EXPLICIT DECONVOLUTION OF

WELLBORE STORAGE DISTORTED WELL TEST DATA

A Thesis

by

OLIVIER BAHABANIAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2006

Major Subject: Petroleum Engineering

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Approved by:

Chair of Committee,	Thomas A. Blasingame
Committee Member,	Jerry L. Jensen
	Wayne M. Ahr
Head of Department,	Stephen A. Holditch

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ABSTRACT

Explicit Deconvolution of Wellbore Storage Distorted Well Test Data. (December 2006)

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Chair of Advisory Committee: Dr. Thomas A. Blasingame

The analysis/interpretation of wellbore storage distorted pressure transient test data remains one of the most significant challenges in well test analysis. Deconvolution (*i.e.*, the "conversion" of a variable-rate distorted pressure profile into the pressure profile for an equivalent constant rate production sequence) has been in limited use as a "conversion" mechanism for the last 25 years. Unfortunately, standard deconvolution techniques require accurate measurements of flow-rate and pressure — at *downhole* (or sandface) conditions. While accurate pressure measurements are commonplace, the measurement of sandface flow-rates is rare, essentially non-existent in practice.

As such, the "deconvolution" of wellbore storage distorted pressure test data is problematic.
In theory, this process is possible, but in practice, without accurate measurements of flowrates, this process can not be employed. In this work we provide *explicit* (direct) deconvolution of wellbore storage distorted pressure test data using only those pressure data. The underlying equations associated with each deconvolution scheme are derived in the Appendices and implemented via a computational module.

The value of this work is that we provide *explicit* tools for the analysis of wellbore storage distorted pressure data; specifically, we utilize the following techniques:

- Russell method (1966) (very approximate approach),
- "Beta" deconvolution (1950s and 1980s),
- "Material Balance" deconvolution (1990s).

Each method has been validated using both synthetic data and literature field cases and each method should be considered valid for practical applications.

Our primary technical contribution in this work is the adaptation of various deconvolution methods for the *explicit* analysis of an arbitrary set of pressure transient test data which are distorted by wellbore storage — without the requirement of having measured sandface flowrates.

DEDICATION

We must never be afraid to go too far, for truth lies beyond.

— Marcel Proust

He who loves practice without theory is like the sailor who boards ship without a rudder and compass, and never knows where he may cast.

— Leonardo da Vinci

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CHAPTER I

INTRODUCTION

1.1 Research Problem

Production logging tools have been greatly improved since their introduction. However, the measurement of sandface flowrates is still infrequent (and often inaccurate). For instance, during the beginning of a buildup test (often referred to as "afterflow"), wellbore storage affects the pressure and flowrate in such a way that these rates rapidly fall below the measurement threshold of the tools, which then record a no-flow period. This scenario causes a loss of information with regard to the behavior in the wellbore and in the reservoir.

In the presence of such limitations, well test interpretation techniques have been developed to analyze the wellbore storage distorted pressure response — using *only* pressure transient data (which are recorded with higher accuracy than the well flowrates). This study presents the most promising of the methods found in the petroleum engineering literature, and provides an explicit formulation for each technique. These explicit interpretation techniques have been implemented into a Microsoft Excel module constructed in Visual Basic.

1.2 Research Objective

The objective of this work is to provide a comprehensive study of the analytical techniques that can be used to *explicitly deconvolve wellbore storage distorted well test data using only the given pressure data and the well/reservoir information*. No sandface flowrate history is required.

1.3 Previous Work

For the elimination of *wellbore storage effects* in pressure transient test data, a variety of methods using different techniques have been proposed. An approximate "direct" method by Russell[1] "corrects" the pressure transient data distorted by wellbore storage into an equivalent pressure function for the constant rate case (see Appendix A for detail). This approach has several shortcomings such as limited accuracy and erroneous skin factor estimation — in short, it should not be used in practice.

Rate normalization techniques [Gladfelter *et. al.*[2], Fetkovich and Vienot[3]] have also been employed to correct for wellbore storage effects and these rate normalization methods were successful in some cases. The most appropriate application of rate normalization is its use *for pressure transient data influenced by continuously varying flowrates*.

This thesis follows the style and format of the *SPE Journal*.

The application of rate normalization requires measured sandface rates, and generally yields a shifted results trend that has the correct slope (which should yield the correct permeability estimate), but incorrect intercept in a semilog plot (which will yield an incorrect skin factor).

Material balance deconvolution (an enhancement of rate normalization) is also thought to require continuously varying sandface flowrate measurements. We will show that sandface flowrates can be approximated from the observed pressure data to yield reasonably accurate results based on the material balance deconvolution approach.

Essentially, rate normalization (and material balance deconvolution) techniques are restricted when the lack of rate measurement exists. van Everdingen[4] and Hurst[5] demonstrated *empirically* that the sandface rate profile can be modeled approximately using an exponential relation for the duration of wellbore storage distortion during a pressure transient test. The van Everdingen/Hurst exponential rate model is given in dimensionless form as:

$$q_D(t_D) = 1 - e^{-\beta t_D} \quad \text{(during wellbore storage distortion)} \tag{1.1}$$

Further, van Everdingen and Hurst showed that the "rate-time" relationship during afterflow (for a pressure buildup test) or unloading (in a pressure drawdown test) is a function of the pressure drop change with respect to time and a relatively *constant* wellbore storage coefficient.

Based on a material balance in the wellbore, the sandface flowrate is calculated by the following relation given in dimensionless form: (this relation is exact for the conditions from which it was derived)

$$q_D = 1 - C_D \frac{dp_{wD}}{dt_D} \quad \left[\text{or } q_D = 1 - \frac{\frac{d}{dt}\left[\Delta p_w\right]}{m_{wbs}} \text{ in field units} \right] \tag{1.2}$$

Where we note that in the development of wellbore storage models/solutions (*e.g.*, type curves), we *always* assume a constant wellbore storage coefficient (C_s).

Eqs. 1.1 and 1.2 laid the groundwork for "β-deconvolution" — Joseph and Koederitz[6] and Kuchuk[7] applied "β-deconvolution" for the analysis of wellbore storage distorted pressure transient data. In Appendix B, we provide a detailed derivation of the "β-deconvolution" relations that we use in our work. The β-deconvolution formula, which computes the undistorted pressure drop function directly from the wellbore storage affected data, is given as:

$$p_{sD}(t_D) = p_{wD}(t_D) + \frac{1}{\beta}\frac{dp_{wD}(t_D)}{dt_D} \tag{1.3}$$

We note that Eq. 1.3 *is only valid when the sandface flowrate profile follows an exponential trend as prescribed by* Eq.1.1. In this work, our objective is to generalize Eq. 1.3 by treating β as a variable [$\beta(t)$ or $\beta(t_D)$], rather than as a constant. We develop several schemes to perform "β-deconvolution" directly using pressure derivative and/or pressure integral and integral-derivative functions. We describe these schemes in detail in Appendix C.

Once we obtain the $\beta(t_D)$ function, we utilize Eq. 1.3 as the mechanism for directly estimating the "undistorted pressure drop" function. The obvious advantage of "β-deconvolution" is that the wellbore storage effects are eliminated using *only* the given pressure data.

1.4 Summary

This study begins with an in-depth analysis of the wellbore storage condition — theoretical developments, simplified and rigorous relations, and practical issues. The methods introduced previously are then derived explicitly (specifically — the Russell method (Appendix A), the β-deconvolution model (Appendix B), the β-deconvolution coefficients (Appendix C), and the explicit rate normalization and material balance deconvolution methods (Appendix D). These formulations are then implemented into Microsoft Excel computation/interface module (description provided in Appendix E).

Synthetic and field data cases are used within the computation module to assess the behavior, performance, and possible shortcomings of each explicit deconvolution method. The primary product of this thesis is a workflow for the correction of well test data distorted by wellbore storage without the requirement of measured flowrates. The individual deconvolution methods are relevant for discussion and evaluation and the computational module is a major accomplishment as well, but (again) the most important aspect of this work is the process (or workflow) that one must consider in order to perform deconvolution of well test data distorted by wellbore storage effects.

For the purposes of inventory, we note that in this work we utilize the explicit deconvolution methods given below:

- Rate Normalization (approximate)
- Material Balance Deconvolution (rigorous for monotonic rate functions)
- Beta Deconvolution (rigorous for exponential rate functions)

CHAPTER II

THE WELLBORE STORAGE DISTORTION OF WELL TEST DATA

In this chapter we provide a complete treatment of the wellbore storage condition — theoretical developments, simplified and rigorous relations, as well as practical issues. This comprehensive analysis will provide the basis for the introduction of flowrate estimation functions during the wellbore storage dominated part of a well test (which is the key element to performing deconvolution without measured sandface flowrate data).

2.1 Wellbore Effects on a Well Test

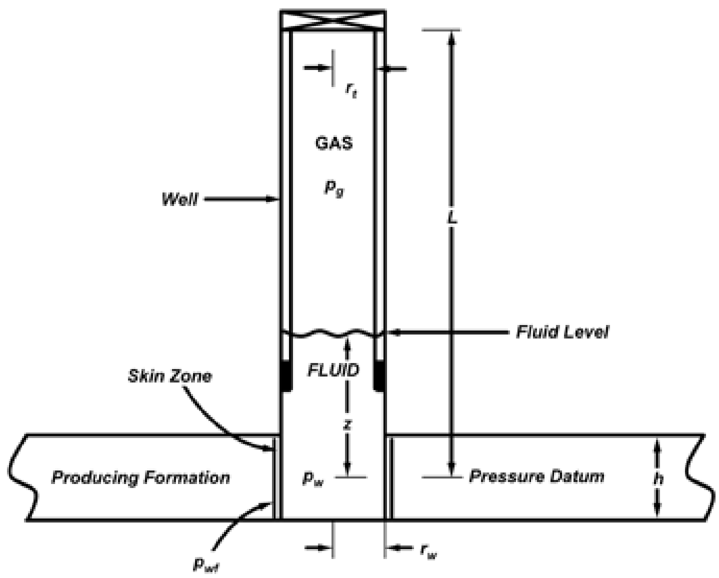

Figure 2.1 — Schematic diagram of well and formation during pressure build-up (from Russell[1])

A pressure recorder, as accurate as it may be (nowadays the error can be less than 1/100 of a psi), generally performs its measurements in somewhere between the sandface and wellbore (shown as p_w on Fig. 2.1). This must be acknowledged when using pressure data for the characterization of a reservoir, since the pressure transient test data is the result of a combination of wellbore and reservoir effects. For most of the life of a reservoir, reservoir effects dominate the pressure response of the system, and the conventional pressure transient test equations and analyses apply accurately. However, for cases of transient flow, wellbore effects (*i.e.*, storage of the fluid in the wellbore or wellbore storage) distorts and even dominates the reservoir pressure and rate response particularly at early times.

A schematic case of wellbore storage effects imposed on a system is illustrated in the pressure response shown in Fig. 2.2. A complete knowledge of these "wellbore effects" would permit the "correction" of these effects (using a process known as "deconvolution") which would provide interpretation and analysis of well tests for early and very early data (as these are the most distorted data). Simply put, the goal of this work (and of deconvolution in general) is to correct the pressure data taken at early times which are affected by "wellbore storage." Lacking the ability to "correct" these data means that we must wait for the distortion of the data to diminish (sometimes only a few hours, but possibly months or years for very low permeability gas reservoirs). As well tests are often run for as short as economically feasible for a particular well, many well tests are often completely distorted by wellbore storage effects.

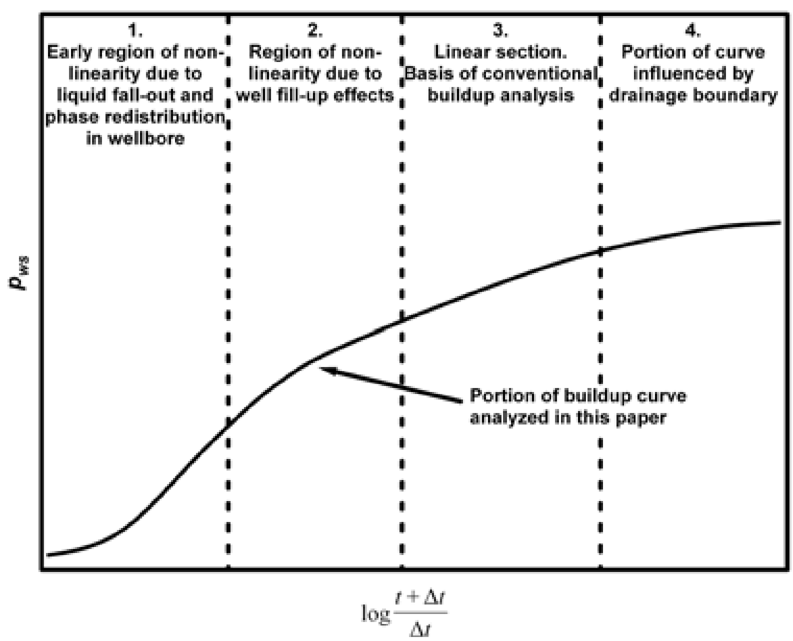

Figure 2.2 — Typical pressure buildup plot (from Russell[1]).

These "wellbore" effects have been labeled as "wellbore dynamics" by Mattar and Santo[8], and these effects include the following components: (one or more effects may act at any given time)

- Liquid influx/efflux.
- Phase redistribution.
- Wellbore and near-wellbore cleanup.
- Plugging.
- Recorder effects: drift, hysteresis, malfunction, temperature sensitivity, and fluid PVT changes.
- Gas/oil solution/liberation.
- Retrograde condensation.
- Diverse effects such as leaks, geotidal/microseismic.

2.2 The Wellbore Storage Effect

Since its introduction by van Everdingen and Hurst[9] in 1949, the issue of wellbore storage distortion has been extensively treated in the Petroleum Engineering literature. In 1970, Agarwal *et al.*[10] and Wattenbarger and Ramey[11] provided the theoretical detail (as well as analytical and numerical solutions) to support the base relations put forth by van Everdingen and Hurst[9]. The theoretical issues are relatively straightforward, the wellbore and reservoir are separate models coupled together, influences in the wellbore affect the reservoir and vice-versa. For the purpose of this work we treat the "simple" case of a constant wellbore storage behavior. This condition should be applicable in the vast majority of cases in practice, and it provides us a basis for extending beyond the constant wellbore storage case in later work.

2.2.1 Theoretical Developments

Whenever a well is shut in, fluid from the formation will flow into the wellbore until equilibrium conditions are reached. Similarly, a part of the fluid produced when a well is put on production is the fluid that was present is the wellbore prior to the opening of the well. This "ability of the well to store and unload fluids" (Raghavan[12]) is the definition of wellbore storage.

$$q_{wb} = -\frac{C}{B}\frac{dp_{wf}}{dt} \tag{2.1}$$

Where q_{wb} represents the rate at which the wellbore "unloads" fluids, and C represents the storage constant of the well. In the specific case where the wellbore unloading is entirely due to fluid expansion, then the wellbore storage constant is defined by: (Ramey[13])

$$C = \frac{\Delta V}{\Delta p} \tag{2.2}$$

Where ΔV is the change in volume of fluid in the wellbore — at *wellbore* conditions — and Δp is the change in bottomhole pressure.

When the wellbore is filled with a single fluid phase, Eq. 2.2 becomes

$$C = V_w c \tag{2.3}$$

where V_w is the total wellbore volume and c is the compressiblility of the fluid in the wellbore at wellbore conditions. The use of dimensionless pressure functions in most of the derivations of this work leads to the use of a dimensionless wellbore storage coefficient, C_D.

$$C_D = 0.894\frac{C}{\phi c_t h r_w^2} \tag{2.4}$$

As such, wellbore storage affects the sandface flowrate, causing a lag in the sandface flowrate relative to any change in the surface flowrate. The surface flowrate is the sum of the wellbore rate (q_{wb}) and the sandface rate (q_{sf}) — *i.e.,* the sum of the wellbore (unloading) rate and the sandface flowrate:

$$q = q_{sf} + q_{wb} \tag{2.5}$$

van Everdigen and Hurst[9] expressed the rigorous sandface flowrate relation for wellbore storage and skin using constant wellbore storage coefficient. The relation is given in dimensionless form as:

$$q_D(t_D) = 1 - C_D \frac{dp_{wD}}{dt_D} \tag{1.2}$$

We will make frequent use of this relation in this study, since it directly links the sandface flowrate (for which we do not have any direct measurements) to the wellbore pressure (for which we typically do have direct and accurate measurements).

2.2.2 Practical Issues

For more than 40 years, a time-dependent wellbore storage profile has been reported in the technical literature [Hegeman *et al.*[14]]. When this phenomenon occurs, it makes the application of well test analysis techniques which are based on the constant wellbore storage assumption — such as type-curve matching — very difficult. A changing wellbore storage condition occurs when the fluid compressibility in the wellbore (c, defined in Eq. 2.3) varies with changing pressure (or more appropriately, time). Fortunately, *such variations in the wellbore storage coefficient are most often negligible*. Well tests strongly affected by this phenomenon include occurrences of wellbore phase redistribution (segregation), and injection well testing.

2.3 Sandface Flowrate Estimators

Blasingame *et al.*[15] proposed five different methods of calculating sandface rates from pressure data for the *constant wellbore storage case*. These methods will be useful in the implementation of the computational module since most of the implemented methods require the knowledge (or an estimate) of the sandface flowrates.

<u>Method 1:</u> Definition of sandface flowrate (exact)

$$q_D = 1 - C_D \frac{dp_{wD}}{dt_D} = 1 - \frac{\frac{d}{dt}\left[\Delta p_w\right]}{m_{wbs}} \tag{1.2}$$

<u>Method 2:</u> Alternative calculation of sandface flowrate based on Method 1 (exact)

$$Q_D = t_D - C_D p_{wD} = t - \frac{\Delta p_w}{m_{wbs}} \tag{2.6}$$

$$q_{D_1} = \frac{d}{dt}\left[Q_D(t)\right] \tag{2.7}$$

<u>Method 3:</u> Average sandface flowrate calculation (exact)

$$q_{Di} = 1 - C_D \frac{p_{wD}}{t_D} = 1 - \frac{1}{m_{wbs}} \frac{\Delta p_w}{t} \tag{1.2}$$

$$t_{D_i} = \frac{t_D}{2} \tag{2.8}$$

Method 4: Semi-empirical sandface flowrate calculation — assumes $C_D = t_D/p_{wD}$ (approximate)

$$q_{D_2} = 1 - \frac{t_D}{p_{wD}} \frac{dp_{wD}}{dt_D} = 1 - \frac{t}{\Delta p_w} \frac{d}{dt}\left[\Delta p\right] \tag{2.9}$$

$$t_{D_2} = \frac{t_D}{2} \tag{2.10}$$

Method 5: Semi-empirical sandface flowrate calculation — assumes $C_D = t_D/p_{wDi}$ (approximate)

$$q_{D_3} = 1 - \frac{p_{wDid}}{p_{wDi}} = 1 - \frac{p_{wD} - p_{wDi}}{p_{wDi}} = 1 - \frac{t}{\Delta p_{wi}} \frac{d}{dt}\left[\Delta p_{wi}\right] \tag{2.11}$$

$$t_{D_3} = \frac{t_D}{4} \tag{2.12}$$

2.4 Theoretical Development: Superposition Principle and Convolution

Convolution is a mathematical operator which, using two functions f and g, produces a third function commonly noted as $f*g$ representing the amount of overlap between f and a reversed and shifted version of g. The convolution operation is defined as:

$$(f * g)(t) = \int_0^t f(\tau)\, g(t-\tau) d\tau \tag{2.13}$$

The convolution operation can by expressed in discrete form as:

$$(f * g)(t) \approx \sum_{i=1}^{n} f(\tau_{i-1}) g(t - \tau_{i-1}) \Delta\tau \tag{2.14}$$

The principle of superposition (or convolution) states that, for a linear system, a linear combination of solutions for a system is also a solution to the same linear system. The superposition (or convolution) principle applies to linear systems of algebraic equations, and for our field of study — linear partial differential equations (*i.e.*, the diffusivity equation for flow in porous media)

In well test analysis, the superposition principle is used to construct reservoir response functions, to represent various reservoir boundaries (by superposition in space), and to determine variable rate reservoir responses (using superposition in time). However, we must always keep in mind when applying this principle that it is only valid for *linear* systems that is when nonlinearities are present (*e.g.* gas flow), principle of superposition is not directly applicable. In those cases linearization (*via* the pseudopressure transform) must be performed in order to apply the superposition principle to the tranformed system.

The early work by Duhamel[16] on heat transfer has since then been used in numerous engineering domains. Adapted to our domain, petroleum engineering, Duhamel's principle states that the observed pressure drop is the convolution of the input rate function and the derivative of the constant-rate pressure response — at t=0 the system is assumed to be in equilibrium (*i.e.*, $p(r,t=0) = p_i$).

For reference, the convolution integral is defined as:

$$\Delta p(t) = \int_0^t q(t-\tau)p_u'(\tau)d\tau \tag{2.15}$$

Eq. 2.15 can be written in a discrete form by assuming that the rate change can be discretized as a series of rate changes:

$$\Delta p(t) = \sum_{i=1}^{n}(q_i - q_{i-1})(p_u(t - t_{i-1})) \tag{2.16}$$

van Everdingen and Hurst[8] introduced the use of Duhamel's principle in the analysis of variable-rate well-test data and they utilized Duhamel's principle to obtain dimensionless wellbore pressure-drop responses for a continuously (smoothly) varying flowrate. The underlying idea was to introduce a method to convolve/superimpose the constant rate pressure response with a continuous (smooth) rate profile to produce the variable rate wellbore pressure-drop response.

Odeh and Jones[17], Agarwal[18], Soliman[19], Stewart, Wittman and Meunier[20], Fetkovich and Vienot[3], among others, applied the convolution guidelines in various settings. However, these methods are inherently restricted by the use of a particular model for the constant rate pressure function (*i.e.*, presumed reservoir model) used in the convolution integral.

CHAPTER III

EXPLICIT METHODS FOR THE ANALYSIS OF WELLBORE STORAGE DISTORTED WELL TEST DATA

This work was put forth as an attempt to provide a set of simple, explicit deconvolution formulas that could be used on wellbore storage distorted pressure transient test data. We evaluated a very old "correction" method by Russell[1] and found this method to be *unacceptable for all applications*. We also evaluated the "material balance deconvolution" [Johnston[21]] for the purpose of evaluating pressure transient test data *without* any sandface rate information. This approach was successful and should be considered sufficiently accurate to be used as a standard tool for field applications.

The other "major" method considered was the direct β-deconvolution algorithm modified to estimate the β-parameter from pressure rather than flowrate data as originally proposed by van Everdingen[4] and Hurst [5]. The modification of the β-deconvolution algorithm (given only in terms of pressure variables) was also successful.

3.1. Russell Method (1966): The pressure "correction" function given by Russell[1] is given as:

$$\frac{[p_{ws}(\Delta t) - p_{wf}(\Delta t = 0)]}{\left[1 - \dfrac{1}{C_2 \Delta t}\right]} = f(\Delta t = 1\,\text{hr}) + m_{sl} \log(\Delta t) \tag{3.1}$$

Where the C_2-term is derived rigorously using Russell's assumptions of the system. The C_2-term is used as an arbitrary constant to be optimized. In short, the Russell method has an elegant mathematical formulation, but ultimately, we believe that this formulation does not represent the wellbore storage condition, and hence, we do not recommend the Russell method under any circumstances.

3.2. Rate Normalization

Gladfelter, Tracy and Wilsey[2] introduced the "rate normalization" deconvolution approach — which, in their words "permits direct measurement of the cause of low well productivity." The objective of rate normalization is to remove/correct the effects of the variable rate from the observed pressure data. Rate normalization can also be defined as an approximation to convolution integral (Raghavan[11]).

$$\Delta p(t) \approx q(t)\, p_u(t) \tag{3.2}$$

Where p_u is the constant rate pressure response. Rate normalization has been employed for a number of applications in well test analysis. For the specific application of "rate normalization" deconvolution, we must recognize that the approach is *approximate* — and while this method does provide some "correction" capabilities, it is basically *a technique that can be used for pressure data influenced by continuously varying flowrates*. Most notably, Fetkovich and Vienot[3], Winestock and Colpitts[22] (1965, pressure transient test analysis) and Doublet *et al.*[23] (1994, production data analysis) have demonstrated the effectiveness of "rate normalization" deconvolution (albeit for specialized cases). In particular, for the wellbore storage domination and distortion regimes, rate normalization can provide a reasonable approximation of the no wellbore storage solution. For this inifinite-acting radial flow case, rate normalization yields an erroneous estimate of the skin factor by introducing a shift on the semilog straight line (obvioulsy, the sandface rate profile must be known). This last point, however, makes the application of rate normalization techniques very limited in our particular problem — we do not have measurements of sandface flowrate. Therefore, this method must be applied using an estimate of the downhole rate (see rate estimation relations in Chapter II) — which will definitely introduce errors in the deconvolution process. Such issues make rate normalization a "zero-order" approximation — that is, rate normalization results should be considered as a guide, but not relied upon as the best methodology.

3.3. Material Balance Deconvolution

The relations for the deconvolution of wellbore storage distorted well test data using material balance deconvolution are provided in Appendix D. The wellbore storage-based, *material balance time function* for the pressure buildup case is given as:

$$\Delta t_{mb,BU} = \frac{N_{p,wbs,BU}}{1-q_{wbs,BU}} = \frac{\Delta t - \dfrac{1}{m_{wbs}}\Delta p_{ws}}{1-\dfrac{1}{m_{wbs}}\dfrac{d}{d\Delta t}[\Delta p_{ws}]} \tag{3.3}$$

And the wellbore storage-based, *rate-normalized pressure drop function* for the pressure buildup case is given as:

$$\Delta p_{s,BU} = \frac{\Delta p_{ws}}{1-q_{wbs,BU}} = \frac{1}{1-\dfrac{1}{m_{wbs}}\dfrac{d}{d\Delta t}[\Delta p_{ws}]}\Delta p_{ws} \tag{3.4}$$

In the material balance deconvolution formulation the $\Delta t_{mb,BU}$ function is used in place of the time function, in whatever fashion is required — plotting data functions, modeling, etc. And the $\Delta p_{s,BU}$ function is used as a pressure drop function — in any appropriate manner that pressure drop would be employed.

3.4. β ("Beta") Deconvolution

We also present the application of our new β-deconvolution algorithm derived from wellbore-storage distorted pressure functions (see Appendices B and C). The final result developed for application in our present work is given by: (this is the general form for pressure drawdown or buildup cases).

$$\Delta p_s = \Delta p_w + \frac{\Delta p_{wd}}{(\Delta p_w - \Delta p_{wd})}\Delta p_{wid} \tag{3.5}$$

Where, for the pressure buildup case, we have:

$$\Delta p_w = p_{ws} - p_{wf}(\Delta t = 0) \qquad \text{(pressure drop)} \tag{3.6}$$

$$\Delta p_{wd} = \Delta t \frac{d\Delta p_w}{d\Delta t} \qquad \text{(pressure drop derivative)} \tag{3.7}$$

$$\Delta p_{wi} = \frac{1}{\Delta t}\int_0^{\Delta t} \Delta p_w d\tau \qquad \text{(pressure drop integral)} \tag{3.8}$$

$$\Delta p_{wid} = \Delta t \frac{d\Delta p_{wi}}{d\Delta t} \qquad \text{(pressure drop integral-derivative)} \tag{3.9}$$

The more "rigorous" β-deconvolution algorithm [*i.e.*, where an exponential rate profile is required (Eqs. 1.1 and 1.3), and the β-term is constant (*i.e.*, *not* time-dependent as we have derived in this case)], could be applied [Kuchuk[7]] — but the constant β formulation will not perform as well as the time-dependent (and approximate) β-deconvolution algorithm that we have proposed in this work (see Appendix B for full details of the β-deconvolution algorithms).

Of the methods reviewed/developed in this work, we believe that our modifications of the "material balance deconvolution" approach and the β-deconvolution algorithm should perform well in field applications. We note that both of these methods have been specifically formulated for the analysis of wellbore storage distorted pressure transient test data — the relations in this chapter are presented for the purpose of field analysis. For a complete treatment of the β-deconvolution algorithm, see Appendices B and C; and for a complete treatment of the material balance deconvolution method (for wellbore storage applications), see Appendix D.

CHAPTER IV

EXAMPLE APPLICATIONS

4.1 Demonstration using a Synthetic Data Case

In this example we provide a synthetic case for a well producing at a constant flowrate in an infinite-acting reservoir, with wellbore storage effects. In this synthetic example case the dimensionless wellbore storage coefficient (C_D) is set at 1x10^6, and the results of this model are shown by the solid red line in Fig. 4.1. The "no wellbore storage" solution is shown as the solid black line in Fig. 4.1.

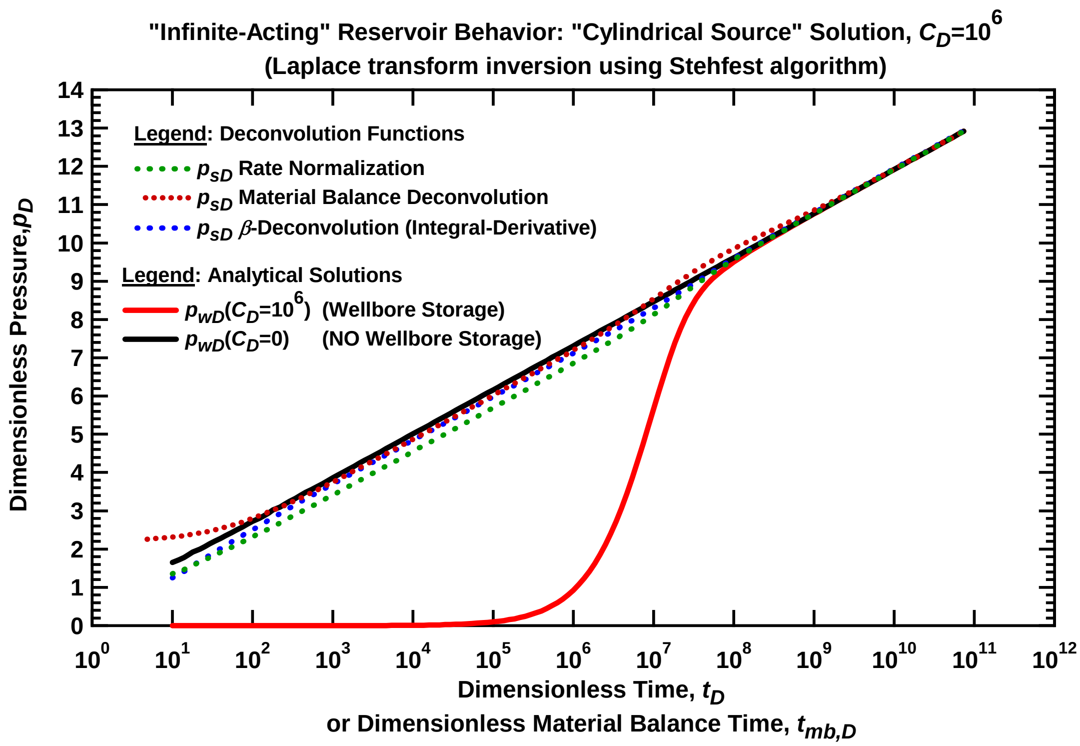

Figure 4.1 — Synthetic example using various deconvolution techniques (infinite-acting reservoir case with wellbore storage effects)

In this example we present the performance of the various deconvolution techniques in Fig. 4.1, and we provide a synopsis of the performance of each technique below.

- *Rate Normalization*: In this case the rate normalization process yields excellent results (see dashed green line in Fig. 4.1), with the exception of the fact that (as expected) the rate normalized data trend is shifted from the exact solution (the black line trend). This implies that, for this case (i.e., a well in an infinite-acting homogeneous reservoir), the estimated permeability (from the slope of the trend) should be quite accurate — however; the skin factor (which is estimated from the intercept of the pressure trend) will be in error. The level of error in the skin factor estimated from the rate normalization technique will depend on the level of wellbore storage imposed on the system (more wellbore storage, more error). The material balance deconvolution approach should resolve the error in the skin factor as this approach provides a time correction as well as the pressure drop correction provided by the rate normalization approach.

- *Material Balance Deconvolution*: The material balance deconvolution technique performs extremely well for this case, with only minor discrepancies at the start of the data set and at the point where the wellbore storage and no wellbore storage solutions merge. This performance of this method is excellent, and suggests that, based on the simplicity of the material balance deconvolution method, this is probably the most practical approach for the analysis of pressure transient test data distorted by wellbore storage.
- *β-Deconvolution*: The β-deconvolution technique also performs very well for this case (surprisingly well, in fact). This performance is most likely due to the analytic nature of the "data" (*i.e.*, the synthetic dimensionless pressure and auxiliary functions). In other words, the fact that we used the analytical (*i.e.*, exact) solutions in this process most likely accounts for the remarkable success of the β-deconvolution technique for this example.

4.2. Demonstration using a Field Case

This example is taken from the literature (Bourdet[24]). In this case we provide the explicit deconvolu-tion of field well test data using the methods presented in this work. The data are taken from a pressure buildup test and should be considered reasonably well behaved for field data (*i.e.*, average or a little better than average). The deconvolution "conversion" results are shown in Fig. 4.2 (semilog format) and Fig. 4.3 (log-log format) — different plotting formats (semilog and log-log) are used to emphasize the character in the data.

The most positive aspect of the application of the explicit deconvolution methods in this example is that we gain approximately 1.5 log cycles of results which can be analyzed using conventional well test interpretation methods (*i.e.*, the data in the range from $0.01 < \Delta t < 4$ hr are effectively deconvolved, and can be analyzed using "traditional" semilog or log-log analysis/interpretation methods for well test data)

As comment, we have reviewed the given data and believe that the data are of sufficient quality to provide a reasonably competent deconvolution using explicit methods (*i.e.*, rate normalization, material balance deconvolution, and β-deconvolution). We note that these data are clearly distorted (if not dominated) by wellbore storage effects, and that the data have a "typical" quality profile for field data.

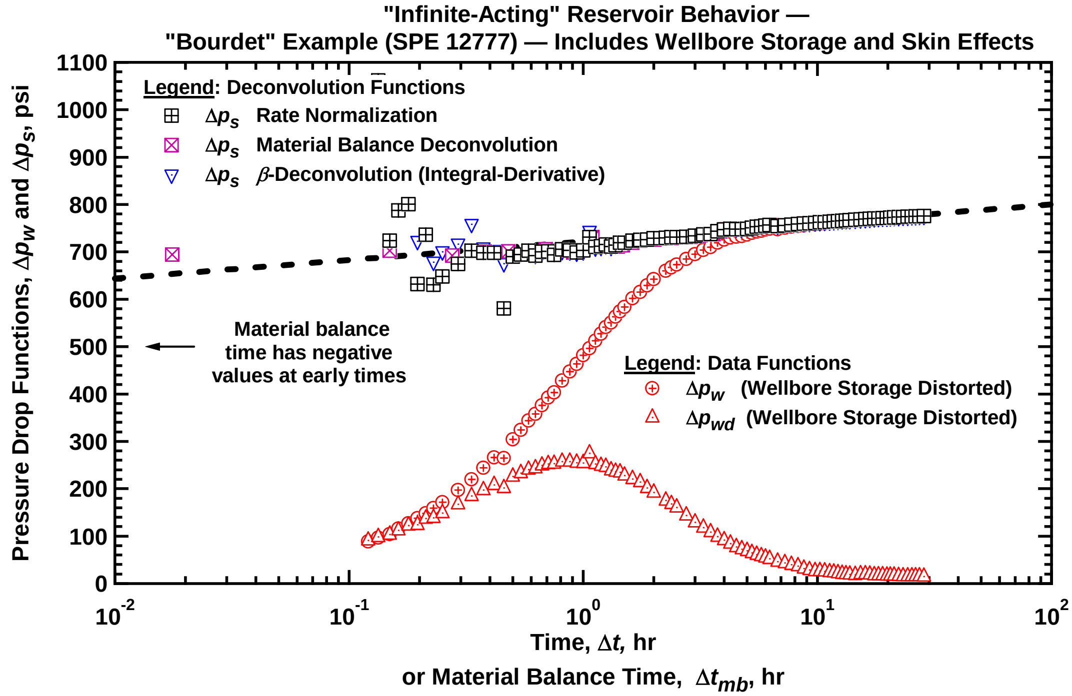

Figure 4.2 — (Semilog plot) Bourdet[24] field example using various deconvolution techniques (infinite-acting reservoir case with wellbore storage effects)

- *Rate Normalization*: From Figs. 4.2 and 4.3 we note that the rate normalization profile is more stable than the β-deconvolution profile, but is not as accurate as the material balance deconvolution profile. In particular, the rate normalization profile is slightly unstable at early times. In the context of comparison, we would rank the performance of the rate normalization method for this case as good.

- *Material Balance Deconvolution*: The response of the material balance deconvolution method as shown in Figs. 4.2 and 4.3 appears to be the most accurate deconvolution. We will note that we encountered negative values in the material balance time function (due to the negative "rates" computed from the wellbore storage-distorted data — these negative rates also affected the rate normalization and β-deconvolution results, as indicated by the off-trend performance at early times). Phenomena such as the calculation of negative rates should be considered "normal" given the quality of data. From a conventional analysis of these data (not presented), the pressure derivative function (distorted data) suggests a slightly changing wellbore storage scenario — which is one plausible explanation of the issues with the calculation of the rates at early times.

- *β-Deconvolution*: The β-deconvolution results shown in Figs. 4.2 and 4.3 are reasonably stable, and suggest a good performance of this method for this particular data set. We had hoped for more stability in the β-deconvolution at early times, but all of the explicit deconvolution methods were affected at early times for this case and the β-deconvolution will not be immune to such effects.

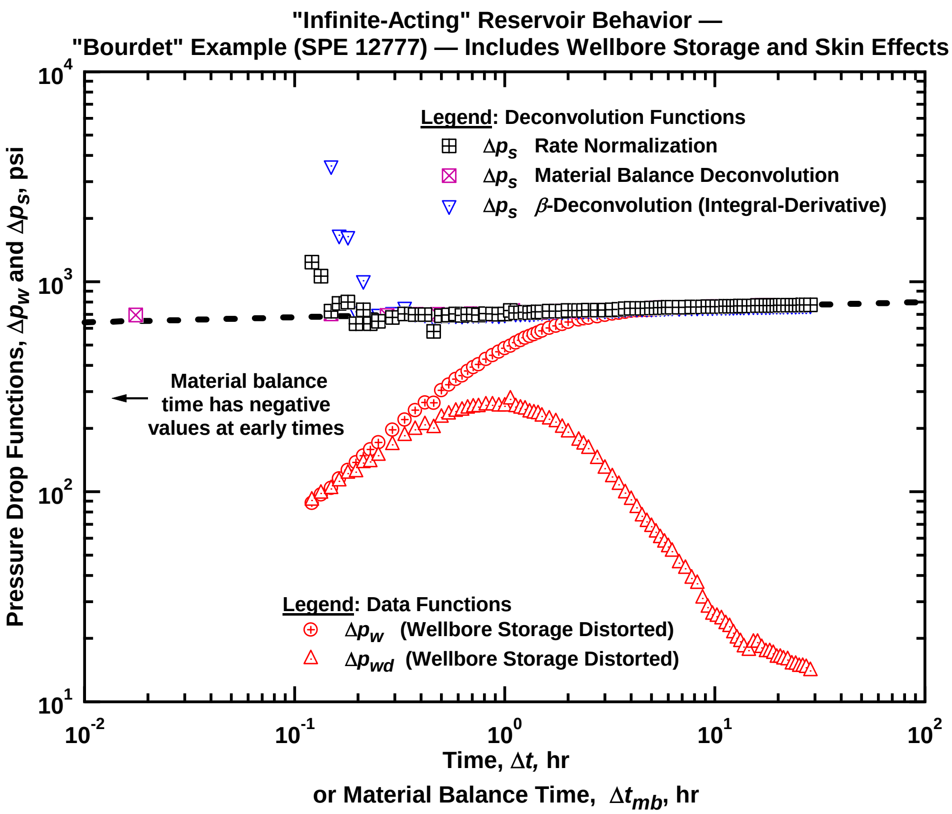

Figure 4.3 — (Log-log plot) Bourdet[24] field example using various deconvolution techniques (infinite-acting reservoir case with wellbore storage effects)

As closure commentary regarding this example, we believe that this example does indicate success for the methods employed. Obviously the degree of success for any particular case will rely on the quality and relevance of the data. As for a general recommendation, we encourage vigilance in data acquisition, and care in the application of the methods used in this work. While these methods are theoretically supported, these methods are highly susceptible to data errors and bias.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

5.1 Summary and Conclusions

We summarize this work as follows — the expectation of success for the deconvolution of pressure transient test data using *explicit* deconvolution techniques (rate normalization, material balance deconvolution, and β-deconvolution) must be tempered with the knowledge that *we create an inherent bias when we do not use the rate profile* — but rather, we infer the rate profile from a wellbore storage model imposed (in some manner) on the pressure data.

Having made those qualifying comments, we should also recognize that the theory for each method does provide confidence that these methods should perform well in practice. The primary concern must be the quality and relevance of the pressure data. The following conclusions have been derived from this work:

Wellbore Storage Rate Models:

Governing relation(s): [$m_{wbs} = qB/(24C_s)$, where C_s is estimated from early time pressure data]

Pressure Drawdown Case:

$$\Delta p_{wf} = p_i - p_{wf} \tag{5.1a}$$

$$q_{wbs,DD} = 1 - \frac{1}{m_{wbs}} \frac{d}{dt}[\Delta p_{wf}] \tag{5.1b}$$

$$N_{p,wbs,DD} = \int_0^t q_{wbs,DD}\, dt = t - \frac{1}{m_{wbs}} \Delta p_{wf} \tag{5.1c}$$

Pressure Buildup Case:

$$\Delta p_{ws} = p_{ws} - p_{wf}(\Delta t = 0) \tag{5.2a}$$

$$q_{wbs,BU} = \frac{1}{m_{wbs}} \frac{d}{d\Delta t}[\Delta p_{ws}] \tag{5.2b}$$

$$N_{p,wbs,BU} = \int_0^{\Delta t} (1 - q_{wbs,BU})\, d\Delta t = \Delta t - \frac{1}{m_{wbs}} \Delta p_{ws} \tag{5.2c}$$

Conclusion(s):

- Strength: Models are rigorous (based on consistent theory).
- Weakness: Assumption of C_s = constant.

Rate Normalization:

Governing relation(s):

$$\frac{\Delta p_{wf}}{q_{wbs,DD}} \; vs. \, t \qquad \text{(pressure drawdown case)} \qquad (5.3)$$

$$\frac{\Delta p_{ws}}{1-q_{wbs,BU}} \; vs. \, \Delta t \qquad \text{(pressure buildup case)} \qquad (5.4)$$

Conclusion(s):

- Strength: Rate normalization is a reasonably approximate correction.
- Weakness: Pressure drop function is in error by a "shift" (*i.e.*, a constant value).

Material Balance Deconvolution:

Governing relation(s):

$$\frac{\Delta p_{wf}}{q_{wbs,DD}} \; vs. \, \frac{N_{p,wbs,DD}}{q_{wbs,DD}} \qquad \text{(pressure drawdown case)} \qquad (5.5)$$

$$\frac{\Delta p_{ws}}{1-q_{wbs,BU}} \; vs. \, \frac{N_{p,wbs,BU}}{1-q_{wbs,BU}} \qquad \text{(pressure buildup case)} \qquad (5.6)$$

Conclusion(s):

- Strength: Very good correction, essentially best approximate method for practice.
- Weakness: Slight "bump" in correction near end of wellbore storage trend (steep rate change).

β-Deconvolution:

Governing relation(s): (integral-derivative formulation for $\beta(t)$ approximation)

$$\Delta p_s \approx \Delta p_w + \frac{\Delta p_{wd}}{(\Delta p_w - \Delta p_{wd})} \Delta p_{wid} \qquad \text{(general — pressure drawdown or buildup case)} \qquad (5.7a)$$

where:

$$\Delta p_{wd} = t \frac{d\Delta p_w}{dt} \qquad \text{(pressure drawdown case)} \qquad (5.7b)$$

$$\Delta p_{wd} = \Delta t \frac{d\Delta p_w}{d\Delta t} \qquad \text{(pressure buildup case)} \qquad (5.7c)$$

$$\Delta p_{wid} = t \frac{d\Delta p_{wi}}{dt} \quad \text{where} \quad \Delta p_{wi} = \frac{1}{t}\int_0^t \Delta p_w d\tau \qquad \text{(pressure drawdown case)} \qquad (5.7d)$$

$$\Delta p_{wid} = \Delta t \frac{d\Delta p_{wi}}{d\Delta t} \quad \text{where} \quad \Delta p_{wi} = \frac{1}{\Delta t}\int_0^{\Delta t} \Delta p_w d\tau \qquad \text{(pressure buildup case)} \qquad (5.7e)$$

β-Deconvolution: (continued)

Conclusion(s):

- Strength: The "integral-derivative" formulation (Eq. 5.7a) appears to be most accurate.
- Weakness: Erratic at very early times, also needs an exhaustive validation.

5.2 Recommendations for Future Work

The future work on this topic should consider mechanisms for further improvements in the material balance deconvolution and β-deconvolution methods as these methods are applied to wellbore storage distorted well test data.

NOMENCLATURE

Dimensionless Variables:

C_D = dimensionless wellbore storage coefficient

t_D = dimensionless time

p_D = dimensionless pressure

q_D = dimensionless rate

Field Variables

B_o = oil formation volume factor, RB/STB

c = fluid compressibility, 1/psi

C_2 = arbitrary constant, hr^{-1}

h = net pay thickness, ft

k = formation permeability, md

m_{wbs} = slope of wellbore storage dominated regime, psi/hr

N_p = cumulative oil production, vol

p = reservoir pressure, psi

$p_{wf(\Delta t=0)}$ = wellbore pressure at the time of shut-in, psia

q = volumetric production rate, STB/D

r = radial distance, ft

s = skin factor

u = Laplace variable

t = producting time, hr

Δt = shut-in time, hr

Greek

γ = Euler's constant, $\gamma \approx 0.557216 \ldots$

β = "*beta-deconvolution*" variable, hr^{-1}

μ = viscosity, cp

ρ = fluid density, lb/cuft

Subscripts

a = after production period

d = "well-testing" derivative

D = dimensionless quantity

f = to pressure in the formation

i = initial reservoir conditions

i = "well-testing" pressure integral function

n = index number

w = conditions at wellbore radius

Supercripts

$'$ = derivative of a function

i = integral of a function

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APPENDIX A

RUSSELL METHOD FOR "CORRECTION" OF WELL TEST DATA DISTORTED BY WELLBORE STORAGE (RUSSELL, 1966)

The purpose of this Appendix is to summarize the work of Russell[1] regarding an approximation to "correct" well test data distorted by wellbore storage. We begin by noting that *this method does not provide results which can be considered useful in the context of modern well test analysis and interpretation methods*.

As a starting point, we consider the well/reservoir configuration as defined by Russell for this case — schematic for this case is shown in Fig. A.1:

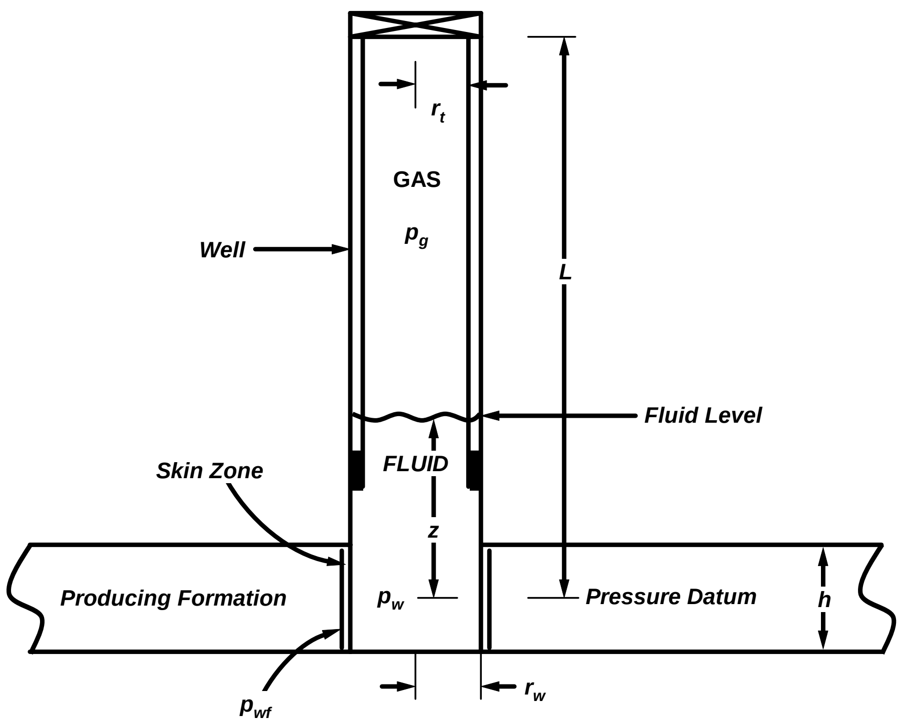

Figure A.1 — Schematic diagram of well and formation during pressure build-up (Russell[1]).

Russell made the following assumptions in the derivation of his wellbore storage "correction" solution:

- Completely penetrating well in an infinite reservoir.
- Slightly compressible liquid (constant compressibility).
- Constant fluid viscosity.
- Single-phase liquid flow in the reservoir.
- Gravity and capillary pressure neglected.
- Constant permeability.
- Horizontal radial flow (no vertical flow).
- Ideal gas (for the gas cushion in the well).

Although the Russell method was derived from analytical considerations, the problem actually solved is a variation of the true wellbore storage problem, derived using Russell's representation of the gas and liquid volume in the wellbore as the "wellbore storage" term. This formulation is not based on the same physics as the wellbore storage problem where the wellbore production (at the start of production or shut-in) is inversely proportional to the compressibility of the fluids the wellbore (or the influence of a rising/falling liquid level).

In short, Russell[1] approximated the wellbore storage concept in order to develop his "storage" function, presumably for the correction of wellbore storage distortion in pressure buildup tests. In field units, Russell's wellbore storage correction is given as:

$$p_{ws}(\Delta t) - p_{wf}(\Delta t = 0) = 162.6\frac{qB\mu}{kh}\left[1 - \frac{1}{C_2\Delta t}\right]\left[\log(\Delta t) + \log\frac{k}{\phi\mu c r_w^2} - 3.23 + 0.87s\right] \quad \text{(A.1)}$$

Where the C_2-term is defined as:

$$C_2 = 0.000528\frac{kh}{r_t^2\mu}\left[\rho g + \frac{1}{L}p_{wf}(\Delta t = 0)\right] \quad \text{(A.2)}$$

Combining Eqs. A.1 and A.2 into a plotting function format, we obtain:

$$\frac{[p_{ws}(\Delta t) - p_{wf}(\Delta t = 0)]}{\left[1 - \frac{1}{C_2\Delta t}\right]} = f(\Delta t = 1\,\text{hr}) + m_{sl}\log(\Delta t) \quad \text{(A.3)}$$

Russell treated the C_2-term as an arbitrary constant to be optimized for analysis — in other words, the C_2-term is the "correction" factor for the Russell method.

As prescribed by Russell, the C_2-term is obtained using a trial-and-error sequence which yields a straight line when the left-hand-side term of Eq. A.3 is plotted versus log(Δt). Where the general form of the y-axis correction term prescribed by Eq. A.3 is:

$$y = \frac{[p_{ws}(\Delta t) - p_{wf}(\Delta t = 0)]}{\left[1 - \dfrac{1}{C_2 \Delta t}\right]} \tag{A.4}$$

A schematic of the Russell method is shown in Fig. A.2, where we note Russell's interpretation of the effect of the C_2-term (*i.e.*, where C_2 is too large and C_2 is too small).

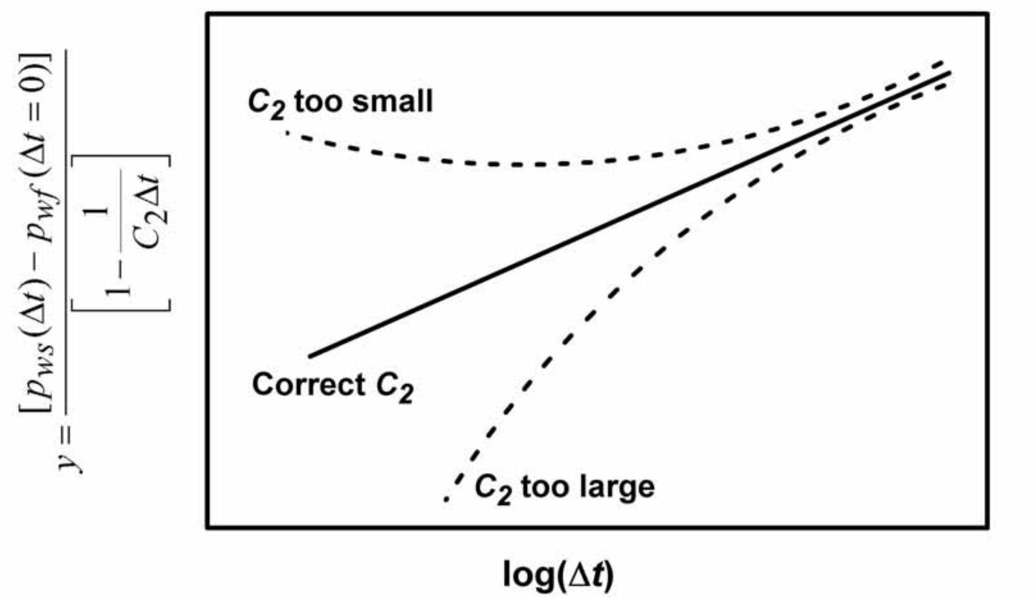

Figure A.2 — Schematic plot showing determination of the correct C_2 value (Russell[1]).

Once the C_2-term is established, the *kh*-product is estimated using:

$$kh = 162.6 \frac{qB\mu}{m_{sl}} \tag{A.5}$$

And the skin factor can be estimated using:

$$s = 1.151 \left[\frac{f(\Delta t = 1\,\text{hr})}{m_{sl}} - \log \frac{k}{\phi \mu c r_w^2} + 3.23 \right] \tag{A.6}$$

Russell[1] also proposed a methodology to obtain the "extrapolated" pressure using the results of his correction procedure. We chose not to demonstrate this methodology; the interested reader is referred to Russell[1] for more detail.

We present two example cases to demonstrate the shortcomings of the Russell method (lack of accuracy, limited range of application). The first example is for "Well B," an example taken from the original Russell reference [Russell[1]]. The second example is taken from data in the reference paper by Meunier, *et al.*[24].

Example 1: (Well B, Wilcox Sand formation) Russell presented the data and analysis for the "Well B" case as a "typical" example application of his wellbore storage correction method. We have reproduced this example and extended the results by presenting a large set of values for the C_2-term to illustrate the influence of this term on the performance of the Russell correction.

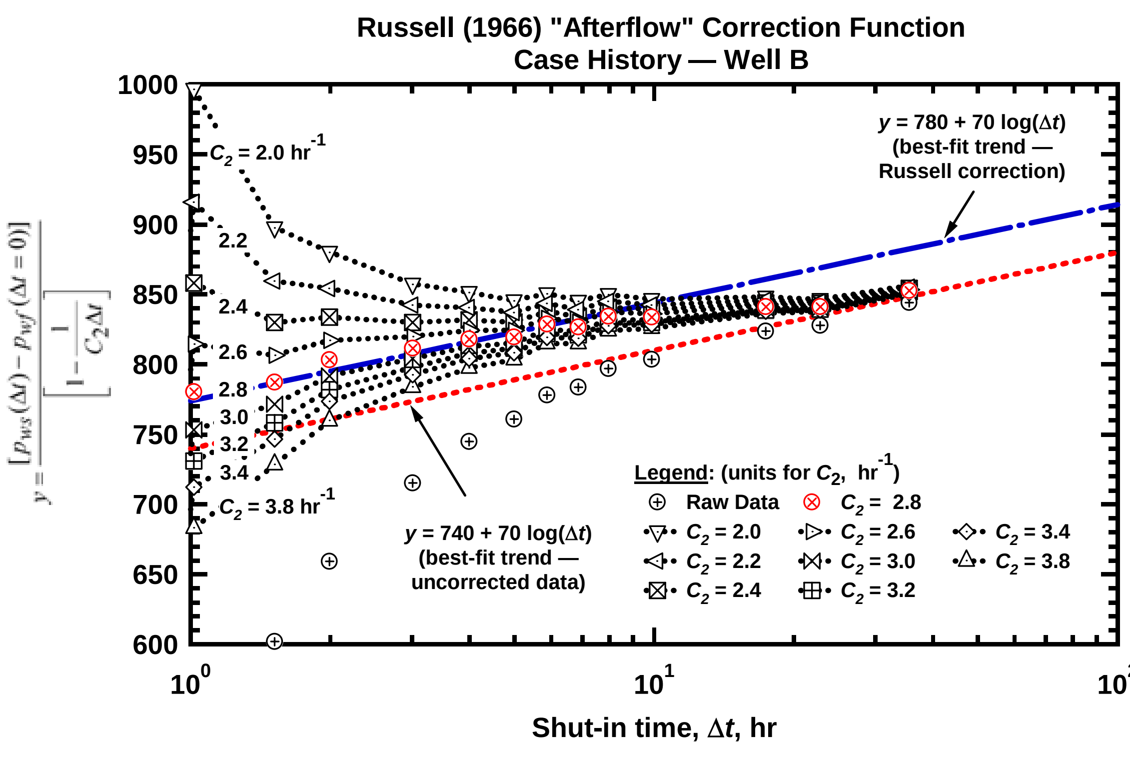

Figure A.3 — Afterflow analysis, Well B (data from Russell[1]). Approximate best fit obtained using $C_2 = 2.8\ \text{hr}^{-1}$.

For our reproduction of this case, we use C_2={2.0 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.8 hr^{-1}} in Eq. A.4, and we plot the results of this exercise on Fig. A.3. The value of the C_2-term for which most of the points form a straight line [y versus log(Δt)] is 2.8 hr^{-1}, and we obtain a straight-line slope (m_{sl}) of about 70 psi/log cycle.

A comparison of our results and those obtained by Russell is shown below.

Analysis	Conventional Analysis* p_{ws} versus log(Δt) m_{sl} (psi/log cycle)	Russell Correction Eq. A.4 versus log(Δt) m_{sl} (psi/log cycle)
Russell[1]	70	67 (C_2=3.0 hr^{-1})
This Study	70	70 (C_2=2.8 hr^{-1})

* Conventional analysis based on using the p_{ws} vs. log(Δt) for data which are not affected by wellbore storage effects. The "conventional" straight-line trend is constructed using the data in the region of $10 < \Delta t < 40$ hours.

As shown in Fig. A.3, our selection of C_2 = 2.8 hr^{-1} as the approximate best fit value appears to be the case for which the Russell correction yields an apparent straight line trend. Russell[1] noted that that C_2=2.75 hr^{-1} "might well have been chosen instead [of 3.0]."

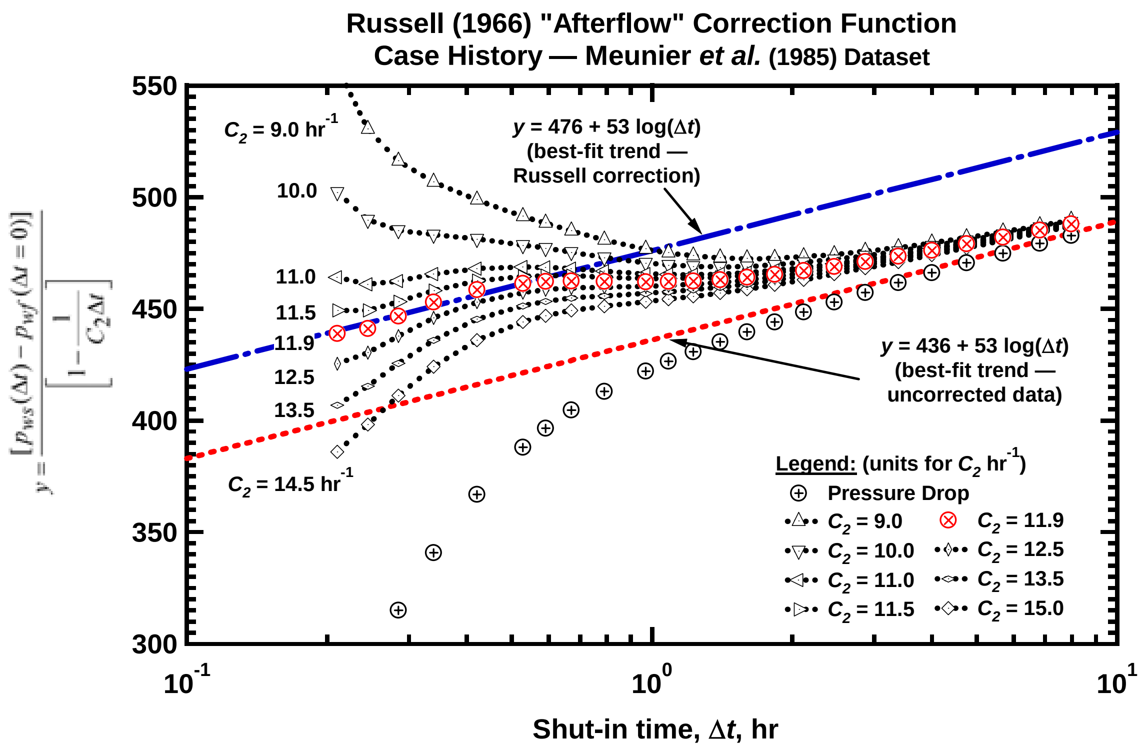

Figure A.4 — Afterflow analysis, Meunier *et al.*[25] data set. Approximate "best" fit obtained using C_2 = 11.9 hr^{-1}.

Example 2: The following example is the field case given by Meunier *et al.*[25]. We have applied the Russell "correction" method in this example and we used several values for the C_2-term to illustrate the influence of this term on the performance of the Russell correction. We use C_2={9.0 10.0, 11.0, 11.5, 11.9, 12.5, 13.5, 14.5 hr^{-1}} and we present our results in Fig. A.4. We obtained a slope value (m_{sl}) of about 53 psi/log cycle using the "best fit" value of the C_2-term 11.9 hr^{-1}.

In the analysis of Meunier *et al.*[25], value of the slope was reported as 57 psi/log cycle using the "sandface rate convolution" method.

If we consider the performance of the Russell method objectively as applied to the data of Meunier *et al.*[25], we would conclude that the "corrected" pressures (the symbols in Fig. A.4) are of little practical use. Obviously such data could not be used for pressure derivative analysis — even if we could accept the (very) approximate straight-line (*i.e.*, the corrected data) such data would yield very erroneous pressure derivative profiles.

APPENDIX B

DERIVATION OF THE β-DECONVOLUTION FORMULATION

We note that the lack of accuracy in flowrate measurements (when these exist) narrows the range of application of Gladfelter deconvolution method (*i.e.*, rate normalization). Van Everdingen[4] and Hurst[5] (separately) introduced an exponential model for the sandface rate during the wellbore storage distortion period of a pressure transient test. The exponential formulation of the flowrate function is given as:

$$q_D(t_D) = 1 - e^{-\beta t_D} \tag{B.1}$$

Eq. (B-1) is based on the empirical observations made by Van Everdingen and Hurst — and as extended by others such as Kuchuk[7] and Joseph and Koederitz[6].

Recalling the convolution theorem, we have:

$$p_{wD}(t_D) = \int_0^{t_D} q_D'(\tau)\, p_{sD}(t_D - \tau)\, d\tau \tag{B.2}$$

Taking the Laplace transform of Eq. B.2 yields:

$$\overline{p}_{wD}(u) = u\,\overline{q}_D(u)\,\overline{p}_{sD}(u) \tag{B.3}$$

Rearranging Eq. B.3 for the equivalent constant rate pressure drop function, $\overline{p}_{sD}(u)$, we obtain:

$$\overline{p}_{sD}(u) = \overline{p}_{wD}(u)\,\frac{1}{u\,\overline{q}_D(u)} \tag{B.4}$$

The Laplace transform of the rate profile (Eq. B.1) is:

$$\overline{q}_D(u) = \frac{1}{u} - \frac{1}{u+\beta} \tag{B.5}$$

Substituting Eq. B.5 into Eq. B.4, and then taking the inverse Laplace transformation of this result yields the "beta" deconvolution formula:

$$p_{sD}(t_D) = p_{wD}(t_D) + \frac{1}{\beta}\frac{dp_{wD}(t_D)}{dt_D} \tag{B.6}$$

Where we note that Eq. (B-6) is specifically valid *only for the exponential sandface flowrate profile* given by Eq. B-1. This may present a serious limitation in terms of practical application of the β-deconvolution method.

To alleviate the issue of the exponential sandface flowrate, we propose that Eq. B-6 be solved for the β-term. Once this identity is established, we will then develop methods for estimating the β-term from data.

After that we will use the identity (Eq. B.6) to estimate the pressure drop function for a constant production rate. Solving Eq. B.6 for the β-term, we have:

$$\beta = \frac{1}{p_{sD}(t_D) - p_{wD}(t_D)} \frac{dp_{wD}(t_D)}{dt_D} \tag{B.7}$$

Or, multiplying through Eq. B.7 by the C_D-term, we have

$$\beta C_D = \frac{1}{p_{sD}(t_D) - p_{wD}(t_D)} C_D \frac{dp_{wD}(t_D)}{dt_D} \tag{B.8}$$

Recalling the definition of the wellbore storage model, we have:

$$q_D(t_D) = 1 - C_D \frac{dp_{wD}(t_D)}{dt_D} \tag{B.9}$$

Assuming wellbore storage domination (*i.e.*, $q_D \approx 0$) at early times, then Eq. B.9 becomes:

$$C_D \frac{dp_{wD}(t_D)}{dt_D} \approx 1 \quad \text{(early time)} \tag{B.10}$$

Separating and integrating Eq. B.10 (our early time, wellbore storage domination result), we have:

$$p_{wD}(t_D) \approx \frac{t_D}{C_D} \quad \text{(early time)} \tag{B.11}$$

Substituting Eqs. B.10 and B.11 into Eq. B.8, we obtain:

$$\beta C_D = \frac{1}{p_{sD}(t_D) - \dfrac{t_D}{C_D}} \quad \text{(early time)} \tag{B.12}$$

Eq. B.12 suggests that we can "correlate" the βC_D product with t_D/C_D — this observation becomes the basis for our use of these plotting functions to compare the β-deconvolution relations. The "master" plot of the β-deconvolution function for the case of a single well in an infinite-acting, homogeneous reservoir is derived using Eq. B.8 and is shown in Fig. B.1.

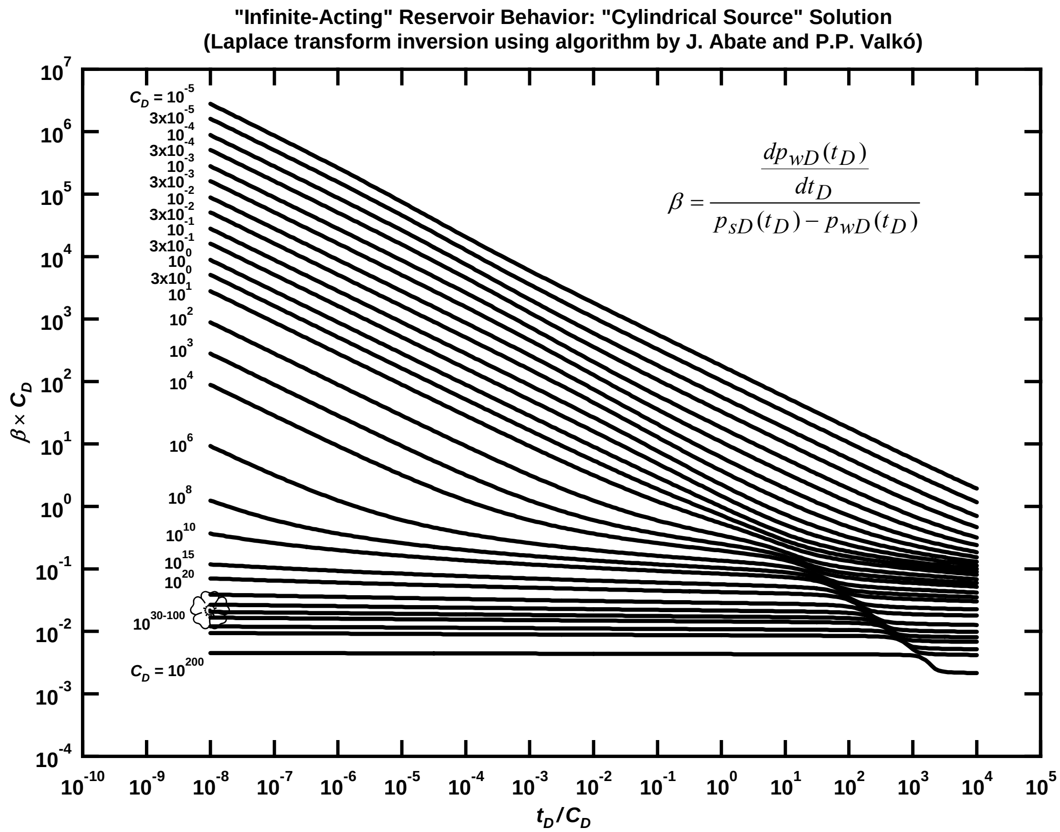

Figure B.1 — Correlation of the β-deconvolution definition for the case of wellbore storage (single well in an infinite-acting, homogeneous reservoir; Laplace transform inversion using algorithm by Abate and Valkó[26]).

APPENDIX C

DERIVATION OF THE COEFFICIENTS FOR β-DECONVOLUTION

C.1 β-Deconvolution — Derivative Approach

Although our stated goal is to develop a deconvolution approach which does not use the pressure derivative function, we can at least develop such a methodology as it may be of practical use in the future. Considering this problem only in terms of dimensionless solutions (and variables), we propose to use the derivative of the $p_{wD}(t_D)$ function as a mechanism to compute the rate function (in our case the $\beta(t_D)$ function from the van Everdingen[4] and Hurst[5] exponential approximation for sand-face flowrate). Recalling this exponential rate model, we have:

$$q_D(t_D) = 1 - e^{-\beta(t_D)t_D} \tag{C.1}$$

Taking the time derivative of Eq. C.1 gives:

$$q_D'(t_D) = \frac{dq_D}{dt_D} = b(t_D)e^{-\beta(t_D)t_D} \tag{C.2}$$

Where the $b(t_D)$-term is defined as:

$$b(t_D) = \beta(t_D) + \beta'(t_D)t_D \tag{C.3}$$

Recalling the definition of the wellbore storage model, we have:

$$q_D(t_D) = 1 - C_D \frac{dp_{wD}}{dt_D} \tag{C.4}$$

Taking the time derivative of Eq. C.4 gives:

$$q_D'(t_D) = \frac{dq_D}{dt_D} = -C_D \frac{d^2 p_{wD}}{dt_D^2} = -C_D p_{wD}'' \tag{C.5}$$

Equating Eqs. C.2 and C.5 gives

$$C_D p_{wD}''(t_D) = C_D \frac{d^2 p_{wD}}{dt_D^2} = -b(t_D)e^{-\beta(t_D)t_D} \tag{C.6}$$

Equating Eqs. C.1 and C.4 gives

$$e^{-\beta(t_D)t_D} = C_D \frac{dp_{wD}}{dt_D} = C_D p_{wD}'(t_D) \tag{C.7}$$

Combining Eqs. C.6 and C.7, and solving for $b(t_D)$

$$\begin{aligned} b(t_D) &= -\frac{p''_{wD}}{p'_{wD}} \\ &= -\frac{1}{t_D}\frac{p_{wDdd}}{p_{wDd}} \\ &= \beta(t_D) + \beta'(t_D)t_D \end{aligned} \qquad \text{(C.8)}$$

Where the p_{wDd} and p_{wDdd} terms are defined as:

$$p_{wDd} = t_D \frac{dp_{wD}}{dt_D} \qquad \text{(C.9)}$$

$$p_{wDdd} = t_D^2 \frac{d^2 p_{wD}}{dt_D^2} \qquad \text{(C.10)}$$

We can use Eq. C.8 to determine $\beta(t_D)$ and $\beta'(t_D)$ — a graphical representation of this technique is shown in Fig. C.1.

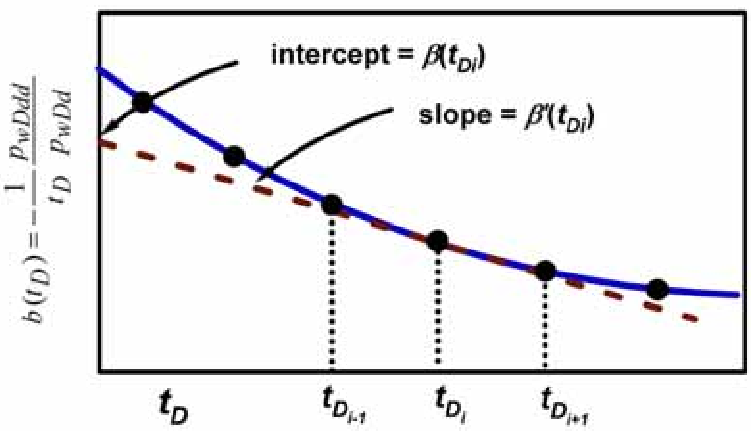

Figure C.1 — β-deconvolution via the derivative approach — $\beta(t_D)$ and $\beta'(t_D)$ determination.

The intercept and slope values [$\beta(t_D)$ and $\beta'(t_D)$, respectively] could be approximated by numerical methods such as least squares — we do not suggest that this approach is functional, we simply present the details for possible use in the future.

C.2 β-Deconvolution — Integral Approach

In this case, we assume $\beta(t_D) = \beta$ (constant) for the purposes of integration and differentiation. We will use integrals and integral-difference (derivative) functions to estimate $\beta(t_D)$.

Recalling Eq. C.7, we have:

$$C_D p'_{wD}(t_D) = e^{-\beta(t_D)t_D} \tag{C.7}$$

Assuming $\beta(t_D) = \beta$ (constant), and integrating Eq. C.7 with respect to t_D, we obtain

$$C_D p_{wD}(t_D) = \frac{1}{\beta}\left[1 - e^{-\beta t_D}\right] \tag{C.11}$$

Integrating Eq. C.11 with respect to t_D yields

$$C_D p^i_{wD}(t_D) = \frac{1}{\beta}\left[t_D - \frac{1}{\beta}\left[1 - e^{-\beta t_D}\right]\right] \tag{C.12}$$

Where the $p^i_{wD}(t_D)$ function is given by:

$$p^i_{wD}(t_D) = \int_0^{t_D} p_{wD}(\tau)d\tau \tag{C.13}$$

Substituting Eq. C.11 into Eq. C.12, we obtain

$$C_D p^i_{wD}(t_D) = \frac{1}{\beta}[t_D - C_D p_{wD}(t_D)] \tag{C.14}$$

Dividing through Eq. C.14 by t_D gives

$$C_D p_{wDi}(t_D) = \frac{1}{\beta}\left[1 - C_D \frac{p_{wD}(t_D)}{t_D}\right] \tag{C.15}$$

Where the $p_{wiD}(t_D)$ function in Eq. C.15 is given by:

$$p_{wDi}(t_D) = \frac{1}{t_D}\int_0^{t_D} p_{wD}(\tau)d\tau \tag{C.16}$$

Taking the derivative of Eq. C.15 with respect to t_D yields:

$$C_D \frac{dp_{wDi}(t_D)}{dt_D} = -\frac{C_D}{\beta}\left[\frac{p'_{wD}(t_D)}{t_D} - \frac{p_{wD}(t_D)}{t_D^2}\right] \tag{C.17}$$

Dividing through by C_D, and multiplying both sides by t_D^2

$$p_{wDid}(t_D) = -\frac{1}{\beta}\frac{1}{t_D}[p_{wDd}(t_D) - p_{wD}(t_D)] \tag{C.18}$$

Where the $p_{wDid}(t_D)$ function in Eq. C.18 is given by:

$$p_{wDid}(t_D) = t_D \frac{dp_{wDi}(t_D)}{dt_D} \tag{C.19}$$

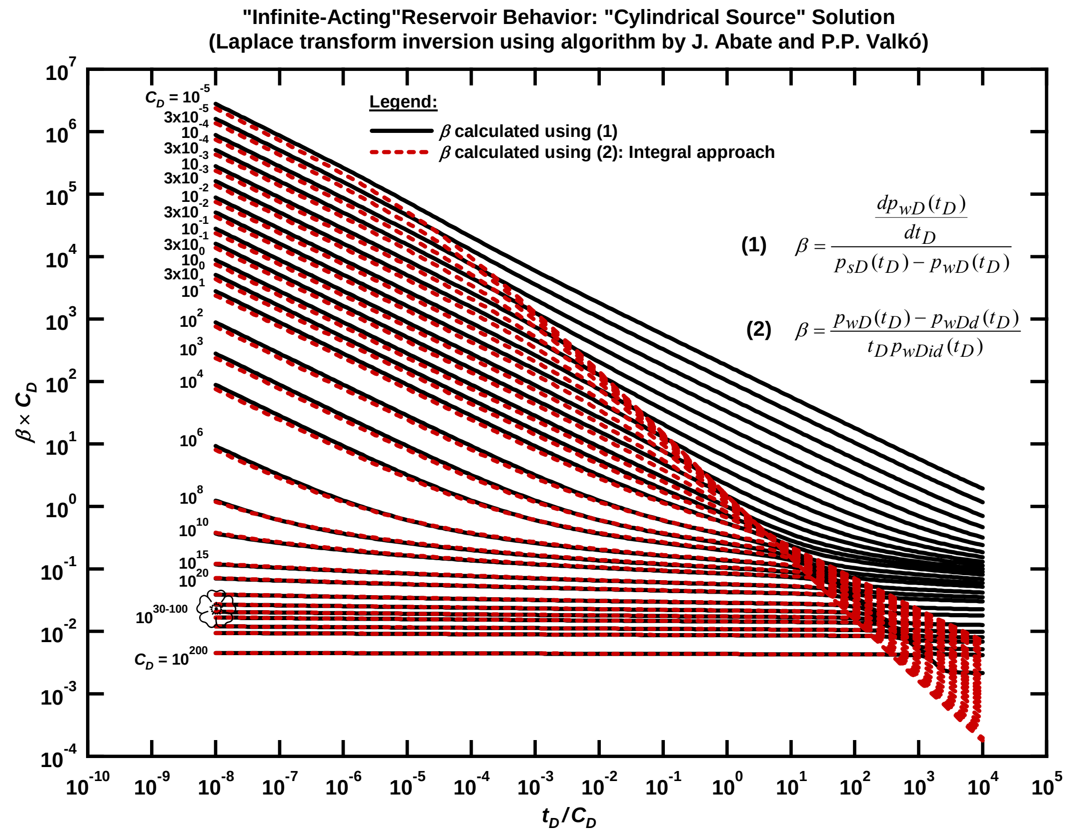

Figure C.2 — β-deconvolution via the integral-derivative approach (approximation of β using Eq. C-20). (for wellbore storage effects in a single well in an in-finite-acting, homogeneous reservoir; Laplace transform inversion using algorithm by Abate and Valkó[26])

Solving Eq. C.18 for β gives us:

$$\beta(t_D) \approx \beta = \frac{1}{t_D} \frac{[p_{wD}(t_D) - p_{wDd}(t_D)]}{p_{wDid}(t_D)} \tag{C.20}$$

Where we assume $\beta \approx \beta t_D$. Eq. C.18 is compared to the analytical formulation for β (Eq. B.7) in Fig. C.2 (βC_D versus t_D/C_D) — and we note a very good correlation at "early" values of t_D/C_D (which is where wellbore storage effects are most important).

Recasting Eq. C.20 into any consistent set of units, we have the following results for the "field units" form of the β-parameter, which we will express as β_f. The result for β_f is.

$$\beta_f = \frac{1}{t} \frac{(\Delta p_w - \Delta p_{wd})}{\Delta p_{wid}} \quad \text{(pressure drawdown)} \tag{C.21a}$$

$$\beta_f = \frac{1}{\Delta t}\frac{(\Delta p_w - \Delta p_{wd})}{\Delta p_{wid}} \qquad \text{(pressure buildup)} \qquad \text{(C.21b)}$$

Where the Δp_w, Δp_{wd}, Δp_{wi}, and Δp_{wid} functions are defined as:

$$\Delta p_w = p_i - p_{wf} \qquad \text{(pressure drawdown)} \qquad \text{(C.22a)}$$

$$\Delta p_w = p_{ws} - p_{wf}(\Delta t = 0) \qquad \text{(pressure buildup)} \qquad \text{(C.22b)}$$

$$\Delta p_{wd} = t\frac{d\Delta p_w}{dt} \qquad \text{(pressure drawdown)} \qquad \text{(C.23a)}$$

$$\Delta p_{wd} = \Delta t\frac{d\Delta p_w}{d\Delta t} \qquad \text{(pressure buildup)} \qquad \text{(C.23b)}$$

$$\Delta p_{wi} = \frac{1}{t}\int_0^t \Delta p_w d\tau \qquad \text{(pressure drawdown)} \qquad \text{(C.24a)}$$

$$\Delta p_{wi} = \frac{1}{\Delta t}\int_0^{\Delta t} \Delta p_w d\tau \qquad \text{(pressure buildup)} \qquad \text{(C.24b)}$$

$$\Delta p_{wid} = t\frac{d\Delta p_{wi}}{dt} \qquad \text{(pressure drawdown)} \qquad \text{(C.25a)}$$

$$\Delta p_{wid} = \Delta t\frac{d\Delta p_{wi}}{d\Delta t} \qquad \text{(pressure buildup)} \qquad \text{(C.25b)}$$

The Δp_s functions for the pressure drawdown and buildup cases are defined in field units as: (based on Eq. B-6)

$$\Delta p_s = \Delta p_w + \frac{1}{\beta_f}\frac{d\Delta p_w}{dt} \qquad \text{(pressure drawdown)} \qquad \text{(C.26a)}$$

$$\Delta p_s = \Delta p_w + \frac{1}{\beta_f}\frac{d\Delta p_w}{d\Delta t} \qquad \text{(pressure buildup)} \qquad \text{(C.26b)}$$

Substituting the for β_f definitions (Eqs. C.21a and C.21b) into the appropriate Δp_s functions (Eqs. C.26a and C.26b) gives the final "field" relation for β-deconvolution using the "integral-derivative" approach (a single relation is obtained for both the pressure drawdown and pressure buildup cases).

$$\Delta p_s = \Delta p_w + \frac{1}{\frac{1}{t}\frac{(\Delta p_w - \Delta p_{wd})}{\Delta p_{wid}}}\frac{d\Delta p_w}{dt}$$

$$\text{or} \quad \Delta p_s = \Delta p_w + \frac{\Delta p_{wd}}{(\Delta p_w - \Delta p_{wd})}\Delta p_{wid} \qquad \text{(C.27)}$$

APPENDIX D

MATERIAL BALANCE DECONVOLUTION RELATIONS FOR WELLBORE STORAGE DISTORTED PRESSURE TRANSIENT DATA

Material balance deconvolution is an extension of the rate normalization method. Johnston[21] defines a new *x*-axis plotting function (material balance time) which provides an approximate deconvolution of the variable-rate pressure transient problem. There are numerous assumptions associated with the "material balance deconvolution" methods — one of the most widely accepted assumptions is that the rate profile must change smoothly and monotonically. In practical terms, this condition should be met for the well-bore storage problem.

The general form of material balance deconvolution is provided for the pressure drawdown case in terms of the material balance time function and the rate-normalized pressure drop function. The material balance time function is given as:

$$t_{mb} = \frac{N_p}{q} \tag{D.1}$$

The rate-normalized pressure drop function is given by:

$$\frac{\Delta p}{q} = \frac{(p_i - p_{wf})}{q} \tag{D.2}$$

The wellbore storage rate function for the pressure drawdown case, $q_{wbs,DD}$, is given as:

$$q_{wbs,DD} = 1 - \frac{1}{m_{wbs}} \frac{d}{dt}[\Delta p_{wf}] \tag{D.3}$$

The wellbore storage rate function for the pressure buildup case, $q_{wbs,BU}$, is given as:

$$q_{wbs,BU} = \frac{1}{m_{wbs}} \frac{d}{d\Delta t}[\Delta p_{ws}] \tag{D.4}$$

Where the wellbore storage "slope" is defined as:

$$m_{wbs} = \frac{qB}{24C_s} \tag{D.5}$$

And the pressure drop terms are defined as:

$$\Delta p_{wf} = p_i - p_{wf} \tag{D.6}$$

$$\Delta p_{ws} = p_{ws} - p_{wf}(\Delta t = 0) \tag{D.7}$$

The wellbore storage cumulative production for the pressure drawdown case, $N_{p,wbs,DD}$, is given as:

$$N_{p,wbs,DD} = \int_0^t q_{wbs,DD}\, dt = t - \frac{1}{m_{wbs}} \Delta p_{wf} \tag{D.8}$$

The wellbore storage cumulative production for the pressure buildup case, $N_{p,wbs,BU}$, is given as:

$$N_{p,wbs,\,BU} = \int_0^{\Delta t} (1 - q_{wbs,BU})\, d\Delta t = \Delta t - \frac{1}{m_{wbs}} \Delta p_{ws} \tag{D-9}$$

The wellbore storage-based, material balance time function for the pressure drawdown case is given as:

$$\Delta t_{mb,DD} = \frac{N_{p,wbs,DD}}{q_{wbs,DD}} = \frac{t - \dfrac{1}{m_{wbs}} \Delta p_{wf}}{1 - \dfrac{1}{m_{wbs}} \dfrac{d}{dt}[\Delta p_{wf}]} \tag{D.10}$$

The wellbore storage-based, rate-normalized pressure drop function for the pressure drawdown case is:

$$\Delta p_{s,DD} = \frac{\Delta p_{wf}}{q_{wbs,DD}} = \frac{1}{1 - \dfrac{1}{m_{wbs}} \dfrac{d}{dt}[\Delta p_{wf}]} \Delta p_{wf} \tag{D.11}$$

The wellbore storage-based, material balance time function for the pressure buildup case is given as:

$$\Delta t_{mb,BU} = \frac{N_{p,wbs,\,BU}}{1 - q_{wbs,BU}} = \frac{\Delta t - \dfrac{1}{m_{wbs}} \Delta p_{ws}}{1 - \dfrac{1}{m_{wbs}} \dfrac{d}{d\Delta t}[\Delta p_{ws}]} \tag{D.12}$$

The wellbore storage-based, rate-normalized pressure drop function for the pressure buildup case is:

$$\Delta p_{s,BU} = \frac{\Delta p_{ws}}{1 - q_{wbs,BU}} = \frac{1}{1 - \dfrac{1}{m_{wbs}} \dfrac{d}{d\Delta t}[\Delta p_{ws}]} \Delta p_{ws} \tag{D.13}$$

Plotting the rate-normalized pressure function versus the material balance time function (on log (t_{mb}) scales) shows that the material balance time function does correct the erroneous shift in the semilog straight-line obtained by rate normalization. We believe that the material balance deconvolution technique is a practical approach (perhaps the most practical approach) for the explicit deconvolution of pressure transient test data which are distorted by wellbore storage and skin effects.

APPENDIX E

IMPLEMENTATION STRUCTURE AND VALIDATION OF THE COMPUTATIONAL MODULE

E.1 Specifications

The contribution of this study is a processing tool for well test data. Before the beginning of its implementation, we define the specification of this tool:

- Input is a table of wellbore storage distorted pressure vs. time.
- Output is an accurate characterization of the reservoir based on a corrected pressure vs. time profile.
- All of the explicit deconvolution methods will be independently applied.

E.2 Structured Analysis

We decided to use the SADT® (Structured Analysis and Design Technique[1], introduced by Ross and Schoman[27] in 1977) to describe in depth the structure of computational module.

E.2.1 Level A-0

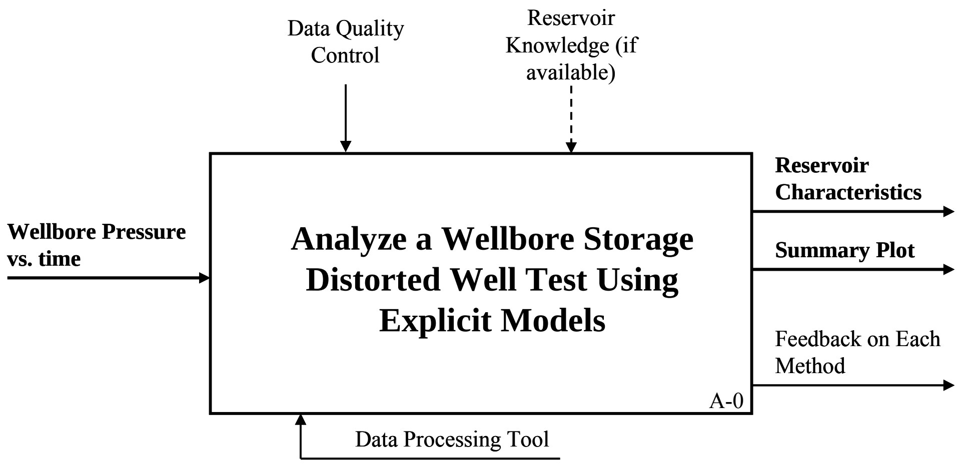

Figure E.1 — Structured analysis of the data processing tool — level A-0.

[1] SADT is a registered trademark of SofTech.

E.2.2 Level A0

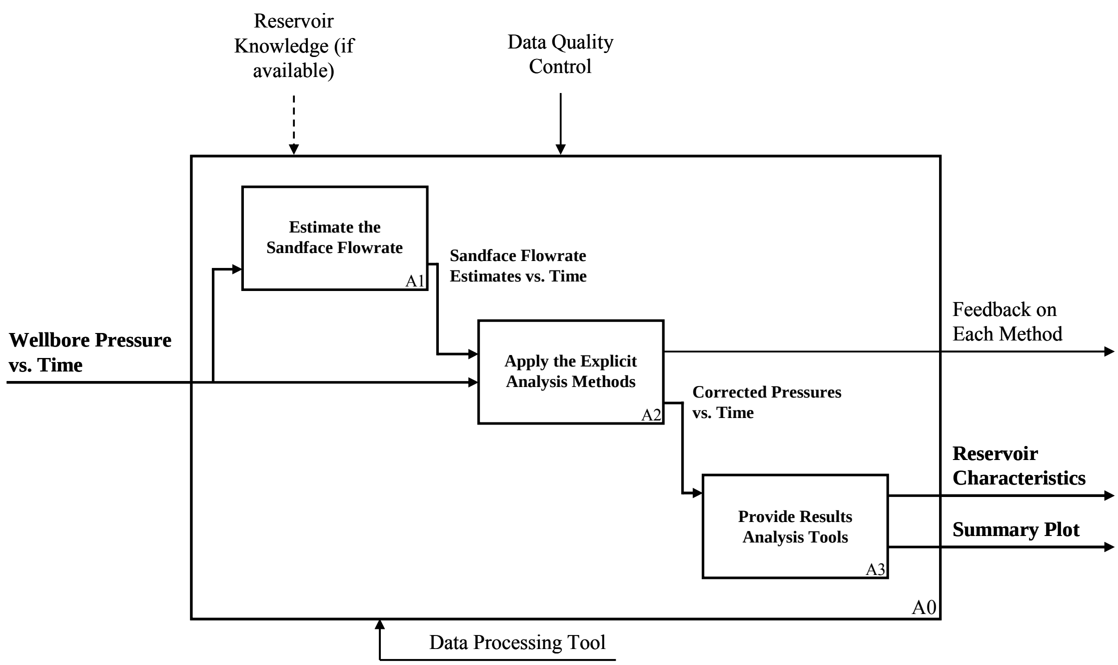

Figure E.2 — Structured analysis of the data processing tool — level A0.

E.2.3 Level A1

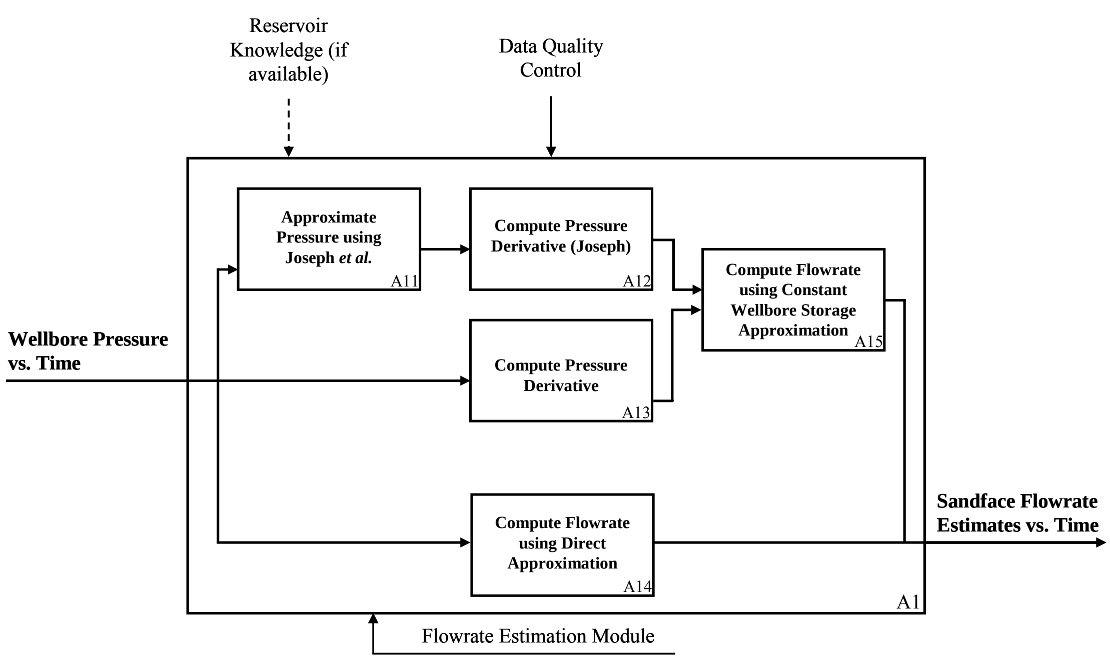

Figure E.3 — Structured analysis of the data processing tool — level A1.

E.2.4 Level A2

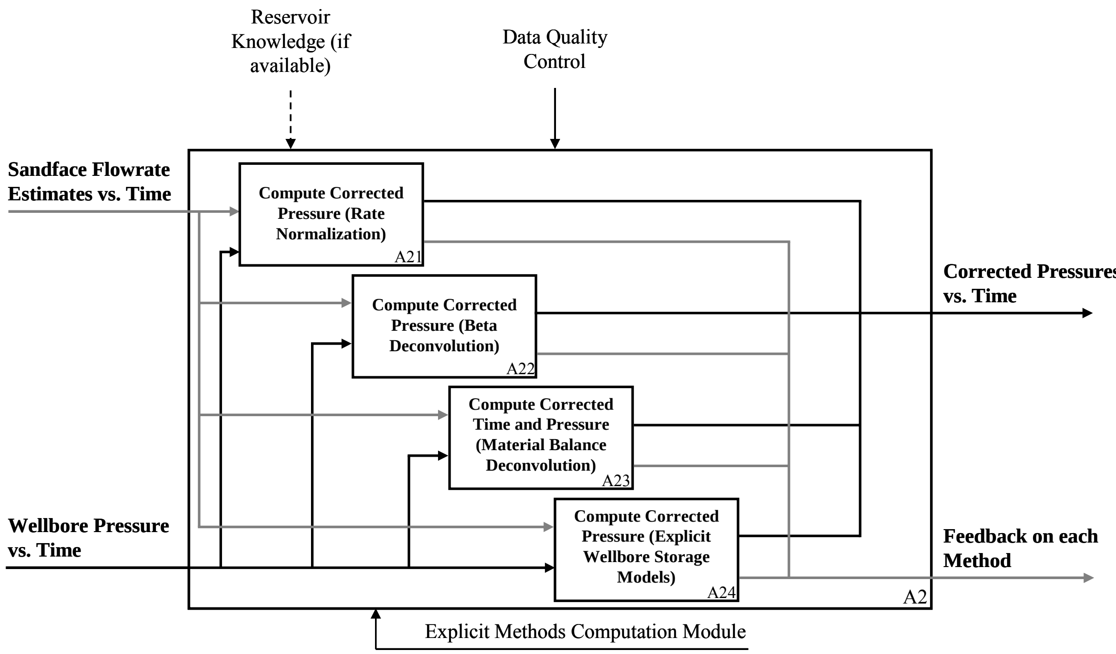

Figure E.4 — Structured analysis of the data processing tool — level A2.

E.3 Output Structure of User Interface

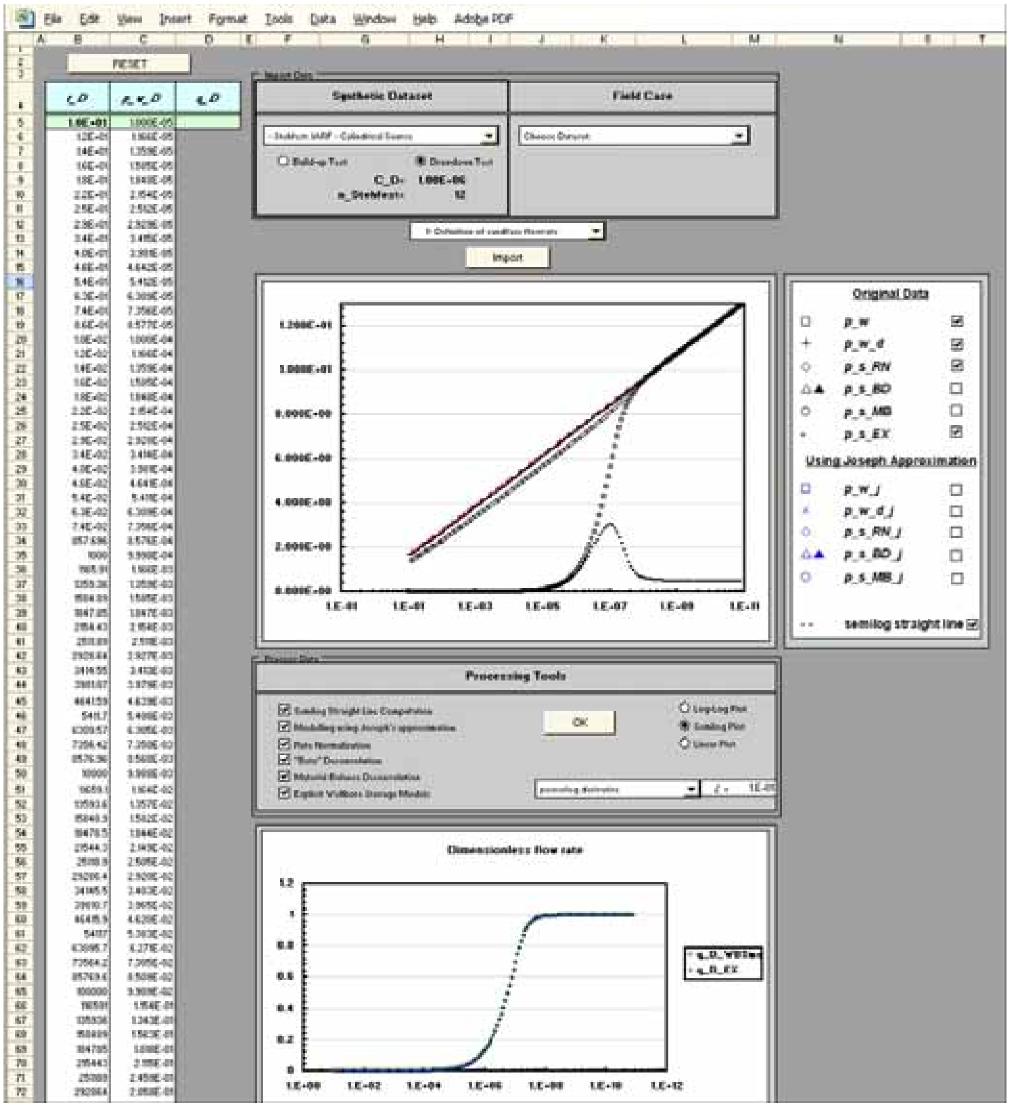

Figure E.5 — Computational module — user interface.

VITA

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