



Fig. 2. Performance.

equalization noticeably improved the quality of DATA-PHONE facsimile transmission. The frequency characteristics of this channel are shown in Fig. 2. The details of the mathematics and implementation will appear in a paper in preparation.

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### Single-Sideband Hybrid AM-PM Signal Models

The purpose of this letter is to disclose eight unique, nontrivial models for a hybrid AM-PM signal modulated by a single sinusoid and having a one-sided frequency spectrum. Previous publications<sup>1,2</sup> have disclosed one model. The importance of the additional models is in the phase relationships of the sidebands to the carrier.

Let the carrier amplitude and radian frequency be denoted by  $A$  and  $\omega_c$ , respectively, and let the modulating sinusoid be

$$f(t) = \Delta\phi \sin[\omega_m t + \phi(t)]. \quad (1)$$

$\Delta\phi$  is the amplitude of the modulating sinusoid,  $\omega_m$  its rest frequency, and  $\phi(t)$  its angle modulation. Denote the Hilbert transform of  $f(t)$  by  $\hat{f}(t)$ . Then, by MacLaurin series expansion of the "analytic signals"<sup>1</sup> may be written

$$\begin{aligned} s_1(t) &= A \exp[-\hat{f}(t)] \cos[\omega_c t + f(t)] \\ &\equiv A \sum_{n=0}^{\infty} \frac{\Delta\phi^n}{n!} \cos[(\omega_c + n\omega_m)t + n\phi(t)] \end{aligned} \quad (2)$$

$$\begin{aligned} s_2(t) &= A \exp[\hat{f}(t)] \cos[\omega_c t - f(t)] \\ &\equiv A \sum_{n=0}^{\infty} (-1)^n \frac{\Delta\phi^n}{n!} \cos[(\omega_c + n\omega_m)t + n\phi(t)] \end{aligned} \quad (3)$$

$$\begin{aligned} s_3(t) &= A \exp[f(t)] \cos[\omega_c t + \hat{f}(t)] \\ &\equiv A \sum_{n=0}^{\infty} (-1)^n \frac{\Delta\phi^n}{n!} \cos[(\omega_c + n\omega_m)t + n(\phi(t) + \pi/2)] \end{aligned} \quad (4)$$

$$\begin{aligned} s_4(t) &= A \exp[-f(t)] \cos[\omega_c t - \hat{f}(t)] \\ &\equiv A \sum_{n=0}^{\infty} \frac{\Delta\phi^n}{n!} \cos[(\omega_c - n\omega_m)t + n(\phi(t) + \pi/2)] \end{aligned} \quad (5)$$

$$\begin{aligned} s_5(t) &= A \exp[-\hat{f}(t)] \cos[\omega_c t - f(t)] \\ &\equiv A \sum_{n=0}^{\infty} \frac{\Delta\phi^n}{n!} \cos[(\omega_c - n\omega_m)t - n\phi(t)] \end{aligned} \quad (6)$$

$$\begin{aligned} s_6(t) &= A \exp[\hat{f}(t)] \cos[\omega_c t + f(t)] \\ &\equiv A \sum_{n=0}^{\infty} (-1)^n \frac{\Delta\phi^n}{n!} \cos[(\omega_c - n\omega_m)t - n\phi(t)] \end{aligned} \quad (7)$$

$$\begin{aligned} s_7(t) &= A \exp[f(t)] \cos[\omega_c t - \hat{f}(t)] \\ &\equiv A \sum_{n=0}^{\infty} \frac{\Delta\phi^n}{n!} \cos[(\omega_c - n\omega_m)t - n(\phi(t) - \pi/2)] \end{aligned} \quad (8)$$

$$\begin{aligned} s_8(t) &= A \exp[-f(t)] \cos[\omega_c t + \hat{f}(t)] \\ &\equiv A \sum_{n=0}^{\infty} (-1)^n \frac{\Delta\phi^n}{n!} \cos[(\omega_c - n\omega_m)t - n(\phi(t) - \pi/2)]. \end{aligned} \quad (9)$$

Inspection of the equations shows that the first four signals have upper sideband structure while the last four have lower. From a total-power standpoint the signals exhibit AM rather than PM behavior in that the residual carrier power is constant. Modulation simply adds power to the signal in the form of sidebands.

Recovery of the first-order side-product (subcarrier) is possible by product demodulation using a reference which is phase-coherent with the residual carrier, provided the modulation on the subcarrier is not so great as to cause the product spectra to overlap. For the case where the subcarrier is binary phase-shift keyed, it is interesting to note that the second harmonic of the subcarrier is also recovered directly. With normal double-sideband phase-modulated carriers it is usually necessary to frequency double the recovered subcarrier and noise to obtain a second harmonic which is then frequency divided to produce a subcarrier demodulation reference. The doubling process, which exhibits a sharp threshold effect, is not necessary when the hybrid AM-PM signal is employed.

Possibilities for future effort are seen in combining various of the eight signals to obtain multiple data channels (subcarriers) which are in phase and/or frequency diversity and occupy essentially the same transmission bandwidth as double sideband phase-modulated signals.

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### An Improved Telemetry Encoding Circuit with Square Loop Cores and SCR's

In a recent correspondence,<sup>1</sup> the author described a new magnetic amplifier telemetry encoder circuit which is suitable for application in artificial satellites and rockets. The encoder circuit, though it gives excellent performance, has some limitations in channel capacity and precision performance, because of the complicated biasing mechanism of transistors in the sequential set circuits. The present letter describes an improved encoding circuit in which transistors have been replaced by SCR's in the set circuits. The resulting advantages are as follows: 1) design complexities of transistor biasing arrangement have been simplified by SCR's, which operate into the simple "trigger into conduction" mode; 2) better reliability and reduced weight have been achieved due to a fewer number of compo-

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<sup>1</sup> E. Bedrosian, "The analytic signal representation of modulated waveforms," *Proc. IRE*, vol. 50, pp. 2071-2076, October 1962.  
<sup>2</sup> R. M. Glorioso and E. H. Brazeal, "Experiments in 'SSB FM' communication systems," *IEEE Trans. on Communication Technology*, vol. COM-13, pp. 109-116, March 1965.

Manuscript received January 17, 1966.  
<sup>1</sup> B. K. Bose, "A magnetic amplifier telemetry encoding circuit," *Proc. IEEE*, vol. 52, p. 1076, September 1964.