# THE EFFECT OF SUPERCRITICAL STRING COSMOLOGY ON THE RELIC DENSITY OF DARK MATTER

A Senior Scholars Thesis

by

PHUONGMAI N. TRUONG

Submitted to the Office of Undergraduate Research
Texas A&M University
in partial fulfillment of the requirements for the designation as

UNDERGRADUATE RESEARCH SCHOLAR

April 2009

Major: Physics
Mathematics

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# ABSTRACT

The Effect of Supercritical String Cosmology
on the Relic Density of Dark Matter. (April 2009)

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Supercritical String Cosmology (SSC) introduces a time-dependent dilaton and a central charge deficit into the history of the development of the universe. To balance the effect of the dilaton and the central charge deficit, the so-called exotic matter, which includes any type of matter but baryonic and dark matters, is also introduced. These three quantities inadvertently alter the relic density of all particles, including dark matter candidates. In this work, we are interested in the correlation between the dark matter density and the equation of state of exotic matter. Using numerical method, we show that there can only be a dilution of dark matter, which ranges approximately between 0.02 – 0.1, depending on how strictly we require the various constraints to be satisfied.

# DEDICATION

To my parents for always guiding, supporting, and believing in me. To Ms. Baker, Mom Louise, and Dad Herb, for being my family.

# ACKNOWLEDGMENTS

I am deeply indebted to Bhaskar Dutta for introducing me to such an interesting project and patiently guiding me through it. I would like to thank Dimitri Nanopoulos for showing me the basics and the beauty of string cosmology. My gratitude also goes out to Sheldon Campbell and Abram Krislock for helping me write the computer program for my research.

# NOMENCLATURE

| | |
|---|---|
| LHC | Large Hadron Collider |
| LSP | Lightest Supersymmetric Particle |
| ODE | Ordinary Differential Equation |
| PAMELA | Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics |
| RWF | Robertson-Walker-Friedman |
| SSC | Supercritical String Cosmology |
| SUSY | Supersymmetry |
| WIMP | Weakly Interacting Massive Particle |
| WMAP | Wilkinson Microwave Anisotropy Probe |

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# CHAPTER I

# INTRODUCTION

In astronomy, there is overwhelming evidence that most of the mass in the universe is some nonluminous ``dark matter,'' of yet unknown composition. The two most convincing evidences for the existence of dark matter are: the Bullet cluster, which shows the separation of dark matter from luminous matter; and the rotation curves of spiral galaxies, which indicates that the density of luminous matter is not large enough to account for the observed galactic dynamics. Furthermore, recent observational results, such as those obtained from the Wilkinson Microwave Anisotropy Probe (WMAP) [1,2], indicate that baryonic matter comprises only 4% of the universe, while 22% is composed of dark matter. There are many candidates for the composition of dark matter, among which the most probable ones are nonbaryonic, that is, that they are some new elementary particles. Among the nonbaryonic candidates, an important categorization scheme is the “hot” versus “cold” classification. A dark matter candidate is called “hot” if it was moving at relativistic speeds at the time galaxies could just start to form, and it is called “cold” if it was moving nonrelativistically at the time. Simulations of structure formation in a universe dominated by hot dark matter, however, do a poor job of reproducing the observed structure. The cold dark matter candidates are basically elementary particles which have not yet been discovered, such as weakly interacting

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This thesis follows the style of Physics Letter B.

massive particles (WIMPs). These are stable particles which arise in extension of the standard model of electroweak interactions. WIMP masses are typically in the range from 10 GeV to a few TeV, and they have characteristic of weak interactions with ordinary matter. The most promising WIMP candidate is the neutralino [3].

In particle physics, supersymmetry (SUSY) asserts the existence of a hypothetical symmetry between bosons and fermions, such that every particle in the standard model would have a supersymmetric partner. SUSY predicts that the lightest supersymmetric particle (LSP) is stable, having a mass less than a few TeV and having weak interactions with ordinary matter. This particle is named the neutralino, a linear combination of the SUSY partners of the photon, Z0, and Higgs boson [3]. If such a WIMP exists, then it has a cosmological abundance of almost 1 as required to fit with the observational data mentioned above, and could therefore account for dark matter in the universe.

At present, there is no direct accelerator evidence to confirm the existence of neutralinos. However, if SUSY models are correct, then the Large Hadron Collider (LHC), which will begin operating in September 2009, is expected to produce the lightest neutralino particles. Many models have been developed in order to provide a parameter space, which can be detected at the LHC and satisfies the constraint placed by WMAP data on dark matter content. Dissipative Liouville Cosmology, or Q-Cosmology, is among those various models.

As various type Ia supernovae projects [4,5] and the WMAP data [1,2] have continually confirmed the existence of dark energy, the cause of universe expansion, Supercritical String Cosmology (SSC) [6,7] arises as a model attempting to formulate correctly an expanding Robertson-Walker-Friedman (RWF) Universe. In both SSC and its sister model, the dissipative Super-noncritical (Liouville) String Cosmology, the dilaton field is time-dependent, which inadvertently introduces new terms in the Bolztmann equation describing relic abundances and the associated particle-physics phenomelogy. In fact, the changes in the Boltzmann equation lead to a factor of 10 difference in the neutralino density. Nonetheless, the baryonic matter density is unaffected, which does not contradict with observational data. Hence, the SSC model provides a wider range in the parameter space for the dark matter search at the LHC [8,9].

In our work we will further explore the behavior of the observables such as the Hubble parameter and the densities of different particle species under the effect of the time-dependent dilaton. Chapter II is a review of SSC and its affect on the Boltzmann equation, also a presentation of our method of solving the modified cosmological equations. Chapter III shows the result of our work. A conclusion and discussion of future directions for this research are provided in chapter IV.

# CHAPTER II

# SOLVING FOR THE NEW RELIC DENSITY

**Overview of Supercritical String Cosmology and the modified Boltzmann equation**

In supercritical string cosmology, after identifying the Liouville mode with the target time, including matter background into the solution, and compactifying the dimensions of the string world volume into four target-space dimensions, one arrive at the following modified Einstein equations [10]:

$$3H^2 - \rho - \rho_\phi = \frac{e^{2\phi}}{2}\,\tilde{\mathcal{G}}_\phi$$

$$2\dot{H} + \rho + \rho_\phi + p + p_\phi = \frac{\tilde{\mathcal{G}}_{ii}}{a^2} \tag{1}$$

$$\ddot{\phi} + 2H\dot{\phi} + \frac{1}{4}\frac{\partial V_{all}}{\partial \phi} + \frac{1}{2}(\rho - 3p) = -\frac{3}{2}\frac{\tilde{\mathcal{G}}_{ii}}{a^2} - \frac{e^{2\phi}}{2}\,\tilde{\mathcal{G}}_\phi$$

Where $\tilde{\mathcal{G}}_\phi$ and $\tilde{\mathcal{G}}_{ii}$ are the noncritical string off-shell terms:

$$\tilde{\mathcal{G}}_\phi = e^{-2\phi}(\ddot{\phi} - \dot{\phi}^2 + Qe^{\phi}\dot{\phi})$$

$$\tilde{\mathcal{G}}_{ii} = 2a^2\left(\ddot{\phi} + 3H\dot{\phi} + \dot{\phi}^2 + (1-q)H^2 + Qe^{\phi}\left(\dot{\phi} + H\right)\right) \tag{2}$$

Here $q = -\frac{\ddot{a}}{\dot{a}^2}$ is the standard deceleration parameter. In the above equations, $\phi$ and $Q$ denote the dilaton and the central charge deficit, respectively. H denotes the Hubble parameter, $\rho$ and $p$ are the density and pressure, respectively, of all matter and radiation except the dilaton, for which is accounted by $\rho_\phi$ and $p_\phi$. The dotted quantities are derivatives with respect to the dimensionless Einstein time. We will later explain these notations in more details before we discuss solving the equations. The scalar potential $V_{all}$ is dependent on the central charge deficit $Q$:

$$V_{all} = 2Q^2 e^{2\phi} + V_0 \tag{3}$$

Although we have assumed a spatially flat universe, the terms on the right hand side of (3), which manifest departure from the criticality, act similarly to curvature terms since they are non-zero at certain epochs. The dilaton energy density and pressure are defined as follows:

$$\rho_\phi = \frac{1}{2}\left(2\dot{\phi}^2 + V_{all}\right)$$
$$p_\phi = \frac{1}{2}\left(2\dot{\phi}^2 - V_{all}\right) \tag{4}$$

For the renormalizability of the theory, the dependence of the central charge deficit on the cosmic time, to leading order of the Regge slope, is expressed via the Curci-Paffuti equation [11]:

$$\frac{d\tilde{\mathcal{G}}_\phi}{dt_E} = -6e^{-2\phi}\left(H + \dot{\phi}\right)\frac{\tilde{\mathcal{G}}_{ii}}{a^2} \tag{5}$$

For completeness, we shall also display the continuity equation of the energy stress tensor here which can be obtained from the set of Einstein equations:

$$\frac{d\rho}{dt_E} + 3H(\rho + p) + \frac{\dot{Q}}{2}\frac{\partial V_{all}}{\partial Q} - \dot{\phi}(\rho - 3p) = \frac{6\left(H + \dot{\phi}\right)\tilde{\mathcal{G}}_{ii}}{a^2} \tag{6}$$

In Hamiltonian mechanics, the Boltzmann equation is often written in general form as:

$$\hat{\mathbf{L}}[f] = \mathbf{C}[f] \tag{7}$$

where **L** is the Liouville operator, describing the evolution of a phase space volume, and **C** is the collision operator. After careful derivation [9] of the dilaton source and noncritical string induced modifications, for a given species $\chi$, the new Boltzmann equation for a 4-dimensional effective field theory after string compactification (or restriction on the D3 brane), in the presence of off-shell string background, differs from the standard cosmological Boltzmann equation just by the contribution of the graviton:

$$\frac{dn}{dt} + 3Hn - \dot{\phi}n = \frac{1}{2}\eta\left(e^{-\phi}\, g^{\mu\nu}\, \tilde{\beta}^{Grav}_{\mu\nu}\right)n + \int \frac{d^3p}{E}\mathbf{C}[f] \tag{8}$$

with $\tilde{\beta}^{Grav}_{\mu\nu}$ denoting the graviton Weyl anomaly coefficient. Let us consider only the physical scheme, in which $\eta = -1$, and define $\Gamma(t) \equiv \dot{\phi} + \frac{1}{2}\, e^{-\phi}\, g^{\mu\nu}\, \tilde{\beta}^{Grav}_{\mu\nu}$, then

$$\dot{n} + 3Hn = \Gamma(t)n + \int \frac{d^3p}{E}\,\mathbf{C}[f] = \Gamma(t)n - \langle\sigma v\rangle(n^2 - n_{eq}^2) \tag{9}$$

Before the freeze-out time, i.e. when $t < t_f$, equilibrium is maintained and the number density $n = n_{eq}$. Assume that $n = n_{eq}^{(0)}$ at a very early epoch $t_0$. Then the solution of the modified Boltzmann equation at all times $t < t_f$ is:

$$na^3 = n^{(0)}a^3(t_0)\exp\left(\int_{t_0}^{t} \Gamma dt\right) \tag{10}$$

We can further assume, reasonably, that $t_0$ is the time right after the inflationary period [9], since soon after the exit from inflation, all particles are in thermal equilibrium. Let $x \equiv T/m_\chi$ be the rescaled dimensionless temperature for the specific particle $\chi$, which for our purpose would be the neutralino. We will also assume the usual correlation between the redshift $z$ and the temperature $T$

$$z + 1 = \left(\frac{g(T)}{g(T_{CMB})}\right)^{\frac{1}{4}}\frac{T}{T_{CMB}} \tag{11}$$

where g is the relativistic degrees of freedom and $T_{CMB} = 2.725K$ is the measured temperature of cosmic microwave background. Detailed derivation from this point would lead to the modified relic density caused by an effect of the dilaton and noncritical string background:

$$\Omega_{\tilde{\chi}} = \Omega_{\tilde{\chi}_{no\ source}} \times \left(\frac{\tilde{g}_*}{g_*}\right)^{\frac{1}{2}} \left(1 + \int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{\psi(x)} dx\right) \quad (12)$$

where Ω denotes the relic abundance of a particular species, $g_*$ is the number of relativistic degrees of freedom of particles at their freeze-out temperature,

$$\psi(x) = x \exp\left(-\int_{x_0}^{x} \frac{\Gamma H^{-1}}{x} dx\right) \quad (13)$$

The relic abundance without the source term caused by string effect is:

$$\Omega_{\tilde{\chi}_{no\ source}} = \frac{1.066 \times 10^9\ GeV^{-1}}{M_{Pl}\sqrt{g_*}\ \int_{x_0}^{x_f} \langle \sigma v \rangle dx} \quad (14)$$

And finally, the freeze-out temperature is:

$$x_f^{-1} = \ln\left(0.03824\ g_s \frac{M_{Pl} m_{\tilde{\chi}}}{\sqrt{g_*}} \sqrt{x_f}\ \langle \sigma v \rangle_f\right) + \frac{1}{2}\ln\left(\frac{g_*}{\tilde{g}_*}\right) + \int_{x_f}^{x_{infl}} \frac{\Gamma H^{-1}}{x} dx \quad (15)$$

which differs from the standard equation by the last term, and for all practical purpose, the last term with the source has a very small contribution to the value of $x_f$, therefore can be omitted during calculation.

We also denote the ratio between the string-effected relic density and the no-source one by $R$:

$$R = \sqrt{\frac{\tilde{g}_*}{g_*}} \exp\left(\int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{x} dx\right)$$

$$\Gamma = \dot{\phi} + \frac{1}{2} e^{-\phi} g^{\mu\nu} \tilde{\beta}_{\mu\nu}^{Grav} \sim \dot{\phi} \tag{16}$$

$$\frac{\tilde{g}_*}{g_*} = \frac{3{H_*}^2}{\rho_{r*}}$$

where again, the stars denote quantities measured at freeze-out temperature.

**Solving the equations numerically**

In order to solve for the multiplication factor *R*, we need to solve for the Hubble parameter *H*, the rate of change of the dilaton, and the evolution of radiation density from the freeze-out time to present. The behavior of cosmological parameters, such as the Hubble parameter and different energy densities, are again given by the dynamical equations [8]:

$$\ddot{\phi} = -2\dot{H}^2 - 3H\dot{\phi} - e^{\phi} Q\left(\dot{\phi} + H\right) + \frac{1}{2}(\rho + p) \tag{17}$$

$$3\dot{H} = -H^2 - 2\dot{\phi}^2 + e^{\phi} Q\left(\dot{\phi} + H\right) - \frac{1}{2}(3\rho + p) \tag{18}$$

$$\begin{aligned} \dot{\rho} + 2Q\dot{Q}e^{2\phi} &= -3H(\rho + p) + \dot{\phi}(\rho - 3p) \\ &+ 4\left(H + \dot{\phi}\right)\left(-H^2 + \dot{\phi}^2 + e^{\phi} Q\left(\dot{\phi} + H\right) + p\right) \end{aligned} \tag{19}$$

$$\begin{aligned}(e^{\phi}Q+H)e^{\phi}\dot{Q} \\ &= \frac{\dot{p}}{2}+\frac{\rho}{2}\left(e^{\phi}Q-2H\right)+\frac{1}{6}\left(e^{\phi}Q+22H+18\dot{\phi}\right)p \\ &-\frac{1}{3}e^{2\phi}Q^2\left(\dot{\phi}+H\right)+e^{\phi}Q\left(\frac{23}{3}\dot{\phi}^2+8H^2+\frac{47}{3}\dot{\phi}H\right) \\ &+10\dot{\phi}^3+\frac{62}{3}\dot{\phi}^2H+12\dot{\phi}H^2+\frac{4}{3}H^3\end{aligned} \tag{20}$$

Recall that in the above equations, H denotes the rescaled Hubble parameter, $\rho$ and $p$ are the rescaled density and pressure, respectively, of all matter and radiation except the dilaton. Rescaling means these quantities differ from the usual ones by a factor of $\rho_c = 3H_0^2/(8\pi G_N)$, where $H_0 = 1.022 \times 10^{-10} h_0\ yr^{-1}$ is the Hubble constant of today, and $\rho_c$ is the critical density of the universe, the determine factor of whether the universe will contract or keep expanding. Also, $\phi$ and $Q$ denote the dilaton and the central charge deficit, respectively. The dotted quantities are derivatives with respect to the dimensionless Einstein time $t_E \equiv \sqrt{3}\, H_0\, t$, with $t$ being the cosmic time in the RWF metric. In this system of units, one year of cosmic time corresponds to $t_E = 1.292 \times 10^{-10}$ and one second corresponds to $t_E = 4.097 \times 10^{-18}$. This means with $t \sim 2$ we can encompass the whole history of the universe. Thus, when solving the differential equations numerically with respect to the Einstein time, we would need very fine time steps to get a reliable result. To get around this complication, we convert the quantities to functions of redshift, via the following relation:

$$\frac{dt_E}{dz} = -\frac{1}{(1+z)H} \tag{21}$$

We also have to separate the density and pressure into different densities of baryonic matter (including dark matter and all non-relativistic matters), radiation, and the so-called exotic matter, which is any kind of matter not grouped with baryonic matter, and is assumed to be effected by the dilaton. The reason for doing so is that these components develop differently throughout the course of the universe, and combining them together would lead to loss of information on how each component evolves. The densities and pressures are related to each other in the following way:

$$p_b = 0$$
$$p_r = \frac{1}{3}\rho_r \tag{22}$$
$$p_e = w_e \rho_e$$

where $w_e$ is our control parameter, to be determined numerically. Quite trivially, $\rho_b, \rho_r, \rho_e$ denote baryonic matter, radiation, and exotic matter densities. In previous work by Lahanas et al [8], the evolution of baryonic matter and radiation are:

$$\dot{\rho}_b = (\dot{\phi} - 3H)\rho_b \tag{23}$$

$$\dot{\rho}_r = -4H\rho_r \tag{24}$$

It is interesting to note that the dilaton does not have any direct effect on the radiation density. Upon using (21) to change the dependence on Einstein time to redshift, we find that radiation is completely independent of dilaton or any other string cosmological quantity:

$$\frac{d\rho_r}{dz} = \dot{\rho}_r \frac{dt_E}{dz} = \frac{4}{1+z}\,\rho_r \tag{25}$$

This independence is a useful tool to check the reliability of our numerical solver, since one can solve the system of differential equations analytically in the case of normal cosmology (the usual RWF metric) and obtain:

$$\rho_r \propto (1+z)^4 \tag{26}$$

Also note that the effect of the dilaton on the matter density, demonstrated in (23), causes the dilution of dark matter species, which is the motivation of our research.

We now need to find the rate of change of exotic matter density and the central charge deficit as functions of $\phi, H, \rho$ and $Q$. From equations (19) and (20), we have:

$$\begin{gathered} \dot{\rho}_e + 2e^{2\phi} Q\dot{Q} = L \\ -\frac{w_e}{2}\,\dot{\rho}_e + \left(e^{\phi}Q + H\right)e^{\phi}\dot{Q} = M \end{gathered} \tag{27}$$

where:

$$\begin{aligned} L = {} & -3(1+w_e)H\rho_e + (1-3w_e)\dot{\phi}\rho_e + 4(\dot{\phi}+H)(-H^2 \\ & + \dot{\phi}^2 + e^{\phi}Q\left(\dot{\phi}+H\right) + \frac{1}{3}\rho_r + w_e\rho_e) \end{aligned}$$

$$\begin{aligned} M = {} & -\frac{2}{3}H\rho_r + \frac{1}{2}\left(e^{\phi}Q - 2H\right)(\rho_r + \rho_b + \rho_e) \\ & + \frac{1}{6}\left(e^{\phi}Q + 22H + 18\dot{\phi}\right)\left(\frac{1}{3}\rho_r + w_e\rho_e\right) \\ & - \frac{1}{3}e^{2\phi}Q^2\left(\dot{\phi}+H\right) \\ & + e^{\phi}Q\left(\frac{23}{3}\dot{\phi}^2 + 8H^2 + \frac{47}{3}H\dot{\phi}\right) + 10\dot{\phi}^3 \\ & + \frac{62}{3}\dot{\phi}^2 H + 12\dot{\phi}H^2 + \frac{4}{3}H^3 \end{aligned} \tag{28}$$

Therefore, we obtain:

$$\dot{\rho}_e = \frac{\left(e^{\phi} Q + H\right) L - 2e^{\phi} QM}{(1 + w_e)e^{\phi} Q + H} \tag{29}$$

$$\dot{Q} = \frac{\frac{w_e}{2} L + M}{e^{-\phi}\left((1 + w_e)e^{\phi} Q + H\right)} \tag{30}$$

Now that we are equipped with differential equations of all the necessary quantities (equations (17), (18), (23), (24), (28), and (29)), let us look at the initial condition. From observational WMAP data, we know the following present day values: $H_0 = 1/\sqrt{3}$, ${\rho_b}_0 = 0.238$, ${\rho_r}_0 = 7.826603 \times 10^{-5}$. We assume that there is no dilaton and exotic matter in our current universe. To find the present day values of $\dot{\phi}$ and $Q$, we employ the following equations, which are also derived from the dynamical equations of the theory:

$$2Q^2 - e^{-\phi} QH + e^{-2\phi}\left(\dot{\phi}^2 - 8H^2 - 3H\dot{\phi} + \frac{5}{2}\rho + \frac{1}{2}p\right) = 0 \tag{31}$$

$$q = -\frac{1}{H^2}\left(\frac{2}{3}H^2 - \frac{2}{3}\dot{\phi}^2 + \frac{e^{\phi} Q}{3}\left(H + \dot{\phi}\right) - \frac{\rho}{2} - \frac{p}{6}\right) \tag{32}$$

Recall that $q$ is the deceleration parameter, and $q_0 = -0.61$. Upon solving (30) and (31), we get $Q_0 = 1.066747$ and $\dot{\phi}_0 = -0.211678$. Notice that the quadratic equation (30) gives two sets of solution, only one of which provides a physically plausible solution of the quantities in question. The mentioned solution is checked by various constraints, which we will discuss in chapter III.

Combining equations (16) and (21), we have the density factor to be:

$$R = \frac{3H_*^2}{\rho_{r*}}\, e^{\phi_*} \tag{33}$$

Again, the star denotes quantities measured at freeze-out. The variable of integration is redshift z, which goes from $z_0 = 0$ to $z_f \sim 10^{16}$. The value of redshift at freeze-out temperature is calculated like in standard cosmology, using (11).

The system of equations is extremely sensitive to the input value of $w_e$, the constant in the equation of state of exotic matter. This constant is also our only controlled parameter.

**Method of integration**

The algorithm presented in this section is adapted from [12] and [13]. The equations (17), (18), (23), (24), (28) and (29) are numerically solved using the embedded Runge-Kutta pair algorithm. Although software such as Matlab and Mathematica employ such algorithms in their ODE solver, the system of equations we have proves to be too stiff to be solved by the software. Hence, we constructed a numerical solver in Fortran 90 for this work. Consider the equations in the form:

$$\dot{\mathbf{y}}(z) = f\big(z, \mathbf{y}(z)\big) \tag{34}$$

where $\mathbf{y} = \big(\phi, \dot{\phi}, H, \rho_b, \rho_r, \rho_e, Q\big)$ is the phase space vector.

A Runge-Kutta algorithm estimates the values $\mathbf{y}_n \approx \mathbf{y}(z_n)$ for a finite set of values of z. The finite step size of step $n$ is defined to be $k_n = z_{n+1} - z_n$, which changes accordingly to produce the most precise approximation possible with a fixed tolerance. Thus, the initial step size does not really matter, and we choose $k_0 = 1$. The non-autonomous equations can be expressed in the form:

$$\mathbf{y}_{n+1} = \mathbf{y}_n + k_n \sum_{i=1}^{s} b_i f_i \tag{35}$$

where

$$f_1 = f(z_n)$$

$$f_i = f\left( z_n + d_i k_n + k_n \sum_{j=1}^{i-1} a_{ij} f_j \right), \quad i = 2,3, \dots, s \tag{36}$$

and $s$ is the number of stages of evaluating the derivatives, $a_{ij}$, $b_i$, and $d_i$ are parameters of the algorithm. The values of these coefficients are given in a Butcher table (Table 1). When we allow adaptive step size for each step, we need a method to determine their appropriate values. The Runge-Kutta algorithm of higher order usually gives better accuracy, but is less efficient because it has more stages. So we can choose between accuracy and efficiency by measuring the difference between the calculation in algorithm of order $p$ and that in algorithm of order $p + 1$, and compare that difference with the tolerance level **T**. This procedure would also determine which appropriate step size to use. To avoid evaluating the derivatives $f_i$ with 2 sets of values of $a_{ij}$ for the two orders, we use a Runge-Kutta embedded pair:

$$\hat{\mathbf{y}}_{n+1} = \hat{\mathbf{y}}_n + k_n \sum_{i=1}^{s} \hat{b}_i f_i$$

$$\mathbf{y}_{n+1} = \mathbf{y}_n + k_n \sum_{i=1}^{s} b_i f_i \tag{37}$$

$$f_i = f\left( \hat{\mathbf{y}}_n + k_n \sum_{j=1}^{i=1} a_{ij} f_j \right), \quad i = 1,2, \dots, s$$

The hat indicates that the quantities are calculated using the higher order method (order $p+1$), and quantities without the hat is calculated using the $p^{th}$ order algorithm. We take the norm of the local error of step $n$, which is defined as:

$$\delta_{n+1} = \mathbf{y}_{n+1} - \hat{\mathbf{y}}_{n+1} \tag{38}$$

to check against the specified tolerance $\mathbf{T}$. Any norm can be used, and we use the max norm here. If $\|\delta_{n+1}\| \leq \mathbf{T}$, then the step is successfully calculated within tolerance, and the step size for the next step is:

$$k_{n+1} = \min\left(10k_n, 0.9k_n\left(\frac{\mathbf{T}}{\|\delta_{n+1}\|}\right)^{\frac{1}{p+2}}\right) \tag{39}$$

If $\|\delta_{n+1}\| > \mathbf{T}$, then the step is rejected and recalculated using a smaller step size, which is determined by:

$$k_n^{NEW} = 0.9\, k_n^{OLD}\left(\frac{\mathbf{T}}{\|\delta_{n+1}\|}\right)^{\frac{1}{p+2}} \tag{40}$$

In our work, we find that the program with $\mathbf{T} = 1 \times 10^{-13}$ gives the best performance. We also use $p = 4$, i.e. the algorithm RK5(4) for non-autonomous equations.

| $a_{ij}$ | | | | | | $\hat{b}_i$ | $b_i$ | $d_i$ |
|---|---|---|---|---|---|---|---|---|
| | | | | | | $\frac{35}{384}$ | $\frac{5149}{57600}$ | |
| $\frac{1}{5}$ | | | | | | 0 | 0 | $\frac{1}{5}$ |
| $\frac{3}{40}$ | $\frac{9}{40}$ | | | | | $\frac{500}{1113}$ | $\frac{7571}{16695}$ | $\frac{3}{10}$ |
| $\frac{44}{45}$ | $-\frac{56}{15}$ | $\frac{32}{9}$ | | | | $\frac{125}{192}$ | $\frac{393}{640}$ | $\frac{8}{10}$ |
| $\frac{19372}{6561}$ | $-\frac{25360}{2187}$ | $\frac{64448}{6561}$ | $-\frac{212}{729}$ | | | $-\frac{2187}{6784}$ | $-\frac{92097}{339200}$ | $\frac{8}{9}$ |
| $\frac{9017}{3168}$ | $-\frac{355}{33}$ | $\frac{46732}{5247}$ | $\frac{49}{176}$ | $-\frac{5103}{18656}$ | | $\frac{11}{84}$ | $\frac{187}{2100}$ | 1 |
| $\frac{35}{384}$ | 0 | $\frac{500}{1113}$ | $\frac{125}{192}$ | $-\frac{2187}{6784}$ | $\frac{11}{84}$ | 0 | $\frac{1}{40}$ | 1 |

Table 1 – Embedded Runge-Kutta pairs for RK5(4).

# CHAPTER III

# RESULTS

## The *w*-*R* correlation and cosmological constraints

After various calculations, we observed that the reduction factor $R$ (equation 33) is very sensitively dependent on the exotic matter ratio $w_e$ (Table 2). This is true for both hadronic matter (with typical mass of 1GeV, freeze-out redshift $z \sim 6 \times 10^{10}$) and the neutralino as the lightest supersymmetric particle (LSP) candidate. The value of $R_{hadron}$ (reduction factor of hadron density) provides a stringent constraint on what value of $w_e$ is physically plausible, as we know from cosmological data that the dilaton should not affect the relic density of baryonic matter. Hence the values $w_e \sim 0.38$, which gives $R_{hadron} \sim 1$, are the most preferable choices. From that, we obtain a range of acceptable values of $R_{LSP}$, the reduction factor of the neutralino density:

$$R_{LSP} = 0.027 - 0.116$$

This result is certainly not a fixed range, since various fine tunings can be made to improve the precision. The seemingly random values of $w_e$ were chosen for inspection as we were looking for the values of which one can find an enhancement in the relic density of dark matter, to suit the new discovery by PAMELA [14]. But obviously from the calculation, such enhancement from the rolling dilaton is made impossible by the constraint of $R_{hadron}$. For the same reason, we stopped the calculation at $w_e = 0.4$, where $R_{hadron} \sim 0.1$.

| w_e | R_LSP | R_hadron |
|---|---|---|
| 0.2 | 1.49E+08 | 1.34E+06 |
| 0.3 | 8927 | 2226 |
| 0.3185 | 1046 | 549.2 |
| 0.319 | 984.7 | 528.0 |
| 0.32 | 872.7 | 488.1 |
| 0.35 | 17.71 | 38.97 |
| 0.36 | 4.158 | 15.32 |
| 0.37 | 0.881 | 5.666 |
| 0.38 | 0.165 | 1.938 |
| 0.382 | 0.116 | 1.546 |
| 0.383 | 9.69E-02 | 1.379 |
| 0.385 | 6.75E-02 | 1.093 |
| 0.39 | 2.67E-02 | 0.598 |
| 0.4 | 3.97E-03 | 0.159 |

Table 2 – Values of reduction factors $R$ of LSP and hadron for corresponding $w\_e$. $R_{hadron}$ is measured at $z = 10^{16}$. $R_{LSP}$ is measured at $z = 6 \times 10^{10}$.

Another constraint given by cosmological data is that at primordial nucleosynthesis ($T \sim 1MeV$), radiation must significantly dominate matter. Hence, using equation (11), we need to check if $\rho_b \ll \rho_r$ at $z \sim 10^9$. A quick look at the behavior of the densities revealed that all $w_e$'s in the displayed range in Table 1 satisfy this condition in acceptable degree.

We do not examine negative values of $w_e$ since that would imply that exotic matter has negative pressure, acting like a dark energy term. Also, notice that the factor R increases as $w_e$ decreases. When $w_e = 0$, $R_{LSP} = 3.8 \times 10^{14}$. Such high enhancement factor is obviously ruled out by observational data, so exotic matter cannot have zero pressure like normal matter.

**The behavior of various quantities in specific cases of $w_e$**

*First case*: $w_e = 0.382$

As motivated by Lahanas et al, we started this project with the intention to look for a reduction factor of 10 of dark matter density. Hence we shall first pay attention to the case where $w_e = 0.382$, and correspondingly, $R_{LSP} = 0.116$.

Here we have reproduced the result from Lahanas et al., with perfect agreement up to $z = 2$ (Figure 1). In figures 2 and 3, we show the behavior of the dilaton $\phi$, Hubble parameter $H$, and central charge deficit $Q$ as a function of redshift from today to freezeout time of neutralino ($z \sim 1 \times 10^{16}$). As these quantities differ drastically from one another, it is most convenient to plot them separately to observe their behavior. One can see that the dilaton stays relatively constant although its small change leads to significant growth of the central charge deficit as one goes further back in time. In figure 4, it is clear that radiation becomes predominant over matter at $z \sim 1 \times 10^4$, which satisfies the primordial nucleosynthesis constraint.

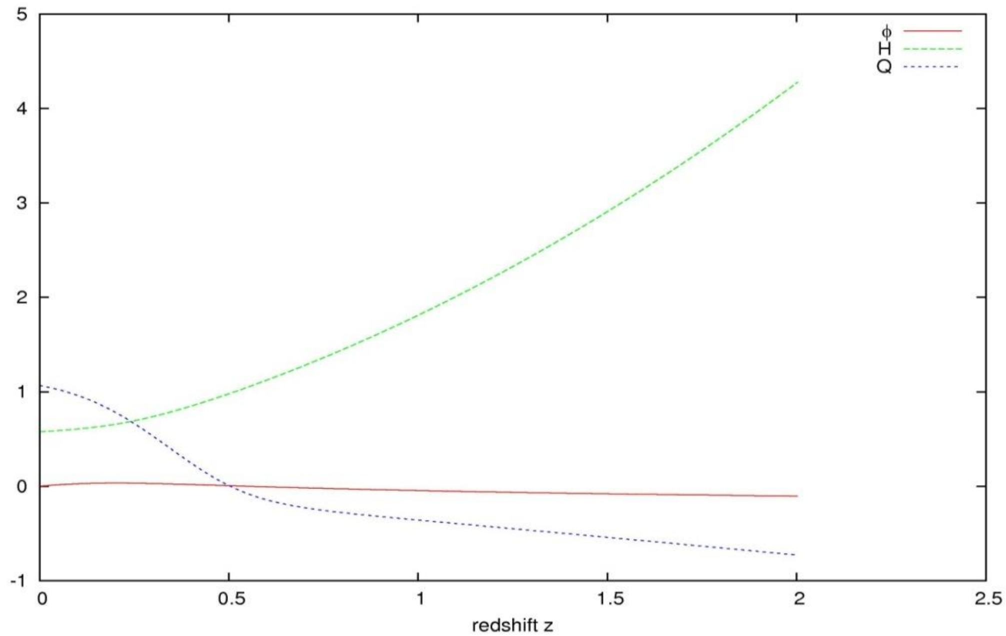


Figure 1 – The behavior of $\phi$, $H$, and $Q$ up to $z = 2$, for the case of $w_e = 0.382$.

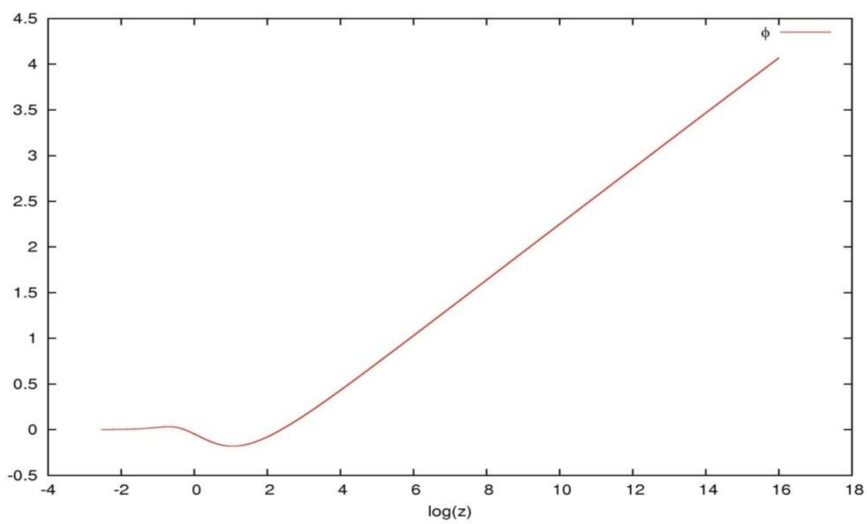


Figure 2 – The behavior of the dilaton $\phi$ with respect to redshift from today to neutralino freezeout temperature. Notice that the $x$-axis is measured in logarithm of $z$.

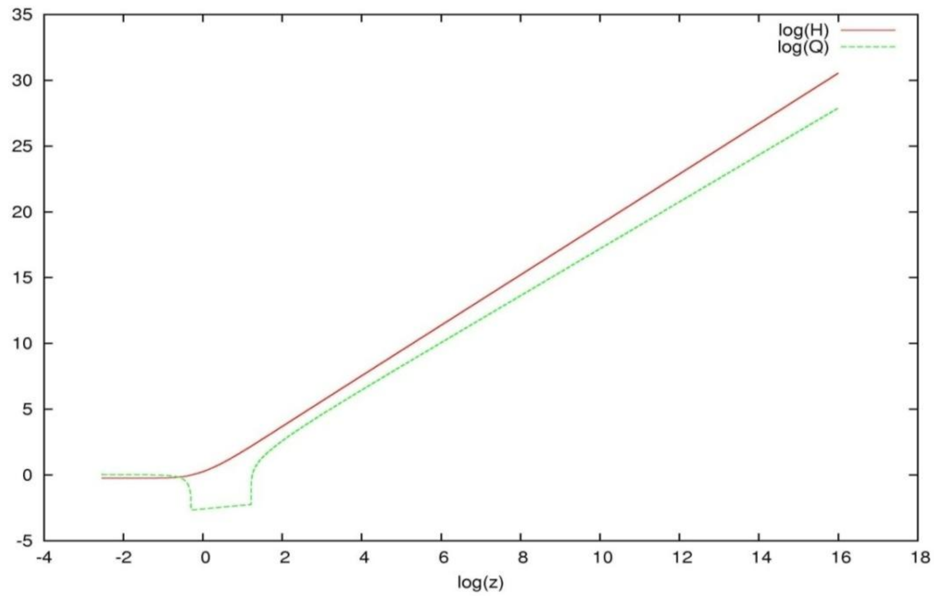


Figure 3 – The behavior of Hubble parameter $H$ and central charge deficit $Q$ with respect to redshift from today to neutralino freezeout temperature. Notice that both axes are measured in logarithmic scale.

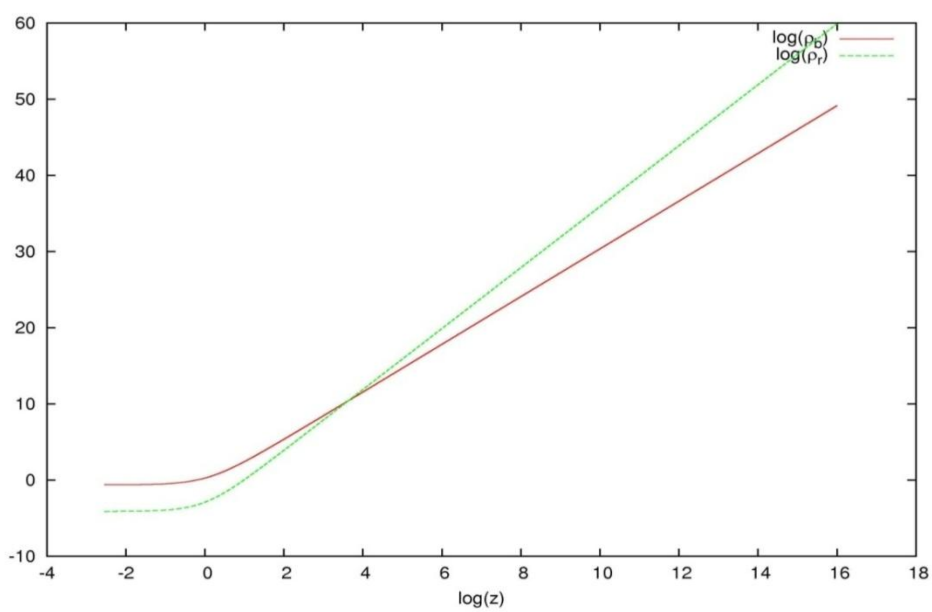


Figure 4 – The behavior of matter density $\rho_b$ and radiation density $\rho_r$ with respect to redshift from today to neutralino freezeout temperature. Both axes are measured in logarithmic scale.

*Second case*: $w_e = 0.385$

For completeness, we should look at the case where $R_{hadron}$ is closest to unity. Again, we examine the behavior of $\phi, H, Q, \rho_b, \rho_r$ with respect to redshift (Figure 5). Surprisingly, there is not much visible difference between this case and the previous, as the quantities grow to approximately the same order of magnitude in both cases. We can deduce that the drastic change in $R_{LSP}$ must come from the tiny fluctuations of $\phi$ (see equation (33)). This reaffirms the sensitivity of the reduction factor on the equation on state of exotic matter, shown in Table 2.

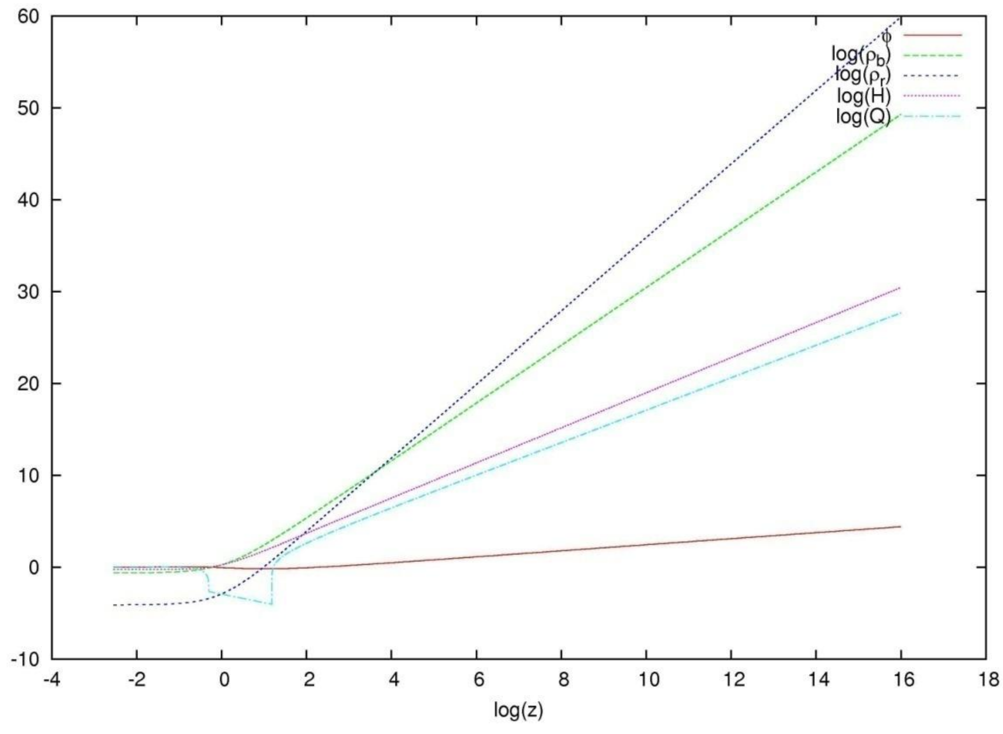


Figure 5 – The behavior of $\phi$, $H$, $Q$, $\rho_b$, and $\rho_r$ with respect to redshift up to neautralino freezeout temperature for $w_e = 0.385$. Notice that all quantities, except $\phi$, are measured in logarithmic scale.

*Third case*: $\rho_e = 0$ *at all times*

This case is an experiment on the importance of exotic matter in this model. When we set the rate of change of exotic matter density to zero at all time, i.e. prevent exotic matter to enter the picture altogether, the program does not work for large value of *z*. The set of differential equations becomes too stiff to be solved, as $\phi$ and Q grow to large negative values too quickly. Thus, one can see that exotic matter plays a vital role of balancing the effect of the dilaton and the central charge deficit on the behavior of other observables in the universe.

**Error analysis**

From equation (26), we see that theoretically $\rho_r \sim \mathcal{O}(60)$ at the freeze-out redshift $z_f = 10^{16}$ (since ${\rho_r}_0 \sim \mathcal{O}(-5)$. Our calculation does satisfy this condition. Furthermore, equation (31), which must hold at all time, provides a way to check the precision of our calculation. Let us call the right hand side of (31) *E* (for "error"), then theoretically $E = 0$ for all *z* in the range of interest. At each step of our numerical calculation, we calculate *E*. The more *E* deviates from zero, the less precise our result becomes. For $w_e = 0.385$ and $w_e = 0.382$, *E* grows to as large as $\mathcal{O}(44)$ in order of magnitude.. Since in both cases, the calculated quantities and their products, such as radiation density $\rho_e$, are as large as $\mathcal{O}(60)$, we can say that the error is relatively small (16 orders of magnitude smaller). Thus, the result is reliable.

# CHAPTER IV

# SUMMARY AND CONCLUSIONS

In our work we have investigated the effect of the established theory of Supercritical String Cosmology (SSC) [6, 8, 9] on the relic density of dark matter in our universe. Currently, the Einstein model with the Robertson-Walker-Friedmann metric does not explain the so-called cosmological constant Λ. Thus, SSC introduces new quantities, i.e. the scalar field dilaton, the central charge deficit, and exotic matter, into the theoretical picture to explain Λ. SSC also allows the dilaton to be time-dependent; and we have shown that the time-dependent dilaton, which indeed has a profound effect on the changing of matter density, can be manipulated by a fine tuning of the equation of state of exotic matter. More specifically, using the same assumptions made by Lahanas et al [8, 9], such as the effect of SSC on the freeze-out temperature of a type of particle is negligible, we have produced a more precise measurement of the density factor $R$ corresponding to the ratio $w_e$ between the density and pressure of exotic matter. We require that SSC does not alter the density of hadronic (normal) matter, as this quantity can be measured via astronomical observations. This requirement provides a constraint on physically acceptable values of $R$ and $w_e$. Therefore, we have shown here that the neutralino, a strong candidate for dark matter composition, can only be diluted, as enhancement factors of $R_{LSP}$ are not allowed. Given that the numerical calculation is only to a good approximation, and with various assumptions, we can accept values of $w_e$ which give $R_{hadron} \sim \mathcal{O}(1)$. Correspondingly, the range of acceptable values of

$R_{LSP}$ $(\sim 0.026 - 0.116)$ opens a new window in the parameter space for direct detection and LHC detection of dark matter. Nonetheless, we are currently investigating different possibilities to obtain an enhancement factor of neutralino density. One can have a time-dependent, and/or dilaton-dependent $w_e$, or a time-dependent CP-violation to counterbalance the enhancement factor of hadron density. There are many interesting routes to continue this study on SSC and what makes our universe today.

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