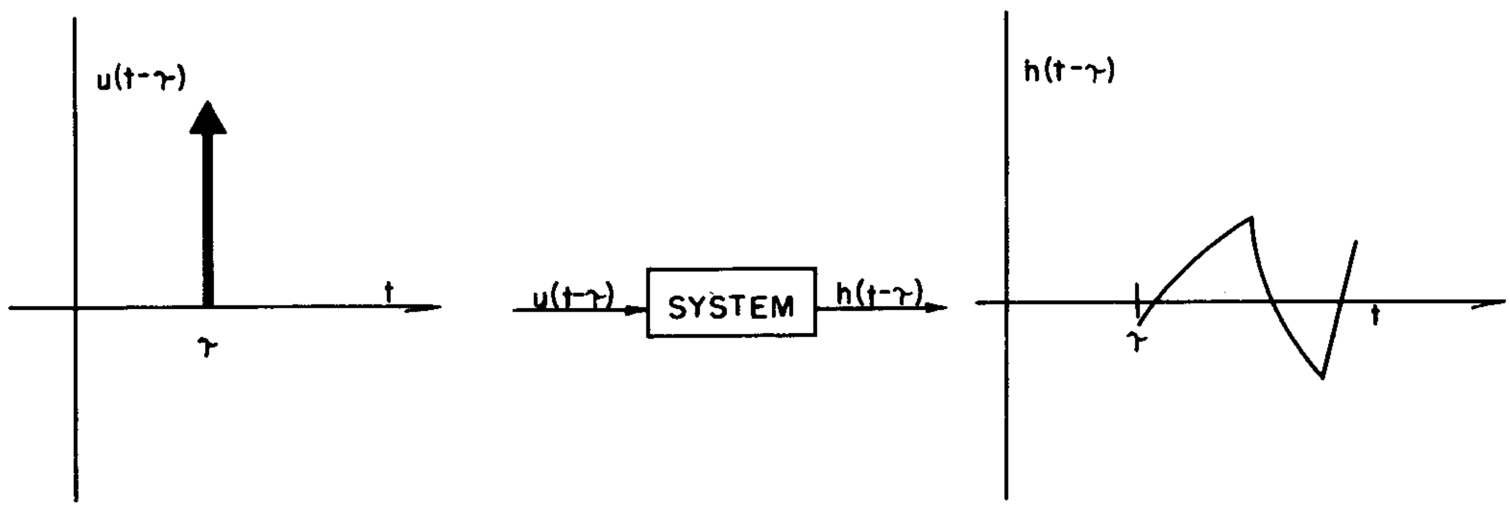


Figure 3.3. Illustrative system response to a delayed unit-impulse.

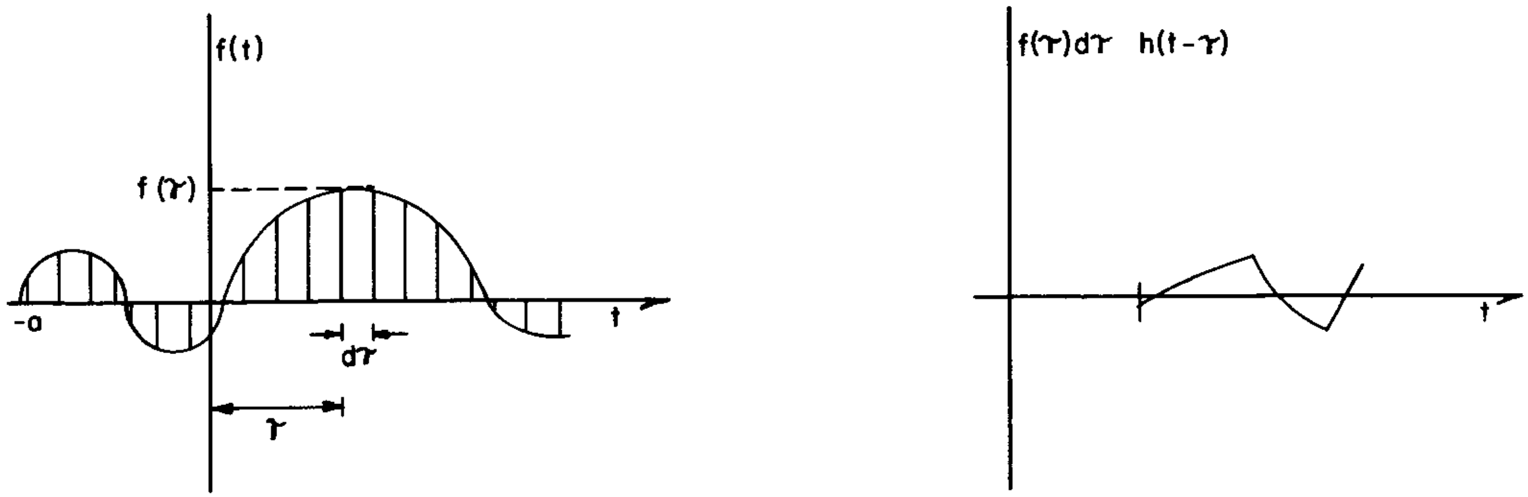


Figure 3.4. Illustrative random function with infinitesimal response to a unit-impulse.

same, we are assuming that the ratio of the output due to the infinitesimal impulse to the output due to the unit-impulse is equal to the ratio of the respective heights of the impulses. Since the height of the infinitesimal impulse is $f(\tau)$ and that of the unit-impulse is $1/d\tau$, the assumption can be stated mathematically as

$$\frac{dg(t)}{h(t-\tau)} = \frac{f(\tau)}{1/d\tau} \quad . \qquad 3.3$$

Equation 3.3 is simply Equation 3.2 rearranged. This agrees with the basic assumption of linearity between the input and the output. From Equation 3.2 it can be seen that if the input is multiplied by a real constant k then the output also is multiplied by k. Since it is a property of a set of linear equations that superposition holds, the output of a linear system is the superposition of the component outputs produced by the respective component inputs acting individually. Therefore, by summing all the infinitesimal outputs due to all the elements into which f(t) has been divided, from the beginning to time t, we shall have, at time t, the output g(t) due to cumulative effects of f(t); this expression is

$$g(t) = \int_{-a}^{t} f(\tau)\, h(t-\tau)\, d\tau \quad . \qquad 3.4$$

Equation 3.4 is designated the convolution integral or superposition integral expressing the so-called convolution of functions $f(t)$ and $h(t)$. $g(t)$ is the result of this convolution.

We will now consider the role of $\tau$, which was introduced as a dummy variable in Equation 3.4, and will investigate the process defined by this integral. Also, $h(\tau)$ is assumed to be one of the functions to be convolved as represented graphically in Figure 3.5. In order to utilize the convolution integral we must now form the function $h(\tau-t)$. On the $\tau$-scale this is simply the function $h(\tau)$ delayed by t seconds. Graphic representation of $h(\tau-t)$ is shown in Figure 3.6 by the dotted curve. Since the argument $(t-\tau)$ is the negative of $(\tau-t)$, we can see that the function $h(t-\tau)$ is the mirror image of $h(\tau-t)$ about a vertical line erected at the point $\tau = t$. The solid curve in Figure 3.6 which is $h(t-\tau)$ and the dotted curve are such a pair of images. It should be pointed out again that $\tau$ is the independent variable and t is a parameter.

$f(\tau)$ can be represented graphically, as shown in Figure 3.7. In Figure 3.8 the functions $f(\tau)$ and $h(t-\tau)$ are shown dotted. The integrand appearing in Equation 3.4 is the product of these two functions and is depicted by the solid curve in Figure 3.8. The value of the integral of Equation 3.4 or the output is the area under this solid curve. We can see from the graphical representation and Equation 3.4 that $f(\tau)$ is multiplied into the past history of $h(t-\tau)$ up to the present moment t, and the product is

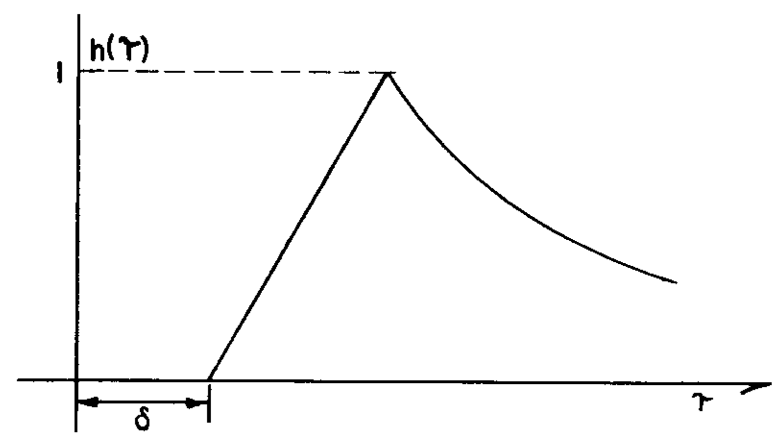


Figure 3.5. Illustrative function, $h(\tau)$, to be convolved.

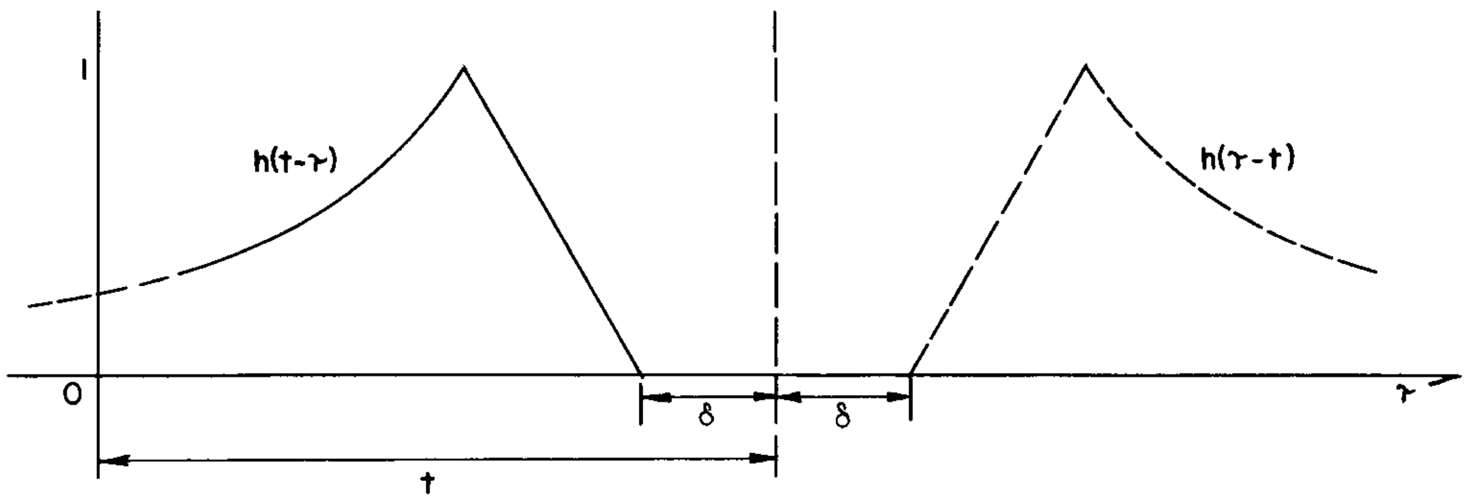


Figure 3.6. Illustration of the formation of h(t-τ).

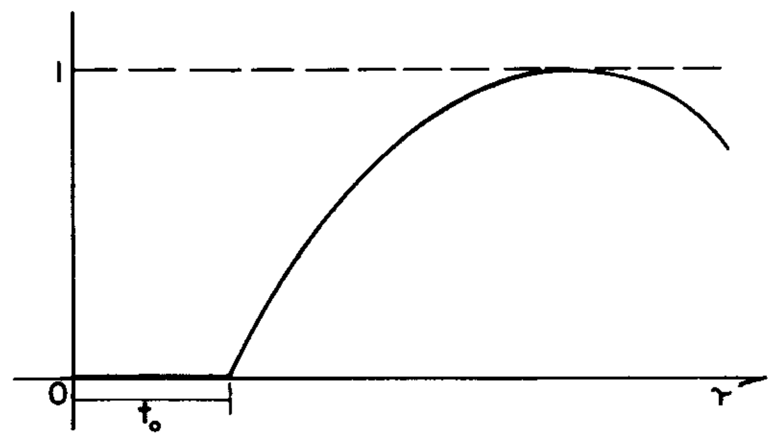


Figure 3.7. Illustration of other convolving function, $f(\tau)$.

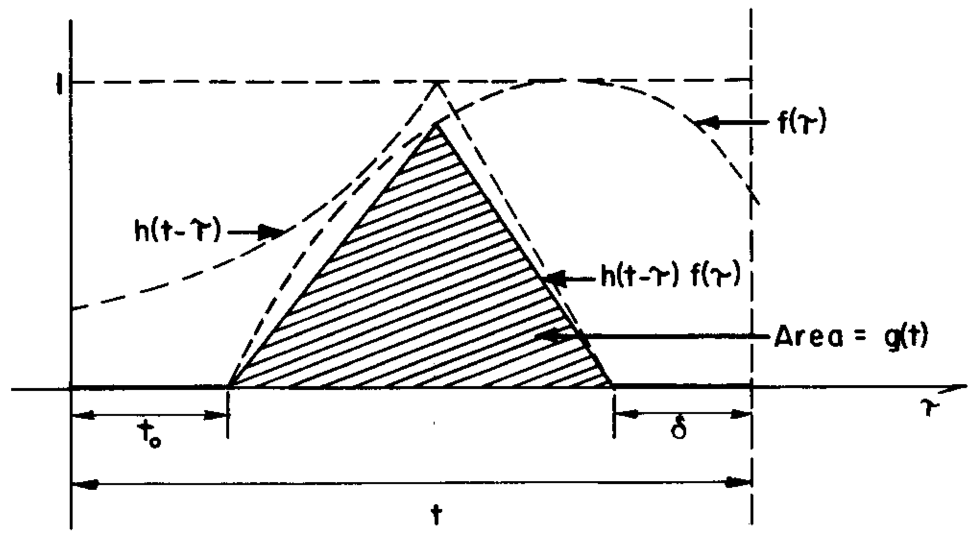


Figure 3.8. Illustration of convolution.

integrated over the interval of the past history to yield the output for the time t. Therefore, the linear system depends upon what has happened to the input from $\tau = -a$ to $\tau = t$ to give its output. The output is, therefore, the summation of the weighted past. The weighted function is the system unit-impulse response. In other words, a linear system scans the history of the input with its unit-impulse response to yield its output.

Referring back to Equation 3.4 we note that, since f(t) is assumed to be zero from $-\infty$ to $-a$ and since $h(t) = 0$ for $\tau < 0$, the product of $h(t-\tau)\ f(\tau)$ is actually zero outside the limits of integration. Furthermore, if f(t) starts from $-\infty$, then $a = \infty$. Therefore, nothing will be added to the integral if we replace the limits by $(-\infty, \infty)$ and write the convolution integral in the more general form as it is usually presented:

$$g(t) = \int_{-\infty}^{\infty} f(\tau)\ h(t-\tau)\ d\tau \qquad 3.5$$

The basic problem is to solve Equation 3.5 for a meaningful h(t) when g(t) and f(t) are related by a system that is not truly linear. With f(t) and g(t) as known analytic continuous functions representing the transient input and output of a truly linear monotone, h(t) may be determined analytically by use of Laplace transforms (10, 14, 55).

In our application we will be concerned with the output (runoff) at some time t due to the input (rainfall) starting at time t = 0. Therefore, the a in Equation 3.4 will equal zero and the expression for g(t) will be

$$g(t) = \int_0^t f(\tau)\, h(t-\tau)\, d\tau$$

or equivalently, with $\nu = t-\tau$ , 3.6

$$g(t) = \int_0^t h(\nu)\, f(t-\nu)\, d\nu \quad ,$$

where $\tau$ or $\nu$ are independent time variables.

This integral can be defined in discrete form. If we define discrete time values i and j as $i > 0$, $j > 0$ and increment the argument of h, Equation 3.6 can be written in the discrete form:

$$g(i) = \sum_{j=1}^{i} f(j)\, h(i-j+1)\, \Delta j$$

or equivalently, 3.7

$$g(i) = \sum_{j=1}^{i} h(j)\, f(i-j+1)\, \Delta j \quad ,$$

where

i = 1, 2, 3, . . .m, and m is the number of elements in the observed output. It is this form of the convolution integral that we will utilize in this work.

For a truly linear system, Equation 3.7 can be solved for h(j) by the method of "back substitution",

$$h(1) = \frac{g(1)}{f(1)}$$

$$h(2) = \frac{g(2) - f(2)\ h(1)}{f(1)} = [g(2) - \frac{f(2)}{f(1)}\ g(1)]\ \frac{1}{f(1)} \qquad 3.8$$

$$\vdots$$

$$h(m) = [g(m) - \sum_{k=2}^{m} f(k)\ h(m-k+1)]\ \frac{1}{f(1)}$$

or by other methods of matrix inversion (13, 21, 46).

For a truly linear system, the unit-impulse response obtained through the solution of Equation 3.7 is a unique characterization of the system which is independent of the particular f(t) and g(t) used in its determination. However, our problem is to solve Equation 3.7 for a meaningful h(t) when g(t) and f(t) are related by a system that is not truly linear. With this system we can still obtain an exact solution of Equation 3.7 for any given pair of input and output signals, but the h(t) so obtained is no longer unique and will vary with each pair of f(t) and g(t) used. As an example, suppose that for a truly linear system the impulse response is known to be

$$h(i) = [2, 1],$$

with an input

$$f(i) = [2, 6, 1].$$

If $\Delta j = 1$, Equation 3.7 gives

$$g(1) = (2)\,(2) = 4$$

$$g(2) = (2)\,(6) + (1)\,(2) = 14$$

$$g(3) = (2)\,(1) + (1)\,(6) = 8$$

$$g(4) = (2)\,(0) + (1)\,(1) = 1$$

$$g(5) = g(6) = \ldots\ldots = 0$$

or

$$g(i) = [4, 14, 8, 1].$$

Suppose that an error is made in measuring g(i) such that

$$\underline{g}(i) = [4, 11, 8, 1],$$

then by Equation 3.8

$$\underline{h}(1) = \frac{4}{2} = 2$$

$$\underline{h}(2) = \frac{11 - (6)(2)}{2} = -1/2$$

$$\underline{h}(3) = \frac{8 - (6)(-1/2) - (1)(2)}{2} = 9/2$$

$$\underline{h}(4) = \qquad = -5\ 1/4$$

or

$$\underline{h}(i) = [2, -1/2, 9/2, -5\ 1/4, \ldots],$$

which is both unstable and oscillating. With nonlinearity we would expect this result from the linear approximation. Thus, a technique must be devised for averaging several h(t)'s in order to obtain an acceptable unit-impulse response of the system. It easily can be demonstrated that the averaging process to obtain h(t) often fails due to the loss of stability and physical realizability in the solution (14). The use of Equation 3.7 as an approximation to a nonlinear system and in turn Equation 3.8 for a solution of h(t) can result in an unstable and oscillating solution.

If we accept the fact that we can only provide, with the derived linear system, an arbitrarily close approximation of the observed output from a given input, then there are a multiple of methods by which we can obtain a stable solution to the system unit-impulse response. Let

g(i) = observed output,

f(i) = observed input, and

*g(i) = predicted output.

Now, utilizing Equation 3.7 and requiring the predicted output be an exact solution of the convolution relation, we obtain

$$*g(i) = \sum_{j=1}^{i} h(j)\ f(i-j+1)\ \Delta j. \qquad 3.9$$

Letting the time interval $\Delta j = 1$, the error in the predicted output is:

$$E(i) = g(i) - \sum_{j=1}^{i} h(j)\ f(i-j+1). \qquad 3.10$$

We will adopt the minimization of the mean-square error

$$\varepsilon = \sum_{i=1}^{\infty} E^2(i) = \sum_{i=1}^{\infty} [g(i) - \sum_{j=1}^{i} h(j)\ f(i-j+1)]^2 \qquad 3.11$$

as the performance criteria of the prediction system.

Seeking to minimize $\varepsilon$, the differential of $\varepsilon$ with respect to the predicting function h(j) is taken as zero, yielding the usual linear normal equations

$$\frac{\partial \varepsilon}{\partial h\ (k)} = 0,\ k = 1,\ 2,\ .\ .\ .\ \quad . \qquad 3.12$$

Note that k is restricted to the positive range of time. Since the system is at rest before the application of the unit-impulse response at t = 0, and since the output (response) of the system cannot precede the input, this condition must hold in order to obtain a unit-impulse response of our physical linear system. Hence, we have

$$\frac{\partial \varepsilon}{\partial h\ (k)} = 0 = 2 \sum_{i=1}^{\infty} [g(i) - \sum_{j=1}^{i} h(j)\ f(i-j+1)]\ f(i-k+1) \quad ,$$

$$k \geq +1 \quad . \qquad 3.13$$

Rearranging Equation 3.13 yields

$$\sum_{i=1}^{\infty} g(i)\ f(i-k+1) = \sum_{i=1}^{\infty} f(i-k+1) \sum_{j=1}^{i} h(j)\ f(i-j+1) \quad ,$$

$$k \geq +1 \quad . \qquad 3.14$$

Letting i-k+1 = j on the left side of Equation 3.14 and recognizing that $f(\sigma)$ and $g(\sigma)$ are zero for $\sigma \geq 0$, Equation 3.14 becomes

$$\sum_{j=1}^{\infty} f(j)\ g(j+k-1) = \sum_{j=1}^{\infty} h(j) \sum_{i=1}^{\infty} f(i-k+1)\ f(i-j+1) \quad ,$$

$$k \geq +1. \qquad 3.15$$

Letting $\nu = i-k+1$ on the right-hand side of Equation 3.15 gives

$$\sum_{j=1}^{\infty} f(j)\ g(j+k-1) = \sum_{j=1}^{\infty} h(j) \sum_{\nu=1}^{\infty} f(\nu)\ f(\nu+k-j) \quad ,$$

$$k \geq +1 \quad . \qquad 3.16$$

By definition, we can write the discrete-time, cross-correlation function as

$$\phi_{fg}(i) = \sum_{j=1}^{\infty} f(j)\ g(j+i)\ \Delta j \quad , \qquad 3.17$$

for all integer i's, and the auto-correlation function as

$$\phi_{ff}(i) = \sum_{\nu=1}^{\infty} f(\nu)\ f(\nu+i)\ \Delta\nu \qquad 3.18$$

for all integer i's.

Now letting $\Delta j$ and $\Delta\nu$ equal one in Equations 3.17 and 3.18 and utilizing Equation 3.16, we can write what is known as the discrete time form of the Wiener-Hopf equation, or

$$\phi_{fg}(k-1) = \sum_{j=1}^{\infty} h(j)_{opt}\ \phi_{ff}(k-j),\ k \geq +1 \quad ,$$

which in matrix form becomes 3.19

$$[\phi_{fg}] = [h_{opt}]\ [\phi_{ff}] \quad .$$

It can be shown (27) that satisfaction of Equation 3.12 in the form of Equation 3.19 will lead to the minimization of the mean-square error. Therefore, the unit-impulse response, h(j), determined by Equation 3.19 is optimum and stable and will be designated by the subscript "opt". Lee (27) also shows that as long as $\phi_{fg}(k)$ is neither constant nor zero, Equation 3.19 is both a necessary and a sufficient condition for defining the optimum linear system. By comparing Equation 3.19 and Equation 3.7 it is easily seen that Equation 3.19 has the form of the convolution equation with the important exception that it applies only to positive arguments of $\phi_{fg}$. This restriction entered the development of Equation 3.19 when the positive range of time was specified over which the output error is to be minimized (Equation 3.12). This leads to major difficulties in the solution to either the continuous or discrete form of Equations 3.19.

From only a superficial look at Equations 3.19 it can be seen that even the simplest set defined by these equations contains more than one unknown h(j). Therefore, the system may not be solved by synthetic division, and other means of matrix inversion must be used.

Before we continue in the development of a discrete solution of Equations 3.19 we must consider the physical problem at hand. Our prime objective is to predict output (runoff) via Equation 3.7. It is easily concluded that if h(j) is allowed to take on negative values, then the possibility of predicting a negative output (runoff) exists and this is physically unrealistic. Therefore, we will add an additional constraint to Equation 3.19, namely,

$$h(j)_{opt} \geq 0 \qquad 3.20$$

We now limit the number of Equations 3.19 to m nonhomogeneous equations resulting from the m nonzero values of the observed output. We will include all j elements of $h(j)_{opt}$ that have nonzero coefficients in Equations 3.19. Let the observed input have a duration of n time units. Then there are 2n-1 nonzero elements in $\phi_{ff}$ and $j_{max} = m+n-1$. Equations 3.19 can now be written as the undetermined set

$$\phi_{fg}(k-1) = \sum_{j=1}^{m+n-1} h(j)_{opt}\ \phi_{ff}(k-j),\ k = 1, 2, \ldots m. \qquad 3.21$$

We now need a criterion for the selection of the best solution to Equations 3.21. We will specify that the optimum response, $h(j)_{opt}$, has a length, $L$, consistent with the observed duration of the input and output, $L = m-n+1$. This criterion for $L$ is easily seen by examination of Equation 3.7. Then, we will logically seek the solution to Equations 3.21 that satisfies the condition that

$$\sum_{j=m-n+2}^{m+n-1} h(j)_{opt} \quad \text{is a minimum.} \qquad 3.22$$

Equations 3.20, 3.21 and 3.22 constitute the statement of a normal linear programming problem, and a solution to this type of problem can be obtained by the simplex method (18).

The simplex method assures that Equation 3.20 is always satisfied while an initial solution to Equations 3.21 is being sought. In the absence of a starting basis (unit matrix), the simplex method will introduce "artificial variables" into the basis.

When the artificial variables fail to vanish, the solution to Equations 3.21 is infeasible. When this happens we will specify that the vector of constant terms, $[\phi_{fg}]$, constitutes a limiting value which may be approached but not exceeded and write Equations 3.21 as

$$\phi_{fg}(k-1) \geq \sum_{j=1}^{m+n-1} h(j)_{opt}\ \phi_{ff}(k-j),\ k = 1, 2, \ldots m. \qquad 3.23$$

We can now introduce slack variables $h(\phi)$ to Equations 3.23 as a starting basis. With the introduction of the slack variables, we are relaxing the Wiener-Hopf equations to admit approximate positive solutions. The values of the individual slack variables will represent the degree to which the original Wiener-Hopf equations were not satisfied.

We now express our system mathematically as

$$\phi_{fg}(k-1) = \sum_{j=1}^{m-n+1} h(j)_{opt}\ \phi_{ff}(k-j) + h(m-n+1+k)$$

$$k = 1, 2, \ldots m \quad , \qquad 3.24$$

with the conditions that

$$\sum_{\sigma=m-n+2}^{2m-n+1} h(\sigma) \text{ is minimum} \quad , \qquad 3.25$$

and

$$h(j)_{opt} \geq 0,\ h(\sigma) \geq 0. \qquad 3.26$$

A solution to the linear programming problem posed by Equations 3.24, 3.25, and 3.26 was obtained by utilizing an International Business Machines (IBM) application program entitled "Mathematical Programming System/360 (360A-CO-14X); Linear and Separable Programming (32)". The linear programming procedures of MPS/360 use the variable-product form of the inverse-revised simplex method (32, 18). MPS/360 is a very versatile computer program and was used in combination with the IBM application program READCOMM (33) to obtain the solution to Equations 3.24, 3.25 and 3.26. The actual job-control language (JCL) and Fortran IV Program are not reported herewithin. The JCL for these application programs is specific to the Texas A&M University installation and is being updated quite frequently; therefore, the program used for this study undoubtedly will be outdated in the near future.

The procedures being utilized have considerable similarities to the least-squares procedures utilized by Snyder (50), Singh (46) and other investigators. In 1962, Singh (46) presented a rigorous solution of the least-squares analysis for deriving unit hydrographs. The important differences between this approach and a normal least-squares approach are:

1) There is no restriction on the time duration of the transfer function (unit hydrograph). In a least-squares approach this time duration would

be limited to the number of output intervals less the number of input intervals plus one.

2) There is no restriction on the number of equations to be solved.

The logical but arbitrary selection of the time duration of the transfer function and the number of equations requiring solution has been explained earlier in this chapter.

# CHAPTER IV

## DESCRIPTION OF BASINS AND DATA COLLECTION

Two small drainage basins which have been instrumented by the cooperative efforts of Texas A&M University, the U. S. Geological Survey, and the Office of Water Resources Research, the Department of the Interior, were utilized in the evaluation of the hydrologic model proposed in Chapter III. Burton Creek Watershed is located within the city limits of Bryan, Texas. Figure 4.1 depicts the Burton Creek Watershed. Approximately 16 percent of the watershed has not been urbanized. These areas are indicated in Figure 4.1. The watershed encompasses a total area of approximately 890 acres. Impervious area is estimated to be approximately 210 acres, or about 23.5 percent of the total area.

Hudson Creek Watershed, depicted in Figure 4.2, is located approximately three miles east of Bryan, Texas. The basin encompasses approximately 1,270 acres and is completely in pasture.

The predominant soil over the two basins is Lufkin-Tabor Clay. Lufkin-Tabor Clay is a montmorillonite clay with a very high water-holding capacity, but a low infiltration capacity. This soil cracks very severely when dry.

Bryan, Texas is approximately 150 miles inland from the Gulf of Mexico in a northwest direction and is contained within the upper portion of the Gulf Coastal Plains of Texas. The general

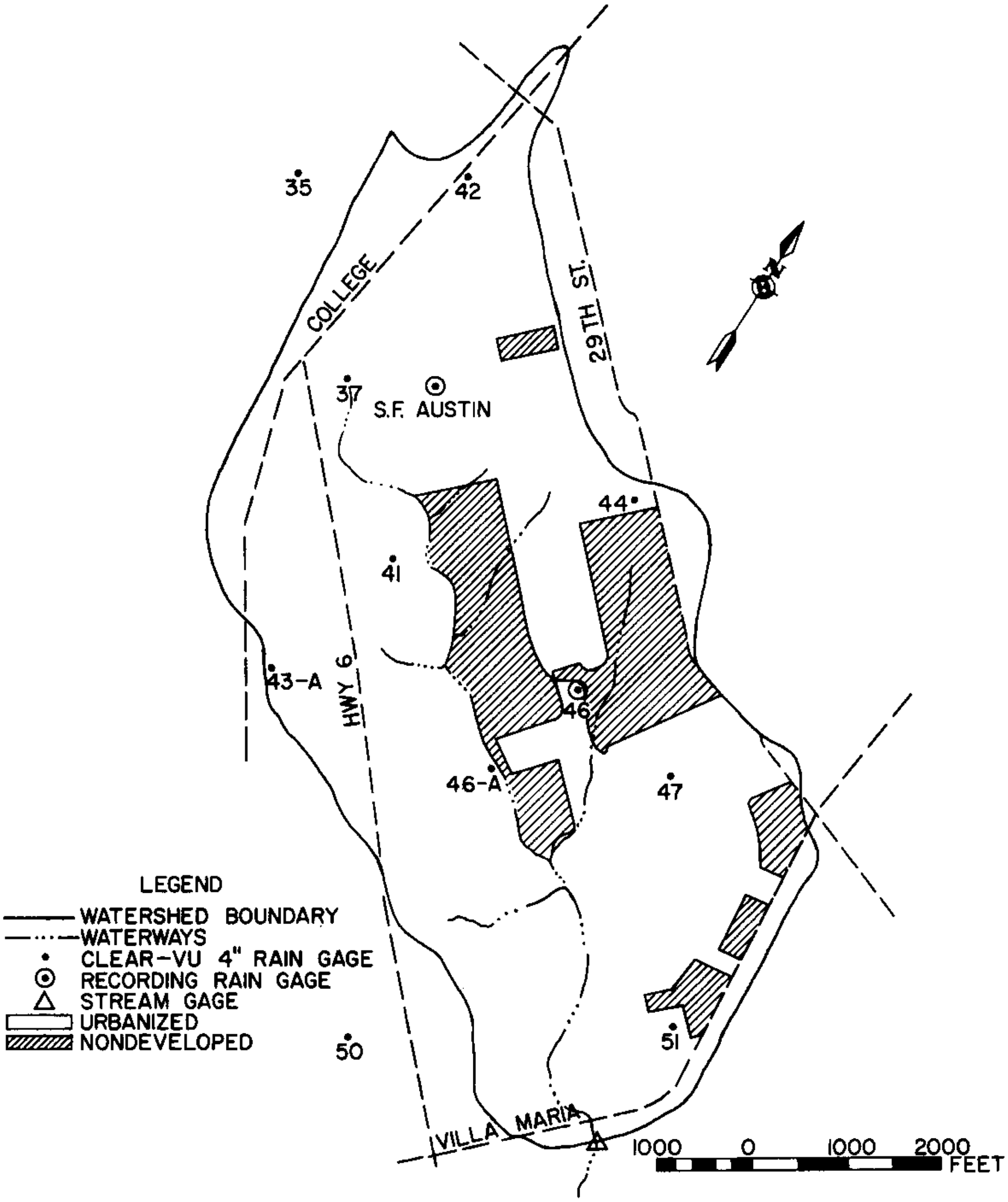


Figure 4.1. Burton Creek Watershed, Bryan, Texas.

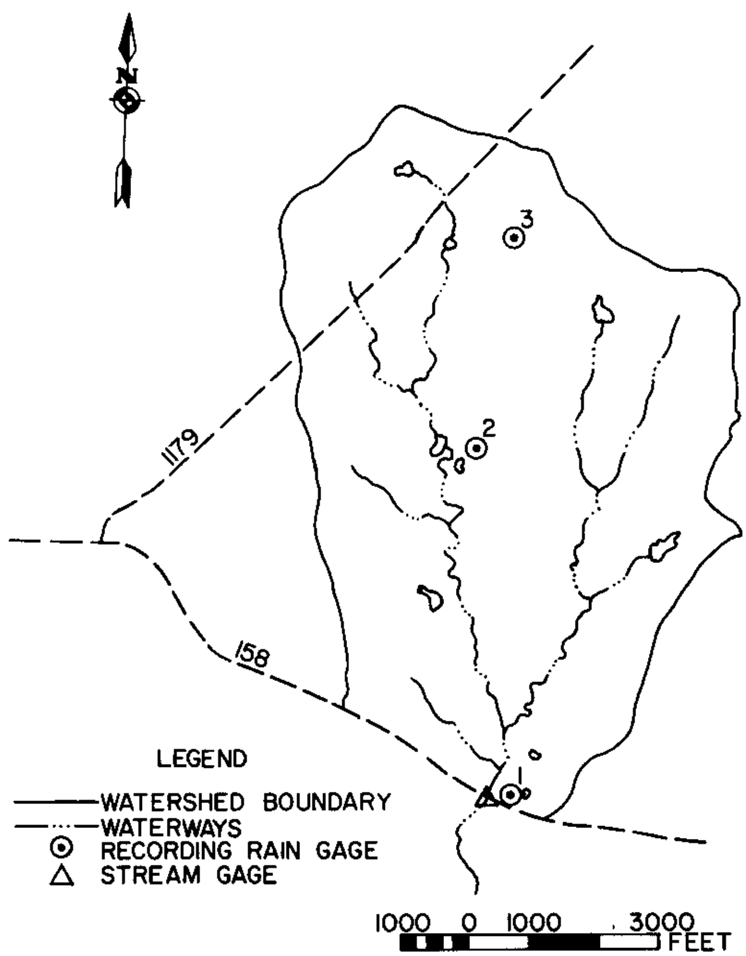


Figure 4.2. Hudson Creek Watershed, Bryan, Texas.

topography of the area is gently undulating coastal prairies and hills. Elevation at Bryan, Texas is 360 feet, and the mean annual precipitation is 39 inches per year. Frontal activity is at a minimum during the summer and most of the rain produced during this period is provided by cumuliform clouds. Maritime air flowing off the Gulf of Mexico is the dominate feature of the climate of the area during late spring and early summer. Maximum thunderstorm activity generally is observed during the mid-afternoon and early evening.

The rainfall network on each basin is maintained by the Department of Meteorology at Texas A&M University. In this investigation, the most centrally located recording rain gage was assumed representative for the basin. Recording rain gage number 46 was used for Burton Creek, and recording gage number 2 was used for Hudson Creek. Both gages are U. S. Weather Bureau *weighing-type* gages -- gage number 46 on Burton Creek is a six-hour gage, and gage number 2 on Hudson Creek is a 24-hour gage.

The stage recording installation is a standard U. S. Geological Survey installation utilizing an A-35 Stevens stage recorder and also a five-minute, punched-tape stage recorder. These installations are maintained by the U. S. Geological Survey. The data were made available to Texas A&M University through a cooperative agreement.

# CHAPTER V

## PRESENTATION AND DISCUSSION OF RESULTS

The major objective of this study was to evaluate the ability of the time-invariant convolution relationship to predict the output (runoff) of a hydrologic system when only the input (rainfall) is known.

Appendix B presents the results of the solution for the transfer function for the Burton Creek Watershed. The cross-correlation and auto-correlation functions were calculated by utilizing equations 3.17 and 3.18. The solution for the transfer function was obtained by solving equations 3.24, 3.25 and 3.26, as explained in Chapter III.

The important difference between this application of the linear convolution model to natural hydrologic systems and other applications (21, 26, 46, 49) is that the input (rainfall) has not been adjusted for antecedent moisture conditions. Many researchers have demonstrated that hydrologic systems do not perform in a strictly linear nature. Therefore, this investigation was based on the premise that the linear approximation model should include the complete hydrologic system and utilize recorded rainfall intensities as its input. The investigation of the effect of antecedent moisture conditions on the transfer function is one of the major considerations in this study and will be discussed later.

Figure 5.1 exhibits graphically the solution for the transfer function for six independent events occurring from May 10, 1968 to June 18, 1968 on the Burton Creek Watershed in Bryan, Texas. To allow better graphical comparison of the six events, the elapsed time duration has been arbitrarily truncated at 525 minutes. However, the solution was obtained with the input-output time interval specified in Chapter III. The complete linear portion of the solution to each event is given in Appendix B. Figures 5.2 and 5.3, based on two separate events, exhibit the recorded hydrographs and the hydrographs obtained by convolving the computed transfer function of length m with the recorded rainfall intensities. Similar results were obtained for all events analyzed. These results indicate that the linear approximation model does an adequate job of representing the system.

In order for the model to have any usefulness, a method of obtaining a generalized transfer function (instantaneous unit hydrograph) for a particular basin must be available. In order to develop this general transfer function for Burton Creek it was assumed that the six events reported in Figure 5.1 were representative and that the only variable basin characteristic was antecedent moisture conditions. It would have been desirable to utilize a longer record for development of the transfer function; unfortunately, less than two years of record are available on the watersheds. In addition, data from this same period were used to

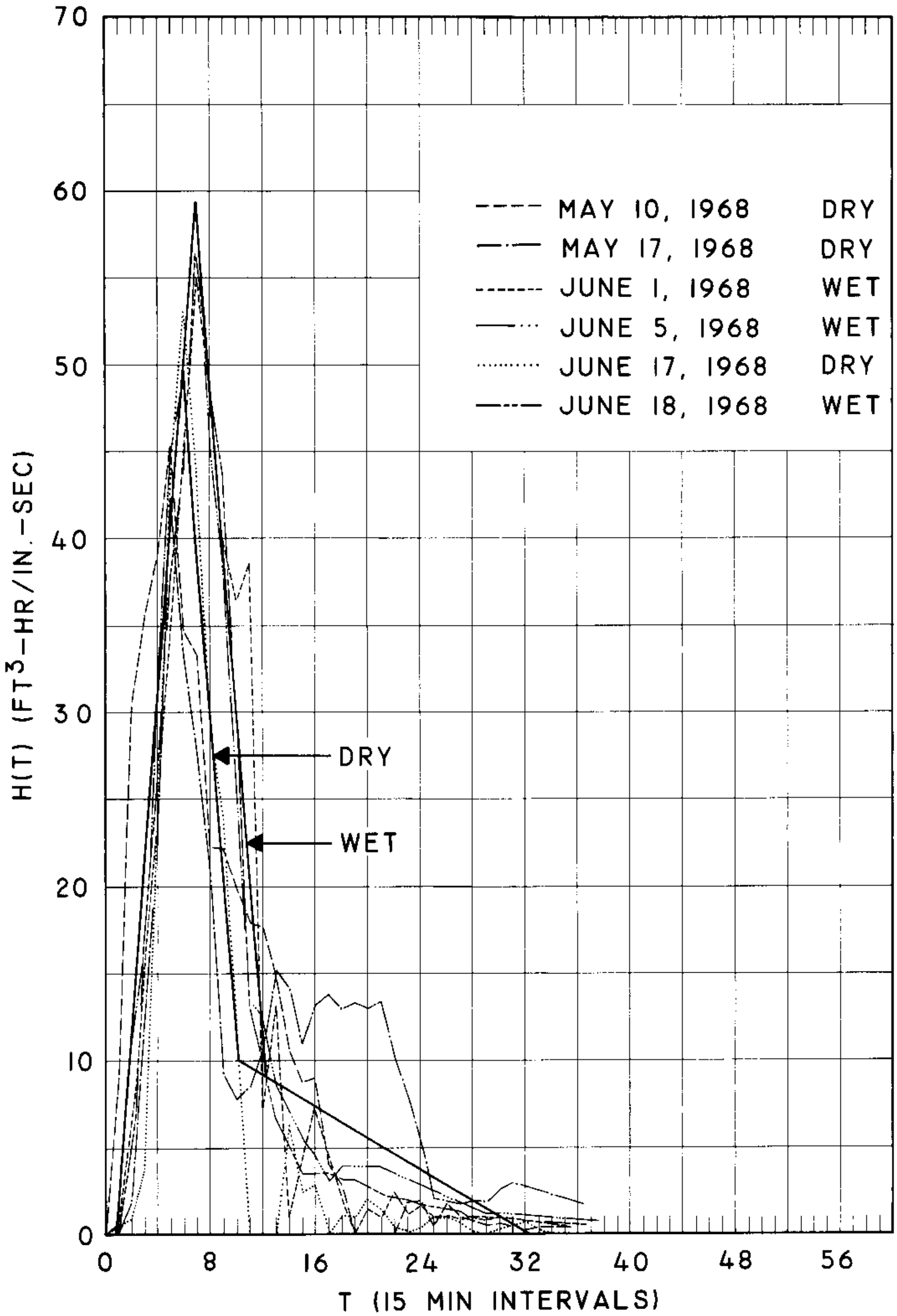


Figure 5.1. Transfer function for Burton Creek Watershed, Bryan, Texas.

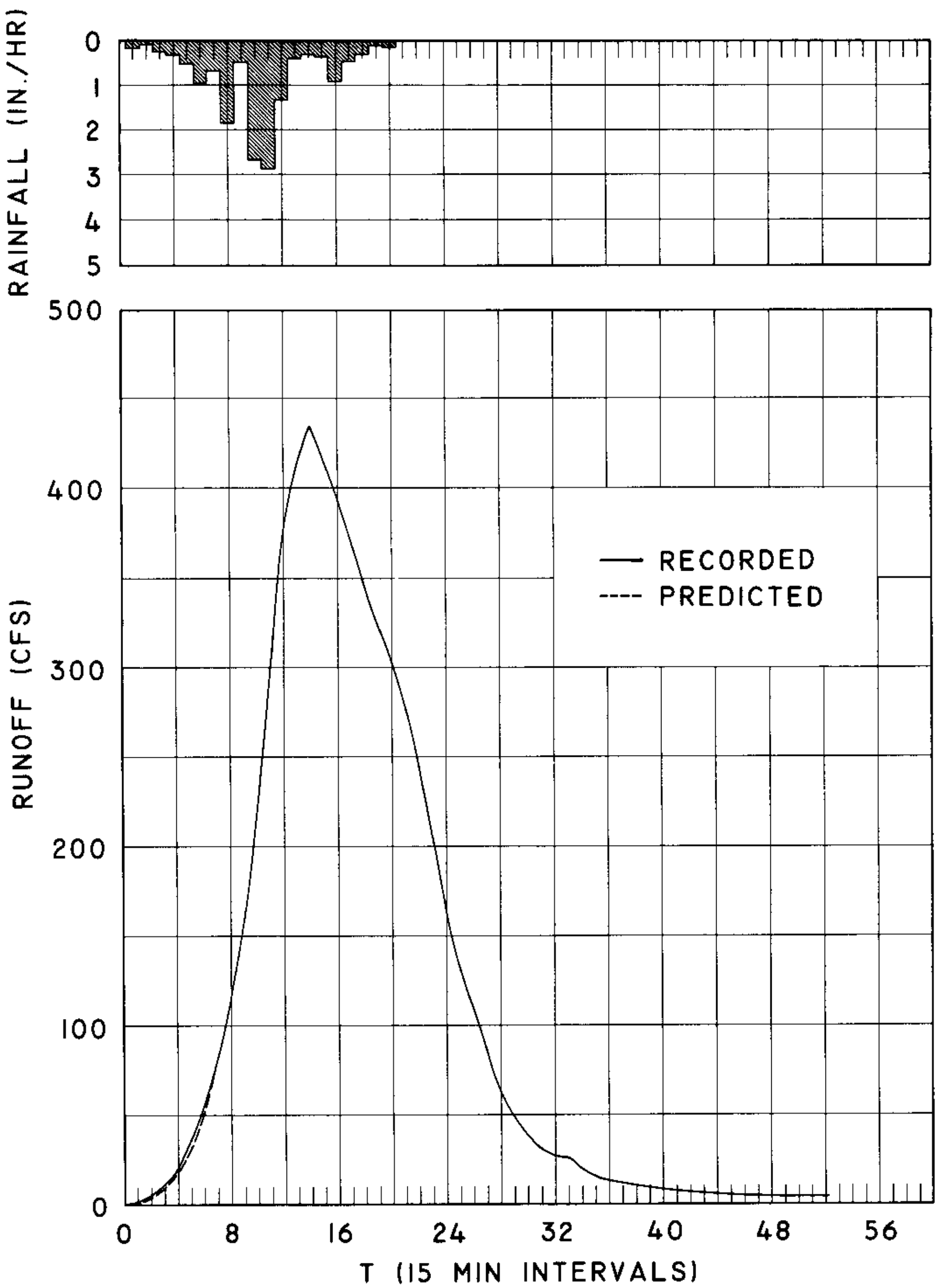


Figure 5.2. Recorded and predicted hydrograph from the derived transfer function, May 10, 1968, Burton Creek, Bryan, Texas.

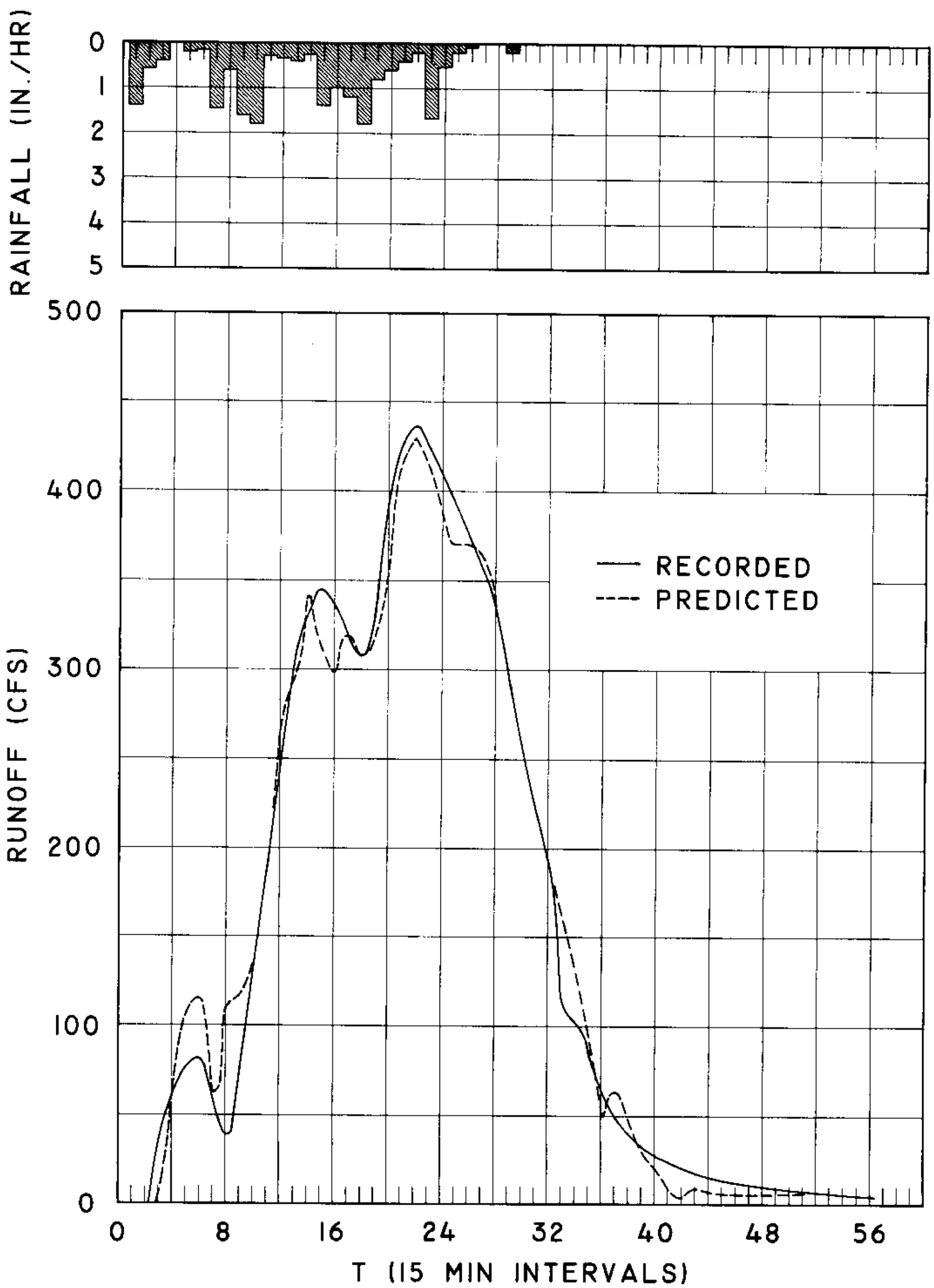


Figure 5.3. Recorded and predicted hydrograph from the derived transfer function, July 9, 1968, Burton Creek, Bryan, Texas.

develop the transfer function and evaluate its performance.

From examination of Figure 5.1 and associated rainfall records, it was surmised that the first portion of the receding limb of the transfer function moved to the right as antecedent moisture conditions increased. From the premise that antecedent moisture conditions did in fact affect the transfer function, the six events were classified as either *wet* or *dry* events. The classification was made upon examination of the rainfall records and was based on the author's judgment as to the antecedent moisture conditions at the time the event occurred. Since Burton Creek is located in Bryan, Texas and the author has resided there for the past five years, an intuitive knowledge of the soils in the area and their performance may have influenced the event classifications; however, this very general classification could be made by anyone who had descriptive knowledge of the soils and rainfall records. The events occurring on May 10, 1968, May 17, 1968 and June 17, 1968 were classified *dry* and the events occurring on June 1, 1968 and June 5, 1968 and June 18, 1968 were classified *wet*. Each transfer function exhibited a tendency to have an increasing limb to the peak and a decreasing limb made up of a rather steep slope for a short period of time and then a much flatter slope. Therefore, each transfer function for the six events was divided by visual inspection into these three sections. The data of these six events indicate that as antecedent moisture conditions move from *wet* to *dry* the peak of the transfer function is lowered, but the slopes of the three sections described above do not change appreciably.

Keeping in mind the intent of the research to develop a simple workable model for predicting runoff, the transfer function was generalized by assuming a straight line for the three above mentioned sections. The straight line equation for the increasing limb and the lower receding limb of the transfer function were obtained by utilizing the data from all six events. Two equations were obtained for the upper receding limb -- one by using the three events classified *wet* and one with the *dry* events. The generalized transfer function for Burton Creek is shown in Figure 5.1, and the equations for the four straight lines are as follows:

Increasing limb

$$H(T) = 9.60\ T - 7.81, \qquad 5.1$$

Upper receding limb

$$H(T) = -\ 9.21\ T + 122.03,\ \textit{wet}, \qquad 5.2$$

$$H(T) = -\ 6.80\ T + 80.30,\ \textit{dry},\ \text{and} \qquad 5.3$$

Lower receding limb

$$H(T) = -\ 0.46\ T + 14.69, \qquad 5.4$$

where

$H(T)$ = generalized transfer function and

$T$ = the number of 15 minute intervals since the beginning of rainfall.

An available least-squares regression program was utilized for obtaining the *best fit* straight lines for the generalized transfer function.

In order to further simplify the generalized transfer function, the *dry* upper receding limb was assumed to have the same slope as the *wet* limb but moved to the left by two time intervals (30 minutes). Therefore, the equation used in the predicting model for the *dry* upper receding limb was adjusted so that

$$H(T) = - 9.21\ T + 103.61. \qquad 5.5$$

Table 5.1 gives the generalized transfer function for Burton Creek.

A computer program which utilizes equation 3.7 for discrete convolution, with f(j) as the rainfall intensity input (in./hr) and equations 5.1, 5.2 or 5.3 or 5.5, and 5.4 as the convolved transfer function, was developed in order to evaluate the prediction model. From the data available for Burton Creek, 20 events of appreciable runoff were selected for evaluating the model. The six events used in developing the generalized transfer function are included in the 20 events. A summary of the results for Burton Creek is given in Table 5.2. Comparisons of the recorded versus the predicted total volume, the time to peak from the start of rainfall, and the peak flow rate are given in Table 5.2. The event classification, total rainfall, and the rainfall

TABLE 5.1. GENERALIZED TRANSFER FUNCTION FOR BURTON CREEK

| *Time Interval | Generalized Transfer Function ($ft^3$-hr/in.-sec) | |
|---|---|---|
| | *Wet* | *Dry* |
| 1 | 1.79 | 1.79 |
| 2 | 11.39 | 11.39 |
| 3 | 20.99 | 20.99 |
| 4 | 30.59 | 30.59 |
| 5 | 40.18 | 40.18 |
| 6 | 49.78 | 49.78 |
| 7 | 59.38 | 39.16 |
| 8 | 48.37 | 29.95 |
| 9 | 39.16 | 20.74 |
| 10 | 29.95 | 11.54 |
| 11 | 20.74 | 9.67 |
| 12 | 11.54 | 9.21 |
| 13 | 8.75 | 8.75 |
| 14 | 8.30 | 8.30 |
| 15 | 7.84 | 7.84 |
| 16 | 7.38 | 7.38 |
| 17 | 6.93 | 6.93 |
| 18 | 6.47 | 6.47 |
| 19 | 6.02 | 6.02 |
| 20 | 5.56 | 5.56 |

Table 5.1. Continued.

| *Time Interval | Generalized Transfer Function ($ft^3$-hr/in.-sec) | |
| --- | --- | --- |
| | *Wet* | *Dry* |
| 21 | 5.10 | 5.10 |
| 22 | 4.65 | 4.65 |
| 23 | 4.19 | 4.19 |
| 24 | 3.73 | 3.73 |
| 25 | 3.28 | 3.28 |
| 26 | 2.82 | 2.82 |
| 27 | 2.37 | 2.37 |
| 28 | 1.91 | 1.91 |
| 29 | 1.45 | 1.45 |
| 30 | 1.00 | 1.00 |
| 31 | 0.54 | 0.54 |
| 32 | 0.08 | 0.08 |

* Time Interval = T = the number of 15 minute intervals since the beginning of rainfall.

TABLE 5.2. SUMMARY OF THE PREDICTION MODEL RESULTS FOR BURTON CREEK, BRYAN, TEXAS.

| Date | Time | Rainfall (inches) | Rainfall Duration (minutes) | Total Volume (acre-feet) Recorded | Total Volume (acre-feet) Predicted | Time to Peak (minutes) Recorded | Time to Peak (minutes) Predicted | Peak Flow (cfs) Recorded | Peak Flow (cfs) Predicted | Event Class |
|---|---|---|---|---|---|---|---|---|---|---|
| 05/10/68 | 0245 | 3.81 | 300 | 124.7 | 114.4 | 210 | 225 | 434.0 | 413.3 | *Dry* |
| 05/11/68 | 1400 | 0.66 | 285 | 31.7 | 24.7 | 330 | 330 | 115.0 | 70.4 | *Wet* |
| 05/17/68 | 1415 | 0.75* | 150 | 24.1 | 22.5 | 90 | 105 | 79.2 | 100.6 | *Dry* |
| 05/26/68 | 0115 | 1.55 | 30 | 36.8 | 46.5 | 135 | 90 | 276.0 | 279.1 | *Dry* |
| 06/01/68 | 2015 | 2.08 | 150 | 71.0 | 77.7 | 165 | 165 | 347.0 | 351.5 | *Wet* |
| 06/05/68 | 1715 | 1.95 | 180 | 70.5 | 72.8 | 120 | 120 | 391.0 | 394.3 | *Wet* |
| 06/17/68 | 1715 | 0.80 | 90 | 17.3 | 24.0 | 150 | 105 | 120.0 | 113.4 | *Dry* |
| 06/18/68 | 1715 | 1.49 | 195 | 48.1 | 55.7 | 120 | 120 | 284.0 | 297.5 | *Wet* |
| 06/20/68 | 1230 | 1.17* | 345 | 57.2 | 43.7 | 180 | 180 | 153.0 | 126.6 | *Wet* |
| 07/09/68 | 0400 | 4.78* | 435 | 175.3 | 143.5 | 330 | 330 | 436.0 | 326.8 | *Dry* |
| 11/26/68 | 1900 | 2.30* | 510 | 67.3 | 67.6 | 180 | 210 | 187.0 | 172.6 | *Dry* |
| 11/27/68 | 0600 | 1.11* | 300 | 63.5 | 41.5 | 195 | 225 | 203.0 | 163.5 | *Wet* |
| 11/30/68 | 0300 | 1.74* | 1155 | 95.8 | 71.9 | 1035 | 1080 | 180.0 | 134.4 | *Wet* |

Table 5.2. Continued.

| Date | Time | Rainfall (inches) | Rainfall Duration (minutes) | Total Volume (acre-feet) | | Time to Peak (minutes) | | Peak Flow (cfs) | | Event Class |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Recorded | Predicted | Recorded | Predicted | Recorded | Predicted | |
| 02/21/69 | 0515 | 1.51 | 480 | 77.3 | 56.4 | 300 | 345 | 181.0 | 141.5 | *Wet* |
| 03/07/69 | 2315 | 0.68 | 75 | 28.1 | 25.4 | 90 | 105 | 166.0 | 148.1 | *Wet* |
| 04/04/69 | 0930 | 2.79 | 120 | 70.8 | 83.8 | 300 | 300 | 392.0 | 423.2 | *Dry* |
| 04/09/69 | 2015 | 1.90 | 150 | 53.7 | 57.1 | 135 | 120 | 304.0 | 299.5 | *Dry* |
| 04/27/69 | 0830 | 1.31* | 180 | 31.9 | 39.3 | 90 | 105 | 172.0 | 193.8 | *Dry* |
| 05/01/69 | 1515 | 0.75* | 255 | 24.7 | 28.0 | 270 | 315 | 105.0 | 80.0 | *Wet* |
| 05/05/69 | 0830 | 0.63 | 165 | 21.2 | 23.5 | 195 | 240 | 128.0 | 128.7 | *Wet* |

* Rainfall event produced a multi-peak hydrograph.

duration for each event also are given in Table 5.2. The information presented in Table 5.2 was provided for general comparison; however, based on this information no inferences should be made with regard to rainfall intensities. The rainfall intensities generally are varied, and it is not feasible to average these values for use with the model. Rainfall intensities do not appear to cause an appreciable effect on the prediction model. Due to limited funds for computer time, the longer duration events were not used in developing the generalized transfer function. The solution to the higher density matrices produced by the longer duration rainfalls was estimated to require in excess of fifteen minutes of computer time. In any event, the shorter-duration, higher-intensity events would be the most logical events to use for developing the generalized transfer function for the assumed lumped-linear, time-invariant system. Intuition would indicate that the longer-duration, lower-intensity events would result in a much more oscillating transfer function with considerable more activity in the slack variables.

In order to further exemplify the performance of the prediction model for Burton Creek, Figures 5.4, 5.5, 5.6 and 5.7 are presented as typical plotted comparisons of the recorded and predicted hydrographs. The particular events reported in these four figures were selected for their diversity in antecedent moisture conditions, total rainfall, and rainfall intensity.

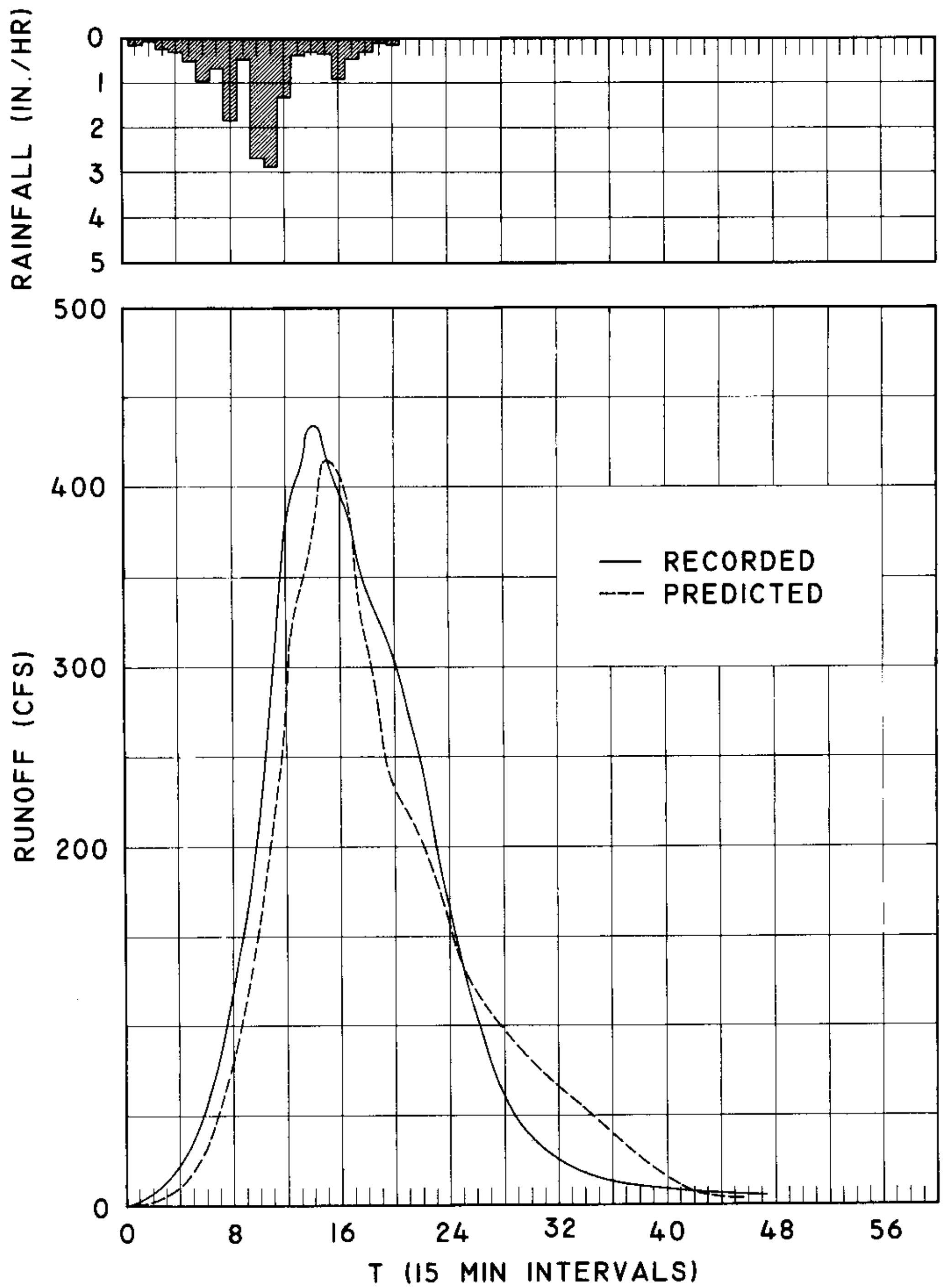


Figure 5.4. Recorded and predicted hydrograph from the generalized transfer function, May 10, 1968, Burton Creek, Bryan, Texas.

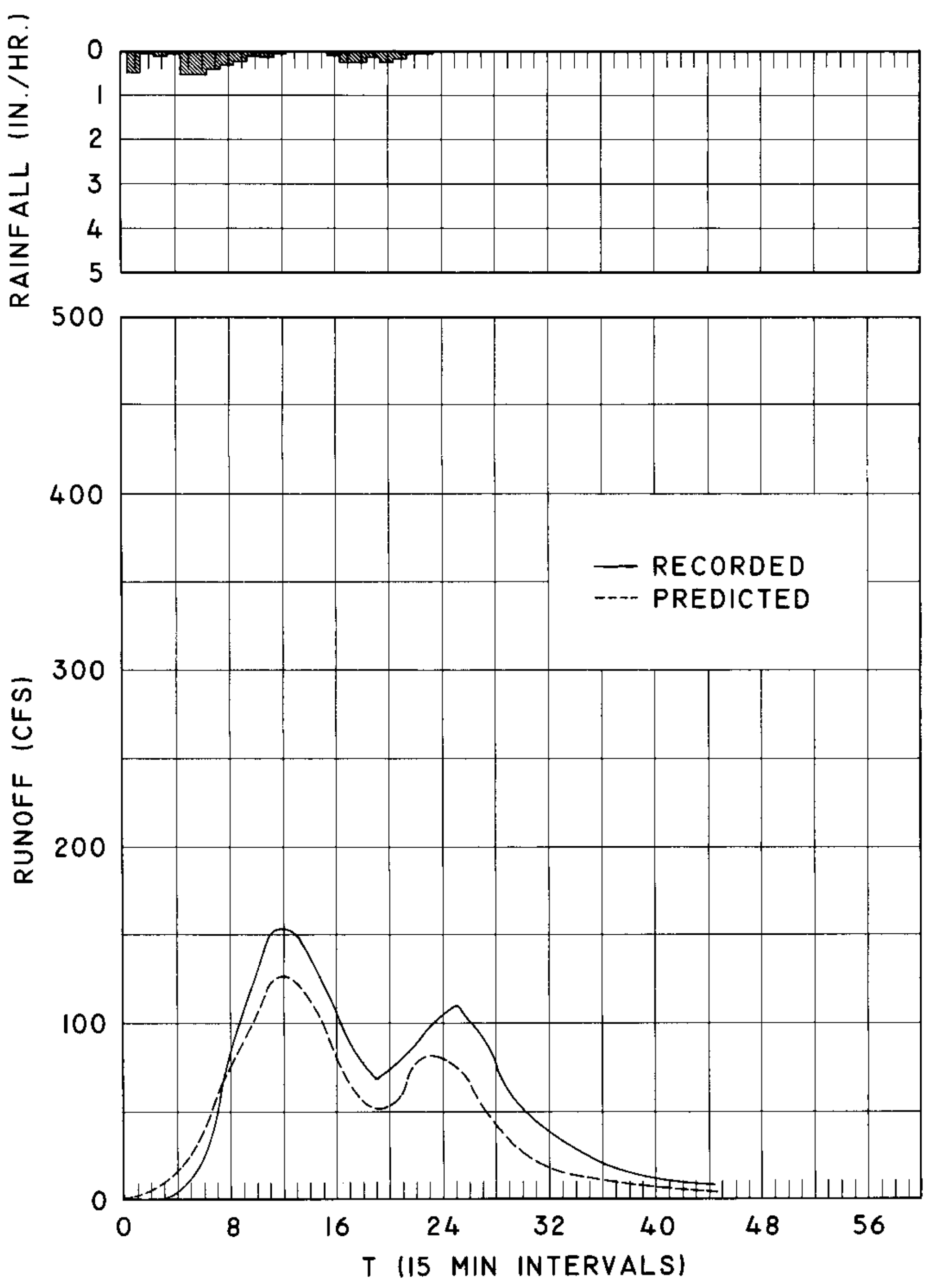


Figure 5.5. Recorded and predicted hydrograph from the generalized transfer function, June 20, 1968, Burton Creek, Bryan, Texas.

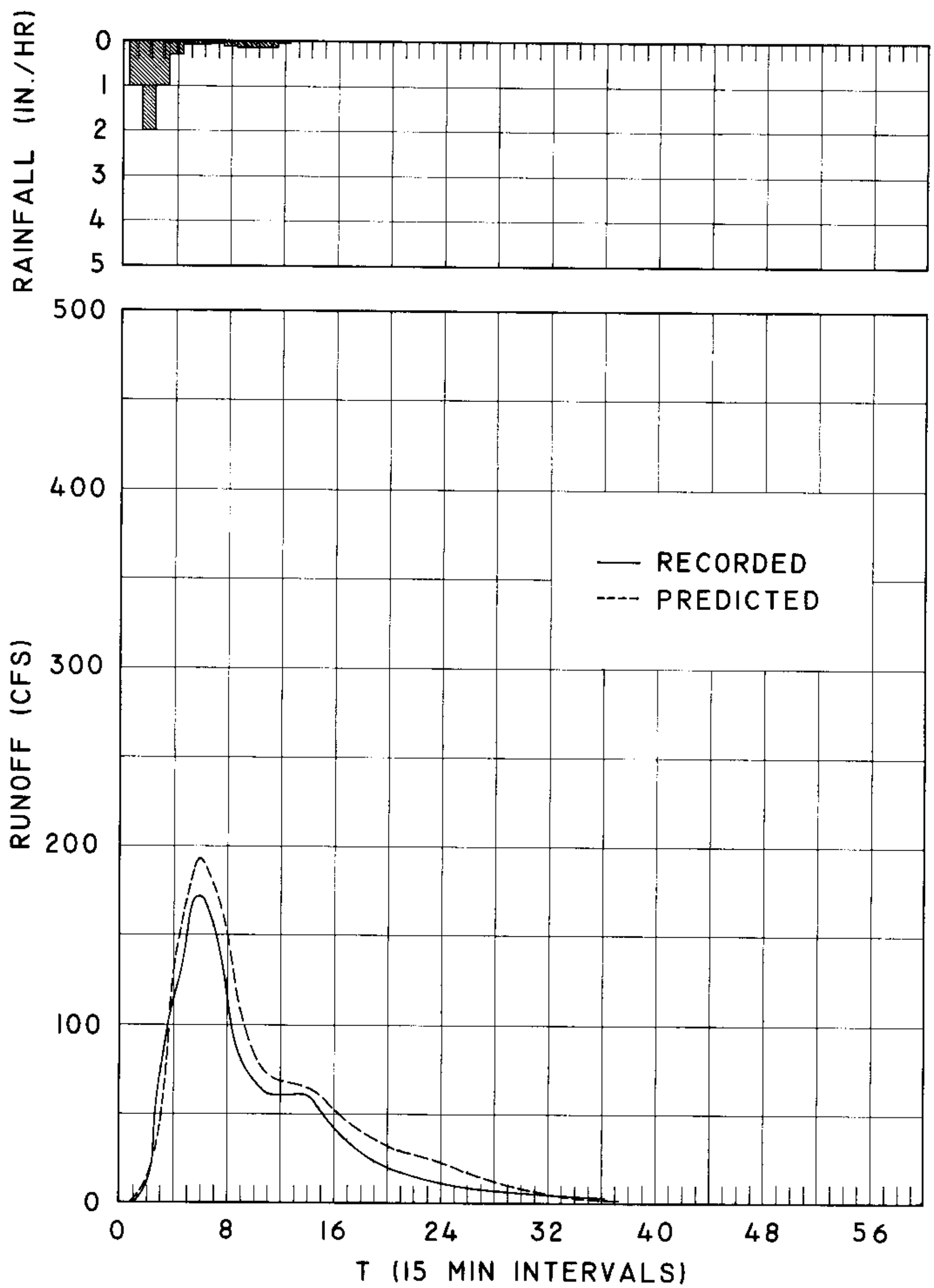


Figure 5.6. Recorded and predicted hydrograph from the generalized transfer function, April 27, 1969, Burton Creek, Bryan, Texas.

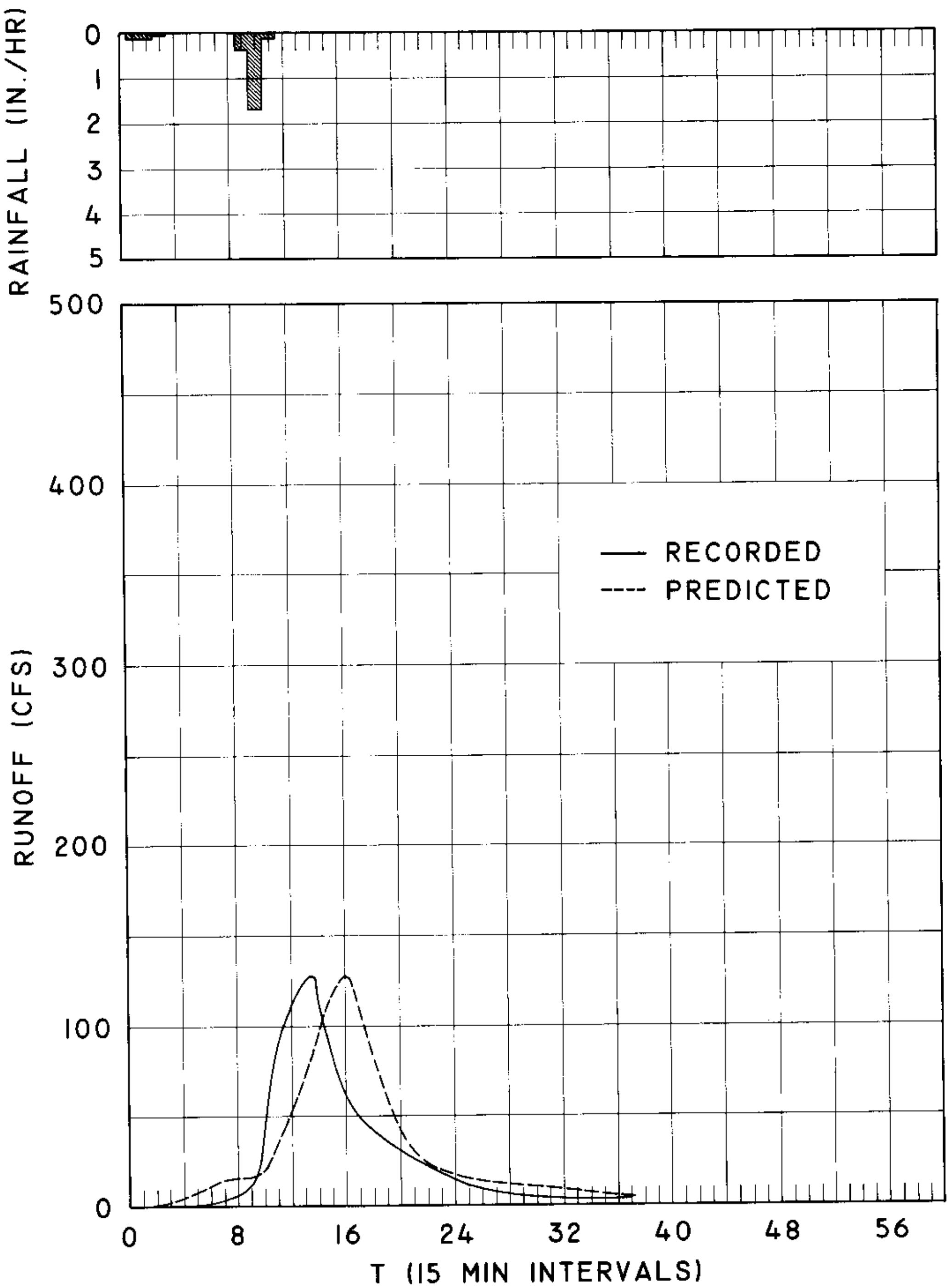


Figure 5.7. Recorded and predicted hydrograph from the generalized transfer function, May 5, 1969, Burton Creek, Bryan, Texas.

Figures 5.4 and 5.5 are for events occurring during 1968, and Figures 5.6 and 5.7 are for events occurring in 1969.

Most of the rainfall events producing appreciable runoff during 1968 and 1969 on Burton Creek occurred in the late spring and early summer. The generalized transfer function was developed for events occurring over only a 40-day period in late spring of 1968. Keeping in mind the events used for developing the transfer function, it is interesting to note the performance of the prediction model for the rainfall event of November 26, 1968. Figure 5.8 gives a plotted comparison of the recorded and predicted hydrographs for this event. In light of the rather diverse rainfall intensity, the long duration of the rainfall (510 minutes), and the time of year of this event, the agreement between the recorded and predicted hydrograph is excellent. From the data presently available, it appears that the only basin characteristic that appreciably affects the Burton Creek generalized transfer function is antecedent moisture condition.

It should be pointed out that the base flow was assumed negligible. This affected the agreement between the recorded and predicted hydrograph on some of the events classified *wet*. The intent of this research was to evaluate the prediction model and, therefore, the effect of base flow was ignored. A correction for base flow would refine the model and improve the accuracy for the *wet* events on Burton Creek.

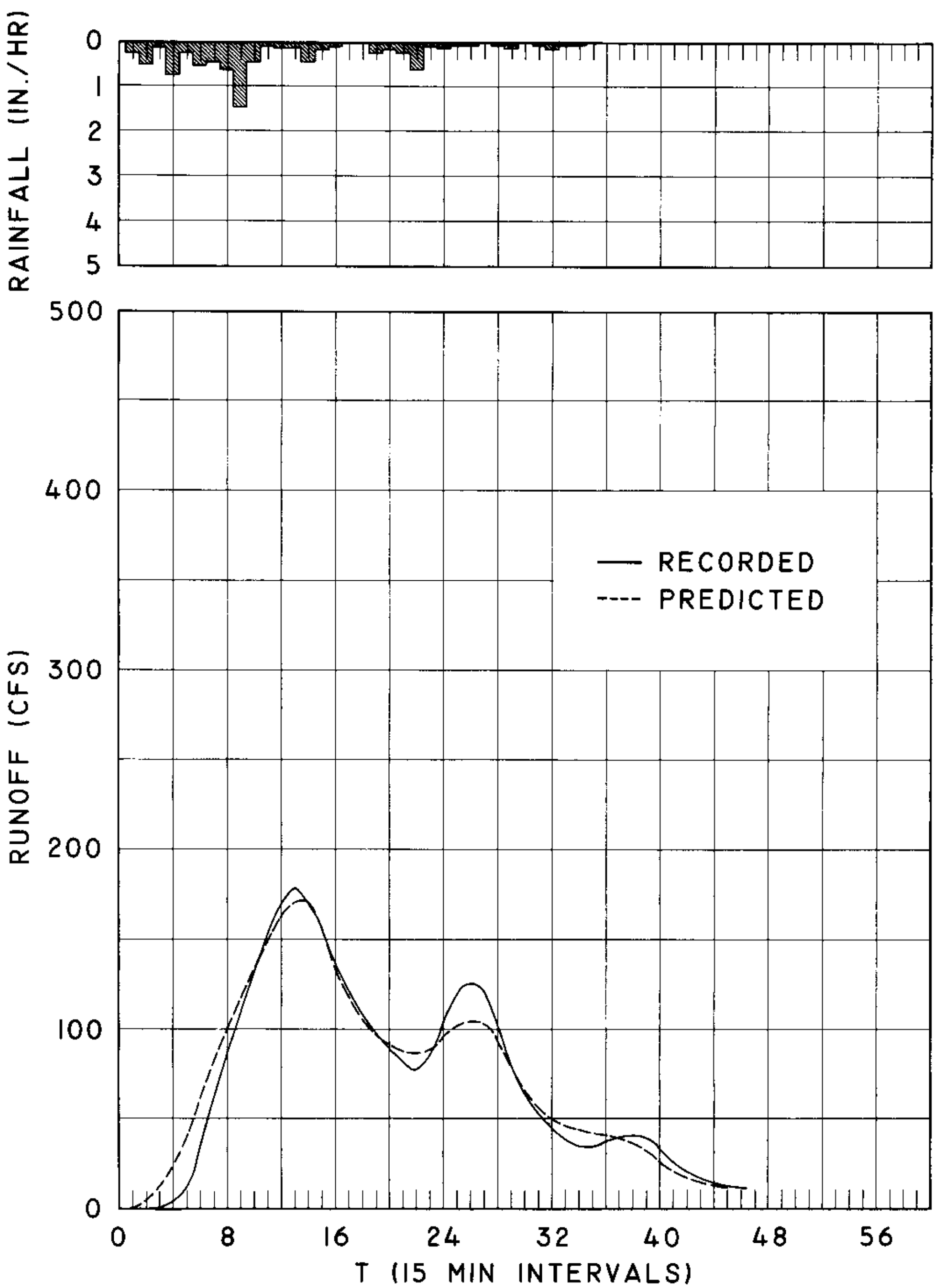


Figure 5.8. Recorded and predicted hydrograph from the generalized transfer function, November 26, 1968, Burton Creek, Bryan, Texas.

Figure 5.9 exhibits the event of November 27, 1968 and is presented to illustrate the effect of the *wet* or *dry* classification on the prediction model. The November 27 event began two and one-half hours after the November 26 event and, therefore, would be logically classified *wet*. The event was also analyzed as a *dry* event and the effect of the event classification can be examined in Figure 5.9. In this particular case, the effect of base flow was quite apparent and, therefore, the predictions were adjusted upward 10 cfs.

To continue the evaluation of the prediction model, attention was directed to the rural Hudson Creek Watershed which is located approximately three miles east of the Burton Creek Watershed. The original intent was to utilize the same six events that were used on Burton Creek in developing the generalized transfer function for Hudson Creek. Rain gages number 2 and 3 were not installed on Hudson Creek until June 12, 1968. Rain gage number 1 is a remote recording gage which is integrated with the stream-stage recorder, and unfortunately the data obtained from this gage are not reliable. The gage appears to report intensities that are too high for the first part of the rainfall, and the total amount reported is not consistent with other more reliable records. Therefore, the intended direct comparison between the two watersheds was not possible.

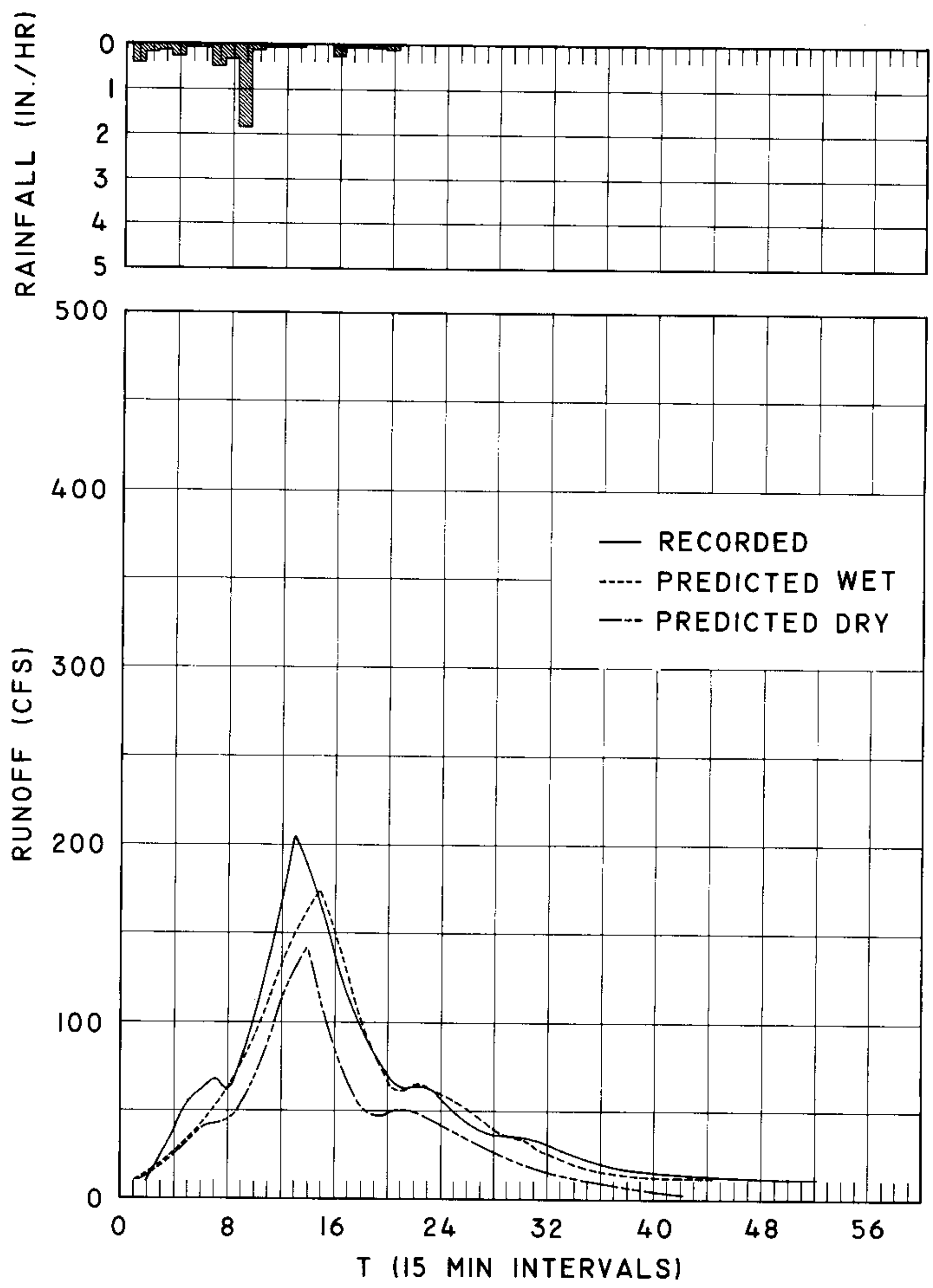


Figure 5.9. Recorded and predicted hydrograph from the generalized transfer function, November 27, 1968, Burton Creek, Bryan, Texas.

Reliable rainfall-runoff data were not available on Hudson Creek until after June 15, 1968 and 12 events producing appreciable runoff were selected. From these 12 events only six events had short enough rainfall durations to make feasible the computation of the transfer function with the computer program being used. Figure 5.10 exhibits graphically the solution for the transfer function for these six events which occurred between February 14, 1969 to May 1, 1969. In Figure 5.10 the elapsed time duration has been arbitrarily truncated at 750 minutes to allow better graphical comparison. The complete solution was obtained with the time interval specified in Chapter III. The complete solution is not reported herein due to its voluminous nature; however, the complete solutions appear very similar to those reported for Burton Creek in Appendix B. Figures 5.11 and 5.12 exhibit the recorded hydrographs and the hydrographs obtained by convolving the computer transfer function of length m with the recorded rainfall intensities for two different events. Figure 5.11 is typical of all of the six events reported in Figure 5.10.

The transfer function for the July 9, 1968 event was obtained in order to evaluate the model under rather extreme conditions. This rainfall event occurred over a period of seven hours with a total rainfall of 6.62 inches and intensities ranging from 0.04 to 3.20 inches per hour. Figure 5.13 exhibits the complete optimum solution for the transfer function for this event obtained from

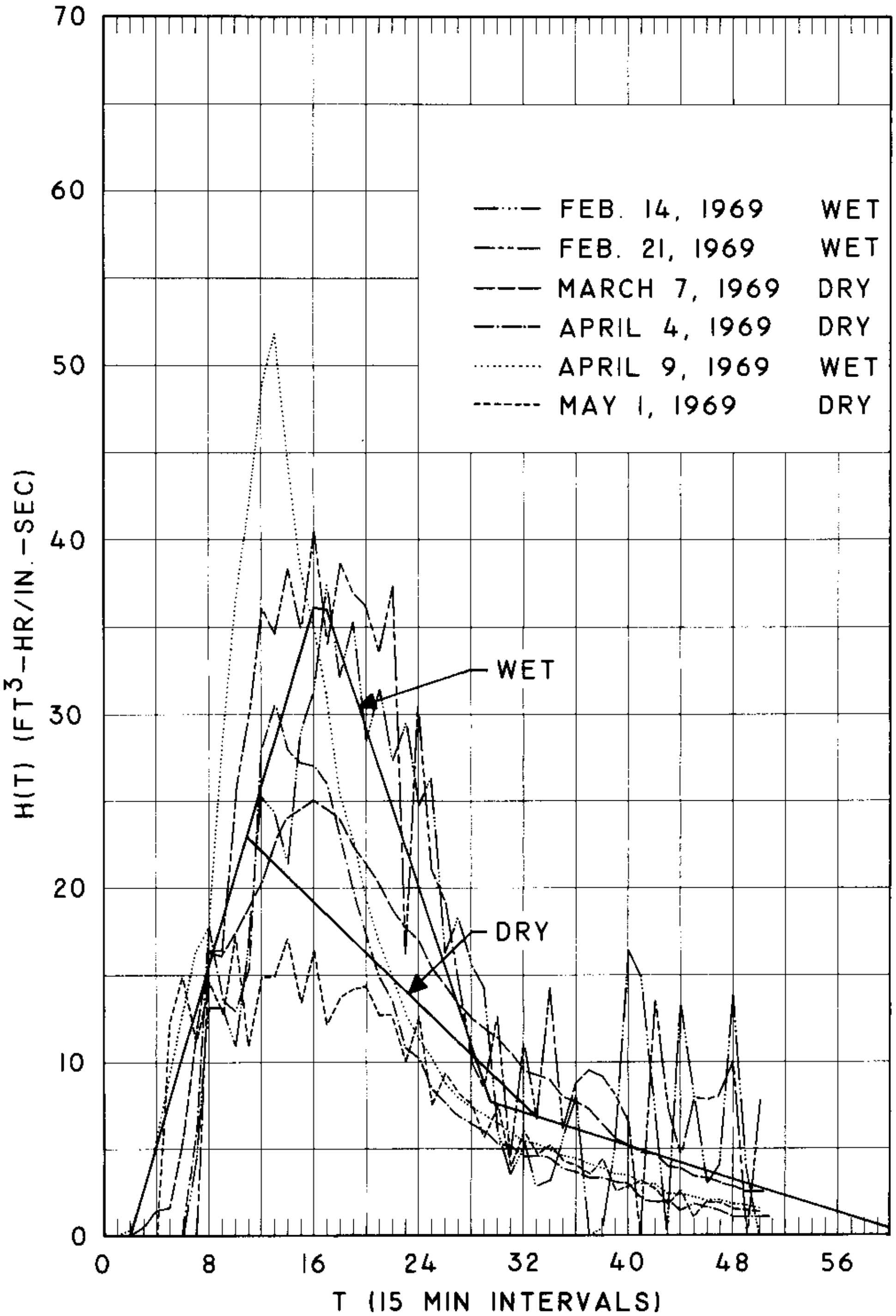


Figure 5.10. Transfer function for Hudson Creek Watershed, Bryan, Texas.

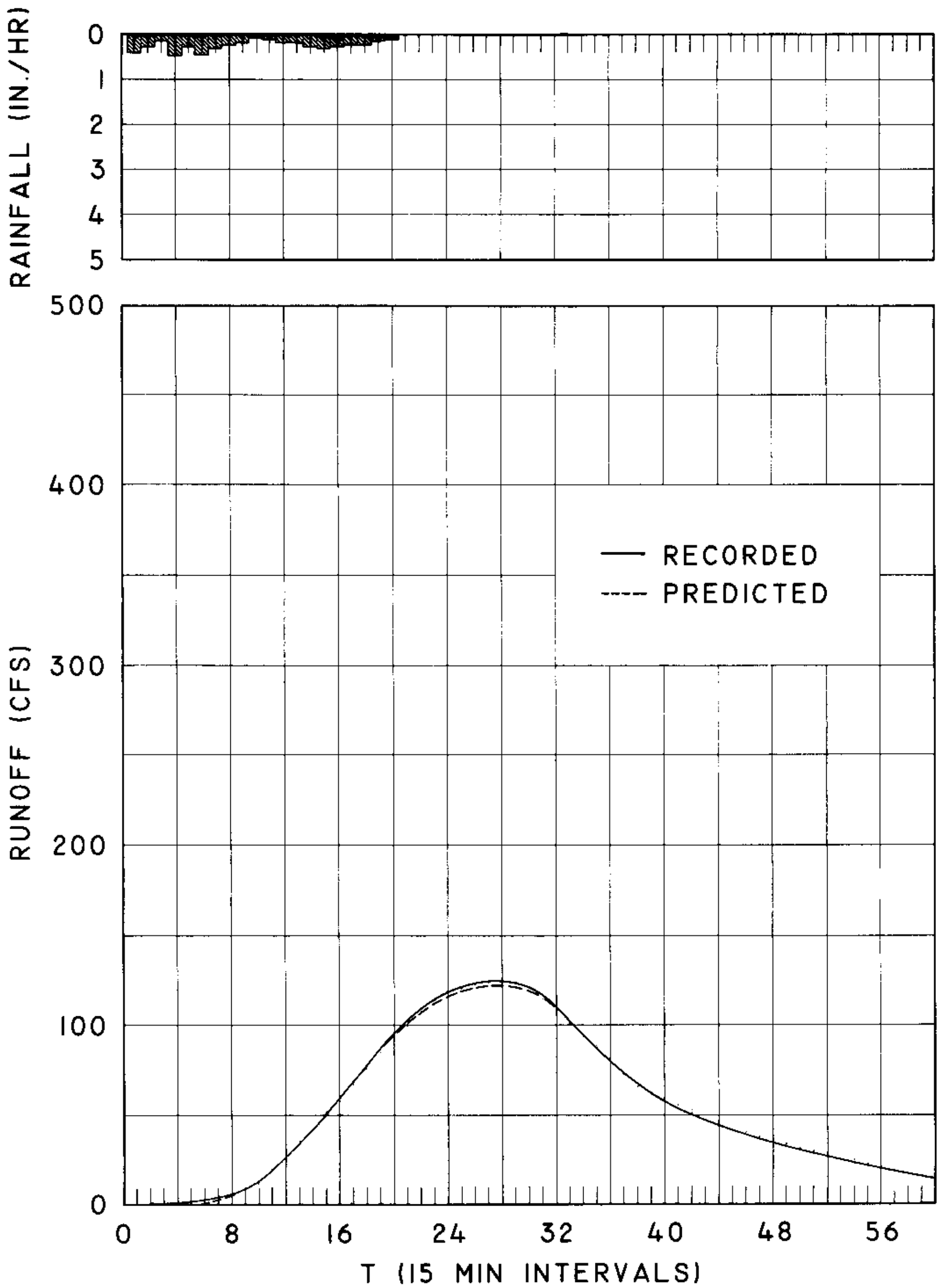


Figure 5.11. Recorded and predicted hydrograph from the derived transfer function, February 14, 1969, Hudson Creek, Bryan, Texas.

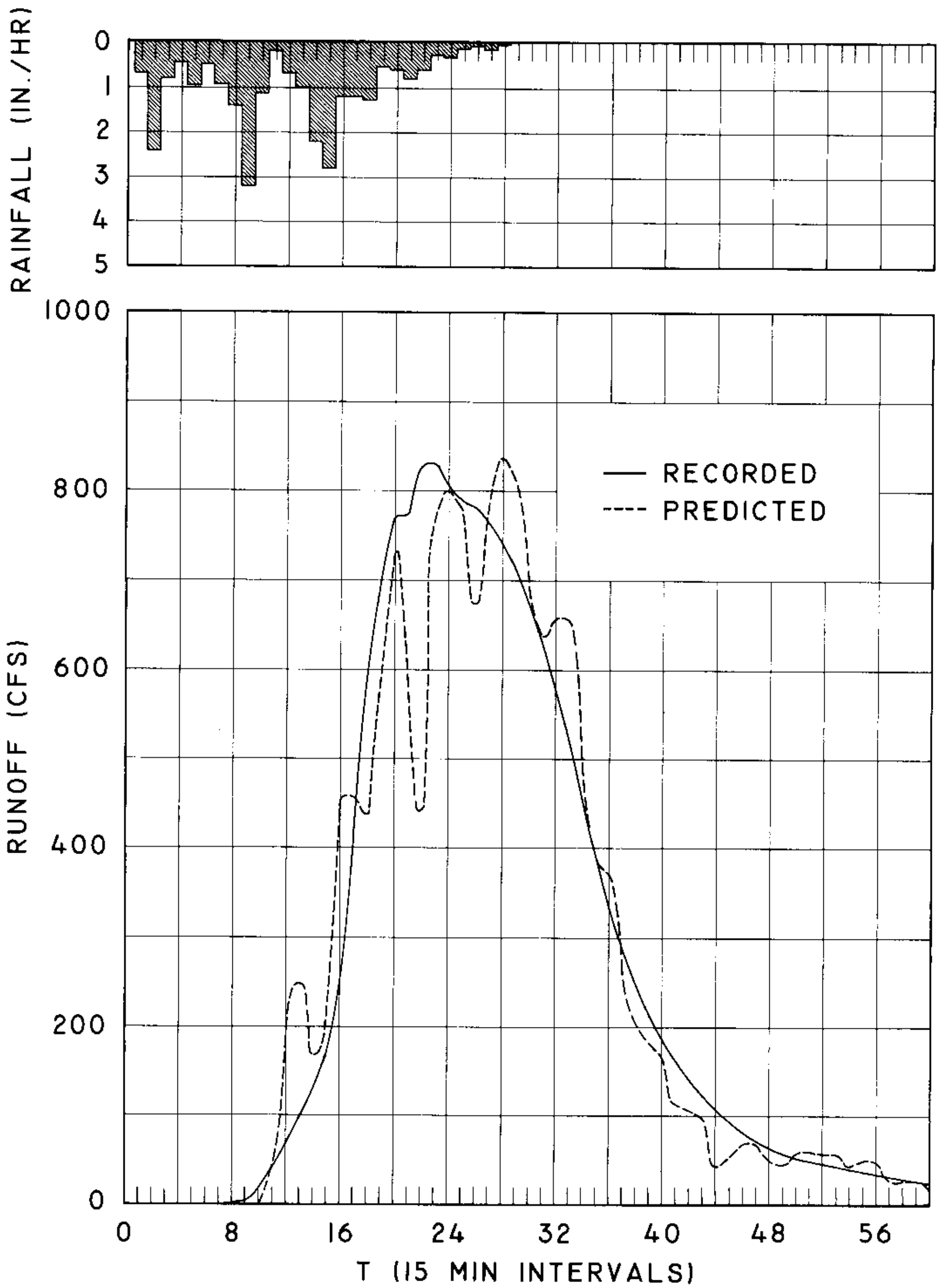


Figure 5.12. Recorded and predicted hydrograph from the derived transfer function, July 9, 1968, Hudson Creek, Bryan, Texas.

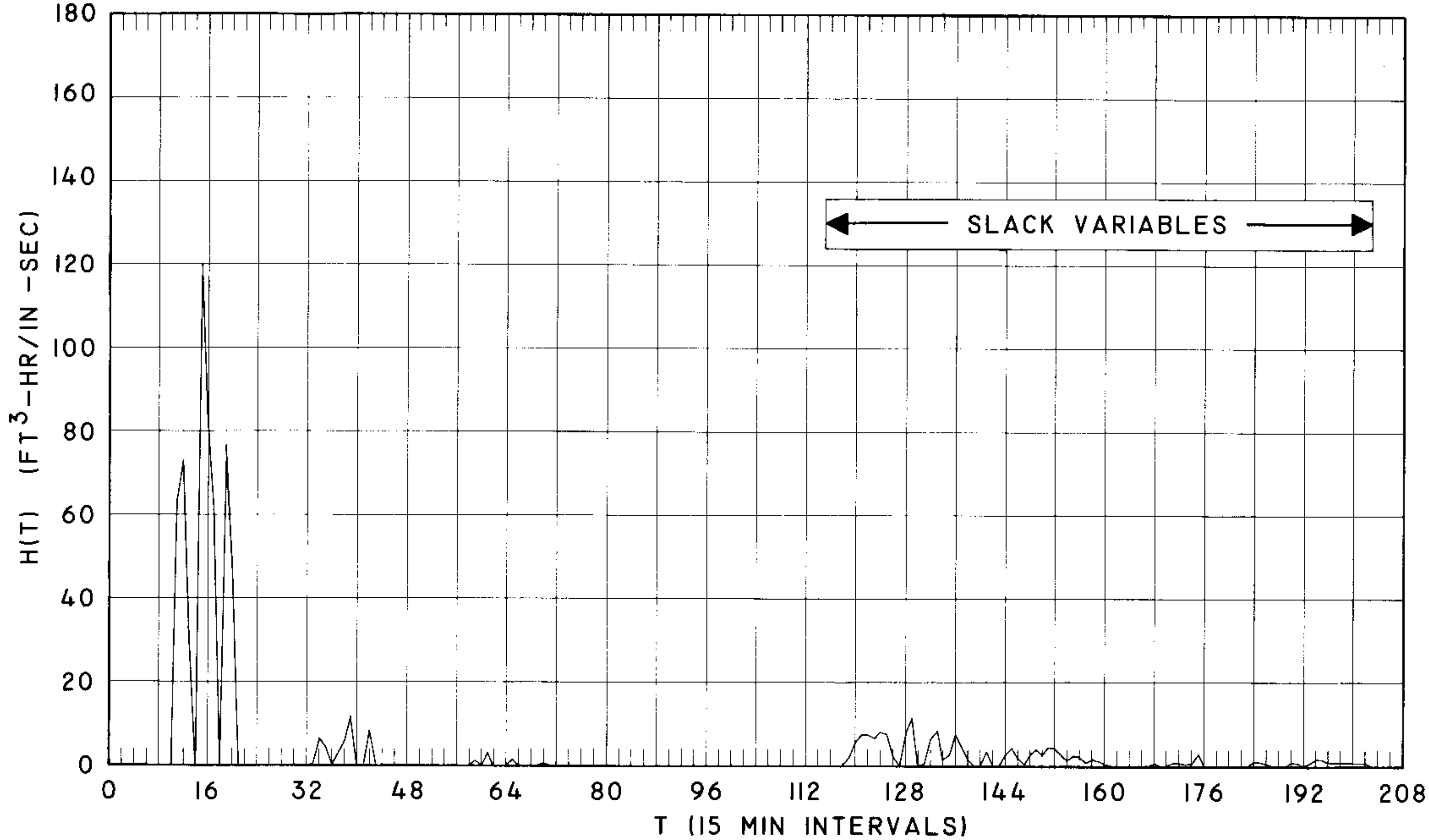


Figure 5.13. Transfer function for Hudson Creek, July 9, 1968, Bryan, Texas.

equations 3.24, 3.25 and 3.26. The recorded runoff had a duration of 88 intervals (1320 minutes) and, therefore, the slack variables introduced to assure feasibility start with interval 115 and continue to the end of the solution length. Much of the activity involved in this solution takes place in the slack variables which gives an intuitive indication that the linear model would not produce a satisfactory prediction result. The predicted hydrograph in Figure 5.12 is the result of convolving the linear portion of the solution with the recorded rainfall. The linear prediction is surprisingly good in light of the fact that only a small portion of the optimum transfer function was used. This particular event was selected because the transfer function for it was the most *misbehaved* of all events that were analyzed. We will look at this event again with the generalized transfer function, but it was felt that this illustration would help to exemplify the apparent power of the proposed model in representing hydrologic rainfall-runoff events.

In developing a generalized transfer function for Hudson Creek, the inferences and results obtained from Burton Creek were utilized. It is apparent that the *wet* and *dry* event classification is not warranted if the six events in Figure 5.10 are the only information available. However, it is felt that as more data become available on Hudson Creek this phenomenon will be substantiated. Furthermore, the greater temporal variability in basin characteristics in the rural basin, Hudson Creek, will complicate the isolation of the effects of any specific characteristic on the transfer function. In any event, the results of the evaluation of the model with the limited data available on Hudson Creek are presented. It is felt that the results warrant further and more detailed study.

The six events reported in Figure 5.10 were handled exactly as the Burton Creek events with the February 14, 1969, February 21, 1969 and April 9, 1969 events classified as *wet* and the March 7, 1969, April 4, 1969 and May 1, 1969 events classified *dry*. The developed generalized transfer function for Hudson Creek is shown in Figure 5.10 and presented in Table 5.3. The equations for the four straight lines are as follows:

Increasing limb

$$H(T) = 2.58\ T - 5.18, \tag{5.6}$$

Upper receding limb

$$H(T) = -\ 2.27\ T + 74.01,\ \textit{wet}, \tag{5.7}$$

$$H(T) = -\ 0.73\ T + 30.96,\ \textit{dry},\ \text{and} \tag{5.8}$$

Lower receding limb

$$H(T) = -\ 0.24\ T + 14.80, \tag{5.9}$$

where

$H(T)$ = generalized transfer function, and

$T$ = the number of 15 minute intervals since the beginning of rainfall.

TABLE 5.3. GENERALIZED TRANSFER FUNCTION FOR HUDSON CREEK

| *Time Interval | Generalized Transfer Function ($ft^3$-hr/in.-sec) | |
|---|---|---|
| | *Wet* | *Dry* |
| 1 | 0.00 | 0.00 |
| 2 | 0.00 | 0.00 |
| 3 | 2.57 | 2.57 |
| 4 | 5.15 | 5.15 |
| 5 | 7.73 | 7.73 |
| 6 | 10.32 | 10.32 |
| 7 | 12.90 | 12.90 |
| 8 | 15.48 | 15.48 |
| 9 | 18.07 | 18.07 |
| 10 | 20.65 | 20.65 |
| 11 | 23.24 | 23.24 |
| 12 | 25.82 | 22.14 |
| 13 | 28.40 | 21.40 |
| 14 | 30.99 | 20.67 |
| 15 | 33.57 | 19.94 |
| 16 | 36.15 | 19.20 |
| 17 | 36.10 | 18.47 |
| 18 | 33.82 | 17.73 |
| 19 | 31.55 | 17.00 |
| 20 | 29.28 | 16.27 |

Table 5.3. Continued.

| *Time Interval | Generalized Transfer Function ($ft^3$-hr/in.-sec) *Wet* | *Dry* |
|---|---|---|
| 21 | 27.01 | 15.53 |
| 22 | 24.74 | 14.80 |
| 23 | 22.47 | 14.06 |
| 24 | 20.20 | 13.33 |
| 25 | 17.93 | 12.59 |
| 26 | 15.66 | 11.86 |
| 27 | 13.38 | 11.12 |
| 28 | 11.11 | 10.39 |
| 29 | 8.84 | 9.66 |
| 30 | 7.61 | 8.92 |
| 31 | 7.37 | 8.19 |
| 32 | 7.13 | 7.45 |
| 33 | 6.89 | 6.89 |
| 34 | 6.65 | 6.65 |
| 35 | 6.41 | 6.41 |
| 36 | 6.17 | 6.17 |
| 37 | 5.93 | 5.93 |
| 38 | 5.69 | 5.69 |
| 39 | 5.45 | 5.45 |

Table 5.3. Continued.

| *Time Interval | Generalized Transfer Function ($ft^3$-hr/in.-sec) | |
|---|---|---|
| | *Wet* | *Dry* |
| 40 | 5.21 | 5.21 |
| 41 | 4.97 | 4.97 |
| 42 | 4.73 | 4.74 |
| 43 | 4.49 | 4.49 |
| 44 | 4.25 | 4.25 |
| 45 | 4.01 | 4.01 |
| 46 | 3.77 | 3.77 |
| 47 | 3.53 | 3.53 |
| 48 | 3.29 | 3.29 |
| 49 | 3.05 | 3.05 |
| 50 | 2.81 | 2.81 |
| 51 | 2.57 | 2.57 |
| 52 | 2.33 | 2.33 |
| 53 | 2.09 | 2.09 |
| 54 | 1.85 | 1.85 |
| 55 | 1.61 | 1.61 |
| 56 | 1.37 | 1.37 |
| 57 | 1.13 | 1.13 |

Table 5.3. Continued.

| *Time Interval | Generalized Transfer Function ($ft^3$-hr/in.-sec) *Wet* | *Dry* |
|---|---|---|
| 58 | 0.90 | 0.90 |
| 59 | 0.66 | 0.66 |
| 60 | 0.42 | 0.42 |
| 61 | 0.18 | 0.18 |

* Time Interval = T = the number of 15 minute intervals since the beginning of rainfall.

As before, a least-squares regression program was utilized for obtaining the *best fit* straight lines for the generalized transfer function.

A summary of the results of convolving recorded rainfall intensities with the generalized transfer function for Hudson Creek is given in Table 5.4. The agreement between the recorded and the predicted hydrographs is not as good as was experienced with Burton Creek. The fact that less representative data may have been used for developing the generalized transfer function plus the fact that the rural basin appears to be a more complicated hydrologic system may have caused the disagreement between the predicted and recorded events. As with Burton Creek, it was felt that the tabular summary did not fully illustrate the abilities of the model. Therefore, Figures 5.14, 5.15, 5.16 and 5.17, which are reproductions of four selected events, are presented. Figure 5.14 is the event of July 9, 1968 that was discussed in some detail previously. The complicated rainfall histogram and compound hydrograph exhibited in Figure 5.16 explain some of the apparent discrepancies in the summary in Table 5.4.

The hydrograph prediction with the generalized transfer function for Hudson Creek generally produced total volume and peak flow that were too low. It appears that antecedent moisture conditions have definite effect on runoff from this basin. This is suggested upon examination of the differences in the *dry*

TABLE 5.4. SUMMARY OF THE PREDICTION MODEL RESULTS FOR HUDSON CREEK, BRYAN, TEXAS.

| Date | Time | Rainfall (inches) | Rainfall Duration (minutes) | Total Volume (acre-feet) | | Time to Peak (minutes) | | Peak Flow (cfs) | | Event Class |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Recorded | Predicted | Recorded | Predicted | Recorded | Predicted | |
| 06/23/68 | 1245 | 3.65 | 960 | 308.8 | 207.2 | 915 | 990 | 555.0 | 277.1 | *Wet* |
| 07/09/68 | 0415 | 6.62* | 420 | 339.1 | 376.2 | 345 | 435 | 829.0 | 651.9 | *Wet* |
| 11/26/68 | 1945 | 4.18* | 1395 | 131.3 | 178.5 | 945 | 315 | 166.0 | 136.3 | *Wet* |
| 11/30/68 | 0830 | 1.76 | 690 | 116.1 | 102.3 | 750 | 765 | 198.0 | 143.6 | *Wet* |
| 02/14/69 | 0145 | 1.26 | 300 | 72.0 | 71.6 | 420 | 360 | 126.0 | 124.9 | *Wet* |
| 02/21/69 | 0745 | 1.17 | 345 | 79.3 | 66.5 | 345 | 345 | 136.0 | 118.0 | *Wet* |
| 03/07/69 | 2315 | 0.82 | 75 | 41.4 | 35.9 | 240 | 165 | 81.0 | 72.9 | *Dry* |
| 03/14/69 | 1900 | 3.04* | 1335 | 195.4 | 172.8 | 1215 | 1275 | 230.0 | 158.3 | *Wet* |
| 04/04/69 | 0930 | 2.06 | 135 | 79.4 | 90.1 | 270 | 240 | 225.0 | 176.0 | *Dry* |
| 04/09/69 | 2030 | 1.95 | 135 | 110.5 | 110.3 | 210 | 270 | 383.0 | 275.3 | *Wet* |
| 04/12/69 | 2445 | 4.60 | 930 | 369.6 | 261.4 | 810 | 930 | 691.0 | 364.9 | *Wet* |
| 05/01/69 | 1515 | 1.15* | 255 | 39.1 | 50.3 | 330 | 345 | 63.8 | 78.2 | *Dry* |

* Rainfall event produced a multi-peak hydrograph.

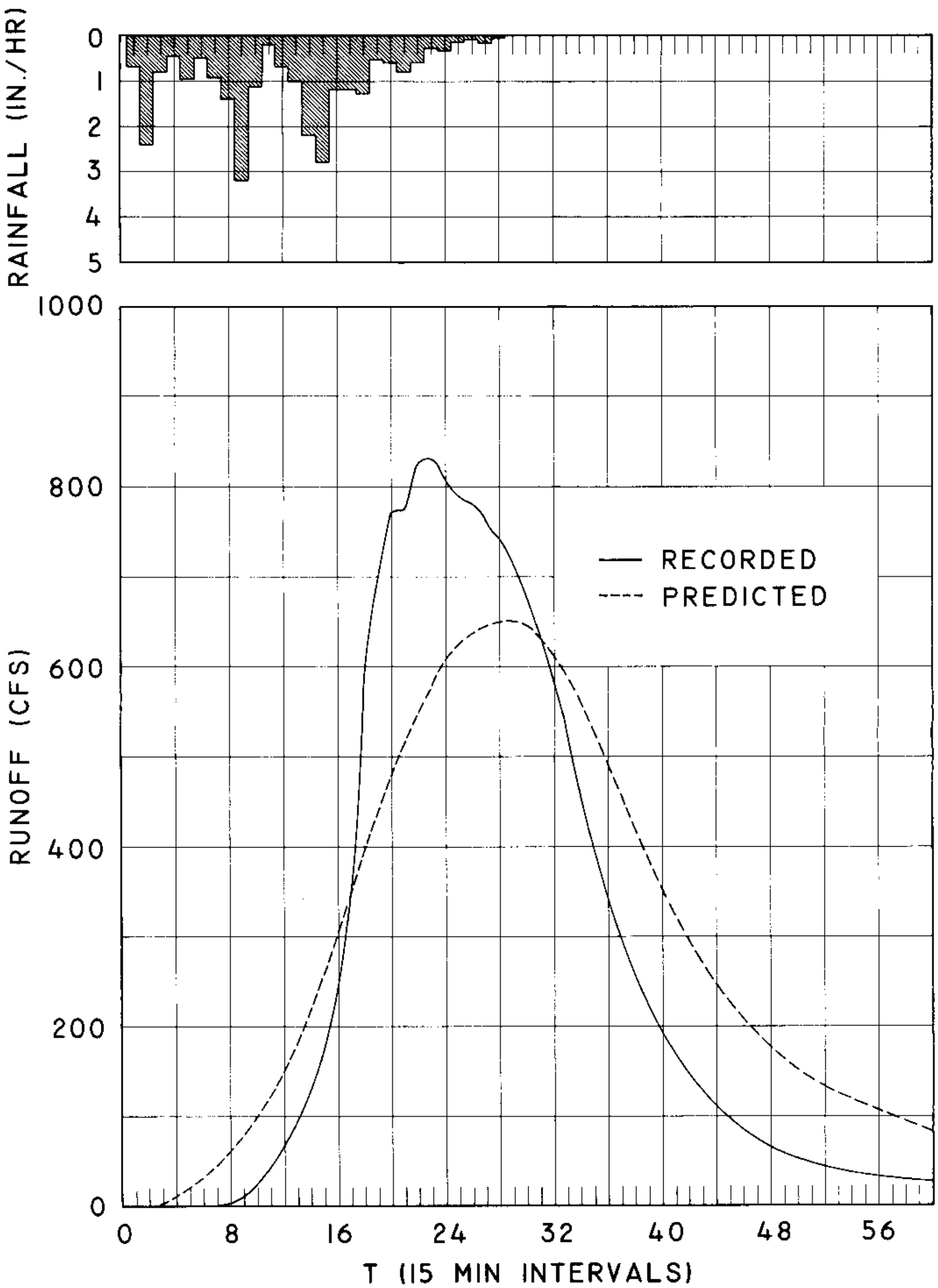


Figure 5.14. Recorded and predicted hydrograph from the generalized transfer function, July 9, 1968, Hudson Creek, Bryan, Texas.

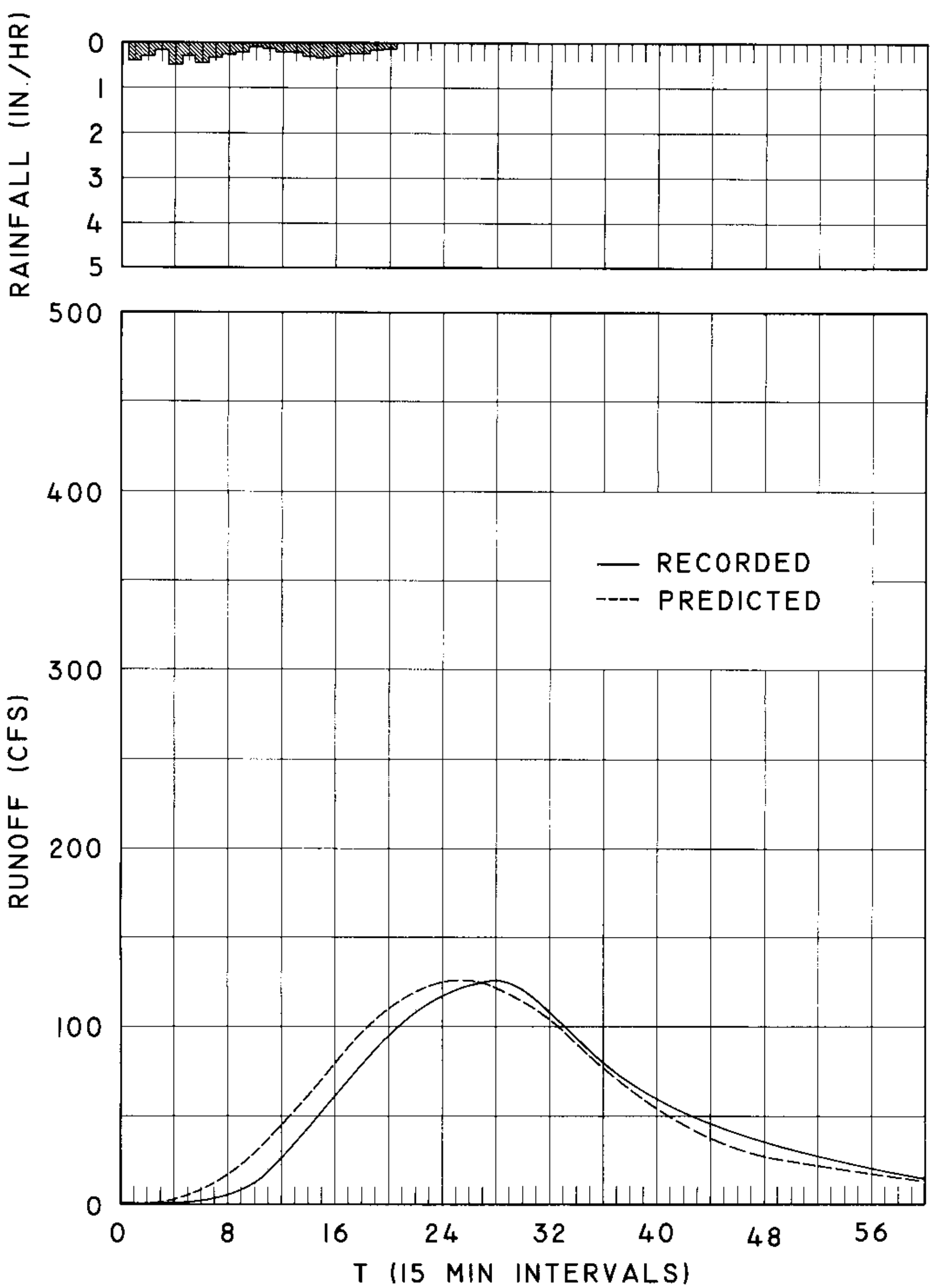


Figure 5.15. Recorded and predicted hydrograph from the generalized transfer function, February 14, 1969, Hudson Creek, Bryan, Texas.

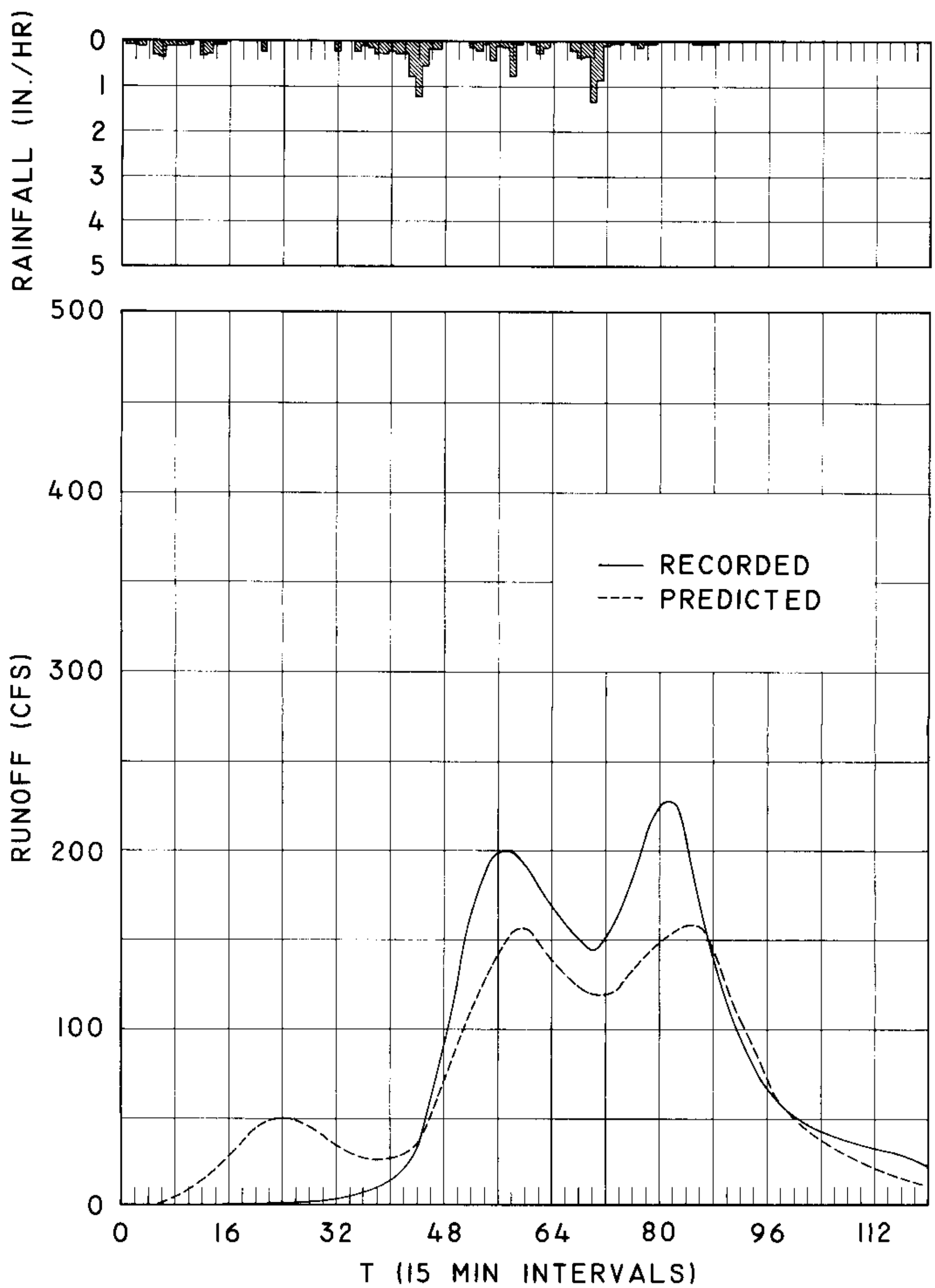


Figure 5.16. Recorded and predicted hydrograph from the generalized transfer function, March 14, 1969, Hudson Creek, Bryan, Texas.

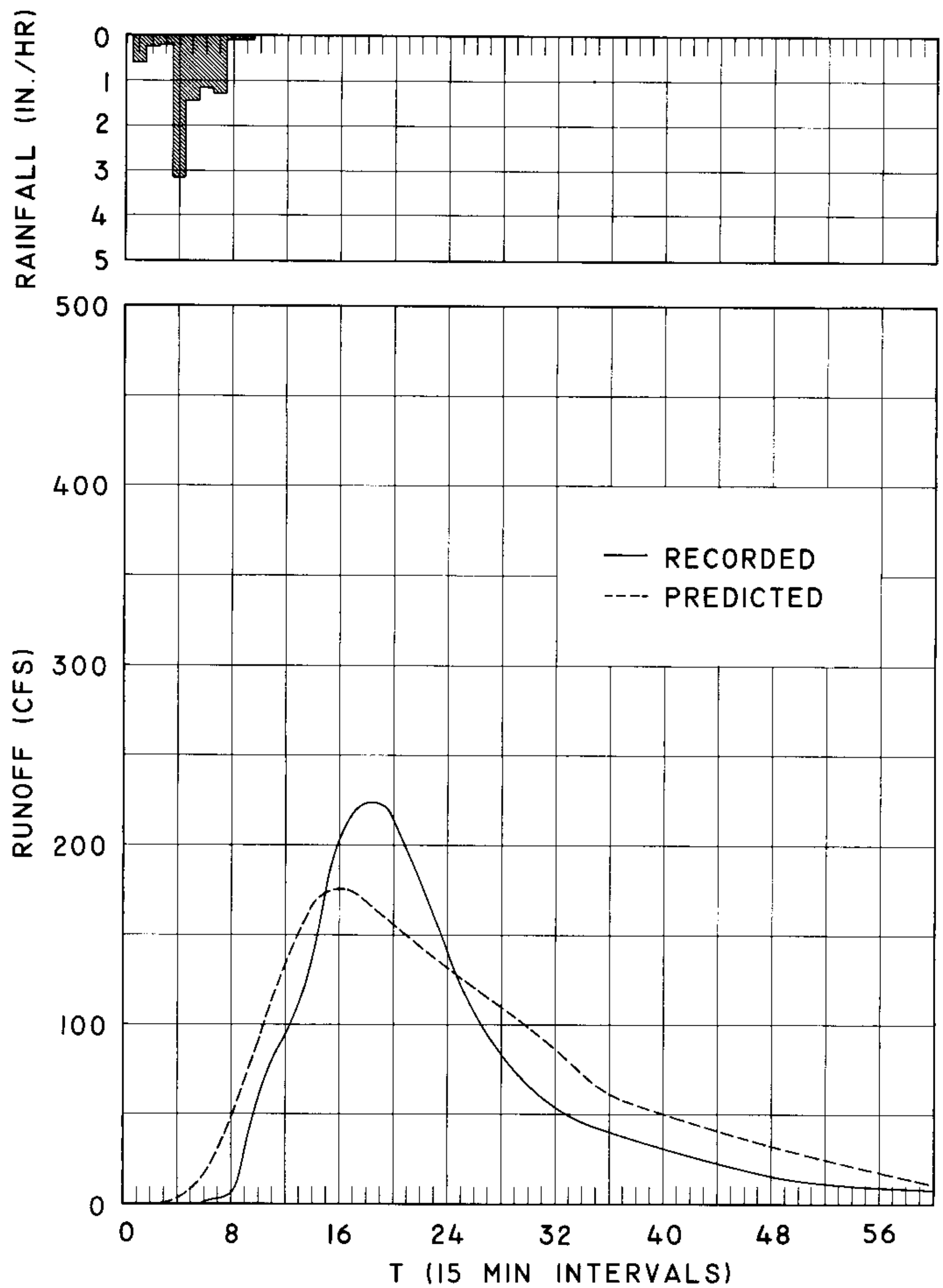


Figure 5.17. Recorded and predicted hydrograph from the generalized transfer function, April 4, 1969, Hudson Creek, Bryan, Texas.

transfer functions in Figure 5.12. Some winter rainfall events of one-half to one and one-half inches produced no recorded runoff while other events, such as the event on November 30, 1968, produced runoff that was predictable with the generalized transfer function. The watershed cover conditions in the winter should be conducive to greater runoff; therefore, the Hudson Creek basin may be more responsive to antecedent moisture conditions than the urban Burton Creek basin. Much additional study of the effect of antecedent moisture conditions on the transfer function for natural rural basins should be made in order to evaluate fully the effect of this basin characteristic.

The relationship between the *wet* and *dry* slopes of the upper descending limb of the transfer function exhibited by the generalized transfer function for Burton Creek was not apparent from the data available on Hudson Creek. If the two February events were removed from Figure 5.10, it is possible to conjecture that the slopes of the upper receding limb for Hudson Creek would be very similar to those reported on Burton Creek. This substantiates the previous conjecture made about Hudson Creek with reference to the performance of the basin during the winter.

Captain Robert G. Feddes is presently analyzing the same events on both watersheds with more conventional hydrologic techniques. This research will be reported in a Master of Science thesis and will be available in January 1970. The examination of the results reported herein with reference to the Feddes investigation will allow a good comparison between the performance of the proposed model and more conventional hydrologic analysis.

# CHAPTER VI

## CONCLUSIONS AND RECOMMENDATIONS

The results of this investigation indicated that reasonably accurate predictions of runoff hydrographs for small drainage basins in the Gulf Coastal Plains of Texas can be obtained by using a linear model to approximate the rainfall-runoff phenomenon. The use of the linear convolution relationship, as described in Chapter III, produced a linear transfer function that quite adequately predicted the recorded runoff when convolved with the recorded rainfall.

The development of a generalized transfer function representing a particular basin with available rainfall-runoff records appears quite feasible. The application of the model in the realization of a generalized transfer function for a basin with no available records is not easily substantiated at this time. For the urban watershed, Burton Creek, the developed generalized transfer function very adequately predicts hydrographs with a large diversity in the input (rainfall). The extension of the generalized transfer function to other urban basins was not undertaken in this study, but should be considered for further study. This model can provide the basis for developing a simple, inexpensive model for predicting the response of urban basins to rainfall events.

The conclusions with regard to the rural basin, Hudson Creek, cannot be stated so efficaciously. The generalized transfer function developed for Hudson Creek did not predict the recorded hydrographs as well as the Burton Creek transfer function. However, the validity of the linear model does appear substantiated. Most events that were analyzed were quite adequately represented by the model, as illustrated and discussed in some detail in Chapter V. It should be pointed out that infiltration is directly related to antecedent moisture conditions and, therefore, included within the generalized transfer function. Limited available data may have caused the results for the rural basin to be rather inconclusive.

The investigation of the hydrologic system as a truly *black box* system warrants further study. The effect of antecedent moisture on the transfer function is apparent in all of the events analyzed. The effect of basin characteristics other than antecedent moisture was not evaluated in this study. The data required for such an evaluation must be inclusive in nature. In addition, long periods of record should be available. Data could be collected and techniques developed to allow the eventual selection of a generalized transfer function from limited basin information. Although the development of these techniques is feasible, it is pointed out that these techniques were not developed in this research. Of course, the proposed model could be used for any basin where rainfall-runoff data are available if the generalized transfer function is obtained by the method described in Chapter III. The direct application of the proposed model is not feasible from an engineering design point of view until simple, inexpensive techniques have been developed for selecting the generalized transfer function. However, it is not unreasonable to assume that selection techniques similar to the selection of C for the rational method could be developed for the selection of the transfer function. Once the generalized transfer function is obtained, the application (Equation 3.7) is analogous to applying an instantaneous unit-graph.

This investigation presents ample justification for continued research on the application of linear time theory to the hydrologic phenomenon. The model presented herein is very simple and easy to use once a representative transfer function has been obtained. Its application to urban basins should be fully substantiated by further testing. Continued research also is recommended on rural basins. It is recommended that subsequent investigations consider the treatment of antecedent moisture conditions as an additional basin characteristic rather than applying an *antecedent index* to the recorded rainfall.

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# APPENDICES

# APPENDIX A

## SYMBOLS

| | |
|---|---|
| $g(t)$, $g(i)$ | observed output (runoff in cfs) |
| $f(t)$, $f(i)$ | observed input (rainfall in in./hr) |
| $h(t)$, $h(i)$ | transfer function |
| $t$, $\tau$ | dummy continuous time variables |
| $i$, $j$, $k$, $\nu$ | dummy discrete time variables |
| $*g(i)$ | predicted output (runoff in cfs) |
| $E$ | error between predicted and observed output |
| $\varepsilon$ | mean-square error |
| $H(T)$ | generalized transfer function |
| $T$ | the number of 15 minute intervals since the beginning of rainfall |
| $h(j)_{opt}$ | optimum stable transfer function ($ft^3$-hr/in.-sec) |
| $u(t)$ | unit impulse |
| $\phi_{ff}(i)$ | auto-correlation function |
| $\phi_{fg}(i)$ | cross-correlation function |
| $L$ | length of optimum transfer function (m-n+1) |
| $n$ | number of time units (15 min intervals) in the observed input (rainfall) |
| $m$ | number of time units (15 min intervals) in the observed output (runoff) |

# APPENDIX B

## DATA AND RESULTS

Table B.1. Data utilized in developing transfer function for Burton Creek, Bryan, Texas.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| 05/10/68 | 0245 | 0.16 | 3.60 | 0.03 | 3545.86 | 8.54 |
| | 0300 | 0.08 | 5.30 | 0.03 | 4180.23 | 30.60 |
| | 0315 | 0.24 | 13.05 | 0.10 | 4662.94 | 35.62 |
| | 0330 | 0.32 | 20.80 | 0.18 | 5017.20 | 39.01 |
| | 0345 | 0.52 | 37.60 | 0.38 | 5246.07 | 45.30 |
| | 0400 | 0.96 | 54.40 | 0.57 | 5269.68 | 34.72 |
| | 0415 | 0.68 | 86.20 | 0.85 | 5230.05 | 33.29 |
| | 0430 | 1.84 | 118.00 | 1.40 | 5043.05 | 22.79 |
| | 0445 | 0.48 | 164.00 | 1.89 | 4821.67 | 22.23 |
| | 0500 | 2.68 | 210.00 | 3.24 | 4544.96 | 19.84 |
| | 0515 | 2.88 | 296.00 | 4.06 | 4210.50 | 17.91 |
| | 0530 | 1.32 | 382.00 | 5.50 | 3832.91 | 17.73 |
| | 0545 | 0.40 | 408.00 | 6.72 | 3411.48 | 14.86 |
| | 0600 | 0.32 | 434.00 | 9.40 | 2977.27 | 10.63 |
| | 0615 | 0.36 | 415.00 | 11.22 | 2575.26 | 8.79 |
| | 0630 | 0.92 | 396.00 | 11.91 | 2197.11 | 9.07 |
| | 0645 | 0.48 | 369.00 | 13.00 | 1826.79 | 4.13 |
| | 0700 | 0.32 | 342.00 | 15.38 | 1497.05 | 2.38 |
| | 0715 | 0.12 | 322.00 | 18.12 | 1210.00 | 0.19 |
| | 0730 | 0.16 | 302.00 | 24.29 | 983.94 | 0.00 |
| | 0745 | | 272.50 | 18.12 | 788.17 | 0.00 |
| | 0800 | | 243.00 | 15.38 | 645.47 | 0.00 |
| | 0815 | | 202.00 | 13.01 | 525.27 | 1.85 |
| | 0830 | | 161.00 | 11.91 | 431.01 | 1.95 |
| | 0845 | | 134.50 | 11.22 | 348.33 | 0.53 |
| | 0900 | | 108.00 | 9.40 | 296.89 | 1.78 |
| | 0915 | | 83.20 | 6.72 | 247.64 | 1.01 |
| | 0930 | | 58.40 | 5.50 | 214.26 | 0.81 |
| | 0945 | | 47.20 | 4.06 | 185.80 | 0.49 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 1000 | | 36.00 | 3.24 | 164.05 | 0.64 |
| | 1015 | | 30.80 | 1.89 | 145.92 | 0.47 |
| | 1030 | | 25.60 | 1.40 | 130.62 | 0.23 |
| | 1045 | | 25.60 | 0.85 | 118.41 | 0.44 |
| | 1100 | | 18.20 | 0.57 | 107.04 | 0.38 |
| | 1115 | | 15.80 | 0.38 | 97.27 | 0.40 |
| | 1130 | | 13.40 | 0.18 | 88.57 | 0.24 |
| | 1145 | | 12.00 | 0.10 | 81.08 | 0.37 |
| | 1200 | | 10.60 | 0.03 | 74.72 | 0.29 |
| | 1215 | | 9.80 | 0.03 | 69.04 | 0.24 |
| | 1230 | | 9.00 | | 64.45 | 0.26 |
| | 1245 | | 8.20 | | 60.49 | 0.24 |
| | 1300 | | 7.40 | | 57.45 | 0.27 |
| | 1315 | | 6.80 | | 54.95 | 0.21 |
| | 1330 | | 6.20 | | 53.09 | 0.24 |
| | 1345 | | 5.60 | | 51.45 | 0.16 |
| | 1400 | | 5.00 | | 49.92 | 0.10 |
| | 1415 | | 4.65 | | 49.18 | 0.04 |
| | 1430 | | 4.30 | | 49.63 | 0.04 |
| | 1445 | | 4.05 | | 51.46 | 0.12 |
| | 1500 | | 3.80 | | 54.44 | 0.36 |
| | 1515 | | 3.60 | | 56.08 | 0.39 |
| | 1530 | | 3.40 | | 56.98 | 0.40 |
| | 1545 | | 3.20 | | 56.93 | 0.43 |
| | 1600 | | 3.00 | | 55.00 | 0.27 |
| | 1615 | | 2.85 | | 53.17 | 0.20 |
| | 1630 | | 2.70 | | 51.15 | 0.09 |
| | 1645 | | 2.85 | | 50.11 | 0.17 |
| | 1700 | | 3.00 | | 49.53 | 0.35 |
| | 1715 | | 3.65 | | 46.85 | 0.29 |
| | 1730 | | 4.30 | | 42.99 | 0.20 |
| | 1745 | | 4.30 | | 39.05 | 0.23 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 1800 | | 4.30 | | 34.52 | 0.11 |
| | 1815 | | 3.95 | | 31.01 | 0.09 |
| | 1830 | | 3.60 | | 27.82 | 0.14 |
| | 1845 | | 3.40 | | 24.28 | 0.15 |
| | 1900 | | 3.20 | | 20.92 | 0.08 |
| | 1915 | | 3.50 | | 18.56 | 0.00 |
| | 1930 | | 3.80 | | 16.80 | 0.00 |
| | 1945 | | 3.30 | | 15.07 | 0.00 |
| | 2000 | | 2.80 | | 12.78 | 0.00 |
| | 2015 | | 2.25 | | 9.20 | 0.00 |
| | 2030 | | 1.70 | | 6.08 | 0.00 |
| | 2045 | | 1.60 | | 5.29 | 0.00 |
| | 2100 | | 1.50 | | 3.26 | 0.00 |
| | 2115 | | 1.35 | | 2.45 | 0.00 |
| | 2130 | | 1.20 | | 1.40 | 0.00 |
| | 2145 | | 1.15 | | 0.84 | 0.00 |
| | 2200 | | 1.10 | | 0.50 | 0.00 |
| | 2215 | | 1.05 | | 0.25 | 0.00 |
| | 2230 | | 1.00 | | 0.16 | 0.00 |
| 05/17/68 | 1415 | 0.40 | 0.10 | 0.03 | 56.57 | 0.00 |
| | 1430 | 1.36 | 0.10 | 0.16 | 75.69 | 1.87 |
| | 1445 | 0.24 | 12.45 | 0.29 | 103.59 | 12.28 |
| | 1500 | 0.24 | 24.80 | 0.54 | 148.24 | 22.94 |
| | 1515 | 0.00 | 52.00 | 0.48 | 195.39 | 42.35 |
| | 1530 | 0.00 | 79.20 | 0.13 | 193.33 | 33.25 |
| | 1545 | 0.28 | 72.00 | 0.19 | 174.62 | 27.68 |
| | 1600 | 0.28 | 64.80 | 0.48 | 148.82 | 21.24 |
| | 1615 | 0.12 | 51.20 | 1.05 | 122.35 | 9.31 |
| | 1630 | 0.08 | 37.60 | 2.30 | 121.53 | 7.77 |
| | 1645 | | 41.20 | 1.05 | 129.92 | 8.49 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 1700 | | 44.80 | 0.48 | 135.87 | 10.62 |
| | 1715 | | 48.40 | 0.19 | 139.30 | 15.20 |
| | 1730 | | 52.00 | 0.13 | 128.32 | 14.17 |
| | 1745 | | 44.40 | 0.48 | 113.53 | 10.99 |
| | 1800 | | 36.80 | 0.54 | 108.26 | 13.14 |
| | 1815 | | 36.80 | 0.29 | 105.16 | 13.80 |
| | 1830 | | 36.80 | 0.16 | 103.69 | 13.03 |
| | 1845 | | 38.40 | 0.03 | 102.45 | 13.35 |
| | 1900 | | 40.00 | | 98.18 | 13.08 |
| | 1915 | | 39.20 | | 93.17 | 13.42 |
| | 1930 | | 38.40 | | 82.61 | 10.32 |
| | 1945 | | 33.60 | | 70.35 | 8.02 |
| | 2000 | | 28.80 | | 57.71 | 5.55 |
| | 2015 | | 22.90 | | 45.75 | 2.14 |
| | 2030 | | 17.00 | | 39.55 | 1.90 |
| | 2045 | | 15.20 | | 35.16 | 1.75 |
| | 2100 | | 13.40 | | 31.48 | 1.90 |
| | 2115 | | 12.00 | | 28.11 | 1.88 |
| | 2130 | | 10.60 | | 25.40 | 2.64 |
| | 2145 | | 9.60 | | 22.98 | 2.91 |
| | 2200 | | 8.60 | | 20.99 | 2.79 |
| | 2215 | | 7.85 | | 19.19 | 2.51 |
| | 2230 | | 7.10 | | 17.63 | 2.26 |
| | 2245 | | 6.50 | | 16.16 | 1.86 |
| | 2300 | | 5.90 | | 14.92 | 1.50 |
| | 2315 | | 5.45 | | 13.78 | 1.22 |
| | 2330 | | 5.00 | | 12.93 | 1.17 |
| | 2345 | | 4.75 | | 12.17 | 1.15 |
| 05/18/68 | 2400 | | 4.50 | | 11.44 | 1.14 |
| | 2415 | | 4.25 | | 10.74 | 1.16 |
| | 2430 | | 4.00 | | 9.98 | 1.14 |
| | 2445 | | 3.70 | | 9.19 | 1.07 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 0100 | | 3.40 | | 8.43 | 0.97 |
| | 0115 | | 3.10 | | 7.72 | 0.83 |
| | 0130 | | 2.80 | | 7.20 | 0.78 |
| | 0145 | | 2.65 | | 6.80 | 0.72 |
| | 0200 | | 2.50 | | 6.47 | 0.66 |
| | 0215 | | 2.35 | | 6.11 | 0.62 |
| | 0230 | | 2.20 | | 5.71 | 0.57 |
| | 0245 | | 2.05 | | 5.37 | 0.57 |
| | 0300 | | 1.90 | | 4.85 | 0.36 |
| | 0315 | | 1.60 | | 4.75 | 0.52 |
| | 0330 | | 1.70 | | 4.64 | 0.53 |
| | 0345 | | 1.65 | | 4.51 | 0.47 |
| | 0400 | | 1.60 | | 4.43 | 0.54 |
| | 0415 | | 1.60 | | 4.32 | 0.67 |
| | 0430 | | 1.60 | | 4.06 | 0.50 |
| | 0445 | | 1.45 | | 3.76 | 0.32 |
| | 0500 | | 1.30 | | 3.51 | 0.49 |
| | 0515 | | 1.25 | | 3.24 | 0.53 |
| | 0530 | | 1.20 | | 2.85 | 0.00 |
| | 0545 | | 1.15 | | 2.48 | 0.00 |
| | 0600 | | 1.09 | | 2.45 | 0.44 |
| | 0615 | | 1.10 | | 2.43 | 0.53 |
| | 0630 | | 1.10 | | 2.11 | 0.00 |
| | 0645 | | 1.05 | | 1.78 | 0.00 |
| | 0700 | | 1.00 | | 0.40 | 0.00 |
| 06/01/68 | 2015 | 1.08 | 0.10 | 0.17 | 736.09 | 0.45 |
| | 2030 | 0.32 | 7.95 | 0.61 | 1049.24 | 8.57 |
| | 2045 | 0.60 | 15.80 | 0.82 | 1379.82 | 15.50 |
| | 2100 | 0.52 | 39.10 | 1.21 | 1689.73 | 25.55 |
| | 2115 | 1.96 | 62.40 | 3.29 | 1980.12 | 34.94 |
| | 2130 | 2.04 | 100.20 | 4.75 | 2243.63 | 44.97 |
| | 2145 | 0.60 | 158.00 | 4.90 | 2384.82 | 55.43 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 2200 | 0.52 | 203.00 | 5.68 | 2358.70 | 48.50 |
| | 2215 | 0.52 | 248.00 | 7.76 | 2237.68 | 39.71 |
| | 2230 | 0.16 | 297.50 | 10.83 | 2068.90 | 36.55 |
| | 2245 | | 347.00 | 7.76 | 1841.57 | 38.79 |
| | 2300 | | 331.00 | 5.68 | 1479.59 | 7.20 |
| | 2315 | | 315.00 | 4.90 | 1177.57 | 13.20 |
| | 2330 | | 270.00 | 4.75 | 901.10 | 1.07 |
| | 2345 | | 252.00 | 3.29 | 704.79 | 4.51 |
| 06/02/68 | 2400 | | 187.00 | 1.21 | 527.47 | 7.34 |
| | 2415 | | 122.00 | 0.82 | 375.17 | 4.60 |
| | 2430 | | 89.00 | 0.61 | 275.03 | 2.17 |
| | 2445 | | 56.00 | 0.17 | 200.15 | 0.00 |
| | 0100 | | 46.80 | | 167.96 | 1.59 |
| | 0115 | | 37.60 | | 140.97 | 1.07 |
| | 0130 | | 27.30 | | 119.03 | 2.52 |
| | 0145 | | 17.00 | | 100.75 | 1.17 |
| | 0200 | | 15.50 | | 92.92 | 1.59 |
| | 0215 | | 14.00 | | 85.52 | 1.02 |
| | 0230 | | 12.90 | | 79.00 | 1.04 |
| | 0245 | | 11.80 | | 73.13 | 0.85 |
| | 0300 | | 11.00 | | 68.07 | 1.00 |
| | 0315 | | 10.10 | | 63.63 | 0.79 |
| | 0330 | | 9.40 | | 59.76 | 0.96 |
| | 0345 | | 8.60 | | 55.78 | 0.87 |
| | 0400 | | 8.00 | | 51.92 | 0.82 |
| | 0415 | | 7.40 | | 47.99 | 0.68 |
| | 0430 | | 7.10 | | 44.35 | 0.57 |
| | 0445 | | 6.80 | | 41.04 | 0.50 |
| | 0500 | | 6.35 | | 37.88 | 0.47 |
| | 0515 | | 5.90 | | 35.29 | 0.46 |
| | 0530 | | 5.35 | | 33.20 | 0.54 |
| | 0545 | | 4.80 | | 30.98 | 0.50 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 0600 | | 4.40 | | 28.85 | 0.39 |
| | 0615 | | 4.00 | | 26.99 | 0.32 |
| | 0630 | | 3.90 | | 25.59 | 0.35 |
| | 0645 | | 3.80 | | 24.45 | 0.28 |
| | 0700 | | 3.50 | | 23.43 | 0.32 |
| | 0715 | | 3.20 | | 22.42 | 0.40 |
| | 0730 | | 3.00 | | 21.54 | 0.29 |
| | 0745 | | 2.80 | | 20.69 | 0.27 |
| | 0800 | | 2.75 | | 19.95 | 0.38 |
| | 0815 | | 2.70 | | 19.28 | 0.22 |
| | 0830 | | 2.60 | | 18.67 | 0.19 |
| | 0845 | | 2.50 | | 17.84 | 0.38 |
| | 0900 | | 2.40 | | 16.95 | 0.14 |
| | 0915 | | 2.30 | | 16.44 | 0.16 |
| | 0930 | | 2.25 | | 15.75 | 0.46 |
| | 0945 | | 2.20 | | 14.72 | 0.22 |
| | 1000 | | 2.05 | | 13.52 | 0.00 |
| | 1015 | | 1.90 | | 12.25 | 0.53 |
| | 1030 | | 2.00 | | 11.10 | 0.00 |
| | 1045 | | 1.70 | | 7.39 | 0.00 |
| | 1100 | | 1.70 | | 4.20 | 0.00 |
| | 1115 | | 1.70 | | 3.32 | 0.00 |
| | 1130 | | 1.65 | | 2.29 | 0.00 |
| | 1145 | | 1.60 | | 1.73 | 0.00 |
| 06/05/68 | 1715 | 0.40 | 0.00 | 0.05 | 407.78 | 0.59 |
| | 1730 | 4.80 | 7.40 | 0.61 | 774.78 | 11.34 |
| | 1745 | 1.40 | 62.70 | 0.60 | 1185.78 | 18.56 |
| | 1800 | 0.36 | 118.00 | 0.76 | 1715.21 | 32.10 |
| | 1815 | 0.04 | 202.00 | 0.63 | 2208.57 | 44.57 |
| | 1830 | 0.12 | 286.00 | 0.81 | 2518.94 | 49.76 |
| | 1845 | 0.16 | 338.50 | 1.05 | 2684.62 | 58.69 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 1900 | 0.12 | 391.00 | 0.90 | 2369.16 | 45.41 |
| | 1915 | 0.08 | 343.50 | 0.61 | 1980.93 | 38.60 |
| | 1930 | 0.12 | 296.00 | 2.46 | 1486.07 | 25.83 |
| | 1945 | 0.08 | 222.00 | 9.24 | 1052.66 | 13.54 |
| | 2000 | 0.12 | 148.00 | 25.39 | 866.39 | 12.53 |
| | 2015 | | 124.50 | 9.24 | 714.99 | 8.66 |
| | 2030 | | 101.00 | 2.46 | 610.48 | 7.22 |
| | 2045 | | 86.90 | 0.61 | 512.56 | 5.58 |
| | 2100 | | 72.80 | 0.90 | 426.00 | 4.66 |
| | 2115 | | 60.40 | 1.04 | 352.05 | 3.23 |
| | 2130 | | 48.00 | 0.81 | 318.71 | 4.06 |
| | 2145 | | 44.00 | 0.63 | 289.36 | 4.00 |
| | 2200 | | 40.00 | 0.76 | 262.48 | 3.96 |
| | 2215 | | 36.40 | 0.60 | 237.07 | 3.79 |
| | 2230 | | 32.80 | 0.61 | 215.78 | 3.49 |
| | 2245 | | 30.00 | 0.05 | 194.99 | 3.22 |
| | 2300 | | 27.20 | | 174.49 | 2.86 |
| | 2315 | | 24.40 | | 154.53 | 2.52 |
| | 2330 | | 21.60 | | 135.87 | 2.18 |
| | 2345 | | 19.00 | | 117.77 | 1.84 |
| 06/06/68 | 2400 | | 16.40 | | 101.76 | 1.52 |
| | 2415 | | 14.10 | | 88.23 | 1.20 |
| | 2430 | | 11.80 | | 82.90 | 1.24 |
| | 2445 | | 11.20 | | 78.30 | 1.19 |
| | 0100 | | 10.60 | | 73.71 | 1.14 |
| | 0115 | | 10.00 | | 68.86 | 1.09 |
| | 0130 | | 9.40 | | 63.21 | 1.00 |
| | 0145 | | 8.60 | | 57.93 | 0.90 |
| | 0200 | | 7.80 | | 54.16 | 0.87 |
| | 0215 | | 7.30 | | 50.59 | 0.81 |
| | 0230 | | 6.80 | | 47.34 | 0.76 |
| | 0245 | | 6.35 | | 44.31 | 0.70 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 0300 | | 5.90 | | 42.05 | 0.67 |
| | 0315 | | 5.60 | | 39.93 | 0.63 |
| | 0330 | | 5.30 | | 38.08 | 0.61 |
| | 0345 | | 5.05 | | 36.36 | 0.58 |
| | 0400 | | 4.80 | | 35.15 | 0.57 |
| | 0415 | | 4.65 | | 34.07 | 0.55 |
| | 0430 | | 4.50 | | 33.19 | 0.54 |
| | 0445 | | 4.40 | | 32.10 | 0.54 |
| | 0500 | | 4.30 | | 30.36 | 0.50 |
| | 0515 | | 4.05 | | 28.77 | 0.46 |
| | 0530 | | 3.80 | | 27.89 | 0.45 |
| | 0545 | | 3.70 | | 26.94 | 0.45 |
| | 0600 | | 3.60 | | 25.56 | 0.41 |
| | 0615 | | 3.40 | | 24.29 | 0.40 |
| | 0630 | | 3.20 | | 23.49 | 0.38 |
| | 0645 | | 3.10 | | 22.67 | 0.38 |
| | 0700 | | 3.00 | | 21.64 | 0.36 |
| | 0715 | | 2.85 | | 20.65 | 0.35 |
| | 0730 | | 2.70 | | 19.93 | 0.32 |
| | 0745 | | 2.60 | | 19.35 | 0.32 |
| | 0800 | | 2.50 | | 19.25 | 0.44 |
| | 0815 | | 2.50 | | 18.96 | 0.00 |
| | 0830 | | 2.50 | | 18.72 | 0.58 |
| | 0845 | | 2.50 | | 18.27 | 0.00 |
| | 0900 | | 2.50 | | 17.44 | 0.15 |
| | 0915 | | 2.40 | | 16.57 | 0.00 |
| | 0930 | | 2.30 | | 15.87 | 0.00 |
| | 0945 | | 2.25 | | 15.19 | 0.00 |
| | 1000 | | 2.20 | | 14.46 | 0.52 |
| | 1015 | | 2.10 | | 13.17 | 0.00 |
| | 1030 | | 2.00 | | 10.16 | 0.00 |
| | 1045 | | 1.95 | | 0.78 | 0.00 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| 06/17/69 | 1715 | 0.20 | 0.00 | 0.04 | 67.80 | 0.50 |
| | 1730 | 1.40 | 0.20 | 0.51 | 116.42 | 0.78 |
| | 1745 | 0.20 | 3.60 | 1.67 | 161.56 | 3.89 |
| | 1800 | 0.04 | 14.00 | 0.34 | 230.24 | 23.70 |
| | 1815 | 1.16 | 44.00 | 0.85 | 294.34 | 43.71 |
| | 1830 | 0.20 | 76.00 | 3.43 | 328.20 | 53.15 |
| | 1845 | | 93.10 | 0.85 | 306.32 | 44.18 |
| | 1900 | | 107.00 | 0.34 | 265.46 | 29.21 |
| | 1915 | | 114.00 | 1.67 | 247.80 | 24.29 |
| | 1930 | | 120.00 | 0.51 | 179.00 | 12.39 |
| | 1945 | | 87.50 | 0.04 | 104.22 | 0.00 |
| | 2000 | | 44.00 | | 69.30 | 0.00 |
| | 2015 | | 29.60 | | 45.36 | 0.00 |
| | 2030 | | 17.00 | | 35.48 | 6.33 |
| | 2045 | | 14.00 | | 29.13 | 2.52 |
| | 2100 | | 11.40 | | 23.83 | 2.87 |
| | 2115 | | 9.40 | | 19.43 | 0.00 |
| | 2130 | | 7.40 | | 16.42 | 1.11 |
| | 2145 | | 6.20 | | 13.80 | 1.00 |
| | 2200 | | 5.00 | | 11.96 | 2.12 |
| | 2215 | | 4.30 | | 10.73 | 1.44 |
| | 2230 | | 3.80 | | 9.57 | 0.42 |
| | 2245 | | 3.40 | | 8.49 | 0.18 |
| | 2300 | | 3.00 | | 7.81 | 0.41 |
| | 2315 | | 2.80 | | 7.12 | 1.07 |
| | 2330 | | 2.50 | | 6.33 | 1.12 |
| | 2345 | | 2.20 | | 5.69 | 0.72 |
| 06/18/69 | 2400 | | 2.00 | | 5.21 | 0.20 |
| | 2415 | | 1.90 | | 4.74 | 0.00 |
| | 2430 | | 1.70 | | 4.36 | 0.36 |
| | 2445 | | 1.50 | | 3.90 | 0.61 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 0100 | | 1.30 | | 3.70 | 0.75 |
| | 0115 | | 1.20 | | 3.34 | 0.28 |
| | 0130 | | 1.20 | | 2.04 | 0.00 |
| | 0145 | | 1.10 | | 1.96 | 0.09 |
| | 0200 | | 1.10 | | 1.62 | 0.00 |
| | 0215 | | 1.00 | | 0.20 | 0.00 |
| 06/18/69 | 1715 | 1.00 | 1.90 | 0.04 | 221.56 | 0.00 |
| | 1730 | 4.00 | 6.50 | 0.20 | 389.16 | 7.00 |
| | 1745 | 0.08 | 46.40 | 0.20 | 610.87 | 17.00 |
| | 1800 | 0.00 | 94.50 | 0.28 | 836.97 | 26.29 |
| | 1815 | 0.08 | 144.00 | 0.53 | 1062.59 | 37.39 |
| | 1830 | 0.12 | 196.00 | 0.38 | 1254.07 | 43.79 |
| | 1845 | 0.20 | 237.00 | 1.01 | 1460.12 | 56.64 |
| | 1900 | 0.20 | 284.00 | 0.97 | 1367.72 | 48.41 |
| | 1915 | 0.04 | 253.00 | 0.61 | 1215.00 | 43.90 |
| | 1930 | 0.12 | 226.00 | 0.39 | 907.69 | 29.62 |
| | 1945 | 0.04 | 159.00 | 0.17 | 575.95 | 12.98 |
| | 2000 | 0.04 | 94.50 | 4.41 | 457.14 | 9.91 |
| | 2015 | 0.04 | 82.40 | 17.13 | 388.83 | 6.69 |
| | 2030 | | 69.60 | 4.41 | 326.79 | 5.15 |
| | 2045 | | 58.40 | 0.17 | 270.30 | 3.47 |
| | 2100 | | 48.00 | 0.39 | 231.38 | 3.61 |
| | 2115 | | 41.60 | 0.61 | 200.24 | 3.58 |
| | 2130 | | 36.00 | 0.97 | 170.78 | 3.20 |
| | 2145 | | 30.50 | 1.02 | 144.11 | 3.18 |
| | 2200 | | 25.60 | 0.38 | 118.72 | 2.71 |
| | 2215 | | 20.80 | 0.53 | 97.72 | 2.33 |
| | 2230 | | 17.00 | 0.28 | 85.75 | 2.25 |
| | 2245 | | 15.20 | 0.20 | 75.93 | 2.02 |

Table B.1. Continued.

| Date | Time | f(t) (in./hr) | g(t) (cfs) | $\phi_{ff}(t)$ | $\phi_{fg}(t)$ | $h_{opt}(t)$ |
|---|---|---|---|---|---|---|
| | 2300 | | 13.40 | 0.20 | 67.02 | 1.77 |
| | 2315 | | 11.80 | 0.04 | 60.08 | 1.57 |
| | 2330 | | 10.60 | | 55.16 | 1.50 |
| | 2345 | | 9.80 | | 49.08 | 1.30 |
| 06/19/69 | 2400 | | 8.60 | | 42.65 | 1.07 |
| | 2415 | | 7.40 | | 38.67 | 1.00 |
| | 2430 | | 6.80 | | 35.36 | 0.90 |
| | 2445 | | 6.20 | | 33.27 | 0.89 |
| | 0100 | | 5.90 | | 30.34 | 0.80 |
| | 0115 | | 5.30 | | 27.54 | 0.73 |
| | 0130 | | 4.80 | | 24.88 | 0.65 |
| | 0145 | | 4.30 | | 23.05 | 0.61 |
| | 0200 | | 4.00 | | 21.04 | 0.54 |
| | 0215 | | 3.60 | | 19.77 | 0.52 |
| | 0230 | | 3.40 | | 18.67 | 0.49 |
| | 0245 | | 3.20 | | 17.59 | 0.47 |
| | 0300 | | 3.00 | | 16.50 | 0.43 |
| | 0315 | | 2.80 | | 16.15 | 0.45 |
| | 0330 | | 2.80 | | 15.60 | 0.43 |
| | 0345 | | 2.70 | | 15.36 | 0.46 |
| | 0400 | | 2.70 | | 14.27 | 0.39 |
| | 0415 | | 2.50 | | 13.92 | 0.43 |
| | 0430 | | 2.50 | | 12.70 | 0.38 |
| | 0445 | | 2.30 | | 12.08 | 0.38 |
| | 0500 | | 2.30 | | 11.44 | 0.36 |
| | 0515 | | 2.20 | | 11.16 | 0.34 |
| | 0530 | | 2.20 | | 10.36 | 0.39 |
| | 0545 | | 2.00 | | 10.00 | 0.03 |
| | 0600 | | 2.00 | | 2.00 | 0.00 |