Comparison of Dynamic Data Analysis Methods for Thermal Property

Measurement of a Building Wall

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Abstract: Energy usage in heating and air-conditioning a building is influenced by several factors. Among them is the building wall, whose production is affected by the planning, design and construction process. This process causes a certain deviation of the theoretical and actual values of the building wall's thermal transmittance. For constructed buildings, if the thermal properties of the building wall, on site, are calculated rapidly and accurately, it will make the testing process easier and the analytical results more accurate. In this paper, two methods for analyzing the building wall's thermal transmittance, where data is measured on site, are presented. They are respectively called the dynamic analysis method and the system identification method. The two methods are based on two different thermal models. Both models of the wall's heat transfer were solved with the system identification algorithm. Analysis and comparison of the two methods shows that the system identification method for analyzing the measured dynamic data is simpler and has a shorter measured time.

Keyword: dynamic analysis method, system identification method, thermal transmittance

1. INSTRUCTION

With the increasing steadily development of the economics, the imbalance between supply and demand of energy is increasingly convex day by day. It, nowadays, has become the bottleneck which has restricted the native economy that develops healthily and continuously. At the same time, along with the development of our native construction, its energy consumption has already become an important co0mponent of the total energy consumption of the society. The building's energy-conservation is defined as: under the terms of guaranteeing and improving comfortableness of the building, designing the building's energy-conservation and specifically construction can utilize energy rationally and improve the energy efficiency. The thermal performance of the building components is one of the main influence factors that effect the energy consumption of the building. Because the influence of the actual process of construction and person, the performance of the building components, eventually, is deviated from the purpose of the designer and can not meet the need of the reality energy-conservation.

Measurement for U of a building component on a site has already become an urgent problem in current society.

Two dynamic analytical methods to calculate the U value of the building component is used to analysis the data measured on a site. Because dynamic analytical method is not confined by the data of steady state, it make the measure apparatus and process simple, and is free of seasons, can shorten the measured time. So it is suitable for the measurement of complicated building components.

2. CALCULATION MEHTOD FOR THE U-VALUE ON A SITE

The thermal transmittance of a building

component (U-value) is defined in ISO7345 as the "Heat flow rate in the steady state divided by area and by the temperature difference between the surroundings on each side of a system. U is thermal conductivity $(Wm^{-1} K^{-1})$.

In principle, the U-value can be obtained by measuring the heat flow rate through an element with a heat flow meter or a calorimeter, together with the temperatures on both sides of the element under steady state conditions.

However, since steady state conditions are never encountered on a site in practice, such a simple measurement is not possible. For keeping steady state, a hot and a cold box are used. This method is commonly used in the laboratory but is cumbersome on a site.

There are several ways of overcoming this difficulty:

(1) The thermal properties of the materials and the heat transfer coefficients are constant over the range of temperature fluctuations occurring during the test.

(2) The change of amount of heat stored in the element is negligible when compared to the amount of heat going through the element.

2.1 The dynamic analysis method[1]

The dynamic analysis method is a sophisticated method which may be used to obtain the steady-state properties of a building element from HFM measurements when large variations occur in temperatures and heat flow rates, it takes into account the thermal variations by the use of the heat equation.

The building element is represented in the model by its thermal transmittance Λ and several time constants τ. The unknown parameters (Λ, $\tau 1$, $\tau 2$, $\tau 3$,…) are obtained by an identification technique using the measured densities of heat flow rates and temperatures.

With this approach, a set of linear equations must be solved which can be done by a microcomputer in a fewer seconds.

The basic algorithms are as follows:

The measurements give N sets of data of the density of heat flow rate (q_i), indoor and outdoor surface temperatures (T_{Ii}, T_{Ei}) taken at the times t_i (i

ranges from 1 to N). The time interval between two measurements is Δt, defined as:

$$\Delta t = t_{i+1} - t_i \qquad (1)$$

Where Δt is time interval of discretization [s or h]

The heat flow rate at time t_i is a function of the temperatures at that time and at all of the preceding times

$$q_i = \Lambda(T_{Ii} - T_{Ei}) + K_1 T_{Ii}' - K_2 T_{Ei}' +$$

$$\sum_n P_n \sum_{j=i-p}^{i-1} T_{Ij}(1-\beta_n)\beta_n(i-j) + \sum_n Q_n \sum_{j=i-p}^{i-1} T_{Ej}(1-\beta_n)\beta_n(i-j) \qquad (2)$$

Where the derivative of the indoor surface temperature is

$$T_{Ii}' = \frac{T_{Ii} - T_{I,i-1}}{\Delta t} \qquad (3)$$

Where T_I is the internal temperature and T_E is the external temperature.

The same formula is valid for the derivative of external temperature T_{Ei}'.

K_1, K_2, as well as P_n and Q_n are dynamic characteristics of the wall without any particular significance. They depend on the time constant τ_n.

The variables β_n are exponential functions of the time constant τ_n

$$\beta_n = \exp(-\frac{\Delta t}{\tau_n}) \qquad (4)$$

The sum over n in equation (2) is over all the time constants, theoretically an infinite number.

This time constants, tn, however, decrease rapidly with n, as β_n increase. Hence only a few time constants (in practice, between 1 and 3 is sufficient) are needed to correctly describe the relationship between q, T_E and T_I.

Assuming that m time constants (τ_1, $\tau_{2,...}\tau_m$) are chosen, equation (2) will contain 2m+3 unknown parameters which are:

$\Lambda, K_1, K_2, P_1, Q_1, P_2, Q_2 ..., P_m, Q_m \qquad (5)$

Writing equation (2) 2m+3 times for 2m+3 sets of data at various times, a system of linear equations can be solved to determine these parameters, particularly Λ. A number of supplementary sets, p, is needed however, for the integration corresponding to the sum over j in equation (2). Finally, in order to eliminate stochastic variations, more measured system of linear

equations which can be solved by a classic least square fit.

This set of more than 2m+3 equations can be written in a matrix form

$$\bar{q} = (X)\bar{Z} \qquad (6)$$

Where $\bar{q}$ is a vector, the M components of which are the last M heat flow density data, qi. The value of M is then greater than 2m+3 and i goes from N-M+1 to N.

$\bar{Z}$ is a vector, the 2m+3 components of which are the unknown parameters listed in equation(5).

(X) is a rectangular matrix with M lines (i=M-N-1 to N) and 2m+3 columns(1 to 2m+3). The matrix elements are:

$$X_{i1} = T_{Ii} - T_{Ei} \qquad (7)$$

$$X_{i2} = T'_{I} = (T_{Ii} - T_{I,i-1})/\Delta t$$

$$X_{i3} = T'_{E} = (T_{Ei} - T_{E,i-1})/\Delta t$$

$$X_{i4} = \sum_{j=i-p}^{i-1} T'_{Ij}(1-\beta_1)\beta_1(i-j)$$

$$X_{i5} = \sum_{j=i-p}^{i-1} T'_{Ej}(1-\beta_1)\beta_1(i-j)$$

$$X_{i6} = \sum_{j=i-p}^{i-1} T'_{Ij}(1-\beta_2)\beta_2(i-j)$$

$$X_{i7} = \sum_{j=i-p}^{i-1} T'_{Ej}(1-\beta_2)\beta_2(i-j)$$

$$\cdots$$

$$X_{i,2m+2} = \sum_{j=i-p}^{i-1} T'_{Ij}(1-\beta_m)\beta_m(i-j)$$

$$X_{i,2m+3} = \sum_{j=i-p}^{i-1} T'_{Ej}(1-\beta_m)\beta_m(i-j)$$

In the sums over j, p is large enough to make the residual sum (j=i-p to minus infinity) negligible. Then, the number of data sets, N, has to be larger than M+p. practically, p=N-M, where N is large enough.

The sets of equations give an estimate, of the vector:

$$\bar{z}_* = [(X)'(X)]^{-1}(X)'q \qquad (8)$$

Where $(X)'$ is the transposed matrix of (X).

2.2 System identification method

System identification is theory and method of mathematical model which is set up through input and output data. It includes modeling and parameter estimation.

Discrete time model is used in this dynamic method.[2]

$$q(k) + a_1 q(k-1) + \ldots + a_n q(k-n) =$$

$$b_1 \Delta T(k-1) + \ldots + b_n \Delta T(k-n) \qquad (9)$$

Equation (10) is z-transfer function as follow:

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}} \qquad (10)$$

Where z is called Arithmetic Operator, also is called the Shift Arithmetic Operator. The relationship of operation of it is expressed in equation (11)

$$z^{-1}q(k) = q(k-1)$$

$$zq(k) = q(k+1) \qquad (11)$$

It's convenient to use equation (11) to turn equation (9) into equation (10).

Model of the dynamic thermal characteristics for the Building component can be set up the discrete time model by the temperature difference of the internal and external room. And because of the effect of the colored noise, the discrete time model of the wall is expressed as:

$$A(z^{-1})Q(k) = B(z^{-1})\Delta T(k) + \frac{1}{D(z^{-1})}e(k) \qquad (12)$$

Where: $A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{an} z^{-an}$

$$B(z^{-1}) = b_1 z^{-1} + \ldots + b_{bn} z^{-bn}$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + \ldots + d_{dn} z^{-dn}$$

e(k) is the white noise, e(k)/d(z⁻¹) is the colored noise.

T_{Oi} is the external surface temperature . T_{Ii} is the internal surface temperature . Q_i is the internal surface heat flow rate w. ΔT_i is the temperature difference. It is defined as: $\Delta T_i = T_{Oi} - T_{Ii}$.

When z=1, Λ-value can be calculated as follows:

$$G(1) = \frac{b_1 + b_2 + ... + b_n}{1 + a_1 + ... + a_n} = \Lambda \qquad (13)$$

3 EXPERIMENT AND DATA

3.1 Experiment

Tab.1 The descriptions of the sample wall

Description	L [mm]	λ [W/(m·K)]	R [m²/(K·W)]
Inside surface film	0	----	0.13
Concrete slurry	20	0.93	0.02
Brick	240	0.41	0.59
Insulation	20	0.08	0.25
Concrete slurry	10	0.93	0.01
Inside surface film	0	----	0.04

The sample wall of the experiment is shown in Table 1. It contains an insulation layer. In the experiment, several temperature meters and heat meters are used during the measurement. The thermocouples are installed on the surface of the wall and are covered by a black thin film in order to avoid the turbulence of wind and radiation. The indoor environment is controlled by the air-conditioner. The metrical data include the internal and external surface temperature and the heat flow rate through the internal surface. The identification time is seven days.

The metrical data is shown in the Fig. 1:

3.2 Analysis of the results

3.2.1 Theory value

Heat transfer of the building component consists of two parts. The first is the heat convection of the air layer closely both sides wall, another is conductive heat through the wall itself.

In general, it can be assumed that structure has constant properties (i.e. k, q and C_p) and the transient heat conduction through it is one-dimensional. Then its heat conduction differential equation can be solved.

U-value contains heat conduction and heat convection. It is expressed as: [3]

$$U = \frac{1}{\frac{1}{h_i} + \frac{\delta_1}{\lambda_1} + ... + \frac{\delta_n}{\lambda_n} + \frac{1}{h_o}} \qquad (14)$$

R is the thermal resistance of the component. It is given as:

$$R = \frac{\delta_1}{\lambda_1} + ... + \frac{\delta_n}{\lambda_n} \qquad (15)$$

Where 1/hi+1/ho=0.17 (m2.K)/W[3]

According to equation (14) and parameters of the sample wall, the theory U_{th} is 0.97 W/(m² K.).

3.2.2 Dynamic analysis method for U-value

The data of the experiment is used in the equation (7) and (8). Λ-value can be calculated. It is 1.136 W/(m² K) and U-value is 0.952 W/(m² K）.

3.2.3 System identification method for U-value

It is assumed that an=4,bn=2,dn=3 according to equation (12). Equation (9) is given as:

$$q(k) = -a_1 q(k-1) - ... - a_4 q(k-4) + b_1 \Delta T(k-1) + ... + b_3 \Delta T(k-n)$$

$$+ d_1 e(k-1) + ... + d_3 e(k-3) \qquad (16)$$
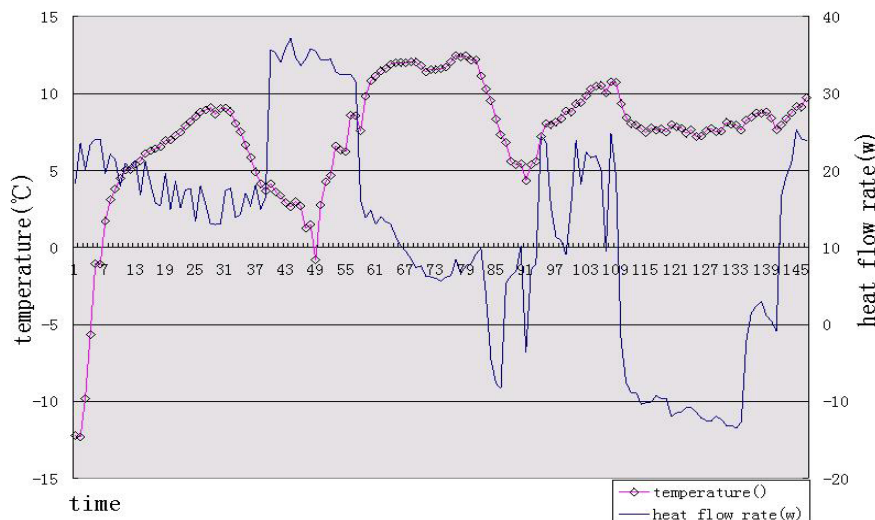
Fig. 1 Temperature difference and heat flow rate

Extension least square method is used to analysis the model. The result is shown bellow:

$$G(z) = \frac{-9.85z^{-1} + 6.17z^{-2} + 3.95z^{-3}}{1 - 1.46z^{-1} + 0.48z^{-2} + 0.47z^{-3} - 0.26z^{-4}} \quad （17）$$

$$\varLambda = G(z)|_{z=1} = 1.174 \text{W}/(\text{m}^2 \text{ K}) \quad （18）$$

4. ANALYSIS OF THE RESULT OF TWO METHODS

4.1 Result

By Tab.2, the results calculated by Dynamic analysis method and System identification method have the certain errors. The result of the system identification method is more precise.

Tab.2 Results

	Dynamic method	System identification
$\varLambda$[Wm^{-2} K^{-1}]	1.136	1.174
U[Wm^{-2} K^{-1}]	0.952	0.979
U_{th}[Wm^{-2} K^{-1}]	0.970	0.970
Error	1.18%	0.89%

4.2 Error analysis.

1) The experimental error. Experimentation is held on a site, not go on in the laboratory. Because of the influence of the environment and apparatus, the measuring value will deviate from actual value.

2) The influence of the algorithm. When two methods are calculated, because the least square method is limited in precision, the results tend to the error.

3) The noise influences. Because of the influence of various signals, the noise is produced in the process of gathering the data. At the same time, since the model of the noise is uncertain during measurement on different environments, the results calculated, finally, cause the error.

5. CONCLUSIONS

1) U-value calculated by the system identification method is more steady and reliable than the one that calculated by dynamic analysis method. U-value is the first coefficient of the model in the dynamic analysis method. While this model is calculating, the little noise from the experimentation can cause the first coefficient change. Change of U-value will fluctuate with sample interval and identification time. U-value of System identification method is obtained from the model of the dynamic thermal performance of the wall, so the impact on U-value of the test data noise is reduced.

2) System identification method reflects dynamic thermal performance of the wall. Its concept is clearer. Q in dynamic analysis method is made up of the steady part and the dynamic part. For further research of the characteristics of the wall, system identification method is more practical.

3) System identification method is simple. When U-value is calculated by System identification method, the measured data need be not any dealt with, just be used as the element of the matrix. The data

used in dynamic analysis method, however, should be dealt with first. Then the data of these results is used as the element of the matrix. The later is more complicated.

4) System identification method is more flexible and changeable. For different thermal inertia and thermal transmittance of the wall, several models can be set up. Through calculation the best model can be confirmed. It's mean that many models and algorithms can be used to improve the precision. It offers more space for further research in measurement of the wall. Studies have suggested that increase of the time constant number have no effect on improvement of the precision of the result. In practice, between 1 and 3 is sufficient.

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