Modeling of Heat Transfer in Geothermal Heat Exchangers [1]

Ping Cui Yi Man Zhaohong Fang
Ph.D Student Student Professor
The Ground Source Heat Pump Research Center
Shandong University of Architecture and Engineering, Jinan, China
Cuip@sdjzu.edu.cn

Abstract: Ground-coupled heat pump (GCHP) systems have been gaining increasing popularity for space conditioning in residential and commercial buildings. The geothermal heat exchanger (GHE) is devised for extraction or injection of thermal energy from/into the ground. This paper summarizes the authors' studies on heat transfer in ground-coupled heat pump systems. Taking the fluid axial convective heat transfer and thermal "short-circuiting" among U-tube legs into account, a quasi-3-D model has been solved for heat transfer inside boreholes. The transient 2-D temperature response in a semi-infinite medium with a line-source of finite length has also been derived for heat conduction outside boreholes. In order to investigate the impact of groundwater advection on the performance of ground heat exchangers, an analytical solution is obtained for a line heat source in an infinite porous medium with groundwater advection. These explicit expressions have more solid theoretical basis, and can be easily incorporated into computer programs for thermal analysis and engineering design of ground heat exchangers.

Key words: geothermal heat exchanger; groundwater advection; heat transfer

1. INTRODUCTION

Due to their reduced energy and maintenance costs ground-coupled heat pump (GCHP) systems, which use the ground as a heat source/sink, have been gaining increasing popul arity for space conditioning in buildings. The efficiency of GCHP systems is inherently higher than that of air source heat pumps because the ground maintains a relatively stable temperature. This system is environment-friendly, causing less CO_2 emission than their conventional alternatives. The geothermal heat exchanger (GHE) is devised for extraction or injection of heat from/into the ground. These systems consist of a sealed loop of pipe, buried in the ground and connected to a heat pump through which water/antifreeze is circulated. In vertical borehole systems the GHE consists of a number of boreholes, each containing single or double U-tube pipes. The borehole annulus should be grouted with backfilling materials that provide thermal contact between the pipe and the soil/rock and to protect groundwater from possible contamination. Despite all the advantages of the GCHP system, commercial growth of the technology

has been hindered by higher capital cost of the system, of which a significant portion is attributed to the GHEs. Heat transfer between a GHE and its surrounding soil/rock is difficult to model for the purpose of sizing the exchanger or energy analysis of the system. Thus, it is crucial to work out appropriate and validated tools, by which the thermal behaviour of GCHP systems can be assessed and then, optimised in technical and economical aspects.

There are roughly two categories of approaches in dealing with the thermal analysis and design of the GHEs. Empirical or semi-empirical formulations are recommended in textbooks and monographs for GHE design purposes, *e.g.* by Bose *et al.* (1985) and Caneta Research Inc. (1995). Containing few parameters of the GHE structure and physical properties, these approaches

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are relatively simple, and may be manipulated by design engineers. However, they do not reveal in detail the impacts of complicated factors on the GHE performance such as configuration of the borehole fields and imbalance in the annual heating and cooling loads. The second kind of approaches involves numerical simulation of the heat transfer in the GHEs. With fast development of computers and computational technologies, numerical simulation has become a routine in heat transfer studies, capable of taking into account details which are ignored in the empirical engineering approaches. Nevertheless, the heat transfer in GHEs is normally transient three-dimensional (3-D) problems, dealing with complicated domains and long durations of tens of years. It takes too substantial computing time to conduct such simulations for practical engineering applications. A notable example of such simulations was carried out by Mei and Baxter (1986), in which the geometry inside the borehole was simplified and, then, a 2-D model in the cylindrical coordinates was solved by the finite difference method. On the other hand, Yavuzturk *et al.* (1999) analysed the transient 2-D heat transfer by the finite element method in the cross-section perpendicular to the borehole axis, ignoring the heat flow along the borehole depth. While having provided important understandings on GHE heat transfer, these studies of numerical simulation have not yet been suitable to design and/or energy analysis of full scale engineering projects.

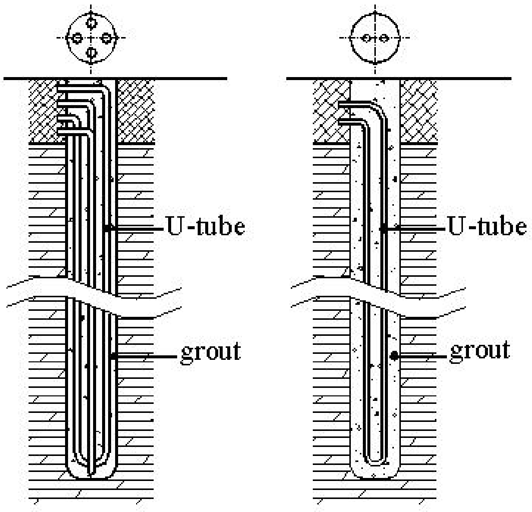
Because of all the complications of this problem and its long time scale, the heat transfer process may usually be analysed in two separated regions. One is the solid soil/rock outside the borehole, where the heat transfer has to be treated as a transient process. The region inside the borehole may also be isolated for analysis, including the backfilling, the U-tube and circulating fluid inside the pipes. Compared with the infinite ground outside it, both the dimensional scale and thermal easily

mass of the borehole are much smaller. Thus, it is a common practice that the heat transfer in this region is approximated as a steady-state process. Such simplification has been proved appropriate for most engineering practices except for analysis dealing with dynamic responses within a few hours.

The concept of thermal resistances and the principle of superimposition have been used in a new approach for GHE analysis initiated by Eskilson (1987). Here the temperature response and, then, the resistance of a single borehole experiencing a constant heating rate are cited repeatedly to obtain the actual GHE transient performance. It is more adequate and accurate than the empirical approaches and yet much more convenient for computations than the numerical simulations. In this regard, better understanding of every thermal resistances of the single-borehole GHE is crucial, and their analytical solutions are especially preferred to facilitate the computation. This paper summarizes the authors' work in this direction in recent years. Three important analytical solutions have been derived for heat transfer processes both inside and outside the boreholes, which can be easily incorporated into computer programs for thermal analysis and engineering design of the GHEs while providing better insight into influences of various factors on the GHE performance.

2. HEAT TRANSFER INSIDE BOREHOLES

The thermal resistance inside the borehole bears strong impact on GHE performance, defined by the thermal properties of the grouting materials and the arrangement of flow channels of the borehole. The main objective of this analysis is to determine the entering and leaving temperatures of the circulating fluid in the exchanger according to the borehole wall temperature and its heat flow.



a) Double U-tube b) Single U-tube

Fig.1 Schematic diagram of boreholes in the vertical GHE exchanger

A few models of varying complexity have been established to describe the heat transfer inside the GHE boreholes. Models for practical engineering designs are often oversimplified in dealing with the complicated geometry inside the boreholes. One-dimensional models have been recommended by Bose *et al.* (1985) and Caneta Research Inc. (1995) for engineering design, conceiving the U-tube pipes as a single "equivalent" pipe. Such an approach seems to be inadequate and unsatisfactory in spite of its simplicity. By a different approach Hellstrom (1991) has derived analytical 2-D solutions of the thermal resistances among pipes in the cross-section perpendicular to the borehole axis, which are superior to empirical expressions. On assumptions of identical temperatures and heat fluxes of all the pipes in it the borehole resistance can then be worked out. However, the fluid circulating through different legs of the U-tubes is, in fact, of varying temperatures. As a result, thermal interference, or thermal "short-circuiting", among U-tube legs is inevitable, which degrades the effective heat transfer in the GHEs. With the assumption of identical temperature of all the pipes, it is impossible for these models to reveal impact of this thermal interference on GHE performances.

Taking the fluid axial convective heat transfer and thermal "short-circuiting" among U-tube legs into account, a quasi-3-D model for boreholes in GHEs has been established, and analytical solutions of the fluid temperature profiles along the borehole depth have been obtained for both single and double U-tube boreholes (Zeng and Fang 2002 and Zeng *et al.* 2003).

The schematic diagram of a borehole with U-tubes in vertical GHEs is illustrated in Figure 1. In the case of a single U-tube in the borehole the energy equilibrium equations can be written for up-flow and down-flow of the circulating fluid. That is

$$\left.\begin{aligned} -Mc\frac{dT_{f1}}{dz} &= \frac{(T_{f1}-T_b)}{R_1^\Delta} + \frac{(T_{f1}-T_{f2})}{R_{12}^\Delta} \\ Mc\frac{dT_{f2}}{dz} &= \frac{(T_{f2}-T_b)}{R_2^\Delta} + \frac{(T_{f2}-T_{f1})}{R_{12}^\Delta} \end{aligned}\right\} \quad (0 \le z \le H) \tag{1}$$

where M is the flow rate of the circulating fluid and c its specific heat, and R_1^Δ and R_2^Δ are defined as $R_1^\Delta = \dfrac{R_{11}R_{22}-R_{12}^2}{R_{22}-R_{12}}$, $R_2^\Delta = \dfrac{R_{11}R_{22}-R_{12}^2}{R_{11}-R_{12}}$, $R_{12}^\Delta = \dfrac{R_{11}R_{22}-R_{12}^2}{R_{12}}$. R_{11} and R_{22} are thermal resistances between the pipe and borehole wall, R_{12} is the resistance between the two pipes, following Hellstrom's (1991) approach and denotation.

The full solution of this problem can be found elsewhere (Zeng and Fang, 2002). At the instance of symmetric disposal of the U-tube inside the borehole, the dimensionless temperature profiles in the two pipes are reduced as

$$\left.\begin{aligned}\Theta_1(Z) &= \cosh(\beta Z) - \frac{\sinh(\beta Z)}{\sqrt{1-P^2}}\left[1 - P\,\frac{\cosh(\beta) - \sqrt{\dfrac{1-P}{1+P}}\sinh(\beta Z)}{\cosh(\beta) + \sqrt{\dfrac{1-P}{1+P}}\sinh(\beta)}\right] \\ \Theta_2(Z) &= \frac{\cosh(\beta) - \sqrt{\dfrac{1-P}{1+P}}\sinh(\beta)}{\cosh(\beta) + \sqrt{\dfrac{1-P}{1+P}}\sinh(\beta)}\cosh(\beta Z) + \frac{\sinh(\beta Z)}{\sqrt{1-P^2}}\left[\frac{\cosh(\beta) - \sqrt{\dfrac{1-P}{1+P}}\sinh(\beta)}{\cosh(\beta) + \sqrt{\dfrac{1-P}{1+P}}\sinh(\beta)} - P\right]\end{aligned}\right\} \quad (2)$$

where the dimensionless parameters are defined as

$\Theta = \dfrac{T_f(z) - T_b}{T'_f - T_b}$, $Z = z/H$, $P = R_{12}/R_{11}$, and

$\beta = \dfrac{H}{Mc\sqrt{(R_{11}+R_{12})(R_{11}-R_{12})}}$.

Typical dimensionless temperature profiles of the single U-tube pipes are plotted in Figure 2. It is noticed that the temperature profiles in the two illustrated examples are of different features due to difference in their pipe-to-pipe resistance, or the dimensionless parameter P, which, of course, results in different exiting temperatures with other conditions unchanged. This demonstrates clearly the impact of thermal interference between the U-tube pipes.

Differing from the single U-tube boreholes, the double U-tube boreholes may have different flow configurations. The two sets of U-tubes may be arranged in parallel or series flow circuits, and each of them includes a few connecting patterns. The fluid temperature profiles in the flow channels and, then, the borehole resistance are affected by borehole configurations. All these options have been analysed separately. Although the temperature profiles for the double U-tube boreholes are even more complicated, these explicit solutions can be computed readily on a computer. Fortunately, the temperature profiles for double U-tubes in parallel are similar to those of the single U-tube configuration. Detailed results about double U-tube borehole heat transfer can be found elsewhere (Zeng *et al.*, 2003).

The concept of effective thermal resistance facilitates heat transfer analysis. The effective borehole thermal resistance defines the proportional relationship between the heat flow rate transferred by the borehole and the temperature difference between the circulating fluid and the borehole wall. It can be determined explicitly according to the analytical solutions of fluid temperature profiles in the borehole. For the single U-tube borehole and double U-tube in series the borehole thermal resistance can be obtained as

$$R_b = \frac{H}{2Mc}\cdot\frac{1+\Theta''_f}{1-\Theta''_f} \quad (3)$$

The expressions of the borehole thermal resistance presented here take into account more factors than previous models ever did before, including the geometrical parameters (borehole and pipe sizes and pipe disposal in the borehole) and physical parameters (thermal conductivity of the materials, flow rate and fluid properties) as well as the flow circuit configurations. The solutions have provided a reliable tool for GHE sizing and performance simulation and a solid basis for technical and economic assessment of different borehole configurations.

3. HEAT CONDUCTION OUTSIDE BOREHOLES

In GHEs the borehole diameter is much smaller compared to their depth, and the ground can be treated as a semi-infinite medium. Then, the one-dimensional (1-D) line-source model in infinite medium is often used in GHE analysis (Bose *et al.*, 1985), often referred to as Kelvin's model. While being simple, this model is inadequate for analysis of long-term performance of the GHEs. In many

GCHP applications the heating loads are not in balance with the cooling loads in a year round basis. In case that the heat injected in summer cannot be extracted in winter, the redundant heat will accumulate in the ground and thus lead to increase in the annual mean temperature in adjacent soil. With the effect of the heat transfer on the ground surface taken into account, the influence of the imbalanced heat will approach a relatively steady state after the ground heat exchanger operates for a long enough period. This process normally takes ten years or even more, depending mainly on the depth of the boreholes. The variation in the annual mean temperature of the ambient soil around the GHE will affect its long-term behaviour; and thus it must be taken into account when the ground loop is designed. The transient heat conduction around boreholes of the GHEs is analysed here in a 2-D model. An analytical solution of the transient temperature response has been derived in a semi-infinite medium with a line-source of finite length. This solution is suitable for sizing the ground loop of GCHP systems because it describes the GHE performance more adequately than the 1-D model does with an infinite line-source.

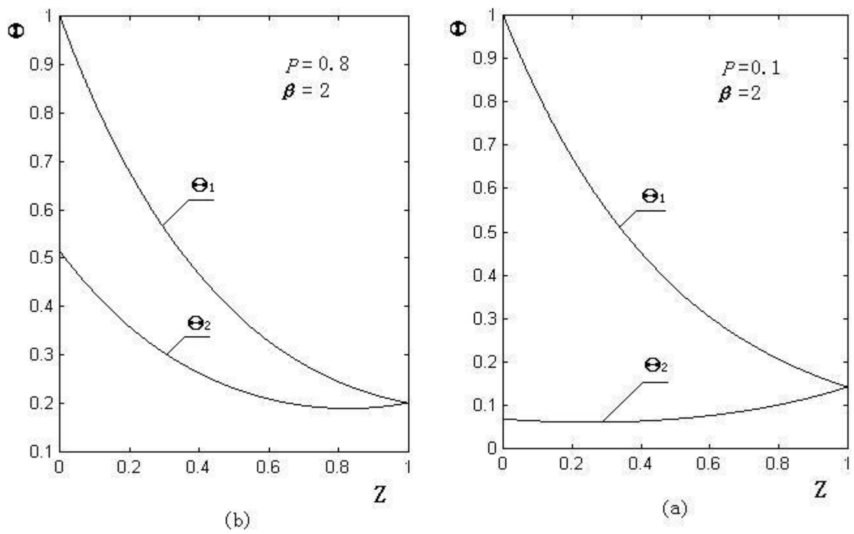


Fig. 2: Fluid temperature profiles along the borehole depth

The 2-D model assumes that the ground is regarded as a homogeneous semi-infinite medium with a uniform initial temperature, and that a line-source stretching vertically from the boundary to a certain depth, *H*, releases heat at a constant rate per length, q_l. Due to the central symmetry of the problem the temperature distribution is two-dimensional in the cylindrical coordinates. The solution of the temperature excess has been obtained by Zeng *et al.* (2002) as

$$\theta(r,z,\tau)=\frac{q_l}{4k\pi}\int_0^H\left\{\frac{\operatorname{erfc}\dfrac{\sqrt{r^2+(z-h)^2}}{2\sqrt{a\tau}}}{\sqrt{r^2+(z-h)^2}}-\frac{\operatorname{erfc}\dfrac{\sqrt{r^2+(z+h)^2}}{2\sqrt{a\tau}}}{\sqrt{r^2+(z+h)^2}}\right\}dh \quad (4)$$

where *k* and *a* denote thermal conductivity and diffusivity of the ground, respectively.

In the design of the GHEs in the GCHP system, special attention is paid to the borehole wall temperature rise. In engineering applications a common practice is to use the temperature at the middle of the borehole wall as its representative temperature. The temperature responses obtained with equation (4) of the 2-D model are plotted in Figure 3 together with that of the Kelvin's 1-D model for comparison. It can be seen in Figure 3 that according to the 2-D model the temperature excess rises rapidly at the early period of heating and, then, turns to a rather gentle increase. Finally the temperature reaches a steady state when the time approaches infinity. In contrast, the temperature response indicated by the 1-D model keeps rising even when time approaches infinity.

Similarly, equation (4) defines the thermal resistance of the soil outside the borehole, and can be used in thermal analysis and design of the GHEs.

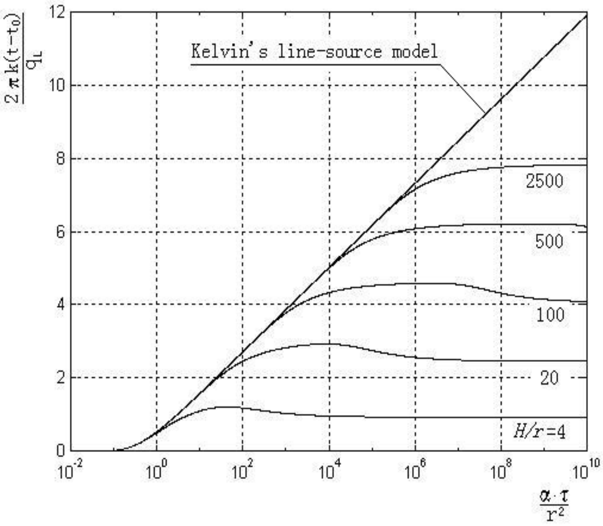


Fig. 3: The temperature responses of a line source in 1-D and 2-D models

4. GROUND HEAT EXCHANGERS WITH GROUNDWATER ADVECTION

Researchers and engineers in the GCHP arena realize that groundwater filtration may exert impact, large or small, on performance of GHEs. Few studies on this problem, however, can be seen in literature except some qualitative discussions. All of the GHE design tools, therefore, are based simply on principles of heat conduction, and do not consider the implications of groundwater flow in carrying away heat. The reasons for such simplification are the difficulties encountered both in modelling and computing the convective heat transfer and in learning about the actual groundwater flow in engineering practice. In general, groundwater flow is beneficial to the thermal performance of GHEs. A moderate groundwater advection is expected to make notable difference in alleviating the possible heat build-up around the borehole over time. As a result, it is desirable to consider the groundwater flow in the heat transfer analysis to avoid over-sizing of the GHEs.

Among the few reports on the effect of the groundwater flow found in literature Eskilson (1987) discussed the problem based on a steady-state analytical solution. Chiasson *et al.* (2000) made a preliminary investigation of the effects of groundwater flow by means of a numerical finite element scheme. No correlations among the influencing parameters were obtained due to the discrete nature of the approach. The authors (Diao *et al.*, 2003) have solved the combined heat transfer of conduction and advection in the vertical GHEs by an analytical approach, and an explicit expression of the temperature response has been derived describing correlation among various factors, which impact on this process.

In this study the ground around the boreholes is assumed to be a homogeneous porous medium saturated by groundwater. Heat is transported through the saturated porous medium in a combined mechanism: by conduction through its solid matrix and liquid in its pores as well as by convection of the moving liquid. Following the

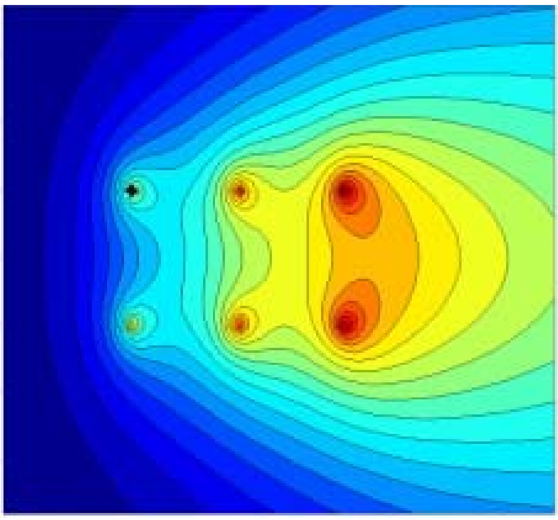


Fig. 4 Isotherms of a GHE field of six boreholes with groundwater advection

Eskilson's (1987) model, a further approximation is accepted that the groundwater velocity is uniform in the whole domain concerned and parallel to the ground surface. The term "advection" is often used to describe such a flow.

Define that the velocity u is in the direction of the x-coordinate. Then, on the assumption of constant thermal properties the governing equation of the heat transfer can be written as

$$\frac{\partial t}{\partial \tau}+U\frac{\partial t}{\partial x}=a\nabla^2 t \qquad (5)$$

Where $U=u\rho_w c_w/(\rho c)$, and the effective thermal diffusivity $a=k/(\rho c)$.

This problem has been solved under following assumptions:

1. The ground is regarded as homogeneous and semi-infinite medium;
2. The heat transfer along the borehole axis is neglected. Then, the problem may be simplified as two-dimensional;
3. The borehole is approximated by a line heat source;
4. The medium has a uniform initial temperature, t0;
5. The heating rate per length of the source, q_l, is constant since a starting instant, $\tau = 0$.

The authors have derived the solution on transient conditions. When a single line-source is deployed at the origin of the coordinates, the temperature rise in the medium, $\theta = t - t_0$, of the discussed case can be written as

$$\theta(x,y,\tau)=\frac{q_l}{4\pi k}\exp\left(\frac{Ux}{2a}\right)\cdot \int_0^{\frac{r^2}{4a\tau}}\frac{1}{\eta}\exp\left[-\frac{1}{\eta}-\frac{U^2r^2\eta}{16a^2}\right]\mathrm{d}\eta \qquad (6)$$

where $r=\sqrt{x^2+y^2}$.

Equation (6) reduces to the solution of the steady condition while the time approaches infinity, that is

$$\theta_s(x,y)=\frac{q_l}{2\pi k}\exp\left(\frac{Ux}{2a}\right)\mathrm{K}_0\left[\frac{Ur}{2a}\right] \qquad (7)$$

where $K_0(z)$ is the modified Bessel function of the second kind of order zero.

On the other hand, for the specific instance when the advection velocity u, and then, $U=u\rho_w c_w/(\rho c)$, equals zero, Equation (6) reduces to the common conductive temperature response of the line-source in an infinite medium

$$\theta_0(r,\tau)=-\frac{q_l}{4\pi k}\mathrm{Ei}\left(-\frac{r^2}{4a\tau}\right) \qquad (8)$$

where Ei(z) is the exponential integral function. As this expression indicates, the temperature in the medium will keep rising and no steady condition will be reached when the time lasts to infinity.

The obtained temperature response to a single line-source heating with water advection considered can be used to compute the response of a GHE with multiple boreholes by superimposition of all temperature rises caused by individual boreholes. Figure 4 demonstrates such isotherms of a six-borehole GHE with groundwater advection. It has been shown that such simulations take much less computing time and assure more reliable precision than numerical solutions do with the finite element or finite difference methods. The influence of the groundwater advection on GHE performance is discussed in more details elsewhere (Diao *et al.*, 2004).

5. CONCLUSION

Describing key processes in GHE heat transfer, analytical expressions play an important role in engineering application of the GCHP technology. This paper presents three analytical solutions derived from more sophisticated models, *i.e.* the quasi-3-D model for heat transfer inside the borehole, the 2-D model for heat conduction of a line-source of limited length and the combined conduction/advection model considering the groundwater flow in porous media. These explicit expressions are helpful to improve design procedures recommended by available literature.

The models mentioned above deal with the temperature response of a single borehole to a step heating. Variation in load and on-off cycling of the

GHEs can be considered by superimposition of a series of heating pulses (Fang *et al*., 2002). For a borehole field of varying configurations its temperature response and thermal resistance can be determined also by the superimposition principle. According to these improved models computer software has been developed for engineering design and performance simulation of the vertical GHEs, and has been proven appropriate for engineering applications (Yu *et al*., 2002).

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