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The Economic Value of Irrigation Water in the Western United States: An Application to Ridge Regression

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ABSTRACT

Reliable estimates of the demand characteristics of irrigation water are crucial to successful water policy formulation in the West. Although various studies concerning irrigation water demand exist in the literature, most are somewhat limited in scope and present their results in varied forms. Thus, comparison of results presents a problem. This study follows a more comprehensive approach by determining the demand characteristics, viz., water value, demand elasticities, etc., for major western irrigated regions. These results should prove useful in water policy formulation and evaluation.

Eleven homogeneous regions were identified as major irrigated areas of the West. Agricultural output (in value terms) in each region was hypothesized to take the form of a multiplicative function with nine domain variables, i.e., irrigation water applied, value of land and buildings, hired labor expenditures, fuel and lubricant expenditures, fertilizer and lime expenditures, feed expenditures, value of machinery inventory, value of livestock inventory and miscellaneous expenditures.

Using *1969 Census of Agriculture* data, each regional function was statistically fit using both ordinary least squares (OLS) and ridge regression. As expected, parameter estimates under OLS were highly unstable due to high correlations among the explanatory variables (multicollinearity). One-third of the estimated coefficients took on nonsensical signs and the standard errors were generally high.

To circumvent the multicollinearity problem ridge regression was employed. While admittedly a biased estimation technique, the credibility of the estimates appeared to increase. All parameter estimates, except for one out of 99, took on the expected positive sign and the standard errors were decreased in every case. Returns to scale were estimated to vary from a high of 1.200 in the Northwestern Ogallala to a low of .887 in the Lower Rio Grande Basin. Overall, the functions estimated with ridge regression were more compatible with theoretical expectations than those based on OLS estimates.

From the fitted production functions, the demand for irrigation water was derived for the long run, two intermediate runs and the short run. Generally, water demand was found to be slightly elastic for all lengths of run considered with the more elastic demand in the Desert Southwest and Upper Colorado Basin, and slightly less elastic demand in the Snake-Columbia, Lower Rio Grande Basin and Northwestern Ogallala. The quantity of water applied was found to be most sensitive to product price in the Central California, Desert Southwest, Upper Colorado Basin and Northwestern Ogallala Regions. In terms of cross-factor effects, water application rates were found to be most responsive to changes in the prices of land and labor for all regions.

Marginal irrigation water values for each length of run considered were estimated for 1969 at the respective regional mean values of water usage, fixed input levels and variable input prices. These estimated values varied from a high of $27.79 for the long run in Central California to a low of $1.71 in the short run for the

Snake-Columbia Basin. It appears that the value estimates may be distorted in some instances due to the influence of livestock variables in the model. Subsequent research should attempt to correct this deficiency.

Projections of values for 1974 (a census year) and 1978 were made with the assumption of no change in technology and level of "fixed" input and water usage since 1969. Though a somewhat gross projection, water values were found to increase until 1974 and then decrease in 1978. These projections should serve as a basis for possible later validation by other researchers.

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CHAPTER I

INTRODUCTION

During the mid-1970s there has been a resurgence of interest in water and water related problems. Various factors--increasing pumping costs, drought conditions in the West and Midwest and declining aquifers--emphasize the need for further evaluation of current water usage. Such an evaluation requires reliable estimates of the economic parameters of water in each of its varied uses, viz., agriculture, residential, industry, recreation, etc.

Since agriculture is the single largest consumer of water supplies in the United States (National Water Commission, p. 7), there is a need to understand more fully the economics of irrigated agriculture. Numerous studies concerned with assessing the value of irrigation water were conducted in the 1960s and early 1970s. Most of these studies were limited in scope to a regional or local situation, and for the most part, involved different estimation techniques and different data sources making comparison of various results difficult.[1] Therefore, it should be helpful for policy makers to have a comprehensive study based on a single estimation technique and data source to discern regional economic characteristics and differences of irrigation water

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[1]Difficulties, conceptual and otherwise, were noted in this regard by Young and Gray in their background work on agricultural water values for the National Water Commission.

demand. These estimates should be more relevant for relative comparison of water demand characteristics among regions.

Basic Approaches to Water Valuation

Considerable economic research has been concerned with the value of water in irrigated agriculture. Previous studies can be categorized as basically following one of four approaches, i.e., (1) use of water-market price-quantity data, (2) linear programming, (3) budgeting and (4) production function estimation with subsequent water value derivation. The following sections briefly discuss each approach and summarize the results of these important previous studies.

Market Observations

Since water for agricultural uses is rarely exchanged in a well defined market, few instances of direct measurement of value are available. Gardner and Fullerton studied the rental market for water in the Sevier Basin of Utah. They reported an average price paid of $9.50 per acre foot for a period from 1950-1964. Anderson also analyzed prices paid data for six irrigation companies in the South Platte Basin of Colorado. His findings revealed water prices (excluding delivery expenses) ranging from $3.50 to $4.85 per acre foot.

Linear Programming

Probably the most extensively used technique of discerning water values is linear programming. In this approach, a profit (cost) function is maximized (minimized), subject to various economic and physical constraints. A solution of this type, in addition to optimal input combinations, yields estimates of the marginal value productivities (shadow prices) of each input. Several studies of this type have been summarized by Young and Gray in a report done for the National Water

Commission (table 1). In particular, they note research utilizing linear programming methodology by Harman and Whittlesey; Anderson, *et.al.;* McLeod; Lindeborg; Sorenson and Clark; and Young. These studies reported irrigation water values ranging from $2.00 per acre foot for Wyoming pastureland to $41.00 per cropland acre in the Lower Colorado Basin.

Table 1. MVP Estimates for Irrigation Water from Selected Linear Programming Studies.[a]

Area	MVP($/ac.ft.)
Western Colorado	$.39-$41[b]
Southeastern Wyoming (Meadow)	$2.00
East Central Wyoming	$12.50-$17.50[c]
Western Oklahoma	$7.50
Idaho	$17.60
North Dakota	$18.38[d]

[a]Summarized from Young and Gray.

[b]High value is for high valued crops grown under near "ideal conditions".

[c]Values due to second and first acre foot applications, respectively.

[d]Includes return to management.

In addition to these early attempts, more recent work has been published using linear programming. Griffin found the value of irrigation water in Southwestern North Dakota to range from $17.09 to $94.62 per acre foot under alternative output price assumptions. Butcher, *et.al.*, calculated the marginal value productivity of water for various crops in the Yakima River Basin in Washington. Water values reported in their study ranged from -$.36 per acre foot for corn to $43.68 per acre foot for pears. Finally, Shumway found values ranging from $3 to $17.00 per acre foot for a subregion of the Western San Joaquin Valley of California.

Budgeting

Another widely used approach for evaluating the value of irrigation water, which is essentially a simple linear program, is budgeting. An example of estimation using this approach is a study by Grubb. Grubb estimated the imputed residual value of irrigation water on the Texas High Plains to be $27 per acre foot. Lacewell, Sprott and Beattie used budgeting methods to compute the ability-to-pay (value) for water in all of the major irrigated regions of Texas, including the High Plains. Individual estimates were made for 20 crops, assuming alternative product and input prices. Their findings for the High Plains region revealed water values ranging from $47 to $121 per acre foot for cotton and from $11 to $59 per acre foot for soybeans.

Production Function Analysis

Yet another approach to water value determination involves derivation from statistically fitted production functions. In this method, a particular functional form of the agricultural production process is specified and statistically estimated using regression procedures. Marginal productivities for each input involved in the process are calculated by differentiating the production function with respect to each specific input. Using this approach on micro level data, Miller and Boersma studied the value of irrigation water in Oregon. Fitting data from controlled agronomic experiments, the marginal productivities of various irrigation levels on corn were determined. Water was valued in excess of $120 per acre foot for the first few inches applied, and reached $0 at approximately 18 acre inches of water applied.

Taking a more comprehensive approach at a more aggregated level, Ruttan used production function analysis to determine the value of an irrigated acre from *Census of Agriculture* data. While Ruttan's theoretical framework was most appropriate, several statistical complications in his empirical methodology (regional heterogeneity, overaggregation and omission of relevant variables) were noted by Hock. No values for water *per se* were given in Ruttan's study; however, the difference between the marginal productivities of irrigated and nonirrigated land may be imputed as a marginal return to water and management.

Applying a modified Ruttan framework, Beattie and co-workers attempted to ascertain the value of irrigation water in the High Plains of Texas and New Mexico based on *1964 Census of Agriculture* data. Results of their study revealed water values of $33.32 per acre foot for cotton and $20.29 per acre foot for all noncotton crops.

Relationship to Previous Research

This study involves direct estimation of production functions and subsequent derivation of the value of water as a productive input in agriculture. A Ruttan-Beattie framework was followed with several methodological improvements. These include (1) improved variable definitions, (2) improved study region delineation, (3) a Cobb-Douglas model specification and (4) parameter estimation by ridge regression. A brief discussion of each improvement follows.

Variable Definition

The use of secondary data in research almost always constrains the analyst in the specification of variable definitions. Data in the *Census of Agriculture* are reported in their least aggregated form for counties. Thus, the reported data are quite aggregated. A problem results in that certain expenditure (input) categories include both crop and livestock components, e.g., the labor reported includes both labor used in crop production *and* that used in livestock production. Beattie, *et.al.* chose to explain the variation in crop output only by using selected aggregated input categories as explanatory variables; some of which included livestock production expenses. Such an approach could lead to imprecise estimates if the livestock components of each input are sufficiently large.

In contrast to the Beattie approach, an aggregate model specification which includes both crop and livestock output and associated inputs was formulated in this study. Even though increased aggregation makes inferences to the firm level difficult, this aggregate model eased the limitations due to the data source.

The Ruttan and Beattie works were further constrained by their data sources in that certain significant inputs, e.g., machinery values and water quantities, were not reported. To compensate for this deficiency several approaches, i.e., proxy variables for the machinery input and/or indirect water productivity measurements, were used.

The *1969 Census of Agriculture* offers an improved data base for the present study, in that it reports directly both the value of machinery and the acre feet of water applied. Thus, not only will the machinery input be improved over the Beattie and Ruttan studies, but a direct measurement of the effect of water applications can be deduced. In these previous studies, water productivities were inferred from the difference between the return from irrigated land versus nonirrigated land. Having this direct measurement of the quantity of water applied also allows for the expression of the land variable in value terms. Using the value of land and buildings rather than acreage should account for quality differences in land which no doubt exist within a study region.

Study Region Delineation

Delineation of study regions is extremely crucial in estimating production functions. Ideally, a study region should be made up of counties having similar output mixes. The estimation of a single production model for such a region should yield useful results by lessening the aggregation bias relative to that for a heterogeneous region.

In the Ruttan work, study regions were chosen to conform to major river basins of the United States (p. 35). Since these basins encompass substantial acreages and cut across diverse type-of-farming areas, it appears that regional heterogeniety was a problem. By contrast, the criteria for study region delineation adopted in this study emphasized homogeneity of type-of-farming, e.g. similar crop-livestock mixes. Thus, the ill-effects of regional heterogeniety should be lessened.

Cobb-Douglas Model Specification

Various functional specifications have been used in the estimation of agricultural production functions. One approach has been to assume a linear relationship (Knight; Beattie, *et. al.*). This assumption infers that all inputs have a constant marginal productivity and that each is technically independent of all other inputs. Alternatively, a multiplicative (Cobb-Douglas) specification allows decreasing marginal productivity and input interaction. On an *a priori* basis the Cobb-Douglas function appears applicable to agricultural production, whereas a linear function is deficient for many research purposes; e.g., factor demand functions are undefined. Ruttan hypothesized a Cobb-Douglas model in his work.

Estimation by Ridge Regression

Assuming rationality on the part of the individual entrepreneurs, all coefficients in a Cobb-Douglas production function should have positive signs, i.e., positive factor elasticities and marginal productivities. However, most production input-output data is characterized by high intercorrelations among the input data series (multicollinearity). In this case estimation by ordinary least squares often gives results which are contrary to *a priori* reasoning, e.g., negative values and nonsignificant parameter estimates.

One commonly used approach to combat this problem is deletion of nonsignificant variables, e.g., see Ruttan. However, assuming that the model specification is correct, variable deletion will necessarily cause increased specification bias (Kmenta). The potential magnitude of this error in parameter estimation when variable deletion is used to circumvent multicollinearity problems can be quite large (Brown). For this reason, variable deletion was not considered as appropriate procedure for purposes of this study.

Recently ridge regression (Hoerl and Kennard; Marquandt and Snee) has been advanced to mitigate the effects of these intercorrelations (large variances, over estimated coefficients and illogical signs) by introducing nominal amounts of bias to the estimation procedure. This technique appears to be applicable in economics as well as in engineering where it was originally advanced (Brown and Beattie). As expected, multicollinearity was a problem in this study and ridge regression rather than variable selection was employed to circumvent this problem.

Purpose, Objectives and Outline of Study

The purpose of this study was to formulate an improved methodology for estimating the ability-to-pay (economic value) for irrigation water and to estimate these values for major irrigated regions in the western United States. Four specific objectives were involved:

1. To identify and delineate major irrigated regions and subregions for the 17 western states.
2. To postulate an appropriate aggregate economic model for production in irrigated agriculture.
3. To estimate the parameters of the model using data from the *U.S. Census of Agriculture*.
4. To determine factor demand, own-, cross- and product-price elasticities and economic value for water as an input in regional agricultural production processes.

These four objectives and results are addressed in the ensuing chapters. Chapter II briefly discusses study region delineation criteria and provides a description of each study region. Chapter III articulates the underlying economic theory involved in the model specification. In Chapter IV multicollinearity and ridge regression are discussed. Results are presented in Chapters V, VI and VII. The final chapter (Chapter VIII) addresses conclusions and limitations of the study with some implications for further research.

CHAPTER II

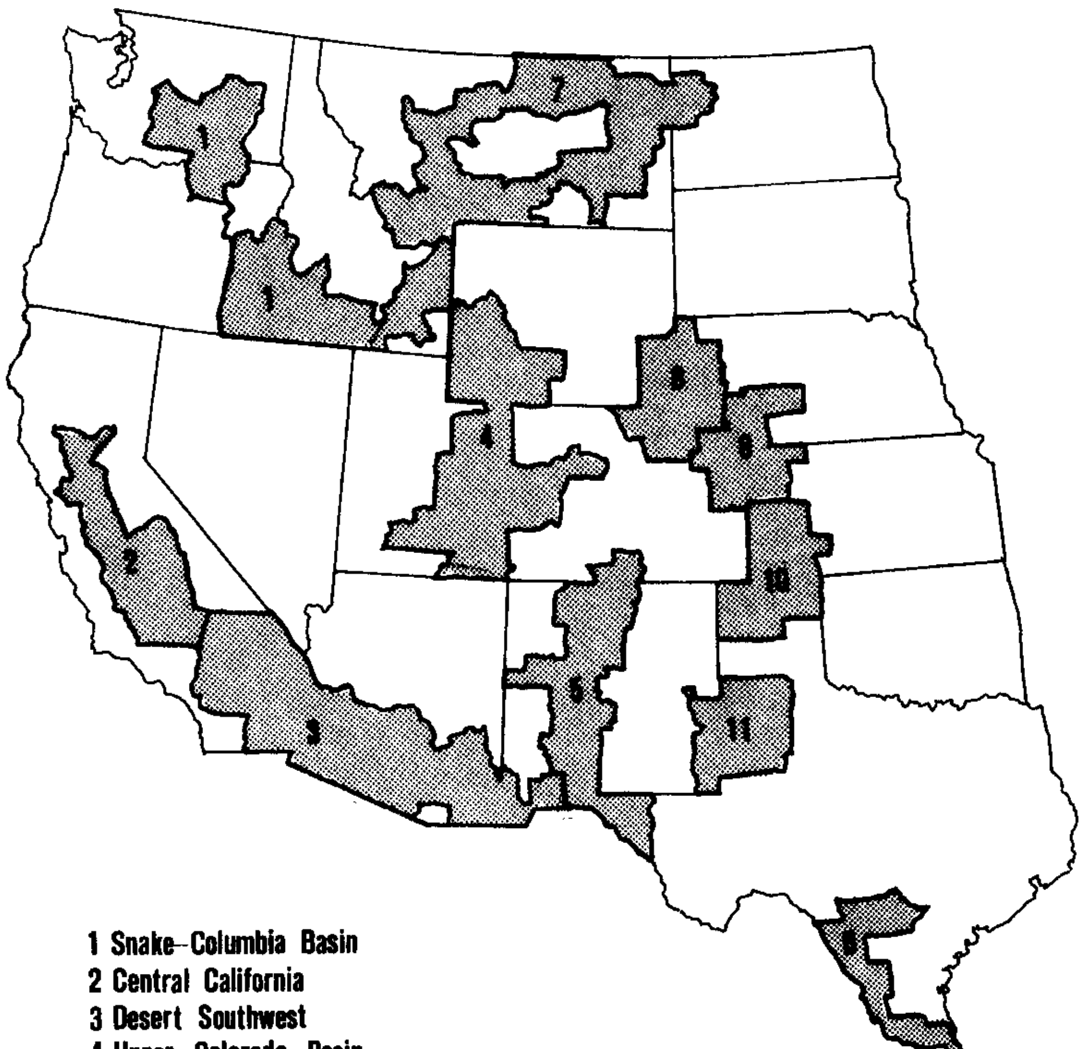
DELINEATION OF STUDY AREA

The selection of study areas stands as one of the more crucial aspects of any functional analysis. If reliable results are to be obtained, the production relationship for the entire region must refer to factors and/or commodities which are homogeneous (Heady). Therefore, especially in cross-sectional analysis, great care should be given to the selection of an area which is relatively homogeneous, but at the same time has sufficient variation in the explanatory variables to yield accurate statistical results.

In this study, relative homogeneity was achieved by careful definition of the term "major irrigated area." To be considered a major irrigated area, a region must be comprised of counties which have a minimum gross application of 5000 acre feet of irrigation water. In addition, regions were delineated such that counties within each region have relatively uniform crop and/or livestock mixes, and somewhat similar topographies and climatic features. A combination of these characteristics should yield a study area which employs essentially the same technology for the production of a relatively homogeneous product mix.

Study Areas

Utilizing the preceding criteria, eleven homogeneous regions were identified for the 17 western states (figure 1). These include:



1 Snake-Columbia Basin
2 Central California
3 Desert Southwest
4 Upper Colorado Basin
5 Upper Rio Grande Basin
6 Lower Rio Grande Basin
7 Upper Missouri Basin
8 Northwestern Ogallala
9 Northeastern Ogallala
10 Central Ogallala
11 Southern Ogallala

FIGURE 1. Major Irrigated Regions of the 17 Western States.

(1) the Snake-Columbia Basin, (2) the Central California Region, (3) the Desert Southwest, (4) the Upper Colorado Basin, (5) the Upper Rio Grande Basin, (6) the Lower Rio Grande Basin, (7) the Upper Missouri Basin, (8) the Northwestern Ogallala, (9) the Northeastern Ogallala, (10) the Central Ogallala and (11) the Southern Ogallala. The agricultural characteristics of each of these regions are discussed in turn.[2]

Snake-Columbia Basin

Outlining the basins of the Columbia and Snake Rivers in Washington, Oregon and Idaho, the Snake-Columbia Basin is characterized by a mountainous relief, a fine soil in the river valleys and annual rainfall varying from 8 to 12 inches. Historically, dryland wheat and livestock evolved as the main agricultural endeavors; however, a series of river diversions have made irrigated agriculture very important. Utilizing this new technology, vast amounts of potatoes, sugar beets, feed grains, vegetables and fruits are produced in the inherently fertile river valleys. For an average county in this area, 34.5% of the cropland, including irrigated pasture, is irrigated. The typical irrigated acre includes 35.3% hay, 17.2% pasture, 13.6% wheat, 12.4% small grains and 15.7% specialty crops. This region is comprised of 30 counties.

Central California

Encompassing the highly productive San Joaquin and Sacramento

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[2]The following discussion of the study regions draws importantly on the writings of Bouge and Beale.

Valleys, Central California is a highly mechanized and irrigation-dependent region. Soil and climatic conditions (210-270 freeze-free days) are near ideal for the production of a varied crop mix. The principal crops of this region are vegetables, fruits, cotton, small grains and pasture. In the northern portion, grapes are the dominant crop, with cotton becoming more important in the southern areas. Of the 31.4% of a typical county which is irrigated, 24.4% is irrigated hay, 14.3% is irrigated pasture, 15.6% is small grains and 26.4% is in specialty crops such as vegetables, fruits and vineyards. Fifteen counties are included in the Central California Region.

Desert Southwest

Composed of 12 counties in the southern parts of California, Arizona and New Mexico, this region is typified by intensively irrigated and highly mechanized agricultural operations. With its sandy soils, scant rainfall (2-15 inches per year), and moderate climate (240-300 freeze-free days), the Desert Southwest Region is almost sub-tropical in nature (U.S. Department of Interior). These climatic conditions coupled with water supplies from the Colorado, Gila and Salt Rivers allow various specialty crops to be produced nearly year-around. The major crops of this area include winter vegetables, cotton, hay and orchards, with limited grazing in the eastern-most areas. A typical Desert Southwest farm has approximately 34.8% of its total harvested acreage under irrigation. Of this, 25.9% is irrigated hay, 29.0% is cotton, vegetables and orchards, with small grains, sorghums and pasture accounting for the balance.

Upper Colorado Basin

Outlining the Colorado River as it flows through the rugged areas of Colorado, Wyoming and Utah, the Upper Colorado Basin is characterized by a livestock economy with limited irrigation in the scattered arable areas. These areas are used mainly to grow crops related to the livestock industry. An average county in this region has an irrigated acreage (18.2%) comprised of 51.6% hay, 38.5% pasture, 5.3% oats-barley-rye, with the remainder in corn and specialty crops. This region includes 18 counties.

Upper Rio Grande Basin

Following the course of the Rio Grande River, this region stretches from the San Luis Valley of Colorado to the two western-most counties of Texas. Fourteen counties are included in the region. Exemplified by its semi-arid environment and sandy soils, the Upper Rio Grande Basin has essentially a livestock-oriented economy with sizable irrigated areas along the river's edge. Due to the relative importance of the livestock industry, hay and pasture are the most intensively irrigated with cotton and potatoes as secondary crops. Of the 21.1% of an average county which is irrigated, 36% is irrigated hay, 23.4% irrigated pasture, with 22.6% accounted for by cotton, potatoes and other vegetables.

Lower Rio Grande Basin

Situated in the southern-most portion of Texas, the Lower Rio Grande Basin (14 counties) is a broad, nearly level plain which supports extensive agricultural operations. This naturally fertile region is characterized by loamy to clayey soils, variable rainfall and an extremely long growing season (more than 300 days). Irrigation water is supplied from the Rio Grande, Nueches and other tributaries. Although water quality and soil alkalinity have become increasing problems in recent years, a highly productive irrigation-based industry thrives. Of the 30.3% of a typical county which is irrigated, cotton, citrus and vegetables account for 50.9% of irrigated acres with sorghum and pasture totaling 28.8% and 9.6%, respectively.

Upper Missouri Basin

This region is comprised of a large portion of the land (26 Montana counties) drained by the Missouri River and its tributaries. The relief is predominately mountainous with little annual precipitation. The grazing of livestock and dryland wheat and hay operations prevail as the major agricultural endeavors. However, in some areas of the Upper Yellowstone and Big Horn Rivers, appreciable irrigated regions may be found yielding sugar beets, alfalfa, canning peas and vegetable crops. Little of an average county is irrigated (8.1%) with irrigated hay and pasture comprising 51.6% and 38.5%, respectively.

Northwestern Ogallala

Underlain by the northwestern portion of the expansive Ogallala aquifer, this region encompasses 15 contigious counties in Colorado, Nebraska and Wyoming. Characterized by a sandy, slightly undulating relief and an average annual rainfall of 15-20 inches, the Northwestern Ogallala is predominantly a grain-livestock region with limited irrigation. A typical county within this region has approximately 12.7% of its total harvested acreage (including pasture) classified as irrigated land. This irrigated portion consists of corn (40.8%), hay (38.2%), pasture (9.1%), oats-barley-rye (7.0%), wheat (2.3%) with sorghum, sugar beets and other specialty crops accounting for the remainder. The majority of these irrigated crops are grown using diverted water from the northern and southern forks of the Platte River, with a smaller acreage irrigated with subsurface water primarily from the Ogallala formation.

Northeastern Ogallala

Comprised of 18 counties in Colorado, Kansas and Nebraska the Northeastern Ogallala resembles a transitional region between the Corn Belt and the Central Plains. Characterized by a gently sloping, silty soil with an annual precipitation of 18-24 inches, it would appear that a predominately corn-livestock industry could not thrive. However, with extensive pumping operations, a corn belt-like economy predominates. Of the 5.2% of an average county which is irrigated, 63.7% is irrigated corn with sorghum and hay representing 8.2% and 15.2%, respectively.

Central Ogallala

The Central Ogallala is a broad level plain representing a large portion of the winter wheat belt. In general, this region is characterized by a fine silty soil, variable rainfall from 15 to 26 inches per year and extremely high evaporation potential. Historically, a dryland wheat-livestock economy has been predominant. However, the development of a pump irrigation-based culture in recent times has induced the introduction of higher valued crops. On average, a Central Ogallala county has an irrigated acreage amounting to 24.6% of the total harvested acres with irrigated sorghum accounting for 45.6% of this fraction, wheat 29.8% and corn 19.3%. Twenty-four counties are included in this region.

Southern Ogallala

Lying directly south of the Canadian River, this region is comprised of 21 north Texas and three eastern New Mexico counties. The region is typically high plains in nature, i.e., a broad level relief, fine silty soils and limited rainfall. Normally dryland farming and livestock would be the major agricultural enterprises. However, by utilizing groundwater from the Ogallala aquifer, a highly productive irrigation based economy has evolved with sorghum and cotton production as the two major enterprises. Of the 39.1% of harvested land which is irrigated, sorghum accounts for 38.2% and cotton 45.7%.

A Comment

The preceding descriptions provide some insight as to the type of irrigated agriculture which predominates each region. As noted, the agriculture of the study regions varies in disparate degree from grazing with sparse acreages of specialty crops in the Upper Missouri Basin to highly intensive farming enterprises with less emphasis on livestock in Central California and the Lower Rio Grande. Although the technical parameters and input levels may vary from region to region, certainly the same basic inputs (land, labor and capital) are required, regardless of type of operation. Therefore, it seems reasonable that a common technical specification (inputs included and functional form) should suffice for estimating each regional production function.

CHAPTER III

THE PRODUCTION FUNCTION MODEL AND FACTOR DEMAND

The problem of model specification for agricultural production is somewhat perplexing. In addressing the two-fold problem of explanatory variables to be included and the functional form to be fitted, one must balance theoretical and empirical (statistical) considerations. Oftentimes model specifications having desirable theoretical properties are accompanied by statistical problems, or simply not supported by the data. This problem is particularly prevelant when using aggregate secondary data. Only with careful selection of relevant variables and functional form can a model be specified that exhibits important theoretical properties, and at the same time yields statistically reliable estimates of the structural coefficients.

Choice of Explanatory Variables

For meaningful results, a particular production function specification should reflect all variables (inputs) relevant to the "real world" situation. The omission of any relevant variable will result in a model which is biased in an economic sense. Certain inputs, though variable in the sample, are impossible or too costly to measure. By specifying a model which includes all essential measurable inputs, the economic bias is lessened. Such a specification, though admittedly incomplete, should yield estimates which are useful in policy-making decisions (Heady and Dillion).

The model specification in this study assumes that agricultural production, like most production processes, is the result of the application of land, labor and capital. These fundamental inputs, however, are extremely aggregate for many policy questions; in particular, further disaggregation of the capital input is needed. Specific inputs of interest for full economic evaluation of irrigated agriculture are land, labor, irrigation water, energy, fertilizer and lime, machinery inventories, livestock inventories and other operating expenses. To account for the effects of quality variation within the sample, the value of, or expenditures on, each input (except water) is used. (Specific variable definitions and data sources are discussed in the final section of this chapter and in appendix B.) Since the main objective of this research is to ascertain the value of irrigation water, this input is measured in physical units, viz., acre-feet.

Functional Form

In addition to guiding the choice of explanatory variables, theory also provides insight in hypothesizing a suitable functional form(s). An appropriate production function model should exhibit several technical characteristics generally believed true of production processes. On an *a priori* basis, agricultural production can be hypothesized to exhibit three essential chatacteristics. First, like most production processes, inputs in an agricultural process will likely follow the law of diminishing marginal productivity, i.e., as successive units of a variable input are applied to a given quantity of other resources, the resultant increments to output (marginal product) will decline. Secondly, the marginal product forthcoming from a decision to increase or decrease a factor level depends on the available quantities of the other factors; e.g., the additional product yielded by an additional unit of fertilizer applied depends greatly upon the quantities of land, labor, etc., combined with it. That is, one expects production inputs to be technically interdependent; specifically, for normal inputs it is expected that an increase in an input level increases the marginal and average productivities of other inputs in the production process. Finally, if one of the requisites for production is absent (i.e., a zero input level) the process would yield no output. This characteristic infers that a production function should pass through the origin of the input-output space.

Properties of a Cobb-Douglas Production Function

Technical

The Cobb-Douglas or power function appears to be quite applicable in estimating agricultural production, in that it intercepts the origin, displays decreasing marginal productivity of inputs and allows input interdependence or interaction. In general, a Cobb-Douglas form is

$$(3.1) \qquad y = \beta_o \prod_{i=1}^{n} x_i^{\beta_i}$$

where y is output, x_i denotes the i^{th} input, β_o is a parameter reflecting the production technology and the parameter, β_i, is the elasticity of the i^{th} factor.[3] The mathematical operator, Π, denotes the product over i.

[3] The factor elasticities for the Cobb-Douglas production function are constant and equal to the exponent coefficients (Heady and Dillon, p. 75).

We note first that when $x_i = 0$, $y = 0$, satisfying the condition of a zero product intercept. Differentiating (3.1) with respect to the k^{th} input, the equation for the marginal productivity is given by

$$(3.2) \qquad \frac{\partial y}{\partial x_k} = \beta_k \beta_o x_k^{(\beta_k - 1)} \prod_{\substack{i=1 \\ i \neq k}}^{n} x_i^{\beta_i}$$

For the usual case of $\beta_o > 0$ and $0 < \beta_i < 1$, it follows that decreasing marginal productivity is exhibited. As x_k approaches zero, $\partial y/\partial x_k$ increases without bound. As x_k tends toward infinity, $\partial y/\partial x_k$ approaches zero. Alternatively, note that the sign of the derivative of the marginal productivity of x_k with respect to x_k is negative. That is,

$$(3.3) \qquad \frac{\partial^2 y}{\partial x_k^2} = \beta_k(\beta_k - 1)\beta_o x_k^{(\beta_k - 2)} \prod_{\substack{i=1 \\ i \neq k}}^{m} x_i^{\beta_i} < 0$$

for $\beta_o > 0$ and $0 < \beta_i < 1$, implying a negative slope and thus diminishing marginal returns.

The third technical property of interest is that of technical complementarity; i.e., we expect the marginal productivity of each input to be increased as the level of each other input is increased for all pairs of inputs. Taking the derivative of (3.2) with respect to the j^{th} input (second-order "mixed" derivative) yields

$$(3.4) \qquad \frac{\partial^2 y}{\partial x_k \partial x_j} \equiv \frac{\partial\left(\frac{\partial y}{\partial x_k}\right)}{\partial x_j} = \beta_j \beta_k \beta_o x_k^{(\beta_k - 1)} x_j^{(\beta_j - 1)} \prod_{\substack{i=1 \\ i \neq j,k}}^{m} x_i^{\beta_i}$$

Again for $\beta_o > 0$ and $0 < \beta_i < 1$, $\partial^2 y/\partial x_k \partial x_j > 0$ implying an increase in the marginal productivity of x_k as x_j is increased or technical complementarity. The technical interdependence property is symmetric, i.e., $\partial^2 y/\partial x_k \partial x_j \equiv \partial^2 y/\partial x_j \partial x_k$--see Chiang, p. 309.

Factor Demand

The demand for a specific input (x_k) in a Cobb-Douglas function can be specified by solving for x_k in terms of its price, the product price, the prices of other variable factors and the quantities of the fixed factors. Underlying this rationale are the assumptions of factor homogeneity and perfect competition (zero price flexibility) in both product and factor markets, as well as a profit maximizing behavior postulate.[4] Letting p and r_i denote the prices of the product and i^{th} factor, respectively, profit (π) may be expressed as

$$(3.5) \qquad \pi = p\beta_o \prod_{i=1}^{n} x_i^{\beta_i} - \sum_{i=1}^{m} r_i x_i - C_o$$

where x_i is an element of a vector of n factors, of which the first m factors are variable and the remaining n-m are fixed, and C_o denotes the fixed cost owing to the fixed factors, i.e., $\sum_{i=m+1}^{n} r_i x_i$.

Setting each partial derivative of (3.5) equal to zero, the first order conditions for profit maximization are

$$(3.6a) \qquad \frac{\partial \pi}{\partial x_1} = \beta_1 p \beta_o x_1^{(\beta_1 - 1)} \prod_{\substack{i=1 \\ i \neq 1}}^{n} x_i^{\beta_i} - r_1 = 0$$

---

[4]The assumptions of factor homogeneity, profit maximization and perfect competition in the k^{th} factor market are essential to the theory of derived demand. However, the assumptions of perfect competition in the product market and in the other factor markets is a matter of convenience. The theory of derived factor demand can accomodate a relaxation of these two assumptions although doing so in effect eliminates product and other price variables as explicit arguments in the resulting k^{th} factor demand equation.

$$(3.6b) \qquad \frac{\partial \pi}{\partial x_2} = \beta_2 p \beta_o x_2^{(\beta_2 - 1)} \prod_{\substack{i=1 \\ i \neq 2}}^{n} x_i^{\beta_i} - r_2 = 0$$

$$\vdots$$

$$(3.6m) \qquad \frac{\partial \pi}{\partial x_m} = \beta_m p \beta_o x_m^{(\beta_m - 1)} \prod_{\substack{i=1 \\ i \neq m}}^{m} x_i^{\beta_i} - r_m = 0$$

Assuming second order conditions are satisfied[5] and solving these equations for $x_1, \cdots, x_m$, an expression for the demand for each input is given in terms of own price, product price and quantities of other inputs; i.e.,

$$(3.7a) \qquad x_1 = \left[\frac{r_1}{\beta_1 p \beta_o \prod_{\substack{i=1 \\ i \neq 1}}^{m} x_i^{\beta_i}} \right]^{\frac{1}{\beta_1 - 1}}$$

$$(3.7b) \qquad x_2 = \left[\frac{r_2}{\beta_2 p \beta_o \prod_{\substack{i=1 \\ i \neq 2}}^{m} x_i^{\beta_i}} \right]^{\frac{1}{\beta_2 - 1}}$$

$$\vdots$$

$$(3.7m) \qquad x_m = \left[\frac{r_m}{\beta_m p \beta_o \prod_{\substack{i=1 \\ i \neq m}}^{n} x_i^{\beta_i}} \right]^{\frac{1}{\beta_m - 1}}$$

---

[5]Second-order conditions are satisfied if $0 < \beta_i < 1$ for all i and $\sum_i \beta_i < 1$. These restrictions assure a non-zero Jacobian which guarantees a solution to the system of first order conditions for profit maximization.

These m equations (3.6a - 3.6m) involve the familiar requirement that factor marginal value productivities (MVP_i) must equal factor prices (r_i) for profit maximization. These equations have sometimes been presented as factor demand equations in applied work (e.g., Ruttan). Unfortunately they represent factor demands only in the limiting case where all factors except one are considered fixed; i.e., assuming m = 1, then all x_i for i = 2, ..., n are fixed and the system of equations (3.7a - 3.7m) reduces a single equation.

When more than one factor is assumed to be variable--a longer run case--the demand for a specific variable factor depends upon its substitutability with other variable factors. That is, assuming economic efficiency, an entrepreneur will substitute one factor for another until a least-cost combination is attained. Therefore, in this case, factor demand may be derived using the expansion path conditions in conjunction with the first-order profit-maximizing condition of interest or by solving equations (3.7a - 3.7m) simultaneously,[6] yielding demand equations of the general form:

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[6]Consider the following profit function

$$\pi = p\beta_o x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} - r_1x_1 - r_2x_2 - r_3x_3$$

where x_1 and x_2 are variable factors and x_3 is fixed at some level. The first order conditions for profit maximization are

$$\frac{\partial\pi}{\partial x_1} = \beta_1 p\beta_o x_1^{(\beta_1-1)} x_2^{\beta_2} x_3^{\beta_3} - r_1 = 0$$

$$\frac{\partial\pi}{\partial x_2} = \beta_2 p\beta_o x_2^{\beta_1} x_2^{(\beta_2-1)} x_3^{\beta_3} - r_2 = 0$$ (continued)

$$(3.8)\quad x_k = r_k^{\left(\sum_{\substack{i=1\\ i\neq k}}^{m}\beta_i - 1 \Big/ 1-\sum_{i=1}^{m}\beta_i\right)} \left[\frac{\beta_k^{\sum_{\substack{i=1\\ i\neq k}}^{m}\beta_i - 1}\prod_{\substack{i=1\\ i\neq k}}^{m}\left(\frac{r_i}{\beta_i}\right)^{\beta_i}}{p\beta_o \prod_{i=m+1}^{n} x_i^{\beta_i}}\right]^{\frac{1}{\sum_{i=1}^{m}\beta_i - 1}}$$

Note that the demand equations exhibit a characteristic downward (negative)

(continue footnote 6)

Assuming the second order conditions hold,

$$x_1 = \left[\frac{r_1}{\beta_1 p\beta_o x_2^{\beta_2} x_3^{\beta_3}}\right]^{\frac{1}{\beta_1 - 1}} \quad \text{and} \quad x_2 = \left[\frac{r_1}{\beta_2 p\beta_o x_1^{\beta_1} x_3^{\beta_3}}\right]^{\frac{1}{\beta_2 - 1}}$$

Substituting the expression for x_2 into that for x_1 and simplifying, the demand for x_1 may be expressed as

$$x_1 = r_1^{\left(\frac{\beta_2 - 1}{1 - \beta_1 - \beta_2}\right)} \left[\frac{\beta_1^{(\beta_2 - 1)}\left(\frac{r_2}{\beta_2}\right)^{\beta_2}}{p\beta_o x_3^{\beta_3}}\right]^{\frac{1}{\beta_1 + \beta_2 - 1}}$$

Similarly for the three variable-one fixed factor case, the demand for x_1 is

$$x_1 = r_1^{\left(\frac{\beta_2 + \beta_3 - 1}{1 - \beta_1 - \beta_2 - \beta_3}\right)} \left[\frac{\beta_1^{(\beta_2 + \beta_3 - 1)}\left(\frac{r_2}{\beta_2}\right)^{\beta_2}\left(\frac{r_3}{\beta_3}\right)^{\beta_3}}{p\beta_o x_4^{\beta_4}}\right]^{\frac{1}{\beta_1 + \beta_2 + \beta_3 - 1}}$$

slope if the sum of the variable factor elasticities (ε) is less than one.[7] If $\varepsilon \geq 1$, second order conditions are not satisfied, and factor demand functions are not defined given assumptions of perfect competition (see p. 24).

Price Elasticities of Factor Demand

An important characteristic of the demand for a factor (x_k) is its price elasticity. This entity is defined as the proportionate rate of change in x_k demanded, divided by the proportionate rate of change in r_k, all other factors constant (Henderson and Quandt, p. 27). Using this definition, an expression for a change in a factor's own price, i.e., *own-price elasticity* can be formed:

$$\eta_{kk} = \frac{\partial x_k}{\partial r_k} \frac{r_k}{x_k} \quad (k = 1,2,3...n) \tag{3.9}$$

The definition of elasticity can be further extended to express the effect of the change in price of a different factor on the demand for x_k , i.e., its *cross-price elasticity*:

$$\eta_{kj} = \frac{\partial x_k}{\partial r_j} \cdot \frac{r_j}{x_k} \quad (j \neq k) \tag{3.10}$$

Two factors of production (x_k, x_j) may be considered economically complementary, independent or substitutable if the partial with resepct to the j^{th} factor price is less than zero, equal to zero, or greater than zero. For the "cross effect" of a change in product price, equation (3.14)

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[7]Though similar to the concept of "returns to scale" the factor elasticity sum, ε, does not include the fixed factor elasticities. In the long-run case, all factors are variable and ε reflects returns to scale for a homogenous production function. Thus, the general shape of the demand curve depends as much on the length of run considered as on factor returns.

defines a product-price elasticity if r_j is replaced with product price.

For the Cobb-Douglas production function, the implied demand elasticities are constants and equal to the respective price coefficients of the factor demand equations (Heady and Hexem). Thus, estimates of all three kinds of elasticities (own, cross-factor and cross-product) may be read directly from the demand functions.

The Hypothesized Production Function Model

At this juncture, the following Cobb-Douglas type model appears to be an appropriate production function specification:

$$(3.11) \qquad y = \beta_o \prod_i x_i^{\beta_i} \qquad (i = 1,2,\ldots,9)$$

where

y = value of agricultural output--value of crops harvested and livestock and livestock product sales (dollars/county)

x_1 = irrigation water applied (acre-feet/county)

x_2 = value of land and service buildings (dollars/county)

x_3 = hired labor expenditures (dollars/county)

x_4 = fuel and lubricant expenditures (dollars/county)

x_5 = fertilizer and lime expenditures (dollars/county)

x_6 = feed expenditures (dollars/county)

x_7 = value of machinery inventory (dollars/county)

x_8 = value of livestock inventory (dollars/county)

x_9 = other operating expenses (dollars/county)

The principal data source for this study was the *1969 Census of Agriculture*. See appendix B for a detailed discussion of variable definitions and data development.

Ommitted-variable-specification-bias should not be a major problem in this production function model as the inputs included virtually exhaust those factor services contributing to output. However, because of the relatively large number of independent variables complications due to multicollinearity will likely be significant. Recently, a method of estimation called ridge regression (Hoerl and Kennard) has been proposed to circumvent the multicollinearity problem. This method, although admittedly a biased estimation technique, allows the formulation of a more complete model (thereby avoiding ommitted variables bias) and results in parameter estimates which have substantially smaller variances than those of ordinary least squares in the presence of multicollinearity. The applicability of ridge regression for agricultural production function estimation has been demonstrated by Brown and Beattie.

Chapter IV presents a brief discussion of multicollinearity, ridge regression, and the empirical methodology used in this study. The reader interested in only the results can move directly to Chapters V and VI without serious disruption in the flow.

CHAPTER IV

MULTICOLLINEARITY AND RIDGE REGRESSION

Multicollinearity, as defined by Silvey, is a "term used in econometrics to denote the presence of linear relationships or 'near linear relationships' among explanatory variables in a linear regression" (p. 539). Least squares estimators of the parameters of such a model are known to posess the desirable properties of unbiasness and efficiency (Kmenta). Unfortunately, with linear dependencies in the model, least squares estimates are imprecise, i.e., highly variant. However, in practical application, multicollinearity has little effect on the predictive qualities of such a model, but due to these large variances greatly affect the quality of the individual (structural) parameter estimates. When obtaining reliable estimates of structural parameters is an important aspect of the research, multicollinearity becomes a crucial problem.

Estimation and Linear Dependencies[8]

Consider the following multiple linear regression model,

$$Y = \beta_0 1 + X\beta + \varepsilon \tag{4.1}$$

where Y is an n x 1 vector of dependent variables, 1 is an n x 1 vector of ones, X is an n x k matrix of independent variables, β_o is an unknown

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[8] The following sections follow closely the writing of Mason, Gunst and Webster.

parameter, β is an $k \times 1$ vector of parameters, and ε is an $n \times 1$ vector of stochastic disturbances. Also, assume that the x's have been standardized, i.e., $1'X = 0$ and $X_j'X_j = 1$ for $j = 1,2,\ldots,k$. For complete specification we must further assume that $\varepsilon_i (i=1,2,\ldots,n)$ is normally distributed, $E(\varepsilon_i) = 0$, $E(\varepsilon_i^2) = \sigma^2$, $E(\varepsilon_i \varepsilon_m) = 0$ $(i \neq m)$, nonstochasticity of the independent variables, independence of X_j's, and that the number of observations is greater than the number of variables (Kmenta).

The effects of multicollinearity on the estimation of (4.1) can be illustrated by examining the elements of the $(X'X)^{-1}$ in the following form:

$$(4.2) \quad C = (X'X)^{-1} = \begin{cases} c_{ii} = (1-R_i^2)^{-1} \\ c_{ij} = -s_{ij\cdot(k-2)}(1-R_i^2)^{-1}(1-R_{j\cdot(k-2)}^2)^{-1} \end{cases}$$

where

$s_{ij\cdot(k-2)}$ = partial "co-variance" of X_i and X_j, adjusted for the remaining explanatory variables

R_i^2 = coefficient of determination of the regression of X_i on the remaining explanatory variables

$R_{j\cdot(k-2)}$ = coefficient of determination of the regression of X_j on the remaining k - 2 explanatory variables, excluding X_i

Note that if X_i is a member of the multicollinearity (linear dependent set), R_i^2 should be large, implying that c_{ii} is necessarily large. Since $\mathrm{Var}(\hat{\beta}_i) = c_{ii}\sigma^2$, it follows that the variance of the estimates are inflated in the presence of multicollinearity. Thus, t-values tend to be nonsignificant. Also notice that if both X_i and X_j are involved in the multicollinearity, s_{ij} should be large and one or both of the coefficients of dermination is (are) large. Thus, $Cov(\hat{\beta}_i \hat{\beta}_j)$ will also be inflated.

The least squares estimates of the regression parameters may be written as

$$(4.3) \qquad \hat{\beta}_i = \sum_{j=1}^{k} c_{ij}(X_j'Y) \qquad (i,j = 1,2,\ldots,k)$$

Now if X_i is a member of the multicollinearity, (4.2) infers that the c_{ij} may be large in magnitude, and will be either positive or negative, dependent upon the sign of s_{ij}. Thus, the magnitude (absolute value) of $\hat{\beta}_i$ will likely be large and/or carry an incorrect sign.[9] This largeness is due mainly to the relationships *among* the regressor variables. Therefore, the true relationship between dependent and explanatory variables can be distorted by a high degree of multicollinearity. Before considering a procedure for mitigating the effect of multicollinearity on structural parameter estimation, a brief diversion will be taken to establish that the predictive qualities of a model are unaffected by multicollinearity.

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[9]For example, consider the function $Y = f(X_1, X_2, X_3)$ where $X_1 = f(X_2, X_3)$. Now assuming that X_1, X_2 and X_3 are all positively related, the first row of $(X'X)^{-1}$ will be $C_1 = (c_{11}, c_{12}, c_{13})$. From (4.2), c_{12} and c_{13} will be negative, and

$$\beta_1 = c_{11}(X_1'Y) - c_{12}(X_2'Y) - c_{13}(X_3'Y)$$

Now if $c_{12}(X_2'Y) + c_{13}(X_3'Y) > c_{11}(X_1'Y)$, β_1 will be negative.

Prediction and Multicollinearity

Although individual estimates of the parameters are imprecise multicollinearity does not necessarily affect the predictive qualities of a model. Suppose the first p variables of (4.1) constitute a linearly dependent set. Although the individual parameters may be poor estimates, the linear combination

$$\sum_{i=1}^{p} X_{ij}\beta_j \qquad j = 1,2,\ldots k \tag{4.4}$$

may be estimated well. This may be illustrated by looking at the $(X'X)^{-1}$ in an alternate form:

$$C = (X'X)^{-1} = \sum_{r=1}^{k} \lambda_r^{-1} V_r V_r' \tag{4.5}$$

where

$\lambda_r = r^{th}$ eigenvalue with $\lambda_1 \leq \lambda_2 \leq \cdots \lambda_r \cdots\cdots \leq \lambda_k$

V_r = eigenvector corresponding to λ_r

If λ_1 is assumed to be substantially less than λ_k, multicollinearity is an inherent problem (Silvey). Now the individual estimate of $Y_i = \sum_{j=1}^{k} X_{ij}\beta_j$ is $\hat{Y}_i = \sum_{j=1}^{k} X_{ij}[(X'X)^{-1}X'Y]$. The variance of which is

$$\mathrm{Var}(\hat{Y}_i) = \sigma^2[X_i'(\sum_{r=1}^{k} \lambda_r^{-1} V_r V_r') X_i + \frac{1}{n}] \tag{4.6}$$

However, since $\underline{X}_i'\underline{V}_1 \approx 0$,[10] the ill effects of the small λ_1 are can-

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[10] By definition $XV = \lambda V$. If $\lambda \approx 0$ then $\lambda V \approx 0$ which in turn implies that $XV \approx 0$.

celled.[11] Thus, $Var(\hat{Y}_i)$ is not inflated and the individual Y_i's are predicted well within the range of the data.

Improved Estimation Under Multicollinearity

As shown in the preceding sections, least squares estimates of highly collinear data are adequate predictors, but are usually very imprecise. According to Kmenta, these complications are a feature of the sample, not the population (p. 380). Therefore, the reliability of the least squares estimates are improved with increased sample size. However, increasing sample size is often impossible or to costly. Recently other techniques for circumventing multicollinearity in relatively small samples have been introduced--one of which is ridge regression.

Ridge Regression

The technique of ridge regression was developed by Hoerl and Kennard (1970a, 1970b), and later applied to economic problems by Brown and Beattie and Vinod, among others. This approach recognizes the fact that data complications, specifically multicollinearity, often

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[11] Consider the case of k = 2, n extremely large, and $\lambda_1 \approx 0$; thus

$$Var(\hat{Y}_i) = \sigma^2[X_i'(\lambda_1^{-1}V_1 V_1' + \lambda_2^{-1}V_2 V_2') X_i]$$

$$= \sigma^2[(\lambda_1^{-1}X_i'V_1 V_1' + \lambda_2^{-1}X_i'V_2 V_2') X_i]$$

Now since $\underline{X_i'V_1} \approx 0$,

$$Var(\hat{Y}_i) = \sigma^2[\lambda_2^{-1}X_i'V_2 V_2' X_i]$$

$$= \sigma^2 X_i'(\lambda_2^{-1}V_2V_2')X_i$$

cause inflation of least squares estimates. For certain applied problems there is a substantial *a priori* basis for believing this inflation is severe enough to distort the true relationship. By augmenting the X'X matrix with small constants (k), ridge regression controls this inflation and yields estimates which, although biased, have a smaller Mean Square Error (MSE) than least squares estimates under multicollinearity (Dempster, *et. al.*; McDonald and Galarneau).

That is, MSE may be defined as

$$(4.7) \qquad E(\hat{\theta} - \theta)^2 = \mathrm{Var}(\theta) + \mathrm{Bias}^2$$

For the ridge estimator this expression may be rewritten as

$$(4.8) \qquad E(\hat{\theta} - \theta)^2 = \sigma^2 \sum_{i=1}^{P} \lambda_i/(\lambda_i + k)^2 + k^2 \sum_{i=1}^{P} \alpha_i^2/(\lambda_i + k)$$

where $\alpha = P\theta$ and P is a matrix of orthonormal eigenvectors (Hoerl and Kennard, 1970a). For $0 \leq k \leq .3$, it appears that $\mathrm{Var}(\Theta)$ should decrease at a greater rate than squared bias increases.[12]

Although exhibiting a smaller total MSE than OLS estimates, ridge estimates do lose the desirable property of zero bias, i.e., as the first increment of k is introduced, $k^2 \sum_{i=1}^{P} \alpha_i^2/(\lambda_i + k)$ takes on non-zero value. The effect of biasness on the sampling distribution with multicollinearity is graphically illustrated in figure 2. Notice that with multicollinearity, the sampling distribution $f(\Theta)$ takes on an

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[12]The choice of .3 as an upper bound has no theoretical basis other than its squared value is less than .1. In practice it appears that values up to this point seem to have the most effect. However, this choice is somewhat arbitrary.

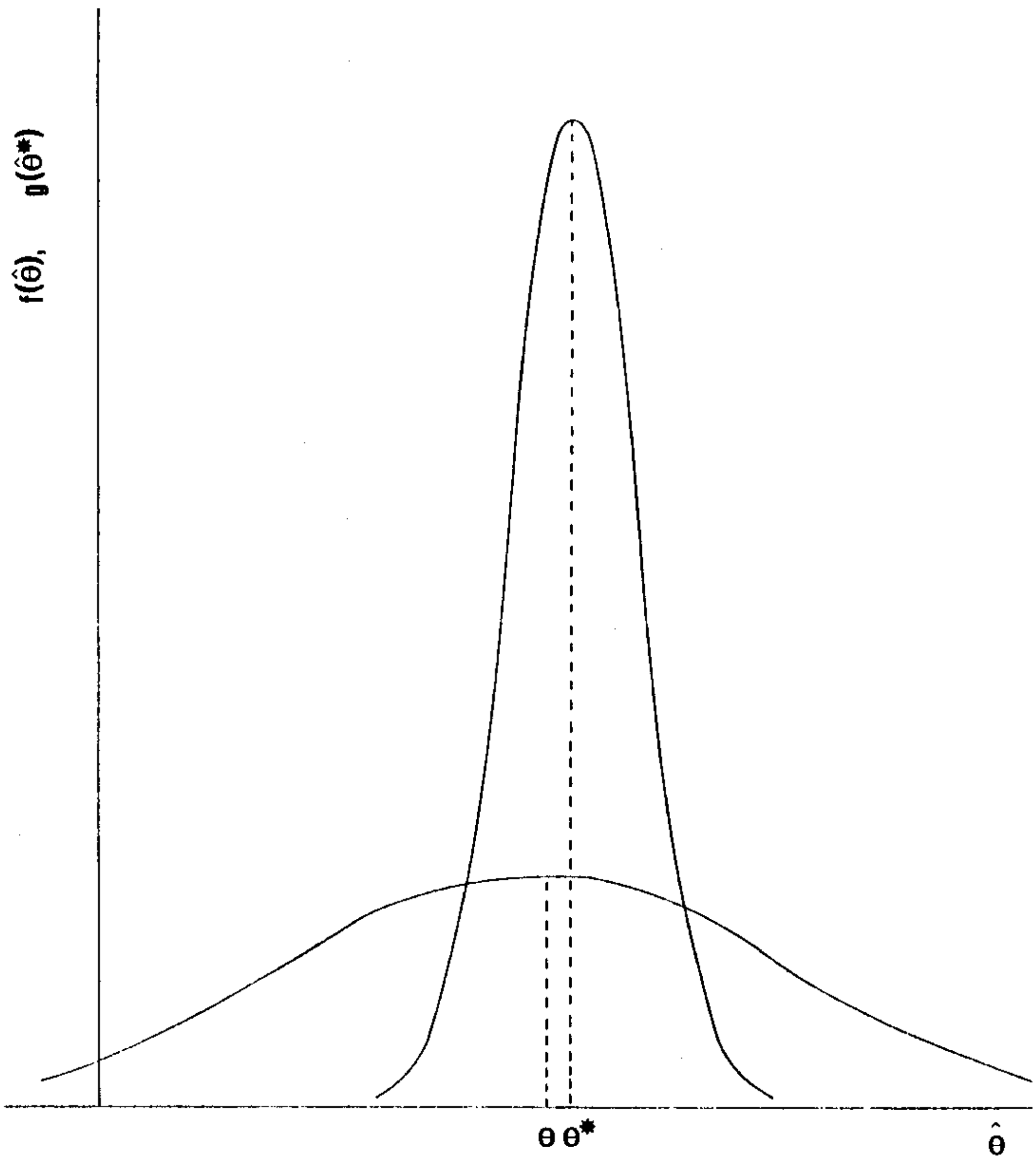

FIGURE 2. Sampling Distributions for the Biased and Unbiased Cases.

extremely "flattened" shape, denoting highly dispersed estimates. This distribution, however, does exhibit the property of unbiasness in that $E(\hat{\Theta}) = \Theta$. Note however, that with $f(\hat{\Theta})$ the probability of having estimates which differ substantially from the "true" parameter is large. Conversely, $g(\hat{\Theta}^*)$ exhibits the sampling distribution of a biased but less dispersed estimator. If bias is kept at a minimum (as shown in figure 2), the probability of having an estimate which differs greatly from Θ is lessened. An estimator with a distribution of the second form would be desirable for certain applications, e.g., policy implications based on structural parameter estimates like water MVPs.

A further advantage of ridge regression is that the analyst can examine the sensitivity of estimates to slight changes in the data, whereas mathematical optimization techniques like least squares yield only point estimates (Hoerl; Hoerl and Kennard 1970a). Before addressing the mathematical properties of ridge regression, a quick review of least squares estimation may prove helpful.

Least Squares Estimators[13]

Consider the multiple linear regression model defined by (4.1). The least squares estimate of β is

$$\hat{\beta} = (X'X)^{-1}X'Y \tag{4.9}$$

which gives the minimum sum of squares of the residuals:

$$\phi_{min} = (Y - X\hat{\beta})'(Y - X\hat{\beta}) \tag{4.10}$$

---

[13]The following sections follow Hoerl and Kennard (1970a).

An important property of $\hat{\beta}$ is its distance from its expected value. Let

(4.11) $$L \equiv \hat{\beta} - \beta$$

and

(4.12) $$L^2 = (\hat{\beta} - \beta)'(\hat{\beta} - \beta)$$

Notice that L^2 is an equivalent expression for the mean square error of $\hat{\beta}$. Since $\hat{\beta}$ is unbiased,

(4.13) $$E(L^2) = \sigma^2 \mathrm{Trace}(X'X)^{-1}$$

and

(4.14) $$E(\hat{\beta}'\hat{\beta}) = \beta'\beta + \sigma^2 \mathrm{Trace}(X'X)^{-1}$$

where the mathematical operator, Trace, denotes the sum of the diagonal elements of a matrix. Thus, as the diagonal elements of the $(X'X)^{-1}$ increase, $\hat{\beta}'\hat{\beta}$ is increased accordingly.

Denote the eigenvalues of $X'X$ as

(4.15) $$\lambda_{max} = \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p = \lambda_{min} > 0$$

Equation (4.13) may be written as

(4.16) $$E(L^2) = \sigma^2 \sum_{i=1}^{p} (1/\lambda_i)$$

since $P'(X'X)^{-1}P = \mathrm{Diag}\ (1/\lambda)$. Hence, for instances where there is a significant spread in the eigenvalues, e.g., highly collinear data, the distance from $\hat{\beta}$ to β will be large. The underlying rationale of ridge regression attempts to decrease this squared distance, L^2, without an appreciable increase in the sum of squares of the residuals, ϕ.

The Ridge Estimator

Let B denote any estimate of β. An expression for the residual sums of squares is

$$(4.17) \qquad \phi = (Y - XB)'(Y - XB)$$
$$= (Y - X\hat{\beta})'(Y - X\hat{\beta}) + (B - \hat{\beta})'X'X(B - \hat{\beta})$$
$$= \phi_{min} + \phi(B)$$

Notice that for an ill-conditioned X'X,[14] a substantial decrease in B (assuming $\hat{\beta}$ is inflated) will not greatly increase ϕ. Thus for B to to be a desirable estimator of β, it must reduce the MSE term in (4.14) without an appreciable increase in $\phi(B)$. An estimator of this type would control the inflation due to ill-conditioning with a minimal loss in fit.

This reasoning may be set up as a simple minimization problem where one minimizes B'B subject[15]

$$(B - \hat{\beta})'X'X(B - \hat{\beta}) = \phi_o$$

That is, form the Lagrangian:

$$(4.18) \qquad L \equiv B'B + k[(B - \hat{\beta})'X'X(B - \hat{\beta}) - \phi_o]$$

[14] Ill-conditioning may be defined as existence of 1 or more extremely small eigenvalues relative to the largest eigenvalue.

[15] For a graphical interpretation of this approach, see Marquant and Snee.

Now,

$$(4.19) \qquad \frac{\partial L}{\partial B} = 2B + k[2(X'X)B - 2(X'X)\hat{\beta}] = 0$$

Solving for B :

$$(4.20) \qquad B = \hat{\beta}* = [X'X + kI]^{-1}X'Y$$

In practice, however, values of $k \geq 0$ are chosen and the resulting ϕ_o is computed. The residual sums of squares for this ridge estimator ($\hat{\beta}*$) is

$$(4.21) \quad \phi* = (Y - X\hat{\beta}*)(Y - X\hat{\beta}*) = \phi_{min} + k^2\hat{\beta}*'(X'X)^{-1}\hat{\beta}*$$

Notice that for values of $0 \leq k \leq .3$, ϕ_{min} is not appreciably inflated. Therefore, B'B may be reduced substantially with a relatively small ϕ_o.

By substituting (4.1) into (4.20), the ridge estimator can be rewritten as[16]

$$(4.22) \qquad \hat{\beta}* = \beta + \omega + (X'X+kI)^{-1} X'\varepsilon$$

where ω denotes the bias of $\hat{\beta}*$. The variance-covariance matrix of (4.22)

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[16]From (4.20)

$$\begin{aligned}\hat{\beta}* &= (X'X + kI)^{-1} X'Y \\ &= (X'X + kI)^{-1} X'(X\beta + \varepsilon) \\ &= (X'X + kI)^{-1} X'X\beta + (X'X + kI)^{-1} X'\varepsilon\end{aligned}$$

and

$$E(\hat{\beta}*) = E[(X'X = kI)^{-1} X'X\beta] + E[(X'X + ki)^{-1} X'\varepsilon]$$

Since by definition, $E(\varepsilon) = 0$, then

$$\begin{aligned}E(\hat{\beta}*) &= E[(X'X + kI)^{-1} X'X\beta] \\ &= (X'X + kI)^{-1} X'X\beta\end{aligned}$$

which can be rewritten as

$$E(\hat{\beta}*) = \beta + \omega$$

is given by

$$(4.23) \qquad \text{Var-Cov}(\hat{\beta}^*) = E\{[\hat{\beta}^* - E(\hat{\beta}^*)]\ [\hat{\beta}^* - E(\hat{\beta}^*)]'\}$$

$$= E\{[(X'X + kI)^{-1}\ X'\varepsilon]\ [(X'X + kI)^{-1}\ X'\varepsilon]\}$$

$$= \sigma^2(X'X + kI)^{-1}\ X'X(X'X + kI)^{-1}$$

It can be shown that the augmentation of (4.20) simply increases each individual eigenvalue by k. Looking at (4.16), this increase causes $E(L^2)$ to decrease monotonically.[17] Therefore, there exists a value of k such that the rate of decrease in $\hat{\beta}'\hat{\beta}$ is minimal. In practice, the desired value of k is one that yields a vector of ridge estimators whose squared distance is nearer the "true" distance *without* an appreciable increase in ϕ. An idea of this point can be inferred from a visual inspection of a 2-dimensional plot of $\hat{\beta}^*$ and ϕ versus k, called a ridge trace.[18]

Ridge Trace

The ridge trace is simply a graphical illustration of the interrelations between the stability of $\hat{\beta}^*$ as k is increased. An additional plot of R^2 for each value of k is sometimes included to give an indication of the general fit of the ridge regression plane. The k-values and resultant $\hat{\beta}^*$ is selected where the change in the β-values is minimal. Although somewhat subjective, choosing k in this manner yields a vector of estimates whose squared distance should be nearer to the true squared distance ($\hat{\beta}'\hat{\beta}$). The Ridge Traces for this study are presented in appendix D.

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[17] A detailed discussion of the properties of the ridge estimator given Hoerl and Kennard (1970a) and Theobald.

[18] Other attempts at selecting optimal k-values are by Hoerl, Kennard and Baldwin; McDonald and Galarneau; Vinod.

A Comment

As this and the preceding chapter have shown, various problems are encountered in production function analysis. However, by accepting two major assumptions, the model and methodology profferred in this study should prove useful in derived water demand estimation. These critical assumptions are:

1. The economic specification of the model is correct with regard to explanatory variables included and functional form.
2. The magnitudes and variances of the least squares estimates are inflated due to interrelations among the independent variables.

The OLS and ridge estimates for each regional production function are presented in the following chapter. Implications for water demand and value are developed in Chapter VI.

CHAPTER V

RESULTS: THE FITTED REGIONAL MODELS

Before addressing the empirical results, it should be noted that the primary intent of this study is not to contrast estimation by ordinary least squares (OLS) and ridge regression. Rather, ridge regression is applied as a viable alternative to OLS under multicollinearity in order to obtain "better" estimates of underlying structural parameters, and hence better description of water demand characteristics. However, it is difficult to accomplish the latter without at least implicitly involving the former. Therefore, both OLS and ridge regression estimates of the regional production function coefficients are presented, and briefly compared.

The statistical model fitted for each of the eleven regions was

$$\ln Y = \ln\beta_o + \beta_1 \ln X_1 + \dots + \beta_9 \ln X_9 + u \tag{5.1}$$

Note that this expression is simply the logarithmic form of equation (3.15) with the error term, u, added.

Ordinary Least Squares Estimates

The OLS estimates are presented in table 2. A cursory examination of table 2 reveals that these estimates are generally inconsistent with theoretical expectations. Notice the large number of negative coefficients, suggesting negative marginal and average returns and hence, zero optimal input levels. In general, negative estimates were found for 34 of the 99 slope parameters. More specifically, these negative estimates occur for *five* of the *nine* inputs in both the Upper Rio Grande Basin and the Central Ogallala regions. Also note the unusually large estimates for β_9. For *eight* of the *eleven* regions this estimate exceeds .50. Note the overall nonsignificance of all the coefficients due to the large standard errors.

Table 2. Production Function Estimates and Related Statistics (OLS).[a,b]

Snake Columbia Basin

$$y = 1.023\, x_1^{-.0142}\, x_2^{-.00468}\, x_3^{.00716}\, x_4^{-.199}\, x_5^{-.153}\, x_6^{.243}\, x_7^{.241}\, x_8^{.0286}\, x_9^{.884}$$

(.0604) (.128) (.102) (.365) (.113) (.0784) (.245) (.105) (.269)

$n = 31$ $R^2 = .968$

Central California

$$y = .104\, x_1^{.183}\, x_2^{.261}\, x_3^{.315}\, x_4^{.170}\, x_5^{-.690}\, x_6^{.0161}\, x_7^{.129}\, x_8^{.141}\, x_9^{.621}$$

(.337) (.362) (.420) (.680) (.456) (.145) (.416) (.227) (.683)

$n = 15$ $R^2 = .994$

Desert Southwest

$$y = 6.281\, x_1^{-.362}\, x_2^{-.320}\, x_3^{.279}\, x_4^{.663}\, x_5^{.0772}\, x_6^{.303}\, x_7^{-.217}\, x_8^{.0286}\, x_9^{.609}$$

(.215) (.298) (.147) (.486) (.103) (.0654) (.271) (.0582) (.310)

$n = 12$ $R^2 = .999$

Upper Colorado Basin

$$y = .0246\, x_1^{.100}\, x_2^{.133}\, x_3^{.170}\, x_4^{-.612}\, x_5^{.0455}\, x_6^{-.0378}\, x_7^{.599}\, x_8^{.422}\, x_9^{.237}$$

(.0806) (.0953) (.149) (.494) (.0483) (.0752) (.381) (.156) (.246)

$n = 18$ $R^2 = .997$

Upper Rio Grande Basin

$$y = 2.79\, x_1^{.404}\, x_2^{-.0444}\, x_3^{.323}\, x_4^{-.647}\, x_5^{-.248}\, x_6^{-.0325}\, x_7^{.754}\, x_8^{-.134}\, x_9^{.598}$$

(.447) (.196) (.279) (.880) (.430) (.0661) (.201) (.249) (.537)

$n = 14$ $R^2 = .993$

Lower Rio Grande Basin

$$y = 171.441\, x_1^{.0812}\, x_2^{-.0773}\, x_3^{.154}\, x_4^{.831}\, x_5^{-.286}\, x_6^{.354}\, x_7^{.121}\, x_8^{-.158}\, x_9^{-.121}$$

(.148) (.426) (.230) (.634) (.508) (.226) (.540) (.282) (.604)

$n = 14$ $R^2 = .960$

Upper Missouri Basin

$$y = .439\, x_1^{.0703}\, x_2^{.0838}\, x_3^{.0363}\, x_4^{-.0122}\, x_5^{-.119}\, x_6^{.237}\, x_7^{.274}\, x_8^{.0267}\, x_9^{.505}$$

(.0211) (.106) (.119) (.223) (.0496) (.0384) (.187) (.0995) (.311)

$n = 26$ $R = .991$

Table 2. Continued

Northwestern Ogallala

$$y = 19.731\ x_1^{-.0793}\ x_2^{-.188}\ x_3^{.069}\ x_4^{-.272}\ x_5^{.0755}\ x_6^{.385}\ x_7^{-.0470}\ x_8^{.105}\ x_9^{.859}$$

(.0874) (.163) (.114) (.157) (.0722) (.0616) (.335) (.146) (.471)

$n = 15$ $\qquad$ $R^2 = .998$

Northeastern Ogallala

$$y = 5.379\ x_1^{.0976}\ x_2^{-0589}\ x_3^{.00345}\ x_4^{.0553}\ x_5^{-.123}\ x_6^{.269}\ x_7^{.599}\ x_8^{.173}\ x_9^{-.0206}$$

(.123) (.410) (.150) (.660) (.151) (.112) (.608) (.199) (.217)

$n = 19$ $\qquad$ $R^2 = .957$

Central Ogallala

$$y = 3.93\ x_1^{-.0219}\ x_2^{.110}\ x_3^{-.0405}\ x_4^{-.0718}\ x_5^{.0897}\ x_6^{.409}\ x_7^{-.0519}\ x_8^{-.00983}\ x_9^{.594}$$

(.0976) (.236) (.173) (.272) (.145) (.0935) (.225) (.170) (.344)

$n = 22$ $\qquad$ $R^2 = .977$

Southern Ogallala

$$y = .761\ x_1^{-.371}\ x_2^{-.430}\ x_3^{.0852}\ x_4^{.156}\ x_5^{.437}\ x_6^{.148}\ x_7^{.257}\ x_8^{.326}\ x_9^{.551}$$

(.0964) (.212) (.163) (.0298) (.132) (.0862) (.192) (.126) (.331)

$n = 21$ $\qquad$ $R^2 = .992$

---

[a]Variables defined as:

- y = Value of agricultural output-- value of crops harvested and livestock sales (dollars/county)
- x_1 = Irrigation water applied (acre-feet/county)
- x_2 = Value of land and service buildings (dollars/county)
- x_3 = Hired labor expenditures (dollars/county)
- x_4 = Fuel and lubricant expenditures (dollars/county)
- x_5 = Fertilizer and lime expenditures (dollar/county)
- x_6 = Feed expenditures (dollars/county)
- x_7 = Value of machinery inventory (dollars/county)
- x_8 = Value of livestock inventory (dollars/county)
- x_9 = Other operating expenses (dollars/county)

[b]Number in parentheses are standard errors

However, at the same time, the R^2-values are generally high, denoting an adequate fit. As a whole, these estimates appear to have little credibility, and no doubt have been distorted by the effects of multicollinearity. The overall severiety of this multicollinearity problem can be assessed by examining the relative magnitudes of the eigenvalues of the X'X matrix.

Regional Eigenvalues

Following Vinod's recommendation (p. 836), the eigenvalues for each X'X matrix (in correlation form) were determined. These values are summarized in table 3. In a completely orthogonal situation, each regional correlation would be an identity matrix. Since the eigenvalues of such a matrix always equal one, the following holds in such an "ideal" situation:

$$(5.2) \quad \sum_{i=1}^{p} \lambda_i = \sum_{i=1}^{p} 1/\lambda_i = p$$

where p is the rank of the X'X matrix. For the model specification in (5.1), orthogonality would require that $\sum_{i=1}^{p} 1/\lambda_i = 9$. A comparison of this index with the last column of table 3, suggests that the regional data sets deviate greatly from an orthogonal situation. Thus, varied degrees of multicollinearity abound in each of the regional data sets, with especailly high levels in the Desert Southwest and Upper Colorado Basin Regions. To avoid likely distortion in parameter estimates ridge regression was employed.

Ridge Regression Estimates

Results of a ridge regression analysis are presented in table 4. Overall, the ill-effects of multicollinearity appear to have been substantially lessened. Notice especially that there is only *one* negative sign and *no* excessively large coefficients. The negative sign and small magnitude of $\hat{\beta}_2$ in the Upper Rio Grande Basin indicates that land exhibits a small

Table 3. Eigenvalues of Regional Correlation Matrices.

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	$\Sigma\lambda_i^{-1}$
Snake-Columbia Basin	6.927	1.175	0.419	0.198	0.114	0.0798	0.0575	0.0206	0.00857	212.290
Central California	7 838	0.952	0.130	0.0282	0.0242	0.0159	0.00632	.00469	.00165	1126.350
Desert Southwest	8.207	0.474	0.194	0.0584	0.0321	0.0168	0.0130	.00355	.000971	1503.690
Upper Colorado Basin	8.013	0.409	0.250	0.172	0.0826	0.0442	0.0243	0.00493	0.000946	1348.410
Upper Rio Grande Basin	6.937	0.916	0.466	0.392	0.124	0.0862	0.0547	0.0229	0.000848	1266.670
Lower Rio Grande Basin	5.585	1.781	0.627	0.570	0.269	0.126	0.0264	0.00874	0.00674	316.410
Upper Missouri Basin	6.238	1.795	0.349	0.321	0.142	0.0999	0.0426	0.00635	0.00505	402.772
Northwestern Ogallala	8.048	0.363	0.277	0.102	0.0791	0.0686	0.0411	0.0196	0.00225	563.331
Northeastern Ogallala	6.935	0.956	0.476	0.235	0.209	0.100	0.0538	0.0255	0.00943	186.226
Central Ogallala	6.747	1.377	0.367	0.249	0.103	0.0577	0.0511	0.0282	0.0197	140.593
Southern Ogallala	6.384	1.365	0.530	0.411	0.173	0.0641	0.0469	0.0165	0.00964	212.201

Table 4. Production Function Estimates and Related Statistics (RR).[a,b]

Snake Columbia Basin

$$y = 6.503\, x_1^{.0170}\, x_2^{.0690}\, x_3^{.108}\, x_4^{.108}\, x_5^{.0531}\, x_6^{.223}\, x_7^{.0879}\, x_8^{.0982}\, x_9^{.226}$$

(.0351) (.0441) (.0364) (.0292) (.0335) (.0333) (.0377) (.0494) (.0311)

$n = 31$ $k = .15$ $R^2 = .927$

Central California

$$y = .829\, x_1^{.0334}\, x_2^{.211}\, x_3^{.175}\, x_4^{.116}\, x_5^{.0460}\, x_6^{.0995}\, x_7^{.117}\, x_8^{.153}\, x_9^{.141}$$

(.0272) (.0416) (.0242) (.0188) (.0189) (.0160) (.0280) (.0235) (.0163)

$n = 15$ $k = .20$ $R^2 = .957$

Desert Southwest

$$y = 1.026\, x_1^{.0476}\, x_2^{.165}\, x_3^{.188}\, x_4^{.0788}\, x_5^{.0477}\, x_6^{.219}\, x_7^{.145}\, x_8^{.0410}\, x_9^{.172}$$

(.0200) (.0202) (.0140) (.0175) (.0138) (.0159) (.0253) (.0277) (.0119)

$n = 12$ $k = .11$ $R^2 = .978$

Upper Colorado Basin

$$y = 1.004\, x_1^{.133}\, x_2^{.148}\, x_3^{.103}\, x_4^{.128}\, x_5^{.0334}\, x_6^{.0682}\, x_7^{.117}\, x_8^{.217}\, x_9^{.144}$$

(.0194) (.0225) (.0174) (.0141) (.0149) (.0238) (.0165) (.0224) (.0108)

$n = 18$ $k = .15$ $R^2 = .975$

Upper Rio Grande Basin

$$y = 12.276\, x_1^{.0172}\, x_2^{-.00806}\, x_3^{.142}\, x_4^{.122}\, x_5^{.0769}\, x_6^{.0422}\, x_7^{.358}\, x_8^{.0752}\, x_9^{.110}$$

(.0200) (.0354) (.0228) (.0229) (.0142) (.0213) (.0332) (.0441) (.0200)

$n = 14$ $k = .30$ $R^2 = .907$

Lower Rio Grande Basin

$$y = 42.168\, x_1^{.0135}\, x_2^{.0703}\, x_3^{.161}\, x_4^{.195}\, x_5^{.0741}\, x_6^{.183}\, x_7^{.122}\, x_8^{.0201}\, x_9^{.0477}$$

(.0416) (.0741) (.0681) (.0471) (.0348) (.0600) (.0350) (.0823) (.0431)

$n = 14$ $k = .25$ $R^2 = .871$

Upper Missouri Basin

$$y = 1.270\, x_1^{.0243}\, x_2^{.162}\, x_3^{.120}\, x_4^{.146}\, x_5^{.00969}\, x_6^{.145}\, x_7^{.132}\, x_8^{.140}\, x_9^{.183}$$

(.0103) (.0185) (.0208) (.0122) (.0177) (.0168) (.0108) (.0241) (.00951)

$n = 26$ $k = .25$ $R^2 = .938$

Table 4. Continued

Northwestern Ogallala

$$y = .160\ x_1^{.0284}\ x_2^{.176}\ x_3^{.125}\ x_4^{.00285}\ x_5^{.0247}\ x_6^{.230}\ x_7^{.203}\ x_8^{.182}\ x_9^{.226}$$

(.0137) (.0274) (.0158) (.0240) (.0144) (.0114) (.0277) (.0206) (.00999)

$n = 15$ $k = .275$ $R^2 = .922$

Northeastern Ogallala

$$y = 7.90\ x_1^{.0211}\ x_2^{.121}\ x_3^{.0291}\ x_4^{.190}\ x_5^{.00256}\ x_6^{.179}\ x_7^{.218}\ x_8^{.163}\ x_9^{.0351}$$

(.0309) (.0543)(.0503) (.0480)(.0395) (.037)(.0615)(.0485)(.0672)

$n = 19$ $k = .25$ $R^2 = .894$

Central Ogallala

$$y = 1.17\ x_1^{.0178}\ x_2^{.0515}\ x_3^{.0831}\ x_4^{.0841}\ x_5^{.0464}\ x_6^{.257}\ x_7^{.202}\ x_8^{.114}\ x_9^{.259}$$

(.0189) (.0521) (.0415) (.0502) (.0231) (.0278) (.0544) (.0601) (.0405)

$n = 22$ $k = .225$ $R^2 = .911$

Southern Ogallala

$$y = 2.443\ x_1^{.0129}\ x_2^{.0343}\ x_3^{.190}\ x_4^{.107}\ x_5^{.0803}\ x_6^{.129}\ x_7^{.217}\ x_8^{.161}\ x_9^{.127}$$

(.0132) (.0316) (.0225) (.0184) (.0123) (.0111) (.0240) (.0180) (.0181)

$n = 21$ $k = .235$ $R^2 = .925$

---

[a]Variables defined as:

- y = Value of agricultural output--value of crops harvested and livestock sales (dollars/county)
- x_1 = Irrigation water applied (acre-feet/county)
- x_2 = Value of land and service buildings (dollars/county)
- x_3 = Hired labor expenditures (dollars/county)
- x_4 = Fuel and lubricant expenditures (dollars/county)
- x_5 = Fertilizer and lime expenditures (dollars/county)
- x_6 = Feed expenditures (dollars/county)
- x_7 = Value of machinery inventory (dollars/county)
- x_8 = Value of livestock inventory (dollars/county)
- x_9 = Other operating expenses (dollars/county)

[b]Number in parentheses are standard errors

negative marginal value product in this region. However, since this region is comprised largely of "desert-like" land, having limited access to the waters of the Rio Grande River, an extremely small or even negative return to these marginal lands seems plausible.

By summing the slope coefficients in table 4, i.e., β_1 through β_9, the regional returns to scale are determined. This sum varies from a low of .887 in the Lower Rio Grande Basin to a high of 1.2 in the Northwestern Ogallala with an average of 1.043. Overall, these results do not differ greatly from those reported by Hoch (1976) and Griliches.

The standard errors have been decreased in every case, denoting an overall decrease in variance. The R^2-values are lower than those under OLS; however, this reduction has not been substantial. These values range from a low of .871 to a high of .978, whereas those under OLS ranged from .957 to .999. Finally, notice that the selected k values are not excessively large, i.e., ranging from .11 to .30.[19] This result suggests that bias is probably minimal (see Chapter IV, footnote 12).

A Comment

At this point it should be noted that classical inferential statistical methodology is not applicable when biased estimators are used. Because the estimators of the structural parameters and their respective variances are biased, usual confidence intervals and significance tests are invalid.[20] Thus, objective evaluation based on statistical criteria is impossible. However, because of their overall

[19] The k-values were selected from the ridge traces given in appendix D.

[20] Although invalid in a biased case, the "t-tests" under OLS are extremely weak due to the large variances of the estimates. Thus, even though the OLS estimates *are* unbiased, their usefulness for inference is limited.

consistency with economic theory and *a priori* reasoning, the ridge estimates appear to be superior to the least squares estimates in describing the production process. In the following chapter, these estimates are used in deriving regional demand functions and associated properties for irrigation water.

CHAPTER VI

RESULTS: REGIONAL WATER DEMAND CHARACTERISTICS

Reliable estimates of the structural parameters of a system are crucial to any economic analysis. Policy formation, if based on unreliable estimates, could lead to gross misallocations of human, natural and financial resources. Estimates which are *less likely* to be substantially incorrect, e.g., the less variant ridge estimates, should be desirable in policy-making situations. Thus, in the following sections, irrigation water demands are derived from the estimated production functions in table 4. Estimates of the water demand functions and their respective demand elasticities are developed for each of the eleven regions for alternative lengths of run, i.e., planning horizon.

Derived Demand For Irrigation Water

The estimated production functions in table 4 depict agricultural production in its most aggregated form. As previously mentioned, such an aggregated formulation should lessen the complications of the data source (see Chapter I, p. 6). However, since direct water demand is probably quite inelastic in the livestock sector, its effect on the overall demand for agricultural water is probably minimal.[21]

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[21]Irrigation water is indirectly used by the livestock sector in hay production and irrigated pasture. This study, however, classifies hay and silage production explicitly as a crop activity. Thus, the only irrigated activity in the livestock sector is pasture irrigation. Since most irrigation on pasture is confined to very localized areas, e.g. small meadows near streams where the water is virtually costless, it appears that an increase in livestock activity will cause little increased irrigation of pasture due to the costs of delivering water to considerably more cost-ineffective sites.

Thus, livestock related inputs are assumed constant in all lengths of run considered.[22] The long-run, intermediate-run and short-run water demand functions are presented in table 5.

Long-Run Demand

When all crop related inputs are considered variable the demand for irrigation water is termed *long run*. Note that the long-run demand (table 5) is defined in all regions except for the Upper Rio Grande region where the negative production elasticity on land causes that function to be undefined. Each of the remaining regions have demands which display the characteristic inverse relationship between irrigation water price and quantity utilized. Similarly, the negative coefficients on the alternative input prices suggest that the other variable inputs are complementary to water in an economic sense for all regions. That is, an increase in the price of a non-water input implies a reduction in water demanded. Since reduced useage of both inputs is implied the inputs are classified as economic complements. Such complementary relationships are characteristic of agricultural inputs.

---

[22]Intermediate economic theory texts, e.g. Ferguson and Gould and Mansfield, define only two distinct lengths of run, i.e., the *long run* where all inputs are considered variable and the *short run* where *one or more* inputs are fixed. The terminology of this study differs from this classical treatment in two ways. First, the long run considered in this study is a long run situation only from the point of view of the crop sector and thus considers livestock related inputs fixed. Second, the concept of an *intermediate* run has been introduced to differentiate between the various short runs which exist under the classical definition.

Table 5. Irrigation Water Demand Functions by Region, 1969.[a]

Snake-Columbia Basin

Long Run

$$x_1 = .0627\ r_1^{-1.0514}\ r_2^{-.209}\ r_3^{-.326}\ r_4^{-.328}\ r_5^{-.161}\ r_7^{-.266}\ r_9^{-.685}\ x_6^{.673}\ x_8^{.297}\ p^{3.0249}$$

Intermediate Run I

$$x_1 = .0794\ r_1^{-1.0425}\ r_3^{-.269}\ r_4^{-.271}\ r_5^{-.133}\ r_7^{-.220}\ r_9^{-.567}\ x_2^{.173}\ x_6^{.557}\ x_8^{.246}\ p^{2.508}$$

Intermediate Run II

$$x_1 = .0932\ r_1^{-1.0348}\ r_3^{-.221}\ r_4^{-.222}\ r_5^{-.109}\ r_9^{-.465}\ x_2^{.142}\ x_6^{.457}\ x_7^{.180}\ x_8^{.201}\ p^{2.052}$$

Short Run

$$x_1 = .106\ r_1^{-1.017}\ x_2^{.0702}\ x_3^{.110}\ x_4^{.110}\ x_5^{.0540}\ x_6^{.226}\ x_7^{.0894}\ x_8^{.0999}\ x_9^{.230}\ p^{1.017}$$

Central California

Long Run

$$x_1 = (3.42 \times 10^{-7})\ r_1^{-1.207}\ r_2^{-1.308}\ r_3^{-1.0882}\ r_4^{-.718}\ r_5^{-.285}\ r_7^{-.723}\ r_9^{-.874}\ x_6^{.617}\ x_8^{.952}\ p^{6.205}$$

Intermediate Run I

$$x_1 = .00056\ r_1^{-1.0897}\ r_3^{-.471}\ r_4^{-.311}\ r_5^{-.124}\ r_7^{-.313}\ r_9^{-.379}\ x_2^{.567}\ x_6^{.267}\ x_8^{.412}\ p^{2.688}$$

Intermediate Run II

$$x_1 = .00247\ r_1^{-1.0683}\ r_3^{-.359}\ r_4^{-.237}\ r_5^{-.0942}\ r_9^{-.288}\ x_2^{.432}\ x_6^{.204}\ x_7^{.238}\ x_8^{.314}\ p^{2.0469}$$

Short Run

$$x_1 = .024\ r_1^{-1.035}\ x_2^{.218}\ x_3^{.181}\ x_4^{.120}\ x_5^{.0478}\ x_6^{.103}\ x_7^{.121}\ x_8^{.159}\ x_9^{.146}\ p^{1.035}$$

Table 5. Continued

Desert Southwest

Long Run

$$x_1 = (1.129 \times 10^{-6})\ r_1^{-1.306}\ r_2^{-1.0592}\ r_3^{-1.208}\ r_4^{-.506}\ r_5^{-.306}\ r_7^{-.931} r_9^{-1.102}\ x_6^{1.406} x_8^{.263}\ p^{6.418}$$

Intermediate Run I

$$x_1 = .000680\ r_1^{-1.148}\ r_3^{-.587}\ r_4^{-.246}\ r_5^{-.149}\ r_7^{-.452}\ r_9^{-.535}\ x_2^{.514}\ x_6^{.683}\ x_8^{.128}\ p^{3.117}$$

Intermediate Run II

$$x_1 = .00467\ r_1^{-1.1022} r_3^{-.404}\ r_4^{-.169}\ r_5^{-.102}\ r_9^{-.369}\ x_2^{.354}\ x_6^{.470}\ x_7^{.311}\ x_8^{.0881}\ p^{2.147}$$

Short Run

$$x_1 = .0420\ r_1^{-1.050}\ x_2^{.173}\ x_3^{.200}\ x_4^{.0828}\ x_5^{.0501}\ x_6^{.230}\ x_7^{.152}\ x_8^{.0431}\ x_9^{.180}\ p^{1.050}$$

Upper Colorado Basin

Long Run

$$x_1 = (2.181 \times 10^{-5})\ r_1^{-1.689}\ r_2^{-.765}\ r_3^{-.531}\ r_4^{-.663}\ r_5^{-.172}\ r_7^{-.602}\ r_9^{-.743}\ x_6^{.352}\ x_8^{1.120}\ p^{5.16}$$

Intermediate Run I

$$x_1 = .00218\ r_1^{-1.390}\ r_3^{-.301}\ r_4^{-.376}\ r_5^{-.0977} r_7^{-.341}\ r_9^{-.421} x_2^{.433}\ x_6^{.200}\ x_8^{.635}\ p^{2.927}$$

Intermediate Run II

$$x_1 = .0107\ r_1^{-1.291}\ r_3^{-.224}\ r_4^{-.280}\ r_5^{-.0728}\ r_9^{-.314}\ x_2^{.323}\ x_6^{.149}\ x_7^{.254}\ x_8^{.473}\ p^{2.182}$$

Short Run

$$x_1 = .0982\ r_1^{-1.154} \cdot x_2^{.171}\ x_3^{.119}\ x_4^{.148}\ x_5^{.0385}\ x_6^{.0787}\ x_7^{.135}\ x_8^{.250}\ x_9^{.166}\ p^{1.154}$$

Table 5. Continued

Upper Rio Grande Basin

Long Run

"undefined"

Intermediate Run I

$$x_1 = 9.336\ r_1^{-1.0988}\ r_3^{-.816}\ r_4^{-.702}\ r_5^{-.442}\ r_7^{-2.0554}\ r_9^{-.630}\ x_2^{-.0463}\ x_6^{.243}\ x_8^{.432}\ p^{5.743}$$

Intermediate Run II

$$x_1 = .270\ r_1^{-1.0323}\ r_3^{-.267}\ r_4^{-.230}\ r_5^{-.145}\ r_9^{-.206}\ x_2^{-.0152}\ x_6^{.0794}\ x_7^{.673}\ x_8^{.141}\ p^{1.880}$$

Short Run

$$x_1 = .206\ r_1^{-1.0175}\ x_2^{-.00820}\ x_3^{.145}\ x_4^{.124}\ x_5^{.0782}\ x_6^{.0430}\ x_7^{.364}\ x_8^{.0765}\ x_9^{.112}\ p^{1.0175}$$

Lower Rio Grande Basin

Long Run

$$x_1 = 19.347\ r_1^{-1.0426}\ r_2^{-.222}\ r_3^{-.507}\ r_4^{-.615}\ r_5^{-.234}\ r_7^{-.385}\ r_9^{-.151}\ x_6^{.579}\ x_8^{.0636}\ p^{3.156}$$

Intermediate Run I

$$x_1 = 8.355\ r_1^{-1.0349}\ r_3^{-.415}\ r_4^{-.503}\ r_5^{-.191}\ r_7^{-.315}\ r_9^{-.123}\ x_2^{.182}\ x_6^{.474}\ x_8^{.0520}\ p^{2.582}$$

Intermediate Run II

$$x_1 = 2.969\ r_1^{-1.0265}\ r_3^{-.315}\ r_4^{-.383}\ r_5^{-.145}\ r_9^{-.0937}\ x_2^{.138}\ x_6^{.360}\ x_7^{.239}\ x_8^{.0396}\ p^{1.964}$$

Short Run

$$x_1 = .573\ r_1^{-1.014}\ x_2^{.0714}\ x_3^{.163}\ x_4^{.200}\ x_5^{.0751}\ x_6^{.186}\ x_7^{.124}\ x_8^{.0204}\ x_9^{.0484}\ p^{1.014}$$

Table 5. Continued

Upper Missouri Basin

Long Run

$$x_1 = .0001\ r_1^{-1.109}\ r_2^{-.722}\ r_3^{-.537}\ r_4^{-.653}\ r_5^{-.0433}\ r_7^{-.589}\ r_9^{-.818}\ x_6^{.647} x_8^{.624}\ p^{4.471}$$

Intermediate Run I

$$x_1 = .00177\ r_1^{-1.0630}\ r_3^{-.312}\ r_4^{-.379}\ r_5^{-.0252}\ r_7^{-.342}\ r_9^{-.475}\ x_2^{.419}\ x_6^{.376}\ x_8^{.362}\ p^{2.596}$$

Intermediate Run II

$$x_1 = .00578\ r_1^{-1.0469}\ r_3^{-.232}\ r_4^{-.283}\ r_5^{-.0187}\ r_9^{-.354}\ x_2^{.312}\ x_6^{.280}\ x_7^{.255}\ x_8^{.270}\ p^{1.9342}$$

Short Run

$$x_1 = .0283\ r_1^{-1.025}\ x_2^{.166}\ x_3^{.123}\ x_4^{.150}\ x_5^{.00993}\ x_6^{.148}\ x_7^{.135}\ x_8^{.143}\ x_9^{.187}\ p^{1.025}$$

Northwestern Ogallala

Long Run

$$x_1 = (6.00 \times 10^{-9})\ r_1^{-1.137}\ r_2^{-.832}\ r_3^{-.588}\ r_4^{-.0134}\ r_5^{-.116}\ r_7^{-.952}\ r_9^{-1.064}\ x_6^{1.080}\ x_8^{.854}\ p^{4.702}$$

Intermediate Run I

$$x_1 = (1.00 \times 10^{-5})\ r_1^{-1.0746}\ r_3^{-.321}\ r_4^{-.00732}\ r_5^{-.0633}\ r_7^{-.520}\ r_9^{.581}\ x_2^{.454}\ x_6^{.589}\ x_8^{.466}\ p^{2.567}$$

Intermediate Run II

$$x_1 = .000340\ r_1^{-1.0491}\ r_3^{-.211}\ r_4^{-.00481}\ r_5^{-.0417}\ r_9^{-.382}\ x_2^{.299}\ x_6^{.388}\ x_7^{.342}\ x_8^{.307}\ p^{1.689}$$

Short Run

$$x_1 = .00397\ r_1^{-1.030}\ x_2^{.182}\ x_3^{.129}\ x_4^{.00294}\ x_5^{.0254}\ x_6^{.236}\ x_7^{.209}\ x_8^{.187}\ x_9^{.233}\ p^{1.030}$$

Table 5. Continued

Northeastern Ogallala

Long Run

$$x_1 = .1933\ r_1^{-1.055}\ r_2^{-.315}\ r_3^{-.0761}\ r_4^{-.497}\ r_5^{-.00669}\ r_7^{-.570}\ r_9^{-.0917}\ x_6^{.467}\ x_8^{.425}\ p^{2.612}$$

Intermediate Run I

$$x_1 = .189\ r_1^{-1.042}\ r_3^{-.0578}\ r_4^{-.378}\ r_5^{-.00508}\ r_7^{-.434}\ r_9^{-.0698}\ x_2^{.240}\ x_6^{.355}\ x_8^{.323}\ p^{1.986}$$

Intermediate Run II

$$x_1 = .154\ r_1^{-1.0293}\ r_3^{-.0403}\ r_4^{-.263}\ r_5^{-.00355}\ r_9^{-.0487}\ x_2^{.167}\ x_6^{.248}\ x_7^{.302}\ x_8^{.225}\ p^{1.385}$$

Short Run

$$x_1 = .161\ r_1^{-1.022}\ x_2^{.123}\ x_3^{.0297}\ x_4^{.194}\ x_5^{.00262}\ x_6^{.183}\ x_7^{.223}\ x_8^{.166}\ x_9^{.0359}\ p^{1.022}$$

Central Ogallala

Long Run

$$x_1 = .0001\ r_1^{-1.070}\ r_2^{-.201}\ r_3^{-.325}\ r_4^{-.329}\ r_5^{-.181}\ r_7^{-.792}\ r_9^{-1.0136}\ x_6^{1.00339}\ x_8^{.444}\ p^{3.912}$$

Intermediate Run I

$$x_1 = .00043\ r_1^{-1.0580}\ r_3^{-.271}\ r_4^{-.274}\ r_5^{-.151}\ r_7^{-.659}\ r_9^{-.844}\ x_2^{.168}\ x_6^{.835}\ x_8^{.370}\ p^{3.256}$$

Intermediate Run II

$$x_1 = .00358\ r_1^{-1.0350}\ r_3^{-.163}\ r_4^{-.165}\ r_5^{-.0911}\ r_9^{-.509}\ x_2^{.101}\ x_6^{.504}\ x_7^{.397}\ x_8^{.223}\ p^{1.963}$$

Short Run

$$x_1 = .0195\ r_1^{-1.018}\ x_2^{.0524}\ x_3^{.0846}\ x_4^{.0856}\ x_5^{.0472}\ x_6^{.261}\ x_7^{.206}\ x_8^{.116}\ x_9^{.264}\ p^{1.018}$$

Table 5. Continued.

Southern Ogallala

Long Run

$$x_1 = .00847\ r_1^{-1.0555}\ r_2^{-.148}\ r_3^{-.819}\ r_4^{-.462}\ r_5^{-.347}\ r_7^{-.939}\ r_9^{-.550}\ x_6^{.557}\ x_8^{.697}\ p^{4.320}$$

Intermediate Run I

$$x_1 = .00186\ r_1^{-1.0483}\ r_3^{-.713}\ r_4^{-.402}\ r_5^{-.302}\ r_7^{-.817}\ r_9^{-.479}\ x_2^{.129}\ x_6^{.485}\ x_8^{.607}\ p^{3.762}$$

Intermediate Run II

$$x_1 = .00882\ r_1^{-1.0266}\ r_3^{-.392}\ r_4^{-.221}\ r_5^{-.166}\ r_9^{-.263}\ x_2^{.0711}\ x_6^{.267}\ x_7^{.450}\ x_8^{.334}\ p^{2.0700}$$

Short Run

$$x_1 = .0300\ r_1^{-1.0130}\ x_2^{.0348}\ x_3^{.192}\ x_4^{.108}\ x_5^{.0813}\ x_6^{.131}\ x_7^{.220}\ x_8^{.163}\ x_9^{.129}\ p^{1.0130}$$

[a] variables defined as:

- x_1 = Irrigation water applied (acre-feet/county)
- x_2 = Value of land and service buildings ($/county)
- x_3 = Hired labor expenditures ($/county)
- x_4 = Fuel and lubricant expenditures ($/county)
- x_5 = Fertilizer and lime expenditures ($/county)
- x_6 = Feed expenditures ($/county)
- x_7 = Value of machinery inventory ($/county)
- x_8 = Value of livestock inventory ($/county)
- x_9 = Other operating expenses ($/county)
- r_1 = Price of irrigation water ($/ac.ft.)
- r_2 = Implicit rental price of land and service building investment ($)
- r_3 = Price of labor expenditures ($)
- r_4 = Price of fuel and lubricant expenditures ($)
- r_5 = Price of fertilizer and lime expenditures ($)
- r_7 = Implicit rental price of machinery inventories ($)
- r_9 = Price of other input expenditures ($)
- p = Composite product price ($)

The exponent on product price (p) displays the expected positive relationship between water used and product price. Note also that the fixed livestock inputs display a direct positive relationship between the quantity of water used and their respective levels.

Intermediate Run (I & II)

The *intermediate run I and intermediate run II* (table 5) characterize production when only land, and then when land and machinery, respectively, are held fixed. These runs are discussed concurrently since, for the most part, the demands appear to vary little when land and land plus machinery inventories are held fixed, in addition to the livestock related factors. Both sets of demand equations generally display the inverse relationship between price and quantity utilized, a positive relationship with the fixed factors and a positive relationship with product price. The only exception to this is in the Lower Rio Grande where the negative sign on land is evident. Overall the major difference between the two runs is that intermediate run I is, as expected, more elastic than the shorter intermediate run II.

Short-Run Demand

In the *short run*, all crop-related inputs except water are considered fixed to the firm. In such a case, the demand function and the marginal value productivity function are one and the same, i.e., the former is merely the inverse of the latter when equated to input price. Again, from table 5, we note that each short-run water demand function exhibits the characteristic downward slope and positive relationship with all fixed inputs (except for, of course, the Upper Rio Grande Basin). In the short-run case the level of irrigation water is dependent only upon its price and the quantities of other associated inputs used, and not explicitly upon the prices of these other inputs.

Water Demand Elasticities

Since all demand functions relate the quantity of a factor to its price, the prices of the associated variable inputs, the product price and fixed factor quantities, expressions for both own and cross-price elasticities are easily deduced from the demand equations. Furthermore, in this case the elasticities can be read directly from the demand equations (table 5) since a power (Cobb-Douglas) production function yields elasticities which are constant and equal to the respective price coefficients of the demand function (see Chapter III).

Since the arguments in the demand functions differ with the length of run considered, the elasticities also differ among lengths of run. As expected, the own-price elasticity becomes larger in absolute value as the length of run is extended, implying increased elasticity. For example, in the Upper Colorado Basin Region the estimated own-price elasticity of water varies from -1.689 in the long run to -1.153 in the short run. This result occurs because as the planning horizon is lengthened, fewer inputs are fixed and decision makers have a broader range of choice in responding to changing economic conditions. This, of course, holds as well for the input cross-elasticities and product-price elasticities.

Own-Price Elasticities

Estimates of the own-price elasticities for irrigation water are given by the coefficients on water price (r_1) in the regional demand functions (table 5). Each estimate displays the characteristic negative sign denoting an inverse price-quantity relationship and absolute magnitudes of near one. This result tends to support an overall elastic relationship between water utilized in irrigated agriculture and price. More specifically, notice that water demand in the long run is extremely elastic in the Desert Southwest and Upper Colorado Basin. This result appears to hold for each length of run. A possible explanation for this responsiveness is that the major irrigated crops in these regions are mainly lower valued, e.g., sorghum, hay and wheat (see appendix A). Conversely, note the relative inelasticity of the Snake-Columbia, Lower Rio Grande and Northeastern Ogallala Regions. Relatively low water prices and/or the existence of supplemental irrigation practices are plausible factors in rationalizing this decreased elasticity.

In the shorter-run cases (intermediate run I & II and short run), own-price water demand elasticity appears to vary little among regions. In general, demand appears to be somewhat elastic. Again, water application is most responsive to water prices in the Desert Southwest and Upper Colorado Regions.

Cross-Price Elasticities

The coefficients of each associated input price in table 5 is an estimate of the cross-price elasticity of these inputs relative to irrigation water. Notice that for the long run the quantity of water applied is more responsive to the price of other inputs. More specifically, the estimated cross-price elasticities for the Central California region are -1.308 for land, -1.0882 for labor, -.718 for energy, and -.285 for fertilizer. For the intermediate run I & II, the magnitudes of these estimates are generally much smaller, denoting a lesser response. As a whole, it appears that changes in land and labor prices have the greatest impact on water applied in agriculture.

Product-Price Elasticities

The estimates of the product-price elasticities, as given by the product price (p) exponents of the demand functions, reveal the dependence of agricultural water demand on product price. In general, the long-run demand is extremely responsive to produce price changes. Specifically, the Central California, Desert Southwest, Upper Colorado Basin, and Northwestern Ogallala Regions appear to be the most sensitive to product price fluctuations, whereas the Northeastern Ogallala Region is the least sensitive. This same result holds generally in each of the shorter-run cases, but to a lesser extent. As a whole, agriculture's water usage depends greatly upon product price changes, except in the extreme short run where the product price elasticity approaches unity.

A Comment

The preceding two chapters have dealt with irrigation water demand in general terms. As mentioned earlier, specific (point) estimates of the demand characteristics (marginal water values) can be deduced by introducing additional information on magnitudes of the economic variables, i.e., input prices, input levels, etc. The following chapter utilizes the results of Chapters V and VI, along with relevant input prices and fixed input levels, to derive specific results relative to the regional demand characteristics.

CHAPTER VII

IRRIGATION WATER VALUES IN THE WEST

Probably the most important parameter needed in water policy evaluation is economic value. Most economists argue that it is this value which best serves as an indicator of the social benefits (in efficiency terms) of a resource in alternative uses and/or regions. In this chapter, estimates of relative water values for the various study regions are derived from the respective demand functions. Further, "projections" of water values are made for 1974 and 1978.

Regional Irrigation Water Values

Point estimates of marginal water values for each of the eleven regions were obtained by substituting relevant input levels and prices into the estimated demand functions reported in table 5 (Chapter VI). Since all input variables (except water) are expressed in value terms, the appropriate output and nondurable input prices are $1.00. For the durable inputs, e.g., land and building and machinery inventories, the appropriate input price is the implicit rental price of the specific capital good (see appendix C). Regional mean values were used for the levels of the fixed resources.

Point estimates of regional marginal water values are presented in table 6. These values are reported *for water usage at the mean level for each region*. There are several points worthy of note in table 6. First, there are important differences in marginal water values for alternative lengths of run and among regions. The marginal value of water in irrigated agriculture is generally low--ranging from a low of

Table 6. Marginal Water Values Under Alternative Lengths of Run, 1969.[a]

Region	Length of Run			
	Long Run	Intermediate Run I	Intermediate Run II	Short Run
Snake-Columbia Basin	1.80	1.80	1.83	1.71
Central California	27.79	6.79	5.30	4.92
Desert Southwest	b	22.74	10.37	7.73
Upper Colorado Basin	8.08	6.87	6.65	7.02
Upper Rio Grande Basin	---	22.07	2.52	2.25
Lower Rio Grande Basin	10.00	8.94	6.54	3.51
Upper Missouri Basin	5.98	4.40	4.27	4.09
Northwestern Ogallala	b	24.88	11.97	9.42
Northeastern Ogallala	16.58	12.71	8.40	6.63
Central Ogallala	27.74	21.53	7.44	4.36
Southern Ogallala	8.81	8.27	3.08	2.03

[a]Based on mean levels of all variables for each region.

[b]Values not reported--see text.

$1.71 per acre foot in the short run for the Snake-Columbia Basin to a high of $27.79 in the long run for Central California at the mean water use level. Also notice that the value of water decreases as the length of run is shortened. For example, in the Lower Rio Grande Basin the marginal value ranges from $3.51 in the short run to $10.00 in the long run.

At first glance this result seems at odds with conventional wisdom (Young and Gray). Normally, one would expect longer-run demand curves to intersect shorter-run curves from below as the quantity of water is increased form zero. Further, if fixed factors are held at optimal levels, (they were *not* in table 6) a common intersection point would occur at optimal water use. In such cases, water values will be greater for shorter-run curves up to the optimal water-use point. However, for water usage exceeding the optimal level, the reverse would be expected. Following this logic the results in table 6 suggest that mean water usage levels are in excess of optimal levels. However, recall that short-run fixed factors were held at regional mean rather than long-run optimal levels in calculating the values in table 6. Thus, the larger long-run values may be due to the fact that when more inputs are variable, entrepeneurs have a greater range of choice in adjusting their input mix in response to changing economic conditions (prices). It is possible that in some cases productivity could be increased as a result. Either or both of these reasons could account for the generally higher long-run values.[23]

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[23]More discussion of this issue is provided in the comment at the end of this chapter.

Finally, notice that the values for the long run for both the Desert Southwest and the Northwestern Ogallala regions are not reported in table 6. It appears the combined effect of the livestock sector and variable land have caused serious upward distortions in value estimates for these regions. These values were estimated at $135.38 for the Desert Southwest and $129.70 for the Northwestern Ogallala. This result emphasizes the possible distortions which can occur for those regions with a hetereogeneous livestock industry.[24]

Comparison of Estimates With Those of Other Studies

Overall the estimated water values appear to compare favorably with several previous studies. However, since earlier studies report their results in various form, i.e., with and without returns to management included, with and without delivery costs accounted for, etc., comparisons are at best gross. Since, for the most part, earlier studies report short-run estimates only, the most relevant comparisons are with intermediate run II and the short-run results. If delivery costs were considered, it appears that the $3.50-$4.85 price range reported by Anderson (1961) compares well with the $9.42-$11.97 values of the Northwestern Ogallala. Also the value of $7.44 found for the intermediate run II is very near to the $7.50 result reported by Anderson, et. al., (1966) for Western Oklahoma.

A possible indirect evaluation of the estimates in table 6 can

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[24]It appears that the relative importance of livestock feeding operations in these regions attributed to the distortion. These effects were not as evident in the two southernmost Ogallala regions since most of these counties with extensive feed lot operations were deleted from the study region.

can be made using cost data. Rational behavior on the part of farmers would cause them to purchase and apply water and/or bear pumping and delivery costs up to the point where the value of water in production was near that cost. Thus, the cost per acre foot of using water should be a good estimate of the value of that water.[25] Shumway and Sults (1964) have published a study which reports agricultural water cost estimates for various regions and subregions of California. The mean cost for a region comparable to this study's Central California Region is $4.95. This cost value lies within the $4.92 to $5.30 values in table 6.

Recently, Sloggett published a nation-wide survey on 1974 pumping costs for agriculture. An average of his cost estimates yield a mean water cost of $8.92 in Arizona and $2.66 in Montana.[26] These cost values are slightly below the water value estimates for the Desert Southwest and Upper Missouri Basin in table 6. Finally, water costs for the Texas Rio Grande Valley range from $4.29 to $9.49 per acre foot.[27] Again, this cost range bounds the $6.54 value for intermediate

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[25]An idea analogous to this approach was attempted by Johnson and Halter. In their study of northwestern feedlots, cost functions were estimated indirectly from expected marginal revenue observations.

[26]Arizona and Montana were chosen because an average of Sloggett's estimates are probably more precise than for the other states. Sloggett reported cost data on both water pumped from groundwater sources and that pumped from surface sources. Since Arizona was reported to have no water pumped from surface sources a weighted average of the costs of pumping groundwater should be an adequate estimate of that state's pumping costs. Montana, on the other hand, has 90.6% of its water drawn from surface sources. Both of the above numbers have been deflated to 1969 dollars.

[27]These costs were determined by dividing irrigation water costs per acre (Extension Economist-Management) by per acre water application rates for various crops (Lacewell, et.al.). The lower cost is for coastal bermuda hay; the higher cost is for onions.

run II for the Lower Rio Grande Basin, which encompasses all of the Lower Rio Grande Valley.

Projected Water Values--1974 and 1978

Projected water values for 1974 and 1978, based on the 1969 data and estimates, are presented in table 7 and 8 , respectively. In order to make these "projections", a critical assumption must be made regarding present agricultural technology, viz., that technology and level of water and nonwater fixed-input usage have not changed significantly since 1969. If this situation holds, then projection is straight forward and should be reasonably accurate. All that is needed is the appropriate price indices. The various 1974 and 1978 input and output prices used were based on indices of prices paid by farmers for production inputs by input categories and prices received by farmers[28] (United States Department of Agriculture).

A comparison of tables 6, 7 and 8 reveals, as expected, an increase in water values between 1969 and 1978. Steadily increasing product prices attribute to this trend. However, projected water values decrease between 1974 and 1978 except for the short-run case. The projected decrease in water values between 1974 and 1978 is due to the fact that input prices increased at a more rapid rate than output prices during this period.

In the short run, marginal water values do not depend on alternative input prices, but rather fixed input quantities and the product price. Therefore, projected MVPs increased between 1974 and 1978 due

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[28]The 1978 prices are based on an average of the first ten months of 1978.

Table 7. Projected Marginal Water Values for Alternative Lengths of Run, 1974.[a,b]

Region	Length of Run			
	Long Run	Intermediate Run I	Intermediate Run II	Short Run
Snake-Columbia Basin	4.02	3.80	3.61	3.04
Central California	84.18	14.75	10.52	8.76
Desert Southwest	c	51.25	20.62	13.76
Upper Colorado Basin	18.76	14.20	12.81	12.47
Upper Rio Grande Basin	---	73.29	4.87	4.02
Lower Rio Grande Basin	23.02	19.31	12.87	6.23
Upper Missouri Basin	16.11	9.65	8.47	7.27
Northwestern Ogallala	c	54.31	22.86	16.76
Northeastern Ogallala	36.93	23.23	15.63	11.79
Central Ogallala	70.22	50.66	14.55	7.75
Southern Ogallala	23.81	20.94	6.10	4.75

[a]Uses indices of prices paid for production inputs by input and prices received by farmers in 1974.

[b]Assumes no change in technology since 1969.

[c]Not reported--see text.

Table 8. Projected Marginal Water Values for Alternative Lengths of Run, 1978,[a,b]

Region	Length of Run			
	Long Run	Intermediate Run I	Intermediate Run II	Short Run
Snake-Columbia Basin	3.17	3.25	3.43	3.33
Central California	44.20	12.25	10.13	9.57
Desert Southwest	c	40.75	19.82	15.04
Upper Colorado Basin	13.94	12.54	12.65	13.64
Upper Rio Grande Basin	---	30.57	4.80	4.39
Lower Rio Grande Basin	17.60	16.19	12.47	6.82
Upper Missouri Basin	9.77	7.89	8.10	7.95
Northwestern Ogallala	c	44.03	22.93	18.33
Northeastern Ogallala	28.39	22.73	16.09	12.89
Central Ogallala	44.53	35.97	13.96	8.47
Southern Ogallala	14.31	13.83	5.87	5.19

[a]Uses average indices of prices paid for production inputs by input catagory and prices received by farmers in the first 10 months of 1978.

[b]Assumes no change in technology since 1969.

[c]Not reported--see text.

to the increasing produce price index, *ceteris paribus*. For example, in the Southern Ogallala (intermediate run II), water values increase from $3.08 in 1969 to $6.10 in 1974, but then decrease to $5.87 in 1978.

A Comment

Two major points must be emphasized in this chapter. First, the marginal water values in table 6 and the projections in table 7 and table 8 are reported at mean levels of all explanatory variables. The reader is cautioned that these point estimates depend importantly on the input levels, input prices and product prices assumed. Of particular concern is the fact that longer-run water values generally exceeded shorter-run values. This suggests that either too much water (on average) was applied in 1969 or that mean levels of short-run fixed factors are not optimal. If all factors were fixed at optimal levels, including water, then the marginal water value would be the same for all lengths of run. Second, the water values in table 7 and table 8 are merely projections which are based on the assumption of an unchanging technology in agriculture since 1969. Their major purpose is to express the values in more current dollar terms and to demonstrate the sensitivity of irrigation water values to changing product and factor prices. However, since 1974 is a census year, comparison of the projections reported here with estimates based on 1974 data should be possible in subsequent research. This would allow an indirect test of the validity of the assumption (hypothesis) of no significant technological change.

CHAPTER VIII

SUMMARY AND LIMITATIONS

This study has attempted to develop an improved approach for assessing the value of water as a productive input in irrigated agriculture. Although numerous previous attempts exist in the literature, most are somewhat limited in scope and present their results in varied forms. Thus, comparisons between studies and therefore regions are difficult. This study differs in that a single methodology and data source is utilized to ascertain water values for the major irrigated regions of the Western United States. These results should prove useful in water policy evaluations involving interregional comparisons.

Summary and Results

Eleven regions were selected as the major irrigated areas of the West. Relative homogeneity within each of these regions was insured by choosing counties (observational units) which have similar aggregate agricultural output. Production of this output (in value terms) was hypothesized to take the form of a multiplicative function with nine domain variables, i.e., irrigation water applied, value of land and buildings, hired labor expenditures, fuel and lubricant expenditures, fertilizer and lime expenditures, feed expenditures, value of machinery inventory, value of livestock inventory and miscellaneous expenditures.

Using *1969 Census of Agriculture* data, each regional function was statistically fit using both ordinary least squares (OLS) and ridge regression. As expected, parameter estimates under OLS were highly unstable due to high correlations among the explanatory variables (multicollinearity). One-third of the estimated coefficients took on nonsensical signs and the standard errors were generally high. This posed a serious problem as the precision of the individual parameter estimates greatly influences inputed values to irrigation water.

To circumvent this problem ridge regression was employed. While admittedly a biased estimation technique, the credibility of the estimates appeared to increase. All parameter estimates, except for one out of 99, took on the expected positive sign and the standard errors were decreased in every case. Returns to scale were estimated to vary from a high of 1.200 in the Northwestern Ogallala to a low of .887 in the Lower Rio Grande Basin. Overall, the functions estimated with ridge regression were more compatible with theoretical expectations than were those based on OLS estimates.

From these production functions the demand for irrigation water was derived for alternative lengths of run. As expected, each demand function became more inelastic as the length of run was shortened. As a whole, water demand was found to be slightly elastic for all lengths of run considered with the more elastic demand in the Desert Southwest and Upper Colorado Basin, and slightly less elastic demand in the Snake-Columbia, Lower Rio Grande Basin and Northeastern Ogallala. The quantity of water applied was found to be most sensitive to product price changes in the Central California, Desert Southwest, Upper Colorado Basin and Northwestern Ogallala Regions. In terms of cross-factor effects, water application rates were found to be most responsive to changes in the prices of land and labor for all regions.

Marginal irrigation water values for each length of run considered were calculated for 1969 at the respective regional mean values of the explanatory variables. Water values generally increased with increasing length of run. These values varied from a high of $27.79 for the long run in the Central California to a low of $1.71 in the short run for the Snake-Columbia Basin. Projections of values for 1974 (a census year) and 1978 were made with the assumption of no change in technology and level of "fixed" input and water usage since 1969. Water values were found to increase until 1974 and then decrease in 1978. These projections should serve as a basis for possible later validation by other researchers.

Study Limitations

Determination of water demand from estimated production functions is no easy task. This study attempted to improve on previous methodology by hypothesizing a more complete production specification, and utilizing ridge regression as an estimation technique. This approach allows a lessening of the level of aggregation of the inputs. Thus, the various cross-effects of specific input categories on the quantity of water utilized may be determined. This inclusion of additional independent variables almost always causes an increase in the effects of multicollinearity. Ridge regression has been shown to be a possible answer to this problem. However, the use of such a biased estimation technique causes the loss of the statistical inferential qualities of a study, and it becomes mainly descriptive. This is an important limitation of this study.

Another limitation lies in the data source. The *Census of Agriculture* reports selected production expenditures as a single value, including both crop and livestock components. Our approach was to sum crop and livestock output into a single output variable to be consistent with the input data. However, this approach may cause distortions in the effect of input variables which are used mainly in the production of just one component of aggregate output, e.g., water for cropland irrigation. This distortion was particularly evident in both the Desert Southwest and Northwestern Ogallala regions.

Possible Additional Research

Two issues need research attention to improve the methodology used in this study. These include (1) purging the livestock components from the data, and (2) utilizing statistical estimation techniques which have inferential qualities. A brief discussion of each of these purported improvements follows.

Since irrigation water is used predominantly in the crop sector, a model specification which includes only crop output and crop related inputs seems desirable. However, as previously mentioned, input expenditures are reported as a sum of both livestock and crop related expenditures. Thus, some method which purges the census data of their livestock components would be needed to specify an exlusively crop related production relationship. One attempt at this has been to deflate each input catagory by the ratio of crop sales to livestock sales (Roseine & Helmberger). The idea of a crop-livestock index seems to be a questionable (gross) approach. Possibly a better approach would be to develop an index (by region) which relates the expenditure of each specific input category to a set amount, say $1.00, of livestock and/or crop output. In this manner, an estimate of that portion of the total expenditure for each component could be determined and the aggregate data more appropriately deflated.

A second possible improvement lies in the estimation technique. As previously noted, standard statistical inferences are not possible using biased estimators like those of ridge regression. Thus, an estimation technique which circumvents the problems of multicollinearity and retains desirable properties would be superior. One possible alternative would be to amass cross-sectional data from two census years, e.g., 1969 and 1974, and derive the Aitken's generalized least squares estimator of the parameters (Kmenta). This estimator is known to be consistent and asymptotically efficient. Another alternative would be to utilize a "mixed estimator" introduced by Theil and Goldberger. This method attempts to combine prior information with sample information. Basically, an assumption is made about a probable interval within which the parameter value lies. Another assumption deals with the distribution of the parameter within that interval. The least squares principle is then applied with the assumed restrictions. The resulting estimators can be shown to be both consistent and asymptotically efficient.

A Final Comment

Although this study has several limitations, leaving room for improvement, it does yield water value estimates which should be useful for *relative* comparisons among regions. However, the water value estimates reported herein should be interpreted with some caution. It appears that, at least in a few instances, the estimates may be distorted due to undue influence of the livestock related variables in the model. Work will soon be undertaken to remedy this deficiency in the model.

Finally, the demand for the several nonwater farm inputs could be deduced from the production elasticity estimates presented in this report, if one were interested in other input valuation and/or allocation policy issues in western regions.

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APPENDICES

APPENDIX A: CROPLAND CHARACTERISTICS OF STUDY REGIONS

Appendix Table A1. Cropland Characteristics of the Study Regions.

(1) State/County	(2) Harvested Acres[a]	Irrigated Acres Harvested (3) Total	(4) Corn	(5) Sorghum	(6) Hay	(7) Wheat	(8) Oats, Rye, Barley	(9) Other[b]	(10) Pasture
Snake-Columbia Basin									
Washington									
Adams	762,960	79,242	2,650	0	15,520	41,391	2,230	9,732	7,719
Benton	487.368	28,994	1,638	173	10,226	2,012	1,044	3,901	10,000
Franklin	406,876	103,299	5,763	151	59,414	16,762	1,196	15,473	3,099
Grant	736,538	171,197	19,394	407	70,235	28,823	4,336	34,037	13,965
Kittitas	438,207	74,704	1,426	46	29,383	2,314	4,176	929	36,430
Lincoln	939,512	27,043	0	0	5,949	16,973	2,183	40	1,898
Walla Walla	488,880	58,695	585	1,022	17,520	14,007	3,711	13,553	8,297
Yakima	548,492	101,825	22,529	314	26,335	9,408	5,654	11,457	26,474
Oregon									
Malheur	1,226,026	174,491	10,607	222	81,058	7,192	14,906	23,537	36,969
Morrow	773,588	16,912	75	0	8,804	4,816	762	40	2,415
Umatilla	957,926	56,689	1,749	222	21,250	11,319	3,402	3,958	14,789
Idaho									
Ada	266,351	63,422	10,788	83	26,935	2,227	6,541	2,347	14,501
Bannock	236,291	41,372	277	0	17,302	4,259	7,781	4,374	7,379
Bingham	471,643	214,687	2,031	58	59,394	31,738	42,574	50,000	28,892
Bonneville	367,990	124,256	454	263	34,681	21,102	20,375	35,227	12,154
Canyon	191,617	134,730	17,482	388	31,219	9,606	25,686	25,659	24,690
Cassia	434,000	131,863	4,472	10	51,762	13,946	23,700	19,057	18,916
Elmore	362,672	38,879	2,424	4	11,237	978	10,738	9,255	4,243
Fremont	264,381	70,407	35	50	16,330	10,785	10,417	17,635	15,153
Gem	288,465	29,436	3,370	55	11,782	949	2,491	670	10,119
Gooding	220,174	66,343	7,737	33	25,144	4,832	3,125	3,075	23,193
Jefferson	307,804	130,570	848	0	57,336	13,780	24,094	20,739	13,773

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Jerome	136,952	82,024	4,165	49	33,440	10,250	8,015	11,300	14,805
Lincoln	96,888	40,225	1,537	12	16,275	3,354	3,824	3,071	12,152
Madison	195,746	77,486	50	60	14,793	11,305	19,494	24,910	6,874
Minnidoka	217,797	104,986	4,145	70	30,941	11,294	27,500	18,637	12,399
Owyhee	549,981	87,530	6,077	359	41,033	4,059	10,120	12,865	13,017
Payette	119,111	32,769	3,501	25	11,782	2,557	2,993	1,823	10,088
Power	268,449	50,118	343	0	8,720	13,253	8,954	15,742	3,106
Twin Falls	363,722	140,222	8,690	92	55,445	22,520	13,550	9,826	30,099
Washington	568,685	23,754	2,297	5	9,269	2,181	3,302	1,266	5,434
Total for Region		2,578,170	147,139	4,173	910,514	349,992	318,874	404,135	443,122
Weighted Percentages[c]		34.5%	5.71%	.16%	35.32%	13.58%	12.37%	15,68%	17.19%
Central California									
California									
Colusa	251,010	34,064	764	8,888	9,252	5,946	3,473	1,234	4,507
Contra Costa	262,348	16,987	4,227	890	3,071	547	1,931	175	6,146
Fresno	1,432,569	575,306	14,861	12,605	97,086	21,479	172,403	226,846	30,026
Kern	3,440,998	510,761	3,171	35,205	124,811	17,803	59,945	225,220	44,603
Kings	468,144	270,290	19,259	17,919	38,765	14,697	81,406	84,059	14,185
Madera	600,080	149,162	11,267	4,976	46,554	5,467	10,967	43,956	25,975
Merced	819,531	221,717	24,290	11,510	74,588	2,634	17,060	24,774	66,861
Sacramento	406,094	74,176	21,284	9,302	15,109	1,620	1,944	29	24,888
San Joaquin	551,033	204,257	43,169	17,032	60,776	5,403	21,914	5,067	50,896
Solano	344,192	49,168	8,225	10,880	12,120	3,658	866	57	13,362
Stanishlaus	518,168	131,312	23,309	4,187	44,042	949	5,971	1,331	51,523
Sutter	175,548	52,301	2,350	20,662	7,834	4,701	1,987	1,379	13,388
Tulare	935,750	325,539	25,137	35,511	73,520	11,818	44,316	109,821	25,416
Yolo	320,686	100,194	15,552	16,602	55,927	4,342	3,072	669	4,030
Yuba	142,248	22,987	337	619	4,139	602	703	5	16,582
Total for Region		2,738,221	217,202	206,788	667,594	101,666	427,961	724,622	392,499
Weighted Percentages[c]		31.4%	7.93%	7.55%	24.38%	37.13%	15.63%	26.46%	14.33%

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Desert Southwest					
Arizona									
Cochise	1,877,533	76,621	1,904	48,364	4,539	5,040	1,552	9,673	5,549
Graham	732,972	49,335	997	20,508	4,260	239	5,080	15,602	2,525
Greenlee	6,013	4,708	188	856	1,680	30	92	1,170	692
Maricopa	1,535,611	360,033	5,782	46,974	82,177	18,917	67,362	129,344	9,477
Pima	1,234,038	38,140	0	7,559	1,044	3,763	5,845	16,275	3,645
Pinal	1,339,560	203,377	1,088	25,151	25,850	7,714	36,893	96,787	9,894
Yuma	238,109	143,615	1,909	15,649	44,601	21,422	15,212	40,223	4,599
California									
Imperial	439,451	322,654	415	35,187	132,955	17,837	64,720	53,166	18,374
Riverside	334,901	91,181	1,686	10,976	35,990	5,669	12,305	18,004	6,551
San Bernadino	2,024,811	26,914	1,391	305	17,750	290	240	635	6,303
New Mexico									
Hidalgo	17,183	16,005	159	7,602	1,479	579	498	3,805	1,883
Luna	37,394	33,803	749	16,945	909	510	1,348	11,831	1,511
Total for Region		1,366,386	16,268	236,440	317,244	82,010	211,156	396,515	71,003
Weighted Percentages[c]		34,79%	1.19%	18.51%	25,85%	6.00%	15.45%	29.02%	5.20%
				Upper Colorado Basin					
Colorado									
Delta	232,665	54.866	4,683	35	22,366	707	5,763	12	21,300
Eagle	204,664	30,227	0	0	16,811	61	271	10	13,074

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Garfield	412,966	52,638	886	214	32,982	352	1,427	53	16,724
Mesa	444,794	67,513	11,446	757	24,147	441	2,589	231	27,902
Montrose	419,280	78,465	7,110	141	23,613	1,845	13,624	241	31,881
Utah									
Carbon	360,002	9,380	621	29	4,340	387	839	21	3,143
Daggett	28,150	8,092	0	0	4,660	0	51	1	3,380
Duchesne	371,795	90,998	2,502	22	32,928	1,040	3,565	286	50,655
Emery	242,892	33,035	1,278	238	13,354	1,182	2,183	201	14,599
Garfield	132,209	16,373	112	0	8,120	306	466	99	7,270
Grand	161,140	1,951	85	0	1,319	10	14	9	514
San Juan	404,279	5,352	2	0	2,743	213	316	8	2,070
Unitah	437,058	59,434	1,989	0	23,957	544	4,320	46	28,578
Wayne	77,334	10,737	0	0	6,923	30	1,515	175	2,094
Wyoming									
Lincoln	627,834	94,163	23	0	63,555	644	9,679	0	20,262
Sublette	600,328	163,589	0	0	117,858	0	290	0	45,441
Sweetwater	684 596	33.183	50	0	17,891	0	987	2	14,253
Uinta	636,589	94,278	174	3	48,573	15	261	6	45,246
Total for Region		904,274	30,961	1,439	466,140	7,777	48,170	1,401	348,386
Weighted Percentages[c]		18.2%	3.42%	.16%	51.55%	.86%	5.33%	.15%	38.53%
Upper Rio Grande Basin									
Colorado									
Alamosa	225,264	81,990	20	0	32,900	174	15,226	4,974	28,696
Conejos	405,446	113,398	341	0	56,750	208	22,347	3,951	29,801
Costilla	301,074	18,212	35	0	8,548	233	3,397	1,049	4,950
New Mexico									
Bernalillo	71,911	4,809	842	366	3,172	4	4	0	421
Dona Ana	448,966	59,744	1,076	2,507	11,109	201	1,653	40,827	2,371

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Rio Arriba	713,672	15,969	263	42	7,481	160	190	4	7,829
Sanddoval	216,902	1,830	15	0	1,211	20	26	0	558
Santa Fe	628,506	9,048	3,713	309	3,455	244	207	100	1,020
Socorro	1,843,851	9,717	782	644	5,511	18	36	1,561	1,165
Sierra	1,215,503	4,141	351	147	1,269	0	111	1,443	820
Taos	118,459	6,544	3	0	3,660	131	255	32	2,463
Valencia	1,588,280	12,991	870	90	8,704	58	124	42	3,103
Texas									
Hudspeth	1,678,557	40,217	1,213	8,949	4,598	237	728	9,854	14,638
El Paso	352,233	54,043	549	5,382	7,554	8	3,232	34,110	3,208
Total for Regions		432,653	10,073	18,436	155,922	1,696	47,5[illegible]6	97,947	101,043
Weighted Percentages[c]		21.1%	2.33%	4.26%	36.04%	.39%	10.99%	22.64%	23.35%
				Lower Rio Grande Plain					
Texas									
Atascosa	607,454	29,026	156	2,413	2,554	486	0	17,396	6,021
Bexar	362,150	21,611	5,403	3,515	2,584	205	226	5,470	4,208
Cameron	361,202	191,537	3,321	75,377	1,983	0	0	98,818	12,038
Dimmit	849,316	17,771	330	7,691	421	1,770	0	3,989	3,570
Frio	551,160	46,561	1,278	13,227	3,100	260	240	23,052	5,404
Hidalgo	611,347	239,641	14,075	60,191	3,162	0	610	151,146	10,457
Maverick	807,498	24,881	2,424	6,767	3,837	1,723	242	3,177	6,711
Medina	612,683	15,824	6,877	3,188	1,056	140	102	3,182	1,279
Stan	475,399	14,047	326	636	885	0	0	11,357	843
Uvadale	1,059,181	29,607	6,732	10,177	935	2,392	157	5,058	4,156
Webb	1,869,297	7,699	87	651	468	0	0	4,338	2,155
Willacy	327,771	20,490	197	9,401	10	0	0	9,605	1,277
Wilson	335,797	15,020	519	1,170	1,388	362	0	6,086	5,495

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Zapata	482,976	4,745	10	662	115	0	0	2,659	1,300
Total for region		678,460	41,735	195,066	22,498	7,338	1,557	345,333	64,914
Weighted Percentages[c]			6.12%	28.79%	3.12%	1.08%	.22%	50.91%	9.57%
				Upper Missouri Basin					
Montana									
Blaine	1,411,474	42,498	1,145	0	29,159	2,050	3,351	105	6,688
Carbon	564,924	65,877	2,528	146	34,124	1,929	7,387	47	19,716
Custer	2,176,105	22,001	5,110	66	9,656	1,247	1,590	594	3,738
Dawson	1,097,167	7,675	1,637	26	3,930	595	901	18	568
Park	788,541	51,313	0	0	35,941	1,696	2,679	2	10,995
Prairie	566,791	5,891	1,976	14	2,143	516	530	114	598
Richland	899,738	17,604	3,201	24	6,748	2,066	4,246	113	1,206
Rosebud	2,622,034	31,555	2,664	0	20,377	2,055	2,089	700	3,670
Stillwater	786,872	24,510	2,110	0	13,932	823	1,357	50	6,238
Sweetgrass	797,508	48,592	60	30	25,702	194	1,579	1	21,026
Treasure	538,749	11,995	2,811	15	4,391	665	2,143	146	1,824
Yellowstone	1,384,364	59,171	10,935	0	18,553	5,759	8,546	28	15,350
Beaverhead	1,604,467	243,082	0	0	141,357	2,219	8,504	0	90,337
Broadwater	437,778	42,068	539	0	27,053	2,439	5,199	350	6,488
Cascade	1,228,231	28,954	1,039	37	16,838	1,999	2,807	182	6,053
Chouteau	1,594,794	8,188	15	0	7,325	270	372	36	170
Gallatin	786,001	80,687	146	141	39,555	7,633	11,152	911	21,149
Jefferson	354,661	21,890	0	0	16,351	789	1,026	27	3,697
Lewis & Clark	963,646	39,246	155	0	29,312	527	2,936	75	6,241
Madison	938,356	99,487	107	0	70,271	918	5,271	320	22,600
McCone	1,131,313	5,069	212	0	3,563	701	153	31	409
Phillip	1,469,872	46,177	313	0	36,246	1,547	1,429	294	6,348
Roosevelt	728,039	6,228	0	0	3,626	847	1,043	40	672
Valley	1,102,594	34,199	876	0	21,676	4,837	2,625	0	4,185

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
North Dakota									
McKenzie	826,669	8,310	294	0	4,457	2,272	935	51	301
Williams	779,591	9,260	287	0	6,265	1,876	447	0	385
Total for Region		1,061,527	38,159	499	628,551	48,469	80,297	4,235	260,652
Weighted Percentages[c]		8.1%	3.59%	.05%	59.21%	4,57%	7.56%	.40%	24.55%
Northwestern Ogallala									
Colorado									
Adams	487,143	36,236	11,172	962	13,261	3,740	3,162	0	3,939
Larimer	507,210	83,147	23,758	167	35,561	1,303	10,110	13	11,059
Logan	810,008	74,999	32,003	528	30,506	1,512	1,829	10	8,611
Morgan	595,211	92,940	54,829	1,009	22,465	3,068	3,717	2,154	5,698
Weld	1,608,732	275,039	136,728	772	87,600	4,659	24,345	5,082	15,853
Nebraska									
Banner	332,447	7,161	782	297	3,450	128	515	896	1,093
Box Butte	465,488	22,653	10,689	445	5,524	730	2,416	1,999	850
Cheyenne	446,957	11,954	5,501	239	4,908	793	256	0	257
Kimball	384,533	8,379	2,968	190	3,653	537	550	110	371
Morrill	628,652	40,850	14,682	192	18,536	587	1,060	788	5,005
Scotts Bluff	324,922	76,224	37,390	575	24,545	685	3,828	1,887	7,314
Sioux	971,572	19,817	4,820	0	12,791	81	538	108	1,428
Wyoming									
Goshen	1,098,533	64,714	18,192	370	28,012	1,521	5,903	1,615	9,102
Laramie	1,519,727	30,412	3,422	0	20,955	539	2,423	1,118	1,955

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Platte	1,390,531	46,709	6,899	215	28,724	244	1,757	0	8,870
Total for Region		891,234	363,935	5,961	340,491	20,127	62,409	15,770	81,405
Weighted Percentages[c]		12.7%	40.83%	.67%	38.20%	2.26%	7.00%	1.77%	9.13%
Northeastern Ogallala									
Colorado									
Kit Carson	913,368	60,187	39,110	2,803	7,657	7,736	366	1	2,514
Phillips	253,204	12,189	7,463	406	2,152	653	273	1	1,241
Sedgwick	220,343	16,841	8,492	447	4,802	266	275	8	2,551
Washington	939,317	18,876	11,550	859	4,289	799	321	100	958
Yuma	1,064,594	66,130	43,437	3,241	9,626	2,393	817	646	5,970
Kansas									
Cheyenne	425,600	26,021	17,235	2,546	2,753	2,116	313	1	1,057
Decatur	379,174	7,508	4,811	1,154	1,193	259	31	1	60
Logan	390,379	9,769	4,775	2,995	582	868	325	70	154
Rawlins	475,568	10,439	4,314	3,092	1,927	753	0	15	338
Sherman	339,675	42,127	31,506	3,877	1,469	3,411	456	4	1,404
Thomas	413,156	29,907	18,266	7,957	532	1,242	262	56	592
Wallace	344,815	23,017	13,243	5,308	613	2,605	305	515	409
Nebraska									
Chase	385,308	40,824	29,508	1,080	5,782	1,256	608	236	2,282
Deuel	187,922	7,794	5,024	339	1,539	527	52	183	130
Dundy	449,291	25,031	17,850	667	4,872	573	96	310	663
Keith	542,477	25,469	13,855	1,679	7,771	1,010	186	36	932
Lincoln	1,377,890	58,333	36,860	1,514	15,075	746	131	1,303	2,593
Perkins	314,566	14,542	7,823	615	2,342	1,293	785	0	1,684
Total for Region		494,894	315,194	40,579	74,994	28,506	5,602	3,486	25,532
Weighted Percentages[c]		5.21%	63.69%	8.20%	15.15%	5.53%	1.13%	.70%	5.24%

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Central Ogallala					
Kansas									
Finney	518,433	123,609	30,495	43,562	15,820	31,079	178	260	2,215
Ford	452,216	31,243	6,280	15,126	2,148	5,975	398	25	1,291
Grant	207,129	77,842	20,437	26,761	1,073	25,095	397	344	3,735
Gray	348,419	76,082	10,769	39,126	4,476	17,819	406	523	2,963
Greeley	235.851	20,301	11,381	5,172	674	2,627	210	0	237
Haskell	249,479	116,211	21,687	49,691	3,442	39,128	174	609	1,480
Hodgeman	367,657	14,832	5,447	5,740	1,770	1,268	53	100	454
Kearney	346,290	40,453	12,206	10,706	5,170	9,828	10	0	2,533
Meade	440,697	49,389	8,128	26,375	1,822	11,773	190	196	905
Lane	290,598	9,385	5,262	2,640	271	1,304	180	0	228
Seward	253,187	47,735	13,373	18,434	2,207	11,825	56	0	1,840
Scott	302,900	64,007	20,413	24,072	2,043	15,557	229	0	1,693
Wichita	270,567	80,311	40,169	25,629	1,213	11,896	151	639	614
Oklahoma									
Beaver	941,510	21,436	2,176	9,781	2,501	4,739	1,199	6	1,034
Cimarron	832,553	75,228	10,188	44,190	2,886	13,951	450	232	3,331
Texas	904,283	152,888	27,064	73,564	1,708	43,456	1,260	282	5,554
Texas									
Dallam	678,426	97,962	18,254	46,366	2,803	16,231	885	300	13,123
Hansford	551,054	183,406	12,664	81,671	1,930	74,120	170	550	12,301
Hartley	823,495	69,320	3,671	32,548	845	26,367	307	124	5,458
Hutchinson	459,412	54,915	3,856	19,594	795	24,570	175	116	5,809
Libscombe	481,472	9,174	1,672	2,725	1,107	2,565	190	85	830
Moore	417,762	135,427	26,204	54,780	1,461	48,650	400	633	3,299
Ochiltree	498,733	84,049	3,881	47,029	570	27,938	353	504	3,774
Sherman	410,265	148,743	15,463	78,382	1,279	45,621	679	1,012	6,307
Total for Region		1,720,441	331,140	733,664	60,014	513,382	8,700	6,870	81,008
Weighted Percentages[c]		21.62%	19.25%	45.56%	3.49%	29.84%	.52%	.39%	4.72%

Appendix Table A1. Continued.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Southern Ogallala					
Texas									
Bailey	314,153	122,345	13,962	43,291	7,163	3,500	429	46,831	7,069
Borden	559,520	2,648	0	65	11	0	2	2,530	40
Briscoe	374,248	47,261	252	22,388	574	7,846	64	14,784	1,353
Castro	403,858	276,376	35,081	117,805	4,907	53,795	662	52,461	11,665
Cochran	343,782	93,601	86	40,799	1,085	626	25	49,673	1,307
Crosby	415,467	160,216	124	51,537	664	8,148	594	95,096	4,053
Dawson	518,873	63,118	1	7,125	384	68	0	54,864	676
Floyd	449,101	216,605	2,497	86,526	1,428	30,999	195	90,506	4,454
Gaines	421,247	145,914	1	41,447	8,718	5,908	1,124	34,083	4,633
Garza	464,265	12,685	9	1,738	70	214	55	10,160	439
Hale	503,271	379,628	9,528	164,808	2,797	26,456	292	164,606	11,141
Hockely	418,062	191,318	456	50,900	771	598	146	133,447	2,534
Lamb	522,957	244,127	12,691	100,161	3,743	3,366	385	116,314	7,467
Lubbock	458,680	291,097	761	90,724	1,539	2,239	571	191,342	3,921
Lynn	416,223	77,697	0	12,693	151	15	0	64,250	588
Parmer	370,716	281,786	18,115	150,775	3,759	53,984	1,137	44,681	9,335
Swisher	411,001	229,114	7,874	116,168	3,700	43,242	1,421	48,571	8,138
Terry	394,174	141,717	14	39,090	363	1,620	55	98,978	1,597
Yoakum	328,024	57,325	0	20,641	1,410	1,137	60	32,268	1,809
New Mexico									
Lea	2,232,430	50,239	1,157	15,258	7,389	129	906	18,536	6,864
Roosevelt	1,343,770	55,993	1,505	25,846	5,276	1,968	489	14,620	6,289
Total for Region		3,140,710	104,114	1,269,785	55,902	245,862	8,612	1,433,601	95,372
Weighted Percentages[c]		39,09%	3.31%	38.20%	1.78%	7.83%	.27%	45.65%	2.94%

[a]Harvested arres includes both dryland and irrigated areas harvested in 1969.

Appendix Table A1. Continued.

[b]Other crops include acres of soybeans, peanuts, tobacco, cotton, strawberries, potatoes, red clover for seed, and alfalfa for seed.

[c]Percentages weighted by amount of irrigated acreage per county.

APPENDIX B: VARIABLE DEFINITIONS

Variable Definitions

In this section, variables included in the production function model are more specifically defined. Procedures for determining their magnitude from the census data are discussed.

Value of Agricultural Output (y)

The value of agricultural output is comprised of two components--the value of crops harvested and the value of livestock and livestock products produced for each county.[1] Since nonbreeding herd replacement livestock are rarely inventoried beyond normal marketing weights, and since a relatively insignificant amount is consumed on the farm, the value of livestock and livestock sales (County, 13)[2] adequately represents the latter component. The value of all crops sold (County, 13), however, does not sufficiently represent the total value of crops which are actually harvested, since a significant portion is often utilized as livestock feed and/or held in inventory. Thus, the value of the crop component was determined indirectly by applying a State Average Price for each major crop to the appropriate quantity produced (County, 21). Using an average price for each individual state instead of say, a national average price, allows for price differentials which may exist between states.

The State Average Prices were calculated by dividing the appropriate value of each crop harvested in the state (State, 8) by its aggregate

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[1]County observations are based on the total output of those farms with an income of greater than $2,500 in 1969.

[2]The notation (County, 13) denotes that the required data are found in table 13 of the "Statistics for Counties" section of the census.

yield (State, 8).[3] These prices were then multiplied by county crop output data (County, 21) to obtain the value of each crop harvested. The summation of each individual crop output value gave an aggregate value of crops harvested for each county.

Irrigation Water Applied (x_1)

The variable, irrigation water applied, is defined as the quantity of water in acre-feet applied to either cropland or pasture by sprinklers, furrows or flooding (County, 11).

Value of Land and Service Buildings (x_2)

The value of land and service buildings is defined as the estimated market value of all land and buildings engaged in agricultural activities in each county (County, 9).

Hired Labor Expenditures (x_3)

Hired labor expenditures are defined as all money paid in cash for farm labor (County, 14). This includes payments to family members but does not take into account expenditures for housework or contract work.

Fuel and Lubricant Expenditures (x_4)

This variable is defined as the countywide total of all petroleum products purchased for the farm business (County, 14). This term includes those expenditures on diesel fuel, LP gas, butane, propane, piped gas, kerosine, fuel oil, motor oil and grease.

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[3]Crop values are not reported in the 1969 census at the county level.

Fertilizer and Lime Expenditures (x_5)

Fertilizer and lime expenditures are defined as the total cost of all commercial fertilizer and lime (including rock phosphate and gypsum) used in ordinary agricultural activities (County, 14).

Feed Expenditures (x_6)

Feed expenditures are the total cost of all feed purchased for livestock and poultry (County, 14.) This total includes expenditures on grain, hay, silage, mixed feeds, concentrates, etc.

Value of Machinery Inventory (x_7)

The value of machinery inventory is defined as the current market value of all machinery usually kept and used for the farm business (County, 15). This term includes the value of automobiles, motortrucks, wheel tractors, crawler tractors, garden tractors, grain and bean combines, cornpickers and picker-shellers, pickup balers, windrowers and field forage harvestors.

Value of Livestock Inventory (x_8)

The value of livestock inventory is defined as the total value based on state average prices of all cows and calves, hogs and pigs, sheep and goats, and poultry per county (County, 16-18).

Other Operating Expenses (x_9)

Other operating expenses includes those costs which when taken alone comprise an insignificant part of the total inputs (County, 14). This other category includes those expenditures on seeds, bulbs and trees, all contract labor, machine hire, customwork, agricultural chemicals purchased, purchases of irrigation water, costs of operating and maintaining irrigation systems, farm electricity, veterinary supplies, hauling and other market changes, farm taxes and interest on farm debts.

APPENDIX C: THE IMPLICIT RENTAL COST OF CAPITAL

The Implicit Rental Cost of Capital

Since the firm is oftentimes unable to economically purchase certain services in a market, it must maintain a stock of these capital goods from which to derive needed services, e.g., land, machinery inventory and livestock inventory. The implicit rental cost of a unit of this capital stock for any production period must include the opportunity costs of financing the stock, plus depreciation, less capital-gains (Ott, *et.al.*). This may be written as

(C.1) $$c_k = \rho + \delta - \gamma$$

where

c_k = implicit rental cost of capital services

ρ = opportunity costs of financing over the production period

δ = depreciation during the production period

γ = rate of capital-gains for the production period

Although both the depreciation and the rate of capital-gains are easily determined,[1] the opportunity cost of financing poses a problem. Since the purchase of any capital good can be financed by either borrowed funds or cash, the value of ρ should reflect a weighted cost of capital, i.e.,

(C.2) $$\rho = r_d \cdot \frac{D_t}{D_t + D_e} + r_e \cdot \frac{D_e}{D_t + D_e}$$

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[1]Because the machinery inventory is comprised of various types of equipment (see appendix B), an estimate of the rate of depreciation on all machinery may be calculated by weighting the depreciation rate for each individual machinery type (U. S. Department of Agriculture, 1977) by its contribution to total value (U. S. Department of Agriculture, 1969b).

where

r_d = interest rate for loans

r_e = rate of return on savings accounts

D_t = total debt

D_e = total equity[2]

Since by definition the productive lifetime of capital stock is more than one production period, tax laws also affect the user cost of capital. When taxes are considered, equation (C.1) may be written as[3]

$$c_k = \frac{(1-u)\rho + \delta - \gamma(1-gu)\ (1-k-uz)q}{1-u} \tag{C.3}$$

where

u = personal income tax rate

g = tax rate applicable to capital gains income

k = investment tax credit

z = present value of a one dollar deduction for depreciation[4]

To facilitate the usage of equation (C.3) several simplifying

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[2]The appropriate interest rate for land and buildings was assumed to be that charged by the Federal Land Bank. For shorter term assets such as machinery and livestock, the appropriate rate was assumed to be that charged by the Production Credit Association. The rate of return on savings accounts was based on the interest rate paid by commercial banks for time and savings deposits. These values were obtained from the *1969 FDIC Annual Report*. The debt-equity values used are from the *Balance Sheet of the Farming Sector*.

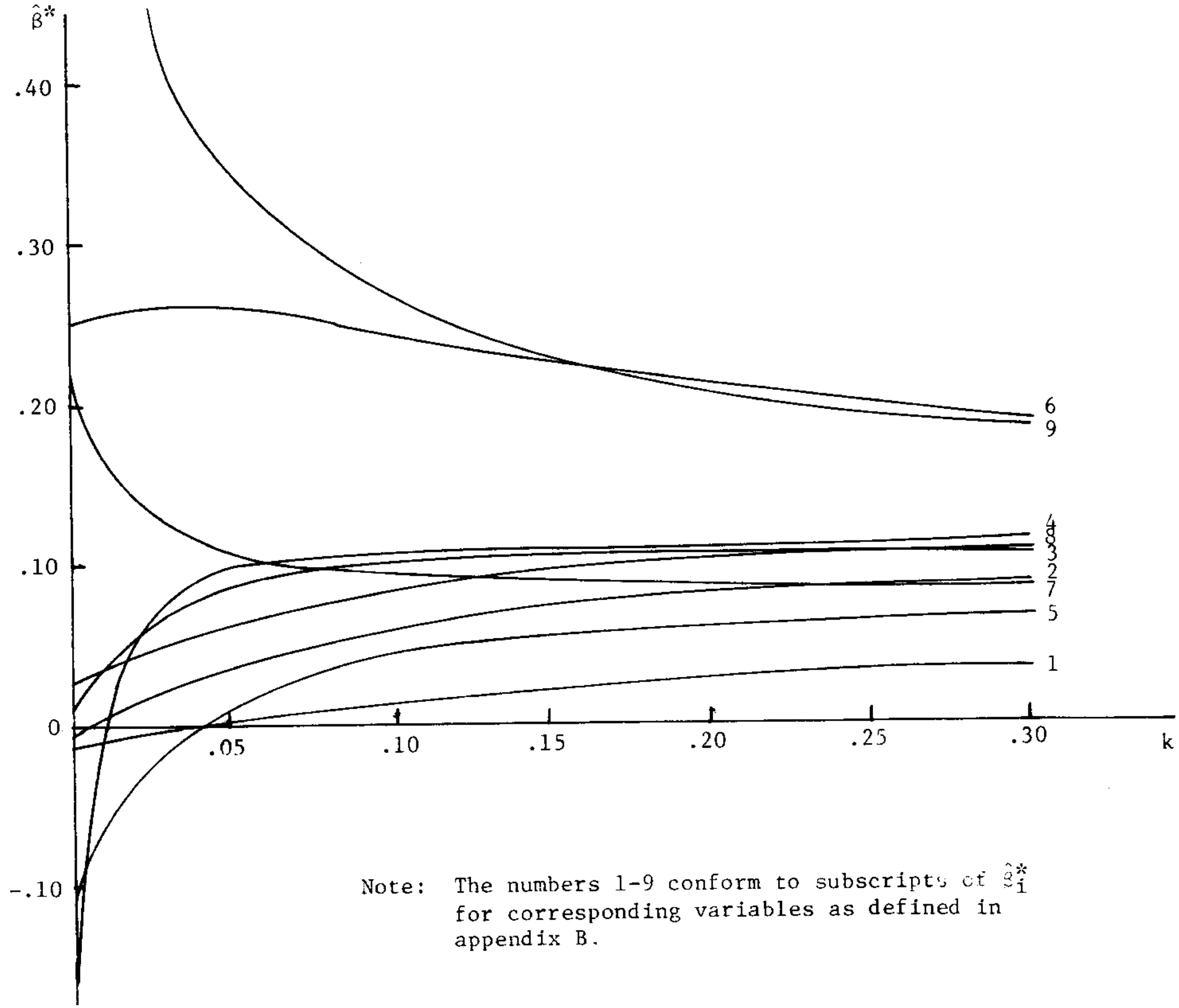
[3]The values of each of the tax variables may be found in *Farm Income Statistics*.

[4]In calculating the value of z, machinery was assumed to have a seven year service life. The sum-of-the-years digits method of depression was used.

assumptions must be made. Since the values of machinery and livestock inventories are closely tied to market conditions, it is difficult to assess a definite rate of capital gain (loss) for each of these stocks. Thus, these factors were assumed to be negligible. In addition, land was assumed to have no depreciation.

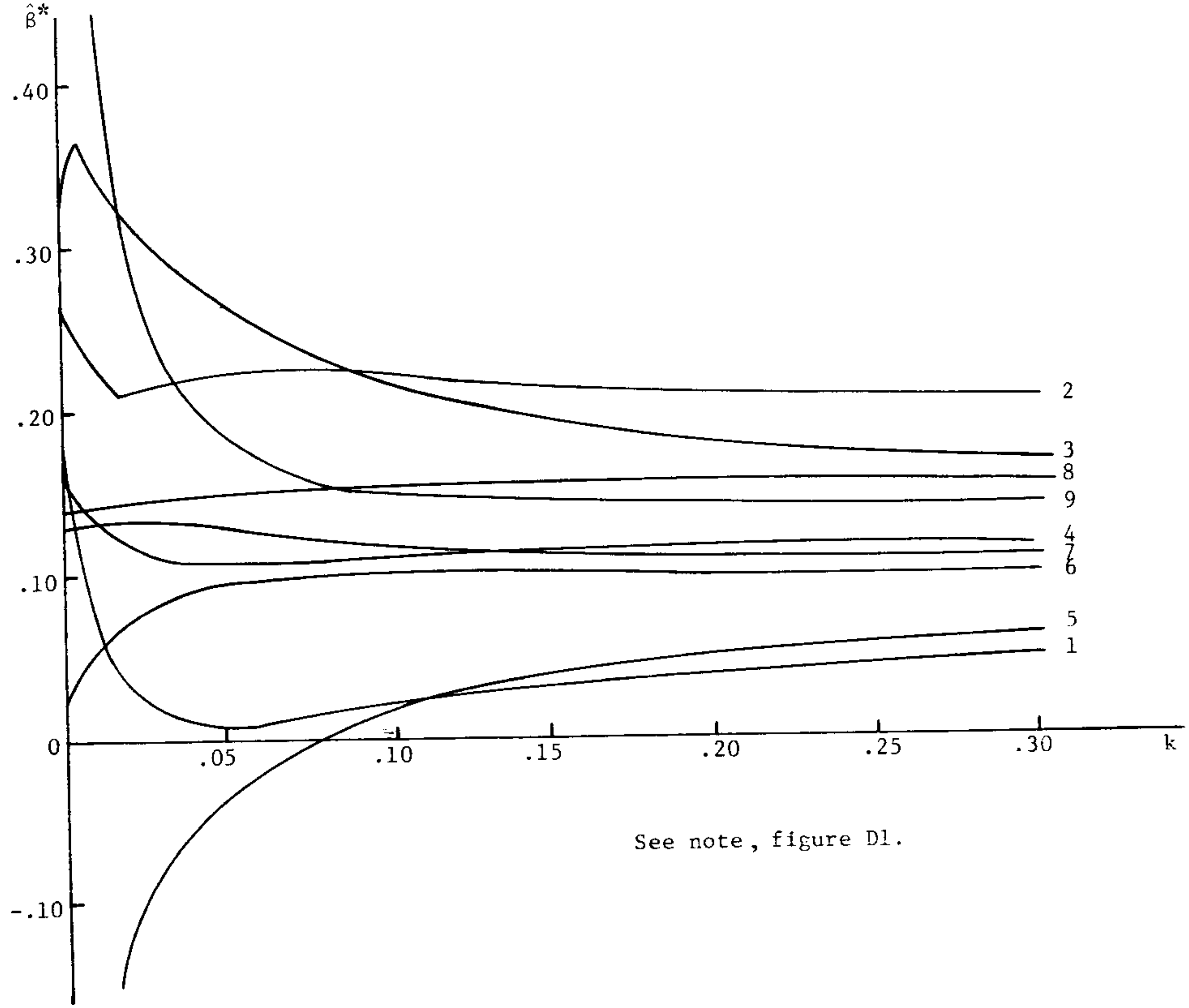
Using the preceeding methodology, estimates of the implicit rental cost for land and building, machinery inventory, and livestock inventory were determined as .02308, .2157 and .05357, respectively.

APPENDIX D: REGIONAL RIDGE TRACES



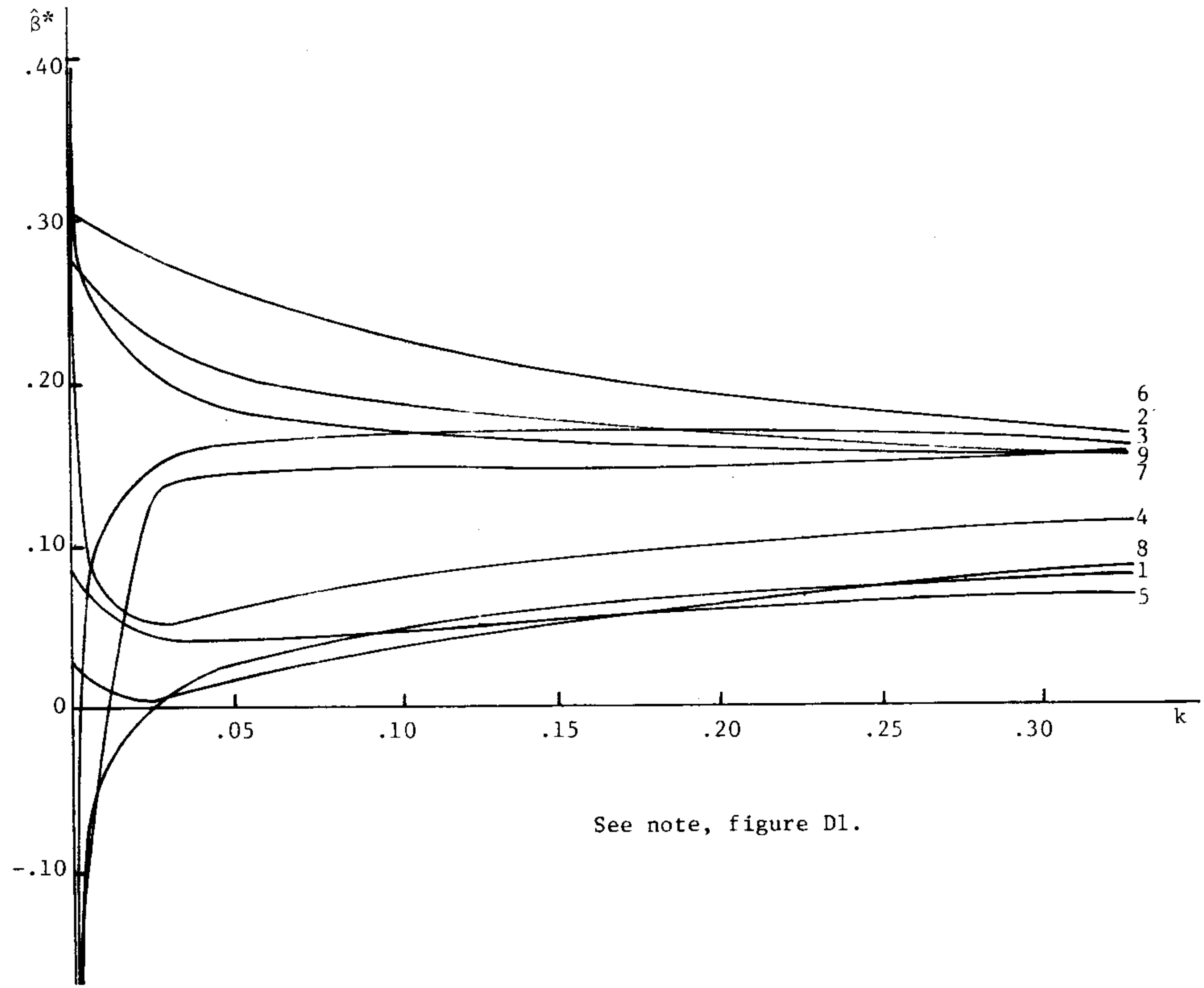
Note: The numbers 1-9 conform to subscripts of $\hat{\beta}_i^*$ for corresponding variables as defined in appendix B.

Appendix Figure D1. Ridge Trace for Snake-Columbia Basin.

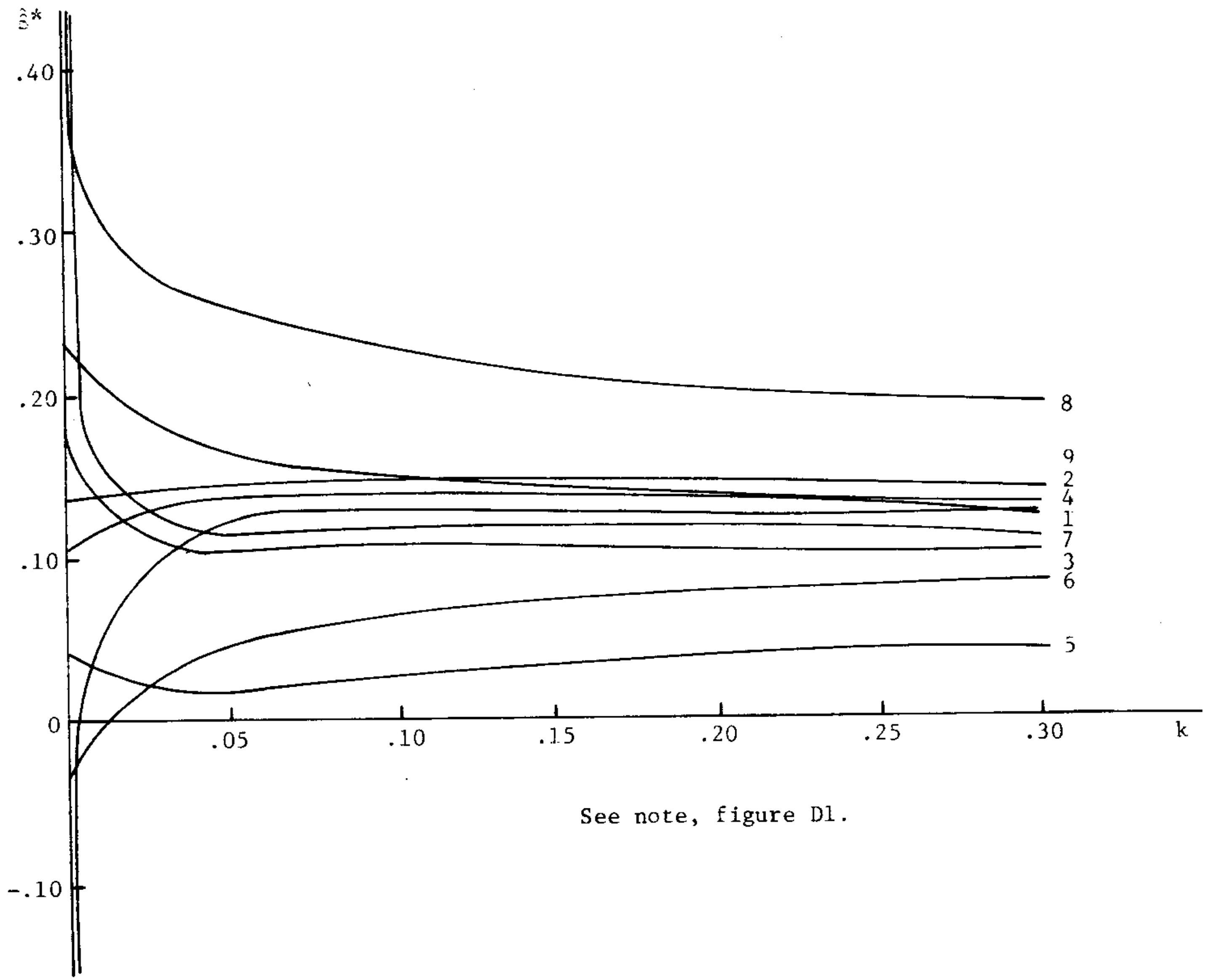


See note, figure D1.

Appendix Figure D2. Ridge Trace for Central California.

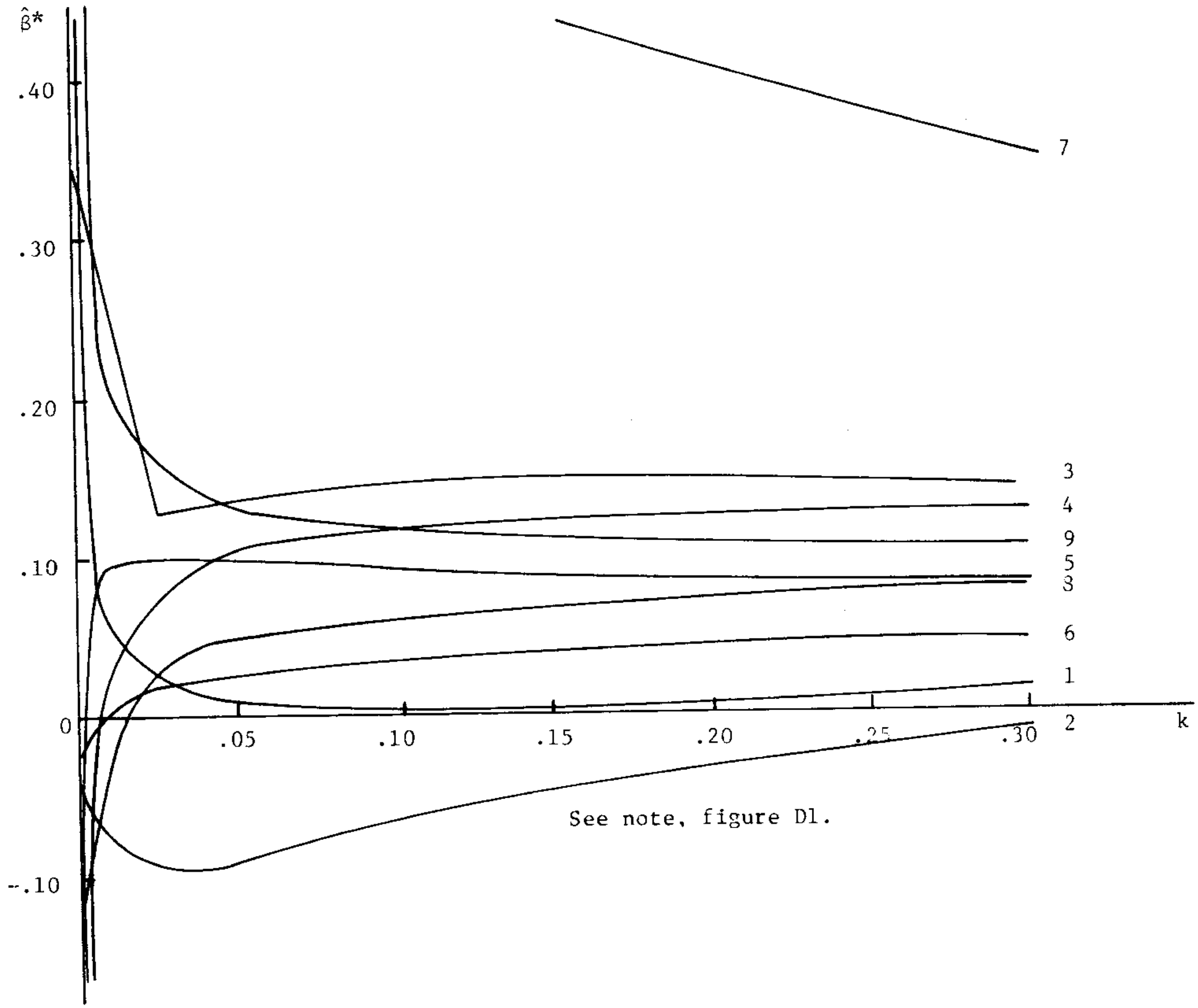


Appendix Figure D3. Ridge Trace for Desert Southwest.



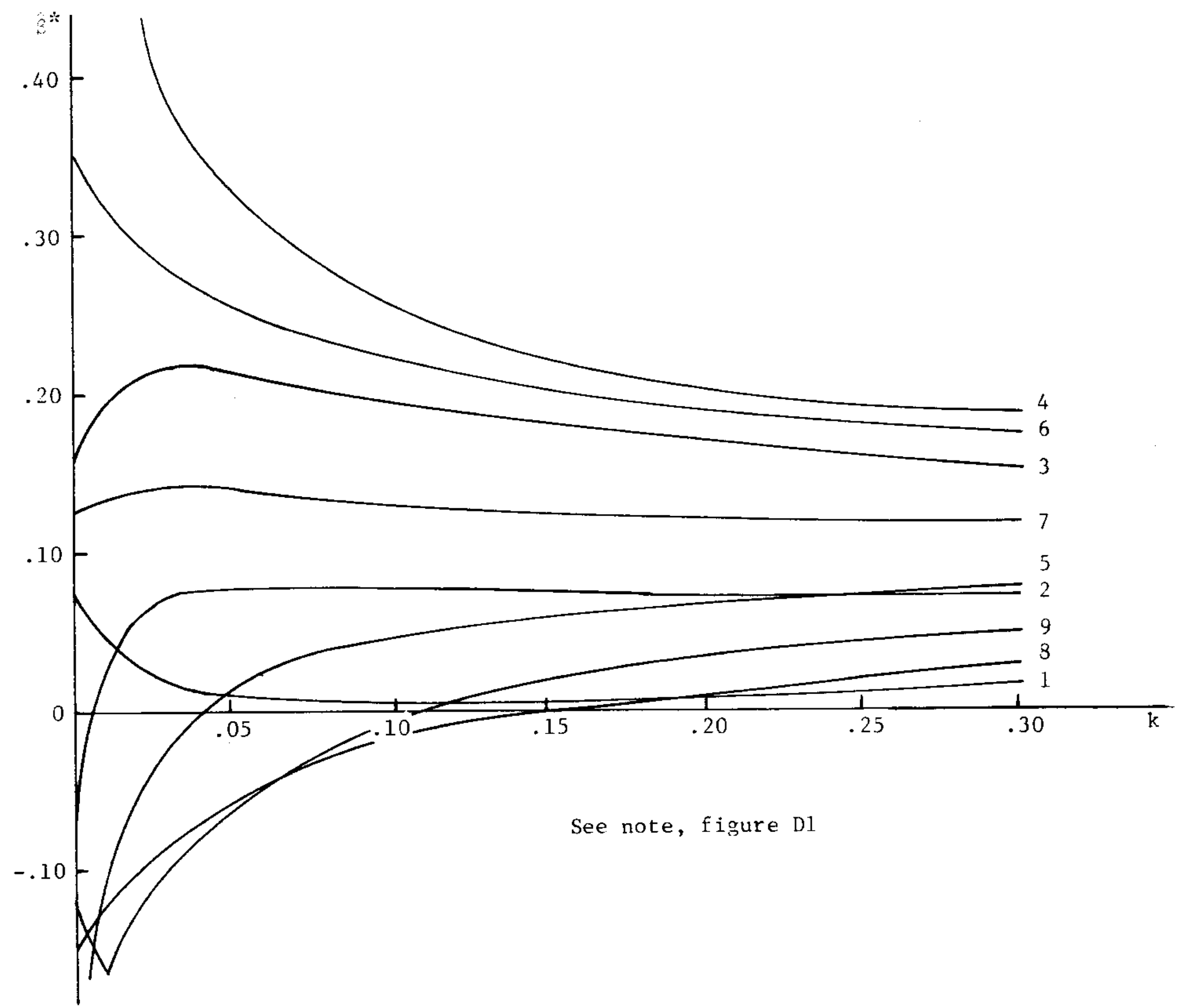
See note, figure D1.

Appendix Figure D4. Ridge Trace for Upper Colorado Basin.

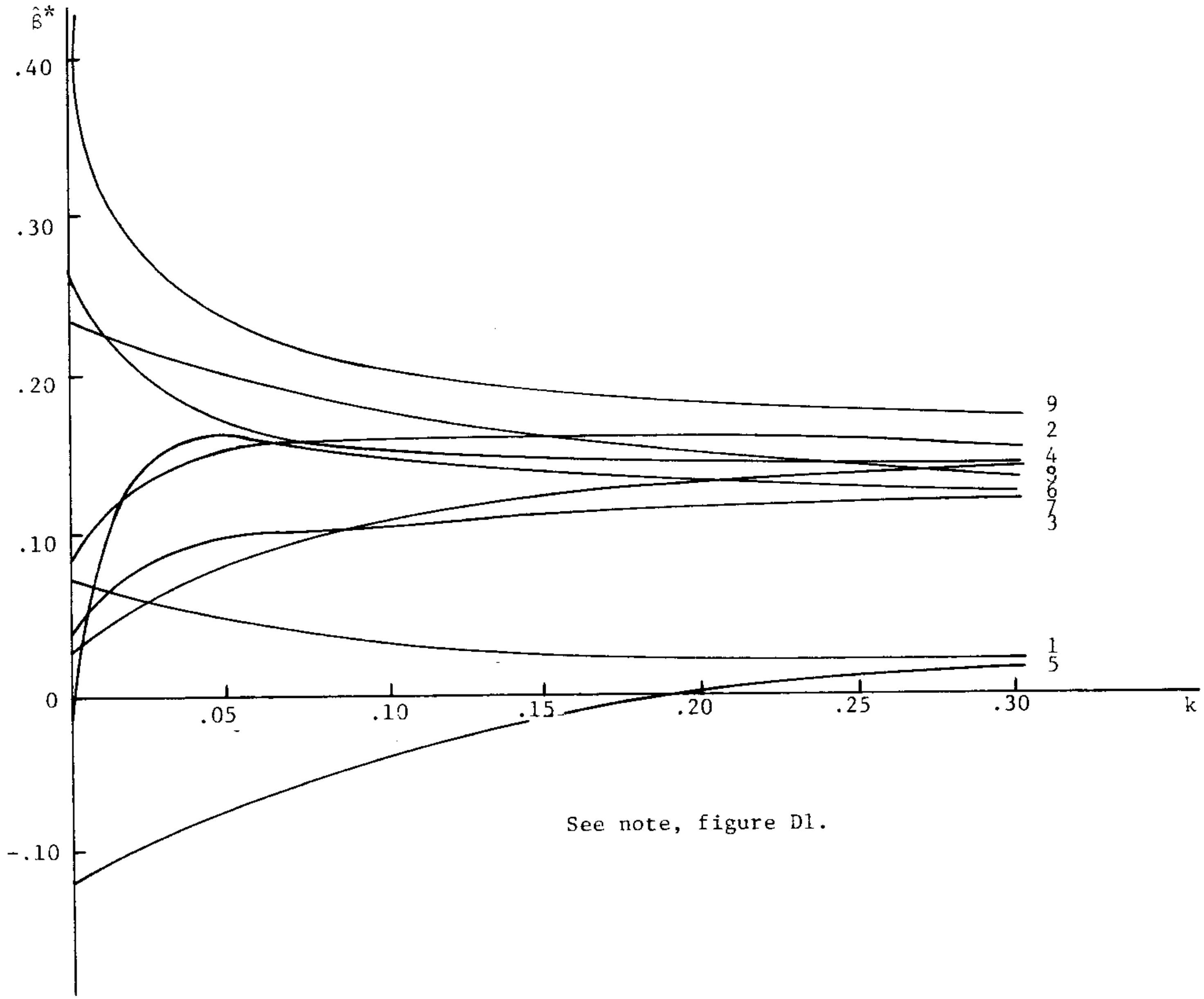


See note, figure D1.

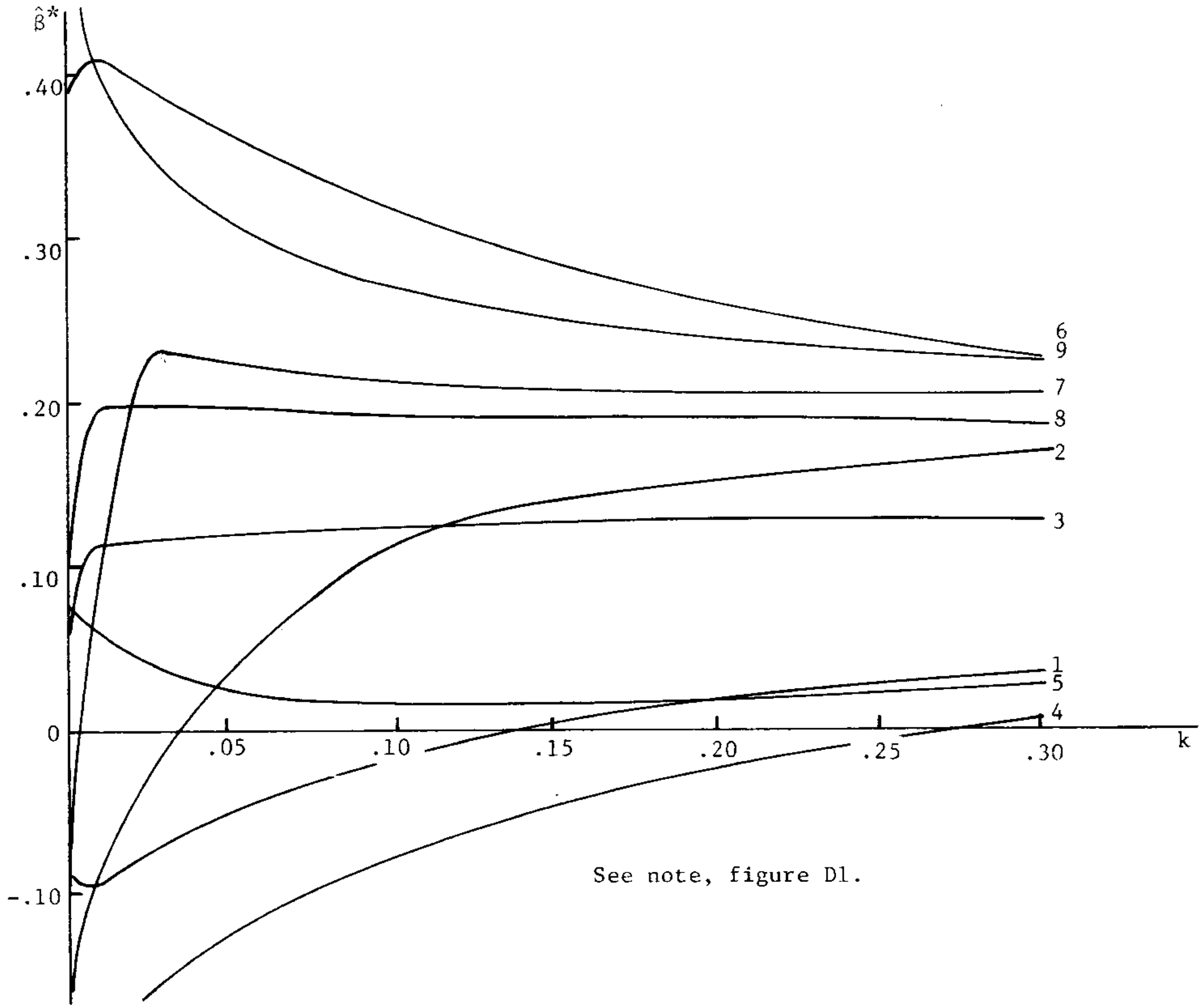
Appendix Figure D5. Ridge Trace for Upper Rio Grande Basin.



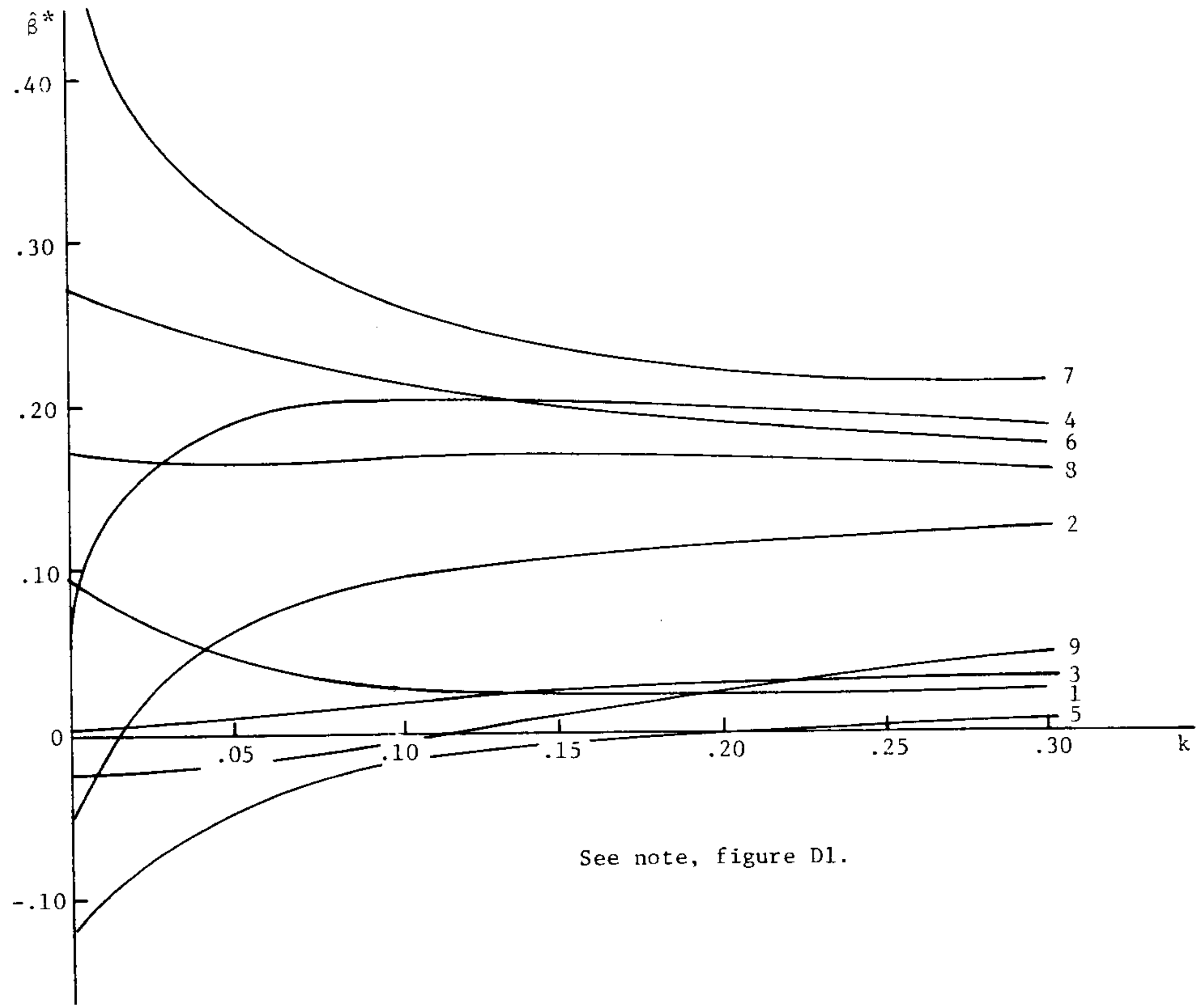
Appendix Figure D6. Ridge Trace for Lower Rio Grande Basin.



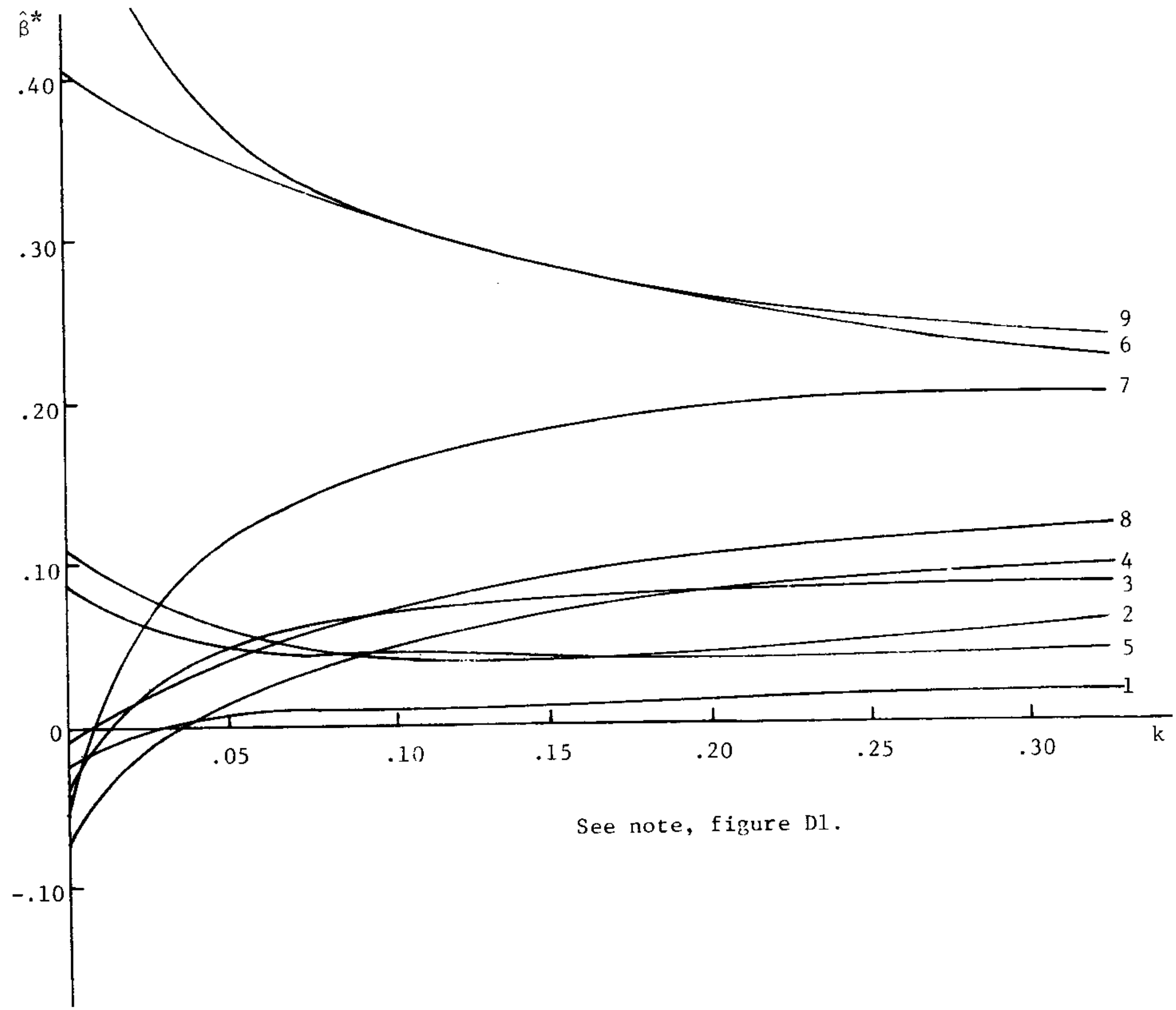
Appendix Figure D7. Ridge Trace for Upper Missouri Basin.



Appendix Figure D8. Ridge Trace for Northwestern Ogallala.

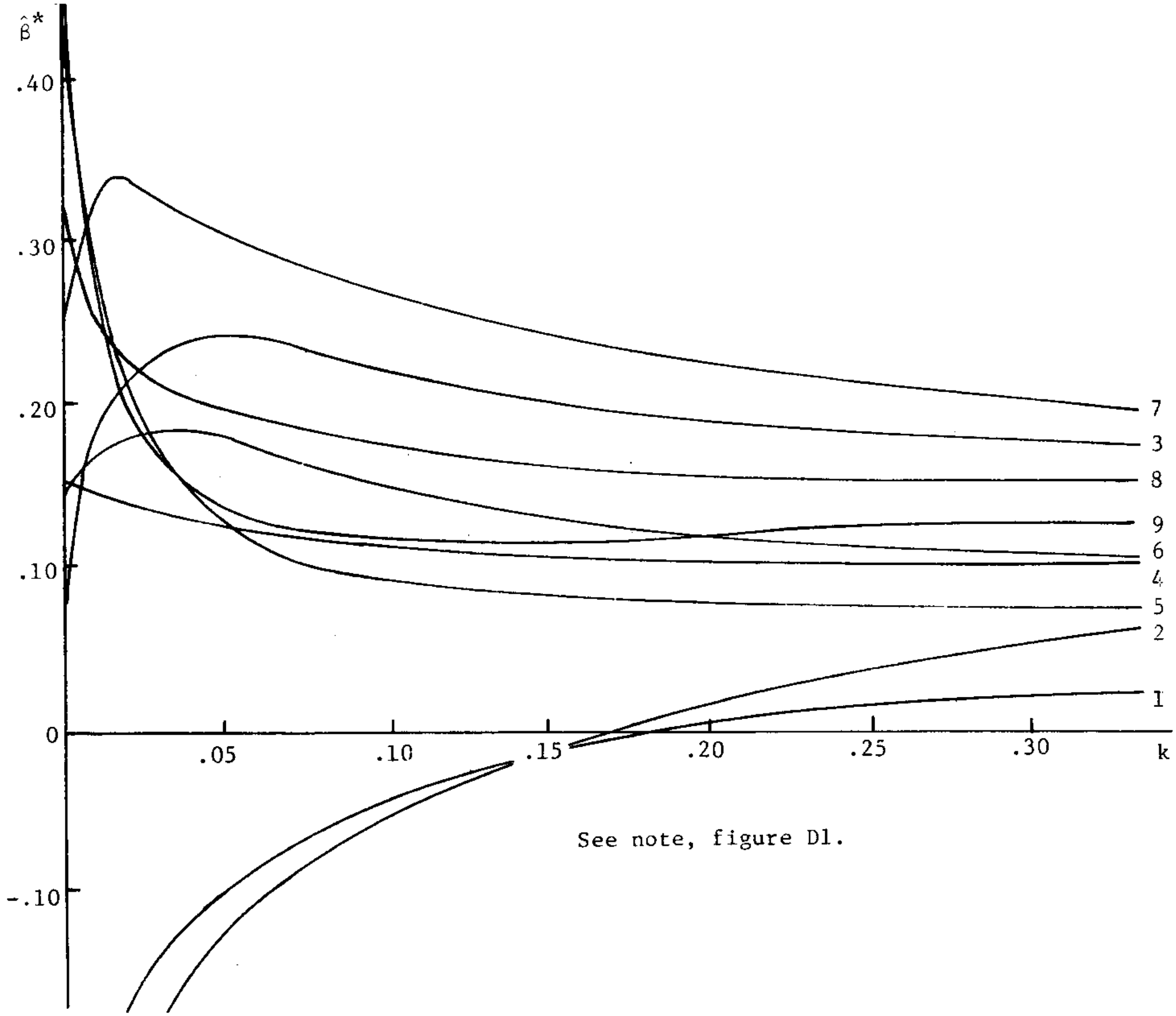


Appendix Figure D9. Ridge Trace for Northeastern Ogallala.



See note, figure D1.

Appendix Figure D10. Ridge Trace for Central Ogallala.



See note, figure D1.

Appendix Figure D11. Ridge Trace for Southern Ogallala.

APPENDIX E: REGIONAL CORRELATIONS MATRICES

Appendix Table E1. Correlation Coefficients for Snake-Columbia Basin.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
				Simple Correlation Coefficients					
x_1	1	.452	.624	.604	.606	.641	.513	.740	.737
x_2		1	.859	.931	.856	.587	.908	.521	.819
x_3			1	.902	.851	.747	.815	.639	.907
x_4				1	.930	.625	.968	.595	.926
x_5					1	.584	.918	.511	.943
x_6						1	.481	.832	.711
x_7							1	.452	.858
x_8								1	.638
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E2. Correlation Coefficients for Central California.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
					Simple Correlation Coefficients				
x_1	1	.791	.884	.947	.979	.556	.967	.568	.963
x_2		1	.964	.932	.871	.794	.887	.817	.902
x_3			1	.966	.928	.788	.947	.802	.957
x_4				1	.979	.730	.979	.726	.987
x_5					1	.596	.982	.608	.992
x_6						1	.633	.980	.643
x_7							1	.639	.984
x_8								1	.654
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E3. Correlation Coefficients for Desert Southwest.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
					Simple Correlation Coefficients				
x_1	1	.124	.412	.806	.838	.410	.672	.347	.719
x_2		1	.883	.580	.494	.793	.696	.492	.681
x_3			1	.763	.740	.889	.858	.442	.871
x_4				1	.913	.699	.960	.552	.872
x_5					1	.594	.875	.293	.947
x_6						1	.805	.534	.706
x_7							1	.518	.857
x_8								1	.327
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E4. Correlation Coefficients for Upper Colorado Basin.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
				Simple Correlation Coefficients					
x_1	1	.816	.736	.857	.835	.834	.852	.890	.877
x_2		1	.936	.904	.769	.935	.883	.942	.949
x_3			1	.910	.798	.942	.883	.896	.958
x_4				1	.893	.953	.995	.944	.970
x_5					1	.914	.894	.799	.885
x_6						1	.946	.917	.974
x_7							1	.928	.955
x_8								1	.966
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E5. Correlation Coefficients for Upper Rio Grande Basin.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
					Simple Correlation Coefficients				
x_1	1	.394	.777	.914	.926	.430	.500	.550	.839
x_2		1	.621	.654	.527	.814	.820	.713	.703
x_3			1	.923	.905	.761	.814	.658	.926
x_4				1	.939	.690	.737	.650	.964
x_5					1	.601	.693	.596	.941
x_6						1	.959	.693	.772
x_7							1	.719	.821
x_8								1	.691
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E6. Correlation Coefficients for Lower Rio Grande Basin.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
				Simple Correlation Coefficients					
x_1	1	.377	.632	.512	.592	.056	.460	-,313	.564
x_2		1	.715	.879	.761	.584	.859	.154	.701
x_3			1	.808	.809	.320	.724	-.155	.838
x_4				1	.945	.423	.970	-.087	.932
					1	.469	.946	-.104	.941
x_6						1	.477	.710	.222
x_7							1	-.093	.905
x_8								1	-.311
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E7. Correlation Coefficients for Upper Missouri Basin.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
					Simple Correlation Coefficients				
x_1	1	-.005	.287	-.189	.209	.438	-.268	.507	-.057
x_2		1	.874	.905	.693	.578	.873	.698	.967
x_3			1	.751	.811	.731	.703	.810	.875
x_4				1	.754	.481	.985	.537	.966
x_5					1	.687	.730	.631	.779
x_6						1	.412	.829	.599
x_7							1	.431	.939
x_8								1	.680
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E8. Correlation Coefficients for Northwestern Ogallala.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
					Simple Correlation Coefficients				
x_1	1	.830	.878	.841	.935	.662	.839	.918	.732
x_2		1	.939	.919	.828	.812	.913	.877	.857
x_3			1	.941	.892	.810	.924	.896	.878
x_4				1	.897	.765	.992	.867	.965
x_5					1	.758	.885	.902	.798
x_6						1	.716	.848	.681
x_7							1	.837	.965
x_8								1	.763
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E9. Correlation Coefficients for Northeastern Ogallala.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
				Simple Correlation Coefficients					
x_1	1	.620	.855	.598	.821	.131	.606	.395	.537
x_2		1	.795	.954	.627	.297	.942	.702	.516
x_3			1	.766	.755	.207	.753	.561	.572
x_4				1	.671	.376	.978	.785	.400
x_5					1	.051	.687	.521	.504
x_6						1	.391	.592	.022
x_7							1	.731	.418
x_8								1	.426
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E10. Correlation Coefficients for Central Ogallala.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
				Simple Correlation Coefficients					
x_1	1	.436	.760	.348	.942	.404	.449	.361	.794
x_2		1	.796	.472	.556	.687	.847	.804	.712
x_3			1	.445	.808	.710	.756	.748	.860
x_4				1	.425	.315	.492	.372	.468
x_5					1	.462	.599	.426	.862
x_6						1	.643	.917	.676
x_7							1	.672	.815
x_8								1	.589
x_9									1

[a]For definition of x_1 - x_9, see appendix B.

Appendix Table E11. Correlation Coefficients for Southern Ogallala.[a]

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
				Simple Correlation Coefficients					
x_1	1	.895	.871	.701	.956	.599	.876	.423	.877
x_2		1	.951	.780	.915	.597	.944	.385	.934
x_3			1	.716	.940	.499	.945	.251	.950
x_4				1	.711	.363	.765	.186	.736
x_5					1	.471	.935	.242	.947
x_6						1	.462	.933	.457
x_7							1	.220	.977
x_8								1	.205
x_9									1

[a]For definition of x_1 - x_9, see appendix B.