A Distributed Converging Overland Flow Model
2. Effect of Infiltration

BERNARD SHERMAN AND VIJAY P. SINGH

New Mexico Institute of Mining and Technology, Socorro, New Mexico 87801

The overland flow on an infiltrating converging surface is studied. Mathematical solutions are developed to study the effect of infiltration on nonlinear overland flow dynamics. To develop mathematical solutions, infiltration and rainfall are represented by simple time and space invariant functions. For complex rainfall and infiltration functions, explicit solutions are not feasible.

INTRODUCTION

Overland flow and infiltration have been extensively studied as separate components of the hydrologic cycle [*Woolhiser and Liggett*, 1967; *Woolhiser*, 1969; *Kibler and Woolhiser*, 1970; *Singh*, 1974; *Lane*, 1975; *Philip*, 1957; *Hanks and Bowers*, 1962; *Whisler and Klute*, 1965; *Rubin*, 1966]. A combined study of these phases is required for modeling overland flow. With a few exceptions, notably the work by *Smith* [1970] and *Smith and Woolhiser* [1971], the conventional approach [*Wooding*, 1965; *Eagleson*, 1972; *Singh*, 1975] to combining these phases has been through the familiar notion of so-called rainfall excess. In this approach, infiltration is independently determined and subtracted from rainfall; the residual is termed rainfall excess, which forms input to the overland flow model. It seems to us that this concept of rainfall excess is more an artifice than a reality. The processes of rainfall, infiltration, and runoff occur concurrently in nature and therefore warrant a combined study. The purpose of this paper, part 2 of a series, is to consider infiltration in the converging overland flow model and then develop mathematical solutions for overland flow. The mathematical treatment developed here is useful in studying the effect of infiltration on nonlinear watershed dynamics.

EFFECT OF INFILTRATION ON OVERLAND FLOW: MATHEMATICAL SOLUTIONS

In a previous paper [*Sherman and Singh*, 1976], hereafter referred to as part 1, the infiltration of water through the ground was disregarded. Now we include such a term in the model. Let $f(x, t)$ be the rate of infiltration per unit area; f is dependent on the depth of flow h in the following sense:

$$f(x, t) > 0 \quad \text{if} \quad h(x, t) > 0$$

$$f(x, t) = 0 \quad \text{if} \quad h(x, t) = 0$$

We will assume further that

$$q(x, t) > f(x, t) \qquad 0 \le t \le T \qquad 0 \le x \le L(1 - r)$$

where q is the lateral inflow per unit area, T is the duration of q, L is the length of the converging section, r is the degree of convergence, and x and t are space and time coordinates. Then the continuity and momentum equations are

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = q(x, t) - f(x, t) + \frac{uh}{L - x} \qquad (1)$$

$$Q = uh = \alpha(x)h^n \qquad (2)$$

where u is the local mean velocity and α and n are kinematic wave parameters. As before, $q(x, t) = 0$ when $t > T$ and $n > 1$. The boundary conditions are

$$h(x, 0) = 0 \qquad 0 \le x \le L(1 - r)$$
$$h(0, t) = 0 \qquad 0 \le t \le T \qquad (3)$$

It is plausible on physical grounds that there will be a curve $t = t^0(x)$ in $\{t \ge T, 0 \le x \le L(1 - r)\}$ starting at $x = 0$, $t = T$ such that $h(x, t^0(x)) = 0$. This curve gives the time history of the water edge as it recedes from $x = 0$ to $x = L(1 - r)$. Equations (1) and (2) are satisfied in $S = \{0 < t < t^0(x), 0 < x < L(1 - r)\}$. Thus $t = t^0(x)$ is a free boundary, and (1)-(3) and $h(x, t^0(x)) = 0$ constitute a free boundary problem. In the domain above the curve $t = t^0(x)$, $h(x, t) = 0$. The determination of the free boundary $t = t^0(x)$ is, as we shall see, relatively simple when q and f are constant (see Figure 1); in this paper we will discuss only that case.

If we eliminate u between (1) and (2) we get

$$\frac{\partial h}{\partial t} + n\alpha(x)h^{n-1}\frac{\partial h}{\partial x} = q(x, t) - f(x, t) + \frac{\alpha(x)h^n}{L - x} - \alpha'(x)h^n \qquad (4)$$

The characteristics of (4) are

$$dt/ds = 1 \qquad dx/ds = n\alpha(x)h^{n-1}$$

$$\frac{dh}{ds} = q(x, t) - f(x, t) + \frac{\alpha(x)h^n}{L - x} - \alpha'(x)h^n$$

and the solution of (4) and (3) is the surface $h(x, t)$ formed by all the characteristic curves through the segment $t = 0, 0 \le x \le L(1 - r)$ and the segment $x = 0, 0 \le t \le T$. The free boundary $t = t^0(x)$ is the locus $h(x, t) = 0$ in the (x, t) plane.

If we take x as a parameter, then the characteristic curves are

$$dt/dx = [n\alpha(x)h^{n-1}]^{-1} \qquad (5)$$

$$\frac{dh}{dx} = \frac{q(x, t) - f(x, t)}{n\alpha(x)h^{n-1}} + \frac{h}{n(L - x)} - \frac{\alpha'(x)h}{n\alpha(x)} \qquad (6)$$

and the initial conditions are

$$t(0) = t_0 \quad h(0) = 0 \qquad 0 \le t_0 \le T \qquad (7)$$

or

$$t(x_0) = 0 \qquad h(x_0) = 0 \qquad 0 \le x_0 \le L(1 - r) \qquad (8)$$

We assume that the curves $t = t(x, t_0)$, which are the solutions

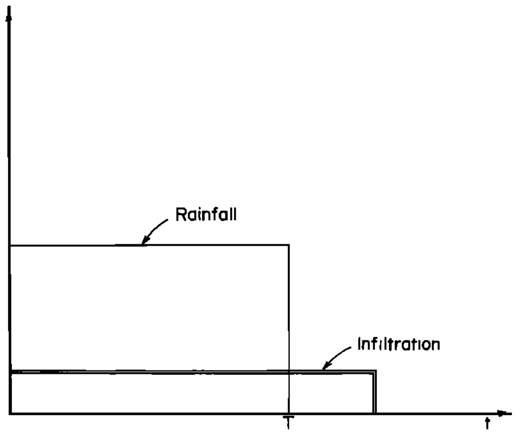

Fig. 1. Rainfall and infiltration, constant in space and time.

of (5), (6), and (7), do not intersect for distinct values of t_0. Similarly, we assume that the curves $t = t(x, x_0)$, which are the solutions of (5), (6), and (8), do not intersect for distinct values of x_0. This is true when q and f are constant and $(L - x)/\alpha(x)$ is a decreasing function of x; it is known from part 1 when $t \leq T$, i.e., in domains D_2 and D_3 (Figure 2), and it is proved in Appendix A when $t > T$.

We distinguish three cases A, B_1, and B_2 (Figure 2) which depend on the relative disposition of the three curves $t = t^0(x)$, $t = T$, and $t = t(x, 0)$; $t = t(x, x^*)$ is the prolongation of $t = t(x, 0)$ to the right of $x = x^*$. In case A, $t^0(x) > T > t(x, 0)$, $0 < x \leq L(1 - r)$. In case B_1, $t^0(x) > T$, and $t^0(x) > t(x, 0)$, but $t = T$ and t / $t(x, 0)$ intersect at x / x^*; i.e., $T = t(x^*, 0)$, and $0 < x^* < L(1 - r)$. In case B_2, $t^0(x) > T$, but $t = T$ and $t = t(x, 0)$ intersect at $x = x^*$, and $t = t^0(x)$ and $t = t(x, x^*)$ intersect at $x = \bar{x}$; i.e., $t^0(\bar{x}) = t(\bar{x}, x^*)$, and $0 < \bar{x} < L(1 - r)$.

Since $t^0(x)$ and $t(x, 0)$ are not known until we have solved the problem, it appears that we cannot distinguish these cases beforehand. But in the special case which we consider in this paper, $q(x, t)$ and $f(x, t)$ both constant, we can distinguish the three cases beforehand. The domains D_1, D_2, and D_3 in case A and the domains D_{11}, D_{12}, D_2, and D_3 in cases B_1 and B_2 are indicated in Figure 2.

In case A the solutions in D_2 and D_3 when q and f are constant are obtained from the discussion in part 1. Let

$$q^* = q - f \qquad \beta^* = \left(\frac{q^*}{2}\right)^{1/n} \qquad \gamma^* = \frac{1}{n}\left(\frac{2}{q^*}\right)^{(n-1)/n}$$

Then in D_2 the solution is given by (13) and (14) of part 1, β and γ being replaced by β^* and γ^*:

$$h(x, t_0) = \beta^*\left[\frac{L^2 - (L - x)^2}{\alpha(x)(L - x)}\right]^{1/n} \tag{9}$$

$$t(x, t_0) = t_0 + \gamma^* \cdot \int_0^x \frac{1}{\alpha(\eta)^{1/n}}\left[\frac{L - \eta}{L^2 - (L - \eta)^2}\right]^{(n-1)/n} d\eta \tag{10}$$

In D_3 the solution is given by (18) and (19) of part 1, β and γ being replaced by β^* and γ^*:

$$h(x, x_0) = \beta^*\left[\frac{(L - x_0)^2 - (L - x)^2}{\alpha(x)(L - x)}\right]^{1/n} \tag{11}$$

$$t(x, x_0) = \gamma^* \int_{x_0}^x \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(L - x_0)^2 - (L - \eta)^2}\right]^{(n-1)/n} d\eta \tag{12}$$

In D_1 we solve (5) and (6) with $q(x, t) = 0$ and $f(x, t) = f$, subject to

$$t(x_0^*) = T \qquad h(x_0^*) = \beta^*\left[\frac{L^2 - (L - x_0^*)^2}{\alpha(x_0^*)(L - x_0^*)}\right]^{1/n}$$

The solution is (here $\rho = f/q$)

$$h(x, x_0^*) = \beta\left[\frac{(1 - \rho)L^2 - (L - x_0^*)^2 + \rho(L - x)^2}{\alpha(x)(L - x)}\right]^{1/n} \tag{13}$$

$$t(x, x_0^*) = T + \gamma \int_{x_0^*}^x \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(1 - \rho)L^2 - (L - x_0^*)^2 + \rho(L - \eta)^2}\right]^{(n-1)/n} d\eta \tag{14}$$

The curves $t = t(x, t_0)$ do not intersect in D_2, the curves $t = t(x, x_0^*)$ do not intersect in D_1, and on the assumption

$$\frac{d}{dx}\frac{L - x}{\alpha(x)} < 0 \tag{15}$$

the curves $t = t(x, x_0)$ do not intersect in D_3 (Appendix B of part 1). The free boundary $t = t^0(x)$ is now determined by

$$(1 - \rho)L^2 - (L - x_0^*)^2 + \rho(L - x)^2 = 0 \tag{16}$$

and (14). Eliminating x_0^* between (16) and (14), we get (here $\omega = n^{-1}(2/f)^{(n-1)/n}$)

$$t^0(x) = T + \omega \int_{\chi(x)}^x \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(L - \eta)^2 - (L - x)^2}\right]^{(n-1)/n} d\eta \tag{17}$$

where

$$\chi(x) = L - [(1 - \rho)L^2 + \rho(L - x)^2]^{1/2}$$

As in part 1, for fixed x, $h(x, t)$ is an increasing function of t in D_3, independent of t in D_2, and a decreasing function of t in D_1 (Figure 3).

The criterion for distinguishing between case A and cases B_1 and B_2 is, as in part 1, obtained from

$$T = \gamma^* \int_0^x \frac{1}{\alpha(\eta)^{1/n}}\left[\frac{L - \eta}{L^2 - (L - \eta)^2}\right]^{(n-1)/n} d\eta \tag{18}$$

If (18) does not have a root between 0 and $L(1 - r)$, then we are in case A; if there is such a root x^*, then we are in case B_1 or case B_2. If $F(x)$ is the right side of (18), then case A occurs if and only if $F[L(1 - r)] \leq T$, and case B_1 or case B_2 occurs if and only if $F[L(1 - r)] > T$. To distinguish between cases B_1 and B_2, we note, referring to (13), that

$$(1 - \rho)L^2 - (L - x^*)^2 + \rho(L - x)^2 = 0 \tag{19}$$

does not have a root between 0 and $L(1 - r)$ in case B_1 and does have such a root $\bar{x}$ in case B_2. Such a root exists if and only if

$$L^2r^2 < \rho^{-1}(L - x^*)^2 - [(1/\rho) - 1]L^2$$

or

$$1 - \rho(1 - r^2) < [(L - x^*)/L]^2 \tag{20}$$

Thus if (20) is true, we are in case B_2, and otherwise we are in case B_1. In case B_2 the intersection of the curves $t = t(x, x^*)$

and $t = t^0(x)$ occurs at

$$\bar{x} = L - \{(1/\rho)(L - x^*)^2 - [(1/\rho) - 1]L^2\}^{1/2} \quad (21)$$

$$\bar{t} = T + \omega \int_{x^*}^{\bar{x}} \frac{1}{\alpha(\eta)^{1/n}} \left[\frac{L - \eta}{(L - \eta)^2 - (L - \bar{x})^2}\right] d\eta \quad (22)$$

We discuss now the solution in cases B_1 and B_2. In both cases the solution in D_{11} is given by (13) and (14), in D_2 by (9) and (10), and in D_3 by (11) and (12). It remains to determine the solution in D_{12}. As in part 1, we define x_0^* by $T = t(x_0^*, x_0)$; here $x^* \leq x_0^* \leq L(1 - r)$. Thus from (12),

$$T = \gamma^* \int_{x_0}^{x_0^*} \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(L - x_0)^2 - (L - \eta)^2}\right]^{(n-1)/n} d\eta \quad (23)$$

Then from (16) and (17) of part 1,

$$h(x; x_0^*, x_0) = \beta\left[\frac{(1 - \rho)(L - x_0)^2 - (L - x_0^*)^2 + \rho(L - x)^2}{\alpha(x)(L - x)}\right]^{1/n} \quad (24)$$

$$t(x; x_0^*, x_0) = T + \gamma \int_{x_0^*}^{x} \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(1 - \rho)(L - x_0)^2 - (L - x_0^*)^2 + \rho(L - \eta)^2}\right]^{(n-1)/n} d\eta \quad (25)$$

It is proved in Appendix A that the curves defined by (23) and (25) do not, on condition (15), intersect in D_{12}.

In case B_2, part of the boundary of D_{12} is $t = t^0(x)$. This is obtained by eliminating x_0 and x_0^* between (23) and (25), and from (24),

$$(1 - \rho)(L - x_0)^2 - (L - x_0^*)^2 + \rho(L - x)^2 = 0 \quad (26)$$

From (26) we get $x_0^* = \chi(x, x_0)$, where

$$\chi(x, x_0) = L - [(1 - \rho)(L - x_0)^2 + \rho(L - x)^2]^{1/2} \quad (27)$$

Thus $t = t^0(x)$ is defined by

$$T = \gamma^* \int_{x_0}^{\chi(x, x_0)} \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(L - x_0)^2 - (L - \eta)^2}\right]^{(n-1)/n} d\eta \quad (28)$$

$$t(x, x_0) = T + \omega \int_{\chi(x, x_0)}^{x} \frac{1}{\alpha(\eta)^{1/n}} \cdot \left[\frac{L - \eta}{(L - \eta)^2 - (L - x)^2}\right]^{(n-1)/n} d\eta \quad (29)$$

In (28) and (29), $\bar{x} \leq x \leq L(1 - r)$; when $0 \leq x < \bar{x}$, $t^0(x)$ is defined by (16).

The behavior of $h(x, t)$ as a function of t for fixed x, $0 < x < x^*$, is the same in cases B_1 and B_2 as it is in case A (Figure 3). In cases B_1 and B_2, $h_t(x, t) > 0$ when $(x, t) \in D_3$, and $h_t(x, t) < 0$ when $(x, t) \in D_{11}$; the arguments are the same as they are in case A. The maximum of $h(x, t)$ occurs therefore when $(x, t) \in D_{12}$ (Figure 3), but it can occur on the boundary of D_{12} as in part 1 ($\rho = 0$). The case $\alpha(x) = \alpha$ is discussed in greater detail in Appendix B; Figure 4 illustrates the possibilities.

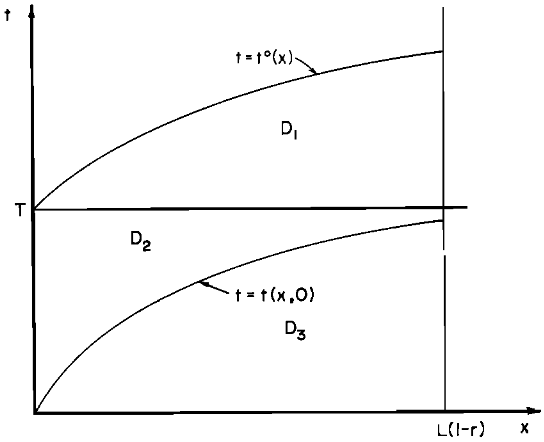

Fig. 2*a*. Solution domain for case A.

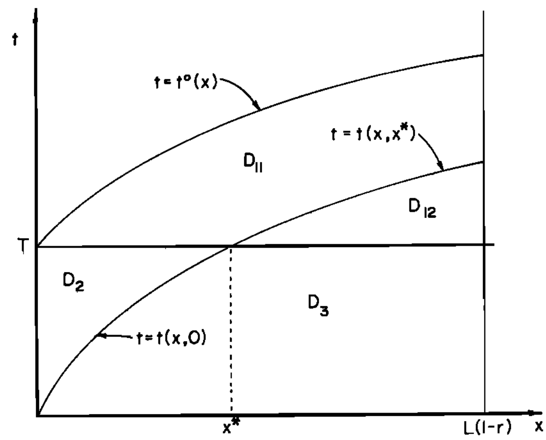

Fig. 2*b*. Solution domain for case B_1.

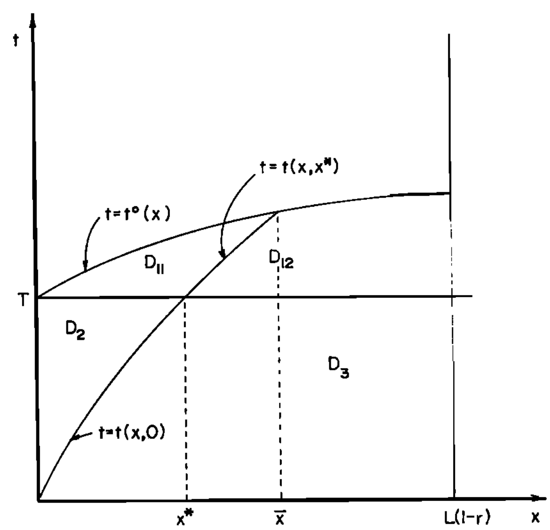

Fig. 2*c*. Solution domain for case B_2.

CONCLUDING REMARKS

The mathematical solutions developed above demonstrate that simultaneous consideration of the rainfall and infiltration phases of the hydrologic cycle alters the character of overland flow dynamics. To stimulate the watershed response realistically, their combined study is essential, although we do recognize that this enhances the mathematical complexity.

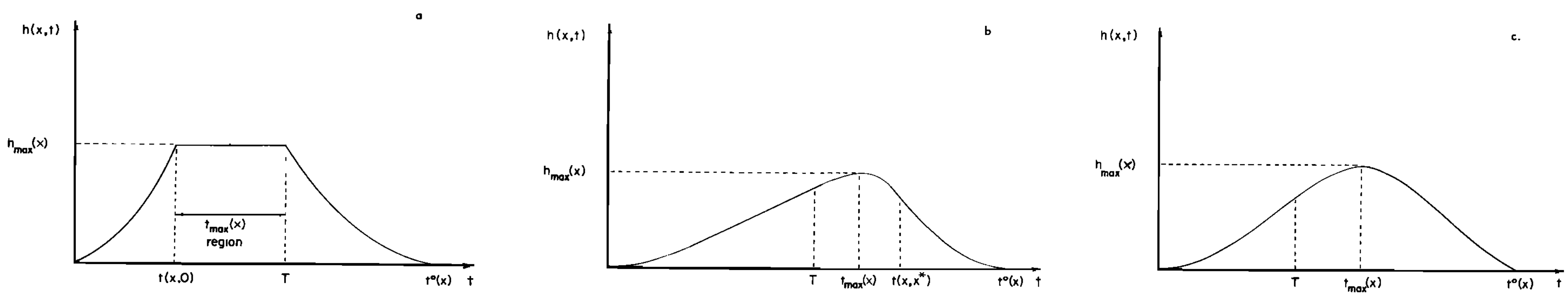

Fig. 3. Depth of flow $h(x, t)$ as a function of t for fixed x. (*a*) Case A occurs if $0 < x \leq L(1 - r)$; case B_1 or case B_2 occurs if $0 < x < x^*$. (*b*) Case B_1 occurs if $x^* \leq x \leq L(1 - r)$; case B_2 occurs if $x^* \leq x \leq \bar{x}$. (*c*) Case B_2 occurs if $\bar{x} < x \leq L(1 - r)$.

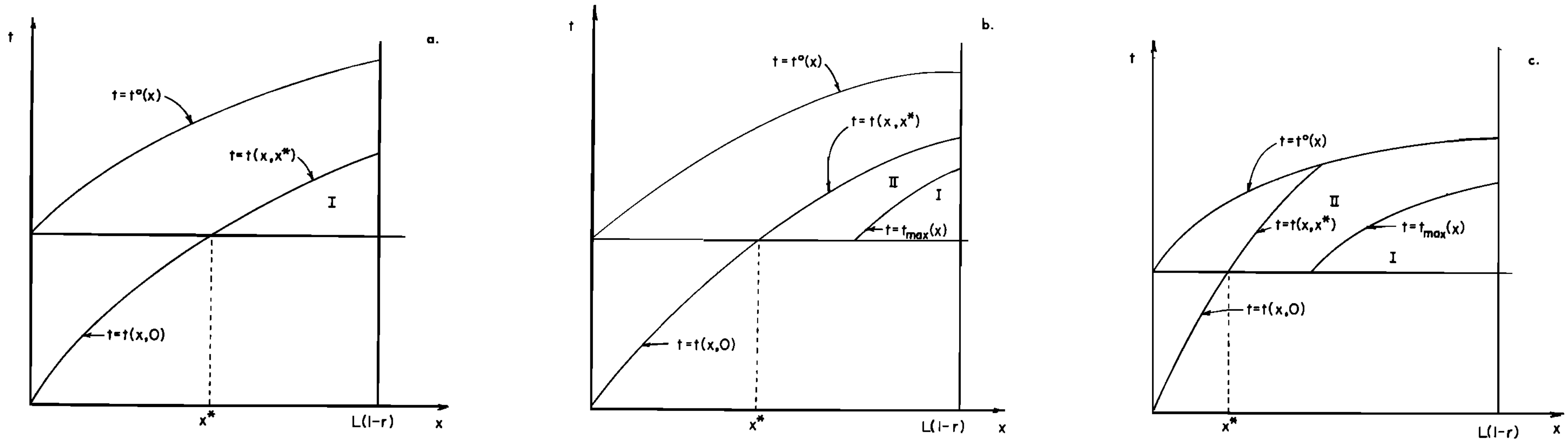

Fig. 4. Behavior of $h(x, t)$ as a function of t for fixed x in D_{12}. (*a*) Case B_1 occurs if ρ is small and $\alpha(x) = \alpha$, and I is a region where $h_t(x, t) > 0$. (*b*) Case B_1 occurs if ρ is intermediate and $\alpha(x) = \alpha$; I is a region where $h_t(x, t) > 0$, and II is a region where $h_t(x, t) < 0$. (*c*) Case B_2 occurs if $\alpha(x) = \alpha$; I is a region where $h_t(x, t) > 0$, and II is a region where $h_t(x, t) < 0$.

As was noted in part 1, the analysis in this paper can be carried out on the assumption that $q(x, t) = q(x)$ and $f(x, t) = f(x)$ with only a slight increase in mathematical complexity. But the main features are already contained in the case $q(x, t) = q$ and $f(x, t) = f$ discussed above and in the appendices.

APPENDIX A

It follows from Appendix B of part 1 that the curves $t = t(x, t_0)$ do not intersect in D_2 and from assumption (15) that the curves $t = t(x, x_0)$ do not intersect in D_3. It follows from (14) that $t_{x_0*}(x, x_0^*) < 0$, so the curves $t = t(x, x_0^*)$ do not intersect in D_1 (case A) or in D_{11} (cases B_1 and B_2). We prove now with assumption (15) that the curves $t = t(x; x_0^*, x_0)$ do not intersect in D_{12}. We introduce the change of variable $\xi = (L - \eta)/(L - x_0)$ in (23) and (25):

$$T = \gamma^* \int_z^L \left\{\frac{L - x_0}{\alpha[L - \xi(L - x_0)]}\right\}^{1/n} \cdot \left(\frac{\xi}{1 - \xi^2}\right)^{(n-1)/n} d\xi \qquad \text{(A1)}$$

$$t(x; x_0^*, x_0) = T + \gamma \int_{(L-x)/(L-x_0)}^{z} \left\{\frac{L - x_0}{\alpha[L - \xi(L - x_0)]}\right\}^{1/n} \cdot \left(\frac{\xi}{1 - \rho - z^2 + \rho\xi^2}\right)^{(n-1)/n} d\xi \qquad \text{(A2)}$$

where $z = (L - x_0^*)/(L - x_0)$. As in Appendix B of part 1, it follows from (A1) that $dz/dx_0 < 0$. We have therefore, from (A2), that $t_{x_0}(x; x_0^*(x_0), x_0) < 0$.

APPENDIX B

In this appendix we discuss the behavior, when $\alpha(x) = \alpha$, of $h(x, t)$ for fixed x in D_{12}. The discussion is parallel to that of Appendix C of part 1. From (A1) we have

$$T = \gamma^* \alpha^{-1/n}(L - x_0)^{1/n} \int_z^1 \left(\frac{\xi}{1 - \xi^2}\right)^{(n-1)/n} d\xi \qquad \text{(B1)}$$

From (24) it is clear that we need be concerned only with

$$G(x_0, z) = (1 - \rho)(L - x_0)^2 = (L - x_0)^2(1 - \rho - z^2) \qquad \text{(B2)}$$

as a function of x and t. On eliminating x_0 between (B1) and (B2) we see that (B2) is, except for a constant positive multiplier,

$$g(z) = (1 - \rho - z^2)\left\{\left[\int_z^1 \frac{\xi}{1 - \xi^2}\, d\xi\right]^{(n-1)/n}\right\}^{-1} \qquad \text{(B3)}$$

The relationship between (x, t) and (x, z) in D_{12} is obtained from (A2):

$$t(x, z) = T + \gamma\alpha^{-1/n}(L - x_0)^{1/n} \cdot \int_{(L-x)/(L-x_0)}^{z} \left[\frac{\xi}{1 - \rho - z^2 + \rho\xi^2}\right]^{(n-1)/n} d\xi \qquad \text{(B4)}$$

Here x_0 is a function of z through (B1). Since $z'(x_0) < 0$ and $t_{x_0}(x; x_0^*(x_0), x_0) < 0$, $t_z(x, z) > 0$. The correspondence between (x, t) and (x, z) in D_{12} is one to one. The curve $z = z_0$ coincides with $t = t(x; x_0^*, x_0)$, where x_0 and x_0^* are determined by $(L - x_0^*)/(L - x_0) = z_0$ and (B1).

A simple calculation shows that the sign of $g'(z)$ and therefore also the sign of $h_t(x, t)$ is determined by

$$k(z) = n(1 - \rho - z^2) - [z(1 - z^2)^{n-1}]^{1/n} \cdot \int_z^1 \left(\frac{\xi}{1 - \xi^2}\right)^{(n-1)/n} d\xi \qquad \text{(B5)}$$

For a fixed x, $T \le t \le t(x, x^*)$ in case B_1 and also in case B_2 when $x \le \bar{x}$; when $x > \bar{x}$, $T \le t \le t^0(x)$. Correspondingly,

$$(L - x)/(L - x_0) \le z < z_0(x) \qquad \text{(B6)}$$

where $z_0(x) = (L - x^*)/L$ in case B_1 and also in case B_2 when $x \le \bar{x}$; when $x > \bar{x}$, we determine $z_0(x)$ from (B4) by replacing the left side of (B4) by $t^0(x)$ and then solving (B4) and (B1) for z. Thus the problem is to determine the sign of $k(z)$ in the interval (B6). If $k(z_0) = 0$ and z_0 is in the interval of (B6), then the maximum occurs for $z = z_0$; the corresponding value of t is determined from (B4). Since the locus $z = z_0$ is one of the curves $t = t(x; x_0^*, x_0)$, the maximum of $h(x, t)$ for fixed x occurs on this curve when these maxima are interior to the t interval corresponding to the given x. Various possibilities are indicated in Figure 4.

REFERENCES

Eagleson, P. S., Dynamics of flood frequency, *Water Resour. Res.*, *8*(4), 878–898, 1972.

Hanks, R. J., and S. A. Bowers, Numerical solution of the moisture flow equation for infiltration into layered soils, *Soil Sci. Soc. Amer. Proc.*, *26*(6), 530–534, 1962.

Kibler, D. F., and D. A. Woolhiser, The kinematic cascade as a hydrologic model, *Hydrol. Pap. 39*, Colo. State Univ., Fort Collins, 1970.

Lane, L. J., Influence of simplifications of watershed geometry in simulation of surface runoff, Ph.D. dissertation, 198 pp., Colo. State Univ., Fort Collins, June 1975.

Philip, J. R., The theory of infiltration, 1, The infiltration equation and its solution, *Soil Sci.*, *83*(5), 105–116, 1957.

Rubin, J., Theory of rainfall uptake by soils initially drier than their field capacity and its application, *Water Resour. Res.*, *2*(4), 739–749, 1966.

Sherman, B., and V. P. Singh, A distributed converging overland flow model, 1, Mathematical solutions, *Water Resour. Res.*, *12*, this issue, 1976.

Singh, V. P., A nonlinear kinematic wave model of surface runoff, Ph.D. dissertation, 282 pp., Colo. State Univ., Fort Collins, June 1974.

Singh, V. P., A distributed approach to kinematic wave modeling of watershed runoff, paper presented at National Symposium on Urban Hydrology and Sediment Control, Univ. of Ky., Lexington, July 1975.

Smith, R. E., Mathematical simulation of infiltrating watersheds, Ph.D. dissertation, Colo. State Univ., Fort Collins, June 1970.

Smith, R. E., and D. A. Woolhiser, Overland flow on an infiltrating surface, *Water Resour. Res.*, *7*(4), 899–913, 1971.

Whisler, F. D., and A. Klute, The numerical analysis of infiltration, considering hysteresis, into vertical soil column at equilibrium under gravity, *Soil Sci. Soc. Amer. Proc.*, *29*(5), 489–494, 1965.

Wooding, R. A., A hydraulic model for the catchment-stream problem, 1, Kinematic wave theory, *J. Hydrol.*, *3*(3), 254–267, 1965.

Woolhiser, D. A., Overland flow on a converging surface, *Trans. ASAE*, *12*(4), 460–462, 1969.

Woolhiser, D. A., and J. A. Liggett, Unsteady one-dimensional flow over a plane—The rising hydrograph, *Water Resour. Res.*, *3*(3), 753–771, 1967.

(Received October 1, 1975;
revised March 15, 1976;
accepted April 9, 1976.)