# ESSAYS ON RISK AVERSION

A Dissertation

by

PAAN JINDAPON

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Economics

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Approved by:

| | |
|---|---|
| Chair of Committee, | William S. Neilson |
| Committee Members, | Hae-Shin Hwang |
| | Brit Grosskopf |
| | W. Douglass Shaw |
| Head of Department, | Amy J. Glass |

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## ABSTRACT

Essays on Risk Aversion. (August 2006)

Paan Jindapon, B.A., Thammasat University;

M.A., Southern Methodist University;

M.B.A., Thammasat University

Chair of Advisory Committee: Dr. William S. Neilson

This dissertation contains three essays on risk aversion. In the first essay, we analyze comparative risk aversion in a new way, through a comparative statics problem in which, for a cost, agents can shift from an initial probability distribution toward a preferred distribution. The Ross characterization arises when the original distribution is riskier than the preferred distribution and the cost is monetary, and the Arrow-Pratt characterization arises when the original distribution differs from the preferred distribution by a simple mean-preserving spread and the cost is a utility cost. Higher-order increases in risk lead to higher-order generalizations, and the comparative statics method yields a unified approach to the problem of comparative risk attitudes.

In the second essay, we analyze decisions made by a group of terrorists and a government in a zero-sum game in which the terrorists minimize a representative citizen's expected utility and the government maximizes it. The terrorists' strategy balances the probability and the severity of the attack while the government chooses the level of investment reducing the probability and/or mitigating the severity. We find that if the representative citizen is risk neutral, the terrorists' response is not associated with the government's action and the representative citizen's risk attitudes affect the strategies of the government and the terrorists. Risk aversion always increases equilibrium severity but does not always increase equilibrium expenditure of

the government.

In the last essay, we consider a situation in which an individual has to pay for a good before he realizes the state-dependent surplus of the good. This ex-ante willingness to pay is called the option price and the difference between the option price and the expected surplus is the option value. We find that the option value actually is the buying price for a fixed payment of the expected surplus, and there is a special case in which the option value equals the negative of the compensating risk premium. We also find the effects on the option price and the option value when the expected utility assumption is replaced by a rank-dependent expected utility.

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# CHAPTER I

## INTRODUCTION

This dissertation contains three chapters on risk aversion. In the next chapter, we consider a problem in which two agents are endowed with one payoff probability distribution, but for a set monetary cost can move toward another, less risky payoff distribution. Which individual spends more on improving his payoff distribution? Intuition suggests that the more risk averse individual is willing to spend more to improve the distribution. The literature, however, contains two notions of more risk averse. The Arrow-Pratt characterization is consistent with one individual with non-stochastic initial wealth being willing to pay more than another to avoid a mean-zero risk, while the Ross characterization is consistent with one individual with random initial wealth being willing to pay more than another to avoid a conditionally mean-zero risk. But which characterization is consistent with the more risk averse individual choosing a less risky final payoff distribution in the comparative statics problem?

In Chapter III, we study the impact of risk aversion of a representative citizen in a game played by a government and a terrorist organization. Suppose that the terrorist organization believes that if a representative member of the target population becomes sufficiently dissatisfied, a policy change is enacted. The well-being of the representative citizen is measured by her expected utility. Therefore the terrorist uses his resources to minimize the target citizen's expected utility. In response to threat of the terrorists, the government of the target country may undertake some costly actions that help prevent attacks or mitigate their severity. The strategic variable of the terrorists is the severity level of the attack, which is negatively related

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This dissertation follows the style of Journal of Economic Theory.

to the probability of an attack because of resource constraints. The government chooses an optimal level of action that mitigates the severity and/or reduces the attack probability.

In Chapter IV, we consider a situation in which an individual has to pay for a good before he knows whether or not he will consume the good in the future, or before he realizes the true surplus of consuming the good. If the surplus depends on the state of nature which is unknown at the moment, what will be his ex-ante willingness to pay for this good? This ex-ante willingness to pay is called the option price because it can be thought of the price for an option to consume this good in the future. It is possible that the individual's option price is greater or less than the expected surplus. The difference between the option price and the expected surplus is the option value. The goals of this chapter are to find a relationship between the option value and individual's risk premium, and to see how the option price and the option value change when the individual is not an expected utility maximizer. We present the summary of the dissertation in Chapter V.

# CHAPTER II

## HIGHER-ORDER GENERALIZATIONS OF RISK AVERSION

### A. Introduction

Consider a problem in which two agents are endowed with one payoff probability distribution, but for a set monetary cost can move toward another, less risky payoff distribution. Which individual spends more on improving his payoff distribution? Intuition suggests that the more risk averse individual is willing to spend more to improve the distribution. The literature, however, contains two notions of more risk averse. The Arrow-Pratt characterization is consistent with one individual with non-stochastic initial wealth being willing to pay more than another to avoid a mean-zero risk, while the Ross characterization is consistent with one individual with random initial wealth being willing to pay more than another to avoid a conditionally mean-zero risk.[1] But which characterization is consistent with the more risk averse individual choosing a less risky final payoff distribution in the comparative statics problem?

We find that the stronger, Ross characterization of comparative risk aversion governs behavior in the comparative statics problem, thereby providing a new method of deriving the Ross characterization. This derivation raises two other issues. First, since the Arrow-Pratt characterization is not sufficiently strong to govern behavior in the simple comparative statics problem given above, is there a different comparative statics problem for which the Arrow-Pratt characterization does govern behavior?[2] The answer turns out to be affirmative, and the Arrow-Pratt characterization can

[1] See Pratt [33], Arrow [1], and Ross [36]. Machina and Neilson [26] extend the notions to a differentiable non-expected utility setting.

[2] Chiu [13] provides a different approach to generating both the Arrow-Pratt and Ross characterizations of risk aversion.

be recovered if the cost of moving from the risky initial distribution toward the less risky distribution is a utility cost instead of a monetary one. The problem leading to the Ross characterization is more natural than the one leading to the Arrow-Pratt characterization, though, because the former requires that two individuals with different utility functions face the same monetary cost but the latter requires that two individuals with different utility functions face the same utility cost, which may be problematic. Second, can similar comparative statics problems be used to derive characterizations of higher-order risk attitudes? Again the answer is affirmative, and the paper derives higher-order versions of both the Ross and the Arrow-Pratt characterizations.

The comparative statics approach to deriving characterizations of comparative risk attitudes has several advantages. The primary one is that the approach provides a unified method of getting all of the higher orders of both the Ross and the Arrow-Pratt characterizations. This is not the only advantage, though. The higher-order analysis provides a link between the properties of probability distributions (such as mean preserving spreads) and the derivatives of the utility functions. Ekern [18] and Eeckhoudt and Schlesinger [16] also establish links between properties of probability distributions and the derivatives of the utility functions, but in an absolute, as opposed to comparative, sense.[3] Eeckhoudt and Schlesinger provide a unified framework for constructing the different higher order notions of risk aversion based on the change in expected utility from alternately adding noise and disaggregating risks in successive gambles. We provide a different unified framework for achieving the same

[3]For example, they provide links analogous to that between a desire for mean-preserving decreases in risk and the concavity of the utility function, while this paper provides links analogous to that between one agent having more of a desire for mean-preserving decreases in risk and one agent having a more concave utility function (in the Ross sense).

result, based on a comparative statics exercise.

A second advantage of the comparative statics approach is that Machina [25] shows that comparative statics results like these extend to differentiable non-expected utility preferences. Finally, the approach can be thought of as a higher-order version of Ehrlich and Becker's [17] self-protection problem. In their original problem agents faced a binary payoff distribution and could pay to reduce the probability of the low outcome and raise the probability of the high outcome. Dionne and Eeckhoudt [15], and Briys and Schlesinger [5] show that the willingness of one agent to pay for more self-protection than another is not monotonically related to risk attitudes. The negative result is driven by the fact that the individuals pay for a first-order stochastically dominating shift in the original self-protection problem, but our paper shows that when individuals pay for higher-order improvements behavior is governed by risk attitudes.

The paper proceeds as follows. In Section 2 we illustrate how comparative statics problems can uncover the Ross and Arrow-Pratt measures of risk aversion. Section 3 generalizes the Ross measure to higher order risks, and Section 4 does the same for the Arrow-Pratt measure. Section 5 offers some concluding remarks.

## B. Two Measures of Comparative Risk Aversion

An expected utility maximizing agent's initial monetary payoff is determined by the distribution function $F$ with support contained in $[0, M]$. Let $G$ be another distribution with support contained in $[0, M]$ and which the agent strictly prefers to $F$. The betweenness property of expected utility implies that for any $t \in [0, 1]$ he prefers $G$ to the mixture $(1-t)F + tG$, which in turn he prefers to $F$. Furthermore, repeated application of the betweenness property implies that if $t_1 > t_2$ then he strictly prefers

$(1-t_1)F+t_1G$ to $(1-t_2)F+t_2G$. Now suppose that the agent can choose to improve his payoff distribution from $F(\cdot)$ to $H(\cdot,t)=(1-t)F(\cdot)+tG(\cdot)$ for a cost of $c(t)$, where $c(0)=0$, $c(1)=M$,[4] and $c'>0$. We are interested in conditions governing when one agent chooses a higher value of $t$ than another. The agent's objective function is

$$U(t)=\int_0^M u(x-c(t))dH(x,t),$$

and he chooses $t$ to maximize $U(t)$. The utility function $u$ is assumed to be strictly increasing, concave, and twice continuously differentiable. We assume throughout that $U''<0$ so that the first-order condition identifies a maximum. The first-order condition is

$$\int_0^M u(x-c(t^*))d[G(x)-F(x)]-\int_0^M u'(x-c(t^*))c'(t^*)dH(x,t^*)=0. \tag{2.1}$$

Suppose that $G$ differs from $F$ by a mean-preserving decrease in risk, as in Rothschild and Stiglitz [37]. Integration by parts yields

$$\int_0^M u(x-c(t^*))d[G(x)-F(x)]=-\int_0^M u''(x-c(t^*))\int_0^x[F(y)-G(y)]dydx,$$

and $\int_0^x[F(y)-G(y)]dy>0$, for all $x\in[0,M]$. Consequently, (2.1) can be rewritten

$$-\frac{\int_0^M u''(x-c(t^*))\int_0^x[F(y)-G(y)]dydx}{\int_0^M u'(x-c(t^*))dH(x,t^*)}-c'(t^*)=0. \tag{2.2}$$

Compare two agents with utility functions $u$ and $v$ and optimal values $t_u$ and $t_v$,

[4]The sole purpose of this assumption is to guarantee the existence of an interior solution.

respectively. Agent $u$ is *more Ross risk averse* than $v$ if

$$-\frac{u''(y)}{u'(z)} \geq -\frac{v''(y)}{v'(z)}$$

for all $y$, $z \in [-M, M]$. Under this condition,

$$-\frac{u''(y)}{\int_0^M u'(z)dH(z)} \geq -\frac{v''(y)}{\int_0^M v'(z)dH(z)}$$

for all $y \in [-M, M]$, and therefore the first term in (2.2) is larger for $u$ than it is for $v$, and

$$-\frac{\int_0^M v''(x - c(t_u)) \int_0^x [F(y) - G(y)]dydx}{\int_0^M v'(x - c(t_u))dH(x, t^*)} - c'(t_u) \leq 0.$$

It follows that $t_u \geq t_v$, and the more Ross risk averse agent chooses a less risky, but costlier distribution.

While the Ross measure of risk aversion is sufficient for comparing the chosen $t$ for any $F$ that is a mean-preserving increasing in risk of $G$, the Arrow-Pratt measure of risk aversion is not. Agent $u$ is *more Arrow-Pratt risk averse* than $v$ if

$$-\frac{u''(x)}{u'(x)} \geq -\frac{v''(x)}{v'(x)}$$

for all $x \in [-M, M]$. For example, let $u(w) = -e^{-9w}$ and $v(w) = -e^{-7w}$, so that $u$ is more Arrow-Pratt risk averse than $v$. Let $x \in [0, 1]$, $F(x) = x$, and

$$G(x) = \begin{cases} 0 & \text{if } 0 \leq x < 0.4; \\ 5x - 2 & \text{if } 0.4 \leq x < 0.6; \\ 1 & \text{if } 0.6 \leq x \leq 1, \end{cases}$$

with $c(t) = t^2$, which yields an interior solution for both agents. Agents $u$ and $v$ maximize expected utility when $t_u = 0.087$ and $t_v = 0.103$, so that the more Arrow-

Pratt risk averse agent chooses the lower value of $t$.

Now consider an alternative problem characterized by the objective function

$$\bar{U}(t) = \int_0^M u(x)dH(x,t) - c(t),$$

where, as before, it is assumed that $\bar{U}'' \leq 0$ so the first-order condition identifies a maximum. The first-order condition is

$$\int_0^M u(x)d[G(x) - F(x)] = c'(t). \tag{2.3}$$

This time assume that $F$ differs from $G$ by a simple mean-preserving spread as in Rothschild and Stiglitz [37], so that $F$ and $G$ have the same mean and there exists a payoff $x_0$ such that $F(x) \geq G(x)$ for all $x \in [0, x_0]$ and $F(x) \leq G(x)$ for all $x \in [x_0, M]$, and the cost function is increasing and convex. Let $t_u$ and $t_v$ be the optimal values of $t$ for $u$ and $v$, respectively, and scale $u$ and $v$ so that $u'(x_0) = v'(x_0) > 0$ . Consider the expression

$$\theta = \left[\frac{\int_0^M u(x)d[G(x) - F(x)]}{u'(x_0)}\right] - \left[\frac{\int_0^M v(x)d[G(x) - F(x)]}{v'(x_0)}\right].$$

If $\theta \geq 0$, then $c'(t_u)/u'(x_0) \geq c'(t_v)/v'(x_0)$. Because $u'(x_0) = v'(x_0)$ and $c(t)$ is convex, it follows that $t_u \geq t_v$. Integrating by parts yields

$$\theta = -\int_0^M \left[\frac{u'(x)}{u'(x_0)} - \frac{v'(x)}{v'(x_0)}\right][G(x) - F(x)]dx.$$

Pratt [33] proves that $u$ is more Arrow-Pratt risk averse than $v$ if and only if $u'(x)/u'(y) \leq v'(x)/v'(y)$ for all $y < x$. Consequently, when $x < x_0$ the term in large brackets is nonnegative but $G(x) - F(x)$ is nonpositive, while when $x > x_0$ the term in large brackets is nonpositive but $G(x) - F(x)$ is nonnegative, and therefore the integrand is always nonpositive. Therefore $\theta \geq 0$ and $t_u \geq t_v$, so that the more Arrow-Pratt risk averse agent chooses a less risky, but more costly distribution.

The first maximization problem leads to Ross risk aversion, while the second problem leads to Arrow-Pratt risk aversion. In the first optimization problem costs are monetary and subtracted from the monetary payoff inside the utility function. In the second problem costs are utility costs and subtracted outside the utility function, which is troubling because the problem assumes that the utility cost function is the same for both agents. Monetary costs must therefore be different for the two individuals, making the second problem far less natural than the first one.

### C. Higher-Order Ross Measures of Risk Aversion

In this section, we generalize the problem of maximizing $U(t)$ to higher orders of risk. Let $u^{(n)}$ denote the $n^{th}$ derivative of $u(x)$. For convenience, define $F_1(x) = F(x)$ and $F_k(x) = \int_0^x F_{k-1}(z)dz$ for $k = 2, 3, \ldots, n$. If $F_k(M) = G_k(M)$ for $k = 2, 3, \ldots, n$, then integration by parts yields

$$\int_0^M u(x - c(t^*))d[F(x) - G(x)] = (-1)^{n-1} \int_0^M u^{(n)}(x - c(t^*))[F_n(y) - G_n(y)]dx.$$

We can rewrite (2.1), the first-order condition for maximizing $U(t)$, as

$$\frac{(-1)^{n-1} \int_0^M u^{(n)}(x - c(t^*))[F_n(x) - G_n(x)]dx}{\int_0^M u'(x - c(t^*))dH(x, t^*)} - c'(t^*) = 0. \tag{2.4}$$

Adopting Ekern's [18] definition of having more $n^{th}$ degree risk, for $n \geq 2$, we say that $F$ has *more $n^{th}$ degree risk than* $G$ if (i) $F_k(M) = G_k(M)$ for $k = 1, \ldots, n$; and (ii) $F_n(x) \geq G_n(x)$ for all $x \in [0, M]$ and $F_n(x) > G_n(x)$ for some $x \in (0, M)$. Note that if $F$ has more $n^{th}$ degree risk than $G$, then the first $n - 1$ moments of $F$

and $G$ are equal. In this and the next section, we assume that $F(0) = G(0) = 0$ and, where applicable, $F$ is the distribution with more $n^{th}$ degree risk.

The utility function $u$ is assumed to be strictly increasing and infinitely continuously differentiable. An agent $u$ is $n^{th}$ *degree risk averse* if $(-1)^n u^{(n)}(x) < 0$ for all $x \in [-M, M]$. This definition also follows Ekern [18].[5] In his paper, Ekern shows that $F$ has more $n^{th}$ degree risk than $G$ if and only if every $n^{th}$ degree risk averter prefers $G$ to $F$. Similar to the previous section, we assume that $U$ is concave in $t$ so that the second-order condition holds, and that the cost function $c(t)$ is increasing with $c(0) = 0$ and $c(1) = M$ so that the chosen $t$ falls in the $[0, 1]$ interval. Consequently, $H(\cdot; t) = (1 - t)F(\cdot) + tG(\cdot)$ is a distribution function that has more $n^{th}$ degree risk than $G$ and less $n^{th}$ degree risk than $F$.

**Proposition 1** *Let $t^*$ maximize $U(t) = \int_0^M u(x - c(t))dH(x, t)$. For any distribution function $F$ which is $n^{th}$ degree riskier than $G$, $t^* > 0$ if and only if the agent is $n^{th}$ degree risk averse.*

**Proof.** The result follows immediately from Proposition 2 below.

Proposition 1 identifies the relationship between $n^{th}$ degree risk aversion and the chosen value of $t$ when $F$ has more $n^{th}$ degree risk than $G$. For $n = 2$, Proposition 1 implies that for any distribution function $F$ that is a mean-preserving increase in risk of $G$, $t^* > 0$ if and only if the agent is risk averse.[6]

[5] The utility function $u$ exhibits mixed risk aversion, as defined by Caballe and Pomansky [7], if it exhibits $n^{th}$ degree risk aversion for every $n$. Caballe and Pomansky claim that most utility functions used in examples exhibit mixed risk aversion.

[6] Meyer and Ormiston [29] use comparative statics analysis to address a different problem, identifying distributional changes preferred by every risk averse expected utility maximizer. Their result takes the form: [Assume the agent is risk averse. Then $F$ differs from $G$ by a shift of type _ if and only if $t^* > 0$.] Our result, in contrast, takes the form: [Assume that $F$ is riskier than $G$. Then $u$ is risk averse if and only if $t^* > 0$.]

For $n = 3$, Menezes, Geiss, and Tressler [27] define $F$ having more $3^{rd}$ degree risk than $G$ as $F$ having more downside risk than $G$, and $3^{rd}$ degree risk aversion as downside risk aversion. Therefore, from the Theorem 1, for any distribution function $F$ that has more downside risk than $G$, $t^* > 0$ if and only if the agent is downside risk averse. The definition of $3^{rd}$ degree risk aversion also coincides with Kimball's [22] definition of prudence, by which he means the motive for precautionary saving. Eeckhoudt and Schlesinger [16] state that a prudent agent prefers adding zero-mean risk to a higher wealth level than to a lower wealth level. Similarly, our result shows that a prudent agent always prefers a probability distribution function that is less risky at the lower wealth level, in other words, a distribution function with less downside risk.

For $n = 4$, Menezes and Wang [28] define $F$ having more $4^{th}$ degree risk than $G$ as $F$ having more outer risk than $G$. Theorem 1, then, shows how outer risk aversion is governed by the fourth derivative of the utility function. Bigelow and Menezes [4] provide a comparative statics approach, albeit one different from the one used here, to analyzing outer risk.

This section's main result compares the optimal $t$ chosen by different agents. Theorem 2 describes the necessary and sufficient condition for one agent to choose to shift the probability distribution closer to $G$ than another, or in other words, choose a larger $t^*$.

**Definition 1** *$u$ is more $n^{th}$ degree Ross risk averse than $v$ if*

$$(-1)^{n-1}\frac{u^{(n)}(x)}{u'(y)} \geq (-1)^{n-1}\frac{v^{(n)}(x)}{v'(y)}$$

*for all $x$, $y \in [-M, M]$.*

**Proposition 2** *Let $u$ and $v$ be $n^{th}$ degree risk averse, and let $t_u$ and $t_v$ maximize*

*$U(t) = \int_0^M u(x - c(t))dH(x,t)$ and $V(t) = \int_0^M v(x - c(t))dH(x,t)$, respectively. For any distribution function $F$ which is $n^{th}$ degree riskier than $G$, $t_u \geq t_v$ if and only if $u$ is more $n^{th}$ degree Ross risk averse than $v$.*

**Proof.** If: Let $H_u$ and $H_v$ denote the distributions chosen by agents $u$ and $v$, respectively, and let $y_u$ solve $u'(y_u) = \int_0^M u'(x - c(t_u))dH_u(x)$. Rescale the utility function $v$ so that $v'(y_u) = \int_0^M v'(x - c(t_u))dH_u(x)$. The first-order conditions of agents $u$ and $v$ can be written as

$$(-1)^{n-1} \int_0^M \frac{u^{(n)}(x - c(t_u))}{u'(y_u)}[F_n(x) - G_n(x)]dx - c'(t_u) = 0 \tag{2.5}$$

and

$$(-1)^{n-1} \frac{\int_0^M v^{(n)}(x - c(t_v))[F_n(x) - G_n(x)]dx}{\int_0^M v'(x - c(t_v))dH_v(x)} - c'(t_v) = 0 \tag{2.6}$$

respectively.

Consider the expression

$$\theta = \left[(-1)^{n-1} \int_0^M \frac{u^{(n)}(x - c(t_u))}{u'(y_u)}[F_n(x) - G_n(x)]dx\right]$$
$$- \left[(-1)^{n-1} \int_0^M \frac{v^{(n)}(x - c(t_u))}{v'(y_u)}[F_n(x) - G_n(x)]dx\right].$$

We have $t_u \geq t_v$ if and only if $\theta \geq 0$. Since $(-1)^{n-1}\frac{u^{(n)}(x)}{u'(y_u)} \geq (-1)^{n-1}\frac{v^{(n)}(x)}{v'(y_u)}$ for all $x \in [-M, M]$ and $F_n(x) \geq G_n(x)$ for all $x \in [0, M]$, then $\theta \geq 0$, and hence $t_u \geq t_v$.

Only if: Suppose that there exist $y$, $z \in (-M, M)$ such that $(-1)^{n-1}\frac{u^{(n)}(z)}{u'(y)} < (-1)^{n-1}\frac{v^{(n)}(z)}{v'(y)}$. Because $u \in C^\infty$ , this must hold for all $z$ in some neighborhood $Z$. Construct $\bar{F}$ and $\bar{G}$ so that $\bar{F}$ is $n^{th}$ degree riskier than $\bar{G}$ and $\bar{F}_n - \bar{G}_n$ is a function whose support is in $Z$. Choose $t$ so that $u'(y) = \int_{-M}^M u'(z)d[(1-t)\bar{F}(z) + t\bar{G}(z)]$. Fix $c$ so that the chosen $t$ is $t_u$. Let $x = z + c(t_u)$, and let $X$ be the open set obtained by adding $c(t_u)$ to all $z \in Z$. Then $(-1)^{n-1}\frac{u^{(n)}(x-c(t_u))}{u'(y)} < (-1)^{n-1}\frac{v^{(n)}(x-c(t_u))}{v'(y)}$ all $x \in X$. Define $F(x) = \bar{F}(x - c(t_u))$. Then $F$ is $n^{th}$ degree riskier than $G$ and the support of $F_n - G_n$ is in $X$. It follows that $\theta < 0$. By construction $u'(y) = \int_0^M u'(x-c(t_u))dH_u(x)$ and $v'(y) = \int_0^M v'(x - c(t_u))dH_u(x)$, hence $t_u < t_v$. □

For $n = 2$, $F$ differs from $G$ by a mean-preserving increase in risk. The necessary and sufficient condition for $t_u \geq t_v$ is $u$ more Ross risk averse than $v$, as discussed in the previous section. For $n = 3$, the definition of $F$ having more $3^{rd}$ degree risk than $G$ coincides with $F$ having more downside risk than $G$. Following Modica and Scarsini [30], $u$ is more downside risk averse than $v$ when $u'''(x)/u'(y) \geq v'''(x)/v'(y)$ for all $x$ and $y$. Proposition 2 states that when $u$ and $v$ are downside risk averse agents, for any distribution function $F$ having more downside risk than $G$, $t_u \geq t_v$ if and only if $u$ is more downside risk averse than $v$. In the context of self-protection, an agent who is more downside risk averse will protect himself more by reducing further the amount of downside risk, thereby bearing higher cost.

## D. Higher-Order Arrow-Pratt Measures of Risk Aversion

Next we consider the problem of maximizing $\bar{U}(t)$, in which the cost of shifting the probability distribution is disutility subtracted from the expected utility of the outcomes. The disutility $c(t)$ is assumed to be increasing and convex with $c(0) = 0$, $c(1) = u(M)$. In Section 2 we found the sufficient condition for comparing the optimal $t$ for $F$ differing from $G$ by a simple mean-preserving spread, that is, a mean-preserving increase in risk satisfying the single crossing property. Here we generalize this property to $n^{th}$ degree risk. We say that $F$ differs from $G$ by a simple increase in $n^{th}$ degree risk if (i) $F$ has more $n^{th}$ degree risk than $G$; and (ii) there exists $x_0 \in [0, M]$ such that $F_{n-1}(x) \geq G_{n-1}(x)$ for all $x \leq x_0$ and $F_{n-1}(x) \leq G_{n-1}(x)$ for all $x \geq x_0$.

**Definition 2** *u is more $n^{th}$ degree Arrow-Pratt risk averse than v if*

$$-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \geq -\frac{v^{(n)}(x)}{v^{(n-1)}(x)}$$

*for all $x \in [0, M]$.*

**Proposition 3** *Let u and v be $n^{th}$ and $n-1^{th}$ degree risk averse, and let $t_u$ and $t_v$ maximize $\bar{U}(t) = \int_0^M u(x)dH(x,t) - c(t)$ and $\bar{V}(t) = \int_0^M v(x)dH(x,t) - c(t)$, respectively. For any distribution function F which differs from G by a simple increase in $n^{th}$ degree risk, $t_u \geq t_v$ if and only if u is more $n^{th}$ degree Arrow-Pratt risk averse than v.*

**Proof.** If: Rescale the utility function $u$ so that $u^{(n-1)}(x_0) = v^{(n-1)}(x_0)$. The solution of $\bar{U}(t)$ maximization is the same as the solution from maximizing $(-1)^n \frac{\bar{U}(t)}{u^{(n-1)}(x_0)}$. The first-order condition yields

$$(-1)^n \frac{E_G u(x) - E_F u(x)}{u^{(n-1)}(x_0)} = (-1)^n \frac{c'(t_u)}{u^{(n-1)}(x_0)} \tag{2.7}$$

Define

$$\theta = \left[(-1)^n \frac{E_G u(x) - E_F u(x)}{u^{(n-1)}(x_0)}\right] - \left[(-1)^n \frac{E_G v(x) - E_F v(x)}{v^{(n-1)}(x_0)}\right].$$

Integration by parts yields

$$\theta = \int_0^M \left[\frac{u^{(n-1)}(x)}{u^{(n-1)}(x_0)} - \frac{v^{(n-1)}(x)}{v^{(n-1)}(x_0)}\right] [F_{n-1}(x) - G_{n-1}(x)]dx.$$

Following an argument in Pratt [33], since $\frac{u^{(n)}(x)}{u^{(n-1)}(x)} = \frac{d}{dx} \log u^{(n-1)}(x)$, then from $\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \leq \frac{v^{(n)}(x)}{v^{(n-1)}(x)}$, integrating from $x_0$ to $x$, we have $\log \frac{u^{(n)}(x)}{u^{(n-1)}(x)} \leq \log \frac{v^{(n)}(x)}{v^{(n-1)}(x)}$, and hence $\frac{u^{(n-1)}(x)}{u^{(n-1)}(x_0)} \leq \frac{v^{(n-1)}(x)}{v^{(n-1)}(x_0)}$, for all $x \geq x_0$. Similarly $\frac{u^{(n-1)}(x)}{u^{(n-1)}(x_0)} \geq \frac{v^{(n-1)}(x)}{v^{(n-1)}(x_0)}$, for all $x \leq x_0$. Since $F_{n-1}$ and $G_{n-1}$ cross only once at $x_0$, then $\theta \geq 0$. Since $u^{(n-1)}(x_0) = v^{(n-1)}(x_0)$, then $c'(t_u) \geq c'(t_v)$ and hence $t_u \geq t_v$.

Only if: Suppose that there exists some $x \in (0, M)$ such that $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} < -\frac{v^{(n)}(x)}{v^{(n-1)}(x)}$. Because $u \in C^\infty$ there exists a neighborhood $Z$ of $x$ such that $-\frac{u^{(n)}(z)}{u^{(n-1)}(z)} < -\frac{v^{(n)}(z)}{v^{(n-1)}(z)}$ for all $z \in Z$. Choose $F$ and $G$ such that $F_{n-1}(z) - G_{n-1}(z)$ is a function with support in $Z$, and $F$ differs from $G$ by a simple increase in $n^{th}$ degree risk with a crossing at $x_0 = x$. For all $z \in Z$ such that $z < x_0$, $\frac{u^{(n)}(z)}{u^{(n-1)}(x_0)} < \frac{v^{(n)}(z)}{v^{(n-1)}(x_0)}$ and $F_{n-1}(z) > G_{n-1}(z)$.For all $z \in Z$ such that $z > x_0$, $\frac{u^{(n)}(z)}{u^{(n-1)}(x_0)} > \frac{v^{(n)}(z)}{v^{(n-1)}(x_0)}$ and $F_{n-1}(z) < G_{n-1}(z)$. Therefore $\theta < 0$. It follows that $t_u < t_v$. $\square$

When $n = 2$, Proposition 3 yields the same result as in Section 2, a more risk averse agent chooses a larger value of $t$ for any $F$ that is a simple mean-preserving spread of $G$. For $n = 3$, Kimball [22] defines the absolute prudence of agent $u$ as $p_u(x) = -u'''(x)/u''(x)$ to measure the strength of the precautionary saving motive. We obtain the following results from the Proposition 3: if $u$ and $v$ are risk averse and prudent, and $F_2$ and $G_2$ cross only once, then for any $F$ having more downside risk than $G$, $t_u \geq t_v$ if and only if $p_u(x) \geq p_v(x)$ for all $x \in [0, M]$.[7]

## E. Conclusion

We characterize comparative measures of Arrow-Pratt and Ross risk aversion through a comparative statics problem. The Ross characterization arises when two risk averse agents optimally choose their shifts in probability distribution toward a preferred distribution that differs from the original one by a mean preserving decrease in risk, and the cost of shifting probabilities is monetary. The Arrow-Pratt characterization arises when the original distribution differs from the preferred distribution by a simple

[7] Chiu [12] shows that a more prudent individual invests in more self-protection under certain circumstances.

mean-preserving spread, and the cost is a utility cost.

Using the same approach, we generalize the comparative statics problem to the difference in $n^{th}$ degree risk between two probability distributions and the $n^{th}$ degree of risk aversion. For the third degree risk comparison in the monetary cost problem, our characterization of more Ross risk averse in the third degree coincides with more downside risk averse, as recently defined by Modica and Scarsini [30]. On the other hand, in the utility cost problem, the Arrow-Pratt risk aversion measures in the 3rd degree coincide with the absolute prudence measures defined by Kimball [22].

# CHAPTER III

# OPTIMAL TERRORISM AND COUNTERTERRORISM

## A. Introduction

Terrorist organizations use violent attacks, and the threat of further violent attacks, to provoke fear and intimidation among the target population with the ultimate goal of affecting policy change. In response to a threat of the terrorists, the government of the target country may undertake some costly actions that help prevent attacks or mitigate their severity. In this paper, the decisions of terrorism and counterterrorism are modeled in a two-person zero-sum game. The terrorists aim to minimize the representative citizen's expected utility, while the government maximizes it. The strategic variable of the terrorists is the severity level of the attack, which is negatively related to the probability of an attack because of resource constraints. The government chooses an optimal level of an activity that mitigates the severity and/or reduces the attack probability. The problem that the government faces is similar to Ehrlich and Becker's [17] self-insurance and self-protection problem.

We examine the effect of changes in the representative citizen's degree of risk aversion on the choice variables of the terrorists and the government. Then we find an equilibrium of this game from the best response functions of both players when the representative citizen is risk neutral, and then extend the result to the case of risk aversion. We find that risk aversion increases the the terrorists' choice of severity and the government's choice of mitigation, but does not always increase the level of prevention. When the representative citizen is risk neutral, the terrorists always choose the level of severity that maximizes the cost of counterterrorism activity and does not depend on the nature of the action chosen by the government.

In the next section, we discuss related literature in the economics of terrorism. In Section 3 and 4, we analyze the effect of risk aversion on the terrorists' and the government's strategies, respectively. We present the equilibrium of the game in Section 5 and conclude in Section 6.

## B. Economics of Terrorism

Enders and Sandler [19] define terrorism as "the premeditated use, or threat of use, of extra-normal violence or brutality to obtain a political objective through intimidation or fear directed at a large audience." These political objectives of terrorists include the promotion of religious freedoms, economic equality, income equality, income redistribution, nationalism, etc. The motive of the terrorist attacks can be explained by Berry's [3] theory of overreaction. Berry argues that terrorists can achieve their political goal by weakening the government. Terrorists want the government to overreact to the attack because when the government overreacts to the terrorist attack, it weakens itself. Overreaction, for example, a very strict homeland security policy resulting in a loss of liberty and privacy, can lessen public support and can be very expensive. As a result, the choice of policy can be easily influenced.

The first economic analysis of terrorism is the study of Sandler, Tschirhart, and Cauly [39]. Using interaction between government and terrorists who have their own objective functions and constraints, Sandler et al examine the negotiation process between government and terrorists when hostages are seized and demands are issued. However, this negotiation model cannot explain bombings and assassinations which are most incidents pursued by terrorists. Since there are strategies involved in terrorism and counterterrorism decisions, and each player's payoff depends on the chosen strategy of the other player, a game theoretical approach to the terrorism problem has recently received more attention.

Konrad [23] applies a simple game of extortion to the investment problem in terrorism. Terrorists decide whether or not to invest in an attack, which is not observed by government, before they use the threat of violence to pursue political or monetary goals. The paper discusses the credibility of terrorist threats in a repeated game but does not bring up any counterterrorism policy. On the other hand, there are two papers that do focus on counterterrorism actions taken by target firms and target governments. Kunreuther and Heal [24] examine the interdependent security problem in which two target firms consider whether to invest in security precautions whose effects depend on the actions of others. Sandler [38] studies a game in which the two players are governments choosing whether to pursue preemption policy or deterrence policy against terrorists. Since the threat of terrorism is global, the payoff from the adopted policy depends on the policy adopted by another government.

A few other studies of counterterrorism aim to empirically investigate the impact of enhanced security policy on terrorism. Cauley and Im [9] apply the intervention analysis to the data on international terrorist events from 1968 to 1979 and find that the installation of metal detectors has reduced the number of skyjackings but increased other types of terrorist events. Enders and Sandler [20] suggest two appealing points based on a vector autoregressive analysis (VAR). First, terrorists try to achieve a greater impact from fewer events. Despite a decline in terrorism since the post-cold war era, each incident is more likely to result in death or injury. Second, there is evidence that if the government responds by installing metal detectors, the terrorists will substitute to less-protected targets with more deadly consequences.

## C. The Terrorist's Decision in a World without Counterterrorism

The United States Code of Regulations defines terrorism as "...the unlawful use of force and violence against persons or property to intimidate or coerce a government, the civilian population, or any segment thereof, in furtherance of political or social objectives." The U.S. Department of Defense defines it as "the calculated use of unlawful violence or threat of unlawful violence to inculcate fear; intended to coerce or to intimidate governments or societies in the pursuit of goals that are generally political, religious, or ideological." Terrorist organizations use violent attacks, and the threat of further violent attacks, to provoke fear and intimidation among the target population with the ultimate goal of affecting policy change.

For example, purported political objectives behind al-Qaeda attacks against the U.S. include ending the U.S. military presence in the Middle East, ending U.S. support of Israel, and ending U.S. support for corrupt regimes in the Muslim world (Byman [6]). In this paper we focus on situations in which terrorists try to intimidate or coerce a democratically-elected government, because in those cases it makes the most sense for the terrorists to target the population. After all, in a democratic system, policy changes can occur when the electorate becomes dissatisfied with the current policy choices.

To that end, suppose that the terrorist organization (referred to simply as the terrorist in the remainder of the paper) believes that if a representative member of the target population becomes sufficiently dissatisfied, a policy change is enacted. The well-being of the representative citizen is measured by her expected utility. The terrorist does not know how far the representative citizen's expected utility must fall before she votes for a policy change, and therefore uses his resources to minimize the target citizen's expected utility.

Because fear and intimidation work only against those who are not the direct victims of terrorist attacks, the threat of a future attack is the primary terrorism tool. Like everyone else, the terrorist has limited resources to devote to the production of fear. More severe attacks require more resources, and more frequent attacks require more resources. Thus, the terrorist faces a tradeoff between the severity and the frequency of attacks. This is reminiscent of Becker's [2] classic treatment of a government choosing between the certainty and severity of punishment to deter crime. Borrowing his terminology, the terrorist chooses the certainty and severity of an attack in order to minimize the expected utility of a representative citizen in the target country.

To capture the tradeoff induced by the terrorist's limited resources, let $p(s)$ be the probability of attack that fully utilizes the terrorist's resources when the chosen severity of attack is $s$. Thus, $p(s)$ describes a production possibility set for the terrorist, and $p_s(s) < 0$. There is a least severe attack that can be considered a terrorist attack, and let $\underline{s}$ denote the least severe attack. Also, assume that there is a most severe attack to which the representative citizen assigns positive probability, and denote that level of severity by $\bar{s}$. Then $p(\bar{s}) > 0$, and let $\underline{p} = p(\bar{s})$ . Finally, let $\bar{p} = p(\underline{s}) < 1$. The production possibility frontier is shown in Figure 1.

The terrorist chooses a combination of certainty and severity to minimize the expected utility of a representative citizen in the target country. Let her initial wealth be $w$, and let $u(\cdot)$ be her von Neumann-Morgenstern utility function, with $u' > 0, u'' \leq 0$. Then her expected utility is given by

$$EU = (1 - p(s))u(w) + p(s)u(w - D(s)), \tag{3.1}$$

where $D(s)$ captures the monetary equivalent of the damage caused by an attack of severity $s$, with $0 < D(s) < w$, $D_s(s) > 0$, and $D_{ss}(s) \leq 0$.

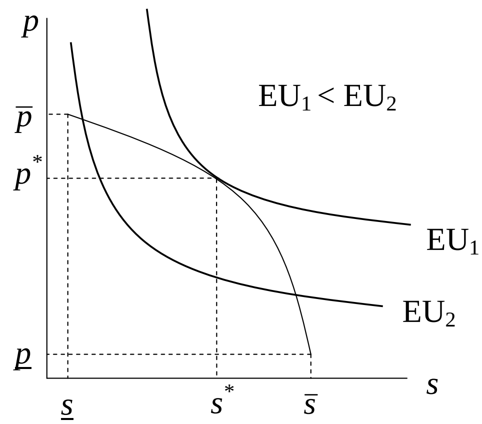


Fig. 1. Optimal solution of the terrorist

In Figure 1, the terrorist chooses the point on the production possibilities frontier that minimizes the representative citizen's expected utility. To find the shape of the terrorist's (and the citizen's) indifference curves, fix $\bar{U}$ and let $\pi(s)$ satisfy $(1-\pi)u(w) + \pi u(w - D(s)) = \bar{U}$. Then one can compute

$$\frac{d\pi}{ds} = -\frac{u(w) - \bar{U}}{[u(w) - u(w - D(s))]^2} \cdot u'(w - D(s))D_s(s) < 0,$$

and

$$\frac{d^2\pi}{ds^2} = -\frac{u(w) - \bar{U}}{[u(w) - u(w - D(s))]^2} \cdot [(u'(w - D(s))D_{ss}(s) - u''(w - D(s))(D_s(s))^2) - \frac{2[u'(w - D(s))(D_s(s))]^2}{u(w) - u(w - D(s))}].$$

If the agent is risk neutral, then $d^2\pi/ds^2 > 0$. Therefore, as long as the agent is not too risk averse her iso-expected-utility curves in $p - s$ space are convex, as in Figure 1.

The terrorist chooses $s$ to minimize the agent's expected utility. The first-order condition is

$$\frac{\partial EU}{\partial s} = -p_s(s)[u(w) - u(w - D(s))] - p(s)u'(w - D(s))D_s(s) = 0. \tag{3.2}$$

The second-order condition of the terrorist's minimization problem is

$$\begin{aligned}\frac{d^2EU}{ds^2} = &- p_{ss}(s)[u(w) - u(w - D(s))] - 2p_s(s)u'(w - D(s))D_s(s) \\ &+ p(s)[u''(w - D(s))D_s(s))^2 - u'(w - D(s))D_{ss}(s)].\end{aligned}$$

If the agent is risk neutral, we have

$$\frac{d^2EU}{ds^2} = -u'(\cdot)[p_{ss}(s)D(s) + 2p_s(s)D_s(s) + p(s)D_{ss}(s)] > 0.$$

Therefore if the representative citizen is not too risk averse the value of $s$ that satisfies (3.2) minimizes the representative citizen's expected utility. This is exactly the intuition from the graphical approach.

Our first result concerns how the representative citizen's risk attitudes affect the choice made by the terrorist.

**Proposition 4** *The level of severity chosen by the terrorist is higher when the representative citizen is more risk averse.*

**Proof.** Let $u$ and $v$ be two utility functions with $u$ a concave transformation of $v$. Let $s_u$ and $s_v$ be the values of $s$ that satisfies the first-order condition (3.2) for the two different utility functions. Rearrange the first-order condition for the utility function $v$ to get

$$p_s(s_v)D(s_v) \cdot \frac{v(w) - v(w - D(s_v))}{v'(w - D(s_v))D(s_v)} + p(s_v)D_s(s_v) = 0. \tag{3.3}$$

Normalize the two utility functions so that $u(w - D(s_v)) = v(w - D(s_v)) = k > 0$ and $u'(w - D(s_v)) = v'(w - D(s_v)) = 1$. Define the function $\rho$ to satisfy $u(x) = \rho(v(x))$ for all $x$. Then by hypothesis $\rho$ is concave, and by construction $\rho'(k) = 1$. Consequently,

$\rho'(z) \leq 1$ for all $z > k$. Now note that

$$\begin{aligned} u(w) &= k + \int_0^{D(s_v)} u'(w - D(s_v) + t)dt \\ &= k + \int_0^{D(s_v)} \rho'(v(w - D(s_v) + t)) \cdot v'(w - D(s_v) + t)dt \\ &\leq k + \int_0^{D(s_v)} v'(w - D(s_v) + t)dt \\ &= v(w), \end{aligned}$$

and therefore, remembering that $p_s < 0$,

$$p_s(s_v)D(s_v) \cdot \frac{u(w) - u(w - D(s_v))}{u'(w - D(s_v))D(s_v)} + p(s_v)D_s(s_v) \geq 0. \tag{3.4}$$

Plugging (3.4) back into (3.2) yields $\partial EU/\partial s \leq 0$ at $s = s_v$, and because $\partial EU/\partial s$ is increasing in $s$ and equal to zero at $s = s_u$, it follows that $s_u \geq s_v$. □

Thus, when compared with a risk neutral population, a risk averse population in the target country leads terrorists to commit more damaging, but less frequent, attacks.

This result is similar in spirit to Becker's [2] observation that risk-averse expected-utility-maximizing criminals are more sensitive to changes in the severity than certainty of punishment. Under these circumstances, in order to make a criminal worse off in an attempt to deter him from committing a crime, the authorities should decrease the likelihood of capturing the criminal but increase the punishment if caught.

## D. Optimal Counterterrorism

The analysis of the preceding section covered only part of the story. The government of the target country is unlikely to sit idly under the threat of terrorist attacks. Instead, the government can undertake costly actions that either help prevent attacks, mitigate their severity, or both. To that end, assume that the government can take action $a \geq 0$ to reduce the damage from and the probability of a successful attack. Hence the level of damage and the probability of success are functions of both $a$ and $s$.

We assume that the representative citizen's share of the cost of the action is $c(a)$, where $c'(a) > 0$ and $c''(a) > 0$. The costs include the financial cost of the action as well as costs associated with any possible loss of liberty and privacy to the agent resulting from the chosen action. The government's objective is to choose $a$ to maximize the representative citizen's expected utility. In addition, we assume that $D_a(a,s) \leq 0$, $D_{aa}(a,s) \geq 0$, $p_a(a,s) \leq 0$, $p_{aa}(a,s) \geq 0$. These conditions say that increases in $a$ make a given attack both less likely to succeed and less damaging if it does succeed, and that the impact of further increases in $a$ diminishes as $a$ rises. Finally, $D(a,s) > 0$ and $p(a,s) > 0$ for all values of $a$ and $s$, so that the government can never completely eliminate the threat of terrorism.

The representative citizen's expected utility can be written as

$$EU = (1 - p(a,s))u(w - c(a)) + p(a,s)u(w - D(a,s) - c(a)). \tag{3.5}$$

The government chooses $a$ to maximize (3.5). The first-order condition is

$$\begin{aligned} \frac{\partial EU}{\partial a} = &- p_a(a,s)[u(w - c(a)) - u(w - D(a,s) - c(a)] \\ &- (1 - p(a,s))u'(w - c(a))c'(a) \\ &- p(a,s)u'(w - D(a,s) - c(a))[D_a(a,s) + c'(a)] = 0. \end{aligned} \tag{3.6}$$

If the agent is risk neutral, then the first-order condition becomes

$$-[p_a(a,s)D(a,s) + p(a,s)D_a(a,s)] = c'(a), \tag{3.7}$$

which has the standard interpretation that the marginal benefit of the counterterrorism activity equals the marginal cost.

Differentiating the government's objective function a second time yields

$$\begin{aligned}\frac{\partial^2 EU}{\partial a^2} =& - p_{aa}(a,s)[u(w-c(a)) - u(w-D(a,s)-c(a)] \\ &+ 2p_a(a,s)[u'(w-c(a))c'(a) - u(w-D(a,s)-c(a))(D_a(a,s)+c'(a)] \\ &+ (1-p(a,s))[u''(w-c(a))(c'(a))^2 - u'(w-c(a))c''(a)] \\ &+ p(a,s)[u''(w-D(a,s)-c(a))(D_a(a,s)+c'(a))^2 \\ &- u'(w-D(a,s)-c(a))(D_{aa}(a,s)+c''(a))].\end{aligned}$$

If the representative agent is risk neutral, we have

$$\frac{\partial^2 EU}{\partial a^2} = -u'(\cdot)[p_{aa}(a,s)D(a,s) + 2p_a(a,s)D_a(a,s) + p(a,s)D_{aa}(a,s) + c''(a)] < 0.$$

Therefore if the representative citizen is not too risk averse the second-order condition for a maximum is satisfied.

The government's problem is analogous to the self-insurance and self-protection problem introduced by Ehrlich and Becker [17]. The mitigation of the damage from terrorism and the reduction in the probability of a successful attack are analogous to self-insurance and self-protection, respectively. Dionne and Eeckhoudt [15] and Briys and Schlesinger [5] show that an increase in risk aversion increases the optimal level of self-insurance, but does not always increase the optimal investment in self-protection.

We say that the government's action is purely mitigating if it affects the damages $D$ but not the success probability $p$, that is, $D_a(a,s) < 0$ but $p_a(a,s) = 0$. In contrast, the action is purely preventive if it affects the success probability $p$ but not the damages $D$. We now treat the two cases separately.

### 1. Mitigating activities

In this subsection assume that the action $a$ affects only the level of damages from an attack but not the probability of success. In particular, assume that $D(a,s) = g(a)D(s)$, with $g'(a) \leq 0, g''(a) \geq 0$, and $p(a,s) = p(s)$. Under these circumstances, the first-order condition (3.6) can be written

$$-\frac{[g'(a)D(s) + c'(a)]p(s)}{c'(a)(1-p(s))} = \frac{u'(w - c(a))}{u'(w - g(a)D(s) - c(a)}. \tag{3.8}$$

**Proposition 5** *Assume that the government's action is purely mitigating. The level of counterterrorism activity chosen by the government rises when the representative agent becomes more risk averse.*

**Proof.** By Pratt [33], when the representative citizen becomes more risk averse, the right-hand side of (3.8) falls. Differentiating the left-hand side by $a$ yields

$$\frac{\partial LHS}{\partial a} = -\frac{[c'(a)g''(a) - c''(a)g'(a)]D(s)p(s)}{(c'(a))^2(1-p(s))} \leq 0.$$

Therefore, when the representative citizen becomes more risk averse, $a$ must increase to maintain equality in (3.8). □

Proposition 5 states that increased risk aversion leads to increased counterterrorism activities when those activities mitigate the damages of an attack but do not prevent an attack. It is reminiscent of existing results that increased risk aversion leads to increased self-insurance. The intuition behind the result is that by increasing the mitigating activity the government increases the representative citizen's payoff in the bad state (an attack occurs) but reduces it in the good state (no attack). When the representative citizen becomes more risk averse, the government desires the two payoff levels to be closer together, which it achieves by spending more on mitigating the damages from an attack.

## 2. Preventive activities

Now assume that the government's actions affect only the attack's success probability and not the extent of damages. In particular, assume that $D(a,s) = D(s)$ and $p(a,s) = f(a)p(s)$ with $f'(a) \leq 0, f''(a) \geq 0$. Now the first-order condition in (3.6) reduces to

$$\begin{aligned}\frac{\partial EU}{\partial a} =& - f'(a)p(s)[u(w-c(a)) - u(w-D(s)-c(a)]\\ &- (1-f(a)p(s))u'(w-c(a))c'(a)\\ &- f(a)p(s)u'(w-D(s)-c(a))c'(a) = 0.\end{aligned}$$

This can be rearranged to get

$$-\frac{c'(a)}{f'(a)p(s)} = \frac{u(w-c(a)) - u(w-D(s)-c(a))}{(1-f(a)p(s))u'(w-c(a)) + f(a)p(s)u'(w-D(s)-c(a))}. \tag{3.9}$$

**Proposition 6** *Assume that the government's action is purely preventive. If the probability of attack is sufficiently low, the level of counterterrorism activity chosen by the government rises when the representative agent becomes more risk averse.*

**Proof.** Consider two utility functions $u$ and $v$ with $u$ more risk averse than $v$. Fix $s$, and let $a_v$ be the value that maximizes $v$'s expected utility. Assume, without loss of generality, that $u(w-c(a_v)) = v(w-c(a_v)) = 1$ and that $u(w-D(s)-c(a_v)) = v(w-D(s)-c(a_v)) = 0$. Furthermore, because $u$ is a concave transformation of $v$, $u'(w-D(s)-c(a_v)) \geq v'(w-D(s)-c(a_v)) \geq v'(w-c(a_v)) \geq u'(w-c(a_v))$. Then there exists $p^*$ such that

$$(1-p^*)u'(w-c(a_v))+p^*u'(w-D(s)-c(a_v)) = (1-p^*)v'(w-c(a_v))+p^*v'(w-D(s)-c(a_v)),$$

and for any $p < p^*$,

$$(1-p)u'(w-c(a_v))+pu'(w-D(s)-c(a_v)) \leq (1-p)v'(w-c(a_v))+pv'(w-D(s)-c(a_v)).$$

Consequently, if $p(s)$ is so low that $f(a_v)p(s) < p^*$ then

$$\frac{u(w-c(a_v)) - u(w-D(s)-c(a_v))}{(1-f(a_v)p(s))u'(w-c(a_v)) + f(a_v)p(s)u'(w-D(s)-c(a_v))} \geq$$

$$\frac{v(w-c(a_v)) - v(w-D(s)-c(a_v))}{(1-f(a_v)p(s))v'(w-c(a_v)) + f(a_v)p(s)v'(w-D(s)-c(a_v))}.$$

Therefore, when $p(s)$ is sufficiently small, the increase in risk aversion from $v$ to $u$ makes the right-hand side of equation (3.9) rise. The left-hand side of (3.9) increases in $a$ because both $c$ and $f$ are convex, and it follows that $a_u \geq a_v$. $\square$

Proposition 6 states that if an attack is not too likely, when the representative citizen becomes more risk averse the government spends more to prevent an attack. Intuitively, when an attack is not too likely, expected marginal utility of income is closer to $u'(w-c(a))$ than to $u'(w-D(s)-c(a))$, and the former is smaller than the latter. An increase in risk aversion makes the utility function more concave, which increases marginal utility close to the left endpoint of the interval $[w-D(s)-c(a), w-c(a)]$ and decreases marginal utility near the right endpoint. When the probability of attack is small, expected marginal utility decreases as the citizen becomes more risk averse. Because the citizen values income less, the government finds it optimal to spend more of the income on preventing terrorism attacks.

Conversely, if the probability of attack is high, an increase in risk aversion increases expected marginal utility of income, and the government spends less on preventing attacks. The ambiguity of the result is consistent with the findings of Dionne and Eeckhoudt [15] and Briys and Schlesinger [5] that the optimal level of self-protection may or may not increase with risk aversion.

## E. Equilibrium Terrorism and Counterterrorism

This section contains our primary results, which concern the equilibrium of the game between the terrorist organization and the target government. In light of the results of Sections 3 and 4, we can begin the analysis with the risk neutral case and then use Propositions 1 through 3 to discuss how the equilibrium behavior changes when the representative citizen becomes risk averse.

In Section 4 we found that the impact of increased risk aversion on the government's choice of the level of counterterrorism activity depends on the nature of that activity. In particular, the direction of the effect is unambiguous if the activity is purely mitigating but ambiguous if the activity is purely preventive. To aid in the identification of the counterterrorism activity into one of these two categories, we assume that the probability-of-attack function $p(a, s)$ and the damage function $D(a, s)$ are both multiplicatively separable:

$$p(a, s) = f(a)m(s),$$

and

$$D(a, s) = g(a)n(s),$$

where $f$ and $g$ are decreasing and convex, $m$ is decreasing and concave, and $n$ is increasing and concave. If $f' < 0$ but $g' = 0$ the counterterrorism activity is purely preventive, but if $g' < 0$ and $f' = 0$ it is purely mitigating.

Begin with the terrorist's decision. When the representative citizen is risk neutral, the first-order condition from Section 3 becomes

$$p(a, s)D_s(a, s) + p_s(a, s)D(a, s) = 0,$$

which implicitly defines the terrorist's best-response function $s^*(a)$. As the next proposition shows, in the risk-neutral case the terrorist's best-response function is single-valued.

**Proposition 7** *Assume that the representative citizen is risk neutral and the probability-of-attack function $p(a, s)$ and the damage function $D(a, s)$ can be written as $f(a)m(s)$ and $g(a)n(s)$, respectively. Then the terrorist organization has a dominant strategy, that is, the severity level it chooses does not depend on the action chosen by the government.*

**Proof.** The first-order condition of the terrorists can be written as

$$f(a)g(a)[m'(s)n(s) + m(s)n'(s)] = 0$$

Implicitly differentiating with respect to a and rearranging yields

$$\frac{ds^*}{da} = -\frac{(f'g + fg')(mn' + m'n)}{fg(m''n + 2m'n' + mn'')}$$

and the numerator is zero by the first-order condition. □

The result that the terrorist's activities are independent of the government's attempts to combat them is surprising. After all, if the government undertakes purely mitigating activities, for example, thereby raising the price of severity, one would expect the terrorists to respond by "purchasing" more certainty and less severity. Proposition 7 shows that this is not the case when the representative citizen of the target country is risk neutral. Instead, the terrorists choose the same severity level, call it $s^*$, no matter what the target government does.

Turning attention to the target government's decision, when the representative citizen is risk neutral the first-order condition is given by equation (3.7). The next proposition describes the shape of the government's best-response function.

**Proposition 8** *Assume that the representative citizen is risk neutral and the probability-*

*of-attack function* $p(a, s)$ *and the damage function* $D(a, s)$ *can be written as* $f(a)m(s)$ *and* $g(a)n(s)$*, respectively. Then the government's best-response function* $a^*(s)$ *is increasing when* $s < s^*$ *and decreasing when* $s > s^*$*.*

**Proof.** Implicitly differentiating (3.7) with respect to $s$ and rearranging yields

$$\frac{da^*}{ds} = -\frac{(f'g + fg')(mn' + m'n)}{c'' + mn(f''g + 2f'g' + fg'')} \tag{3.10}$$

The terrorist organization chooses $s$ to minimize

$$EU(a, s) = w - p(a, s)D(a, s) = w - f(a)m(s)g(a)n(s).$$

Then $\partial EU(a, s)/\partial s = -fg[m'n + mn']$, and by the second-order condition for a minimum, $m'n + mn' > 0$ when $s < s^*$ and $m'n + mn' < 0$ when $s > s^*$. The second-order condition also guarantees that the denominator of (3.10) is positive, and so the sign of $da^*/ds$ is the opposite of the sign of the numerator of (3.10). Finally note that, by construction, $f'$ and $g'$ are both negative, and so $da^*/ds$ has the same sign as $m'n + mn'$. □

Figure 2 shows the best-response curves for the multiplicatively-separable risk neutral case described in Propositions 7 and 8. The terrorist organization's best-response curve is vertical, consistent with the dominant strategy found in Proposition 7. The target government's best-response curve is hump-shaped, and the Nash equilibrium of the game lies at the intersection of the two curves.

As shown in the figure, in equilibrium the terrorist chooses the level of severity that provokes the highest level of counterterrorism activity. At first glance this may seem counterintuitive. After all, if the terrorist wants the attacks to succeed, why would they try to generate a large amount of counterterrorist activity? The answer lies in the nature of the terrorist organization's objective function. The goal of the

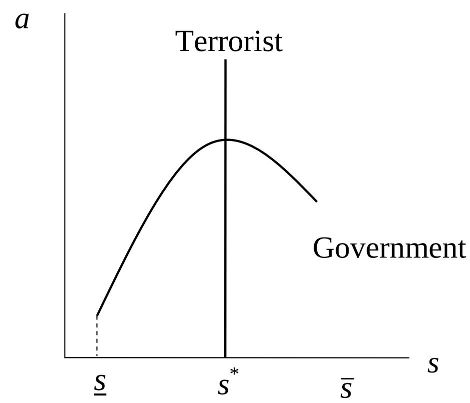


Fig. 2. Best response functions when the representative agent is risk neutral

terrorist is to make the representative citizen as poorly off as possible, and this can be achieved either through the threat of attack or by making the citizen bare a large counterterrorism burden.

The impact of risk aversion on the levels of terrorism and counterterrorism activities can now be easily derived using the results of Propositions 4 through 8. By Proposition 4, when the representative citizen becomes risk averse the terrorist's best response curve shifts rightward, and therefore risk aversion leads to more severe, but less likely attacks (holding the government's response fixed). By Propositions 5 and 6, if counterterrorism is purely mitigating or if it is purely preventive but attacks are sufficiently unlikely, risk aversion causes the government's best-response curve to shift upward. Because the risk-neutral equilibrium was at the peak of the government's best-response curve, however, the effect of risk aversion on the level of counterterrorism is ambiguous in this case, as shown in panel (a) of Figure 3. If, on the other hand, counterterrorism is purely preventive and attacks are sufficiently likely, risk aversion causes the government's best-response curve to shift downward, in which case risk aversion unambiguously causes the level of severity to rise and the level of

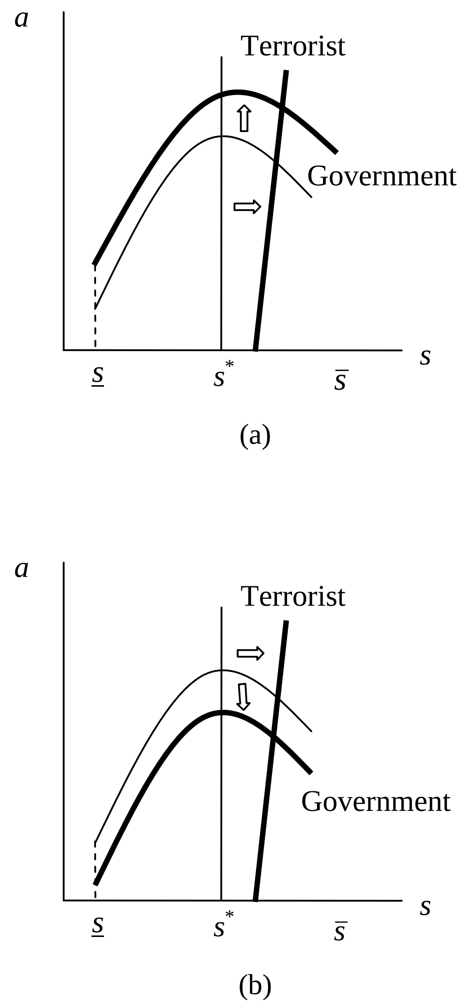


Fig. 3. Best response functions when the representative agent is risk averse

counterterrorism activity to fall, as shown in panel (b) of Figure 3.

### F. Conclusion

We analyze decisions made by a group of terrorists and a government in a zero-sum game where the terrorists minimize a representative citizen's expected utility and the government maximizes it. The terrorists' strategy balances the probability and the severity of the attack while the government chooses the level of investment reducing the probability and/or mitigating the severity. We find that, if the representative agent is risk neutral, the terrorists choose the level of severity that does not depend on the government's choice of activities, but provokes the highest level of counterterrorism activity. We also find that the citizen's risk attitudes affect the strategies of the terrorists and the government. When compared with a risk neutral population, a risk averse population in the target country leads the terrorists to commit more damaging, but less frequent, attacks.

When the government's activities mitigate the damages of an attack but do not prevent an attack, increased risk aversion leads to increased counterterrorism activities. However, when the government's activities only prevent an attack but do not mitigate the damages, the effect of risk aversion on the counterterrorism activities depends on the probability of an attack. If an attack is not too likely, the government spends more to prevent an attack when the representative citizen becomes more risk averse. If an attack is likely, then the government spends less to prevent an attack when the representative citizen becomes more risk averse.

# CHAPTER IV

## A RISK ANALYSIS OF EX-ANTE WILLINGNESS TO PAY

### A. Introduction

Consider a situation in which an individual has to pay for a good before he knows whether or not he will consume the good in the future, or before he realizes the true surplus of consuming the good. If the surplus depends on the state of nature which is unknown at the moment, what will be his ex-ante willingness to pay for this good? This ex-ante willingness to pay is called the option price because it can be thought of the price for an option to consume this good in the future.

Ex-ante welfare measures are important. For example, Presidential executive orders and federal regulations urge policy makers to consider benefits and costs of various policy programs that involve risks and unknowns. If policy makers collect information on the wrong set of ex-ante welfare measures, then programs will not reflect social desires, let alone be in any sense efficient. The concept of option price has received much attention since Graham [21] adopted it in cost-benefit analysis and suggested that the option price is an appropriate welfare measure for a public good under uninsurable risk. Following Graham's framework, economists have used the option price as an ex-ante measure of benefits in several health and environmental studies.[1]

Cicchetti and Freeman [14] show that an option price exists for a private good in a market of a perfectly discriminating monopolist who sells options for the good in advance by charging each individual his willingness to pay. For example, if there are two states of nature, a sunny or a rainy day, and a perfectly discriminating monopolist

[1] For example, Smith and Devousges [45]; Cameron [8]; Riddel and Shaw [35].

knows each individual's willingness to pay for a baseball ticket for a game in each weather condition, the option price is the willingness to pay for the ticket when it is sold in advance–the true state is not yet known. If the individual does not buy the ticket in advance, on the game day he will be charged the willingness to pay given that it shines or it rains. Specifically, the monopolist knows both the ex-ante willingness to pay (or the option price) and the ex-post surplus in each state. It is possible that the individual's option price is greater or less than the expected surplus. The difference between the option price and the expected surplus has been called option value.

Even though the appropriate ex-ante benefit measure is the option price, the option value may play a crucial role in a cost-benefit analysis. Smith [44] suggests that it is possible that expected surplus might be measured more easily than option price and one can use option value in order to gauge the error in using expected surplus in an ex-ante analysis. Cicchetti and Freeman[14] state that the option value is an individual's risk premium, and thus it is positive for any risk-averse agent. Even though the latter part of the statement has been proved to be wrong by showing that the option value can take either a positive or a negative sign (see Schmalensee [40] and Graham [21]), no one to my knowledge has demonstrated how the option value and the risk premium are related.

It is obvious that the option value cannot equal the risk premium because a risk-averse agent must have a nonnegative risk premium, but may have a negative option value. Nonetheless, economists still use Cicchetti and Freeman's interpretation of the option value (for example, Chavas and Mullarkey [10] p.23-24). In this paper, I examine the relationship between the option value and the risk premium by applying a methodology suggested by Nau [31] to Graham's model of option price and option value. I find that the option value actually is similar to Nau's buying price for a fixed payment of the expected surplus, and there is a special case where the option value is a negative of the compensating risk premium.

Another goal of this paper is to relax the expected utility assumption in Graham's model. Since many experiments show that individual behavior does not always agree with the independence axiom of expected utility theory, it is important to determine how the option price and the option value change when one assumes a nonexpected utility theory rather than the conventional expected utility theory. I find that if an expected utility maximizer and a rank-dependent expected utility agent who is pessimistic have the same state-dependent surplus for a good, the pessimistic agent's option value is larger in magnitude.

In the next section, I discuss how the option value is related to the risk premium. In Section 3, I generalize Graham's model to rank-dependent expected utility. I conclude in Section 4.

## B. The Relationship between Risk Premium and Option Value

### 1. Background

Weisbrod [48] introduces the concept of option demand in a classic example, the willingness to pay for preserving a national park that an individual may or may not visit in the future. Cicchetti and Freeman [14] formulate the option demand in a problem of two periods. First an individual decides whether to buy an option to consume a good, and later, learns whether he demands the good. Cicchetti and Freeman define *option price* as the willingness to pay for the option, and define *option value* as the difference between the option price and the expected surplus. They interpret the option value as a risk premium and suggest that it is nonnegative for a risk-averse person.

Assuming expected utility theory, Schmalensee [40] defines option price as the maximum amount an individual is willing to pay with certainty to guarantee that a preferred price system will prevail in all states of nature. Let $\mathbf{P}$ and $\mathbf{P}^*$ denote two price systems where $\mathbf{P}$ is preferred to $\mathbf{P}^*$. Assume that there are $n$ states of nature, and the probability that state $i$ occurs is $p_i$. Let $w_i$ and $u_i$ denote income and utility function in state $i$, respectively. The option price, $T$, is defined by

$$\sum_{i=1}^{n} p_i u_i(w_i - T, \mathbf{P}) = \sum_{i=1}^{n} p_i u_i(w_i, \mathbf{P}^*).$$

If $s_i$ denotes the consumer's surplus (the Hicksian compensating variation) in state $i$, then

$$u_i(w_i - s_i, \mathbf{P}) = u_i(w_i, \mathbf{P}^*).$$

Similar to Cicchetti and Freeman's definition of option value, Schmalensee's option value, $t$, is defined by

$$t = T - \sum_{i=1}^{n} p_i s_i.$$

Schmalensee argues that the option value can be positive or negative for a risk-averse agent.

Graham [21] adopts Schmalensee's model for cost-benefit analyses. If $u_i^*(w_i)$ and $u_i(w_i)$ are utility levels of income $w_i$ when a good (or a public project) is unavailable and available, respectively, then the willingness to pay for such a good in state $i$, $s_i$, satisfies this condition:

$$u_i(w_i - s_i) = u_i^*(w_i).$$

Let $\mathbf{p} = [p_1, \ldots, p_n]$ be a vector of probabilities, $\mathbf{w} = [w_1, \ldots, w_n]$ be a vector of income levels, and $\mathbf{s} = [s_1, \ldots, s_n]$ be a vector of surplus levels. Let $\mathbf{u}[\mathbf{w}] = [u_1(w_1), \ldots, u_n(w_n)]$ be a utility vector. The expected surplus of the good is then $E_{\mathbf{p}}(\mathbf{s})$, and denote the (state-dependent) expected utility of the income vector $\mathbf{w}$ by

$U(\mathbf{w})$. Let $\mathbf{1}$ be a unit vector. The option price given the surplus vector $T(\mathbf{s})$ satisfies

$$U(\mathbf{w}-\mathbf{s}) = E_{\mathbf{p}}(\mathbf{u}[\mathbf{w}-\mathbf{s}]) = E_{\mathbf{p}}(\mathbf{u}[\mathbf{w}-T(\mathbf{s})\mathbf{1}]). \tag{4.1}$$

Since the sign of the option value (or risk premium as interpreted by Cicchetti and Freeman) is ambiguous, the role of risk premia in the ex-ante payment problem deserves further explanation. If the option value is akin to the risk premium, how can a risk averse agent have a negative risk premium? After Cicchetti and Freeman's exposition, no other paper has explicitly discussed this matter. [2]

Kimball [22] calls the premium that an agent is willing to pay for avoiding a zero-mean risk an *equivalent risk premium*, and the premium that an agent requires as a compensation for bearing a zero-mean risk a *compensating risk premium*.[3] Nau [31] generalizes Kimball's framework to allow for state-dependent utility and derives a selling risk premium and a buying risk premium that relate to Kimball's equivalent risk premium and compensating risk premium, respectively. However, in his article, Nau uses the buying risk premium rather than the selling risk premium in measuring local risk aversion because it directly measures the local quasi-concavity of utility.

Nau defines a risk-neutral probability distribution, a buying price and a buying risk premium as follow. The normalized gradient of $U$ at wealth $\mathbf{w}$, which is known as the *risk-neutral probability distribution*, is defined as

$$\mathbf{q}(\mathbf{w}) = \frac{\nabla U(\mathbf{w})}{\|\nabla U(\mathbf{w})\|},$$

where $\|\cdot\|$ is the $L_1$ norm. The *buying price* for a risky asset $\mathbf{z}$, denoted $B(\mathbf{z})$, is

[2]Schlee and Schlesinger [41] show a similarity between risk premium and option price for accessing a contingent claim market.

[3]Pratt's [33] risk premium is the equivalent risk premium.

determined by

$$E_{\mathbf{p}}(\mathbf{u}[\mathbf{w}+\mathbf{z}-B(\mathbf{z})\mathbf{1}]) = E_{\mathbf{p}}(\mathbf{u}[\mathbf{w}]).$$

In the above expression, if $B(\mathbf{z})\mathbf{1}$ is instead added in the brackets in the right hand side, $B(\mathbf{z})$ would be interpreted as the certainty equivalent for the asset $\mathbf{z}$. Finally, the *buying risk premium* for the asset $\mathbf{z}$, $\mathrm{b}(\mathbf{z})$, is defined by

$$b(\mathbf{z}) = E_{\mathbf{q}(\mathbf{w})}(\mathbf{z}) - B(\mathbf{z}).$$

Nau adopts Yarri's [49] definition of risk aversion, specifically, a risk-averse individual has preferences that are payoff-convex.[4] He concludes that the individual is risk averse if and only if the buying risk premium is nonnegative for every asset at every wealth distribution. In the ex-ante payment problem, I also use the buying risk premium in the analysis because it fits the situation better than the selling risk premium. Specifically, the individual is assumed to pay a constant payment in advance and he is asked how much he needs to be compensated to make him indifferent to when he pays the state-dependent surplus.

## 2. Results

I assume that the utility function $u_i$ is twice differentiable. In this paper, I follow Nau's [31] approach in identifying buying price and risk premium, but slightly change notations as follows. The risk-neutral probability distribution at surplus $\mathbf{s}$, is defined as

$$\mathbf{q}(\mathbf{s}) = \frac{\nabla U(\mathbf{w}-\mathbf{s})}{\|\nabla U(\mathbf{w}-\mathbf{s})\|}.$$

[4] The preference relation $\succsim$ is payoff-convex if $\mathbf{x} \succsim \mathbf{z}$ and $\mathbf{y} \succsim \mathbf{z}$ imply $\alpha\mathbf{x} + (1-\alpha)\mathbf{y} \succsim \mathbf{z}$ for $\alpha \in (0,1)$.

For some payment vector $\mathbf{y}$, if an individual's payment vector is changed from $\mathbf{s}$ to $\mathbf{y}$, then his buying price for this change, denoted $B(\mathbf{y})$, is determined by

$$E_{\mathbf{p}}(\mathbf{u}[\mathbf{w} - \mathbf{y} - B(\mathbf{y})\mathbf{1}]) = E_{\mathbf{p}}(\mathbf{u}[\mathbf{w} - \mathbf{s}]). \tag{4.2}$$

The buying risk premium for the change from $\mathbf{s}$ to $\mathbf{y}$ is defined by

$$b(\mathbf{y}) = E_{\mathbf{q(s)}}(\mathbf{s} - \mathbf{y}) - B(\mathbf{y}). \tag{4.3}$$

In the ex-ante payment problem, I assume that initially an individual must pay the state-dependent surplus for a good, represented by the payment vector $\mathbf{s}$. Then the problem is changed so that the individual is assumed to pay the constant amount $E_{\mathbf{p}}(\mathbf{s})$ before he knows the state of nature. Defining $\bar{\mathbf{s}} = E_{\mathbf{p}}(\mathbf{s})\mathbf{1}$, this is equivalent to saying that the individual's payment vector has been changed from $\mathbf{s}$ to $\bar{\mathbf{s}}$, which might increase or decrease the individual's expected utility from the initial position. To keep this individual's expected utility unchanged, the individual must pay a buying price in addition to the constant amount $E_{\mathbf{p}}(\mathbf{s})$. Proposition 1 describes how the option value is related to the buying price.

**Proposition 9** *The option value is equal to the buying price for changing the payment from $\boldsymbol{s}$ to $\bar{\mathbf{s}}$, i.e., $t(\boldsymbol{s}) = B(\bar{\mathbf{s}})$.*

**Proof.** Equating (4.1) and (4.2) yields:

$$E_{\mathbf{p}}(\mathbf{u}[\mathbf{w} - T(\mathbf{s})\mathbf{1}]) = E_{\mathbf{p}}(\mathbf{u}[\mathbf{w} - \mathbf{y} - B(\mathbf{y})\mathbf{1}]).$$

Let $\mathbf{y} = \bar{\mathbf{s}}$. Then, $T(\mathbf{s}) = E_{\mathbf{p}}(\mathbf{s}) + B(\bar{\mathbf{s}})$, and therefore $B(\bar{\mathbf{s}}) = t(\mathbf{s})$. □

Following Graham's [21] approach, Proposition 9 can be illustrated in the 2-state payment space in Figures 4 and 5. For example, assume that the state is determined by the individual's health, with 1 for sick and 2 for healthy. For simplicity, let

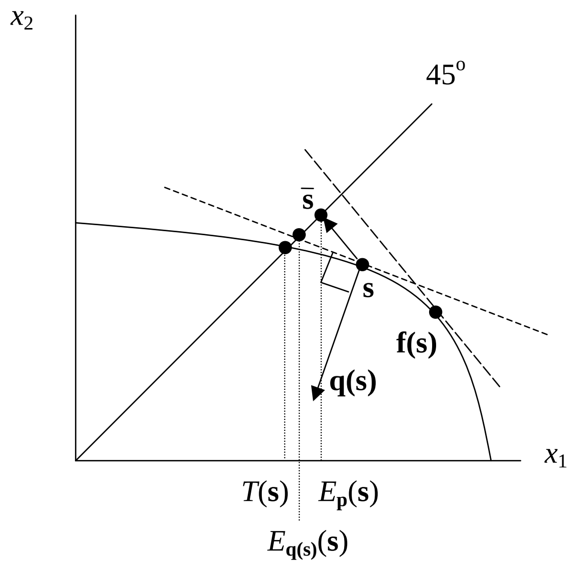


Fig. 4. Willingness-to-pay locus for a hospital ($s_1 > s_2$)

$w_1 = w_2 = w$. Based on a previous survey by Viscusi and Evans [47], $u_1'(w) < u_2'(w)$ for all $w$. If the good in this problem is a hospital or a health care service, we should expect that $s_1 > s_2$. The willingness-to-pay locus $(x_1, x_2)$, defined by

$$p_1 u_1(w - x_1) + p_2 u_2(w - x_2) = p_1 u_1(w - s_1) + p_2 u_2(w - s_2),$$

is shown in Figure 4. Along the willingness-to-pay locus, $\frac{\partial x_2}{\partial x_1} = -\frac{p}{1-p} \cdot \frac{u_1'(w_1 - x_1)}{u_2'(w_2 - x_2)}$, and $\frac{\partial^2 x_2}{\partial x_1^2} = \frac{p}{1-p} \cdot \frac{u_1''(w_1 - x_1)}{u_2'(w_2 - x_2)}$. Any payment in the upper contour set (between the willingness-to-pay locus and the origin) is preferred to the willingness-to-pay locus. The expected surplus and the option price denoted $E_{\mathbf{p}}(\mathbf{s})$ and $T(\mathbf{s})$ yield a negative option value. In Figure 5 the good is a sport facility in which I assume that $s_1 < s_2$. In this case, the option value is positive. The sign of the option value can be explained by Nau's buying price. In the hospital example, $\mathbf{s} \succsim \bar{\mathbf{s}}$ and the individual must be compensated by a positive amount of money if he has to pay $\bar{\mathbf{s}}$. Therefore the buying price (or the option value) must be negative. In the sport facility example, $\bar{\mathbf{s}} \succsim \mathbf{s}$, the individual's

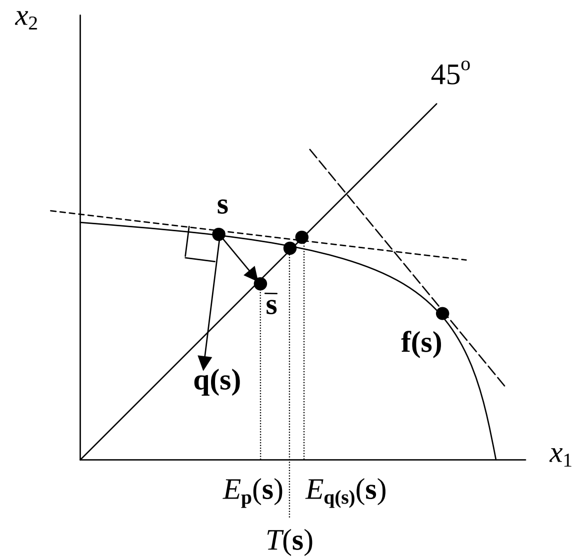


Fig. 5. Willingness-to-pay locus for a sport facility $(s_1 < s_2)$

expected utility would increase if he did not pay for this change. To keep his expected utility unchanged, the buying price must be positive. In both cases, the risk premia, $E_{\mathbf{q(s)}}(\mathbf{s}) - T(\mathbf{s})$, are positive.

From Proposition 9, it is clear that the option value should not be interpreted as a risk premium. However, there is a special case where the option value can be derived directly from the buying risk premium. To get the result, following Graham's definition, I define the *fair-bet payment* of surplus $\mathbf{s}$, $\mathbf{f}(\mathbf{s})$, as a vector $\mathbf{x}$ that maximizes $E_{\mathbf{p}}(\mathbf{x})$ subject to $E_{\mathbf{p}}(\mathbf{u}[\mathbf{w} - \mathbf{x}]) = E_{\mathbf{p}}(\mathbf{u}[\mathbf{w} - \mathbf{s}])$. If $\mathbf{f}(\mathbf{s}) = [f_1, \ldots, f_n]$, then from the maximization problem, $u_i'(w_i - f_i) = u_j'(w_j - f_j)$ for all $i, j = 1, \ldots, n$. The surplus vector is a fair-bet vector of itself when $\mathbf{s} = \mathbf{f}(\mathbf{s})$.

**Proposition 10** *Assume that the surplus vector is a fair-bet vector of itself. The option value is the negative of the risk premium, i.e.,* $t(\boldsymbol{s}) = -b(\bar{\mathbf{s}})$*, and the individual is risk averse if and only if* $t(\boldsymbol{s}) \leq 0$*.*

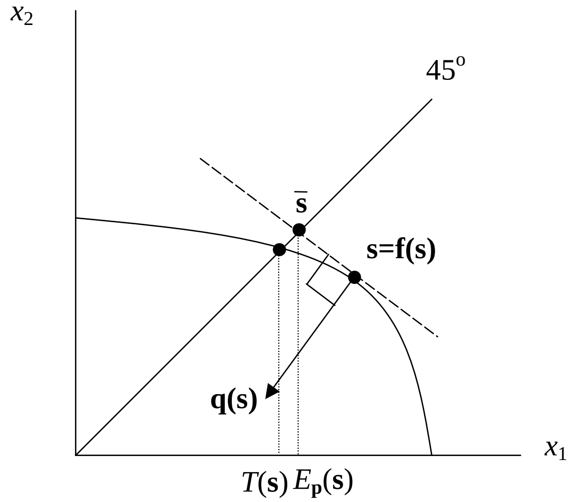


Fig. 6. Willingness-to-pay locus with $\mathbf{s} = \mathbf{f}(\mathbf{s})$

**Proof.** Let $\mathbf{z} = \mathbf{s} - \bar{\mathbf{s}}$. If $\mathbf{s} = \mathbf{f}(\mathbf{s})$, then $E_{\mathbf{q}(\mathbf{s})}(\mathbf{z}) = E_{\mathbf{p}}(\mathbf{z}) = 0$. Hence, from (4.3), $b(\bar{\mathbf{s}}) = -B(\bar{\mathbf{s}})$. The second part of the proposition follows immediately from Proposition 1 in Nau [31]. □

Now I assume that $\mathbf{s} = \mathbf{f}(\mathbf{s})$. The result can be illustrated in Figure 6. The option value is $T(\mathbf{s}) - E_{\mathbf{p}}(\mathbf{s})$ which is negative, and the risk premium is $E_{\mathbf{p}}(\mathbf{s}) - T(\mathbf{s})$.

## C. Rank-Dependent Expected Utility

### 1. Background

Economists and psychologists agree that individuals tend to substitute decision weights for probabilities so that expected utility theory cannot always explain individuals' behavior. An appropriate utility theory that embodies probability transformation without violating the first-order stochastic dominance is a rank-dependent expected utility proposed by Quiggin [34]. Let $\mathbf{x} = (x_1, \ldots, x_n)$ and, without loss of generality,

$x_1 < \ldots < x_n$. The rank-dependent expected utility function is given by

$$V(\mathbf{x};\mathbf{p}) = h(p_1)u(x_1) + \sum_{i=2}^{n} \left[ h(\sum_{j=1}^{i} p_j) - h(\sum_{j=1}^{i-1} p_j) \right] u(x_i), \tag{4.4}$$

where $h : [0,1] \to [0,1]$ is a *probability transformation function* with $h(0) = 0$, $h(1) = 1$, and $h' \geq 0$. Tversky and Kahneman [46] suggest a single parameterized form of the probability transformation function,

$$h(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}},$$

where $\gamma \in (0,1)$. With certain values of the parameter, one obtains the popular inverted-S weighting function which implies that the individual overweights extreme outcomes with small probabilities. As a result, the four-fold pattern of risk attitudes, risk-seeking for small-probability gains and large-probability losses, and risk aversion for small-probability losses and large-probability gains, can be explained.

## 2. Results

First I assume that expected utility (EU) theory holds. There are two states and the utility function is the same in both states, but income varies. Assume throughout this section that $w_1 < w_2$ and surplus $s_i$ depends on $w_i$, for $i = 1, 2$, with $0 \leq w_1 - s_1 \leq w_2 - s_2$. This is a special case of state-dependent utility. Therefore the option value is the buying price in the sense of Nau [31]. The fair-bet payment in this case is the vector $\mathbf{f}$ such that $w_1 - f_1 = w_2 - f_2$. Changing the payment vector from $\mathbf{s} = \mathbf{f}$ to $\bar{\mathbf{s}}$ is an increase in risk in the sense of Rothschild and Stiglitz [37]. Therefore $\mathbf{s} \succsim \bar{\mathbf{s}}$ and the option value $t(\mathbf{s})$ is negative.

Two other interesting cases are a normal good and an inferior good. For a normal good, $s_1 \leq s_2$, and paying the expected surplus is an increase in risk. Thus the option value is negative. In contrast, for an inferior good, $s_1 \geq s_2$, paying the

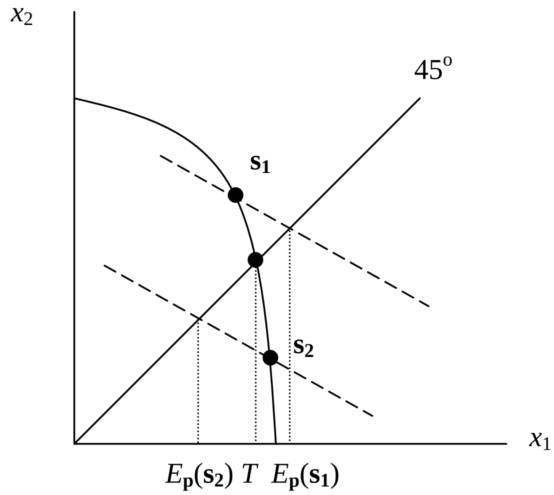


Fig. 7. Willingness-to-pay locus of the EU agent

expected surplus is an increase in risk and the option value is positive. These results are illustrated in Figure 7. The payment vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ are the surplus vectors of a normal good and an inferior good, respectively. Both goods have the same option price ($T$). The option values $t(\mathbf{s}_1)$ is negative and $t(\mathbf{s}_2)$ is positive.

Now let the agent be a rank-dependent expected utility (RDEU) maximizer, and the probabilities that the wealth level is $w_1$ and $w_2$ are $p$ and $(1 - p)$, respectively. Then (4.4) can be written as

$$V(w_1, w_2; p) = h(p)u(w_1) + [1 - h(p)]u(w_2). \tag{4.5}$$

If $h(p) > p$, the individual overweights the lower outcome, and the individual is said to be *pessimistic*. If $h(p) < p$, the individual overweights the higher outcome, and the individual is said to be *optimistic* (see also Quiggin [34] and Neilson [32]). Suppose that two agents, an EU agent and a RDEU agent have the same surplus. Who has a higher option value?

**Proposition 11** *If an EU agent and a pessimistic (an optimistic) agent have the same surplus, then the pessimistic (optimistic) agent's option value is larger (smaller) in magnitude.*

**Proof.** Define $u(s_1, s_2) = pu(w_1 - s_1) + (1-p)u(w_2 - s_2)$ and $v(s_1, s_2) = h(p)u(w_1 - s_1) + (1 - h(p))u(w_2 - s_2)$ so that $u$ is an EU agent and $v$ is a pessimistic agent. Since $w_1 - s_1 < w_2 - s_2$ and $h(p) > p$, then $u(s_1, s_2) > v(s_1, s_2)$. Let $T_u$ and $T_v$ be the option price of agent $u$ and $v$, respectively. Then $u(s_1, s_2) = u(T_u, T_u)$ and $v(s_1, s_2) = v(T_v, T_v) \equiv \bar{v}$. Hence $u(T_v, T_v) > v(T_v, T_v)$. We have the following equations.

$$u(s_1, s_2) - \bar{v} = [h(p) - p][u(w_2 - s_2) - u(w_1 - s_1)] \tag{4.6}$$

$$u(T_v, T_v) - \bar{v} = [h(p) - p][u(w_2 - T_v) - u(w_1 - T_v)] \tag{4.7}$$

Let $g(x_1)$ be a continuous function such that $v(x_1, g(x_1)) = \bar{v}$. We know that $\frac{dg(x_1)}{dx_1} < 0$. Hence $\frac{d}{dx_1}[u(w_2 - g(x_1)) - u(w_1 - x_1)] > 0$. If the good is normal for both agents, i.e., $s_2 > s_1$, then $T_v > s_1$ and the RHS of (4.7) is greater than the RHS of (4.6). Therefore $u(T_v, T_v) > u(s_1, s_2) = u(T_u, T_u)$, and $T_v < T_u$. Since the good is normal for both agents, $T_v < T_u < E(\mathbf{s})$. Let $t_u$ and $t_v$ be the option value of agent $u$ and $v$, respectively, it follows that $t_v < t_u < 0$. Similarly, if the good is inferior, $t_v > t_u > 0$. In either case, $|t_v| > |t_u|$. If the agent is optimistic, then $h(p) > p$ and the signs are reversed. $\square$

Fix the level of the RDEU at $\bar{v} = h(p)u(w_1 - s_1) + [1 - h(p)]u(w_2 - s_2)$. The willingness-to-pay locus $(x_1, x_2)$ for a RDEU agent is derived from

$$\bar{v} = \begin{cases} h(p)u(w_1 - x_1) + [1 - h(p)]u(w_2 - x_2) & \text{if } w_1 - x_1 \leq w_2 - x_2; \\ h(1-p)u(w_2 - x_2) + [1 - h(1-p)]u(w_1 - x_1) & \text{if } w_1 - x_1 > w_2 - x_2. \end{cases}$$

The willingness-to-pay locus now has a kink at $(k_1, k_2)$, where $w_1 - k_1 = w_2 - k_2$.

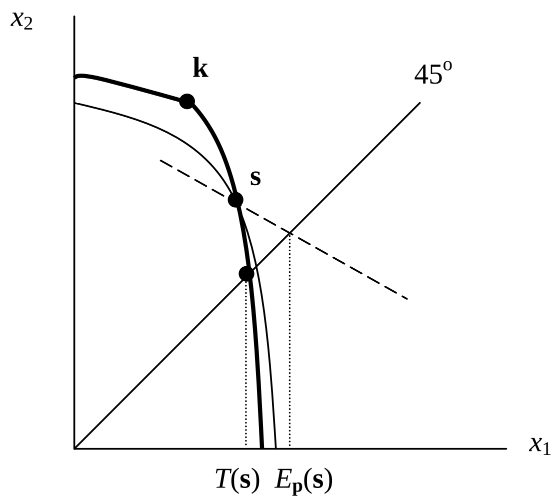


Fig. 8. Willingness-to-pay locus of the RDEU agent

Along the willingness-to-pay locus, $\frac{\partial x_2}{\partial x_1} = -\frac{h(p)}{1-h(p)} \cdot \frac{u'(w_1-x_1)}{u'(w_2-x_2)}$ and $\frac{\partial^2 x_2}{\partial x_1^2} = \frac{h(p)}{1-h(p)} \cdot \frac{u''(w_1-x_1)}{u'(w_2-x_2)}$ when $x_1 \geq k_1$; and $\frac{\partial x_2}{\partial x_1} = -\frac{1-h(1-p)}{h(1-p)} \cdot \frac{u'(w_1-x_1)}{u'(w_2-x_2)}$ and $\frac{\partial^2 x_2}{\partial x_1^2} = \frac{1-h(1-p)}{h(1-p)} \cdot \frac{u''(w_1-x_1)}{u'(w_2-x_2)}$ when $x_1 < k_1$. If I assume that the surplus of a normal good for the RDEU agent is the same as for the EU agent, the RDEU willingness-to-pay locus is the bold curve with a kink at **k** crossing the EU willingness-to-pay locus at **s** as shown in Figure 8. This is similar to the kink on a indifference curve of an individual who is risk averse of order 1 defined by Segal and Spivak [42].[5] Segal and Spivak [43] prove that the first-order risk aversion at **k** is equivalent to the local utility function that is not differentiable at **k**.

The result described above is based on state-independent utility, while Graham's model is based on state-dependent utility. To apply the RDEU assumption to a state-dependent utility, I follow Chiu's methodology. Chiu [11] ranks the prospective out-

[5]For a random variable $\varepsilon$ with $E(\varepsilon) = 0$. Let $\mu(t\varepsilon)$ be the risk premium that the agent is willing to pay to avoid the risk $t\varepsilon$. The agent is risk averse of order 1 if $\partial\mu(t\varepsilon)/\partial t|_{t=0} \neq 0$.

comes according to their state-dependent utility levels instead of their state-dependent income levels. Now assume that there are two states of nature, $u_1(w) < u_2(w)$ for all $w_1 = w_2 = w$, and $u_1(w_1 - s_1) \leq u_2(w_2 - s_2)$. If the utility function is state-dependent, the RDEU can be written as

$$W(w_1, w_2; p) = h(p)u_1(w_1) + [1 - h(p)]u_2(w_2), \tag{4.8}$$

where $u_1(w_1) < u_2(w_2)$. Fix the level of the RDEU at $\bar{w} = h(p)u_1(w_1 - s_1) + [1 - h(p)]u_2(w_2 - s_2)$. The willingness-to-pay locus $(x_1, x_2)$ is derived from

$$\bar{w} = \begin{cases} h(p)u_1(w_1 - x_1) + [1 - h(p)]u_2(w_2 - x_2) & \text{if } u_1(w_1 - x_1) \leq u_2(w_2 - x_2); \\ h(1-p)u_2(w_2 - x_2) + [1 - h(1-p)]u_1(w_1 - x_1) & \text{if } u_1(w_1 - x_1) > u_2(w_2 - x_2). \end{cases}$$

Let $(k_1, k_2)$ be a payment vector on the willingness-to-pay locus such that $u_1(w_1 - k_1) = u_2(w_2 - k_2)$. If $w_1 < w_2$, then $k_1 < k_2$. The analysis is similar to the state-independent case (see Figure 8), except that with state-dependent utility, $k_1$ may be negative.

## D. Conclusion

The option price is the ex-ante willingness to pay for a good whose surplus is dependent on state of nature. The difference between the option price and the expected surplus is called option value. The option price has been used as an appropriate welfare measure under risk under the assumption of expected utility, while the option value has been interpreted as the risk premium. This interpretation is arguable since the option value can be positive or negative, but the risk premium must be positive for a risk-averse agent. I show that actually the option value is the buying price for changing the payment from the state-dependent surplus to the expected surplus, and it can be positive or negative.

I also show that Graham's model can be generalized to rank-dependent expected utility. When there are two states of nature and two agents, an EU agent and a pessimistic agent. If both have the same surplus, then the pessimistic agent's option value is larger in magnitude. In contrast, if an EU agent and an optimistic agent have the same surplus, then the optimistic agent's option value is smaller in magnitude.

# CHAPTER V

## SUMMARY

In Chapter II, we characterize comparative measures of Arrow-Pratt and Ross risk aversion through a comparative statics problem. The Ross characterization arises when two risk averse agents optimally choose their shifts in probability distribution toward a preferred distribution that differs from the original one by a mean preserving decrease in risk, and the cost of shifting probabilities is monetary. The Arrow-Pratt characterization arises when the original distribution differs from the preferred distribution by a simple mean-preserving spread, and the cost is a utility cost.

Using the same approach, we generalize the comparative statics problem to the difference in $n^{th}$ degree risk between two probability distributions and the $n^{th}$ degree of risk aversion. For the third degree risk comparison in the monetary cost problem, our characterization of more Ross risk averse in the third degree coincides with more downside risk averse, as recently defined by Modica and Scarsini [30]. On the other hand, in the utility cost problem, the Arrow-Pratt risk aversion measures in the 3rd degree coincide with the absolute prudence measures defined by Kimball [22].

In Chapter III, we analyze decisions made by a group of terrorists and a government in a zero-sum game where the terrorists minimize a representative citizen's expected utility and the government maximizes it. The terrorists' strategy balances the probability and the severity of the attack while the government chooses the level of investment reducing the probability and/or mitigating the severity. We find that, if the representative agent is risk neutral, the terrorists choose the level of severity that does not depend on the government's choice of activities, but provokes the highest level of counterterrorism activity. We also find that the citizen's risk attitudes affect the strategies of the terrorists and the government. When compared with a risk

neutral population, a risk averse population in the target country leads the terrorists to commit more damaging, but less frequent, attacks.

When the government's activities mitigate the damages of an attack but do not prevent an attack, increased risk aversion leads to increased counterterrorism activities. However, when the government's activities only prevent an attack but do not mitigate the damages, the effect of risk aversion on the counterterrorism activities depends on the probability of an attack. If an attack is not too likely, the government spends more to prevent an attack when the representative citizen becomes more risk averse. If an attack is likely, the government spends less to prevent an attack when the representative citizen becomes more risk averse.

In Chapter IV, we discuss the option price which is defined by Cichetti and Freeman [14] as an ex-ante willingness to pay for a good whose surplus is dependent on state of nature. The difference between the option price and the expected surplus is called option value. The option price has been used as an appropriate welfare measure under risk under the assumption of expected utility, while the option value has been interpreted as the risk premium. This interpretation is arguable since the option value can be positive or negative, but the risk premium must be positive for a risk-averse agent. We show that actually the option value is the buying price for changing the payment from the state-dependent surplus to the expected surplus, and it can be positive or negative.

We also show that Graham's [21] model can be generalized to rank-dependent expected utility. When there are two states of nature and two agents, an EU agent and a pessimistic RDEU agent. If both have the same surplus, then the pessimistic RDEU agent's option value is larger in magnitude. In contrast, if an EU agent and an optimistic RDEU agent have the same surplus, then the optimistic RDEU agent's option value is smaller in magnitude.

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# VITA

Paan Jindapon was born in Bangkok , Thailand, in 1975. He received his Bachelor of Arts degree in Economics from Thammasat University in 1995, Master of Arts degree in Applied Economics from Southern Methodist University in 1998, and Master of Business Administration degree in Finance from Thammasat University in 2002. He entered the graduate program in Economics at Texas A&M University in August 2002, and received his Doctor of Philosophy degree in August 2006. Under the supervision of Dr. William S. Neilson, his research interests include behavioral decision theory, game theory, and industrial organization.

Dr. Jindapon may be reached at Department of Economics, Finance and Legal Studies, The University of Alabama, Tuscaloosa, AL 35487. His email address is jindapon@gmail.com.