Decoupled Modeling of Chilled Water Cooling Coils Using a Finite Element Method

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Abstract

Chilled water cooling coils are important components in air handling unit systems. Generally the cooling coil removes both moisture and sensible heat from entering air. Since the sensible and latent heat transfer modes are coupled and the saturation humidity ratio vs. temperature curve on the psychrometric chart is non-linear, it is very difficult to solve cooling coil heat transfer differential equations across the entire coil. However, the sensible and latent heat transfer modes can be decoupled using a constant sensible heat ratio (SHR) and the saturation humidity ratio vs. temperature curve can be treated as linear in a small area corresponding to a finite element of the coil. This paper presents the decoupled cooling coil model using the finite element method with an application to the simulation of a particular cooling coil case.

Introduction

Modeling of chilled water coils plays an important role in analyzing air handler operation as well as in fault detection and diagnosis. The cooling coil performance can be simulated using either a steady state model or a dynamic model for different applications. A steady state model is sufficient to simulate the cooling coil thermal performance in most circumstances (Chow 1997).

Basic steady state heat and mass transfer differential equations or governing equations are used in steady state cooling coil models. The transfer of heat and mass in a cooling coil includes air side sensible and latent heat transfer (from moist air to coil surface) and water side heat transfer (from coil surface to chilled water). The governing equations were discussed in detailed by Mirth and Ramadhyani (1993) and by Khan (1994). Since the sensible and latent heat transfer modes are coupled and the saturation humidity ratio vs. temperature curve on the psychrometric chart is non-linear, it is very difficult to solve these differential equations along the entire coil. An analytical model was developed and a numerical method was used to obtain solutions of these governing equations.

The simple analytical models use the heat exchanger analogy method, which is suitable for coils with only sensible heat transfer. The heat exchanger analogy method is well known as documented by Incropera and DeWitt (2002) with the detailed solution for coils with only sensible heat transfer given by ASHRAE (2000). However, some assumptions must be applied to simplify these coupled heat transfer differential equations to match the standard format in the heat exchanger analogy method. Mirth and Ramadhyani (1994) compared several simple analytical models.

Elmahdy and Mitalas (1977) developed a single-potential model that uses an enthalpy difference as the sole driving force to calculate the total heat transfer. This model assumes that the slope of the enthalpy-saturation temperature curve is constant along the entire coil and the Lewis number is unity. The single potential model is recommended by ASHRAE (ASHRAE 2000, Bourdouxhe et al. 1998).

McQuiston (1975 and 1978) developed a dual-potential model that uses temperature difference which drives the sensible heat transfer and humidity ratio difference which drives the latent heat transfer. This model assumes that the entire cooling process line is a straight line on the psychrometric chart corresponding to the SHR being constant along the entire coil. The SHR is used to decouple the sensible and latent heat along the entire coil in this model.

It is well known that a cooling coil may operate at partially wet conditions, being dry at the inlet and wet at the outlet of the coil. The SHR then varies from unity to a much smaller value as the air passes through the coil. On the other hand, the saturation specific heat varies with the wet bulb temperature due to the nonlinear enthalpy-saturation temperature curve on the psychrometric chart (ASHRAE 2001). Figure 1 shows the saturation specific heat versus the wet bulb temperature. The saturation specific heat varies from 0.5 to 1.1 Btu/lb-F in a wet bulb temperature range between 40ºF and 80ºF.

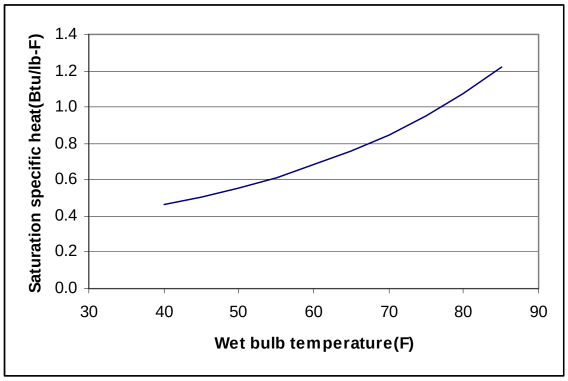

Figure 1: Saturation specific heat of moist air

The numerical method does not use these assumptions. Khan (1994) used a numerical model to analyze the cooling coil performance at partial load conditions. A cooling coil is divided into a number of control volumes and the differential equations are converted into finite-difference equations in each control volume. Since these finite difference equations are deduced directly from the complicated governing equations rather than using the existing heat exchanger analogy method, the simulation process is complicated.

In this paper, a numerical cooling coil model is developed using the existing heat exchanger analogy theory. First the basic cooling coil heat transfer differential equations are given. Then the sensible and latent heat transfer modes in a cooling coil are decoupled assuming a constant value of SHR and slope of the saturation humidity ratio vs. temperature curve within each element. Now both the sensible and the latent heat transfer modes have a standard format in the heat exchanger analogy method and the SHR value controls the element conditions, whether completely wet, completely dry or partially wet. Finally the element equations are deduced from the decoupled differential equations for the sensible heat transfer and latent heat transfer separately. Since both the SHR and the curve slope are determined by the unknown conditions of the air, coil surface and chilled water, an iterative method must be used. It first assumes SHR and curve slope values and then determines the air, coil and chilled water conditions. The SHR and curve slope values are then changed to values corresponding to the conditions calculated and the calculation is repeated until the values converge for all elements.

Modeling

<u>Cooling coil governing equations</u>

Figure 2(a) illustrates a schematic of a cooling coil with fins. The coil can be simplified as a counterflow heat exchanger, as shown in Figure 2(b). Sensible and latent heat transfer occurs between the moist air and the unfinned tube surface on the air side and heat transfer occurs between the unfinned tube surface and the chilled water.

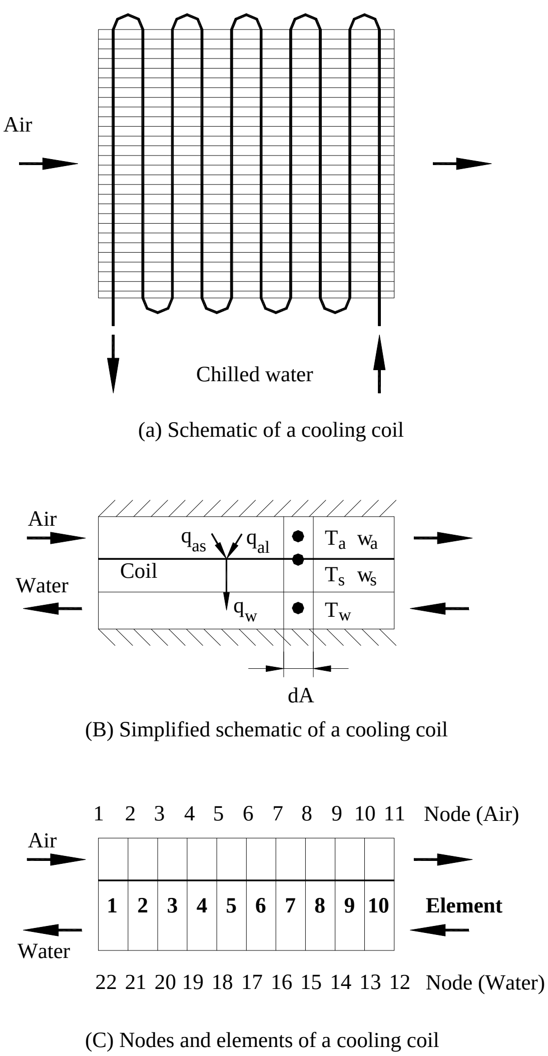

Figure 2: Cooling coil model

Dry air and water vapor are treated as a mixture of ideal gases and the heat and mass transfer coefficients are treated as constants in the derivation of the governing equations.

The air loses sensible heat when in contact with a surface cooler than the air. The sensible heat transfer in a differential surface area dA can be expressed as:

$$dq_{as} = -m_a c_{pa} dt_a = U_a \cdot (t_a - t_s) dA \qquad (1)$$

where m_a is the dry air mass flow rate, C_{pa} is the humid air specific heat, and U_a is the overall sensible heat transfer coefficient between the moist air and the coil surface based on the coil surface area.

The removal of latent heat through condensation occurs only on the portion of the coil where the surface temperature is lower than the dew point of the air passing over it. The latent heat transfer on dA can be expressed as:

$$dq_{al} = -m_a h_g dw_a = K_M \cdot h_g \cdot (w_a - w_s) dA \qquad (2)$$

where h_g is the latent heat of vaporization and K_M is the overall mass transfer coefficient between the moist air and the coil surface based on the coil surface area.

The saturation humidity ratio at the coil surface is a function of the surface temperature based on the saturation humidity ratio vs. temperature curve, given by ASHRAE (2001).

$$w_s = w_s(t_s) \qquad (3)$$

The water side heat gain is expressed as:

$$dq_w = m_w c_{pw} dt_w = U_w (t_s - t_w) dA \qquad (4)$$

where m_w is the chilled water mass flow rate, C_{pw} is the chilled water specific heat and U_w is the overall heat transfer coefficient between the coil surface and the chilled water based on the coil surface area.

Finally the water side heat gain should be balanced with the total heat transfer, the sum of sensible heat and latent heat transfer on the air side if the enthalpy of condensate removed is neglected.

$$dq_w = dq_{as} + dq_{al} \qquad (5)$$

Equations (1) to (5) are basic differential equations, which completely describe the cooling coil heat and mass transfer processes. The five unknown variables, air drybulb temperature (t_a), air humidity ratio (w_a), coil surface temperature (t_s), coil surface humidity ratio (w_s) and the water temperature (t_w) can be obtained by solving the five equations above.

<u>Simplification for a finite element</u>

Unfortunately, it is very hard to solve those differential equations across the entire coil due to the coupled and nonlinear heat and mass transfer processes. However, for a small piece of the cooling coil, it is reasonable to assume that the sensible heat ratio is constant. As a result, the heat transfer and mass transfer can be decoupled by using a constant sensible heat ratio. Then

$$dq_w = a_e \cdot dq_{as} = b_e \cdot dq_{al} \qquad (6)$$

where:

$$SHR_e = \frac{q_{as,e}}{q_{w,e}} = \frac{c_{pa}(t_{a,e} - t_{a,l})}{h_g(w_{a,e} - w_{a,l}) + c_{pa}(t_{a,e} - t_{a,l})}$$

$$a_e = \frac{1}{SHR_e}$$

$$b_e = \frac{1}{1 - SHR_e}$$

The decoupled sensible heat transfer equation can be obtained by substituting Eq. (1) into Eq. (6).

$$dq_w = a_e dq_{as} = -(a_e \cdot m_a) c_{pa} dt_a = (a_e \cdot U_a)(t_a - t_s) dA \qquad (7)$$

The decoupled sensible heat transfer equivalent mass flow rates and heat transfer coefficients can be defined as:

$$m'_a = a_e \cdot m_a \qquad (8a)$$

$$U'_a = a_e \cdot U_a \qquad (8b)$$

$$m'_w = m_w \qquad (8c)$$

$$U'_w = U_w \qquad (8d)$$

The decoupled sensible heat transfer differential equations are deduced from Eqs. (4) and (7).

$$dq_w = -m'_a c_{pa} dt_a = U'_a (t_a - t_s) dA \qquad (9)$$

$$dq_w = m'_w c_{pw} dt_w = U'_w (t_s - t_w) dA \qquad (10)$$

Decoupled differential equations (8) and (9) describe the sensible heat transfer process in a cooling coil driven by the temperature difference with an assumed constant SHR. Since these equations have the standard format of the heat exchanger analogy theory, the element equations can be easily obtained using the heat exchanger analogy method (see Appendix).

Similarly the decoupled latent heat transfer differential equations on the air side can be obtained by substituting Eq. (2) into Eq. (6).

$$dq_w = -(b_e \cdot m_a \frac{h_g}{c_{pa}}) c_{pa} dw_a$$
$$= (b_e \cdot K_M h_g)(w_a - w_s) dA \qquad (11)$$

However, the driving force on the air side of Eq. (11) is the humidity ratio and the driving force on the water side in Eq. (4) is the temperature. In order to convert the temperature to the humidity ratio, the saturation humidity ratio vs. temperature curve, which is described by Eq. (3), should be used.

Along a small portion of the coil, the saturation humidity ratio vs. temperature curve can be simplified as a straight line. This straight line passes through the coil surface entering and leaving points. The slope of the straight line can be expressed as:

$$\tan\alpha_e = \frac{w_{s,e} - w_{s,l}}{t_{s,e} - t_{s,l}} = \frac{w - w_{s,e}}{t - t_{s,e}} \qquad (12)$$

where $w_{s,e}$ and $w_{s,l}$ are determined by $t_{s,e}$ and $t_{s,l}$ using Eq. (3). The coil surface humidity ratio is a linear function of the surface temperature.

$$w_a = (t_s - t_{s,e}) \tan\alpha_e + w_{s,e} \qquad (13)$$

Equation (12) is also used to convert the chilled water temperature to the fictitious chilled water humidity ratio.

$$w_w = (t_w - t_{s,e}) \tan\alpha_e + w_{s,e} \qquad (14)$$

The water side heat transfer can be rewritten by substituting Eqs. (13) and (14) into Eq. (4).

$$dq_w = (m_w \frac{1}{\tan\alpha_e}) \cdot c_{pw} dw_w$$
$$= (U_w \frac{1}{\tan\alpha_e}) \cdot (w_s - w_w) dA \qquad (15)$$

The equivalent decoupled latent heat transfer mass flow rates and heat transfer coefficients can be defined as:

$$m_a'' = b_e \cdot m_a \frac{h_g}{c_{pa}} \qquad (16a)$$

$$U_a'' = b_e \cdot K_M h_g \qquad (16b)$$

$$m_w'' = m_w \frac{1}{\tan\alpha_e} \qquad (16c)$$

$$U_w'' = U_w \frac{1}{\tan\alpha_e} \qquad (16d)$$

The decoupled latent heat transfer differential equations are deduced from Eqs. (11) and (15).

$$dq_w = -m_a'' c_{pa} dw_a = U_a''(w_a - w_s) dA \qquad (17)$$
$$dq_w = m_w'' c_{pw} dw_w = U_w''(w_s - w_w) dA \qquad (18)$$

Decoupled differential equations (17) and (18) describe the latent heat transfer process in a cooling coil driven by humidity ratio difference with two parameters assumed constant: SHR and the slope of the saturation humidity ratio vs. temperature curve. Similarly, the element equations can be easily obtained using the heat exchanger analogy method (see Appendix).

Simulation Procedure

Heat transfer area and heat transfer coefficients on both the air side and water side are given in a cooling coil simulation. The leaving water temperature and the leaving air dry bulb temperature and humidity ratio need to be simulated based on the given supply water temperature and mass flow rate, and the supply air temperature, humidity ratio and mass flow rate.

The cooling coil performance can be easily simulated for a given sensible heat ratio and coil surface saturation humidity ratio vs. temperature slope using the finite element method developed and shown in the Appendix. However, the sensible heat ratio and coil surface saturation humidity ratio slope are unknown parameters before simulation. So the trial and error method has to be used in simulation. The simulation should use the following procedure.

- The cooling coil is discretized into a number of elements, each with four nodes, as shown in Figure 1c, where ten elements are used.
- The initial SHR and coil surface saturation humidity ratio vs. temperature curve slope

values can be assumed for all finite elements in the first trial.

- The sensible and latent heat transfer modes are decoupled using Eqs. (8) and (9).
- The node temperature and humidity ratio on both the air side and water side are calculated with these initial SHRs and curve slopes using the method in the Appendix.
- The SHR of each element can be updated based on calculated air and water node conditions along the entire coil using Eq. (6).
- The coil surface temperature at each node can be calculated using the SHR as well as air and chilled water temperatures at each node. Finally the coil surface saturation humidity ratio vs. temperature curve slope at each element is updated based on the calculated coil surface temperatures at both the entering node and the leaving node of each element using Eq. (12).
- The simulation is repeated until all SHR and slope changes are sufficiently small.

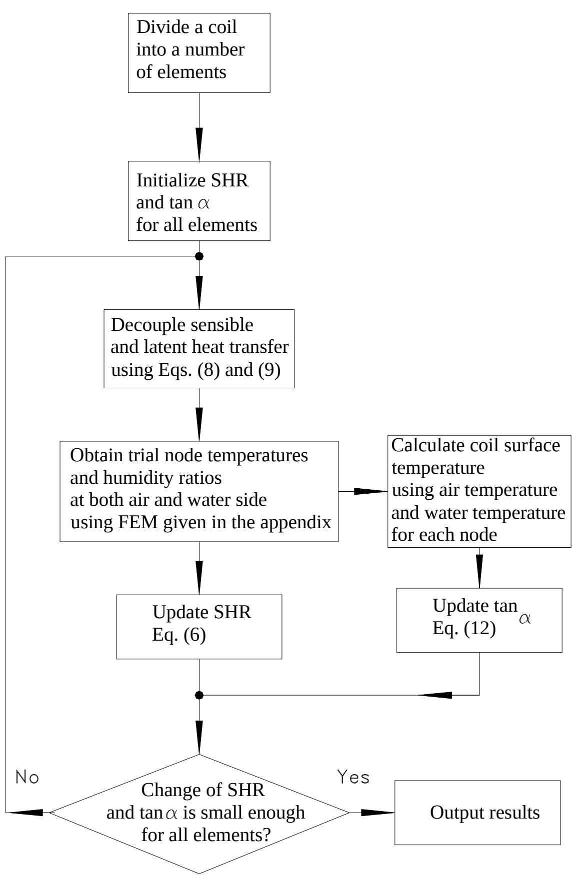

Figure 3: Flow chart of decoupled cooling coil model

Figure 3 presents the flow chart of the trial and error process.

Application and Results

The simulation is done on a cooling coil with a UA value of 12,000 Btu/h-F on the air side and a UA value of 60,000 Btu/h-F on the water side. The air flow rate is 4,500 CFM, which is less than the design airflow of 6,000 CFM, and the water flow rate is 20 GPM. The chilled water supply temperature is 42ºF and the supply air temperature is 86ºF with a humidity ratio of 0.0121 lb/lb. The mass transfer coefficient is calculated with the Lewis number of 1 and the coil is a counter flow configuration. The entire coil is separated into ten elements in the simulation, as shown in Figure 1c.

Figure 4 shows the simulated cooling process with node temperatures and humidity ratios in the coil on the psychrometric chart. The process is described by the air, chilled water and coil surface conditions. The coil surface temperature at the entering air node is higher than the entering air dew point due to the partial load, so the coil surface is dry at this node. It can be seen that the air humidity ratio does not change in this area.

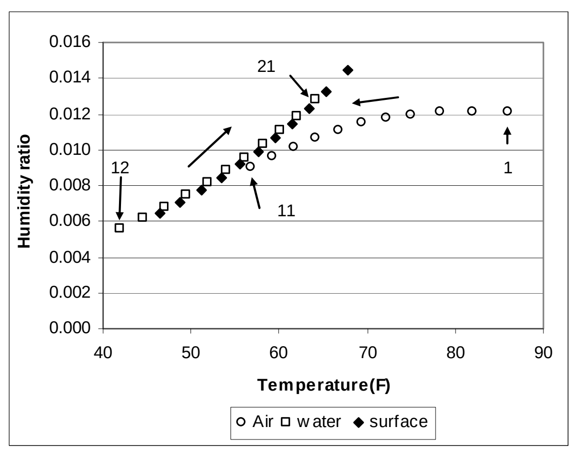

Figure 4: Cooling process in a coil using node conditions

The decoupled sensible heat transfer is shown using temperatures in Figure 5 and the decoupled latent heat transfer is shown using humidity ratio in Figure 6. Since the SHR value varies with location, the temperature and humidity ratio distributions are different. At the air entrance, the cooling coil operates as a dry coil, so the air temperature changes rapidly but the humidity ratio does not change.

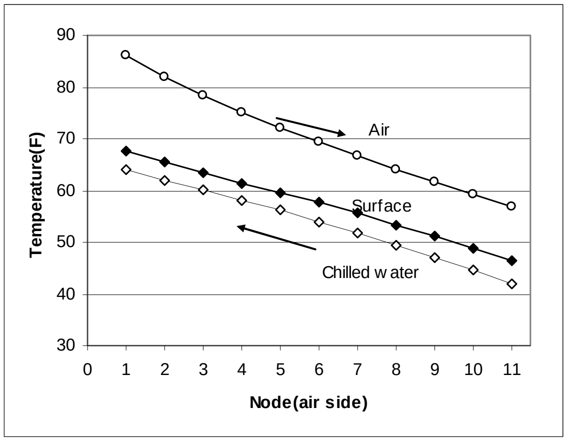

Figure 5: Temperature change due to sensible heat transfer

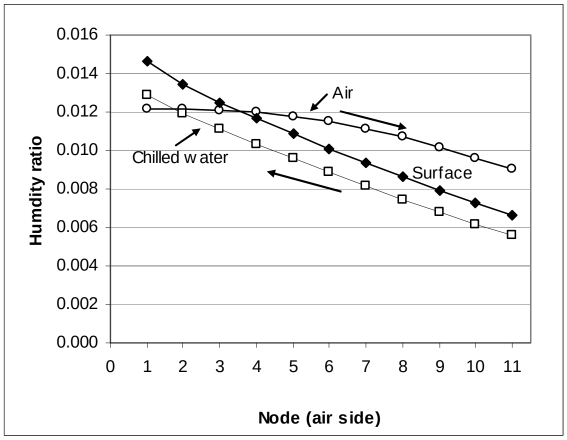

Figure 6: Humidity ratio change due to latent heat transfer

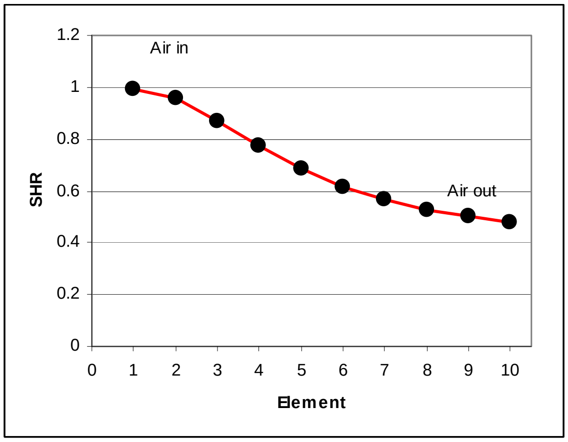

Figure 7: SHR distribution over the coil

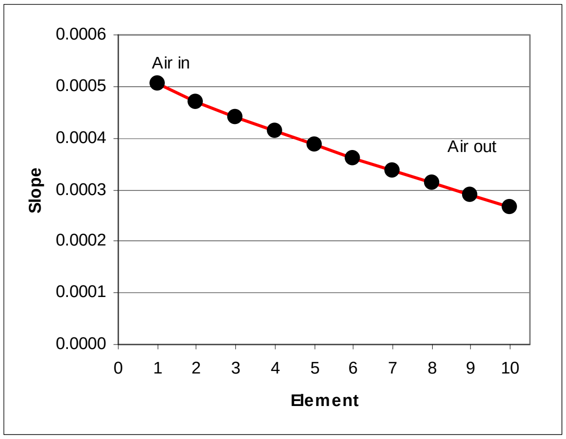

Figure 8: Slope change over the coil

Figure 7 shows the SHR distribution over the entire coil and Figure 8 shows the slope of the saturation humidity ratio vs. temperature curve over the entire coil. It can be seen that the actual variable SHR and slope of the coil surface saturation humidity ratio vs. temperature curve are considered in the model. The simulation result will be close to the actual cooling process.

Conclusions

A decoupled cooling coil model has been developed for the finite element method using the element sensible heat ratio. The element equation for both sensible and latent heat transfer has a standard format for all coil conditions. The variable element SHR determines the coil condition, whether it is completely dry, completely wet or partially wet. The actual SHR and coil surface saturation temperature curve are considered in the model, making the model more accurate than models that assume constant SHR across the coil.

The decoupled chilled water cooling coil model can also be used to develop the simulation engine for fault detection and diagnosis in air handling units. More detailed work on this topic will be done in the future.

Nomenclature

A - coil surface area, m^2

a - decoupled sensible heat transfer factor

b - decoupled latent heat transfer factor

c_p - constant pressure specific heat

h_g - specific enthalpy for saturated water vapor, J/kg

K_M - mass transfer coefficient, kg/m^2-s

m - mass flow rate, kg/s

q - heat transfer rate, W

t - temperature, ºC

SHR - sensible heat ratio

U - heat transfer coefficient, W/m^2-ºC

w - humidity ratio of moist air

Subscripts:

a - dry air for mass flow rate and humid air for specific heat

al - air latent heat

as - air sensible heat

e -element or entering

l -leaving

s -coil surface

w -chilled water

Superscripts**:**

' - decoupled equivalent sensible heat transfer

" - decoupled equivalent latent heat transfer

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Appendix: Finite element method based on effectiveness-NTU method

1. Differential equations for heat exchanger

Water side:

$$dq_w = m_w c_{pw} dt_w = U_w (t_s - t_w) dA \qquad \text{(A1)}$$

Air side

$$dq_a = -m_a c_{pa} dt_a = U_a (t_a - t_s) dA \qquad \text{(A2)}$$

Heat balance

$$dq_t = dq_w = dq_a \qquad \text{(A3)}$$

2. Effectiveness-NTU method

$$U = \frac{U_a \cdot U_w}{U_a + U_w} \qquad \text{(A4)}$$

$$C_a = m_a c_{pa} \qquad \text{(A5)}$$

$$C_w = m_w c_{pw} \qquad \text{(A6)}$$

$$C_{\min} = \min(C_a, C_w) \qquad \text{(A7)}$$

$$C_{\max} = \max(C_a, C_w) \qquad \text{(A8)}$$

$$C_r = \frac{C_{\min}}{C_{\max}} \qquad \text{(A9)}$$

$$NTU = \frac{UA}{C_{\min}} \quad \text{(A10)}$$

$$\varepsilon = f(NTU, C_r) \quad \text{(A11)}$$

$$q_t = \varepsilon C_{\min}(t_{a,i} - t_{w,i}) \quad \text{(A12)}$$

3. Element equations

Figure A1 shows the schematic of an element and the variable locations.

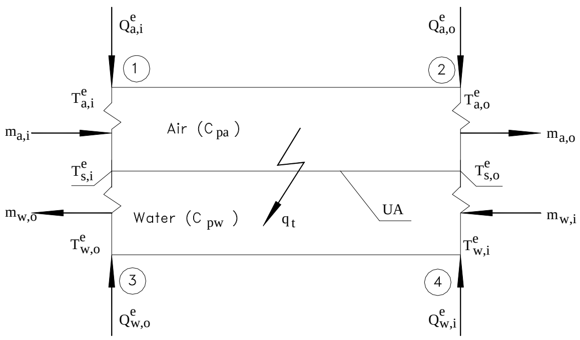

Figure A1: Schematic of an element

The node heat flux inside an element without mass loss ($m_{a,o} = m_{a,i} = m_a$ and $m_{w,o} = m_{w,i} = m_w$) on both sides can be expressed as:

$$Q^e_{a,i} = -m_a c_{pa} t^e_{a,i} \quad \text{(A13)}$$

$$Q^e_{a,o} = m_a c_{pa} t^e_{a,i} - \varepsilon C_{\min}(t^e_{a,i} - t^e_{w,i}) \quad \text{(A14)}$$

$$Q^e_{w,o} = m_w c_{pw} t^e_{w,i} - \varepsilon C_{\min}(t^e_{a,i} - t^e_{w,i}) \quad \text{(A15)}$$

$$Q^e_{w,i} = -m_w c_{pw} t^e_{w,i} \quad \text{(A16)}$$

The node heat flux inside an element with mass loss ($m_{a,o} \neq m_{a,i}$ or $m_{w,o} \neq m_{w,i}$) on either side can be expressed as the following equations, where the average mass flow rates are used to calculate the overall heat flux term ($\varepsilon C_{\min}$).

$$Q^e_{a,i} = -m_{a,i} c_{pa} t^e_{a,i} \quad \text{(A17)}$$

$$Q^e_{a,o} = m_{a,o} c_{pa} t^e_{a,i} - \varepsilon C_{\min}(t^e_{a,i} - t^e_{w,i}) \quad \text{(A18)}$$

$$Q^e_{w,o} = m_{w,o} c_{pw} t^e_{w,i} - \varepsilon C_{\min}(t^e_{a,i} - t^e_{w,i}) \quad \text{(A19)}$$

$$Q^e_{w,i} = -m_{w,i} c_{pw} t^e_{w,i} \quad \text{(A20)}$$

Generally the basic element equations can be written in matrix format.

$$\begin{bmatrix} Q^e_{a,i} \\ Q^e_{a,o} \\ Q^e_{w,o} \\ Q^e_{w,i} \end{bmatrix} = \begin{bmatrix} -m_{a,i}c_{pa} & 0 & 0 & 0 \\ m_{a,o}c_{pa} - \varepsilon C_{\min} & 0 & 0 & \varepsilon C_{\min} \\ \varepsilon C_{\min} & 0 & 0 & m_{w,o}c_{pw} - \varepsilon C_{\min} \\ 0 & 0 & 0 & -m_{w,i}c_{pw} \end{bmatrix} \cdot \begin{bmatrix} t^e_{a,i} \\ t^e_{a,o} \\ t^e_{w,o} \\ t^e_{w,i} \end{bmatrix} \quad \text{(A21)}$$

4. Integrate equations over the entire coil

The total heat flux to a node is contributed by all elements, which include this node. Finally the integrated node heat flux equation can be obtained using the superposition method.

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} \quad \text{(A22)}$$

5 Boundary conditions

Since no net heat flux flows into or out of the interior nodes, the heat flux can be set to zero. However, the entering and leaving nodes have different boundary conditions.

Entering nodes

For the water and air entering node, the temperatures ($t_{a,i}$ and $t_{w,i}$) are given.

It is assumed that the node number in the integrated equations is m. In order to force constant temperatures at this node, the following changes can be done on the integrated equation.

$$k_{mm} = \infty \quad \text{(A23)}$$

$$Q_m = \begin{cases} k_{mm} t_{a,i} & air \\ k_{mm} t_{w,i} & water \end{cases} \quad \text{(A24)}$$

Leaving nodes

For the water and air leaving node, heat balance is applied to determine the temperature. It is assumed that the node number in the integrated equation is n.

$$Q_n = \begin{cases} m_{a,o} c_{pa} t_n & air \\ m_{w,o} c_{pw} t_n & water \end{cases} \quad \text{(A25)}$$

Since the temperature is unknown, heat balance can be satisfied by changing the matrix and setting the heat flux to zero.

$$Q_n = 0 \quad \text{(A26)}$$

$$k_{nn} = \begin{cases} k_{nn} - m_{a,o} c_{pa} & air \\ k_{nn} - m_{w,o} c_{pw} & water \end{cases} \quad \text{(A27)}$$