A Thermal Sensation Index for Real-Time Tuning and Energy-Optimal Control of HVAC Systems

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Abstract

In this paper we discuss and develop a thermal comfort index that addresses the limitations of applying thermal comfort indices to control applications. The derivation closely follows the derivation of PMV, but certain changes and simplifications make the index an explicit, linearly parameterized function of environmental variables. We show that the differences between the derivation of this index and the derivation of PMV do not reduce the accuracy of the index in comparison to PMV.

Since this index is linearly parameterized, the parameters can be quickly and efficiently tuned in real time to reflect the thermal sensation of the specific occupant. Parameter tuning makes it possible to accurately predict the thermal sensation of the occupant without exact knowledge of the activity level or clothing insulation of the occupant when these two quantities are known to be constant. Additionally, the tuning process makes the thermal sensation prediction relatively insensitive to sensor location because biases and scaling errors are absorbed by the estimated parameters. Real-time parameter tuning is demonstrated experimentally for a seated, stationary occupant.

The feasibility of using variable air flow and variable heat flow to regulate the thermal sensation index in a way that minimizes power consumption is investigated. The simplified index provides a quantitative means for determining the most energy efficient comfortable conditions. The analysis demonstrates that for low to moderate outdoor relative humidity there is an energy optimal combination of air flow and heat flow.

1 Introduction

Thermal comfort indices have been used for decades for the analysis of indoor climates and the design of HVAC systems. The two most comprehensive and well-known comfort indices are the Predicted Mean Vote (PMV) developed in [4], and the Effective Temperature (ET*) developed in [8]. Recently, it has been shown that controllers that directly regulate a thermal comfort index have advantages over the conventional thermostatic controller. Such controllers have been proposed in [17, 13, 22]. The advantages of directly regulating a thermal comfort index rather than air temperature alone are increased comfort with the possibility of energy savings. However, PMV and ET* were developed for the purpose of analysis of indoor climates, not feedback control, so controllers based directly on PMV or ET* suffer from certain limitations.

Thermal comfort indices are based on the statistical average of the traits of a large population. However, not all occupants are alike, and the designer of an HVAC system cannot exactly know the traits of the specific occupants during the design stage. While it was shown in [5] that inter individual differences in the desired ambient temperature are relatively small when all other variables affecting comfort are held constant, it was also shown in [4, 19] that the sensitivity to perturbations from the desired condition is relatively large. In [18], the sensitivity of thermal sensation to a perturbation in air temperature was shown to be twice as large for women as it was for men. Since many HVAC systems are designed to perturb the indoor climate from the desired conditions (i.e., on-off control) and since it is desirable to know the effect of perturbations from the desired conditions for evaluating the tradeoff between comfort and energy savings, it is important that the control system is based on a comfort index that accurately reflects both the desired conditions and the perturbations from the desired conditions for the specific occupants.

A second limitation of controllers based directly on PMV or ET* is that neither PMV nor ET* is an explicit function of the six variables that affect thermal comfort. Calculation of PMV and ET* requires an iterative solution. For feedback control applications, the requirement of iterative solutions is at odds with nearly all control system design methods, which require an explicit input-output relationship. Furthermore, iterative solutions introduce a computational burden that may not be suitable for feedback control systems.

Another limitation of controllers based on PMV or ET* is that the clothing insulation and activity level (i.e., rate of bodily heat produced) must be known. While these two factors can be estimated based on the type of space for which the HVAC system is being designed, they are never exactly known.

Finally, to compute a value of a comfort index such as PMV or ET*, values of the environmental variables must be measured near the occupant. In most practical applications, this is not possible.

In this paper, we develop a thermal sensation index for the application of feedback control. This index directly addresses the four limitations of PMV-based or ET*-based controllers described above. The derivation closely follows the derivation of PMV, but certain changes, simplifications, and assumptions result in a PMV-like index that is an explicit, linearly parameterized function of environmental variables. We show experimentally that the differences between the derivation of this index and the derivation of PMV do not significantly reduce the accuracy of the index in comparison to PMV. Since this index is linearly parameterized, the parameters can be quickly and efficiently tuned in real time from thermal sensation ratings provided by the occupant.

In [17, 22] it was shown that PMV-based controllers can offer energy savings over the traditional thermostatic control methodology. In this paper, we analyze the feasibility of energy savings for systems with variable air flow and variable heat flow. Based on a single-zone model, we show that at low to moderate outdoor humidity, there is an energy-optimal combination of air flow and heat flow that will maintain thermally neutral conditions.

In the next section, we develop the main result of this paper, which is the derivation of a PMV-like thermal sensation index. In the following sections, we demonstrate the applicability of this simplified index to the real-time tuning and energy-optimal control of HVAC systems.

2 Thermal Sensation Index

This section describes the critical points of the derivation of the simplified index. The details of the derivation are omitted. The interested reader is referred to the paper by Federspiel and Asada (1991) for the details of the derivation. Since the derivation of this index closely follows the derivation of PMV, we include a brief summary of PMV in this section.

2.1 PMV

The PMV index is a prediction of the average thermal sensation rating of the members of a large population at steady-state conditions. The PMV index is based on the steady-state heat balance between a clothed human and the environment, shown schematically in Figure 1, and the seven-point psycho-physical rating scale shown below.

$+3 \Leftrightarrow$ hot
$+2 \Leftrightarrow$ warm
$+1 \Leftrightarrow$ slightly warm
$0 \Leftrightarrow$ neutral or comfortable
$-1 \Leftrightarrow$ slightly cool
$-2 \Leftrightarrow$ cool
$-3 \Leftrightarrow$ cold

The heat balance and the rating scale are related empirically from a large quantity of experimental data. Since PMV is based on the heat balance between a clothed human and the environment, it is dependent on all six variables that affect the heat balance: bodily heat production, clothing insulation, air temperature, mean radiant temperature, humidity and air velocity.

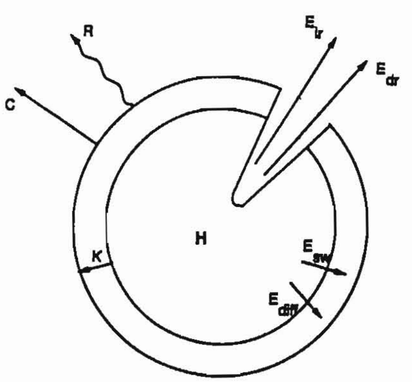

Figure 1: Single-state lumped-parameter model of the heat balance between a clothed human and the environment. H is the internal heat production, E_{lr} is the evaporative heat loss from the lungs, E_{dr} is the convective heat loss from the lungs, E_{sw} is the heat loss due to sweating, E_{diff} is the heat loss due to diffusion of vapor through the skin, K is the heat loss through the clothing, C is the convective heat loss from the clothing, and R is the radiative heat loss from the clothing.

The accuracy of PMV has been criticized because it does not account for skin wettedness [11] or moisture permeability of the clothing [2]. It is therefore believed that PMV is not accurate in hot and humid environments where sweating dominates the human thermoregulatory response. However, PMV provides an accurate assessment of moderate climates and has been used in an ISO standard for evaluating moderate climates [12]. Therefore, PMV can provide an accurate assessment of controlled indoor climates even when the outdoor climate is hot and humid.

2.2 Derivation

While the derivation of the simplified index is similar to that of PMV, there are four key differences in the derivation of the simplified index that lead to great mathematical simplification.

1. In the derivation of PMV, the Stefan-Boltzmann law is used to model the radiative heat transfer between the outer clothing surface and the walls of an enclosure. To simplify the derivation, we used the linear radiative law as suggested in [2].

$$R = h_r(t_{mrt} - t_a) \tag{1}$$

where h_r is considered constant. A linear approximation is valid for the low temperature differences encountered in indoor climates.

2. In the derivation of PMV, the convective heat transfer coefficient was modeled as the maximum of the natural and forced convective heat transfer coefficients. We modeled the convective heat transfer coefficient as the sum of the natural and forced convective heat transfer coefficients.

$$C = (h_{c_n} + h_{c_f})(t_{cl} - t_a) \tag{2}$$

and we assume that the natural convective heat transfer coefficient, h_{c_n} is a constant. The forced convective heat transfer coefficient h_{c_f} depends on the air velocity. This approach was used in [3] . Since the conditions in a room are often in the range where the convective heat transfer mechanisms between the human body and the air are mixed, we feel that the summation of coefficients is at least as accurate as the maximization of coefficients.

3. The thermal load, L, of PMV differs from the thermal difference, D, of the simplified index. The thermal load, L, is defined as the difference between the internal heat production and the heat loss to the environment when the sweat rate results in neutral thermal sensation and when the clothing outer surface temperature is determined from the heat balance at the clothing outer surface temperature assuming a skin temperature that results in neutral thermal sensation. The thermal difference, D, is defined as the difference between the internal heat production and the heat loss to the environment when the clothing outer surface is temperature determined from the heat balance at the body surface assuming that the sweat rate and skin temperature are those resulting in neutral thermal sensation.

$$D = \bar{Q}_B - \bar{Q}_C \tag{3}$$

when

$$t_{cl} = \bar{t}_s + 0.155 I_{cl} \bar{Q}_B \tag{4}$$

4. We assume that the bodily heat production, mechanical efficiency and clothing insulation are constant. This assumption is valid for a variety of applications because spaces are typically designed and used for a specific purpose. This assumption combined with the changes to the derivation of PMV result in an index that is an explicit, linearly parameterized function of the four environmental variables:

$$\begin{aligned} \hat{V} &= \hat{\theta}_0 + \hat{\theta}_1 p_a + \hat{\theta}_2 t_a + \hat{\theta}_3 t_{mrt} + \hat{\theta}_4 v^{\frac{2}{3}} \\ &\quad + \hat{\theta}_5 p_a v^{\frac{2}{3}} + \hat{\theta}_6 t_a v^{\frac{2}{3}} \end{aligned} \tag{5}$$

where $\hat{\theta}_0$ through $\hat{\theta}_6$ are dependent on clothing insulation and bodily heat production. When clothing insulation and bodily heat production are constant, the parameters of $\hat{V}$ are constant.

2.3 Comparison of $\hat{V}$ with PMV

We compared the ability of each index to predict the thermal sensation ratings of the data published in [15]. In their experiments, six men were exposed to a total of 15 different conditions, where the air temperature, air velocity and mean radiant temperature were varied independently. The relative humidity for all 15 experimental conditions was 50%. All six men were seated for one hour in the test chamber, and all were wearing between-season clothing (I_{cl} = 0.6 clo). In our evaluation, we used 10 of these 15 conditions so that the range of thermal sensation ratings provided by the subjects uniformly covered the entire range of the rating scale and so that the mean value of all of the ratings was zero (neutral).

Table 1 shows the mean and standard deviation of the prediction errors for PMV and two cases of $\hat{V}$. In Case I, the parameters of $\hat{V}$ are determined based on the data of [4] a way analogous to the determination of the parameters of PMV. In Case II, the parameters were determined from a least squares fit to the data. From this table we can conclude that the accuracy of $\hat{V}$ is comparable to that of PMV when the parameters are determined for each index by the same method because the standard deviation of $\hat{V}$ in Case I is nearly equal to the standard deviation of PMV. Furthermore, we can conclude that the accuracy of $\hat{V}$ may be made better than that of PMV if the parameters of $\hat{V}$ are determined from a least squares estimate because the standard deviation of $\hat{V}$ in Case II is less than that of PMV.

Table 1: Mean value and standard deviation of the prediction error of $\hat{V}$ and PMV using data from [15]. Case I is when the parameters of $\hat{V}$ are based on predetermined heat balance relations. Case II is when the parameters of $\hat{V}$ are determined using a least squares estimate.

	$\hat{V}$		PMV
	case I	case II	
mean	-0.029	0	0.187
std	0.916	0.797	0.923

We have derived the mathematical relationship between $\hat{V}$ and PMV. The details of the derivation are included in the Appendix. If the difference between the Stefan-Boltzmann and linear radiative heat transfer coefficients is negligible, and if the difference between the convective heat transfer coefficients used in each index is negligible, then the following relationship between D and L exists

$$D = L(1 + 0.155 I_{cl}(h_r + h_c)) \tag{6}$$

In the next section we will discuss the application of $\hat{V}$ to the control of the thermal sensation of an occupant in a room. We will demonstrate how the index can be used to calibrate the combined sensor and occupant system so that the controller learns the parameters of $\hat{V}$ that most accurately reflect the thermal sensation of the occupant.

3 User-Adaptable Comfort Control

If we can measure the four environmental variables, then Equation 5 tells us how those variables are to be combined to predict the thermal sensation of an occupant. However, there may be considerable uncertainty in the parameters $\hat{\theta}_0$ through $\hat{\theta}_6$. These parameters are dependent on the traits of the occupant, which are unknown to the controller. They are also dependent on the activity and clothing insulation, which cannot be estimated accurately. Furthermore, we may only be able to measure the environmental variables at some remote point such as the air inlet rather than in the vicinity of the occupant. If we use these remote measurements in the computation of $\hat{V}$, then the parameters of $\hat{V}$ must be modified to reflect the differences between the remotely measured variables and the conditions experienced by the occupant.

In this section we describe a controller based on the simplified index of Equation 5. The objectives of the controller are to regulate and calibrate the system simultaneously. This is accomplished by driving the index to the neutral value of zero by adjusting the heat flow into the room, and by learning from actual thermal sensation ratings and measurements of environmental variables the parameters of $\hat{V}$ that most accurately reflect the actual occupants' thermal sensation. We call this controller a User-Adaptable Comfort Controller (UACC).

To simplify the description and implementation of the UACC, but not to limit its applicability, we consider the case where there is a single, stationary occupant in the room. Figure 2 shows a flowchart of the operation of the UACC. After initializing the parameters of $\hat{V}$ to those of the average or standard occupant, the system begins operation by measuring or computing from measurements the values of air temperature, mean radiant temperature, vapor pressure, and air velocity. The system then computes a value of $\hat{V}$ and decides the appropriate control input based on the new value of $\hat{V}$. This decision is dependent on the type of feedback (e.g., on-off or PID). The system then checks for a prompt from the user indicating that the user desires to inform the system of his or her thermal sensation rating. If there is no prompt, then the feedback control loop resumes. If there is a prompt, then the system acquires the thermal sensation rating from the occupant. There are many different types of user interfaces that may be used for acquiring a thermal sensation rating. For example, the user interface may simply be a knob or dial displaying the seven-point psycho-physical rating scale upon which $\hat{V}$ is based. Next, the system computes the error between the actual thermal sensation rating and the predicted thermal sensation rating. Finally, the error and the measured or computed environmental variables are used to adjust the parameters of $\hat{V}$ in such a way that $\hat{V}$ more closely matches the actual thermal sensation ratings. Once the

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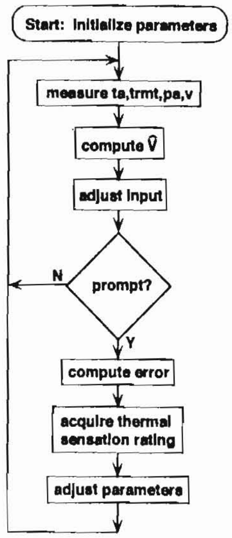

Figure 2: Flowchart of the operation of the UACC.

parameters have been changed, the feedback control loop resumes.

Changing the parameters of $\hat{V}$ on-line can have a profound effect on the dynamical system behavior. In particular it may lead to system instability. In this paper we will not discuss the stability issues of the UACC. Instead we refer the interested reader to the paper by [7]. However, we will demonstrate the behavior of the UACC with an example experiment.

3.1 Experimental Conditions

To demonstrate the feasibility of the UACC, we conducted experiments involving a single, seated, stationary occupant in a space cooled by a heat pump. The heat pump was a split system with a variable-speed-drive compressor. The indoor unit was located on one wall of the room. There was no mixing of room air with outside air, and because there were no ducts to transport air, there was no significant time delay. The occupant was seated facing the outlet of the air conditioner, approximately two meters from the wall. A layout of the room is shown in Figure 3.

The compressor speed was controlled with a digital computer using a PID controller with a sampling rate of 10 seconds. Thermal sensation ratings were acquired at thirty minute intervals beginning thirty minutes after the commencement of the experiment. The occupant placed the rating by typing a number at the keyboard of the computer. Ratings were not restricted to integer values. Instead, the occupant was presented with the rating scale and was allowed to place any real-valued rating between -3 and +3.

The parameters of $\hat{V}$ were estimated with the constrained recursive least squares algorithm described in [9]. The constraints on the parameters are determined by observing that the signs of some of the parameters are known *a priori*. Constraining the parameter estimates ensures the stable operation of the controller [7].

The air temperature, air velocity, and humidity were all measured at the air inlet. The air temperature was measured with a thermistor with an accuracy of ±0.1°C. The air velocity was measured with a heated wire anemometer with an accuracy of ±0.02 m/s and a range of 0 - 1 m/s. The relative humidity was measured with a thin film polymer capacitor device with an accuracy

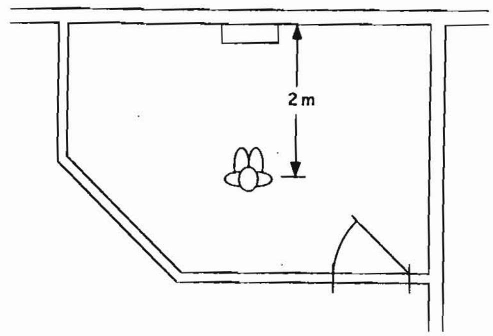

Figure 3: Plan view of room showing the positions of the human subject and the air-conditioner. Dimensions are in meters.

of ±2% relative humidity. From the measurement of air temperature and relative humidity, the vapor pressure was computed from the following relation:

$$p_a = \phi p_s \quad (7)$$

where ϕ is the relative humidity as a fraction, and p_s is the saturated vapor pressure. The saturated vapor pressure is dependent on air temperature, and was computed from an empirical relationship. The mean radiant temperature was determined in accordance with [1]. The temperature of each surface was measured with a thermistor identical to that used for the air temperature measurement, and the area factors were computed based on the knowledge of the position of the occupant.

3.2 Experimental Results

Figure 4 shows the predicted thermal sensation as a function of time, and Figure 5 shows the air temperature, mean radiant temperature and vapor pressure as a function of time. The air velocity measured at the inlet was initially 0.71 m/s, but decreased to 0.67 m/s after 10 minutes due to the formation of condensate on the heat exchanger surface. The inlet air velocity remained at 0.67 m/s for the duration of the experiment. The jumps in the predicted thermal sensation are the result of the parameter estimation process, and the magnitude of the jumps is equal to the magnitude of the prediction error at that time. Note that the magnitude decreases each time. At 30 minutes the occupant provides the first thermal sensation rating, and the magnitude of the prediction error is about one, but by the third time the parameters are changed (90 minutes) the prediction error is negligible. This fast convergence is partly due to the rate at which least squares estimators converge and partly due to the fact that the outdoor conditions do not change during this experiment. With changing outdoor conditions, it will generally be necessary to provide more than three thermal sensation ratings before the prediction errors remain small. Since there are seven parameters in $\hat{V}$, a rule of thumb is that seven thermal sensation ratings must be provided before the prediction errors remain small. The reader is referred to [9] for the mathematical details of the convergence properties of least squares estimators.

4 Energy-Optimal Control

In this section we consider the advantage of regulating $\hat{V}$ with more than one control variable. The additional degree of freedom

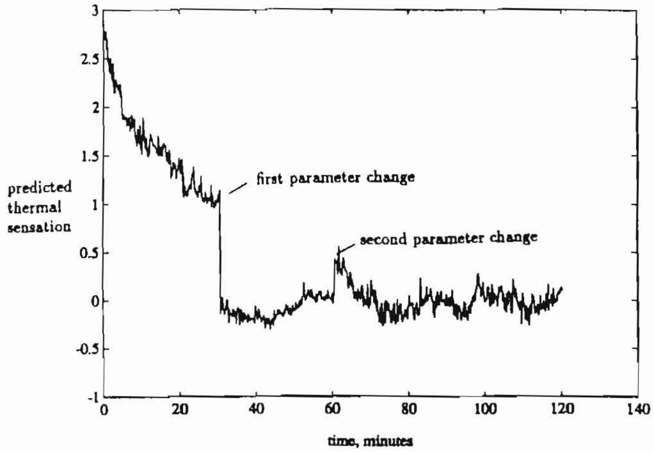

Figure 4: Predicted thermal sensation when cooling. Discontinuities are due to parameter adjustments. At 90 minutes, the prediction was equal to the rating of the user.

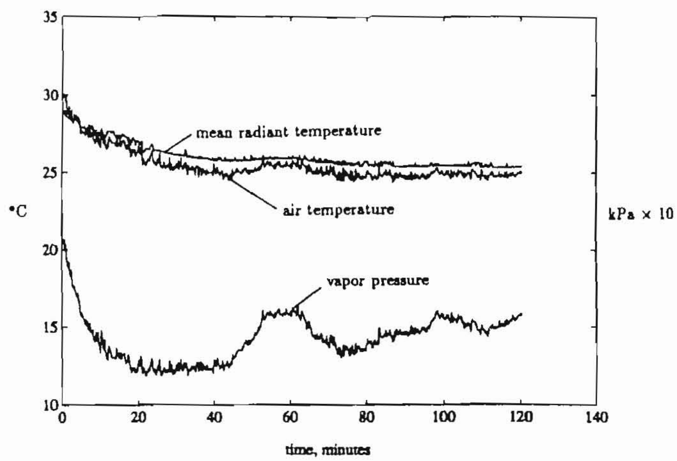

Figure 5: Air temperature, mean radiant temperature, and vapor pressure when cooling.

may be used to minimize the power consumption while maintaining the output at the desired value of zero. The discussion is oriented towards systems with both variable air flow and variable heat flow (e.g., variable speed fans and variable speed compressors).

Typically, energy optimal control problems are posed as a minimization of a functional such as

$$J = \int_{\tau_i}^{\tau_f} f(x(\tau), u(\tau), \tau) d\tau + g(x(\tau_f)) \tag{8}$$

The objective is to determine the function $u(\tau)$ given an initial time, a final time, and a final condition. The solution to the optimal control problem is well known for linear systems. For nonlinear systems, there are few analytical solutions available. Most actual implementations of optimal controllers for nonlinear systems involve discretization of the continuous system, and then numerical solution by means of dynamic programming. Both the analytical and the numerical solutions are highly dependent on the accuracy of the dynamic model of the system being controlled. It is well known that the solutions are extremely sensitive to modeling uncertainties.

There are numerous uncertainties in the models of HVAC systems. In reality, HVAC systems are high-dimensional, nonlinear systems involving the complex behavior of convection-driven fluid flow. However, the models of such systems are often low dimensional and often ignore the flow field behavior by making assumptions about the flow such as perfect mixing or filling. Therefore, a controller that minimizes a functional such as that of Equation 8 is not appropriate for HVAC systems. Instead we propose to control the system so that at steady-state the minimum power is consumed while maintaining $\hat{V} = 0$.

In the remainder of this section we will show the results of our analysis of the feasibility of saving energy at steady-state conditions during cooling. The analysis is based on a single-zone, bilinear model of the system of Figure 6 during cooling. The model is bilinear (if the air velocity to a power is considered as an input) because the air velocity is coupled with the state variables. There are few cases where the nonlinear effects of air velocity have been included in models of HVAC systems that are used for control system design. Most models of HVAC systems either assume constant air flow rate [23, 10, 16] or linearize the nonlinear model about a point in the state space [14]. In the model, the heat transfer coefficient between the air and the walls is the sum of the natural and forced convective heat transfer coefficient [illegible] forced convective heat transfer coefficients are modeled proportional to the air velocity to the $\frac{2}{3}$ power to maintain consistency with the derivation of $\hat{V}$. It is assumed that the air velocity in the occupied space is 10 percent of the air velocity at the inlet point. The dynamic equations are then written in terms of the air velocity in the occupied space. Correspondingly, the forced convective heat transfer coefficient between the air and the heat exchanger is multiplied by $10^{\frac{2}{3}}$. It is assumed that the energy exchange due to condensation is proportional to the difference between the saturated vapor pressure evaluated at the heat exchanger surface temperature and the vapor pressure when this pressure difference is negative, and zero when it is positive. The state equation is as follows:

$$\dot{x} = Ax + bq + Bxv^{\frac{2}{3}} + f(x, v) \tag{9}$$

where x is the state vector, A and B are matrices, q is the heat source, v is the average air velocity in the occupied space and $f(x, u)$ is a vector function describing the condensation of vapor on the indoor heat exchanger surface.

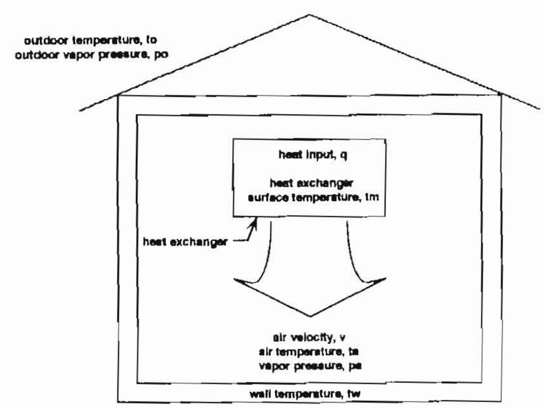

Figure 6: Schematic diagram of HVAC system and room. Energy storage elements are the heat pump, air, water vapor, and walls.

The optimal steady-state values of v and q can be determined by setting the time derivatives of Equation 9 to zero, setting Equation 5 equal to zero and applying the method of Lagrange multipliers. In this lumped parameter model, the mean radiant temperature is equal to the wall temperature. Since there are only two variables in the optimization, we will present a graphical view of the constraint in the two-dimensional input space. Figure 7 shows the constraint $\hat{V} = 0$ for the following system parameters

$$T_o = 32°C \tag{10}$$

$$h_m v^{\frac{2}{3}} = 1500 v^{\frac{2}{3}} \frac{W}{°C} \tag{11}$$

$$h_{a_n} = 300 \frac{W}{°C} \tag{12}$$

$$h_{a_f} v^{\frac{2}{3}} = 900 v^{\frac{2}{3}} \frac{W}{°C} \tag{13}$$

$$h_w = 100 \frac{W}{°C} \tag{14}$$

$$h_v = \mathcal{L} h_w = 1650 \frac{W}{kPa} \tag{15}$$

and for three different values of outdoor relative humidity, where q is plotted in units of kilowatts. The curves for low and moderate outdoor humidity have a knee that is caused by the transition from condensation to no condensation as the air velocity increases. Below the knee the cooling load is much higher because considerable energy goes into dehumidification with a minimal effect on thermal sensation. Above the knee, there is no condensation. The cooling load decreases with increasing air velocity due to the cooling effect of air velocity. As the outdoor humidity increases, the cooling load increases, and the air velocity required to cease condensation increases.

Figure 7 also shows lines of constant power consumption. The total power consumption, P, is related to the heat flow rate and the air velocity in the occupied space as follows:

$$P = 0.38q + 463v^3 \tag{16}$$

where the units of P are in kilowatts. Equation 16 is based on the experimentally determined coefficient of performance of a commercially available room air conditioner and the assumption that the inlet air velocity is 10 times the air velocity in the occupied space. For low and moderate outdoor humidity, the optimal occurs at or near the knee in the constraint. For high outdoor relative humidity, the analysis predicts that the optimal air flow rate will be zero. However, the model is not valid for extremely low air flow rates. Therefore, at high outdoor humidity, the results are inconclusive.

The analysis also predicts that there is a maximum total power consumption on the constraint $\hat{V} = 0$. At 50% outdoor relative humidity, the minimum total power consumption along the constraint is 63% of the maximum value. Thus an optimally operating system could be as much as 37% more energy efficient than a poorly operating system under these outdoor conditions.

5 Discussion

In Section 2 we derived an index that is a modification of the PMV index. We are not the first to recognize that a simplification of PMV may be advantageous for certain applications. In [21], PMV was simplified so that the resulting index could be computed explicitly in closed form. It was suggested that the simplified index could be used in a control algorithm. However, the simplified index was not linearly parameterized, and therefore not suitable for

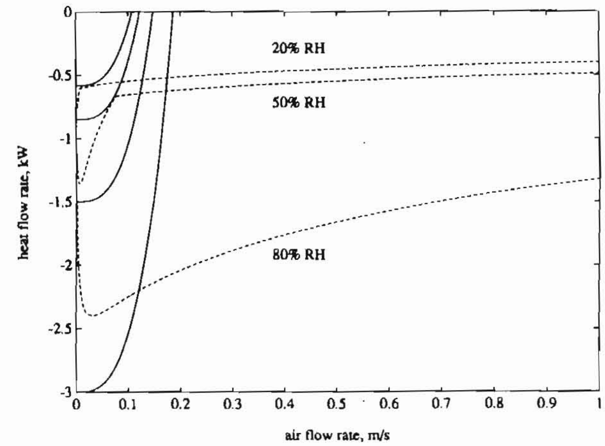

Figure 7: Combinations of air flow rate and heat flow rate producing neutral thermal sensation when cooling (dotted lines). As the outdoor humidity increases, the cooling load increases and the air flow rate necessary to prevent condensation increases. Solid lines indicate constant power consumption.

on-line calibration. With this goal in mind, our derivation results in an index that is both explicitly dependent on environmental variables and linearly parameterized.

We are not the first to use a linear regression to predict thermal sensation ratings. In [18, 20] a linear regression on the air temperature and relative humidity to predict thermal sensation ratings. However, we believe that $\hat{V}$ is the first linear regression predictor of thermal sensation ratings in which the basis functions are chosen based on heat balance relations rather than intuition. Therefore, $\hat{V}$ is dependent upon all four of the environmental variables that affect thermal sensation in a way that is consistent with the physics of the heat exchange process.

Although our experimental validation indicates that $\hat{V}$ is as accurate as PMV, we do not propose $\hat{V}$ as a substitute for PMV. The PMV index was developed as a design and analysis tool. The limitations of using PMV for feedback control are not as significant when PMV is used for design and analysis problems.

Instead we propose $\hat{V}$ for use as the controlled output of an HVAC system. We have described an adaptive version called the UACC. The UACC requires the measurement of air temperature, mean radiant temperature, humidity and air velocity. While measurement of air temperature, humidity and air velocity at a point is straightforward, the measurement of mean radiant temperature is fairly complicated. In our experiments we computed the mean radiant temperature from the direct measurement of individual wall temperatures. An alternative way of measuring mean radiant temperature is with a globe thermometer. The mean radiant temperature is related to the globe temperature, air velocity, and air temperature as follows

$$t_{mrt}^4 = t_g^4 + cv^{\frac{1}{2}}(t_g - t_a) \tag{17}$$

Under moderate conditions, we can use the linear radiative relation to model the radiant heat exchange and modify the model of forced convective heat exchange to be consistent with that of [3] so that Equation 17 becomes

$$t_{mrt} = t_g + cv^{\frac{2}{3}}(t_g - t_a) \quad (18)$$

If we substitute Equation 18 into Equation 5, we get the following alternative formulation for $\hat{V}$

$$\begin{aligned}\hat{V} &= \hat{\theta}_0 + \hat{\theta}_1 p_a + \hat{\theta}_2 t_a + \hat{\theta}_3 t_g + \hat{\theta}_4 v^{\frac{2}{3}} + \hat{\theta}_5 p_a v^{\frac{2}{3}} \\ &+ (\hat{\theta}_6 - \hat{\theta}_3 c) t_a v^{\frac{2}{3}} + (\hat{\theta}_3 c) t_g v^{\frac{2}{3}} \quad (19)\end{aligned}$$

An important feature of Equation 19 is that, like Equation 5, it is linearly parameterized. Therefore, the UACC can be implemented using either Equation 5 or 19.

In our experiments, we measured the air temperature, humidity and air velocity at the air inlet point rather than in the vicinity of the occupant. As we discussed previously, the adaptive mechanism makes the UACC robust to sensor relocation. For example, if the air velocity in the vicinity of the occupant is proportional to the air velocity at the air inlet point, then we can rewrite Equation 5 as

$$\begin{aligned}\hat{V} &= \hat{\theta}_0 + \hat{\theta}_1 p_a + \hat{\theta}_2 t_a + \hat{\theta}_3 t_{mrt} + (\hat{\theta}_4 k^{\frac{2}{3}}) v_{in}^{\frac{2}{3}} \\ &+ (\hat{\theta}_5 k^{\frac{2}{3}}) p_a v_{in}^{\frac{2}{3}} + (\hat{\theta}_6 k^{\frac{2}{3}}) t_a v_{in}^{\frac{2}{3}} \quad (20)\end{aligned}$$

where v_{in} is the inlet air velocity, and k is the proportionality constant relating the inlet air velocity to the air velocity near the occupant. Again, Equation 20 is linearly parameterized. If the average air velocity in the occupied space is a more complex function of the inlet air velocity, then Equation 20 will be less accurate than Equation 5, but it will be more accurate than if the air velocity was not included in the prediction at all because the average air velocity in the occupied space must be correlated with the inlet air velocity. Other uncertainties resulting from sensor relocation, such as a bias in temperature measurement, can be accommodated similarly.

The UACC is based on the assumption that the clothing insulation and activity of the occupant are constant. Clothing insulation and activity level will not always be constant, but because spaces are typically used for a specific purpose the variations will often be small.

In our description of the UACC, we considered only one occupant in the space. For certain spaces such as small offices and some automobiles this will be the case. However, in many cases it is not true. When it is not true, the implementation of the UACC is more complicated because each occupant must place the same number of ratings or else the preference of one occupant will be weighted more heavily in the prediction than the preference of another. Also, as the number of occupants in the spaces becomes large, the uncertainty related to the average traits of the occupants will be small because they will approach the average traits of the general population. However, the uncertainty related to knowledge of activity level and clothing insulation and the effect of sensor locations will exist even if there are a large number of occupants in the space.

It is assumed in the derivation of the UACC that the occupant does not move about in the room. Again, this assumption is valid for certain cases, such as automobiles, but is not valid in general. When this assumption is violated, and when the environmental variables are not measured next to the occupant, then the applicability of the UACC is reduced if the spatial variation in conditions is large for the region of the space in which the occupant moves about.

Finally, the UACC is based on a model of thermal sensation at steady-state conditions. Therefore transients have the potential of biasing the parameter estimates. However, if the prediction errors are uncorrelated, then the parameter estimates will be unbiased. Since the occupant will generally provide ratings to the system at widely spaced intervals of time, the prediction errors will be uncorrelated, so the parameter estimates will not be biased by transients.

In the section on energy-optimal control, we demonstrated that for a variable air flow, variable heat flow system, there is an energy optimal, steady-state combination of air flow and heat flow that will result in $\hat{V} = 0$ when the relative humidity is not too high. This result is not surprising, since it is common practice to use a fan for cooling as an alternative to air conditioning. For the case of heating, we believe that in most cases there will also be an energy optimal steady-state combination of air flow and heat flow, but for a different reason than when cooling. When heating, warm air escapes the occupied zone due to buoyancy, so it is necessary to circulate the air to drive the warm air back into the occupied zone. It is difficult to develop a simple and accurate model to show that an energy-optimal solution exists, because the model must account for the effects of buoyancy. However, it is well-known that a ceiling fan aids in heating a room with high ceilings. We are currently pursuing an experimental approach to demonstrate that an energy optimal, steady-state solution exists when heating.

We have demonstrated that an energy-optimal, steady-state equilibrium point exists, but we have not addressed the problem of using a feedback controller to drive the system to the optimal equilibrium point. This is a difficult problem since the plant is nonlinear, and the controller must be able to handle disturbances, errors in the dynamic model of the system, and errors in the algebraic relationship between the control inputs, state variables and the power consumption. We are presently developing a controller that will drive the system to the optimal steady-state operating conditions in the presence of these uncertainties.

6 Conclusions

The following are the contributions of this paper:

1. We derived a thermal sensation index that is better suited to the application of feedback control than previous thermal sensation indices.

2. We demonstrated how the simplified index could be used to calibrate the combined sensor and occupant system so that the controller could establish comfortable conditions for an arbitrary occupant.

3. We demonstrated the feasibility of using the simplified index for optimally establishing comfortable conditions in a space for variable air flow and variable heat flow systems.

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Notation

C: convective heat transfer between a clothed human and the air, $\frac{W}{m^2}$.
D difference between actual conditions and neutral conditions, $\frac{W}{m^2}$.

h_{a_n}: natural convective heat transfer coefficient between the room air and the wall surfaces, $\frac{W}{^\circ C}$.

$h_{a_f} v^{\frac{2}{3}}$: forced convective heat transfer coefficient between the room air and the wall surfaces, $\frac{W}{^\circ C}$.

h_c: convective heat transfer coefficient between the clothed human body and the air, $\frac{W}{m^2 {}^\circ C}$.

h_{c_n}: natural convective heat transfer coefficient between the clothed human body and the air, $\frac{W}{m^2 {}^\circ C}$.

$h_{c_f} v^{\frac{2}{5}}$: forced convective heat transfer coefficient between the clothed human body and the air, $\frac{W}{m^2 {}^\circ C}$.

$h_m v^{\frac{2}{3}}$: convective heat transfer coefficient between the indoor heat exchanger and the air, $\frac{W}{^\circ C}$.

h_r: linear radiant heat transfer coefficient, $\frac{W}{m^2 {}^\circ C}$.

h_w: heat transfer coefficient between the wall inner surfaces and the outdoors, $\frac{W}{^\circ C}$.

J: cost functional.

L: thermal load, $\frac{W}{m^2 {}^\circ C}$.

$\mathcal{L}$: Lewis number, $\frac{^\circ C}{kPa}$.

p_a: vapor pressure, kPa.

PMV: predicted mean vote.

p_o: outdoor vapor pressure, kPa.

p_s: saturated vapor pressure, kPa.

Q_B: heat flow at the outer surface of the human body, $\frac{W}{m^2 {}^\circ C}$.

Q_C: heat flow at the outer surface of the clothing, $\frac{W}{m^2 {}^\circ C}$.

R: radiant heat transfer from the clothing surface, $\frac{W}{m^2 {}^\circ C}$.

t_a: air temperature, $^\circ C$.

t_g: globe temperature, $^\circ C$.

t_m: heat exchanger surface temperature, $^\circ C$.

t_o: outdoor temperature, $^\circ C$.

t_{mrt}: mean radiant temperature, $^\circ C$.

t_w: wall temperature, $^\circ C$.

$u(\tau)$: input vector.

$\hat{V}$: predicted thermal sensation.

v: air velocity, $\frac{m}{s}$.

$x(\tau)$: state vector.

ϕ: relative humidity.

τ: time.

$\hat{\theta}$: parameter of $\hat{V}$.

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Appendix

First define the difference, Δ_c, between the convective heat transfer coefficient used to derive PMV and the convective heat transfer coefficient used to derive $\hat{V}$ as

$$\Delta_c \triangleq h_c^{PMV} - h_c \tag{21}$$

where the superscript refers to the relation used in the derivation of PMV. The difference, Δ_R is defined as

$$\Delta_R \triangleq R^{PMV} - h_r(t_{cl} - t_{mrt}) \tag{22}$$

Now D and the thermal load L can be written as

$$\begin{aligned} D &= \bar{Q}_B - \bar{Q}_C \\ &= \bar{Q}_B - h_r(\bar{t}_{cl} - t_{mrt}) - h_c(\bar{t}_{cl} - t_a) \qquad (23) \\ L &= \bar{Q}_B - \bar{Q}_C^{PMV} \\ &= \bar{Q}_B - \bar{R}^{PMV} - \bar{h}_c^{PMV}(\bar{t}_{cl}^{PMV} - t_a) \qquad (24) \end{aligned}$$

where the overbar implies the use of the ideal skin temperature and sweat rate used to derive PMV. Solving for $\bar{Q}_B$ in Equation 23 and substituting into Equation 24 gives

$$\begin{aligned} L = \; & D + h_r(\bar{t}_{cl} - t_{mrt}) + h_c(\bar{t}_{cl} - t_a) - \\ & \bar{R}^{PMV} - h_c^{PMV}(\bar{t}_{cl}^{PMV} - t_a) \end{aligned} \tag{25}$$

Substituting Equations 21 and 22 into Equation 25 gives

$$L = D + (\bar{t}_{cl} - \bar{t}_{cl}^{PMV})(h_r + h_c) - \Delta_R - \Delta_c(\bar{t}_{cl}^{PMV} - t_a) \tag{26}$$

The difference $\bar{t}_{cl} - \bar{t}_{cl}^{PMV}$ is

$$\begin{aligned} \bar{t}_{cl} - \bar{t}_{cl}^{PMV} &= \bar{t}_s - 0.155 I_{cl}\bar{Q}_B - (\bar{t}_s - 0.155 I_{cl}\bar{Q}_C^{PMV})) \\ &= -0.155 I_{cl} L \end{aligned} \tag{27}$$

Substituting Equation 27 into Equation 26 and simplifying, the relationship between D and L is

$$D = L(1 + 0.155 I_{cl}(h_r + h_c)) - \Delta_R - \Delta_c(\bar{t}_{cl}^{PMV} - t_a) \tag{28}$$

Since Δ_R and Δ_c result from different models of the same physical phenomena, the magnitudes of Δ_R and Δ_c should be small. Therefore D can be approximately related to L as

$$D \approx L(1 + 0.155 I_{cl}(h_r + h_c)) \tag{29}$$