Investigation of the Accuracy of Calculation Methods for Conduction Transfer Functions of Building Construction

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Abstract: Conduction transfer functions (CTFs) are widely used to calculate conduction heat transfer in building cooling load and energy calculations. They can conveniently fit into any load and energy calculation techniques to perform conduction calculations. There are three methods: the Laplace transform (LP) method, the state-space (SS) method and the frequency-domain regression (FDR) method to calculate CTF coefficients. The limitation of methodology possibly results in imprecise or false CTF coefficients. This paper investigates the accuracy of the three methods applied to the material properties of a single-layer and a multilayer heavyweight building construction.

Key words: CTF coefficients; Laplace transform method; state-space method; frequency-domain regression method; error

1. INTRODUCTION

Conduction heat transfer through the building envelope is one of the principal components of space cooling/heating loads and energy requirements [1]. In current building simulation programs such as [2-5], as well as space cooling load calculations [6], the dynamic thermal behavior data of building constructions including thermal response factors, conduction transfer function (CTF) coefficients or periodic response factors are calculated by various algorithms, and then utilized in conjunction with weather data to calculate the heat flow through the constructions [7]. The accuracy of the dynamic thermal behavior data directly affects the accuracy of the building load and/or energy calculations. However, there are various potential factors such as too big of an iteration step, low calculation precision, unconverged computational results, the limitation of calculation methods, etc., which may lead to incorrect results in calculating the dynamic thermal behavior data. As pointed out by Spitler and Fisher [8], computational inaccuracy sometimes occurs in calculating the dynamic thermal behavior data. Therefore, it is necessary to seek a precise calculation dynamic thermal behavior data solution for getting the accuracy space cooling/heating loads.

In cooling load and energy calculation, building simulation and energy analysis, conduction heat transfer is usually modeled as a one-dimensional, transient process with constant material properties [9]. The simplified heat diffusion equation in Cartesian coordinates is shown in equation (1) [10].

$$\frac{\partial^2 T(x,\tau)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,\tau)}{\partial \tau} \tag{1}$$

where T, x, α and τ are the temperature, heat flow direction, thermal diffusivity and time, respectively. Fourier's law, equation (2), specifies the conduction heat flux in terms of the thermal conductivity of the material and temperature gradient across a differential thickness.

$$q = -k\frac{\partial T(x,\tau)}{\partial x} \tag{2}$$

where q and k are heat flux, thermal conductivity, respectively.

Since equation (1) is a partial differential equation, the system is usually solved numerically, often by means of conduction transfer function

methods. CTFs represent the material's thermal response as determined by its material properties. The method results in a simple linear equation that expresses the current heat flux in terms of the current temperature and temperature and heat flux histories as shown in equations (3) and (4) [9].

$$q_{o,\theta} = -\sum_{n=0}^{N_y} Y_n T_{is,\theta-n\delta} + \sum_{n=0}^{N_x} X_n T_{os,\theta-n\delta} + \sum_{n=1}^{N_\phi} \phi_n q_{o,\theta-n\delta} \quad (3)$$

$$q_{i,\theta} = -\sum_{n=0}^{N_z} Z_n T_{is,\theta-n\delta} + \sum_{n=0}^{N_y} Y_n T_{os,\theta-n\delta} + \sum_{n=1}^{N_\phi} \phi_n q_{i,\theta-n\delta} \quad (4)$$

where q_o and q_i are heat flux at exterior and interior surface, respectively. X_n, Y_n and Z_n are surface-to-surface exterior, cross and interior CTF coefficient, respectively. T_{is} and T_{os} are interior and exterior surface temperature, respectively. N_x, N_y and N_z are number of exterior, cross and interior CTF terms, respectively. ϕ_n is flux coefficient. N_ϕ is the number of flux history terms. The subscript θ represents the current time, and δ is time step. The zero subscript represents a current value.

The linear relationship can greatly reduce computational effort and facilitate computer implementation of the method. Since CTFs are temperature independent, they are usually pre-determined. Pre-calculated CTFs of some typical constructions are available in the literature (ASHRAE, 1997) [11].

2. OVERVIEW OF CALCULATING METHODS FOR CTF

While there are number of numerical methods for solving equations (1) and (2), the Laplace transform method and the state-space method are the most widely used in cooling load and energy calculations. The frequency-domain regression method is developed recently (Chen, et al., 2003).

2.1 Laplace Transform Method

Hittle [12] introduced a procedure to solve the conduction heat transfer governing equations (1) and (2) by using Laplace transform method. The system in the Laplace domain is shown in equation (5).

$$\begin{bmatrix} q_i(s) \\ q_o(s) \end{bmatrix} = \begin{bmatrix} \dfrac{D(s)}{B(s)} & \dfrac{-1}{B(s)} \\ \dfrac{1}{B(s)} & \dfrac{-A(s)}{B(s)} \end{bmatrix} \begin{bmatrix} T_i(s) \\ T_o(s) \end{bmatrix} \quad (5)$$

where $A(s)$, $B(s)$ and $D(s)$ are overall transmission matrices that depend on material properties, and/or film coefficients.

Response factors are generated by applying a unit triangular temperature pulse to the inside and outside surface of the multilayered slab. The response factors are defined as the discretized heat fluxes on each surface due to both the outside and inside temperature pulse. The response factors are an infinite series. Hittle also described an algebraic operation to group response factors into CTFs, and to truncate the infinite series of response factors by the introduction of flux histories coefficients. A convergence criterion shown in equation (6) is used to determine whether the numbers of CTFs and flux history terms are sufficient such that the resulting CTFs accurately represent the response factors. Therefore, through Laplace inverse transform of transfer function $D(s)/B(s)$, $-1/B(s)$ and $-A(s)/B(s)$ with unit triangle temperature, response factors and CTF coefficients can be worked out.

$$\sum_{n=0}^{N_x} X_n = \sum_{n=0}^{N_y} Y_n = \sum_{n=0}^{N_z} Z_n = U(1 - \sum_{n=1}^{N_\phi} \phi_n) \quad (6)$$

where U is the overall heat transfer coefficient.

2.2 State-space Method

The use of the state-space method in solving the governing equations (1) and (2) was introduced by Jiang (1982) [13], Seem (1987) [14]. The state-space expression relates the interior and exterior boundary temperatures to the inside and outside surface heat fluxes at each node of a multi-layered slab as shown in equations (7) and (8).

$$\begin{bmatrix} dT_{si}/d\tau \\ \vdots \\ dT_{so}/d\tau \end{bmatrix} = \boldsymbol{a} \begin{bmatrix} T_{si} \\ \vdots \\ T_{so} \end{bmatrix} + \boldsymbol{b} \begin{bmatrix} T_i \\ T_o \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} q_i \\ q_o \end{bmatrix} = \boldsymbol{c} \begin{bmatrix} t_{si} \\ \vdots \\ t_{so} \end{bmatrix} + \boldsymbol{d} \begin{bmatrix} T_i \\ T_o \end{bmatrix} \quad (8)$$

where T_s is the temperature of each code. ***a***, ***b***, ***c***, ***d*** are coefficient matrices that depends on material properties, and/or film coefficients.

Coefficients matrices ***a***, ***b***, ***c***, ***d*** can be worked out through numerical computation. And then the CTF coefficients can be obtained by Leverrier's algorithm[15] from coefficients matrices ***a***, ***b***, ***c***, ***d***.

2.3 Frequency-domain Regression Method

The use of the frequency domain regression (FDR) method in solving the governing equations (1) and (2) was introduced by Chen, et al. [16]. Base on Laplace transform, transmission matrix of a single-layer slab can be obtained. The frequency characteristics interior, cross and exterior heat transfer of a multilayer slab are calculated through transmission matrix multiplication by substituting s with $j\omega$ ($j = \sqrt{-1}$). It is very easy to calculate the frequency characteristics of a multilayer slab. The FDR method is used to estimate some simple *s*-transfer functions in the form of equation (9) from the interior, cross and exterior frequency characteristics.

$$G(s) = \frac{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_r s^l}{1 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_m s^m} \quad (9)$$

Where α_i and β_i are real coefficients. l and m are the orders of the numerator and denominator, respectively.

The simple *s*-transfer functions are in the forms of the polynomial ratio of variable s. For short, they are called polynomial *s*-transfer functions. In the FDR method, by minimizing the sum of the square error between the frequency characteristics of the multilayer slab and the polynomial *s*-transfer function at all frequency points, the coefficients of the polynomial *s*-transfer function are easily obtained by solving a set of linear equations. The frequency characteristic of the polynomial *s*-transfer functions is evaluated by substituting s with $j\omega$. Through Laplace inverse transform of interior, cross and exterior transfer functions $G(s)$ with unit triangle temperature, response factors and CTF coefficients can be worked out. Therefore, through the polynomial *s*-transfer functions, the response factors and CTF coefficients of a multilayer slab are easily and accurately calculated.

3. VERIFICATION OF THE ACCURACY OF CTFS AND THE TEST PROCEDURE

A conventional verification check for CTFs is to check whether or not they give the correct heat transfer in steady-state.

$$\frac{\sum_{n=0}^{N_x} X_n}{(1-\sum_{n=1}^{N_\phi} \phi_n)} = \frac{\sum_{n=0}^{N_y} Y_n}{(1-\sum_{n=1}^{N_\phi} \phi_n)} = \frac{\sum_{n=0}^{N_z} Z_n}{(1-\sum_{n=1}^{N_\phi} \phi_n)} = U \quad (10)$$

The above relationships are valid for the case when frequency $\omega = 0$, but do not address the accuracy for dynamic thermal behavior.

There is a method [1] for verification of dynamic CTFs, based on the equivalence of dynamic models and the frequency characteristics of linear systems.

Certainly there are many methods for verification of the accuracy of CTFs. This paper employs the method by means of comparing the heat flux. This method verifies the accuracy of CTFs by checking the equivalence of the heat flux calculated by numerical methods and the analytical method. The analytical method is explained as follows:

The governing conduction equations (1) and (2) are usually not solved analytically in building thermal load and energy calculations due primarily to the computational intensity of the implementation. However, with a periodic temperature boundary condition on one side of the slab and a constant temperature boundary condition on the other side, the analytical solution is tractable. Spitler et al. [17] presents an analytical solution for a multi-layered slab subject to a sinusoidal outside temperature and a constant inside temperature. For single-layered slabs of thickness *L*, the inside temperature and heat flux are related to the outside temperature and heat flux by the following set of equations:

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix} \begin{bmatrix} T_o \\ q_o \end{bmatrix} \tag{11}$$

where

$$m_1 = \cosh(p + jp) \tag{12}$$

$$m_2 = \frac{L \sinh(p + jp)}{k(p + jp)} \tag{13}$$

$$m_3 = \frac{k(p + jp)\sinh(p + jp)}{L} \tag{14}$$

for a 24-hour cycle

$$p = \left(\frac{\pi L^2 \rho c_p}{86400k} \right)^{0.5} \tag{15}$$

where ρ and c_p are density and specific heat, respectively.

$$j^2 = -1 \tag{16}$$

For a resistive layer, equation (12) to (14) can be simplified as follows.

$$m_1 = 1 \tag{17}$$

$$m_2 = R \tag{18}$$

$$m_3 = 0 \tag{19}$$

The matrix formulation shown in equation (11) can be extended for multi-layered slabs with appropriate changes on the *m* matrix, equation (20).

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_1 \end{bmatrix} \begin{bmatrix} T_o \\ q_o \end{bmatrix} \tag{20}$$

where

$$\begin{bmatrix} M_1 & M_2 \\ M_3 & M_1 \end{bmatrix} = \begin{bmatrix} 1 & R_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix}_{layer,1} \cdots \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix}_{layer,n} \begin{bmatrix} 1 & R_o \\ 0 & 1 \end{bmatrix} \tag{21}$$

where n is the number of layers. R_i and R_o are thermal resistance from inside and outside film coefficient, respectively.

As a result, the so-called decrement factor f and time lag ψ can be calculated as follows:

$$f = \left| \frac{1}{UM_2} \right| \tag{22}$$

$$\psi = -\frac{1}{\omega} \tan^{-1} \left[\frac{\mathrm{Im}(1/UM_2)}{\mathrm{Re}(1/UM_2)} \right] \tag{23}$$

where ω is the frequency of temperature boundary condition. The arctangent should be evaluated in the range of $-\pi$ to 0 radians. For a sinusoidal outside temperature and a constant inside temperature, the inside heat flux can be formulated as:

$$q_i(\tau) = UfT_A \sin[\omega(\tau - \psi)] \tag{24}$$

where T_A is the amplitude of the sinusoidal temperature variation.

The heat flux calculated from equation (24) is thus the exact solution of equation (1) and (2). If conduction is the only heat transfer in a control volume, and with constant inside and outside film coefficients, equation (24) can be used to calculate the exact cooling load values. The test procedure [18] uses the analytical solution to benchmark the accuracy of conduction calculations by the state-space, Laplace and frequency-domain regression CTFs.

The error is calculated as the percent deviation of the numerical, CTF calculated heat flux from the analytical solution as follows:

$$error = \frac{q_{num} - q_{exact}}{q_{exact}} \times 100\% \tag{25}$$

where q_{exact} is the peak value of the 24-hour conduction heat fluxes calculated from analytical solution. q_{num} is the conduction heat flux at the same time that q_{exact} is calculated.

In the test, the ASHRAE Loads Toolkit (2001) [6] algorithm was used to calculate the Laplace and state-space CTF solutions. The procedure adopts the default Toolkit CTF algorithm settings in the Tab. 1. The boundary conditions are assumed as follows: the inside and outside film coefficients are treated as resistive layers and the resistances R_i and R_o are

respectively equal to 0.120 and 0.044 (m^2-K)/W [6]. The sinusoidal outside air temperature profile is approximated for 1-hour time steps and 24-hour

Tab. 1 Default Toolkit settings for CTF solution

Error types	Error sources	Parameter settings	
		State-space CTF	Laplace CTF
CTF numerical error	(a) No. of state-space nodes	10 ~ 19	NA
	(b) Root finding tolerance	NA	1E-10
	(c) No. of CTF terms	TL = 1E-13 NI = Total no. of nodes<19	TL = 1E-4 NI = No. of roots found < 5
	(d) Solution time step	1 hour	1 hour
Application error	(e) Solution convergence*	TL = 1E-6 NI = 100	TL = 1E-6 NI = 100
	(f) No. of flux history terms	24	24

Note: NA = Not applicable TL = Tolerance NI = Number of iterations * = User defined parameters

period as shown below. The inside air temperature is the mean air temperature, T_m.

$$T_o = T_m + T_A \sin(\frac{\pi}{12}\tau) \quad (26)$$

The mean air temperature is 20℃, and the amplitude temperature is also 20℃.

4. WALL CHARACTERISTIC PARAMETERS

To show the relationship between the wall material fabric and the error of CTFs calculation, this paper presents Fourier number and thermal structure factor.

Fourier number is defined as:

$$Fo = \Delta\tau / RC \quad (27)$$

where R and C are the thermal resistance and capacity, respectively.

$$R = L / k \quad (28)$$

$$C = L\rho c_p \quad (29)$$

Since the time step $\Delta\tau$ is constant in the CTF calculations, changes in the Fourier number represent changes in material properties only. Equation (27) shows that Fourier numbers for heavy weight materials (larger R and C) are smaller than Fourier numbers for light weight materials. In other words, the value of *1/Fo* is larger for heavy weight materials. As the layer of thickness (L), specific heat (c_p), density (ρ) increasing and the layer of thermal conductivity (k) decreasing, the reciprocal of Fourier number is larger and larger. At the same time the slab becomes more thermally massive. So, Fourier numbers can be used for representing the heavyweight characteristic of the slabs.

However, the relationship of *1/Fo* to CTF solution errors shown above is only suitable for single-layered slabs. For multi-layered slabs, not only material properties, but also layer arrangement influences the CTF calculations. In order to take the layer arrangement into account, the thermal structure factor [19] is introduced. The thermal structure factor S_{ie} is related to the thermal resistance and capacity as shown below, where layer 1 is the interior layer.

$$S_{ie} = \frac{1}{R_T^2 C_T} \sum_{m=1}^{n} C_m \left(-\frac{R_m^2}{3} + \frac{R_m R_T}{2} + R_{i-m} R_{m-o} \right) \quad (30)$$

$$R_{i-m} = R_i + \sum_{k=1}^{m-1} R_k \quad (31)$$

$$R_{m-o} = R_o + \sum_{k=m+1}^{n} R_k \quad (32)$$

where R_T and C_T are the total thermal resistance and capacity of the slab, respectively.

For single-layered slabs, the thermal structure factor can be simplified as:

$$S_{ie} = \frac{1}{6} + \frac{R_i R_o}{R^2} \quad (33)$$

For taking Fourier number and thermal structure factor into account, this paper takes $1/(FoS_{ie})$ as the wall characteristic parameter.

5. RESULTS AND DISCUSSIONS

5.1 Single-layered Slabs

The test procedure starts with a single-layered slab described in Tab. 2. It is ASHRAE (2001) [6] wall 24. The slab is subject to changes of layer thickness (L), thermal conductivity (k), density (ρ) and specific heat (c_p) respectively until the CTF solutions fail to converge.

Fig. 1(a) shows that the error of SS and LP remains within 5% when the parameter of $1/(FoS_{ie})$ is less than 600. The error range can be accepted. As the parameter of $1/(FoS_{ie})$ increasing, the absolute error values appear the tendency of proportion increasing for SS and are stable for LP. When the parameter of $1/(FoS_{ie})$ is more than 600, i.e., the material properties become more and more thermally massive, the absolute error values of SS and LP are wide-range increasing and even exceed 100%. This error is unacceptable. However, for FDR case, no matter how the slabs are lightweight or heavyweight and no matter how the parameter of $1/(FoS_{ie})$ is varied, the absolute error values of the FDR method always remain within 1% and are equal to 0.55% in the majority. So for single-layer slabs, the calculation precision of the FDR method is the highest. FDR method is an excellent calculation CTFs method.

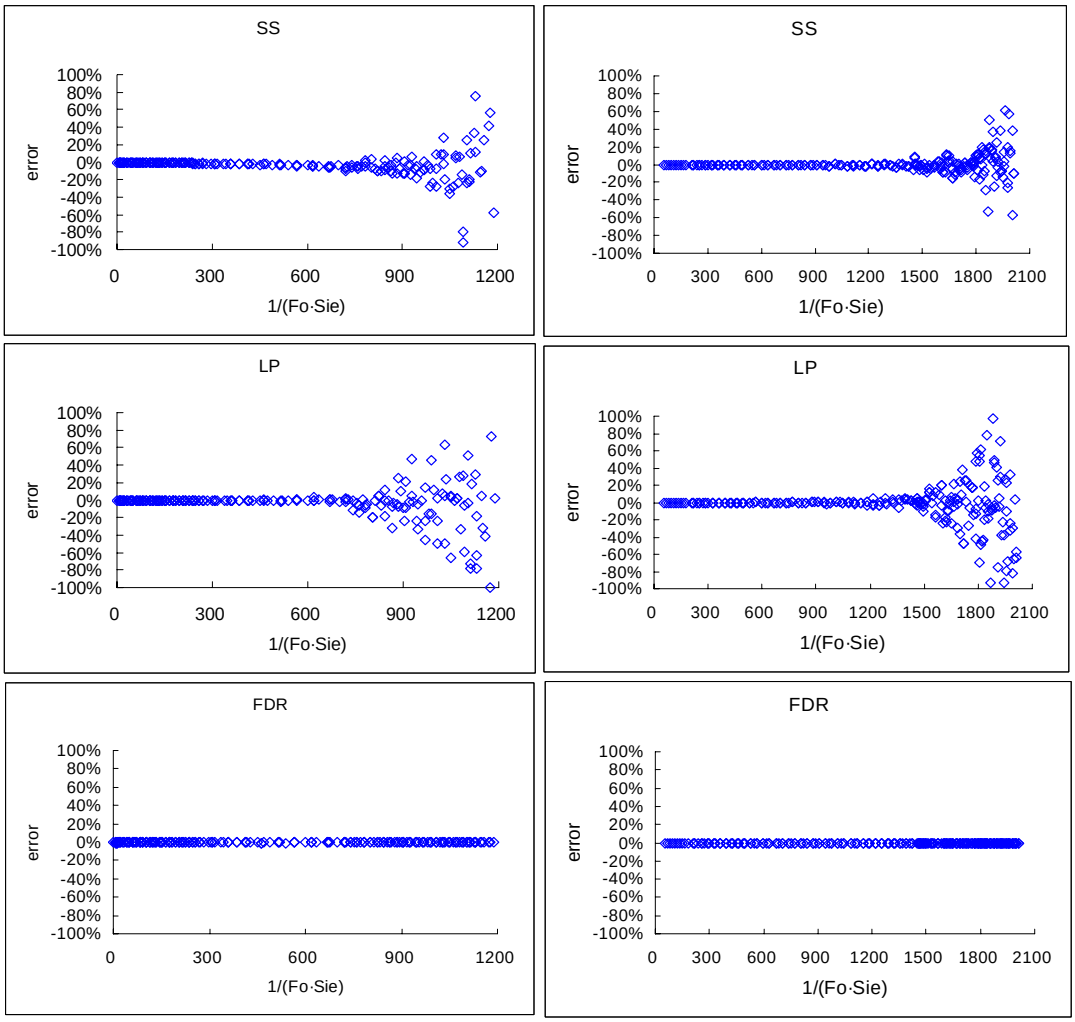
5.2 Multi-layered Slabs

Tab. 2 Details of ASHRAE wall 24

Description	Thickness and thermal properties				
	L (m)	k ($Wm^{-1}K^{-1}$)	ρ (kg m^{-3})	c_p (J $kg^{-1}K^{-1}$)	R (m^2 K W^{-1})
Outside surface film	-	-	-	-	0.044
LW concrete block (filled)	0.2032	0.26	465	879	0.783
Inside vertical surface film	-	-	-	-	0.120

Tab. 3 Details of ASHRAE wall 19

Description	Thickness and thermal properties				
	L (m)	k ($Wm^{-1}K^{-1}$)	ρ (kg m^{-3})	c_p (J $kg^{-1}K^{-1}$)	R (m^2 K W^{-1})
Outside surface film	-	-	-	-	0.044
Brick	0.1016	0.894	1922.2	795.5	0.114
Wall air space resistance	-	-	-	-	0.153
Insulation board	0.0254	0.029	43.2	1214.2	0.881
Heavyweight concrete	0.3048	1.947	2242.6	921.1	0.157
Wall air space resistance	-	-	-	-	0.153
Gyp board	0.0159	0.160	800.9	1088.6	0.099
Inside vertical surface film	-	-	-	-	0.120



(a) (b)

Fig. 1 Single- and 6-layered CTF solution errors

ASHRAE (2001) wall 19 is investigated. Wall 19 is described [6] in Tab. 3. Both brick layer and heavyweight concrete layers of the wall are simultaneous subject to changes of layer thickness (L), thermal conductivity (k), density (ρ) and specific heat (c_p) respectively until the CTF solutions fail to converge.

Fig. 1(b) shows the CTF solution errors for wall 19. The results are consistent with the single layered case. As the material properties become more thermally massive, the magnitude of error increases and the error becomes increasing random for the state-space and Laplace CTF solution. Fig. 1 also shows that the range of the FDR error values remains between -0.7% and 0.7% no matter how to change the condition. So the FDR method is the most accurate calculation CTFs method.

6. SOURCES OF ERRORS IN CTF SOLUTION

Since CTFs are the products of numerical solution, numerical errors exist in the CTF solution [18]. As the numerical methods: Laplace transform and state-space methods are concerned, the error sources are categorized into followings:

- Root finding tolerance**:** This error applies only to the Laplace transform method. In order to calculate response factors, it is necessary to find the root of $B(s) = 0$ in equation (5) [12]. Since the expression for $B(s)$ becomes complicated for slabs with more than one layer, the root finding procedures rely on numerical method. The procedures iteratively continue until the root is found within a root finding tolerance or the maximum number of iterations is reached. The

tolerance value and number of iterations can cause error in the CTF calculation.

- Number of nodes**:** This error applies only to the state-space method that uses state-space nodes to discretize the transient conduction equations. Seem[14] demonstrated that the CTF accuracy is dependent on the number of nodes specified. The CTF accuracy is proportional to the number of nodes used in each material layer in the calculation. So the number of nodes can cause error in the CTF calculation.
- Number of CTF terms**:** In the Laplace transform method, CTFs are derived from response factors and it is necessary to determine the number of CTF terms so that the resulting CTFs can equivalently represent the response factors. Equation (6) is used to check the equivalence of response factors and CTFs. While in the state-space method, the number of CTF terms is determined by tracking the ratio of the last CTF flux term to the first term until the value is negligible [14]. The number of CTF terms is determined with an iterative process until the conditions are satisfied within a tolerance limit or until the maximum number of iterations is reached. The tolerance value and number of iterations can introduce errors in the CTF calculation.

7. CONCLUSIONS

Conduction transfer function (CTF) is widely used to calculate conduction heat transfer in building cooling load and energy calculations. There are a number of methods to solve transient heat transfer through a building construction, such as LP, SS and FDR methods, etc. In this paper, the CTFs of single- and 6-layer building constructions are calculated using LP, SS and FDR methods and compared through the heat flux calculated by analytical and numerical solutions to investigate the accuracy of the three methods. The amplitude errors for LP and SS methods increase with the increasing of the parameter of $1/(FoS_{ie})$. However, no matter how to change the thickness, thermal conductivity, material density and specific heat, the program for FDR method can calculate the CTFs with a very good accuracy. The FDR method has proven to be more robust and reliable than the existing methods, in particular giving good accuracy for very heavyweight constructions.

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