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Optimal Operation of Large Agricultural Watersheds with Water Quality Constraints

J.R. Williams
R.W. Hann, Jr.

OPTIMAL OPERATION OF LARGE AGRICULTURAL WATERSHEDS WITH WATER QUALITY CONSTRAINTS[1/]

J. R. Williams and R. W. Hann, Jr.[2/]

[1/] Contribution from the USDA, Science and Education Adminstration, Federal Research, in cooperation with Texas Agricultural Experiment Station, Texas A&M University.

[2/] Hydraulic Engineer, USDA, SEA, FR, P. O. Box 748, Temple, TX 76501, and Professor, Civil Engineering Department, Texas A&M University, College Station, TX 77843.

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INTRODUCTION

Improved technology is needed for use in properly managing large agricultural watersheds. Proper watershed management means selecting land uses that are appropriate for each subarea, using erosion control measures where necessary, and applying fertilizers at rates that maximize agricultural production without polluting the environment. Watershed runoff and industrial and municipal effluents pollute streams and reservoirs. Point source pollution (industries and municipalities) can be monitored. Nonpoint-source pollution (watersheds) is widely dispersed and not easily measured. Mathematical models are needed to predict nonpoint-source pollution as affected by watershed characteristics, land use, conservation practices, chemical fertilizers, and climatic variables. Routing models are needed to determine the quality of water as it flows from nonpoint sources through streams and valleys to rivers and large reservoirs. Models are also needed to determine optimal strategies for planning land use, conservation practices, and fertilizer application to maximize agricultural production subject to water quality constraints.

Three of the most important agricultural pollutants are suspended sediment, phosphorus, and nitrogen. Robinson [1971] pointed out that sediment is the greatest pollutant of water in terms of volume. Sediment also transports other pollutants, like phosphorus and nitrogen. These two elements are principally involved in lake eutrophication. Frequently algae blooms develop in nutrient-laden water and cause it to have an off-taste and an unpleasant odor. The odor of decaying plants becomes offensive; fish are killed because of reduced dissolved oxygen in the water, and recreation is deterred.

The objective of this research was to develop models for use in managing large agricultural watersheds to obtain maximum agricultural production and to maintain water quality standards. The models were designed to:

(1) Simulate daily runoff, and sediment, phosphorus, and nitrogen yields from small watersheds (areas $\leq 40 \text{ km}^2$) and determine frequency relationships.

(2) Route various frequency hydrographs, and sediment, phosphorus, and nitrogen yields from subwatersheds through streams and valleys of large agricultural watersheds (areas $\leq 2500 \text{ km}^2$) to obtain frequency relationships at the entrance of a river or reservoir.

(3) Determine strategies that are acceptable to the decision makers (land owners and operators) for planning land use, fertilizer application, and conservation practices on subwatersheds.

(4) Determine the optimal strategy for each subwatershed to maximize agricultural production for the entire watershed subject to water quality constraints.

Generally, water-quality models are developed by adding chemical modeling components to existing runoff and sediment models because runoff and sediment provide transportation for chemicals. Several conceptual models for predicting chemical yields from small watersheds have been presented [Crawford and Donigian, 1973; Donigian and Crawford, 1976; Frere, et al., 1975; Hagin and Amberger, 1974; Kling, 1974; Johnson and Straub, 1971]. However, these models are not applicable to large watersheds because they have no routing mechanism. For this reason, runoff, sediment, and nutrient models were refined and developed here for application to large watersheds.

Probably, the most widely used and accepted model for predicting runoff volume is the Soil Conservation Service (SCS) curve number system [U.S. Soil Conservation Service, 1972]. The SCS model was modified by adding a soil-moisture-index accounting procedure [Williams and Laseur, 1976]. The modified water yield model is considerably more accurate than the original SCS model. On a watershed near Riesel, Texas, the modified model explained 95% of the variation in monthly runoff as compared with 65% for the original model. The water-yield model was refined here by replacing the climatic index (lake evaporation) with daily consumptive water use for individual crops.

Besides predicting individual storm runoff volumes, it is also necessary to predict hydrographs and to perform flood routing for water quality modeling on large agricultural watersheds. HYMO, a problem-oriented computer language for building hydrologic models [Williams and Hann, 1973] was selected to compute hydrographs and perform flood routings. Worldwide use has shown that HYMO is convenient and reliable for extremely varied hydrologic conditions. The Variable Travel Time (VTT) flood routing method [Williams, 1975a] used in HYMO is about as accurate as an implicit solution of the unsteady flow equations of continuity and motion and is free of convergence problems.

The USLE [Wischmeier and Smith, 1965] is the most widely used and accepted erosion model. It can be used to predict long-term average annual sediment yields for watersheds by applying a delivery ratio. However, the USLE was not designed for application to individual storms and is, therefore, not appropriate for individual storm water quality modeling. The USLE was modified [Williams, 1975c] by replacing the rainfall energy factor with a runoff factor. The modified universal soil loss equation (MUSLE) increased sediment-yield-prediction accuracy, eliminated the need for delivery ratios, and is applicable to individual storms. In tests with data from Riesel, Texas; Chickasha, Oklahoma; Oxford, Mississippi; Treynor, Iowa; Hastings, Nebraska; and Boise, Idaho, MUSLE generally explained 80% or more of the variation in individual storm sediment yield for each watershed. These tests included 60 watersheds with areas ranging from 0.01 to 234 km^2 and slopes ranging from less than 1 to about 30%. The MUSLE was combined with the modified SCS water-yield model and HYMO to form a daily runoff-sediment prediction model [Williams and Berndt, 1976]. Satisfactory results were obtained when the runoff-sediment model was tested with data from 26 watersheds in Texas.

The MUSLE is useful in predicting sediment yield from small watersheds (area $\leq$ 40 km^2), but sediment routing is needed to maintain prediction accuracy on large watersheds with nonuniformly distributed sediment sources. A sediment routing model was developed for large agricultural watersheds [Williams, 1975b] and has had limited testing. The sediment routing model was refined here and combined with nutrient-loading functions to develop a sediment-phosphorus-nitrogen routing model.

Nitrogen and phosphorus loading functions [McElroy, et al., 1976] were developed for use on small agricultural watersheds. The loading functions were designed for predicting long-term average annual phosphorus and nitrogen yields based on predicted sediment yield, nutrient concentration in the soil, and enrichment ratios. However, there is no provision for predicting nitrate yield, since it is not attached to the sediment. There are no functions provided for determining nutrient concentrations in the soil as affected by fertilizer application. Also, relations were not developed for predicting enrichment ratios. Here the loading functions were adapted to individual storm prediction of phosphorus and nitrogen yields from small watersheds. A nitrate component was added and the enrichment ratios were related to particle-size distributions of the soil and the sediment.

Since water quality models are not well developed for large agricultural watersheds, little has been done to develop models to determine optimal watershed management strategies subject to water quality constraints. Onishi and Swanson [1974] used linear programming to determine crop systems and practices that are economically optimal on a 4.86-km^2 watershed subject to sediment and nitrogen constraints. Wade, et al. [1974] described a model that uses linear programming to minimize national agricultural production costs subject to meeting agricultural production demands and sediment yield constraints. Miller and Gill [1976] used a linear programming model to maximize net revenue to farm firms constrained by acreage limits and soil loss limits. Heady [1976] developed a national model to minimize the cost of producing and transporting farm commodities subject to soil loss and other constraints.

None of these models are directly applicable to large agricultural watershed management, because only soil loss or nutrient losses are considered constraints. By including routing models, yields of sediment, phosphorus, and nitrogen can be determined and used as constraints. Considering yields to rivers or reservoirs provides more flexibility in management and higher potential agricultural production for the large watershed. Soil loss may not contribute to pollution because it may never reach a point to cause damage (permanent stream or reservoir).

Yields of sediment, nitrogen, and phosphorus depend upon location of the source within a watershed, hydraulic efficiency of the channels, and the particle-size distribution as well as soil loss. If only soil loss is considered as a constraint, agricultural production cannot be truly maximized.

The model presented here uses linear programming to maximize agricultural utility subject to constraints of sediment, phosphorus, and nitrogen yields at the watershed outlet. Decision analysis, as described by Raiffa [1970], is used to determine strategies that are acceptable to the decision makers (landowners and land operators) and to calculate the utility of the strategies. A strategy specifies land use, fertilization rate, and conservation practice. Utility is described with a multiattribute utility function based on gross income, production cost, dependability, and disease, insect, and weed control. Utility theory expresses the decision makers' preferences on a scale from zero to one. This provides for easier and clearer decisions because attributes with various units can be compared and combined directly.

To apply decision analysis, each subwatershed is subdivided according to land capability classes. This simplifies the selection of strategies for the decision maker because different land classes have different production and pollution potentials. The number of possible strategies for operating each land capability class within a subwatershed approaches infinity, but the number can be reduced greatly by considering only strategies that are acceptable to the decision makers.

Generally, crop production records are not adequate to evaluate the attributes for all strategies. However, the analyst or modeler can evaluate the attributes through the use of subjective probability distributions. Raiffa [1970] suggested special techniques for developing subjective probability distributions by interviewing the decision makers. Each year as the crops are harvested, the probability distributions are revised using Bayes' Theorem [1763] to include the observed data.

Since there is usually more than one decision maker per subwatershed, most decisions concerning utility functions and probability distributions are group decisions. Raiffa [1970] suggested using Pareto-optimality in making group decisions. A joint action is Pareto-optimal if no alternative action exists that is at least as acceptable to all and definitely preferred by some. Decision analysis has been used very little in water-resources planning. McCuen [1973] used decision analysis to determine benefits from recreation facilities; Dean and Shih [1973] showed the advantages of subjective decision making for urban water resources development; and Russell [1974] applied decision theory to reservoir operation.

MODEL FOR SIMULATING DAILY RUNOFF, SEDIMENT, PHOSPHORUS, AND NITROGEN

Large agricultural watersheds (areas < 2500 km^2) are subdivided into many subwatersheds with relatively homogeneous characteristics when flood routing is performed. Besides delineating areas with relatively uniform characteristics, subdivision also provides routing reaches that are hydraulically similar from one end to the other. For water-quality modeling, the subwatersheds were further divided according to their land capability class. This division expedites calculations and improves prediction accuracy because land classes have characteristic nutrient levels, land use, and erosion potentials.

Runoff and sediment, phosphorus, and nitrogen yields must be predicted for each subwatershed before routing can be accomplished. Because watershed cover, soil moisture, and phosphorus and nitrogen concentrations in the soil change considerably with time, a time increment of 1 day was selected for simulation. Frequency curves are developed for daily runoff, sediment, phosphorus, and nitrogen using 30 to 40 years of simulated data for each subwatershed.

Runoff-Volume Model

A model for predicting daily water yield from mixed land-use agricultural watersheds [Williams and Laseur, 1976] was modified for use in the water-quality model. The modification consisted of replacing the climatic index (average monthly lake evaporation) with a climatic index based on average monthly consumptive use of the crops growing on the watershed.

The model is based on the SCS curve number technique [U.S. Soil Conservation Service, 1972] and a soil moisture index accounting procedure. Runoff is predicted with the SCS equation

$$Q = \frac{(R - .2s)^2}{R + .8s} \qquad (1)$$

where Q is the daily runoff, R is the daily rainfall, and s is a retention parameter. A soil moisture index was linked to the retention parameter by the equation

$$SM = s_m - s \qquad (2)$$

where SM is a soil moisture index and s_m is the maximum value of moisture storage in the soil.

The soil moisture index is determined by the equation

$$SM_\tau = \frac{SM + R}{1 + \alpha(SM + R) \sum_{\tau=1}^{\Lambda} CI_\tau} - Q \qquad (3)$$

where SM is the soil moisture index at the beginning of the first storm; SM_τ is the soil moisture index at time τ; CI_τ is the climatic index for day, τ; Λ is the number of days between beginning of storms; α is the depletion coefficient, and R and Q are the rainfall and runoff for the first storm. The model must be calibrated on a gaged watershed to determine the depletion coefficient, α. Once calibrated, the model can be used to extend runoff records for the calibrated watershed or to predict runoff for nearby ungaged watersheds.

The climatic index is determined by weighting average monthly consumptive use for each crop according to the area covered by the crop.

$$CI_i = \frac{\sum_{j=1}^{N} CU_{ij} \times DA_j}{DA}, \quad i = 1, 12 \qquad (4)$$

where CU_{ij} is the average consumptive use for month, i, by the crop, j; DA_j is the area covered by crop, j; N is the number of crops; and DA is the total drainage area. Consumptive use can be predicted with various models, but in Texas CU values have been determined for major crops [McDaniels, 1960]. The new climatic index gives more accurate runoff predictions and reflects differences in runoff caused by land use that lake evaporation did not show.

Peak Runoff Model

HYMO [Williams and Hann, 1973] is used to predict the peak runoff rate. The COMPUTE HYD command computes storm hydrographs by convolving source runoff with an instantaneous unit hydrograph (IUH). Equations for computing the IUH are:

$$q = q_p \left(\frac{\Psi}{\Psi_p}\right)^{(n-1)} e^{(1-n)\left(\frac{\Psi}{\Psi_p} - 1\right)} \qquad (5)$$

$$0 \leq \Psi \leq \Psi_0$$

$$q = q_0 \, e^{\left(\frac{\Psi_0 - \Psi}{\lambda}\right)} \qquad (6)$$

$$\Psi_0 \leq \Psi \leq \Psi_0 + 2\lambda$$

$$q = 0.135\, q_0\, e^{\left(\frac{\Psi_0 + 2\lambda - \Psi}{\lambda_1}\right)} \qquad (7)$$

$$\Psi_0 + 2\lambda \leq \Psi \leq \infty$$

where q is the flow rate at time Ψ; Ψ_p is the time to peak; n is a dimensionless parameter dependent upon $\frac{\lambda}{\Psi_p}$; q_0 is the flow rate at the inflection point; Ψ_0 is the time at the inflection point, and λ and λ_1 are recession constants. Since n is a function of $\frac{\lambda}{\Psi_p}$, the IUH can be computed if Ψ_p, q_p, λ, and λ_1 are known. The shape parameters Ψ_p, λ, and λ_1 can be estimated by the previously developed equations [Williams, 1972].

$$\Psi_p = 1.96\ DA^{0.39}\ ZL^{-0.50} \qquad (8)$$

$$\lambda = 3.17\ DA^{0.24}\ ZL^{-0.84} \qquad (9)$$

$$\lambda_1 = 3\lambda \qquad (10)$$

where Ψ_p, λ, and λ_1 are in h, DA is in km^2, and ZL is the relief-length ratio in m/km. The IUH peak flow rate can be determined from the equation

$$q_p = \frac{G\ DA\ Q}{\Psi_p} \qquad (11)$$

where G is determined from the dimensionless IUH.

$$G = \frac{J}{\int_0^\infty \frac{q}{q_p} \, d\,\Psi} \tag{12}$$

where J is a constant expressing units (J = 2.778 if q = m^3/s, Q = cm, DA = km^2, and Ψ = h).

A retention function [Snyder, 1971] is used to predict incremental source runoff. The form of the retention function used in HYMO is

$$r_\Psi = r_c + (r_0 - r_c)\, e^{-W\Psi} \tag{13}$$

where r_Ψ is the retention rate at time Ψ; r_c is the basic low retention rate; r_0 is the retention rate at $\Psi = 0$, and W is a scaling coefficient expressed by the equation

$$W = \xi \left(\frac{g_\Psi + r_u - r_0}{g_\Psi + r_u - r_c} \right) \left(\frac{g_\Psi - r_c}{g_\Psi - r_c} \right) \tag{14}$$

where ξ is a dimensionlesss parameter; g_Ψ is the rainfall intensity at time, Ψ; and r_u is the maximum retention rate. The value of r_c is dependent upon the hydrologic soils group and r_u was arbitrarily set to a value of 50 cm/h. Thus the only unknown parameters are ξ and r_0. Slack and Snyder [1977] related the parameters to the runoff curve number with the equations

$$\log_{10} \xi = -1.575 - 0.147\, r_c - 0.00624\, D + 0.0491\, D\, r_c + 0.0171\, CN \tag{15}$$

and

$$r_0 = 1.45\ g_\psi + r_c\ (1.0 - 0.315\ g_\psi) \tag{16}$$

where r_c, r_0, and g_ψ are in cm/h, D is the duration in h, and CN is the SCS curve number.

Since hydrograph computation is quite time consuming, only three or four hydrographs with a wide range in volumes are predicted. Peak rate is related to runoff volume with the function

$$q_p = b_1\ Q^{b_2} \tag{17}$$

where b_1 and b_2 are constants determined by least squares. Peak rate is predicted for each day that runoff occurs by applying equation (17) to the daily runoff volumes predicted with the water yield model.

The Sediment Model

The USLE was modified for predicting sediment yield from watersheds [Williams, 1975c]. MUSLE is expressed as

$$Y = 11.8\ (V\ q_p)^{0.56}\ (K)\ (C)\ (P)\ (LS) \tag{18}$$

where Y is the sediment yield from an individual storm in t; V is the storm runoff in m^3; q_p is the peak runoff rate in m^3/s; K is the soil erodibility factor; LS is the slope length and gradient factor; C is the crop management factor, and P is the erosion-control-practice factor.

Values of V and q_p are obtained from the runoff model. The average watershed slope can be computed with the grid-contour method [Williams and Berndt, 1977]. The length of each grid line within the watershed is measured and the contours crossing or tangent to the line are counted. The land slope in any direction is computed by the equation:

$$S_d = \frac{N_d\ H}{X_d} \qquad (19)$$

where S_d is the land slope in one direction; N_d is the total number of crossings for all lines in one direction; H is the contour intervals, and X_d is the total length of lines in that direction. The average watershed slope can be determined by computing the slope in both grid directions with equation (19) and calculating the resultant:

$$S = \sqrt{S_L^2 + S_W^2} \qquad (20)$$

where S is the average watershed slope; S_L is the average slope of the watershed along its length, and S_W is the average slope along the width. The average watershed slope length can be computed with the contour-extreme point method [Williams and Berndt, 1977] using the equation

$$L = \frac{LC}{2\ EP} \qquad (21)$$

where L is the average watershed slope length; LC is the total length of contours, and EP is the number of extreme points on the contour (extreme points on the contour occur where a channel crosses the contour).

Previously the K, C, P, and LS factors of MUSLE were determined separately on an area weighted basis [Williams and Berndt, 1972]. In later work [Williams and Berndt, 1976], K and LS were determined separately, but the product of C and P was calculated using the same area weighting technique. Both of these approaches are short cuts that usually produce acceptable results on mixed land-use watersheds. However, these short cut approaches can introduce considerable error in watersheds that have subareas with great differences in combinations of K, C, P, and LS because

$$\Sigma K_i DA_i \times \Sigma C_i DA_i \times \Sigma P_i DA_i \times \Sigma LS_i DA_i \neq \Sigma K_i C_i P_i LS_i DA_i \qquad (22)$$

The righthand side of equation (22) was used here for water-quality modeling to eliminate as much error as possible. To expedite calculations, watersheds are divided into land capability classes, because land classes have characteristic land uses, soils, and slopes.

Information from soils maps and estimates by Soil Conservation Service (SCS) personnel can be used to compute the average slope length and gradient for each land class within a subwatershed. The equations necessary for determining the average slope for each land class are:

$$\sum_{i=1}^{N} \gamma_i S_i = S \qquad (23)$$

$$S_i = S_1 \frac{E_i}{E_1} \qquad (24)$$

where γ_i is the portion of the watershed covered by land class, i; S_i is the average slope of land class, i; E_i is the estimated slope for land class, i (taken from soils maps), and N is the number of land classes within the subwatershed. Similar equations are used to compute L for each land class. The LS factor for each land class is computed with the equation

$$LS_i = \left(\frac{L_i}{22.1} \right)^{\phi} (.065 + .0454\ S_i + .0065\ S_i^2) \qquad (25)$$

where ϕ is 0.5 for slopes greater than 3% and 0.3 for flatter slopes.

The value of the C factor for each crop is determined for each month from tables prepared by Wischmeier and Smith [1965].

The P factor can be estimated for the cultivated areas of the watershed using information contained in Technical Release No. 51 [U.S. Soil Conservation Service, 1975]. The product of the K, C, P, and LS factors is determined for the entire watershed with the equation

$$K\ C\ P\ LS = \sum_{i=1}^{N} \gamma_i\ K_i\ LS_i \sum_{j=1}^{M} \eta_j\ C_{\ell j}\ P_j\ , \quad \ell = 1,\ 12 \qquad (26)$$

where $C_{\ell j}$ is the C value for month, ℓ, for the crop on area, j; P_j is the P value for area, j; M is the number of areas in the land class, and η_j is the portion of land class, i, contained in area, j.

The Phosphorus Model

The phosphorus model is based on a phosphorus-loading function [McElroy, et al., 1976] for predicting long-term average annual phosphorus yields. Here the loading function was adapted to individual storm prediction by simulating daily phosphorus concentration in the soil, using daily predicted sediment yields, and computing enrichment ratios. It is necessary to simulate daily soil phosphorus concentration to determine the effect of fertilizer rate on phosphorus yield. Also, McElroy, et al. [1976] suggested estimating enrichment ratios from limited data but did not provide a technique for computing them.

The phosphorus loading function for individual storms is

$$YP = 0.001\ (Y)\ (c_p)\ (ER_p) \tag{27}$$

where YP is the phosphorus yield in kg; Y is the sediment yield in t; c_p is the soil phosphorus concentration in ppm, and ER_p is the enrichment ratio for phosphorus. Y is predicted with MUSLE for each day that runoff occurs.

The enrichment ratio, ER_p (concentration of phosphorus in the sediment divided by the concentration of phosphorus in the soil), is determined by considering the specific surface area of the soil and the sediment because physical adsorption of chemicals depends on the specific surface area. Types of clay are also significant because of their adsorption capacities. Young and Onstad [1976] pointed out that montmorillonite will adsorb considerably more herbicides than kaolinite because it has a larger surface area and higher cation exchange capacity. They related specific surface areas of 61 soils to the percent sand, silt, clay, and montmorillonite in the equation

$$SS = 0.16\ (Sa) + 0.185\ (Si) + 0.33\ (Cl) + 0.107\ (MM)\ (Cl) \tag{28}$$

where SS is the specific surface area of the soil particles in m^2/g; Sa is the percent sand; Si is the percent silt; Cl is the percent clay, and MM is the percent montmorillonite.

Here the particle-size distribution was divided into nine parts, instead of the general divisions of sand, silt, and clay. Thus, it was necessary to relate SS to particle size and montmorillonite. By assuming the average particle size of sand, silt, and clay is 350, 15, and 1 μ, and solving equation (28) three times, assuming 100% sand, 100% silt, and 100% clay, the relationship was developed. The three points were plotted on log paper and the best fit is expressed by the equation

$$SS = 33\ (d)^{-0.1785} + 10.7\ MM \qquad (29)$$

where d is the sediment particle diameter in μ. Thus, the specific surface can be computed for each of the nine particle sizes being considered. Since the technique for computing the sediment particle-size distribution is based on sediment routing, it is described in the sediment-routing section. The specific surface of the sediment-particle distribution is determined by weighting the SS values calculated with equation (29), according to the portion of the distribution each represents.

$$SSY = \sum_{i=1}^{N} SS_i\ \omega_i \qquad (30)$$

where SSY is the specific surface of the particle-size distribution; SS_i is the specific surface of the portion, ω_i, represented by particle size d_i.

The specific surface of the soil is determined similarly and the phosphorus enrichment ratio is calculated with the equation

$$ER_p = \frac{SSY}{SSS} \tag{31}$$

where SSS is the specific surface of the soil.

The concentration of soil phosphorus can be predicted daily by attaching a phosphorus balance model to the runoff and sediment model. The phosphorus balance model was designed to operate within the top 15 cm of soil because 15 cm is a common plow depth and represents a maximum erosion depth even with severe rilling. The phosphorus balance is expressed as

$$PT_{\tau+1} = PT_{\tau} + PF_{\tau} + RSP_{\tau} - UP_{\tau} - YP_{\tau} \tag{32}$$

where PT is the total phosphorus contained in the top 15 cm of soil; τ is time in days; PF is the amount of phosphorus fertilizer applied; RSP is the amount of phosphorus contained in crop residue; UP is the amount of phosphorus used by the crop, and YP is the phosphorus yield. Phosphorus input from rainfall was neglected because the contribution is relatively small.

The model is designed so that phosphorus fertilizer can be applied at any rate as many as six times annually. Fertilizer is assumed to be mixed within the top 15 cm of cropland areas and the top 2.5 cm of grassland areas.

Use of phosphorus by the crop depends upon stage of growth if phosphorus is available in the soil throughout the growing season.

Vanderlip [1972] showed that the ratio of daily phosphorus use to annual use was greater than the ratio of daily grain sorghum growth to annual growth during the early growing season. The ratios gradually approach each other until about midseason when they become equal. After midseason, the growth ratio exceeds the phosphorus ratio. Walsh and Beaton [1973] showed similar relationships for other crops. A polynomial was fitted to Vanderlip's data to relate phosphorus use to growth of grain sorghum. The relationship is

$$up = 2.03\ (sg) - 2.33\ (sg)^2 + 1.30\ (sg)^3 \qquad (33)$$

where up is a dimensionless unit (ranges from 0-1) phosphorus-use curve, and sg is a dimensionless unit grain sorghum growth curve. Equation (33) was differentiated to obtain the dimensionless phosphorus use rate.

$$\frac{d\ (up)}{d\ (sg)} = 2.03 - 4.66\ (sg) + 3.90\ (sg)^2 \qquad (34)$$

Growth rate was assumed proportional to water use. Thus, the value of sg at any time during the growing season is calculated with the equation

$$sg_N = \frac{\sum_{i=1}^{N} CU_i}{CU} \qquad (35)$$

where sg_N is the accumulated dimensionless growth through day, N; CU_i is the average consumptive use for day, i, and CU is the total average consumptive use for the crop. The amount of phosphorus use on any day can be computed by substituting sg from equation (35) into equation (34) and multiplying by the growth for the day and the potential annual phosphorus use.

$$UP_i = (2.03 - 4.66 (sg_i) + 3.90 (sg_i)^2) (SG_i) (PUP) \qquad (36)$$

where SG_i is the amount of plant growth on day, i, and PUP is the maximum potential phosphorus use during the growing season of the crop. Since plant growth was assumed proportional to water use, the plant growth on any day is computed by the relationship

$$SG_i = \frac{(UW_i) (PSG)}{CU} \qquad (37)$$

where UW_i is the water use on day, i, determined from equation (3), and PSG is the maximum potential annual plant growth (aboveground and roots). Maximum potential phosphorus use and crop growth can be estimated satisfactorily by SCS personnel, who are familiar with the watershed.

The amount of phosphorus contained in the crop residue is computed by the equation

$$RSP = (RS) (PP) \qquad (38)$$

where RS is the total crop residue including roots; PP is the percent phosphorus in the residue, and RSP is the amount of phosphorus in the residue. Values of PP have been determined for several crops [Walsh and Beaton, 1973; Bassett, et al., 1970].

The Nitrogen Model

The nitrogen model simulates both organic and inorganic nitrogen yields associated with sediment and runoff. Like the phosphorus model, the organic nitrogen model is based on a loading function [McElroy, et al., 1976]. The nitrogen-loading function was modified for use on individual storms by including refinements similar to those of the phosphorus model. However, to predict total nitrogen and to determine the effects of fertilizer, a nitrate model was added. The loading function was not applicable to nitrate prediction because nitrate is not attached to sediment.

The organic nitrogen loading function for individual storms is

$$YON = 0.001\ (Y)\ (c_{ON})\ (ER_N) \qquad (39)$$

where YON is the organic nitrogen yield in kg; Y is the sediment yield in t; c_{ON} is the soil organic nitrogen concentration in ppm, and ER_N is the nitrogen-enrichment ratio.

The nitrogen-enrichment ratio is determined by considering only the finest sediment particle size of the distribution (1 μ) because organic nitrogen is associated with fine clay. This is slightly different than the phosphorus-enrichment ratio because phosphorus tends to associate with coarse clay and silt as well as fine clay. The nitrogen-enrichment ratio is computed with the equation

$$ER_N = \frac{\zeta_y}{\zeta_s} \qquad (40)$$

where ζ_y is the percent of the sediment, and ζ_s is the percent of the soil with a 1-μ particle size.

To predict soil nitrogen concentration, a nitrogen balance model was developed and attached to the runoff and sediment model. The nitrogen-balance model is also useful in predicting nitrate yield. The nitrogen balance is expressed by the equations

$$ON_{\tau+1} = ON_\tau + IM_\tau + RSON_\tau - MN_\tau - YON_\tau \qquad (41)$$

$$\begin{aligned} NO3_{\tau+1} = {} & NO3_\tau + NF_\tau + MN_\tau + RN_\tau - IM_\tau - DN_\tau - YNO3_\tau \\ & - DRN_\tau - UN_\tau \end{aligned} \qquad (42)$$

where ON is the amount of organic nitrogen contained in the top 90 cm of soil; τ is time in days; IM is the amount of nitrate nitrogen immobilized; RSON is the amount of organic nitrogen contained in crop residue; MN is the amount of organic nitrogen that becomes nitrate nitrogen through mineralization; NO3 is the amount of nitrate nitrogen contained in the top 90 cm of soil; NF is the amount of nitrate nitrogen fertilizer applied; RN is the amount of nitrate nitrogen contributed by rainfall; DN is the amount of nitrate nitrogen lost through denitrification; YNO3 is the nitrate nitrogen yield; DRN is the amount of nitrate nitrogen that drains below 90 cm, and UN is the amount of nitrate nitrogen used by the crop.

The 90-cm depth was selected because most root growth, nutrient uptake, and water use occurs within this depth. To adequately simulate the dynamics of the processes involved, the soil profile was divided into seven storages. The top storage is 2.5 cm deep; the second one is

12.5 cm, and the five storages below are 15 cm each. Submodels were developed to simulate each of the nitrogen balance components. The loss of organic nitrogen in sediment is predicted with equation (39). Individual descriptions of the other submodels follow.

Organic Nitrogen in Crop Residue

The amount of organic nitrogen contained in the crop residue is computed with an equation similar to equation (38) for estimating phosphorus in crop residue.

$$RSON = (RS)\ (PN_R) \tag{43}$$

where PN_R is the percent organic nitrogen contained in the crop residue. Values of PN_R have been determined for several crops [Walsh and Beaton, 1973; Bassett, et al., 1970; Stanford, 1973].

At the end of a crop season the residue is added to the soil storages. The aboveground residue is added to the top storage for grass and to the top two storages for cultivated crops. Root residue is simply added to the storages according to the root location at the end of the growing season. Equation (43) is used to determine the amount of organic nitrogen added to each storage.

Leaching

The leaching model assumes that nitrate moves through the soil with the drainage water. Water flow through the storages is computed by modifying the variable storage coefficient routing equation [Williams, 1969]. The modified equation is

$$0 = \sigma \left(F + \frac{ST}{\Delta\Psi} \right) \tag{44}$$

$$\left(F + \frac{ST}{\Delta\Psi} \right) > .75 \text{ UL}$$

where 0 is the outflow rate in cm/h; F is the infiltration or inflow rate in cm/h; ST is the storage volume in cm; σ is the storage coefficient; Ψ is time in h; and UL is the storage capacity in cm. If $F + \frac{ST}{\Delta\Psi} \leq .75$ UL, then 0 = 0.0. The storage coefficient is a function of the travel time through the storage expressed by the equation

$$\sigma = \frac{2\ \Delta\Psi}{2TT + \Delta\Psi} \tag{45}$$

where TT is the travel time through the storage in h. Travel time is determined with the equation

$$TT = \frac{UL}{r_c} \tag{46}$$

where r_c is the basic low retention rate of the soil, as expressed in equation (13).

As water flows through a soil storage, the nitrate concentration of the storage changes continuously. For a very short time interval or a very small outflow, the change in nitrate content of the soil can be closely approximated by the equation

$$NO3_{\Psi+\Delta\Psi} = NO3_{\Psi} - (c_{NO3_{\Psi}})\ (0)\ (\rho_w) \tag{47}$$

where NO3 is the weight of nitrate in the soil in kg/ha; c_{NO3} is the concentration of nitrate nitrogen in the soil at time, Ψ; O is the amount of outflow in cm that occurs during time, $\Delta\Psi$; and ρ_w is the weight of water in kg/cm • ha. Dividing equation (47) by the weight of the soil gives an expression for the change in nitrate concentration.

$$c_{NO3_{\Psi+\Delta\Psi}} = c_{NO3_{\Psi}} - c_{NO3_{\Psi}} \ (O)\left(\frac{\rho_w}{\rho_s}\right) \qquad (48)$$

where ρ_s is the weight of a particular soil storage in kg/ha. Equation (48) is a finite difference approximation of the first order decay function

$$c_{NO3_{\Psi+\Delta\Psi}} = c_{NO3_{\Psi}} \ e^{-(O)\ (\frac{\rho_w}{\rho_s})} \qquad (49)$$

Equation (49) accounts for the change in concentration with flow. Thus, the daily amount leached can be computed more accurately than by assuming that the concentration remains constant as in equation (48). The amount of nitrate nitrogen leached for a specified amount of drainage can be computed by subtracting the weight of nitrate nitrogen in the soil at the end of flow from the beginning weight.

$$DRN = (\rho_s)\ (c_{NO3})\left(1 - e^{-(O)\ (\frac{\rho_w}{\rho_s})}\right) \qquad (50)$$

The model uses equations (44) and (50) to simulate daily water and nitrate movement through each soil storage. The amount that drains below the bottom storage is accumulated for use in the nitrogen balance (equation (42)).

Nitrate Yield

The yield of nitrate in runoff water is predicted by assuming that the concentration of the runoff water is the same as that of the top soil storage. The two concentrations are the same because runoff mixes with detached soil particles and also enters the soil and returns to the surface on a microscale (not classic return flow). As runoff mixes and temporarily infiltrates the soil, the soil nitrogen concentration is continuously reduced. Thus, equation (49) can be used to define concentration by replacing drainage with runoff.

To estimate total nitrate nitrogen yield, the change in the upper storage nitrate weight during the storm is added to the nitrate contributed by rainfall.

$$YNO3 = (\rho_s)\ (c_{NO3}) \left(1 - e^{-(Q)\ (\frac{\rho_w}{\rho_s})} \right) + (c_R)\ (Q) \qquad (51)$$

where c_R is the concentration of nitrate nitrogen in the rainfall.

Denitrification

Denitrification, the microbial reduction of nitrate and nitrite, results in gaseous loss. Quantification of this process has been limited. According to Frere [1976], most estimates have been based on nitrogen budgets where all unaccounted-for nitrogen is assigned to denitrification. Average losses have been estimated at 10 to 30% of the total annual mineral nitrogen input [Broadbent and Clark, 1965]. Alexander [1961] stated that the rate of denitrification depends upon the presence of a carbon source, anaerobic conditions, temperature, and nitrate concentration. No losses occur at moisture levels below 60%

of the water-holding capacity of the soil. Above 60%, the rate of denitrification varies directly with the moisture regime. The optimum temperature range is 25°C and above. Upper and lower temperature limits are about 70° and 2°C. Alexander's concept of denitrification is approximated here with the equation

$$DN = NO3 \quad 1 - e^{-(CDN)\ (T^4)\ (c_{ON})\ (\frac{ST}{UL})} \qquad (52)$$

$$.6\ UL \leq ST \leq UL$$

where DN is the daily amount of denitrification; NO3 is the weight of nitrate nitrogen in the soil at the beginning of the day; CDN is the denitrification constant; T is the daily average air temperature, and c_{ON} is the concentration of organic nitrogen in the soil at the beginning of the day. If the water content of a storage is less than 60% of the upper limit, denitrification does not occur. Temperature is raised to the fourth power to expand the scale for a more appropriate description of microbial activity. As the temperature approaches 0°C, denitrification is greatly reduced and ceases at 0°. At higher temperatures, denitrification increases rapidly. Generally, soil temperatures would never approach the upper limit (70°C). The concentration of organic nitrogen is used to indicate the presence of a carbon source in the soil.

To estimate an initial value of the denitrification constant, average daily values of the other variables are estimated, and CDN is computed using equation (52).

$$CDN = \frac{\ln\left(1 - \frac{DN}{NO3}\right)}{(T^4)\ (c_{ON})\ \left(\frac{ST}{UL}\right)} \tag{53}$$

Average annual denitrification can be estimated as 10 to 30% of the annual mineral nitrogen input. The average daily amount of denitrification is greater than the annual amount divided by 365 days, because denitrification does not occur on all days. The average annual number of days of rainfall can be used to estimate the number of days of denitrification. Of course, this is only a rough approximation. Average values of NO3, ST, and c_{ON} can be estimated more accurately and average temperature can be determined quite accurately. The value of CDN obtained from equation (53) can be adjusted if necessary, after observing the results of the first simulation.

Immobilization

Alexander [1961] described immobilization as the microbial assimilation of inorganic nutrients. It occurs under aerobic or anaerobic conditions and basically involves the uptake of the mineral forms by microorganisms in the synthesis of cell tissue. When organic residues low in nitrogen are being decomposed, mineral nitrogen is used. Kissel [1977] observed that the immobilization rate increased with temperature. In the presence of a carbon source, immobilization increased rapidly when the temperature exceeded 23°C.

Crop residue has a carbon:nitrogen (C:N) ratio of about 70. According to Alexander [1961], about 40% of the residue is carbon. Microorganisms assimilate carbon and nitrogen to form cells with a C:N ratio of about 10:1. About 30% of the carbon is assimilated and 70% is liberated as CO_2. Crop residue usually contains a small amount of nitrogen, but additional inorganic nitrogen is needed by the microorganisms. The additional nitrate nitrogen required is the immobilization component of equation (42). The daily carbon concentration of the soil is expressed here by the equation

$$c_{c(\tau+1)} = c_{c\tau} \, e^{-(CIM)\,(T^4)\,(\frac{ST}{UL})} \qquad (54)$$

where c_c is the concentration of carbon in the soil, and CIM is the immobilization constant. Daily carbon decomposition is computed by multiplying the daily difference in concentration $(c_{c\tau} - c_{c(\tau+1)})$ by the weight of the soil.

$$DC = (\rho_s)\,(c_c)\;\; 1 - e^{-(CIM)\,(T^4)\,(\frac{ST}{UL})} \qquad (55)$$

where DC is the daily amount of decomposed carbon.

The daily amount of immobilization is computed by subtracting the amount of nitrogen contained in the crop residue from the amount assimilated by the microorganisms.

$$IM = \frac{(0.3)\,(DC)}{10} - \frac{(DC)\,(PN_R)}{0.4} \qquad (56)$$

where 0.3 is the portion of carbon assimilated; 10 is the C:N ratio of the cells, and 0.4 is the portion of the residue composed of carbon.

Since immobilization is a better defined process than denitrification, the immobilization constant can be estimated more accurately. CIM is estimated by using average daily values for the variables of equation (54) and assuming that 99% of the carbon is decomposed annually.

$$\mathrm{CIM} = \frac{\ln\ (.01)}{(T^4)\ (\frac{ST}{UL})\ (365)} \qquad (57)$$

After observing results of the first simulation, it may be necessary to adjust CIM.

The amount of carbon contained in each storage is determined daily using equation (55). At the end of a crop season, the amount of carbon added to each storage is estimated to be 40% of the added crop residue.

Use by Crop

Like phosphorus, nitrogen use by the crop depends upon stage of growth, if nitrogen is available in the soil throughout the growing season. Vanderlip [1972] provided data from grain sorghum tests that related crop growth to nitrogen use. The nitrogen curve is similar to the phosphorus curve, except phosphorus use lags nitrate use. A polynomial was fitted to Vanderlip's data to relate nitrogen use to growth of grain sorghum. The relationship is

$$un = 2.47\ (sg) - 3.01\ (sg)^2 + 1.50\ (sg)^3 \qquad (58)$$

where un is a dimensionless unit nitrogen use curve. Differentiating equation (58) gives the dimensionless nitrogen-use rate equation

$$\frac{d\ (un)}{d\ (sg)} = 2.47 - 6.02\ (sg) + 4.5\ (sg)^2 \qquad (59)$$

The amount of nitrogen use on any day is computed by using equation (59) to develop a relationship similar to equation (36).

$$UN_i = (2.47 - 6.02\ (sg_i) + 4.5\ (sg_i)^2)\ (SG_i)\ (PUN) \qquad (60)$$

where PUN is the maximum potential nitrogen use during the growing season of the crop. The daily crop growth, SG, is computed with equation (37).

Total nitrogen use is distributed through the soil storages, according to root growth within each storage. Root growth, like total plant growth, is assumed proportional to water use. Root depth and root weight are estimated with the equations

$$RD = \frac{\sum_{i=1}^{N} UW_i}{CU} \qquad (61)$$

$$RW = 0.2 \sum_{i=1}^{N} SG_i \qquad (62)$$

where RD is the root depth expressed as a fraction of the 90-cm storage depth; RW is the root weight in kg/ha; 0.2 is the fraction of the total plant growth made up of roots, and N is the number of days of growth. Since root weights are not easily determined, little data is available. However, 20% of the total plant growth was assumed for the weight of corn roots [Stanford, 1973].

A model was developed to determine the distribution of water use (root growth) by depth. The top two storages were lumped together to obtain six 15-cm storages. The total water use is made up of the uses in the six storages.

$$UW = \sum_{i=1}^{6} uw_i \tag{63}$$

where uw_i is the water use by the crop in soil storage, i. The water-use rate as a function of depth is expressed by the equation

$$v = v_0 \, e^{-(\chi)\,(RD)} \tag{64}$$

where v is the water use rate by the crop at depth RD; v_0 is the rate at the surface, and χ is the water-use rate constant. The total water use within any depth can be computed by integrating equation (64) to obtain the equation

$$UW = \frac{v_0}{\chi}\left(1 - e^{-(\chi)\,(RD)}\right) \tag{65}$$

The water use within any storage can be calculated by solving equation (65) for the depth at the top and bottom of the storage and taking the difference.

$$uw_i = \frac{v_0}{\chi}\left(e^{-(\chi)\,(RD_{i-1})} - e^{-(\chi)\,(RD_i)}\right) \tag{66}$$

Since a large portion of root growth occurs within the top 15 cm of soil, the water use in the top storage was assumed to be twice as large as the use in the second storage. This assumption allows the determination of χ.

$$uw_1 = 2\ uw_2 \qquad (67)$$

$$uw_1 = \frac{v_0}{\chi} \quad 1 - e^{-\frac{1}{6}\chi} \qquad (68)$$

$$uw_2 = \frac{v_0}{\chi} \quad e^{-\frac{1}{6}\chi} - e^{-\frac{1}{3}\chi} \qquad (69)$$

The value of χ was determined to be 4.16 by substituting equation (68) and (69) into equation (67) and solving with Newton's classical method for solving nonlinear equations.

Since v_0 is the only unknown in equation (65), it can be determined and equation (66) is used to calculate water use in each soil storage. Water use is determined separately for all storages including the top (2.5 cm) storage. The 2.5- and 12.5-cm storages were lumped only for developing equation (67). The nitrogen use computed with equation (60) is distributed with depth in proportion to the water use.

Mineralization

Mineralization, the conversion of organic nitrogen to inorganic nitrogen, is a microbial process that is most active when soil is warm and moist. Alexander [1961] stated that the rate of mineralization is governed by moisture, temperature, inorganic nutrient supply and other variables. He added that production of inorganic nitrogen is closely correlated with the total nitrogen content of the soil. Frere [1976] indicated that annual mineralization ranges from about 20 to 135 kg/ha. Here, the reduction in organic nitrogen caused by mineralization is expressed by the equation

$$ON_{\tau+1} = ON_{\tau}\, e^{-(CMN)\ (T^4)\ (\frac{ST}{UL})} \tag{70}$$

where CMN is the mineralization constant. The daily amount of mineralization is the difference in ON at the beginning and ending of a day as determined with equation (70).

$$MN_{\tau} = ON_{\tau} \left(1 - e^{-(CMN)\ (T^4)\ (\frac{ST}{UL})}\right) \tag{71}$$

The mineralization constant can be estimated by considering immobilization and nonfertilized crop yields. Annual mineralization can be approximated as the sum of immobilization and the nitrogen content of unfertilized crops. This approximation neglects nitrogen contribution from rainfall and losses by denitrification, runoff, and leaching. However, these components are generally relatively small.

Annual nitrogen use by a crop can be estimated by the equation

$$UN = (YLD)\ (PN_y) + RSON \tag{72}$$

where UN is the annual nitrogen use by the crop; PN_y is the percent nitrogen contained in the harvested yield, and YLD is the harvested yield. Values of PN_y have been determined for several crops [Walsh and Beaton, 1973; Bassett, et al., 1970; Stanford, 1973]. For crops like grain sorghum, corn, small grain, and cotton, the residue can be estimated by assuming that the aboveground weight is 50% yield and that the roots are 20% of the total plant. Substituting these assumptions into equation (72) gives an expression for UN based on yield.

$$UN = YLD\ (PN_y + 1.5\ PN_R) \tag{73}$$

Crop yield can be estimated more accurately than residue because yields are determined annually at harvest time. However, equation (73) does not apply to pasture and forage crops because the harvested yield varies up to almost 100% of the aboveground weight and the root weight is more than 20% of the total weight.

The annual amount of immobilization can be computed with equation (56), using annual values of DC and RSON. Annual mineralization can then be estimated using the equation

$$MN = UN + IM \tag{74}$$

The annual MN value is divided by 365 days and substituted, along with other average daily values, into equation (71) to determine the mineralization constant.

$$CMN = \frac{\ell n\ (1 - \frac{MN}{ON})}{(T^4)\ (\frac{ST}{UL})} \tag{75}$$

CMN can be adjusted if necessary after the first simulation.

Nitrogen from Rainfall

The average annual contribution of inorganic nitrogen from rainfall can be estimated for any location in the United States from an isohyetal map [McElroy, et al., 1976]. Better estimates can be obtained if data are available on or near the watershed. The nitrogen contribution from an individual storm is predicted with the equation

$$RN = (c_R)\ (R) \qquad (76)$$

where c_R is the average concentration of inorganic nitrogen in the rainfall, and R is the amount of rainfall for the storm.

Fertilizer

Like phosphorus, nitrogen fertilizer can be applied at any rate as many as six times annually. Nitrogen fertilizer is assumed to be mixed within the top two storages (15 cm) for cropland and the top storage (2.5 cm) for grassland.

MODEL FOR ROUTING SEDIMENT, PHOSPHORUS, AND NITROGEN

A model was developed for routing sediment, phosphorus, and nitrogen through streams and valleys of large watersheds. The model is based on a sediment routing model [Williams, 1975b] and phosphorus and nitrogen loading functions [McElroy, et al., 1976]. Output from a flood routing model and the daily runoff, sediment, phosphorus, and nitrogen simulation model is used as input to the routing model. Flood routing can be accomplished conveniently using HYMO [Williams and Hann, 1973]. The Variable Travel Time flood routing method [Williams, 1975a] used in HYMO is about as accurate as an implicit solution of the unsteady flow equations of continuity and motion, and is free of convergence problems. Since flood routing is quite time consuming, only a few storms of various frequencies are routed. These storms are selected from runoff, sediment, phosphorus, and nitrogen frequency distributions prepared for each subwatershed with output from the daily simulation model.

Once the flood routing is completed, sediment, phosphorus, and nitrogen are routed simultaneously for the selected frequencies. To make the routings more convenient, a problem-oriented computer language called SPNM (Sediment-Phosphorus-Nitrogen Model) was developed using the routing model as its main component. Like HYMO, SPNM is written in the language of the discipline, and the input is entirely familiar to hydrologists and environmental engineers. No conventional programming experience is necessary to describe a problem or to interpret the results. A SPNM program is written for every problem. The system makes no assumption about the sequence of operations, and the user has full freedom to specify the sequence of basic processes best suited for each problem.

The Routing Model

The routing model was designed to route individual storm sediment, phosphorus and nitrogen yields from subwatershed outlets through streams and valleys of large agricultural watersheds. Routing increases prediction accuracy on large watersheds and allows determination of subwatershed contributions to the total yield. Since calibration is not required, the model is directly applicable to ungaged watersheds.

Sediment Routing

A sediment-routing model [Williams, 1975b] was developed for application to large watersheds with nonuniformly distributed sediment sources. The model was based on the assumption that sediment deposition depends upon settling velocities of the sediment particles, length of travel time, and the amount of sediment in suspension. These assumptions were expressed by the sediment routing equation

$$RY = \sum_{i=1}^{N} Y_i \, e^{-\beta \, TT_i \sqrt{d_i}} \qquad (77)$$

where RY is the routed sediment yield from an individual storm for the entire watershed; Y is the sediment yield for subwatershed, i (from simulated frequency distribution); β is the routing coefficient; TT is the travel time from subwatershed, i, to the watershed outlet; d_i is the median particle diameter of sediment for subwatershed, i; and N is the number of subwatersheds. Y_i and TT_i are obtained from simulation and flood routing, and d_i can be estimated from soils information. Thus, only the total sediment, RY, and the routing coefficient, β, are unknown. RY can be predicted fairly accurately with equation (18) if K, C, P, LS, and d_i are uniformly distributed over the entire watershed. To determine β for an individual storm on a particular watershed, uniform distributions of K, C, P, LS, and d_i are assumed, and Y is computed with equation (18). The predicted Y is used to replace RY in equation (77). Or setting the righthand sides of equations (18) and (77) equal yields the equation:

$$11.8\ (V\ q_p)^{0.56}\ (K)\ (C)\ (P)\ (LS) = 11.8 \sum_{i=1}^{N} (V_i\ q_{pi})^{0.56}\ (K_i)\ (C_i)\ (P_i)\ (LS_i)\ e^{-\beta\ (TT_i)\ \sqrt{d_i}} \qquad (78)$$

If K, LS, C, and P are equal for all subwatersheds, they cancel in equation (78), thus producing the equation for determining β.

$$(V\ q_p)^{0.56} = \sum_{i=1}^{N} (V_i\ q_{pi})^{0.56}\ e^{-\beta\ (TT_i)\ \sqrt{d_i}} \qquad (79)$$

β is determined using Newton's classical method for solving nonlinear equations. Once β has been determined, sediment yield for the watershed can be predicted with equation (77), using the actual values of K, C, P, LS, and d_i for each subwatershed.

Floodplain scour for each routing reach is estimated with equation (18), written in the form

$$YFP = 2045\ (DA\ Q\ q_p)^{0.56}\ (K)\ (C)\ (P)\ (LS) \qquad (80)$$

where YFP is the sediment yield from floodplain scour of a routing reach in t; DA is the area flooded in km^2; Q is the runoff from the watershed above the reach in cm, and q_p is the average of the inflow and outflow peak flowrates in m^3/s. Relatively high peak flowrates provide tremendous energy for scour. Also, P is usually 1.0 because there are few erosion-control practices in the floodplain. However, LS is quite low because slopes are flat. The crop factor, C, is the dominant factor in floodplain scour. Scour is very minor on a well-covered floodplain but can be severe on poorly covered floodplains.

Equation (80) can also be used to estimate channel scour. The area flooded is simply the product of the channel width and length. The peak flowrate contained in the channel for a particular storm is q_p. The LS factor is usually high for channels because the slope is the resultant of the side slopes and the channel slope. The crop factor, C, is usually high because most natural channels are not covered with vegetation. As in the floodplain, P is 1.0 because there are no erosion-control practices. The most critical factor in channel scour is the soils factor, K. If the channel is stable, K must be very low because all the other factors are high.

The sediment-routing model was refined here by replacing the median particle size with the entire particle-size distribution. Also a technique was developed for determining β for each routing reach, instead of using one value of β for the entire watershed as in the original model. The amount of sediment that outflows from a routing reach consists of the amount that originates within the reach plus the portion of upstream inflow that is transported through the reach. Sediment sources within the reach are the subwatershed that drains into the channel, the floodplain, and the channel (Figure 1). For these sources, sediment is predicted at the downstream end of the reach so routing is

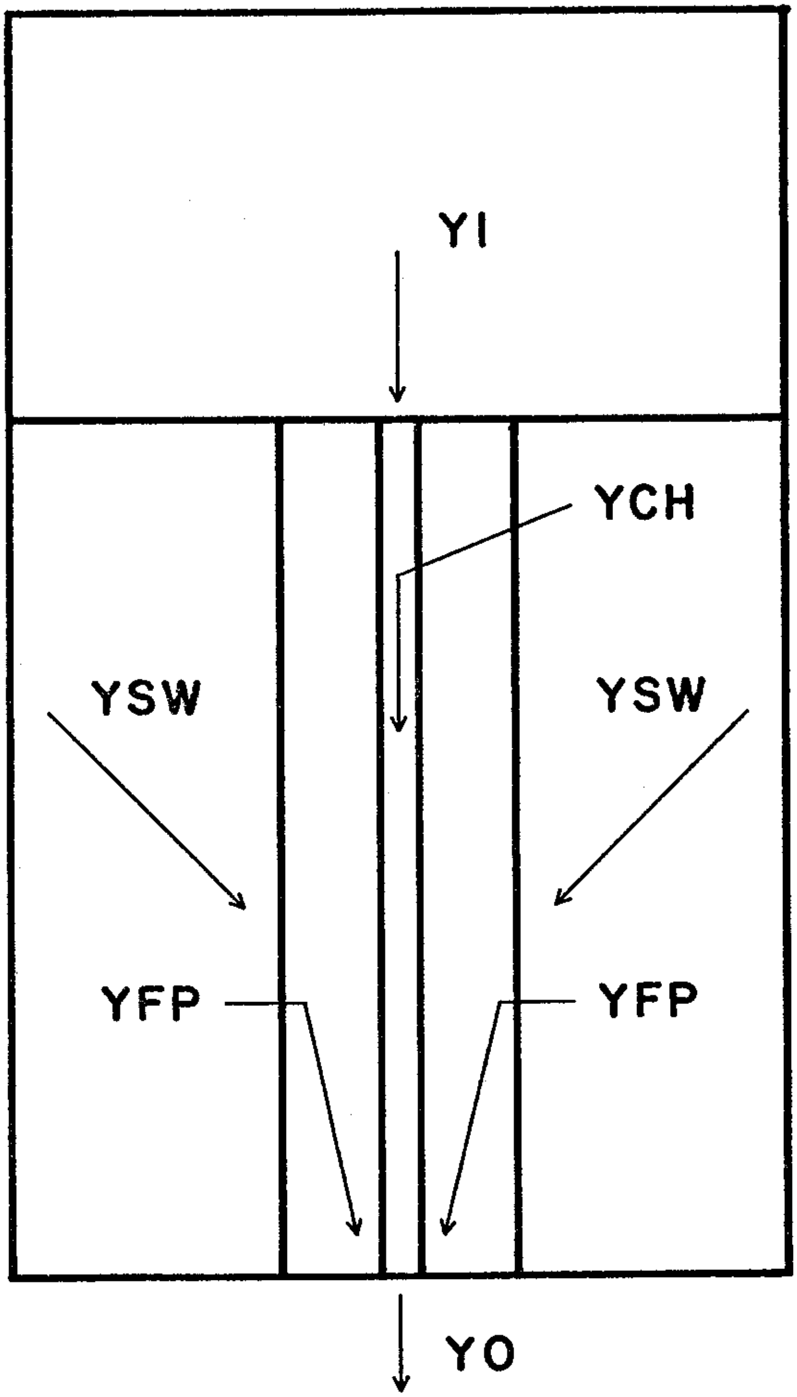

Figure 1. Schematic of a Routing Reach Showing Inflow, YI, Outflow, YO, and Contributions from Subwatershed, YSW, Floodplain, YFP, and Channel, YCH.

not required. Thus, only the inflow to the upstream end of the reach is routed. The new sediment-routing equation used to express these relationships is

$$YO = YFP + YCH + YSW + YI \sum_{i=1}^{M} \omega_i \, e^{-\beta (TT) \sqrt{d_i}} \tag{81}$$

where YO is the outflow sediment yield at the downstream end of the reach; YCH is the sediment yield from the channel within the reach; YSW is the sediment yield from the subwatershed contributing to the channel within the reach; YI is the inflow sediment yield at the upstream end of the reach; TT is the travel time through the reach, and M is the number of particle sizes used to define the particle-size distribution. To determine the routing parameter, β, the MUSLE factors K, C, P, and LS are assumed equal for all areas as in the original method. Canceling K, C, P, and LS and rearranging equation (81) to solve for β gives

$$\sum_{i=1}^{M} \omega_i \, e^{-\beta (TT) \sqrt{d_i}} = \frac{11.8(V\ q_p)_O^{0.56} - 2045(DA\ Q\ q_p)_{FP}^{0.56} - 2045(DA\ Q\ q_p)_{CH}^{0.56} - 11.8(V\ q_p)_{SW}^{0.56}}{11.8\ (V\ q_p)_I^{0.56}} \tag{82}$$

where subscripts are used to indicate runoff energy factors that apply to outflow, floodplain, channel, subwatershed, and inflow. Since travel time is constant for all particle sizes, it can be eliminated from equations (81) and (82) without changing the results obtained with equation (81). The constant travel time is simply combined with the constant β. β can be determined from equation (82) using the actual particle-size distribution and Newton's classical method for solving nonlinear equations. However, if the actual particle-size distribution is used in equation (82) to determine β, particle size will cancel when β is substituted into equation (81). Thus, particle size would have no effect on the amount of inflow sediment that is transported through the reach. Instead of using the actual particle-size distribution to determine β, a base (low value) of particle size is used. The base value of d (20 μ) is the mean particle size of Houston Black clay, a soil with an extremely fine particle-size distribution. Using one value of d also simplifies equation (82) to give a direct solution for β.

$$\beta = \frac{-\ln \dfrac{11.8(V\ q_p)_O^{0.56} - 2045(DA\ Q\ q_p)_{FP}^{0.56} - 2045(DA\ Q\ q_p)_{CH}^{0.56} - 11.8(V\ q_p)_{SW}^{0.56}}{11.8(V\ q_p)_I^{0.56}}}{4.47} \tag{83}$$

Once β is determined, it is substituted into the new routing equation (equation (81) without TT) to perform the routing.

$$YO = YFP + YCH + YSW + YI \sum_{i=1}^{M} \omega_i\, e^{-\beta \sqrt{d_i}} \tag{84}$$

As routing proceeds downstream, β and the inflow particle-size distribution must be determined for each reach. The outflow particle-size distribution (inflow to the downstream reach) is computed with the equation

$$\omega_{0i} = \frac{\omega_i \, (YFP) + \omega_i \, (YCH) + \omega_i \, (YSW) + \omega_i \, e^{-\beta \sqrt{d_i}} \, (YI)}{YO} \qquad (85)$$

$i = 1, M$

A simpler version of the new sediment-routing model is used to determine the outflow sediment particle-size distribution for each subwatershed. These distributions are used in determining enrichment ratios for phosphorus and nitrogen simulation and for routing sediment downstream. Upland sediment production can be predicted with equation (18) using the source runoff peak flowrate, instead of the outflow peak rate.

$$YU = 11.8 \, (V \, Q_p)^{0.56} \, (K) \, (C) \, (P) \, (LS) \qquad (86)$$

where YU is the upland sediment production in t, and Q_p is the source runoff peak rate in m^3/s. Q_p is determined by applying three point numerical differentiation to accumulated source runoff predicted with the retention function.

For most small upland watersheds, the channel and floodplain sediment contributions are relatively small. Thus, for the subwatersheds equation (84) is written

$$YO = YU \sum_{i=1}^{M} \omega_i \, e^{-\beta \sqrt{d_i}} \qquad (87)$$

Since K, C, P, LS, and V are equal for predicting sediment yield or upland sediment production on the same watershed, they cancel in equations (18) and (87) to give

$$\left(\frac{q_p}{Q_p}\right)^{0.56} = \sum_{i=1}^{M} \omega_i \, e^{-\beta \sqrt{d_i}} \qquad (88)$$

Using the base low value to represent the particle-size distribution as in equation (83) produces the equation for determining the routing coefficient, β.

$$\beta = \frac{-\ell n \left(\frac{q_p}{Q_p}\right)^{0.56}}{4.47} \qquad (89)$$

The particle-size distribution of the sediment at the watershed outlet is computed using the particle-size distribution of the soil and β from equation (89).

$$\omega_{0i} = \frac{\omega_i \, e^{-\beta \sqrt{d_i}}}{\left(\frac{q_p}{Q_p}\right)^{0.56}} \qquad (90)$$

Phosphorus Routing

The phosphorus-loading function (equation (27)) was attached to the sediment-routing model to develop the phosphorus-routing equation.

$$YP_0 = 0.001\ (c_p)\ (ER_p)\ (YI) \sum_{i=1}^{M} \omega_i\ e^{-\beta \sqrt{d_i}} \qquad (91)$$

where YP_0 is the routed outflow phosphorus yield; c_p is the concentration of phosphorus in the sediment inflowing to the upstream end of the routing reach; ER_p is the phosphorus enrichment ratio; YI is the sediment yield inflow; β is the sediment routing coefficient, and ω_i and d_i describe the particle-size distribution of the inflow sediment. The values of YI, c_p, and the particle-size distrbituion are obtained from the upstream routing reach. YI is the upstream reach YO, and the particle-size distribution of YO, computed with equation (85), becomes the inflow distribution.

The outflow phosphorus concentration in the sediment is computed with the equation

$$c_{p0} = \frac{c_p\ (YFP) + c_p\ (YCH) + c_p\ (YSW) + c_p\ e^{-\beta \sqrt{d_i}}\ (YI)}{YO} \qquad (92)$$

The resulting outflow phosphorus concentration is used as c_p in equation (91) for the downstream-reach routing.

The enrichment ratio is determined from the specific surface areas of the inflow sediment before and after routing. The specific surface of the inflow particle-size distribution before routing is computed with equation (30). After routing the specific surface can be determined with the equation

$$SSY_R = \frac{\sum_{i=1}^{M} SS_i \; \omega_i \; e^{-\beta \sqrt{d_i}}}{\sum_{i=1}^{M} \omega_i \; e^{-\beta \sqrt{d_i}}} \tag{93}$$

where SSY_R is the specific surface of the routed inflow; SS is the specific surface for particle size, i; and ω_i and d_i define the inflow particle-size distribution. The phosphorus-enrichment ratio is computed by dividing SSY_R by the specific surface of the inflow before routing.

$$ER_P = \frac{\sum_{i=1}^{M} SS_i \; \omega_i \; e^{-\beta \sqrt{d_i}}}{\sum_{i=1}^{M} \omega_i \; e^{-\beta \sqrt{d_i}} \; \sum_{i=1}^{M} \omega_i \; SS_i} \tag{94}$$

Nitrogen Routing

The organic nitrogen-routing model has the same structure as the phosphorus-routing model. Equation (39), the nitrogen-loading function, was attached to the sediment-routing model to develop the nitrogen-routing equation

$$YON_O = 0.001 \; (c_{ON}) \; (ER_N) \; (YI) \; \sum_{i=1}^{M} \omega_i \; e^{-\beta \sqrt{d_i}} \tag{95}$$

where YON_O is the routed outflow organic nitrogen yield; c_{ON} is the concentration of organic nitrogen in the inflow sediment, and ER_N is the nitrogen-enrichment ratio.

Outflow concentration is computed with equation (92), using organic nitrogen concentration to replace phosphorus concentration. The enrichment ratio is determined from the percent of 1-μ sediment contained in the inflow sediment before and after routing. Before routing the percent 1 μ, ζ_y, is 100 × ω_1. After routing the percent 1 μ is

$$\zeta_{YR} = \frac{100\ \omega_1\ e^{-\beta}}{\sum_{i=1}^{M} \omega_i\ e^{-\beta \sqrt{d_i}}} \qquad (96)$$

where ζ_{YR} is the percent 1-μ sediment contained in the routed sediment. The nitrogen enrichment ratio is determined by the equation

$$ER_N = \frac{\zeta_{YR}}{\zeta_y} \qquad (97)$$

Or substituting (96) for ζ_{YR} and 100 ω_1 for ζ_y gives

$$ER_N = \frac{e^{-\beta}}{\sum_{i=1}^{M} \omega_i\ e^{-\beta \sqrt{d_i}}} \qquad (98)$$

Nitrate that is carried by runoff water is considered a conservative material for the duration of an individual flood. Actually, a small amount of nitrate is probably lost as water flows downstream. However, since nitrate is not attached to sediment (subject to deposition), the loss should be quite small. Thus, nitrate routing is simply a matter of determining the outflow concentration of nitrate based on the concentrations and amounts of flow of the nitrate sources.

$$c_{NO3O} = \frac{c_{NO3}\ V_{FP} + c_{NO3}\ V_{CH} + c_{NO3}\ V_{SW} + c_{NO3}\ V_I}{V_O} \qquad (99)$$

where c_{NO3O} is the nitrate concentration of the outflow, and V is the volume of flow from sources denoted by subscripts (floodplain, channel, subwatershed, inflow, and outflow).

SPNM

SPNM is a problem-oriented computer language for modeling sediment, phosphorus, and nitrogen yields from large agricultural watersheds. The language is called SPNM from the words "sediment-phosphorus-nitrogen model." It consists of a main program and 12 subroutines written in FORTRAN IV, but it can be used by hydrologists and environmental engineers with little knowledge of computer programming. The language provides 11 commands that can be used in any sequence for application to any watershed.

SPNM was designed to predict sediment, phosphorus, and nitrogen yields for individual storms on small watersheds and to route these yields through streams and valleys of large watersheds. The language should be useful in planning water resources projects and in research. Since the SPNM commands can be used in any sequence, the language will be convenient for water resources planning because all watersheds present different routing problems. In research, SPNM can be used to determine the effects of various inputs on sediment, phosphorus, and nitrogen yields. Probably, a more important feature, however, is the convenience of testing and refining new submodels. Submodels can be modified or new ones can be added quite easily, because each submodel is simply a FORTRAN subroutine. When a new submodel is added, inputs are supplied by the existing SPNM commands. As an example, if new routing models were tested, source inputs would be available because they are predicted by SPNM.

Operation of SPNM

For normal operation, the main program and 12 subroutines written in FORTRAN are not changed. However, the user writes a new SPNM program for each problem. Changes in the FORTRAN program require experience with FORTRAN programming and are generally not necessary, except in research to improve submodels. The SPNM card deck is set up in the following order:

(1) Main program.

(2) Subroutine.

(3) Command table.

(4) User's SPNM program consisting of commands and appropriate data.

The main program reads the command table and then calls the HONDO subroutine to read a SPNM program card. Subroutine HONDO determines the command name and number by comparing the first 20 columns of the SPNM card with the command table. Then, HONDO determines the value of individual data items contained in columns 21 through 80 of the SPNM card. The data are placed in an array and returned to the main program. Based on the command number, the main program calls the proper subroutine to do the desired calculations. When the calculations are complete, control is returned to the main program and HONDO is called again to read the next SPNM card.

Definitions of Commands

The 11 SPNM commands and the required inputs are shown in Table 1. A brief description of each command follows.

TABLE 1. SPNM Commands and Required Inputs.

Command	Required Input
START	c_R = 0.4 ppm; ρ_s = 287,400 kg/ha; particle-size distribution: 1 μ = 28%, 3 μ = 3%, 6 μ = 2%, 12 μ = 4%, 23.5 μ = 2%, 46.5 μ = 4%, 93.5 μ = 5%, 187.5 μ = 8%, 375 μ = 44%
SED YLD	ID = 3, K = 0.34, C = 0.25, P = 0.1, LS = 0.28, Q = 3.02 cm, q_p = 10.4 m^3/s, DA = 4.17 km^2, Q_p = 33 m^3/s, MM = 90%
PHOS YLD	ID = 3, c_p = 600 ppm
NIT YLD	ID = 3, c_{ON} = 1,000 ppm, c_{NO3} = 3 ppm
ADD SED	ID = 1, ID1 = 3, ID2 = 2
ADD PHOS	ID = 1, ID1 = 3, ID2 = 2
ADD NIT	ID = 1, ID1 = 3, ID2 = 2
ROUTE SED	ID = 5, IDH = 1, q_{pO} = 12.6 m^3/s, IDSW = 2, IDFP = 4, q_{pI} = 10.4 m^3/s, q_{pSW} = 5 m^3/s.
ROUTE PHOS	ID = 5
ROUTE NIT	ID = 5
FINISH	

START. This is the first command for any watershed. Data associated with START are the average concentrations of nitrate in the rainfall; the weight of the top 2.5 cm of soil in kg/ha, and the particle-size distribution of the soil for the first subwatershed. Up to nine particle sizes can be used to define the particle-size distribution. If the soils of the subwatersheds change as routing proceeds downstream, a new START command is used to specify the soil weight and the particle-size distribution. If all subwatersheds have similar soils, only one START command is needed.

SED YLD. This command is used to predict sediment yield from subwatersheds, channels, and floodplains. Also the outflow particle-size distribution is determined. Sediment yield is predicted with equation (18) and the particle-size distribution is computed with equation (90). Required inputs are the ID number, values of the MUSLE factors (K, C, P, and LS determined with equation (26)), the runoff volume in cm, the peak outflow rate in m^3/s, the watershed area in km^2, the source runoff peak rate in m^3/s, and the montmorillonite content of the clay in percent. The ID number is the storage location of the predicted sediment yield. Storage location numbers are necessary because output from certain commands is used as input to other commands. In example, in the ROUTE SED command the ID of the inflow sediment yield specifies where the inflow information is stored in the computer. Numbers 1-6 can be used as ID numbers. Thus, information from a total of six subwatersheds, floodplains, and routing reaches can be stored at one time. However, no more than six ID's are ever needed at one time because SPNM programs begin at the upstream end of a watershed and proceed downstream through one reach at a time. When a storage location number is used, whatever was previously stored in that location is lost. The user should be sure that he is finished with the information stored in a particular location before he reuses that number.

PHOS YLD. This command is used to predict the phosphorus yield from subwatersheds, channels, and floodplains. Also the concentration of phosphorus in the outflow sediment is determined. Phosphorus yield is predicted with equation (27), and the outflow concentration is the product of the soil concentration and the enrichment ratio. The only inputs required are the ID number and the concentration of phosphorus in the soil. It is essential that the ID number is the same as the one used in SED YLD for the area, because SED YLD supplies information to PHOS YLD.

NIT YLD. This command operates like PHOS YLD, except it predicts organic nitrogen and nitrate. Equation (39) is used to predict organic nitrogen yield, and equation (51) is used to predict nitrate yield. The outflow concentration of organic nitrogen in the sediment is the product of the soil concentration and the enrichment ratio. Nitrate concentration in the water is the nitrate yield divided by the runoff volume. Input requirements are storage location number, concentration of organic nitrogen in the soil, and the nitrate concentration in the soil. Like PHOS YLD, the ID number must be the same as the one used with SED YLD.

ADD SED. The ADD SED command is used to add the sediment yield from two sources. It also adds the runoff volumes and calculates the particle-size distribution of the combined sediment yields. The percent of a particular particle size contained in the combined sediment yields is the total weight of that size divided by the total sediment yield.

Storage-location numbers of the two yields to be added and of the total yield are the only input data.

ADD PHOS. This command is similar to the ADD SED command. It adds phosphorus yields from two sources and calculates the phosphorus concentration contained in the total sediment yield. Storage-location numbers of the two yields to be added and of the total yield are the only input data.

ADD NIT. This command performs the same function as ADD PHOS, except that it adds organic nitrogen yields from two sources and calculates the organic nitrogen concentration of the total sediment yield. ADD NIT also adds the nitrate yields from two sources and calculates the nitrate concentration of the total runoff. Inputs required are the same as required with ADD PHOS (storage-location numbers of the two yields to be added and of the total yield).

ROUTE SED. This command routes the sediment yield through a routing reach to obtain the outflow yield and particle-size distribution. Equation (84) performs the routing, and equation (85) calculates the outflow particle-size distribution. Necessary inputs are the storage-location numbers of the outflow and inflow sediment yields, the peak outflow rate in m^3/s, the storage-location numbers of the subwatershed and floodplain contributing to the downstream end of the reach, the peak inflow rate in m^3/s, and the peak outflow rate of the subwatershed in m^3/s.

ROUTE PHOS. This command is used to route the phosphorus yield through a routing reach and to calculate the outflow concentration of phosphorus contained in the sediment. Equation (91) is used for routing phosphorus, and equation (92) is used to determine the outflow concentration. The only input required is the storage-location number of the outflow phosphorus yield. This ID must be the same as the ID of the outflow sediment yield from ROUTE SED.

ROUTE NIT. This command routes organic nitrogen yield through routing reaches using equation (95). It also determines outflow concentration of organic nitrogen in the sediment by replacing c_p with c_{ON} in equation (92). The nitrate concentration of the outflow water is computed with equation (99). Like ROUTE PHOS, the only input is the storage-location number and it must be same as the ROUTE SED outflow ID.

FINISH. This command ends the program when all computations are completed. There is no input data.

SPNM Program Format

SPNM commands are written in the first 20 columns of the data card, and columns 21 through 79 are used for numeric data and keywords. Column 80 is reserved for a page change code (an asterisk in column 80 causes the card to be printed on a new page). Continuation cards are allowed when 59 characters are insufficient to express the data.

The data can be written in any format, but at least one blank space or a comma must be placed between data items. A decimal is required for numbers containing fractions but not for whole numbers. Keywords can be written with the data to describe individual data items. Comment cards may be used at any point in a SPNM program by punching an asterisk in column 1 and the comment in columns 2 through 79. Table 2 shows an example of a SPNM program for routing through the reach shown in figure 2. Comment cards and keywords are used liberally to aid the reader in

TABLE 2. Example SPNM Program for Routing Sediment, Phosphorus, and Nitrogen Through One Reach.

```
 * LITTLE ELM CREEK WATERSHED REACH 1.  ONE YEAR FREQUENCY STORM.                 *

 * THE FIRST COMMAND FOR ALL SPNM PROGRAMS IS START.

 *

 START                NITRATE CONC IN RAIN = .4 PPM    TOP STOR WT = 291900 KG/HA
                      PARTICLE SIZE DIST (DIAMETER (U), FRACTION)
                      1 .2763   3 .0305   6 .0163   12 .0368   23.5 .0224
                      46.5 .0406   93.5 .048   187.5 .0808   375 .4484
                      THIS COMPLETES INPUT FOR START, OUTPUT IS
     MEAN PARTICLE SIZE = 191.110 U

 *

 * THE NEXT STEP IS TO PREDICT THE SEDIMENT, PHOSPHORUS AND NITROGEN YIELDS FOR

 * THE SUBWATERSHED AT THE UPSTREAM END OF LITTLE ELM CR

 *

 SED YLD              ID = 3   K = .34   C = .11   P = .4   LS = .54   Q = 3.02CM
                      QP(OUTFLOW) = 5 CMS   DA = .75 SQ KM   QP(SOURCE) = 5.95CMS
                      MONTMORILLONITE CLAY = 90 %   END OF INPUT   OUTPUT FOR
                      VS-19 IS
     MEAN PARTICLE SIZE = 191.110 U
     OUTFLOW PARTICLE SIZE DISTRIBUTION
0.3391   0.0368   0.0194   0.0428   0.0253   0.0439   0.0488   0.0752   0.3688
     MEAN PARTICLE SIZE = 160.665 U
     SEDIMENT YIELD =        64.5 T      =    0.860 T/HA
     DELIVERY RATIO =  0.797
     ROUTE COEF =     0.217920E-01
```

Table 2. (continued)

```
PHOS YLD              ID=3   PHOS CONC = 600 PPM                                      *
                      VS-19 OUTPUT
    TOTAL PHOSPHORUS YIELD =       47.1 KG      =       0.628 KG/HA
    DELIVERY RATIO =  0.971
    OUTFLOW P CONC
         SEDIMENT =    730.5 PPM
         WATER =   2.080 PPM
    ENRICHMENT RATIO =   1.218



NIT YLD               ID = 3   ORG N CONC = 1177 PPM   NITRATE CONC = 3.1 PPM
                      VS-19 OUTPUT IS
    ORG NITROGEN YIELD =       93.1 KG      =       1.242    KG/HA
    DELIVERY RATIO =  0.978
    OUTFLOW ORG N CONC
         SEDIMENT =   1444.4 PPM
         WATER =   4.112 PPM
    ENRICHMENT RATIO =   1.227
    NO3 YIELD =       52.8 KG      =       0.704 KG/HA
    OUTFLOW CNO3 =   2.331 PPM
```

Table 2. (continued)

```
  * NEXT PREDICT SED. PHOS. AND NIT YLDS FOR THE SUBWATERSHED BETWEEN VS-19 AND  *

  * VS-18

  *

  SED YLD                ID = 1    K = .32    C = .20    P = .24    LS = .53
                         Q = 3.02 CM    QPO = 10.4 CMS    DA = 4.17 SQ KM
                         QPS = 34 CMS    MM = 90 %  OUTPUT VS-18
      MEAN PARTICLE SIZE = 191.110 U
      OUTFLOW PARTICLE SIZE DISTRIBUTION
0.6470   0.0641   0.0308   0.0598   0.0296   0.0401   0.0310   0.0288   0.0688
      MEAN PARTICLE SIZE =  38.399 U
      SEDIMENT YIELD =       256.0 T      =    0.614 T/HA
      DELIVERY RATIO =  0.368
      ROUTE COEF =     0.148401E 00



  PHOS YLD               ID = 1   PHOS CONC = 600 PPM
                         VS-18 OUTPUT
      TOTAL PHOSPHORUS YIELD =      350.6 KG      =       0.841 KG/HA
      DELIVERY RATIO =  0.840
      OUTFLOW P CONC
           SEDIMENT =   1369.7 PPM
           WATER =   2.784 PPM
      ENRICHMENT RATIO =   2.283
```

Table 2. (continued)

```
  NIT YLD                ID = 1   ORG N CONC = 1177 PPM   NITRATE CONC = 3.1 PPM    *
                         VS-18 OUTPUT
      ORG NITROGEN YIELD =      705.5 KG      =       1.692    KG/HA
      DELIVERY RATIO =  0.862
      OUTFLOW ORG N CONC
           SEDIMENT =   2756.2 PPM
           WATER =   5.602 PPM
      ENRICHMENT RATIO =   2.342
      NO3 YIELD =       293.5 KG      =      0.704 KG/HA
      OUTFLOW CNO3 =   2.331 PPM



  * NEXT PREDICT SED, PHOS, AND NIT YLDS FOR THE FLOODPLAIN AND CHANNEL BETWEEN

  * VS-19 AND VS-18.

  *

  SED YLD                ID = 4   K = .34   C = .003   P = 1   LS = .11
                         Q = 3.02 CM   QPO = 2.29 CMS   DA = .013 SQ KM
                         QPS = 5 CMS   MM = 90 %   OUTPUT FP(VS-19 TO VS-18)
      MEAN PARTICLE SIZE = 191.110 U
      OUTFLOW PARTICLE SIZE DISTRIBUTION
0.5478   0.0563   0.0280   0.0573   0.0305   0.0456   0.0408   0.0463   0.1475
      MEAN PARTICLE SIZE =  72.190 U
      SEDIMENT YIELD =        0.1 T      =   0.046 T/HA
      DELIVERY RATIO =  0.457
      ROUTE COEF =     0.978291E-01
```

Table 2. (continued)

```
PHOS YLD              ID = 4    PHOS CONC = 600 PPM                                  *
                      OUTPUT FP(VS-19 TO VS-18)
    TOTAL PHOSPHORUS YIELD =          0.1 KG      =      0.053 KG/HA
    DELIVERY RATIO =  0.887
    OUTFLOW P CONC
         SEDIMENT =   1164.0 PPM
         WATER =   0.177 PPM
    ENRICHMENT RATIO =   1.940


NIT YLD               ID = 4    ORG N CONC = 1177 PPM   NITRATE CONC = 3.1 PPM
                      OUTPUT FP(VS-19 TO VS-18)
    ORG NITROGEN YIELD =          0.1 KG      =      0.107    KG/HA
    DELIVERY RATIO =  0.907
    OUTFLOW ORG N CONC
         SEDIMENT =   2333.6 PPM
         WATER =   0.355 PPM
    ENRICHMENT RATIO =   1.983
    NO3·YIELD =         0.9 KG      =      0.704 KG/HA
    OUTFLOW CNO3 =   2.331 PPM
```

Table 2. (continued)

```
  * THE OUTPUTS FROM VS-19 MUST BE ROUTED THRU REACH 1 (VS-19 TO VS-18).          *

  *

  ROUTE SED            ID = 5   INFLOW ID = 3   QPO = 12.6 CMS   SUBWS ID = 1
                       FP ID = 4   QPI = 5 CMS   QPSW = 10.4 CMS
                       ROUTED SED REACH 1
      OUTFLOW PARTICLE SIZE DISTRIBUTION
0.5339   0.0551   0.0276   0.0567   0.0304   0.0459   0.0418   0.0486   0.1600
      MEAN PARTICLE SIZE =  77.430 U
      SEDIMENT YIELD =        38.2 T      =   0.509 T/HA
      DELIVERY RATIO =  0.592
      ROUTE COEF =     0.701724E-01



  ROUTE PHOS           ID = 5   ROUTED PHOS REACH 1
      TOTAL PHOPHORUS YIELD =       43.3 KG      =      0.578 KG/HA
      DELIVERY RATIO =  0.920
      OUTFLOW P CONC
           SEDIMENT =   1135.1 PPM
           WATER =   1.913 PPM
      ENRICHMENT RATIO =   1.554
```

Table 2. (continued)

```
 ROUTE NIT             ID = 5   ROUTED NIT REACH 1                                  *
     ORG NITROGEN YIELD =      86.8 KG     =      1.158    KG/HA
     DELIVERY RATIO =  0.932
     OUTFLOW ORG N CONC
          SEDIMENT =   2274.3PPM
          WATER =   3.834PPM
     ENRICHMENT RATIO =   1.575
     NO3 YIELD =        52.8 KG      =      0.704 KG/HA
     OUTFLOW CNO3 =   2.331 PPM



 * TO OBTAIN THE TOTAL YIELDS FROM REACH 1, YIELDS FROM THE SUBWS AND THE

 * CHANNEL AND FLOODPLAIN MUST BE ADDED TO THE ROUTED YIELD.  FIRST ADD THE

 * FLOODPLAIN AND THE OUTPUT DATA FROM VS-18

 *

 ADD SED               ID = 2   IDFP = 4   IDSWS = 1(VS-18 OUTPUT)
                       OUTPUT(FP + SWS 18)
     OUTFLOW PARTICLE SIZE DISTRIBUTION
0.6470   0.0641   0.0308   0.0598   0.0296   0.0401   0.0311   0.0288   0.0688
     MEAN PARTICLE SIZE =  38.407 U
     DRAINAGE AREA =       4.183 SQ KM
     RUNOFF VOLUME =   3.020 CM
     SEDIMENT YIELD =      256.0 T     =   0.612 T/HA
```

Table 2. (continued)

```
  ADD PHOS               ID = 2   IDFP = 4   IDSWS = 1    OUTPUT(FP+SWS18)          *
      TOTAL PHOSPHORUS YIELD =       350.7 KG     =      0.838 KG/HA
      PHOSPHORUS CONC
           SEDIMENT =   1369.7 PPM
           WATER =  2.7758 PPM



  ADD NIT                ID = 2   IDFP = 4   IDSWS = 1    OUTPUT(FP+SWS18)
      ORG NITROGEN YIELD =      705.6 KG     =      1.687 KG/HA
      ORG N CONC
           SEDIMENT =   2756.1 PPM
           WATER =   5.586 PPM
      NO3 YIELD =       294.4 KG      =      0.704 KG/HA
      OUTFLOW CNO3 =    2.331 PPM



  * FINALLY ADD THE TOTAL (FP + SWS 18) AND THE ROUTED AMOUNTS TO OBTAIN THE

  * TOTAL YIELDS OUTFLOWING FROM REACH 1.

  ADD SED                ID = 1   IDRT = 5   IDAD = 2     SED YLD VS-18
      OUTFLOW PARTICLE SIZE DISTRIBUTION
0.6323   0.0629   0.0304   0.0594   0.0297   0.0408   0.0324   0.0313   0.0807
      MEAN PARTICLE SIZE =  43.471 U
      DRAINAGE AREA =      4.933 SQ KM
      RUNOFF VOLUME =   3.020 CM
      SEDIMENT YIELD =       294.2 T     =   0.596 T/HA
```

Table 2. (continued)

```
ADD PHOS            ID = 1   IDRT = 5   IDAD = 2                                *
                    PHOS YLD VS-18
    TOTAL PHOSPHORUS YIELD =      394.0 KG     =      0.799 KG/HA
    PHOSPHORUS CONC
         SEDIMENT =   1339.2 PPM
         WATER =  2.6447 PPM



ADD NIT             ID = 1   IDRT = 5   IDAD = 2
                    NIT YLD VS-18
    ORG NITROGEN YIELD =      792.4 KG     =      1.606 KG/HA
    ORG N CONC
         SEDIMENT =   2693.6 PPM
         WATER =   5.319 PPM
    NO3 YIELD =      347.2 KG     =      0.704 KG/HA
    OUTFLOW CNO3 =   2.331 PPM



* THIS COMPLETES THE ROUTING THROUGH THE FIRST REACH.  THE YIELDS FROM VS-18

* BECOME INFLOW VALUES FOR REACH 2, AND ROUTING CONTINUES DOWNSTREAM.

* THE LAST COMMAND FOR ANY SPMN PROGRAM IS FINISH.

FINISH
```

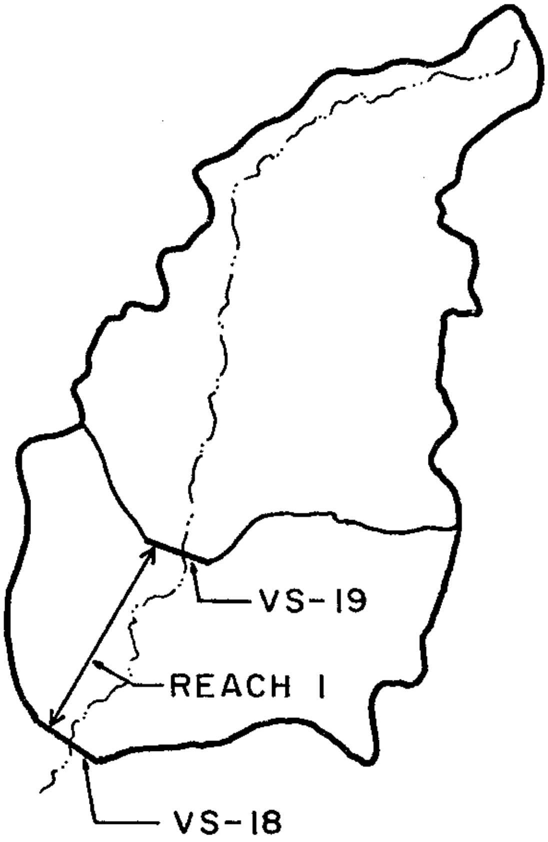

Figure 2. Routing Reach Used to Demonstrate SPNM.

following the process. This detailed description is not necessary after a user becomes familiar with SPNM.

MODEL FOR DETERMINING OPTIMAL OPERATING POLICY FOR THE WATERSHED

A model based on decision analysis and linear programming (LP) was developed to determine optimal operating policies for large agricultural watersheds. The objective of the model is to maximize a multiattribute utility function within constraints imposed by sediment, phosphorus, and nitrogen yield; watershed area; and cropping system. Strategies that are acceptable to the decision makers (landowners and land operators) are selected for each land capability class within each subwatershed. A strategy specifies the crop to be grown on the area, the fertilizer application rate, and the type of conservation system. The utility of each strategy is determined by analyzing attributes that are important to the decision makers. Utility values allow the combination of various attributes into a single function. For example, money, weight, risk, and environmental impact could be combined in a multiattribute utility function. This approach has considerable advantage over traditional approaches that consider only monetary benefits and costs.

Since there is no direct means of solving constrained problems with decision analysis, linear programming is used to maximize the constrained objective function. Linear programming is convenient, has a user-oriented program available (MPS-360) [IBM, 1971], and allows more than one strategy per land class in the optimal solution. It is important to allow more than one strategy per land class because crops are normally rotated. The selection of one strategy per land class would suggest the continuous use of a single crop on that area.

Inputs for the optimization model are provided by the model for simulating daily sediment, phosphorus, and nitrogen yields and by SPNM.

Long-term average annual values are used in the optimization model because crop production and costs are determined on an annual basis. Also annual values are more meaningful in expressing sediment, phosphorus, and nitrogen constraints because the effects of these pollutants are chronic rather than accute.

Decision Analysis

Decision analysis, sometimes called Bayesian decision theory, is a systematic solution procedure for solving complex decision problems under uncertainty. Preferences for consequences are numerically scaled in terms of utility values, and judgments about uncertainties are numerically scaled in terms of probabilities. Solutions to large problems are simplified by considering individual strategies. Strategies are evaluated by calculating their expected utility value. The utility value may be a function of one or more attributes. A utility function is developed for each attribute to express the decision maker's preferences on a scale from zero to one. Zero is the worst consequence of the alternatives being considered, and one is the most desirable. Each utility function is assigned a weighting factor to express the relative importance of the attribute. The overall utility of a particular strategy is determined by summing the products of the individual attribute utilities and their weighting factors.

Strategies

The landowners and operators in each subwatershed are the decision makers for that particular area. They must select a range of strategies for each land class within the subwatershed. The analyst (modeler) should encourage the decision makers to include at least one strategy for each land class that has low sediment, phosphorus, and nitrogen yield potential. Otherwise, it may not be possible to obtain a feasible solution to the LP problem, because the constraints would be violated with any combination of strategies. Also, the decision makers should include strategies that produce high gross income because gross income is quite important in determining the maximum utility for the watershed. Some decision makers may be reluctant to include strategies that specify high fertilization rates because of the costs involved. However, high fertilization rates increase gross income and should be included. Generally, the range of strategies should be wide enough to include the maximum utility and a low pollutant potential. The fewer strategies the better because additional strategies increase computing time. There is no point in including strategies that are obviously poor choices, like growing grain sorghum on class IV land. Table 3 shows an example of strategies that are typical choices of decision makers in the Texas Blackland area. Normally there are 50 or more strategies for each subwatershed. Only a few of these are shown in Table 3 to save space.

Utility Functions

The decision makers in each subwatershed must determine a multiattribute utility function for their particular subwatershed. The first step is to decide which attributes are important to the decision makers. Since there is usually more than one decision maker on each subwatershed, all decisions leading to the development of the utility function are group decisions. Thus, the analyst must persuade the group to compromise in making these decisions. Raiffa [1970] suggested Pareto-

TABLE 3. Example Strategies Selected by the Decision Makers for a Subwatershed.

Land Class	Strategy No.	Crop	Fertilizer Applied (kg/ha)		Conservation Practice
			Nitrogen	Phosphorus	
I	1	Grain sorghum	135	45	Terraces and waterways
	2	" "	90	45	" " "
	3	" "	135	45	None
	4	" "	90	45	"
	5	Cotton	90	45	Terraces and waterways
	6	"	45	45	" " "
	7	Wheat	135	45	None
	8	"	90	45	"
	9	Pasture	150	45	None
	10	"	45	45	"
II	1	Grain sorghum	135	45	Terraces and waterways
	2	" "	90	45	" " "
	3	" "	135	45	Contouring
	4	" "	90	45	"
	5	Cotton	90	45	Terraces and waterways
	6	"	45	45	" " "
	7	Wheat	135	45	None
	8	"	90	45	"
	9	Pasture	135	45	None
	10	"	45	45	"
III	1	Grain sorghum	90	45	Terraces and waterways
	2	" "	90	45	Contouring
	3	Wheat	90	45	None
	4	"	45	45	"
	5	Pasture	90	45	None
	6	"	90	45	"

TABLE 3. (continued)

Land Class	Strategy No.	Crop	Fertilizer Applied (kg/ha) Nitrogen	Fertilizer Applied (kg/ha) Phosphorus	Conservation Practice
IV	1	Wheat	90	45	Terraces and waterways
	2	"	45	45	None
	3	Pasture	90	45	None
	4	"	45	45	"

optimality as an aid in arriving at group decisions. He described a joint action as Pareto-optimal if there does not exist an alternative action that is at least as acceptable to all and definitely preferred by some.

Attributes selected for the optimization model include gross income, production cost, dependability, and weed, insect, and disease control. The decision makers in each subwatershed may choose to omit some of these attributes and add others. The next step is to assign weighting factors to the attributes, according to their importance to the decision makers. Values of the weighting factors can be determined by assigning a value of one to the most important attribute and fractions to the other attributes expressing their relative importance. Shih and Dean [1973] described the application of Keeney's additive utility function [Keeney, 1969].

$$u(\underline{v}) = \sum_{i=1}^{M} \delta_i \, u_i \, (v_i) \tag{100}$$

where $u(\underline{v})$ is the utility for the attribute vector $\underline{v}$; δ_i is the weighting factor for attribute v_i, and u_i is the utility for attribute v_i. Keeney's additive utility function assumes mutual utility independence of the attributes. Mutual utility independence means that preferences depend solely on the marginal distribution functions and not on the joint distribution function. The weighting factors in equation (100) must be scaled so that

$$0 \leq \delta_i \leq 1 \quad \text{and} \quad \sum_{i=1}^{M} \delta_i = 1 \tag{101}$$

Thus, the weighting factors obtained by assigning one to the most important attribute and fractions to the others must be scaled to conform with equation (101). Scaling is accomplished with the equation

$$\delta_i = \frac{w_i}{\sum_{i=1}^{M} w_i} \qquad (102)$$

where w_i is the weight assigned to attribute, i.

The next step is to develop utility functions for each attribute. Many decision makers tend to be averse to risk. This aversion to risk influences the shape of their utility function. For example, consider net income. If the decision makers are averse to risk, they might feel that it is very important to break even. Thus, the point of zero net income would have a fairly high utility value, and the curve would be similar to the one shown in figure 3. If the decision makers are not risk averse, their utility functions may be linear.

About five points are adequate to define most utility functions. Two of these points are known (the worst consequence is zero and the most desirable, one). To obtain the value of consequence that has a utility of 0.5, the analyst proposes a gamble to the decision makers. He says what value of the attribute would you exchange for a 50-50 chance of receiving the most desirable or the least desirable consequence. If the decision makers are risk-averse as in figure 3, they would be just as willing to have no profit as to gamble with equal probabilities of obtaining $500 or losing $100. Thus, the 0.5 utility has a value of $0. To get the 0.75 utility value, the analyst asks what value of the attribute would you exchange for a 50-50 chance at $0 or

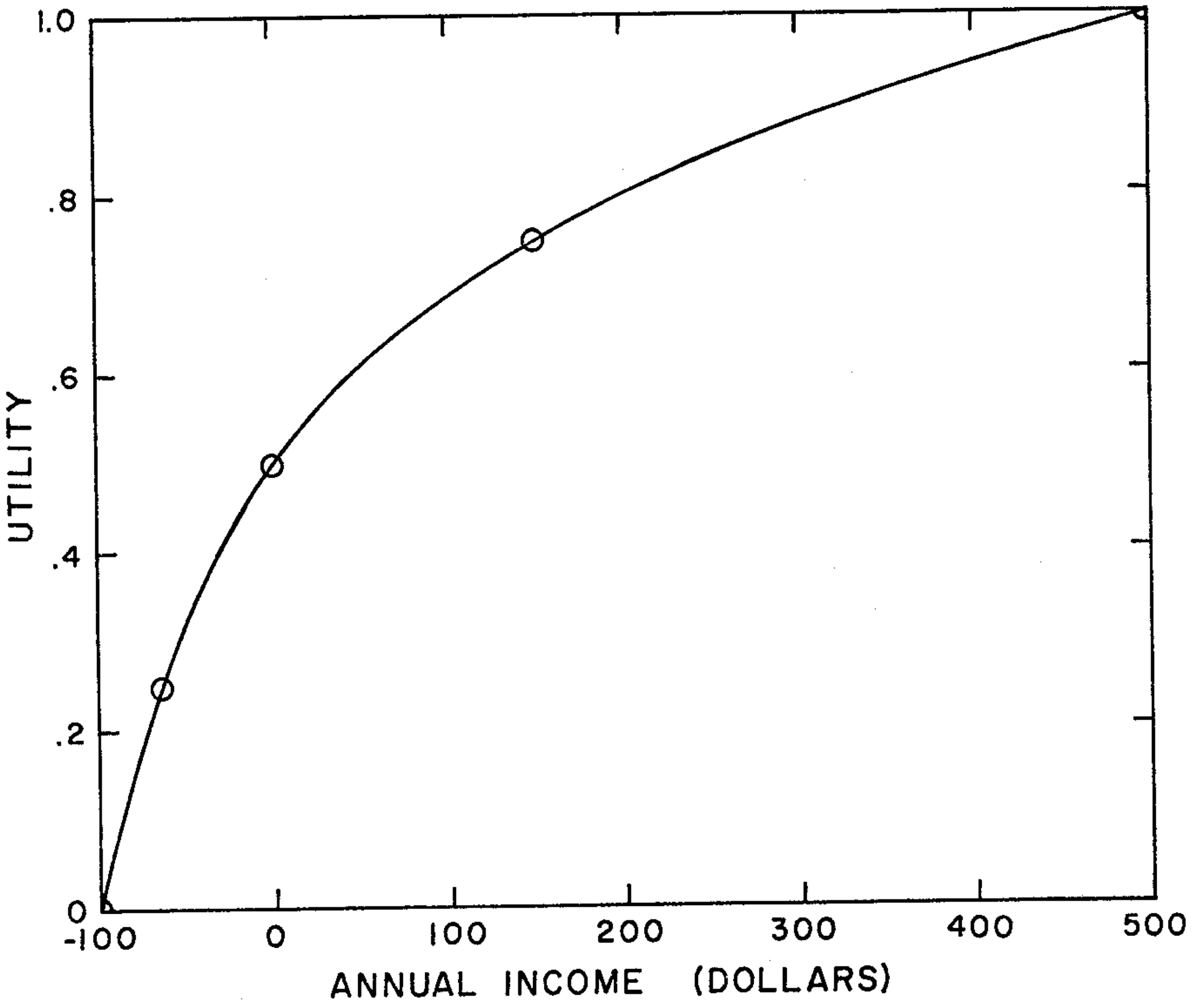

Figure 3. Net Income Utility Function for Risk Averse Decision Makers.

$500. The decision makers answer $150. Thus, the value of the 0.75 utility, $150, is plotted and the same procedure is repeated for the 0.25 utility. The analyst then passes a smooth curve through the points.

The utility function concerned with monetary values was used as an example because it is easier to relate to money than other more qualitative attributes, like disease control. However, these qualitative attributes must be assigned numerical values to develop the utility function. For example, disease control could be rated on the basis of percent crop loss caused by plant diseases. The same technique that was used to determine the relationship between monetary values and utility is used to relate percent crop loss to utility. A utility of one is assigned to the crop with the lowest percent loss, and a zero utility is assigned to the crop with the highest percent loss. The percent loss for the 0.5 utility is determined by proposing a gamble to the decision makers, similar to the gamble used in the monetary example. The analyst asks the decision makers what percent crop loss they would be willing to accept instead of taking a 50-50 chance at obtaining the smallest or the greatest loss. Assuming the decision makers are risk-averse, as they were in the monetary example, they might be willing to accept a 30% crop loss to avoid a 50% chance at a 50% loss. Utilities of 0.25 and 0.75 are related to percent crop loss similarly and plotted to give the disease control utility function shown in figure 4.

Utility functions for other attributes are developed using the same technique. The overall utility function for the decision makers is computed using equation (100).

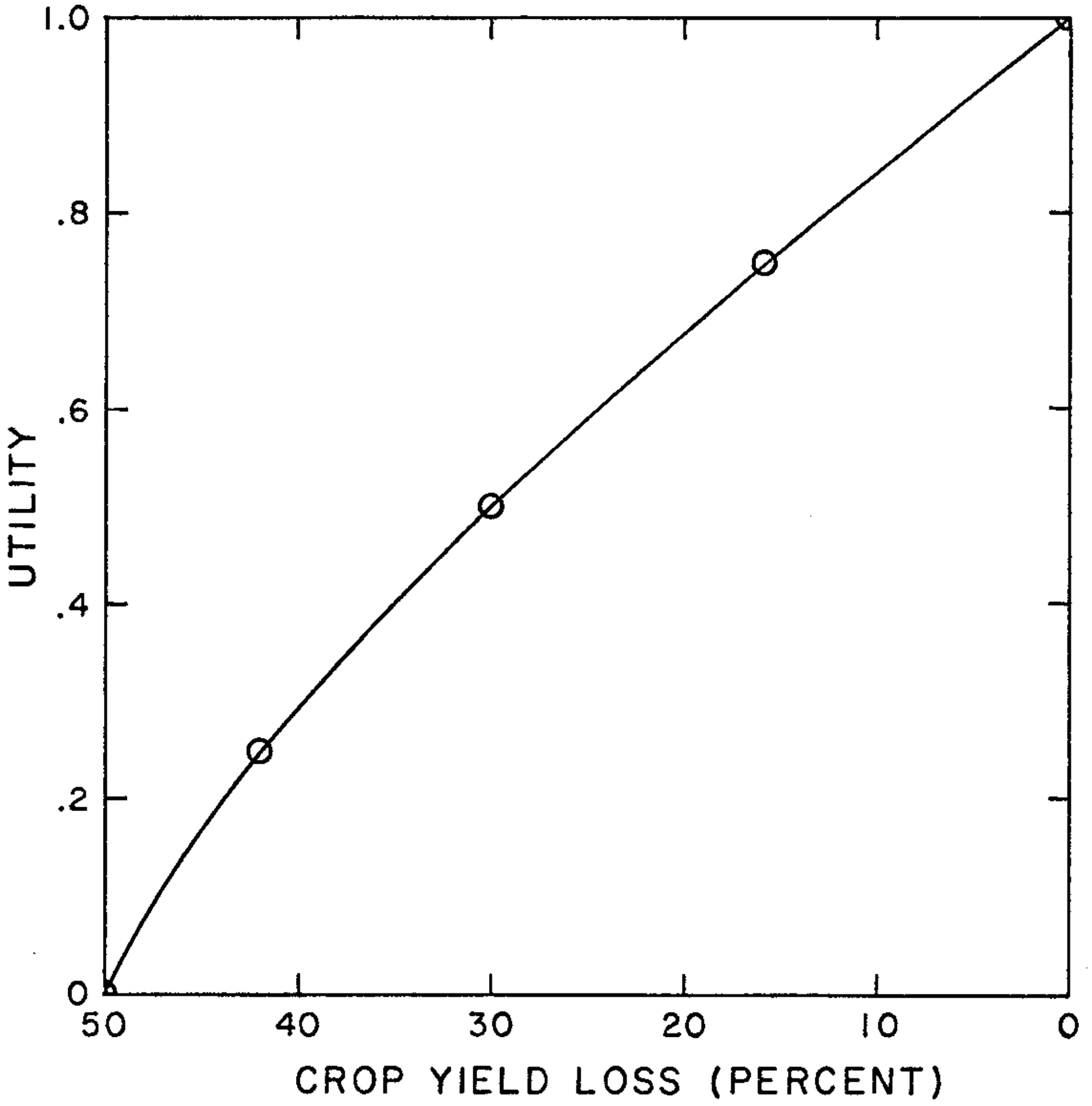

Figure 4. Crop Disease Control Utility Function of Risk Averse Decision Makers.

Evaluating Strategies

If adequate data are available, the utility of strategies can be evaluated easily. However, there is usually little or no data available for evaluating some of the attributes for many of the strategies. Thus, a considerable amount of subjective information must be used. The information necessary to evaluate a strategy is the average annual value of each attribute. Some data are available from agricultural experiment stations and the state extension service. The analyst should use as much of this objective data as he can obtain in determining the average annual values of the attributes. However, these data may not be applicable to the watershed being modeled, and they probably do not cover the range in attribute values for many strategies. Thus, the decision makers must estimate the missing data.

The average annual value of an attribute is the mean or expected value of the attributes probability distribution. Raiffa [1970] suggested a technique for determining the decision maker's subjective probability distribution for any attribute. Since gross income is probably the easiest attribute to evaluate of those chosen for the optimization model, it is used to describe the procedure. Consider a strategy for class II land that specifies grain sorghum, 135 kg/ha nitrogen fertilizer, 45 kg/ha phosphorus fertilizer, and terraces and waterways. Gross income can be estimated by assuming current prices or predicted future prices, etc., and multiplying by the average annual yield of the crop. Thus, the problem is to determine the average annual crop yield subjectively. If the decision makers have maintained production records over a period of several years, they should be able to determine their production probability distribution easier and more accurately. However, records are not necessary.

The analyst does not ask the decision makers to estimate the mean annual crop yield directly. Instead he determines their judgmental density function and calculates the mean. To arrive at the density function, the analyst determines the cumulative judgmental probability distribution by questioning the decision makers. He first asks them for a number that they feel gives equal probabilities of the yield being less than or greater than the number. For the example strategy, the decision makers answer 4500 kg/ha. Thus, the analyst has established one point ($x_{.5} = 4500$) on the cumulative distribution. He next asks the decision makers to choose a number that gives equal probabilities of the yield being less than or greater than the number, if they know that the yield is greater than 4500 kg/ha. The answer, 4800 kg/ha, gives the point $x_{.75} = 4800$. The analyst can continue splitting the intervals in half to obtain as many points on the cumulative distribution curve as are needed to define the shape. Of course, he also splits the intervals on the lower end (below 4500) similarly. Figure 5 shows the points that the analyst determined and the smooth curve drawn through the points (the cumulative judgmental probability distribution).

The judgmental density function is determined by differentiating the cumulative distribution numerically. Figure 6 shows the density function after the adjustment to assure that the area under the curve is 1.0. The mean annual crop yield can be calculated with the integral

$$\overline{x} = \int_{x_{mn}}^{x_{mx}} x\, f(x)\, d\, x \qquad (103)$$

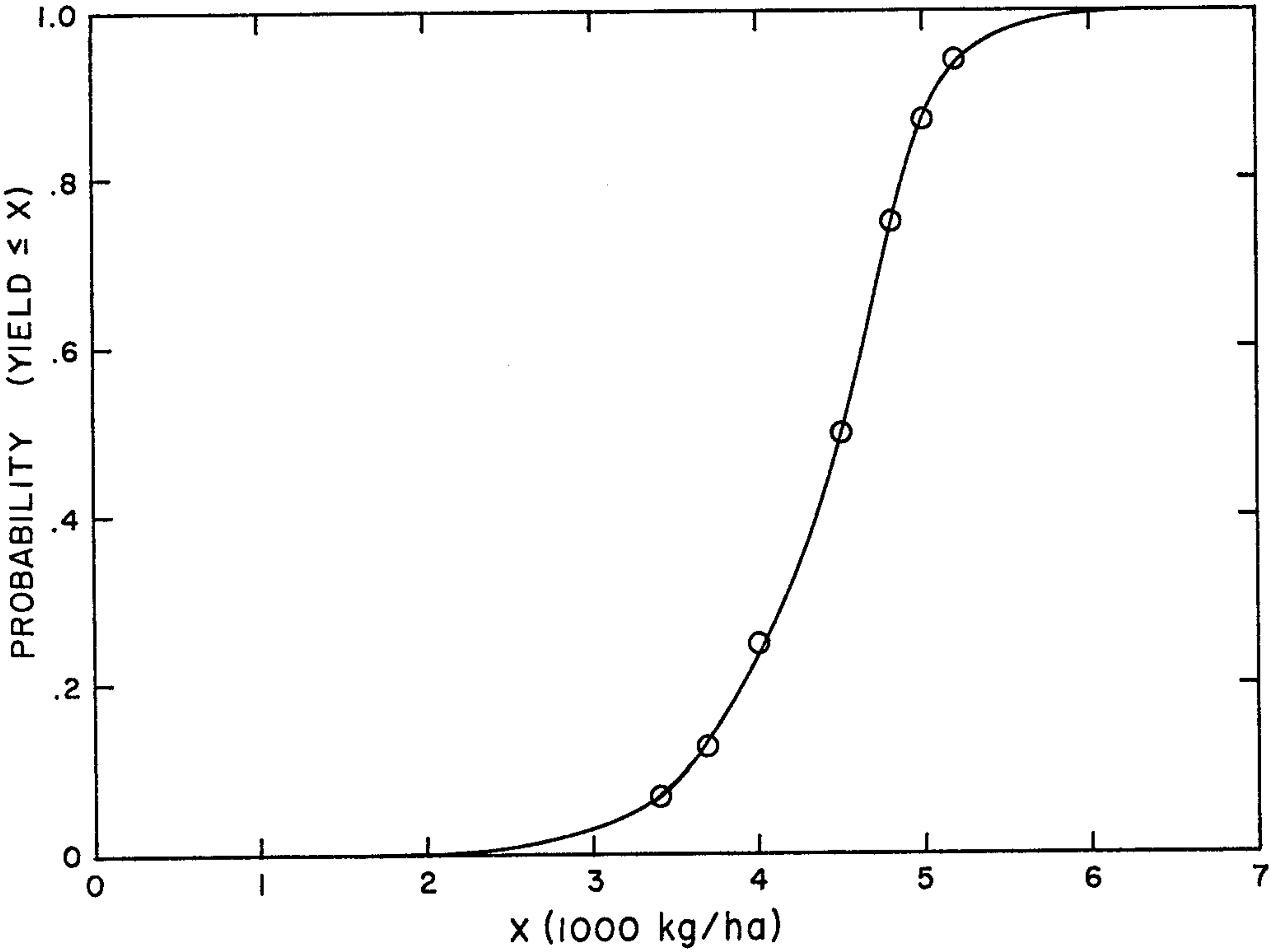

Figure 5. Example Cumulative Judgmental Probability Distribution for Crop Yield.

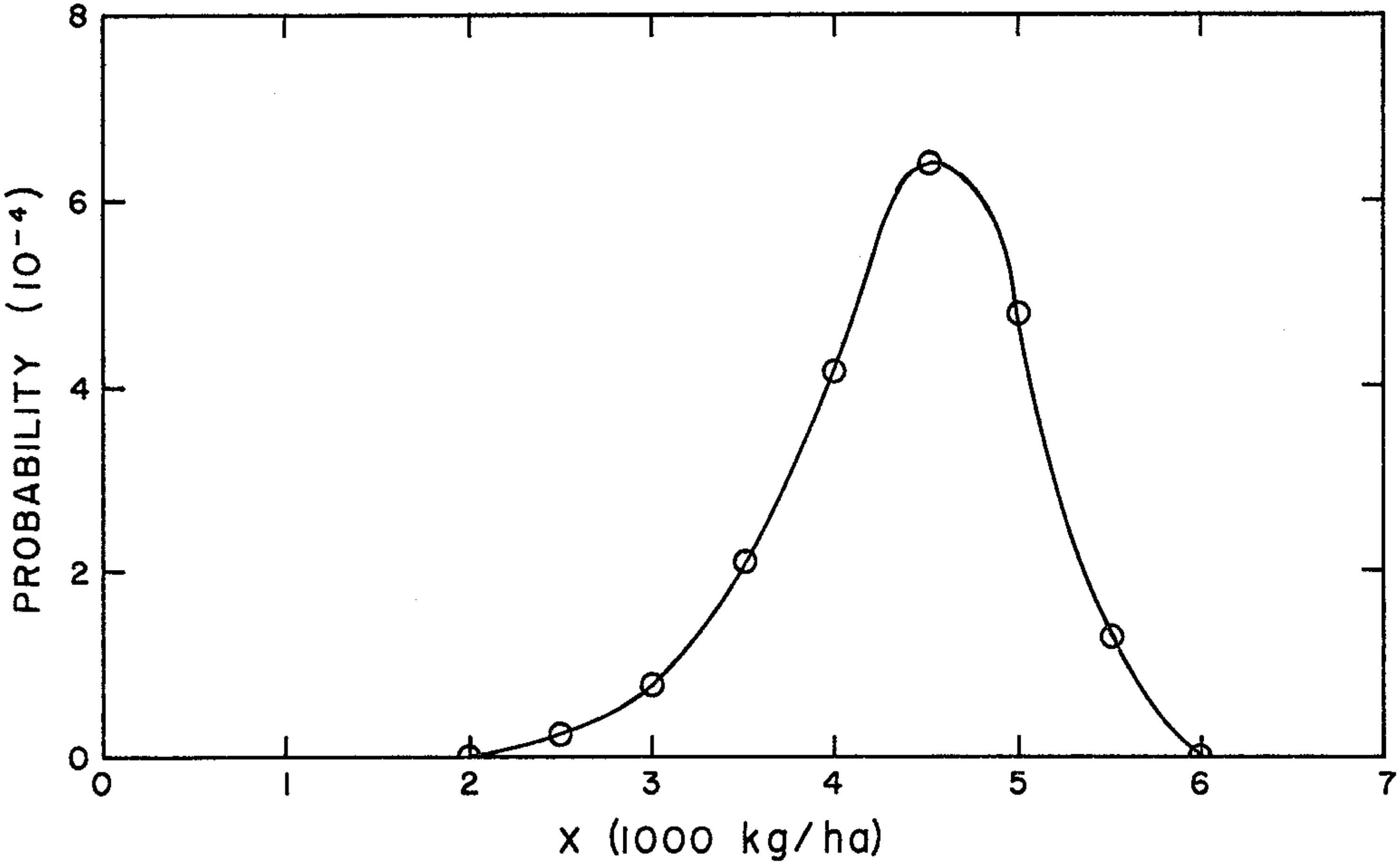

Figure 6. Example Judgmental Density Function for Crop Yield.

where $\bar{x}$ is the mean of the density function; x is the variable expressing the magnitude of crop yield; x_{mx} is the maximum crop yield; x_{mn} is the minimum crop yield; and f(x) is the relative probability of x. The mean can also be determined conveniently by numerically integrating the cumulative distribution. However, the density function is needed for revising the subjective estimates after additional data have been obtained.

The means obtained from the subjective probability distributions are used to evaluate the strategies. However, the subjective probability distributions are refined at the end of each year by including data from that crop year. Probability distributions are adjusted using Bayes' Theorem [1763] written in the form

$$\Phi(x_i|\theta) = \frac{\Phi(\theta|x_i)\ \Phi(x_i)}{\sum_{j=1}^{M} \Phi(\theta|x_j)\ \Phi(x_j)}, \quad i = 1, M \tag{104}$$

where $\Phi(x|\theta)$ is the probability of the value x given a sample described by θ; $\Phi(\theta|x)$ is the probability of obtaining the sample θ given x; $\Phi(x)$ is the subjective probability of x occurring; and M is the number of points used to define the subjective probability distribution.

The probability of obtaining the sample θ given x is computed with the equation

$$\Phi(\theta|x_i) = \Omega_\ell^k \prod_{j=1}^{N} \Phi(\theta_j|x_i) \tag{105}$$

where Ω_ℓ^k is the number of combinations contained in the sample, and N is the number of years sampled. The number of combinations is not important because Ω_{ℓ}^{k} cancels when equation (105) is substituted into equation (104). To determine the probability of obtaining an individual sample of size θ from the population, x, a normal distribution was assumed. The normal distribution with a coefficient of variation of 0.1 generally describes sample variations in crop yields within a particular field for a specified year. Since all of the attributes of the optimization model are related to crop yield, the normal distribution with mean x and coefficient of variation 0.1 was used in evaluating each attribute. Other distributions can be used if they are considered more appropriate for a particular attribute. If samples are taken from several locations that use the same strategy, the distribution can be developed with these data.

Each year as new data are collected and the probability distributions are updated, the model should be rerun to refine the operating policy.

Linear Programming

Once the utility of the strategies has been evaluated, linear programming can be used to determine the optimal combination of strategies for the entire watershed. The LP model is

$$\text{maximize} \quad z_0 = \underline{u}\ \underline{z} \tag{106}$$

subject to

$$(\underline{A},\ \underline{I})\ \underline{z} = \underline{p_0} \tag{107}$$

$$\underline{p_0} \geq 0 \tag{108}$$

$$\underline{z} \geq 0 \tag{109}$$

where z_0 is the value of the objective function to be maximized; $\underline{u}$ is the utility vector; $\underline{z}$ is a vector that specifies the amount of watershed area devoted to each strategy; $\underline{A}$ is the constraint coefficient matrix; $\underline{I}$ is the identity matrix; and $\underline{p_0}$ is a vector that gives the magnitude of the constraints.

To describe the model in more detail, the objective function is written as

$$\text{maximize} \quad z_0 = \sum_{i=1}^{M} u_i \, z_i \tag{110}$$

where the subscript, i, is the strategy number, and M is the total number of strategies. Constraints requiring the amount of land used to equal the watershed area are written

$$\Sigma \, a_{jk} \, z_k = p_{oj} \tag{111}$$

where subscript, j, gives the land class number within a subwatershed and subscript, k, indicates the strategy number for all strategies within a land class. These constraints simply require that all the area within each land class be assigned to some strategy. Thus, the value of p_{oj} is the subwatershed area covered by land class, j. Since crops are normally rotated to a certain extent, it is necessary to constrain the model so that more than one crop is grown on each land class. These constraints are

$$\Sigma\, a_{jk}\, z_k \geq p_{oj} \tag{112}$$

where subscript, j, refers to a particular crop within a land class and subscript, k, refers to the strategies in a land class that specify crop, j. Here p_{oj} is 0.1 of the particular land class area. Thus, each crop must occupy at least 10% of the area of any land class. Ten percent was chosen arbitrarily and can be adjusted for any subwatershed if the decision makers feel another fraction is more appropriate. The final four constraints force the model to operate within water-quality standards imposed at the watershed outlet. These constraints are

$$\Sigma\, a_{jk}\, z_k \leq p_{oj} \tag{113}$$

where subscript, j, indicates the water quality parameter being constrained, and subscript, k, refers to the strategies. The water quality standards for sediment, phosphorus, organic nitrogen, and nitrate are the values of p_o.

For all constraints, except the last four, values of the coefficients, a_{jk}, are one if the strategy is being constrained and zero if not. Coefficients of the sediment constraint are calculated with the equation

$$a_{Y,i} = \frac{(AQE_i)\ (K_\ell)\ (C_i)\ (P_i)\ (LS_\ell)\ (DR_k)}{DA_k} \tag{114}$$

where $a_{Y,i}$ is the sediment yield constraint coefficient for strategy, i; AQE_i is the average annual runoff energy factor for strategy, i; K_ℓ is the soils factor for land class, ℓ; C_i is the crop management factor for strategy, i; P_i is the erosion-control-practice factor for strategy, i; LS_ℓ is the slope length and gradient factor for land class, ℓ; DR_k is the portion of sediment yield from subwatershed, k, that is delivered to the watershed outlet; and DA_k is the drainage area of subwatershed, k. AQE is calculated by integrating the runoff energy ($11.8\ (V\ q_p)^{0.56}$) frequency distribution obtained from the simulation model. DR is determined with sediment routing.

The phosphorus constraint coefficients are computed with the equation

$$a_{Yp,i} = 0.001\ (a_{Y,i})\ (c_{p,i})\ (ER_{p,k}) \qquad (115)$$

where $a_{Yp,i}$ is the phosphorus yield coefficient for strategy, i; $c_{p,i}$ is the phosphorus concentration in the soil for strategy, i; and $ER_{p,k}$ is the phosphorus enrichment ratio. $ER_{p,k}$ is determined by considering the particle-size distribution of the soil and of the sediment routed to the watershed outlet. The phosphorus routing component of SPNM is used to calculate the value of ER_p.

Constraint coefficients for organic nitrogen are computed with an equation similar to equation (115)

$$a_{YN,i} = 0.001\ (a_{Y,i})\ (c_{ON,i})\ (ER_{N,k}) \qquad (116)$$

Like phosphorus, the nitrogen routing component of SPNM is used to determine ER_N.

The nitrate constraint coefficients are computed with the equation

$$a_{YNO3,i} = 0.0001\ (Q_i)\ (c_{NO3,i})\ (\rho_w) \qquad (117)$$

where $a_{YNO3,i}$ is the nitrate yield constraint coefficient for strategy, i; Q_i is the runoff volume from the area allocated to strategy, i; and $c_{NO3,i}$ is the nitrate concentration of the soil for strategy, i.

EXAMPLE APPLICATION

Little Elm Creek watershed near Aubrey, Texas, was selected to demonstrate the model. The entire watershed is located in the Blackland Prairie and has fine-to-medium textured blackland soils. Watershed slopes average about 3.5%. Average annual rainfall is about 86 cm, and the average annual runoff is about 19.6 cm.

Runoff has been measured at the Aubrey gaging station since June 1956, and sediment has been measured since February 1966. There are no nitrogen or phosphorus data available for the watershed, so data from other watersheds were used in checking the model results. Besides being quite scarce, most of the available phosphorus and nitrogen data have been collected at large river gaging stations or from research plots. Data from large gaging stations contain contributions from sources other than agriculture, and small plot data cannot be used to evaluate routing models. However, a limited amount of phosphorus and nitrogen data has been collected from eight small watersheds (area ≤ 1.25 km^2) near Riesel, Texas. The eight watersheds are subwatersheds of the Brushy Creek watershed and are operated by the Agricultural Research Service. Soils, watershed characteristics, and climatic conditions for the six watersheds are similar to those of Little Elm Creek.

About 94 km^2 of the 195 km^2 Little Elm Creek watershed was partially controlled by 16 floodwater-retarding structures during the period when sediment data were collected. The structures have little effect on the long term water yield of the watershed, but are quite important in reducing sediment yield. Since the Soil Conservation Service estimated that about 95% of the sediment from the subwatersheds draining into the structures is trapped, only the area below the structures was considered in modeling sediment, phosphorus, and nitrogen.

The 101-km^2 uncontrolled area was divided into 22 subwatersheds for flood routing and sediment routing. Average watershed slopes were determined using the grid-contour method (equations (19) and (20)). Slope lengths were computed with equation (21). Equations (23) and (24) were used to determine the slope of each land class within each subwatershed. Slope lengths of the land classes were determined similarly. The values of the LS factors were calculated with equation (25). LS was combined with K, C, and P to obtain the value of KCPLS for each subwatershed. Of course, C varies each month giving 12 KCPLS values per year for use in the simulation model. An average C value was obtained by weighting individual storm C values, according to sediment yield predicted with the simulation model. These average C values were used to determine KCPLS values for use in SPNM because SPNM is not a continuous model, i.e., it routes one storm at a time with given initial conditions. Average KCPLS values for land classes, and subwatersheds are shown in Table 4, along with other watershed characteristics.

Once the watershed has been subdivided and the basic characteristics obtained, the model can be applied. Figure 7 shows the three sets of models involved, the sequence of application, and inputs and outputs. A detailed description of the application follows.

Calibrating the Runoff Volume Model

The calibration period selected was 1957 through 1972. Average annual rainfall during the period was 92.5 cm and average annual runoff was 18.9 cm. Monthly runoff volume measured at the Aubrey gage was used

TABLE 4. Little Elm Creek Subwatershed Characteristics.

Subwatershed No.	Drainage Area -(km^2)--	Average Slope --(%)--	Relief-Length Ratio ---(m/km)----	Average Value of KCPLS			
				Land Class			Subwatershed
				II	III	IV	
1	0.75	3.15	19.94	0.0122	0.0178	0.0123	0.0140
2	4.17	2.80	9.11	0.0109	0.0217	0.0123	0.0144
3	1.35	3.75	8.36	0.0121	0.0196	0.0074	0.0130
4	2.02	3.77	6.53	0.0111	0.0244	0.0143	0.0165
5	9.74	3.66	7.08	0.0153	0.0121	0.0071	0.0127
6	2.07	4.35	8.55	0.0140	0.0103	0.0147	0.0132
7	1.84	3.02	7.29	0.0079	0.0327	0.0221	0.0165
8	25.59	3.21	6.95	0.0098	0.0104	0.0107	0.0101
9	2.18	3.76	12.32	0.0096	0.0123	0.0209	0.0121
10	6.11	3.33	5.87	0.0064	0.0272	0.0109	0.0132
11	4.58	3.47	12.46	0.0071	0.0212	0.0076	0.0106
12	1.84	5.56	18.28	0.0218	0.0106	0.0348	0.0275
13	2.62	3.68	11.52	0.0059	0.0106	0.0131	0.0092
14	4.17	4.16	12.94	0.0037	0.0039	0.0118	0.0059
15	3.16	4.50	11.61	0.0098	0.0118	0.0094	0.0104
16	2.15	3.69	9.55	0.0041	0.0096	0.0089	0.0078
17	2.87	3.64	7.70	0.0087	0.0152	0.0155	0.0119
18	1.79	4.23	10.98	0.0083	0.0233	0.0188	0.0165
19	4.77	3.06	5.15	0.0101	0.0080	0.0139	0.0099
20	5.36	3.45	5.49	0.0082	0.0149	0.0153	0.0121
21	4.51	3.81	8.71	0.0088	0.0150	0.0053	0.0097
22	7.51	2.27	3.92	0.0070	0.0207	0.0161	0.0102

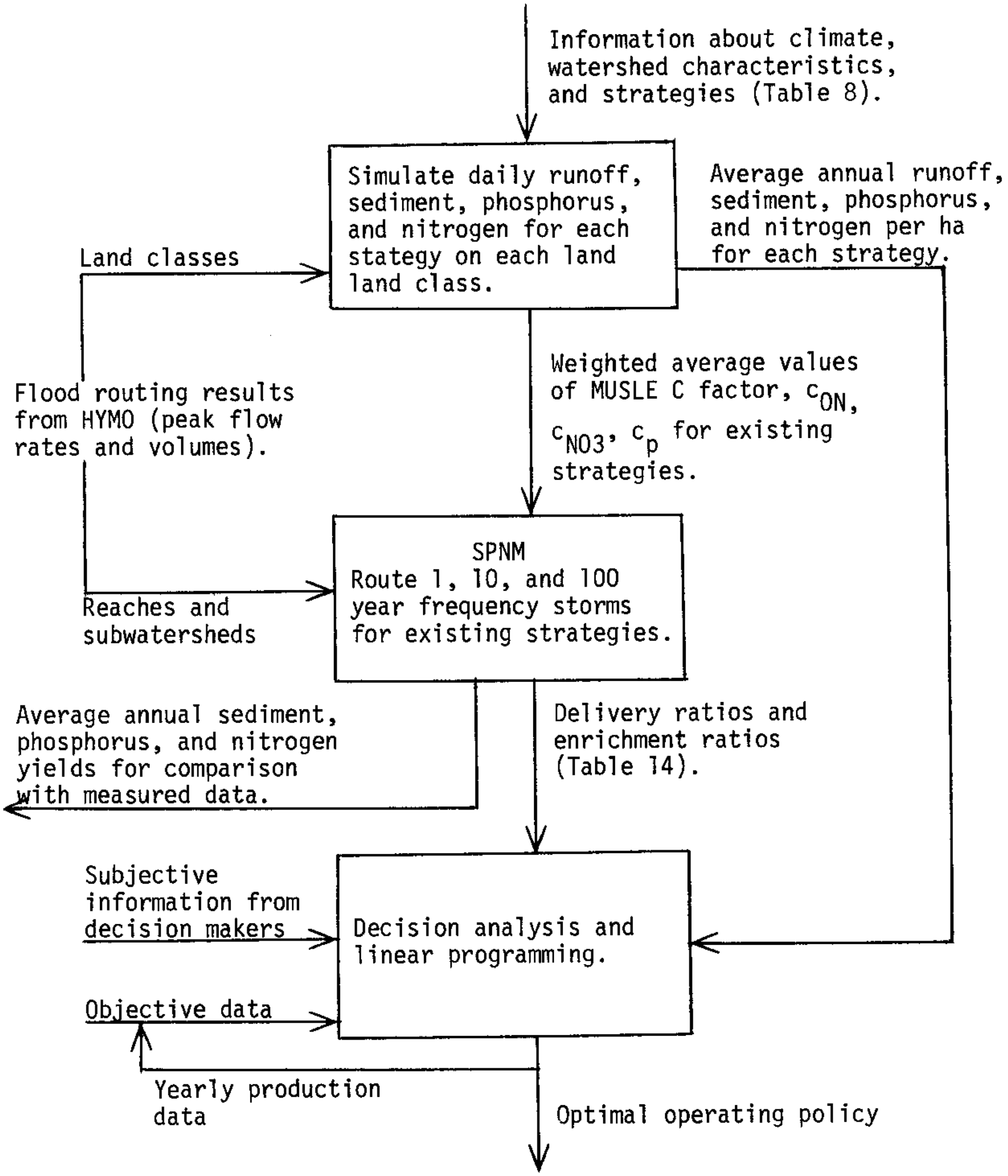

Figure 7. Sequence of Model Application with Inputs and Outputs.

to calibrate the model to the 195-km^2 watershed. The entire watershed area was used in calibrating the runoff volume model because the floodwater retarding structures have little effect on runoff volume. Daily rainfall was obtained from Denton, about 15.5 km southwest of Aubrey.

The average monthly climatic index numbers were calculated with equation (4). Table 5 shows the consumptive use for each crop, the portion of the watershed growing the crop, and the calculated climatic index numbers. Consumptive use was determined with the Texas Board of Water Engineers method [McDaniels, 1960] using Denton lake evaporation. The II-condition SCS curve number for the entire watershed was estimated to be 83 using SCS procedures [U.S. Soil Conservation Service, 1972].

The average curve number for the runoff events simulated in calibrating the model was 76. The average curve number and the II-condition curve number will be used later in predicting water yield for ungaged watersheds with various land uses. Results of the calibration are shown in Table 6. The R^2 used here approximates the variation in the measured variable explained by the model and is expressed by the equation

$$R^2 = 1 - \frac{\Sigma\ \nu^2}{\Sigma\ y^2} \qquad (118)$$

where R^2 is the multiple correlation coefficient squared; $\Sigma\ \nu^2$ is the error sum of squares, and $\Sigma\ y^2$ is the corrected sum of squares of the measured variable. Equation (118) reflects the differences in the magnitude of the two variables being compared, while the simple correlation coefficient does not. The R^2 values shown in Table 6 could be improved if data were available from several raingages located on the Little Elm Creek watershed. With only one raingage located 15.5 km from the

TABLE 5. Determining the Climatic Index for Little Elm Creek Watershed.

Month	Average Daily Lake Evaporation -----(cm/d)-----	Consumptive Use (cm/d)				Climatic Index -(cm/d)-
		Portion of Watershed				
		0.2	0.1	0.2	0.5	
		Grain Sorghum	Small Grain	Cotton	Pasture	
1	0.15	0.06	0.12	0.06	0.08	0.08
2	0.20	0.08	0.21	0.08	0.11	0.11
3	0.33	0.13	0.50	0.13	0.24	0.22
4	0.43	0.17	0.64	0.17	0.29	0.28
5	0.46	0.30	0.42	0.18	0.47	0.37
6	0.61	0.66	0.24	0.24	0.55	0.48
7	0.64	0.52	0.25	0.66	0.58	0.55
8	0.64	0.25	0.25	0.53	0.41	0.39
9	0.51	0.20	0.20	0.31	0.47	0.36
10	0.36	0.14	0.14	0.14	0.33	0.23
11	0.23	0.09	0.13	0.09	0.17	0.13
12	0.15	0.06	0.09	0.06	0.07	0.07

TABLE 6. Results of Runoff Volume Model Calibration.

Depletion coefficient $\alpha = 4.44 \times 10^{-4}$

Average curve number = 76.1

Comparison of measured and predicted runoff

	R^2
Monthly	0.74
Annual	0.80

	Standard Deviations (cm)	
	Measured	Predicted
Monthly	3.51	3.69
Annual	13.99	15.94

watershed, the results are satisfactory. Although the model did not duplicate the measured record with great precision, the predictions fluctuated properly as shown by the measured and predicted standard deviations in Table 6.

Predicting Peak Flow Rate

The relation between peak flow rate and runoff volume was determined for each subwatershed. HYMO was used to route three floods through the Little Elm Creek watershed. Flood frequencies of 1, 10, and 100 years were selected for the routing. For each subwatershed, the three (Q, q_p) points were plotted on log paper to determine the values of b_1 and b_2 in equation (17). Table 7 shows the values of b_1 and b_2 obtained for each subwatershed. The flood-routing results will be discussed further in demonstrating sediment routing. The next step after calibration and peak flowrate determination is to simulate sediment, phosphorus, and nitrogen yields.

Simulating Daily Sediment, Phosphorus, and Nitrogen Yields

The main purpose for simulating daily sediment, phosphorus, and nitrogen yields is to determine the average values of the MUSLE C factor and the concentrations of nitrogen and phosphorus in the soil for each strategy. These average values are used in SPNM and in the optimization model. Thus, the simulation model provides a means for predicting differences in sediment, phosphorus, and nitrogen yields for various strategies.

The simulation period generally should be 25 years or longer to allow a range in climatic conditions. Here a 40-year period was used.

TABLE 7. Values of b_1 and b_2 Determined for Equation 17.

Subwatershed No.	b_1	b_2
1	2.59	0.77
2	3.16	0.94
3	1.27	0.93
4	1.26	0.95
5	3.62	0.99
6	2.07	0.92
7	1.92	0.92
8	8.10	1.00
9	2.83	0.91
10	2.98	0.97
11	5.38	0.90
12	3.16	0.89
13	2.96	0.91
14	4.11	0.92
15	3.47	0.91
16	1.72	0.93
17	2.84	0.92
18	3.37	0.91
19	3.50	0.94
20	2.69	0.97
21	3.52	0.94
22	2.89	0.98

Simulations were performed for all strategies within each land class. Since each particular land class has quite similar characteristics for all subwatersheds, simulations were not repeated for each subwatershed. This is an important reason for dividing the subwatersheds into land classes. For the example problem, there are 88 strategies considered for each subwatershed. This requires 88 simulations of 40 years of daily sediment, phosphorus, and nitrogen yields. Although these 88 simulations require considerable computing time, it is relatively small as compared with 1936 simulations if all subwatersheds were simulated. The time saved is much greater for large watersheds that may have 100 or more subwatersheds.

Since each land class on all subwatersheds is not simulated, the same drainage area (1 ha) was used for all simulations to assure comparable results. The 1-ha drainage area was selected because the effect of deposition is minimized and also most results are reported on a per hectare basis.

Example input data used in simulating a strategy are shown on Table 8. The output from this simulation is shown on Table 9. The steps required for simulating daily sediment, phosphorus, and nitrogen yields are:

Predicting Daily Runoff

The II-condition and the average curve number from the calibration results were combined with the II-condition curve number for each strategy to predict the average curve numbers for the strategies. The average and the II-condition curve numbers were assumed proportional [Williams and LaSeur, 1976] in the equation

TABLE 8. Example Input Data Used in Simulating Daily Sediment, Phosphorus, and Nitrogen Yields.

Watershed -- Little Elm Creek

Land Class -- II

Crop -- Grain Sorghum

Beginning Year -- 1933

Number of Years -- 40

Initial SCS Curve Number -- 83

II-Condition SCS Curve Number -- 81

Average SCS Curve Number -- 74.3 (from equation 118)

Soil Moisture Depletion Coefficient -- 0.003966 (from iterative solution of runoff model requiring average CN = 74.3)

Month	MUSLE C	Temperature (C°)	Climatic Index (cm/d)
1	0.42	7	0.06
2	0.44	9	0.08
3	0.44	13	0.13
4	0.43	19	0.17
5	0.32	22	0.30
6	0.20	27	0.66
7	0.11	29	0.52
8	0.19	29	0.25
9	0.32	25	0.20
10	0.40	20	0.14
11	0.42	13	0.09
12	0.43	9	0.06

MUSLE Factors
K -- 0.34
P -- 0.10
LS -- 0.35

Day Crop Growth Begins -- 106

Day Crop Growth Ends -- 228

Potential Crop Yield -- 5,700 kg/ha

Potential Water Use by Crop -- 51 cm

TABLE 8. (continued)

Initial Soil Phosphorus Concentration -- 550 ppm

Nitrate Concentration in Rainfall -- 0.4 ppm

CIM -- 1.69×10^{-7}

CMN -- 9.95×10^{-10}

CDN -- 2.94×10^{-11}

Enrichment Ratios
Nitrogen -- 1.2
Phosphorus -- 1.2

Initial Contents of Storages

Storage Depth (cm)	Water (cm)	Nitrate-N (kg/ha)	Organic Nitrogen (kg/ha)	Carbon (kg/ha)
0-2.5	0.66	2.92	292	134
2.5-15	3.25	14.59	1460	202
15-30	3.53	17.74	887	112
30-45	2.90	18.28	457	106
45-60	2.44	19.12	239	67
60-75	2.16	20.87	130	45
75-90	1.96	22.01	69	28

Annual Fertilizer Application

Day No.	Amount (kg/ha)	
	Phosphorus	Nitrogen
60	20	65
120	0	65

TABLE 9. Example Output from Simulation Model Using Input Data Shown in Table 8.

Average Annual Values

Rainfall = 84.5 cm

Runoff = 13.5 cm

Sediment Yield = 2.91 t/ha

Organic Nitrogen Yield = 3.71 kg/ha

Nitrate Yield = 6.61 kg/ha

Phosphorus Yield = 2.05 kg/ha

Crop Yield = 4,880 kg/ha

Mineralized Nitrogen = 88.06 kg/ha

Water Drained Below 90 cm = 4.49 cm

Nitrate Leached Below 90 cm = 12.90 kg/ha

Immobilization = 32.56 kg/ha

Denitrification = 24.47 kg/ha

Phosphorus Use = 18.92 kg/ha

Nitrogen Use = 146.45 kg/ha

Phosphorus in Residue = 3.76 kg/ha

Nitrogen in Residue = 55.50 kg/ha

Nitrate in Rainfall = 3.38 kg/ha

Average Values Weighted on Predicted Sediment Yield Basis

Concentration Organic Nitrogen (Top 2.5 cm) = 1,063 ppm

Concentration Phosphorus (Top 15 cm) = 587 ppm

MUSLE C Factor = 0.41

Average Value Weighted on Predicted Runoff Basis

Concentration Nitrate (Top 2.5 cm) = 4.91 ppm

$$CN_u = \frac{(CNII_u)\ (CN_G)}{CNII_G} \tag{119}$$

where CN is the average curve number; CNII is the II-condition curve number, and the subscripts, u and G, refer to ungaged and gaged watersheds.

The soil moisture depletion coefficient, α, was determined by simulating runoff for the calibration period and requiring the average simulated curve number to equal CN_u from equation (119). This iterative solution can be accomplished conveniently with Newton's method for solving nonlinear equations. Once α has been determined, daily runoff volume can be predicted and peak runoff rate is predicted using equation (17).

Predicting Daily Sediment Yield

Equation (18) uses the predicted runoff volume and peak rate to predict daily sediment yield. Values of K and LS in equation (18) are dictated by the land class being considered. P is assigned a value according to the conservation practice of the strategy, and C varies with crop and season.

Predicting Phosphorus Yield

Equation (27) is used to predict daily phosphorus yield, based on the daily sediment yield from equation (18). The phosphorus enrichment ratio is computed by considering the specific surface of the soil and the sediment in equation (31). Equation (32) is used to determine the daily concentration of phosphorus in the soil. Fertilizer application, PF, is specified by the strategy. Phosphorus returned to the soil as crop residue is computed with equation (38) at the end of each growing season. Daily crop growth is determined from daily water use in equation (37). Of course, daily water use is computed with the soil moisture depletion equation (3). Daily phosphorus use by the plant is predicted with equation (36).

Predicting Nitrogen Yield

Daily organic nitrogen yield is predicted with equation (39), based on daily predicted sediment yield. Equation (40) is used to calculate the nitrogen-enrichment ratio considering the percent sediment and soil with particles $\leq 1\ \mu$. The concentration of organic nitrogen in the soil is computed with equation (41). Equation (43) is used to determine the amount of organic nitrogen returned to the soil in the crop residue at the end of each crop season. The immobilization constant, CIM, is estimated with equation (57). Then, daily decomposed carbon is computed with equation (55), and immobilization is computed with equation (56). The mineralization constant, CMN, is estimated with equation (75), and daily mineralized nitrogen is predicted with equation (71).

The daily nitrate yield is predicted with equation (51) based on daily runoff. Nitrate concentration in rainfall was estimated to be 0.4 ppm. Equation (42) is used to determine the daily nitrate concentration in the soil. Fertilizer application is specified by the strategy. Water flow through the seven soil storages is computed with equation (44), and nitrate leaching is predicted with equation (50). The denitrification constant, CDN, is estimated with equation (53), and daily denitrification is computed with equation (52). Daily nitrate use by the crop is predicted with equation (60) and distributed through the soil storages according to water use defined by equation (66).

The 88 simulations produced the inputs for SPNM and the optimization model shown in Table 10.

Routing Sediment, Phosphorus, and Nitrogen Yields

SPNM was used to route the sediment, phosphorus, and nitrogen yields from the subwatersheds to the watershed outlet. Results of flood routing the 1-, 10-, and 100-year floods with HYMO were used as inputs to SPNM. Table 2 contains an example of routing through one reach with SPNM.

The main purpose of routing is to determine the portions of sediment, phosphorus, and nitrogen from each subwatershed that is delivered to the watershed outlet. These ratios are used in the optimization model to evaluate the strategies. Since these delivery ratios are affected only by the hydraulic characteristics of the individual storm runoff and not by the magnitude of subwatershed inputs, any land use arrangement may be assumed for routing. However, when measured data are available, the actual land use should be input to evaluate the accuracy of the routing model. Table 11 shows a comparison of measured and routed (based on actual land use) average annual sediment yields for Little Elm Creek. The routed average annual sediment yield was computed by integrating the cumulative frequency distribution and dividing by the largest return period [Williams, 1974]. Table 11 also shows the routed values for the 1-, 10-, and 100-year frequencies and the measured and routed sediment particle-size distributions at the Aubrey gaging station. The measured particle-size distribution is an average of 11

TABLE 10. Strategy Descriptions and Simulation Output Used to Determine Optional Operating Policy for Little Elm Creek Watershed.

			Fertilizer Applied (kg/ha)			Simulation Output				
Strategy Number	Land Class	Crop	Phosphorus	Nitrogen	P Factor	Average C Factor	Runoff -(cm)-	Average Top Storage Soil Concentration (ppm)		
								Phosphorus	Organic Nitrogen	Nitrate
1	II	Grain sorghum	0	0	0.1	0.420	18.5	429	812	0.78
2	"	"	20	45	0.1	0.410	16.4	640	969	2.21
3	"	"	20	90	0.1	0.410	14.4	609	1121	3.43
4	"	"	20	135	0.1	0.410	13.5	587	1217	4.91
5	"	"	0	0	1.0	0.440	19.6	332	690	0.68
6	"	"	20	45	1.0	0.420	17.4	456	801	1.97
7	"	"	20	90	1.0	0.410	15.4	444	923	3.17
8	"	Grain sorghum	20	135	1.0	0.410	14.4	428	1002	4.95
9	"	Cotton	0	0	0.1	0.440	18.5	400	750	1.00
10	"	"	20	45	0.1	0.430	16.4	597	895	2.83
11	"	"	20	90	0.1	0.430	14.4	568	1035	4.40
12	"	"	20	135	0.1	0.420	13.5	547	1124	6.30
13	"	"	0	0	1.0	0.460	19.6	310	637	0.87
14	"	"	20	45	1.0	0.450	17.4	425	740	2.53
15	"	"	20	90	1.0	0.440	15.4	414	853	4.06
16	"	Cotton	20	135	1.0	0.430	14.4	399	925	6.35
17	"	Wheat	0	0	0.1	0.170	18.5	435	815	0.50
18	"	"	20	45	0.1	0.160	16.4	645	970	1.50
19	"	"	20	90	0.1	0.150	14.4	615	1125	3.00
20	"	"	20	135	0.1	0.150	13.5	590	1225	4.00
21	"	"	0	0	1.0	0.170	18.5	335	700	0.45
22	"	"	20	45	1.0	0.160	17.4	460	810	1.40

TABLE 10. (continued)

			Fertilizer Applied (kg/ha)			Simulation Output				
								Average Top Storage Soil Concentration (ppm)		
Strategy Number	Land Class	Crop	Phosphorus	Nitrogen	P Factor	Average C Factor	Runoff -(cm)-	Phosphorus	Organic Nitrogen	Nitrate
23	II	Wheat	20	90	1.0	0.150	15.4	450	930	2.90
24	"	Wheat	20	135	1.0	0.150	14.4	435	1010	3.90
25	"	Hay	0	0	0.1	0.330	19.6	250	600	0.80
26	"	"	20	45	0.1	0.330	16.4	373	716	2.20
27	"	"	20	90	0.1	0.320	14.4	355	828	3.38
28	"	"	20	135	0.1	0.320	13.5	342	899	4.93
29	"	"	0	0	1.0	0.340	19.6	225	510	0.70
30	"	"	20	45	1.0	0.330	16.4	266	592	1.99
31	"	"	20	90	1.0	0.320	14.4	259	682	3.20
32	"	Hay	20	135	1.0	0.320	14.4	249	740	4.93
33	"	Pasture	0	0	0.1	0.023	19.6	511	1568	0.60
34	"	"	20	45	0.1	0.007	16.4	763	1871	1.69
35	"	"	20	90	0.1	0.002	14.4	726	2165	2.62
36	"	"	20	135	0.1	0.001	13.5	700	2350	3.75
37	"	"	0	0	1.0	0.027	19.6	396	1332	0.52
38	"	"	20	45	1.0	0.009	17.4	544	1547	1.51
39	"	"	20	90	1.0	0.002	14.4	529	1782	2.42
40	II	Pasture	20	135	1.0	0.002	14.4	510	1935	3.78
41	III	Grain Sorghum	0	0	0.1	0.470	23.3	405	710	0.61
42	"	"	20	45	0.1	0.460	22.0	663	780	2.14
43	"	"	20	90	0.1	0.450	20.8	656	910	3.45
44	"	"	0	0	1.0	0.480	23.3	313	603	0.53
45	"	"	20	45	1.0	0.470	22.0	472	663	1.87
46	"	Grain Sorghum	20	90	1.0	0.460	22.0	467	773	3.01

TABLE 10. (continued)

Strategy Number	Land Class	Crop	Fertilizer Applied (kg/ha)		P Factor	Simulation Output				
			Phosphorus	Nitrogen		Average C Factor	Runoff -(cm)-	Average Top Storage Soil Concentration (ppm)		
								Phosphorus	Organic Nitrogen	Nitrate
47	III	Cotton	0	0	0.1	0.480	23.3	360	675	0.95
48	"	"	20	45	0.1	0.470	22.0	537	806	2.69
49	"	"	20	90	0.1	0.460	20.8	511	932	4.18
50	"	"	0	0	1.0	0.490	23.3	279	573	0.83
51	"	"	20	45	1.0	0.480	22.0	383	666	2.40
52	"	Cotton	20	90	1.0	0.470	22.0	373	768	3.86
53	"	Wheat	0	0	0.1	0.200	19.6	391	734	0.48
54	"	"	20	45	0.1	0.190	17.4	581	873	1.43
55	"	"	20	90	0.1	0.180	16.4	554	1013	2.85
56	"	"	0	0	1.0	0.210	19.6	302	630	0.43
57	"	"	20	45	1.0	0.200	18.5	414	729	1.33
58	"	Wheat	20	90	1.0	0.190	17.4	405	837	2.76
59	"	Hay	0	0	0.1	0.350	20.7	225	540	0.76
60	"	"	20	45	0.1	0.330	18.5	336	644	2.09
61	"	"	20	90	0.1	0.320	17.4	320	745	3.21
62	"	"	0	0	1.0	0.360	20.7	203	459	0.67
63	"	"	20	45	1.0	0.340	19.6	239	533	1.89
64	"	Hay	20	90	1.0	0.330	17.4	233	614	3.04
65	"	Pasture	0	0	0.1	0.045	20.7	460	1411	0.57
66	"	"	20	45	0.1	0.025	19.6	687	1684	1.61
67	"	"	20	90	0.1	0.010	17.4	653	1949	2.49
68	"	"	0	0	1.0	0.050	20.7	356	1199	0.49
69	"	"	20	45	1.0	0.030	19.6	490	1392	1.43
70	III	Pasture	20	90	1.0	0.013	18.5	476	1604	2.30

TABLE 10. (continued)

			Fertilizer Applied (kg/ha)			Simulation Output				
								Average Top Storage Soil Concentration (ppm)		
Strategy Number	Land Class	Crop	Phosphorus	Nitrogen	P Factor	Average C Factor	Runoff -(cm)-	Phosphorus	Organic Nitrogen	Nitrate
71	IV	Wheat	0	0	0.1	0.210	20.7	371	697	0.46
72	"	"	20	45	0.1	0.200	19.6	552	829	1.36
73	"	"	20	90	0.1	0.190	18.5	526	962	2.71
74	"	"	0	0	1.0	0.220	20.7	287	599	0.41
75	"	"	20	45	1.0	0.210	19.6	393	693	1.26
76	"	Wheat	20	90	1.0	0.200	18.5	385	795	2.62
77	"	Hay	0	0	0.1	0.360	20.7	214	513	0.72
78	"	"	20	45	0.1	0.340	19.6	319	612	1.99
79	"	"	20	90	0.1	0.330	18.5	304	708	3.05
80	"	"	0	0	1.0	0.370	20.7	193	436	0.64
81	"	"	20	45	1.0	0.350	19.6	227	506	1.80
82	"	Hay	20	90	1.0	0.340	18.5	221	583	2.89
83	"	Pasture	0	0	0.1	0.045	20.7	437	1340	0.54
84	"	"	20	45	0.1	0.025	19.6	653	1600	1.53
85	"	"	20	90	0.1	0.014	18.5	620	1852	2.37
86	"	"	0	0	1.0	0.045	20.7	338	1139	0.47
87	"	"	20	45	1.0	0.025	19.6	466	1322	1.36
88	IV	Pasture	20	90	1.0	0.014	18.5	452	1524	2.19

TABLE 11. Comparison of Measured and Routed Sediment Yields for Little Elm Creek Watershed.

Frequency of Occurrence ---(yrs)---	Routed Sediment Yield ----(t/ha)----
1	0.50
10	1.52
100	2.65

	Average Annual Sediment Yield ----(t/ha)----
Routed	3.62
Measured	3.85

Particle Size Distribution at Aubrey Gaging Station

Sediment Diameter (μ)	Percent of Sediment with Specified Diameter	
	Measured	Routed (10-year frequency)
1.0	79.0	79.1
3.0	7.0	6.7
6.0	3.0	2.8
12.0	5.0	4.5
23.5	2.0	1.8
46.5	2.0	1.9
93.5	1.0	1.1
187.5	0.5	0.8
375.0	0.5	1.3

distributions, determined from 11 daily samples, collected during 1966-1976. Only the 10-year frequency routed partice-size distribution is shown in Table 11 because the 1 and 100 year distributions are quite similar. The results indicated that the sediment-routing model is operating properly because both the sediment yield and the particle-size distribution compared closely with the measured values.

Table 12 shows routed phosphorus and nitrogen yields for the 1-, 10-, and 100-year frequency storms and the average annual yields. Kissel et al. [1976] reported average annual yields of nitrate (3.2 kg/ha) and organic nitrogen (5.1 kg/ha) for two small watersheds at Riesel for 1970 to 1974. Average annual runoff was 16.2 cm, and average annual sediment yield was 4.5 t/ha. During 1976, the ranges in nutrient yields from six small watersheds at Riesel were:

Phosphorus, 0.22 - 2.15 kg/ha
Organic nitrogen, 1.54 - 10.93 kg/ha
Nitrate nitrogen, 1.33 - 6.66 kg/ha

The average 1976 runoff for the six watersheds was 15.5 cm.

More data are needed to determine the accuracy of the phosphorus- and nitrogen-routing models, but the Riesel data indicated that the models are giving realistic results. Generally, the routed values are slightly higher than the Riesel data. However, the average annual runoff for Little Elm (19.6 cm) is higher than the Riesel runoff for the two periods (16.2 and 15.5 cm).

The sediment delivery ratios and the phosphorus- and nitrogen-enrichment ratios computed with SPNM for the 1-year frequency storm, are shown in Table 13. Similar output was obtained for 10 and 100 year storms. To determine the average sediment, phosphorus, and nitrogen

TABLE 12. Routed Phosphorus and Nitrogen Yields for Little Elm Creek Watershed.

Frequency of Occurrence ---(yrs)----	Routed Yields (kg/ha)		
	Phosphorus	Organic Nitrogen	Nitrate-N
1	0.41	1.15	0.45
10	1.24	3.47	0.81
100	2.15	6.02	1.01
Average Annual Yields	2.88	8.01	4.09

TABLE 13. Delivery Ratios and Nitrogen and Phosphorus Enrichment Ratios for One-Year Frequency Storm on Little Elm Creek Watershed.

Subwatershed No.	Portion of Sediment Delivered to Outlet	Enrichment Ratios	
		Phosphorus	Nitrogen
1	0.19	3.30	3.49
2	0.33	3.98	4.20
3	0.44	3.11	3.28
4	0.49	3.28	3.43
5	0.51	3.49	3.62
6	0.56	2.77	2.83
7	0.59	2.65	2.71
8	0.64	3.19	3.28
9	0.68	2.25	2.29
10	0.73	2.85	2.92
11	0.79	2.25	2.30
12	0.31	3.74	3.89
13	0.41	3.20	3.32
14	0.47	3.09	3.19
15	0.58	2.63	2.72
16	0.65	2.73	2.83
17	0.71	2.49	2.55
18	0.79	1.66	1.68
19	0.87	2.44	2.50
20	0.91	2.56	2.63
21	0.95	2.31	2.37
22	1.00	2.56	2.64

delivery ratios for use in the optimization model, the average annual yields delivered to the watershed outlet by a particular subwatershed were compared to those leaving the subwatershed. Of course, average annual yields are computed by integrating the cumulative frequency distributions. The cumulative frequency distributions of the sediment, phosphorus, and nitrogen yields leaving a subwatershed are obtained directly from the SPNM results. Frequency distributions of the yields contributed to the watershed outlet by a particular subwatershed are determined by multiplying the 1-, 10-, and 100-year yields by their delivery ratios. As an example, the 1-, 10-, and 100-year sediment yields from subwatershed 3 are 127, 402, and 721 t. A frequency distribution developed with these three points gives an average annual sediment yield of 824 t. The 1-, 10-, and 100-year sediment delivery ratios for subwatershed 3 are 0.44, 0.39, and 0.37. Multiplying the yields by the delivery ratios gives the three points 56, 157, and 267 t that define the frequency distribution of sediment delivered from subwatershed 3. Integrating the frequency distribution gives an average annual delivered yield of 381 t. The average delivery ratio for use in the optimization model, 0.46, is simply the ratio of the delivered amount to the subwatershed yield. Table 14 shows the average sediment delivery ratios and enrichment ratios obtained by this process for use in the optimization model.

Determining Optimal Operating Policy

For the example problem, the same set of strategies (Table 10) were used on all subwatersheds. The decision makers were not contacted to determine their preferences in strategies or utility functions, or to

TABLE 14. Average Delivery Ratios and Nitrogen and Phosphorus Enrichment Ratios for Use in Determining Optimal Strategies on Little Elm Creek Watershed.

Subwatershed No.	Portion of Sediment Delivered to Outlet	Enrichment Ratios	
		Phosphorus	Nitrogen
1	0.18	3.55	3.93
2	0.33	3.79	4.21
3	0.46	3.18	3.49
4	0.43	3.39	3.69
5	0.46	3.53	3.84
6	0.51	2.71	2.94
7	0.54	2.60	2.79
8	0.64	3.05	3.27
9	0.68	2.13	2.25
10	0.73	2.71	2.87
11	0.78	2.12	2.21
12	0.23	4.13	4.46
13	0.37	3.44	3.71
14	0.43	3.22	3.49
15	0.56	2.64	2.83
16	0.63	2.69	2.86
17	0.70	2.41	2.54
18	0.78	1.59	1.64
19	0.86	2.44	2.42
20	0.90	2.47	2.56
21	0.95	2.20	2.28
22	1.00	2.48	2.55

determine subjective probability distributions. Since the problem is only an example, typical values were assumed and they should suffice for illustrative purposes. Most of the information, used in selecting and evaluating strategies and multiattribute utility functions, was obtained from M. J. Norris, Agronomist at the Blackland Research Center, Temple, Texas. Mr. Norris provided objective data and his long-term experience in agronomic research in the Texas Blacklands was most helpful in supplying subjective information.

Utility Functions

Attributes selected for developing the multiattribute utility function were gross income, production costs, dependability, disease control, insect control, and weed control. Gross income was chosen as the most important attribute and thus assigned a weighting factor of 1.0. The other attributes were assigned smaller factors, according to their importance. Equation (102) was used to scale the weighting factors for use in equation (100). The values of the scaled weighting factors δ_i appear in Table 15.

In developing individual utility functions for the attributes, it was assumed that the decision makers were not risk-averse and, therefore, had linear utility functions. Such decision makers are interested only in the expected monetary value of a gamble. They are indifferent to a 50-50 chance of gaining \$500 or losing \$100 and a sure income of \$200 (the expected monetary value of the gamble). The linear utility function probably would not be acceptable to most decision makers, but it simplifies computations slightly and will serve the purpose of illustration in the absence of real utility functions.

TABLE 15. Attribute Values and Multiattribute Utilities for Each Strategy on Little Elm Creek.

	Weighting Factors δ =						
	0.357	0.321	0.179	0.036	0.036	0.071	
				Yield Reduction			
Strategy Number	Gross Income ($/ha)	Production Cost --($/ha)--	Coefficient of Variation ----(%)-----	Disease --(%)--	Insect -(%)--	Weed -(%)-	Utility
1	180	289	18.8	10	7.5	15	0.44
2	225	324	16.1	10	7.5	15	0.45
3	269	343	13.5	10	7.5	15	0.48
4	297	363	12.0	10	7.5	15	0.49
5	161	217	19.8	10	7.5	15	0.47
6	203	252	17.5	10	7.5	15	0.48
7	242	272	15.1	10	7.5	15	0.50
8	267	292	13.7	10	7.5	15	0.51
9	425	474	35.3	20	25	7.5	0.35
10	531	509	31.0	20	25	7.5	0.41
11	625	529	27.2	20	25	7.5	0.47
12	687	549	24.7	20	25	7.5	0.50
13	381	435	37.1	20	25	7.5	0.34
14	477	469	33.2	20	25	7.5	0.39
15	563	489	29.7	20	25	7.5	0.45
16	618	509	27.5	20	25	7.5	0.47
17	74	163	25.4	8	5	1	0.51
18	109	198	21.3	8	5	1	0.53
19	148	217	16.7	8	5	1	0.56
20	163	237	15.0	8	5	1	0.52
21	67	151	26.3	8	5	1	0.51
22	99	185	22.6	8	5	1	0.52
23	133	240	18.5	8	5	1	0.53
24	148	314	16.7	8	5	1	0.50

TABLE 15. (continued)

	Weighting Factors δ =						
	0.357	0.321	0.179	0.036	0.036	0.071	
				Yield Reduction			
Strategy Number	Gross Income ($/ha)	Production Cost --($/ha)--	Coefficient of Variation ----(%)-----	Disease --(%)--	Insect -(%)--	Weed -(%)-	Utility
25	131	368	22.5	9	5	7.5	0.38
26	324	403	16.3	9	5	7.5	0.50
27	467	423	11.7	9	5	7.5	0.59
28	514	442	10.2	9	5	7.5	0.61
29	116	292	22.9	9	5	7.5	0.43
30	292	326	17.3	9	5	7.5	0.53
31	415	346	13.3	9	5	7.5	0.60
32	457	366	12.0	9	5	7.5	0.62
33	131	84	18.1	4	2.5	3	0.64
34	259	119	14.9	4	2.5	3	0.70
35	415	138	11.0	4	2.5	3	0.79
36	457	158	10.0	4	2.5	3	0.81
37	116	69	18.4	4	2.5	3	0.64
38	235	104	15.5	4	2.5	3	0.69
39	373	124	12.1	4	2.5	3	0.77
40	415	143	11.0	4	2.5	3	0.79
41	86	289	24.2	10	7.5	15	0.36
42	109	324	22.9	10	7.5	15	0.36
43	128	343	21.7	10	7.5	15	0.36
44	77	217	24.7	10	7.5	15	0.40
45	96	252	23.5	10	7.5	15	0.39
46	116	272	22.4	10	7.5	15	0.40
47	205	474	44.2	20	25	7.5	0.19
48	259	509	42.0	20	25	7.5	0.20
49	309	529	40.1	20	25	7.5	0.23
50	183	435	45.0	20	25	7.5	0.20

TABLE 15. (continued)

Strategy Number	Weighting Factors δ =						Utility
	0.357	0.321	0.179	0.036	0.036	0.071	
				Yield Reduction			
	Gross Income ($/ha)	Production Cost --($/ha)--	Coefficient of Variation ----(%)-----	Disease --(%)--	Insect -(%)--	Weed -(%)-	
51	232	469	43.1	20	25	7.5	0.21
52	272	489	41.4	20	25	7.5	0.23
53	54	163	27.7	8	5	1	0.49
54	82	198	24.5	8	5	1	0.50
55	111	217	21.1	8	5	1	0.52
56	49	151	28.3	8	5	1	0.49
57	74	185	25.4	8	5	1	0.50
58	99	240	22.5	8	5	1	0.49
59	62	368	24.7	9	5	7.5	0.36
60	156	403	21.7	9	5	7.5	0.38
61	250	423	18.7	9	5	7.5	0.43
62	52	292	25.0	9	5	7.5	0.38
63	141	326	22.2	9	5	7.5	0.42
64	225	346	19.5	9	5	7.5	0.47
65	62	84	19.7	4	2.5	3	0.59
66	124	119	18.2	4	2.5	3	0.61
67	217	138	15.9	4	2.5	3	0.66
68	52	69	20.0	4	2.5	3	0.59
69	104	104	18.7	4	2.5	3	0.60
70	188	124	16.7	4	2.5	3	0.65
71	37	163	29.7	8	5	1	0.47
72	54	198	27.7	8	5	1	0.46
73	74	217	25.4	8	5	1	0.47
74	35	151	30.1	8	5	1	0.47
75	44	185	28.3	8	5	1	0.46
76	67	240	26.3	8	5	1	0.45

TABLE 15. (continued)

Strategy Number	Weighting Factors δ = 0.357	0.321	0.179	0.036	0.036	0.071	Utility
				Yield Reduction			
	Gross Income ($/ha)	Production Cost --($/ha)--	Coefficient of Variation ----(%)-----	Disease --(%)--	Insect -(%)--	Weed -(%)-	
77	52	368	25.0	9	5	7.5	0.33
78	131	403	22.5	9	5	7.5	0.36
79	208	423	20.0	9	5	7.5	0.40
80	47	292	25.2	9	5	7.5	0.38
81	99	326	23.5	9	5	7.5	0.39
82	175	346	21.0	9	5	7.5	0.43
83	52	84	20.0	4	2.5	3	0.58
84	104	119	18.7	4	2.5	3	0.59
85	183	138	16.8	4	2.5	3	0.63
86	52	69	20.0	4	2.5	3	0.59
87	104	104	18.7	4	2.5	3	0.60
88	183	124	16.8	4	2.5	3	0.64

To evaluate equation (100), the additive multiattribute utility function, individual utility functions must be developed for each attribute. Since the functions are linear, they can be determined from only two points (the highest and lowest values of the attribute). The highest value has a utility of 1.0, and the lowest value has a utility of 0.0. Utility functions for each attribute are expressed in the equations

$$u\ (GI) = -0.053 + 0.00153\ GI \qquad (120)$$

$$u\ (PC) = 1.144 - 0.00208\ PC \qquad (121)$$

$$u\ (DP) = 1.293 - 0.0287\ DP \qquad (122)$$

$$u\ (DC) = 1.25 - 0.0625\ DC \qquad (123)$$

$$u\ (IC) = 1.11 - 0.0444\ IC \qquad (124)$$

$$u\ (WC) = 1.071 - 0.0714\ WC \qquad (125)$$

where GI is gross income in \$/ha; PC is production cost in \$/ha; DP is the coefficient of variation of crop yield; DC is the percent crop loss caused by diseases; IC is the percent crop loss caused by insects, and WC is the percent crop loss caused by weeds. All variables in equations (120 to 125) are average annual values, except the coefficient of variation.

Equation (100) can be written as

$$u\ (\underline{v}) = 0.357\ u\ (GI) + 0.321\ u\ (PC) + 0.179\ u\ (DP) + 0.036\ u\ (DC) + 0.036\ u\ (IC) + 0.071\ u\ (WC) \qquad (126)$$

or by substituting equations (120 to 125) into equation (126) the relationship is reduced to

$$u\ (\underline{v}) = 0.741 + 0.000546\ GI - 0.000668\ PC - 0.00514\ DP - 0.00225\ DC - 0.00160\ IC - 0.00507\ WC \qquad (127)$$

Equation (127) is the multiattribute utility function that was used to evaluate the strategies. Table 15 contains the values of the attributes and the computed utilities of each strategy.

Values of the attributes were determined from objective and subjective information. Gross income was computed by multiplying current prices by average annual crop production. Considerable data were available for determining average annual crop production, but subjective information was inserted for strategies where little or no data were available. Production cost information was provided for all strategies by the Texas Agricultural Extension Service. The dependability attribute was evaluated on the basis of the coefficient of variation of annual crop production. The same crop production information used in evaluating gross income was used to determine the coefficient of variation. The disease, insect, and weed control attributes were evaluated on the basis of average annual percent yield reduction. This information provided by M. J. Norris was entirely subjective.

Linear Programming

MPS-360 [IBM, 1971] was used to solve the LP problem. The utilities shown in Table 15 are the coefficients for the objective function, equation (110). Since there are 88 strategies for each of the 22 subwatersheds, the LP problem has 1936 columns. There are 66 constraints requiring the amount of land used to equal land class areas (equation (111)), 286 constraints requiring mixed land use for all areas (equation (112)), and four water-quality constraints (equation (113)). Thus, the LP problem has 356 rows.

For the first 66 constraints, the coefficients, a_{jk}, have the value 1.0 for strategies that apply to the land class specified by drainage area in $\underline{p}_o$. All other coefficients are zero. As an example, for the first row, strategies 1 to 40 have coefficients of 1.0 because they apply to class-II land of the first subwatershed. All other coefficients of the first row are zero. Coefficients of strategies 41 to 70 are 1.0 for the second row because they apply to class-III land of the first watershed. The righthand side of equation (111), p_{oj}, contains the drainage areas of the land classes II to IV for each subwatershed.

Each constraint (67 to 352) has coefficients of 1.0 for all strategies within the specified land class that have the same crop. For example, on row 67, strategies 1 to 8 have coefficients of 1.0 because they specify grain sorghum growing on class-II land. The righthand side, p_{067}, is equal to 0.1 of the drainage area of class-II land in subwatershed 1. Row 68 has coefficients of 1.0 for strategies 9 to 16 (cotton) and p_{068} is the same as p_{067}. Thus, land classes II and III require five rows, and land class IV requires three rows (one row per crop).

Coefficients of the sediment yield constraint (row 353) are computed with equation (113). As an example, the coefficient for strategy one was computed using the inputs

K = 0.34 (Soils factor for soils in class-II land of the first subwatershed.)

C = 0.41 (Average crop management factor for the strategy--taken from the simulation model.)

P = 0.1 (Specified by the strategy--Table 10.)

LS = 0.35 (Computed with equations (23 to 25) for class-II land in the first subwatershed.)

DR = 0.18 (Computed with SPNM--Table 14.)

DA = 0.75 km^2 (Table 4)

AQE = 54,676 (Runoff volume frequency distribution obtained from simulation for strategy number one gave the values

Frequency (yrs)	Q (cm)
1	3.48
10	10.38
100	17.89

The peak runoff rate was computed for each of the three frequencies using Q and equation (17). $b_1 = 2.59$, $b_2 = 0.77$--Table 7. AQE was calculated by integrating the runoff energy ($11.8\ (V\ q_p)^{0.56}$) frequency distribution).

Substituting these values into equation (113) gives the constraint coefficient for strategy 1 (64.02).

Equations (114) and (115) are used to calculate the phosphorus and organic nitrogen constraint coefficients. The soil concentrations of phosphorus and organic nitrogen are the weighted average values computed with the simulation model and shown in Table 10. Phosphorus and nitrogen enrichment ratios were determined with SPNM and appear in Table 14.

The nitrate constraint coefficients were calculated using equation (116). Q, the runoff volume, is the average annual runoff for the strategy computed with the simulation model (Table 10). The nitrate concentration of the soil is the weighted average concentration determined with the simulation model.

Examples of the solutions to equation (114 to 116) for the first strategy are

$$a_{Yp,1} = (0.001)\ (64.02)\ (429)\ (3.55) = 97.50$$

$$a_{YN,1} = (0.001)\ (64.02)\ (812)\ (3.93) = 204.3$$

$$a_{YNO3,1} = (0.0001)\ (18.5)\ (0.78)\ (100{,}000) = 144.3$$

The righthand side of the water-quality constraints should have values determined by water quality standards. Since standards have not been established, values were selected arbitrarily at 3.0 t/ha for sediment, 2.25 kg/ha for phosphorus, 5.5 kg/ha for organic nitrogen, and 2.0 ppm for nitrate.

The optimal solution to the LP problem gave a total utility value of 23.73. Values of the attributes expressed in their original units can be obtained by accumulating the product of the optimal area allocated to each strategy and the attribute value for the strategy. The two most important attributes, gross income and production cost, gave average values of $251/ha and $193/ha. Thus, the decision makers could net $58/ha/yr, and the runoff water would be relatively low in sediment, phosphorus, and nitrogen. The optimal strategies for each land class of each subwatershed are shown in Table 16.

The active water quality constraints were nitrate and phosphorus. Sediment and organic nitrogen had slack values of 12,351 t and 13,069 kg indicating that yields were only 1.58 t/ha and 4.32 kg/ha. One reason for the relatively low sediment yield is that the optimal solution specified a large portion of pasture land. With current prices, pasture is the most profitable land use and also gives the lowest sediment yield. Thus, the optimal solution is obvious (put as much land in pasture as possible). However, if crop prices increased so that crops were more profitable than pasture, the solution would not be so obvious. If crops were more profitable, the solution would specify as much crop land as possible without violating the water-quality constraints.

Fluctuating prices are another reason for the constraints that provide for mixed land use. If these constraints were not imposed, land use might change drastically from year to year. Such changes are not practical or acceptable to the decision makers. For example, perennial grasses are expensive and slow to establish. Thus, the decision makers would not want to switch from crop to grass and back again year after year. Also, as mentioned previously, crop rotation is practiced generally, so more than one crop should be included in the optimal operating policy of areas that consider cropping strategies. Here the mixed-land-use constraints require that at least 0.1 of each land class is used for each crop considered. The 0.1 value is arbitrary and would vary with different decision makers, land classes, and probably with

TABLE 16. Optimal Solution of Linear Programming Problem for Little Elm Creek Watershed.

Subwatershed No.	Land Class	Optimal Strategies--Crop Area (ha), Nitrogen and Phosphorus Fertilizer (kg/ha), Conservation or Nonconservation (C or N)				
		Grain Sorghum	Cotton	Wheat	Hay	Pasture
1	II	4.1, 0-0, N	4.1, 90-20, C	4.1, 0-0, C	4.1, 90-20, N	25.9, 90-20, C
	III	2.1, 0-0, C	2.1, 0-0, C	2.1, 0-0, C	2.1, 90-20, N	13.7, 0-0, N
	IV	-------------	---------------	1.0, 0-0, C	1.0, 0-0, N	8.6, 0-0, N
2	II	25.6, 0-0, N	25.6, 90-20, C	25.6, 0-0, C	25.6, 90-20, N	152.6, 90-20, C
	III	15.3, 0-0, C	15.3, 0-0, C	15.3, 0-0, C	15.3, 90-20, N	92.7, 0-0, N
	IV	-------------	---------------	0.8, 0-0, C	0.8, 0-0, C	6.0, 0-0, N
3	II	3.4, 0-0, N	3.4, 90-20, C	3.4, 0-0, C	3.4, 90-20, N	21.0, 90-20, C
	III	4.7, 0-0, N	4.7, 0-0, C	4.7, 0-0, C	4.7, 90-20, N	27.7, 0-0, N
	IV	-------------	---------------	5.4, 0-0, C	5.4, 0-0, N	42.7, 0-0, N
4	II	12.4, 0-0, N	12.4, 90-20, C	12.4, 0-0, C	12.4, 90-20, N	74.9, 90-20, C
	III	5.4, 0-0, C	5.4, 0-0, C	5.4, 0-0, C	5.4, 90-20, C	32.1, 90-20, C
	IV	-------------	---------------	2.3, 0-0, C	2.3, 0-0, C	18.1, 0-0, N
5	II	45.1, 0-0, N	45.1, 90-20, C	45.1, 0-0, C	45.1, 90-20, N	269.6, 90-20, C
	III	31.1, 0-0, C	31.1, 0-0, C	31.1, 0-0, C	31.1, 90-20, N	186.2, 0-0, N
	IV	-------------	---------------	21.2, 0-0, C	21.2, 0-0, C	170.7, 0-0, N
6	II	3.9, 0-0, C	3.9, 90-20, C	3.9, 0-0, C	3.9, 90-20, C	23.3, 90-20, C
	III	7.5, 0-0, C	7.5, 0-0, C	7.5, 0-0, C	7.5, 90-20, C	46.1, 90-20, C
	IV	-------------	---------------	9.3, 0-0, C	9.3, 0-0, C	74.1, 0-0, N
7	II	8.3, 0-0, N	8.3, 90-20, C	8.3, 0-0, C	8.3, 90-20, N	50.3, 90-20, C
	III	7.0, 0-0, N	7.0, 0-0, C	7.0, 0-0, C	7.0, 90-20, N	41.4, 0-0, N
	IV	-------------	---------------	3.1, 0-0, C	3.1, 0-0, C	24.9, 0-0, N
8	II	152.3, 0-0, C	152.3, 90-20, C	152.3, 0-0, C	152.3, 90-20, C	913.5, 90-20, C
	III	95.3, 0-0, C	95.3, 0-0, C	95.3, 0-0, C	95.3, 90-20, C	570.6, 90-20, C
	IV	-------------	---------------	8.8, 0-0, C	8.8, 0-0, C	69.4, 0-0, N
9	II	6.2, 0-0, N	6.2, 90-20, C	6.2, 0-0, C	6.2, 90-20, N	37.8, 90-20, C
	III	7.3, 0-0, C	7.3, 0-0, C	7.3, 0-0, C	7.3, 90-20, C	43.8, 90-20, C
	IV	-------------	---------------	8.3, 0-0, C	8.3, 0-0, C	65.8, 0-0, N

TABLE 16. (continued)

Subwatershed No.	Land Class	Optimal Strategies--Crop Area (ha), Nitrogen and Phosphorus Fertilizer (kg/ha), Conservation or Nonconservation (C or N)				
		Grain Sorghum	Cotton	Wheat	Hay	Pasture
10	II	31.3, 0-0, N	31.3, 90-20, C	31.3, 0-0, C	31.3, 90-20, N	188.3, 90-20, C
	III	17.1, 0-0, C	17.1, 0-0, C	17.1, 0-0, C	17.1, 90-20, C	102.8, 90-20, C
	IV	-------------	---------------	12.7, 0-0, C	12.7, 0-0, N	101.3, 0-0, N
11	II	26.9, 0-0, C	26.9, 90-20, C	26.9, 0-0, C	26.9, 90-20, C	162.4, 90-20, C
	III	15.3, 0-0, C	15.3, 0-0, C	15.3, 0-0, C	15.3, 90-20, C	91.2, 90-20, C
	IV	-------------	---------------	3.6, 0-0, C	3.6, 0-0, C	29.0, 0-0, N
12	II	0.8, 0-0, N	0.8, 90-20, C	0.8, 0-0, C	0.8, 90-20, N	4.9, 90-20, C
	III	7.3, 0-0, C	7.3, 0-0, C	7.3, 0-0, C	7.3, 90-20, C	44.8, 90-20, C
	IV	-------------	---------------	10.1, 0-0, C	10.1, 0-0, C	82.1, 0-0, N
13	II	10.1, 0-0, N	10.1, 90-20, C	10.1, 0-0, C	10.1, 90-20, C	60.9, 90-20, C
	III	9.3, 0-0, N	9.3, 0-0, C	9.3, 0-0, C	9.3, 90-20, N	55.9, 0-0, N
	IV	-------------	---------------	6.7, 0-0, C	6.7, 0-0, C	53.6, 0-0, N
14	II	6.5, 0-0, N	6.5, 90-20, C	6.5, 0-0, C	6.5, 90-20, N	37.8, 90-20, C
	III	18.4, 0-0, C	18.4, 0-0, C	18.4, 0-0, C	18.4, 90-20, C	109.6, 90-20, C
	IV	-------------	---------------	17.1, 0-0, C	17.1, 0-0, N	136.0, 0-0, N
15	II	4.7, 0-0, C	4.7, 90-20, C	4.7, 0-0, C	4.7, 90-20, C	27.2, 90-20, C
	III	11.9, 0-0, C	11.9, 0-0, C	11.9, 0-0, C	11.9, 90-20, C	71.7, 90-20, C
	IV	-------------	---------------	15.0, 0-0, C	15.0, 0-0, C	120.7, 0-0, N
16	II	8.8, 0-0, N	8.8, 90-20, C	8.8, 0-0, N	8.8, 90-20, N	51.5, 90-20, C
	III	7.0, 0-0, C	7.0, 0-0, C	7.0, 0-0, C	7.0, 90-20, C	42.7, 90-20, C
	IV	-------------	---------------	5.7, 0-0, C	5.7, 0-0, C	46.1, 0-0, N
17	II	8.8, 0-0, C	8.8, 90-20, C	8.8, 0-0, C	8.8, 90-20, C	53.9, 90-20, C
	III	9.8, 0-0, C	9.8, 0-0, C	9.8, 0-0, C	9.8, 90-20, C	57.8, 90-20, C
	IV	-------------	---------------	10.1, 0-0, C	10.1, 0-0, C	81.3, 0-0, N
18	II	5.7, 0-0, N	5.7, 90-20, C	5.7, 0-0, C	5.7, 90-20, C	34.4, 90-20, C
	III	5.9, 0-0, C	5.9, 0-0, C	5.9, 0-0, C	5.9, 90-20, C	35.7, 90-20, C
	IV	-------------	---------------	6.2, 0-0, C	6.2, 0-0, C	49.5, 0-0, N

TABLE 16. (continued)

Subwatershed No.	Land Class	Optimal Strategies--Crop Area (ha), Nitrogen and Phosphorus Fertilizer (kg/ha), Conservation or Nonconservation (C or N)				
		Grain Sorghum	Cotton	Wheat	Hay	Pasture
19	II	25.6, 0-0, C	25.6, 90-20, C	25.6, 0-0, C	25.6, 90-20, C	153.3, 90-20, C
	III	19.7, 0-0, C	19.7, 0-0, C	19.7, 0-0, C	19.7, 90-20, C	118.1, 90-20, C
	IV	-------------	---------------	2.3, 0-0, C	2.3, 0-0, C	19.2, 0-0, N
20	II	26.4, 0-0, C	26.4, 90-20, C	26.4, 0-0, C	26.4, 90-20, C	157.5, 90-20, C
	III	18.9, 0-0, C	18.9, 0-0, C	18.9, 0-0, C	18.9, 90-20, C	113.7, 90-20, C
	IV	-------------	---------------	8.3, 0-0, C	8.3, 0-0, C	67.1, 0-0, N
21	II	14.7, 0-0, N	14.7, 90-20, C	14.7, 0-0, C	14.7, 90-20, N	89.1, 90-20, C
	III	15.0, 0-0, C	15.0, 0-0, C	15.0, 0-0, C	15.0, 90-20, C	89.9, 90-20, C
	IV	-------------	---------------	15.3, 0-0, C	15.3, 0-0, C	121.2, 0-0, N
22	II	61.9, 0-0, N	61.9, 90-20, C	61.9, 0-0, C	61.9, 90-20, N	371.9, 90-20, C
	III	4.4, 0-0, C	4.4, 0-0, C	4.4, 0-0, C	4.4, 90-20, C	26.7, 90-20, C
	IV	-------------	---------------	8.8, 0-0, C	8.8, 0-0, C	69.4, 0-0, N

different crops. As an example, one group of decision makers might want to specify at least 0.5 pasture, 0.2 hay, and 0.2 wheat for class-II land. These decision makers would likely be in the cattle business and need a considerable amount of land for grazing and growing hay each year. However, they allow some area for crop production if crop prices are good, and wheat can be used for grazing or harvested for grain. Other decision makers may have a large investment in equipment used for crop production, their land may not be fenced, or there may be other reasons that they would want to allocate a large portion of their land to crop production.

SUMMARY AND CONCLUSIONS

Models were developed for predicting daily sediment, phosphorus, and nitrogen yields from small watersheds; routing the yields through large watersheds; and determining the optimal operating policy of the large watershed. The model for predicting daily sediment, phosphorus, and nitrogen was developed by refining existing models and building new ones when existing models were considered inadequate. Daily sediment yield, predicted with MUSLE, requires estimates of daily runoff volume and peak runoff rate. A water-yield model, based on SCS curve numbers and a soil moisture index, is used to predict daily runoff volumes. The water-yield model was refined by replacing the climatic index (lake evaporation) with daily consumptive water use for individual crops. This model must be calibrated on a gaged watershed and can then be used to extend short periods of record for the calibrated watershed or to predict water yield for nearby ungaged watersheds. Peak runoff rate is predicted with HYMO. To save computing time, peak runoff rates are predicted for only a few storms and related to runoff volume. Thus, peak rate can be determined rapidly for each daily runoff volume.

Sediment-yield prediction with MUSLE was refined by dividing subwatersheds into land-capability classes and computing the product of the K, C, P, and LS factors for each land class. This allows the determination of an area weighted K C P LS for each subwatershed. The technique is convenient because land-capability classes have characteristic soils, land uses, and slopes. It is also more accurate than short-cut methods that determine area weighted values of the individual factors and compute their product.

The phosphorus yield model is based on a phosphorus-loading function for predicting long-term average annual phosphorus yields. Here the loading function was adapted to individual storm prediction by simulating daily phosphorus concentration in the soil, using daily predicted sediment yields, and computing enrichment ratios. The enrichment ratio (the ratio of phosphorus concentration in the sediment to that of the soil) is estimated by the ratio of the specific surface area of the sediment to that of the soil. Specific surface area is a function of the particle-size distribution and the percent montmorillonite in the soil or sediment.

A phosphorus balance model was developed to predict the daily concentration of phosphorus in the soil. The model was designed to operate within the top 15 cm of soil. Components of the model include phosphorus addition through fertilizer and crop residue, phosphorus loss in runoff, and phosphorus use by the crop. Phosphorus use was related to crop growth, and crop growth was assumed proportional to consumptive water use.

The nitrogen model simulates both organic and inorganic nitrogen yields associated with sediment and runoff. Like the phosphorus model, the organic nitrogen model is based on a loading function. The loading-function enrichment ratio is estimated as the ratio of the percent of the sediment to the percent of the soil with particle sizes $\leq 1\ \mu$. To predict soil nitrogen concentration, a nitrogen balance model was developed and attached to the runoff and sediment model. The nitrogen balance model is also used in predicting nitrate yield. Components of the nitrogen balance include immobilization, mineralization, denitrification, leaching, fertilization, nitrogen in crop residue, losses in runoff, use by the crop, and rainfall contributions. The top 90 cm of soil is divided into seven storages for modeling the nitrogen balance. Flow through the storages is computed with a modified, variable, storage-coefficient routing equation. As water flows through a soil storage, the nitrate concentration changes continuously, reflecting the amount of nitrate leaving the storage. Nitrate yield is a function of runoff volume and top-soil-storage nitrate concentration. Denitrification, immobilization, and mineralization are microbial processes that are related to soil moisture and temperature. Like phosphorus use by the crop, nitrogen use was related to crop growth. Nitrogen use was distributed according to root growth in the various storages. Root growth in each storage was assumed proportional to water use within the storage. Water use was distributed exponentially with root depth.

A model was developed for routing sediment, phosphorus, and nitrogen through streams and valleys of large watersheds. The model is based on a sediment-routing model and phosphorus- and nitrogen-loading functions. The sediment-routing model, derived from MUSLE and a first-order decay function of travel time and particle size, was refined here by replacing the median, sediment-particle size with the entire particle-size distribution. Also, a technique was developed for determining the routing coefficient for each routing reach, instead of using one routing coefficient for the entire watershed as in the original method. The new routing equation eliminates the need for determining travel time and simplifies the determination of the routing coefficient.

Phosphorus and nitrogen routing is accomplished by using the routed sediment particle-size distributions to calculate the enrichment ratios in the loading functions. Concentrations of phosphorus and nitrogen are computed at the end of each routing reach by weighting the concentrations from various sources according to their sediment yields. Sources considered are the inflow at the upstream end of the reach and contributions within the reach from the subwatershed, channel, and floodplain. Nitrate that is carried by the runoff water is considered a conservative material for the duration of an individual flood. Thus, nitrate routing is simply a matter of determining the outflow concentration of nitrate, based on the concentrations and amounts of flow of the nitrate sources.

SPNM, a problem-oriented computer language, was developed for modeling sediment, phosphorus, and nitrogen yields through large agricultural watersheds. The language consists of a main program and 12 subroutines written in FORTRAN IV, but it can be used by hydrologists and environmental engineers with little knowledge of computer programming. SPNM was designed to predict sediment, phosphorus, and nitrogen yields for individual storms on small watersheds, and to route these yields through streams and valleys of large watersheds. These features makes SPNM useful in water resources planning and in research. Submodels used in SPNM include MUSLE, the phosphorus- and nitrogen-loading functions, the nitrate prediction equation (51), and sediment, phosphorus, and nitrogen routing. SPNM provides 11 commands that can be used in any sequence for application to any watershed. Data are input with the commands in a free format.

A model, based on decision analysis and linear programming, was developed to determine optimal operating policies for large agricultural watersheds. The objective of the model is to maximize a multiattribute utility function within constraints imposed by sediment, phosphorus, and nitrogen yield; watershed area; and cropping system. Strategies that are acceptable to decision makers are selected for each land class within each subwatershed. A strategy specifies the crop to be grown on the area, the fertilizer application rate, and the type of conservation system. The utility of each strategy is determined by analyzing attributes that are important to the decision makers. Utility values allow the combination of attributes with various units into a single function.

Inputs for the optimization model are provided by the model for simulating daily sediment, phosphorus, and nitrogen yields and by SPNM. Long-term average annual values are used in the optimization model because crop production and costs are determined on an annual basis. Also annual values are more meaningful in expressing sediment, phosphorus, and nitrogen constraints.

Strategies are evaluated, according to the utility of their attributes. Utility functions are developed for each attribute and combined into an additive multiattribute utility function. To determine the utility of a strategy, the average annual value of each attribute must be estimated for the strategy. Usually, a considerable amount of subjective information must be used. Techniques are provided for obtaining subjective probability distributions from the decision makers. As each season's crops are harvested, the subjective probability distributions are adjusted with Bayes' Theorem using the additional objective information.

Linear programming is used to determine the optimal combination of strategies for the entire watershed. The LP model determines the amount of land allocated to each strategy to obtain maximum utility for the entire watershed. The LP problem is constrained so that all the land in the watershed is assigned a strategy and more than one crop is grown on each land class. Other constraints include sediment, phosphorus, organic nitrogen, and nitrate yields at the watershed outlet. LP is convenient and has a user-oriented program available.

The overall combination of models was demonstrated with data from Little Elm Creek watershed. The model was applied to the 101-km^2 uncontrolled drainage area, and results were compared with measured values. In calibrating the runoff volume model, measured and predicted volumes were compared and gave $R^2 = 0.74$ for monthly amounts and $R^2 = 0.80$ for annual amounts. The routed average annual sediment yield (3.62 t/ha) compared closely with the measured yield (3.85 t/ha). Also, the routed and measured particle-size distributions were very similar. More data are needed to determine the accuracy of the phosphorus and nitrogen routing models. There is no phosphorus and nitrogen data available for Little Elm Creek. However, data from eight watersheds near Riesel, Texas, indicated that the models are giving realistic results.

In demonstrating the optimization model, gross income, production costs, dependability, and disease, insect, and weed control were the attributes selected for developing the multiattribute utility function. Individual attribute utility functions were assumed linear. Both objective and subjective information was used in evaluating the attributes for each strategy. There were 88 strategies for each of 22 subwatersheds and 356 constraints. Thus, the LP problem had 1936 columns and 356 rows. The optimal solution gave a total utility value of 23.73. Converting utility back to original units showed that the decision makers could net $58/ha/yr. The active water quality constraints were nitrate (2.0 ppm) and phosphorus (2.25 kg/ha). Sediment yield (1.58 t/ha) and organic nitrogen yield (4.32 kg/ha) were below their constraint levels of 3.0 t/ha and 5.5 kg/ha.

The combination of models should be useful in managing large agricultural watersheds to obtain maximum utility and to meet water-quality standards. Use of the models should lead to improved agricultural efficiency because soil will be conserved, fertilizer will not be applied at excessive rates, and crops will be grown on the proper soils and locations within the watershed. Individual components of the model should be useful for other purposes. MUSLE is useful in designing small reservoirs, and the refinements added here should increase its effectiveness. The model for simulating daily sediment, phosphorus, and nitrogen should be useful in determining the optimal time, rate, and depth of fertilizer application. It should also be useful in research involving plant response to fertilizer, residue management, the MUSLE crop management factor, etc. SPNM is a convenient tool for use in water resources planning. Two important applications are in designing large reservoirs and in assessing environmental effects. Probably, the most important feature of SPNM is the convenience of testing and refining new submodels. When a new submodel is added, inputs are supplied by the existing SPNM commands.

In its present form, the combined simulation-routing-optimization model is operational and gave realistic results for the Little Elm Creek watershed. More tests are needed to properly evaluate the model's effectiveness. Future plans include continued testing and refinement of individual model components.

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NOTATION

a	constraint coefficient.
$\underline{A}$	constraint coefficient matrix.
AQE	average annual runoff energy factor.
b_1	coefficient used in relating runoff volume to peak rate.
b_2	exponent used in relating runoff volume to peak rate.
c_c	concentration of carbon in the soil, parts per million.
c_{NO3}	concentration of nitrate, parts per million.
c_{ON}	concentration of organic nitrogen in the soil, parts per million.
c_p	concentration of phosphorus in the soil, parts per million.
c_R	concentration of nitrate in rainfall, parts per million.
C	crop management factor.
CDN	denitrification constant.
CI	climatic index, centimeters.
CIM	immobilization constant.
Cl	percent clay in a particular soil.
CMN	mineralization constant.
CN	SCS curve number.
CNII	II-condition SCS curve number.
CU	consumptive use of water of a crop in centimeters.
d	sediment particle diameter, microns.
D	rainfall duration, hours.
DA	drainage area, square kilometers.
DC	average annual crop loss caused by diseases in percent.
DN	amount of nitrate lost through denitrification, kilograms per hectare.

DP	annual coefficient of variation of crop yield.
DR	portion of sediment yield from a subwatershed that is delivered to the watershed outlet.
DRN	amount of nitrate that drains below 90 centimeters, kilograms per hectare.
E	estimated average percent slope for a particular land class.
EP	number of extreme points on a contour.
ER_N	enrichment ratio for organic nitrogen.
ER_P	enrichment ratio for total phosphorus.
F	infiltration rate, centimeters per day.
g	rainfall intensity, centimeters per hour.
G	peak flowrate prediction parameter dependent upon unit hydrograph shape.
GI	average annual gross income in dollars per hectare.
H	contour interval, meters.
$\underline{I}$	identity matrix.
IC	average annual crop loss caused by insects in percent.
IM	amount of nitrate immobilized, kilograms per hectare.
J	peak flowrate prediction constant.
K	soil erodibility factor.
L	average watershed slope length, meters.
LC	total length of contours on a watershed, meters.
LS	watershed slope length and gradient factor.
MM	montmorillonite contained in clay, percent.
MN	amount of organic nitrogen converted to nitrate through mineralization, kilograms per hectare.
MN_a	average daily mineralization, kilograms per hectare.

n	dimensionless unit hydrograph shape parameter.
N_d	number of times grid lines in one direction cross a contour.
NF	amount of nitrate fertilizer applied, kilograms per hectare.
NO3	amount of nitrate contained in the top 90 centimeters of soil, kilograms per hectare.
O	soil storage outflow rate, centimeters per day.
ON	amount of organic nitrogen contained in the top 90 centimeters of soil, kilograms per hectare.
ON_a	average daily content of organic nitrogen in the top 90 centimeters of soil, kilograms per hectare.
$\underline{P_o}$	vector that gives the magnitude of the constraints.
P	erosion control practice factor.
PC	average annual production cost in dollars per hectare.
PF	amount of phosphorus fertilizer applied, kilograms per hectare.
PN_R	organic nitrogen in crop residue, percent.
PN_y	organic nitrogen in crop yield, percent.
PP	total phosphorus in crop residue, percent.
PSG	maximum potential annual plant growth, kilograms per hectare.
PT	total phosphorus contained in the top 15 centimeters of soil, kilograms per hectare.
PUN	maximum potential annual nitrate use by the crop, kilograms per hectare.
PUP	maximum potential annual phosphorus use by the crop, kilograms per hectare.
q	runoff rate, cubic meters per second.

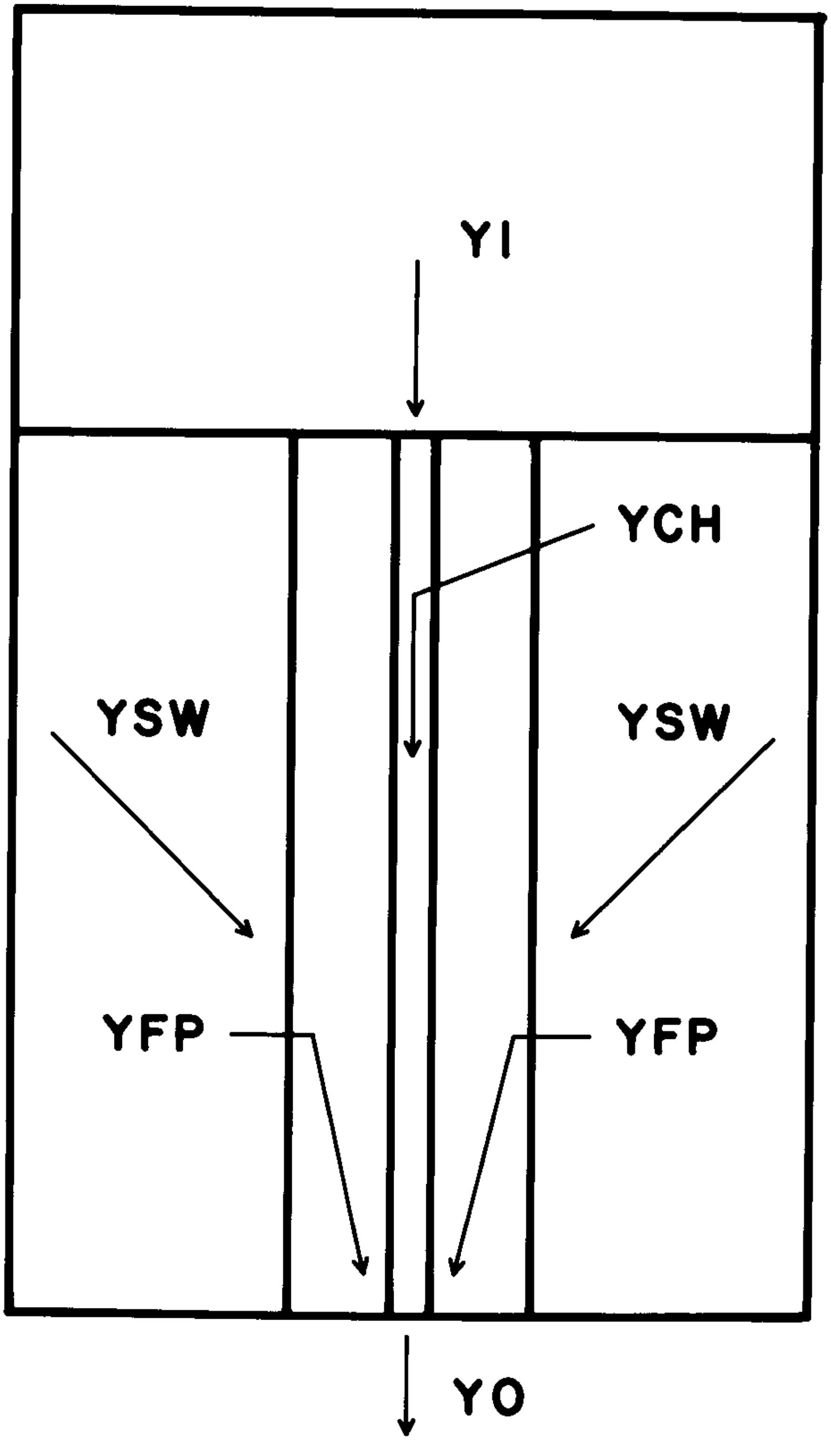
q_0	runoff rate at the inflection point on the recession limb of the unit hydrograph, cubic meters per second.
q_p	peak runoff rate, cubic meters per second.
Q	runoff volume, centimeters.
r	retention rate, centimeters per hour.
r_c	basic low retention rate, centimeters per hour.
r_u	maximum retention rate, centimeters per hour.
r_0	retention rate at the beginning of a storm, centimeters per hour.
R	amount of rainfall, centimeters.
R^2	multiple correlation coefficient squared.
RD	root depth, fraction of 90 centimeter storage depth.
RN	amount of nitrate contributed by rainfall, kilograms per hectare.
RS	total crop residue including roots, kilograms per hectare.
RSON	amount of organic nitrogen contained in crop residue, kilograms per hectare.
RSP	amount of phosphorus contained in crop residue, kilograms per hectare.
RW	root weight, kilograms per hectare.
RY	routed sediment yield for the entire watershed, tonnes.
s	curve number retention parameter, centimeters.
s_m	maximum retention capacity of the soil, centimeters.
sg	stage of crop growth, dimensionless.
S	average watershed slope, percent.
S_d	land slope in one direction, percent.
S_L	average land slope along the length of the watershed, percent.

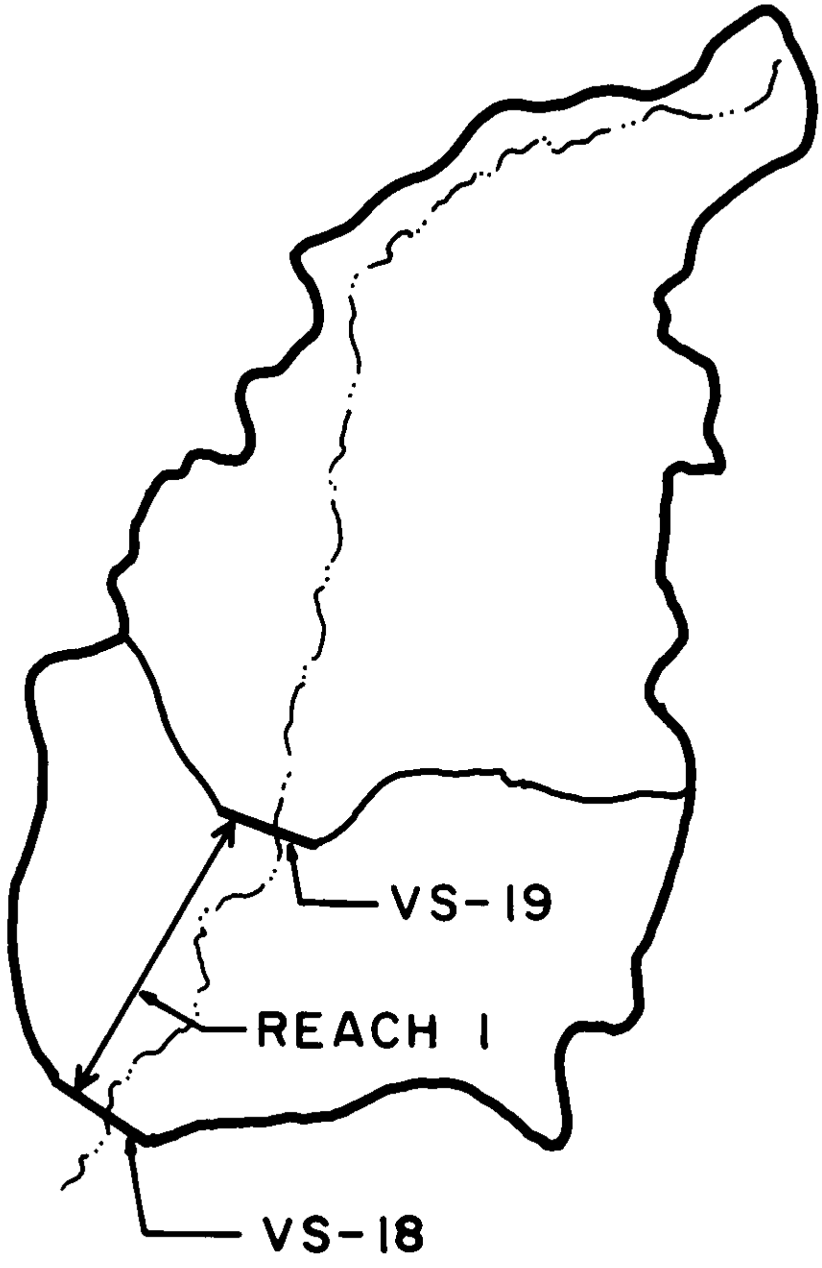
S_w	average land slope along the width of the watershed, percent.
Sa	sand content of the soil, percent.
SG	daily amount of crop growth, kilograms per hectare.
Si	silt content of the soil, percent.
SM	soil moisture index, centimeters.
SS	specific surface area of sediment particles of a particular size, square meters per gram.
SSS	specific surface area of the entire soil particle-size distribution, square meters per gram.
SSY	specific surface area of the entire sediment particle-size distribution, square meters per gram.
ST	water content of the soil, centimeters.
ST_a	average daily water content of the soil, centimeters.
T	average air temperature for each day, degrees centigrade.
T_a	average annual air temperatures, degrees centigrade.
TT	travel time of water through a soil storage or routing reach, hours.
un	accumulated nitrate use by the crop, dimensionless.
up	accumulated phosphorus use by the crop, dimensionless.
u ($\underline{v}$)	utility of attribute vector v, 0-1.
uw	water use by crop within a particular soil storage, centimeters.
UL	storage capacity of the soil, centimeters.
UN	nitrate use by the crop, kilograms per hectare.
UP	phosphorus use by the crop, kilograms per hectare.
UW	water use by the crop, centimeters.
V	runoff volume, cubic meters.

w	weighting factor assigned to an attribute, one is assigned to the most important attribute and fractions to the other attributes.
W	retention function scaling coefficient.
WC	average annual crop loss caused by weeds in percent.
x	variable expressing the magnitude of events in a probability distribution.
X_d	total length of grid lines within the watershed in one direction, meters.
Y	sediment yield, tonnes.
YCH	sediment yield from the channel, tonnes.
YFP	sediment yield from the floodplain, tonnes.
YI	sediment yield at the reach inlet, tonnes.
YLD	crop yield, kilograms per hectare.
YNO3	nitrate yield, kilograms.
YO	sediment yield at the reach outlet, tonnes.
YON	organic nitrogen yield, kilograms.
YP	total phosphorus yield, kilograms.
YSW	sediment yield from subwatershed between upper and lower end of routing reach, tonnes.
YU	upland sediment production, tonnes.
z	watershed area allocated to each strategy.
z_o	utility of the objective function of the LP model.
ZL	subwatershed relief length ratio, meters per kilometer.
α	soil moisture index depletion parameter.
β	sediment-routing parameter.

γ	portion of the watershed covered by a particular land capability class.
δ	attribute weighting factor scaled such that $0 \leq \delta_i \leq 1$ and $\sum_{i=1}^{M} \delta_i = 1$.
ζ_s	percent of soil with a one micron particle size.
ζ_y	percent of sediment with a one micron particle size.
η	portion of a particular land class covered by a particular crop.
λ	unit hydrograph recession constant from the inflection point, Ψ_0, to $\Psi = \Psi_0 + 2\lambda$, hours.
λ_1	unit hydrograph recession constant from $\Psi = \Psi_0 + 2\lambda$ to ∞, hours.
Λ	accumulated time between storms, days.
ξ	dimensionless retention function parameter.
ρ_s	weight of soil in kilograms per hectare.
ρ_w	weight of water in kilograms per centimeter covering one hectare.
σ	storage routing coefficient, dimensionless.
τ	time, days.
ϕ	length exponent in LS prediction equation.
$\Phi(x)$	the probability of x occurring.
$\Phi(x\|\theta)$	the probability of the value x given the sample θ.
$\Phi(\theta\|x)$	the probability of obtaining a sample θ given the value of x.
χ	crop water use rate constant.
Ψ	time, hours.
Ψ_p	time to peak of the unit hydrograph, hours.

Ψ_0	time at the inflection point on the recession limb of the unit hydrograph, hours.
ω	portion of the particle-size distribution represented by a particular diameter.
Ω_ℓ^k	the number of combinations that the sample may be composed of.
$\Sigma\ \nu^2$	error sum of squares.
$\Sigma\ y^2$	corrected sum of squares of the measured variable.


YI
YCH
YSW
YSW
YFP
YFP
YO


VS-19
REACH 1
VS-18

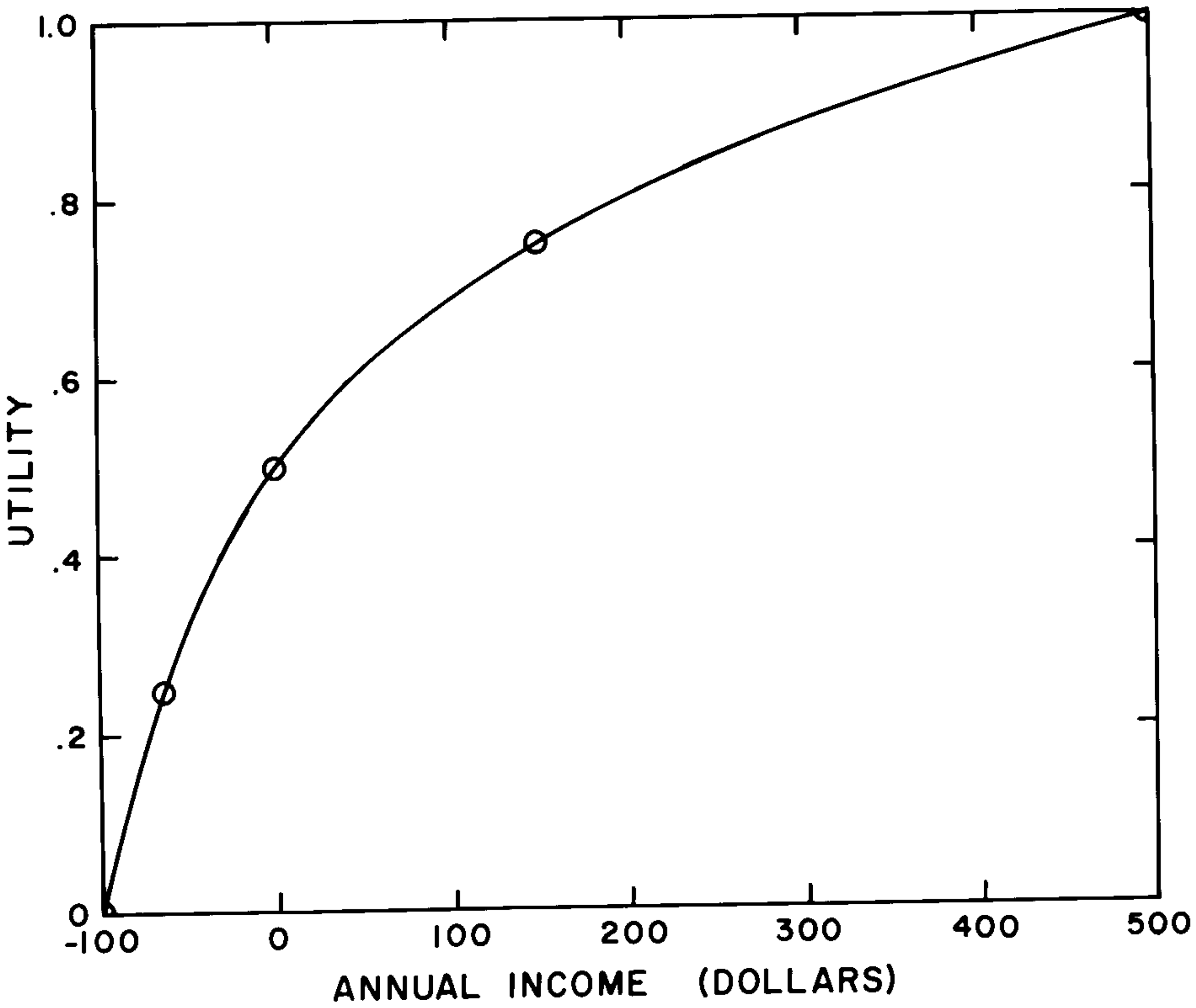
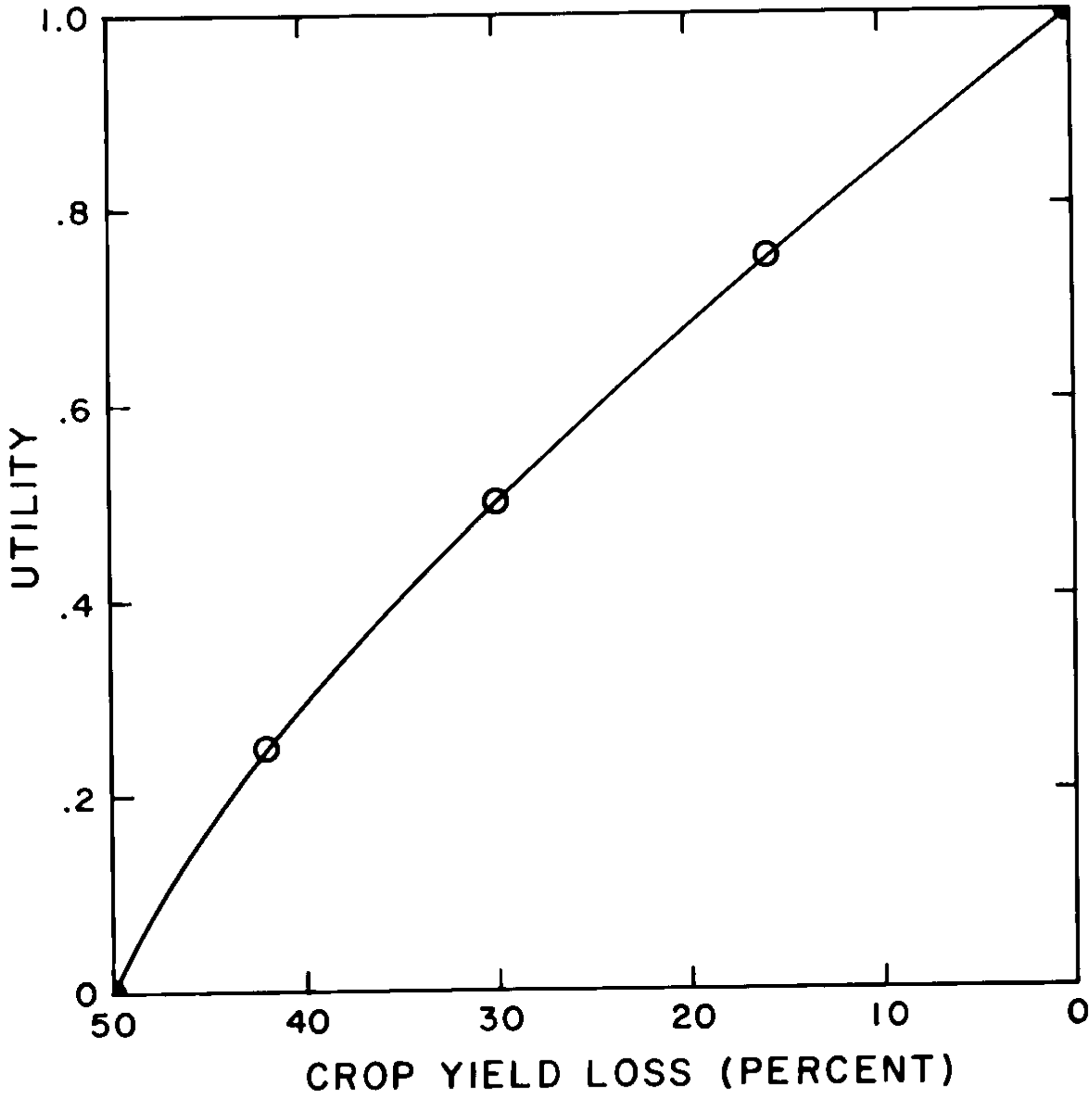
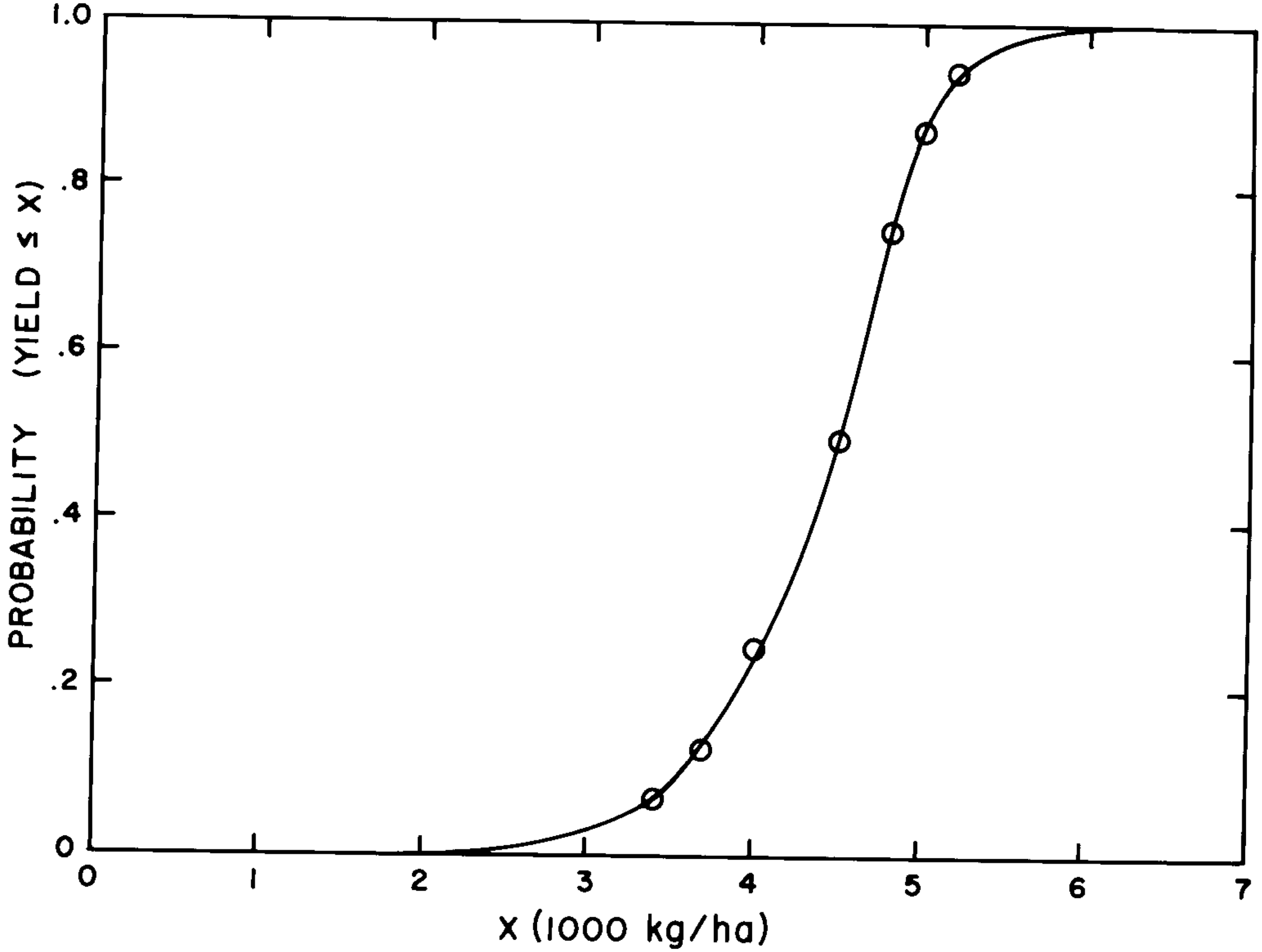
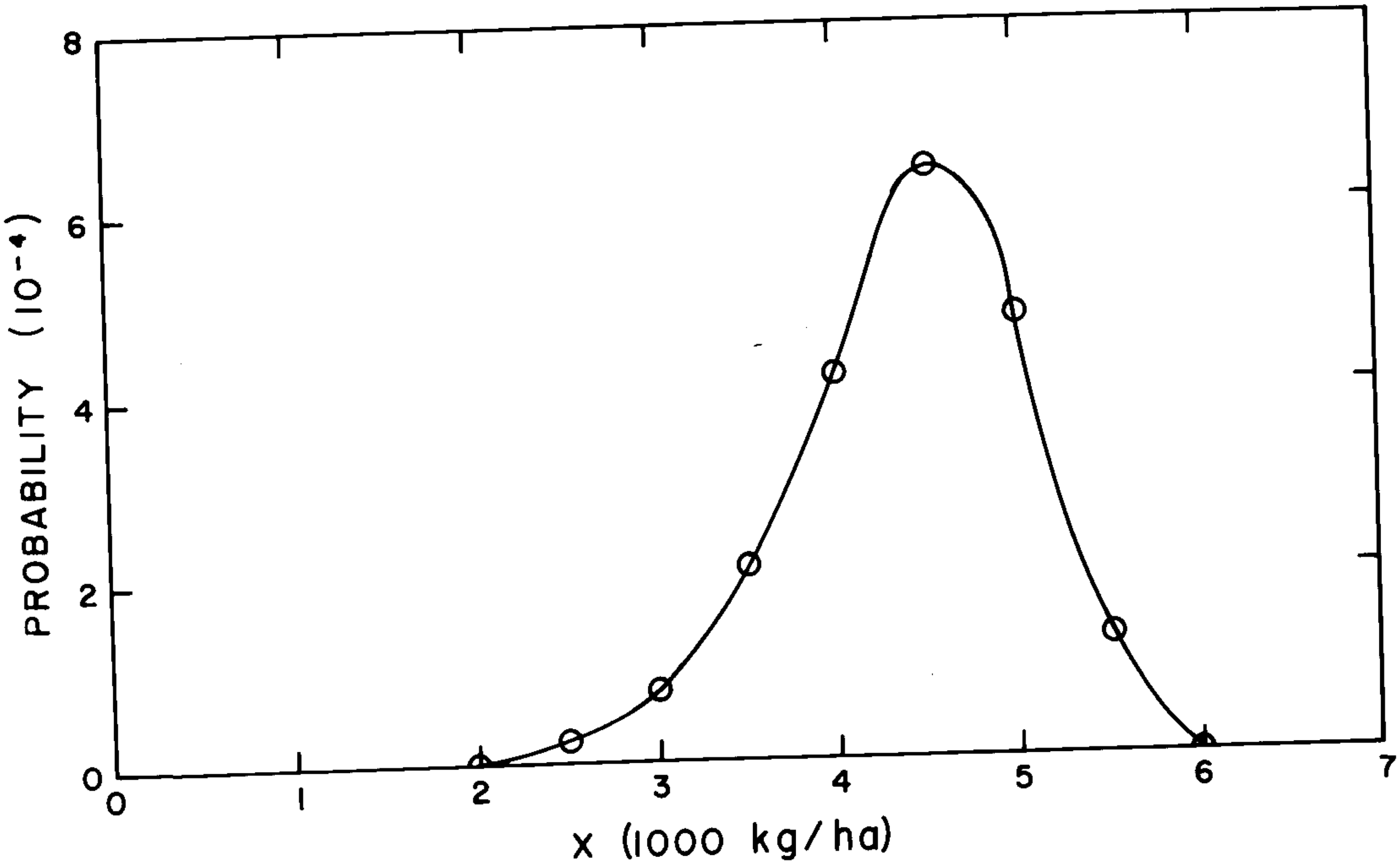


Figure 3. Net Income Utility-Function for Risk Averse Decision Makers.



1.0
.8
.6
.4
.2
0
UTILITY
50
40
30
20
10
0
CROP YIELD LOSS (PERCENT)


PROBABILITY (YIELD ≤ X)
1.0
.8
.6
.4
.2
0
0
1
2
3
4
5
6
7
X (1000 kg/ha)



PROBABILITY (10⁻⁴)
0
2
4
6
8
0
1
2
3
4
5
6
7
x (1000 kg/ha)