ORDER ACCEPTANCE AND SCHEDULING AT A MAKE-TO-ORDER

SYSTEM USING REVENUE MANAGEMENT

A Dissertation

by

ANSHU JALORA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Industrial Engineering

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Approved by:

Chair of Committee,	Brett A. Peters
Committee Members,	Guy L. Curry
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ABSTRACT

Order Acceptance and Scheduling at a Make-to-Order System Using Revenue
Management. (August 2006)

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Chair of Advisory Committee: Brett A. Peters

Make-to-order (MTO) systems have been traditionally popular in manufacturing
industries that either seek to provide greater variety to their customers or make
products that are unique to their customers. More recently, with shrinking product
life cycles, there is an increasing interest in operating as MTO systems. With the
tremendous success of revenue management techniques in the service industries over
the last three decades, there is a growing interest in applying these techniques in
MTO manufacturing industries.

In the present work, we consider three problems that apply revenue management
(RM) to *on-date* delivery MTO systems. In the first problem, we assume that all
orders completed in advance of their due-dates are stored at third party warehouses
and apply RM in computing efficient order acceptance and scheduling policies. We
develop an optimal solution scheme, and based on the insights gained on the structural
properties of the optimal solution, we develop a stochastic approximation scheme for
finding efficient solutions. Through computational studies on simulated problems, we
illustrate the potential of RM in improving net profits over popular practices.

In our second problem, we extend the RM model to consider presence of a certain
amount of *first party* warehousing capacity for storing the orders completed in advance
of their due-dates. We study the conditions under which it is desirable to consider the
holding cost aspects in the RM model. In our third problem, we develop a scheme

for determining an efficient capacity of the *first party* warehouse that is used for storing the orders completed in advance of their due-dates at an *on-date* delivery MTO system. This scheme captures the completed orders storage demand resulting from a RM based order acceptance and scheduling policy. We illustrate that when booking horizon is large, considerable amount of savings in the holding costs can be made with an efficiently sized *first party* warehouse.

To my Dad

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CHAPTER I

INTRODUCTION

Make-to-order (MTO) and *on-date* deliveries are two features that are increasingly gaining popularity in manufacturing industry. After the tremendous success with revenue management (RM) over the last three decades in various segments of the service industry such as airlines, hotels, car-rentals, media, etc., there is a huge interest in understanding and applying RM in the manufacturing industry. Kroll [1] cites several examples of manufacturing organizations that are making efforts in this direction. Recently, General Motors has expressed interest in utilizing RM in their pricing policies [2]. In this dissertation, we develop a RM framework for designing efficient order acceptance and scheduling policies for an *on-date* delivery MTO system.

Traditionally, there have been two main reasons for the popularity of MTO manufacturing. The first is that in many industry segments (for example, print and electronic media) the products offered by the firms are unique to each customer [3]. The other reason is that inspired by the success of companies such as Dell Computer Corporation using an *on-demand* manufacturing model [4], many firms seek to offer greater product variety at a low cost by eliminating finished goods inventories and operating in an MTO fashion [5, 6, 7].

Recently, various factors have contributed to increases in the popularity of MTO practices. Many segments in the manufacturing industry are experiencing shrinking product life cycles (for example, semi-conductor manufacturing) and increasing demand for customized products (for example, personal computers, garments, automobiles). As a result, there is a growing interest in exploring an MTO or a hybrid of

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an MTO and make-to-stock (MTS) practices to gain operational efficiencies [8, 9, 10].

By following an MTO approach, organizations benefit by eliminating finished goods inventory carrying and obsolescence costs [11]. However, this benefit comes at the cost of increased response time in meeting customer demand and/or cost of keeping higher manufacturing capacities to accommodate variations in customer demand [12]. When it is not economical to keep spare manufacturing capacities and there is little flexibility in terms of due-dates, it becomes very critical for the MTO manufacturer to selectively accept and schedule customer orders, so that neither the manufacturing capacity gets wasted because too few jobs have been accepted nor high profit earning jobs are turned down because low profit earning jobs have been previously accepted.

On-date deliveries of raw material from suppliers allow customers to reduce their raw material inventories, as a result minimizing the raw material inventory holding costs for customers. Therefore, an increasing number of customers are demanding *on-date* deliveries from their suppliers [13]. However, it should be noted that *on-date* deliveries do not totally eliminate the holding costs, but transfer the holding costs from the customers to the suppliers. This is because, under an *on-date* delivery system, suppliers bear the holding costs incurred for the orders that are completed in advance of their due-dates. Therefore, it becomes very important for the suppliers to consider the holding costs aspect while accepting and scheduling customer orders.

To develop efficient order acceptance and scheduling policies, RM techniques can be employed. In the following section, we present a brief overview of RM. RM has been extensively studied in the context of the airline industry [14]. Section B presents a comparison between airline and MTO manufacturing RM models, and highlights that solutions developed for airline RM cannot be directly applied to MTO manufacturing RM problem. This motivates us to explore efficient solutions to a version of the

MTO manufacturing RM model. Section C outlines the scope of this dissertation, while Section D describes the organization of the remainder of the dissertation.

A. Overview of Revenue Management

Revenue management (RM) is an area of operations research that is concerned with demand management by finding at *what* price, *how much* of a limited resource should be made available to the customers [14]. Cross [15] describes revenue management as 'The art and science of predicting real-time customer demand at the micromarket level and optimizing the price and availability of products.'

RM is also called by the following names - *yield management*, *pricing and revenue management*, *pricing and revenue optimization*, *revenue process optimization*, *demand management*, and *demand-chain management* [14].

Efficient demand management can have significant impact on the total revenue generated out of utilizing a limited amount of resource. This is illustrated by an example reported in Cross [15], where the author cites that selling just one seat per flight at full price rather than at a discount rate could add over $50 million to the annual revenues of Delta Airlines.

After the deregulation of the airlines industry in the 1970s, the established carriers were faced with the difficult situation of competing with the newer low-priced carriers. It was during this time that airlines adopted revenue management to stay competitive [15]. This led to interest in the study, research, and application of revenue management. One of the first RM models was developed by Littlewood [16]. It was for a basic case with only two fare classes and was based on the concept of expected marginal seat revenue. Since then, a number of researchers have considered different extensions of the basic case and solution approaches. A detailed review of the

developments in the area of revenue management is beyond the focus of the present work, and we refer the reader to Talluri and van Ryzin [14] for a recent and extensive review.

After its popularity in the airline industry, RM is now applied in various other industries [14, 17]. RM applications can be classified into traditional and nontraditional categories [17]. Traditional applications are similar to the airline model at a mathematical level. Examples of traditional applications would include hotel and car rental industries. The nontraditional applications use models that are sufficiently different from the airline model and warrant separate categorizations [17]. Examples of nontraditional applications would include retail, media and broadcasting, casino, theaters and sporting events, manufacturing, cruise ships and ferry lines, passenger railways, electricity generation and transmission, air cargo, freight, etc. Talluri and van Ryzin [14] describe revenue management in various nontraditional applications.

RM is a micro-management practice and ignores the long term effects on customer relations. Therefore, before it is applied to any situation, its long term benefits should be analyzed. The following list gives insight into the various conditions that are conducive to RM.

- *Customer segmentation*: Based on factors like time of purchase, quantity of purchase, etc., different customers may value the same resource differently. RM can exploit the variations in willingness to pay by segmenting customers based on criteria that are closely related to their willingness to pay and controlling the amount of resource made available to each segment [14, 15].

- *Demand variability and uncertainty*: Demand-management becomes more difficult when there is uncertainty in future demand. In such cases, the potential to make bad decisions rises, and it becomes important to use sophisticated tools

to evaluate the resulting complex tradeoffs [14].

- *Resource inflexibility*: When capacity is fixed over the short term and is perishable if left unused after a certain period of time, and if the marginal cost of consuming the capacity is low, then it is very important to utilize the available capacity efficiently such that neither the available capacity is left unused nor a high price customer is turned away due to the allocation of capacity to a low price customer [1, 15].

B.   Comparison of Airline and Manufacturing RM Models

Although RM has been extensively studied and researched in the context of airline seat inventory control, the airline RM models cannot be directly applied in manufacturing capacity control, due to the difference between the two RM models, as highlighted below.

- In the case of manufacturing, the time horizon considered by the RM model determines the underlying resource capacity, and is equal to the useful machine hours available for production, while the airline seat capacity (for a specific flight) remains fixed irrespective of the time horizon considered by the RM model.

- Manufacturing capacity is available over a time continuum in comparison to the airline seat capacity which is available at a specific point in time, (i.e., flight departure). Therefore, manufacturing capacity control has to simultaneously plan the capacity utilization, while the airline capacity is fully utilized at the end of the time-horizon involved, which is linked with the flight departure.

- In the case of the manufacturing industry, there is flexibility in scheduling cus-

tomer orders, as long as it allows meeting their due-dates. While in the case of the airlines industry, there is limited flexibility in substituting a seat reserved in a flight with another seat in some other flight. Such substitutions are done to meet overbookings, but incur significant penalty costs. The order scheduling aspects in manufacturing capacity control presents new challenges in RM.

C. Scope of the Dissertation

The scope of this dissertation can be summarized as follows:

1. Study the Potential of RM at an On-Date Delivery MTO System

We study the potential of RM for an *on-date* delivery MTO system under the following contexts:

- *Significance of efficient order acceptance policies.* We isolate the two problems of efficient order acceptance and efficient order scheduling and study the significance of each of these problems individually. To study and gain insights into the extent of the impact of efficient order acceptance policies, we consider single period systems in which the scheduling element is not present as all the order classes are due at the end of the current period. An efficient order acceptance policy will protect manufacturing capacities for future high profit earning orders, while ensuring that manufacturing capacity is not getting wasted because too few orders have been accepted. We show that substantial improvements can be made in net profits with efficient order acceptance policies.

- *Significance of efficient scheduling policies.* Duenyas [18] has shown that when all incoming orders are accepted as long as there is sufficient manufacturing capacity available to process them, then under pre-emptive scheduling in the

absence of holding costs, earliest due-date scheduling maximizes the total profits. However, in the presence of holding costs or non-pre-emptive scheduling, this result does not hold. To study and gain insights into the extent of the significance of efficient scheduling policies, we consider multi-period systems with identical profit earnings and processing requirements for all order classes. In such systems, any efficient order acceptance policy will not prioritize any order class over another, since all order classes earn the same profits and consume the same amount of manufacturing capacity. However, an efficient order scheduling policy will schedule orders in less busy periods, while saving the manufacturing capacities in busier periods for future orders. We identify situations under which efficient order scheduling policies have significant impact on net profits.

- *Effects of loading factor on the efficiency of RM model.* It has been shown for service industry that RM models are most effective when demand is higher than the available capacity. For the manufacturing industry, we study the potential of RM at different levels of capacity overloading.

- *Extent of the effects of overlooking the holding costs aspects while accepting and scheduling customer orders.* Consideration of the holding costs aspects while accepting and scheduling customer orders for an *on-date* delivery MTO system, makes the RM model very complicated. Therefore, it is tempting to overlook the holding costs aspects in the RM model. We gain insights into the extent of impact on total profits earned if holding costs aspects are overlooked in a RM model.

2. Evaluate First Party and Third Party Warehousing Systems

For storing the orders completed in advance of their due-dates, the manufacturer can either setup his own warehouse (called *first party* warehousing) or use the option of *third party* warehousing. *First party* warehousing involves a high amount of initial investment comprising costs related to building, equipment, utilities, personnel, etc., whereas *third party* warehousing costs will be directly linked with the amount of storage space utilized or the number of items stored. For both of these warehousing options, we develop RM models for accepting and scheduling customer orders, and gain insights into the structural properties of these models and the potential of these models in improving net profits.

3. Efficient First Party Warehouse Capacity Planning

First party warehouses involve a large initial setup cost part of which is independent of the warehouse capacity, for example cost of information technology systems for warehouse management, and part of which is directly dependent on the size of the warehouse, for example, cost of land and storage infrastructure. Therefore, it is very important to efficiently plan the *first party* warehouse capacity, such that the warehouse is used at high utilization, while still minimizing capacity shortages.

We develop a *first party* warehouse capacity design model that determines the capacity, which along with an RM model for accepting and scheduling customer orders, maximizes expected net profits for *on-date* delivery MTO systems.

D. Organization of the Dissertation

The dissertation is organized as follows. Chapter II provides a review of the order acceptance and scheduling literature for MTO systems. Chapter III presents an RM

model for accepting and scheduling customer orders for an *on-date* delivery MTO system, under the assumption that all orders completed in advance of their due-dates are stored in *third party* warehouses. Chapter IV presents a similar RM model, but under the assumption that there is a certain amount of *first party* warehousing capacity is available for storing customer orders completed in advance of their due-dates. While Chapter V extends this RM model and develops a scheme for finding efficient *first party* warehousing capacities. The contributions of the dissertation are summarized in Chapter VI.

CHAPTER II

LITERATURE REVIEW

Miller [19] and Lippman and Ross [20] are examples of some of the earliest models for selective order acceptance policies that can be applied in MTO systems [21]. These models assume that the order service times are exponentially distributed and there are no due-date restrictions or lateness penalties in serving the orders. Miller [19] studies the order acceptance problem as an admission control problem to a queue. Lippman and Ross [20] extend Miller's model by allowing service times that are dependent on the customer classes and a general arrival process. One of the key insights from these models is that in an MTO system with exponentially distributed processing times, a cμ policy gives optimal results. Consider a single machine system with exponentially distributed service time. Jobs ($i \in I$, where I is the set of jobs) arrive randomly and their mean service time ($1/\mu_i$) and revenues (c_i, earned when the jobs are completed before their due-date) are known in advance. A cμ policy states that if he job with largest value of $c_i\mu_i$, amongst all available jobs, is chosen for scheduling, it maximizes the total expected returns (for details, see [22, 23]). Other significant early contributions to the field of selective order acceptance polices are the works of Stidham [24], Matsui [25], and Matsui [26]. They consider systems with Poisson arrival process and exponentially distributed service time and analyze the properties of the optimal order acceptance policies and derive structural results.

The next phase of research on order acceptance policies addresses strict due-date restrictions with all orders. Some examples from this phase of research are the works of Wester et al. [27], Tenkate [28], and Nandi and Rogers [29]. They develop policies that accept a new order if a feasible production schedule without tardiness is possible for all previously accepted orders and the new order. A shortcoming with

these acceptance policies is that no consideration is given to the future order arrivals. Guerrero and Kern [30] claim that significant revenue gains might be possible if future order arrivals are considered in making order acceptance decisions.

Over the last decade, several RM models have been reported in literature that account for possible future orders in making order acceptance and scheduling decisions in MTO systems. Table I classifies these models based on several criteria relevant to MTO manufacturing systems.

Table I. Classification of RM Models

Criteria	Classification	Examples
Scope	Pure MTO System	[3], [31], [32], [33], [34]
	Hybrid (MTO+MTS) System	[21], [35], [36], [37], [38], [39], [40], [41], [42], [43]
Time Horizon	Single Period	[36], [44], [45]
	Multi Period	[34], [46]
	Infinite Horizon	[3], [19], [33], [42]
Order Service Times	Deterministic	[3], [33], [44], [45]
	Stochastic	[20], [22], [24], [42], [47]
Order Arrival Process	Homogenous	[3], [33], [36], [42]
	Non-Homogenous	[34], [46], [48]
Delivery Process	On-Date	[34]
	Due-Date	[3], [36], [42]
Scheduling Rule	Static, Non-preemptive	[33], [34]
	Dynamic, preemptive	[3], [42], [49]

Below we provide details on the various RM models reported in literature for

accepting and scheduling customer orders at pure MTO systems with deterministic order service times.

Lewis and Slotnick [46] consider the long term effects of the order selection policies on future orders from each customer and develop a dynamic programming model for selecting the optimal set of orders in each period. One of the key assumptions with their model is the availability of complete information on the future orders from each customer, which is an over simplification of the dynamic arrival process for customer orders. Balakrishnan et al. [50, 51] and Sridharan and Balakrishnan [52] propose a capacity rationing model for practicing RM at an MTO facility. Their model is extended by Barut and Sridharan [44, 45] into a dynamic capacity apportionment procedure (DCAP) for determining short term nested protection levels when an MTO system experiences bursts of demand in excess of capacity. However, both the capacity rationing approach as well as the DCAP neglect the dynamic nature of the order arrival process and prescribe a static acceptance policy irrespective of demand realization. Our RM models overcome this shortcoming and prescribe dynamic acceptance and scheduling policies that evolve with demand realization.

Celik and Maglaras [49] develop a diffusion model for quoting due-dates on customer orders, with the objective of maximizing the total net revenues after accounting for the order expediting costs. They assume, however, that order scheduling is preemptive and can be resumed any number of times without any losses. This is an over simplification and may not represent actual practice, since usually there are fixed setup costs in resuming preempted orders. Another model that considers preemptive scheduling of customer orders is the model by Gallien et al. [3]. In their model, the authors assume a homogenous arrival process for customer orders, which fails to capture the seasonal variations in the demand. They show that in their RM model, an earliest due-date (EDD) scheduling of customer orders maximizes the total rev-

enues generated at the MTO system. They develop two policies, the fluid policy and the look-ahead policy, for accepting customer orders at MTO systems. The computational performance of their look-ahead policy is questionable since it considers a number of future order arrival scenarios and solves an NP-hard problem for each of them in arriving at the order accept/reject decision. A shortcoming with their fluid policy is that it considers the arrival process at an aggregate level and permits acceptance of fractional solutions, which results in overestimation of the expected revenue function. In comparison, our model neither allows preemptive scheduling of customer orders, nor permits acceptance of fractional orders.

Kniker and Burman [33] develop a Markov Decision Process for accepting customer orders at an MTO system, and Defregger and Kuhn [31] outline a heuristic for this approach. They assume that the orders are scheduled according to the first-come-first-served policy, and thus their model is not able to exploit the manufacturing capacity by efficient order scheduling. Our model overcomes this shortcoming by considering a joint order acceptance and scheduling policy.

Perry [34] describes an application of RM for accepting orders in an MTO system manufacturing semi-conductor products. He models the multi-period problem as a stochastic knapsack problem. However, he assumes that all the orders are scheduled in the period in which they are due. Therefore, his model is able to satisfy on-date deliveries of customer orders, but it is not able to fully exploit the manufacturing capacity by efficient order scheduling. In addition, he assumes that for all the orders received in a period, the accept/reject decisions are made at the end of the period, which is an over simplification and does not necessarily represent actual practices. Our model overcomes these shortcomings by considering a dynamic order acceptance and scheduling policy.

With recent advances in 'lean thinking' practices, many organizations (cus-

tomers) are interested in receiving their orders in an *on-date* delivery fashion. Most of the RM models have not looked at this feature of the MTO system and the challenges offered by it in simultaneously considering both the manufacturing capacities and the holding costs in deciding the order acceptance and scheduling policies.

In this research, we apply RM for accepting and scheduling customer orders at an MTO system that makes *on-date* deliveries of customer orders and follows non-preemptive scheduling rule.

CHAPTER III

THIRD PARTY WAREHOUSING OPTION

A. Problem Description

Order acceptance and scheduling policies play a very important role in MTO systems. These policies have a direct impact on not only the delivery performance [27, 53, 54, 55] but also the profits generated [3, 31, 32, 33] by the MTO system.

In the current chapter, we focus on the revenue aspects of the order acceptance and scheduling policies, and develop and analyze an RM model for MTO systems that make *on-date* deliveries of their customer orders. This RM model segments the possible future order arrivals into classes $\{i \in I\}$ that are based on the due-dates (represented by d_i), processing time requirements (represented by Q_i) and profit margins (represented by r_i, reflecting the revenue earnings by processing the order minus all except holding costs). Consider a case where an MTO system is offering 1 product (processing requirement Q units of processing time) at 3 different profit margins $(r1, r2, r3)$ over the next 2 due-dates $(d1, d2)$. The set of order classes in this case is $\{(d1, r1, Q), (d1, r2, Q), (d1, r3, Q), (d2, r1, Q), (d2, r2, Q), (d2, r3, Q)\}$. Using the forecast information for the arrival distributions of the order classes (whose mean is represented by λ_{ic}, where $i \in I$ is the product class and c is the arrival period), the RM model prescribes efficient order acceptance and scheduling policies that are used to decide if a new order arriving at time τ and belonging to a certain order class i should be accepted or not, and if accepted, in which period it should be scheduled.

To get a clear idea of the problem, consider Figure 1. It is shown that an order belonging to order class i (that is, earns profits r_i, requires Q_i amount of processing time, and due at the end of period d_i) arrives in period 2. At the time of its arrival, we
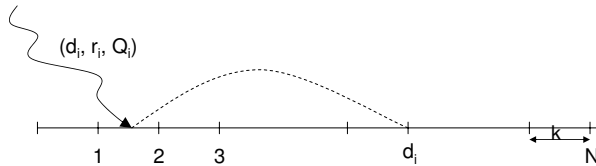
Fig. 1. Order Acceptance and Scheduling

are interested in deciding if this order should be accepted, which can be done only if there is sufficient manufacturing capacity in any period between periods 2 (the period of order arrival) and d_i (the due-date period). If we decide to accept this order, the next decision is in which of the periods between period 2 and d_i should this order be scheduled for manufacturing.

If an order is scheduled in any period prior to its due-date, it is stored in a third party warehousing facility and incurs holding costs. Thus, this RM model has to simultaneously consider manufacturing capacity utilization and holding cost in determining efficient order acceptance and scheduling policies. For a new order arrival, using the information about the arrival distributions over the remaining time in the planning horizon for different order classes, the RM model computes the opportunity cost of committing manufacturing capacity (in time units) to it. A comparison of this opportunity cost with the profits earned by this new order guides the RM model in deciding if this new order should be accepted, which is done when profits earned by the new order are higher than the opportunity cost, or if it should be rejected, which is done when the profits earned by the new order are less than the opportunity cost.

The key assumptions made in our RM model are as follows:

ASSUMPTION 1 *Single Machine Model.* An MTO manufacturing system can involve a number of machines. However, there is usually a machine, called as the

bottleneck machine, that determines the capacity of the manufacturing facility [56]. Therefore, we model the MTO manufacturing facility as a single machine, which may be the bottleneck in a larger system [3, 33, 36, 42, 43, 44, 45, 47].

ASSUMPTION 2 *Finite Horizon Problem.* Researchers in the past have developed both finite horizon models [34, 36, 44, 45, 46, 50, 51] and infinite horizon models [3, 31, 33, 42] for applying RM in accepting customer orders. RM is effective when demand is higher than the available capacity [15]. Manufacturing industries typically face non-homogenous and seasonal demand. Kurawarwala and Matsuo [57] cite end of quarter and Christmas season effects on demand. Kevin Rollins, CEO of Dell Computer Corporation, highlights end of quarter volume ramp up as one of the challenges faced by Dell [4]. Therefore, given the seasonality patterns in the demand faced at many manufacturing industries, the present work considers a finite horizon model to effectively utilize RM during the peak demand periods. The planning horizon is divided into periods of equal length with homogenous demand within each period.

ASSUMPTION 3 *Fixed Manufacturing Capacity.* In various manufacturing industries, adding manufacturing capacity is a complicated process, involving changes in the building infrastructure and utility supplies (for example, compressed air and electricity). Procurement and receipt of machines is also a lengthy process involving negotiations and budgeting. Therefore, it is assumed that the manufacturing capacity is fixed over the planning horizon.

ASSUMPTION 4 *Identical Storage.* It is assumed that all storage units are identical and each order is assigned a unique storage unit, which could be a group of stacking rows in a stack storage system or a group of cells in a rack storage system.

ASSUMPTION 5 *Independence.* It is assumed that the order arrival processes between different classes are independent of each other [3] and are not affected by the acceptance and scheduling policy followed by the manufacturer [14].

ASSUMPTION 6 *Demand Forecast.* It is assumed that a probabilistic demand forecast is available with the manufacturer. With the use of electronic media for storing sales information, it is easier for manufacturers to maintain systems that facilitate demand forecasts [1]. In addition, we assume that the order arrival process for different classes is Poisson [3, 31, 33, 36, 42, 43].

ASSUMPTION 7 *Deterministic Processing Times.* It is assumed that the processing times for orders is deterministic but differs by product classes [3, 31, 33, 34, 36, 43, 44, 45, 50, 51]. A few researchers have considered exponentially distributed processing times [42, 47], however with automation and advances in manufacturing technologies, the manufacturing processes have become more reliable and often offer nearly fixed processing times [58]. It is further assumed that all supplies will be coordinated to meet the processing requirements of different orders without causing any delays or time losses.

ASSUMPTION 8 *Non-Preemptive Scheduling.* It is assumed that the scheduling of orders is non-preemptive. For every new item/order, manufacturing processes typically require a certain amount of setup time/cost. Therefore, it is desirable to complete processing on the current job before starting a new one. This may lead to profit losses since it might be possible to make extra profits by scheduling a new high profit earning order and postponing the order currently in processing to a future period. It is further assumed that order processing does not span periods, which is reasonable as long as the length of each period is significantly larger than the

processing time requirements of orders. However, this has the disadvantage of letting small amounts of processing capacities remain unutilized.

B. Research Motivation

The key research motivation behind this problem can be summarized as follows:

- *Stochastic system.* The order acceptance and scheduling decisions are being made under incomplete information on future arrivals. This is because, due to the stochastic arrival process of different order classes, a number of scenarios of future order arrivals are possible. In addition, the set of possible scenarios for future arrivals grows exponentially with the length of the planning horizon. As a result, it is difficult to determine optimal order acceptance and scheduling decisions.

- *Simultaneous consideration of manufacturing capacity utilization and holding cost in an RM model is challenging.* Under an *on-date* delivery system, optimal order scheduling is a complicated problem, since the manufacturer has to not only consider the manufacturing capacity utilization, but also the resulting holding costs for storing the orders until their due-dates. Duenyas [18] and Gallien et al. [3] have shown that an earliest due-date scheduling scheme maximizes the manufacturing capacity utilization, while a latest due-date scheduling scheme minimizes holding cost when the manufacturer is responsible for the holding costs incurred for the orders completed in advance of their due-dates. As a result, simultaneous consideration of manufacturing capacity utilization, and holding costs makes optimal order scheduling a challenging problem.

- *Optimal order acceptance and scheduling problem is mathematically difficult.* As

shown later in this chapter in Theorem 1, the order acceptance and scheduling problem is NP-hard.

C.   Mathematical Formulation

Analogous to the optimal policies for *Single-Resource Dynamic Capacity Control* identified in Talluri and Van Ryzin [14] (pg. 59), the optimal order acceptance and scheduling policies for a single machine on-date delivery MTO system can be stated as Remarks 1 and 2 below.

REMARK 1 *Optimal Order Acceptance Policy.* Accept an order if the expected opportunity cost of scheduling it in any period between its arrival and its due-date is less than its profit earnings. Opportunity cost is defined as the difference between the expected profits with the uncommitted manufacturing capacity when the order is accepted and scheduled and when it is not.

REMARK 2 *Optimal Order Scheduling Policy.* Schedule an order in the period that has the least opportunity cost for processing the order.

Discrete time formulations based on stochastic dynamic programming (SDP) approach are common in RM literature [14]. The way this approach is executed is as follows. Under the assumptions of a Poisson arrival process, the time horizon under study is divided into small time slots, such that the probability of arrival of more than one order in each time slot is very small. SDPs are then constructed over the discrete space of the time slots, with the objective of computing the expected returns from the available manufacturing capacity. In the sense of RM, when a decision needs to be made on accepting or rejecting an order, the difference between the expected returns from the available manufacturing capacities when an order is rejected and is

accepted is compared with the profits earned by processing the order at hand. If the comparison is in favor of the order, the order is accepted, otherwise it is rejected. Following this general approach, we develop SDP formulation for our problem, as described below. The notation followed in this formulation is presented in the next subsection.

1.   Notation

The following notation is used throughout the formulation:

Sets

I	Set of order classes.
D	Set of due-dates; $D = \{1, ..., N\}$.

Indices

τ	Index for time slots over the planning horizon.
i	Index for order class; $i \in I$.
c	Index for the time periods; $c \in \{1, ..., N\}$.

Parameters

t_i	Period in which class i order arrives.
d_i	Period in which class i order is due.
Q_i	Processing time of class i order, expressed in time slots; $Q_i \geq 1$.
r_i	Profits earned by processing an order belonging to class i.
N	Total number of time periods in the planning horizon.
λ_{ic}	Arrival rate of order class i in period c, expressed in units per period.

k	Total time per period, expressed in number of time slots; $k \gg \sum_{i \in I} \lambda_{ic}, \forall c.$
h	Third party holding cost per order per time period.
P_{ic}	Probability of an arrival of a class i order in period c; $P_{ic} = \frac{\lambda_{ic}}{k} < 1.$
C_w	Cost per unit of first party warehousing capacity.

State Variables

$\widehat{S}$	Vector of available manufacturing capacity (expressed in number of time slots) in different periods.
$S(c)$	The cth element of vector $\widehat{S}$; $\widehat{S} = \{S(1), S(2), ..., S(N)\}.$
$\widehat{S} \setminus c$	Vector $\widehat{S}$, excluding the cth element; $\widehat{S} = \{S(c)\} \cup \widehat{S} \setminus c.$
$\widehat{W}$	Vector of committed storage volume in different periods at an order arrival.
$W(c)$	The cth element of vector $\widehat{W}$.
$\widehat{W} \setminus c$	Vector $\widehat{W}$, excluding the cth element.

Decision Variables

X_{ic}	1 if an order of class i is scheduled in period c, 0 otherwise.
W	First party warehousing capacity; $W \in Z^+.$

2. Optimal Acceptance Policy

Let an order from class i arrive in time slot τ, when the vector of available capacities is $\widehat{S}$. This order should be accepted if condition (3.1) is satisfied. $V(\tau, \widehat{S})$ in (3.1) is the expected profit function, and is computed by solving the stochastic dynamic program (3.7) described later in the section.

$$\min_{\substack{X_{ic} \\ s.t.(3.2)-(3.5)}} [V(\tau+1, \widehat{S'}) - V(\tau+1, \sum_c X_{ic}(\{S(c) - Q_i\} \cup \widehat{S'} \setminus c))+$$

$$h_i \sum_c X_{ic}(d_i - c)] \leq r_i \qquad (3.1)$$

where,

$$S'(c) = \begin{cases} S(c) & c \neq \lceil \tau/k \rceil \\ \min(S(c), k - (\tau \bmod k) - 1) & \text{otherwise} \end{cases} \qquad (3.2)$$

$$\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} \leq 1 \qquad (3.3)$$

$$S(c) - X_{ic}Q_i \geq 0, \forall c \in \{\lceil \tau/k \rceil, ..., d_i\} \qquad (3.4)$$

$$X_{ic} \in \{0,1\}, \forall c \in \{\lceil \tau/k \rceil, ..., d_i\} \qquad (3.5)$$

The first term of the expression on the left hand side of (3.1) is the expected profits if the order at hand is not accepted, while the second term is the expected profits from the remaining manufacturing capacity after accepting and scheduling the order in period c. The third term is the holding cost incurred for storing the order from period c until d_i. Therefore, the expression on the left hand side of (3.1) is the smallest of the opportunity costs of scheduling the order at hand in different periods between the period of the order arrival and due-date. $\widehat{S'}$ in (3.1) is the vector of available manufacturing capacity in different periods if the order at hand is not accepted and is computed by expression (3.2). Constraint (3.3) and (3.4) ensure that the order is accepted only when there is sufficient manufacturing capacity available in one of the periods between the period of order arrival and due-date.

3. Optimal Scheduling Policy

The optimal scheduling policy, represented by Remark 2 can be mathematically expressed by (3.6), where c^* is the best period to schedule an order from class i arriving in time slot τ when $\widehat{S}$ is the vector of available manufacturing capacity in different periods.

$$c^* = \arg\min[(V(\tau+1, \widehat{S}') - V(\tau+1, \{S(c) - Q_i\} \cup \widehat{S}' \setminus c) + h_i(d_i - c)$$
$$: c \in \{\lceil \tau/k \rceil, ..., d_i\}; (3.2); S(c) - Q_i \geq 0] \qquad (3.6)$$

(3.6) determines the period with the least opportunity cost for scheduling the order. Opportunity costs for scheduling the order in a feasible period c is computed by evaluating the difference in the expected profits between rejecting the order and accepting and scheduling the order in period c. The expected profits are computed by SDP (3.7).

4. Expected Profit Function

The expected profit function $V(\tau, \widehat{S})$ represents the net expected profit that can be generated by the available manufacturing capacity, represented by the vector $\widehat{S}$, with the orders arriving during and after time slot τ. An SDP for computing $V(\tau, \widehat{S})$ is expressed by (3.7) with boundary condition (3.8). Feasibility of the order acceptance and scheduling policy for an order that arrives in time slot τ is ensured by (3.9) - (3.13). Constraints (3.9) and (3.13) restrict the scheduling of the order from class i to at most one of the periods between the period of its arrival $\lceil \tau/k \rceil$ and due-date d_i. Constraints (3.10), (3.11), and (3.12) ensure that sufficient manufacturing capacity is available in the period in which the order is scheduled. Constraints (3.11) and (3.12)

account for the consumption and/or loss of manufacturing capacity with accepting and rejecting orders. Constraint (3.12) accounts for loss in manufacturing capacity in the period of order arrival if sufficient workload is not available. The boundary condition used is (3.8), where $N \times k$ represents the last time slot in the time horizon under study. Any manufacturing capacity available after completion of the last time slot in the planning horizon is lost, and so it does not generate any revenues.

$$V(\tau, \widehat{S}) = \sum_{i \in I : d_i \geq \lceil \tau/k \rceil} (P_{i \lceil \tau/k \rceil} \max_{\substack{X_{ic} \\ s.t.(3.9)-(3.13)}} [\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic}(r_i - h_i(d_i - c)) + V(\tau+1, \widehat{S}')]) \quad (3.7)$$

with boundary condition (3.8),

$$V((N \times k) + 1, \widehat{S}) = 0, \qquad \forall \widehat{S} \qquad (3.8)$$

where,

$$\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} \leq 1 \qquad (3.9)$$

$$S'(c) \geq 0, \forall c \in \{\lceil \tau/k \rceil, ..., d_i\} \qquad (3.10)$$

$$S'(c) = S(c) - X_{ic}Q_i, \forall c \in \{\lceil \tau/k \rceil + 1, ..., d_i\} \qquad (3.11)$$

$$S'(\lceil \tau/k \rceil) = \min(S(\lceil \tau/k \rceil) - X_{i\lceil \tau/k \rceil}Q_i, k - (\tau \bmod k) - 1) \qquad (3.12)$$

$$X_{ic} \in \{0, 1\}, \forall c \in \{\lceil \tau/k \rceil, ..., d_i\} \qquad (3.13)$$

5.   FCES and FCLS Policies

To evaluate the performance of our RM model, we consider two simple policies, first come earliest served (FCES) and first come latest served (FCLS). Under an FCES policy, all incoming orders are accepted if there is sufficient manufacturing capacity available, at the time of order arrival in any of the periods between order arrival and

its due-date, that can process this order. An accepted order is scheduled in the earliest possible period with sufficient capacity to process the order. FCLS policy follows the same order acceptance rule as FCES policy, but schedules an accepted order in the latest possible period with sufficient capacity to process the order. These policies are not only simple to implement but also have their own strengths which make them attractive and gives us a reference against which we can compare the profits earned by adopting the optimal order acceptance and scheduling policies determined by our RM model. FCES policy ensures that as long as there is any pending order in the system, the manufacturing capacity will not be wasted idling, while FCLS policy ensures that orders are processed as close to their due-date as possible, thereby minimizing the holding costs incurred in storing the orders completed in advance of their due-dates.

The expected value function under FCES scheme can be expressed by SDP (3.14), where c is chosen as in (3.15). (3.15) determines the earliest possible period with sufficient manufacturing capacity available to process an order from class i. Feasibility of scheduling an order from class i arriving in time slot τ when the vector of available manufacturing capacities is $\widehat{S}$ in period c is ensured by (3.16) - (3.19). (3.16) and (3.17) ensure that there is sufficient manufacturing capacity available in period c to process an order from class i. (3.17) accounts for loss in manufacturing capacity in the period of order arrival if sufficient workload is not available. (3.18) states that the available manufacturing capacities in all periods besides the period of order arrival and the period in which the order is scheduled remain unchanged at the values prior to the order arrival. The expression $\mathbf{1}_y$ is used as an indicator function and takes value 1 if condition y is true, and 0 otherwise.

$$V(\tau, \widehat{S}) = \sum_{i \in I : d_i \geq \lceil \tau/k \rceil} P_{i\lceil \tau/k \rceil}(\mathbf{1}_{\lceil \tau/k \rceil \leq c \leq d_i}(r_i - h_i(d_i - c)) + V(\tau + 1, \widehat{S}'')) \qquad (3.14)$$

with boundary condition (3.8), where

$$c = \min\{s : S(s) \geq Q_i; \lceil \tau/k \rceil \leq s \leq d_i; (3.16) - (3.19)\} \tag{3.15}$$

$$S''(c) \geq 0, \forall c \in \{\lceil \tau/k \rceil, ..., d_i\} \tag{3.16}$$

$$S''(c) = \widehat{S}(c) - Q_i, \text{if } c \neq \lceil \tau/k \rceil \tag{3.17}$$

$$S''(\lceil \tau/k \rceil) = \min(S(\lceil \tau/k \rceil) - \mathbf{1}_{c=\lceil \tau/k \rceil} Q_i, k - (\tau \bmod k) - 1) \tag{3.18}$$

$$S''(l) = S(l), \forall l \neq c, l \neq \lceil \tau/k \rceil \tag{3.19}$$

Similarly, the expected value function under FCLS scheme can also be expressed by SDP (3.14), where c is chosen as in (3.20). (3.20) determines the latest possible period with sufficient capacity to process an order from class i.

$$c = \max\{s : \widehat{S}(s) \geq Q_i; \lceil \tau/k \rceil \leq s \leq d_i; (3.16) - (3.19)\} \tag{3.20}$$

D.   Solution Approaches

In the next subsection, we show that order acceptance and scheduling problem is an NP-hard problem. Therefore, it is unlikely to be possible to solve large size problems to optimality. Thus, we develop two approaches as outlined later in this section. The first approach is based on solving the SDP (3.7) by value iteration scheme. This scheme can be used to generate near optimal solutions. However, the state space in our SDP (3.7) has size $O(Nk^{N+1})$, which is very large even for small size problems. Thus, our value iteration scheme is computationally intensive. Therefore, we develop a second solution approach, which is based on insights gained from the properties of the optimal policies. This approach employs a stochastic approximation techniques for finding efficient solutions to the order acceptance and scheduling problem.

1. Complexity of Order Acceptance and Scheduling Problem

As shown in Theorem 1, the optimal order acceptance and scheduling problem is NP-hard.

THEOREM 1 *The optimal order acceptance and scheduling problem for a single machine MTO system is NP-hard.*

Proof: Consider a deterministic system such that at the time of an order arrival all future order arrivals are known and are indexed by j. For each future order j, let a_j, d_j, Q_j, r_j represent the arrival time, the period in which the order is due, the processing time requirements of the order, and the profits earned by processing the order, respectively. In addition, ignore the holding costs incurred for storing any orders that are processed in advance of their due-dates and assume that orders can be processed in pre-emptive scheduling without incurring any penalties.

The optimal order acceptance problem in this deterministic system can be mathematically expressed as:

$$\max \sum_j r_j x_j \tag{3.21}$$

s.t.

$$\sum_{\substack{l \in \{i:a_i \geq a_j, \\ P_i \leq g\}}} Q_l x_l \leq \left[\min(S(\lceil \tfrac{a_j}{k} \rceil), k - (a_j \bmod k)) + \sum_{e=\lceil \frac{a_j}{k} \rceil + 1}^{g} S(e) \right], \forall j, g \tag{3.22}$$

$$x_j \in \{0, 1\}, \forall j \tag{3.23}$$

where e, g are indices for due-dates and x_j is the decision variable that takes value 1 if the order j is accepted and 0 otherwise. This is a multiple 0-1 knapsack problem, which is NP-hard [59].

In the problem of order acceptance and scheduling, we do not know the future arrivals in advance, and in addition to acceptance/rejection decisions, we make optimal scheduling decisions in the presence of holding costs. These features make the optimal order acceptance and scheduling problem more difficult than the NP-hard optimal order acceptance problem for the deterministic system described above. Therefore, the optimal order acceptance and scheduling problem is NP-hard. ∎

2. Heuristic Scheme based on Value Iteration (HSVI)

Value iteration is a popular scheme used for solving SDPs [60, 61, 62]. If this scheme is executed for a sufficiently long time, then it converges to optimal solutions, while if it is terminated when the improvements in the objective function are less than a certain fraction (ϵ) it gives near optimal solutions, also called as ϵ-optimal solutions [60]. We use value iteration to solve the SDP expression (3.7). At each iteration, using (3.25) as the boundary condition, the value function at different states $(\tau, \widehat{S})$ is updated as follows,

$$V^{n+1}(\tau, \widehat{S}) = \sum_{i \in I: d_i \geq \lceil \tau/k \rceil} (P_{i \lceil \tau/k \rceil} \max_{\substack{X_{ic} \\ s.t.(3.9)-(3.13)}} [\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic}(r_i - h(d_i - c)) + V^n(\tau+1, \widehat{S}')])$$

(3.24)

$$V^n((N \times k) + 1, \widehat{S}) = 0, \qquad \forall n, \widehat{S}$$

(3.25)

In this solution approach, the number of states in the SDP (3.7) is $O(Nk^{N+1})$, which grows exponentially with the number of periods in the planning horizon. Consider a 3 period problem with number of time slots in each period equal to 250. The number of possible states in the SDP (3.7) is $O(1.17 \times 10^{10})$, which shows that it is difficult to apply value iteration scheme to even small size problems. To condense the

number of states in SDP (3.7), we compute the greatest common factor b of the processing requirements of all order classes, and scale down the processing requirements of all jobs and available manufacturing capacity vector by factor b and change (3.18) in SDP (3.7) to (3.26).

$$S'(\lceil \tau/k \rceil) = \min(S(\lceil \tau/k \rceil) - X_{i\lceil \tau/k \rceil}Q_i, \lfloor \frac{k - (\tau \bmod k) - 1}{b} \rfloor) \qquad (3.26)$$

These changes reduce the number of states in SDP (3.7) to $O(N\frac{k^{N+1}}{b^N})$. Therefore, in our earlier example where we considered $N = 3$ and $k = 250$, if $b = 25$ then the number of states in SDP (3.7) reduces to $O(7.5 \times 10^5)$, which is much easier to manage than the original number of states. Therefore, while applying HSVI approach, we condense the state space as described above.

3.   Heuristic Scheme based on Stochastic Approximation (HSSA)

To gain insights into the structural properties of the optimal order acceptance policies, consider a one period problem. In a one period problem, all the orders received are due at the end of the period. If accepted, the orders are scheduled in the period in which they arrive, therefore the scheduling element is not present in this problem.

Consider that an order from class i arrives in time slot τ when the amount of manufacturing capacity available (MCA) is S. Then based on Remark 1, this order should be accepted if and only if condition (3.27) holds.

$$r_i \geq V(\tau + 1, S) - V(\tau + 1, S - Q_i) \qquad (3.27)$$

If the function $V(\tau, S)$ is concave in S, then according to Theorem 2, the optimal acceptance policy can be expressed in terms of threshold values of MCA for each order class i at all times τ, such that if the MCA at an order arrival is higher than the

threshold value, then the order is accepted, otherwise the order is rejected.

THEOREM 2 *For an order from class i that arrives at time τ, if the function $V(\tau, S)$ is concave in S, then there exists a threshold value of MCA, $S^*_{i\tau}$, such that it is optimal to accept the order if the MCA at order arrival is higher than the threshold value, and to reject it otherwise.*

Proof: Case 1: (3.27) is satisfied for some values of S. Let (3.27) be satisfied at equality at $S^*_{i\tau}$. Due to concavity of $V(\tau, S)$ in S, for all values of S less than $S^*_{i\tau}$ (3.27) is not satisfied. Similarly, for all values of S greater than $S^*_{i\tau}$ (3.27) is satisfied. Therefore, $S^*_{i\tau}$ is the threshold value.

Case 2: (3.27) is never satisfied for any value of S. This is the case when it is never optimal to accept a job from class i that arrives at time τ for any amount of MCA. The threshold value in this case is $S^*_{i\tau} = \infty$. ∎

In Theorem 3 we show that $V(\tau, S)$ is non-concave in S. Unfortunately, when this happens, we may not be able to express the optimal order acceptance policy in terms of threshold values, as illustrated by Example 1.

Example 1 Let $V(1, 10) = \$99, V(1, 20) = \$150, V(1, 30) = \$275$. Let an order that requires 10 units of processing capacity, earns \$100 in profits and due at the end of current period arrive in time slot $\tau = 0$. Based on condition (3.27), this order can be accepted when MCA is 10 or 20 units. However, we cannot express the optimal order acceptance policy in terms of threshold values of MCA as expressed in Theorem 2, since condition (3.27) is violated when MCA is 30 units.

THEOREM 3 *The function $V(\tau, S)$ is non-concave in S.*

Proof: Let's assume that $V(\tau, S)$ is concave in S, where S is the manufacturing capacity available at time slot τ. Consider a single period, single product class with

the following parameters:

k $\quad = 100$ minute

τ $\quad = 0$

Q_1 $\quad = 24$ minute

r_1 $\quad = 10$

λ_1 $\quad = 10$ per period

At $S_1 = 0$, $V(\tau, S_1) = 0$, and at $S_2 = 24$, $V(\tau, S_2) = (1 - e^{-0.1 \times 76}) \times 10 = 9.995$. However, at $S = 0.5S_1 + 0.5S_2 = 12$, $V(\tau, S) = 0 < (0.5V(\tau, S_1) + 0.5V(\tau, S_2))$, which is a violation of our earlier assumption on concavity of $V(\tau, S)$ in S. Therefore, $V(\tau, S)$ is a non-concave function of S. ∎

Since $V(\tau, S)$ is non-concave in S, an order acceptance policy expressed in terms of threshold values of MCA may be sub-optimal. For a one period problem, the acceptance rule based on threshold values can be expressed as the following policy, called as threshold policy. Accept an order from class i arriving in time slot τ if and only if the amount of MCA at the order arrival is higher than the threshold value corresponding to class i and time slot τ. Threshold policy controls the acceptance of orders at different level of MCA. Therefore, if an order class earns less profit in comparison to other order classes, its threshold value should be kept high to ensure that such orders are accepted only when there is ample amount of MCA, while for high profit earning order classes, the threshold values should be kept low to ensure that such orders are always accepted, as long as there is manufacturing capacity available to process them. Under certain conditions, as shown in Theorems 4 and 5, $V(\tau, S)$ is concave, in which case the threshold policy is optimal. We present a scheme for computing efficient threshold values later in this section.

THEOREM 4 *In a single period problem, if fractional orders can be accepted, then*

$V(\tau, S)$ *is concave in* S.

Proof: Let's assume that even if fractional orders can be accepted, $V(\tau, S)$ is a non-concave function of S. This implies that there exists a set of $\tau, S, \Delta S$, such that:

$$V(\tau, S + \Delta S) - V(\tau, S) > V(\tau, S) - V(\tau, S - \Delta S) \tag{3.28}$$

where, $\Delta S > 0, S - \Delta S \geq 0$, and $S + \Delta S \leq k$.

Consider the smallest S for which (3.28) holds and the following order acceptance policy: Accept an order and compute the state transitions based on the optimal acceptance and scheduling decisions determined by solving the SDPs (3.1) and (3.6) as if the amount of manufacturing capacity available at time τ is $S + \Delta S$. However, amongst the orders that are determined as acceptable, do not accept the fraction of orders that are in part responsible for the reduction in the amount of available manufacturing capacity from S to $S - \Delta S$. But instead, use this piece of available manufacturing capacity for processing the fraction of orders that were originally responsible for the reduction in the available manufacturing capacity from $S + \Delta S$ to S. Let $g(S, \Delta S)$ be the amount of profits lost in not accepting the fraction of orders that are in part responsible for the reduction in the amount of available manufacturing capacity from S to $S - \Delta S$. Due to optimality of the value function,

$$V(\tau, S) \geq V(\tau, S - \Delta S) + g(S, \Delta S)$$

or,

$$V(\tau, S) - V(\tau, S - \Delta S) \geq g(S, \Delta S) \tag{3.29}$$

Therefore, by adopting the order acceptance policy described above, it is possible to generate an expected profits equal to $V(\tau, S + \Delta S) - (g(S, \Delta S))$, which from (3.29) has a lower bound of $V(\tau, S + \Delta S) - (V(\tau, S) - V(\tau, S - \Delta S))$. From (3.28), we know

that this quantity is higher than $V(\tau, S)$. But this is a violation, since $V(\tau, S)$ is the maximum expected profits that can be earned when the amount of manufacturing capacity available at time τ is S. This shows that our earlier assumption on the non-concavity of $V(\tau, S)$ in S when fractional orders can be accepted is incorrect. ∎

THEOREM 5 *In a single period problem, if for all order class, the arrival rates are very high and processing time requirements are very small then $V(\tau, S)$ is concave in S.*

Proof: Let's assume that for the given system, $V(\tau, S)$ is a non-concave function of S. This implies that there exists a set of $\tau, S, \Delta S$, such that:

$$V(\tau, S + \Delta S) - V(\tau, S) > V(\tau, S) - V(\tau, S - \Delta S) \qquad (3.30)$$

where, $\Delta S > 0, S - \Delta S \geq 0$, and $S + \Delta S \leq k$.

Consider the smallest S for which (3.30) holds and the following order acceptance policy: Accept an order and compute the state transitions based on the optimal acceptance and scheduling decisions determined by solving the SDPs (3.1) and (3.6) as if the amount of manufacturing capacity available at time τ is $S + \Delta S$. However, amongst the orders that are determined as acceptable, do not accept the orders that are responsible for the reduction in the amount of available manufacturing capacity from S to $S - \Delta S$. But instead, use this piece of available processing capacity for processing the fraction of orders that were originally responsible for the reduction in the available manufacturing capacity from $S + \Delta S$ to S. Since the arrival rates for all order classes are very high, there will be a large number of complete orders that are responsible for reduction in the available manufacturing capacity from S to $S - \Delta S$. However, there will be at most two orders that are in fraction responsible for reduction in the available manufacturing capacity from S to $S - \Delta S$. Therefore, the effects of

these two orders can be overlooked with minimal effects on the expected profits. Let $g(S, \Delta S)$ be the amount of profits lost in not accepting the fraction of orders that are in part responsible for the reduction in the amount of available manufacturing capacity from S to $S - \Delta S$. Due to optimality of the value function,

$$V(\tau, S) \geq V(\tau, S - \Delta S) + g(S, \Delta S)$$

or,

$$V(\tau, S) - V(\tau, S - \Delta S) \geq g(S, \Delta S) \qquad (3.31)$$

Therefore, by adopting the order acceptance policy described above, it is possible to generate an expected profit equal to $V(\tau, S + \Delta S) - g(S, \Delta S)$, which from (3.31) has a lower bound of $V(\tau, S + \Delta S) - (V(\tau, S) - V(\tau, S - \Delta S))$. From (3.30), we know that this quantity is higher than $V(\tau, S)$. But this is a violation, since $V(\tau, S)$ is the maximum expected profits that can be earned when the amount of manufacturing capacity available at time τ is S. This shows that our earlier assumption on the non-concavity of $V(\tau, S)$ in S when fractional orders can be accepted is incorrect. ∎

Based on Theorem 2 and 5 it can be seen that in a one period problem if the orders consume a very small amount of processing capacity and their arrival rates are very high then there exists an optimal threshold policy for accepting customer orders. Theorem 6 shows that if fractional orders can be accepted then there exists an optimal threshold policy for accepting customer orders.

THEOREM 6 *In a one period problem, if fractional orders can be accepted, then there exists an optimal threshold policy.*

Proof: Let an order from class i arrive during time slot τ. A threshold policy that accepts this order completely if the amount of capacity available at the time of arrival

is greater that $S_{i\tau}^*$ is optimal, where $S_{i\tau}^*$ is the solution of the following expression.

$$\frac{dV(\tau, S - Q_i)}{dS} = \frac{r_i}{Q_i}$$

Since $V(\tau, S)$ is a concave function of S, $\frac{dV(\tau, S - Q_i)}{dS} < \frac{r_i}{Q_i}$ for all $S > S_{i\tau}^*$. Hence, it would always be optimal to accept an order from class i completely if the amount of manufacturing capacity available at the time of order arrival τ is greater than the corresponding threshold value $S_{i\tau}^*$.

Similarly, a threshold policy that accepts a fraction $\alpha_{i\tau}$ of an order from class i that arrives during time slot τ if the amount of manufacturing capacity available at the time of order arrival is equal to $S_{i\tau\alpha_{i\tau}}^{**}$ is optimal, where $S_{i\tau\alpha_{i\tau}}^{**}$ is the solution to the following expression:

$$\frac{dV(\tau, S - \alpha_{i\tau}Q_i)}{dS} = \frac{r_i}{Q_i}$$

Let a fraction equal to β of the order be accepted. If $\beta < \alpha_{i\tau}$ then from the manufacturing capacity $S = S_{i\tau\alpha_{i\tau}}^{**}$ that was available to us during time slot τ, we can generate expected profit equal to

$\beta r_i + V(\tau + 1, S - \beta Q_i)$

$= \beta r_i + V(\tau + 1, S - \beta Q_i) - \alpha r_i - V(\tau + 1, S - \alpha_{i\tau}Q_i) + \alpha_{i\tau}r_i +$

$\qquad\qquad\qquad\qquad\qquad\qquad V(\tau + 1, S - \alpha_{i\tau}Q_i)$

$= \alpha_{i\tau}r_i + V(\tau + 1, S - \alpha_{i\tau}Q_i) - r_i(\alpha_{i\tau} - \beta) + V(\tau, S - \beta Q_i) - V(\tau, S - \alpha_{i\tau}Q_i)$

$= \alpha_{i\tau}r_i + V(\tau + 1, S - \alpha_{i\tau}Q_i) - r_i(\alpha_{i\tau} - \beta) + r'(\alpha_{i\tau} - \beta)$

(where, r' is the average slope of the expected profit function between $S - \alpha_{i\tau}Q_i$ and $S - \beta Q_i$; $r' < r_i$ due to concavity of $V(\tau, S)$ in S)

$< \alpha_{i\tau}r_i + V(\tau + 1, S - \alpha_{i\tau}Q_i)$

If $\beta > \alpha_{i\tau}$, then from the manufacturing capacity S that was available to us during time slot τ, we can generate an expected profit equal to

$$\beta r_i + V(\tau + 1, S - \beta Q_i)$$

$$= \beta r_i + V(\tau + 1, S - \beta Q_i) - \alpha_{i\tau} r_i - V(\tau + 1, S - \alpha_{i\tau} Q_i) + \alpha_{i\tau} r_i +$$

$$V(\tau + 1, S - \alpha_{i\tau} Q_i)$$

$$= \alpha_{i\tau} r_i + V(\tau + 1, S - \alpha_{i\tau} Q_i) - r_i(\alpha_{i\tau} - \beta) + V(\tau, S - \beta Q_i) - V(\tau, S - \alpha_{i\tau} Q_i)$$

$$= \alpha_{i\tau} r_i + V(\tau + 1, S - \alpha_{i\tau} Q_i) + r_i(\beta - \alpha_{i\tau}) - r'(\beta - \alpha_{i\tau})$$

(where, r' is the average slope of the expected profit function between $S - \alpha_{i\tau} Q_i$ and $S - \beta Q_i$; $r' > r_i$ due to concavity of $V(\tau, S)$ in S)

$$< \alpha_{i\tau} r_i + V(\tau + 1, S - \alpha_{i\tau} Q_i)$$

Since if a fraction $\beta \neq \alpha_{i\tau}$ is accepted the expected profit is reduced, therefore if an order from class i arrives in time slot τ when the available manufacturing capacity is $S^{**}_{i\tau\alpha}$, then it is optimal to accept an $\alpha_{i\tau}$ fraction of this order. ∎

As shown in Theorems 2, 5, and 6, under certain special cases, threshold policies give optimal order acceptance solution. Thus, we are motivated to extend the idea of threshold policies to general cases. In general cases, threshold policies will give sub-optimal results. But, as shown later, the ease with which threshold policies can be computed for large size problems and their solution quality makes them attractive in application.

In a general setting of multi-period problems and when only complete orders can be accepted, the threshold policy for accepting and scheduling customer orders when only *third party* warehousing option is available for storing orders completed in advance of their due-dates is as follows: Accept an order if the amount of MCA at the time of order arrival in any of the periods between the order arrival and due-date is higher than a pre-determined threshold value corresponding to the order class, time of arrival and period under consideration; otherwise, reject the order. If the order is

accepted, schedule the order in the period that has the largest excess of MCA over the threshold value.

To find efficient threshold values, a gradient search scheme based on stochastic approximation [63] is followed. In each iteration of this scheme, we generate one scenario of future order arrivals and compute approximate gradients using finite difference approximation [64, 65, 66] and move in the direction of steepest descent in small steps [63]. The iterations are continued until the improvement in the objective function value between successive iterations is less than a certain fraction, or certain number of iteration is completed. Algorithm 1 describes each step of the stochastic approximation scheme for computing efficient threshold values for solving the RM problem in this chapter. Since this procedure is based on stochastic approximation approach, we refer to it as the heuristic scheme based on stochastic approximation (HSSA). The notation used in Algorithm 1 is as follows.

i Index for order class; $i \in I$.

m Iteration counter.

p Index for the period in which orders can be scheduled; $p \in \{1, ..., N\}$.

τ Index for time slots over the planning horizon.

Δ Difference interval in gradient approximation; $\Delta > 0$.

η Step size in the stochastic approximation iteration.

m_{max} Maximum number of iterations.

ω^m Future order arrivals scenario used in the mth iteration.

$F(\omega_m, \widehat{Z}^m)$ Expected profits generated by using threshold policy $\widehat{Z}$ in iteration m.

∇_{itp}^{m} Approximate gradient w.r.t. the threshold value for order class i,

arriving at time τ for scheduling in period p, based on the

scenario for future order arrivals used in the mth iteration.

$\widehat{Z}$ Vector of threshold values.

$Z(i, \tau, p)$ Threshold value for an order of class i, that arrives at time τ,

for scheduling in period p.

$\widehat{Z} \setminus (i, \tau, p)$ Vector of threshold values, excluding the threshold value

corresponding to order class i, arriving at time τ for scheduling

in period p; $\widehat{Z} = \{Z(i, \tau, p)\} \cup \widehat{Z} \setminus (i, \tau, p)$.

$\widehat{Z}^{m}$ Vector of mth iteration of threshold values.

ALGORITHM 1 Step by step procedure in HSSA is as follows

1. *Initialize the threshold policy. Set $m = 0$.*

2. *Simulate $(m + 1)^{st}$ scenario of future order arrivals.*

3. *Using the most recent threshold policy, approximate the steepest descent gradient by evaluating expression (3.32), which represents the finite difference approach for computing approximate gradients ([64, 65]).*

$$\nabla_{i\tau p}^{m+1} = \frac{\begin{array}{c} F(\omega^{m+1}, \{Z^m(i, \tau, p) + \Delta\} \cup \widehat{Z}^m \setminus (i, \tau, p)) - \\ F(\omega^{m+1}, \{Z^m(i, t, p)\} \cup \widehat{Z}^m \setminus (i, t, p)) \end{array}}{\Delta}, \forall i, \tau, p \qquad (3.32)$$

4. *Update the threshold values as shown in (3.33).*

$$Z^{m+1}(i, \tau, p) = Z^m(i, \tau, p) + \nabla_{i\tau p}^{m+1} \eta, \forall i, \tau, p \qquad (3.33)$$

5. *If $m = m_{max}$ stop. Else goto Step 2.*

At Step 4 a constant step size η is used. Another step size rule that can be used for choosing step sizes is $\{\sum \eta_k \longrightarrow 0; \sum \eta_k^2 < \infty\}$ [64]. The latter step size rule ensures asymptotic convergence, but the rate of convergence could be very slow [67, 68]. For constant step size rule, which we have used in our stochastic approximation heuristic, weak convergence results have been shown by Benaim and Hirsch [69] and Yin and Yin [70]. Pflug [67] and Gaivoronski [68] have highlighted that with proper selection of constant but small step sizes, the decision variables come close to the best solution at a fast rate but may ossicilate in close vicinity, in which case a secondary stopping criteria should be applied. The secondary stopping criteria we have built in our heuristic is to truncate the iterations after a certain limit. We found that after a certain number of iterations, on the order of 100-500 iterations, the changes in the threshold values between successive iterations were very small, which resulted in differences of less than 0.1% between the expected values computed at successive iterations. Therefore, truncating the iterations after a certain limit had a minimal effect on the threshold policy computed, while reducing the computational times.

There are three major benefits with solving the RM problem by using a threshold policy approach. First is that the threshold policy can be applied to general arrival process problems, thus relaxing the Poisson arrival assumption inherent in the SDP formulation. The second benefit is that the threshold policies can be computed in polynomial time ($O(N^3 k^2 |I| m_{max})$, where N is the number of periods, k is the length of each period (in time slots), and I is the set of order classes.), which makes this approach suitable for industry size problems. The computational time can be further improved by consolidating the threshold values over the time index. The third benefit with this approach is that since the threshold values are computed for all time slots in the planning horizon, they can be stored and used as static values for as long as the problem parameters do not change. In this way, the threshold values need to be

computed only once.

E. Computational Results

There are two objectives of the computational experiments. The first objective is to study the potential of the RM model developed in this chapter in improving profits over simple FCES and FCLS policies at an *on-date* delivery MTO system. The second objective is to study the computational performance of the solution approaches presented in this chapter and evaluate the suitability of these approaches for solving large size problems. In all computational studies presented in this section, manufacturing capacity units refer to time units available for processing in the MTO system. The computing environment used is Pentium 4, 2.2 GHz, 512 MB RAM Optiplex-GX240 system.

1. Significance of Efficient Scheduling Policies

To study the significance of efficient scheduling policies independent of the efficient order acceptance policies, we consider three sets of test cases, such that for each test case in these sets, all the order classes earn the same amount of profit and require the same amount of manufacturing capacity for processing. Thus, all order classes are equally attractive for acceptance. However, efficient scheduling plays an important role by prioritizing scheduling of orders in less busy periods and saving manufacturing capacity in busier periods so that more orders can be processed.

The first set of test cases considers three period problems, where each period has 150 units of manufacturing capacity available, with 3 order classes, one for each due-date, each requiring 25 units of processing capacity. The second set of test cases considers four period problems, where each period has 100 units of manufacturing

capacity available, with 4 order classes each requiring 25 units of processing capacity. The third set of test cases considers five period problems, where each period has 75 units of manufacturing capacity available, with 5 order classes each requiring 25 units of processing capacity.
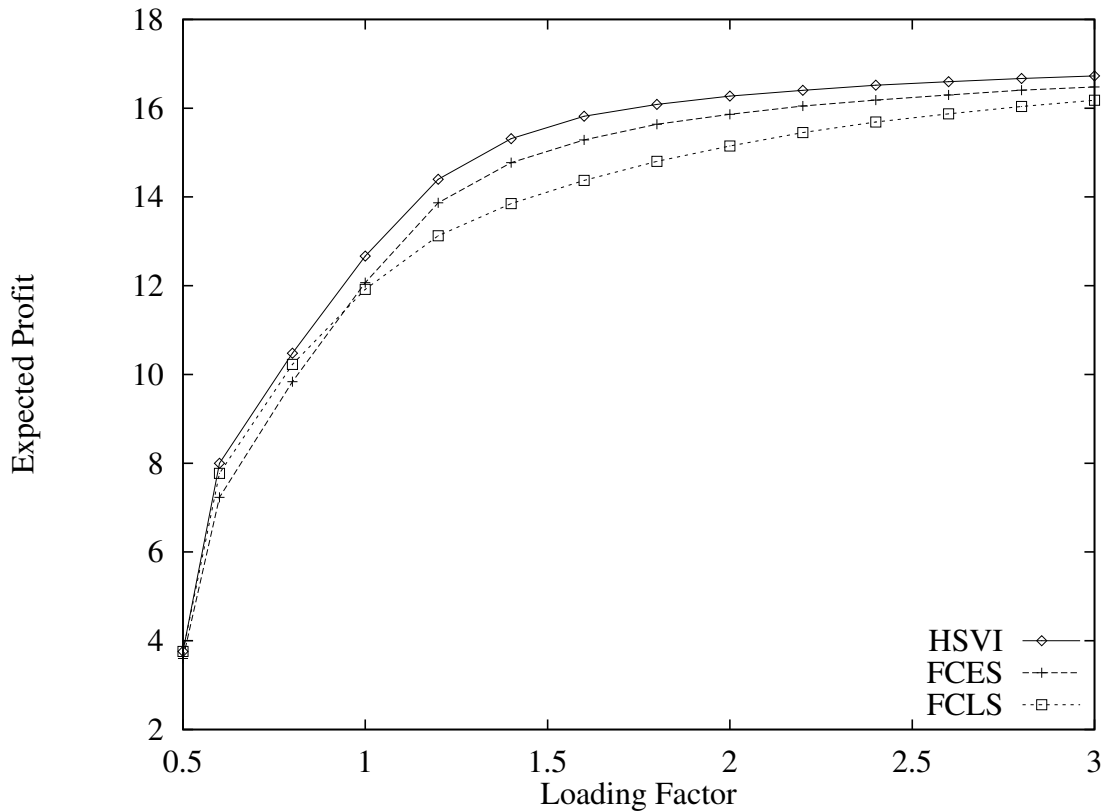
Fig. 2. Efficient Scheduling Policies - First Set

Figures 2, 3 and 4 show plots of expected profits, expressed in number of orders processed, generated by adopting the optimal order acceptance and scheduling policies determined by the HSVI approach and FCES and FCLS policies at different levels of loading factors, which refers to the ratio of total expected demand for processing divided by the available processing capacity. From these results, it can be observed that when the loading factor is between 0.8 and 1.6, efficient scheduling policies itself contribute to about 3.5% to 12% gain in expected profits over simple FCES and FCLS
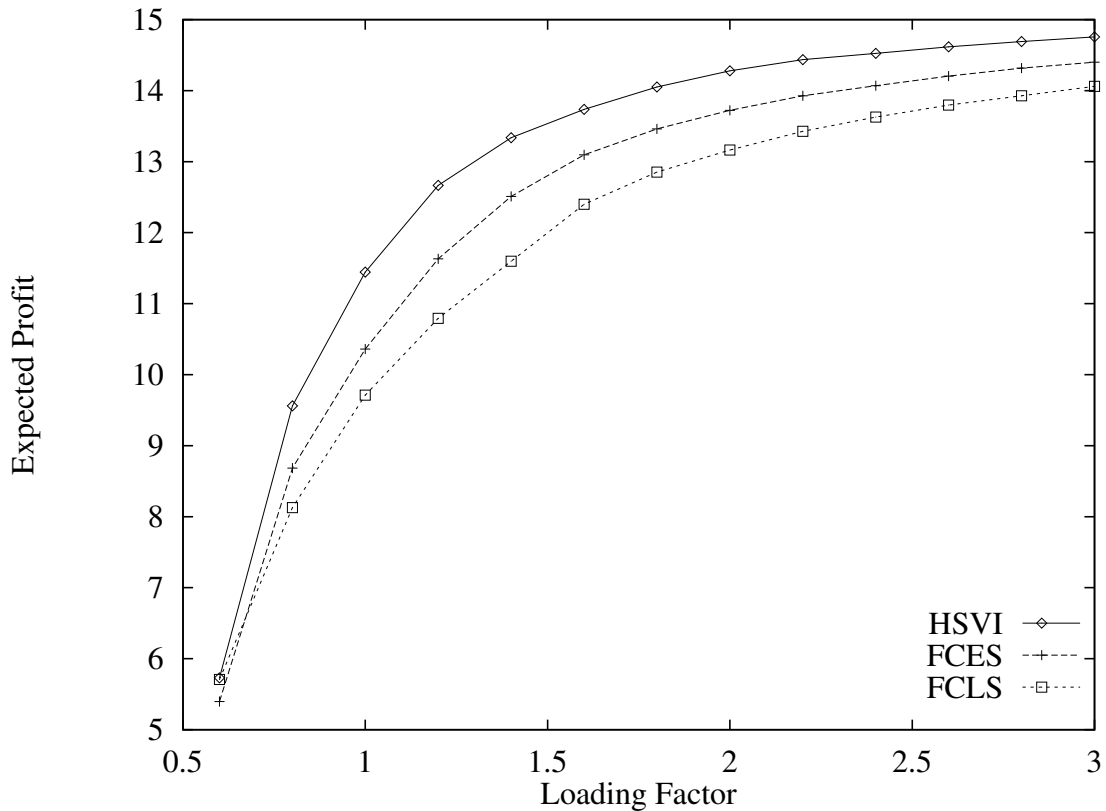
Fig. 3. Efficient Scheduling Policies - Second Set

policies. However, scheduling policies are not very effective when either the loading factor is very low or very high. When the loading factor is very low, there is an excess of manufacturing capacity in comparison to the demand, and therefore, any scheduling policy can be adopted without leading to shortage of processing capacity for any order request. When loading factor is very high, there is an excess of demand, and so manufacturing capacity is seldom wasted by idling, regardless of the scheduling policy.

2. Significance of Efficient Order Acceptance Policies

To study the significance of efficient order acceptance policies independent of the scheduling policies, we consider single period test problems with 500 units of man-
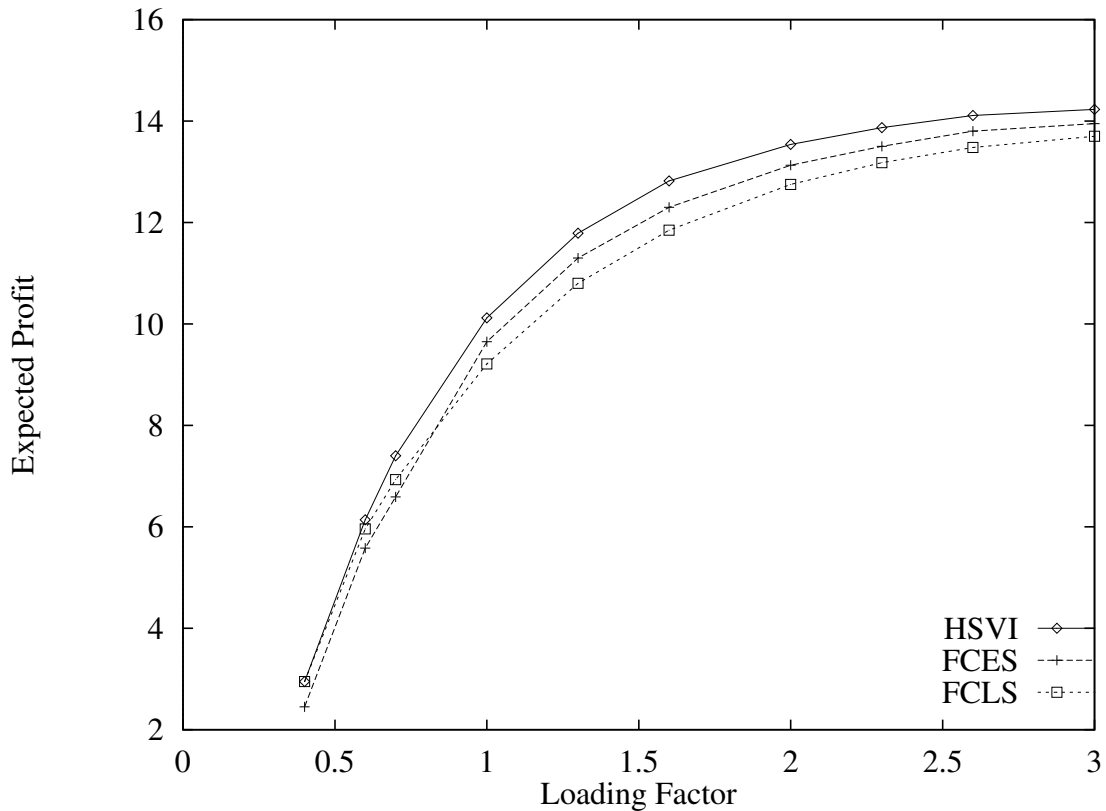
Fig. 4. Efficient Scheduling Policies - Third Set

ufacturing capacity available at various levels of loading factors. In a single period problem all orders are scheduled in the current period, therefore scheduling policies become irrelevant and the profit gains reflected by the RM model can be attributed to efficient order acceptance policies. Table II shows the results for the test problems considered, and illustrates the gains in profits with the order acceptance policies determined by the HSVI approach over FCES policy. In a single period problem, FCES policy is identical to FCLS policy. Therefore we compare results only with FCES policy.

It is intuitive to expect that when loading factor is small, almost all orders will be accepted, and hence the significance of efficient order acceptance policies will be minimal. Also, at high loading factor, it makes tremendous economic sense to

selectively accept orders and protect manufacturing capacities for future higher profit earning orders. The results in Table II are in agreement with our intuition and show that, at low loading factor, the results with HSVI are comparable to FCES, while the gap between the HSVI and FCES results grows large as the loading factor increases.

Table II. Efficient Order Acceptance

$|I|$	Loading Factor	Expected Profits		Diff. (%)	Comp. Time HSVI (sec)
		HSVI	FCES		
4	0.6	1184	1183	0.1	12.6
4	0.9	1571	1555	1.0	13.6
5	1.1	1722	1678	2.6	15.3
5	1.2	1795	1727	4.0	15.5
5	1.4	1936	1804	7.3	13.9
10	1.5	1961	1814	8.1	23.7
10	1.9	2118	1853	14.3	24.0

3.   Performance of HSVI Approach

We next study the joint impact of efficient order acceptance and scheduling policies computed by the HSVI approach on the net profits (revenue earnings of accepted orders minus holding costs incurred to satisfy *on-date* deliveries). To accomplish this, we consider two sets of test problems consisting of 2 and 3 period problems as shown in Tables III and IV, respectively.

In Table III(IV), the second column shows the number of order classes considered in the test problems, third column shows the distribution of the mean arrival rates of the order classes, and the next 2(3) columns show the amounts of manufacturing

Table III. Test Problems Set I

Test Case Number	$\|I\|$	Avg Arr Rate Dist. (Num per pd)	S(1)	S(2)
1	12	UNIF(1.5, 3.5)	250	250
2	12	UNIF(1.5, 3.5)	225	250
3	12	UNIF(2, 4)	250	250
4	12	UNIF(2, 4)	225	250
5	12	UNIF(1.75, 3.75)	250	250
6	12	UNIF(1.75, 3.75)	225	250
7	30	UNIF(0.5, 2)	250	250
8	30	UNIF(0.5, 2)	225	250
9	30	UNIF(0.75, 2.25)	250	250
10	30	UNIF(0.75, 2.25)	225	250
11	30	UNIF(0.75, 2.25)	200	250
12	30	UNIF(1, 2.5)	250	250
13	30	UNIF(1, 2.5)	225	250
14	30	UNIF(1, 2.5)	200	250
15	30	UNIF(1, 2.5)	175	250
16	30	UNIF(1, 2.5)	250	225
17	30	UNIF(1, 2.5)	250	200
18	30	UNIF(1, 2.5)	250	175

Table IV. Test Problems Set II

Test Case Number	$|I|$	Avg Arr Rate Dist. (Num per pd)	S(1)	S(2)	S(3)
19	24	UNIF(1, 3)	250	250	250
20	24	UNIF(1, 3)	225	250	250
21	24	UNIF(1.25, 3.5)	250	250	250
22	24	UNIF(1.25, 3.5)	225	250	250
23	24	UNIF(1.25, 3.5)	200	250	250
24	24	UNIF(1.25, 3.5)	175	250	250
25	30	UNIF(1, 2.25)	250	250	250
26	30	UNIF(1, 2.25)	225	250	250
27	30	UNIF(1, 2.25)	200	250	250
28	30	UNIF(1, 2.25)	175	250	250
29	30	UNIF(1.25, 2.5)	250	250	250
30	30	UNIF(1.25, 2.5)	225	250	250

capacity available in different periods. We compare the results for expected profits obtained with the HSVI approach with the results obtained with FCES and FCLS polices in Tables V and VI. It can be observed that the efficient order acceptance and scheduling policies computed by the HSVI approach can improve the net expected profits, in comparison to the net profits earned by following FCES and FCLS policies, by 22% - 34%. Typically, manufacturing systems operate at low profit margins. Therefore, an increase in the net profits on the magnitude demonstrated above shows that efficient order acceptance and scheduling policies might make a significant impact on the net profits earned at *on-date* delivery MTO systems.

Table V. Performance of HSVI Approach - Test Problem Set I

Test Case	Expected Profits			HSVI vs. (%)		Comp. Time
Number	HSVI	FCES	FCLS	FCES	FCLS	HSVI (sec)
1	2261	1743	1754	22.9	22.4	77
2	2255	1736	1751	23.0	22.4	77
3	2388	1728	1767	27.6	26.0	73
4	2381	1719	1763	27.8	26.0	74
5	2328	1735	1762	25.5	24.3	75
6	2322	1727	1758	25.6	24.3	75
7	2390	1837	1867	23.1	21.9	146
8	2383	1827	1863	23.3	21.8	145
9	2477	1820	1871	26.5	24.5	137
10	2467	1808	1865	26.7	24.4	138
11	2374	1714	1785	27.8	24.8	134
12	2540	1810	1872	28.7	26.3	129
13	2529	1796	1865	29.0	26.3	129
14	2428	1702	1779	29.9	26.7	126
15	2315	1607	1680	30.6	27.4	122
16	2422	1718	1777	29.1	26.6	125
17	2302	1627	1680	29.3	27.0	102
18	2180	1536	1583	29.5	27.4	89

Table VI. Performance of HSVI Approach - Test Problem Set II

| Test Case | Expected Profits | | | HSVI vs. (%) | | Comp. Time |
Number	HSVI	FCES	FCLS	FCES	FCLS	HSVI (sec)
19	3954	2901	2980	26.6	24.6	2008
20	3946	2890	2978	26.8	24.5	2008
21	4095	2876	3020	29.8	26.3	1893
22	4084	2862	3017	29.9	26.1	1897
23	3987	2762	2956	30.7	25.9	1856
24	3874	2659	2864	31.4	26.1	1784
25	3920	2685	2792	31.5	28.8	2631
26	3912	2674	2790	31.6	28.7	2621
27	3825	2583	2745	32.5	28.2	2566
28	3722	2488	2670	33.2	28.3	2472
29	4026	2662	2811	33.9	30.2	2506
30	4016	2650	2809	34.0	30.1	2504

The number of states in SDP (3.7) is $O(Nk^{N+1})$, which grows exponentially with the number of periods (N) in the problem. This poses a limitation on the applicability of the HSVI approach for large size problems, which might typically consider 5 to 10 periods in the planning horizon. It can be observed from the results in Tables V and VI that the computational time grows very fast between 2 period and 3 period problems, and even for small problems consisting of 3 periods and 30 order classes, it takes roughly 2500 seconds for computing efficient policies. Therefore, this approach cannot be adopted for large size problems, which motivates exploring less computationally intensive heuristic schemes.

Table VII. Performance of HSSA Approach - Test Problem Set I

Test Case	Exp. Profits	HSSA vs. (%)			Avg. Comp. Tm (sec)	
Number	HSSA	HSVI	FCES	FCLS	HSVI	HSSA
1	2008	-11.2	16.6	15.0	77	2
2	1969	-12.7	15.2	13.2	77	2
3	2143	-10.3	23.6	21.0	73	2
4	2125	-10.7	23.7	20.5	74	2
5	2084	-10.5	20.3	18.1	75	3
6	2065	-11.0	20.3	17.5	75	3
7	2139	-10.5	15.8	13.5	146	9
8	2118	-11.1	15.8	13.0	145	10
9	2229	-10.0	21.7	18.7	137	10
10	2236	-9.4	23.6	19.8	138	10
11	2103	-11.4	22.6	17.8	134	10
12	2325	-8.5	28.2	24.0	129	10
13	2283	-9.7	27.6	22.6	129	10
14	2162	-11.0	27.2	21.5	126	11
15	2045	-11.7	27.5	21.8	122	11
16	2072	-14.4	20.3	16.5	125	10
17	1962	-14.8	20.2	16.7	102	11
18	1844	-15.4	19.7	16.5	89	10

Table VIII. Performance of HSSA Approach - Test Problem Set II

Test Case Number	Exp. Profits HSSA	HSSA vs. (%)			Avg. Comp. Tm (sec)	
		HSVI	FCES	FCLS	HSVI	HSSA
19	3345	-15.4	16.6	11.8	2008	10
20	3331	-15.6	16.9	11.5	2008	9
21	3501	-14.5	21.0	15.2	1893	10
22	3491	-14.5	21.7	15.1	1897	10
23	3360	-15.7	21.4	13.1	1856	10
24	3216	-17.0	20.5	11.9	1784	10
25	3237	-17.4	20.4	15.4	2631	18
26	3232	-17.4	21.2	15.5	2621	18
27	3086	-19.3	19.8	12.1	2566	18
28	2968	-20.2	19.4	10.9	2472	18
29	3237	-19.6	20.4	15.4	2506	18
30	3232	-19.5	21.2	15.5	2504	18

4. Performance of HSSA Approach

Tables VII and VIII compare the quality of results obtained with HSSA versus HSVI, FCES and FCLS using the test problems in Tables III and IV. It can be observed that the order acceptance and scheduling policies determined by the HSSA approach perform significantly better than FCES and FCLS policies, with about 13% - 28% higher expected profits. A comparison with the results obtained with HSVI approach reveals that HSVI approach generates about 8% - 20% higher profits than HSSA approach.

Since HSVI approach computes ϵ-optimal solutions to SDPs [62], the expected

profits with HSVI approach represent upper bounds on the expected profits. The comparison between the results obtained with HSVI and HSSA approaches indicate that HSSA approach is able to capture about 50%-75% of the maximum possible gains in expected profits over FCES and FCLS policies. A comparison between HSVI and HSSA approaches based on computational times, shown in Table IX, reveals that HSSA approach takes significantly less time in determining efficient order acceptance and scheduling policies. Table X shows that HSSA approach is able to solve large size problems consisting of 5 to 10 periods, 30 to 75 order classes in manageable amounts of time. Column 3 in Table X refers to the mean arrival rate distribution for order classes. This is because, HSSA approach has a polynomial time worst case computational complexity of $O(N^3 k^2 |I|m)$, where N is the number of periods, k is the length of each period (in time slots), I is the set of order classes, and m is the number of scenarios considered in the HSSA approach.

Another advantage with HSSA approach is that it needs to be applied only once during the planning horizon or until there are changes in the problem parameters or forecasts. The threshold policy results computed by HSSA approach can be stored and accessed without the necessity of re-computations.

The HSSA approach is attractive for industry applications, since it is able to capture a large portion of the maximum possible gains in profits over the FCES and FCLS polices, has a polynomial time worst case computational complexity leading to reasonable computational times for large size problems, and can be used as a static policy that needs to be computed only once during the planning horizon or until the problem parameters change.

Table IX. Performance of HSSA Approach

N	$|I|$	Number of Sample Problems	Avg. Comp. Time (sec) HSVI	HSSA
2	12	21	67.4	2.4
2	30	21	123.7	9.9
3	24	30	1757.1	9.6
3	36	30	2311.2	17.8

Table X. Performance of HSSA Approach - Large Problems

N	$|I|$	Arr. Rt. Dist. (Num per pd)	Expected Profits HSSA	FCES	FCLS	Comp. Time HSSA (sec)
5	30	UNIF(1.5, 3.5)	4276	3326	3663	25
5	45	UNIF(1.5, 2.5)	3684	2956	3250	48
5	75	UNIF(1, 2)	3945	3052	3367	119
8	36	UNIF(1.5, 3.)	8163	5772	7583	68
8	72	UNIF(1, 2)	8158	5920	7634	206
10	55	UNIF(1.5, 3.)	10374	6966	9646	193

5. Effects of Loading Factor on the Efficiency of RM Model

To gain insights into the impact of loading factor on the efficiency of RM model for accepting and scheduling orders at an *on-date* delivery MTO system, we consider three sets of problems (2 period, 12 order classes; 2 period, 24 order classes, 3 period, 24 order classes) at different levels of loading factors, as shown in Table XI.

Table XI reveals that with increase in loading factor, the efficiency of RM model for accepting and scheduling orders improves, as shown by the increase in the expected profits generated by adopting HSVI approach versus expected profits generated by FCES or FCLS schemes. This is intuitive, since an increase in demand makes it more attractive to adopt efficient schemes for accepting and scheduling orders, so that both the manufacturing capacity is not wasted in idling and also sufficient amount of manufacturing capacity is protected for future high profit earning orders.

Table XI. Effects of Loading Factor on the Efficiency of RM Model

Loading Factor	Problem Set 1 HSVI vs.		Problem Set 2 HSVI vs.		Problem Set 3 HSVI vs.	
	FCES (%)	FCLS(%)	FCES (%)	FCLS(%)	FCES (%)	FCLS(%)
0.6	3.4	1.7	3.4	1.6	6.7	4.0
0.8	3.7	4.3	3.6	4.3	8.0	9.2
1.0	4.1	7.2	3.9	7.1	10.0	14.6
1.2	5.1	9.9	4.7	9.5	14.1	20.0
1.4	7.2	12.6	6.5	11.9	20.0	25.2
1.6	10.6	15.4	9.5	14.3	26.3	29.3
1.8	14.8	18.4	13.4	17.0	31.8	32.1
2.0	19.5	21.6	17.7	19.8	36.3	34.0

Table XI also shows that when loading factors are close to 1.0, FCES policy performs better than FCLS policy. This is because when FCLS policy is used, the current periods' processing capacity can be used for a future due-date order only when the manufacturing capacity available in all periods following the current period upto the due-date is used up. Therefore, when the loading factor is close to 1.0, the chances are high that in the current period we might be idling our manufacturing capacity, although there might be pending orders scheduled in some future periods. FCES ensures that as long as there is a pending order, the manufacturing capacity available is not wasted in idling.

6.   Effects of Overlooking Holding Costs in the RM Model

Consideration of the holding costs aspects while accepting and scheduling customer orders at an *on-date* delivery MTO systems makes the RM model very complicated. Therefore, it is tempting to overlook the holding cost aspects in the RM model. To gain insights into the extent of the effect of overlooking holding costs in the RM model, we consider test problems comprising of 5 periods, 250 units of available manufacturing capacity, processing requirements of order classes ranging between 25 and 50 units, and number of job types, arrival rate distribution for order classes, and holding costs as shown in Table XII.

The fourth column in Table XII shows the expected profit when holding costs are considered in making the acceptance and scheduling decision, the fifth and sixth columns show the expected revenues with FCES and FCLS policies, the seventh column shows the expected profit when holding costs are ignored in making the acceptance and scheduling decision, while the eighth column compare the expected profits when holding costs are considered in making the acceptance and scheduling decision and when they are not. It is intuitive to expect the net profits to be higher

Table XII. Effects of Overlooking Holding Costs in the RM Model

| $|I|$ | Arr Rate Dist. (Num per Pd) | h_i/r_i (%) | Expected Profit | | | | HSSA vs. |
			HSSA	FCES	FCLS	HSSA*	HSSA* (%)
15	UNIF(2, 4)	1	4677.5	4202.4	4573.7	4664.3	0.28
15	UNIF(2, 4)	2	4626.4	4158.9	4572.6	4611.8	0.32
15	UNIF(2, 4)	3	4606.5	4115.5	4571.5	4589.3	0.37
15	UNIF(2, 4)	6	4552.8	3985.0	4568.2	4521.9	0.68
15	UNIF(2, 4)	8	4511.9	3898.0	4566.0	4477.0	0.78
15	UNIF(2, 4)	9	4491.7	3854.5	4564.9	4454.5	0.84
15	UNIF(2, 4)	10	4484.4	3811.0	4563.7	4432.0	1.18
30	UNIF(1, 3)	1	3958.7	3523.8	3664.1	3951.7	0.18
30	UNIF(1, 3)	3	3914.0	3455.1	3654.6	3905.1	0.23
30	UNIF(1, 3)	6	3860.4	3352.1	3640.4	3835.3	0.66
30	UNIF(1, 3)	9	3811.6	3249.0	3626.3	3765.4	1.23
30	UNIF(1, 3)	10	3805.1	3214.7	3621.5	3742.1	1.68

HSSA: Considering holding costs in the RM model.

HSSA*: Overlooking holding costs in the RM model.

when holding costs are considered in making the acceptance and scheduling decisions.

The results in Table XII gives insights into the extent of the impact of ignoring the holding costs while making the acceptance and scheduling decisions. It can be observed that even when holding costs are very small, ignoring the holding costs in RM model leads to a loss in profits on the order of 0.2%, and considering that manufacturers typically operate at tight profit margins, this small gap of 0.2% could translate into large amounts of money. When holding costs are high, the loss in profits is on the order of 1%-2%, which might represent significant amount of money.

F.   Summary and Conclusions

In this chapter, we develop an RM model for accepting and scheduling customer orders at an *on-date* delivery MTO system. We study and gain insights into the significance of efficient acceptance and scheduling policies and incorprating holding cost aspects into the RM model. We demonstrate that the RM model has potential for significantly improving the expected profits at MTO systems. We develop an efficient heuristic scheme for solving large size problems in manageable amount of time.

CHAPTER IV

FIRST PARTY WAREHOUSING OPTION

A.   Problem Description

In Chapter III, we study the potential of an RM model in improving net profits at an *on-date* delivery MTO system. We illustrate that order acceptance and scheduling decisions can significantly impact the profit earnings, and the holding cost aspects must not be overlooked to realize maximum potential of RM.

The objective of the problem considered in this chapter is similar to Chapter III, which is to develop an RM model for efficient order acceptance and scheduling at an *on-date* delivery MTO system. In Chapter III, we consider the case where any order completed in advance of its due-date is stored at a third party warehouse until its due-date. In this chapter, we assume that there is a certain amount of *first party* warehousing capacity available with the manufacturer, which gets priority for storage requirements of completed orders. Storage requirements in excess of the *first party* warehouse capacity are met through third party warehousing.

The motivation for employing *first party* warehousing facility for storing completed orders is that typically such facilities will be located on the same premises as the manufacturing facility. Therefore, if orders are stored in a *first party* warehouse, there will be less material handling and transportation activities in comparison to storing at third party warehouses. In addition, if the variability in storage requirements between different periods is low, and the *first party* warehouse is consistently used at high utilization levels, it can be argued that under such cases *first party* warehousing is less expensive than *third party* warehousing. There are other benefits with *first party* warehousing, such as it makes it easy to ensure that proper care is taken

in handling and storing all items, which is necessary to minimize any damages.

Our RM model segments the possible future order arrivals into classes $\{i \in I\}$ based on their due-dates (represented by d_i), processing time requirements (represented by Q_i) and profit margins (represented by r_i). Using the information on the forecast for arrival distributions (whose mean is represented by λ_{ic}, where $i \in I$ is the product class and c is the arrival period) for order classes, the RM model prescribes efficient order acceptance and scheduling policies that are used to decide if a new order arriving at time τ and belonging to a certain order class i should be accepted or not, and if accepted, in which period it should be scheduled.

The key assumptions in the RM model developed in this chapter include Assumptions 1 - 8 described in Section A of Chapter III and Assumption 9 outlined below.

ASSUMPTION 9 *Third party warehousing availability.* We assume that there is sufficient amount of third party warehousing capacity accessible to the MTO manufacturer such that there is never a shortage of storage space. We also assume that the third party warehousing costs are fixed for all orders and identical for all periods.

Analogous to Chapter III, the optimal order acceptance and scheduling policies can be stated as follows.

REMARK 3 *Optimal Order Acceptance Policy.* Accept an order if the expected opportunity cost of scheduling it in any period between its arrival and its due-date is less than its profit earnings.

REMARK 4 *Optimal Order Scheduling Policy.* Schedule an order in the period that has the least opportunity cost for processing the order.

B. Mathematical Formulation

We follow the notation described in Section C of Chapter III throughout this chapter. Under the assumption of a Poisson arrival process, the time horizon under study is divided into small time slots, such that the probability of arrival of more than one order in each time slot is very small. SDPs are then constructed over the discrete space of the time slots, with the objective of computing the expected returns from the available manufacturing capacity. In the sense of RM, when a decision needs to be made on accepting or rejecting an order, the difference between the expected returns from the available manufacturing capacities when an order is rejected and is accepted is compared with the profits earned by processing the order at hand. If the comparison is in favor of the order, the order is accepted, otherwise it is rejected. Following this general approach, we develop SDP formulations for our problems, as described below.

1. Optimal Order Acceptance Policy

An order from order class i arriving in time slot τ when the vector of available manufacturing capacities in different periods is $\widehat{S}$ and the vector of committed storage volumes in different periods is $\widehat{W}$ should be accepted if it satisfies expression (4.1). In (4.1), $V(\tau, \widehat{S}, \widehat{W})$ is the expected profit function and can be computed by SDP (4.5). (4.1) compares the minimum of the opportunity cost for committing manufacturing capacity in different periods with the profits earned by processing the order and is a mathematical representation of the optimal acceptance policy stated in Remark 3.

$$\min_{\substack{X_{ic} \\ s.t.(3.2)-(3.5),(4.2)}} [V(\tau+1,\widehat{S}',\widehat{W}) - V(\tau+1,\sum_c X_{ic}(\{S(c)-Q_i\} \cup \widehat{S}' \setminus c),\widehat{W}')$$

$$+h_i(d_i-c)\sum_{c=\lceil \tau/k \rceil} X_{ic}(1-\prod_{l=c}^{d_i}(1-\mathbf{1}_{W(l)+1>W}))] \leq r_i \quad (4.1)$$

where,

$$W'(l) = \begin{cases} W(l) + \sum_{c=\lceil \tau/k \rceil}^l X_{ic} \prod_{l=c}^{d_i}(1-\mathbf{1}_{W(l)+1>W}) & \text{if } l < d_i \\ W(l) & \text{o.w.} \end{cases} \quad (4.2)$$

In (4.1), the first term is the expected profit when the order at hand is not accepted, the second term is the expected profit when the order at hand is accepted, while the third term is the warehousing cost incurred for storing the order in a third party warehouse if sufficient storage capacity is not available in any of the periods between the processing c and due-date d_i. (4.2) refers to the change in the committed storage volume in different periods.

2.   Optimal Order Scheduling Policy

An order from order class i arriving in time slot τ when the vector of available manufacturing capacities in different periods is $\widehat{S}$ and the vector of committed storage volumes in different periods is $\widehat{W}$ should be in period c^* determined by (4.3). (4.3) is the mathematical representation of Remark 4.

$$c^* = \arg\min[V(\tau,\widehat{S}',\widehat{W}) - V(\tau+1,\{S(c)-Q_i\} \cup \widehat{S}' \setminus c,\widehat{W}') + h_i(d_i-c)(1-$$

$$\prod_{l=c}^{d_i}(1-\mathbf{1}_{W(l)+1>W})) : c \in \{\lceil \tau/k \rceil,...,d_i\};(3.2);(4.4);S(c)-Q_i \geq 0] \quad (4.3)$$

where,

$$W'(l) = \begin{cases} W(l) + \prod_{l=c}^{d_i}(1 - \mathbf{1}_{W(l)+1>W}) & \text{if } c \leq l < d_i \\ W(l) & \text{o.w.} \end{cases} \quad (4.4)$$

In (4.3), $V(\tau, \widehat{S}, \widehat{W})$ is the expected profit function and can be computed by solving SDP (4.5). (4.3) determines the least opportunity cost period for scheduling the order. (4.4) refers to the change in the committed storage volume in different periods.

3. Expected Profit Function

The expected profit function $V(\tau, \widehat{S}, \widehat{W})$ represents the net expected profit that can be generated by the available manufacturing capacity represented by the vector $\widehat{S}$, with the orders arriving during and after time slot τ. An SDP for computing $V(\tau, \widehat{S}, \widehat{W})$ is expressed by (4.5) with boundary condition (4.6).

$$V(\tau, \widehat{S}, \widehat{W}) = \sum_{i \in I: d_i \geq \lceil \tau/k \rceil} P_{i\lceil \tau/k \rceil} \Big(\max_{\substack{X_{ic} \\ s.t.(3.9)-(3.13),(4.7)}} \Big[\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} r_i -$$

$$h_i(d_i - c) \sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic}(1 - \prod_{l=c}^{d_i-1}(1 - \mathbf{1}_{W(l)+1>W})) + V(\tau+1, \widehat{S}', \widehat{W}')\Big]\Big) \quad (4.5)$$

$$V(N \times k + 1, \widehat{S}, \widehat{W}) = 0, \qquad \forall \widehat{S}, \widehat{W} \quad (4.6)$$

where,

$$W'(l) = W(l) + \sum_c X_{ic}\mathbf{1}_{c \leq l}\mathbf{1}_{d_i > l}\prod_{l'=c}^{d_i}(1 - \mathbf{1}_{W(l')+1>W}), \forall l \quad (4.7)$$

Feasibility of the successive order acceptance decisions in (4.5) is ensured by constraints (3.9)-(3.13) and(4.7). Constraint (4.7) accounts for the storage volume

committed in different periods by accepting and scheduling manufacturing orders.

4. FCES and FCLS Policies

To evaluate the performance of our RM model, we consider two policies, first come earliest served (FCES) and first come latest served (FCLS). Under an FCES policy, all incoming orders are accepted if there is sufficient manufacturing capacity available, at the time of order arrival in any of the periods between order arrival and its due-date, that can process this order. An accepted order is scheduled in the earliest possible period with sufficient capacity to process the order. FCLS policy follows the same order acceptance rule as FCES, but schedules an accepted order in the latest possible period with sufficient capacity to process the order. Section C.5 in Chapter III describes the strengths and weaknesses of these policies.

Expected value function with the FCES scheduling is represented by the SDP expression (4.8) with c chosen as in (4.9). (4.9) determines the earliest period with sufficient available manufacturing capacity to process an order from class i.

$$V(\tau, \widehat{S}, \widehat{W}) = \sum_{i \in I: d_i \geq \lceil \tau/k \rceil} P_{i\lceil \tau/k \rceil}(r_i \mathbf{1}_{\lceil \tau/k \rceil \leq c \leq d_i} - [h_i(d_i - c)(1 -$$
$$\prod_{l=c}^{d_i - 1}(1 - \mathbf{1}_{W(l)+1>W}))] + V(\tau + 1, \widehat{S}', \widehat{W}')) \tag{4.8}$$

$$c = \min\{s : S(s) > Q_i; \lceil \tau/k \rceil \leq s \leq d_i; (3.16) - (3.19); (4.10)\} \tag{4.9}$$

$$W'(l) = W(l) + \mathbf{1}_{c \leq l}\mathbf{1}_{d_i > l}\prod_{l=c}^{d_i}(1 - \mathbf{1}_{W(l)+1>W}), \forall l \tag{4.10}$$

Similarly, for the FCLS scheduling policy, the expected value function in this

case can also be expressed by the SDP expression (4.8) with c chosen as in (4.11). (4.11) determines the latest period with sufficient available manufacturing capacity to process an order from class i.

$$c = \max\{s : S(s) > Q_i; \lceil \tau/k \rceil \leq s \leq d_i; (3.16) - (3.19); (4.10)\} \qquad (4.11)$$

C.   Solution Approaches

The SDP (4.5) is intractable even for problems of small size. To get an idea of the size of state space for a small problem, consider Example 2.

Example 2 Let the number of periods in the planning horizon be 3 with 5 units of *first party* warehousing capacity and length of each period equal to 250 units. Discretize the time horizon into time slots of width equal to 1 time unit. Since the number of feasible states in the SDP (4.5) is $O(Nk^{N+1}W^N)$ the number of states in this problem is $O(1.46 \times 10^{12})$. If we can scale down the available manufacturing capacity and the processing requirements of all order classes by a factor of 10 (as described in Section D.2 of Chapter III), the number of states in our SDP (4.5) is $O(9.4 \times 10^7)$, which is still difficult to handle. ■

Since the number of states in the SDP (4.5) is very high, none of the methods currently available in the literature [71] can be used to solve this SDP efficiently. Therefore, we explore heuristic approaches for finding efficient order acceptance and scheduling policies.

1.   Heuristic Scheme based on Stochastic Approximation-II (HSSA-II)

In Chapter III, we motivate use of a threshold policy for finding efficient order acceptance and scheduling policies. It was demonstrated that efficiently designed threshold

policies give high quality results while it takes a manageable amount of time for computing such policies for large size problems. Therefore, we adopt threshold policy approach for making efficient order acceptance and scheduling decisions in the RM model presented in this chapter.

The threshold policy for accepting and scheduling customer orders when a certain amount of *first party* warehousing capacity is available is as follows: Accept an order if the amount of MCA at the time of order arrival in any of the periods between the order arrival and due-date is higher than a pre-determined threshold value corresponding to the order class, time of arrival, and period under consideration; otherwise, reject the order. If the order is accepted, schedule the order in the period that has the largest excess of MCA over the threshold value.

Similar to Chapter III, we adopt a stochastic approximation scheme for computing efficient threshold policy. Below, we describe Algorithm 2 for computing efficient threshold values for solving the RM problem in this chapter. The notation described in Section D of Chapter III is used in this formulation and the rest of the chapter. Since this procedure is based on stochastic approximation approach, we refer to it as heuristic scheme based on stochastic approximation (HSSA-II).

ALGORITHM 2 Step by step procedure in HSSA-II is as follows:

1. *Initialize the threshold policy. Set $m = 0$.*

2. *Simulate $(m + 1)^{st}$ scenario of future order arrivals.*

3. *Using the most recent threshold policy, approximate the steepest descent gradient by evaluating the expression (4.12), which represents the finite difference approach for computing approximate gradients [64, 65].*

$$F(\omega^{m+1}, \{Z^m(i, \tau, p) + \Delta\} \cup \widehat{Z}^m \setminus (i, \tau, p)) -$$

$$\nabla_{i\tau p}^{m+1} = \frac{F(\omega^{m+1}, \{Z^m(i, \tau, p)\} \cup \widehat{Z}^m \setminus (i, \tau, p))}{\Delta}, \forall i, \tau, p \qquad (4.12)$$

4. *Update the threshold policy as shown in (4.13).*

$$Z^{m+1}(i, \tau, p) = Z^m(i, \tau, p) + \nabla_{i\tau p}^{m+1}\eta, \forall i, \tau, p \qquad (4.13)$$

5. *If $m = m_{max}$ stop. Else goto Step 2.*

For the reasons described in the previous chapter, we use a constant step size in each iteration of HSSA-II. The computational performance of HSSA-II is discussed in the following section.

D. Computational Results

In the previous chapter, we carry out computational experiments to demonstrate the significance of efficient order acceptance and scheduling policies. We expect the performance of the RM model presented in this chapter to be similar to the RM model presented in the previous chapter, since the only difference between the two models is in the holding cost model. Therefore, in this section we focus the computational experiments on gaining insights into the importance of considering the warehousing aspects in the RM model when a certain amount of *first party* warehousing capacity is available for storing completed orders, and on studying the computational performance of the HSSA-II approach.

1. Significance of the Holding Cost Model in the Overall RM Model

In the problem presented in this chapter, third party warehousing costs are incurred only when the *first party* warehousing capacity is used up. Therefore, to extract

maximum benefit out of the RM model, the warehousing aspects should be considered along with the manufacturing capacity utilization.

Table XIII. Effects of Overlooking Storage Aspects in RM Model

N	$\|I\|$	$WQ = \frac{w}{\left(\frac{k}{\min_i Q_i}\right)}$	HSSA-II vs. HSSA-II* (%)
5	15	0.3	0.52
5	30	0.3	0.95
5	15	0.6	0.28
5	30	0.6	0.50
5	15	0.9	0.04
5	30	0.9	0.06

HSSA-II: Considering the warehousing aspects in the RM model.

HSSA-II*: Overlooking the warehousing aspects in the RM model.

To gain insights into the significance of considering the warehousing aspects in the RM model, we consider two sets of 8 test cases each and three different levels of availability of *first party* warehousing capacity. We compare results between cases when warehousing aspects are considered and when they are ignored in the RM model for making the acceptance and scheduling decisions. Table XIII summarizes our results. Column 3 shows the different levels of *warehousing quotient*(WQ) considered. Warehousing quotient is the ratio of the available *first party* warehousing capacity and the maximum number of orders that can be processed in a period. In Column 4, we show the average difference (computed over 8 test cases each) in profit (expressed in percentage) resulting from neglecting the holding cost model. It can be observed that at low levels of WQ, there is a little impact of neglecting the warehousing aspects

on the expected profits, while the impact further diminishes as the WQ increases. HSSA-II* corresponds to the case when the holding costs are ignored, while HSSA-II corresponds to the case when they are not. From these results, we gain the insights that if the WQ is high, the RM model can ignore the warehousing aspects with minimal effects on the expected profits.

2. Performance of HSSA-II Approach

We study the performance of HSSA-II approach from two perspectives. The first is the improvements in the expected profits with the threshold policy computed by HSSA-II approach over FCES and FCLS policies. The other is the computational time for solving the problems. In this regard, we consider the test problems shown in Table XIV, which includes problems of 2 to 10 periods and 12 to 180 order classes. Column 5 in Table XIV shows the distribution for the mean of arrival rates of order classes. Table XV shows the results. From the results it can be observed that significant improvements, up to 38%, can be made in the total profits over FCES and FCLS policies by HSSA-II approach. It can be further observed that even for large problems, consisting of 8 - 10 periods with 250 - 500 manufacturing capacity units (time slots), the computational time is manageable. This shows the suitability of HSSA-II for industry applications.

E. Summary and Conclusions

In this chapter, we develop an RM model for accepting and scheduling customer orders at an *on-date* delivery MTO system, while assuming that a certain amount of *first party* warehousing capacity is available for storing orders completed in advance of their due-dates and storage requirements in excess of the *first party* warehousing

Table XIV. Test Problem Set

Problem Set Num.	N	k	$|I|$	Mean Arr. Rate Dist. (Num per pd.)
1	2	250	12	UNIF(1.5, 3.5)
2	2	250	12	UNIF(2, 4)
3	2	250	12	UNIF(1.75, 3.75)
4	2	250	30	UNIF(0.5, 2)
5	2	250	30	UNIF(0.75, 2.25)
6	2	250	30	UNIF(1, 2.5)
7	3	250	24	UNIF(1, 3)
8	3	250	24	UNIF(0.75, 2.75)
9	3	250	24	UNIF(1.25, 3.5)
10	3	250	36	UNIF(0.5, 2.25)
11	3	250	36	UNIF(1, 2.25)
12	3	250	36	UNIF(1.25, 2.5)
13	5	500	30	UNIF(1, 3)
14	5	500	45	UNIF(1, 2)
15	5	500	75	UNIF(0.5, 1.5)
16	8	500	72	UNIF(1, 3)
17	8	500	108	UNIF(1, 2)
18	8	500	180	UNIF(0.5, 1.5)
19	10	250	55	UNIF(1, 3)

Table XV. Computational Performance HSSA-II

| Problem | Expected Profits | | | HSSA-II vs. (%) | | Comp |
Set Num.	HSSA-II	FCES	FCLS	FCES	FCLS	Time (sec)
1	2002	1751	1749	14.4	14.5	2.3
2	2164	1763	1774	22.7	22.0	2.4
3	2094	1760	1767	18.9	18.5	2.3
4	2122	1880	1887	12.8	12.4	9.8
5	2258	1864	1881	21.2	20.1	10.2
6	2344	1847	1879	26.9	24.8	10.7
7	3417	2989	3005	14.3	13.7	10
8	3233	2928	2954	10.4	9.5	9.4
9	3587	3015	3055	19.0	17.4	10.7
10	3139	2815	2832	11.5	10.8	18
11	3276	2806	2820	16.7	16.1	18.9
12	3492	2786	2829	25.3	23.4	19.7
13	6208	5065	5718	22.6	8.6	66.6
14	5899	4899	5481	20.4	7.6	132
15	6303	5230	5841	20.5	7.9	329.8
16	14366	11030	13933	30.2	3.1	745.7
17	14041	10780	13691	30.2	2.6	1495.8
18	13223	10398	12456	27.2	6.2	4225.9
19	10205	7381	9309	38.3	9.6	190.3

capacity is met through third party warehousing. We study and gain insights into the effects of neglecting the warehousing and holding cost aspects on the efficiency of this RM model. We find that at high warehousing quotient, the effect of ignoring the warehousing related costs is minimal. We illustrate that this RM model has potential for significantly improving the profits over FCES and FCLS policies at MTO systems. We develop an efficient heuristic scheme that solves large size problems in manageable amounts of time.

CHAPTER V

FIRST PARTY WAREHOUSE CAPACITY PLANNING

A. Problem Description

In this chapter, we address the problem of determining an efficient capacity or size of a *first party* warehousing facility, which will be used for storing completed orders until their due-dates at an MTO system. Our objective is to maximize the net profit at the MTO system, which consists of profits earned by processing customer orders minus the warehousing costs. Warehousing costs include *first party* warehouse setup and operating costs and third party warehousing costs incurred to meet the storage requirements in excess of the *first party* warehouse storage capacity.

Since warehouses are typically designed to have a long service life, we model the warehouse design problem as an infinite horizon problem. For ease of modeling, we apportion all warehousing related costs, including costs that depend on the storage capacity (for example, cost of storage infrastructure) and costs that do not depend on the storage capacity (for example, costs associated with warehouse support IT infrastructure), into an approximate storage cost per unit storage per period (C_W). To determine the storage volume requirements, we consider the storage volume requirement resulting from an RM based order acceptance and scheduling policy. This will allow us to design a warehousing capacity that can reap maximum benefits from an RM model for accepting and scheduling customer orders at an *on-date* delivery MTO system, leading to maximization of net profit earnings.

The key assumptions in our warehouse capacity design model developed in this chapter include Assumptions 1 and 3-8 described in Section A of Chapter III, in addition to Assumptions 10 and 11 described below.

ASSUMPTION 10 *Once determined, the first party warehousing capacity will remain fixed during the planning horizon.* Setting up a *first party* warehouse requires a large investment and takes a long lead time, typically on the order of months [72]. Therefore, it is not feasible to change the warehousing capacity to meet the period-to-period fluctuations in storage capacity requirements.

ASSUMPTION 11 *Homogenous arrival process in all the periods during the planning horizon [3, 33].* For ease of modeling, we assume that the order arrival process is identical in different periods. The order classes in this case are classified according to number of periods after which the order is due, processing requirements, and profit earnings.

B.   Mathematical Formulation

1.   Design Model

For the mathematical formulation, we use the notation described in Section C of Chapter III. We define booking horizon as the number of periods in future for which the orders are being accepted in the current period. As we are considering an infinite horizon design model, the concept of booking horizon provides us with a rolling horizon for accepting future orders. Moreover, due to Assumption 11, all booking horizons are statistically equivalent. To develop a mathematical model of the problem, we divide a period into small time slots indexed by τ, such that the probability of arrival of more than one order in a time slot is very small and can be neglected. The *first party* warehouse design problem can now be represented by (5.1), in which $g(W)$ is the optimum average profit earned per time slot at a given level W of *first party* warehousing capacity and C_W is the long run apportioned cost of a unit of storage per time slot.

$$\max_{W} g(W) - C_W W \qquad (5.1)$$

$g(W)$ can be computed by solving the average reward stochastic dynamic program (5.2) - (5.3). In (5.2) and (5.3), $H(\tau, \widehat{S}, \widehat{W})$ refers to the state value function. $H(\tau, \widehat{S}, \widehat{W})$ assigns a value to a state to reflect its relative value in comparison to other states. The average profit per time slot is computed by accounting for the expected change in the state values plus the expected profit realized in the state transition. Due to a rolling booking horizon, when a period ends and the next one starts, a new period is introduced for which orders can be accepted. This new period corresponds to the last period in the booking horizon. (5.3) accounts for the changes in the state vectors $\widehat{S}$ and $\widehat{W}$ resulting from end of one period and start of another, as indicated in (5.4) and (5.5). (5.4) and (5.5) move the state values in vectors $\widehat{S}$ and $\widehat{W}$ forward by one period, while resetting the state values corresponding to the new period. (5.4) sets the amount of manufacturing capacity available in the new period at k, indicating that no orders have been scheduled in this period, while (5.5) sets the committed storage volume in the new period at 0, indicating that the entire *first party* warehousing capacity is available for storage. Note that $V(\tau, \widehat{S}, \widehat{W})$ is the expected value function and computes the optimal expected profit realized by processing future orders with the available manufacturing capacity. For an infinite horizon problem, the expected value function will always return value equal to infinity. The values of $g(W)$ and $H(\tau, \widehat{S}, \widehat{W})$ can be determined by solving (5.2) - (5.3) using value iteration or policy iteration schemes [61].

If $\tau < k$,

$$g(W) + H(\tau, \widehat{S}, \widehat{W}) = \sum_{i \in I: d_i \geq \lceil \tau/k \rceil} P_{i\lceil \tau/k \rceil} \left(\max_{\substack{X_{ic} \\ s.t.(3.9)-(3.13),(4.7)}} \left[\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} r_i - \right. \right.$$

$$h_i \sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} (1 - \prod_{l=c}^{d_i-1} (1 - 1_{W(l)+1>W})) + H(\tau+1, \widehat{S}', \widehat{W}')]), \forall \widehat{S}, \widehat{W} \qquad (5.2)$$

If $\tau = k$,

$$g(W) + H(\tau, \widehat{S}, \widehat{W}) = \sum_{i \in I: d_i \geq \lceil \tau/k \rceil} P_{i\lceil \tau/k \rceil} \left(\max_{\substack{X_{ic} \\ s.t.(3.9)-(3.13),(4.7)}} \left[\sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} r_i - \right. \right.$$

$$h_i \sum_{c=\lceil \tau/k \rceil}^{d_i} X_{ic} (1 - \prod_{l=c}^{d_i-1} (1 - 1_{W(l)+1>W})) + H(((\tau+1) \bmod k), \widehat{S}'', \widehat{W}'')]), \forall \widehat{S}, \widehat{W} \quad (5.3)$$

where $\widehat{S}''$ and $\widehat{W}''$ are defined by (5.4) and (5.5), respectively.

$$S''(c) = \begin{cases} S'(c+1) & \forall c \in \{1, ..., N-1\} \\ k & c = N. \end{cases} \qquad (5.4)$$

$$W''(c) = \begin{cases} W'(c+1) & \forall c \in \{1, ..., N-1\} \\ 0 & c = N. \end{cases} \qquad (5.5)$$

2. Properties of the First Party Warehouse Design Problem

To establish the properties of the first party warehouse design objective function (5.1), we will need Theorem 7 and Corollary 1.

THEOREM 7 $g(W)$ *has decreasing marginal gains in* W.

Proof: Assume that $g(W)$ does not have decreasing marginal gains in W. This implies that there exists some $W = w$, such that expression (5.6) holds.

$$g(w+1) - g(w) > g(w) - g(w-1) \qquad (5.6)$$

After re-arrangement of the terms in (5.6), we get expression (5.7).

$$g(w) < g(w+1) - (g(w) - g(w-1)) \tag{5.7}$$

Now consider an order acceptance and scheduling policy that makes the acceptance and scheduling decisions by treating $W = w+1$ in (5.2) and (5.3), but with the difference that instead of storing the order that raises its number of items stored from $w-1$ to w (that is, consumes the wth unit of warehousing capacity) in the *first party* warehouse, stores it at the third party warehouse while in the state transitions shows a unit of *first party* storage capacity consumed. In this manner, only a maximum of w orders will be stored in the *first party* warehouse in any given period, which ensures that the *first party* capacity is not violated. The average expected profit per time slot generated by this policy, $g'(w)$ can now be expressed by (5.8).

$$g'(w) = g(w+1) - (g(w) - g(w-1)) \tag{5.8}$$

If we compare expressions (5.7) and (5.8), it indicates that $g'(w) > g(w)$, which is a contradiction since $g(w)$ is the optimum average reward per time slot, thus our earlier assumption was incorrect. Therefore, $g(w)$ has decreasing marginal gains in w. ∎

COROLLARY 1 *The expression $g(W) - C_W W$ has decreasing marginal gains in W.*

Proof: Consider any $W = w$. Then,

$g(w+1) - C_W(w+1) - (g(w) - C_W w)$

$= g(w+1) - g(w) - C_W w$

$< g(w) - g(w-1) - C_W w + C_W(w-1)$

(from Theorem 7)

$$= g(w) - C_W w - (g(w-1) - C_{W(w-1)}) \ \blacksquare$$

THEOREM 8 *The smallest W that satisfies expression (5.9) is optimal.*

$$(g(W+1) - C_W(W+1)) - (g(W) - C_W W) \le 0 \qquad (5.9)$$

Proof: Let $W = w$ be the smallest quantity that satisfies expression (5.9). Consider any $w' > w$. Corollary 1 shows that $(g(W) - C_W W)$ has decreasing marginal gains in W. Therefore, by recursive induction, we get expression (5.10).

$$(g(w') - C_W w') - (g(w) - C_W w) \le 0 \qquad (5.10)$$

This shows that none of the w's greater than w are better.

Now consider any $w' < w$. Since w is the smallest warehousing capacity at which expression (5.9) holds, therefore we can write expression (5.11).

$$(g(w) - C_W w) - (g(w+1) - C_W(w+1)) > 0 \qquad (5.11)$$

Therefore for any $w' < w$, due to induction expression (5.12) holds.

$$(g(w) - C_W w) - (g(w') - C_W w') > 0 \qquad (5.12)$$

This shows that none of the w's smaller than w are better. $\blacksquare$

C. Solution Approaches

According to Theorem 8, it is possible to carry out a linear search to find the optimal warehouse capacity. Such search would require computation of $g(W)$ at different values of W. Since $g(W)$ does not have a closed form, in order to compute the exact values of $g(W)$, we need to solve SDP (5.2)-(5.3). As the state space in (5.2) - (5.3)

becomes unmanageable even for small size problems, we explore a heuristic scheme for estimating the values of $g(W)$. This heuristic scheme is based on estimating the net expected profit per period by developing an efficient threshold policy for accepting and scheduling customer orders. Since this heuristic scheme computes the threshold policy by using stochastic approximation scheme, we refer to it as HSSA-III. The following subsection describes HSSA-III.

1. Heuristic Scheme based on Stochastic Approximation-III

In HSSA-III, we use the notation described in Section D.3 in Chapter III.

ALGORITHM 3 HSSA-III proceeds as follows.

1. *Compute an upper bound on the first party warehousing capacity as $\frac{k}{\min_i Q_i} B$, where B is the length (in number of periods) of the booking horizon.*

2. *Do a Fibonacci search for the optimum warehousing capacity level between 0 and the upper bound computed in the previous step. In each iteration of the search, evaluate a warehouse capacity, as follows.*

   (a) *Initialize the threshold policy. Set $m = 0$.*

   (b) *Simulate $(m+1)^{st}$ scenario of future order arrivals in l successive periods.*

   (c) *Using the most recent threshold policy, approximate the steepest descent gradient by evaluating the expression (5.13), which represents the finite difference approach for computing approximate gradients [65].*

$$\nabla_{i\lceil\frac{\tau}{k}\rceil p}^{m+1} = \frac{\begin{aligned} & F(\omega^{m+1}, \{Z^m(i, \lceil\tfrac{\tau}{k}\rceil, p) + \Delta\} \cup \widehat{Z}^m \setminus (i, \lceil\tfrac{\tau}{k}\rceil, p)) - \\ & F(\omega^{m+1}, \{Z^m(i, \lceil\tfrac{\tau}{k}\rceil, p)\} \cup \widehat{Z}^m \setminus (i, \lceil\tfrac{\tau}{k}\rceil, p)) \end{aligned}}{\Delta}, \forall i, \tau, p$$

$$(5.13)$$

(d) *Update the threshold policy as shown in (5.14).*

$$Z^{m+1}(i, \lceil \tfrac{\tau}{k} \rceil, p) = Z^m(i, \lceil \tfrac{\tau}{k} \rceil, p) + \nabla^{m+1}_{i \lceil \tfrac{\tau}{k} \rceil p} \eta, \forall i, \tau, p \qquad (5.14)$$

(e) *If $m = m_{max}$ goto Step (f). Else goto Step (b).*

(f) *Approximate:*

$$g(W) \approx \frac{F(\omega^{m+1}, \widehat{Z}^{m_{max}}, p)}{kl} \qquad (5.15)$$

D.   Computational Results

The objective of the computational experiments in this section is to study the impact of first party warehousing capacity on the efficiency of the RM model. In addition, this section studies the computational performance of HSSA-III.

1.   Impact of Efficient First Party Warehouse Capacity Planning on the RM model

In Table XVI we compare the performance of the RM models used for accepting and scheduling customer orders between first party warehousing at efficient capacity levels determined by HSSA-III and third party warehousing for storing orders completed in advance of their due-dates. When the booking horizon is small (2 periods), the difference in net profits between the two RM models is small, with a maximum difference of 1%. However, for longer booking horizons (5 periods), the difference in the net profits is larger (between 7.5% and 8.6%).

Based on these results, we gain the insight that if booking horizon is large, then an efficiently sized *first party* warehousing option can have a considerable impact on the net profits in comparison to third party warehousing option.

Table XVI. Warehouse Design Results

Booking Horizon	$|I|$	Avg Arr. Rate Dist. (Num per pd)	Expected Profits TPW*	EFPW**	Difference (%)
2	4	UNIF(3,9)	74849	75124	0.4
2	8	UNIF(1.5,4.5)	63821	64066	0.4
2	12	UNIF(1,3)	59120	59736	1.0
2	16	UNIF(1,3)	65994	66588	0.9
3	6	UNIF(2,6)	82956	85964	3.6
3	12	UNIF(1,3)	78886	81710	3.6
3	18	UNIF(0.67,2)	68113	70936	4.1
3	24	UNIF(0.5,1.5)	58146	60692	4.4
5	10	UNIF(0,5)	78485	85226	8.6
5	10	UNIF(1,4)	73230	78751	7.5

*TPW: Third Party Warehousing

**EFPW: Efficient First Party Warehousing

2. Performance of HSSA-III Approach

The computational performance of HSSA-III is presented in Table XVII. It can be observed that even for long (in terms of number of periods) booking horizon problems and large set of order classes, the HSSA-III approach is able to compute solutions in manageable amounts of time.

E. Summary and Conclusions

In this chapter we develop a new approach for designing *first party* warehousing facilities at MTO systems. The purpose of this warehouse is to store the completed orders

Table XVII. Computational Performance HSSA-III

Booking Horizon	$|I|$	Arr. Rate Dist. (Num per pd)	W*	Comp. Time (sec)
2	4	UNIF(3,9)	9	118
2	8	UNIF(1.5,4.5)	12	259
2	12	UNIF(1,3)	11	384
2	16	UNIF(1,3)	11	657
3	6	UNIF(2,6)	29	298
3	12	UNIF(1,3)	19	703
3	18	UNIF(0.67,2)	20	1462
3	24	UNIF(0.5,1.5)	14	1348
5	10	UNIF(0,5)	42	1580
5	10	UNIF(1,4)	50	1445

until their due-dates. The objective of the design problem is to achieve maximum benefits with the RM model used for accepting and scheduling customer orders. Because of the selective order acceptance and scheduling policies, it is necessary to capture the resulting effect on the warehousing demand in the design problem. We model the problem as an infinite horizon problem with homogenous demand and develop an average reward SDP representation of the problem. We identify the structural properties of the problem and use the insights gained to develop an efficient heuristic for solving large size problems.

CHAPTER VI

CONTRIBUTIONS AND CONCLUSIONS

Manufacturing organizations have exploited technology and automation, quality improvement, customer relations and operational efficiencies in their attempts to increase their bottom lines. Although there can still be potential areas for improvement with these approaches, many of the gains have already been achieved, and organizations are looking for new ways to improve profitability. One area that presents tremendous opportunities to the manufacturing sector and presents many unexplored areas is the application of RM in operational and tactical level planning in manufacturing and supply chain systems. RM has had a tremendous impact in the service industry, and there is an increasing interest in studying and understanding new areas of RM application.

This dissertation demonstrates the significance of applying RM based techniques for accepting and scheduling customer orders in improving net profits at *on-date* delivery MTO manufacturing systems over popular practices. This could have big implications for the MTO industry, since this industry is facing an increasing global competition, which is leading to shrinking profit margins.

In this dissertation, we develop RM models for determining efficient order acceptance and scheduling policies. The key insights gained from our RM models can be summarized as follows:

- There is a potential for large improvements in net profits with applying RM on-date delivery MTO systems.

- Both efficient order acceptance and efficient order scheduling are crucial components in these RM models.

- The efficiency of these RM models improves with loading factor.

- When first party warehousing capabilities are not present, but third party warehousing costs are low, then the warehousing aspects can be ignored in RM model with little effect on profits.

- When first party warehousing capabilities are present, warehousing aspects can be ignored in RM model with little effect on profits.

- In addition to the operational benefits of employing *first party* warehouses for storing completed orders, a properly sized *first party* warehouse can give considerably superior economic performance in comparison to employing third party warehousing.

This dissertation opens an important new research area in applying RM in accepting and scheduling customer orders at *on-date* delivery MTO systems. While it provides useful results and insights, it also provides a foundation for further study and highlights several avenues for future research, as shown below:

- *Alternative solution approaches*: Solution approaches based on stochastic programming and DP approximations can be explored for alternative solution approaches for order acceptance and scheduling at *on-date* delivery MTO systems.

- *Multiple machines models*: When an MTO manufacturer offers a wide range of processing capabilities, the orders received can have significantly different processing requirements. Therefore, instead of a unique bottleneck machine, there could be a floating bottleneck, depending upon the jobs accepted and the scheduling policy employed. Since different jobs could have different processing sequences, even the stand along job-scheduling problem is challenging and NP-

hard. Therefore, a rigorous treatment will be required to develop a RM based approach for selective order acceptance and scheduling.

- *MTO-RM with multiple facilities*: The current model can be extended to consider a network of facilities, such that any one of the facilities can be used to satisfy the customer orders. A RM based approach can be developed that not only considers the manufacturing capacity and holding costs but also the transportation costs in order acceptance and scheduling.

- *Due-date and price setting*: The order acceptance problem presented in the current research can be extended to a due-date setting or a pricing problem, where at the time of accepting the order the MTO manufacturer determines and negotiates efficient due-dates and prices for different customer orders.

- *Overbooking system*: Analogous to airline industry, there is a significant potential with an overbooking system, that accepts high revenue earning orders even when sufficient production capacity is unavailable. Such a system would either re-arrange the production schedule or reject a previously accepted low revenue earning order with a penalty cost in favor of the new high revenue earning order.

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