# ENHANCED SOFTWARE FOR DISPLAYING ORTHOGRAPHIC, STEREOGRAPHIC, GNOMIC AND 

 CYLINDRICAL PROJECTIONS OF THE SUNPATH DIAGRAM AND SHADING MASK PROTRACTOR.John Kie Whan Oh, Ph.D.<br>Department of Architecture<br>Texas A\&M University<br>College Station, Texas

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#### Abstract

The well-known versions of the sun-path diagrams and shading mask protractors that appear in the AIA's Architectural Graphics Standards (Ramsey and Sleeper 1994) are based on the equidistant projections and use a shading mask protractor developed by Olgyay and Olgyay at Princeton University in the 1950s. In the previous papers by McWatters and Haberl (1994a; 1994b; 1995) and Oh and Haberl $(1996 ; 1997)$ the development of a computerized display of the equidistant projection of the sunpath diagram and shading mask protractor was presented. This paper describes enhancements to the display of the sunpath diagram and shading mask protractor that include orthographic, stereographic, gnomic and cylindrical projections. Descriptions of the new algorithms and examples of the different displays are also provided.


## INTRODUCTION

The sun-path diagram and shading mask protractor are well known graphic formats that have traditionally been used by architects and engineers to analyze whether or not a solar shading device will block direct sunlight on a given point in the plane of an exterior window. The sun-path diagram is a two-dimensional graphical representation of the path of the sun across the sky's hemispherical vault for a given latitude. In the sun-path diagram the threedimensional sky dome is condensed onto a twodimensional circular display where the sun's path becomes a series of elliptical curves. When a shading mask protractor template is super-imposed on the sun-path diagram, and oriented with respect to off-south orientation to represent a window in a building's facade, the combined diagrams indicate those times of the year when direct sunlight does or does not strike a point at the centered at the lower edge of the window.

The diagrams also allow for the impact of shading devices, such as overhanging protrusions from the building (i.e., fins or eyebrows), to be accurately plotted upon the shading mask protractor and superimposed on the sun path diagram. A designer using the AIA's Graphics Standards
book, or other printed versions of the sun path diagram must select the nearest latitude, make photocopies of the appropriate sun-path diagram and shading mask protractor, and then overlay the shading mask protractor upon the diagram in the proper orientation. The outline of the shading device is then transcribed upon the shading mask, aligned at the proper orientation for the facade in which the window is being analyzed, and placed on top of the sun-path diagram to determine if a point centered at the base of the window is exposed to direct sunlight. Obviously, teaching this process to architects and engineers is tedious and error-prone since the students must calculate angles to ascertain whether or not a given shading device is going to work as planned and then painstakingly transcribe their calculations onto the printed sun path diagram and shading mask protractor. The inconvenience of the traditional methods of photocopying the traditional equidistant projection, sun-path diagrams can now be bypassed with the use of a series of simple algorithms that lend themselves readily to a computer program (McWatters and Haberl, 1994a; 1994b; 1995). There are several other analysis programs that are indirectly related to this work including: the OPAQUE (Abouella and Milne 1990), SOMBRERO (Schnieders et al. 1997), AWNSHADE (McCluney 1995), SOLAR-2 (Sheu 1986), SUNPATH (McCluney 1995), and SUNSPEC (McCluney 1995) programs.

OPAQUE (Abouella and Milne 1990), developed by the Department of Architecture at UCLA, draws a detailed wall or roof section, calculates the U-value, time-lag, and decrement factor. It also plots daily and annual outdoor and sol-air temperatures, normal and total surface radiation, and the heat flow through an opaque envelope (i.e., a building wall section which excludes the window).

The computerized shadow calculation program SOMBRERO (Schnieders et al. 1997) was developed by the University of Siegen, Germany for calculating the GSC (geometrical shading coefficient), that is the proportion of shaded area of an arbitrarily oriented surface surrounded by shading elements as a function of time and location. In SOMBRERO the reduction of isotropic diffuse radiation due to different kinds of obstacles is calculated by means
of view factors. SOMBRERO contains a geometry file generator that facilitates the input of 3 dimensional external objects such as a standardized house and trees. Finally, McCluney (1995) developed a shading device design program called AWNSHADE that calculates the sunlit fraction of a vertical window that is irradiated by direct beam solar radiation as well as the approximate effective sunlit fraction by isotropic radiation from the sky and from the ground. AWNSHADE can simulate awnings with horizontal or inclined side-walls, overhangs, and vertical side-fins with horizontal or inclined top edges, and combinations of these.

Several shading and window design programs that provide graphical methods to display the solar analysis, including SOLAR-2 (Sheu 1986), SUNPATCH (Taylor 1987), SUNPATH (McCluney 1995), SUNSPEC (McCluney 1995), and SOLRPATH (McWatters and Haberl 1994a; 1994b; 1995; Oh and Haberl 1996; 1997). SOLAR-2 (Sheu 1986), developed by the Department of Architecture at UCLA, displays sunlight penetrating through a window with any combination of rectangular fins and overhangs. It also plots on an hourly-basis, an animated, 3-D sun's-eye-view of the building, and provides annual tables of the percent of the window in full sun, and radiation on the glass. SUNPATH, developed by McCluney (1995), calculates the position of the sun in the sky at any given time and location on earth as well as sequences of positions to display the sun's path. It also calculates the dates and times that the sun will be within a specified range of directions in the sky, and the times of sunrise and sunset for any location and date. The linked program PATHPLOT is used to plot a sunpath chart for any location. McCluney (1995) also developed a solar radiation calculation program called SUNSPEC that calculates the direct beam and diffuse sky spectral and broadband solar irradiance incident on a vertical surface for a cloudless sky. It allows the user to specify the concentrations of various atmospheric gases and particles. SUNSPEC was developed based on Gueymard's SMARTS solar spectral irradiation algorithm (1993). It also integrates these spectra to determine the total irradiance and illuminance from the sun, sky, and ground incident upon a window aperture. The linked program SPECPLOT can be used to plot solar spectral irradiance distributions calculated by SUNSPEC. The remainder of this paper describes new developments to enhance the computerize display of the sunpath diagram and shading mask protractor through the development of orthographic, stereographic, gnomic and cylindrical projections.

## GRAPHICAL DISPLAY TECHNIQUES

The most widely used projection methods are equidistant, orthographic, stereographic, gnomonic, and cylindrical projection. Each of these techniques has advantages and disadvantages over the other techniques. The following sections describe each of the projection techniques. Equation for plotting these displays can be found in $\mathrm{Oh}(2000)$.

Equidistant Projection. The equidistant projection has become the most widely used method for constructing a sunpath diagram and shading mask protractor in the United States. In this method, the 3-D sky dome is flattened onto a 2-D circular display where the sun's path becomes a series of elliptical curves (Figure 1). Unlike the case of orthographic or stereographic projection, in the equidistant projection solar altitude lines are not geometrically projected but are equally spaced as concentric circles on a 2-D diagram. This method was developed by Irving F. Hand (1948; ref. in Olgyay and Olgyay 1957) and was used in the 'LOF Sun Calculator' which was created by the Libbey-Owens-Ford Glass Co (LOF 1974). The advantage of this method is that it enables the users to plot both low and high sun altitude angles with the same visual accuracy.

The equidistant, orthographic, stereographic, and gnomonic projection methods use concentric circles to display solar altitude and straight radial lines to display solar azimuth. However, because of the difference in the projection method of the 3-D solar altitude onto 2-D Cartesian X-Y coordinates, the radii of circles displaying solar altitude are determined differently. Thus, if we know the equations that calculate the different radius of solar altitude circle, then the location of the sun can be easily calculated as an X and Y coordinate pair on a sunpath diagram regardless of the projection method. In the equidistant projection method, solar altitude lines are equally spaced as concentric circles on a two-dimensional $X-Y$ coordinate. Thus, the radius of the solar altitude circle decreases in direct proportion to an increase in the solar altitude angle (Figure 4). The radius ' $L$ ' of a given altitude angle $\left(\alpha_{S}\right)$ is equal to the difference between the altitude angle and the radius $(R)$ of the solar altitude circle when the angle is $0^{\circ}$. Thus, the radius of the solar altitude angle can be calculated by
$L=R-\alpha_{s}$.


Figure 1: Sunpath Diagram with the Solar Altitude and Azimuth Lines Plotted Using the Equidistant Projection Method (Figure 1.a). This figure shows the four sets of lines and curves including the concentric circles of the solar altitude angles (Figure 1.b), radial lines of the off-south solar azimuth angles (Figure l.c), sunpath lines for the months (Figure l.d) and hours (Figure l.e).


Figure 2: Shading Mask Protractor Plotted Using the Equidistant Projection Method (Figure 2.a). This figure shows the four sets of lines and curves including the concentric semicircles of solar altitude angles (Figure 2.b), radial lines of the off-south solar azimuth angles (Figure 2.c), horizontal ellipses for the forward edge of horizontal shade (Figure 2.d) and vertical ellipses for describing the upper edge of a side fin (Figure 2.e).


Figure 3. $X$-Y Coordinate Calculation in Different Quadrants for the Equidistant Projection.

Then, $X$ and $Y$ coordinates of the sun's location are determined as the product of the radius $L$ and the sine and cosine values of solar azimuth angle $\left(\gamma_{S}\right)$. To begin, one calculates the $X-Y$ coordinate when the sun is located in the third quadrant of sunpath diagram (Figure 3). The sun is west of due south and the solar azimuth angle should be a positive number between $0^{\circ}$ and $90^{\circ}$. The $X$ and $Y$ coordinates are then

$$
\begin{align*}
& x=-\Delta x=-L \sin \left(\gamma_{s}\right)  \tag{2a}\\
& y=-\Delta y=-L \cos \left(\gamma_{s}\right) \tag{2b}
\end{align*}
$$

where $0^{\circ} \leq \gamma_{S} \leq 90^{\circ}$.


Figure 4: Calculation of Solar Altitude Angle Using the Equidistant Prajection Method.

When the sun's location is west of due south and the solar azimuth angle has a positive value between $90^{\circ}$ and $180^{\circ}$, the sun may be located in the second quadrant of sunpath diagram (Figure 3). Using the fact that $0^{\circ} \leq 180^{\circ}-\gamma_{S}$ ) $\leq$ $90^{\circ}$, the $X$ and $Y$ coordinates can be calculated by

$$
\begin{align*}
& x=-\Delta x=-L \sin \left(\gamma_{s}\right)  \tag{3a}\\
& y=\Delta y=-L \cos \left(\gamma_{s}\right) \tag{3b}
\end{align*}
$$

where $90^{\circ} \leq \gamma_{S} \leq 180^{\circ}$.
In the same way, we can calculate the $X$ and $Y$ coordinates, when the sun is located east of due south which yields

$$
\begin{align*}
& x=-L \sin \left(\gamma_{s}\right)  \tag{4a}\\
& y=-L \cos \left(\gamma_{s}\right) . \tag{4b}
\end{align*}
$$

Finally, if we replace radius $(L)$ of the solar altitude circle with Equation 1, we can calculate the coordinates of the sunpath diagram directly from solar altitude and azimuth angles as follows

$$
\begin{align*}
& x=-\left(R-\alpha_{s}\right) \times \sin \left(\gamma_{s}\right)  \tag{5a}\\
& y=-\left(R-\alpha_{s}\right) \times \cos \left(\gamma_{s}\right) . \tag{5b}
\end{align*}
$$

Orthographic Projection. The orthographic sunpath diagram is an exact projection of the sky's hemispherical vault onto a 2-D circular plot (Figure 8b). This method was first developed by Molesworth (1902; ref. in Olgyay and Olgyay 1957) and later suitably modified by Waldram (1933; ref. in Olgyay and Olgyay 1957). Even though this method is geometrically correct, accurate interpretation of the plots is nearly impossible in the outer rim of the circular plot (i.e., the sunrise or sunset times) because of the severe contraction of the solar altitude angles toward the edges of the diagram. The advantage of the orthographic plot is that it can be directly compared to $180^{\circ}$ globoscope photographs (Olgyay and Olgyay 1957). Hence, it can be helpful in tracing the outline of shading caused by surrounding obstructions

The calculation of the radius ( $L$ ) of solar azimuth circles in orthographic projection (Figure 5) is slightly more complicated than the equidistant projection. In the orthographic projection method, the location of the sun is vertically projected from a point on a celestial sphere onto a two dimensional circular diagram which represents the exact geometric projection. In the orthographic method the length of $L$ for a given solar altitude angle $\left(\alpha_{s}\right)$ is calculated by
$L=R \cos \left(\alpha_{s}\right)$.



Figure 5: Calculation of Solar Altitude Angle Using the Orthographic Projection Method.

The remaining calculations then proceed the same as for equidistant projection, with the radius ' $L$ ' in Equations 4 a and 4 b replaced with ' $R \cos \left(\alpha_{S}\right)$ '. We can calculate the $X$ and $Y$ coordinates in an orthographic sunpath diagram as follows
$x=-R \cos \left(\alpha_{s}\right) \cdot \sin \left(\gamma_{s}\right)$
$y=-R \cos \left(\alpha_{s}\right) \cdot \cos \left(\gamma_{s}\right)$.
Stereographic Projection. In a stereographic projection (Figure 8c), any point on the sky vault is first connected to the nadir at the center. Then, the intersection of the lines and the equatorial plane of the sphere are projected down to make solar altitude lines in a 2-D projected plane (Figure 6). In this way each sunrise-tosunset sunpath line is represented as a circle, with the diameter increasing as it approaches winter solstice (i.e., low altitude angles). This method is well researched by noted Swedish architect Gunnar Pleijel (Aronin 1953, ref.
in Olgyay and Olgyay 1957). It has the advantage of reducing the edge distortion of the sunpath in the diagram. Furthermore, since sunpath lines appear as circles, this projection is the easiest to draw. The diagram does have one drawback, namely, the circles that represent the solar altitude angles become densely packed toward the center of the diagram, thus making it hard to read the solar altitude angles greater than 60 degrees (i.e., summertime values for lower latitudes around noon time).

In the stereographic sunpath diagram (Figure 6), the intersection of the equatorial plane of a celestial sphere is used as the projection line. In this method, a line is first drawn from the sun's location ( $S^{\prime}$ ) on the celestial sphere to the Nadir ( $N^{\prime}$ ). The intersection point ( $P^{\prime}$ ) of this line ( $S^{\prime} N^{\prime}$ ) and the horizon line $\left(O^{\prime} H^{\prime}\right)$ is vertically projected down to determine the radius $(L)$ of the solar altitude circle.


Figure 6: Calculation of Solar Altitude Angle Using the Stereographic Projection Method.

As we can see in Figure 6, the angle ( $\angle O^{\prime} S^{\prime} N^{\prime}$ ) between the celestial sphere center and Nadir at the sun's location (symbol ' $\beta$ ') is the same as the angle ( $\angle O^{\prime} N^{\prime} S^{\prime}$ ) between the Zenith and the sun's location on the celestial sphere at Nadir. In other words,
$\left(\alpha_{s}+90^{\circ}\right)+2 \beta=180^{\circ}$.
Thus,
$\beta=\left(90^{\circ}-\alpha_{s}\right) / 2$.
From the triangle $\Delta O^{\prime} P^{\prime} N^{\prime}$, the radius $L$ can be calculated as follows
$L=R \tan (\beta)$.

Using the Equations 9 and 10 , the radius $L$ is

$$
\begin{equation*}
L=R \tan \left[\frac{\left(90^{\circ}-\alpha_{s}\right)}{2}\right] \tag{11}
\end{equation*}
$$

Therefore, using Equations $4 \mathrm{a}, 4 \mathrm{~b}$, and 11 , the coordinates of solar altitude and azimuth angles in the stereographic sunpath diagram are calculated by

$$
\begin{equation*}
x=-R \tan \left[\frac{\left(90^{\circ}-\alpha_{s}\right)}{2}\right] \cdot \sin \left(\gamma_{s}\right) \tag{12a}
\end{equation*}
$$

and

$$
\begin{equation*}
y=-R \tan \left[\frac{\left(90^{\circ}-\alpha_{s}\right)}{2}\right] \cdot \cos \left(\gamma_{s}\right) \tag{12b}
\end{equation*}
$$

Gnomonic Projection. The gnomonic projection (Figure 8d) is derived from the sundial. The location of the observer is at the center of the celestial sphere facing the vernal equinox sunpath (i.e., the horizon in a celestial sphere). Any point on the sphere is projected on a line that is parallel with the horizon at the center of the zenith (Figure 7). The solar lines are, therefore, compressed near the zenith and greatly expanded at low altitudes. The sunrise and sunset cannot be presented on this projection because the projection line becomes infinite. Because of this inability to display the entire sky vault, this method is rarely used for sunpath diagrams but it is used frequently for building shadow studies and for constructing sundials. In the gnomonic projection (Figure 7), the projection line is a horizontally extended line from the Zenith of the celestial sphere instead of the horizon line in a stereographic projection.

From the triangle $\Delta O^{\prime} Z^{\prime} P^{\prime}$, the length $L$ can be calculated by

$$
\begin{equation*}
L=R \tan \left(90^{\circ}-\alpha_{s}\right) \tag{13}
\end{equation*}
$$

or, it can be given from the triangle $\triangle O^{\prime} P^{\prime} H^{\prime}$ as follow

$$
\begin{equation*}
L=\frac{R}{\tan \left(\alpha_{s}\right)} \tag{14}
\end{equation*}
$$

Finally, using Equations $4 \mathrm{a}, 4 \mathrm{~b}$, and 13 , the X and Y coordinates are calculated by

$$
\begin{align*}
& x=-R \tan \left(90^{\circ}-\alpha_{s}\right) \cdot \sin \left(\gamma_{s}\right)  \tag{15a}\\
& y=-R \tan \left(90^{\circ}-\alpha_{s}\right) \cdot \cos \left(\gamma_{s}\right) \tag{15b}
\end{align*}
$$



Figure 7: Calculation of Solar Altitude Angle Using the Gnomonic Projection Method.


Figure 8: Sunpath Diagrams Plotted with Various Projection Methods. This figure shows sunpath diagrams plotted using five different projection methods including: equidistant (a), orthographic (b), stereographic (c), gnomonic (d) and cylindrical (e)

Cylindrical Projection. The cylindrical projection (Figure 8e) method plots the sun's path onto an equally spaced $\mathrm{x}-\mathrm{y}$ grid of solar angles where values of the y -axis represent the solar altitude angles and values of the $x$-axis represent the off-south solar azimuth. The $x-y$ grid of sun angles effectively maps the sun's 3-D hemispherical path onto cylindrical coordinates that can be imagined as being unrolled and displayed in a 2-D format. This method has been extensively used in the area of architectural design. The method was popularized by Edward Mazria (1979) and has been frequently used to visualize the site skyline with solar obstructions that block direct sun from reaching
any point on the site (Duffie and Beckman 1991). The research of Robert Bennett (1978) gives helpful examples of the cylindrical diagrams for various latitudes with overlay of the shading objects. This method provides very easy reading of solar angles owing to the application of straight lines for the solar altitude and azimuth angles. Unfortunately, the sunpath lines and accompanying shading mask protractor lines lead to some rather obscure shapes with this method.

In the cylindrical projection method, we don't need a function to calculate the $X$ and $Y$ coordinates because the $X$ and $Y$-axes represent the solar azimuth and solar altitude angles respectively. However, we do need to define the maximum and minimum ranges of the $X$ and $Y$ coordinates differently from the other projection methods. For most of the displays of the sunpath, the cylindrical sunpath diagram usually does not use the maximum range $\left(-180^{\circ}\right.$ to $180^{\circ}$ ) of solar azimuth angle for the $X$-axis. For example, Bennett (1978) used ranges from $-120^{\circ}$ to $120^{\circ}$ for the solar azimuth angle ( $X$-axis) in his research. However, the maximum solar azimuth range will be used in this research because the software can zoom into any portion of the diagram whenever it is needed.

Shading Mask Protractor. Traditionally, the shading mask protractor has been used to trace the image of an actual shading device onto the sunpath diagram to determine those periods of the day when a point at the middle of the bottom of the window is shaded. The shading mask protractor consists of a series of construction lines that are used to represent the edge of rectangular shades. The equations used to produce the shading mask protractor lines in the equidistant projection method were first described in McWatters and Haberl (1994a).

Figure 2 shows that the shading mask protractor is composed of four different sets of curves and lines. The first set of curves is a series of semicircles (Figure b) in the semicircular region of shading mask protractor. The orientation of the semicircular region is determined by the given azimuth angle ( $\gamma$ ) of the surface being analyzed. To generate the concentric circles, we apply the same equations (Equations 2a and 2b) used to calculate the $X$ and $Y$ coordinates of the solar altitude curves. However, the start and end angles of each semicircle should be defined such as

$$
\begin{equation*}
A_{\text {start }}=\gamma+90^{\circ} \tag{16a}
\end{equation*}
$$

$A_{e n d}=\gamma-90^{\circ}$.
The radius of each semicircular line is then increased by $10^{\circ}$ at a time from $10^{\circ}$ to $90^{\circ}$ for each FOR loop statement. The second group of lines in the shading mask protractor is a series of radial lines (Figure 2.c). The third group of lines in the shading mask protractor is a set of horizontal ellipses (Figure 2.d) that are used to determine the projected line of the horizontal edge of an overhanging shade. Given that surface azimuth angle of the wall is $\gamma$, the window width is 2 W , the window height is H , and the
projected length of overhang is D (Figure 9), the azimuth angle ( $\gamma^{\prime}$ ) of the Point C against the surface outward normal is

$$
\begin{equation*}
\gamma^{\prime}=\tan ^{-1}\left(\frac{D}{W}\right)=\tan ^{-1}\left(\frac{H \tan \beta}{H \tan \alpha}\right)=\tan ^{-1}\left(\frac{\tan \beta}{\tan \alpha}\right) . \tag{17}
\end{equation*}
$$

Thus, the solar azimuth angle $\left(\gamma_{s}\right)$ is
$\gamma_{s}=\gamma+\tan ^{-1}\left(\frac{\tan \beta}{\tan \alpha}\right)$.

Again, from the triangle $\triangle \mathrm{OCB}$,

$$
\begin{equation*}
90^{\circ}-\alpha_{s}=\tan ^{-1}\left(\frac{L}{H}\right)=\tan ^{-1}\left(\frac{\sqrt{W^{2}+D^{2}}}{H}\right) \tag{19a}
\end{equation*}
$$

$$
\begin{align*}
& 90^{\circ}-\alpha_{s}=\tan ^{-1}\left(\frac{\sqrt{H^{2} \tan \alpha^{2}+H^{2} \tan \beta^{2}}}{H}\right)= \\
& \tan ^{-1} \sqrt{\tan \alpha^{2}+\tan \beta^{2}} \tag{19b}
\end{align*}
$$

Thus, the solar altitude angle $\left(\alpha_{s}\right)$ is
$\alpha_{s}=90^{\circ}-\tan ^{-1} \sqrt{\tan \alpha^{2}+\tan \beta^{2}}$.

The fourth group of lines in the shading mask protractor is a set of vertical ellipses (Figure 2.e) that are used to determine the projected line of the vertical edge of a side fin. To plot the vertical ellipses of the shading mask protractor onto a sunpath diagram, the equations and functions used for plotting horizontal ellipses can be used. In this case, a loop statement of horizontal angles $(\alpha)$ of ellipses should be nested within the vertical angles ( $\beta$ ).

Figure 10 shows examples of shading mask protractor plotted using various projection methods. The projected lines of shading devices (lines CD, AC, BD , and CG in Figure 4.11) are highlighted on the different sunpath diagrams. In the cylindrical sunpath diagram, left side-fins are displayed in the right hand side of the diagram because the right hand side is west.


Figure 9: Calculation of the Angles of Shading Mask Protractor. This figure shows the calculation of solar azimuth $\left(\gamma_{S}\right)$ and solar altitude angles ( $\alpha_{S}$ ) when the horizontal ( $\alpha$ ) and vertical ( $\beta$ ) angles for the shading mask ellipses are given.

## DISCUSSION

This paper has described the methods that can be used to plot the sunpath diagram using equidistant, orthographic, stereographic, gnomic and cylindrical coordinates. Each of these methods produces a different version of the sun path diagram. A description of the method used to produce the shading mask protractor has also been described. Figure 10 shows a comparison of the sun path diagrams and shading mask protractors for the shaded window shown in Figure 9. When the different methods are presented side-by-side the similarities and differences in the methods becomes more clear. Clearly,
the equidistant (Figure 10a), orthographic (Figure 10b) and stereographic (Figure 10c) displays are the most similar and have circular shapes with small differences in the display of the solar altitude angle that lead to contractions of the sunpath at the center or the edges of the diagram. The orthographic display produces an expansion of the altitude angles directly overhead, whereas the stereographic display expands the altitude angles near the horizon. The equidistant display appears visually to be in between the orthographic and stereographic displays as one would expect.

The gnomic display (Figure 10d) is difficult to use as a sunpath diagram. However, it does have the advantage
when one is constructing a sundial since the lines that are displayed on the gnomic chart represent the position of a shadow that is cast by a vertical peg in the center of the diagram. The cylindrical sun path diagram presents a convenient image of the path of the sun. However, the projection of the shading mask protractor produces a complex image that is not intuitive. This is further complicated when one attempts to display an off-south image as shown in Figure 10e because the positioning of the shading mask protractor must slide horizontally the one side or the other to align with the off-south azimuth angle as shown.

Prior to this work, it has been difficult to compare the different display methods because the technique for computerizing all the methods had not been published. This forced the reader to attempt to visually evaluate the different charts from the different references that were published in different sizes and/or sometimes did not describe how to perform the various functions (i.e., rotation and/or superimposing the shading mask protractor for an off-south azimuth). Additional features for the sun path diagram have also been developed by $\mathrm{Oh}(2000)$, including: the display of the intensity of the solar data on vertical, development of moveable reference point, offsouth surface and includes the transmittance of the glazing, and the validation of the thermal calculations against measured data from a test bench.


Figure 10: Shading Mask Protractor Plotted Using Various Projection Methods. This figure shows the shading mask protractor for a surface $30^{\circ}$ west of due south for equidistant (a), orthographic (b), stereographic (c), gnomonic (d) and cylindrical (e) projections.

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path diagram for varying latitudes. John Oh developed the Visual Basic code shown in this paper for his Ph.D. dissertation. Copies of the public domain DOS or MSWindows version of the SOLRPATH program can be obtained by writing to the authors.

