ESL-IC-06-11-313

Performance Optimization of an Irreversible Heat Pump with Variable-temperature Heat Reservoirs

Yuewu Huang
Ph.D.
Donghua University
Shanghai & China
huangyaowu@sina.com

Dexing Sun
Ph.D. Professor
Harbin Institute of Technology
Harbin &China
sundx0331@sina.com.cn

Abstract: An irreversible cycle model of a heat pump operating between two variable-temperature heat reservoirs is established and used to analyze the performance of the heat pump affected by heat resistances, heat leakage and internal dissipation of the working substance. The coefficient of performance of the heat pump is optimized for a given heating load. The characteristic curves of the coefficient of performance versus power input are generated. The influence of intake temperatures of heat reservoirs, thermal capacity of heat reservoirs, efficiency of heat exchangers, heat leak and internal irreversibilities on the performance of the system is discussed. The optimal ratio of the times spent on two processes of heat transfer to and from the working substance is determined. Some new results which are conducive to the optimal design and operation of real heat pump systems are obtained.

Key words: irreversibility; Heat pump; optimization; variable-temperature heat reservoir

1. INTRODUCTION

According to classical thermodynamics, the coefficient of performance of a reversible heat pump cycle, given by $\varepsilon_{2,C} = T_H / (T_H - T_L)$, provides a bound on the optimal operation of heat pumps operating between the heat reservoirs at temperatures T_H and T_L. Since it corresponds to infinitely slow operation and zero heating capacity, the manufacture of such a heat pump is not an attractive proposition. Therefore, it is necessary and significant to discuss the optimal performance for an irreversible heat pump.

Some studies[1-4] the external fluid temperatures of heat reservoirs were assumed constants even the heat is transferred from system to reservoir or vice versa during the thermodynamic cycle. Practically, the heat reservoirs have finite thermal capacitance rates. Consequently, the temperature in the reservoirs is not constant[5]. For this purpose, we will extend heat pump cycle models of constant temperature heat reservoirs to the irreversible heat pump cycle model, affected by irreversibility of heat resistances, heat leak and internal dissipation of the working substance, and use it to analyze the optimal performance of a class of heat pumps operating between variable-temperature heat reservoirs.

2. THE THERMODYNAMIC MODEL

Fig. 1 shows a thermodynamic cycle with finite temperature differences existing between system fluid temperatures and the external fluid temperatures; these temperatures have the relationship $T_H > T_x > T_y > T_L$. The heat released to the high-temperature heat exchanger is Q_1 and the heat absorbed from the low-temperature heat exchanger is Q_2 per heat pump cycle. To obtain the equations for the heat transfer rates of both heat exchangers that do not involve any of the outlet temperatures, we introduce the heat exchange effectiveness ε_H and ε_L:

$$Q_1 = C_H \varepsilon_H (T_H - T_{x1}) t_1 \qquad (1)$$

$$Q_2 = C_L \varepsilon_L (T_{y1} - T_L) t_2 \qquad (2)$$

In both heat exchangers, since the thermal capacitance rates of the system fluid are almost infinite (because of the phase change), C_H represents the capacitance rate of cooling flow in the high-temperature heat exchanger and C_L is the counterpart of C_H at the low- temperature heat exchanger. T_{x1} and T_{y1} are respectively the inlet temperatures of heat reservoirs, t_1 and t_2 are respectively the times taken to complete the heat transfer processes at the high- and low-temperature exchangers. The effectiveness of the high-temperature heat exchanger, ε_H, and low-temperature heat exchanger, ε_L, are defined

$$\varepsilon_H = \mathbf{1} - \exp(-NTU_H) \quad (3)$$

$$\varepsilon_L = \mathbf{1} - \exp(-NTU_L) \quad (4)$$

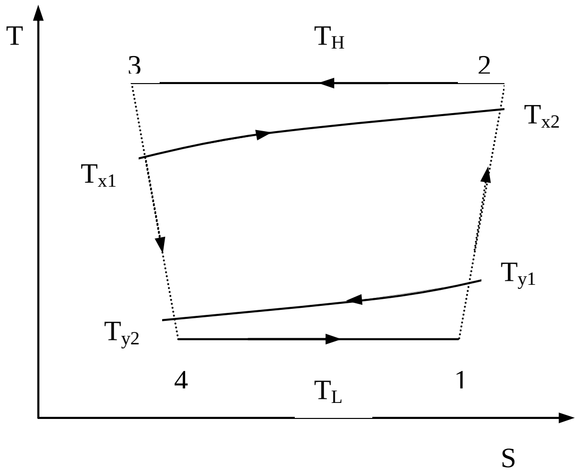

Fig. 1 The T-S diagram of an irreversible heat pump cycle

The associated numbers of heat transfer units of heat exchangers are NTU_H and NTU_L, which are based on the minimum thermal capacitance rates, and are defined as $NTU_H = A_H U_H / C_H$ and $NTU_L = A_L U_L / C_L$. Q_i, which is the heat leaking from the hot reservoir to the cold reservoir per cycle, may be expressed as

$$Q_i = q_i \tau \quad (5)$$

where q_i is the heat leaking rate. According to fig. 1, the net heats Q_L and Q_H transferred from the cold reservoir and to the hot reservoir per cycle are

$$Q_L = Q_2 - Q_i \quad (6)$$

$$Q_H = Q_1 - Q_i \quad (7)$$

To obtain the simple expressions of the performance coefficient and heating rate, two adiabatic processes are often assumed to proceed in negligible time, such that the cycle time may be approximately given by

$$\tau = t_1 + t_2 \quad (8)$$

Owing to the internal dissipation of the system fluid, all processes are irreversible. According to the second law of thermodynamics, one has

$$Q_1/T_H - Q_2/T_L > 0 \quad (9)$$

In order to obtain the quantitative relationship amongst the parameters Q_1, Q_2, T_H and T_L, we introduce a parameter

$$I = (Q_1/T_H)/(Q_2/T_L) \quad (10)$$

It is seen clearly from equation (10) that when $I = 1$, the heat pump cycle is endoreversible; when $I > 1$, the heat pump cycle is internally irreversible. Thus, the parameter I is a measure of internal irreversibility resulting from the system fluid. From equations (1), (2), (5)-(10) and the definitions of the performance coefficient and heating rate, we find that the expressions of the performance coefficient ε and the heating rate q_H are given by

$$\varepsilon = \frac{Q_1 - Q_i}{Q_1 - Q_2} = \left\{ IE_H E_L xy(x + T_{x1}) + q_i \left[E_H x(y - T_{y1}) - IE_L y(x + T_{x1}) \right] \right\} \left\{ E_H E_L xy \left[I(x + T_{x1}) + (y - T_{y1}) \right] \right\}^{-1} \quad (11)$$

and

$$q_H = \frac{Q_H}{\tau} = \frac{IE_H E_L xy(x + T_{x1})}{IE_L y(x + T_{x1}) + E_H x(T_{y1} - y)} - q_i \quad (12)$$

respectively, where $E_H = C_H \varepsilon_H$, $E_L = C_L \varepsilon_L$,

$x = T_H - T_{x1}$, and $y = T_{y1} - T_L$. Starting from equations (11) and (12), we can determine the maximum performance coefficient of a class of irreversible heat pumps for a given output heating rate and obtain the performance coefficient versus power input characteristics of such heat pumps.

3. THE MAXIMUM PERFORMANCE COEFFICIENT AND THE CORRESPONDING POWER INPUT

To maximize the coefficient of performance for a given heating rate, the Lagrangian is introduced

$$L = \varepsilon + \lambda q_H \tag{13}$$

From the Euler-Lagrange equations

$$\left(\frac{\partial L}{\partial x}\right)_y = \mathbf{0} \;\; ; \;\; \left(\frac{\partial L}{\partial y}\right)_x = \mathbf{0} \tag{14}$$

and equation (13), we obtain

$$x = \frac{\left(\sqrt{E_H} + \sqrt{IE_L}\right)\left(q_i + q_H\right)}{E_H \sqrt{IE_L}} \tag{15}$$

$$y = \frac{\sqrt{E_H}\left(\sqrt{E_H} + \sqrt{IE_L}\right)\left(q_i + q_H\right)T_{y1}}{\left(\sqrt{E_H} + \sqrt{IE_L}\right)^2\left(q_i + q_H\right) + IE_H E_L T_{x1}} \tag{16}$$

when the heat pump operates in the state of maximum performance coefficient for a given heating rate, the temperatures of the system fluid in the high- and low-temperature heat exchangers

$$T_H = T_{x1} + \frac{\left(\sqrt{E_H} + \sqrt{IE_L}\right)\left(q_i + q_H\right)}{E_H \sqrt{IE_L}} \tag{17}$$

$$T_L = T_{y1} - \frac{\sqrt{E_H}\left(\sqrt{E_H} + \sqrt{IE_L}\right)\left(q_i + q_H\right)T_{y1}}{\left(\sqrt{E_H} + \sqrt{IE_L}\right)^2\left(q_i + q_H\right) + IE_H E_L T_{x1}} \tag{18}$$

can be derived from (15) and (16). Substituting (15) and (16) into (11), we find the maximum performance coefficient for a given heating rate

$$\varepsilon = \left\{ q_H \left[\left(\sqrt{E_H} + \sqrt{IE_L}\right)^2 \left(q_i + q_H\right) + IE_H E_L T_{x1} \right] \right\} \left\{ \left(q_i + q_H\right) \left[\left(\sqrt{E_H} + \sqrt{IE_L}\right)^2 \left(q_i + q_H\right) + E_H E_L \left(IT_{x1} - T_{y1}\right) \right] \right\}^{-1} \tag{19}$$

of a class of heat pumps operating between the variable-temperature heat reservoirs and having three irreversibilities mentioned above. The corresponding power input to the heat pump system is given by:

$$P = \left(q_i + q_H\right) \left\{ \left[\left(\sqrt{E_H} + \sqrt{IE_L}\right)^2 \times \left(q_i + q_H\right) + E_H E_L \left(IT_{x1} - T_{y1}\right) \right] \right\} \times \left[\left(\sqrt{E_H} + \sqrt{IE_L}\right)^2 \left(q_i + q_H\right) + IE_H E_L T_{x1} \right]^{-1} \tag{20}$$

Using (19) and (20), we can easily generate the curves of the relative performance coefficient varying with power input, as shown in fig. 2. It is seen clearly from fig. 2 that the relative performance coefficient is a monotonically decreasing function of power input. Curves (3) and (4) in fig. 2 respectively correspond to the case of $I > 1, q_i = 0$ and to the case of $I > 1, q_i \neq 0$. The two curves indicate that when the degree of internal irreversibilities are equal, the heat leaking rate has an effect on the relative performance coefficient; When $I = 1$ and $q_i \neq 0$, Curve (2) is just the optimal relation between the relative performance coefficient and power input of an endoreversible heat pump; There only exist the irreversibilities due to thermal resistance in curve (1). Besides $I = 1$ and $q_i = 0$, if $E_L \to \infty$ and $E_H \to \infty$, the heat pump becomes reversible. The reversible performance coefficient is given by $\varepsilon_r = \frac{T_{x1}}{T_{x1} - T_{y1}}$.

When $C_H \to \infty$ and $C_L \to \infty$, variable-temperature heat reservoirs change to constant- temperature heat reservoirs. This shows that the theory of variable-temperature heat reservoirs heat pump cycle established in this paper is also suitable for constant-temperature heat reservoirs heat pump cycle.

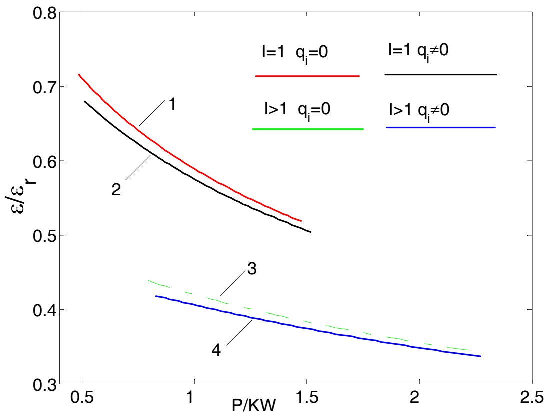

Fig.2 Variation of the coefficient of performance of an irreversible heat pump versus power input

Using (15) and (16), we can prove that the optimal ration of the times spent on the heat transfer processes at the high- and low-temperature exchangers is given by

$$\frac{t_1}{t_2} = \frac{\sqrt{IE_L}}{\sqrt{E_H}} \tag{21}$$

Equation (21) shows that the optimal ratio of the times spent on the heat transfer processes at the high- and low-temperature exchangers is not affected by the heat leak, but it depends on the parameters E_L/E_H and I .

4. RESULTS AND DISCUSSION

In order to have a numerical appreciation of the theoretical analysis of the heat pump cycle, we have studied the effect of various input parameters on the performance of the heat pump system. These results are shown in figures 3-6. During the variation of any one parameter, all other parameters are assumed to be constant.

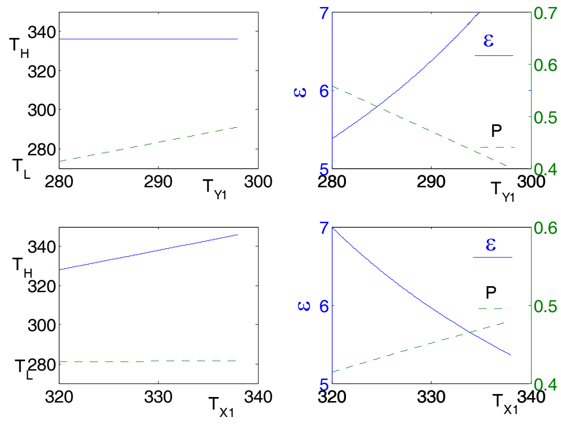

Fig.3 Effect intake temperatures of heat reservoirs on the performance of the heat pump

It can be seen from fig. 3 that increasing the inlet temperature of external heat reservoir (T_{y1}), decreases the power input to the system and consequently the coefficient of performance increases. However, increasing the sink inlet temperature (T_{x1}), of the working fluid on the high temperature side increases the power input to the system and hence decreases the performance coefficient, while the working fluid temperature on the low temperature side remains almost constant.

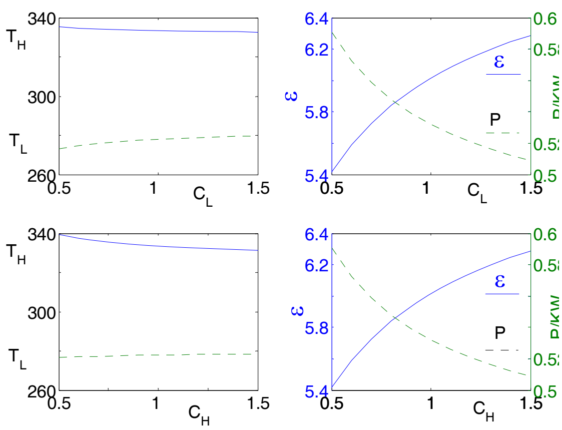

Fig. 4 Effect thermal capacity of heat reservoirs on the performance of the heat pump

Fig. 4 shows similar effects for the thermal capacitance rates of the external reservoir or sink side fluid (C_L, C_H). By increasing either or both of them, the sink side working fluid temperature and the power input to the system decrease and consequently the system performance coefficient increases. The source side working fluid temperature also increases.

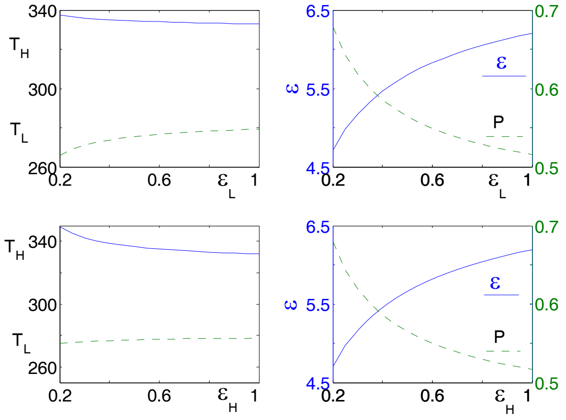

Fig. 5 Effect efficiency of heat exchangers on the performance of the heat pump

Fig. 5 shows the effect of effectiveness of source or sink side heat exchanger of the heat pump system ($\varepsilon_L, \varepsilon_H$). When one of them is increased while the other is kept constant, the power input to the system decreases and consequently the performance coefficient of the system increases because heating load is given. The temperature of the sink side working fluid decreases while the source side working fluid temperature increases with increasing effectiveness.

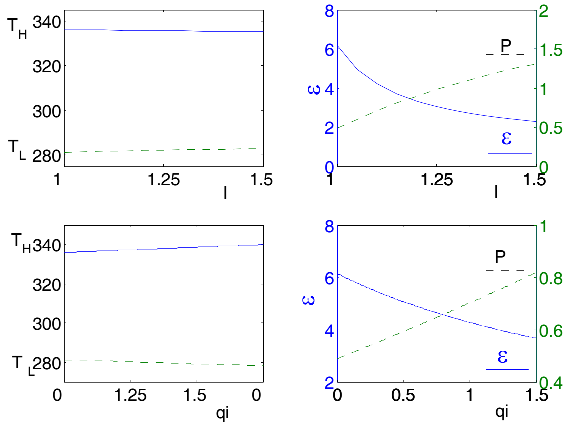

Fig. 6 Effect internal irreversibility and heat leak on the performance of the heat pump

Fig. 6 shows effects of the internal irreversibility parameter (I) and the heat-leaking rate (q_i). As every parameter is increase, the power input to the system increases, and hence the system performance coefficient decreases. Be contrary to the internal irreversibility parameter, increasing the heat-leaking rate, the source side working temperature decreases. However, the sink side working fluid temperature increases. The case of $I = 1$ corresponds to the endoreversible case.

5. CONCLUSIONS

The important feature of the irreversible cycle model adopted in the present paper is that it can include three main irreversibilities often existing in real heat pump systems. Using the cycle model to analyze the performance of irreversible heat pumps operating between two variable temperature heat reservoirs, the results obtained will play a more instructive role in optimal design and operation of real systems than could those derived from the reversible heat pump cycle models.

REFERENCES

[1] Bejan, A. Entroy generation minimization: The new thermodynamics of finite-size devices and finite-time processes. J. Appl. Phys., 1996, 79: 1191-1218

[2] F Angulo-Brown, LA Arias-Hernandez and R Paez-Hernandez. A general property of non-endoreversible thermal cycles. J. Phys. D: Appl. Phys., 1999, 32: 1415-1420.

[3] D. Bedeaux and F. Standaert. Optimization of processes by equipartition. J. Non-Equilib. Thermodyn., 1999, 24: 242-259

[4] Giuseppe Grazzini and Rinaldo Rinaldi. Thermodynamic optimal design of heat exchangers for an irreversible refrigerator. Int. J. Therm. Sci., 2001, 40: 173-180.

[5] M.M. Salah El-Din. Second law analysis of irreversible heat engines with variable temperature heat reservoirs. Energy Conversion & Management, 2001, 42:189-200.