

ROBOTIC LOCALIZATION OF HOSTILE NETWORKED RADIO SOURCES
USING A DIRECTIONAL ANTENNA

A Thesis
by
QIANG HU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

December 2005

Major Subject: Electrical Engineering

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Approved by:

Co-Chairs of Committee,	Dezhen Song Deepa Kundur
Committee Members,	Scott Miller Jim Ji
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ABSTRACT

Robotic Localization of Hostile Networked Radio Sources

Using a Directional Antenna. (December 2005)

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Co-Chairs of Advisory Committee: Dr. Dezhen Song
Dr. Deepa Kundur

One of the distinguishing characteristics of *hostile networked radio sources* (e.g., enemy sensor network nodes), is that only physical layer information and limited *medium access control* (MAC) layer information of the network is observable. We propose a scheme to localize hostile networked radio sources based on the radio signal strength and communication protocol pattern analysis using a mobile robot with a directional antenna. We integrate a *Particle Filter* algorithm with a new sensing model which is built on a directional antenna model and *Carrier Sense Multiple Access* (CSMA)-based MAC protocol model. we model and analyze the channel idle probability and busy collision probability as a function of the number of radio sources in the CSMA protocol modeling. Based on the sensing model, we propose a particle-filter-based scheme to simultaneously estimate the number and the positions of networked radio sources. We provide a localization scheme based on *the method of steepest descent* for the purpose of performance comparison. Simulation results demonstrate that our proposed localization scheme has a better success rate than the scheme based on the steepest descent at different tolerant distances.

To my mother and father

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CHAPTER I

INTRODUCTION AND RELATED WORK

In this chapter, we introduce the motivation and approach of problem. The motivation of this work is described in Section. A. A brief discussion of our scheme is given in Section. B. Related work is introduced and discussed in Section. C. Finally, an overview of the rest chapters of the thesis is given in Section. D.

A. Motivation

We address the problem of localizing *hostile networked radio sources* in this work. By networked radio sources, we refer to a large class of devices that communicate with each other via wireless radio, e.g. cellular phones and wireless sensor networks [1]. A key characteristics of networked radio sources is that they are networked according to specific communication protocols. Hostile networked radio sources, such as enemy sensor network nodes, are characterized by that we can only obtain the physical layer and limited MAC layer information of the network. An example of the hostile networked radio sources is enemy sensor networks.

Imagine a scenario, there is a sensor network deployed by enemy. We dispatch a mobile robot equipped with directional Radio Frequency (RF) antenna to search and destroy the networked radio sources. We need an effective scheme to guide the robot to accomplish this job. This scenario is shown in Fig. 1. We have several networked radio sources (e.g., sensor nodes in Fig. 1). A mobile robot with a directional antenna is used to localizing these radio sources. The mobile robot navigates in the environment and measures the received RF signal. An algorithm is needed to automatically generate

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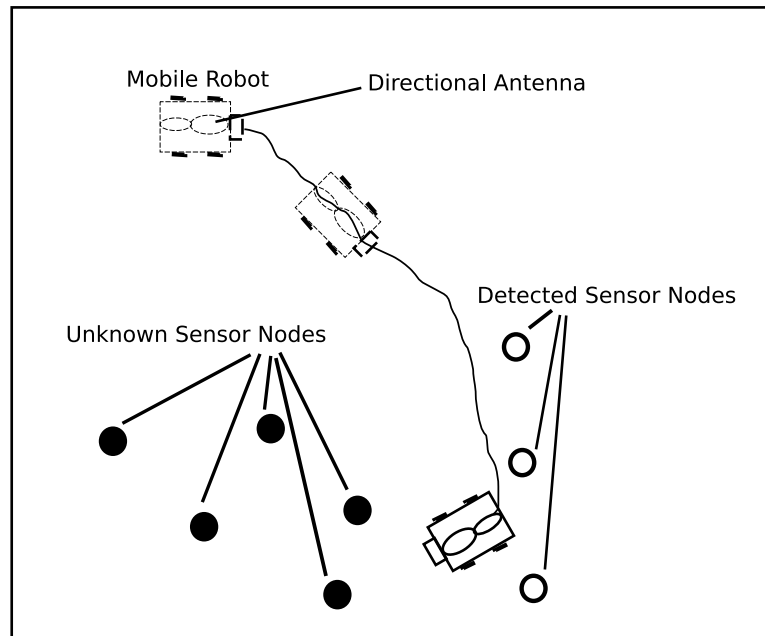


Fig. 1. Robotic localization of hostile radio sources: scenario.

motion command and drive the robot navigating the environment to search and locate the radio sources according to the received information and current knowledge about the environment.

In a hostile environment, we cannot decode the received signal to get detailed information about the network. Therefore, we have no detailed information about the target network. We do not even know the source of the signal (e.g., the identification of the sending sensor node). The information we can obtain is the signal strength and the communication pattern. Our scheme use a mobile robot with a directional antenna to to localize the networked radio sources based on the received signal strength and communication pattern analysis. To the best of our knowledge, our scheme is the first one that combines the RF signal strength and the communication protocol analysis in the multiple-source localization problem.

This scheme can also be used for applications such as search and rescue, and

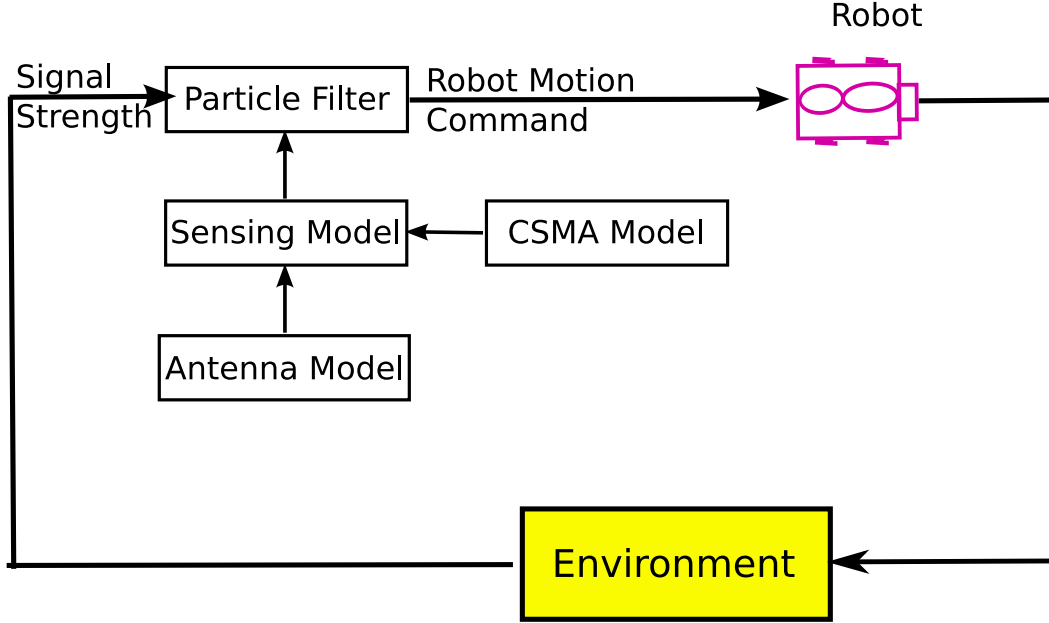


Fig. 2. Robotic localization of hostile radio sources: particle filter-based approach.

protecting privacy [2], [3]. Typical applications include enemy detection in battlefield and object tracking for disaster response.

B. Approach

Our localization scheme will generate a sequence of robot motion command to guide the robot, navigating in the environment and report newly changed signal strength. To effectively process the information, we integrate a *Particle Filter* (PF) [4] with a new sensing model that is built on the characteristics of the directional antenna and a *Carrier Sense Multiple Access* (CSMA) [5] protocol model. A system diagram of our approach is shown in Fig. 2.

The robot used in the scheme is shown in Fig. 3, which is equipped by a *BrainStem Moto 1.0 Module* from Acroname Inc. [6]. The *BrainStem Moto 1.0 Module* is used to control received commands from laptop computer to drive two wheels of the

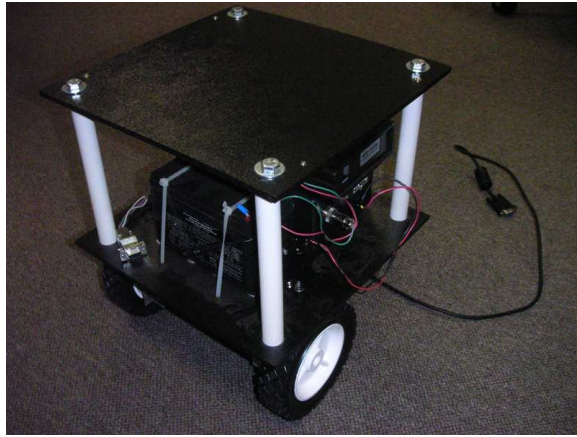


Fig. 3. The mobile robot.

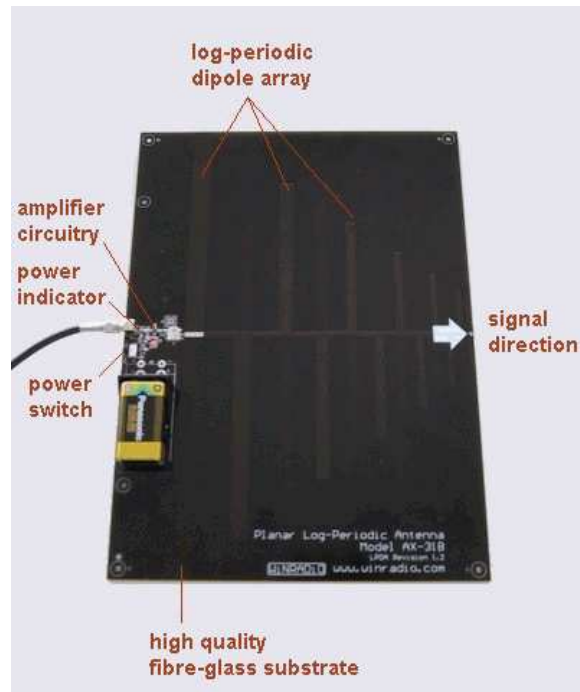


Fig. 4. The log periodic dipole array antenna (LPDA).



Fig. 5. The Crossbow Motes sensor nodes.

robot. Each of the two wheels is equipped with a *Nubotics WW-02 Wheel Watcher* [7] to measure the actual speed to the wheel. In addition, the robot carries a *Devantech Digital Compass Module* [8] to measure the direction of the robot. The robot is equipped with a directional antenna which is shown in Fig. 4. The antenna is a *WinRadio AX-31B Planar Log-periodic Dipole Array* [9] (LPDA) antenna which is a type of broadband directional antenna. The antenna can work in a broad range of radio frequency from 230 to 1600 MHz. In our problem, the antenna is working in the frequency of 433 MHz since our target radio sources work in this frequency. Our target radio sources are the MICA2 and MICA2DOT sensor network nodes, often referred as *Motes*, which are shown in Fig. 5 from Crossbow Technology Inc. [10] operating at 433MHz.

In our work, the robot navigates in a outdoor environment without obstacles. Therefore, in the theoretical modeling, the change of signal strength is determined only by the distance between the robot and the radio source, and the relative antenna direction in terms of the radio source location. Our scheme integrates a particle filter with a new sensing model which is built on the LPDA antenna model and CSMA protocol model to process the sequence of measured signal strength. The LPDA

antenna model is obtained by a curve fitting based on the numerical simulation. The main advantage of our antenna model is its computational efficiency. We construct the sensing model on top of the CSMA-based Medium Control Access (MAC) layer protocol because our target radio sources (Motes sensor nodes) use a CSMA-based protocol, also because that a large class of wireless communication networks use a CSMA-based protocol.

The main contributions of this work include:

1. **Propose a computationally efficient antenna model and a corresponding single-source sensing model for the LPDA antenna.** To localize the radio sources with signal strength, we need to model the received signal strength and the reading of the radio receiver for specific position and direction of the antenna and the position of the radio source. Since the theoretical model of the LPDA antenna is expensive in computation, we use numerical modeling software SuperNEC [11] to model the antenna through simulation. We further fit the result with a computational efficient function in the sense of minimum mean square error (MMSE). We derive a single-source sensing model based on the antenna model by modeling the measurement noise as an additive zero-mean Gaussian distribution.
2. **Design a scheme to localize single radio source.** A scheme to localize single radio source is the basis for a general scheme to localize multiple radio sources. We design the scheme based on Particle Filtering algorithm. We integrate the Particle Filtering algorithm with the single-source sensing model which is derived from our proposed antenna model and a motion model for the robot.
3. **Analyze the general sensing model for multiple radio sources.** For

the purpose of multiple-source localization, we define a *collision type* to be a set of concurrent sending radio sources. The general sensing model could be obtained by computing the conditional probability of signal strength for any given collision type and the probability of the collision type. The conditional probability is analyzed based on wave superposition. The probability of any given collision type could be obtained by the CSMA modeling.

4. **Give a detailed modeling and analysis of the CSMA-based MAC layer protocol.** We give a simplified model of the CSMA protocol which can be embedded into the particle filter framework. We model the channel idle probability and busy collision probability as a function of the number of the radio sources, the propagation delay and the message generating rate. We also derive the number of radio sources as a function of the measured idle probability. The model and analysis is verified by simulations.
5. **Design a scheme to localize multiple radio sources.** We extend the single-source localization scheme to a multiple-source localization scheme. We design a scheme based on the general sensing model which is built on the directional antenna model and the CSMA model. We also propose a simultaneous number estimation and localization scheme for networked radio sources. A scheme based on the method of steepest descent is designed and used for comparison. The performances of the two schemes are compared by simulations.

C. Related Work

This work relates to a variety of research topics. The closest one is to localize friendly radio sources with beacons. The existing work either limits to individual radio source [12] [13] [14] [15] or assumes receiver is part of the network which knows the detailed

packet information of the network [16] [17] [18] [19] [20]. However, such information is not always available in our case in hostile environment. Most RF-based systems use position-static *beacon nodes* (i.e., nodes that are aware of their locations) to help localize sensor nodes. While in our research, we use a mobile robot to localize the sensor nodes. A scheme similar to our idea is to localize sensor network nodes with a mobile beacon [19]. However, the scheme mentioned in [19] requires communication between the mobile beacon and the sensor nodes to identify the origin (e.g., sensor node identification (ID)) of the radio signal. Since we cannot communicate with enemy sensor nodes, this type of communication cannot be applied in hostile environment. In recent work, the authors of [13] propose a localization scheme based on wireless signal strength and apply a spatial connectivity graph to help localizing radio sources. This work is similar to our scheme because it is based on Particle Filter and RF signal strength. However, our scheme differs from [13] in the following aspects. First, our work use a mobile robot to localize hostile radio sources, while [13] assumes static beacon nodes and known signal source. Second, we apply a time-variant communication pattern analysis while [13] employs pre-determined spatial information to help localizing radio sources.

In robotics research, Simultaneous Localization and Mapping (SLAM) is defined as the process to map the environment and localize the robot position at the same time [21] [22] [23] [24] [25] [26]. Although SLAM is based on similar Bayesian approach [26] we used in this work, it assumes that the environment is static or close to static. In our case, networked radio sources can be viewed as an environment, but directly adopting SLAM methods cannot make use of basic communication pattern, which will limit the effectiveness of the method and suggest new development for the problem of localizing networked radio sources.

Several methods are studied as implementations of the Bayesian approach, such

as Kalman Filtering[21], Grid-based Localization [27] and Particle Filter [28], [4]. Among which, Particle Filtering is a new class of method to solve general non-Gaussian, non-linear SLAM problems [4] [28] [29] [30] [31]. Particle Filter is based on the Bayesian rule and represent the probability distribution with samples, each of which represent an unique status of the robot pose, called a particle. In this work, we expand particle filter with a new sensing model that built on the new directional antenna model and the CSMA protocol model.

Since the LPDA directional antenna is the primary sensor of our robot, we need to understand the antenna. In antenna research field, detailed physical model of the LPDA antenna has been constructed more than 20 years ago [32] [33]. However, the detailed physical model is constraint by random factors in the antenna such as material, surface and shape. None of the physical model can provide accurate prediction of signal field. On the other hand, physical model is computational expensive and only useful for antenna design. For application that requires dynamic understanding of signal field, a simplified computation model is needed. However, existing research in sensor network often overly simplify the signal field as inverse quadratic functions which cannot capture the directivity of the antenna. Our work try to approximate the directional antenna with a tractable computation model. A similar research work discussed in [14] uses directional antennas to estimate the position of a radio source. However, the scheme uses multiple static beacon nodes with directional antennas and derives the position of a single radio source by *triangulation*. In our proposed scheme, we use a mobile robot to collect RF signal information from several radio sources. Unlike the deterministic method used in [14], we use a probabilistic method which is based on particle filter and a new sensing model which is built on the directional antenna model and CSMA-type MAC protocol model.

Since majority of the networked radio sources use CSMA-based MAC layer pro-

tol. To incorporate this knowledge into localization will be very helpful. However, existing MAC layer models [34] [35] focus on detailed modeling of the MAC protocol behavior and channel capacity, and cannot be directly applied to localization models. A simplified approximation that captures the main characteristics of the CSMA protocol and can be integrated into localization framework is needed. Our sensing model is based on the CSMA model as well as the antenna model.

D. Thesis Overview

The rest of this thesis is organized as follows.

In Chapter II, we introduce our problem definition. The problem is defined rigorously and some assumptions used in our scheme and modeling are made.

In Chapter III, we propose our scheme to localize hostile networked radio sources. We start with a review of Particle Filter in Section A. In Section B, we introduce an computationally efficient antenna model and corresponding single-source sensing model. We introduce a deterministic motion model in Section C. Combining the models, we design a scheme to localize single radio source in Section D. We generalize our sensing model for multiple radio sources in Section E. In order to complete our general sensing model, we model and analyze the CSMA protocol model in Section F. We integrate the models with particle filter and propose the multiple-source localization scheme in Section G.

In Chapter IV, we verify our modeling and scheme with simulation experiments. We design a localization scheme based on the method of steepest descent. This proposed scheme is used to compare with our proposed scheme based on Particle Filter. The performances of the two schemes are compared through simulation results.

Finally, Chapter V concludes this research work with a summary and some di-

rections for future work.

CHAPTER II

PROBLEM DEFINITION

Our work is to use a mobile robot with a directional antenna to localize hostile networked radio sources. In this chapter, we define our problem rigorously. We start our discussion from several assumptions we made in the modeling and scheme design in Section. A. We then define our problem in Section. B.

A. Assumptions

Before we discuss our problem definition, we make following assumptions in our modeling and scheme.

1. **The received RF signal does not contain the information for the signal source.** This is the common characteristics of a hostile network environment. We cannot decode the packets received, thus we do not know the detail of the network, including the sender information of the received signal.
2. **The radio sources are networked sparsely.** We assume sparse networked radio sources so that most of the collision cases are happened between two radio sources.
3. **The radio sources are communicate with a CSMA-type MAC protocol.** Since a large class of wireless networks use CSMA-based protocol, we assume the MAC layer protocol is CSMA-based to address a broad range of scenarios. It is also because that the target radio sources in our approach, Motes sensor nodes, are using CSMA-based MAC protocol. Although we focus on CSMA-based protocol in this work, our localization framework can be extended for non-CSMA type MAC protocol such as TDMA.

4. **The antenna has a high sensitivity.** We assume the directional antenna carried by the robot is highly sensitive so that the antenna can listen to any signal transmitting in the air.
5. **The radio sources are static nodes that do not move over time.** We assume our target radio sources are static in terms of their positions, which is the common case of sensor networks since usually the sensors will not move after they are deployed.

B. Problem Definition

The definitions of variables used in the scheme is shown in Table. I. With the variable definitions, we can define our problem rigorously.

Problem Definition:

Given,

1. the received RF signal strength up to time t , $z_k, k = 1, 2, \dots, t$;
2. the robot motion measurement up to time t , $u_k, k = 1, 2, \dots, t$;
3. the measured channel idle probability P_{idle} .

Find,

1. the number of the radio sources, m ;
2. the position estimation of each radio source, $\{(x_i, y_i)\}, i = 1, 2, \dots, m$.

Table I. Definitions of variables used in the scheme.

Variable	Description
(x, y)	A point in the 2-D space.
m	The number of radio sources.
l	The number of remaining particles.
n_i	The i -th radio source, $i = 1, 2, \dots, m$.
(x_i, y_i)	The position of the i -th radio source.
ϕ_i	The initial transmission phase of the i -th radio source.
r_i	The distance between the robot and the i -th radio source.
θ_i	The angle between the robot antenna direction and the line connecting the robot and the i -th radio source.
z	The received signal strength. It is a random variable.
z_0	The mean value of received signal strength.
z_t	The received signal strength at time t .
u	The measured motion of the robot.
u_t	The measured motion of the robot at time t .
(D, T)	A robot motion command where D is the required moving distance and T is the required turn angle.
P_{idle}	The channel idle probability.
P_{bc}	The busy collision probability.

CHAPTER III

RADIO SOURCES LOCALIZATION SCHEME

The purpose of this chapter is to design a scheme to localize hostile networked radio sources. In Section. A, we review the Bayesian rule and the Particle Filter (PF) which is the basis of our scheme. A single-source sensing model based on a directional antenna model is introduced in Section. B. A deterministic motion model is introduced in Section. C. Based on the sensing model and the motion model, a single-source localization scheme is designed and described in Section. D. In Section. E, a general sensing model for localizing multiple radio sources are proposed. To complete the sensing model, we analyze and model the CSMA-based MAC protocol in Section. F. Finally, we detailed our scheme to localize multiple networked radio sources in Section. G.

A. Review of Particle Filter

Since our scheme is based on Particle Filter. We start with a review of Particle Filter, which is a new probabilistic method based on Bayes filter to solve the *robot mapping* problem.

1. Robotic Mapping

Robotic mapping [21] addresses the problem of acquiring spatial models of physical environments through mobile robots. Our problem is to estimate the positions of unknown radio sources, which can be viewed as a problem of environment mapping and thus fits in the category of robotic mapping. The physical environments can be static or dynamic, structured or unstructured, of limited size or large-scale. In our work, the target radio sources are static in positions but dynamic in communication pattern

over time. Furthermore, the sensor network nodes are assumed to be unstructured in terms of placement and of limited size.

To acquire a map, robots must possess sensors that enable it to perceive the outside world. In our work, the sensor is a radio signal receiver with a directional antenna which is shown in Fig. 4. The motion commands (controls) issued during environment exploration carry important information for building maps, since they convey information about the locations at which different sensor measurements were taken. Sometimes, the mobile robot need to learn the maps as well as the location of the robot itself. This problem is referred as Simultaneous Localization and Mapping (SLAM) problem.

A key challenge in robotic mapping arises from the nature that robotic mapping is characterized by uncertainty and sensor noise. Therefore, probabilistic methods are often used to address the robotic mapping problem. Traditional probabilistic methods such as *Kalman Filter*, assume a linear motion model with a Gaussian noise. However, in our problem, the motion model cannot be viewed as linear. *Particle Filter* algorithm, also referred as *Monte Carlo Localization (MCL)* [29] has attracted great research interest recently since it can solve non-Gaussian, non-linear localization problem.

2. Bayes Filter

Bayes filters address the problem of estimating the state X of a dynamical system from sensor measurements. For example, in single-source mobile robot localization problem the dynamical system is a mobile robot, the state is the robot's pose (specified by a position in a two-dimensional Cartesian space (x_0, y_0) and an antenna direction θ), and measurements are the radio receiver readings. Bayes filters assume that the environment is *Markov*, that is, past and future data are (conditionally) independent

if one knows the current state.

The key idea of Bayes filter is to estimate the posterior probability density over the state space conditioned on the data. Simply speaking, we need to take the best guess for each step given the current information and known knowledge. Mathematically, Bayes rule is often used to compute the posterior possibility $p(X|d)$ according to the prior possibility $p(X)$.

$$p(X|d) = \eta p(d|X)p(X). \quad (3.1)$$

Suppose we want to learn about a quantity X (e.g., a map), based on measurement data d (e.g., sensor readings, odometry). The Bayes rule tells us that the problem can be solved by multiplying two terms: $p(d|X)$ and $p(X)$. The term $p(d|X)$ specifies the probability of observing the measurement d under the hypothesis X . Thus, $p(d|X)$ is a generative model, in that it describes the process of generating sensor measurements under different conditions X . The term $p(X)$ is called the prior. It specifies our willingness to assume that X is the case in the world before the arrival of any data. Finally, η is a normalizer that is necessary to ensure that the left hand side of Bayes rule is indeed a valid probability distribution. In the robotics and AI literature, this posterior is typically called *belief*.

In robotic mapping, data arrives over time. Two types of data are used in the mapping process: sensor measurements and controls. Let us denote sensor measurements by the variable z , and the controls by u . For convenience, let us assume that the data is collected in alternation:

$$z_1, u_1, z_2, u_2, \dots,$$

Here subscripts are used as time index. In particular, z_t is the sensor measure-

ment taken at time t , and u_t specifies the robot motion command asserted in the time interval $[t - 1, t)$. In our work, the feedbacks from the robot wheel encoders are used for u instead of controls, since they can more accurately reflect the actual robot motion. We further follow common notation by using a superscript t to refer to all data leading up to time t , that is:

$$\begin{aligned} z^t &= \{z_1, z_2, \dots, z_t\} \\ u^t &= \{u_1, u_2, \dots, u_t\}. \end{aligned}$$

Our goal is to study the probability of the state X given a sequence of measurements and controls, i.e., $p(X_t|z^t, u^t)$. According to the Bayes rule shown in Eqn. (3.1), it can be expressed as,

$$p(X_t|z^t, u^t) = \eta p(z_t|X_t, z^{t-1}, u^t) p(X_t|z^{t-1}, u^t) \quad (3.2)$$

The Markov assumption states that measurements z_t are conditionally independent of past measurements and odometry readings given knowledge of the state X_t :

$$p(z_t|X_t, z^{t-1}, u^t) = p(z_t|X_t). \quad (3.3)$$

This allows us to conveniently simplify Eqn. (3.2) to:

$$p(X_t|z^t, u^t) = \eta p(z_t|X_t) p(X_t|z^{t-1}, u^t) \quad (3.4)$$

By integrate throughout the pose X_{t-1} at time $t - 1$, we obtain a recursive form of Eqn. (3.4):

$$p(X_t|z^t, u^t) = \eta p(z_t|X_t) \int p(X_t|X_{t-1}, z^{t-1}, u^t) p(X_{t-1}|z^{t-1}, u^{t-1}) dX_{t-1}. \quad (3.5)$$

The Markov assumption also implies that given knowledge of X_{t-1} and u_{t-1} , the state X_t is conditionally independent of past measurements z_1, \dots, z_{t-1} and odometry readings u_1, \dots, u_{t-2} , that gives:

$$p(X_t|X_{t-1}, z^{t-1}, u^t) = p(X_t|X_{t-1}, u_{t-1}). \quad (3.6)$$

Eqn. (3.5) can be written as a recursive estimator:

$$p(X_t|z^t, u^t) = \eta p(z_t|X_t) \int p(X_t|u_t, X_{t-1}) p(X_{t-1}|z^{t-1}, u^{t-1}) dX_{t-1}, \quad (3.7)$$

which is known as *Bayes filter*. η is a normalizing constant. This equation is the basis for the Particle Filtering algorithm.

3. Particle Filtering

In the robotics literature, the posterior is typically called belief. We use the following notation for a belief:

$$Bel(X_t) = p(X_t|z^t, u^t). \quad (3.8)$$

In order to implement Bayes filter, one needs to know three distributions: the initial belief $Bel(X_0)$ (e.g., uniform), the next state probability $p(s_t|u_t, s_{t-1})$ (called the *motion model*), and the perceptual likelihood $p(z_t|X_t)$ (called the *sensing model*). We will discuss the sensing model and the motion model later in this chapter. When we have these models ready, we can implement the Bayes filter with Particle Filter.

The idea of Particle Filter (Monte Carlo Localization) is to represent the belief

$Bel(X)$ by a set of l weighted samples distributed according to $Bel(X)$

$$Bel(X) = \{X^{(i)}, p^{(i)}\}_{i=1, \dots, l}.$$

Here each $X^{(i)}$ is a sample (a state), and $p^{(i)}$ are non-negative numerical factors called *importance factor*, which sum up to one. As the name suggests, the importance factors determine the weight (=importance) of each sample.

Initially, the beliefs of the pose is a uniform distribution over the robot's space, annotated by the uniform importance factor $\frac{1}{l}$. The recursive update is realized in three steps,

1. Sample a state X_{t-1} from $Bel(X_{t-1})$, by drawing a random $X^{(i)_{t-1}}$ from the sample set representing $Bel(X_{t-1})$ according to the discrete distribution defined through the importance factors $p_{t-1}^{(i)}$.
2. Use the sample $X^{(i)_{t-1}}$ and the action u_{t-1} to sample $X_t^{(j)}$ from the distribution $p(X_t|X_{t-1}, u_{t-1})$. The predictive density of $X_t^{(j)}$ is now given by the product $p(X_t|X_{t-1}, u_{t-1})Bel(X_{t-1})$.
3. Finally, weight the sample $X_t^{(j)}$ by the importance factor $p(y_t|X_t^{(j)})$, the likelihood of the sample $X_t^{(j)}$ given the measurement y_t .

This procedure implements Eqn. (3.42), from right to left.

B. Antenna Model and Single-source Sensing Model

The sensing model is essential in constructing a Particle Filter. Our sensing model is based a directional antenna model. In antenna research, the signal strength received by a directional antenna can be modeled as

$$S_0(r, \theta) = Cr^{-\beta}f(\theta), \quad (3.9)$$

where C is a constant, $r^{-\beta}$ is the signal decay in terms of the distance, and the directivity of the antenna is captured by the term $f(\theta)$. β is a factor between 2 and 4. $\beta = 2$ is well accepted to be the model for the ideal radio decay in free space. In our scheme, we assume $\beta = 2$ through all of our following modeling. $\beta = 4$ is often used to model signal decay in a long distance which is bigger than a certain distance.

We need to model $f(\theta)$ to understand the antenna. $f(\theta)$ is referred in antenna research as *Radiation Pattern* [36]. The exact radiation pattern model of LPDA antenna [37] is given in [32] but it is computationally expensive. In our work, we use a numerical method to compute the radiation pattern of the LPDA antenna. NEC-2 [38] is widely accepted as a antenna modeling toolkit. We use NEC-2 based SuperNEC [11] software to model and compute the radiation pattern of the LPDA. The real antenna as shown in Fig. 4 has a structural model in SuperNEC as shown by Fig. 6. The resulting *radiation pattern* at an operating frequency of 433MHz is given by Fig. 7. The antenna gain as shown in Fig. 7 is represented by *DBi*, which is defined as the number of decibels of gain of an antenna referenced to the zero *dB* gain of a free-space *isotropic* (i.e. direction independent) radiator. SuperNEC toolkit provides an application programming interface (API) to access the raw data of the antenna gain G_{dBi} in *dBi*. We convert the value of *dBi* to the absolute gain value G as follows.

$$G = 10^{G_{dBi}/20}, \quad (3.10)$$

The resulting antenna gain in terms of the direction θ is shown in Fig. 8. Since we need to dynamically understand the antenna behavior in each step of robot movement

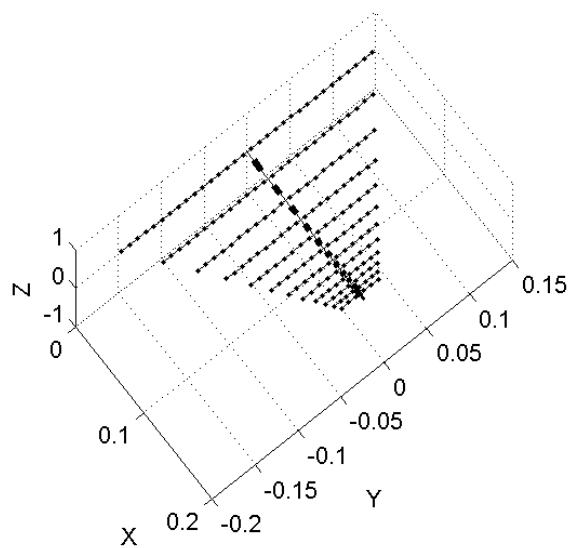


Fig. 6. LPDA antenna structure model in SuperNEC.

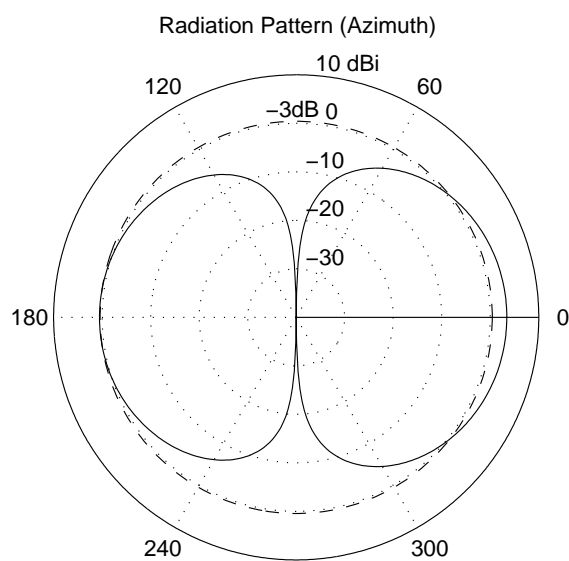


Fig. 7. LPDA radiation pattern.

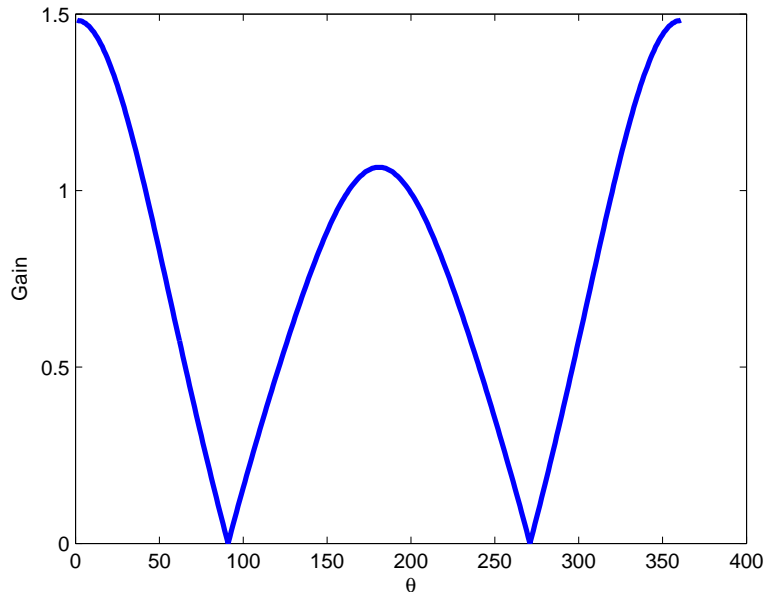


Fig. 8. LPDA antenna gain.

and compute the receiver reading for each particle, we need a efficient computation model for the LPDA directional antenna to achieve the system efficiency. The gain function shown in Fig. 8 could be approximated with a function as follows.

$$f(\theta) = \begin{cases} a \cos \theta, & 0 \leq \theta < \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \leq \theta < 2\pi, \\ -b \cos \theta, & \frac{\pi}{2} \leq \theta < \frac{3\pi}{2}. \end{cases} \quad (3.11)$$

By fitting the curve to minimize the mean square error (MSE), we get $a = 1.4825$ and $b = 1.0654$. The resulting function is given by,

$$f(\theta) = \begin{cases} 1.4825 \cos \theta, & 0 \leq \theta < \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \leq \theta < 2\pi, \\ -1.0654 \cos \theta, & \frac{\pi}{2} \leq \theta < \frac{3\pi}{2}. \end{cases} \quad (3.12)$$

the original antenna gain (blue line) and the fitting function (red line) are shown together in Fig. 9. The resulting MSE is 0.0044. As we can see, the fitted function

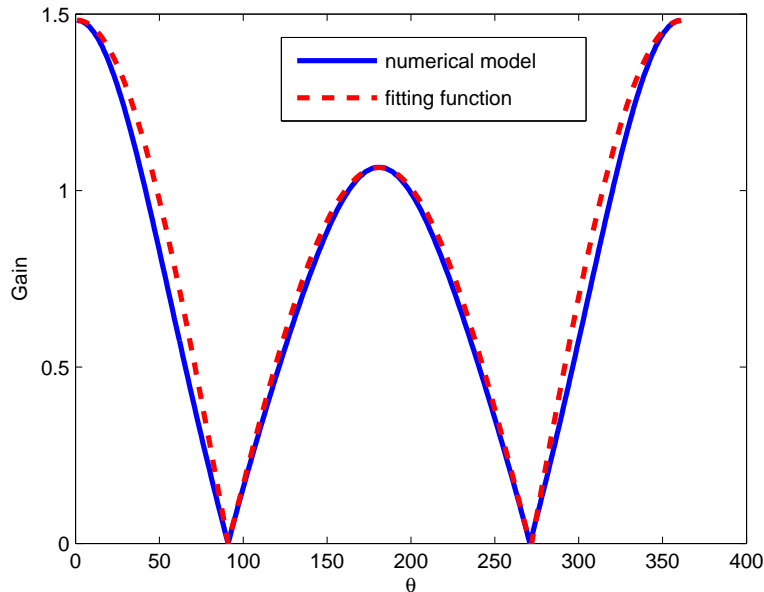


Fig. 9. LPDA antenna gain curve fitting.

(Eqn. (3.12)) is close to the numerical result and computationally efficient.

The antenna model in Eqn. (3.9) gives a deterministic signal strength at a position determined by the pair of (r, θ) . But due to measurement error, we cannot guarantee the received signal strength at a specific position is given by the Eqn. (3.9). The result signal strength given by Eqn. (3.9) gives a reference value of signal strength, or the mean value of the signal strength, which is a deterministic value. The actual received signal strength and the corresponding reading are random variables. The receiver we used is the WR-1550e Radio Receiver from WinRadio Ltd. The reading of radio signal is presented by a number between 0 and 120 and approximately equal to the dB value of the signal strength. To convert the signal strength into the reading of the radio receiver, we need to map the expression of signal strength S_0 into a number in the range between 0 and 120. We called this number as the *expected sensor reading* $z_0(r, \theta)$, which can be obtained by,

$$z_0(r, \theta) = C_1 + C_2 \log S_0 = C_1 + C_2 \log Cr^{-\beta} f(\theta). \quad (3.13)$$

The actual radio receiver reading z is not always equal to the expected sensor reading z_0 because of measurement error. The measurement error is modeled by an additive Gaussian noise N with zero mean and standard deviation σ , i.e. $N \sim \mathcal{N}(0, \sigma^2)$. The actual reading z of the received signal strength is given by,

$$z = z_0(r, \theta) + N. \quad (3.14)$$

C_1 , C_2 , C and σ are the physical parameters determined by the receiver.

Given all of the parameters, we can compute the sensing model $p(z_r|X_t) = p(z_r|r, \theta)$, where z_r is the received signal reading. We assume the reading in the range of $[z - \Delta z, z + \Delta z]$ will be rounded to the reading z . Since our receiver gives integer reading, we use $\Delta z = 0.5$. The probability distribution of received signal strength z is a normal distribution with mean $z_0(r, \theta)$ and standard deviation σ , i.e. $z \sim \mathcal{N}(z_0(r, \theta), \sigma^2)$. Therefore, the probability that the receiver gets the integer reading z_t can be computed as the shaded area in Fig. 10, which is given by,

$$\begin{aligned} p(z_t|r, \theta) &= P(z_t - 0.5 \leq z \leq z_t + 0.5) \\ &= p(z \leq z_t + 0.5) - p(z \leq z_t - 0.5) \\ &= \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{z_t + 0.5 - z_0(r, \theta)}{\sigma} \right) \right] - \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{z_t - 0.5 - z_0(r, \theta)}{\sigma} \right) \right] \\ &= \frac{1}{2} \left[\operatorname{erf} \left(\frac{z_t + 0.5 - z_0(r, \theta)}{\sigma} \right) - \operatorname{erf} \left(\frac{z_t - 0.5 - z_0(r, \theta)}{\sigma} \right) \right] \end{aligned} \quad (3.15)$$

We use physical experiments to fit the parameters of the Gaussian distribution $\mathcal{N}(0, \sigma^2)$. By collecting the receiver readings for a specific sensor position for 200 times, a histogram showing the experimental received signal reading frequency is

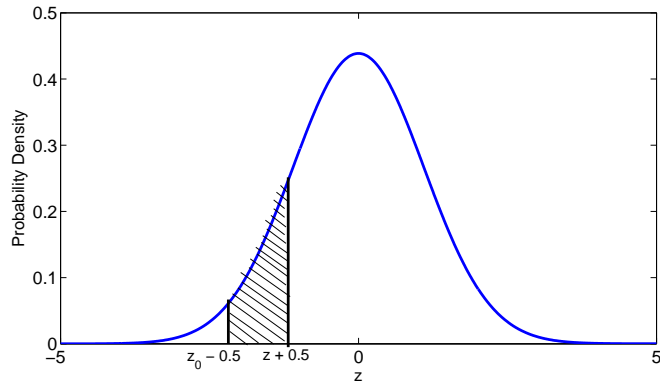


Fig. 10. Compute the probability of received signal strength reading given (r, θ) and the measurement error distribution.

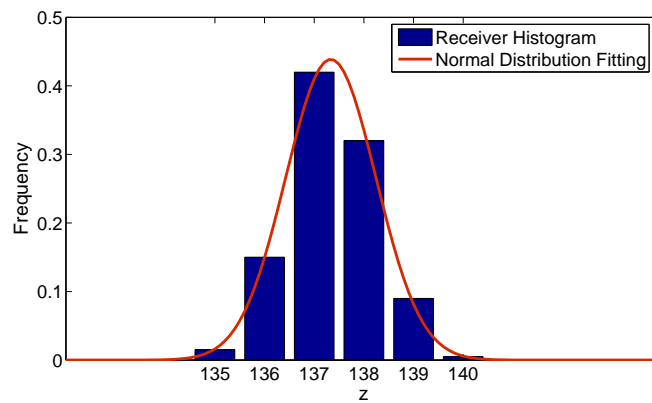


Fig. 11. A histogram showing the distribution of the received signal reading and the best-fit normal distribution.

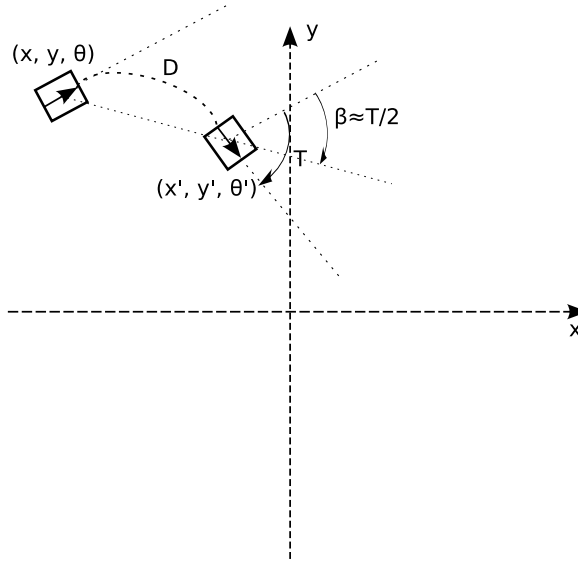


Fig. 12. Robot motion model.

given by Fig. 11. The curve is best fitted by a Gaussian distribution with the standard deviation 0.9093. Thus, the sensing model is given by,

$$p(z_t|r, \theta) = \frac{1}{2} \left[\operatorname{erf} \left(\frac{z_t + 0.5 - z_0(r, \theta)}{0.9093} \right) - \operatorname{erf} \left(\frac{z_t - 0.5 - z_0(r, \theta)}{0.9093} \right) \right] \quad (3.16)$$

C. Motion Model

We model the robot motion as a deterministic function in terms of the initial pose (x, y, θ) of the robot, the moving distance D travelled by the robot, and the turn T performed by the robot. We assume the motion error can be ignored since we have wheel encoders equipped in both of the robot wheels to detect the actual wheel speeds as well as a digital compass to detect the initial and ending directions of the robot. An improved version of the robot carries a GPS receiver which guarantee better accuracy of the knowledge of the robot position is in construction and will be used in the future. Therefore, we assume the movement of the robot can be accurately estimated based

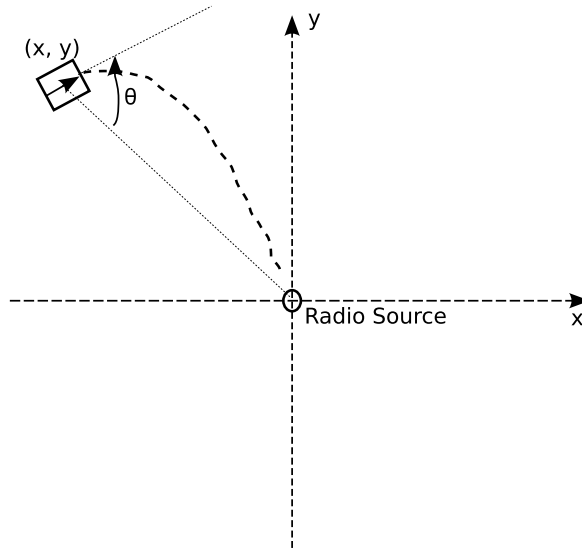


Fig. 13. Single radio source localization.

on the given parameters. The robot movement is illustrated in Fig. 12. Although the robot head turns for an angle of T , the actual angle β that the robot body turns is approximately $T/2$. The motion model used in this thesis is given by [39],

$$\begin{cases} x' = x + D \cos(\theta + \frac{T}{2}) \\ y' = y + D \sin(\theta + \frac{T}{2}) \\ \theta' = (\theta + T) \bmod 2\pi. \end{cases} \quad (3.17)$$

D. Single-source Localization Scheme

By integrating the Particle Filtering algorithm with the sensing model in Section. B and the motion model in Section. C, we design our scheme to localize single radio source.

The scheme is illustrated in Fig. 13. The robot is initially at (x_0, y_0) . An angle θ is defined as the angle from the line connecting the robot and the radio source to the antenna direction which is used to estimate the receiver reading. The initial value of

this angle is θ_0 . The static single radio source remains at position $(0, 0)$, unchanged through the whole localization process. Our scheme should drive the robot toward the radio source gradually.

The particle is defined as the pair of *state* and its *probability*, where the state is defined by $(x^{(i)}, y^{(i)}, \theta^{(i)})$ for the i -th particle. The total number of particles is N . Assume our robot lies between a range within the space. We first generate N particles randomly in the whole state space, with the probability of each particle equals to $\frac{1}{N}$. The system updates its particles iteratively. In each iteration,

1. Get a sensor reading z_t .
2. weight the sample $(x^{(i)}, y^{(i)}, \theta^{(i)})$ with the non-normalized importance factor $p(z_t|(x^{(i)}, y^{(i)}, \theta^{(i)}))$. Based on Eqn. (3.18), with $r = \sqrt{(x^{(i)})^2 + (y^{(i)})^2}$, we know,

$$p[z_t|(x^{(i)}, y^{(i)}, \theta^{(i)})] = \frac{1}{2} \left[\text{erf} \left(\frac{z_t + 0.5 - z_0(r, \theta)}{\sigma} \right) - \text{erf} \left(\frac{z_t - 0.5 - z_0(r, \theta)}{\sigma} \right) \right], \quad (3.18)$$

where $\sigma = 0.9093$, and

$$z_0(r, \theta) = C_1 + C_2 \log C \frac{1}{(x^{(i)})^2 + (y^{(i)})^2} f(\theta^{(i)}).$$

3. After we weight the samples, normalize the weight of samples. Thus,

$$p^{(i)} = \frac{p[z_t|(x^{(i)}, y^{(i)}, \theta^{(i)})]}{\sum_i p[z_t|(x^{(i)}, y^{(i)}, \theta^{(i)})]}. \quad (3.19)$$

4. Then, we resample the sample set from $Bel(x_t)$ according to the probability distribution defined by the importance factor $p_t^{(i)}$.

5. Compute the average among all of the remaining particles,

$$\begin{cases} x_{ave} &= \frac{1}{N_{remain}} \sum_i x_t^{(i)} p_t^{(i)} \\ y_{ave} &= \frac{1}{N_{remain}} \sum_i y_t^{(i)} p_t^{(i)} \\ \theta_{ave} &= \frac{1}{N_{remain}} \sum_i \theta_t^{(i)} p_t^{(i)}, \end{cases} \quad (3.20)$$

where N_{remain} is the number of the remaining particles.

6. Generate a motion command (D, T) with,

$$\begin{cases} D &= D_c \\ T &= (2\pi - \theta_{ave}) \bmod \pi, \end{cases} \quad (3.21)$$

where $D = D_c$ is a constant distance the robot will move in a step. T is the angle of the turn that the robot should make.

7. Update the resampled sample set $x_t^{(i)}$ with the motion model according to .

$$\begin{cases} x_{t+1}^{(i)} &= x_t^{(i)} + D \cos(\theta_t^i + \frac{T}{2}) \\ y_{t+1}^{(i)} &= y_t^{(i)} + D \sin(\theta_t^i + \frac{T}{2}) \\ \theta_{t+1}^{(i)} &= (\theta_t^i + T) \bmod 2\pi. \end{cases} \quad (3.22)$$

8. Stop if the remaining particles fit the *stop condition*, otherwise, go to Step 1.

The stop condition is defined as,

$$\begin{cases} d_{ave} &= \sqrt{x_{ave}^2 + y_{ave}^2} \leq d_{th} \\ MSE_d &= \frac{1}{N_{remain}} \sum_i [(x_{ave} - x_t^{(i)})^2 + (y_{ave} - y_t^{(i)})^2] \leq MSE_{th}. \end{cases} \quad (3.23)$$

The first condition $d_{ave} \leq d_{th}$ denotes that the estimated distance of the robot

and the radio source is less than a threshold value d_{th} . The second condition $MSE_d \leq MSE_{th}$ denotes that the certainty of the estimation is less than a threshold MSE_{th} .

E. General Sensing Model

When we have m radio sources to be localized, since we do not know the exact origin of the received signal, it's possible that the received signal is a signal from several radio sources transmitting radio signals at the same time. We define a *collision type* \mathcal{A} which is a set of concurrent sending radio sources to denote the different types of collision.

We define the set M of all the radio sources as,

$$M = n_1, n_2, \dots, n_m,$$

the combination of the sending sources at any time is a subset of M . Thus, the set of all the combination of the sending sources is the *power set* of M , which is denoted by $\mathcal{P}(M)$. The power set $\mathcal{P}(M)$ of the set S is defined by the set of all the subsets of M . Thus,

$$\mathcal{P}(M) = \{\{\}, \{n_1\}, \dots, \{n_m\}, \{n_1, n_2\}, \dots, \{n_1, n_2, \dots, n_m\}\}.$$

The power set $\mathcal{P}(M)$ contains 2^m elements. We use $|A|$ to denote the number of elements in set A . Thus,

$$|\mathcal{P}(M)| = 2^m.$$

$\mathcal{A} \in \mathcal{P}(M)$ is a set of sending radio sources, i.e., a collision type. The set of $\mathcal{P}(M)$ includes all possible collision types. Using the Bayesian rule, a general probabilistic

sensing model is given by,

$$p(z_t|X_t) = \sum_{\mathcal{A} \in \mathcal{P}(M)} p(z_t|\mathcal{A}, X_t)p(\mathcal{A}). \quad (3.24)$$

There are 2^m items in the right side of this equation, among which, m items with $|\mathcal{A}| = 1$ can be obtained by the single source sensing model. $|\mathcal{A}| = 0$ corresponds the channel idle time, which can not be used to localize the radio sources directly. It is shown later, the channel idle probability can be used to estimate the number of the radio sources. When $|\mathcal{A}| \geq 2$, several radio sources are sending together, we need to model $p(z_t|\mathcal{A}, X_t)$ which is the conditional probability of received signal strength reading for a given collision type. In addition, we need to update our model of $p(\mathcal{A})$ for all elements of $\mathcal{A} \in \mathcal{P}(S)$, which is the probability of a given collision type. We discuss the models for $p(\mathcal{A})$ in the next section of CSMA protocol model. Now we focus on the discussion of the model of the conditional probability $p(z_t|\mathcal{A}, X_t)$.

Assume a collision type \mathcal{A} contains v radio sources n_1, n_2, \dots, n_v send together at the same frequency. Each radio source $n_i, i = 1, 2, \dots, v$ has amplitude A_i and phase ϕ_i . Each wave Ψ_i can be described with a wave function as follows.

$$\Psi_i = A_i \cos(\omega t + \phi_i) \quad (3.25)$$

where ω is the sending frequency. The combined wave Ψ is given by,

$$\Psi_i = \sum_{i=1}^v A_i \cos(\omega t + \phi_i) = A \cos(\omega t + \phi) \quad (3.26)$$

where

$$A^2 = \sum_{i=1}^v A_i^2 + 2 \sum_{i=1}^v \sum_{j>i}^v A_i A_j \cos(\phi_i - \phi_j). \quad (3.27)$$

The signal strength S is proportional to the square of amplitude A^2 . Thus, the received signal strength can be obtained from Eqn. (3.27) and is given by,

$$S = \sum_{i=1}^v S_i + 2 \sum_{i=1}^v \sum_{j>i}^v \sqrt{S_i S_j} \cos(\phi_i - \phi_j). \quad (3.28)$$

where S_i is the signal strength caused by the i -th radio source received by the antenna if only this radio source exists. S_i can be computed by Eqn. (3.9). Based on Eqn. (3.28), we can get the receiver reading by Eqn. (3.13). Similarly, the sensing model is given by Eqn. (3.18).

With the assumption that the radio sources are sparse distributed in the 2-D space, the most cases of collision is happened between two radio sources. Thus, the general sensing model can be approximately simplified as,

$$p(z_t|X_t) = \sum_{i=1}^m p(z_t|n_i, X_t)p(n_i) + \sum_{i=1}^m \sum_{j>i}^m p(z_t|n_i, n_j, X_t)p(n_i, n_j). \quad (3.29)$$

Thus, the two-source collision type is the most important in our scheme. If $v = 2$, Eqn. (3.28) becomes

$$S = S_1 + S_2 + 2\sqrt{S_1 S_2} \cos(\phi_1 - \phi_2), \quad (3.30)$$

where S_1 and S_2 are the signal strengths of the two sources received by the antenna individually, ϕ_1 and ϕ_2 are the phases of the two sources, respectively.

Since the combined field of two radio sources is important in the following modeling, we discuss it in detail.

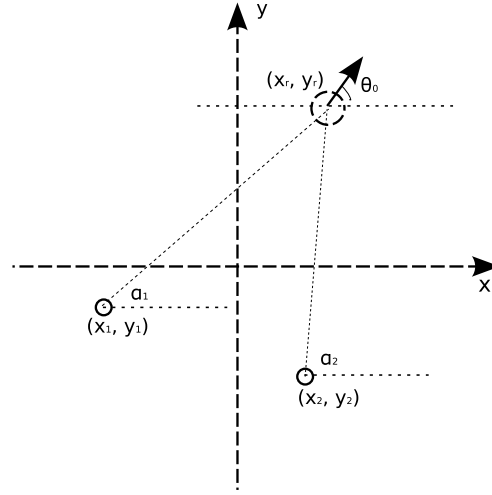


Fig. 14. Combined radio signal of two sources.

As shown in Fig. 14, suppose the positions of the two radio sources are (x_1, y_1) and (x_2, y_2) . The robot is in (x_r, y_r) and the antenna direction is θ , which is the absolute angle between the antenna and the x axis. r_1 and r_2 can be computed as,

$$r_1 = \sqrt{(x_r - x_1)^2 + (y_r - y_1)^2},$$

$$r_2 = \sqrt{(x_r - x_2)^2 + (y_r - y_2)^2}.$$

θ_1 and θ_2 could be computed as,

$$\theta_1 = \pi - \theta_0 + \alpha_1,$$

$$\theta_2 = \pi - \theta_0 + \alpha_2.$$

where

$$\alpha_1 = \arctan\left(\frac{y_r - y_1}{x_r - x_1}\right),$$

$$\alpha_2 = \arctan\left(\frac{y_r - y_2}{x_r - x_2}\right).$$

So we have,

$$S_1 = \frac{Cf(\pi - \theta_0 + \arctan(\frac{y_r - y_1}{x_r - x_1}))}{(x_r - x_1)^2 + (y_r - y_1)^2},$$

$$S_2 = \frac{Cf(\pi - \theta_0 + \arctan(\frac{y_r - y_2}{x_r - x_2}))}{(x_r - x_2)^2 + (y_r - y_2)^2},$$

combined with Eqn. (3.30), we can get the expression of expected signal strength for given collision type, which can be used to compute the conditional probability of sensing for the given collision type.

F. CSMA Model

In this section, we discuss the modeling of $P(\mathcal{A})$ for each collision type $\mathcal{A} \in \mathcal{P}(M)$ in Eqn. (3.24). If $\mathcal{A} = \emptyset$, the channel is idle. Thus, we define the idle probability P_{idle} as $P_{idle} = p(\mathcal{A}|\mathcal{A} = \emptyset)$. The channel busy probability is defined as the probability that the channel is not idle, which is given by $P_b = 1 - P_{idle}$. If $|\mathcal{A}| = 1$, i.e., The channel is occupied by one and only one radio source, the transmission is called successful. If $|\mathcal{A}| > 1$, the channel has collision. We define the collision probability P_c as $P_c = p(\mathcal{A}||\mathcal{A}| > 2)$. We further define busy collision probability P_{bc} which is given by,

$$P_{bc} = \frac{P_c}{P_b} = \frac{P_c}{1 - P_{idle}}. \quad (3.31)$$

When the total number m of sensor nodes increases, the number of elements in the set of $\mathcal{P}(M)$ increases exponentially. Use the sparse network assumption, we know that the probability that more than two radio sources collide is very small. Thus, we get the simplified general sensing model which is shown in Eqn. (3.29). So, we could decrease the number of items of Eqn. (3.24) from an exponential scale to a polynomial scale.

Before we consider the model of multiple-source localization problem, we first consider the collision model of the MAC protocol used in the sensor nodes. The CSMA-based MAC protocol is given in [40]. The protocol can be described as follows. Upon receiving a frame to transmit the sensor node generates a random *initial_backoff* interval, uniformly distributed in the range [15, 68.3] ms, and starts a timer. Then, it enters a loop in which it performs the following actions. Upon timer expiration the channel is sensed. If it is found idle and no incoming frame is detected the frame is transmitted. On the other hand, if the channel is found busy the sensor node generates a further random time interval *congestion_backoff*, uniformly distributed in the range [12.08, 193.3] ms, and starts the backoff timer again. The above actions are repeated until the channel is found free and the frame is thus transmitted.

We use the *transmission period* analysis [35] as shown in Fig. 15 to model this protocol. The variable definitions used in this section is shown in Table. II.

Assume we have m Poisson source nodes, each of which has a packet generation rate of λ packets/second. Thus, the aggregate mean packet generation rate of the traffic is given by $S = m\lambda$ packets/second. The actual packet arrival rate on the transmission channel will be larger than the packet generation rate due to the retransmission. The real traffic arrival rate offered to the channel is called *offered traffic rate* which is denoted by G where $G \geq S$. The key of our modeling is to find out the relationship between S and G , since S is related to the sender's characteris-

Table II. Definitions of variables used in the CSMA protocol modeling.

Variable	Description
P_{idle}	Channel idle probability.
P_b	Channel busy probability.
P_c	Channel collision probability.
P_{bc}	Busy collision probability.
$P_{bc}^{(n)}$	The probability that n radio sources collide in busy periods.
m	The number of radio sources.
λ	The Poisson packet generation rate for each radio source.
S	The aggregate packet generation rate for the m radio sources.
G	The offered traffic rate.
T	Packet transmission time, $T = 1$.
X	Transmission delay between two packets.
τ	Propagation delay, $\tau \ll T$.
δ	Normalized average transmission delay, $\delta = \bar{X}/T$.
a	Normalized propagation delay, $a = \tau/T \ll 1$.
t	The start of a busy period.
$t + a$	Vulnerable period.
$t + Y$	The time that the last packet arrives between t and $t + a$.
B	The duration of busy period.
I	The duration of idle period.
U	The time during a cycle that the channel is used without conflicts.

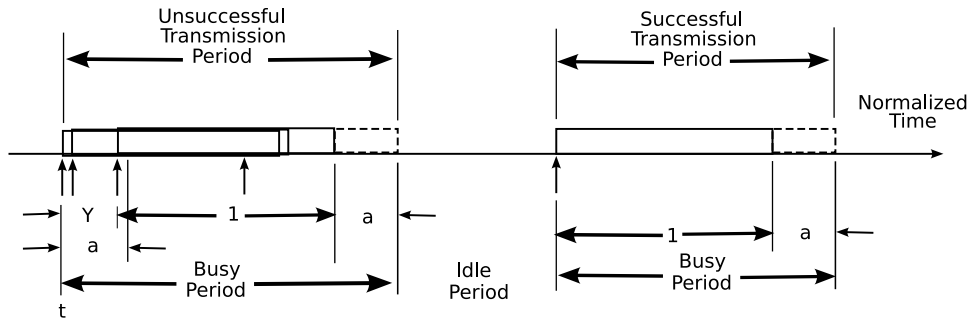


Fig. 15. CSMA: Busy and idle period.

tics and G is related to the channel statistics. Our goal is to represent the channel statistics in terms of the number of sender and the sending rate of each radio source.

Without loss of generality, we choose packet transmission time $T = 1$. We express \bar{X} and propagation time τ in these normalized time units as $\delta = \bar{X}/T$ and $a = \tau/T$.

We should make two assumptions to qualify the following theoretical analysis.

1. Assumption 1: The average retransmission delay \bar{X} is large compared to T .
2. Assumption 2: The interarrival times of the point process defined by the start times of all the packets plus retransmissions are independent and exponentially distributed.

It is clear that Assumption 2 is violated in the protocols we consider. However, it is shown in [35] the assumption gives the problem excellent performance approximation and analytic simplicity.

G denotes the arrival rate of new and rescheduled packets. All arrivals, in this case, do not necessarily result in actual transmissions (a packet which finds the channel in a busy state is rescheduled without being transmitted). Thus, G constitutes the “offered” channel traffic and only a fraction of it constitutes the channel traffic itself. Consider the time axis (See Figure. 15) and let t be the time of arrival of a

packet which senses the channel idle and such that no other packet arriving between t and $t + a$ will find (sense) the channel as unused, will transmit, and hence will cause a conflict. If no other terminal transmits a packet during these a seconds (the “vulnerable” period), then the first packet will be successful.

Let $t + Y$ be the time of occurrence of the last packet arriving between t and $t + a$. The transmission of all packets arriving in $(t, t + T)$ will be completed at $t + Y + 1$. Only a seconds later will the channel be sensed unused. Now, any terminal becoming ready between $t + a$ and $t + Y + 1 + a$ is called a *transmission period* (TP). There can be at most one successful transmission during a TP. Define an *idle period* to be the period of time between two consecutive TP’s (also called busy periods in this simple case). A busy period plus the following idle period constitute a cycle. Let \bar{B} be the expected duration of the busy period, \bar{I} the expected duration of the idle period, and $\bar{B} + \bar{I}$ the expected length of a cycle. Let U denote the time during a cycle that the channel is used without conflicts. Using renewal theory arguments, the average channel utilization is simply given by,

$$S = \frac{\bar{U}}{\bar{B} + \bar{I}}. \quad (3.32)$$

The probability that a TP is successful is simply the probability that no terminal transmits during the first a seconds of the period and is equal to e^{-aG} . Therefore

$$\bar{U} = e^{-aG}. \quad (3.33)$$

The average duration of an idle period is simply $1/G$. The average duration of a busy interval is $1 + \bar{Y} + a$, where \bar{Y} is the expected value of Y .

The distribution function for Y is

$$\begin{aligned}
F_Y(y) &= Pr\{Y \leq y\} \\
&= Pr\{\text{no arrival occurs in an interval of length } a - y\} \\
&= \exp[-G(a - y)], \quad (y \leq a).
\end{aligned} \tag{3.34}$$

The average of Y is therefore given by

$$\bar{Y} = a - \frac{1}{G}(1 - e^{-aG}). \tag{3.35}$$

Therefore, Eqn. (3.32) could be written as

$$\begin{aligned}
S &= \frac{\bar{U}}{\bar{B} + \bar{I}} \\
&= \frac{e^{-aG}}{1/G + 1 + \bar{Y} + a} \\
&= \frac{Ge^{-aG}}{G(1 + 2a) + e^{-aG}}.
\end{aligned} \tag{3.36}$$

The idle probability is given by,

$$\begin{aligned}
P_{idle} &= \frac{1/G}{1/G + 1 + \bar{Y} + a} \\
&= \frac{1}{G(1 + 2a) + e^{-aG}}.
\end{aligned} \tag{3.37}$$

When a is very small, we get from Eqn. (3.36),

$$\lim_{a \rightarrow 0} S = \frac{G}{1 + G},$$

thus,

$$G \approx \frac{S}{1-S} = \frac{m\lambda}{1-m\lambda}.$$

Therefore, the idle probability could be expressed as a function of m and λ as,

$$\begin{aligned} P_{idle} &= \frac{1}{G(1+2a) + e^{-aG}} \\ &= \frac{1}{\frac{m\lambda}{1-m\lambda}(1+2a) + e^{-a\frac{m\lambda}{1-m\lambda}}}. \end{aligned} \quad (3.38)$$

Furthermore, if we can detect P_{idle} , we can estimate the number of radio sources m . In order to express m using P_{idle} , we further simplify Eqn. (3.37) based on the fact that $\bar{Y} \approx a \ll 1$.

$$\begin{aligned} P_{idle} &= \frac{1/G}{1/G + 1 + \bar{Y} + a} \\ &\approx \frac{1/G}{1/G + 1 + 2a} \\ &= \frac{1}{G(1+2a) + 1} \\ &= \frac{1}{\frac{m\lambda}{1-m\lambda}(1+2a) + 1} \\ &= \frac{1-m\lambda}{1+2am\lambda}, \end{aligned} \quad (3.39)$$

which can be rewritten to

$$m = \frac{1}{\lambda} \frac{1 - P_{idle}}{2aP_{idle} + 1}. \quad (3.40)$$

From this equation, we can estimate the number of the sensor nodes based on the detected channel idle probability given the knowledge of the propagation delay a and the packet generation rate λ .

The busy collision probability P_{bc} is defined as the probability that the busy period has a collision, which is equal to the probability that a TP is not successful and can be simply given by,

$$P_{bc} = 1 - e^{-aG} = 1 - e^{am\lambda/(1-m\lambda)}.$$

If we have n sensor nodes with Poisson arrival rate λ , the probability that the n nodes conflict simultaneously is given by,

$$P_{bc}^{(n)} = \frac{e^{-aG}(aG)^{n-1}}{(n-1)!}. \quad (3.41)$$

The transmission rate S is always less than 1. For an efficiently working condition, the retransmission should not be very significant. Thus, G is a little bigger than S , which causes a very small number of aG because of $a \ll 1$. Therefore, $P_{bc}^{(n)} \ll 1$ in an effectively working radio source network. This is an additional verification that most cases of collision will happen between two radio sources, other than the sparse network assumption.

G. Multiple-source Localization Scheme

Combining the general sensing model built on the antenna model and the CSMA protocol model, we propose our particle filter based scheme to localize hostile networked radio sources in this section.

1. Particle Definition

When we need to localize multiple radio sources, we define the location of robot as the origin $(0,0)$. Assume there are m radio sources to be localized, for each radio source, a state of the radio source is characterized by its position (x_i, y_i) and

its initial radio transmission phase ϕ_i . We define the state of a radio source as $s_i = (x_i, y_i, \phi_i)$, $i = 1, 2, \dots, m$. Thus, the particle in the multiple-source localization problem is defined as the states for all the radio sources and the absolute direction of the antenna θ . A pose of the system X is defined as,

$$X = (s_1, s_2, \dots, s_m, \theta).$$

Assume we have l_t remaining particles in a given time t . A particle is defined as a pair of a pose and its corresponding probability, i.e. $(X^{(j)}, p^{(j)})$, $j = 1, 2, \dots, l_t$, where $X^{(j)}$ is the pose of the j -th particle, $p^{(j)}$ is the probability of this particle. With this particle definition, we can rewrite the Eqn. (3.42) to,

$$p(X_t|z^t, u^t) = \eta p(z_t|X_t) \int p(X_t|u_t, X_{t-1}) \cdot p(X_{t-1}|z^{t-1}, u^{t-1}) dX_{t-1}. \quad (3.42)$$

With the new particle definition, we need to adjust our motion model and sensing model accordingly, which is described as follows.

2. Motion Model

Since we fixed the robot position as the origin, we need to update the robot direction and the relative particle positions for each movement. The new motion model is shown as follows.

For each particle, its state $X^{(j)}$ should be updated by,

$$\begin{cases} x_i'^{(j)} &= x_i^{(j)} - D \cos(\theta^{(j)} + \frac{T}{2}) \\ y_i'^{(j)} &= y_i^{(j)} - D \sin(\theta^{(j)} + \frac{T}{2}) \end{cases}, i = 1, 2, \dots, m; j = 1, 2, \dots, l_t. \quad (3.43)$$

and

$$\theta^{(j)} = (\theta^{(j)} + T) \bmod 2\pi, j = 1, 2, \dots, l_t. \quad (3.44)$$

3. Multiple-Source Localization Scheme

We extend our scheme to localize single radio source according to new particle definition, motion model and sensing model.

In the multiple-source localization problem, we assume the robot position is the origin of the Cartesian coordinates $(0, 0)$. Although the robot is moving during the localization process, we can keep a corresponding moving coordinate system to guarantee the robot position is always $(0, 0)$. In addition, we assume the antenna direction θ is originally heading to the positive x -axis. To model the robot movement, we keep moving the Cartesian coordinates. Correspondingly, we need to update the position estimations of the radio sources since they are assumed to be relatively static to the Cartesian coordinates. We model the turn of robot by rotate the antenna direction θ .

Assume we have m networked radio sources n_1, n_2, \dots, n_m to be localized. We define their initial positions as (x_i, y_i) , $i = 1, 2, \dots, m$. We also define the initial radio transmission phase of each radio source as ϕ_i , $i = 1, 2, \dots, m$. The system state is given by the position and phase information of all the radio sources plus the antenna direction. We use $s_i = (x_i, y_i, \phi_i)$, $i = 1, 2, \dots, m$, to characterize a radio source. Since the positions of the radio sources are changing with the coordinates, we denote a radio source at time t as,

$$(s_i)_t = [(x_i)_t, (y_i)_t, (\phi_i)_t], i = 1, 2, \dots, m.$$

where $(\phi_i)_t$ will not change with time t , so we can denote the state of a radio source as,

$$(s_i)_t = [(x_i)_t, (y_i)_t, \phi_i], i = 1, 2, \dots, m.$$

A particle is defined as the pair of state and its probability, where the state is defined by $\{(x_1^{(j)}, y_1^{(j)}, \phi_1^{(j)}), (x_2^{(j)}, y_2^{(j)}, \phi_2^{(j)}), \dots, (x_m^{(j)}, y_m^{(j)}, \phi_m^{(j)}), \theta\}$ for the j -th particle. For the value of the j -th particle at time t , we denote it as follows.

$$X_t^{(j)} = \{[(x_1^{(j)})_t, (y_1^{(j)})_t, \phi_1^{(j)}], [(x_2^{(j)})_t, (y_2^{(j)})_t, \phi_2^{(j)}], \dots, [(x_m^{(j)})_t, (y_m^{(j)})_t, \phi_m^{(j)}], \theta\},$$

$$j = 1, 2, \dots, l_t.$$

This definition of particle results a higher number of parameters used to describe the system. Thus, we need more particles for a good space coverage, which causes a higher time and space complexity for the problem. In addition to use a higher number of particles, we design a dynamic particle updating algorithm to multiply the number of particles in the middle of the localization process. By doing this, we increase the localization accuracy with moderate space complexity. In addition, we update the estimation of the number of the radio sources during the process of localization, we need an algorithm to increase and decrease the variables in a particle definition dynamically. We will not detail the algorithm to manage the particles in this thesis.

The initial total number of particles are l_0 . We first generate l_0 particles randomly in the state space, with the probability of each particle equals to $\frac{1}{l_0}$. The system updates its particles iteratively. In each iteration,

1. Initialize the propagation delay a , the busy collision probability P_{bc} and the number of sensor nodes m .

2. Scan for 100 times in one time unit (one second in our simulation). Update the idle probability P_{idle} and average reading of received signal strength z_t according to the readings of all the scanning tries.

$$P_{idle} = \frac{(t_{idle})_{total}}{t_{total}},$$

$$z_t = \frac{\sum_{t_s} z_{t_s}}{t_s},$$

where $(t_{idle})_{total}$ is the total idle time, t_{total} is the total simulated time. t_s is the times that a non-zero signal is detected within the 100 scanning times, z_{t_s} is the signal strength reading at time t_s .

3. Use idle probability P_{idle} and the estimation value of a to get a estimation of sensor number m .

$$m = \frac{1}{\lambda} \frac{1 - P_{idle}}{2aP_{idle} + 1}. \quad (3.45)$$

where λ is the Poisson arrival rate of each source which is assumed to be a constant in the scheme. If the estimated number m is different with the previous estimation, we need to modify our particle definition accordingly by adding or removing corresponding items in the particle definition and update all the remaining particles.

4. We weight the sample

$$X_t^{(j)} = \{[(x_1^{(j)})_t, (y_1^{(j)})_t, \phi_1^{(j)}], [(x_2^{(j)})_t, (y_2^{(j)})_t, \phi_2^{(j)}], \dots, [(x_m^{(j)})_t, (y_m^{(j)})_t, \phi_m^{(j)}], \theta\}$$

with the non-normalized importance factor $p(z_t|X_t)$.

We use the simplified general sensing model (Eqn. (3.29)) in the multiple-source localization problem.

$$p(z_t|X_t) = \sum_{i=1}^M p(z_t|n_i, X_t)p(n_i) + \sum_{i=1}^M \sum_{j>i}^M p(z_t|n_i, n_j, X_t)p(n_i, n_j). \quad (3.46)$$

where $p(n_i)$ is the probability that the sensor node n_i is transmitting given the channel is busy (i.e., the receiver reading indicates that signal exists.), $p(n_i, n_j)$ is the probability that the sensor node n_i and n_j is transmitting in the same time given the channel is busy.

We assume all the sensor nodes are identical and the communication protocol is fair. Thus, the probability that each sensor node transmits is equal, and the probability that any two sensor nodes transmit in a same transmission period is equal. Thus,

$$\begin{aligned} p(n_1) &= p(n_2) = \cdots = p(n_m) = p, \\ p(n_1, n_2) &= p(n_1, n_3) = \cdots = p(n_{m-1}, n_m) = q. \end{aligned} \quad (3.47)$$

Since the probability that more than 2 sensor nodes transmits together is very small, we have,

$$p(n_1) + p(n_2) + \cdots + p(n_m) + p(n_1, n_2) + p(n_1, n_3) + \cdots + p(n_{m-1}, n_m) = 1,$$

which is,

$$mp + \frac{m(m-1)}{2}q = 1, \quad (3.48)$$

where $\frac{m(m-1)}{2}q = P_{bc}$ is the busy collision probability. From Eqn. (3.48), we have,

$$\begin{aligned} p &= \frac{1}{m} - \frac{m-1}{2}q = \frac{1}{m}(1 - P_{bc}), \\ q &= \frac{2P_{bc}}{m(m-1)}. \end{aligned} \quad (3.49)$$

The busy collision probability is modeled as,

$$P_{bc} = 1 - e^{am\lambda/(1-m\lambda)}, \quad (3.50)$$

where λ is a given constant, a and m are parameters to be estimated. Propagation delay a is proportional to the largest distance between any two sensor nodes. Thus,

$$a = C_a d_{max}^{(k)}$$

where C_a is a constant and $d_{max}^{(k)}$ is the maximum distance of the k -th particle, which is computed by,

$$d_{max}^{(k)} = \max_{i,j} (\sqrt{[x_i^{(k)} - x_j^{(k)}]^2 + [y_i^{(k)} - y_j^{(k)}]^2}), k = 1, 2, \dots, l_t.$$

In the localization scheme, we assign an initial value to a and update the estimation of a during the iteration. The estimation \hat{a} of a in each iteration is computed by the average of the largest distances of all the remaining particles,

$$\hat{a} = \frac{C_a}{N_{remain}} \sum_k d_{max}^{(k)}. \quad (3.51)$$

The estimation \hat{m} of m is according to the idle probability.

$$\hat{m} = \frac{1}{\lambda} \frac{1 - P_{idle}}{2aP_{idle} + 1},$$

which requires the estimation of a and the detected idle probability P_{idle} .

The other items in Eqn. (3.29) need to be modeled are $p(z_t|n_i, X_t)$ and $p(z_t|n_i, n_j, X_t)$.

$p(z_t|n_i, X_t)$ is the single radio source sensing model which is given by,

$$\begin{aligned} p(z_t|n_i, X_t) &= p[z_t|x_i^{(j)}, y_i^{(j)}, \theta^{(j)}] \\ &= \frac{1}{2} [erf(\frac{z_t + 0.5 - z_0(r, \theta^{(j)})}{\sigma}) - erf(\frac{z_t - 0.5 - z_0(r, \theta^{(j)})}{\sigma})], \end{aligned} \quad (3.52)$$

where $\sigma = 0.9093$,

$$z_0(r, \theta) = C_1 + C_2 \log C \frac{1}{(x_i^{(j)})^2 + (y_i^{(j)})^2} f(\theta^{(j)}).$$

The item $p(z_t|n_i, n_j, X_t)$ corresponds the general sensing model from a collision type which contains two radio sources n_i and n_j ,

$$S_0(n_i, n_j, X_t) = S_i + S_j + 2\sqrt{S_i S_j} \cos(\phi_i - \phi_j), \quad (3.53)$$

where,

$$S_i = \frac{Cf(\pi - \theta + \arctan(\frac{y_1}{x_1}))}{(x_1)^2 + (y_1)^2},$$

$$S_j = \frac{Cf(\pi - \theta + \arctan(\frac{y_2}{x_2}))}{(x_2)^2 + (y_2)^2},$$

Similarly, we need to consider the measurement error. Thus,

$$p(z_t|n_i, n_j, \theta) = \frac{1}{2} \left[\operatorname{erf} \left(\frac{z_t + 0.5 - z_0(n_i, n_j, \theta^{(j)})}{\sigma} \right) - \operatorname{erf} \left(\frac{z_t - 0.5 - z_0(n_i, n_j, \theta^{(j)})}{\sigma} \right) \right]. \quad (3.54)$$

By examining Eqn. (3.49), Eqn. (3.50), Eqn. (3.52) and Eqn. (3.54), we can compute $p(z_t|X_t)$ with Eqn. (3.29).

5. After we weight the samples, normalize the weight of samples. Thus,

$$p^{(i)} = \frac{p[z_t|X_t]}{\sum_i p[z_t|X_t]}. \quad (3.55)$$

6. Then, we resample the sample set from $Bel(x_t)$ according to the probability distribution defined by the importance factor $p_t^{(i)}$.

7. Compute the average position of all of the sensor nodes in all of the particles,

$$\begin{cases} x_{ave} &= \frac{1}{l_t \hat{m}} \sum_i \sum_j (x_i^{(j)})_t (p_i^{(j)})_t \\ y_{ave} &= \frac{1}{l_t \hat{m}} \sum_i \sum_j (y_i^{(j)})_t (p_i^{(j)})_t \\ \theta_{ave} &= \frac{1}{l_t \hat{m}} \sum_i \sum_j (\theta_i^{(j)})_t (p_i^{(j)})_t, \end{cases} \quad (3.56)$$

where l_t is the number of the remaining particles at time t .

8. Generate a motion command (D, T) with,

$$\begin{cases} D = D_c \\ T = 2\pi - \theta_{ave} \bmod \pi, \end{cases} \quad (3.57)$$

where $D = D_c$ is a constant distance the robot will move in a step. T is the requested angle that the robot should change.

9. Update the resampled sample set $x_t^{(i)}$ with the motion model according to .

$$\begin{cases} (x_i^{(j)})_{t+1} = (x_i^{(j)})_t - D \cos((\theta_i^{(j)})_t + \frac{T}{2}) \\ (y_i^{(j)})_{t+1} = (y_i^{(j)})_t - D \sin((\theta_i^{(j)})_t + \frac{T}{2}) \\ (\theta_i^{(j)})_{t+1} = ((\theta_i^{(j)})_t - T) \bmod 2\pi. \end{cases} \quad (3.58)$$

10. Update a according to the average of the estimation of the maximum distance within a particle.

$$\hat{a} = \frac{C_a}{N_{remain}} \sum_k d_{max}^{(k)} p^{(k)}.$$

11. Update P_{bc} according to a and m .

$$P_{bc} = 1 - e^{am\lambda/(1-m\lambda)}.$$

12. Stop if the remaining particles fit the *stop condition*. Otherwise, go to step 3.

The stop condition is defined as,

$$\begin{cases} d_{ave} = \sqrt{x_{ave}^2 + y_{ave}^2} \leq d_{th} \\ MSE_d = \frac{1}{t} \sum_i [(x_{ave} - (x_{ave})_t)^2 + (y_{ave} - (y_{ave})_t)^2] \leq MSE_{th}. \end{cases} \quad (3.59)$$

where $((x_{ave})^i t, (y_{ave})^i t)$ is the center position of the i -th particle which is defined as,

$$\begin{cases} ((x_{ave})^i t = \sum_{j=1}^m ((x_j)^i t \\ ((y_{ave})^i t = \sum_{j=1}^m ((y_j)^i t \end{cases} \quad (3.60)$$

The first condition $d_{ave} \leq d_{th}$ denotes that the estimated distance of the robot and the radio source is less than a threshold value d_{th} . The second condition $MSE_d \leq MSE_{th}$ denotes that the certainty of the estimation is less than a threshold MSE_{th} .

CHAPTER IV

SIMULATION EXPERIMENTS

In this chapter, we verify our modeling and scheme with simulation experiments. In Section. A, we verify the CSMA-based MAC protocol model with simulations. In Section. B, we validate our algorithm of radio source number estimation. We evaluate our particle filter based localization scheme with simulations in Section. C.

All simulation programs are written with *C++* compiling in *Microsoft Visual Studio .NET 2003* and running in *Windows XP* operating system. The computer used to run the simulation codes is a PC Laptop with 1.6 GHz Centrino CPU and 512MB RAM.

A. CSMA Protocol Model Verification

We first verify our CSMA modeling by looking at the model of channel idle probability. We fix the value of source sending rate $\lambda = 0.01$ and simulate the CSMA protocol for different number of radio sources m . The idle probability P_{idle} in different values of propagation delay a is shown in Fig. 16. The blue lines are the simulation results and the red lines are the theoretical results from our CSMA model for the idle probability. As we can see from the figure, the two classes of lines are pretty close. In Fig. 17, we fix the value of propagation delay $a = 0.04$, and simulate the CSMA protocol for different number of radio sources m . The resulting blue lines are the simulation results and the red lines are the theoretical results from our CSMA model for the idle probability. As we can see from the figure, the two types of lines are close.

We then fix $a = 0.01$, $\lambda = 0.01$, and the simulation results of P_{idle} with different values of m are shown in Fig. 18. Blue line is the simulation result and red line is the CSMA modeling result.

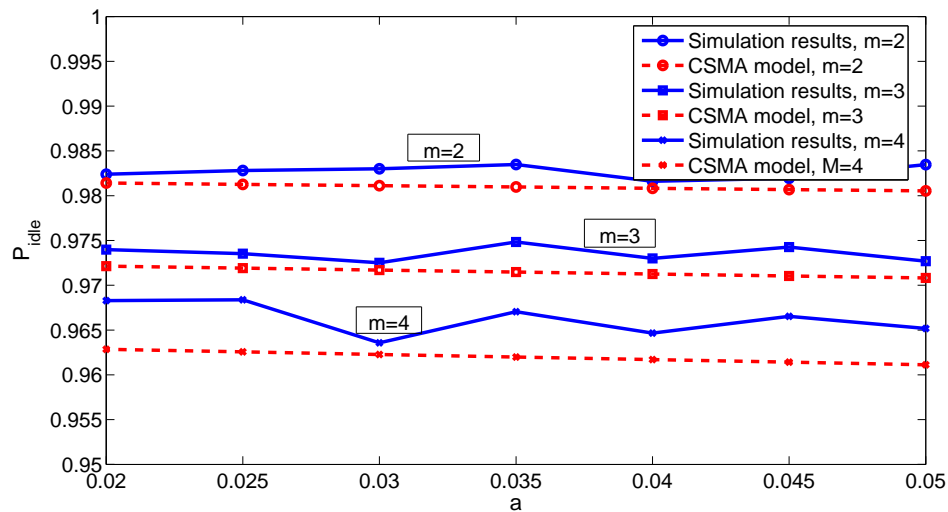


Fig. 16. The idle probability simulation results and the theoretical model with different values of a , for $m = 2, 3$ and 4 radio sources. $\lambda = 0.01$.

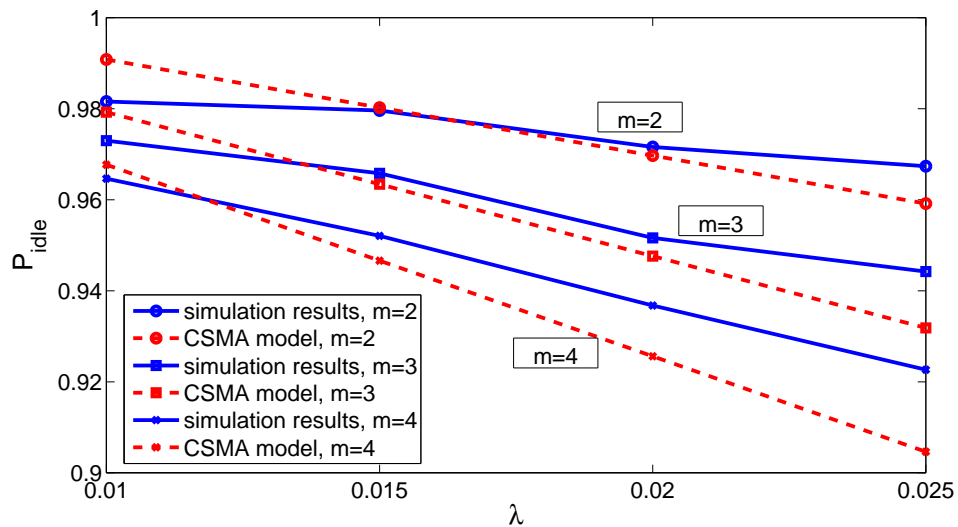


Fig. 17. The idle probability simulation results and the theoretical model with different values of λ , for $m = 2, 3$ and 4 radio sources. $a = 0.04$.

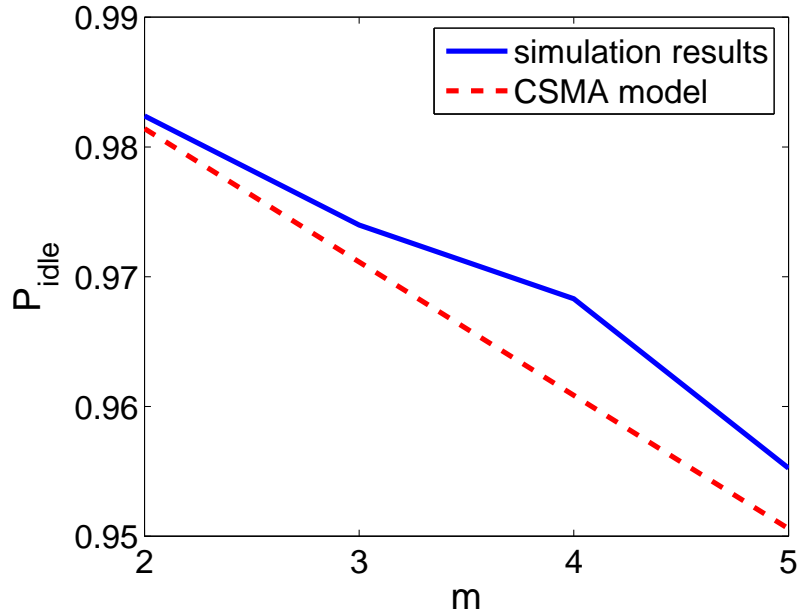


Fig. 18. The idle probability simulation results and the theoretical model with different m (Number of sensor nodes). $a = 0.01$, $\lambda = 0.01$.

We then look at the model of busy collision probability. We fix the value of source sending rate $\lambda = 0.01$ and simulate the CSMA protocol for different number of radio sources m . The busy collision probability P_{bc} in different values of propagation delay a is shown in Fig. 19. The blue lines are the simulation results and the red lines are the theoretical results from our CSMA model for the busy collision probability. As we can see from the figure, the two classes of lines are pretty close. In Fig. 20, we fix the value of propagation delay $a = 0.04$, and simulate the CSMA protocol for different number of radio sources m . The resulting blue lines are the simulation results and the red lines are the theoretical results from our CSMA model for the busy collision probability. As we can see from the figure, the two types of lines are close.

We then fix $a = 0.01$, $\lambda = 0.01$, and the simulation results of P_{bc} with different values of m are shown in Fig. 21. Blue line is the simulation result and red line is the

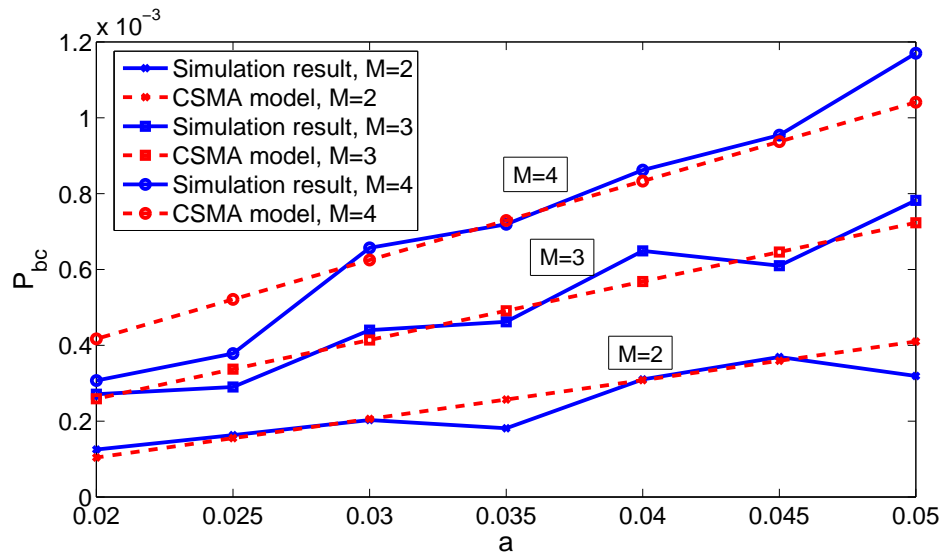


Fig. 19. The busy collision probability simulation results and the theoretical model with different values of a , for $m = 2, 3$ and 4 radio sources. $\lambda = 0.01$.

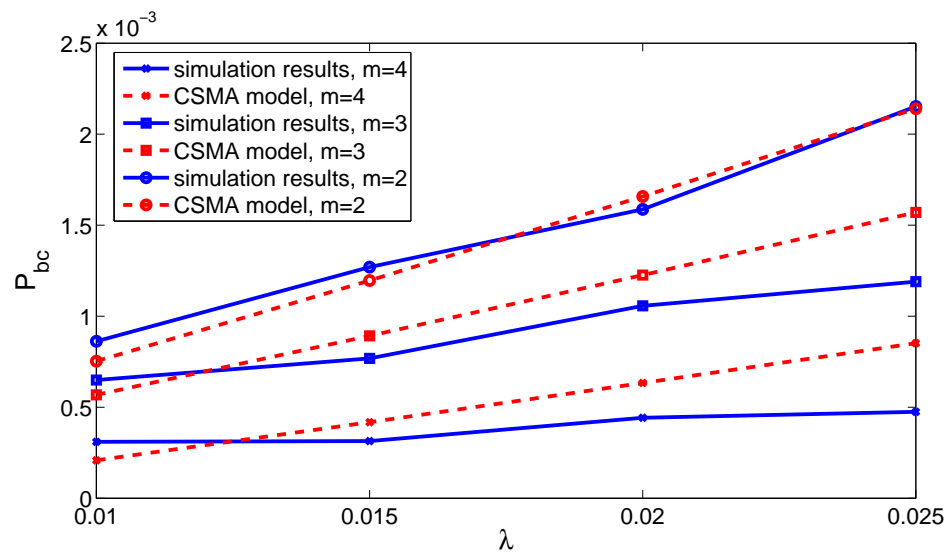


Fig. 20. The busy collision probability simulation results and the theoretical model with different values of λ , for $m = 2, 3$ and 4 radio sources. $a = 0.04$.

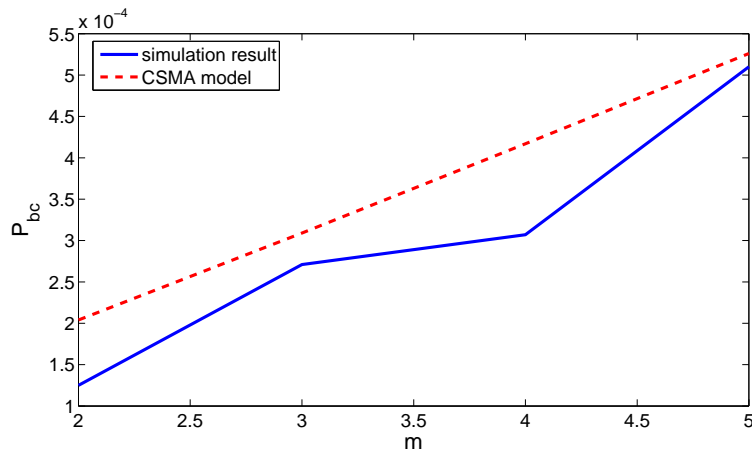


Fig. 21. The busy collision probability simulation results and the theoretical model with different m (Number of sensor nodes). $a = 0.01$, $\lambda = 0.01$.

CSMA modeling result.

B. Radio Source Number Estimation Verification

We also verify the accuracy of our algorithm to estimate the number of the radio sources according to Eqn. (4). We randomly generate a number for the number of radio sources m between 2 and 10, then we construct a network by uniformly insert all radio sources into a 2-D space. We use the measured channel idle probability to estimate the number of the radio sources. If the estimated number is correct, we say it is a successful case. We repeat it for 5000 tries for each combination of a and λ , We compute the average success rate for all the tries and obtain a success rate of 86.6%.

C. Localization Scheme Performance Evaluation

1. Performance Metric

We use success rate to evaluate the performance of our localization scheme. The success rate is defined as the probability of successful localization in a given maximum

allowed steps T_m . The success rate is a function of the tolerant distance d_{th} . Specifically, if the robot approaches a radio source such that the distance of the robot and the radio source is less than d_{th} , we say this radio source is localized. If all the radio sources are localized in a steps less than the step limit T_m , we say this is a successful localization case. We compute the possibility of successful case in all the localization tries and get the success rate.

2. The Method of Steepest Descent

In order to provide a comparison scheme, we construct a framework of localization scheme based on the method of steepest descent. We first introduce the method of steepest descent.

The method of steepest descent, also called gradient descent, is a well-studied numerical method to find a local minimum of an arbitrary function g . The method can be described as follows:

1. Evaluate g at an initial approximation $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$;
2. Determine a direction from $x^{(0)}$ that results in a decrease in the value of g ;
3. Move an appropriate distance in this direction and call the new vector $x^{(1)}$;
4. Repeat steps 1) through 3) with $x^{(0)}$ replaced by $x^{(1)}$.

The algorithm runs iteratively. The key of the algorithm is to determine the direction in step 2) and the distance in step 3). Theoretically, the direction of movement must be the negative direction of the gradient of function g (i.e., $-\nabla g(x)$) to guarantee a steepest descent of function value. The distance of movement can be decided by minimize the resulting function value.

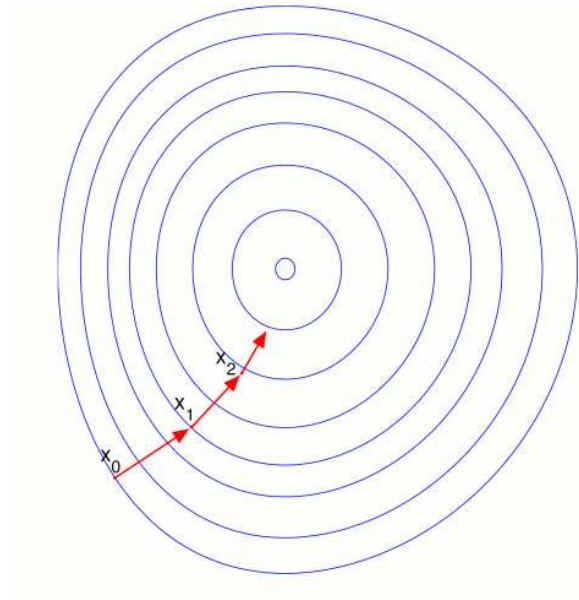


Fig. 22. A demonstration of the steepest descent for a one-node case.

In the localization problem, our goal is to find the maximum of the RF signal strength reading. For one sensor node, the local maximum is the global maximum, thus, we can use a variation of steepest descent algorithm, also called hill climbing algorithm, to solve this problem.

we can obtain the function value $g(x)$ in position x by the sensor reading of the robot, but the position of x is unknown and need to be estimated. We assume a constant movement distance in each step. The problem need to be solved is to find the direction to move. Since we cannot compute the gradient directly, an gradient detection method is essential for our hill climbing algorithm. A demonstration of the method of steepest descent for a one-node case is shown in Fig. 22.

Based on our antenna model, the signal strength is a function of the angle between the antenna and the signal source.

The gain of the antenna in terms of the angle θ is shown in Fig. 23. The direction with the highest gain is the direction of the sensor node. The problem becomes finding

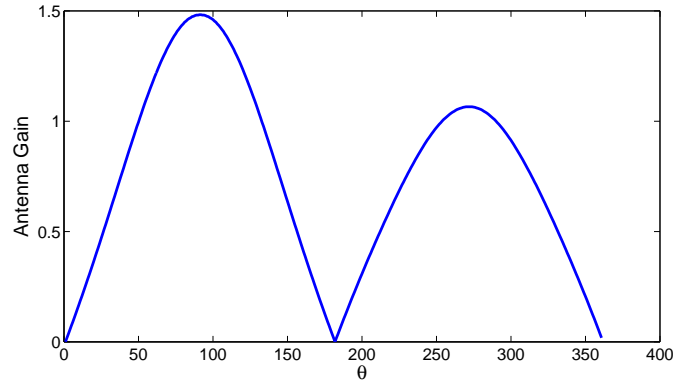


Fig. 23. Antenna gain vs. direction, shifted by $\pi/2$.

a local maximum. It is obviously in Fig. 23, we have two local maxima. Fortunately, the two maxima is in an opposite direction. So, we just need to find a local maximum and drive the robot heading to this direction for a given distance. If the signal strength increases, we know this is the direction of sensor nodes, otherwise, we know the opposite direction is our target. Since we don't know the exact direction of the antenna, we need to find a local maximum through a search algorithm. We design and implement a binary search algorithm to find a local maximum. The search algorithm is described as follows.

1. Put the robot randomly at a pose (x_0, y_0, θ_0) .
2. Get a reading z_0 from the receiver.
3. Assign an initial turning angle α .

The following process is iterative. In each iteration:

4. Turn the robot to a pose $(x_{t-1}, y_{t-1}, \theta_{t-1} + \alpha)$ and get a new reading z_t .
5. If $|z_t - z_{t-1}| < e$, where e is a tolerance error, a local maximum is found. Stop.
6. If $z_t > z_{t-1}$, go back to Step 4) with $t = t + 1$.

7. If $z_t < z_{t-1}$ and, there is no reading for the time $t - 2$, i.e., this is the second reading in a given robot position, we discard this reading, change α to $-\alpha$ and go back to Step 4).
8. If z_{t-2} exists, we need to compare z_{t-2} and z_t . If $z_t < z_{t-2}$, we know a local maximum is in the range of $[z_{t-2}, z_{t-1}]$, otherwise, a local maximum is in the range of $[z_t, z_{t-1}]$. In each case, change the α to $\alpha/2$ and search in the new range. Go back to Step 4).

When this algorithm stops, we have an estimation of the local maximum of direction. We drive the robot toward this direction, and compare the new reading with the current one to decide if this local maximum is the correct one we are finding. If the new reading is bigger, we know the movement is correct and start to find a new direction in the new position. If the new reading is smaller, we need to drive the robot back and to the opposite direction. By repeating this algorithm, we can localize the radio source.

We apply the single-source localization algorithm based on the steepest descent several times in different initial points to localize multiple networked radio sources.

3. Performance Comparison

We did simulations for the scheme based on the method of steepest descent and our scheme based on the particle filtering in a same condition. We use a two-node scenario in the simulations. We changed the threshold of localization distance error d_{th} from 0.4 to 1.6. The maximum allowed number of steps is $T_m = 400$. Simulation results for performance comparison are shown in Fig. 24. The resulting success rates of our proposed scheme based on the particle filtering is shown as the solid blue line. The success rates of the scheme based on steepest descent is shown as the dashed red line.

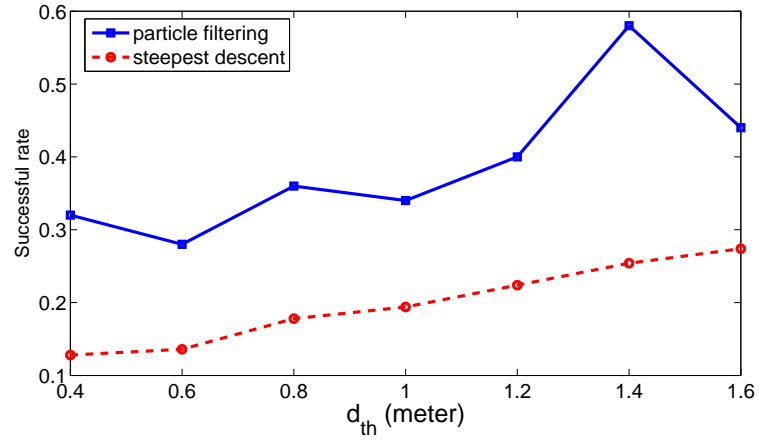


Fig. 24. Performance comparison between the localization scheme based on particle filtering and that based on steepest descent.

As we can see from the figure, our proposed scheme can perform better than a scheme based on the steepest descent we introduced in this chapter.

CHAPTER V

CONCLUSION AND FUTURE WORK

The ideas presented in this document have been expressed in terms of a Particle Filtering based localization framework, a sensing model which is built on a directional antenna model and a CSMA-based MAC protocol model, with the final goal of localizing hostile networked radio sources. In this chapter, we summarize the major work we did and indicate some future directions for extension of this work.

A. Conclusion

Localization of hostile networked radio sources such as sensor network nodes is important in applications like search and rescue, and protecting privacy. The focus of previous work has been to a large extent to localize the sensor nodes based on the signal source, e.g., sensor identification, and the received RF signal strength. In our work, we proposed a particle filter based localization scheme to localize hostile networked radio sources based on a new sensing model which combines the received RF signal strength and the measured communication pattern.

In Chapter II, we made assumptions for our modeling and scheme design and give a mathematic definition of our problem.

The localization scheme is introduced and detailed in Chapter III. First, we give a review of particle filter in Section. A. Then we introduce a directional antenna model and a single-source sensing model based on the antenna model. By combining the single-source sensing model with a robot motion model, we introduce a single-source localization scheme. Then we generalize the sensing model for multiple radio sources. A CSMA-based MAC protocol model is then detailed as a component of the sensing model. Finally, we proposed the localization scheme based on the sensing

model which is built on the antenna model and the CSMA protocol model.

We verified our modeling and evaluated the scheme performance by simulation experiments in Chapter IV. For the purpose of performance comparison, we designed a localization scheme based on the method of steepest descent. We compared the performance of our particle filter based method and the method based on the steepest descent. Simulation results show that, our method has a superior performance in terms of the successful rate within a limited maximum allowed step than the method based on the steepest descent.

B. Future Work

1. Multiple-Robot Multiple-Source Localization

In future work, we are interested in designing multiple-robot multiple-source localization scheme, where we use collaborative robots to localize hostile networked radio sources. In a multiple-robot environment, the key problems to be solved are the information synchronization and fusing between different robots. A distributed Particle Filtering algorithm is often referred to address these problems. However, there are research topics on problems such as how to improve the performance of localization by using multiple robots, and how to analyze the communication pattern information from different robots.

2. Localizing Moving Radio Sources

Localizing moving radio sources is an interesting topic as a future direction and will pose new challenge to the localization framework.

3. Experiment Validations

This work is mainly based on theoretical modeling and simulation. Both the antenna model and the localization scheme require validations from real world experiments.

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