EVALUATION AND ANALYSIS OF AN INTEGRATED PEM FUEL CELL WITH ABSORPTION COOLING SYSTEM FOR SUSTAINABLE BUILDING OPERATION

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ABSTRACT

In this paper, a parametric study of a PEM fuel cell integrated with a double effect absorption system is carried out in order to study the effect of different operating conditions on the efficiency of the PEM fuel cell, utilization factor of the over all system, COPs of the double effect cooling and heating system, and power and heat output of the PEM fuel cell. It is found that the efficiency of the cell decreases, ranging from 46.2% to 24.4% with increase in membrane thickness and current density, and at the same time the COP increases ranging from 0.65 to 1.52. The heat and power output of the fuel cell decreases from 10.54 kW to 5.12 kW, and 9.12 kW to 6.99 kW, respectively for the increase in membrane thickness. However, when the temperature of the cell is increased the heat and power output increases from 5.12 kW to 10.54 kW, and 6.9 kW to 7.02 kW, respectively. The COP is found to be decreasing ranging from 1.53 to 0.33 with the increase in temperature of the cell and heat input to the HTG. As for the utilization factor, it increases ranging from 17% to 87% with increase in the temperature of the cell and heat input to the HTG. This study reveals that an integrated PEM fuel cell with a double effect absorption cooling systems has a very high potential to be an economical and environmental solution as compared with conventional systems of high electricity and natural gas prices which emit lots of harmful gasses and are not that efficient.

KEYWORDS

PEM fuel cell, energy, exergy, efficiency.

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NOMENCLATURE

COP	coefficient of performance
$\dot{E}x$	rate of exergy kW
F	Faraday's constant (96,485 Col mol^{-1})
h	specific enthalpy, kJ kg^{-1}
i	current density (A cm^{-2})
i_0	exchange current density (A cm^{-2})
i_{max}	limiting current density (A cm^{-2})
$\dot{m}$	mass flow rate, kg s^{-1}
$\dot{n}$	molar flow rate, mol s^{-1}
n	number of electrons involved
P	PEM fuel cell pressure
$\dot{Q}$	heat transfer rate, kW
r	ratio
R	universal gas constant (8.314 J mol K^{-1})
s	specific entropy, kJ kg^{-1}.K^{-1}
$\dot{S}_{gen}$	rate of entropy generation, kW K^{-1}
t_{mem}	membrane thickness (cm)
T	temperature, K
$\dot{W}$	work rate, kW
x_A	anode dry gas mole fraction
x_C	cathode dry gas mole fraction
x_{H2O}	hydrogen mole fraction
x_{O2}	oxygen mole fraction

Greek letters

α_A	anode transfer coefficient
α_C	cathode transfer coefficient
β_1, β_2	concentration of overvoltage constants
ε	utilization factor
η	efficiency
ξ_A	anode stoichiometry
ξ_C	cathode stoichiometry

Subscripts

abs	absorber
CHX	condenser heat exchanger
CHP	combined heat and power
con	condenser
EN	energy
Ex	exergy
eva	evaporator
FC	fuel cell
HTG	high temperature generator
HHX	high temperature heat exchanger
HL	heat loss
H,2	hydrogen input
H_{2O}	water
LHX	low temperature heat exchanger
LTG	low temperature generator
PEM	Polymer Electrolyte Membrane
O_2	oxygen
1...28	state numbers
0	ambient or reference condition
o	fuel cell condition
req	required
w	water

INTRODUCTION

The rate of increase in the consumption of energy to run the air-conditioning system in the gulf region has become a major concern for the governments of these countries. In these countries more than 60% of the power produced is being utilized for the cooling of a space. Also, the major source of energy for the countries situated in the gulf region is fossil fuel because of the huge reserves of oil and gas available in these countries. The use of fossil fuels has become an important issue internationally because of their un-friendly behavior with the environment. Fossil fuels are well known for producing CO_2, NOXs, PMs and so on. These harmful emissions from fossil fuels have raised the issue of global warming, air pollution, acid rain, and Ozone depletion. Due to all these harmful effects of using fossil fuels, it has become very important to come up with an air-conditioning system integrated with alternative energy source which is more efficient and more eco-friendly than conventional systems.

In the past decade fuel cell has shown the capability of succeeding fossil fuels as the major energy source in order to cater the ever growing demand of energy. Also, fuel cells are considered as one of the best alternatives for meeting the thermal and electrical demand of buildings. The PEM fuel cells have appeared as a promising source for the provision of energy to the air-conditioning systems, so that they can be made more efficient and eco-friendly. The PEM fuel cell has a capability of generating both power and heat; hence it can be utilized as a co-generation source in order to enhance the efficiency of the system. The Hydrogen fuel cell systems have the potential to reach 60% peak efficiency on lower heating value (LHV) basis [1]. In addition, the source of energy for the PEM fuel cell is hydrogen which is both, energy efficient and eco-friendly. The main characteristics of PEM fuel cells are that they produce water as a byproduct, they have higher efficiency when compared with heat engines, they operate at low temperatures (up to 90°C), which allows a fast start-up, they use a solid polymer as the electrolyte, which reduces concerns related to construction, transportation, and safety [2].

However, many new systems are being tested in order to come up with a new air-condition system which requires less energy to run and is eco-friendly as well. The two prominent systems which are being studied at present are adsorption system and absorption system. Lots of efforts have been made to make absorption system as efficient as eco-friendly as possible. In addition, there are works being done in order to integrate absorption system with renewable energy sources such as solar energy, and fuel cells. Absorption refrigeration systems appear to be a key solution to meet the energy requirement [3]. At present work is being done in order to increase the COP of the absorption system by enhancing the heat recovery through the addition of more generators and heat exchangers. There is huge amount of work done in literature on the single effect system, and on the integration of single effect system with renewable energy sources.

Hydrogen fuel cells are seen as the major source of energy for the new air-conditioning systems. PEM fuel cell integrated with absorption system can be considered as a trigeneration system, because trigeneration systems include those processes of production and simultaneous use of electricity, heat and cold, from a single fuel source [4]. Good amount of work is being published by different researchers in the field of fuel cells. These researchers have tried to study the effect of different parameters on the performance of a fuel cell and have tried to optimize the operation of the fuel cells. The work of these researchers [5-7] are being published in many different journals. The integration of PEM fuel cell with air-condition system has attracted the work of many researchers [4, 8, 9-11]. Moreover, Fuel cell-based CHP systems are very attractive for stationary energy generation, since they allow production of electricity and heat in a decentralized, quiet, efficient and environmentally friendly way [9].

There are very limited studies related to the investigation of the performance of an absorption system integrated with a PEM fuel cell available in

the literature. Also, the most amounts of studies which are being done are on the integration of PEM fuel cell with the single effect systems or integration of solar system absorption system as mentioned above. This is a motivation behind this paper, the main goal of this paper is to develop a relationship for an integrated PEM fuel cell with double effect absorption system and evaluate the performance of the system based on efficiency, COPs, cooling load and utilization factor.

SYSTEM DESCRIPTION AND ANLYSIS

In this paper we have considered a PEM fuel cell integrated with double effect absorption system as shown in Fig. 1. Hydrogen enters the fuel cell from anode side flow plate. After passing through the channels in the plate hydrogen comes in contact with the membrane which allows only protons to pass through. The electrons left behind are then carried to a power source through circuits. These electrons after giving out power comes back to the cathode side flow plate to have a closed loop electrical circuit. On the cathode side oxygen is provide which reacts with the returning electron and protons of the hydrogen to produce water and steam. The power and heat produced is than supplied to the high temperature generator of the double effect system. Small amount of power is also provided to pump. The strong solution enters the high temperature generator and gets heated up. The mixture of ammonia-water vapor in which majority is ammonia goes to the low temperature generator. The weak solution leaves the generator at state 19 and goes through the heat exchanger and acts as a pre-heater for a strong solution till it reaches the absorber. The vapor of ammonia-water leaving through state 7 goes to the LTG and works as a power source for the LTG. The vapor exiting LTG comes out from state 4 and goes through the CHX and comes out at state 3 after pre-heating the strong solution and getting cooled down. The other vapor ammonia-water comes out of the LTG through state 6 and goes into the condenser where it mixes with mixture coming from state 3 to give out the heat to the cooled water flowing into the condenser in order to work as a water heater. The cooled mixture then goes to the expansion valve where the temperature is dropped drastically and comes out at state 2a as a cold mixture. This mixture then goes to the evaporator takes the heat out of the space being cooled and goes to the absorber state 1.

The equations used in order to analyze the overall system and its components and asses the performance of the system through energy and exergy based COPs, efficiency of the cell and utilization factor are given in table 1.

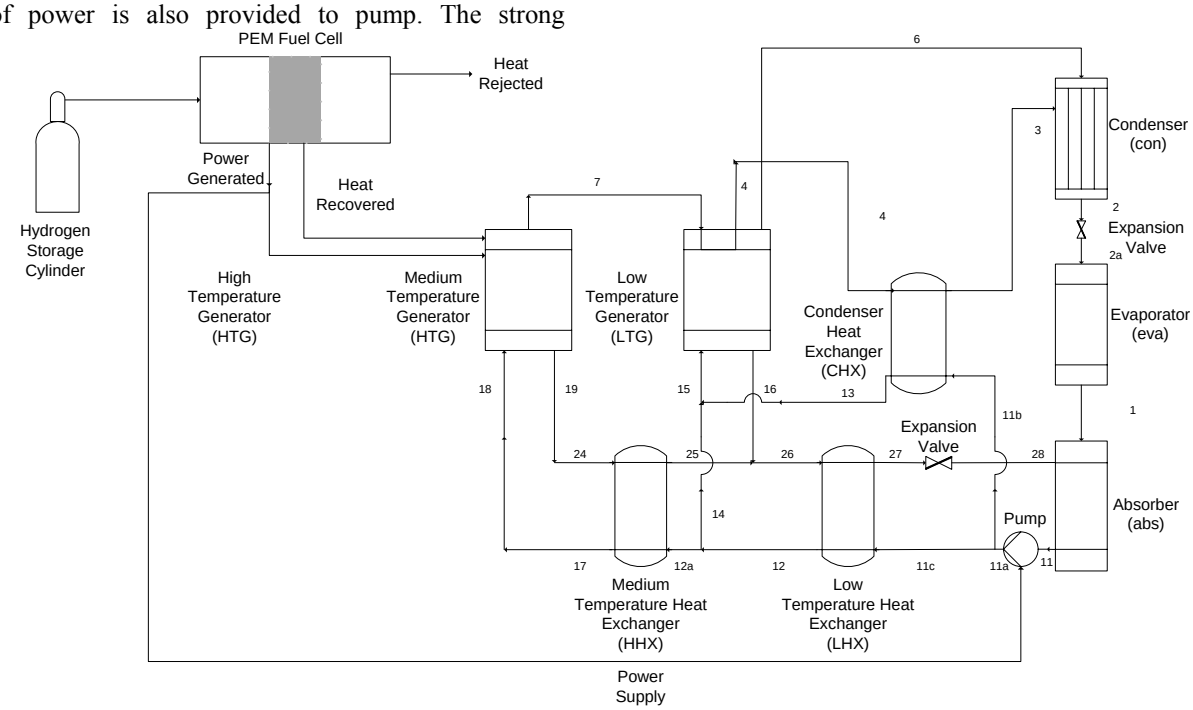

Fig. 1. Schematic of PEM fuel cell integrated with double effect absorption cooling and heating system

RESULTS AND DISCUSSION

After conducting comprehensive parametric studies for different operating conditions and plotting these parameters and linkages, different trends for various operating conditions are observed. It is found that when membrane thickness is increased the efficiency of the cell decreases. This decrease in efficiency is small. The decrease in efficiency ranges from 46.2%

to 35.4% when membrane thickness is increased from 0.016 cm to 0.018 cm for different fuel cell temperatures of 250 K, 300 K and 350 K. This decrease is a result of higher resistance created by the membrane thickness. The membrane acts as a resistor and the increase in thickness increases the resistance. Due to this increase in resistance protons take more time to reach the cathode side and therefore rate of reaction decreases and the efficiency of the cell decreases. This relationship can be seen in Fig. 2.

In Fig. 3 the reduction in power and heat generation of the system is plotted for change in membrane thickness. The heat and power output decreases, ranging from 10.54 kW to 5.12 kW and 9.12 kW to 6.99 kW, respectively as the membrane thickness is increased from 0.016 cm to 0.018 cm for different fuel cell temperatures of 250 K, 300 K and 350 K. This decrease in heat and power output of the fuel cell is a result of increase in resistance of the membrane due to the increase in the thickness. The increase in thickness makes proton take more time to reach cathode side and as a result the reaction on the cathode side gets delayed which affects the heat and power production of the cell.

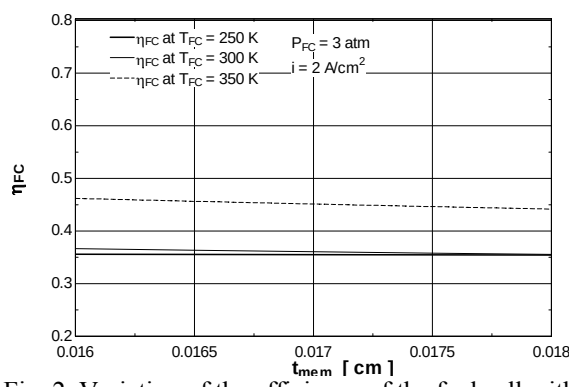

Fig. 2. Variation of the efficiency of the fuel cell with the membrane thickness

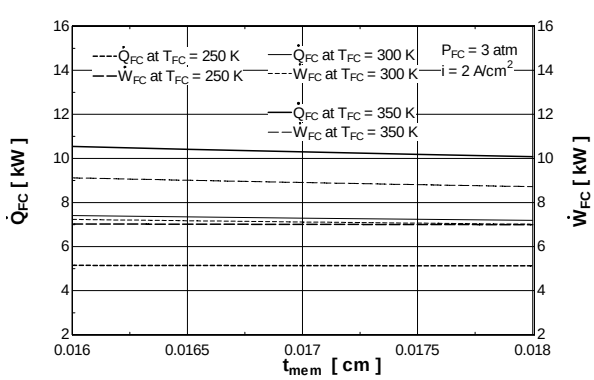

Fig. 3. Variation of the power and heat output of the fuel cell with the membrane thickness

In Fig. 4 the energetic and exergetic COP of the double effect system is plotted against the thickness of the membrane. The energetic and exergetic COP of the system range from 0.97 to 1.52 and 0.65 to 1.25, respectively when membrane of the thickness is increased for different operating temperatures of fuel cell of 200 K, 300 K, and 350 K. This increase in COP is a result of decrease in the performance of a fuel cell. As the membrane thickness increases the power and heat production of fuel cell decreases and for a constant cooling load of a 25 kW, lower amount of energy is being provided to the HTG and as a result the COPs increase.

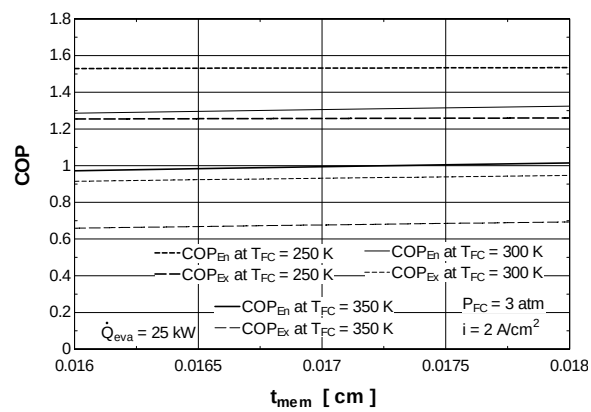

Fig. 4. Variation of the COP with the membrane thickness

In Fig. 5 and Fig. 6 the efficiency of the cell, utilization factor of the overall system, and the heat and power output of the cell is plotted against the temperature of the cell, respectively. The increase in temperature of the cell results in the increase of the efficiency, utilization factor, and heat and power out of the cell. This increase in efficiency, utilization factor, and heat and power output range from 35% to 46%, 36% to 75%, 5.12 kW to 10.54 kW, and 6.9 kW to 7.02 kW, respectively. This behavior is seen because increase in temperature increases the temperature difference between the surroundings and the fuel cell; therefore, increasing the heat and power output of the cell. However, it is also seen the increase in the thickness of the membrane affects the parameters in a negative way but this affect is minute due to the fact that operation of fuel cell is more dependent on temperature than on membrane thickness.

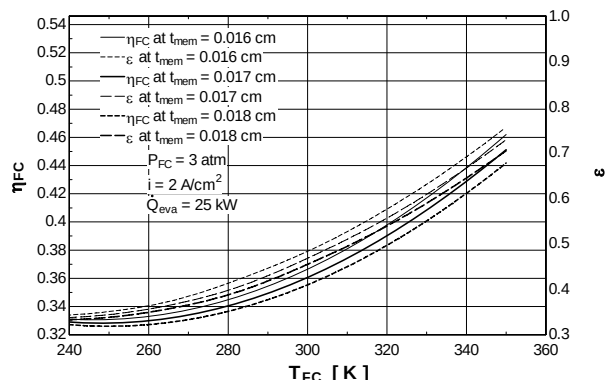

Fig. 5. Variation of the efficiency of the fuel cell and utilization factor of the system with the temperature of the fuel cell

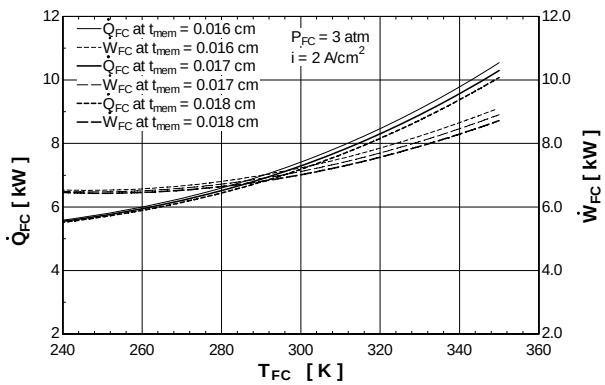

Fig. 6. Variation of the power and heat output of the fuel cell with the temperature of the fuel cell

In Fig. 7 the energetic and exergetic COP of the absorption system are plotted against the temperature of the fuel cell for different membrane thickness. It is found that as the temperature increases the COPs decrease. The decreases in energetic and exergetic COP range from 1.535 to 0.97, and 1.26 to 0.73, respectively. This decrease is noted because the increase in temperature increases the output of the fuel cell, and as this output is fed to the absorption system, for a constant cooling load of 25 kW more energy is provided to the system and therefore the COP of the system decreases as the performance shows the signs of degrading.

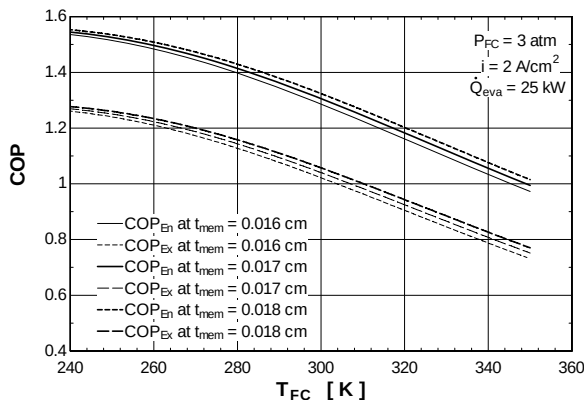

Fig. 7.Variation of the COP with the temperature of the fuel cell

In Fig. 8 effect of temperature of the cell on the utilization factor of the system is being studied. It is found that the increase in temperature results in increase of a utilization factor ranging from 36% to 75%. This increase is a result of more heat being rejected by the condenser as the evaporator doesn't require that heat in order to meet the cooling demand. This rejected heat is then utilized to heat the water and therefore the utilization factor of the system increases when at the same time the COP of the system decreases as explained in the paragraph above.

In Fig. 9 and Fig. 10 the energetic and exergetic COP of the absorption system and the utilization factor of the overall system are studied against the changing heat input to the HTG. It is noticed that energetic and exergetic COP decrease ranging from 1.19 to 0.48, and 0.89 to 0.33, respectively. On the other hand, the utilization factor is seen to be increasing ranging from 17% to 87% when the heat input to HTG is increased for different cooling load of 15 kW, 20 kW and 25 kW. Higher heat input results in a lower COP because system is provided with more energy than it requires in achieving the desired cooling load. The excess energy is then rejected through the condenser which is later utilized to heat the water and hence increases the utilization factor of the overall system.

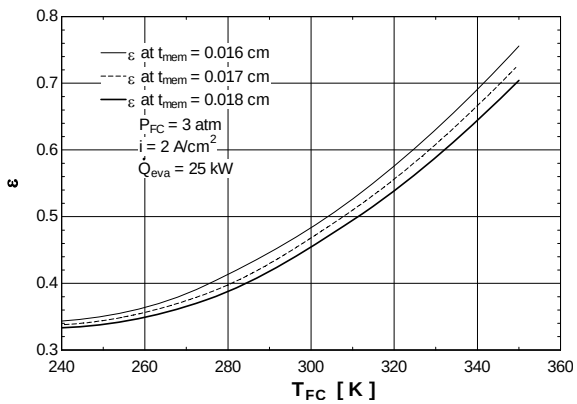

Fig. 8. Variation of the utilization factor with the temperature of the fuel cell

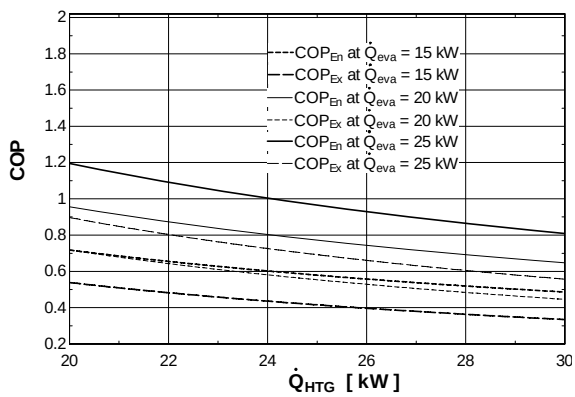

Fig. 9. Variation of the COP with the heat input to the HTG

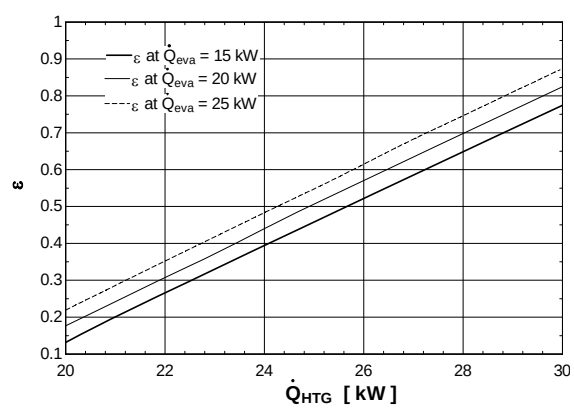

Fig. 10.Variation of the utilization factor with the heat input to the HTG

In Fig. 11 effect of change in current density on the efficiency of the cell is studied. It is found that as current density increases the efficiency of the cell decreases ranging from 33% to 24.4% for different membrane thickness. The increase in the current density results in the voltage lost and therefore the heat and power output of the cell decreases.

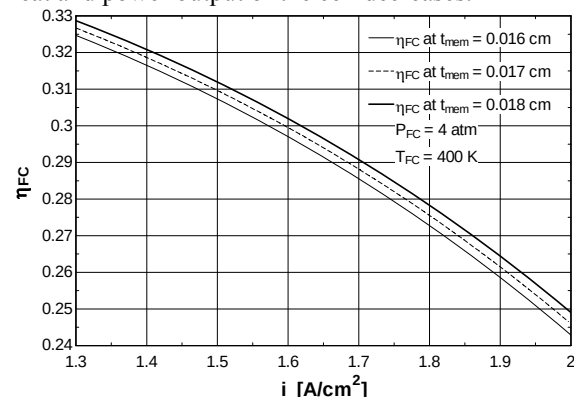

Fig. 11.Variation of the efficiency with the current density

CONCLUSION

In this paper, we have developed and analyzed a PEM fuel cell integrated with double-effect absorption-cooling system to evaluate the performance of fuel cell through studying the effect of several parameters such as temperature, membrane thickness, and current density on the fuel cell efficiency, heat and power output from the fuel cell, the COPs, and utilization factor. Also, parametric study is carried out in order to observe the effect of partial loads for varying heat input into the HTG on the COPs and utilization factor. However, for the case based on the present results, the following conclusions may be drawn:

- The efficiency of the fuel cell decreases with the increase in the membrane thickness and current density. The decrease range from 46.2% to 35.4%, and 33% to 24.4%, respectively.
- The efficiency of the cell increases, ranging from 35% to 46% with increase in the temperature of the cell for different fixed membrane thickness.
- The heat and power output of the fuel cell is found to be decreasing, ranging from 10.54 kW to 5.12 kW, and 9.12 kW to 6.99 kW, respectively for the increase in the membrane thickness for different fixed fuel cell temperature.
- For varying fuel cell temperature and different fixed membrane thickness the heat and power output increase, ranging from 5.12 kW to 10.54 kW, and 6.9 kW to 7.02 kW, respectively.
- In case of increase in membrane thickness the energetic and exergetic COP are found to be increasing, ranging

from 0.97 to 1.52 and 0.65 to 1.25, respectivel.

- However, the energetic and exergetic COP are found to be decreasing with increases in temperature of the cell and heat input to the HTG. This decrease range from 1.53 to 0.97, and 1.26 t 0.73, respectively for increase in temperature of the cell and from 1.19 to 0.48 and 0.89 to 0.33, respectively for increase in heat input to the HTG.

- The utilization factor of the system is found to be increasing, ranging from 17% to 87% with increase in temperature of the cell and heat input to the HTG.

The procedure developed and applied in this paper may be extended to other integrated absorption systems and is expected to improve the understanding of such systems and be useful to the designers. It is also hoped that the analysis presented in this paper lays a groundwork that may easily be extended to other integrated multiple-effect absorption systems.

REFERENCES

[1] R. K. Alhuwalia, X. Wang, ”Fuel cell systems for transportation: Status and trends,” *Journal of Power Sources*, vol. 177, pp. 167-176, Oct. 2007.

[2] G. Karimi, J. J. Baschuk, X. Li, “ Performance analysis and optimization of PEM fuel cell stacks using flow network approach,” *Journal of Power Sources*, vol. 147, pp. 162-177, March 2005.

[3] I. Dincer, and S. Dost, “Energy Analysis of an Ammonia-Water Absorption Refrigeration System”, *Energy Sources*, vol. 18, 727-733, 1996.

[4] J.I. San Martín, I. Zamora , J.J. San Martín, V. Aperribay, P. Eguía, “ Trigeneration Systems with Fuel Cells,” Dept. Elec. Eng., Univ. of the Basque Country, Spain, Sci. Rep.

[5] M. M. Hussain, J. J. Baschuk, X. Li, and I. Dincer, “Thermodynamic analysis of a PEM fuel cell power system,” *International Journal of Thermal Sciences*, vol. 44,pp.903-911, Apr. 2005.

[6] M. Ay, A. Midilli, and I. Dincer, “Exergetic performance analysis of a PEM fuel cell,” *International Journal of Energy Research*, vol. 30, pp.307-321, Aug. 2005.

[7] M. Ay, A. Midilli, and I. Dincer, “Thermodynamic modeling of a proton exchange membrane fuel cell”. *International Journal of Exergy*, vol. 3, 2006

[8] J.M. Olivares-Ramı´rez, R.H. Castellanos, A´. Marroquı´n de Jesu´s, E. Borja-Arco, R.C. Pless, “Design and development of a refrigeration system energized with hydrogen produced from scrap aluminum,” *International Journal of Hydrogen Energy*, vol. 33, pp.2620-2626, Aug. 2004.

[9] G. Gigliucci, L. Petruzzi, E. Cerelli, A. Garzisi, A. La Mendola, “ Demonstration of a residential CHP system based on PEM fuel cells,” *Journal of Power Sources*, vol. 131, pp.62-68, 2004.

[10] I. Pilatowskya, R.J. Romerob, C.A. Isazac, S.A. Gamboaa, W. Riveraa, P.J. Sebastiana,d, J. Moreirad, “ Simulation of an air conditioning absorption refrigeration system in aco-generation process combining a proton exchange membrane fuel cell,” *Intenational Journal of Hydrogen Energy*, vol. 32, pp.3174-3182, April 2007.

[11] M. A. Darwish, “Building air conditioning system using fuel cell: Case study for Kuwait,” *Applied Thermal Engineering*, vol. 27, pp.2869-2876, May 2007.

Table 1. Energy and exergy equations for the system and its components as well as both energetic and exergeti COPs

Component/ System	Mass, Enegry, Entropy, and Exergy Balance Equations
High Temperature Generator	$\dot{Q}_{MTG} = \dot{W}_{FC} + \dot{Q}_{FC}$ $s_{7,ammonia} = \mathbf{s}(\text{'Ammonia'}, P = P_7, h = h_7)$ $s_{7,water} = \mathbf{s}(\text{'Water'}, P = P_7, h = h_7)$ $s_7 = x_7 \cdot s_{7,ammonia} + (1 - x_7) \cdot s_{7,water}$ $h_{19,ammonia} = \mathbf{h}(\text{'Ammonia'}, T = T_{19}, P = P_{19})$ $h_{19,water} = \mathbf{h}(\text{'Water'}, T = T_{19}, P = P_{19})$ $h_{19} = x_{19} \cdot h_{19,ammonia} + (1 - x_{19}) \cdot h_{19,water}$ $h_{18,ammonia} = \mathbf{h}(\text{'Ammonia'}, T = T_{18}, P = P_{19})$ $h_{18,water} = \mathbf{h}(\text{'Water'}, T = T_{18}, P = P_{19})$ $h_{18} = x_{18} \cdot h_{18,ammonia} + (1 - x_{18}) \cdot h_{18,water}$ $\dot{m}_{18} \cdot x_{18} = \dot{m}_{19} \cdot x_{19} + \dot{m}_7 \cdot x_7$ $\dot{m}_{18} = \dot{m}_{19} + \dot{m}_7$ $\dot{m}_{18} \cdot h_{18} + \dot{Q}_{MTG} = \dot{m}_7 \cdot h_7 + \dot{m}_{19} \cdot h_{19}$ $\dot{Ex}_{MTG,th} = \left[1 - \frac{T_0}{T_{MTG}}\right] \cdot \dot{Q}_{MTG}$
High Temperature Heat Exchanger	$\dot{m}_{24} \cdot h_{24} + \dot{Q}_{mhx} = \dot{m}_{25} \cdot h_{25}$ $\dot{m}_{12a} \cdot h_{12a} = \dot{m}_{17} \cdot h_{17} + \dot{Q}_{mhx}$
Absorber	$s_{28} = x_{28} \cdot s_{28,ammonia} + (1 - x_{28}) \cdot s_{28,water}$ $h_{11} = x_{11} \cdot h_{11,ammonia} + (1 - x_{11}) \cdot h_{11,water}$ $\dot{m}_1 \cdot h_1 + \dot{m}_{28} \cdot h_{28} = \dot{m}_{11} \cdot h_{11} + \dot{Q}_{abs}$ $\dot{Ex}_{11} = \dot{m}_{11} \cdot (h_{11} - h_0 - T_0 \cdot (s_{11} - s_0))$ $\dot{Ex}_{28} = \dot{m}_{28} \cdot (h_{28} - h_0 - T_0 \cdot (s_{28} - s_0))$

Evaporator	$h_1 = x_1 \cdot h_{1,ammonia} + (1 - x_1) \cdot h_{1,water}$ $s_1 = x_1 \cdot s_{1,ammonia} + (1 - x_1) \cdot s_{1,water}$ $s_{2a} = x_{2a} \cdot s_{2a,ammonia} + (1 - x_{2a}) \cdot s_{2a,water}$ $\dot{m}_{2a} \cdot h_{2a} + \dot{Q}_{eva} = \dot{m}_1 \cdot h_1$ $\dot{m}_{2a} \cdot s_{2a} + \dot{S}_{gen,eva} = \dot{m}_1 \cdot s_1 + \frac{\dot{Q}_{eva}}{T_0}$ $\dot{E}x_{dest,eva} = T_0 \cdot \dot{S}_{gen,eva}$ $\dot{E}x_{2a} = \dot{m}_{2a} \cdot (h_{2a} - h_0 - T_0 \cdot (s_{2a} - s_0))$ $\dot{E}x_1 = \dot{m}_1 \cdot (h_1 - h_0 - T_0 \cdot (s_1 - s_0))$ $\dot{E}x_{eva,th} = \left[1 - \frac{T_{eva}}{T_0}\right] \cdot \dot{Q}_{eva}$
Condenser	$\dot{m}_6 + \dot{m}_3 = \dot{m}_2$ $\dot{m}_6 \cdot h_6 + \dot{m}_3 \cdot h_3 = \dot{m}_2 \cdot h_2 + \dot{Q}_{con}$ $\dot{m}_6 \cdot s_6 + \dot{m}_3 \cdot s_3 = \dot{m}_2 \cdot s_2 + \frac{\dot{Q}_{con}}{T_0} + \dot{S}_{gen,cond}$ $\dot{E}x_{dest,cond} = T_0 \cdot \dot{S}_{gen,cond}$ $\dot{E}x_6 = \dot{m}_6 \cdot (h_6 - h_0 - T_0 \cdot (s_6 - s_0))$ $\dot{E}x_3 = \dot{m}_3 \cdot (h_3 - h_0 - T_0 \cdot (s_3 - s_0))$ $\dot{E}x_2 = \dot{m}_2 \cdot (h_2 - h_0 - T_0 \cdot (s_2 - s_0))$
Pump	$\dot{m}_{11} = \frac{\dot{W}_p}{h_{11a} - h_{11}}$ $\dot{E}x_{11a} = \dot{m}_{11a} \cdot (h_{11a} - h_0 - T_0 \cdot (s_{11a} - s_0))$
Fuel Cell	$x_{H,2} = \frac{1 - x_{H,2,O,A}}{1 + \frac{x_A}{2} \cdot \left[1 + \frac{\zeta_A}{\zeta_A - 1}\right]}$ $x_{O,2} = \frac{1 - x_{H,2,O,C}}{1 + \frac{x_C}{2} \cdot \left[1 + \frac{\zeta_C}{\zeta_C - 1}\right]}$ $x_{H,2,O,A} = \frac{P_{sat}}{P_A}$

$$x_{H,2,O,C} = \frac{P_{sat}}{P_C}$$

$$P_{sat} = 10^{(-2.1794 + 0.02953 \cdot T - 9.1837 \cdot 10^{-5} \cdot T^2 + 1.4454 \cdot 10^{-7} \cdot T^3)}$$

$$a_l = x_{H,2,O} \cdot \frac{P}{P_{sat}}$$

$$\lambda_{mem} = 0.043 + 17.81 \,.\, a_l - 39.85 \,.\, a_l^2 - 39.85 \,.\, a_l^3$$

$$\beta_1 = (8.66 \cdot 10^{-5} \cdot T_{FC} - 0.068) \cdot \left[\frac{p_{O,2}}{0.1173} + P_{sat}\right] - 1.6 \cdot 10^{-4} \cdot T_{FC} + 0.54$$

$$\dot{W}_{FC,i} = i \cdot (1.229 - 8.5 \cdot 10^{-4} \cdot (T_{FC} - 298.15) + 4.3085 \cdot 10^{-5} \cdot T_{FC} \cdot (\ln(p_{H,2}) + 0.5 \cdot \ln(p_{O,2}))) + F + g + d$$

$$F = -i \cdot \left[\frac{\alpha_A + \alpha_C}{\alpha_A \cdot \alpha_C}\right] \cdot \frac{R \cdot T_{FC}}{n \cdot F} \cdot \ln\left[\frac{i}{i_0}\right]$$

$$g = -i \cdot \frac{i \cdot \left[(0.005139 \cdot lamda_{mem} - 0.00326) \cdot \exp\left(1268 \cdot \left[\frac{1}{303} - \frac{1}{T_{FC}}\right]\right)\right]}{t_{mem}}$$

$$d = -i \cdot i \cdot \left[\beta_1 \cdot \frac{i}{i_{max}}\right]^{\beta_2}$$

$$\dot{n}_{H,2,i} = \frac{i}{2 \cdot F} \cdot \left|0.001 \cdot \frac{kmol/s}{mol/s}\right|$$

$$\dot{n}_{O,2,i} = \frac{i}{4 \cdot F} \cdot \left|0.001 \cdot \frac{kmol/s}{mol/s}\right|$$

$$\dot{m}_{H,2,O,o} = MW_{H,2,O} \cdot \dot{n}_{H,2,O,o}$$

$$\dot{m}_{H,2,reacted} = MW_{H,2} \cdot \dot{n}_{H,2,reacted}$$

$$\dot{m}_{O,2,reacted} = MW_{O,2} \cdot \dot{n}_{O,2,reacted}$$

$$\dot{m}_{H,2,i} = \dot{n}_{H,2,i} \cdot MW_{H,2}$$

$$\dot{m}_{O,2,i} = \dot{n}_{O,2,i} \cdot MW_{O,2}$$

$$\dot{m}_{H,2,i} = \dot{m}_{H,2,reacted} + \dot{m}_{H,2,o}$$

$$\dot{m}_{O,2,i} = \dot{m}_{O,2,reacted} + \dot{m}_{O,2,o}$$

$$\dot{Q}_{FC} = \left[\frac{T_0 \cdot (\dot{m}_{H,2,o} \cdot s_{H,2,o} + \dot{m}_{H,2,O,o} \cdot s_{H,2,O,o} - \dot{m}_{H,2,i} \cdot s_{H,2,i} - \dot{m}_{O,2,i} \cdot s_{O,2,i}) + \dot{W}_{FC,m} + x}{b}\right] \cdot \left|0.001 \cdot \frac{kW}{W}\right|$$

$$x = \dot{m}_{H,2,o} \cdot Ex_{H,2,o} + \dot{m}_{H,2,O,o} \cdot Ex_{H,2,O,o} - \dot{m}_{H,2,i} \cdot Ex_{H,2,i} - \dot{m}_{O,2,i} \cdot Ex_{O,2,i}$$

$$b = r_{HL} + (1 - r_{HL}) \cdot \frac{T_0}{T_{FC}}$$

$$\dot{E}_{req} = \dot{W}_{FC} + \dot{Q}_{FC} - \dot{Q}_{HTG} - \dot{W}_p$$

Energetic and Exergetic COPs	
COP	$COP_{En} = \frac{\dot{Q}_{eva}}{\dot{Q}_{HTG} + \dot{W}_p}$ $COP_{Ex} = \frac{\dot{Ex}_{eva,th}}{\dot{Ex}_{HTG,th}}$
Utilization Factor and Efficiency	$\varepsilon = \frac{\dot{Q}_{eva} + \dot{Q}_{con}}{\dot{n}_{H,2,i} \cdot 286000}$ $\eta_{FC} = 0.95 \cdot \frac{\dot{W}_{FC,i}}{i \cdot 1.25}$