ASSESSING EC-4 PRESERVICE TEACHERS' MATHEMATICS KNOWLEDGE

FOR TEACHING FRACTIONS CONCEPTS

A Thesis

by

KIMBERLY BODDIE WRIGHT

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2008

Major Subject: Curriculum and Instruction

# ASSESSING EC-4 PRESERVICE TEACHERS' MATHEMATICS KNOWLEDGE FOR TEACHING FRACTIONS CONCEPTS

A Thesis

by

KIMBERLY BODDIE WRIGHT

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Co-Chairs of Committee, Dianne S. Goldsby
Yeping Li
Committee Members, G. Donald Allen
B. Stephen Carpenter, II
Head of Department, Dennie L. Smith

August 2008

Major Subject: Curriculum and Instruction

# ABSTRACT


Assessing EC-4 Preservice Teachers' Mathematics Knowledge for Teaching Fractions Concepts. (August 2008)

Kimberly Boddie Wright, B.S., Texas A&M University

Co-Chairs of Advisory Committee: Dr. Dianne Goldsby
Dr. Yeping Li

Recognizing the need for U.S. students' mathematics learning to be built on a solid foundation of conceptual understanding, professional organizations such as the National Council of Teachers of Mathematics (2000) and the Conference Board of the Mathematical Sciences (2001) have called for an increased focus on building conceptual understanding in elementary mathematics in several domains. This study focuses on an exploration of two aspects of Hill, Schilling, and Ball's (2004) mathematics knowledge for teaching: specialized content knowledge (SCK) and knowledge of content and students (KCS) related to fractions concepts, an area that is particularly challenging at the elementary level and builds the foundation for understanding more complex rational number concepts in the middle grades. Eight grades early childhood through four preservice teachers enrolled in a mathematics methods course were asked to create concept maps to describe their knowledge of fractions and interpret student work with fractions.

Results showed the preservice teachers to be most familiar with the part-whole representation of fractions. Study participants were least familiar with other fraction representations, including fractions as a ratio, as an operator, as a point on a number line, and as a form of division. The ratio interpretation of a fraction presented the greatest difficulty for study participants when asked to describe student misconceptions and create instructional representations to change students' thinking.

# DEDICATION

To my students whose curiosity about mathematics made
me want to become a better teacher through study. Thank you!

ACKNOWLEDGEMENTS

I would like to thank my committee co-chairs, Dr. Dianne Goldsby and Dr. Yeping Li, and my committee members, Dr. G. Donald Allen and Dr. B. Stephen Carpenter, II, for their guidance, support, and inspiration throughout the course of my research.

Thanks also to my fellow members of the Math TEKS Connections Project for imparting wisdom in all areas of research and academia. I would especially like to thank Dr. Jackie Stillisano for being a mentor in every sense of the word and Dr. Robert Capraro for giving me a great research opportunity. Also, thank you to the EC-4 preservice teachers of Texas A&M University, whose willingness to share their thinking made this research possible. I would also like to thank all graduate students along the way who imparted advice, experience, and deadline reminders to help me get through.

Finally, I would like to thank my family and friends. Thank you to my wonderful husband Terrell for love, understanding, and support. Thank you to my mom for answering late night phone calls as I walked home from class. A final thank you is owed to the faculty, staff, and students of Olmos Elementary School in San Antonio, who inspired me to become a better teacher.

# TABLE OF CONTENTS

# LIST OF FIGURES

FIGURE Page

# LIST OF TABLES

# CHAPTER I

# INTRODUCTION

With mathematics scores of U. S. elementary students falling behind those of other industrialized nations (National Center for Education Statistics, 2005), professional organizations such as the National Council of Teachers of Mathematics (2000) and the Conference Board of the Mathematical Sciences (2001) have called for an increased focus on building conceptual mathematics understanding in elementary students in several domains. An area that is particularly challenging at the elementary level is that of fraction concepts, which builds the foundation of understanding more complex rational number concepts in the middle grades. For both elementary and middle school students, the rational number system, ". . . constitutes what is undoubtedly the most challenging number system of elementary and middle school mathematics." (Kilpatrick, Swafford, & Findell; 2001, p. 231)

Many students in the U. S. understand fractions concepts in terms of a limited store of representations and algorithmic procedures. Because of this, students are often unsuccessful when asked to consider fractional concepts in different contexts. According to the 2005 results of the National Assessment of Educational Progress (NAEP), 83% of U.S. fourth graders were able to choose which of four pictorial representations correctly showed $\frac{3}{4}$, but only 53% were able calculate a student's total number of apple pieces if he had two apples cut into fifths (NAEP, 2005). Elementary students' difficulties with

---

This thesis follows the style of *Journal for Research in Mathematics Education.*

fraction concepts are well-documented in the research literature, with 20 years of NAEP data highlighting the fact that only about one-third of students up to the seventh grade have been successful in determining fraction equivalency since the 1970s (Kamii & Clark, 1995). With a significant body of literature documenting elementary students' difficulties with fraction concepts, mathematics education research in the last 15 years has begun to focus on the role teacher knowledge might play in contributing to students' limited understanding of fractions.

Beginning with Shulman's (1986) conception of pedagogical content knowledge, early research focused on how teachers' knowledge of pedagogy might influence instruction, including the most useful representations and analogies to present content, as well as student conceptions and misconceptions. Recent research has added additional dimensions to Shulman's early work, focusing more directly on the influence of teacher knowledge of mathematics on instruction and how that knowledge might be characterized and measured (Ball, 2003; Hill, Schilling, & Ball, 2004; Ma, 1999).

Because teachers often possess the same limited store of procedures and representations for fractions concepts from their own schooling, the problem becomes one of "helping teachers transcend their own school experiences with mathematics in order to create new practices of mathematical pedagogy" (Ball, 1992, p. 395). Teachers are not likely to encourage conceptual understanding of mathematics in students if they do not possess conceptual understanding of mathematics themselves.

## *Statement of the Problem*

If U.S. elementary students' knowledge of fraction concepts is to move beyond mainly procedural knowledge toward deep conceptual understanding that will allow students to flexibly solve mathematics problems in a variety of situations, U.S. elementary teachers must enter the field prepared to teach fraction concepts conceptually and with an understanding of various uses and representations of fractions. It is important elementary teachers be able to understand and use fractions flexibly in various constructs as elementary teachers are expected to communicate multiple representations of concepts, including representing fractions as ". . . part of a whole, as an expression of division, as a point on a number line, as a rate, or as an operator" (Conference Board of the Mathematical Sciences, 2001, p. 19). In addition, elementary teachers must be able to communicate the various constructs of fractions using instructional representations and contexts that both activate and challenge students' currently held fraction understandings.

Preservice teacher preparation programs provide a critical link between the mathematics understanding that preservice teachers held as students and the types of mathematics understanding they are responsible for conveying to their students. Preservice mathematics programs should help preservice teachers build a conceptual understanding of the mathematics concepts they will teach, including various interpretations of fractions and how students may confuse them. This type of multi-faceted understanding often differs greatly from the mainly procedural knowledge of fraction concepts they bring with them to higher education. Though many preservice teachers do not enter teacher preparation programs equipped with conceptual understandings of fraction concepts, teacher education programs remain a ". . . strategically critical period in which change can be made" (Ma; 1999, p. 149) because preservice teacher education programs provide preservice teachers with opportunities to consider mathematical knowledge in terms of how it can be successfully conveyed to students, linking preservice teachers' knowledge of elementary fraction concepts from their own schooling and that learned in college-level elementary mathematics courses to the best ways to transmit knowledge to students.

The purpose of this study was to determine the depth of elementary preservice teachers' knowledge of fractions. The study conducted two-part interviews with a group of eight preservice teachers enrolled in a mathematics methods course at a large university in the southern U. S. The teachers were all seeking certification in grades pre-kindergarten through four (EC-4). The primary purpose of the study was to capture the depth and breadth of the preservice teachers' specialized content knowledge of the concept of a fraction. During part one of the two-part interview, the preservice teachers were asked to create a concept map representing their knowledge of fractions.

A secondary purpose of the study was to explore the EC-4 preservice teachers' knowledge of students and content related to student misconceptions of fraction concepts including equivalence of part-whole fractions, ordering of fractions on a number line, interpreting a ratio, and a fraction as an operator on a whole number. Part two of the interview was designed to elicit EC-4 preservice teachers' understanding of students' fraction misconceptions in problem solving situations.

Participants were presented with student misconceptions in various types of fraction problems and were asked to create hypothetical learning trajectories (Simon, 1995) for student. The *hypothetical learning trajectory* is part of Simon's (1995) *mathematics teaching cycle* theory that attempts to explain the thinking process of mathematics teachers as they diagnose, plan for, and observe student thinking. Participants were asked to identify a mathematical goal for a student to help them develop a more complete understanding of a particular fraction concept and discuss instructional representations they would use to help the student to facilitate deeper understanding.

### *Research Questions*

This study explored two aspects of Hill, Schilling, and Ball's (2004) *mathematics knowledge for teaching*: specialized content knowledge (SCK) and knowledge of content and students (KCS), in a group of EC-4 preservice teachers enrolled in a mathematics methods course. These two aspects of mathematics knowledge for teaching were the focus of the study because they are most closely aligned with the goal of teacher preparation programs, which is to connect preservice teachers' current mathematical knowledge with a concern for how they will teach that mathematics to students (Ma, 1999). The research questions were the following:

1. What specialized content knowledge (SCK) is present in EC-4 preservice teachers' representations of their fractional knowledge?

a. Which interpretations of fractions, such as part-whole, ratio, division, operator, and point on a number line, are present in EC-4 preservice teachers' representations of their fractional knowledge?

b. What additional information, such as fractional models, notations, and terminology, is present in EC-4 preservice teachers' representations of their fractional knowledge?

2. What knowledge of content and students (KCS) is present in EC-4 preservice teachers' analysis of student misconceptions related to fractions?

    a. To what do EC-4 preservice teachers attribute student misconceptions when working with various interpretations of fractions?

    b. What specific types of instructional representations of fractions content do EC-4 preservice teachers identify to help students correct their misconceptions with fractions?

# CHAPTER II

# BACKGROUND LITERATURE

It is typically at the elementary level where students transition from operating only with whole numbers to operating with rational numbers in the form of fractions. At this level, students are introduced to a new set of rules regarding the relationship between quantities represented by rational numbers and encounter a new set of symbols and representations for this set of numbers. Adding to the confusion of dealing with a new type of number is the fact that many of these students' teachers learned about fractions themselves according to a set of rote procedures and operations and are ill-equipped to provide a conceptual foundation for fraction understanding. Mathematics education reform recommendations (CBMS, 2000; NCTM, 1991, 2000, 2006) call for teachers to teach fraction concepts in ways often fundamentally different from the way they learned them, in ways that cannot be successfully accomplished without deep understanding of the subject matter and the ability to represent and encourage conceptually sound representations of fractions concepts.

This study sought to create a detailed picture of EC-4 preservice teachers' fraction knowledge by exploring the preservice teachers' specialized content knowledge of the concept of fraction using a participant-created concept map. The study also explored the knowledge of content and students of the preservice teachers through their diagnosis of student misconceptions and choice of representations to instruct students.

*Student Difficulties with Fraction Concepts*

In a study of third through sixth-grade students' rational number understanding, Smith, Solomon, and Carey (2005) hypothesized that a fundamental lack of understanding of the properties of rational numbers by elementary students is what makes rational number concepts so difficult for students. In clinical interviews with 50 elementary school students, researchers elicited student understanding about the infinite nature of rational numbers through questions in two different areas: the divisibility of matter and the divisibility of numbers between zero and one. Although 62% of students agreed there were numbers between zero and one, only 36% acknowledge the number of numbers was infinite. Of those students who initially identified the infinite nature of rational numbers, 50% of students said yes when asked whether infinite division would eventually reach zero, revealing their rational number understanding to be only partial.

The results of the Smith, Solomon, and Carey study were consistent with those found in a similar study with 16 ninth-grade students (Vamvakoussi & Vosniadou, 2004). Researchers considered students to have what they referred to as "deep understanding of rational numbers" if they were, ". . . able to answer that between any two different rational numbers, no matter of the way they are represented, there are infinitely many numbers" (p. 460). No students in the study were able to maintain there were an infinite number of rational numbers between any two numbers when presented with different types of rational number problems.

In addition to exploring students' understanding of the nature of rational numbers, Smith, Solomon, and Carey (2005) also examined students' reasoning in comparing fractions. When asked to compare $\frac{1}{75}$ and $\frac{1}{56}$, 46% of students chose 1/75 as the larger fraction because 75 was larger than 56. Students were using whole number properties to reason about rational numbers, a form of reasoning not uncommon for students to employ when working with fractions. In previous work with third and fourth-grade students, Mack (1995) also found a tendency in elementary students to over generalize the meanings of symbolic whole number representations to fractions.

In the area of fraction equivalency, Kamii and Clark (1995) interviewed 120 fifth and sixth-grade students, asking them to compare two pictorial representations of the fraction one-half. Both representations showed rectangles of the same size divided into two equal parts, however one was divided horizontally and the other divided diagonally. Thirty-eight percent of fifth-graders said that the diagonal half was greater. Forty-four percent of fifth-graders correctly identified that the indicated pieces in both contexts were halves. However, when questioned about which half was greater, 5% of students who were initially correct said that even though both pieces were halves of the same-sized whole, the amounts contained in each piece were different. The sixth-grade students showed slightly more correct reasoning, with 51% initially correct and 32% of the remaining students changing their answer to correct when questioned about whether one half was greater than the other.

Niemi (1996) assessed 540 fifth-grade students' conceptual understanding of equivalency through their interpretation, use, and creation of graphic and symbolic representations with both equivalent and non-equivalent fractions. Students were given the fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{3}{2}$ 3/2 separately and asked to circle which of 18 representations were equivalent to the given fraction. Most students showed only moderate levels of fraction knowledge, with means ranging from 13.5 out of 18 for the easiest fraction $\frac{1}{2}$ to 10.55 for $\frac{3}{2}$. Equivalent representations of the fraction ½ were found to be significantly easier ($p<.01$) than those of $\frac{2}{3}$. The same difference was found in the level of student difficulty in identifying representations for $\frac{2}{4}$ and $\frac{4}{6}$. In problem solving and explanation tasks, the researcher found that, "students with higher levels of representational knowledge produced more principled justifications and explanations and were able to generate and use fractions more effectively in problem solving contexts" (p. 355).

Though much of the research on rational number and, more specifically, fraction knowledge of elementary students, has documented an impartial and often disconnected understanding, instruction with different types of representations for fractions has shown to increase students' understanding of fraction concepts. In their work with fourth and fifth-grade students, Olive (2002) and Steffe (2004) found that whole number knowledge could be reorganized for rational number understanding. In the context of commensurate fractions, both researchers used computer software to introduce students to the concepts of iterating and partitioning a unit into equal pieces. They found that the ability to iterate or partition a unit to create and break apart wholes into fractional pieces was a critical component of students' development of deep understanding of fractions.

In a longitudinal study with five classes of third through sixth-grade students, Lamon (2001) hypothesized that traditional fraction instruction, with its common $\frac{a}{b}$ part-whole representation of fractions, also noted by Carraher (1996), was a factor limiting students' development of rational number concepts. Building on Kieren's work (1976, 1980) with different interpretations of fractions, Lamon provided each of the five classes with a different initial interpretation of the traditional $\frac{a}{b}$ part-whole representation for fractions. The interpretations consisted of part-whole comparisons with unitizing, fractions as operators, fractions as ratios and rates, fractions as quotients, and fractions as measures. A control class received traditional fraction instruction focusing mainly on the part-whole interpretation of fractions. After four years in the study, all experimental groups exceeded the control group on the number of interpretations shown in the representations given to solve fraction problems.

*Preservice Teachers' Subject Matter Knowledge of Rational Numbers*

In the last two decades, research has examined the role teacher knowledge might play in contributing to students' limited understanding of mathematics. However well-meaning they are, teachers are influenced by their own lack of conceptual understanding of the mathematics concepts they teach. Because teachers are often products of procedural instruction from their own schooling, a challenge of teacher education becomes, "helping teachers transcend their own school experiences with mathematics in order to create new practices of mathematical pedagogy" (Ball, 1993, p. 395).

In the mid-eighties, Leinhardt and Smith (1985) examined the relationship between the subject matter knowledge and teaching of novice and expert elementary teachers using semantic nets to describe teacher knowledge. Hypothesizing high levels of student achievement would correspond to high levels of teacher knowledge; they found high levels of student achievement were not necessarily equivalent to teacher knowledge. In fact, there was great variability in teacher knowledge, even among those teachers labeled as experts. Researchers found the distinctions between teachers with high and low levels of knowledge lay in the connections between topics and forming broad categories of knowledge. In other words, teachers with the ability to connect topics to form broad categories of understanding were more likely to have higher levels of understanding of the mathematics they were teaching.

Building on this research, Ball (1988) charged teacher education with the task of becoming, “a more effective intervention in preparing elementary teachers to teach mathematics” through examining “the influence of different kinds of teacher education experiences on teacher candidates’ knowledge about and orientations toward mathematics and mathematics teaching and learning” (p. 16). In her work with developing preservice teachers’ understanding of rational numbers, Tirosh (1992) echoed this sentiment calling for, “more information about the impact of focusing on different forms of knowledge in teacher education” (p. 237).

Through a year-long case study, Borko et al (1992) also looked at the role teacher preparation programs play in the knowledge development of a difficult rational numbers topic, division of fractions. Researchers followed one preservice teacher from a mathematics methods course through student teaching in order to understand the development of her ability to teach division of fractions. Classroom observations showed that the preservice teacher entered her student teaching semester with “only a rote understanding of the division of fractions algorithm” (p. 207). When asked by a student why one must invert the divisor and multiply to divide fractions, the preservice teacher could not provide a correct answer. Though the topic was addressed in the mathematics methods course, researchers suspected that her previous algorithmic knowledge of “invert and multiply” from her own K-12 schooling may have interfered with her construction of a more complete understanding of the concept.

With regard to preservice teachers’ understanding of fractions in non-standard representations, Khoury and Zaskis (1994) analyzed the written and oral responses of preservice elementary and secondary teachers to tasks requiring teachers to compare fractions and decimals in different bases. Preservice teachers were asked to compare $(0.2)_3$ to $(0.2)_5$ and $\frac{1}{2}$ in base 3 to $\frac{1}{2}$ in base 5. Sixty-three percent of the elementary preservice teachers and one hundred percent of the secondary teachers correctly noted through various forms of representation that $(0.2)_3$ and $(0.2)_5$ were not equivalent. However, when asked whether the two fractions were the same, only 26% of the elementary teachers and 17% of the secondary teachers said yes. Researchers noted that an analysis of the strategies used to determine equivalency showed a disconnected knowledge of place value and rational number concepts.

Lubinski, Fox, and Thomason (1998) described the development of one student’s reasoning about division of fractions in a mathematics methods course. Researchers traced a preservice teacher's reasoning through first and third-person descriptions of how the student arrived at conceptual understanding of division of fractions through a three-week assignment that required preservice teachers to develop an explanation for how to solve a division of fractions problem. Through her struggle, authors illustrated common difficulties many preservice teachers have explaining fraction concepts conceptually. Like the teacher in Borko et al's study (1992), the preservice teacher was able to provide a correct pictorial model for multiplication of fractions, but had a difficult time developing a similar model to represent division of fractions. In addition, the preservice teacher could correctly apply the multiplicative inverse rule to divide fractions by inverting the divisor and multiplying, but she could not explain why this procedure worked.

Ma (1999) compared U. S. and Chinese elementary teachers' subject matter knowledge and found the U. S. teachers lacked "fundamental understanding of mathematics" (p. 118) largely because the teachers lacked underlying connections between topics that would allow for conceptual explanations. In a division of fractions problem, 43% of the U.S. teachers studied provided a correct procedure for solving the problem, but only one teacher provided a conceptually correct explanation of division of fractions (p. 83). Ma concluded that along with K-12 schooling and professional development for practicing teachers, teacher preparation programs are critical sites for development of preservice teachers' subject matter knowledge. According to Ma (1999), teacher preparation programs are the point in a preservice teacher's education where "mathematical competence starts to be connected to a primary concern about teaching and learning school mathematics" (p. 145).

*Improving Preservice Teachers' Subject Matter Knowledge*

In conjunction with studies on teachers' subject matter knowledge, various professional education organizations and others call for reforms in the mathematical education of teachers. In 1991, the National Council of Teachers of Mathematics (NCTM) released the *Professional Standards for Teaching Mathematics* which included recommendations for the kind of knowledge mathematics teachers should have. NCTM emphasized the need for a deep, connected understanding of mathematics and its principles and concepts as well as an ability to instruct students beyond a narrow set of algorithmic procedures. With regard to rational numbers, NCTM stated that, "in setting the view of these ideas in the curriculum, teachers should be able to extend the number systems from the whole numbers to fractions and integers, then rationals and real numbers" (p. 136). The *Professional Standards* also called for preservice teacher preparation programs and inservice professional development activities to include research from mathematics education in their curricula

In 1995, the Interstate New Teacher Assessment and Support Consortium (INTASC), a group of state and national education stakeholders, including state education agencies and national education organizations, developed a set of standards specifically for preservice and novice teachers. The INTASC Standards echoed NCTM's earlier recommendations, calling for "a deep understanding of the critical mathematical ideas, processes, and perspectives needed to help all students develop mathematical power" (p. 7).

Following the *Professional Standards for Teaching Mathematics (1991)*, NCTM released another set of standards specifying what kinds of knowledge K-12 students should have and ways in which teachers could promote such knowledge. The *Principles and Standards for School Mathematics* (NCTM, 2000) divided mathematics into five content areas: number and operations, algebra, geometry, measurement, and data analysis and probability. Information regarding rational number and fraction concepts was contained in the number and operations content strand. The *Principles and Standards* document noted that the development of rational number concepts should be a major focus at the elementary level, particularly at grades three through five where students build a foundation for navigating between fractions, decimals, and percents in the middle grades.

In 2000, the Conference Board of the Mathematical Sciences (CBMS), a consortium of various organizations dedicated to increasing knowledge of the mathematical sciences, released *The Mathematical Education of Teachers*. This set of recommendations for preservice teachers was divided by level of certification, with lists of recommendations for elementary, middle, and high school teachers. CBMS highlighted a conventional belief of many elementary preservice teachers that "elementary school mathematics is simple and to teach it requires only prescribed facts and computational algorithms" (p. 56).

According to *The Mathematical Education of Teachers*, the primary job of preservice teacher education programs should be to provide courses that give elementary preservice teachers an opportunity to learn and make connections in the same ways that they are expected to transmit material to students. With regard to developing fraction concepts, CBMS noted that fractions are indeed a difficult concept for elementary students and that preservice teachers must be able to provide students instruction with fractions beyond the common part-whole relationships, including looking at fractions as expressions of division, as points on a number line, as rates, and as operators.

The most recent efforts in mathematics education reform have also called for improvement and continued research in preservice teacher knowledge of mathematics (Kilpatrick, Swafford, & Findell, 2001; Ball, 2003; RAND, 2003). In addition, NCTM recently released *Curriculum Focal Points: A Quest for Coherence* (2006), specifying grade-level specific knowledge at each grade level for each of its five content strands. The *Focal Points*' suggestions for fractions for students in grades two through five focus on comparing fractions using multiple models in problem solving contexts.

*Preservice Teachers' Pedagogical Content Knowledge of Representations*

Teaching is an elaborate process that includes not only knowing one's subject matter, but knowing how to communicate that subject matter to students. Shulman (1986) coined the term "pedagogical content knowledge" to refer to "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (p. 9), including the most useful representations and analogies to present content, as well as student conceptions and misconceptions.

Pedagogical content knowledge relies heavily on a teacher's ability to represent knowledge to others in some form that is comprehensible to them. Cai (2005) used the term "pedagogical representations" to refer to the knowledge of representations that teachers and students use in their classrooms "as expressions of mathematical knowledge that help them explain concepts, relationships, connections, or problem-solving processes" (p. 139). Anyone who has ever tried unsuccessfully to teach a concept understands that the idea of such a pedagogical representation is complicated. For example, take the fraction ¾, which is actually a symbolic representation that itself *represents* the relationship between 2 numbers. Depending on the context, the fraction ¾ can represent 1($\frac{3}{4}$ piece), 3 ($\frac{1}{4}$ pieces), or $\frac{1}{4}$ or 3 (Lamon, 2005). This and other ideas of representation, including who represents, what is represented, and how it is represented have been topics of discussion and debate in mathematics education research for 20 years.

According to Kaput (1987), "representation and symbolization are at the heart of the content of mathematics and are simultaneously at the heart of cognitions associated with mathematical activity" (p. 22). Representations in mathematics involve two worlds: the representing world also referred to as an external representation (Goldin & Kaput, 1996) and the represented world, or internal representation in the mind of a learner. Also involved in any representation of mathematics content is the interplay between the representing and represented worlds. Goldin and Kaput (1996) referred to these interactions as a learner's interpretation of the external representation, or symbols, pictures, diagrams, or manipulatives, with internal representations already held for a concept.

For example, in the case of $\frac{3}{4}$, an external representation for $\frac{3}{4}$ may be provided in the form of the symbolic fraction itself, in the form of a set of four objects, three of which are circled, or in the form of a manipulative such as a fraction strip showing a unit whole divided into four parts with three parts shaded. A kindergarten student, when shown the symbol for $\frac{3}{4}$, having no prior experiences with formal fraction notation, will have little interaction with such a representation because they have no internal structure for understanding formal symbols for fractions. In contrast, a fifth-grade student with previous experience with formal notation, might look at $\frac{3}{4}$ and have an internal representation of a circle divided into four pieces with 3 pieces shaded or may say to themselves “three parts out of four total parts.”

Because students’ internal representations can only be interpreted in terms of ways in which they represent their understanding externally, teachers must be able to choose representations that are appropriate to students’ current levels of understanding. Goldin and Kaput (1996) recommended letting the type of expected internal representation guide the selection of external representations to be used in instruction. When considering the expected internal representation a student should have, teachers must keep in mind students’ previous experience with mathematics concepts as “the premature introduction of representations can sometimes explain their uselessness, and furthermore they may even have negative effects on learning” (Dufour, Bednarz, & Belanger, 1987, p. 116).

Dufour, Bednarz, and Belanger (1987) pointed out difficulties elementary students have transitioning between the use of number lines to represent whole numbers to the use of number lines to represent rational numbers and integers. Number lines are initially external representations for whole numbers, which are equally spaced and demarcated on the number line. The space between the numbers means nothing to students. However, when teachers introduce the number line, a familiar external representation to students, but impose rational numbers in what are essentially the empty space between the whole numbers, students often cannot understand that fractional numbers can fall between whole numbers. The premature use of the number line as a representation for fractions could explain why elementary students and secondary students were unable to determine the infinite number of rational numbers between two whole numbers (Smith, Solomon, & Carey, 2005; Vamvakoussi & Vosniadou, 2004).

Studies with elementary preservice teachers (Borko et al, 1992; Goulding, Rowland, & Barber, 2002; Ward, Anhalt, & Vinson, 2004) have shown the preservice teachers to have an overall weak representational knowledge and rely heavily on algorithms and formal symbolism to represent mathematics concepts. In an analysis of the lesson plans of K-8 elementary preservice teachers enrolled in a mathematics methods course, Ward, Anhalt, and Vinson (2004) found trends in the preservice teachers’ choice of representations. Though researchers anticipated high levels of concrete representations in the lessons, they found technical language and explanations to be the most frequently used representation with an infrequent use of real world representations and contexts. In a case study with one elementary preservice teacher, Borko et al (1992) found similar results in the teacher's ability to parrot the familiar "invert and multiply" algorithm, but found that after several tries, she was unable to illustrate how division of fractions worked. These studies help to explain earlier findings (Kieran, 1991) in which children have a tendency to detach algorithms for fraction computations from meaningful representations. According to Kieran (1991), "it would appear that fractions form a symbolic domain in which students learn to operate with the syntax and certain rules of combination" (p. 325).

*Improving Preservice Teachers' Pedagogical Content Knowledge*

In order to break the cycle of algorithms detached from meaningful representations by students, researchers have made some suggestions regarding teachers' use of representations with students, including language, multiple relationships present in representations, and use of nonstandard representations. Siebert and Gaskin (2006) addressed how the use of misleading terminology for describing fractions, including instructional terms such as "out of" (p. 397) and "over" (p. 400) can be problematic because students do not differentiate expressions such as "one out of four" or "one over four" from whole number relationships. Thompson (1995) suggested that instead of relying on an external representation to embody a specific internal representation, such as using a fraction strip partitioned into four parts with three parts shaded to represent $\frac{3}{4}$, teachers should consider exploring with the students various relationships that they see in the representation.

Greeno (1997) found that students who spend most of their time constructing external representations specified by a teacher will begin to see representations not as a tool for learning, but as “ends in and of themselves” (p. 361) He suggested providing students with the opportunity to create nonstandard representations, such as student-created drawings of mathematics concepts devoid of formal mathematics symbols and terminology, because they often better serve the purpose of helping students create immediate understanding in problem situations and reserving standard representations for purposes of communicating mathematical ideas with others.

Results of studies of teachers’ pedagogical content knowledge prompted professional organizations and other researchers to call for improvements in the pedagogical content knowledge of teachers, specifically in an improved and increased use of various types of representations. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) caution teachers against narrowing mathematics content to a set of algorithms and symbolic representations. In agreement with Kaput (1987) who discussed representations as being the heart of mathematics, NCTM noted in the *Professional Standards* with regard to instruction that, “modeling mathematical ideas is central to the teaching of mathematics” (p. 151).

However, modeling alone is not enough. Preservice teachers must carefully select representations that best fit both their students and the mathematical content of a lesson. According to the Conference Board of the Mathematical Sciences, “future teachers will need to connect to a wide variety of situations, models, and representations” (p. 56). It is the job of preservice teacher education to help preservice teachers choose representations with care, as the ways mathematical ideas are represented to students are “fundamental to how people can understand and use those ideas” (NCTM, 2000, p. 67). A careful selection of representations is particularly important when working with rational numbers and fraction concepts, as fractions have many meanings that are all too commonly represented simply as $\frac{a}{b}$.

Following the recommendations and previous work of Kilpatrick, Swafford, and Findell (2001), McGowen and Davis (2002), Ball (2003), Hill, Schilling, and Ball (2004), the present study sought to create a detailed picture of the fraction knowledge of a group of EC-4 elementary preservice teachers. There has been much research conducted on preservice teachers’ knowledge of rational number concepts. However, much of it has focused on specific concepts, such as division of fractions or students’ interpretations of the infinite nature of rational numbers. Though these topics are important, little research has shown what elementary preservice teachers know about various interpretations of the concept of a fraction or how preservice teachers diagnose and plan for instruction related to student misconceptions of the various interpretations of fractions at the elementary level.

# CHAPTER III

# METHOD

The intent of this study was to explore the specialized content knowledge (SCK) and knowledge of content and students (KCS) related to fraction concepts of a group of EC-4 preservice teachers. Topics explored included EC-4 preservice teachers' knowledge of the concept of a fraction as well as their knowledge of student misconceptions related to work with fractions in varying problem contexts and interpretations. The study utilized a cross-sectional survey design (Creswell, 2005) to explore the beliefs and practices of a group of EC-4 preservice teachers at one point in time through a two-part interview.

Part one of the interview asked participants to create a concept map to represent their understanding of the concept of a fraction. Part two of the interview was designed to elicit participants' knowledge of students and content related to student misconceptions of fractions in various forms. In addition to identifying student misconceptions, participants were asked to identify a goal for student learning related to the misconception and discuss instructional representations that could be utilized to improve student understanding.

## *Theoretical Framework*

This study utilized two aspects of Hill, Schilling, and Ball's (2004) *mathematics knowledge for teaching* to explore EC-4 preservice teachers' knowledge of fraction concepts: specialized knowledge of content (SKC) and knowledge of students and content (KCS). Because this study focused on preservice teachers enrolled in a mathematics methods course, the primary goals of which are to increase preservice teachers' knowledge of the content they will teach and their knowledge of how to best teach that content to students, the two remaining aspects of mathematics knowledge for teaching, common content knowledge and knowledge of students and teaching, were not addressed. In addition, an analysis of preservice teachers' knowledge of students and content would require numerous field observations, which was beyond the scope of the present study. Figure 1 provides a description of the study framework.

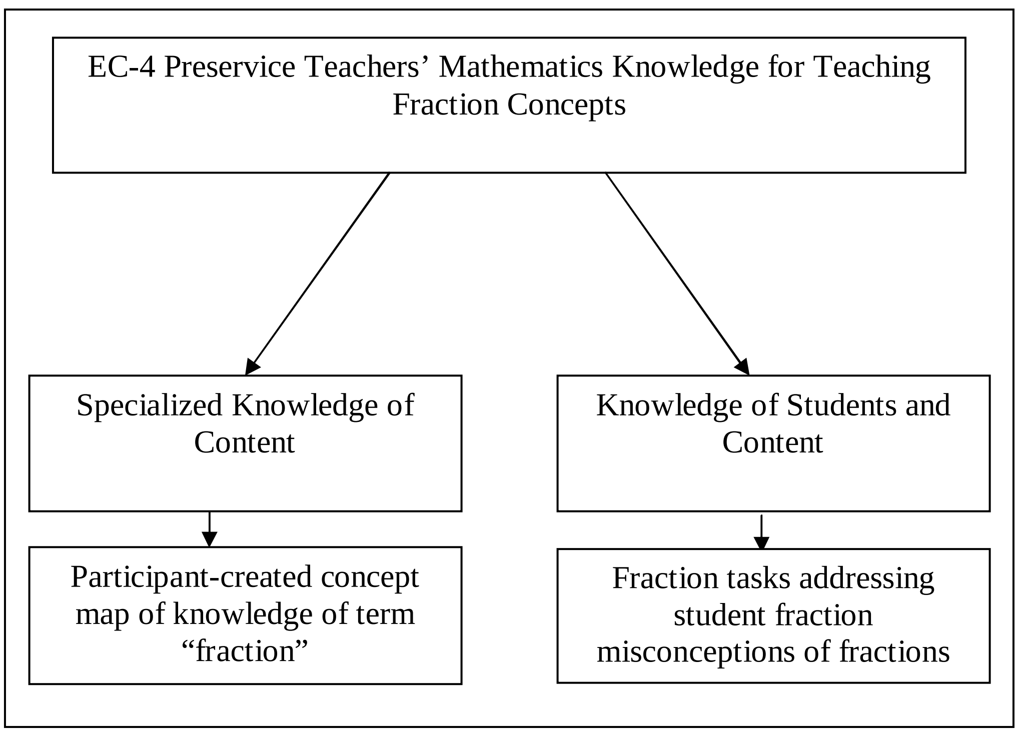



Figure 1. *Study Framework for Assessing EC-4 Preservice Teachers' Mathematics Knowledge for Teaching Fraction Concepts*

*Study Instruments*

The data collection instruments in the study included participant-created concept maps of the term “fraction” and audio-taped interviews with participants regarding their diagnosis and instructional planning when shown student errors with fraction problems.

The use of concept maps as representations of learning were pioneered by Novak and Gowin (1984) as a way for learners to represent their understandings related to particular themes or concepts in science. Concept maps give participants an opportunity to represent their thinking in terms of nodes as sub-themes clustered around a major theme and links that connect related themes to one another. Though pioneered in science education, concept maps are widely used throughout education as a means of helping learners in various disciplines represent their knowledge. Specifically, concept maps have been used in mathematics education research (Chinnappan, 2005; Hough, O’ Rode, Terman, & Weisglass, 2007; Williams, 1998) to represent participant thinking about mathematics concepts, such as algebra, geometry, and functions. The maps allow researchers to analyze the connectedness and depth of participant knowledge related to a particular concept.

In the case of the present study, the major theme was the term “fraction,” which participants were asked to place in the center of the map. The nodes were considered to be any terms circled, boxed, or provided on the concept map with a line connecting either to the major theme or another node. Major nodes were those connecting directly to the major theme, while minor nodes were those separated from the major theme by one or more other nodes.

Since the use of concept maps in mathematics education research is relatively new, the researcher in the present study did not assume participants would be familiar with concept mapping related to mathematics. To help insure participant familiarity with concept mapping, the researcher provided two examples of concept maps to participants on familiar topics at the beginning of each interview. Appendix A contains an example of the concept map template.

In part two of the interviews, participants were shown items adapted from the work of Lamon (2005) that showed various types of fraction problems answered incorrectly by students. The purpose of these tasks was to demonstrate misconceptions of fraction concepts related to different interpretations of fractions, including fractions as representations of part whole relationships, fractions as operators, fractions as ratios, and fractions as points on a number line. If participants correctly identified the student's thinking as erroneous, they were asked to state a learning goal for the student related either to the assessment item or the student's error pattern. In addition, participants were asked to discuss types of instructional representations, such as fraction models and contexts, they would use to help the student attain the stated learning goal and how these models and context would bring about change in the student's thinking. Appendix B contains an example of the fraction tasks.

If participants did not see the student's misconception as an error, the researcher presented them with the correct response of another student to the same problem. The participant was given the opportunity to create a hypothetical learning trajectory for the student whose thinking they believed to be incorrect. This additional task benefited both the participants and the researcher by giving participants an opportunity to challenge their thinking regarding student misconceptions of fractions and giving the researcher an expanded opportunity to collect data on participant thinking.

The hypothetical learning trajectory approach used in part two of the study is an operationalized version of the hypothetical learning trajectory portion of Simon's (1995) mathematics teaching cycle whereby after assessing a student's thinking, teachers or researchers identify a mathematical goal for student learning, create an instructional task to meet that goal, and make hypotheses about how they believe student thinking will progress toward the mathematical goal.

*Study Participants*

The participants in the study were composed of a convenience sample recruited from an accessible population of approximately 160 EC-4 preservice teachers enrolled in ECFB 440 (Early Childhood Education Field Based), Mathematics Methods in Early Childhood Education in the spring of 2008. The EC-4 preservice teachers at a large university in Texas enroll in the mathematics methods course during their senior year in the semester prior to their student teaching semester.

The field-based methods course is the final mathematics class in a sequence of 18 credit hours of mathematics required for a Bachelor's of Science degree in Interdisciplinary Studies, or a EC-4 generalist education degree. The required mathematics coursework prior to enrollment in the field-based mathematics course includes two three-hour calculus courses, two three-hour elementary mathematics courses, and a statistics course. The ECFB 440 course covers mathematics teaching methods for all strands of the National Council of Teachers of Mathematics Standards (2000), as well as the state mathematics standards, the Texas Essential Knowledge and Skills (TEKS). According to the university's course catalog, the ECFB 440 course "analyzes contemporary curricula; implementation of methods relevant for active, authentic learning and age appropriate teaching of mathematics to young learners; considers state and national standards related to teaching and learning mathematics" (Texas A&M University, 2007). The portion of the methods course related to fractions instruction focuses on three main areas: the use of instructional representations, including the use of actual and virtual manipulatives; research on student misconceptions; and fractions instruction in problem solving contexts.

Participants were selected from the four sections of ECFB 440 on a voluntary basis. Course instructors informed students about the purpose of the study and asked for volunteers to participate in interviews. In addition, the researcher visited each ECFB 440 section to personally ask students to participate in the study. There was no reward offered nor any penalty connected to students' choice to participate in the study.

Interested students were asked to sign up and were contacted by the researcher via e-mail or telephone for on-campus interviews. A total of twenty-eight students signed up to participate and were contacted for interviews. Of the twenty-eight students who were contacted, eleven students agreed to be interviewed. Two students dropped out due to scheduling conflicts. One students' interview data was lost due to an audio error. Therefore, a total of nine students participated in interviews between the end of January and the middle of March and the data of eight participants is discussed in the study analysis. The shortest interview was fifteen minutes and the longest was one hour and thirty minutes.

*Study Design*

Study participants were informed of their participation in a project measuring preservice teachers' knowledge of fractions concepts. It was made clear that their participation in the study would have no effect on their course grade or other credentials at their program. Participants were provided with a consent form with details of the study and were informed that they may withdraw from the study at any point. Each interview was audio-taped by the researcher for later transcription. In addition to the audio data, participants' concept maps and the researcher's field notes were included in the analysis.

For purposes of transcript organization and discussion, participants' data was coded with the letter S for "student" and a number, beginning with the number two and ending in the number 12. The participant in the pilot study in the summer of 2007 was assigned the code S1. Numbering was assigned as interviews were scheduled, resulting in gaps in numbering for participants who dropped out (S9 and S11) and for the participant (S7) whose data was lost due to audio error.

During the first portion of the interview, participants worked independently on the concept map portion of the interview without questioning from the researcher. The second portion of the interview was conducted in an interview format with the researcher asking participants open-ended questions about student misconceptions regarding fraction problems. Participants were asked to discuss their thinking about the student's possible misconception as well as make any written notes they wished regarding their thinking.

Both *a priori* and emerging themes were used in the study to analyze participant knowledge of fractions. *A priori* themes from previous research and assigned readings from mathematics methods texts in the ECFB 440 course provided a means for analysis of the completeness of participant concept maps. Kieren's (1976) sub constructs of fractions, including fractions as representations of a part-whole relationship, as a point on a number line, as a ratio, as division, and as an operator, were used as a baseline for analyzing the various understandings of fractions present in participants' SCK of fractions content. In addition, concept maps were analyzed for the presence of fraction concepts which receive a strong focus in mathematics methods texts, particularly types of fractional models, fractional notation, and terminology (Van de Walle, 2007).

Part two of the interview used the constant comparative method of analysis (Glaser & Strauss, 1967) to unitize data collected from participant notes, audio data, and researcher field notes to see what themes emerged from the hypothetical learning trajectories created by study participants. Finally, the emergent themes from participants' creation and discussion of the hypothetical learning trajectories were compared to the *a priori* themes used to analyze the concept maps of fractions to see what themes, if any, were common in the EC-4 preservice teachers' SCK and KSC of fractions.

# CHAPTER IV

# ANALYSIS

*Analysis of Preservice Teachers' SCK Using Concept Maps*

At the beginning of each interview, participants were given a blank concept map following an introduction to the task by the researcher. Appendix A shows an example of the concept map. The purpose of having participants create a concept map was to allow them the opportunity to reflect on and construct a visual representation of their SCK of fractions before analyzing student work on the fraction tasks in part two of the interview.

Following a pilot study of the instrument in the summer of 2007, the researcher decided to provide Kieren's (1976) sub constructs of fractions on the map to maximize the possibility of soliciting varied interpretations of fractions from participants. Specific types of fraction models and terminology were not provided on the concept map. It was the researcher's belief that due to the fact that all participants were required to take two Structure of Mathematics courses prior to entering the mathematics methods courses, they would be somewhat familiar with common fraction terminology and models. The concept maps were analyzed for the presence or absence of fraction sub constructs and for types of fraction models, notations, and terminology using a spreadsheet. As use of sub constructs, models, notation, or terminology was noted for a participant, an "x" was placed in the spreadsheet. Any extra information was typed into the spreadsheet verbatim. The sub constructs provided on the map were part whole, operator, ratio, point on a number line, and division. An overview of the data collected from participants' concept maps can be found in Table 1.

Table 1

*Overview of Kieren's Fraction Sub Constructs Present in Concept Maps*

| | Kieren's Sub Constructs of Fractions | | | | |
|---|---|---|---|---|---|
| | Part-whole | Operator | Ratio | Point on a number line | Division |
| S2 | x | | x | x | x |
| S3 | x | o | | | o |
| S4 | x | | x | | x |
| S5 | x | o | | | x |
| S6 | x | | x | | |
| S8 | o | | | | |
| S10 | x | o | x | | x |
| S12 | x | | x | | x |

x= explicitly stated term
o= showed evidence of terms through symbols or other language

Fractions as Part Whole Relationships

Results of concept map analysis showed that the majority of the EC-4 preservice teachers interviewed (five out of eight) used the part-whole sub construct as a major node on their concept map stemming directly from the word “fraction” in the middle of the map. One participant, S3, labeled the line between the nodes for fraction and “part of a whole” with the word “is” (Figure 2), indicating that a fraction is part of a whole. Another participant connected “part of a whole” to the word “definition” which was then connected to the term fraction in the middle of the concept map.

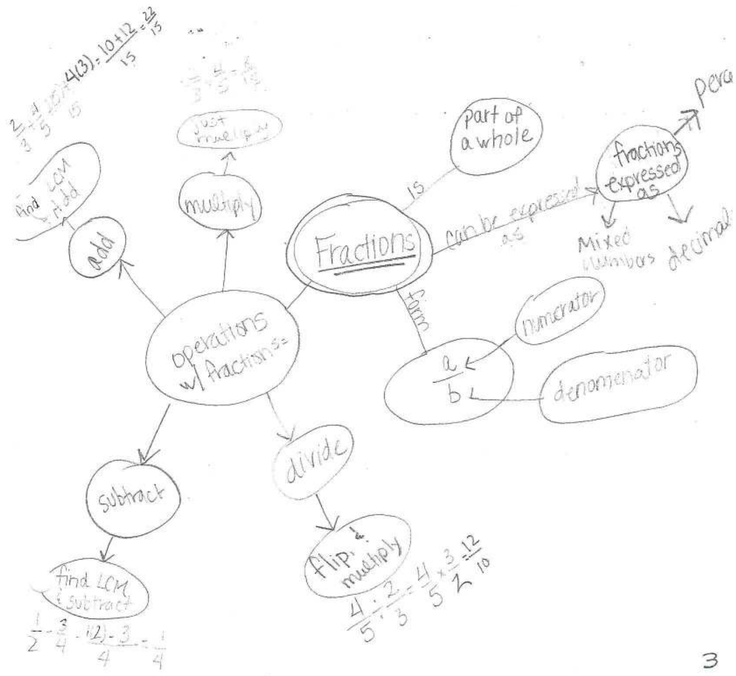



Figure 2: *Fractions Concept Map- S3*

These results lend support to previous research with teachers' knowledge of fraction representations (CBMS, 2000; Lamon, 2001) that part whole representations are those that most teachers are familiar with. Most participants provided this explanation of a fraction as one of the first nodes on their concept map with S8 being the only participant who did not use that sub construct directly. However, S8 provided minor nodes (Figure 3) for the numerator and denominator as "number of parts" and "total number of parts" respectively, indicating her understanding of a fraction as a representation of a part whole relationship. Her use of a rectangular area model divided into thirds could also be understood as a pictorial representation of a part whole relationship.

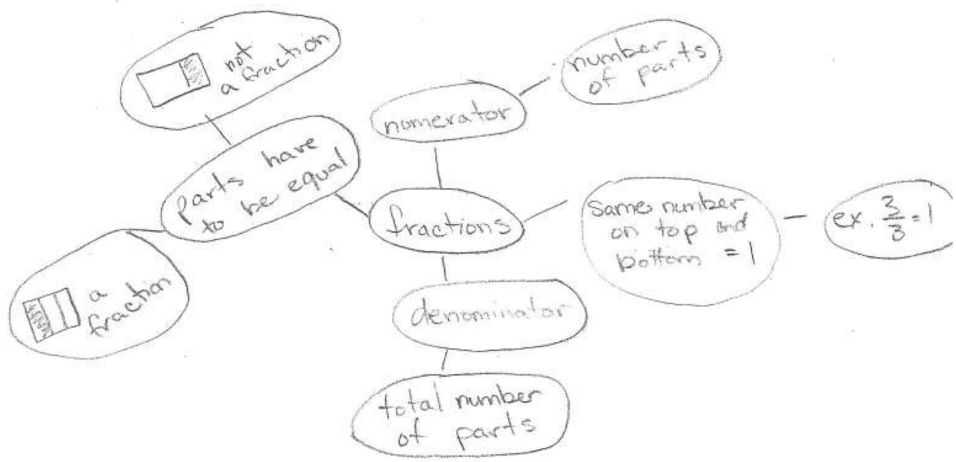



Figure 3: *Fractions Concept Map- S8*

Fractions as Operators

Of the eight students interviewed, none created nodes specifically for fractions as operators. S3's map contained a major node labeled "operations with fractions" with minor nodes branching from that for multiplication, division, addition, and subtraction (Figure 2). Minor nodes will examples of each type of operation were also shown. Though multiplication and division are part of the function of fractions as operators, according to Lamon (2005), the researcher believes that this particular student was referring to algorithmic operations with fractions rather than to using a fractional quantity to perform a function on another quantity. The reason for this is because the nodes connected to each operation contained language such as “flip and multiply” for division, “find LCM (referring to the least common multiple) and subtract” for subtraction, “find LCM and add” for addition, and “just multiply” for the multiply node. The student appeared to be describing an algorithmic process rather than a sub construct of the term fraction.

As a minor node on her map, S5 drew eight hearts divided into groups of two and pointed to two of the hearts with an arrow labeled “1/4 of 8” to indicate ¼ operating on the eight hearts, but used the sub construct division, rather than operator as the major node above it (Figure 4).

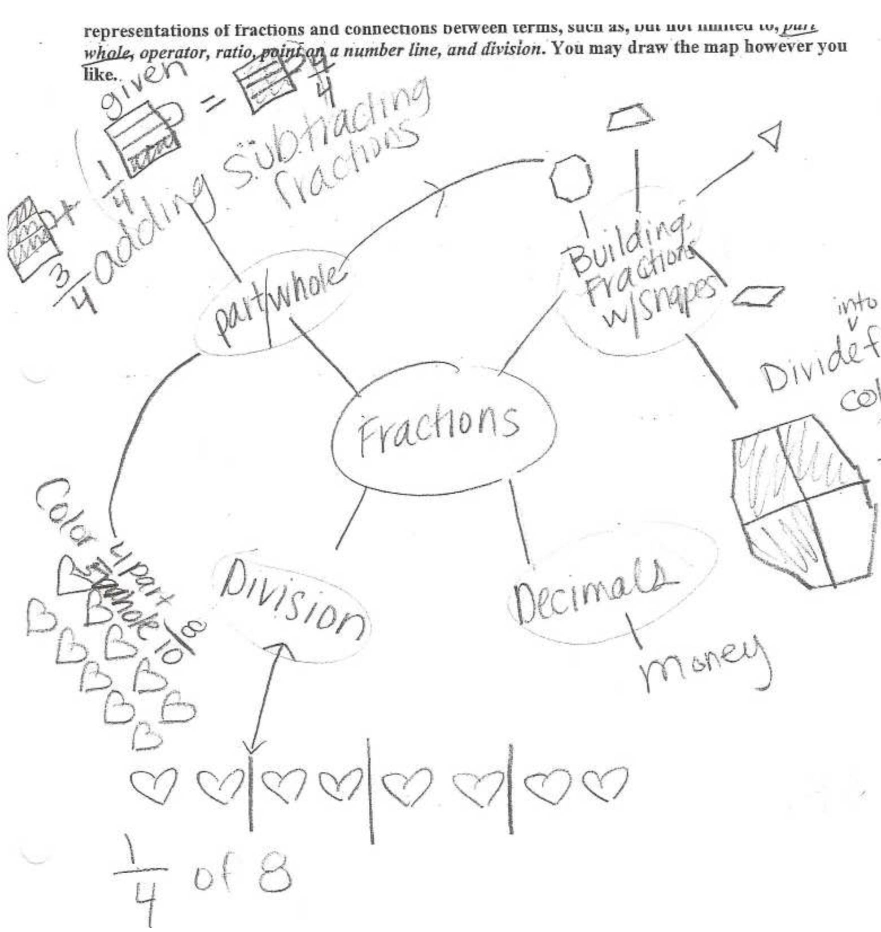



Figure 4: *Fractions Concept Map- S5*

Fractions as Ratios

Four out of eight study participants created concept map nodes for the ratio sub construct. Two students, S4 and S12 (Figures 5 and 6) included ratio as a major node. S4 provided a numerical example of a ratio as “1:3”. S4’s use of colon notation to illustrate a ratio as one interpretation of a fraction highlights the difficulties present when trying to clarify what one means when discussing the concept of a fraction. According to Lamon (2005), though a ratio is one form of a fraction, ratios are not always fractions. For example, a ratio such as ¼ can represent the number of boys in a class to the total number of students in a class. In this way, a ratio is a fraction. However, sometimes ratios are not fractions, such as in the case of $\frac{10}{0}$ representing 10 males to zero females in a room (Lamon, 2005). In this situation, the use of fraction notation would result in an undefined fraction. Therefore, it is clear that S4 understands at least part of the sub construct related to ratio. Further questioning would be necessary to inquire how deeply this understanding was held.

Another (S12) simply made ratio a major node extending directly from the term fraction with no examples (Figure 6). However, this participant did not provide examples for any terms on her map, therefore, a lack of example for ratio cannot necessarily be interpreted as a lack of detailed understanding for this sub construct.

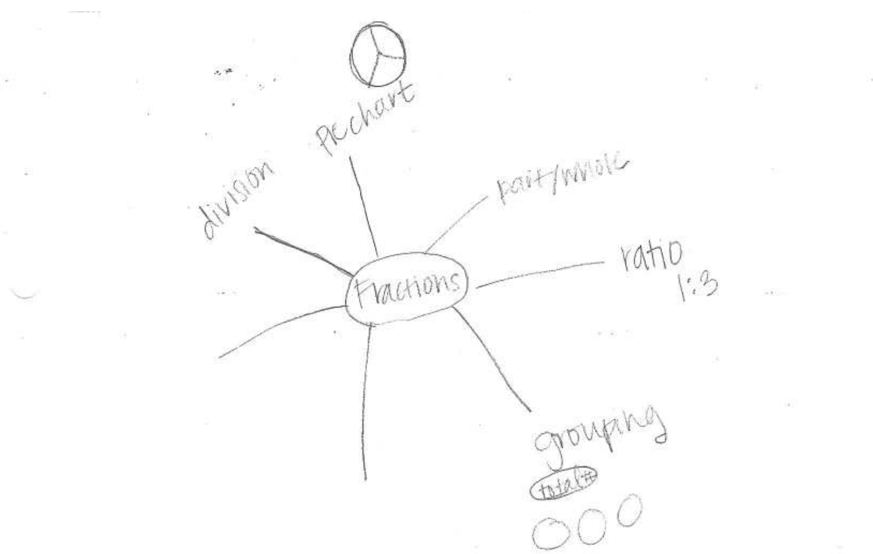



Figure 5: *Fractions Concept Map- S4*

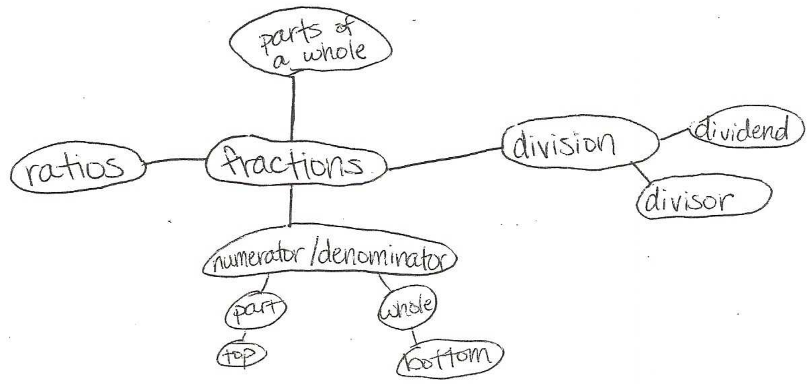



Figure 6: *Fractions Concept Map- S12*

Two participants, S6 and S2, included the ratio sub construct as a minor node (Figures 7 and 8). S6 provided ratio as one of two minor nodes branching from a major node labeled “aka” which the researcher interpreted as “also known as.” This seemed to indicate that S6 understood ratio and division as alternate meanings for the term fraction (Figure 7). S2’s concept map (Figure 8) showed ratio as a minor node connected to two minor nodes labeled as part and whole. The part and whole minor nodes branched from major nodes labeled numerator and denominator, respectively. S2 was the only participant using the ratio sub construct that demonstrated any specific understanding of the relationship of the numerator and denominator of a fraction to a ratio’s representation of two quantities in relationship to one another.

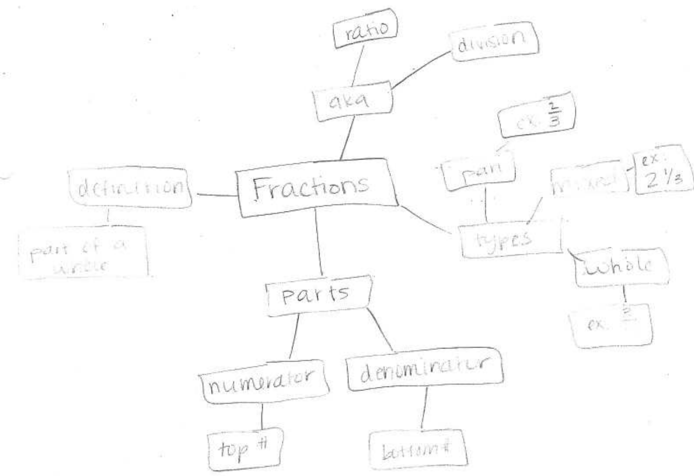



Figure 7: *Fractions Concept Map- S6*

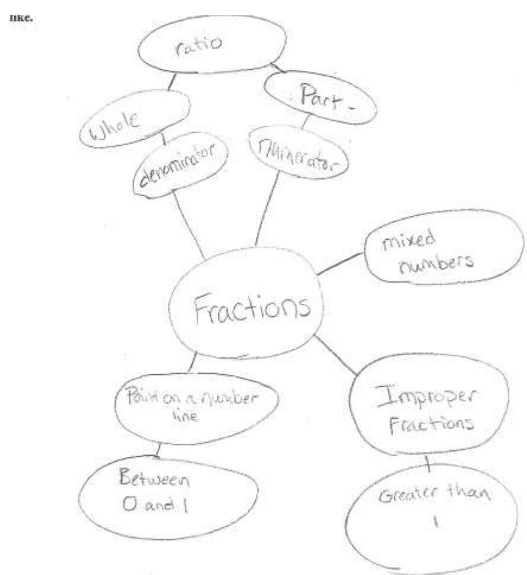



Figure 8: *Fractions Concept Map- S2*

## Fractions as Points on a Number Line

Only one of the eight study participants' concept maps contained a node addressing fractions as a point on a number line. S2 (Figure 8) provided a major node labeled "point a number line" with a minor node connected to it explaining a fraction on a number line as a fraction "between 0 and 1." A fraction on a number line actually "denotes the distance of the labeled point from zero" (Van de Walle, 2007, p. 297) rather than just the distance between zero and one. The nodes did not indicate whether S2 understood that fractional quantities greater than one could also be represented on a number line. The nodes to the right of "point on a number line," and "between 0 and 1" which S2 labeled "improper fractions" and "greater than 1" indicate her understanding of fractions as representations of quantities greater than one. However, there are neither lines labeled nor any nodes indicating connections in her knowledge between the number line representation and improper or mixed fractions.

## Fractions as Division

Six of the eight study participants used the term "division" or "divide" on their concept maps. Two participants provided little elaboration as to how they understood division as a sub construct of fractions. S4 supplied the term division as a major node extending from the term fraction (Figure 5), but did not provide any further nodes or examples to indicate whether they were thinking of fractions as a form of division or that division was an operation one could perform with fractions. S6 provided division as one of two nodes branching from a node labeled "aka" which the researcher interprets as "also known as" with no further explanation (Figure 7). S12 attached nodes for "dividend" and "divisor" to the node labeled "division" but also did not provide any connection to fractions specifically (Figure 6).

Another group of study participants provided some elaboration of their thinking regarding the division sub construct. S3 used the term "divide" rather than division on her concept map (Figure 2). The "divide" node branched from a major node labeled "operations with fractions." From there, S3 created a further node containing the words "flip and multiply". This repetition of a familiar explanation for the division of fractions algorithm found to be common in preservice teachers (Lubinski, Fox, & Thomason, 1998) combined with the node "division" branching from a node labeled "operations with fractions" lead the researcher to believe that S3 was simply stating algorithms one could perform with fractions rather than providing an example of the meaning of a fraction. Similarly, S10 labeled the only node branching from her "division" node as "reciprocal" indicating that she may have been thinking also of division with fractions, as algorithmically one multiplies the dividend by the reciprocal of the divisor to obtain a quotient (Figure 9).

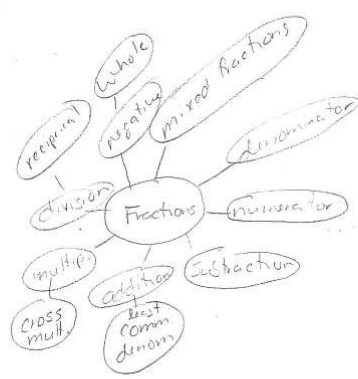



Figure 9: *Fractions Concept Map- S10*

S5 provided a detailed example of her understanding of division by drawing a set model containing eight hearts subdivided by lines into groups of two (Figure 4) and labeled the drawing as “1/4 of 8”. S5’s example of division with fractions revealed that though study participants were able to create concept maps with the given terms, further questioning may have revealed incomplete understanding of relationships between the terms. This is because S5’s set model example was actually an example of a fraction as an operator which, according to Lamon (2005), causes an, “increase or decrease in the number of items in a set of discrete objects” (p. 151). As stated earlier, the researcher did not question participants during this part of the interview, nor were participants asked to explain their completed concept maps.

Fraction Models, Notations, and Terminology

*Fraction Models.* Fraction models provide an important link in the development of fraction understanding by elementary students. Aside from knowledge of different types fraction sub constructs, it is important that preservice teachers have various types of fraction models as part of their specialized content knowledge of fractions. Analysis in this study focused on three types of fraction models commonly used at the elementary level and seen in elementary mathematics methods texts: area or region models, length or measurement models, and set models (Van de Walle, 2007).

Area or region models are those composed of a whole subdivided into equal parts, such as fraction circles or subdivided rectangles or squares. Length or measurement models are those that compare lengths rather than areas. These models include drawn or cut fraction strips or rods and number lines with equal divisions. Set models are composed discrete objects or drawings, with the whole being the total number of objects in the set.

Fraction models were used by only three out of eight study participants. The models used were either set or area models. Participants S4 and S5 used set models (Figures 5 and 4). S4's concept map contained a drawing of three circles next to a major node labeled "grouping" and S5 using hearts divided into four groups to illustrate division. The use of set models helps develop children's understanding of real world uses of fractions, which often involve using discrete objects rather than subdividing a singular whole into parts (Van de Walle, 2007).

Participants S4, S5, and S8 (Figures 5, 4, and 3) used area models, the model most commonly used (Ball, 1993) to represent fractions with denominators of thirds and fourths. S4 and S8 used a fraction circle model and a square model respectively to represent the fraction $\frac{1}{3}$, while S5 subdivided an octagon into four sections and shaded three of them to show $\frac{3}{4}$. Though area models are those most commonly used, they provide a good introduction to the notion of subdividing a unit whole into equal parts, which is a central idea in elementary teachers' work with their students and fractions concepts (CBMS, 2000).

*Fraction Notations.* Previous studies with preservice teachers (Borko et al, 1992; Goulding, Rowland, & Barber, 2002; Ward, Anhalt, & Vinson, 2004) have shown formal fraction notation to be among the most frequently used representations for fraction concepts. Formal fraction notation and symbolism as conventions of the language of rational numbers, however, can be misleading for students (Kieren, 1991; Van de Walle, 2007). For analysis of EC-4 preservice teachers' use of formal fraction notation and symbolism, this study used Brizuela's (2005) definition of notations as that which refers to, "written numerals or symbols" (p. 284).

Similar to results found in previous studies with preservice teachers and fraction notation (Borko et al, 1992; Goulding, Rowland, & Barber, 2002; Ward, Anhalt, & Vinson, 2004) participants in this study showed frequent use of formal symbolism largely unaccompanied by written or pictorial explanations of the notation. Five out of eight participants used notation to represent fractions in various forms. S5 was the only participant who provided extensive explanations of the meaning fraction notation on her concept map (Figure 4). S5's concept map exhibits six instances of the use of fraction notation, all of which are accompanied by either a written or pictorial explanation, such as the fraction $\frac{3}{4}$ accompanied by an illustration of an octagonal pattern block shape subdivided into four parts with three parts shaded. In addition, S5 provided the following written explanation: divide into fourths, color 3.

The remaining four participants using fraction notation did so with little connection to fraction models or terminology. S3 used common fraction notation extensively, with algorithmic examples of fraction division, subtraction, addition, and multiplication (Figure 2). The written explanations provided were "flip and multiply" for division of fractions, "find LCM and subtract" for subtraction, "find LCM and add" for addition, and "just multiply" for multiplication, showing mainly procedural understanding seen in previous studies with preservice teachers (Borko et al, 1992;

Lubinski, Fox, & Thomason, 1998; Ma, 1999). S3 did provide an explanation for a node labeled specifically as $\frac{a}{b}$ with two minor nodes containing the terms "numerator" and "denominator" with arrows pointing to the corresponding letter (Figure 2). S4 provided a "1:3" under the node for ratio, but gave no explanation or illustration as to how 1:3 was representative of the concept (Figure 5). S8 used $\frac{3}{3}=1$ as a minor node emanating from a major node containing the words "same number on top and bottom=1" (Figure 3).

*Fraction Terminology.* As with notation, fraction terminology is part of the language of mathematics that makes fractions difficult for elementary students. Not only do fractions bring different forms of notation, this "new kind of number" (CBMS, 2000) brings a new set of vocabulary to describe what each symbolic part of a fraction represents. For example, when an elementary student enters the domain of fractions, the whole number two can now be called a numerator, meaning that it is counting two parts out of some whole, or a denominator, meaning that a whole is divided into two equal parts. Study analysis focused on terms commonly used to describe fractions, including the formal terms numerator and denominator, and informal terms such as top number and bottom number. According to Van de Walle (2007), the importance in fraction terminology lies not in whether formal or informal terminology is used, but in whether preservice and inservice teachers are providing conceptually correct explanations of the terms to their students.

Six out of the eight study participants used the formal terms numerator and denominator on their concepts map. No one appeared to use the terms incorrectly, however, due to the lack of detail on some of the maps, it was unclear whether the terms were clearly understood by all participants. S2, S3, S8, and S12 (Figures 10, 11, 12 and 13) provided numerator and denominator as major nodes on their concept maps with minor nodes describing the meaning of the terms. S2 and S12 indicated the numerator to represent a part and the denominator to represent the whole. S3 provided the terms connected to a fraction $\frac{a}{b}$ with arrows pointing from numerator to the letter a, and from denominator to the letter b. Though S3 identified the correct location of the terms, the arrows say little about whether S3 could explain the meaning of the terms conceptually. S8 identified distinguished between the numerator as the number of parts and the denominator as the total number of parts. In addition, S8 provided another major node specifying that the parts have to be equal.

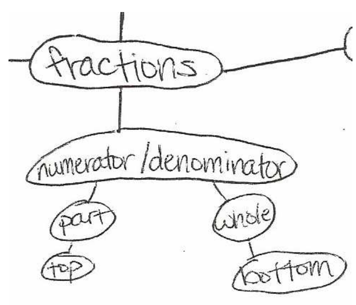



Figure 10: *Fraction Terminology- S2*

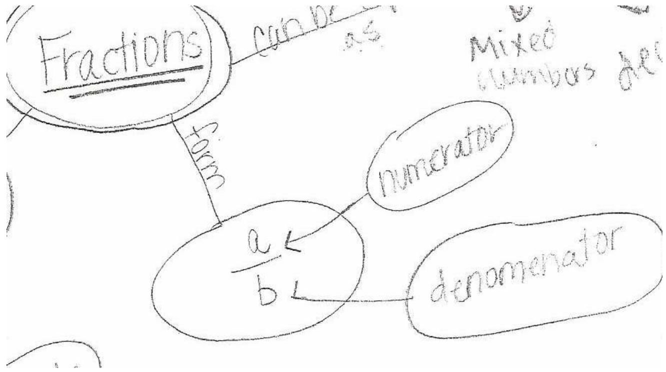



Figure 11: *Fraction Terminology- S3*

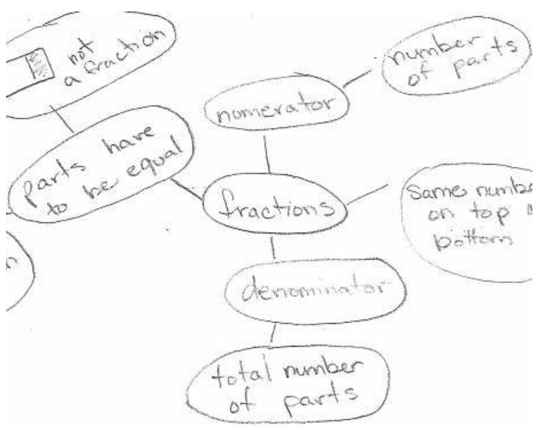



Figure 12: *Fraction Terminology- S8*

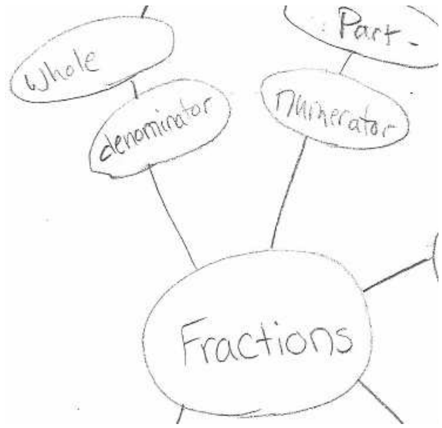



Figure 13: *Fraction Terminology- S12*

Two participants, S6 and S10, differed slightly from the four other participants using the terms numerator and denominator on their concept maps (Figures 14 and 15). S6 used the terms, but provided them as minor nodes under a major node labeled “parts.” S6 described the terms as “top #” and “bottom #” respectively. S10 provided major nodes for the terms, but gave no explanation of what the terms numerator and denominator represent.

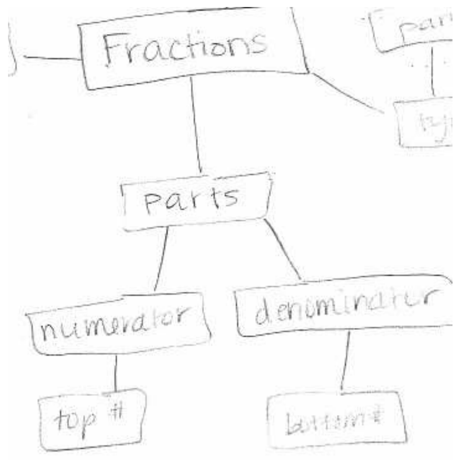



Figure 14: *Fraction Terminology- S6*

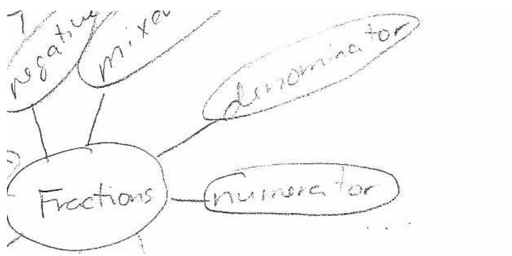



Figure 15: *Fraction Terminology- S10*

*Analysis of Preservice Teachers' KCS Using Fraction Tasks*

Following completion of the concept map, participants were given a brief introduction to part of Simon's (1995) *mathematics teaching cycle* called the *hypothetical learning trajectory*. In creating a *hypothetical learning trajectory*, teachers look at a student's work to determine a learning goal for the student based on the diagnosis of what the student knows about a concept. Teachers then create a plan for learning activities or representations to help the student progress toward the specified goal and hypothesize about how the student's thinking will change based on the learning goal and chosen activities or representations (Figure 16). Simon's *hypothetical learning trajectory* model was a good fit for part two of the present study because it allowed participants to construct their own thinking regarding a student's work, rather than be led by the researcher to choose from a list of possible conclusions. The constructivist nature of Simon's model allowed the preservice teachers to represent their thinking regarding fraction concepts in a similar manner to the open-ended concept maps in part one of the interviews.

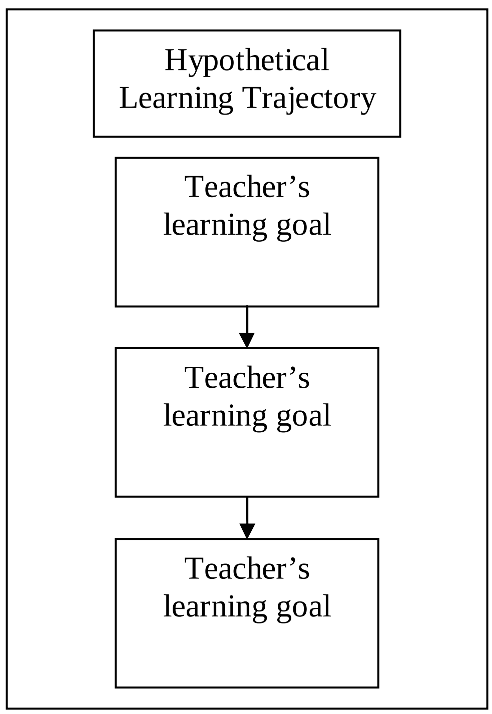



Figure 16: *Hypothetical Learning Trajectory (Simon, 1995)*

Participants were guided through the *hypothetical learning trajectory* for four fraction tasks that were created based on the work of Lamon (2005) showing fraction problems answered incorrectly by students. The tasks showed common misconceptions for four of the five fraction sub constructs (Kieren, 1976) provided for participants on the concept map. Originally there was one task for each sub construct: fractions as representations of part-whole relationships, as operators, as ratios, as points on a number line, and as division or quotients. However, due to a typographical error in the fractions as division task discovered after two interviews had been carried out, the fifth task was omitted, leaving four tasks for analysis. The tasks were created to assess participants' knowledge of content and students (KCS) in the same areas as in part one of the interview to provide a means of comparing the EC-4 preservice teachers' SKC and KCS in an open-ended manner.

Each task provided a problem given to a student and the student's response to the problem. The student responses to the tasks were created based on misconceptions common in elementary students, such as the interference of whole number understanding with fraction knowledge and difficulty with fractional representations seen in previous studies with elementary students (Kamii & Clark, 1995; Mack, 1995; Smith, Solomon, & Clark, 2005). The following was the general format of the interview questions:

1. What do you think about the student's response?
2. What fraction concepts does the student need to know to understand this problem?
3. What instructional situations and/or representations would you use to address the fraction concepts you mentioned?

4. How do you think each instructional situation or representation would change the student's thinking?

In some instances, participants were unable to identify a student's misconception or could identify the existence of some sort of misconception but were unable either to verbalize the misconception or provide a means of helping the student move beyond their current level of understanding. If this type of response occurred, participants were provided with an alternate student's correct explanation and asked to analyze whether the alternate explanation was helpful.

Participant responses were audio-taped and transcribed for analysis. Whereas the concept map analysis focused on the responses of each participant individually and was subject-based, the analysis of the fraction tasks was task-based, looking at overall themes and trends that emerged from participant responses as a whole. Each participant's responses for each task were therefore combined into one transcript for each of the four tasks. Transcripts of participant responses to each task were read several times to see what possible themes emerged from the data. Text segments were bracketed and coded with descriptive labels (Creswell, 2003) and collapsed into themes that provided an overall description of the data. An overview of the data collected from participants' responses can be found in Table 2. Interview questions one and two were collapsed for analysis, as participants often addressed student misconceptions right away when asked what they thought of a student's response.

Table 2

*Overview of Emergent Themes from Fraction Tasks*

| Fraction Tasks | Student misconception | Suggested representations | Change in thinking |
|---|---|---|---|
| Task One | Equivalent fractions misconception, bigger number equals more, common mistake | provide concrete models, go over equivalent fractions, simplifying fractions | moving pieces around, putting pieces together, concrete models will show same size |
| Task Two | Reasonableness of 27 cupcakes, has process without concept, doesn't understand meaning of 2/3 | use concrete objects, have student explain thinking, question student about 27 cupcakes | pictures would challenge erroneous thinking, help student visualize/manipulate groups |
| Task Three | Unsure how to solve problem myself, unsure how to show student, computation off | manipulatives, show 4 boys to 2 girls, cannot mention ratios yet | |
| Task Four | Smaller denominator does not equal smaller fraction, does not understand numerator/denominator, whole numbers different from fractions | use manipulatives, draw pictures, finding common denominator | connect concepts to process, visual representations help clarify numerical representations, young children need concrete representations |

Task One-Fractions as Representations of Part-Whole Relationships

Based on the work of Kamii and Clark (1995) with fifth and sixth grade students and Lamon's work (2001) with fourth graders, Task One showed a student, Marcus, two circular representations of one-half. The first circle was subdivided into four equal sections, with two diagonal sections shaded (see Appendix A). The second circle was subdivided into eight sections, with four sections shaded. Marcus was asked to name the fractional part shaded in each picture and determine whether the fractions named the same amount. Marcus correctly identified the fractions as $\frac{2}{4}$ and $\frac{4}{8}$ but incorrectly identified $\frac{4}{8}$ as the larger fraction due to its larger numerator and denominator. The student's response was modeled after previous research with students' interpretations of $\frac{1}{2}$ (Kamii & Clark, 1995; Lamon, 2001), in which students were able to correctly identify different pictorial representations of the fraction one-half, but incorrectly identified one representation as being larger than the other due either to the orientation of the shaded regions or the number of subdivisions.

Participants picked up quickly on Marcus' misconception in Task One. The preservice teachers were able to follow the student's incorrect line of reasoning and diagnose his misconception that $\frac{4}{8}$ is a larger amount than $\frac{2}{4}$ because of the difference in the numbers representing the numerator and denominator rather than the size of each piece relative to the size of the whole. Three major themes emerged from the preservice teachers' responses to questions one and two of Task One: a lack of understanding of equivalent fractions, the misconception that a bigger number in the numerator or denominator equals a larger fraction, and the overall impression that Marcus' misconception was a common one. Two participants, S6 and S10, solved Task One themselves before looking at Marcus' response (Figures 17 and 18). Participants had the following to say about Marcus' work:

> **S2:** It is the ratio that is important, not the number of pieces. He didn't look at the whole. The size is determined by the ratio. He didn't look at the bottom number.
>
> **S3:** I follow his line of reasoning. I would say this is a pretty common mistake and it's not too far fetched that he thought this.
>
> **S5:** Two out of four (reading Marcus' response out loud) because the second pizza has more pieces. I get that a lot. Definitely the misconception that just because the number is bigger means that they are getting more.
>
> **S6:** Yes, well, he doesn't understand equivalent fractions. And, um, he has a misconception that just because the numerator is a larger or smaller number than the numerator of the other fraction, then they aren't equivalent. He's disregarding that they don't have the same denominator and that they're not cut into the same amount of pieces.
>
> **S8:** Well of course it's kinda wrong. He's just going by 4 is bigger than 2. He's not actually looking at the actual size of the pieces because fourths

are smaller. Or I guess he doesn't understand equivalent fractions because these are equivalent.

**S10:** I mean I can see where they got this one wrong because children have that misconception, they see the pieces and they think that because there are more of them.

**S12:** He got it right, like $\frac{2}{4}$ and $\frac{4}{8}$, but I don't think he realizes that they're both half.

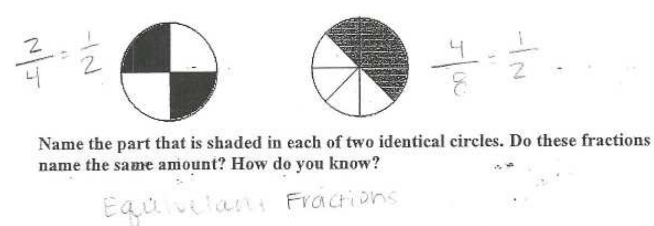



Figure 17: *Task One- S6*

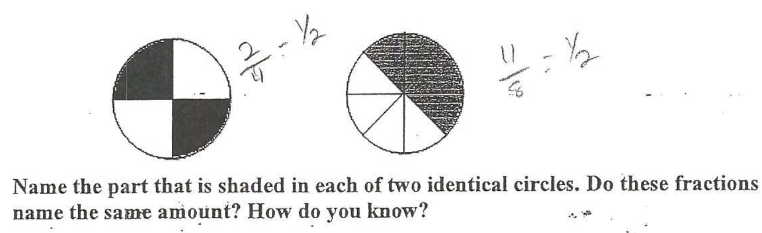



Figure 18: *Task One-S10*

Responses were varied as to what types of instructional representations or situations would help Marcus construct a stronger understanding of equivalent fractions. Seven of the eight participants mentioned using some sort of concrete or pictorial representation, varying from simply drawing a picture of the shaded region and moving the pieces, to using pattern blocks or dry erase boards, to allow Marcus to manipulate the shaded areas to make it clear that the fractions show the same amount.

> **S2:** Use a concrete model so they could move around pieces and let them compare. Color code it. It helps to lay pieces on top.
>
> **S3:** … also maybe using manipulatives because he's saying I would get more pie if I had this one ($\frac{4}{8}$ picture) but you could say okay you'd get two whole pieces from here and four pieces from here but if you push them together then they're the same. Ideally I'm assuming they have something like fraction pies or fraction circles…things like that just so that he could see that this first one divided into four pieces can also look like the one divided into eight.
>
> **S4:** I think the first thing that I would do is I would move this shaded area (points to $\frac{2}{4}$ sections) to where it looked identical to the four pieces and so it would be more like this split into fours and it would look more like this (points between $\frac{2}{4}$ and $\frac{4}{8}$) and show him that it would look the same. It's almost pretty obvious. Maybe try to explain it with a piece of

pizza or something like that. Say I had a piece of pizza and my pizza is this big. Let's say I had an actual representation. And you had two pieces of pizza but they're only this big.

**S5:** They need to actually stop and look at the size of, you know, does this equal this and I would definitely take those pattern blocks and show that two green triangles equal the same as a rhombus. It's the sideways one. Yeah, but it equals that. And just because there are two pieces of green to equal the rhombus, it doesn't mean you are getting more than the rhombus. So I definitely think using those pattern blocks would show them that even thought there's four green triangles and only two rhombuses, that equals the same thing.

**S6:** I'd probably use manipulatives and say if we have four blocks and I took away two, you could see I took away half of the blocks and I would take away four from eight.

**S10:** You would just have to tell them, maybe get different colored pieces and put them all together and say look this is the same amount as this is, we just cut it down into smaller pieces. Uh huh, or just some type of concrete material like the overhead pie chart things, the fraction pieces, that way you can have that, see show them, I have a whole and then I have a half piece, the same thing having $\frac{4}{8}$ is the same things as having a $\frac{1}{2}$ piece.

> **S12:** I think I would just have more pictures and just explain to him that the circles are the same size and if you move these two together, it would look exactly the same as this one.

Three participants said that Marcus needed to learn or relearn how to reduce fractions to lowest terms.

> **S3:** Um…I follow his line of reasoning and I think in this case he would have benefited from knowing how to reduce a fraction, how to get them into the same form…(follows this with suggesting the use of manipulatives).
>
> **S10:** Maybe introducing the reducing type of thing, but not as, I'm thinking of a younger age, you know just starting third grade, fourth grade.
>
> **S12:** So he needs, maybe he could learn how to reduce them like $\frac{1}{2}$, like how to reduce fractions or if like, just looking at the picture a little more carefully because they're both $\frac{1}{2}$ (follows with suggestions of needing more visual aids).

Two participants said they would reteach equivalent fractions to Marcus or discuss fraction equivalency with him.

> **S6:** And so what we need, um I'd probably have to go back over equivalent fractions with him if he like if we've already learned it and

discuss how we take these two things (pointing to $\frac{1}{4}$ pieces) and I'd probably use manipulatives and say if we have four blocks and I took away two, you could see I took away half of the blocks and I would take away four from eight.

**S8:** I guess I would have him to look at the size, to make sure he's looking at the size because both these circles are the same size. To not just look at the numbers.

With regard to how the instructional representations or situations would change Marcus' thinking, participants discussed Marcus' need to develop an understanding of the equivalence of the fractions. Those suggesting the use of concrete manipulatives or pictorial representations said that being able to move the shaded pieces and change the diagonal orientation of $\frac{2}{4}$ would help Marcus see that the two fractions were the same amount. Those who suggested simplifying the fractions suggested that Marcus would see that when simplified, $\frac{1}{2}$ and $\frac{4}{8}$ would be equivalent to the fraction $\frac{1}{2}$. Several participants seemed to go back and forth between the use of concrete manipulatives and simplifying the fractions algorithmically, often asking Marcus' grade level to determine whether the use of manipulatives was appropriate.

**S2:** Helps to lay pieces on top.

**S3:** Um, well maybe, maybe to realize that because he's probably looking at this picture and these shaded regions could be a little confusing cause if they had this darker shaded region here and if they moved it up (meaning to put the fourths adjacent to one another) so they could be equal. So have him make the manipulative as it is shown in the picture and then say look, you can move them around to make them look more similar and it's the same thing, it's not changing the number.

**S4:** Well I think it would change his thinking in that he would have to relate these two to being the same size and this to be the same shaded area regardless of the division. Because if my two pieces of pie are bigger than your $\frac{1}{4}$ piece of pie.

**S5:** So I definitely think using those pattern blocks would show them that even though there's four green triangles and only two rhombuses, that equals the same thing.

**S6:** So but then they'd understand halves and because I took away halves…when you take away a $\frac{1}{2}$, it doesn't matter how many there are, a $\frac{1}{2}$ is a $\frac{1}{2}$ is what he needs to understand.

**S10:** So they can visually see that, I think that would help.

**S12:** Um, more pictures if he could manipulate them like maybe if it was on a dry erase board, like if it was here, he could see, oh wow, they really are the same.

Task Two- Fractions as Operators

Task two was based on Lamon's (2005) description of a fraction as an operator as an, "increase or decrease [in] the number of items in a set of discrete objects" (p. 151). The student, Alishia, was given a real world fraction situation in which 18 cupcakes were baked for a birthday party and $\frac{2}{3}$ of the cupcakes were eaten. She was asked to determine how many cupcakes were eaten (see Appendix A). Alishia's response was based on the researcher's own experience as an elementary teacher whose procedural understanding of this particular sub construct interfered with her ability to explain this type of fraction situation to students conceptually. Alishia's response was the incorrect use of an algorithm whereby she divided the number of cupcakes by the denominator of the fraction and multiplied the resulting quotient by the fraction numerator. However, as students with mainly procedural understanding often do, Alishia used the algorithm incorrectly, dividing 18 by two and multiplying the quotient by three to obtain 27 as the number of cupcakes eaten.

Task Two provided a bit more of a challenge to study participants than Task One. The reason for this may have been that, based on a lack of previous studies with elementary students' thinking on fractions as an operator, the researcher based the student's explanation on her own experience with an algorithm learned in elementary school that may not have been a common one. Because of this, study participants took more time to digest the student's response. Three major themes emerged from the preservice teachers' responses to questions one and two of Task Two: the reasonableness of Alishia's answer of 27 cupcakes eaten, that Alishia had a process without understanding the concept she was working with, and a lack of understanding of the meaning of the fraction $\frac{2}{3}$. Three participants, S3, S5, and S6, solved Task Two on their own as they were trying to make sense of Alishia's response (Figures 19-21).

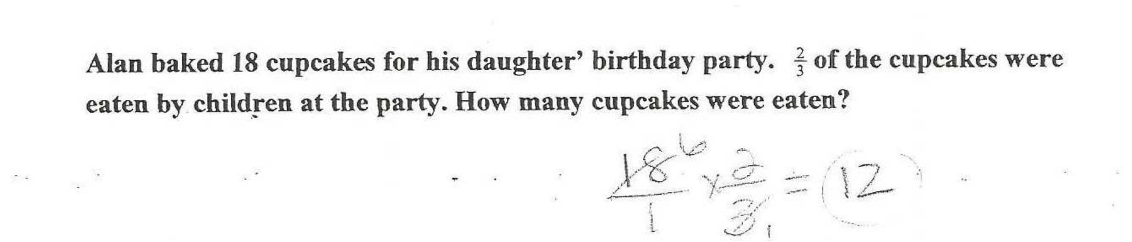



Figure 19: *Task Two- S3*

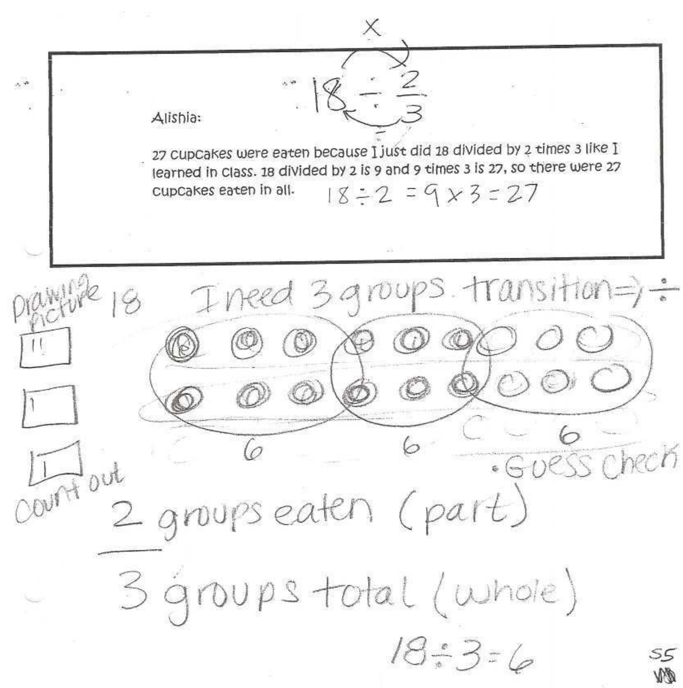
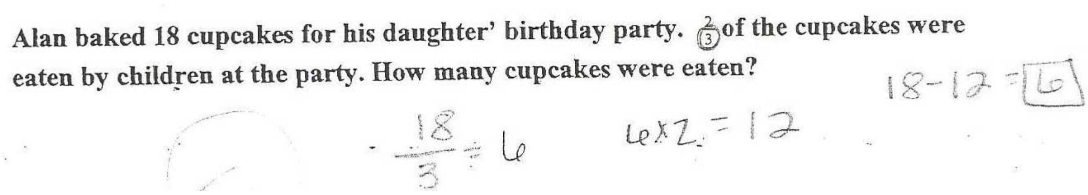



Figure 20: *Task Two- S5*


Alan baked 18 cupcakes for his daughter' birthday party. 2/3 of the cupcakes were eaten by children at the party. How many cupcakes were eaten?

$18-12=\boxed{6}$

$\frac{18}{3} = 6 \qquad 6 \times 2 = 12$

Figure 21: *Task Two- S6*

Participants had the following to say about Alishia's understanding of a fraction as an operator:

> **S2:** Doesn't understand fractions at all. Has a process without a concept.
>
> **S3:** Okay so on this problem I think I see a line of reasoning. So if they see an add they are always going to add, a take away, they're always going to take away. So they have all these words in their head that they automatically think I'm going to do this. So in this case I would say she um, saw the number or the fraction $\frac{2}{3}$ and maybe down the line someone had said that a fraction means to divide it by three because she is seeing a line and sometimes division is written with a line, so I think that was her line of reasoning and it doesn't really see like she, she had no background knowledge to even know how to approach this problem to set up a fraction. It seems like all of her knowledge so far is just the operations, plus, minus, divide, and multiply. Um, because she didn't even think to set it up in a fraction form.

**S4:** Well I mean she did the math correctly so that's great, but I think that I would point out that her answer 27 is more than the initial 18 that were eaten, I mean that were made. So there's no way that 27 could be eaten if 18 were made.

**S5:** I would definitely draw the 18 cupcakes because this is all out of whack (S5 is referring to Task Two student thinking). Definitely. So there are 27 cupcakes eaten in all. They need to stop and think, wait a minute; there were only 18 cupcakes to begin with. How are they going to eat 27?

**S6:** Well, she either forgot or didn't understand that the 3, the denominator is what we're taking out of, we're dividing into three portions, so she divided by two she misunderstood that, if she drew the picture it needs to be cut into three parts.

**S8:** Well their answer doesn't really make sense if there were only 18 cupcakes. (S8 writes $\frac{12}{18}$ on paper.) Teach them to look at answer for reasonableness. Computation is off. Reversed three and two. Knew a process but had it backwards.

**S10:** She, I don't think that she really understands because of the fact that she ended up with 27 cupcakes and there was only 18 to begin with.

**S12:** Whoa. Okay. (laughs) Um, well she gets that fractions are some kind of division. Just a little off.

In order to help Alishia connect her procedural thinking to conceptual understanding of the fraction $\frac{2}{3}$ operating on the whole number quantity 18, four out of eight participants again suggested the use of concrete objects or pictorial models. More than half, five out of eight, remarked that they would begin by questioning Alishia about the reasonableness of her answer of 27 cupcakes eaten if there were only 18 cupcakes to begin with. Two participants mentioned having Alishia explain her process. Though concrete models were mentioned for Task One, Task Two seemed to move most participants in a more student-centered direction, with the goal of helping Alishia challenge her own thinking rather than providing her with an explanation of her misconception or explaining a concept. It was unclear why this task prompted participants to have Alishia explain her thinking rather than provide a process or model for her to follow as was shown in other tasks. Perhaps participants themselves did not understand her solution clearly and move in a student-centered direction to rely on Alishia's explanation as a starting point for further instruction. S3 and S5's responses to this part of Task Two differed from the rest of the participants. S3 explained how she would show Alishia how to correctly use the algorithm she was misusing. S5 gave a detailed example of how she would explain the concepts to the third graders in her methods observation class.

> **S2:** Illustrate $\frac{2}{3}$ of cupcakes. Three groups first then pull two. Two of six out of three groups.

**S4:** I mean are they allowed to have like, okay I would give them manipulatives and I would break it up into, I would have 18 and then I would break it up into three sections and then I would say okay $\frac{2}{3}$ were eaten so you could count each $\frac{2}{3}$ and come up with six.

**S6:** I would talk to her and ask her if she understood, if she could see. I would want her to go through all the steps of her thinking with me. So I would ask her again and then if she would go back to the two, I would be like, oh, did they divide it into two because it's two out of the three cupcakes.

**S8:** I guess divide it into three and then if you take away two of the thirds, two of the sections then that would be how much was eaten. Needs to visualize. Pieces of a group, not just a circle.

**S10:** So more concrete type materials and maybe that would help her to visually see.

**S12:** Okay. I think just maybe, I know a picture would really help on this one just because then there would be no way she would get 27 because if you had 18 of something and you divided it up into three sections, she could just see the two and be like, oh okay, that's 12.

Participant beliefs regarding how the instructional representations or situations would change Alishia's thinking centered around challenging her erroneous thinking and putting the algorithm aside, instead allowing her to visualize or discover the meaning of taking two out of three parts of a discrete set of 18 objects. S8 even went so far as to recognize that Task Two required a different type of model, a set model rather than an area model, to develop Alishia's understanding.

**S2:** Put process aside until they can visualize.

**S3:** Guided manipulation for a problem like this would be excellent.

**S4:** I would have 18 and then I would break it up into three sections and then I would say okay $\frac{2}{3}$ were eaten so you could count each $\frac{1}{3}$ and come up with six.

**S5:** So with my kids, I've see a lot of them do different things and most of the time it's a bigger number and it doesn't work out. They like to do the boxes. They like to draw, they would draw three boxes like this and they would count 18, you know one, two, …and they would figure out how many are in this box (S5 is referring to sharing 18 in three groups using circles or boxes to pass out 18, but does not refer to it as a particular type of division.) and they need to know what 18 divided by threee would be.

**S6:** So I would ask her again and then if she would go back to the two, I would be like, oh, did they divide it into two because it's two out of the three cupcakes. So I'd make sure she understood what the $\frac{2}{3}$ meant, that it's two out of the three.

**S8:** Needs to visualize pieces of a group, not just a circle.

> **S10:** So more concrete type materials and maybe that would help her to visually see.
>
> **S12:** I know a picture would really help on this one just because then there would be no way she would get 27 because if you had 18 of something and you divided it up into three sections, she could just see the two and be like, oh okay, that's 12. Is that okay?

Task Three- Fractions as Ratios

Task three utilized ratios in two forms: a part-part comparison of the ratio of boys to girls in a class and a part-whole comparison of the number of boys in a class compared to the total number of students in the class. Lisa, an elementary student, was asked to determine the number of male students in the class if there were four boys for every two girls in the class. The term ratio was purposefully not used in the problem as ratios are not generally introduced formally in the elementary grades. However, the ratio task was included because preservice elementary teachers' knowledge of content and students should extend beyond that which is expected of elementary students (Conference Board of the Mathematical Sciences, 2000) to include various interpretations of rational numbers.

Task Three presented the greatest challenge to the preservice teachers' own knowledge of fractions. Lamon (2005) defines a ratio as, "a comparison of any two quantities" (p. 183) and notes that due to the varied uses of ratios, they can be difficult to interpret. While four participants identified that Lisa was incorrectly interpreting the ratio of boys to girls in the class, an additional four participants expressed a difficulty solving the problem themselves. Three participants said they could solve the problem themselves, but were unsure how they would explain the problem to a student. These themes composed the majority of participant responses to Task Three. Seven participants, with the exception of S12, showed some amount of work as they tried to solve the problem themselves and/or interpret Lisa's explanation of her answer (Figures 22-27). Below are participant responses regarding their own or Lisa's understanding of Task Three:

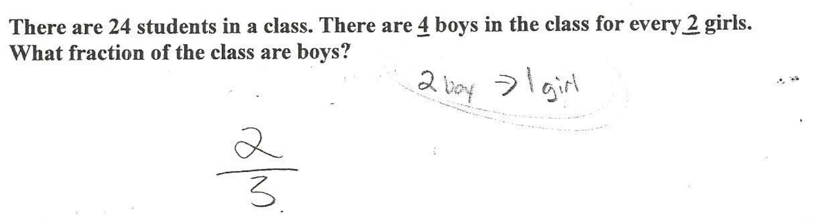
> **S2:** Relationship between part whole. Doesn't have same units in part whole. Ratio within a ratio. Two different units.
>
> **S3:** (S3 creates ratio for task three and solves for x.) Okay, so how do you do this problem? (as she is working it out) I'm so not used to having to explain my reasoning. Okay so for this one I would think it's venturing into my thought, which would be ratios which would be fractions ratios go hand in hand. What his problem was is thinking about a total, so he knows that there's 24 students in the class, but when he saw that there are four boys for every two girls…I'm actually not following his reasoning.
>
> **S4:** Six groups of boys. Well, obviously wrong. Um, I really just think I would explain to them the way I would have to figure it out.
>
> **S5:** Okay, I have to figure this one out. (S5 reads problem a few times**.)**
>
> **S6:** Okay, I don't know how he did it because honestly I had to do it a weird way. Um, um, I honestly don't know where. He shouldn't have gone straight to dividing I don't believe (reads part of problem aloud

again) because it's not four to one. I think that's where he might have made a mistake.

**S8:** I'm trying to remember how to do this myself. I don't know how to get this answer.

**S10:** It's a ratio four to two. I honestly don't know how I would fix that.

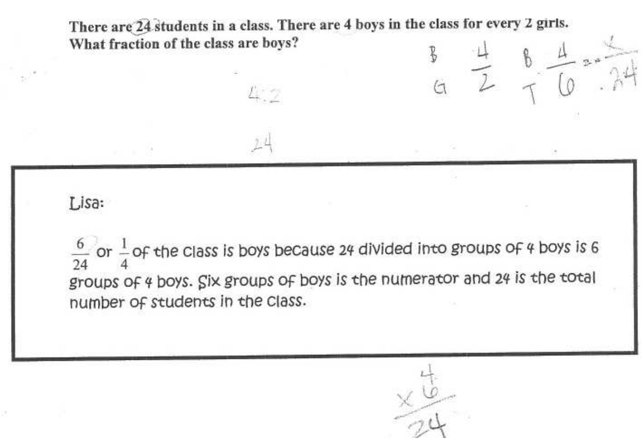
**S12:** This one is going to take me a second to figure out. Okay. Alright well she gets that the top is divided into the bottom which is good. But I think this one is confusing because of the ratio. It even confused me.


There are 24 students in a class. There are 4 boys in the class for every 2 girls.
What fraction of the class are boys?

2 boy → 1 girl

$\frac{2}{3}$.

Figure 22: *Task Three- S2*


There are 24 students in a class. There are 4 boys in the class for every 2 girls.
What fraction of the class are boys?

Lisa:

$\frac{6}{24}$ or $\frac{1}{4}$ of the class is boys because 24 divided into groups of 4 boys is 6 groups of 4 boys. Six groups of boys is the numerator and 24 is the total number of students in the class.

Figure 23: *Task Three- S3*

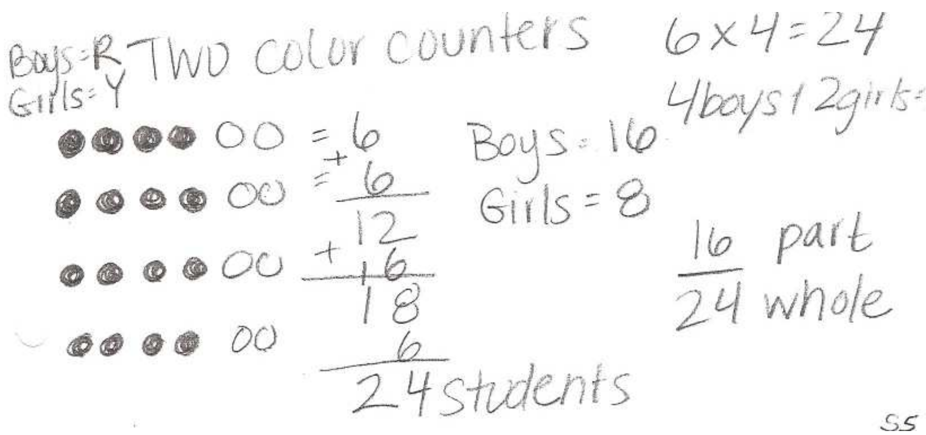



Figure 24: *Task Three- S5*

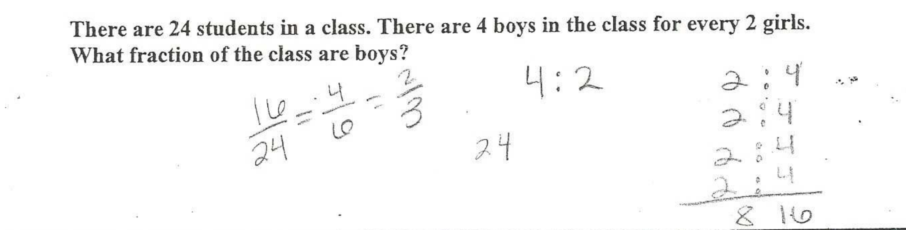

There are 24 students in a class. There are 4 boys in the class for every 2 girls.
What fraction of the class are boys?


Figure 25: *Task Three- S6*

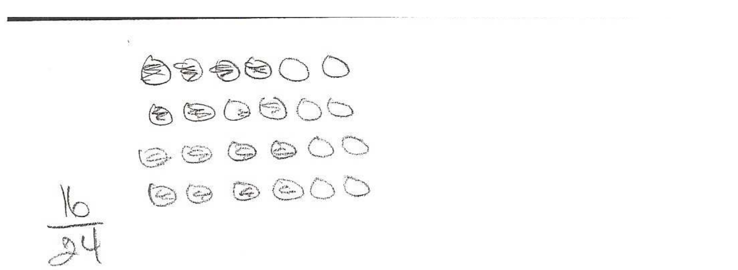
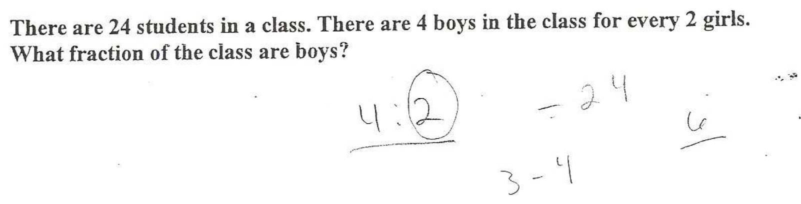



Figure 26: *TaskThree- S8*


There are 24 students in a class. There are 4 boys in the class for every 2 girls. What fraction of the class are boys?

4:2 = 24 6

3-4

Figure 27: *Task Three- S10*

Due to the difficulty most participants had either solving Task Three themselves or articulating how they would explain it to a student, the study provided limited information as to what instructional representations or situations this group of preservice teachers would use to help a study having trouble interpreting fractions within the ratio sub construct. Five of the eight participants were unable to provide any specific representation or situation to change Lisa’s thinking. Three participants, S4, S5, and S12, mentioned the use of various types of concrete manipulatives to count out four boys to two girls to show Lisa the ratio boys to girls in a class of 24. The participant responses of S4, S5, and S12 are below.

> **S4:** Um, I really just think I would explain to them the way I would have to figure it out. And that is maybe have two different colors of manipulatives, one for boys and one for girls and literally sit there and say four and then two and then four and then two.
>
> **S5:** I would get, there are these little chips and I used them today and one side is yellow and one side is red.
>
> (Researcher prompts S5 with correct name for manipulative she is describing.) Two color counters?

Yeah, two color counters. The idea that, um six times four equals 24 is a good idea, that's definitely a good idea. That will help in the long run because four plus two. The four boys plus two girls equals six as well. So I would take those two color counters and we would put down 4 red, so here's four boys (Draws and says boys=red, girls=yellow) and then two girls. Okay, so now we know four boys, 2 girls and that equals six. We have more than six students in the class. Well, we already figured out that six times four equals 24 so we need so again we have four boys for every two girls so there's another six. Six plus six equals 12. Okay, we still need 24. Okay another four boys and two girls. That's six. Okay, 12 plus six, that's 18. Well, we still need more because we need 24. Okay, so another four boys. I'd definitely do this on the overhead while they had their own little manipulative kind of things. Okay that's six more. That's 24. Okay, 24 students. Okay, but that's not our answer. What is the question? Go back and ask yourself what are they asking. Okay, so now we have our 24 students right here, so count up how many are boys…one, two, three, four, five, six, seven, eight, nine, 10, 11, 12, 13, 14, 15, 16, so we know we have 16 boys and two, four, six, eight girls. Okay so how many students do we have in the class? What's your whole? Your whole is 24. Whole goes on the bottom. What are we

looking for? What fraction of the class are boys. There are 16 out of 24 boys.

**S12:** Um, but, I think if you had like manipulatives, like little blocks or cubes they could kind of break it up into four boys, two girls, four boys, two girls.

Task Four- Fractions as Points on a Number Line

Task four asked a student to place three fractions: $\frac{1}{5}, \frac{1}{3}$, and $\frac{5}{6}$, on a number line between zero and one. Two fractions, $\frac{1}{4}$ and $\frac{7}{8}$, were placed on the number line as benchmarks. This task addressed two difficulties explored in previous studies with elementary students' understanding of ordering fractions on a number line. In a study with elementary students, Dufour, Bednarz, and Belanger (1987) discussed the number line representation for fractions as a difficult transition for elementary students. Students' initial use of number lines as external representations for whole numbers proved problematic when students were presented with the empty space between whole numbers as a representation of an infinite number of rational numbers.

In addition to difficulty with the number line as a representation for whole numbers, elementary students also have difficulty correctly interpreting the meaning of fraction denominators to order fractions. Smith, Solomon, and Carey (2005) looked at students' reasoning in comparing fractional denominators and found that students often identified a fraction as larger because the denominator was larger. Therefore, like students in previous studies (Dufour, Bednarz, & Belanger, 1987; Smith, Solomon, & Carey, 2005), the student in Task Four ordered the fractions with respect to the size of the denominators, rather than based on the location of the fractions with respect to zero, one, and the benchmark fractions.

The majority of participants, seven out of eight, correctly attributed Juan's difficulty with ordering the fractions to his incorrect understanding of the relationship between the numerator and denominator of a fraction. Two participants, S2 and S6, commented specifically that Juan did not understand the relationship between the numerator and denominator. Five participants were more detailed, saying that Juan was comparing the denominators without consideration of the relative size of the fraction wholes and the number of pieces being considered in the numerator. One participant, S3, commented about Juan's consideration of fractions in terms of his previous experience with whole numbers. Six participants provided some written explanation of their thinking on Task Four, most of which was related to illustrations of models used to compare the fractions (Figures 28-33). Below are participant responses about Juan's understanding of Task Four.

> **S2:** Didn't understand concept of ratio. Thirds. Fourths. Each part is a different size.
>
> **S3:** Okay so he has the problem with just taking the numbers at face value. What he knows about numbers is that one comes before two and so on, but with fractions its opposite and he doesn't really have that idea yet.
>
> **S4:** So what he's done is just ordered the denominators.

**S5:** Three is smaller than four (emphasis on words as if she's seen this explanation before). Okay, like I said before that just because the numbers are smaller, um, doesn't mean it's smaller by any means.

**S6:** (S6 reads task aloud. Finds CD for some fractions. Spends time solving before focusing on student reasoning.) God bless these kids' souls. I can't handle this. I hate fractions! I didn't realize it until now. Alright. He's probably about as confused as I am. No, I um, I mean he obviously just doesn't get, it's common denominator that he doesn't get. Like there are different denominators and in order to find how to place them easier is to put them in the same denominator, but you can't look at the numerator unless all, without looking at their common denominator first.

**S8:** Yeah he doesn't realize that like one third is really bigger pieces. He's just saying, oh three is smaller.

**S10:** Oh, I see what he did. He automatically thinks that, he just puts the numerators, oh, I'm sorry, the denominators in order. He doesn't have the concept of $\frac{1}{3}$ is actually bigger. In fractions, it's kind of the opposite. You know, the larger the denominator, the smaller the fraction. He doesn't have any representation of what the fraction, what the concept is.

**S12:** Oh, well, he's just going by the denominator four is bigger than three so $\frac{1}{4}$ is bigger, so um.

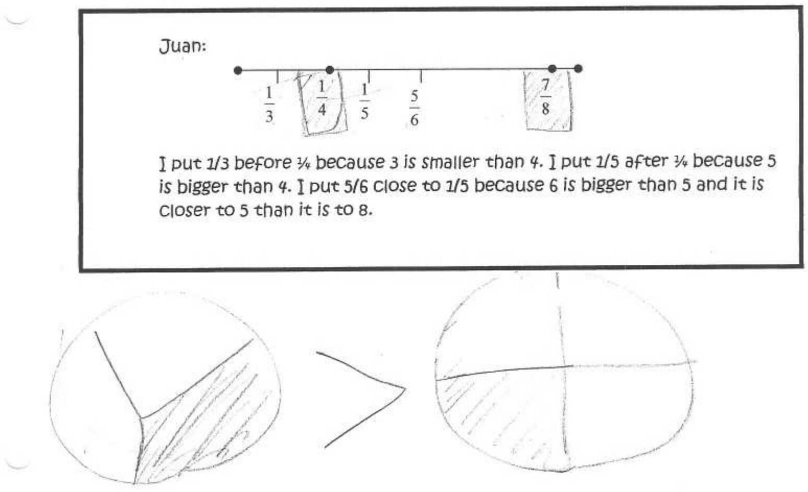



Figure 28: *Task Four- S2*

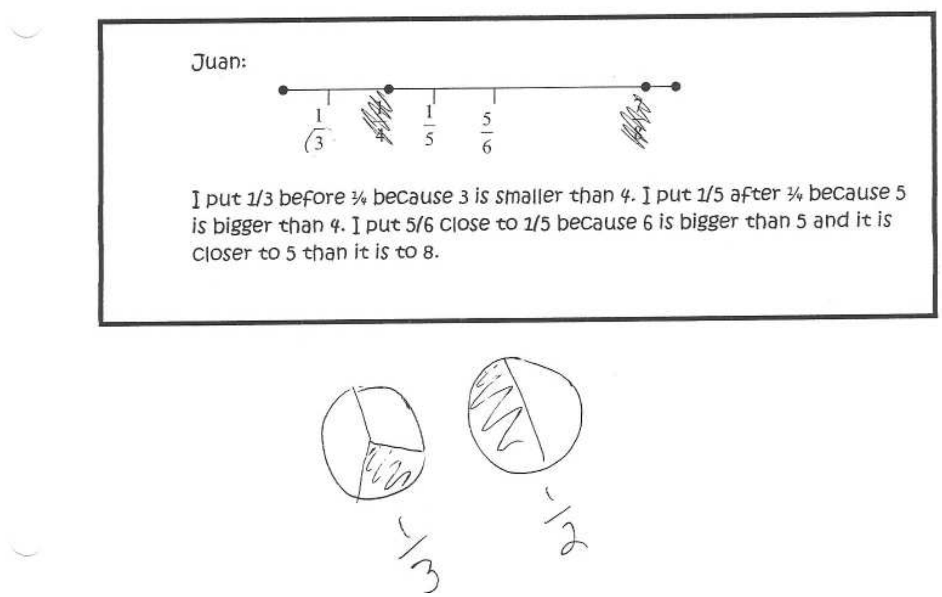



Figure 29: *Task Four- S3*

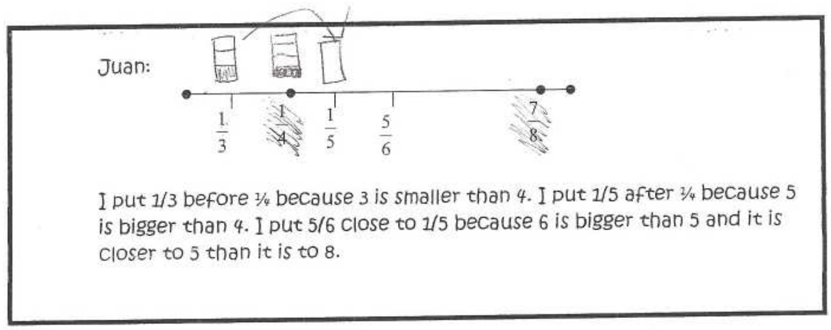



Figure 30: *Task Four- S4*

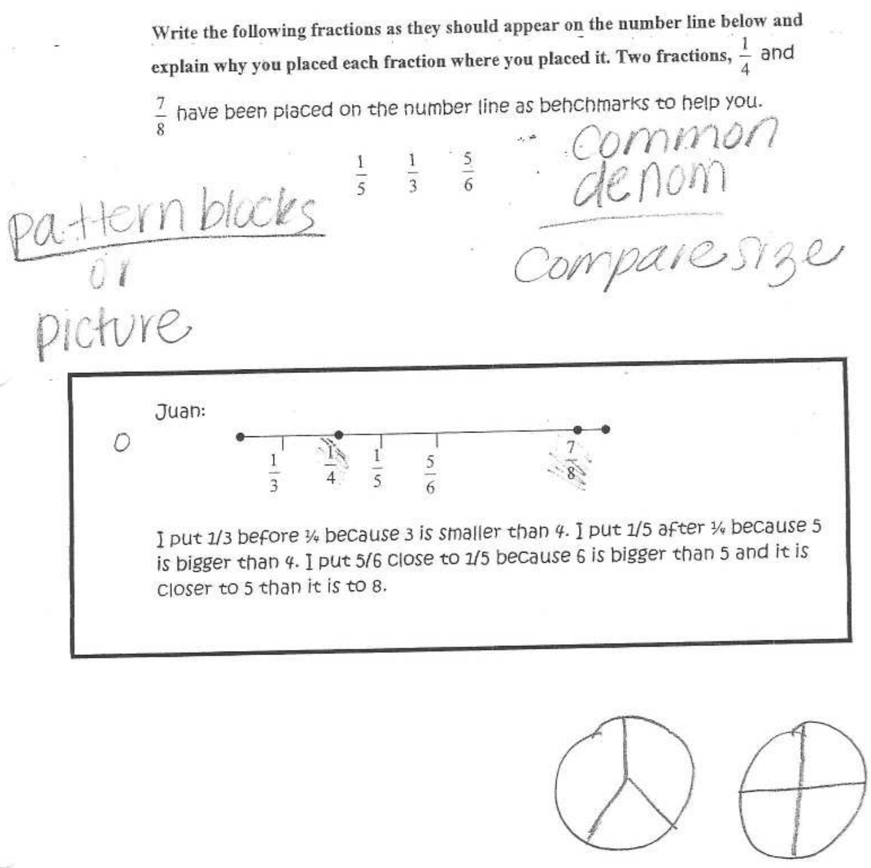



Figure 31: *Task Four- S5*

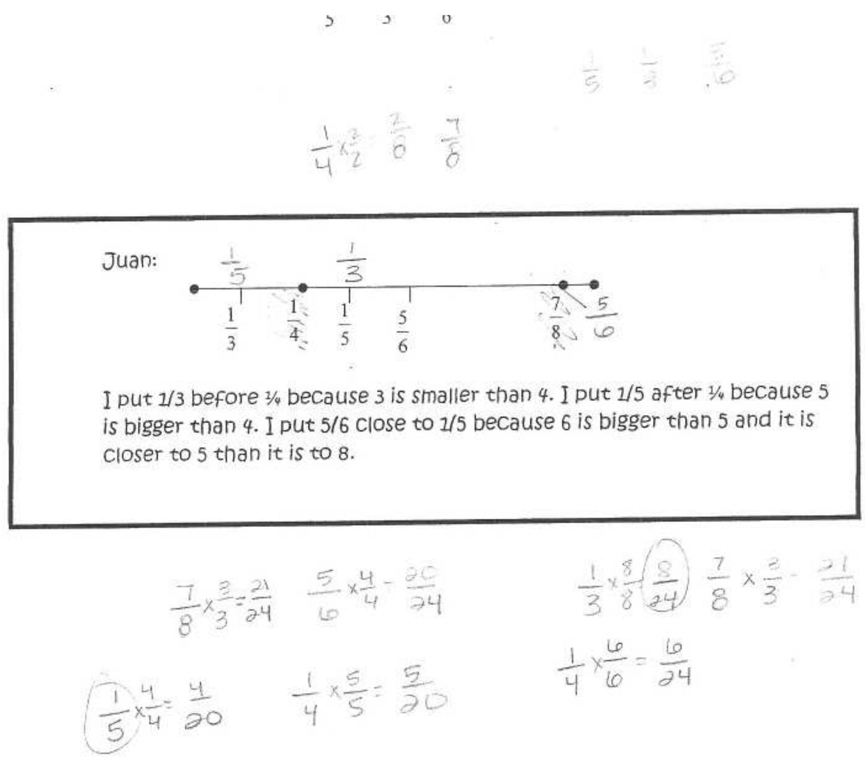



Figure 32: *Task Four- S6*

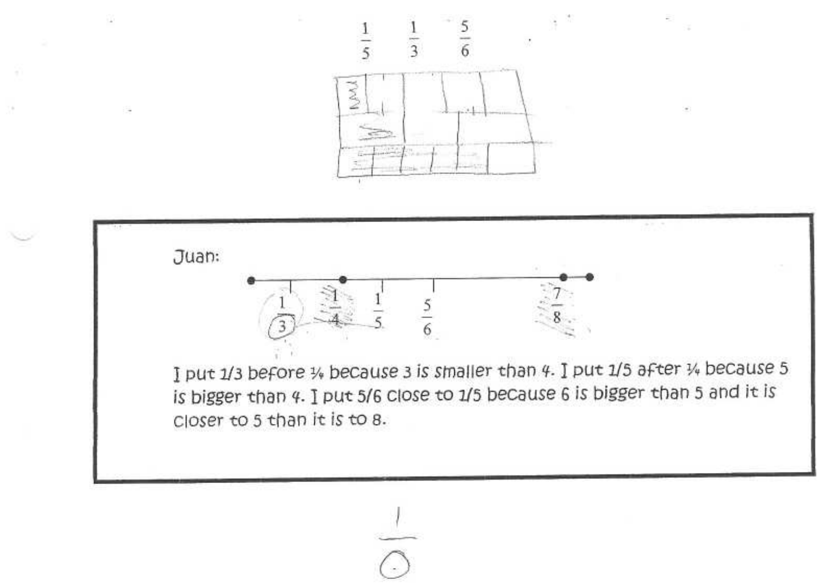



Figure 33: *Task Four- S10*

The majority of study participants easily articulated instructional representations or situations they would use to help clarify Juan's misconception that larger fraction denominators always result in larger fractions. Some participants provided more than one means of clarifying Juan's thinking. Six participants suggested the use of picture models, most of them circular representations of fractions, such as pizzas or pies. Five of the eight participants said they would use various types of manipulatives, including fraction circles, pattern blocks (suggested incorrectly by S5 as representation for fifths), and fraction towers, to let Juan build the fractions and place them on the number line. Three participants, S5, S6, and S8, remarked that they would show Juan how to find a common denominator of the fractions to order them on the number line. Though S5 and S8 mentioned the use of a common denominator, they arrived at that suggestion in different ways. S5 suggested the use of a common denominator when she became confused with her own incorrect explanation of using pattern blocks to create fifths, while S8 was unsure that finding a common denominator was an appropriate strategy for someone Juan's age. S6 was the only study participant on Task Four who did not suggest the use of either a concrete or pictorial model, relying on solely on the use of a common denominator as shown in Figure 33. Participant responses to Task Four are below.

> **S2:** Um Use pizzas with parts for number line. Have them do it visually until they understand (draws on paper). With $\frac{5}{6}$ and $\frac{7}{8}$ …have them draw that out too.

**S3:** To help him, I would probably use some sort of manipulative like a circle (draws a circle) to see, you know, that this is $\frac{1}{3}$ and this is $\frac{1}{2}$ but yet this is bigger. Like have him explore those concepts and have him figure out why that happens and have him explain well, when with fractions we're actually dividing by the bottom number so that would explain…

**S4:** I think that again you would need a visual model. I would use a glass of water instead of a pie or a piece of pizza. (Draws glass $\frac{1}{3}$ full) And this glass is $\frac{1}{3}$. And this glass is $\frac{1}{4}$ full.

**S5:** So I definitely think they should be required to draw a picture for each fraction so they need to, despite the number line, forget the number line, start with $\frac{1}{5}$, and so or they could take the pattern blocks again, they could use the pattern blocks. And I don't know if you can express a fifth, you can express a fifth with the green triangle, pattern blocks. Pattern blocks or picture because it's hard for them to draw an exact picture that would show that it's bigger than another so I definitely think…Um, and use that, okay I would definitely start out with my whole, that's an octagon (referring to the pattern blocks) and it's orange, yeah it's orange. And um start out with that. What I did when I created fractions like that is start with the octagon and then get the triangle. Would that work? No because six triangles fit on that. So how would I express $\frac{1}{5}$? Cause they're not all the same on the bottom. The wholes aren't that same. Okay, that's pretty complicated. I'm making it complicated. Okay, change these all to the same so you can see the numbers on the top. Okay, so what's the common denominator? What can I multiply all of these by to get the same denominator? That's another way. Get a common denominator. And I'm always on a third grade level here so it's hard to, it's hard for me to remember what's required in higher grades.

**S6:** And I think they would have already learned how to find common denominators, so I'd go back through the whole process and we'd go through one by one like I did and have him do each one and have him talk it out with me (gives example of three times eight to find a common denominator between $\frac{1}{3}$ and $\frac{7}{8}$). Sometimes it's easier to multiply each other by the other denominator and so.

**S8:** Or I don't know if you would tell him to get a common denominator. If they're too young for that?

**S10:** So I think again, you know concrete materials. Just show them, you know, I have these blocks here. $\frac{1}{3}$ is this much amount, $\frac{1}{5}$ just because the denominator is bigger doesn't mean, you know, I show him how $\frac{1}{5}$ is bigger than, I mean, smaller than $\frac{1}{3}$. So just more or less concrete visual learning.

**S12: -** Picture models, manipulatives or things like that so they can play around with it and look at it.

Unlike the difficulties study participants had articulating instructional representations that would help the student in Task Three, as well as ways in which those representations or situations would help the student, most of the participants were able to say with much more confidence how their suggested instruction would help correct Juan's thinking about fraction denominators. Seven out of eight participants mentioned that the use of concrete models or pictorial representations would be helpful to Juan because he could see the relative size of the fraction pieces, making problematic his explanation of fractions with larger denominators being larger fractions. S6, whose only instructional situation centered on finding a common denominator, did not provide any particular explanation of why she believed the process of finding a common denominator would be helpful to Juan.

*Comparison of Knowledge Representation on Concept Maps and Fraction Tasks*

A third purpose of the study was to look at connections that existed between the EC-4 preservice teachers' SCK and KSC of fractions. In other words, the researcher wanted to seek a clearer picture of whether knowing fractions in a certain way (SCK) as represented on the concept map affected how study participants interpreted student work with fractions (KCS). There were few direct connections between the *a priori* themes used to analyze the concept maps and emergent themes from participants' discussion of hypothetical learning trajectories related to the fraction tasks.

S2 was the only study participant whose concept map closely mirrored her responses to the fraction tasks with regard to Kieren's sub constructs of fractions. Her concept map focused the most strongly on the ratio sub construct with five of the ten nodes in her map connected to ratio in some way. In addition, her responses to three of the four fractions tasks identified students' misconceptions as related to a misunderstanding of a ratio. In the case of S2, it would appear that her strong connection to the meaning of fractions as a ratio had some influence on her perception of student misconceptions even on tasks that were not designed to measure students' understanding of a fraction as a ratio.

Though the remaining interviews did not exhibit strong connections between the *a priori* and emerging themes, other connections between SKC and KCS of fractions were evident in several of participants' concept maps and hypothetical learning trajectories. For example, S3's concept map placed a strong emphasis on operations with fractions, with nine of the fourteen nodes describing procedural knowledge of how to add, subtract, multiply, and divide with fractions. Likewise, her comments on student work on the fraction tasks often focused on teaching students an algorithm for "reducing fractions" or showing a student how to work through a word problem to "take out what they're asking." In Task Three, S3 solved the ratio herself but was unable to verbalize how she would explain the task to a student, admitting, "I'm not used to having to explain my reasoning." It would appear that S3's understanding of fractions might be related to that of college students who, according to the Conference Board of the Mathematical Sciences, “see fractions only as pairs of natural numbers plugged into arithmetic procedures; hence, to them, fractions is simply a computation with four integers” (2001, p. 19).

S5 approached the concept map and fraction tasks as if she were teaching actual students, providing a significant amount of written and oral detail for the concept map and each of the fraction tasks. She made connections between her methods observation experience in a third grade classroom and the concepts discussed on her map and what she saw in student misconceptions on the fraction tasks. S5 was the only participant who mentioned her mathematics methods class or her observation experience in an elementary school. She seemed grounded in both experiences, using knowledge obtained in her methods course about manipulatives, such as pattern blocks and two color counters, and knowledge of her third graders’ misconceptions about fractions to make suggestions for the students in the fraction tasks. Her responses, both on the concept map and fraction tasks were detailed, with written, pictorial, and symbolic illustrations throughout. It is worth noting, however, that due to scheduling conflicts, S5 was interviewed closer to the midterm of the semester, several weeks after the early interviews. The connectedness of her knowledge may have been influenced more by time spent in the methods and third-grade observation class than participants interviewed earlier in the semester.

The concept map and fraction task responses of S8 were not as detailed many of the study participants. However, she was the only participant who placed any kind of emphasis on fractions as representations of equal-sized shares. Her concept map was the only one that contained a node specifically mentioning that the "parts have to be equal." This idea of fair shares was reiterated by S8 in her discussion of student misconceptions and instructional representations in tasks one and four. According to Van de Walle (2007), "the first goal in the development of fractions should be to help children construct the idea of *fractional parts of the* whole [emphasis in original] (p. 294).

After additional review of the audio, typed transcripts, and written work on the concept map and fraction task responses for participants S4, S6, and S10, no strong connections between the EC-4 preservice teachers' SCK and KCS were found. However, further analysis of interview audio and typed transcripts from S12's interview revealed an affective connection regarding S12's general feelings toward the interview itself and about her ability to represent her own thinking about fractions and produce representations that would help students.

# CHAPTER V

# CONCLUSIONS

## *Summary of Findings*

The first research question looked at participants' specialized content knowledge of fractions using a participant-created concept map. Five of Kieren's sub constructs of fractions: fractions as a representations of a part-whole relationship, fractions as operators, fractions as ratios, and fractions as points on a number line, and fractions as division, were provided to participants on the map and used as a means of *a priori* analysis, along with fraction notation, and terminology.

Participants in this study appeared to be most familiar and/or comfortable with fractions as a representation of a part-whole relationship, with seven out of eight participants including at least one node supporting this sub construct. The EC-4 preservice teachers studied were equally familiar with fractions as a ratio and as a representation of division. Five out of eight participants provided concept map nodes for one or the other of these sub constructs. Fractions as an operator on a quantity and as a point on a number line were the least represented as interpretations of fractions. One participant created a node for a fraction as a point on a number line with one participant specifically mentioning that she did not think of a fraction as a point on a number line. None of the participants in the study used a fraction as an operator explicitly on their concept map, though S5's example related to the node labeled "division" represented taking a fractional amount, $\frac{1}{4}$, of the whole number eight. These results are consistent with previous research (Carraher, 1996; Lamon, 2001) which found part whole relationships to be among the most common representations used by teachers to explain the meaning of a fraction.

The second research question explored participants' knowledge of content and students using student responses based on four of the five sub constructs provided on the concept map. Participants' diagnoses of student misconceptions and plans for instruction were strongest on Task One, fractions as a representation of a part-whole relationship. The strength of the responses on Task One was not surprising, as almost all study participants described this fraction sub construct on their concept maps. The most difficult task for participants, however, was not related to the sub construct exhibited the least on the concept maps. The EC-4 preservice teachers had the greatest difficulty with Task Three, fractions as ratios. Many had a difficult time solving the task themselves and most struggled to provide a student-friendly explanation to the task.

Tasks Two and Four elicited varied reactions from participants. Although no participants created concept map nodes for a fraction as an operator, Task Two, involving taking a fraction amount of 18 cupcakes, presented few problems for the EC-4 preservice teachers. On this task, participants easily solved the task and were able to suggest various types of representations, such as pictures or sets of objects to explain the concept to students. Task Four, the representation of a fraction as a point on a number line, shown by previous research to be the most difficult for students to understand (Dufour, Bednarz, & Belanger, 1987), caused one teacher to say, "God bless these kids' souls. I can't handle this. I hate fractions! I didn't realize it until now!"

The third research question looked at any connections that existed between the concept maps and participant responses to the fraction tasks. Only one participant, S2 showed any connections related to Kieren's sub constructs of fractions. Her familiarity with ratio heavily influenced her diagnosis of student misconceptions on three of the four fraction tasks. Participants S3, S5, and S8 showed consistencies in their general understanding and/or explanation of the meaning of fractions, while S12 showed a connection not in her knowledge necessarily but in her lack of belief in her ability to provide helpful explanations on the concept map and fraction tasks. Though this finding is not directly linked to research questions in the present study, connections have been shown between teachers' attitudes toward mathematics and their conceptual understandings of those topics (Ma, 1999).

S3's strongly algorithmic understanding of fractions as demonstrated on her concept map appeared in her suggested instructional representations for students in Tasks One, Two, and Three. Her suggestions for these tasks centered around teaching or reviewing a procedure, such as simplifying fractions, solving a ratio, or in the case of Task Two, "working through the word problem to take out what they're asking, to realize that it is a fraction problem and it's just more of an algorithm way, just to know that when you say $\frac{2}{3}$ of a number, you have to multiply the fraction." Her strongly procedural knowledge and infrequent use of non-algorithmic representations mirrored findings in studies with other preservice teachers (Borko et al, 1992; Lubinski, Fox, & Thomason, 1998; Ward, Anhalt, & Vinson, 2004) where preservice teachers focused mostly on procedures without much explanation of why such procedures made sense conceptually.

The level of detail in S5's concept map carried over into her interpretations of student misconceptions and instructional representations chosen to correct them. In contrast with Ball's findings (1988) that many times teachers' diagnoses and chosen instructional representational situations failed to emphasize conceptual understanding, S5 went to great lengths to explain her own elementary school experiences with manipulatives and how her experiences in teacher preparation courses strongly emphasized the use of various types of representations to assist students in their developing understanding of mathematics. Again, it is possible that her interview, closer to the midterm of the semester, provided S5 more time to gain a greater connectedness in her knowledge of the concept of fraction and a greater exposure to manipulatives and other representations to help correct students' misconceptions.

S8's concept map, though not as detailed as some, provided the only mention of the same sized shares and/or wholes. This understanding carried over into the instructional representation suggested for students. In the case of S8, it would appear that the concept map and fraction tasks did not adequately capture the connectedness of her knowledge of fractions. Though not lengthy, her thoughtful responses and careful attention to the foundational concepts of fractions, such as the notion of fair shares, indicated that external factors, such as a lack of familiarity with the researcher or the interview environment, impeded a fuller explanation of her conceptual understanding of the fractions.

Largely absent from the concept maps and EC-4 preservice teachers' creation of hypothetical learning trajectories for students on the fraction tasks were several critical aspects of building understanding mathematics understanding using instructional representations as suggested by teacher education organizations such as the National Council of Teachers of Mathematics (2000). The analysis revealed almost no suggestion of real world contexts outside of common pizza or pie representations. In her work with preservice teachers, Ball (1993) suggested that teachers must take into account representational contexts that were relatable to students. Generally, when participants own content knowledge on a task was weak, as in the case of the ratio task, their use of representations was formulaic and rule-bound. This finding supports the results of previous studies with preservice teachers choices of instructional situations and contexts (Goulding, Rowland, & Barber, 2002; Tirosh, 2000; Ward, Anhalt, & Vinson, 2004).

### *Implications of Findings*

The EC-4 preservice teachers SCK of fractions demonstrated the strongest connection to the part whole sub construct, supporting the suggestions of the Conference Board of the Mathematical Science (2000) and Lamon (2005) that teachers tend to focus mostly on one representation of fractions. Though the purpose of the present study was not to determine the effects of either a mathematics or mathematics methods course on EC-4 preservice teachers' understanding of fractions, both types of courses could contribute significantly to widening preservice teachers' views of what fractions are and how they fit into number systems in general.

The findings of this study provide information for mathematics methods instructors regarding what preservice teachers know about fractions, but also about how they hold that knowledge. Mathematics methods instructors can take preservice teachers' existing knowledge of fractions demonstrated on concept maps and through interpretations of student work and challenge and solidify that knowledge to move beyond the part whole representation of fractions. Though mathematics methods classes at this particular university were combined with a classroom observation requirement, there was no guarantee that the preservice teachers would have the opportunity to apply their knowledge of fractions to diagnose student errors and chose instructional representations to correct their thinking. In fact, only one participant, S5, seemed to draw on her classroom observation experience during the interview.

*Limitations of Study*

The limitations of this study are related to the duration and exposure of the researcher to participant knowledge, scheduling during the methods semester, and lack of connections shown between concept maps and hypothetical learning trajectories created by participants. The interviews in the study were conducted only once per participant during the semester and were not combined with any observations of actual instruction in methods observation classrooms. Due to the researcher's primary job as an elementary educator, it was not possible to meet with participants more than once during the semester to follow up or ask further questions related to the concept maps or hypothetical learning trajectories. The one-time interview also resulted in a lack of member-checking to determine the accuracy of the researcher's interpretation of participant responses to interview questions.

Though the one-time interviews yielded significant amounts of data regarding participant knowledge of elementary fractions concepts, the results would have been bolstered by further questioning on the part of the researcher and observations of the EC-4 preservice teachers working with actual students on fraction tasks. Also, the nature of the concept map as a construction of participants' own knowledge would have been more beneficial to participants if follow up and discussion were an additional aspect of the study. This type of follow up with participant-created concept maps has been shown to enhance preservice teacher knowledge in previous research (Bolte, 1999). Again, limited time on the part of the researcher limited the study design to a one-time interview.

The scheduling of the interviews, between the end of January and the end of March of the spring semester may have resulted in unfair comparisons of preservice teachers at differing stages in the mathematics methods course. Though the researcher attempted to schedule interviews as close together in time as possible, conflicts pushed back some interviews significantly. This resulted in some participants having more exposure particularly to knowledge of content and students, both from the mathematics methods course and work with students in methods field observation classrooms, than those interviewed earlier in the semester.

The method of combining concept maps with hypothetical learning trajectories as a means of looking at connections in the specialized content knowledge and knowledge of content and students revealed connections in only half of the participants interviewed. This may have been due to participants' lack of familiarity with concept mapping as a means of explaining thinking or the short duration of the interview with no follow up. A pre-post concept map, like that used in Bolte's (1999) with study with preservice teachers might have been more useful in showing connections in the preservice teachers' specialized content knowledge of fractions and their related knowledge of content and students before and after mathematics methods instruction on elementary fractions concepts.

### *Issues for Further Investigation*

Though the present study provided a detailed picture of one group of EC-4 preservice teachers' mathematics knowledge for teaching fractions, it would be of interest to see how EC-4 preservice teachers' representations of their own knowledge of fractions and their diagnoses of student misconceptions and suggested instructional representations manifested themselves in classroom work with students. Also, it would be helpful to interview more participants in the preservice teachers' learning process, such as the mathematics methods instructor and field observation teachers, to see what connections exist between their understanding of fractions and that of the preservice teachers. S5's repeated mention of both her mathematics methods instructor and her classroom observation teacher suggested an influence of those involved in preservice teacher education on the integration of the specialized content knowledge and knowledge of content and students in preservice teachers.

More work needs to be done with concept maps and the creation of fraction tasks that allow preservice teachers to discuss their own and student thinking and in the determination of what instructional representations are most beneficial for students at the elementary level in various contexts. The methods used in this study would provide valuable data to mathematics methods instructors about the fraction knowledge that preservice teachers bring with them into a mathematics methods course and their beliefs about what instructional representations are meaningful for students. In addition, discourse about concept maps and fraction tasks would benefit preservice teachers in that it would provide them an opportunity to challenge their thinking about fractions concepts.

# REFERENCES

Ball, D. L. (1992). Halves, pieces, and twoths: Constructing and using representational contexts in teaching fractions. In T.P. Carpenter, E. Fennema & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157-195). Hillsdale, NJ: Erlbaum.

Ball, D. L. (2003, February 6). *Mathematics in the 21st century: What mathematical knowledge is needed for teaching mathematics?* Speech presented at the Secretary's Summit on Mathematics, U.S. Department of Education, Washington, DC. Retrieved June 14, 2006 from http://www.ed.gov/rschstat/research/progs/mathscience/ball.html

Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8*(1), 40-48.

Bolte, L. (1999). Enhancing and assessing preservice teachers' integration and expression of mathematical knowledge. *Journal of Mathematics Teacher Education, 2*, 167-185.

Borko, H., Eisenhart, M., Brown, C. A., Underhill R. G., Jones, D. & Agard P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal of Research in Mathematics Education, 23*, 194-222.

Brizuela, B. (2005). Young children's notations for fractions. *Educational Studies in Mathematics, 62*, 281-305.

Cai, J. (2005). U. S. and Chinese teachers' constructing, knowing, and evaluating representations to teach mathematics. *Mathematical Thinking and Learning, 7,* 135-169.

Carraher, D. W. (1996). Learning about fractions. In L. J. Steffe, P. Nesher, P. Cobb, G. Golding, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 241-266). Mahwah, NJ: Lawrence Erlbaum.

Chinnappan, M., & Lawson, M. (2005). A framework for analysis of teacher's geometric content knowledge and geometric knowledge for teaching. *Journal of Mathematics Teacher Education, 8,* 197-221.

Conference Board of the Mathematical Sciences. (2001). Recommendations for elementary teacher preparation. In C. Kessel, J. Epstein, & M. Keynes (Eds.), *CBMS issues in mathematics education series: Vol. 11. The mathematical education of teachers.* Providence, RI & Washington, DC: American Mathematical Society & Mathematical Association of America.

Creswell, J. (2005). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* ($2^{nd}$ ed.). Upper Saddle River, NJ: Merrill-Prentice Hall.

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In D. A. Grouws (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 109-122). Hillsdale, NJ: Lawrence Erlbaum.

Even, R., & Tirosh, D. (2002). Teacher knowledge and understanding of students' mathematical learning. In L. D. English (Ed.), *Handbook of international research on mathematics education* (pp. 219-240). Mahwah, NJ: Erlbaum.

Goldin, G., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. J. Steffe, P. Nesher, P. Cobb, G. Golding, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 397-430). Mahwah, NJ: Lawrence Erlbaum.

Goldin, G. & Shteingold, N. (2001). Systems of representations and the development of mathematics concepts. In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 1-23). Reston, VA: National Council of Teachers of Mathematics.

Goudling, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal*, *28* (5), 689-704.

Greeno, J. G. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan*, *78* (5), 361-368.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*, 330-351.

Hough, S., O' Rode, N., Terman, N., & Weissglass, J. (2005). Using concept maps to assess change in teachers' understandings of algebra: A respectful approach. *Journal of Mathematics Teacher Education, 10*, 23-41.

Interstate New Teacher Assessment and Support Consortium. (1995). *Model standards for beginning teacher licensure and development: A resource for state dialogue.* Washington, DC: Interstate New Teacher Assessment and Support Consortium.

Kamii, C., & Clark, F. B. (1995). Equivalent fractions: Their difficulty and educational implications. *Journal of Mathematical Behavior, 14*, 365-378.

Kaput, J. J. (1987) Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19-26). Hillsdale, NJ: Lawrence Erlbaum.

Khoury, H., & Zazkis, R. (1994). On fractions and non-standard representations: Pre-service teachers' concepts. *Educational Studies in Mathematics*, *27*, 191-204.

Kieran, T. E. (1976). On the mathematical, cognitive, and instructional foundations of rational numbers. In R. Lesh (Ed.), *Number and measurement* (pp. 101-144). Columbus, OH: ERIC-SMEAC.

Kieran, T. E. (1980). The rational number construct: Its elements and mechanisms. In T. Kieran (Ed.), *Recent research on number learning* (pp. 125-149). Columbus, OH: ERIC-SMEAC.

Kieran, T. E. (1991). Rational and fractional numbers as mathematical and personal knowledge: Implications for curriculum and instruction. In G. Leinhardt, R. Putman, & R. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 323-371). Hillsdale, NJ: Lawrence Erlbaum.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Lamon, S. (2001). Presenting and representing: From fractions to rational numbers. In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 146-165). Reston, VA: National Council of Teachers of Mathematics.

Lamon, S. (2005). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Leinhardt, G., & Smith, D. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77,* 247-271.

Lubinski, C. A., Fox, T., & Thomason, R. (1998). Learning to make sense of division of fractions: One K-8 preservice teacher's perspective. *School Science and Mathematics, 98*, 247-259.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates.

Mack, N. C. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education, 26* (5), 422-441.

McGowen, M. A. & Davis, G. E. (2002). Growth and development of pre-service elementary teachers' mathematical knowledge. In John A. Dossey, Jane O. Swafford, Marilyn Parmentie, and Anne E. Dossey (Eds.), *Proceedings of the Nineteenth Annual Meeting of the North American Chapter of the International*

*Group for the Psychology of Mathematics Education* ( Vol. 1, p. 99), Illinois State University, Bloomington/Normal, IL.

National Assessment of Educational Progress. (2005). The nation's report card: Mathematics 2005. Retrieved March 13, 2007 from http://nces.ed.gov/nationsreportcard/pdf/main2005/2006453.pdf

National Center for Education Statistics. (2005). *Highlights from the trends in international mathematics and science study (TIMSS) 2003*. (NCES 2005-005). Washington, D.C.: U.S. Department of Education, Institute of Education Sciences.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points: A quest for coherence.* Reston, VA: NCTM.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89*, 351-363.

Novak, J. D., & Gowin, D. B. (1984). *Learning how to learn.* Cambridge: Cambridge University Press.

Olive, J. (2002). The construction of commensurate fractions. In A. D. Cockburn, & E. Nardi (Eds.), *Proceedings of the Twenty-Sixth Annual Meeting of the*

*International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 1-8.). Norwich, UK: University of East Anglia.

RAND Mathematics Study Panel. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education.* Santa Monica, CA: RAND.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Siebert, D., & Gaskin, N. (2006). Creating, naming, and justifying fractions. *Teaching Children Mathematics, 12*(8), 394-400.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114-145.

Smith, C. L., Solomon, G., & Carey, S. (2005). Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology, 51,* 101-140.

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K., & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Behavior, 4*, 205-225.

Steffe, L. P. (2004). On the construction of learning trajectories of children: The case of commensurate fractions. *Mathematical Thinking and Learning, 6*, 129-162.

Texas A&M University. (2007). *Texas A&M University undergraduate course catalog.* Retrieved June 8, 2008, from Texas A&M University, Office of Admissions Web site: http://www.tamu.edu/admissions/catalogs/07-08_UG_Catalog/course_descriptions/ecfb.htm

Thompson, P. W. (1995). Notation, convention, and quantity in elementary mathematics. In J. T. Sowder & B. P. Schappelle (Eds.), *SUNY series: Reform in mathematics education, Providing a foundation for teaching mathematics in the middle grades* (pp.199-221). Albany, NY: State University of New York Press.

Thompson, P. W. (1995). Notation, convention, and quantity in elementary mathematics. In J. T. Sowder & B. P. Schappelle (Eds.), *SUNY series: Reform in mathematics education, Providing a foundation for teaching mathematics in the middle grades* (pp.199-221). Albany, NY: State University of New York Press.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5-25.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14*, 453-467.

Van de Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Boston, MA: Allyn and Bacon.

Ward, R., Anhalt, C., & Vinson, K. (2004). K-8 teacher candidates' use of mathematical representations and the development of their pedagogical content knowledge as exhibited in their lesson planning. *FOCUS on Learning Problems in Mathematics, 26*(3), 1-11.

Williams, C. G. (1998). Using conceptual maps to assess conceptual knowledge of function. *Journal for Research in Mathematics Education, 29*, 414-421.

# APPENDIX A

## CONCEPT MAP

Before introducing new math topics in her fourth grade class, Mrs. Maciques liked to use concept maps both to organize her own thinking prior to instruction and to elicit her students' prior knowledge about concepts. Below is the beginning of her concept map for a unit on fractions.

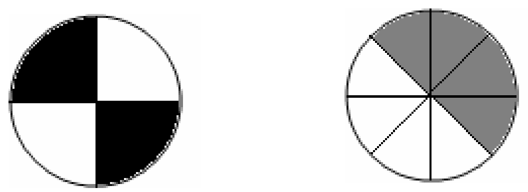
**Complete a concept map describing the concept of a fraction with additional terms and linking words on the lines connecting terms. Possible terms and links may include various meanings and representations of fractions and connections between terms, such as, but not limited to, *part whole, operator, ratio, point on a number line, and division*. You may draw the map however you like.**

# APPENDIX B

## FRACTION TASKS

Task One: Fractions as Part Whole Relationships

Below is an example of an elementary student's work related to the following problem situation involving fractions.



**Name the part that is shaded in each of two identical circles. Do these fractions name the same amount? How do you know?**

Marcus:

$\frac{2}{4}$ is colored in the first circle and $\frac{4}{8}$ is colored in the second circle. They are not the same size because the second circle has more pieces. If the circles showed the amount of pie I was going to get from 2 pies that are the same size, I would only get 2 pieces in the first pie, but I would get 4 pieces from the second pie. Four pieces of pie is more than 2 pieces of pie.

Task Two: Fractions as Operators

Below is an example of an elementary student's work related to the following problem situation involving fractions.

**Alan baked 18 cupcakes for his daughter' birthday party. $\frac{2}{3}$ of the cupcakes were eaten by children at the party. How many cupcakes were eaten?**

Alishia:

27 cupcakes were eaten because I just did 18 divided by 2 times 3 like I learned in class. 18 divided by 2 is 9 and 9 times 3 is 27, so there were 27 cupcakes eaten in all.

$18 \div 2 = 9 \times 3 = 27$

Task Three: Fractions as Ratios

Below is an example of an elementary student's work related to the following problem situation involving fractions.

**There are 24 students in a class. There are 4 boys in the class for every 2 girls. What fraction of the class are boys?**

Lisa:

$\frac{6}{24}$ or $\frac{1}{4}$ of the class is boys because 24 divided into groups of 4 boys is 6 groups of 4 boys. Six groups of boys is the numerator and 24 is the total number of students in the class.

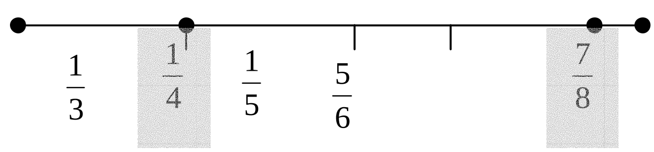
Task Four: Fractions as Points on a Number Line

Below is an example of an elementary student's work related to the following problem situation involving fractions.

**Write the following fractions as they should appear on the number line below and explain why you placed each fraction where you placed it. Two fractions,** $\frac{1}{4}$ and $\frac{7}{8}$ have been placed on the number line as benchmarks to help you.

$$\frac{1}{5} \qquad \frac{1}{3} \qquad \frac{5}{6}$$

Juan:




I put 1/3 before ¼ because 3 is smaller than 4. I put 1/5 after ¼ because 5 is bigger than 4. I put 5/6 close to 1/5 because 6 is bigger than 5 and it is closer to 5 than it is to 8.

# APPENDIX C

## FRACTION TASK INTERVIEW QUESTIONS

Task One: Fractions as Part Whole Relationships

1. What do you think about Marcus' response?

   - If the participant correctly identifies the student's misconception in the problem, questions three through five will be asked to guide the participant through the hypothetical learning trajectory cycle.
   - If the participant incorrectly accepts the student's misconception as the correct answer to the problem, the participant will be presented with the correct thinking of another student to challenge the first student's misconception. The participant will then be asked questions three through five regarding the student whose thinking the participant assesses to be incorrect.

2. In a class discussion, Leisha, Marcus' classmate, had the following response to his answer. What do you think about Leisha's response to Marcus' answer?

   But $\frac{2}{4}$ and $\frac{4}{8}$ are the same size because the circles are the same size and the same amount is shaded in each circle. Even though the second circle has more pieces, each piece is a smaller amount of the whole circle. If you think of the circles as pies, the first pie is cut into 4 pieces and the second pie is cut into 8 pieces. For the second pie, you are just cutting each of the 4 pieces in half to make twice as many smaller pieces. So, 1 piece from the first pie is the same amount of a whole pie as 2 pieces from the second pie.

3. What fraction concepts does the student need to know to understand this problem?

4. What instructional situations and/or representations would you use to address the fraction concepts you mentioned?

5. Specifically, how do you think each instructional situation or representation might change the student's thinking?

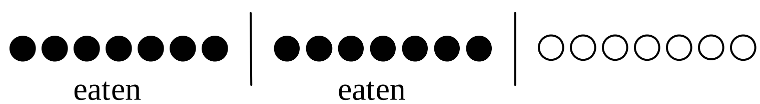
Task Two: Fractions as Operators

1. What do you think about Alishia's response?

    - If the participant correctly identifies the student's misconception in the problem, questions two through four will be asked to guide the participant through the hypothetical learning trajectory cycle.
    - If the participant incorrectly accepts the student's misconception as the correct answer to the problem, the participant will be presented with the correct thinking of another student in the class to challenge the first student's misconception. The participant will then be asked questions two through four regarding the student whose thinking the participant assesses to be incorrect.

2. In a class discussion, Sam, Alishia's classmate, had the following response to her answer. What do you think about Sam's response to Alishia's answer?

    If $\frac{2}{3}$ of the cupcakes were eaten, that means 12 cupcakes were eaten. I know this because the denominator of the fraction tells me that I should divide the whole, 18, into 3 equal amounts. 18 divided into 3 equal parts gives me 6. The numerator tells me that 2 of those three parts were eaten. So I add 6 plus 6 and get 12 cupcakes eaten. It looks like this:

3. What fraction concepts does the student need to know to understand this problem?

4. What instructional situations and/or representations would you use to address the fraction concepts you mentioned?

5. Specifically, how do you think each instructional situation or representation might change the student's thinking?

Task Three: Fractions as Ratios

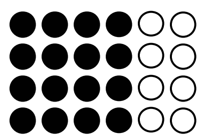
1. What do you think about Lisa's response?

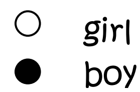
Based on the participant's response to the first question, the interview will proceed as follows:

- If the participant correctly identifies the student's misconception in the problem, questions two through four will be asked to guide the participant through the hypothetical learning trajectory cycle.
- If the participant incorrectly accepts the student's misconception as the correct answer to the problem, questions five and six will be asked to give the participant an opportunity to create a hypothetical learning

2. In a class discussion, Leon, Lisa's classmate, had the following response to her answer. What do you think about Leon's response to Lisa's answer?

If there are 24 kids in the class and there are 4 boys for every 2 girls, I think about it this way:






I 4 boys + 2 girls = 6 students. There are 4 groups of 6 in 24. So I make 4 rows of 6, which is 24 and shade 4 circles for boys and leave 2 circles blank for girls. If I do this for every row, I end up with 16 boys out of 24 students, which is $\frac{16}{24}$ or $\frac{2}{3}$ of the students.

3. What fraction concepts does the student need to know to understand this problem?

4. What instructional situations and/or representations would you use to address the fraction concepts you mentioned?

5. Specifically, how do you think each instructional situation or representation might change the student's thinking?

Task 4: Fractions as Points on a Number Line

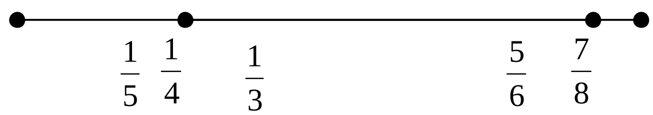
1. What do you think about Juan's response?

   Based on the participant's response to the first question, the interview will proceed as follows:

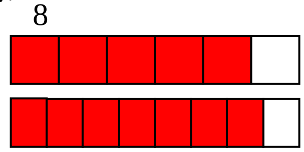
   - If the participant correctly identifies the student's misconception in the problem, questions two through four will be asked to guide the participant through the hypothetical learning trajectory cycle.
   - If the participant incorrectly accepts the student's misconception as the correct answer to the problem, questions five and six will be asked to give the participant an opportunity to create a hypothetical learning

2. In a class discussion, Elmira, Juan's classmate, had the following response to his answer. What do you think about Elmira's response to Juan's answer?

I would put the fractions in this order:




I think $\frac{1}{5}$ is smaller than $\frac{1}{4}$ because if I have 2 candy bars that are the same size and I divide one into 5 pieces and the other one into 4 pieces, one of 4 pieces is a bigger part of the candy bar than one of 5 pieces. I used this same thinking to decide where to put $\frac{1}{3}$. If I divide 1 candy bar into 3 pieces, each piece will be bigger than if I divided it into 4 pieces, so $\frac{1}{3}$ is bigger than $\frac{1}{4}$. With $\frac{5}{6}$, I think it is close to $\frac{7}{8}$, but I think it is smaller. I drew fraction strips to check. My fraction strips help me see that $\frac{5}{6}$ is a little bit smaller than $\frac{7}{8}$.



3. What fraction concepts does the student need to know to understand this problem?

4. What instructional situations and/or representations would you use to address the fraction concepts you mentioned?

5. Specifically, how do you think each instructional situation or representation might change the student's thinking?

APPENDIX D

PARTICIPANT CONSENT FORM FOR INTERVIEWS

# CONSENT FORM
## Elementary Preservice Teachers' Subject Matter and Pedagogical Content Knowledge of Fraction Concepts

You have been asked to participate in a research study of elementary preservice teachers' subject matter and pedagogical content knowledge of fraction concepts. You were selected to be a possible participant because you are the student in the course Mathematics Methods in Early Childhood Education (ECFB 440), at Texas A&M University. The purpose of this study is to explore how preservice teachers understand and represent elementary fraction concepts to students. It will also contribute to fulfillment of the requirements of my Master's thesis in Educational Curriculum and Instruction.

If you agree to participate in this study, you will be asked for an interview. The interview will take place once during the semester and will last 30 minutes to one hour. The interview will be audio taped. There are no risks associated with this study. You will not receive any compensation for participating in this study.

This study is confidential. No real name will be included in the interviews. The records of this study will be kept securely. No identifiers linking you to the study will be included in any sort of reports that might be published. Research records will be stored securely and only Kim Wright will have access to the records which will be erased once this study is completed. Your decision whether or not to participate will not affect your current or future relations with the Department of Teaching, Learning and Culture, College of Education, Texas A&M University. If you decide to participate, you are free to refuse to answer any of the questions that may make you uncomfortable. You can withdraw at any time without your relations with the university, job, benefits, etc., being affected. You can contact Kim Wright, graduate student in the Department of Teaching, Learning and Culture, by phone at (979)575-8947, or by email at (kbwright@tamu.edu or kwright@bryanisd.org) with any questions about this study.

This research study has been reviewed by the Institutional Review Board- Human Subjects in Research, Texas A&M University. For research-related problems or questions regrading subjects' rights, you can contact the Institutional Review Board through Ms. Melissa McIlhaney, IRB Program Coordinator, Office of Research Compliance at (979)-458-4067 (mcilhaney@tamu.edu).

Please be sure you have read the above information, asked questions and received answers to your satisfaction. You will be given a copy of this consent document for your records. By signing this document, you consent to participate in the study.

Signature of participant:________________________________ Date:_____

Signature of investigator:_______________________________ Date:_____

# VITA


| | |
|---|---|
| Name: | Kimberly Boddie Wright |
| Address: | MS 4232 TAMU, College Station, TX, 77843-4232 |
| Email Address: | kim.boddie.wright@gmail.com |
| Education: | B.S., Interdisciplinary Studies, Texas A&M University, 2001<br>M.S., Curriculum and Instruction, Texas A&M University, 2008 |