NOTES 5. APPENDIX A

Physical interpretation of bearing forces during circular whirl motions

The bearing dynamic reaction forces, radial and tangential, add the stiffness ("elastic") and ("viscous") damping forces, i.e.

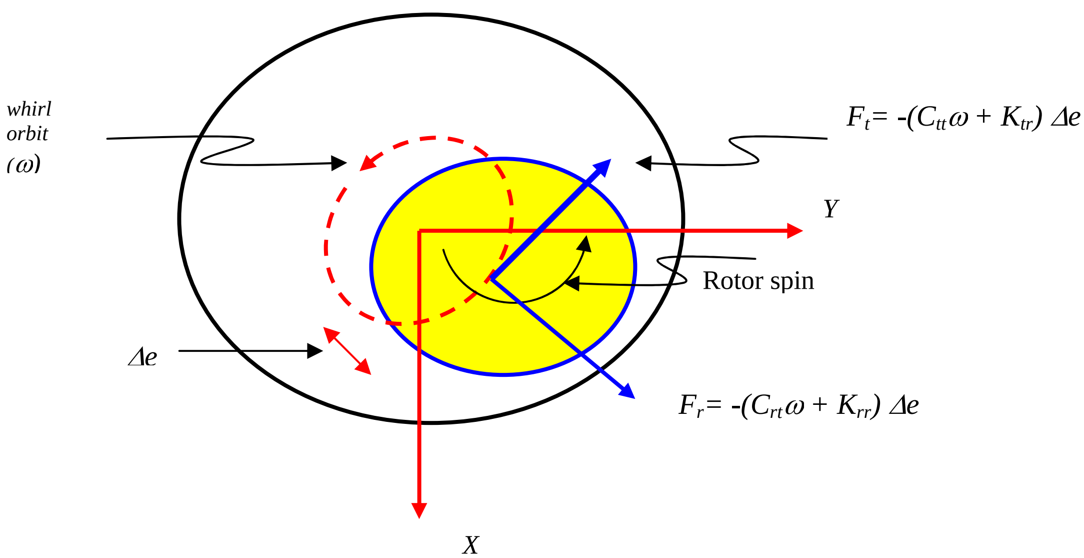
$$\begin{pmatrix} F_r \\ F_t \end{pmatrix}_d = -\begin{bmatrix} K_{rr} & K_{rt} \\ K_{tr} & K_{tt} \end{bmatrix}\begin{pmatrix} \Delta e \\ e_0 \Delta\phi \end{pmatrix} - \begin{bmatrix} C_{rr} & C_{tr} \\ C_{tr} & C_{tt} \end{bmatrix}\begin{pmatrix} \Delta \dot{e} \\ e_0 \Delta\dot{\phi} \end{pmatrix} \tag{5.A.1}$$

For circular journal motions of amplitude Δe around the bearing center position $(e_0=0)$, the radial and tangential journal center velocities are $\Delta\dot{e}=0,\ e_0\Delta\dot{\phi}=\Delta e\,\omega$, with ω as the frequency of whirl motion and in the same direction as the rotor spin speed (Ω).

At the centered position, the short cylindrical bearing force coefficients are given by

$$K_{rr} = K_{tt} = C_{rt} = C_{tr} = 0$$

$$\overline{K} = K_{rt} = -K_{tr} = \frac{\Omega}{2}\overline{C}; \quad \overline{C} = C_{tt} = C_{rr} = \frac{\mu\, R L^3}{C^3}\frac{\pi}{2} \tag{5.A.2}$$



Force diagram for (forward) circular whirl motions

And thus, the bearing radial and tangential force components become

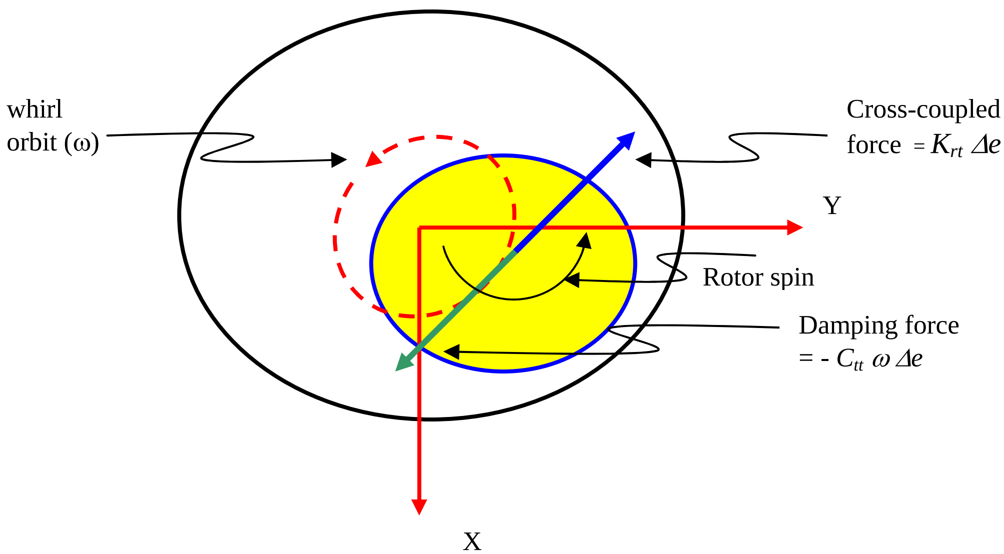
$$\begin{pmatrix} F_r \\ F_t \end{pmatrix}_d = -\begin{bmatrix} 0 & K_{rt} \\ -K_{rt} & 0 \end{bmatrix}\begin{pmatrix} \Delta e \\ 0 \end{pmatrix} - \begin{bmatrix} C_{tt} & 0 \\ 0 & C_{tt} \end{bmatrix}\begin{pmatrix} 0 \\ \Delta e\ \omega \end{pmatrix} = -\begin{pmatrix} 0 \\ C_{tt}\Delta e\ \omega - K_{rt}\ \Delta e \end{pmatrix} \tag{5.A.3}$$

$$F_{r_d} = 0;\ F_{t_d} = -(C_{tt}\,\omega - K_{rt})\,\Delta e \tag{5.A.4}$$

Hence, the radial (centering) force is nil. The tangential force could be destabilizing or stabilizing, depending on its sign. A destabilizing force, also known as a follower force, will drive the journal in the direction of the whirl motion, i.e. $F_t > 0$. For this condition to occur, the equivalent damping coefficient (C_{eq}) for the forward whirl orbit must be negative, i.e.

$$(C_{tt} - \frac{1}{\omega} K_{rt}) = C_{eq} < 0 \tag{5.A.5}$$

At the threshold speed of instability $K_{rt} = \frac{\Omega}{2} C_{tt}$. Thus, unstable forward whirl motions $(C_{eq}<0)$ occur for rotor speeds $\Omega \geq 2\omega_s$, i.e. at twice the natural frequency of the rotor bearing system[1].



Forces driving and retarding whirl motion

In the (X,Y) coordinate system, $\Delta X = \Delta e \cos(\omega t)$ and $\Delta Y = \Delta e \sin(\omega t)$. Thus, the bearing forces are

[1] Note that backward whirl motions (ω<0) would be stable. Backward whirl motions are relatively rare since the external forcing mechanisms to excite them are not present in most rotor-fluid film bearing systems. This is not the case for conditions of rotor rubbing, internal friction and some instances of intermittent contact with rolling bearing elements.

Recall that $K_{XY} = -K_{YX} = \frac{\Omega}{2} C_{XX} = \frac{\Omega}{2} C_{YY}$, and $K_{XX}, K_{YY} = C_{XY} = C_{YX} = 0$. Then

$$\begin{pmatrix} F_X \\ F_Y \end{pmatrix}_d = -\begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix}\begin{pmatrix} \Delta X \\ \Delta Y \end{pmatrix} - \begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix}\begin{pmatrix} \Delta \dot{X} \\ \Delta \dot{Y} \end{pmatrix} = -\begin{bmatrix} 0 & K_{XY} \\ -K_{XY} & 0 \end{bmatrix}\begin{pmatrix} \Delta X \\ \Delta Y \end{pmatrix} - \begin{bmatrix} C_{XX} & 0 \\ 0 & C_{YY} \end{bmatrix}\begin{pmatrix} \Delta \dot{X} \\ \Delta \dot{Y} \end{pmatrix}$$

$$\begin{pmatrix} F_X \\ F_Y \end{pmatrix}_d = -\begin{pmatrix} +K_{XY}\Delta Y + C_{XX}\,\Delta \dot{X} \\ -K_{XY}\Delta Y + C_{YY}\,\Delta \dot{Y} \end{pmatrix} = -\begin{pmatrix} \left(K_{XY} - C_{XX}\,\omega\right)\sin(\omega t) \\ \left(-K_{XY} + C_{YY}\,\omega\right)\cos(\omega t) \end{pmatrix}\Delta e$$

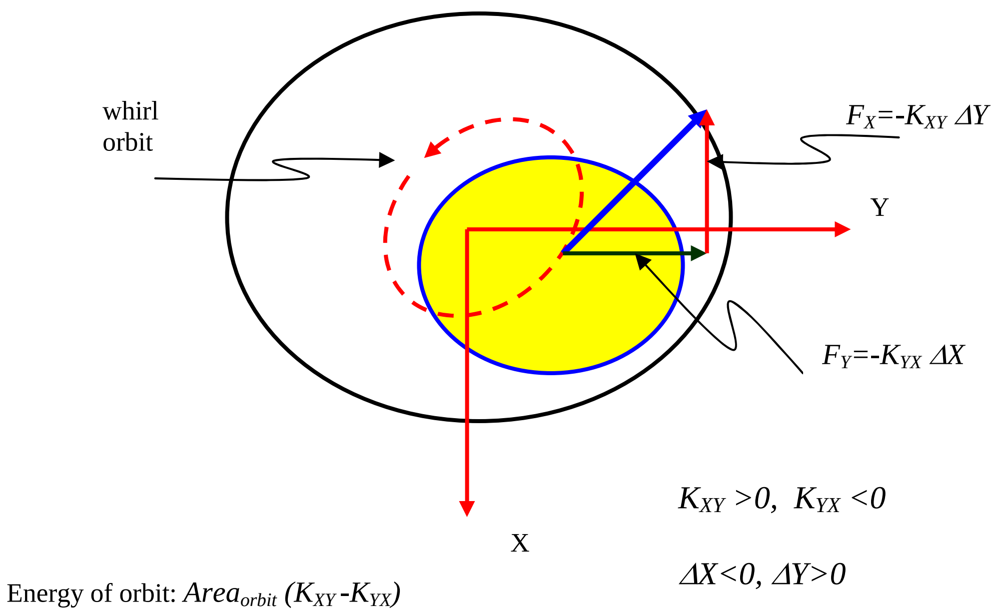
(5.A.6)

$$\begin{pmatrix} F_X \\ F_Y \end{pmatrix}_d = -\begin{pmatrix} +\sin(\omega t) \\ -\cos(\omega t) \end{pmatrix}\left(\frac{\Omega}{2} - \omega\right) C_{XX}\,\Delta e = C_{XX}\left(1 - \frac{\Omega}{2\,\omega}\right)\begin{pmatrix} +\sin(\omega t) \\ -\cos(\omega t) \end{pmatrix}\omega\,\Delta e =$$

(5.A.7)

$$\begin{pmatrix} F_X \\ F_Y \end{pmatrix}_d = -C_{XX}\left(1 - \frac{\Omega}{2\,\omega}\right)\begin{pmatrix} \Delta \dot{X} \\ \Delta \dot{Y} \end{pmatrix}$$

Note that *(F_X, F_Y)* oppose the forward whirl motion for journal speeds $\Omega < 2\omega_s$. For larger rotor speeds, the forces become positive and aid to the growth of the forward whirl amplitude of motion.



Representation of cross-coupled forces

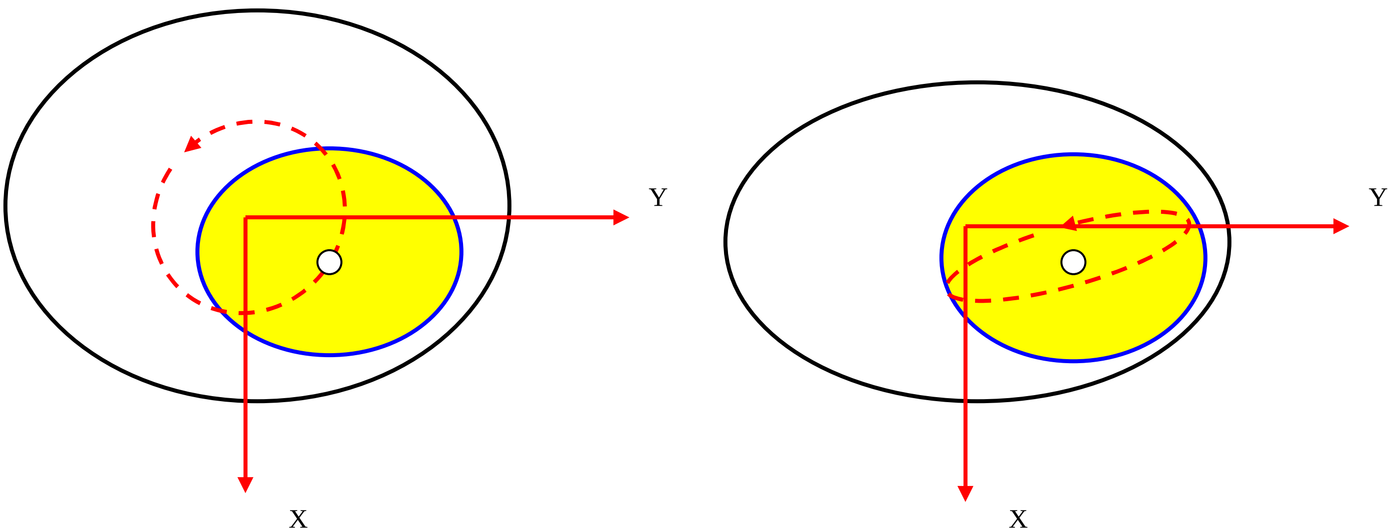
The work performed by the bearing forces during a full period of motion *($T=2\pi/\omega$)* equals

$$E = \oint \left(F_{X_d} \; \Delta\dot{X}_{(t)} + F_{Y_d} \; \Delta\dot{Y}_{(t)} \right) dt = \oint \left(F_t \, \omega \, \Delta e \right) dt = -\left(C_{tt} \; \omega - K_{rt} \right) \omega \, \Delta e^2 \; T =$$

$$E = -\left(2\pi \, \Delta e^2 \right) \left(C_{tt} \; \omega - K_{rt} \right) = -2 \, Area_{orbit} \, C_{eq} \; \omega \tag{5.A.8}$$

Note that E<0 is equivalent to negative work, i.e. energy removed or dissipated from the rotor-bearing system. However when E>0, i.e. for $\Omega \geq 2\omega_s$, the fluid film bearing adds "energy" into the rotor-bearing system thus driving the whirl motion forward.

From this discussion it is easy to deduce that rotor-bearings evidencing whirl orbits with skewed areas (sharp ellipsoids) are less prone to rotordynamic instability. This type of dynamic response is obtained by design of (direct) stiffness asymmetry as given in bearing configurations (elliptic, multiple-lobe with preloads, pressure-dam bearings). However, these bearings are limited to fixed orientation static loads and rotor spin in only one direction.

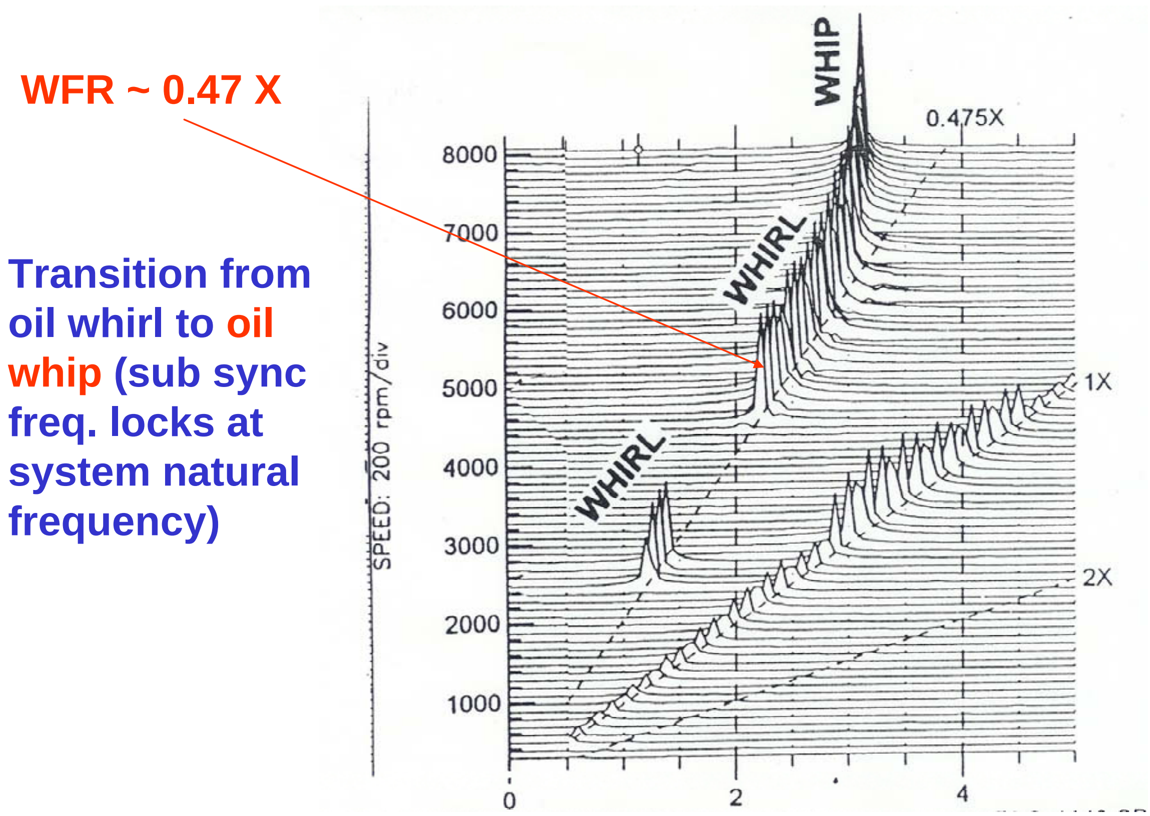


Work from *elastic* forces = $Area_{orbit}\ \ (K_{XY} - K_{YX})$

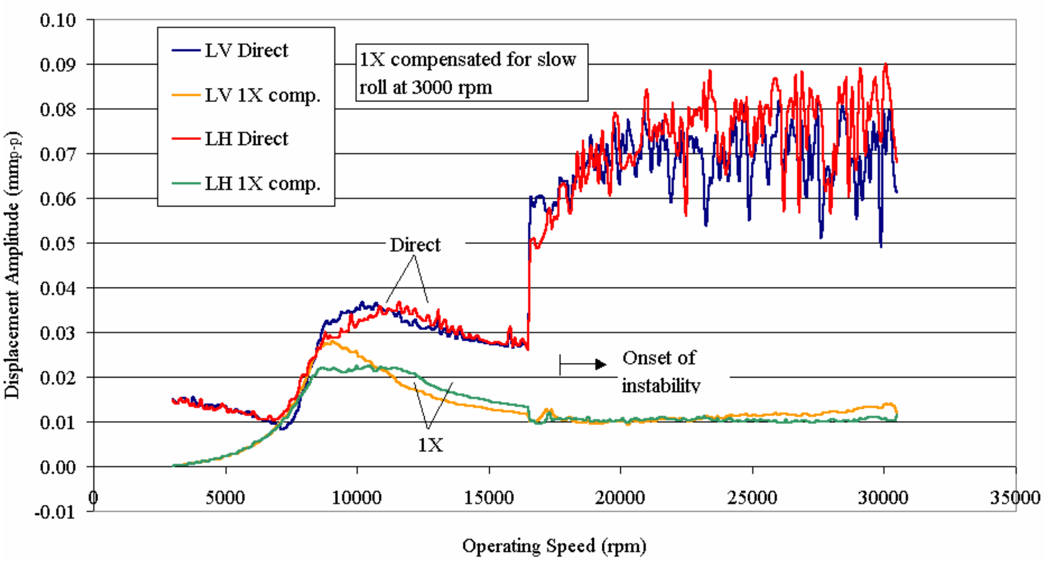
Influence of bearing asymmetry on whirl orbit

Field and experimental evidence of rotor-bearing system instability

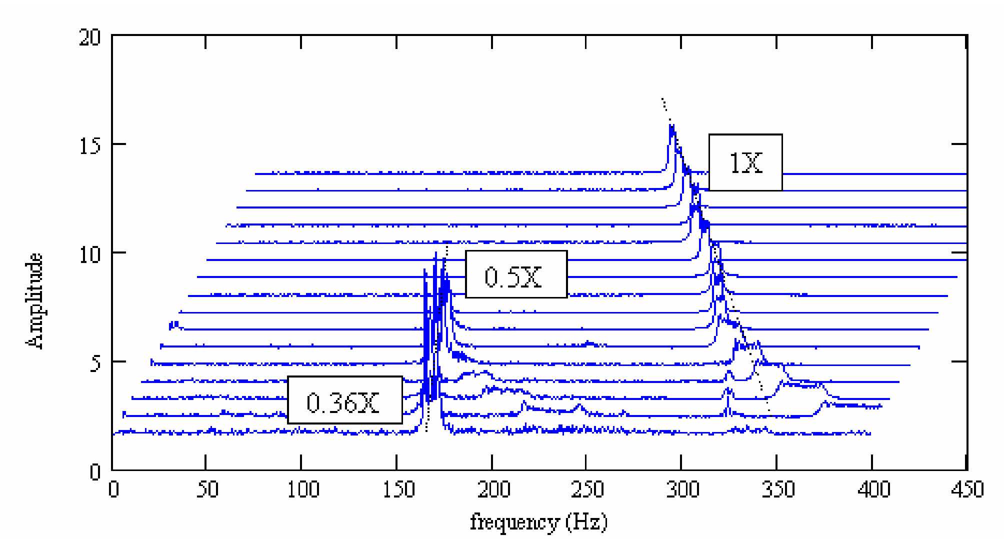
The archival literature is abundant in experimental and field descriptions of severe instabilities induced by fluid film bearings on rotating machinery.



As an example of tests conducted at the author's laboratory on a high speed test rig, the figure below depicts recorded amplitudes of motion versus shaft speed on a rigid rotor supported on plain journal bearings. The displacement measurements correspond to rotor motions along the vertical and horizontal planes (LV, LH). The curves with larger amplitudes denote the total amplitudes of motion while the others in light color show the filtered synchronous (1X) motions with slow roll compensation. The passage through a well-damped critical speed is evident at ~ 8.5 krpm. As the shaft speed increases, the amplitudes of motion decrease. However, at a shaft speed ~ twice the critical speed, the rotor becomes violently unstable with large amplitude motions nearly equaling the bearings' clearances. The second figure depicts the waterfall of the vertical shaft motion. The graph shows the frequency content of the vibration signal as the rotor accelerates. The synchronous motions are denoted by the 1X line. The whirl frequency ratio is 0.50 at the onset of the severe subsynchronous motions. As the speed increases, the whirl frequency locks at the system natural frequency. This phenomenon is known as oil-whip. The rotor was severely damaged upon completion of the experiment.

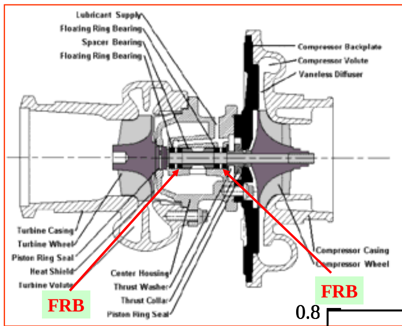


Amplitudes of rotor motion versus shaft speed. Experimental evidence of rotordynamic instability



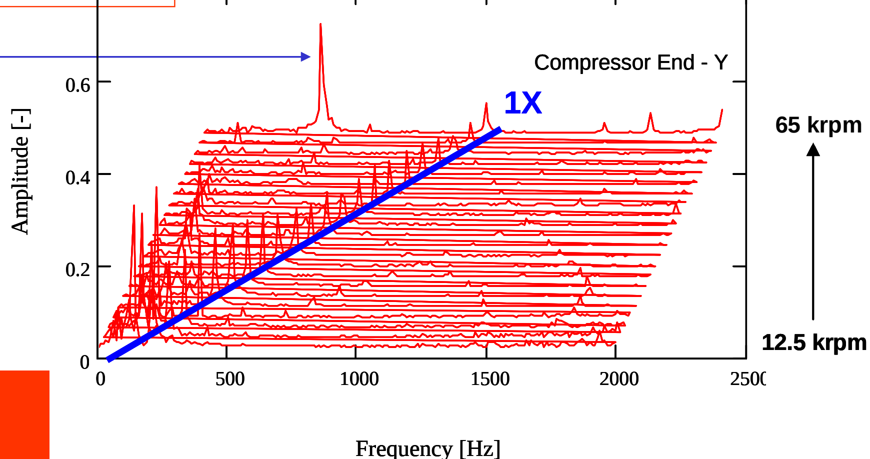
Waterfall of recorded rotor motion demonstrating subsynchronous whirl

Other measurements – HIGH SPEED Turbochargers

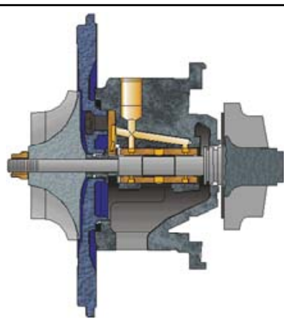


TC supported on floating ring bearings

WFR ~ 0.50 X

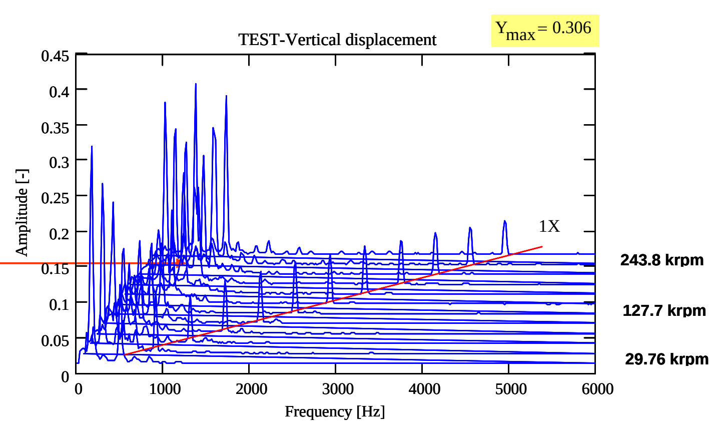


Automotive Turbocharger



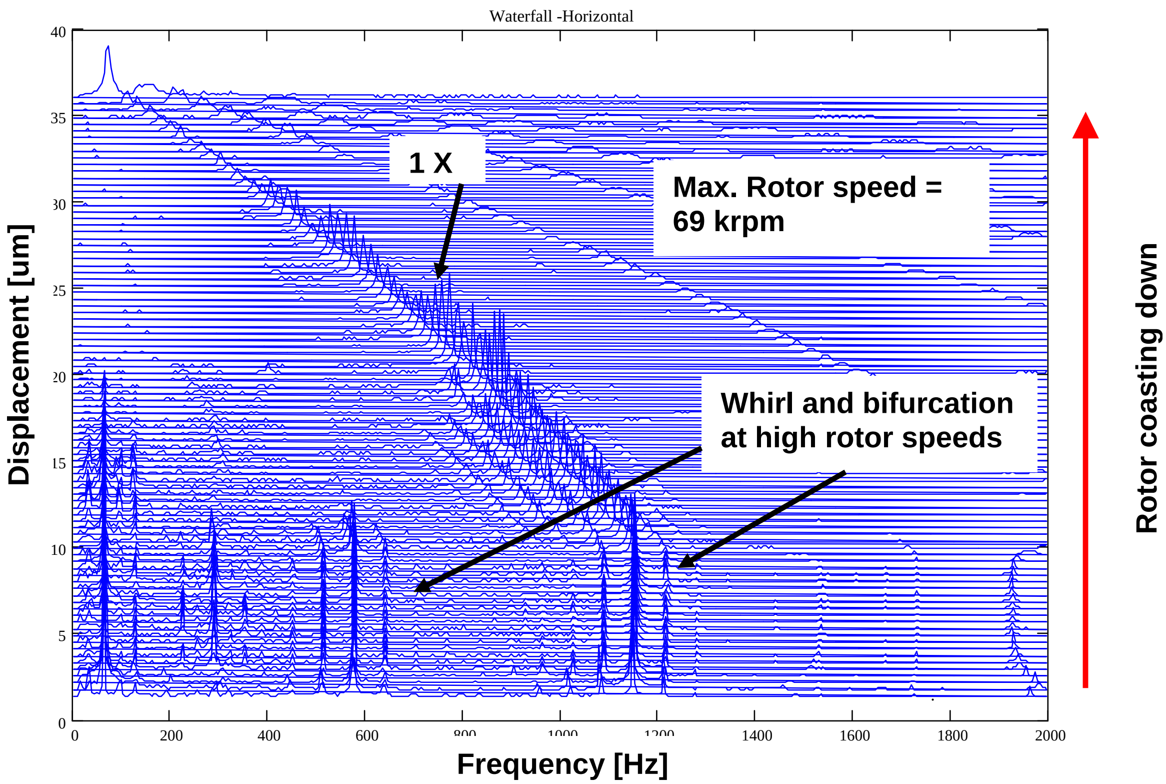
TC supported on semi-floating ring bearings

Multiple sub-synchronous motions



Automotive Turbocharger

Other measurements – HIGH SPEED foil bearing



Waterfall plot of test foil bearing motions. Floating bearing on a journal driven by automotive turbocharger. Rotor coast down from 69 krpm, net load of 16 N acting vertically ups

The following pages show measurements of shaft motion conducted on a rotor-kit – flexible rotor supported on plain journal bearing.

The graphs show Bode plots (amplitude of response versus shaft speed) for TWO static loads (small and large) applied to the rotor near the journal bearing location. The Table shows the recorded threshold speed of instability and whirl frequency ratio of subsynchronous motion. Note that since the rotor is flexible, the whirl frequency corresponds with the natural frequency of the flexible rotor. The waterfalls show the severity of the subsynchronous motions – oil whip – for the unloaded condition.

More discussion in class!