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Kinematic wave model for water movement in municipal solid waste

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Abstract. The movement of water in a large (3.5 m^3) undisturbed sample of 22-year-old municipal solid waste has been modeled using a kinematic wave approximation for unsaturated infiltration and internal drainage. The model employs a two-parameter power expression as macroscopic flux law. The model parameters were determined and interpreted in terms of the internal geometry of the waste medium by fitting the model to one set of infiltration and drainage data. The model was validated using another set of data from a sequence of water input events. The results of the validation show that the model performs satisfactorily, but further development of the model to incorporate spatial variability would increase its capability.

1. Introduction

The concentration of substances such as heavy metals, nutrients, and organic compounds in landfills produces mass and energy gradients within and between the landfill and its surrounding environment. As a result, these substances will leave the landfill with gas or water flow as long as the gradients remain. Landfills therefore constitute a serious environmental problem. The aim of modern landfill management is to level out the energy and concentration gradients between the landfill and the environment in a controlled and sanitary manner. The final product is stabilized waste which may become an integrated part of the natural environment. Since the presence of water in the landfill plays a key role in the biochemical processes [*Augenstein and Pacey*, 1991; *Christensen and Kjeldsen*, 1989; *Ehrig*, 1983; *Klink and Ham*, 1982; *Leuschner and Melden*, 1983; *Straub and Lynch*, 1982], the mapping of water flow and solute transport is of direct interest in engineering the stabilization processes and when developing models for leachate quality and gas production.

A modest number of flow and solute transport models developed for waste media on a laboratory scale have been published. In a review by D. Bendz et al. (Hydrological characteristics of landfills—Implications for modeling, submitted to *Journal of Hydrology*, 1997) (hereinafter referred to as Bendz et al., 1997) it is pointed out that these models all treat the waste as a homogeneous porous medium. It is explicitly or implicitly assumed that the experimental scale is large enough for a representative elementary volume (REV) to exist so that a macroscopic approach can be justified. The water movement has been modeled by the Richards equation, and the transport process, when included, has been modeled by the classical convection-dispersion equation (CDE). Furthermore, the soil water diffusivity, defined by *Klute* [1952], and the forms of hydraulic conductivity and capillary potential suggested by *Clapp and Hornberger* [1978] have been employed. However, as pointed out by Bendz et al. (1997), the process assumptions on which the Richards equation and the ensuing CDE rely for their applicability have not been discussed. *Straub and Lynch* [1982] approached the transport of inorganic contaminants in landfills with a simple well-mixed reactor model and a continuum flow model and applied them to experimental data obtained in the laboratory by others. Influenced by the work of *Straub and Lynch* [1982], *Demetracopoulus* [1986] used the same mathematical framework but employed another numerical scheme to solve the differential transport equations. *Korfiatis et al.* [1984], who investigated water flow on a column scale, and *Straub and Lynch* [1982] found that the capillary diffusivity was of minor importance and could be neglected when the moisture content exceeded the field capacity. *Vincent et al.* [1991] used an artificial model waste and *Lee et al.* [1991] used shredded municipal solid waste of four different ages when performing laboratory experiments to investigate the flow and transport processes.

Straub and Lynch [1982] considered the heterogeneity and channeling effects on an ad hoc basis by simply adjusting the dispersivity parameter in the transport equation. Molecular diffusion was regarded as negligible. In the same way, *Korfiatis et al.* [1984] suggested that the channeling could be taken into account by increasing the exponent in the *Clapp and Hornberger* [1978] expression for the tension head.

An alternative approach to describing the flow in landfills, taking the geometrical configuration into account, was suggested by *Ferguson* [1993]. With the primary objective of estimating the specific surface in a landfill he suggested a hydraulic model where water is present either as a static surface tension film or as a moving film on refuse particles.

Because of the highly heterogeneous nature of a landfill, the flow field is not uniform. The internal geometry of a landfill facilitates fast flow in restricted channels and voids. Further, the field capacity is spatially variable, and some parts of the landfill therefore reach field capacity long before the entire landfill does. Water may be flowing in locally saturated regions, while the largest portion of the landfill may be well below field capacity. Channel flow, which is most significant in young deposits because of their coarser structure, has been observed in several investigations [*Bengtsson et al.*, 1994; *Blakey*, 1982;

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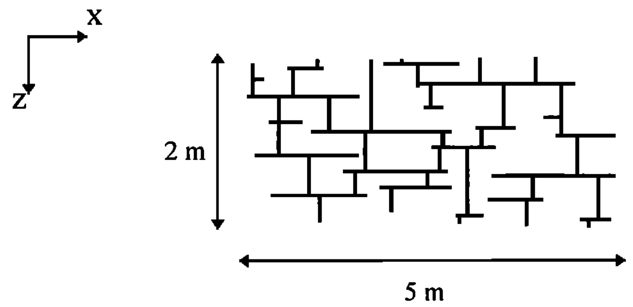

Figure 1. Flow paths in the landfill interior illustrated schematically.

Blight et al., 1992; *Harris*, 1979; *Ham and Bookter*, 1982; *Holmes*, 1983; *Korfiatis et al.*, 1984; *Walsh and Kinman*, 1979]. *Stegmann and Ehrig* [1989] refer to investigations in which excavated landfills have shown evidence of channel flow. As the refuse biodegrades and settles, the dry density increases, and the void volume decreases. This, in turn, limits the fast channel flow to some extent in older deposits. The extent of channel flow is dependent not only on the structure and void volume but also on the rate of precipitation. Additional flow routes may develop during periods of high infiltration rate [*Jasper et al.*, 1985]. The existence of channel flow is very important and is believed to be the reason why existing models are not in agreement with actual field observations [*Ehrig*, 1983; *Stegmann and Ehrig*, 1989; *Zeiss and Major*, 1993]. The existence of channel flow also introduces additional complexity into the dispersion process as the transport process in these flow routes may be different from that within the rest of the medium.

To conclude, the common approach when addressing flow and transport in landfills has been to regard the waste medium as a homogeneous soil with special properties. The prominent phenomena of heterogeneity of the landfill and of channel flow have not been fully addressed in the existing models.

The objective of this paper is to present a framework for the modeling of unsteady water flow in a landfilled municipal solid waste considering the aforementioned characteristics of the landfill. In the model proposed here the heterogeneity of the waste medium is addressed by discretizing the medium into a channel domain, constituted of the flow paths, and a matrix domain. The vertical water flow in the latter domain is assumed to be negligible. In the channel domain the water is assumed to move as a creeping flow in thin layers driven by gravity on solid surfaces, and capillary forces are considered to be minor. The kinematic wave assumption is introduced by employing a strictly convective flux law. Several researchers have investigated the kinematic wave approximation for soil water movement [e.g., *Sisson*, 1980; *Smith*, 1983; *Singh and Joseph*, 1993]. The topic is discussed comprehensively by *Singh* [1997]. The kinematic wave approach to macropore flow in soils proposed by Beven and Germann in a series of papers is followed in this study [*Beven and Germann*, 1981; *Germann*, 1985; *Germann and Beven*, 1985; *Germann*, 1990; *Germann and DiPietro*, 1996].

2. Kinematic Wave Model

2.1. Conceptualization of the Medium and the Flow Regime

Bendz et al. (1997) identified the heterogeneity (variable in time and space), fast gravitational flow in restricted channels and voids, low capillarity, significant horizontal stratification (resulting from the disposal procedure), and impermeable surfaces as major features that govern the flow regime in landfills. The authors proposed that the spatial variability of the landfill may be categorized into three heterogeneity scales which were denoted the lens scale (lens-shaped refuse element), the truckload scale (a group of refuse lenses that derives from the same refuse truckload and may therefore share some characteristics), and the landfill scale. A lens-shaped refuse element is typically a compacted refuse bag/sack with a partially impermeable surface. The lens scale is of the order of 1 m in the horizontal plane and 0.1 m in the vertical plane.

The flow regime was represented as follows. Because of stratification, a significant portion of the flow is taking place in the horizontal direction. These flow paths at different levels are connected by vertical shortcuts. This leads to a network of flow paths, similar to the flow paths in fractured rocks or fissured media. An illustration of this flow pattern in two dimensions is shown in Figure 1. On the basis of the flow pattern described above, the landfill is conceptualized as a dual-domain medium. The channel domain defined by the flow path network constitutes only a fraction of the entire landfill. Gravity is assumed to be dominant here, and capillary force is considered to be negligible. In the matrix domain the capillary force is considered to be significant, and the water movement is slow. In the model presented in this paper the vertical flow in the matrix domain has been neglected.

2.2. Governing Flux Law

In a critical review of the Richards equation, *Germann* [1990] demonstrated macropore flow in soils to be one kind of deviation from the underlying assumption of perfect correlation between capillary diffusivity and capillary potential on the scale of a representative elementary volume (REV). That is, the water movement may be faster than the change in capillary potential. Bendz et al. (1997) acknowledged the similarities between channel flow in landfilled waste and macropore flow in soils and employed the two-parameter power function proposed by *Beven and Germann* [1981] as a macroscopic flux law,

$$q = bw^a \quad (1)$$

where q is the water flux density (m s^{-1}) in the channels per unit cross-sectional area of the medium, a is a dimensionless exponent, and b is the channel conductance (m s^{-1}), which can be interpreted as the lumped effect of surface, geometrical, and spatial characteristics of the flow path. The amount of water in the channels is expressed as a volume fraction of the medium, w, rather than the thickness of a water film. *Germann* [1990] lent the flux law further credibility by treating the flow as a viscous laminar flow in thin layers and deriving a cubic form of (1) considering flow along a planar solid surface. Newton's law of shear stress was employed to that end. Such flow is often called creeping flow and is characterized as a moving fluid where the inertia is dynamically insignificant compared with the viscosity [*Sherman*, 1990]. Slender viscous films with free surfaces constitute a special case of creeping flows. The exponent a was reasoned by *Germann and DiPietro* [1996] to be dependent on the internal geometry of the medium, the initial water content, and the boundary conditions affecting the water content at the surface. The fluid-mechanical interpretation of the parameter a was summarized as a measure of the impact of the stagnant parts of a flow system on the mobile parts. For macropore flow the parameter a was expected to be in the range of $2 \leq a < \sim 8$, where $a = 2$ indicates laminar flow in a cylindrical pipe (which may occur if the medium is saturated) and $a = 3$ indicates flow in planar cracks. Turbulent flow is indicated when $a < 2$. Higher values of a indicate increasingly tortuous flow. Completely dispersive flow corresponds to $a > \sim 10$.

It can be noted that flow in porous media shares certain characteristics with overland flow, such as gravity being the dominating force governing the water movement and viscosity being the only force in opposition [*Singh*, 1997]. Generally, surface flow can be expressed as

$$Q \propto h^{\alpha} \tag{2}$$

where Q is the volume flux, h is water depth, and α is a dimensionless constant. The Chezy and Manning equations constitute special cases, where α equals $\frac{3}{2}$ and $\frac{5}{3}$, respectively.

2.3. Kinematic Wave Equations

The governing equations are the law of conservation of mass and a flux law. The conservation of mass equation can be expressed as

$$\frac{\partial w}{\partial t} + \frac{\partial q}{\partial z} = -S \tag{3}$$

where S is the water loss (s^{-1}) from the system of channels into adjacent pores and voids in the drainable region of the matrix domain, t is the time (s), and z is the depth (m). For a kinematic treatment we may assume that at a fixed z, q is a function of w alone. Equation (3) can then be written as a kinematic wave equation:

$$\frac{\partial w}{\partial t} + c\frac{\partial w}{\partial z} = -S \tag{4}$$

in which

$$c = \frac{\partial q}{\partial w} \tag{5}$$

where c is the wave celerity (m s^{-1}). The average velocity with which w moves, u (m s^{-1}), and the wave celerity (m s^{-1}) follow from (1).

$$u = \frac{q}{w} = bw^{a-1} \tag{6}$$

$$c = \frac{\partial q}{\partial w} = abw^{a-1} \tag{7}$$

Substitution of (7) into (4) gives

$$\frac{\partial w}{\partial t} + abw^{a-1}\frac{\partial w}{\partial z} = -S \tag{8}$$

which can be reduced to the following system of characteristic equations:

$$\frac{dw}{dt} = -S \tag{9}$$

$$\frac{dz}{dt} = abw^{a-1} \tag{10}$$

Table 1. Original Composition of the Waste in the Test Cell

Material	Composition, wt%
Sludge	35
Paper, puitriscable, glass	47
Textiles	6
Metal	6
Plastic	3
Wood, timber	3

From [*Persson and Rylander*, 1977].

For a square pulse input, q_u, starting at $t = 0$ and ending at $t = T$ the following initial and boundary conditions can be assumed:

$$w(z, 0) = f(z) \tag{11A}$$

$$w(0, t_s) = \begin{cases} 0 & t_s \leq 0,\ t_s \geq T \\ w_u & 0 < t_s < T \end{cases} \tag{11B}$$

where $f(z)$ is an arbitrary function describing the initial water content, w_u is the water content in the channels at the upper boundary, $z = 0$ during the square pulse input, and t_s is the point on the time-axis at which the characteristic defined by (10) starts.

3. Experimental Setup and Data

The experimental setup, instrumentation, and the total number of experiments performed on the column have been thoroughly described by H. Rosqvist and D. Bendz (Tracer test in a large scale undisturbed solid waste column, submitted to *Hydrology and Earth Systems Sciences*, 1997). Only the part of the experimental setup that is of direct interest for this study will be discussed here. The data used in this study constitute only part of the data generated by the earlier experiment.

The waste sample was taken in 1995 from a 22-year-old test cell originally containing shredded household waste. The dimensions of the sample are 1.93 m in diameter and 1.20 m in height. The size of the sample is about double the lens scale in the horizontal plane and about 10 times the lens scale in the vertical plane. The composition of the waste was characterized in only a basic way when the test cell was constructed. The composition is given in Table 1. The waste was mixed with sludge, 35% by weight, at the time of disposal. The test cell was compacted to a dry density of 380 kg m^{-3}. With the objective of studying the presence and forms of heavy metals in an old deposit during the stabilization phase, *Flyhammar et al.* [1998] investigated the current composition of the waste in the test cell. It was found that the fraction of paper had decreased by more than 40 wt% and that the easily degradable materials were almost completely degraded. The geometrical forms found in the waste were also investigated and are shown in Table 2. According to our observations, the waste was highly compacted and tightly clustered. The test cell was built in 1973 on a waste disposal site in an area with a very high groundwater table. Since the late 1970s, when the site was covered, closed, and abandoned, the cell has been lying under the groundwater table. The sample may therefore represent waste which can be expected to be found in the saturated well-degraded bottom layer of a landfill.

Table 2. Geometrical Forms Found in the Waste

Geometrical Forms	Size, cm	Dry Waste, wt%	Waste Fraction
Sheets and threads*	>0.2–10^1	12.7	nonrigid plastic, textile leather, rubber
Irregular structures†	>0.2–10^1	28.1	rigid plastic, metal inert (glass, stones) animal (bones) vegetable (including wood)
Cellulose fibers	>0.2–10^0	39.9	paper
Fine residuals	<0.2	19.2	fines

From *Flyhammar et al.* [1998]. Copyright by Munksgaard International Publishers Ltd.; reprinted with permission.
*One- and two-dimensional structures.
†Three-dimensional structures.

After the soil cover of the test cell had been removed a steel cylinder, measuring 1.93 m in diameter and 2 m in height, was carefully driven down 1.2 m into the waste by letting an excavator alternate between applying a pressure on the top of the cylinder and excavating the waste material around the cylinder. To facilitate the procedure, one man cut the waste with a sharp spade along the edge of the cylinder as it was pressed down. When the drainage layer of the test cell had been reached, a steel sheet was forced in under the cylinder and fixed by welding. The cylinder was then lifted up, weighed, and brought to the laboratory, where the bottom steel sheet was removed and the cylinder was installed on a stand. The stand was equipped with a drainage layer of coarse gravel and a drainage pipe. The column was sealed to the stand with silicone. The cylinder was equipped with an irrigation system of 19 microsprinklers evenly distributed on a circular bar which was constantly rotated to ensure that the water was applied evenly. The irrigation flux was measured with an electronic flowmeter, and the outflow flux was measured with a tipping bucket device connected to a data-logger.

The current dry density and field capacity were determined in the laboratory using a smaller, undisturbed sample, measuring 300 mm in diameter and 440 mm in height. The dry density and the field capacity of the waste were found to be 590 kg m^{-3} and 0.41, respectively. The increase in the dry density, compared with the original value, can be attributed to biodegradation and settlement.

The effective porosity was determined by closing the outflow valve of the drainage pipe and saturating the column sample with water and then measuring the water volume until the sample was totally saturated. When saturating the sample with water, there seemed to be voids that were filled immediately and others that required up to 6 hours to fill. The former voids were roughly assumed to constitute the channel volume and are denoted channel porosity in Table 3. By neglecting the presence of air-filled voids the total porosity was calculated as the sum of the effective porosity and field capacity. The column sample was assumed to be at field capacity after allowing it to drain until no outflow was registered.

The data from two of the experiments performed with the column setup were used in the present study. In experiment 1 the time required for the infiltration front to reach the lower boundary of the column was measured when a steady flow of 2.87×10^{-5} m^3 s^{-1}, corresponding to a flux density of 9.80×10^{-6} m s^{-1}, was applied. The water input was then stopped,

Table 3. Measured Properties for the Column Sample

Property	Value
Height, m	1.20
Dry density, kg m^{-3}	590
Porosity, m^3 m^{-3}	0.53
Effective porosity, m^3 m^{-3}	0.12
Channel porosity, m^3 m^{-3}	0.093
Field capacity, m^3 m^{-3}	0.41

and the recession was recorded. These data were used to calibrate the model. In experiment 2 a sequence of square pulses with a water flux of 1.14×10^{-4} m^3 s^{-1}, which corresponds to a flux density of 3.92×10^{-5} m s^{-1}, and duration of 6 min was applied beginning every 24th minute. The time-averaged flow during experiment 2 was set to be the same as the flow in experiment 1. Data from experiment 2 were used for validation of the model.

4. Kinematic Wave Solutions

The solution is derived for the case when a sequence of square pulses of volume flux density q_u with a duration T is supplied to the surface of the waste media. The applied water is assumed to rapidly find its way to the channel domain, where it flows downward. The solution is divided in two main domains, the infiltration domain and the drainage domain, denoted D1 and D2, respectively. A portion of the drainage domain forms a third solution domain, D3. The solution domains are shown in Figure 2. Even though the column sample is at field capacity, the S term is not negligible in experiment 2, which was used for validation, because the water flux during the pulses is high enough to force the water into voids and pores because of the potential gradient that builds up. When infiltration ceases, the water travels back to the channel system, where it propagates downward. The term S plays the role of an exchange term and switches signs depending on the direction of the hydraulic gradient between the matrix and the channel domain. In the infiltration domain, D1, S is positive. The characteristics originate from the t axis on the segment $0 \leq$

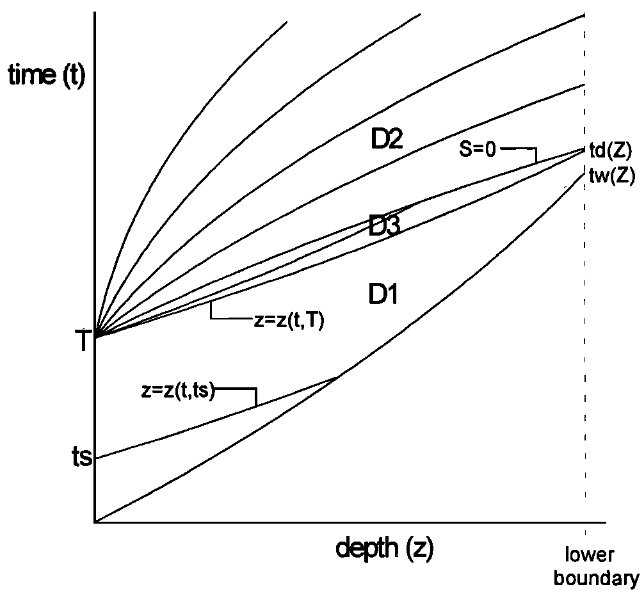

Figure 2. Solution domains for one wetting event.

$t \leq T$. The position on the t axis where the characteristic originates is t_s, and t is the parameter along the characteristics. The characteristic that originates from position T on the t axis and divides domains D1 and D3 is the drainage front. In domain D2, water is flowing back into the channel system, and the S term becomes negative. Domain D3 is a region of internal drainage; however, the water potential is higher than in the storage region, so the S term is still positive. The characteristic originating from the position T on the t axis, which divides domains D2 and D3, carries the water content at which the mobile and immobile regions are in equilibrium, that is, at which S is zero.

The characteristic solution in D1 is given by solving (9) and (10) under boundary condition of (11c) and assuming that S is constant:

$$w(t, t_s) = w_u - S(t - t_s) \qquad 0 < t_s < T \tag{12}$$

$$z(t, t_s) = \frac{b}{S}\{w_u^a - [w_u - S(t - t_s)]^a\} \tag{13}$$

By combining (12) and (13), t and t_s can be eliminated. Solving the resulting equation for $w(t, t_s)$ and inserting into (1) gives the following flux expression:

$$q(z, t) = b\left(w_u^a - \frac{zS}{b}\right) \tag{14}$$

This shows that the flux in domain D1 is dependent only on z. By rearranging (14), S can be determined as

$$S = \frac{q_u - q_z}{Z} \tag{15}$$

where q_u is the applied pulse flux density at $z = 0$ and q_z is the outflow flux density at depth $z = Z$.

The characteristic solution for domain D3 involves a moving boundary along which the equilibrium between the mobile and immobile domains exists, so no interaction takes place. The equilibrium is assumed to appear at a certain water content in the channels, denoted by w_{eq}. The moving boundary gives the time-space history of S as it switches from positive to negative and water starts to flow back into the channels. Equations (12) and (13) become the same here as in D1 except that the characteristics originate from the t axis at the point T with the water content w_s, which varies from one characteristic to the other, from w_u to w_{eq}.

The moving boundary is derived by inserting

$$w_{eq} = w_s - S(t - T) \tag{16}$$

into (13). The solution for the moving boundary then becomes

$$w(t, T) = w_{eq} \tag{17}$$

$$z(t, T) = \frac{b}{S}\{[w_{eq} + S(t - T)]^a - (w_{eq})^a\} \tag{18}$$

In domain D2 the storage differs from the S term in D1 and D3, not only by a shifted sign but also in magnitude, since water is forced into the smaller pores and voids during infiltration, whereas during drainage the flow of water back into the channel domain is governed by a much smaller hydraulic gradient. The flow back into the channels is assumed to be governed by the drainable water volume in the matrix, which has been accumulating in domains D1 and D3. A variable function is therefore suggested, and (9) becomes

$$\frac{dw}{dt} = kw_{\text{mat}} \tag{19}$$

where w is, as previously, the water content in the channels, k is a mass transfer rate coefficient (s^{-1}), and w_{mat} is the drainable volumetric water content in the matrix, stored in domains D1 and D3. The time history of this water content in domain D2 is defined by

$$\frac{dw_{\text{mat}}}{dt} = -kw_{\text{mat}} \tag{20}$$

Solving (20) under the initial condition $w_{\text{mat}}(t_{eq}) = \hat{w}_{\text{mat}}$, where t_{eq} is the time along the moving boundary, defined by (18), and $\hat{w}_{\text{mat}}$ is the maximum water content which has been stored, gives

$$w_{\text{mat}} = \hat{w}_{\text{mat}} e^{-k(t - t_{eq})} \tag{21}$$

Inserting (21) into (19) and solving yields

$$w = w_s + \hat{w}_{\text{mat}}(1 - e^{-k(t - t_{eq})}) \tag{22}$$

By combining (10) and (22) the space and time history of the characteristics in domain D2 becomes

$$\frac{dz}{dt} = ab[w_s + \hat{w}_{\text{mat}}(1 - e^{-k(t - t_{eq})})]^{a-1} \tag{23}$$

As shown in Figure 2, the characteristics originate from the position T on the t axis and carry the water content defined by (22), where w_s varies from w_{eq} to zero. Thus the characteristics intersect the t axis at increasing angles.

The complete model must also include determination of the wetting front in domain D1. *Smith* [1983] formulated the shock velocity in the case where w decreases with depth as

$$\frac{dz}{dt} = \frac{q(w) - q(w^*)}{w - w^*} \tag{24}$$

where w is the water content that propagates down along its characteristic defined by (12) and w^* is the water content in the waste medium ahead of the wetting front.

5. Estimation of Parameters *a* and *b*

For the special case when S is zero, as in experiment 1, the solution domains reduce to D1 and D2, as shown in Figure 3. The characteristics carry a constant water content and therefore become straight lines. The solutions constitute special cases of (12), (13), (22), and (23). The solution in domain D1 becomes

$$w = w_u \tag{25}$$

$$z(t, t_s) = abw_u^{a-1}(t - t_s) \tag{26}$$

and in domain D2 it becomes

$$w = w_s \tag{27}$$

$$z(t, T) = abw_s^{a-1}(t - T) \qquad t > T \tag{28}$$

where w_s varies from w_u to zero.

With $w^* = 0$ the wetting front can be determined from (24) as

$$z(t, t_s) = bw_u^{a-1}t \tag{29}$$

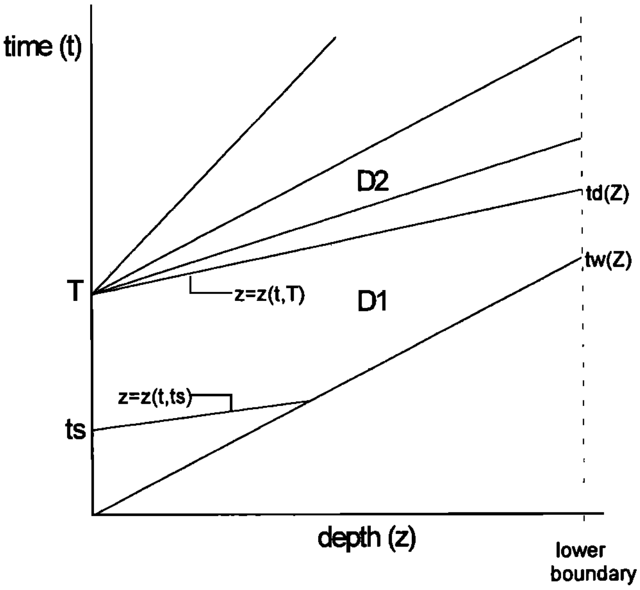

Figure 3. Solution domains for a water input event when S is zero.

The drainage front results when $t_s = T$ is inserted into (26). From (26) and (29) it can be seen that the drainage front travels a times faster than the wetting front.

Following the method of *Germann* [1985], the flow parameters can be determined analytically using the data on the arrival time of the wetting front, the arrival time of the drainage front, and the recession hydrograph for a square pulse water input, in the case of $S = 0$, as follows:

Combining (1) and (29), eliminating w, and solving for b gives

$$b = \left[\frac{Z}{t_w(Z)}\right]^a q_u^{1-a} \qquad (30)$$

where $t_w(Z)$ is the arrival time of the wetting front at depth Z. By rearranging (28) the water content in domain D2 at depth Z and time t can be written as

$$w = Z^{1/a-1}[ab(t-T)]^{1/1-a} \qquad (31)$$

Combining (1), (30), and (31), $q(Z, t)$ in domain D2 becomes

$$q = q_u\left[\frac{t_w(Z)}{a(t-T)}\right]^{a/a-1} \qquad (32)$$

Equation (32) can be transformed into a linear equation by taking the logarithm:

$$q^* = \kappa t^* - v \qquad (33A)$$

where

$$q^* = \ln\left[\frac{q(Z,t)}{q_u}\right] \qquad (33B)$$

$$\kappa = \frac{a}{a-1} \qquad (33C)$$

$$t^* = \ln\left[\frac{t_w(Z)}{t-T}\right] \qquad (33D)$$

$$v = \frac{a}{a-1}\ln(a) \qquad (33E)$$

Solving (26) for t gives the arrival time of the drainage front, $t_d(Z)$. The parameters u and v can be determined by fitting (33A) to the recession data using linear regression. Combining (33C) and (33E), the parameter a is given as

$$a = e^{v/\kappa} \qquad (34)$$

Inserting the value of a into (30) yields b.

6. Results

6.1. Calibration

The data from experiment 1 were used to determine the model parameters a and b, that is, to calibrate the model. The waste was at field capacity, and accumulation of water did not have to be accounted for ($S = 0$). The arrival time of the wetting front at the outflow of the column was 1620 s. The solution for the calibration experiment is given by (25)–(29). The exponent a was determined by fitting (33A) to the recession data using linear regression and by employing (34). The result of the fitting procedure to the recession data is shown in Figure 4. The coefficient of determination for the linear regression r^2 was calculated to be 0.94.

The experimental recession curve is plotted in Figure 5 together with the calibrated model. No data on the rising part of the hydrograph are available, as only the arrival of the infiltration front at the outflow was recorded. The arrival was defined as the point in time at which the outflow rate was equal to the input rate. Result of the calibration was as follows:

$$\begin{cases} a = 3.05 \text{ m s}^{-1} \\ b = 5.24 \text{ m s}^{-1} \end{cases}$$

According to the interpretation of the parameter a by *Germann and DiPietro* [1996] given in section 2.2, the value of a suggests that the flow can be characterized as flow along a planar surface. This interpretation agrees with the aforementioned void structure, involving horizontal stratification and the presence of impermeable surfaces.

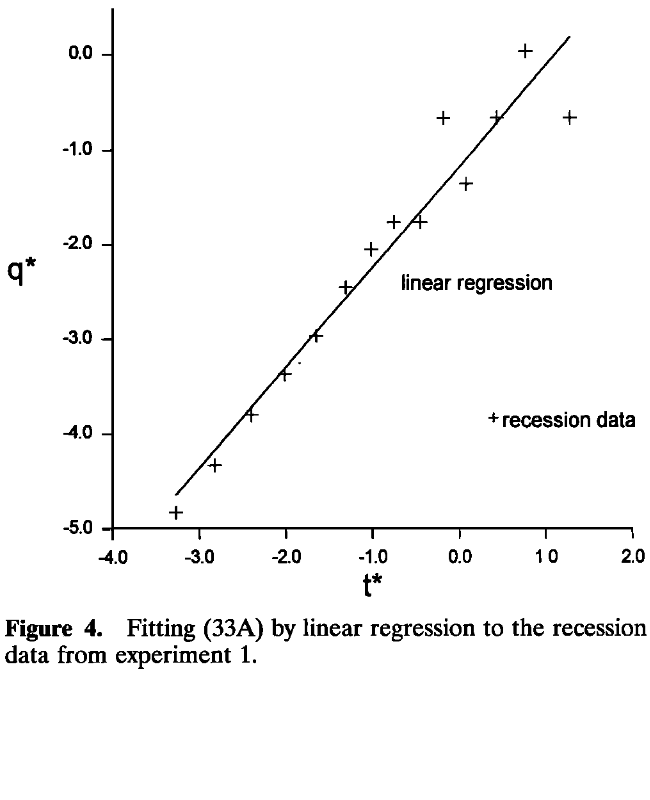

Figure 4. Fitting (33A) by linear regression to the recession data from experiment 1.

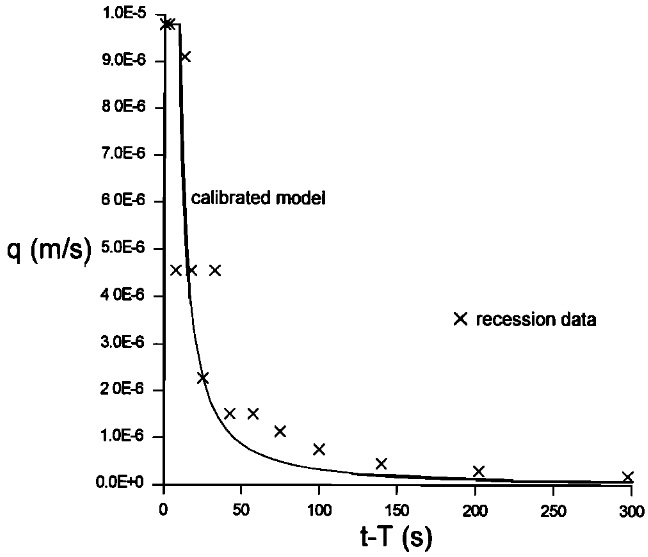

Figure 5. Experimental recession curve and calibrated model.

6.2. Validation

The S term in domains D1 and D3 was determined using (15). The maximum water content that had been stored was determined from the recession curve after irrigation had been shut off. Given $\hat{w}_{\text{mat}}$ and the time it took to drain this volume, the mass transfer rate coefficient k was determined using (21).

The time-space history of the infiltration front was calculated by inserting q^* and w^*, as defined by (22) and (23), and q and w, as defined by (12) and (14), into (24). The final equation was solved by employing a simple numerical scheme. The outflow flux density in domain D1 was calculated using (14), whereas a numerical procedure was the only resort when calculating the outflow flux density in domain D2, by combining (1), (22), and (23). The storage parameters were found to be

$$\begin{cases} S = 2.11 \times 10^{-5} & (\text{s}^{-1})\ (\text{D}_1 \text{ and } \text{D}_3) \\ \hat{w}_{\text{mat}} = 9.97 \times 10^{-3} & (\text{m}^3\ \text{m}^{-3}) \\ k = 1.0 \times 10^{-3} & (\text{s}^{-1})\ (\text{D}_2) \end{cases}$$

The S term represents a flow of about 7×10^{-5} $\text{m}^3\ \text{s}^{-1}$ in D1 and D3. By comparing this value with the applied water flux in the pulses of 1.14×10^{-4} $\text{m}^3\ \text{s}^{-1}$ and the maximum outflow flux of 4.0×10^{-5} $\text{m}^3\ \text{s}^{-1}$ it can be concluded that the water volume applied during the pulses splits approximately equally into flow from the channels into adjacent voids and pores, which may be assumed to be parallel to the horizontal stratification, and vertical flow.

The result obtained with the model is compared with experimental data in Figure 6. The model predicts a net accumulation of about 5% of the applied water during the sequence of pulses. The model predicts this volume to be drained when the water input is shut off and the waste medium is allowed to drain for a longer period of time.

7. Discussion and Conclusions

The kinematic wave model describes the arrival of the wetting front and the drainage front during unsteady flow in the investigated sample fairly accurately. However, the model is not capable of describing the observed dispersion, because of the basic assumption of the kinematic wave model and the flux law employed here, which is strictly convective. There is also a spatial variability in the waste medium, which is not taken into account, and this results in deviations from the modeled hydrograph. Water may infiltrate faster in some channels than the average rate, which can be seen as an earlier rise in the measured hydrograph compared with the model. It is also shown that the flow pattern is dependent on the applied flux density. In the square pulse experiment the horizontal water flow from the channels into adjacent voids and fissures is significant compared with the average vertical flow.

The interpretation of $a = 3.05$ suggests that a limited number of channels is present in which water movement takes place as a creeping flow in thin layers with free surfaces along planar solid surfaces.

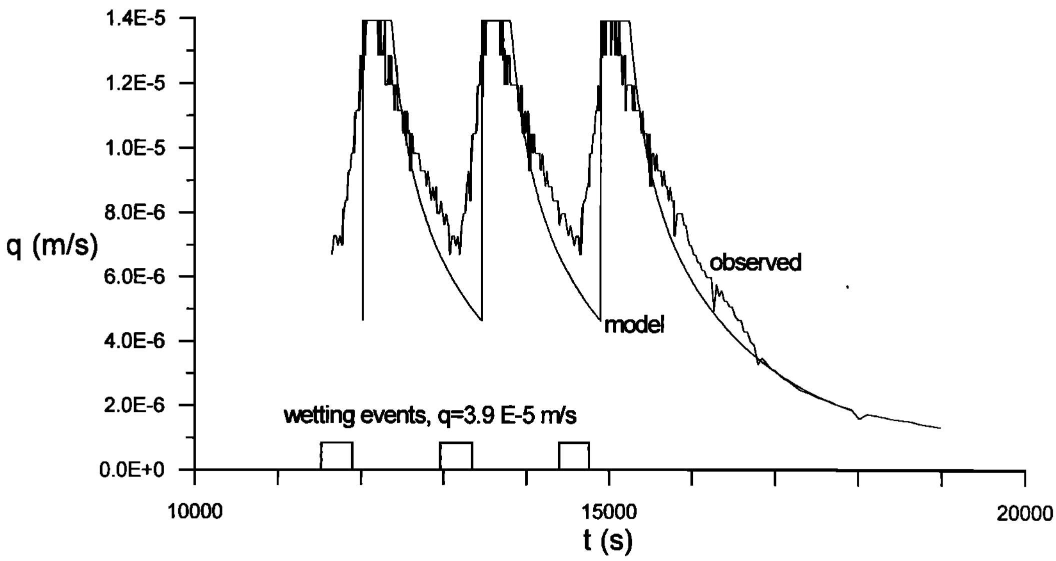

Figure 6. The result obtained with the model compared with data from the column experiment.

Further development of the model, including the incorporation of spatial variability, in order to make it applicable on a larger scale, is planned.

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