TAXES, USER CHARGES AND THE PUBLIC FINANCE OF COLLEGE EDUCATION

A Dissertation

by

DOKOAN KIM

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2003

Major Subject: Economics

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Approved as to style and content by:

Timothy J. Gronberg
(Chair of Committee)

Hae-Shin Hwang
(Member)

Wayne Strayer
(Member)

Arnold Vedlitz
(Member)

Leonardo Auernheimer
(Head of Department)

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ABSTRACT

Taxes, User Charges and the Public Finance of College Education.

(August 2003)

Dokoan Kim, B.A., Busan National University;

M.A., George Washington University

Chair of Advisory Committee: Dr. Timothy J. Gronberg

This paper presents a theoretical analysis of the relative use of general state subsidies (tax finance) and tuition (user charge finance) in the state financing of higher education. State universities across U.S. states are very different among themselves especially in terms of user charges, public finances, and qualities.

In this study, we consider only the State Regime in which the state government decides the user charge, head tax, and expenditure, taking the minimum ability of students as given and the state university simply is treated as a part of government. The households who have a child decide to enroll their children at the university, taking head tax, tuition, and quality of university as given.

The two first-order conditions of the state government's optimization show the redistribution condition and provision condition. For a given marginal household, we show that under certain conditions, we have an interior solution of both head tax and expenditure. In the household equilibrium, the marginal household is determined at the

point where their perceived quality of university is equal to the actual quality of university.

We solve the overall equilibrium, in which the given ability of a marginal household for the state government is the same as the ability of the marginal household from the households' equilibrium. Since it is impossible to derive explicit derivation of comparative statics, we compute the effects of income, wage differential between college graduates and high school graduates, distribution of student ability on head tax, expenditure, tuition, tuition/subsidy ratio, and quality of university.

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CHAPTER I

INTRODUCTION

I.1 Introduction

About three quarters of college students in the United States are enrolled in state higher education institutions. Funding these institutions is a perennial issue for both college-attending households and general taxpayers in the state.

State universities across the United States are highly differentiated especially in terms of user charges, public finances, and qualities. For instance, in 1996, when we compare each flagship university across states, the ratio of tuition to the cost of education varied significantly across states. The highest ratio, 71 percent, comes from state of Vermont, while the lowest ratio, 20 percent, is from the state of Florida.[1] We try to explain why there exist these cross-sectional differences among state universities across states.

Public universities are much more constrained in tuition and admission policy than are private universities. The legal authority to set tuition for public universities and colleges varies by state. Even though there are several different organizations that have authority to set tuition for public four-year institutions, we can divide these groups into two regime types: State Regime and Campus Regime.[2] Regardless of

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This dissertation follows the style and format of the *American Economic Review*.

[1] We view the in-state tuition as a user charge, and state appropriation per student as a subsidy. The ratio of user charge to the cost of education is in-state tuition divided by the sum of in-state tuition and state appropriation per student.

[2] According to Christal (1997), there are different board systems across states such as Legislature,

regime, the state government decides a state appropriation to support higher education. In the State Regime, the state government also chooses the tuition, while the university decides the tuition in the Campus Regime. For example, we claim that Colorado, Florida, Indiana, Oklahoma, South Dakota, Washington, California, New York, North Carolina, and Texas belong to the State Regime. [3] To deal with two regimes, it is easier to start with the State Regime so that we analyze the mix of tuition and tax funding under the institutional arrangement in which the state government chooses both tuition and head taxes.

We consider both tax finance and user charge finance in the model. Every household is to pay a common lump sum tax, while those households who send their children to the state university pay a user charge. The students enrolled at the university enjoy the quality of university, though the benefit of schooling differs as a function of the ability of the student. Quality of university in the model is determined by the average student quality and per student expenditure. According to Cornes and Sandler (1996), a club is defined as a voluntary organization in which the members share some of benefits, such as production costs, characteristics of members, and excludable benefits. Therefore, a club good is what the club members share exclusively. In the public higher education, a club is a public university. The public university produces the quality of the university, which gives the benefit, i.e. higher future income to those enrolled students. Note that only those who pay the tuition can share this quality of university. Therefore, the university quality is a club good.

State Coordinating/Governing Agency, System Governing Board, and Institutional/ Local Board.

[3] In six states, the state legislators have constitutional or statutory authority to set tuition. (Colorado, Florida, Indiana, Oklahoma, South Dakota, Washington). By practice, the legislators in four additional states set tuition. (California, New York, North Carolina, Texas)

In the model, the state government is assumed to choose the user charge, head tax, and expenditure, taking the minimum ability of students as given. The solution requires satisfying a redistribution condition and a provision condition. The redistribution condition shows how to redistribute income among the types of households. The provision condition identifies the tradeoff the state government faces when choosing how much to spend on university quality. This allocation problem involves a modified Samuelson condition. The state government problem is now to combine the two conditions. For a given marginal household, we show that under certain conditions, we have an interior solution of both head tax and expenditure.

The households who have a child decide whether or not to enroll their child. In the household equilibrium, their perceived quality of university is equal to the actual quality of university.

We solve for the overall equilibrium, in which the given ability of a marginal household for the state government is same as the ability of the marginal household from the household equilibrium. We do the comparative statics such as the effect of a change in political weight, and in income. Since it is impossible to do more comparative statics, we use a simulation method to derive several numerical comparative statics result. Using a uniform distribution of students' abilities, we investigate the effect of a change in income, the effect of a change in political weight and the effect of a change in college wage differential. Furthermore, we investigate a change in distribution function from uniform distribution to beta distribution.

I.2 Motivation

It is obvious that education is not a pure public good, because it costs almost nothing to exclude the students from schooling. Since the benefit, mostly higher wage rate, from higher education belongs primarily to those who are enrolled at the university, higher education can be perhaps best classified as a private good. Since we are concerned with the public universities, higher education is either a publicly provided private good or a publicly financed private good. In case of the publicly provided private good, there is no user charge, but exclusive tax finance. In case of the publicly financed private good, there is a mix of both user charges and tax finance.

Tax revenues have supported public higher education around the world. For U.S. public institutions, state and local government appropriation has been one of the main revenue sources, while tuition has been relatively less important.

In order to establish some broad facts about state differences in the relative share of tuition to tax finance, we check the data for state universities. Using Integrated Postsecondary Education Data System (IPEDS) for the past 26 years (1981-1996), we take a look at between-state differences and within-state differences in tuition, subsidy, and tuition/subsidy ratio.[4] In Table I, we report the tuition/subsidy ratio over the period. The tuition is in-state tuition or resident tuition. Since IPEDS provides both the list tuition, and tuition revenue, at first, we calculate total tuition and fee revenue divided by the number of the full-time equivalent students as tuition.

[4] We try to include as many state universities as possible for the 26 year panel. We have 422 universities. There are 291 teaching-oriented universities and 131 research-oriented universities in the data.

Table I. Summary of Tuition/Subsidy Ratio over 26 years

	Year	81	83	85	86	88	89	90	91	92	93	94	95	96
All Types	Gini Index(x100)	31.82	31.85	32.14	31.87	31.42	30.56	29.75	30.20	29.54	28.65	28.70	28.23	27.82
	Theil Index(x1000)	185.17	174.43	178.78	175.92	175.57	157.76	164.88	155.26	150.81	158.08	147.02	142.40	136.20
	p90/p10	4.49	4.38	4.33	4.48	4.37	4.61	3.30	4.23	3.83	3.07	3.66	3.42	3.50
	p75/p25	2.03	2.17	2.06	2.18	2.02	2.07	1.95	2.06	1.96	1.86	1.94	1.97	1.92
Theil Index	*Within States(x1000)*	49.08	53.79	50.61	64.39	58.60	45.33	41.73	39.48	41.10	39.45	42.80	41.68	37.46
	Between States(x1000)	136.09	120.64	128.17	111.53	116.97	112.43	123.15	115.78	109.71	118.63	104.22	100.72	98.74
	Fraction of Between	73.49	69.16	71.69	63.40	66.62	71.27	74.69	74.57	72.75	75.04	70.89	70.73	72.50
	Mean	0.33	0.38	0.33	0.36	0.40	0.42	0.43	0.49	0.54	0.57	0.58	0.59	0.60
	Standard Deviation	0.21	0.25	0.22	0.24	0.27	0.26	0.26	0.30	0.33	0.36	0.37	0.36	0.35
Teaching-Oriented	Gini Index(x100)	31.19	31.33	31.55	31.20	30.54	29.08	28.11	28.26	27.66	27.56	27.03	26.48	26.19
	Theil Index(x1000)	166.08	172.34	175.51	172.95	172.30	147.37	140.92	141.05	137.53	139.86	136.99	131.37	124.47
	p90/p10	3.98	4.05	4.06	4.29	4.21	4.09	3.73	3.63	3.30	3.09	2.94	2.90	3.04
	p75/p25	2.14	2.14	2.12	2.12	2.02	2.02	2.00	1.83	1.77	1.79	1.79	1.76	1.83
Theil Index	*Within States(x1000)*	32.81	50.14	46.30	62.36	55.61	37.28	33.79	29.57	31.07	33.10	32.89	32.93	27.51
	Between States(x1000)	133.27	122.20	129.21	110.59	116.69	110.09	107.13	111.48	106.46	106.76	104.10	98.44	96.96
	Fraction of Between	80.24	70.91	73.62	63.94	67.72	74.70	76.02	79.04	77.41	76.33	75.99	74.93	77.90
	Mean	0.34	0.40	0.35	0.38	0.43	0.44	0.46	0.52	0.58	0.61	0.62	0.62	0.63
	Standard Deviation	0.21	0.27	0.24	0.25	0.29	0.26	0.27	0.30	0.34	0.37	0.38	0.37	0.36
Research – Oriented	Gini Index(x100)	32.58	32.31	32.78	32.37	32.27	32.86	32.45	32.70	31.85	31.27	31.08	30.81	30.19
	Theil Index(x1000)	175.14	169.76	178.47	173.99	170.90	175.02	170.91	177.10	168.41	161.93	159.03	158.12	154.94
	p90/p10	4.97	4.47	4.55	4.54	4.49	4.95	4.36	4.50	4.43	4.11	4.15	4.15	4.18
	p75/p25	2.16	2.33	2.22	2.26	2.26	2.30	2.32	2.17	2.26	2.28	2.24	2.19	2.05
Theil Index	*Within States(x1000)*	25.06	23.59	27.44	31.09	28.01	31.19	27.18	30.64	27.95	29.65	30.26	31.56	32.66
	Between States(x1000)	150.08	146.17	151.03	142.90	142.89	143.83	143.73	146.46	140.46	132.28	128.77	126.56	122.28
	Fraction of Between	85.69	86.10	84.62	82.13	83.61	82.18	84.10	82.70	83.40	81.69	80.97	80.04	79.22
	Mean	0.31	0.34	0.30	0.32	0.35	0.37	0.38	0.42	0.46	0.49	0.51	0.51	0.52
	Standard Deviation	0.20	0.21	0.19	0.21	0.22	0.24	0.24	0.28	0.29	0.31	0.31	0.32	0.33

However, there is no big difference between average tuition and the list tuition. Subsidy is calculated from the per student appropriation, which is total state and local government state appropriation divided by the number of the full-time equivalent students.

We classify two different types of universities: Teaching-Oriented Universities, and Research-Oriented Universities. The reason why we need the classification is that each state provides a different amount of state appropriation to the different types of universities. In terms of Carnegie Foundation Classification Codes, Teaching-Oriented Universities include Comprehensive Universities I, II, and Liberal Arts College I, II, and Research-Oriented Universities include Doctoral Universities I, II, and Research Universities I, II. According to the Carnegie classification, Comprehensive Universities proved a full range of bachelor degree programs and some graduate programs through the master's degrees. Comprehensive Universities I give at least 40 master's degrees in more than three majors every year, while Comprehensive Universities II offer at least 20 master's degrees in more than one major. Liberal Arts Colleges emphasize undergraduate education to give bachelor programs. Liberal Arts College I awards more than 40 percent bachelor degrees in liberal arts with more a relatively selective admission standard, while Liberal Arts College II provide less than 40 percent bachelor degrees in liberal arts with less selective admission policy. Both Doctoral Universities and Research Universities provide a full range of bachelor degree programs with graduate programs toward the doctor degrees. Research Universities emphasize much more research than Doctoral Universities. Depending on the number of doctoral degrees, the Carnegie classifies Doctoral Universities I and Doctoral Universities II. Doctoral

Universities I provides more than 40 doctoral degrees in more than five majors every year, while Doctoral Universities II provide more than 10 doctoral degrees in more than three majors, or more than 20 doctoral degrees in more than one major. Research Universities award more than 50 doctoral degrees every year. Research Universities I receive more than \$40 million research funds from the Federal Government, while Research Universities II receive more than \$15.5 million and less than \$40 million research funds from the Federal Government.

In order to characterize how the tuition/subsidy ratio distribution looks, we use some inequality measures, such as the Gini index, Theil Index, 75/25 percentile ratio, and 90/10 percentile ratio. Referring to Murray, Evans, and Schwab (1998), we know that the Gini index is the average difference in tuition/subsidy ratio between any pair of universities relative to the average tuition/subsidy ratio for all universities in the United States. The Gini index is more sensitive to change around the middle of distribution than to change from the highest to the lowest distribution of the ratio. Since the Gini index cannot be decomposed into between-state and within-state differences, we consider the Theil index. Let R be tuition/subsidy ratio. R_{ij} is the tuition/subsidy ratio of *j* university in state *i*. The Theil index is calculated by

$$T = \frac{1}{N}\sum_{i=1}^{48}\sum_{j=1}^{N_i}\frac{R_{ij}}{\overline{R}}\ln\left(\frac{R_{ij}}{\overline{R}}\right) \tag{1.1}$$

N is the number of total public universities in the U.S. N_i is the number of public universities in state *i*. $\overline{R}$ is the average of tuition/subsidy ratio in the United States. We do not give any weight to the tuition/subsidy ratio. The advantage of using the Theil index is that we can decompose the Theil index into between-state inequality and within-state inequality, as follows.

$$T = \sum_{i=1}^{48} \frac{N_i R_{ij}}{N \overline{R}} \ln\left(\frac{\overline{R_i}}{\overline{R}}\right) + \sum_{i=1}^{48} \frac{N_i \overline{R_i}}{N \overline{R}} \cdot T_i \qquad (1.2)$$

where $T_i = \frac{1}{N_i} \sum_{i=1}^{48} \sum_{j=1}^{N_i} \frac{R_{ij}}{\overline{R_i}} \ln\left(\frac{R_{ij}}{\overline{R_i}}\right)$ is the Theil index for state *i*, and $\overline{R_i}$ is the average tuition/subsidy ratio in state *i*. The first term of (1.2) is between-state inequality, and the second term is within-state inequality, a weighted average of the within-state Theil index.

The 90/10 percentile ratio and 75/25 percentile ratio also measure the inequality of tuition/subsidy ratio. These percentile ratios are not sensitive relatively to some extreme values of tuition/subsidy ratio unlike the Gini index and the Theil index.

From our data, we observe that between-state differences in tuition/subsidy ratio is much larger than the within-state difference in the data. Because the Theil index is decomposable, we calculate the ratio of between-state Theil index to within-state Theil index in Table I. Regardless of classification types of universities, we observe that this ratio is much bigger than 50 percent. After classifying the types of universities, this ratio is bigger in the research-oriented university than in the teaching-oriented university. While within-state differences in tuition/subsidy ratio have fluctuated, between-state differences in tuition/subsidy ratio have decreased over time. We also observe that the national difference in tuition/subsidy ratio has been decreasing by looking at either the Gini index, Theil index, and percentile ratios. The between-state differences in tuition/subsidy ratio are larger than the within-state differences in tuition/subsidy ratio over this period.

Table II. Summary of Tuition over 26 years

	Year	81	83	85	86	88	89	90	91	92	93	94	95	96
All Types	Gini Index(x100)	24.11	24.68	22.68	20.44	21.92	22.56	22.25	22.50	22.90	21.44	21.02	21.19	21.09
	Theil Index(x1000)	98.02	100.75	87.73	73.53	84.19	88.87	85.40	86.57	87.75	76.91	74.58	75.45	74.34
	p90/p10	3.07	3.20	2.87	2.38	2.56	2.63	2.55	2.56	2.64	2.49	2.51	2.47	2.49
	p75/p25	1.75	1.80	1.63	1.54	1.55	1.64	1.67	1.67	1.75	1.68	1.62	1.63	1.66
Theil Index	*Within States(x1000)*	58.75	59.19	52.91	39.98	43.23	49.80	46.60	48.23	49.55	43.32	41.79	41.24	39.56
	Between States(x1000)	59.94	58.75	60.31	54.37	51.35	56.04	54.57	55.71	56.47	56.33	56.03	54.66	53.21
	Fraction of Between	61.15	58.31	68.75	73.95	60.99	63.05	63.90	64.36	64.35	73.24	75.13	72.44	71.58
	Mean	941	1196	1445	1573	1780	1918	2077	2254	2574	2914	3126	3288	3518
	Standard Deviation	435	567	637	649	791	881	937	1019	1163	1226	1299	1370	1451
Teaching Univ.	Gini Index(x100)	22.64	21.77	19.35	16.98	17.81	18.63	18.07	18.39	19.24	18.07	17.18	17.11	17.50
	Theil Index(x1000)	83.66	76.06	62.46	48.46	54.33	61.95	54.38	56.26	60.78	52.19	47.41	46.56	48.57
	p90/p10	2.86	2.90	2.51	2.21	2.16	2.22	2.15	2.25	2.35	2.19	2.12	2.19	2.14
	p75/p25	1.74	1.66	1.52	1.43	1.43	1.49	1.49	1.50	1.60	1.59	1.51	1.54	1.58
Theil Index	*Within States(x1000)*	16.28	14.02	14.79	13.16	17.58	16.82	14.15	13.51	12.69	12.60	9.79	9.90	10.78
	Between States(x1000)	67.38	62.04	47.67	35.30	36.75	45.13	40.23	42.75	48.09	39.59	37.62	36.66	37.79
	Fraction of Between	80.54	81.57	76.32	72.84	67.64	72.85	73.98	75.99	79.12	75.86	79.35	78.74	77.81
	Mean	831	1035	1268	1379	1549	1669	1801	1945	2228	2550	2732	2863	3072
	Standard Deviation	339	411	449	436	529	632	622	683	817	851	866	895	980
Research Univ.	Gini Index(x100)	22.02	23.20	22.46	20.04	22.18	22.34	22.29	22.04	21.98	20.93	21.04	21.35	20.58
	Theil Index(x1000)	82.85	89.47	85.79	70.42	84.22	84.48	84.87	82.03	80.16	72.79	73.22	74.42	70.53
	p90/p10	2.49	2.66	2.63	2.28	2.42	2.41	2.50	2.57	2.76	2.56	2.63	2.48	2.48
	p75/p25	1.65	1.73	1.64	1.51	1.69	1.67	1.66	1.66	1.65	1.68	1.73	1.69	1.61
Theil Index	*Within States(x1000)*	18.78	19.15	16.05	16.78	20.94	19.65	19.69	17.26	16.03	14.62	14.84	15.66	16.05
	Between States(x1000)	64.07	70.32	69.74	53.64	63.28	64.83	65.18	64.77	64.13	58.17	58.38	58.76	54.48
	Fraction of Between	77.33	78.60	81.29	76.17	75.14	76.74	76.80	78.96	80.00	79.91	79.73	78.96	77.24
	Mean	1186	1554	1839	2009	2293	2471	2690	2939	3343	3721	4002	4233	4507
	Standard Deviation	517	691	799	816	1008	1085	1195	1279	1425	1516	1634	1729	1802

Table III. Summary of Subsidy over 26 years

	Year	81	83	85	86	88	89	90	91	92	93	94	95	96
All Types	Gini Index(x100)	22.83	22.85	23.75	23.80	24.67	24.74	23.44	23.77	23.38	22.80	22.52	22.44	21.72
	Theil Index(x1000)	88.33	88.08	96.76	96.37	100.92	101.83	89.52	92.18	89.01	84.84	83.51	83.65	77.29
	p90/p10	2.59	2.54	2.68	2.69	2.89	2.87	2.84	2.93	2.83	2.72	2.66	2.62	2.65
	p75/p25	1.69	1.69	1.76	1.76	1.86	1.83	1.79	1.78	1.76	1.70	1.69	1.69	1.62
Theil Index	*Within States(x1000)*	48.94	50.81	55.47	58.18	54.74	55.00	53.16	55.70	54.68	56.04	56.61	56.11	52.68
	Between States(x1000)	39.39	37.27	41.29	38.19	46.18	46.83	36.36	36.48	34.33	28.80	26.90	27.54	24.61
	Fraction of Between	44.59	42.31	42.67	39.63	45.76	45.99	40.62	39.57	38.57	33.95	32.21	32.92	31.84
	Mean	3106	3448	4225	4448	4691	4837	4911	4924	4935	4966	5127	5392	5532
	Standard Deviation	1409	1545	2026	2123	2267	2355	2199	2242	2197	2152	2212	2340	2286
Teaching Univ.	Gini Index(x100)	22.64	21.77	19.35	16.98	17.81	18.63	18.07	18.39	19.24	18.07	17.18	17.11	17.50
	Theil Index(x1000)	83.66	76.06	62.46	48.46	54.33	61.95	54.38	56.26	60.78	52.19	47.41	46.56	48.57
	p90/p10	2.86	2.90	2.51	2.21	2.16	2.22	2.15	2.25	2.35	2.19	2.12	2.19	2.14
	p75/p25	1.74	1.66	1.52	1.43	1.43	1.49	1.49	1.50	1.60	1.59	1.51	1.54	1.58
Theil Index	*Within States(x1000)*	16.28	14.02	14.79	13.16	17.58	16.82	14.15	13.51	12.69	12.60	9.79	9.90	10.78
	Between States(x1000)	67.38	62.04	47.67	35.30	36.75	45.13	40.23	42.75	48.09	39.59	37.62	36.66	37.79
	Fraction of Between	80.54	81.57	76.32	72.84	67.64	72.85	73.98	75.99	79.12	75.86	79.35	78.74	77.81
	Mean	2731	3023	3701	3851	4067	4179	4203	4172	4200	4215	4373	4622	4755
	Standard Deviation	994	1112	1535	1470	1648	1660	1500	1484	1476	1408	1459	1530	1509
Research Univ.	Gini Index(x100)	23.37	22.69	23.28	23.36	23.58	24.02	22.12	22.31	21.74	21.27	21.40	22.01	20.92
	Theil Index(x1000)	90.43	84.74	90.64	92.77	92.25	95.67	77.92	78.63	74.79	70.67	72.04	76.81	68.96
	p90/p10	2.78	2.73	2.91	2.98	2.96	2.99	2.81	3.08	2.91	2.94	2.90	2.86	2.78
	p75/p25	1.74	1.63	1.66	1.66	1.71	1.87	1.73	1.80	1.73	1.75	1.63	1.69	1.61
Theil Index	*Within States(x1000)*	33.50	32.51	36.04	39.01	33.75	35.36	31.03	32.36	29.67	29.53	29.51	30.92	28.25
	Between States(x1000)	56.93	52.23	54.60	53.76	58.50	60.31	46.89	46.27	45.12	41.14	42.53	45.89	40.71
	Fraction of Between	62.95	61.64	60.24	57.95	63.41	63.04	60.18	58.85	60.33	58.21	59.04	59.74	59.03
	Mean	3940	4392	5391	5774	6077	6301	6483	6596	6569	6635	6802	7102	7260
	Standard Deviation	1791	1915	2460	2685	2788	2948	2651	2699	2622	2551	2646	2870	2733

In Table II, we show the pattern of tuition. Like the tuition/subsidy ratio, between-state difference in tuition is larger than the within-state difference. Note that tuition differences across states are more prominent in those teaching-oriented universities than the research-oriented universities.

In Table III, we show the pattern of state appropriation. Without classifying two different types of universities, within-state differences have dominated between-state differences in state appropriation. However, when we separate the types of universities, we still observe that between-state differences in state appropriation have dominated than within-state differences.

Historically, Goldin and Katz (1998) found that from 1902 to 1940, state and local support for public higher education was quite different across states. They found that these big differences came from the level and distribution of income in a state. We will develop a model to help interpret these sources of differences in tuition/subsidy ratio across states.

I.3 Literature Review

If we classify higher education as a private good, we deal with either a publicly provided private good or a publicly financed private good. In case of a publicly provided private good, there is no user charge but only tax finance. In the literature about public provision of private goods, Besley and Coate (1991) found that the public provision of private goods can redistribute income from the rich households to the poor households, because the rich households will not buy the publicly provided private good, which is of low quality, because quality is assumed to be a normal good. Epple and Romano (1996a), and Epple and Romano (1996b) studied public provision of private goods when the good is supplemented by a privately purchased good, and when a private alternative exists, respectively. Epple and Romano (1996a) found that when the good is supplemented in a private market, a majority voting equilibrium always exists because of single-peaked preferences over public expenditure. Furthermore, they also found that the majority prefers the dual-provision regime to both a market-only and government-only regime. Both Epple and Romano (1996a), and Epple and Romano (1996b) characterize the voting equilibrium in which both the rich households and the poor households oppose the middle-income households who favor an increase in public expenditure or public alternative. Bös (1980) analyzes the exclusive choice between user charges and taxes for publicly provided private goods. In his model, the median voter faces an either/ or choice between the two forms of financing the private goods. The trade-off between taxes and user charges is essentially a trade-off between efficiency and equity. With user charges, the median voter knows that efficiency of the economy is achieved, but that equity is not promoted. In the case of exclusive tax financing, a progressive income tax will lead to a deviation from allocative efficiency because of the welfare cost which arises due to an income tax, but more equity is achieved. Depending on the extent of preferences for redistribution, the median voter chooses either one of the forms to finance the goods.

Several papers view higher education as an exclusive public good, because it costs almost nothing to exclude some students and in our model. The quality of the university is regarded as a congestible public good. In the literature about the exclusive public good, Brito and Oakland (1980) study private provision of exclusive public good under the monopoly market, so that there is a user charge, but no tax in the model. Burns and Walsh (1981) use the demand distribution to provide different pricing strategies than the uniform price. Instead of a profit-maximizing firm, Fraser (1996) assumes that the government maximizes utilitarian social welfare by choosing the level of user charge. Fraser (1996) compares overall welfare of user charge with welfare of tax. The dispersion of income and the degree of inequality aversion determine which financing method is better. Swope and Janeba (2001) explain how society decides the provision of excludable public goods and financing methods. They separate two regimes, in which the median household preference determines the level of provision in a tax regime and a household who has higher preference than the median household determines the user charge in a user charge regime. Like Fraser (1996), they compare the welfare levels of two exclusive financing methods.

Using club theory, Glazer and Niskanen (1997) examine why the public provision of the exclusive public good is of lower quality. Since the rich households are more concerned about the quality of good than the poor households, the rich households will avoid using that good because of an increase in congestion. Therefore, by excluding the rich, the poor households can have benefit due to the decrease in congestion.

Even though both methods of financing higher education are employed simultaneously in all states, most research on financing higher education has assumed either tax finance or user charge finance, but has not considered the choice among mixed financing combinations. In the literature about exclusive tax finance analysis for education, most of the models explain why the economy supported higher education through tax. Johnson (1984) justified tax finance for college education by production externalities, by which relatively low ability people benefit from raising the average human capital of the others. Therefore, there is a possible complementarity relationship between the low ability workers and the high skilled workers. In his model, the expenditure per capita is fixed, and the government decides the subsidy rate. Creedy and Francois (1990) also assumed production externalities for the justification of tax finance, in which those who do not enroll themselves at the universities benefit from the rate of growth of the economy. Unlike Johnson (1984), they assumed that education requires an opportunity cost, forgone earnings, and that the household is different in income, not in ability. The government decides the subsidy rate to maximize the net lifetime income of the median voter in order to obtain majority support. Fernandez and Rogerson (1995) did not assume any externality from education, but assumed an imperfect capital market. They emphasized the subsidy in the role of redistributing income. Because of credit constraints, poor families can be excluded from receiving the education so that they efficiently subsidize the education of rich families. The tax rate is determined by majority vote. In our model, we have a certain feature as described by the above articles. Specifically, holding educational expenditure constant, we assume that the state government chooses head tax, and tuition.

In the literature about exclusive user charge finance analysis, most of the models adopt a university decision-making perspective. They do not differentiate between the state university and private university. Ehrenberg and Sherman (1984) assumed that the university chooses the number of students in different categories and financial aid policies to maximize its utility from diversifying the student groups subject to revenue constraint, given that the (marginal) cost of education is fixed. Similarly, Danziger (1990) modeled the university as deciding the minimum ability of students (admission standard) and tuition to maximize its utility which comes from the student's ability and from tuition level. Rothschild and White (1995) developed a model in which the students are treated as both demanders and inputs. In the competitive market, tuition internalizes the external effect of students on each other, because the higher ability students give an externality to the other students and, hence, can receive scholarships. Using the profit-maximization objective function like Rothschild and White (1995), Epple and Romano (1998) assumed that the students are different in both abilities and income, and that the school quality is determined by the peer group effect, as measured by average ability of enrolled students. There proposes tuition discrimination across students, because of the differentiated contribution of student types to the school quality. Epple, Romano, and Sieg (2001) took a different objective function of university, maximization of school quality. The quality of school depends on both peer quality (student input) and instructional expenditure. The pricing is not different from Epple and Romano (1998). Rey (2001) considered the state university competition to explain why we do have so many different types of state universities. He assumed that there is no tuition and that higher education is solely financed by tax. The funds for universities are supported by the government through both a fixed amount and a per student amount. One of the main differences in previously described models is that the university does include research in the objective function in order to explain the different types of public universities.

Garratt and Marshall (1994) and De Fraja (1999) are among the few papers which allow for both financing methods. Garratt and Marshall (1994) provide a novel explanation for the public financing of higher education by introducing a contract theory of educational finance. The reason why tax finance has spread across states is that every taxpayer agrees to have an implicit lottery over access to higher education. The lottery winners obtain a college education by paying a user charge, while both winners and losers pay a tax to support the publicly provided higher education services. In their model, a lump-sum tax serves as an instrument for common public financing from all taxpayers. The rest of the cost of education is financed by the college lottery winners who pay tuition. The optimal mix depends on the median income level and the cost of education. Though Garratt and Marshall (1994) discuss the optimum quality of university, they do not include student input in the quality of university.

De Fraja (1999) explicitly models a state government which maximizes the unweighted sum of individual household utilities. Without any intervention of government, high-income households are more willing to send their children to college than low-income households. Therefore, the market equilibrium is not equitable if we define equity as equality of opportunity for higher education regardless of income level. The government can pursue two goals of education policies; equality of opportunity and efficiency. Since ability of students is assumed to be unobservable to the state government, the government can only achieve the second best optimal solution by choosing income-based tuition levels, which are set by imposing a separate income tax and giving subsidies to low-income households. The result is that the government cross-subsidizes college education for high-ability and low-income households with higher tuition collected from relatively low-ability and high-income households. While De Fraja (1999) does not consider the quality of university and assumes that the educational expenditure is fixed.

We view the state government as a welfare maximizing government, following De Fraja (1999). Unlike De Fraja (1999), we assume a weighted sum of social welfare because we view that the state government maximizes political support from voters. This is similar to the approach in Peltzman (1971). In this article, Peltzman (1971) divides consumers into several groups and allows the manager of a public enterprise to charge different prices to different groups.

I.4 Overview

In Chapter II, we start to describe the model and households' equilibrium. Then, we explain how the state government chooses head tax, tuition, and expenditure given the marginal household. Since tuition is determined by the state budget constraint, the role of head tax resembles Fernandez and Rogerson (1995). Neither externality assumption nor credit constraint is assumed in our model, but we end up with an exclusive tax finance which is equivalent to the corner solutions. State government is assumed to have an authority to impose the head tax across any households. However, we have a publicly provided private good, which comes from quality of university. When only the first order condition for head tax is considered, the redistribution of income is made between those households who do not enroll their children at the university and those households who send their children to the university. Among the former group, they do not have any children. Unlike the models in which the supply of education is determined by demand, the number of students who are enrolled at the university is determined by both demand and supply of public higher education in our models.

In our model, we include the feature of quality of university which depends on both average student ability and educational expenditure as in Epple, Romano, and Sieg (2001). We do not allow for price discrimination, i.e. we have uniform tuition. We do not consider the objective function of the university, because we are dealing with State Regime in which state government decides most of important variables. Furthermore, our model does not include research, either from a revenue generating or an output dimension.

Even though contract theory of finance is a utilitarian model, our model assumes a non-symmetric weight among the households. Our model is distinguished by the endogenous quality of university, which depends on average student quality and per student expenditure.

We include how the quality of university is determined and the state government chooses the educational expenditure in our model. For simplicity, we assume that the households across types are the same in income, and differ in whether the households have a child or not, and those types of households who have a child are different in the ability of student.

The household decision with respect to college education is a discrete choice problem. The benefit from higher education is, however, assumed to be continuous and depends on both ability of student, and quality of university. This educational production function is similar to educational attainment which depends on both ability of student and peer group in Epple, and Romano (1998). Our model treats quality of university as a publicly provided private good so that those who are enrolled at the university share all benefits from the university. Like Epple, Romano, and Sieg (2002), quality of university is a function of student input (average student quality) and other resources.

We assume that the government forces the households to pay taxes, but there is no rational for this behavior. In general, there are three arguments for the reason why the public finances education; positive externalities, better access to distribution, and imperfect capital markets. Garratt and Marshall (1994) gave an additional reason for public taxation of higher education; gambles and insurance. We view the higher education as a publicly financed private good like Garratt and Marshall (1994), but following Brueckner and Lee (1989), we will interpret quality of university as a club good. Brueckner and Lee (1989) introduced school quality as a club good. In the educational production, implicitly, the lower ability type obtains a peer group effect, but the higher ability type does not receive any peer group effect. In public higher education, a club is a public university and a club developer is state government who can determine the fee (user charge), head tax, and the spending on education. Since head tax is not a club fee, but even non-member should pay it, we cannot explain why we have head tax in terms of a club good theory. Since a club good is an exclusive public good, quality of university is a club good. Only those who enroll their children at the university share this quality of university. Depending on what the ability of the student is, the benefit from a club good is different, because of the educational production function. Because the number of students enrolled is negatively related to average student quality, more students bring less benefit to those who stay in the university due to the lower quality of university. This is equivalent to the notion of congestion. In case of non-anonymous crowding, the crowding cost of each person depends on both the characteristics and number of other members in a club. Therefore, we may think that quality of university is involved in non-anonymous crowding.[5]

The first first-order condition shows how head tax is used as redistributive device in the economy. We view the second first-order condition as how the state government decides the provision level of public good, which is quality of university. The modified Samuelson condition is applied here. Considering both of first-order conditions, we prove that there will be an interior solution under the certain conditions. Then, we explain shortly what the overall equilibrium is. We provide some comparative statics analytically such as the effects of change in income and political weight.

In Chapter III, since we cannot go further to do the comparative statics with our analytical approach, we use some specific functions to examine the comparative statics and to calibrate some parameters to existing empirical evidence in U.S. public universities. Using an additively separable utility function, a Cobb-Douglas return function, and a Cobb-Douglas quality production, we solve the first-order conditions for the state government. Since it is not possible to find the explicit solution for head tax and expenditure, we try to find the expenditure level numerically. Then, substituting the expenditure in one of the first-order conditions, we solve for the head tax. Since we will have a set of combinations of head tax, tuition, and expenditure

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[5] Epple and Romano (1998) regard private schools as clubs with “non-anonymous crowding” due to the existence of peer group effects.

given marginal ability, we find the equilibrium level of marginal ability by checking whether the starting marginal ability is equal to the solved marginal ability. Using a uniform distribution of students' abilities, we investigate the effect of change in income and change in wage differential between college graduates and high school graduates. Change from a uniform distribution to a beta distribution is also added.

In Chapter IV, we summarize the results, some empirical implications, and future research.

CHAPTER II

THE MODEL

II.1 Description of the Model

There are two types of households in the state. N_0 number of Type 0 households have no children and N_1 number of Type 1 households have children who may or may not attend a university. Each household of Type 1 is assumed to have only one child. Let N_{10} and N_{11} denote the number of Type 1 households whose children do not attend and attend a university, respectively, and let $N=N_0+N_1$ be the total number of households.

All households have a common utility function $U(r,x)$, where x is a numeraire composite good and r is the return (human capital) to university education. The return to university education is the present value of future wage income after college graduation divided by the total number of years. The household with a child who has no college education is assumed to have a same annualized income, r_0 for simplicity. The value of educational return to the households without a university-attending child is normalized to zero. The utility function is assumed to be a differentiable and strictly concave increasing function. The return to education is also assumed to be concave in the quality of education (q) and the ability of the student (a),

$$r = r(q,a) \tag{2.1}$$

which is differentiable everywhere and increasing in both quality of education and the ability of student. The quality of education q depends on average level of enrolled students and the per student expenditure (e),

$$q = q\left(\bar{a}, e\right) \tag{2.2}$$

which is assumed to be differentiable and strictly increasing in its arguments.

Children are assumed to have heterogeneous abilities. The distribution of abilities of N_1 children is denoted by a distribution function $F(a)$. We assume that $F(a)$ is a differentiable continuous distribution function over a normalized unit interval [0,1] such that $F(0)=0$ and $F(1)=N_1$. The derivative of $F(a)$ is denoted by $f(a)$ which is nonnegative, $f(a)\geq 0$.

All households have an identical amount of income y and pay a head tax h. When a child of a Type 1 household is enrolled at a university, she has to pay a fixed amount of user charge (tuition) which is denoted by t. Type 1 household makes the enrollment decision by maximizing its utility. Thus, all Type 1 households choose to enroll their child if

$$U\left(r\left(q,a\right), y-h-t\right) \geq U\left(r_0, y-h\right) \tag{2.3}$$

where the left hand side is the utility when they send their child to university and the right hand side the utility when they do not.

The household with a child of ability a_m will be called the *marginal household*. The marginal household is indifferent between university education and no education. All Type 1 households with a child of ability higher than a_m will enroll their child at a university. The average ability of students in the quality function, is given by

$$\bar{a} = \frac{1}{N_{11}} \int_{a_m}^{1} a dF\left(a\right) \tag{2.4}$$

where $N_{11}=N_1-F(a_m)$. N_{11} is the total number of enrollment. It is easy to see that The average ability of students is a monotonically increasing function of a_m.

We develop a public choice interest group type model of state government decision-making. The state government maximizes the non-symmetric utilitarian social welfare function which is defined by the weighted sum of the welfare of all households. The aggregate welfare in each group is defined as the sum of individual household's utility in that group. Let AU_0, AU_{10}, and AU_{11}, respectively, denote the aggregate welfare of Type 0 households, Type 1 households without a university-attending child, and Type 1 households with a university-attending child. These are given by

$$\begin{aligned} AU_0 &= N_0 \cdot U(0, y-h) \\ AU_{10} &= F(a_m) \cdot U(r_0, y-h) \\ AU_{11} &= \int_{a_m}^{1} U\left(r(q,a), y-h-t\right) dF(a) \end{aligned} \tag{2.5}$$

The state government maximizes a weighted sum of the welfare of the households with and without college-attending child

$$Max \quad W = \left(AU_0 + AU_{10}\right) + w \cdot AU_1 \tag{2.6}$$

subject to the state's balanced budget constraint

$$N \cdot h + N_{11} \cdot t = N_{11} \cdot e \tag{2.7}$$

The state government is assumed to choose tuition, head tax, and per student expenditure, taking the marginal household as given. The household decides to send its child to the university or not, taking the decision variables of the state government as given, which is summarized by the following equation:

$$U\left(r_0, y-h\right) = U\left(r\left(q, a_m\right), y-h-t\right) \tag{2.8}$$

II.2 Household Equilibrium of Education Quality and Marginal Ability

Type 1 households are assumed to be quality takers in their enrollment decision. Since both the utility function U and the educational function r are assumed to be monotonically increasing, there exists a unique *strictly interior* minimum ability of child, denoted by a_m, such that

$$U\left(r\left(q,a\right),y-h-t\right)=U\left(r_0,y-h\right) \tag{2.9}$$

if the following conditions are satisfied for a given head tax and tuition

$$\begin{aligned} &U\left(r\left(q,0\right),y-h-t\right)\leq U\left(r_0,y-h\right) \\ &U\left(r\left(q,1\right),y-h-t\right)\geq U\left(r_0,y-h\right) \end{aligned} \tag{2.10}$$

The first inequality of (2.10) indicates that the utility of enrolling a child of lowest ability is lower than the utility of not enrolling the child. The second inequality of (2.10) indicates that the utility of enrolling a child of highest ability is greater than the utility of not enrolling the child. If either inequality is not satisfied, a corner solution arises; either all Type 1 households enroll their child or none of them enroll their child.

Since Type 1 households are assumed to be quality takers in their enrollment decision, equation (2.9) determines the marginal household with ability a_m=$a_m(q;h,t.y)$ as a function of educational quality given income, head tax, and tuition. The marginal ability is a monotonically decreasing function of q. As the educational quality increases, more households of lower ability enroll their child, and this lowers the marginal ability. This relationship will be called the *marginal* *household response function (MHR)* and it is shown as MHR curve in Figure 1.

Since the educational quality depends on the average ability of enrolled students, households' perceived quality of education may not be the same as the quality produced by the quality production function. The quality production function is an increasing function of $\check{a}$ and hence increasing in a_m, which is shown in as QPF curve in Figure 1, where $q_0=q(0,e)$ *and* $q_1=q(1,e)$. Given the state government's decision variables *h, t,* and *e*, the educational quality is determined endogenously where the MHR and QTF curves intersect each other. That is, the equilibrium quality is determined where households' perceived quality turns out to be the realized quality.

An interior equilibrium of marginal ability and educational quality requires inequalities in (2.10) at $q=q_0$ and $q=q_1$, respectively. The households with a child of lower ability ($a=0$) will not enroll their child when the perceived quality of education is at the lowest quality level q_0. Only households of higher ability child will enroll their child, and hence, the marginal ability will be greater than zero, that is, $a_m>0$. This ensures that point A on the MHR curve will be below the QPF curve. On the other hand, the utility of enrolling a child of highest ability is greater than the utility of not enrolling the child when the perceived quality of education is at the highest level $q_.$ Therefore, the households with a child of highest ability ($a=1$) will enroll their child when the perceived quality of education is q_1. This implies that the marginal household will have a child of ability less than one, and it ensures that point B on the MHR curve will be above QPF curve. Define *g* as the gap between the perceived quality and the actual quality. From Figure 1, it is straightforward to know that *g* is a decreasing function of a_m. Then, the two conditions described above assure a unique interior equilibrium by the Brouwer's fixed point theorem. That is, by the

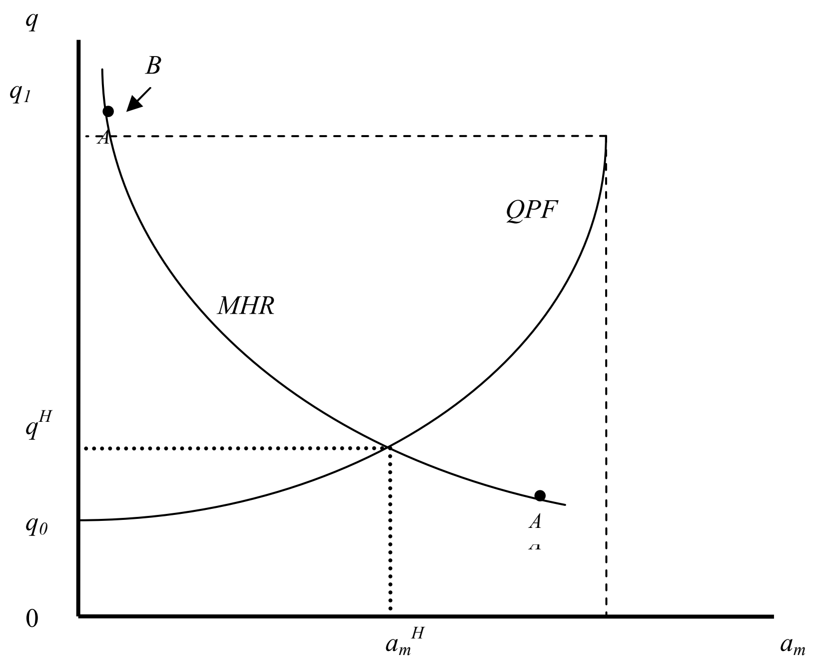

Figure 1. Equilibrium Quality and Marginal Ability

Brouwer's fixed point theorem, there is $a_m{}^H$ such that g($a_m{}^H$)). If either inequality is not satisfied, a corner solution arises; either all Type 1 households enroll their child when the first condition of (2.10) is not satisfied, or none of them enroll their child when the second condition of (2.10) is not satisfied. These results are summarized in the following proposition.

Proposition 1. Given income and state government's decision variables (*h,t,e*), there exists a unique interior equilibrium equality of education and marginal ability if and only if (2.10) is satisfied.

The interior solution will be denoted by a function of state government's decision variables and income

$$a_m^H = a_m\left(h,t,e,y\right) \tag{2.11}$$

$$q^H = q\left(h,t,e,y\right) \tag{2.12}$$

The equilibrium marginal ability then determines the equilibrium number of Type 1 households with a university-attending child

$$N_{11}^H = N_1 - F\left(a_m^H\right) = N_{11}\left(h,t,e,y\right) \tag{2.13}$$

It is easy to see the effect of the educational expenditure *e* on the equilibrium. An increase in *e* attracts more students of lower ability, which reduces the average ability of the students. The net effect is a decrease in the interior equilibrium marginal ability and an increase in the equilibrium quality. Graphically, an increase in *e* shifts the QPF curve upward, resulting in an increase in the equilibrium education quality and a decrease in equilibrium marginal ability, i.e., $\partial a_m{}^H/\partial e<0$ and $\partial q^H/\partial e>0$ as seen Figure 2.

A lower tuition also attracts more students of ability lower than the current marginal ability and it lowers the educational quality. Hence, the *MHR* curve shifts to the left, resulting in a lower equilibrium values of marginal ability and educational equality, $\partial a_m{}^H/\partial t>0$ and $\partial q^H/\partial t>0$ as shown in Figure 3.

Unlike change in tuition and change in expenditure, a change in head tax or income affects all households in the economy. The effect on the household enrollment decision depends on the relative magnitude of the marginal utility of the private good consumption between the households with and without a college-attending child. Consider a case of an *additively separable* strictly concave utility function. Under additively separability, the marginal utility of private consumption does not depend

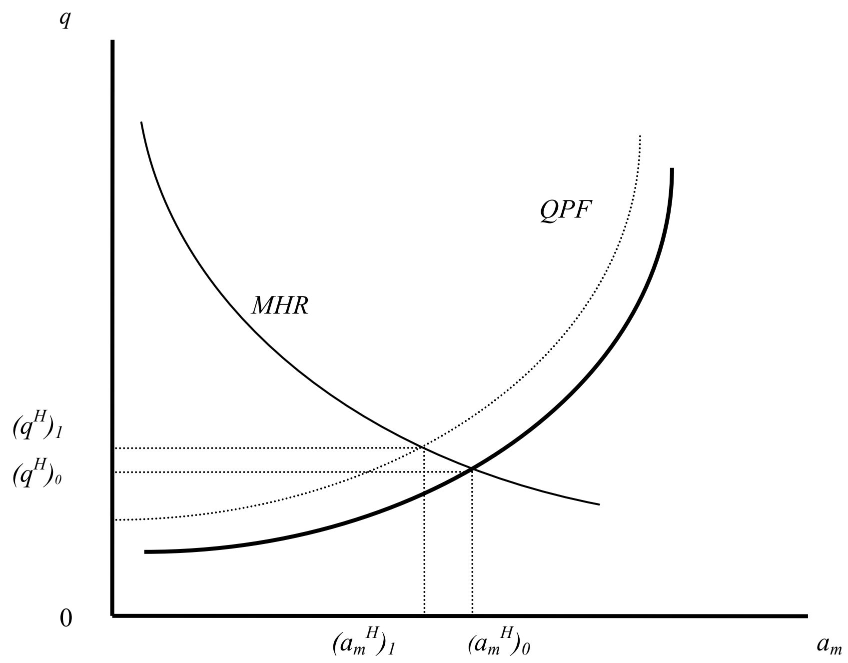

Figure 2. An Increase in Educational Expenditure on Equilibrium Quality and Marginal Ability

on the educational return. Since a decrease in head tax allows every household to have more consumption of private good and marginal utility from an increase in private consumption is higher than that of no enrollment option, the marginal household becomes infra-marginal household. Student of lower ability becomes the marginal ability student. That is, a decrease in head tax decreases the marginal ability and educational ability in this case: equality, $\partial a_m^H/\partial h<0$ and $\partial q^H/\partial h>0$ which is exactly same as in Figure 3. Conversely, a decrease in income raises the marginal ability and educational ability in this case: $\partial a_m^H/\partial y<0$ and $\partial q^H/\partial y>0$.

To show these comparative statics analytically, we substitute the quality production function into (2.9) and totally differentiate it.

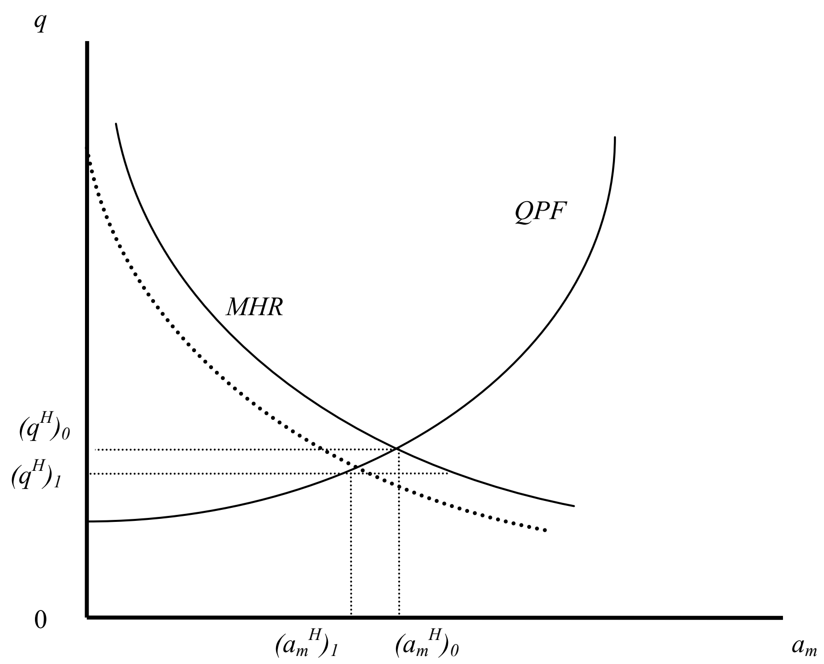

Figure 3. A Decrease in Tuition on Equilibrium Quality and Marginal Ability

$$
\begin{aligned}
&U_r\left(r,x_{11}\right)r_{a_m}da_m + U_r\left(r,x_{11}\right)r_q q_e de + U_x\left(r,x_{11}\right)\left(dy-dh-dt\right) \\
&= U_x\left(r_0,x_0\right)\left(dy-dh\right)
\end{aligned}
\tag{2.14}
$$

where

$$
\begin{aligned}
&x_{11} = y-h-t \\
&x_0 = y-h \\
&r_{a_m} = r_q q_a (d\bar{a}/da_m) + r_a > 0
\end{aligned}
\tag{2.15}
$$

The last term is the derivative of the educational return with respect to a_m. From (2.14), we can have

$$\frac{da_m}{de} = -\frac{r_q q_e}{r_{a_m}} < 0 \tag{2.16}$$

$$\frac{da_m}{dt} = \frac{U_x(r, x_{11})}{U_r(r, x_{11}) r_{a_m}} > 0 \tag{2.17}$$

$$\begin{aligned} \frac{da_{\mathrm{m}}}{dh} &= \frac{U_x(r, x_{11}) - U_x(r_0, x_0)}{U_r(r, x_{11}) r_a(q, a_m)} \\ &= \frac{U_x(r, x_{11}) - U_x(r, x_0)}{U_r(r, x_{11}) r_{a_m}} + \frac{U_x(r, x_0) - U_x(r_0, x_0)}{U_r(r, x_{11}) r_{a_m}} \end{aligned} \tag{2.18}$$

The effects of a head tax are in general indeterminate. The numerator of the first term in the last expression of (2.18) is positive because of the concavity of the utility function, $U_{xr}<0$, and the relationship $x_{11}<x_0$ for all t. The second term may take a positive or negative value: it takes a positive value if x and r are Edgeworth compliments ($U_{xr}>0$), it takes a zero value if the utility function is additively separable ($U_{xr}=0$), and it takes a negative value if x and r are Edgeworth substitutes ($U_{xr}<0$). Therefore, the effect of a change in head tax on a_m is positive if $U_{xr}\geq 0$, and is indeterminate if $U_{xr}<0$. It is easy to see that the effect of income is opposite to the effect of head tax..

Proposition 2. Given the condition of (2.10), we have the following comparative statics results.

(a) the equilibrium marginal ability increases and fewer students attend the university as the per student educational expenditure falls and tuition rises,

(b) the equilibrium educational quality increases as the per student educational expenditure and tuition rise.

(c) Both the equilibrium marginal ability and educational quality increase as head tax rises if x and r are Edgeworth complements ($U_{xr}>0$) or mild substitutes, or if the utility function is additively separable ($U_{xr}=0$). If x and r are strong Edgeworth substitutes ($U_{xr}<0$), the marginal ability and educational quality fall as head tax rises, and

(d) The effects of income on equilibrium are opposite to the effects of head tax.

II.3 State Government's Problem

Taking the marginal ability a_m as given, the state government maximizes the social welfare function.

$$W = N_0 U(0, x_0) + N_{10} U(r_0, x_0) + w \int_{a_m}^{1} U(r, x_{11}) \, dF(a) \tag{2.19}$$

subject to the state's budget constraint

$$t = e - \theta h \tag{2.20}$$

The first order conditions (FOC) are

$$\begin{aligned} W_h &= \frac{\partial W}{\partial h} \\ &= -N_0 U_x(0, x_0) - N_{10} U_x(r_0, x_0) + w \int_{a_m}^{1} U_h(r, x_{11}) \, dF(a) = 0 \end{aligned} \tag{2.21}$$

$$W_e = \frac{\partial W}{\partial e} = w \int_{a_m}^{1} U_e(r, x_{11}) \, dF(a) = 0 \tag{2.22}$$

, where

$$U_h(r, x_{11}) = \frac{\partial U(r, x_{11})}{\partial h} = (\theta - 1) U_x(r, x_{11}) \tag{2.23}$$

$$U_e(r, x_{11}) = \frac{\partial U(r, x_{11})}{\partial e} = U_r(r, x_{11}) r_q q_e - U_x(r, x_{11}) \tag{2.24}$$

The second order conditions are $W_{hh}<0$, $W_{ee}<0$ and $W_{hh} W_{ee} - W_{he}^2>0$, where

$$W_{hh}=\frac{\partial^2 W}{\partial h^2}$$
$$=N_0U_{xx}\left(0,x_0\right)+N_{10}U_{xx}\left(r_0,x_0\right)+w\int_{a_m}^{1}U_{hh}\left(r,x_{11}\right)dF\left(a\right) \tag{2.25}$$

$$W_{he}=\frac{\partial^2 W}{\partial h\partial e}=w\int_{a_m}^{1}U_{he}\left(r,x_{11}\right)dF\left(a\right) \tag{2.26}$$

$$W_{ee}=w\int_{a_m}^{1}U_{ee}\left(r,x_{11}\right)dF\left(a\right) \tag{2.27}$$

,where

$$U_{hh}\left(r,x_{11}\right)=\left(\theta-1\right)^2U_{xx}\left(r,x_{11}\right)<0 \tag{2.28}$$

$$U_{he}\left(r,x_{11}\right)=U_{xr}\left(r,x_{11}\right)r_q q_e-U_{xx}\left(r,x_{11}\right) \tag{2.29}$$

$$U_{ee}\left(r,x_{11}\right)=\frac{\partial^2 U\left(r,x_{11}\right)}{\partial e^2}$$
$$=\begin{Bmatrix}-2U_{rx}\left(r,x_{11}\right)r_q q_e+U_{xx}\left(r,x_{11}\right)+U_{rr}\left(r,x_{11}\right)\left(r_q\right)^2\left(q_e\right)^2\\ +U_r\left(r,x_{11}\right)r_{qq}\left(q_e\right)^2+U_r\left(r,x_{11}\right)r_q q_{ee}\end{Bmatrix} \tag{2.30}$$

We know that W_{hh} is negative under the assumption of strictly concave utility. Under weak Edgeworth Complementarity, we know that $W_{ee}<0$.

In case of an additively separable utility function, we have

$$U\left(r,x\right)=V_1\left(r\right)+V_2\left(x\right) \tag{2.31}$$

. With this additively separable utility function, we have

$$W_{hh}W_{ee}-\left(W_{eh}\right)^2=\left\{\left(N-N_{11}\right)\left(w\left(\theta-1\right)V_{2xx}\left(x_{11}\right)+V_{2xx}\left(x_0\right)\right)\right\}$$
$$\left\{w\int_{a_m}^{1}V_{1ee}\left(r\right)dF\left(a\right)+wN_{11}V_{2xx}\left(x_{11}\right)\right\}-\left\{-\left(N-N_{11}\right)wV_{2xx}\left(x_{11}\right)\right\}^2>0 \tag{2.32}$$

It is straightforward to show that (2.32) is positive. Therefore, the second order condition is satisfied in the case of an additively separable utility function.

From this additively separable utility function, we can rewrite the first FOC

$$wV_{2x}\left(x_{11}\right)=V_{2x}\left(x_{0}\right) \tag{2.33}$$

The left hand side of this equation will be denoted by AMG_h and the right hand side by AML_h. The second F.O.C (2.22) for the additively separable utility function becomes

$$\int_{a_m}^{1} V_{1e}dF\left(a\right)=\mathrm{N}_{11}V_{2x}\left(x_{11}\right) \tag{2.34}$$

where $V_{1e}=V_{1r}r_q q_e$. The left hand side term represents the aggregate marginal gain (AMG_e) to the households with university-attending children as the education quality rises due to an increase in educational expenditure e. The right hand side term is the aggregate marginal loss (AML_e) to the same households as tuition rises with an increase in educational expenditure.

II.3.1 Redistributive Device: Head Tax

Rearranging (2.21), the first FOC, we have

$$N_0U_x\left(0,x_0\right)+N_{10}U_x\left(r_0,x_0\right)=w\left(\theta-1\right)\int_{a_m}^{1}U_x\left(r,x_{11}\right)dF\left(a\right) \tag{2.35}$$

The above equation equates the marginal social welfare of private good among two main different groups; those households who do not have any benefit from the university and those who enroll their children at the university. Since we assume that the state government is a_m-taker, there is no change in the number of households who send their children to the university. Since per student expenditure is fixed, and, hence, quality of university is also constant, the role of head tax is the redistribution from those who do not send any children to the university to those who enroll their children at the university. However, the enrolled households receive an indirect income subsidy from the government through tuition. In fact, given the per student expenditure, they will pay less tuition. Those households who do not have benefit from college education have the same marginal utility from consumption of private goods, while each household among those enrolled households has different marginal utility from private goods, because of the different abilities of the students. Now suppose that the state government increases the head tax. Each non-enrolled household enjoys less utility. Each enrolled household has a net gain, because this household has to pay a head tax, but lower tuition than before. The net gain is

$$(\theta-1)=\left(\frac{N}{N_{11}}-1\right)=\frac{N_0+N_{10}}{N_{11}}>0 \tag{2.36}$$

Therefore, an increase in head tax redistributes the income from those who do not send their children and those who do not have any child to those who enroll their children at the university. Equivalently, we can say

$$\frac{N_0U_x(0,x_0)+N_{10}U_x(r_0,x_0)}{N_0+N_{10}}=w\frac{\int_{a_m}^{1}U_x(r,x_{11})dF(a)}{N_{11}} \tag{2.37}$$

Even though those who do not any benefit from the university have same marginal social welfare, those who do send their children cannot have same marginal social welfare, because of different ability of student from each enrolled household and the restriction to a uniform head tax. Therefore, the state government would like to equate average marginal social welfares from two different groups: those who have no benefit from the university and those who send their children to the university. The left hand side of (2.37) is average marginal social welfare from the former group, and the right hand side of (2.37) is that of the latter group. In case of the additively separable utility function, in the first FOC, (2.33), it is easy to see that $x_{11}<x_0$ for all $t=e-\theta h>0$, and $x_{11}=x_0$ for all $e=\theta h$. Therefore, by the diminishing

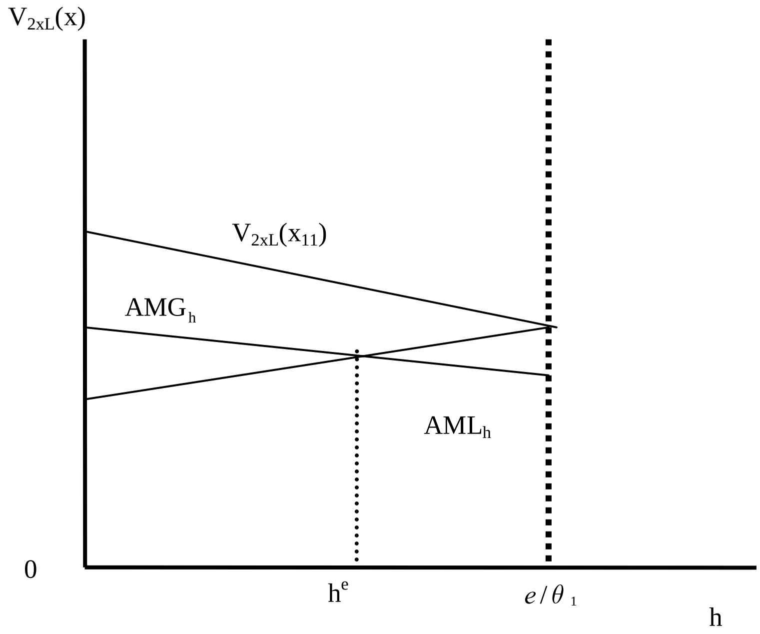

Figure 4. Solution for Head Tax, Given Expenditure

marginal utility, we have $V_{2x}(x_{11})>V_{2x}(x_0)$ for all $e>\theta h$ and $V_{2x}(x_{11})=V_{2x}(x_0)$ for all $e=\theta h$. These relationships are shown in Figure 4 as a function of h for a given value of e. Also shown is AMG_h as a function of h for $w<1$. It is clear that, for a given e, the equilibrium for h is a corner solution $h^e=e/\theta$ for all $w\geq 1$.

On the other hand, if w is sufficiently small, the AMG_h curve may lie below the AML_h curve, and a corner solution of $h^e=0$ arises. Specifically, this corner solution arises if

$$w \leq \underline{w} \text{ , where } \underline{w} = V_{2x}\left(y\right)/V_{2x}\left(y-e\right) \tag{2.38}$$

An interior solution exists under

$$\underline{w} < w < 1 \tag{2.39}$$

It is easy to see that the interior solution for h is an increasing function of w.

Alternatively, for a given weight parameter w, there is a unique level $\bar{e}_h$ of e

such that a positive interior solution for h exists for $e>\bar{e}_h$, a corner solution $h^e=0$ for $e<\bar{e}_h$. This critical value is determined by $wV_{2x}(y-\bar{e}_h)=V_{2x}(y)$.

These results indicate that, if the state government considers the welfare of households with university-attending child more or equally important compared to the welfare of households without university-attending child, all educational expenditure will be financed by the head tax. At the other extreme, educational expenditure will be financed by tuition if the welfare of the university-attending households sufficiently is less important than the welfare of the households with no university-attending child.

In order to find the state government's response function as a_m increases, the AMG_h curve shifts downward and the upper bound of h decreases, while the AML_h curve does not shift. Therefore, the state government decreases the head tax. Figure 5 shows that the interior solution for head tax decreases as a_m increases.

II.3.2 Provision Device: Expenditure

In order to interpret the second FOC, note that social marginal utility of income for those who send their children to the university is

$$u^{1i} = \frac{\partial W}{\partial U(r, x_{11})} \frac{\partial U(r, x_{11})}{\partial x_{11}} = wU_x(r, x_{11}) \tag{2.40}$$

The average value of this is

$$\bar{u}^1 = \frac{1}{N_{11}} \int_{a_m}^{1} \upsilon^{1i} dF(a) = \frac{1}{N_{11}} w \int_{a_m}^{1} U_x(r, x_{11}) dF(a) \tag{2.41}$$

From (2.22), we have

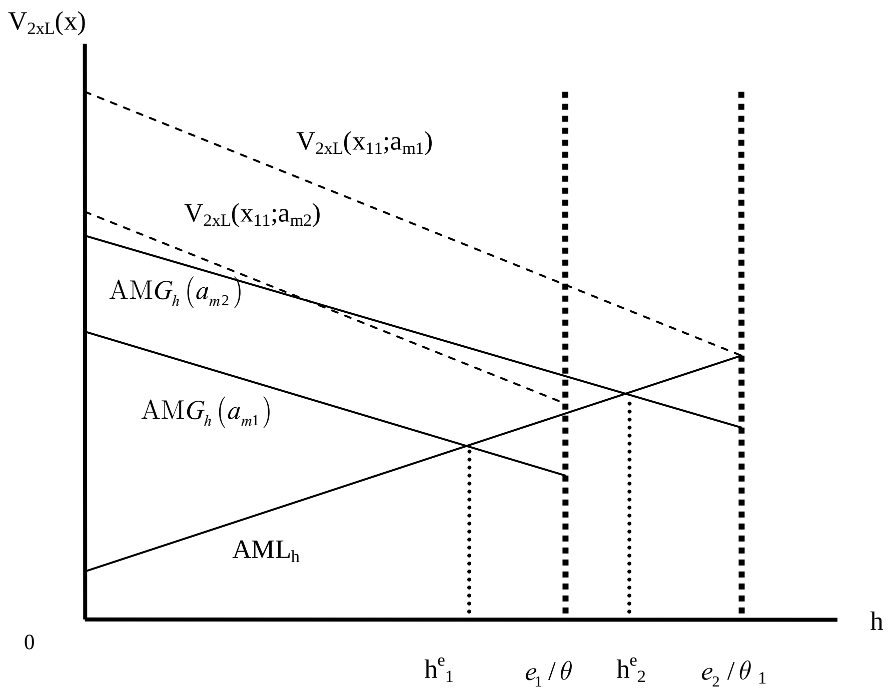

Figure 5. The Effect of an Increase in Marginal Ability *($a_{m1} < a_{m2}$)*

$$\frac{\int_{a_m}^{1}\left[U_r\left(r,x_{11}\right)r_q\right]dF\left(a\right)}{\int_{a_m}^{1}U_x\left(r,x_{11}\right)dF\left(a\right)}=\frac{1}{q_e}\Leftrightarrow\int_{a_m}^{1}\left(\frac{u^{1i}}{\overset{-1}{u}}\right)\left(MRS_{qx}^{1i}\right)dF\left(a\right)=\frac{N_{11}}{q_e} \quad (2.42)$$

where, $MRS_{qx}^{1i}=\dfrac{U_r\left(r,x_{11}\right)r_q}{U_x\left(r,x_{11}\right)}$

The left hand side of this equation is a weighted sum of marginal rates of substitution (MRS). The weights depend on the political weight, *w*. This weighted sum of MRS comes from the fact that the state government uses a uniform head tax. In case of uniform head tax, we cannot apply the usual Samuelson condition, which states that the efficient provision of public good occurs when the sum of the marginal rates of substitution over all households is equal to the marginal rate of transformation

(MRT). According to Atkinson and Stiglitz (1980), we can modify (2.42)

$$\int_{a_m}^{1} MRS_{qx}^{1i}\left(1+\psi_q\right)dF\left(a\right)=\frac{N_{11}}{q_e}$$
$$\text{, where } \psi_q \equiv \operatorname{cov}\left(\frac{u^{1i}}{\overline{u}^{1}},\frac{MRS_{qx}^{1i}}{\overline{MRS}}\right) \tag{2.43}$$

ψ_q is called "distribution characteristic" of public good, quality of university. This depends on the political weight and on the cross partial between private good and public good. For instance, in case of Edgeworth Complementarity, as ability of student falls, the demand for quality of university decreases. This implies that $\psi_q>0$. From (2.43), we know that the level of public good will be produced to the point where summation of MRS is less than MRT.

The right hand side is MRT between the public good and the private good. Define X as the total expenditure of higher education so that

$$X=N_{11}e \tag{2.44}$$

Therefore, the following is true.

$$\frac{d\left(X/N_{11}\right)}{dq}=\frac{1}{q_{X/N_{11}}}\Leftrightarrow\frac{d\left(X/N_{11}\right)}{dq}=\frac{1}{q_e}\Rightarrow\frac{dX}{dq}=\frac{N_{11}}{q_e} \tag{2.45}$$

This equation shows the MRT_{qX} , which measures how much the private good must be given up to produce an additional unit of a public good.

Under assumption of additively separable utility function, because of $u^{1i}=\bar{u}^{1}$, (2.42) becomes

$$\int_{a_m}^{1}\left(MRS_{qx}^{1i}\right)dF\left(a\right)=\frac{N_{11}}{q_e} \tag{2.46}$$

The left hand side is summation of MRS between the public good, q and the private good for the i^{th} household. Therefore, (2.46) shows that the summation of MRS is

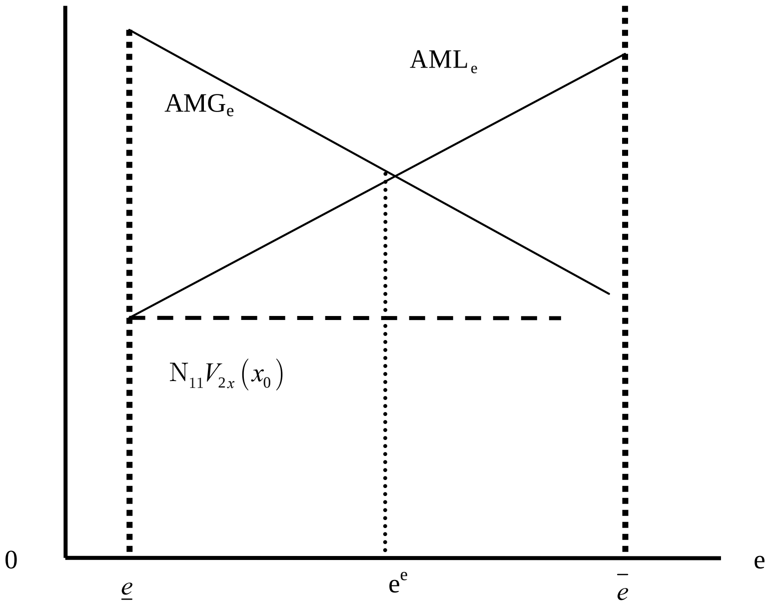

Figure 6. Solution for Expenditure, Given Head Tax and Given Marginal Ability

equal to the MRT.

In case of additively separable utility function, from (2.34), it is easy to show that the AMG_e is a decreasing function of *e* and AML_e is an increasing function of *e*.

$$\frac{\partial V_{1e}}{\partial e} = V_{1rr}\left(r_q q_e\right)^2 + V_{1r} r_{qq} q_e^2 + V_{1r} r_q q_{ee} < 0$$
$$\frac{\partial}{\partial e}\left(N_{11} V_{2x}\left(x_{11}\right)\right) = -N_{11} V_{2xx}\left(x_{11}\right) > 0 \tag{2.47}$$

A nonnegative tuition $t=e-\theta h>0$ imposes a lower bound for *e* is θh, and nonnegative private consumption $x_{11}=y+(\theta-1)h-e\geq 0$ imposes an upper bound for *e,* which is $y+(\theta-1)h$.

Note that, at the lower bound value of *e*, educational expenditure is financed by head tax only ($t=0$) and the private consumption is the same for all households ($x_{11}=x_0$). Hence, the value of AML_e at the lower bound is equal to $N_{11}V_{2x}(x_0)$. At the upper bound value of e, tuition is equal to the disposable income and households with university-attending child have no private consumption (i.e., $x_{11}=0$). These relationships are illustrated in Figure 6 for the case of interior equilibrium for a given $h>0$. Also shown is the benchmark line $N_{11}V_{2x}(x_0)$. An interior solution for e requires

$$\begin{aligned} AMG_e\left(\underline{e}\right) &> N_{11}V_{2x}\left(x_0\right) \\ AMG_e\left(\bar{e}\right) &< N_{11}V_{2x}\left(0\right) \end{aligned} \tag{2.48}$$

The first condition will be satisfied if, at a low level of educational expenditure, its marginal production in quality production function is sufficiently high and/or the marginal utility of educational attainment is sufficiently high. The second condition will be satisfied if the marginal utility of private consumption is sufficiently high at $x=0$. If the first condition is not satisfied, a lower bound corner solution occurs, and an upper bound corner solution occurs if the second condition is not satisfied.

As a_m increases, we will study what would happen to the per student expenditure. An increase in a_m, equivalently, a decrease in N_{11} increases θ, and x_{11}. Hence, as a_m increases, both lower and upper bounds of e increases and the both AML_e curve and the benchmark curve shift downward. The effect of an increase in a_m on the AMG_e curve can be determined from

$$AMG_{ea_m} = \frac{\partial AMG_e}{\partial a_m} = \int_{a_m}^{1} \frac{\partial V_{1e}}{\partial a_m} dF\left(a\right) - V_{1e}\left(a_m\right) \tag{2.49}$$

where $V_{1e}(a_m)$ is the value of V_{1e} evaluated at $a=a_m$, and

$$\frac{\partial V_{1e}}{\partial a_m} = \left(V_{1rr}r_q^2 q_a q_e + V_{1r}r_{qq}q_a q_e + V_{1r}r_q q_{ea}\right)\frac{\partial \bar{a}}{\partial a_m} \tag{2.50}$$

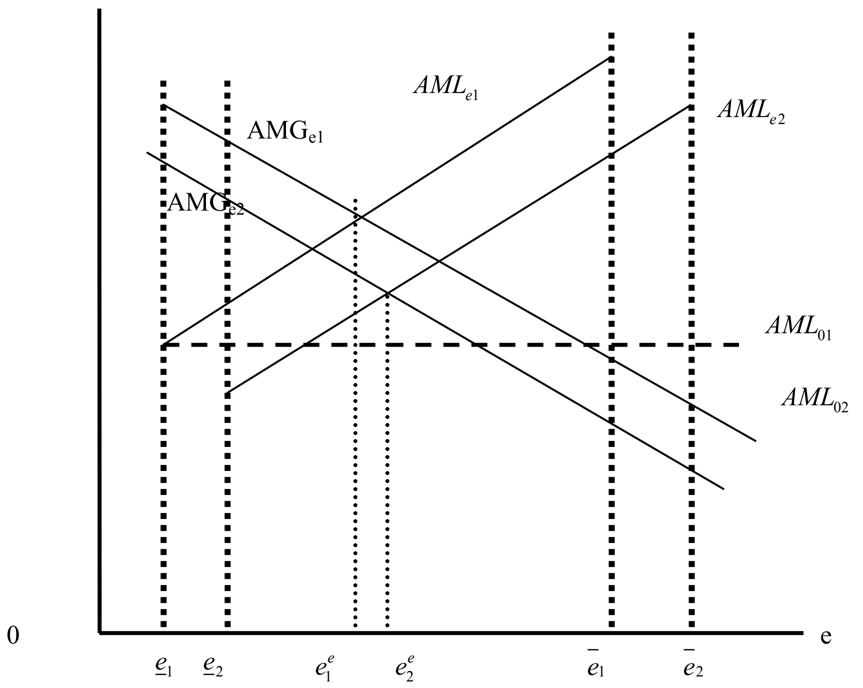

Figure 7. The Effect of an Increase in Marginal Ability on the Solution for Expenditure

$$\frac{\partial \bar{a}}{\partial a_m}=\frac{f\left(a_m\right)}{N_{11}^2}\int_{a_m}^1 af\left(a\right)da-\frac{a_m f\left(a_m\right)}{N_{11}}=\frac{f\left(a_m\right)}{N_{11}^2}\int_{a_m}^1\left(a-a_m\right)f\left(a\right)da>0 \quad (2.51)$$

which is positive as long as $f(a_m)\neq 0$. The effect of a_m on AMG$_e$ is indeterminate in general. The first two terms in parenthesis in (2.50) are negative, and the last term can be positive or negative, depending on the sign of q_{ea}. If $q_{ea}=0$, the derivative in (2.50)takes a negative value, and hence the effect of a_m on the AMG$_e$ is negative. In this case, the AMG$_e$ curve shifts downward as a_m increases, which is illustrated in Figure 7, where AMG$_{e2}$ and AML$_{e2}$ are for a higher value of a_m, and AMG$_{01}$ and AML$_{01}$ are values of benchmark line $N_{11}V_{2x}(x_0)$ for a lower and higher value of a_m, respectively. Even in this case, the effect of an increase in a_m on the solution for e is a priori indeterminate: it can be positive, negative or zero. On the other hand, if the positive effect of a_m on q_e dominates other negative effects so that the AMG_e curve shifts upward, then an increase in a_m increases the solution value for educational expenditure.

II.3.3 Optimal Head Tax and Expenditure

The effect of a decrease in e on the solution for h is illustrated below, where an increase in educational expenditure e does not affect the AML_h curve while the AMG_h curve shifts downward from $AMG_h(e_1)$ to $AMG_h(e_2)$. Figure 8 clearly indicates that the interior solution for h decreases as e decreases. This result can also be shown by taking the total differential of the first order condition (2.33)

$$wV_{2xx}(x_{11})\left[(\theta-1)dh-de\right]=-V_{2xx}(x_0)dh \tag{2.52}$$

which gives

$$\frac{dh}{de}=\frac{wV_{2xx}(x_{11})}{w(\theta-1)V_{2xx}(x_{11})+V_{2xx}(x_0)} \tag{2.53}$$

The effect of an increase in educational expenditure on tuition is ambiguous in general. To show this, we see from $t=e-\theta h$ that

$$\frac{dt}{de}=1-\theta\frac{dh}{de}=\frac{V_{2xx}(x_0)-wV_{2xx}(x_{11})}{w\cdot(\theta-1)V_{2xx}(x_{11})+V_{2xx}(x_0)} \tag{2.54}$$

Note that the interior solution for h is less than e/θ, and hence $x_{11}<x_0$ at $h=h^e$. When the marginal utility is a linear or concave function (i.e. $V_{2xxx}\leq 0$), we have $V_{2xx}(x_0)\leq V_{2xx}(x_{11})$, which implies $V_{2xx}(x_0)-wV_{2xx}(x_{11})\leq 0$ for all $w<1$. In this case, an increase in educational expenditure increases both head tax and tuition. When the marginal utility is a convex function, we have $V_{2xx}(x_0)>V_{2xx}(x_{11})$. Hence, we have

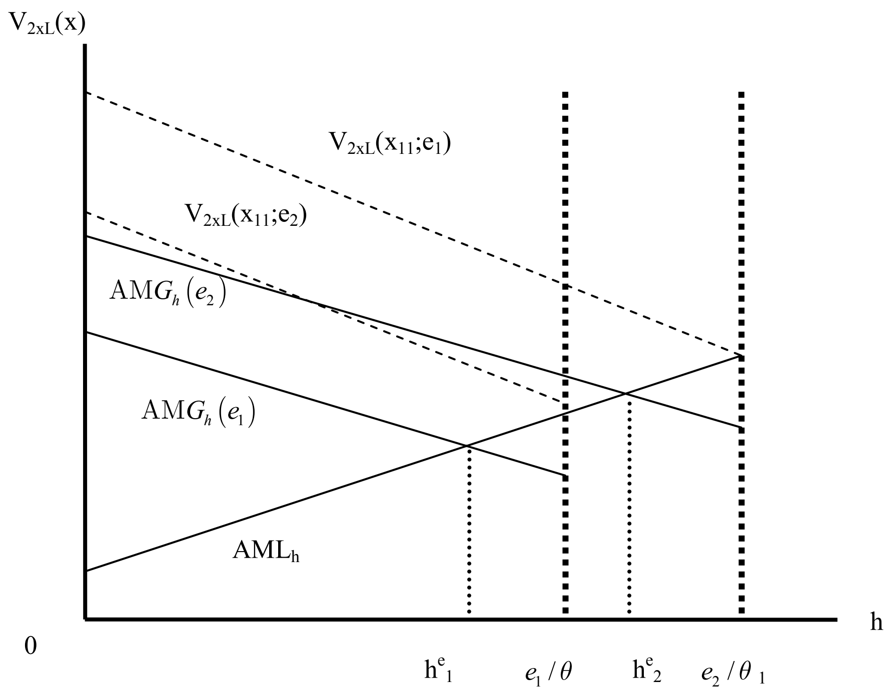

Figure 8. The Effect of an Increase in Expenditure *($e_1<e_2$)*

$V_{2xx}(x_0)-wV_{2xx}(x_{11})<0$ for a sufficiently small *w*, and $V_{2xx}(x_0)-wV_{2xx}(x_{11})>0$ for a sufficiently large *w*. When this term is not positive, an increase in educational expenditure increases both head tax and tuition. When this term is positive, an increase in educational expenditure increases both head tax and tuition if the welfare of the university-attending households is sufficiently less important than the welfare of the households with no university-attending child. Otherwise, an increase in educational expenditure is financed only by an increase in head tax.

An increase in head tax *h* has no effect on the AMG_e curve, but it decreases the AML_e and increase the benchmark line. As illustrated in Figure 9, an increase in *h* increases the interior equilibrium for *e*. This positive relationship between *h* and *e*

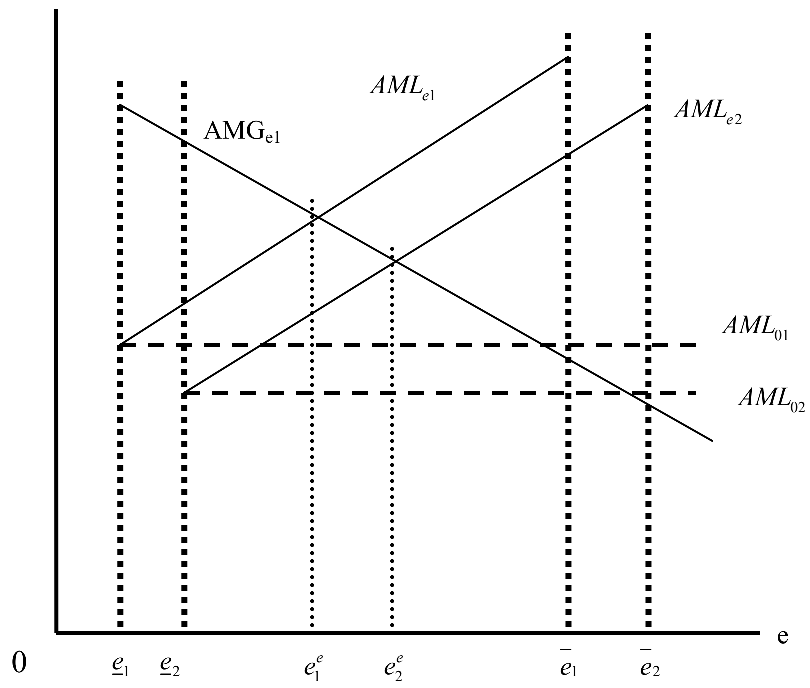

Figure 9. The Effect of an Increase in Head Tax on the Solution for Expenditure

can also be verified from the first order condition in (2.34) by taking the total differential

$$\int_{a_m}^{1} V_{1ee} dF(a) = N_{11} V_{2xx}(x_{11})\left[(\theta-1)dh - de\right] \tag{2.55}$$

which leads to

$$\frac{de}{dh} = \frac{(\theta-1) N_{11} V_{2xx}(x_{11})}{N_{11} V_{2xx}(x_{11}) + \int_{a_m}^{1} V_{1ee} dF(a)} > 0 \tag{2.56}$$

The effect of an increase in head tax on tuition can be analyzed from t=e-θh so that

$$\frac{dt}{dh} = \frac{de}{dh} - \theta = -\frac{N_{11} V_{2xx}(x_{11}) + \theta \int_{a_m}^{1} V_{1ee} dF(a)}{N_{11} V_{2xx}(x_{11}) + \int_{a_m}^{1} V_{1ee} dF(a)} < 0 \tag{2.57}$$

An increase in head tax thus increases per student expenditure, but the total increase in tax collection exceeds the total increase in educational expenditure so that tuition falls.

We showed that there is a positive relation between e and h that satisfies the first order condition (2.33) and (2.34). The boundaries within which an interior solution can exist for the first order conditions are $h \leq e/\theta$ for (2.33) and $\theta h \leq e \leq y+(\theta-1)h$ for (2.34). The set of interior solutions of the first order conditions are inside the triangle between $e=\theta h$ and $e=y+(\theta-1)h$. The boundaries and solution lines are shown in Figure 10, where EL_h and EL_e represent the solution locus of the first order condition (2.33) and (2.34), respectively. The solution lines are drawn such that the EL_h line has a steeper slope than the EL_e line and the intercept of the EL_e line is greater than the intercept of the EL_h line. The EL_h line is drawn to cross the upper boundary line, but not the lower boundary line. Conversely, the EL_e line crosses the lower boundary line, but not the upper boundary line. When these conditions are satisfied, there is a unique interior solution of head tax and educational expenditure for the state government.

It is easy to verify from (2.53) and (2.56) that the slope of EL_h is greater than the slope of EL_e, because

$$\frac{dh}{de} = \frac{wV_{2xx}(x_{11})}{w(\theta-1)V_{2xx}(x_{11}) + V_{2xx}(x_0)} < \frac{1}{(\theta-1)} \tag{2.58}$$

$$\frac{de}{dh} = \frac{(\theta-1)N_{11}V_{2xx}(x_{11})}{N_{11}V_{2xx}(x_{11}) + \int_{a_m}^{1} V_{1ee}\,dF(a)} < (\theta-1) \tag{2.59}$$

where the first derivative is the inverse of the EL_h line.

The intercept term $\bar{e}_h$ and $\bar{e}_e$ are the values of educational expenditure that satisfy the first F.O.C (2.33) when $h=0$, i.e.

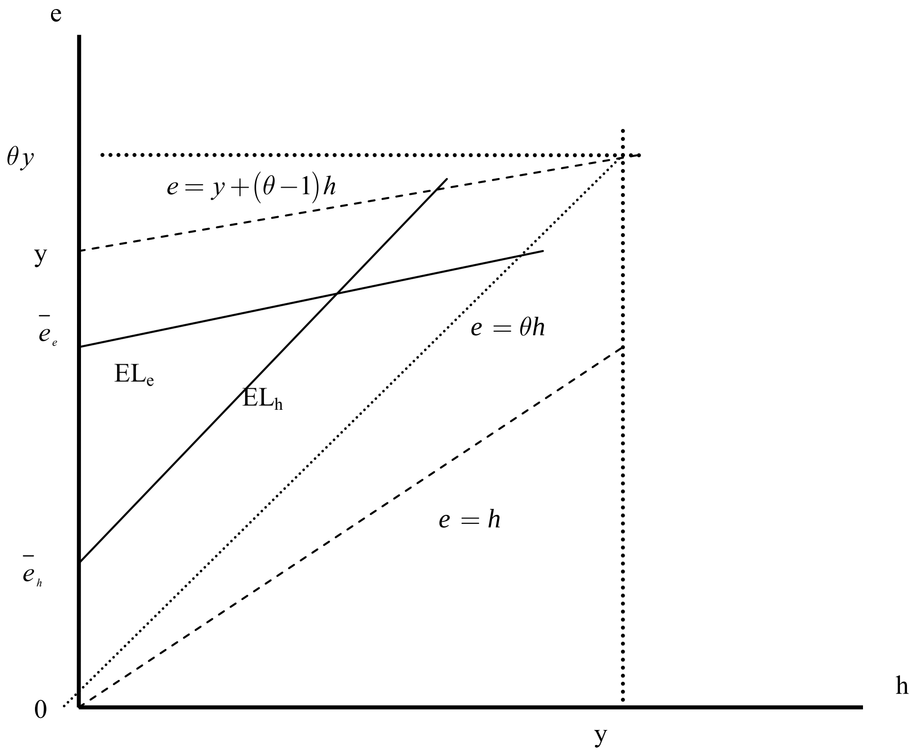

Figure 10. Determination of Both Head Tax and Expenditure

$$wV_{2x}\left(y-\bar{e}_h\right)=V_{2x}\left(y\right) \tag{2.60}$$

$$AMG_e\left(\bar{e}_e\right)=N_{11}V_{2x}\left(y-\bar{e}_e\right) \tag{2.61}$$

where $AMG(\bar{e}_e)$ is the AMG_e evaluated at $e= \bar{e}_e$. These functions are illustrated as a function of educational expenditure in Figure 11. Since the marginal utility of private consumption V_{2x} is an increasing function of educational expenditure and the AMG_e is the decreasing function of e, a sufficient condition for $\bar{e}_e > \bar{e}_h$ is either

$$\begin{aligned} &wV_{2x}\left(y-\bar{e}_e\right)>V_{2x}\left(y\right) \\ \text{or}\quad &AMG_e\left(\bar{e}_h\right)>N_{11}\cdot V_{2x}\left(y-\bar{e}_h\right) \end{aligned} \tag{2.62}$$

Note that $\bar{e}_e$ is the optimum tuition when educational expenditure is financed only by

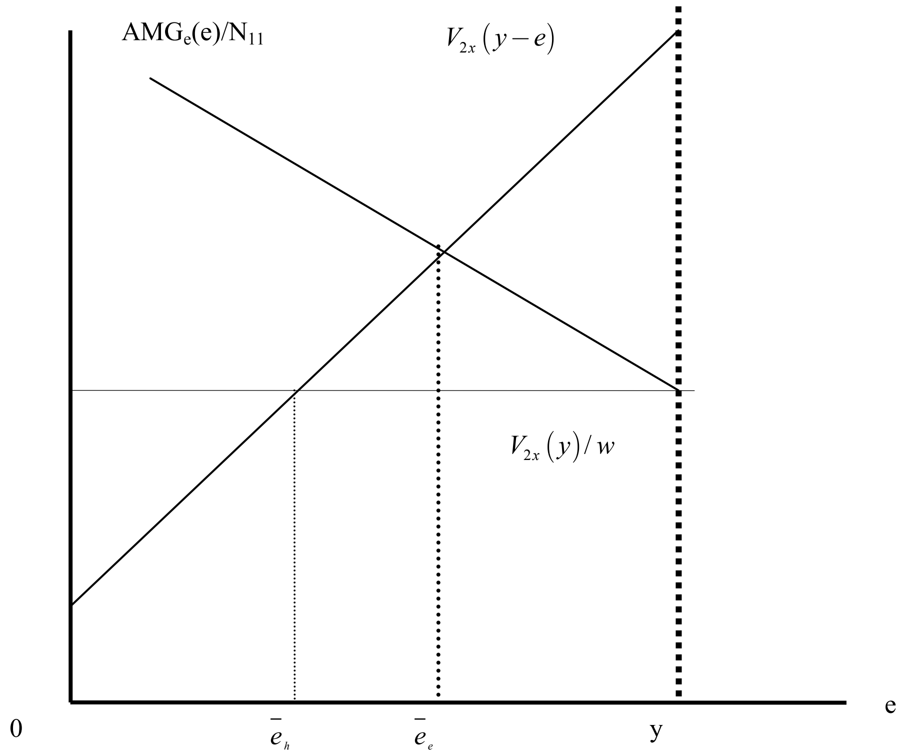

Figure 11. Conditions for Existence of Solution

tuition with zero head tax. The condition (2.62) indicates that the marginal gain of head tax must exceed its marginal loss at the optimum with zero head tax.

Proposition 3. As long as the condition $0< \bar{e}_h< \bar{e}_e<y$, there is an interior solution for head tax and educational expenditure.

Proof) Refer to Lemma 3 and Lemma 4. Q.E.D

In order to prove Proposition 3, we have two Lemmas.

Lemma 3. Given the condition $0<\bar{e}_h<y$**,** the EL_h line cannot cross the lower boundary line, but cross the upper boundary line.

Proof)

To show that the EL_h line crosses the upper boundary line, note that the upper boundary is defined by $e=y+(\theta-1)h$, so that $x_{11}=0$. Therefore, we need to show the existence of h that satisfies the first F.O.C condition,

$$wV_{2x}(0)=V_{2x}(y-h) \tag{2.63}$$

Since $V_{2x}(y-h)$ is increasing in h, there will be a unique h as long as $V_{2x}(y)<wV_{2x}(0)<V_{2x}(0)$. The second inequality is satisfied for all $w<1$. A sufficient condition for the first inequality is $0<\bar{e}_h<y$, which means that the intercept term of the EL_h line is less than income. This is easy to see from (2.60) that $V_{2x}(y)=wV_{2x}(y-\bar{e}_h)<wV_{2x}(0)$, where the last inequality is due to decreasing marginal utility.

The EL_h line cannot cross the lower boundary line $e=\theta h$. This can be shown by noting that $x_{11}=y-h=x_0$ when $e=\theta h$, and hence $wV_{2x}(x_{11})<V_{2x}(x_0)$ for all $w<1$ and the first F.O.C cannot be satisfied. Q.E.D

Lemma 4. Given the condition $0<\bar{e}_e<y$**,** the Eh_e line cannot cross the upper boundary line, but cross the lower boundary line.

Proof)

To show that the EL_e curve cannot cross the upper boundary line, note that $y+(\theta-1)h>y>\bar{e}_e$. The fact that $AMG_{ee}<0$ and (2.61) imply

$$AMG_e\left(y+(\theta-1)h\right)<AMG_e(y)<AMG_e\left(\bar{e}_e\right)=N_{11}V_{2x}\left(y-\bar{e}_e\right)<N_{11}V_{2x}(0) \tag{2.64}$$

The first and the last terms are the AMG_e and AML_e at a point on the upper boundary point e= $y+(\theta-1)h$. The inequality thus indicates that there is no head tax that satisfies $AMG_e=AML_e$.

To show that the EL_e line will cross the lower boundary line, we need to show that there is a head tax level that satisfies the second F.O.C (2.34) at $e=\theta h$, that is

$$AMG_e(\theta h) = N_{11}V_{2x}(y-h) \tag{2.65}$$

Since the AMG_e is decreasing in h and the AML_e is increasing in h, there exists a head tax h that satisfies this equality if the following inequalities are satisfied;

$$AMG_e(0) > N_{11}V_{2x}(y) \text{ and } AMG_e(\theta y) < N_{11}V_{2x}(0) \tag{2.66}$$

Observing $\bar{e}_e<y<\theta y$ and using (2.61), we can write

$$AMG_e(0) > AMG_e(\bar{e}_e) = N_{11}V_{2x}(y-\bar{e}_e) > N_{11}V_{2x}(y) \tag{2.67}$$

$$AMG_e(\theta \cdot y) < AMG_e(y) < AMG_e(\bar{e}_e) = N_{11}V_{2x}(y-\bar{e}_e) < N_{11}V_{2x}(0) \tag{2.68}$$

Both (2.67) and (2.68) prove the required inequalities. Q.E.D

We can do the comparative statics by taking the total differential of the first order conditions in both (2.33) and (2.34).

$$\begin{aligned} V_{2x}(x_{11})dw + wV_{2xx}(x_{11})\{dy + (\theta-1)dh - de + \theta_m h da_m\} \\ - V_{2xx}(x_0)\{dy - dh\} = 0 \end{aligned} \tag{2.69}$$

$$\begin{aligned} AMG_{ee}de + \{AMG_{ea_m} + f(a_m)V_{2x}(x_{11})\}da_m \\ - N_{11}V_{2xx}(x_{11})\{dy + (\theta-1)dh - de + \theta_m h da_m\} = 0 \end{aligned} \tag{2.70}$$

Rearranging the terms and using matrix notation, we have

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} dh \\ de \end{pmatrix} = \begin{pmatrix} C_{1w}dw + C_{1a_m}da_m + C_{1y}dy \\ C_{2a_m}da_m + C_{2y}dy \end{pmatrix} \tag{2.71}$$

where

$$\begin{aligned}
&A_{11} = w(\theta - 1)V_{2xx}(x_{11}) + V_{2xx}(x_0) < 0 \\
&A_{12} = -wV_{2xx}(x_{11}) > 0 \\
&A_{21} = -N_{11}(\theta - 1)V_{2xx}(x_{11}) > 0 \\
&A_{22} = AMG_{ee} + N_{11}V_{2xx}(x_{11}) < 0 \\
&C_{1w} = -V_{2x}(x_{11}) < 0 \\
&C_{1a_m} = -wV_{2xx}(x_{11})\theta_m h > 0 \\
&C_{1y} = V_{2xx}(x_0) - wV_{2xx}(x_{11}) \\
&C_{2a_m} = -AMG_{ea_m} - f(a_m)V_{2x}(x_{11}) + V_{2xx}(x_{11})\theta hf(a_m) \\
&C_{2y} = N_{11}V_{2xx}(x_{11}) < 0
\end{aligned} \tag{2.72}$$

The solution for *dh* and *de* are

$$\begin{pmatrix} dh \\ de \end{pmatrix} = \frac{1}{D}\begin{pmatrix} A_{22}C_{1w}dw + \left(A_{22}C_{1a_m} - A_{12}C_{2a_m}\right)da_m + \left(A_{22}C_{1y} - A_{12}C_{2y}\right)dy \\ -A_{21}C_{1w}dw + \left(A_{11}C_{2a_m} - A_{21}C_{1a_m}\right)da_m + \left(A_{11}C_{2y} - A_{21}C_{1y}\right)dy \end{pmatrix} \tag{2.73}$$

where, $D = A_{11}A_{22} - A_{12}A_{21} > 0$

The effects on tuition are derived from

$$\begin{aligned}
dt &= de - \theta dh - \theta_m h da_m \\
&= \frac{1}{D}\left\{-B_2C_{1w}dw + \left(B_1C_{2a_m} - B_2C_{1a_m} - \theta_m h\right)da_m + \left(B_1C_{2y} - B_2C_{1y}\right)dy\right\}
\end{aligned} \tag{2.74}$$

where

$$\begin{aligned}
&B_1 = A_{11} + \theta A_{12} = C_{1y} \\
&B_2 = A_{21} + \theta A_{22} = N_{11}V_{2xx}(x_{11}) + \theta AMG_{ee} < 0
\end{aligned} \tag{2.75}$$

Therefore, the effects of a change in political weight are

$$\frac{dh}{dw} = \frac{A_{22}C_{1w}}{D} > 0$$
$$\frac{de}{dw} = \frac{-A_{21}C_{1w}}{D} > 0 \tag{2.76}$$
$$\frac{dt}{dw} = \frac{-B_2C_{1w}}{D} < 0$$

Graphically, when the political weight increases, the EL_h line shifts to right, because the state government redistributes the income from those who do not enroll their children at the university to those who are enrolling by increasing the head tax. There is no shift of the EL_e line, because change in the weight parameter does not affect any decision about the expenditure directly, because there is no weight parameter in the second FOC. The result is shown in Figure 12.

The effects of a change in income are

$$\frac{dh}{dy} = \frac{A_{22}C_{1y} - A_{12}C_{2y}}{D}$$
$$\frac{de}{dy} = \frac{A_{11}C_{1y} - A_{21}C_{2y}}{D} \tag{2.77}$$
$$\frac{dt}{dy} = \frac{B_1C_{2y} - B_2C_{1y}}{D}$$

where

$$A_{22}C_{1y} - A_{12}C_{2y} = AMG_{ee}C_{1y} + N_{11}V_{2xx}\left(x_0\right)V_{2xx}\left(x_{11}\right)$$
$$A_{11}C_{1y} - A_{21}C_{2y} = NV_{2xx}\left(x_{11}\right)V_{2xx}\left(x_0\right) > 0 \tag{2.78}$$
$$B_1C_{2y} - B_2C_{1y} = -\theta C_{1y}AMG_{ee}$$

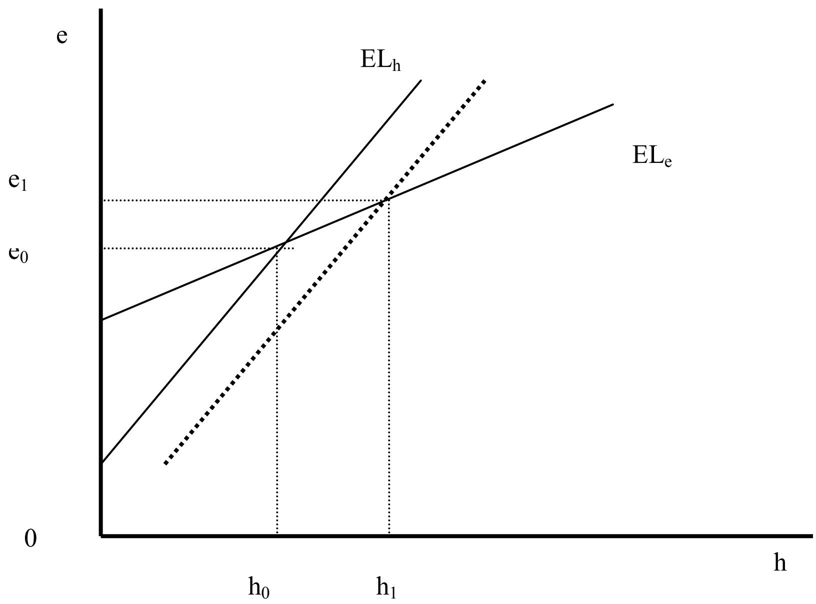

Figure 12. The Effect of an Increase in the Political Weight

An increase in income always increases the educational expenditure. When $C_{1y}=V_{2xx}(x_0)-wV_{2xx}(x_{11})$ is not positive, an increased educational expenditure is funded by an increase in head tax and reduction in tuition. On the other hand, when C_{1y} is positive, tuition rises as income rises, while the head tax may increase or decrease depending on the characteristics of the utility functions, return function and production function. To have the negative effect of change in income on the head tax, the term C_{1y} must be sufficiently large. Graphically, when the term C_{1y} is negative, the EL_h line shifts to the right. Regardless of sign of the term C_{1y}, an increase in income shifts the EL_e line shifts upward. As shown in Figure 13, as income rises, both head tax and expenditure rise. When the term C_{1y} is zero, there is no shift in the

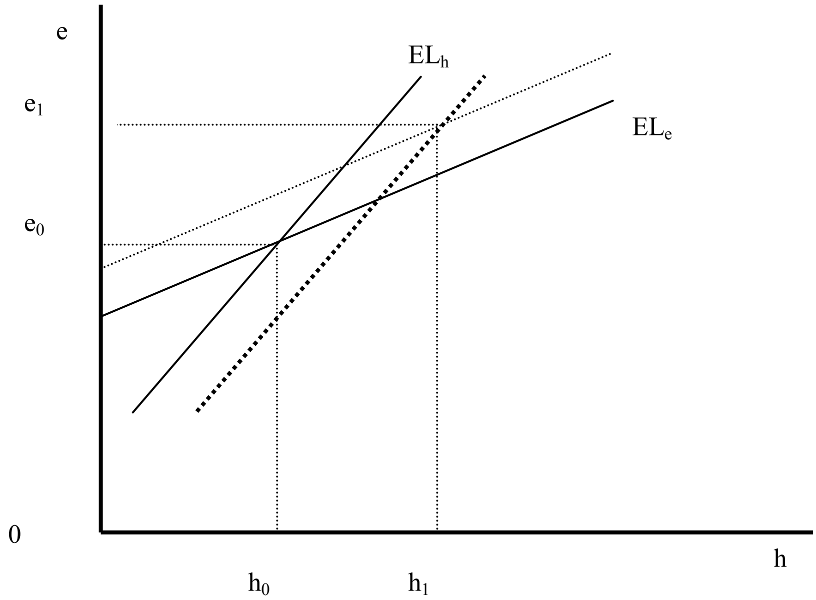

Figure 13. The Effect of an Increase in Income: $C_{1y} < 0$

EL_h line so that the effect of an increase in income on head tax and educational expenditure is positive like the case where the term C_{1y} is positive. When the term C_{1y} is positive, an increase in income shifts the EL_h line to the left. Since the EL_e line is flatter than the EL_h line, as income rises, the educational expenditure rises in this case. Note that the sufficiently larger shift of the EL_h may bring less head tax in the solution.

The effect of a change in marginal ability on head tax, expenditure, and tuition is indeterminate, in general. Using (2.71), we have

$$\frac{dh}{da_m}=\frac{A_{22}C_{1a_m}-A_{12}C_{2a_m}}{D},\ \frac{de}{da_m}=\frac{A_{11}C_{2a_m}-A_{21}C_{1a_m}}{D},$$
$$\frac{dt}{da_m}=\frac{B_1C_{2a_m}-B_2C_{1a_m}-\theta_m h}{D} \tag{2.79}$$

where

$$\begin{aligned}&A_{22}C_{1a_m}-A_{12}C_{2a_m}\\&=-wV_{2xx}(x_{11})\left\{AMG_{ee}\theta_m h+AMG_{ea_m}+f(a_m)V_{2x}(x_{11})\right\}\end{aligned}$$

$$\begin{aligned}A_{11}C_{2a_m}-A_{21}C_{1a_m}=&-A_{11}\left\{AMG_{ea_m}+f(a_m)V_{2x}(x_{11})\right\}\\&+V_{2xx}(x_0)V_{2xx}(x_{11})\theta hf(a_m)\end{aligned}$$

$$\begin{aligned}&B_1C_{2a_m}-B_2C_{1a_m}-\theta_m h\\&=-C_{1y}\left\{AMG_{ea_m}+f(a_m)V_{2x}(x_{11})-V_{2xx}(x_{11})\theta hf(a_m)\right\}\\&+\left\{N_{11}V_{2xx}(x_{11})+\theta AMG_{ee}\right\}\left\{wV_{2xx}(x_{11})\theta_m h\right\}-\theta_m h\end{aligned} \tag{2.80}$$

An increase in the marginal ability reduces the head tax for a given educational expenditure. Therefore, an increase in a_m shifts the EL_h line to the left. On the other hand, the effect of an increase in a_m on the educational expenditure for a given head taxis indeterminate, and hence the EL_e line may shift up or down as a_m increases. The shift of the EL_e line comes from

$$\left.\frac{de}{da_m}\right|_{h=h_0}=-\frac{AMG_{ea_m}+f(a_m)V_{2x}(x_{11})-N_{11}V_{2xx}(x_{11})\theta_m h}{AMG_{ee}+N_{11}V_{2xx}(x_{11})} \tag{2.81}$$

The shift of the EL_e line depends on the numerator. If the numerator is positive, the EL_e line shifts upward. In this case, the head tax decreases and the educational expenditure may increase or decrease. If the numerator is negative, both head tax and educational expenditure will fall as shown in Figure 14. When we assume some specific functions in the next Chapter, we derive (2.81) to find that the numerator is

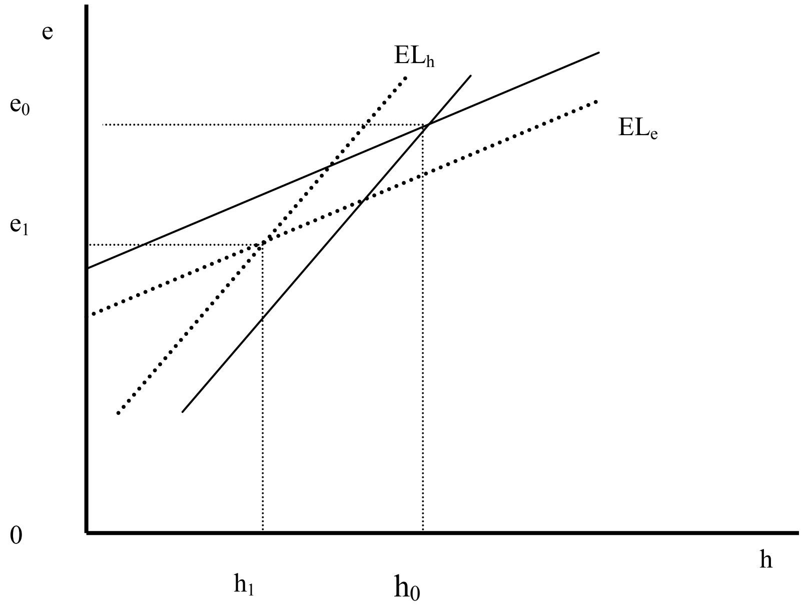

Figure 14. The Effect of an Increase in Marginal Ability

positive. This implies that the EL_e line will shift upward as a_m increases.

II.4 Overall Equilibrium

The state government optimization that is described in the first section of this Chapter yields

$$h^S = h\left(a_m\right) \tag{2.82}$$

$$e^S = e\left(a_m\right) \tag{2.83}$$

Hence, quality production function is

$$q = q\left(\overline{a}\left(a_m\right), e\left(a_m\right)\right) = q\left(a_m\right) \tag{2.84}$$

From t=e-θh, we know that

$$t^S = t\left(a_m\right) \tag{2.85}$$

The household equilibrium gives

$$a_m^H = a_m\left(e, h, t\right) \tag{2.86}$$

Therefore, the overall equilibrium marginal ability is

$$a_m^H = a_m\left(e^S\left(a_m\right), h^S\left(a_m\right), t^S\left(a_m\right)\right) \tag{2.87}$$

Define the overall equilibrium value of a_m as $a_m{}^*$. Then,

$$h^* = h\left(a_m^*\right), e^* = e\left(a_m^*\right), t^* = t\left(a_m^*\right) \tag{2.88}$$

In the next Chapter, we use this procedure to find out the overall equilibrium values of tax, tuition, and expenditure.

II.5 Comparative Statics

Taking total differential of households' equilibrium condition **,** we have

$$\begin{aligned} &V_{1r} r_q q_{a_m} da_m + V_{1r} r_q q_e de + V_{1r} r_{a_m} da_m \\ &= -V_{2x}\left(x_{11}\right)\left\{dy + \left(\theta - 1\right)dh - de + \theta_m h da_m\right\} + V_{2x}\left(x_0\right)\left\{dy - dh\right\} \end{aligned} \tag{2.89}$$

Rearranging (2.89), and combining (2.70) and (2.71), we have

$$\begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix} \begin{pmatrix} dh \\ de \\ da_m \end{pmatrix} = \begin{pmatrix} L_{1w} dw + L_{1y} dy \\ L_{2y} dy \\ L_{3y} dy \end{pmatrix} \tag{2.90}$$

where

$$K_{11} = A_{11} = w(\theta - 1)V_{2xx}(x_{11}) + V_{2xx}(x_0) < 0$$

$$K_{12} = A_{12} = -wV_{2xx}(x_{11}) > 0$$

$$K_{13} = wV_{2xx}(x_{11})\theta_m h < 0$$

$$K_{21} = A_{21} = -N_{11}(\theta - 1)V_{2xx}(x_{11}) > 0$$

$$K_{22} = A_{22} = AMG_{ee} + N_{11}V_{2xx}(x_{11}) < 0$$

$$K_{23} = AMG_{ea_m} + f(a_m)V_{2x}(x_{11}) - V_{2xx}(x_{11})\theta hf(a_m) = -C_{2a_m}$$

$$K_{31} = (\theta - 1)V_{2x}(x_{11}) + V_{2x}(x_0) > 0$$

$$K_{32} = V_{1r}r_q q_e - V_{2x}(x_{11})$$

$$K_{33} = V_{1r}r_q q_{a_m} + V_{1r}r_{a_m} + V_{2x}(x_{11})\theta_m h > 0$$

$$L_{1w} = C_{1w} = -V_{2x}(x_{11}) < 0$$

$$L_{1y} = C_{1y} = V_{2xx}(x_0) - wV_{2xx}(x_{11})$$

$$L_{2y} = C_{2y} = N_{11}V_{2xx}(x_{11}) < 0$$

$$L_{3y} = -\left(V_{2x}(x_{11}) - V_{2x}(x_0)\right)$$

The determinant of the above matrix is

$$D = \overbrace{K_{11}}^{-}\Omega^h - \overbrace{K_{12}}^{+}\Omega^e + \overbrace{K_{13}}^{-}\Omega^{a_m} \tag{2.91}$$

where,

$$\begin{aligned}
\Omega^h &= \overbrace{K_{22}}^{-}\overbrace{K_{33}}^{+} - \overbrace{K_{23}}^{\gtrless}\overbrace{K_{32}}^{\gtrless} \\
\Omega^e &= \overbrace{K_{21}}^{+}\overbrace{K_{33}}^{+} - \overbrace{K_{23}}^{\gtrless}\overbrace{K_{31}}^{+} \\
\Omega^a &= \overbrace{K_{21}}^{+}\overbrace{K_{32}}^{\gtrless} - \overbrace{K_{22}}^{-}\overbrace{K_{31}}^{+}
\end{aligned} \tag{2.92}$$

Since K_{23} and K_{32} are ambiguous, we can have four possible cases. We will consider only one case in which $K_{23}<0$, and $K_{32}>0$, because we have more sufficient condition to have negative determinant than the other cases. In this case, we have definite signs for $\Omega^e>0$, and $\Omega^a>0$, but still ambiguous sign for Ω^h. The sufficient condition for the negative determinant is $\Omega^h>0$. That is, $K_{22}/K_{23}< K_{32}/K_{33}$.

The effects of change in weight parameter are

$$\frac{dh}{dw}=\frac{L_{1w}\Omega^h}{D} \tag{2.93}$$

$$\frac{de}{dw}=\frac{-L_{1w}\Omega^e}{D} \tag{2.94}$$

$$\frac{da_m}{dw}=\frac{L_{1w}\Omega^{a_m}}{D} \tag{2.95}$$

$$\frac{dt}{dw}=\frac{de}{dw}-\theta_m h\frac{da_m}{dw}-\theta\frac{dh}{dw}=\frac{-L_{1w}\left\{\Omega^e+\Omega^{a_m}\left(\theta_m h\right)+\theta\Omega^h\right\}}{D} \tag{2.96}$$

If $K_{23}<0$ $K_{22}/K_{23}< K_{32}/K_{33},$ and $K_{32}>0,$ then $dh/dw>0,$ $de/dw<0,$ $da_m/dw>0,$ and $dt/dw<0.$

The effects of change in income are

$$\frac{dh}{dy}=\frac{1}{D}\left[L_{1y}\Omega^h-L_{2y}\left\{K_{12}K_{33}-K_{13}K_{32}\right\}+L_{3y}\left\{K_{12}K_{23}-K_{13}K_{22}\right\}\right] \tag{2.97}$$

$$\frac{de}{dy}=\frac{1}{D}\left[-L_{1y}\Omega^e+L_{2y}\left\{K_{11}K_{33}-K_{13}K_{31}\right\}-L_{3y}\left\{K_{11}K_{23}-K_{13}K_{21}\right\}\right] \tag{2.98}$$

$$\frac{da_m}{dy}=\frac{1}{D}\left[L_{1y}\Omega^{a_m}-L_{2y}\left\{K_{11}K_{32}-K_{12}K_{31}\right\}+K_{3y}\left\{K_{11}K_{22}-K_{12}K_{21}\right\}\right] \tag{2.99}$$

We cannot go further how to sign the above results so that we use simulation to explain the comparative statics in the model.

CHAPTER III

SIMULATION

In this Chapter, we will use some specific functions in order to examine marginal ability, tuition, head tax, tuition/subsidy ratio, expenditure and quality, and do the comparative statics.

III.1 Specification

We will analyze the equilibrium for the following specific functions.

$$U(r,x) = r^{\alpha} + x^{\beta}, \quad 0 < \alpha < 1, \quad 0 < \beta < 1 \quad (3.1)$$

$$r = \mu q^{\gamma} a^{\delta}, \quad 0 < \gamma < 1, \quad 0 < \delta < 1 \quad (3.2)$$

$$q = \left(\overline{a}\right)^{\lambda} (e)^{\kappa} \quad 0 < \lambda < 1, \quad 0 < \kappa < 1 \quad (3.3)$$

We will consider cases where the student ability takes either uniform distribution or beta distribution.

III.1.1 Uniform Distribution of Student Ability

Assume that the student ability distribution is uniform. Therefore, we have

$$f(a) = N_1 \quad (3.4)$$

These functions give

$$V_{2x}(x) = \beta x^{\beta-1}, \quad V_{2xx}(x) = \beta(\beta-1)x^{\beta-2}$$

$$V_{1e} = V_{1r} r_q q_e = \sigma \mu^{\alpha} \overline{a}^{\nu} e^{\sigma-1} a^{\alpha\delta}$$

$$\text{where } \sigma = \alpha\kappa\gamma, \quad \nu = \alpha\lambda\gamma, \quad \bar{a} = \frac{1+a_m}{2} \tag{3.5}$$

$$N_{11} = N_1\left(1-a_m\right)$$

$$\theta = \frac{N}{N_1\left(1-a_m\right)} \tag{3.6}$$

$$\theta_m = \frac{\partial\theta}{\partial a_m} = \frac{N}{N_1\left(1-a_m\right)^2}$$

The first FOC from state government's optimization gives

$$w\left(y+\left(\theta-1\right)h-e\right)^{\beta-1} = \left(y-h\right)^{\beta-1} \tag{3.7}$$

From (3.7), we can derive

$$h = \frac{w^{-1/(1-\beta)}e+\left(1-w^{-1/(1-\beta)}\right)y}{1+\left(\theta-1\right)w^{-1/(1-\beta)}} \equiv \frac{e-\left(1-\omega\right)y}{\omega+\theta-1} \tag{3.8}$$

where $\omega = w^{1/(1-\beta)}$

The second FOC from state government's optimization AMG_e=AML_e.

$$\begin{aligned} AMG_e &= \int_{a_m}^{1} V_{1e}\,dF\left(a\right) = N_1\mu^{\alpha}\sigma\bar{a}^{\nu}e^{\sigma-1}\int_{a_m}^{1} a^{\alpha\delta}\,da \\ &= N_1\mu^{\alpha}\sigma\left(\frac{1+a_m}{2}\right)^{\nu}e^{\sigma-1}\frac{1-a_m^{1+\alpha\delta}}{1+\alpha\delta} \end{aligned} \tag{3.9}$$

$$AML_e = N_1\left(1-a_m\right)\beta\left\{y+\left(\theta-1\right)h-e\right\}^{\beta-1} \tag{3.10}$$

We can solve for the inverse optimal function which determines the EL_e line.

$$h = \frac{1}{\theta-1}\left(\tau_{uniform}e^{(\sigma-1)(\beta-1)}+e-y\right) \tag{3.11}$$

$$\text{where } \tau_{uniform} = \left(\mu^{\alpha}\sigma\left(\frac{\left(1+a_m\right)}{2}\right)^{\nu}\frac{\left(1-a_m{}^{1+\alpha\delta}\right)}{\beta\left(1+\alpha\delta\right)\left(1-a_m\right)}\right)^{1/(\beta-1)}$$

III.1.2 Beta Distribution of Student Ability

The beta distribution of student ability is defined by

$$f(a) = \frac{N_1}{B(p,q)}(a)^{p-1}(1-a)^{q-1} \quad (3.12)$$

where $a \in [0,1]$, $p,q > 0$

Note that *p, a*nd *q* are the shape parameters. *B(p,q)* is the beta function, which is defined by

$$B(p,q) = \int_0^1 a^{p-1}(1-a)^{q-1}\, da \quad (3.13)$$

The method of moments gives the values of *p*, and *q* from the following equations.

$$p = \breve{a}\left(\frac{\breve{a}(1-\breve{a})}{s^2} - 1\right)$$
$$q = (1-\breve{a})\left(\frac{\breve{a}(1-\breve{a})}{s^2} - 1\right) \quad (3.14)$$

where ă is the sample mean, and *s* is the sample standard deviation. The beta distribution function becomes uniform when *p=q=1*. The formula for the cumulative distribution function of the beta distribution is

$$F(a_m) = \frac{N_1}{B(p,q)} \int_0^{a_m} a^{p-1}(1-a)^{q-1}\, da \quad (3.15)$$

The average student quality is

$$\bar{a} = \frac{1}{N_1 - F(a_m)} \int_{a_m}^1 af(a)\,da \quad (3.16)$$

For the state government optimization, there is no change in the first FOC from this quadratic distribution, but there is a change in the second FOC

$$AMG_e = \int_{a_m}^1 V_{1e}\,dF(a) = N_1\sigma\bar{a}^{\nu} e^{\sigma-1}\int_{a_m}^1 a^{\alpha\delta} f(a)\,da \quad (3.17)$$

Again, there is no change in AML_e. Therefore, we will have the following inverse optimal solution for expenditure, which determines the EL_e line.

$$h = \frac{1}{\theta - 1}\left(\tau_{beta} e^{(\sigma-1)(\beta-1)} + e - y\right) \tag{3.18}$$

where

$$\tau_{beta} = \left(\mu^{\alpha}\sigma\left(\overline{a}\right)^{\nu}\frac{\int_{a_m}^{1} a^{\alpha\delta} f\left(a\right) da}{N_{11}\beta}\right)^{1/(\beta-1)}$$

.

III.2 Simulation

III.2.1 Procedure

We will explain how we obtain our simulation result. Firstly, consider the state government's optimization problem. Given a_m, the solution for the state government should satisfy two first-order conditions. Regardless of student ability distribution, from the first FOC, we have

$$h = \frac{e - (1-\omega) y}{\omega + \theta - 1} \tag{3.19}$$

This will give the EL_h curve for the state government. From the second FOC, we have

$$h = \frac{1}{\theta - 1}\left(\tau e^{(\sigma-1)(\beta-1)} + e - y\right) \tag{3.20}$$

where

$$\tau = \begin{Bmatrix} \tau_{uniform} & \text{if } f\left(a\right) \text{ is uniform distribution} \\ \tau_{beta} & \text{if } f\left(a\right) \text{ is beta distribution} \end{Bmatrix} \tag{3.21}$$

As mentioned in the previous section, (3.20) is the inverse equation of the solution value for educational expenditure EL_e for a given head tax.

Eliminating head tax from these equations, we can derive

$$(\omega+\theta-1)\tau e^{(\sigma-1)(\beta-1)}+\omega e-\omega\theta y=0 \qquad (3.22)$$

which can be solved for the solution value of educational expenditure. Then the solution value for head tax can be computed from the equation of EL_h (3.19).

The household equilibrium is determined by

$$\left(\varphi q^{\gamma} a_m^{\delta}\right)^{\alpha}+\left(y+(\theta-1)h-e\right)^{\beta}=\left(r_0\right)^{\alpha}+\left(y-h\right)^{\beta} \qquad (3.23)$$

This can be expressed by

$$\left(\bar{a}\right)^{\nu} a_m^{\alpha\delta}=\frac{\left\{\left(r_0\right)^{\alpha}+\left(y-h\right)^{\beta}\right\}-\left(y+(\theta-1)h-e\right)^{\beta}}{\varphi^{\alpha}e^{\sigma}} \qquad (3.24)$$

where the average student ability is defined by (3.5) for the uniform distribution and (3.16) for the beta distribution.

The overall equilibrium will include the household's equilibrium so that we put the optimized values of head tax, tuition, and expenditure into the household's equilibrium equation. When the given marginal ability for the government is same as the marginal ability that is obtained from the household equilibrium, we will have the equilibrium marginal ability, which, in turn, gives the equilibrium value of tuition, head tax, and educational expenditure.

III.2.2 Simulation

In order to find out what some of the endogenous variables are, such as educational expenditure per student, tuition, and tuition/subsidy ratio, we consider data set, especially, in year 1996. As mentioned in Chapter I, the reported tuition is sticker resident tuition. Subsidy is the reported state appropriation divided by the number of the full-time equivalent students. In the model, expenditure per student is summed by both tuition and subsidy, but from the real data, since we have the other expenses such as expenditure used for research, and public service. Therefore, we calculate the expenditure per student by summing tuition and subsidy. We report these in Table IV, where we consider only 10 states, which belong to the State Regime.

For the 422 public universities in the U.S., we have \$8378 for the (calculated) expenditure per full-time equivalent student, \$2845 for tuition, and 0.6 for tuition/subsidy ratio. For instance, in Texas, we have \$7358 for the (calculated) expenditure per full-time equivalent student, \$1931 for tuition, and 0.38 for tuition/subsidy ratio.

One possible candidate for y is median household income. Referring to Table V, for the baseline model, we will use the U.S. median income so that y=5.152, measured by \$10,000. The number of total households, and the number of Type 1 households who have children are reported in Table V.[6] The ratio of total number to the number of Type 1 households varies from 2.44 to 3.19. In the U.S., the number of total households is 105,480,101, the number of Type 1 households is 38,022,115, and the ratio between these two numbers is 2.77. For simplicity, those children who are not enrolled at the university, regardless of abilities, we assume that the return of high school graduate is r_0, which is the annualized income for high school graduate. Levy and Murnane (1992) provide annual wage profile among different age groups

[6] We do not think this family ratio changes much so that we use the ratio of the year 2000 instead of year 1996. This is from the CPS 2000.

Table IV. Expenditure, Tuition, Subsidy, and Tuition/Subsidy

States	# of Universities	Reported Expenditure	Tuition	Subsidy	Calculated Expenditure	Tuition/ subsidy
California	25	18845.12	2583.80	6905.56	9489.36	0.37
		(16163.04)	(1071.05)	(2140.70)	(3083.96)	(0.08)
Colorado	12	12233.57	2487.67	3237.25	5724.91	0.77
		(5523.37)	(839.96)	(805.66)	(1543.87)	(0.14)
Florida	8	16095.99	1779.50	7411.79	9191.29	0.26
		(4769.66)	(153.77)	(2014.17)	(1966.69)	(0.07)
Indiana	14	13548.69	2952.43	5281.79	8234.22	0.59
		(6718.52)	(338.87)	(1563.50)	(1794.16)	(0.13)
New York	26	16506.96	3713.81	6364.00	10077.80	0.68
		(8751.16)	(291.15)	(3263.67)	(3365.93)	(0.20)
North Carolina	15	17647.46	1693.73	7857.08	9550.82	0.23
		(8947.58)	(238.45)	(2648.60)	(2772.37)	(0.07)
Oklahoma	11	10238.05	1801.36	4108.21	5909.57	0.47
		(3954.84)	(217.82)	(1537.85)	(1705.47)	(0.10)
South Dakota	3	9671.58	2604.00	3609.24	6213.24	0.81
		(3111.36)	(156.35)	(1582.13)	(1733.51)	(0.30)
Texas	25	12688.36	1931.24	5426.72	7357.96	0.38
		(3701.44)	(293.98)	(1375.63)	(1408.29)	(0.10)
Washington	6	20396.00	2749.83	6094.49	8844.33	0.48
		(14121.71)	(351.54)	(1899.29)	(2223.74)	(0.10)
U.S.	422	15311.56	2845.36	5532.48	8377.84	0.60
		(9528.24)	(1010.25)	(2285.56)	(2596.74)	(0.35)

Notes: Reported expenditure is total current fund expenditure divided by the number of full-time equivalent students. Calculated expenditure is summed by both tuition and subsidy.

for different years in U.S. Averaging the wages of three different age groups among high school graduates for the year 1987, we have \$34,260 for the mean wage of high school graduates in 1996 dollars. Therefore, we set the annualized wage income for those who do not send their children to the university at r_0=3.43. In order to calibrate μ, we consider the average wage of college graduates. From Levy and Murnane (1992), we have \$48,877 for the mean wage of college graduates. From our

Table V. Simulation for Income and Population

States	Median Income	Total Households	Type 1 Households	N/N_1
California	53,807	11,502,870	4,569,910	2.52
Colorado	53,632	1,658,238	585,387	2.83
Florida	44,829	6,337,929	1,986,554	3.19
Indiana	52,962	2,336,306	834,826	2.80
New York	52,799	7,056,860	2,466,483	2.86
North Carolina	46,973	3,132,013	1,104,659	2.84
Oklahoma	43,138	1,342,293	479,275	2.80
South Dakota	45,043	290,245	100,977	2.87
Texas	46,757	7,393,354	3,027,570	2.44
Washington	53,153	2,271,398	799,102	2.84
U.S.	51,518	105,480,101	38,022,115	2.77

Notes: Median household is from http://www.bls.census.gov, households information comes from http://factfinder.census.gov/servlet/BasicFactsServlet.

simulation result, we find that μ=5.7 yields the approximate value of the average annualized income, which is \$49,252. Given quality of university, and using equilibrium marginal ability, we calculate the average annualized income. That is,

$$\begin{aligned} aw &= \frac{1}{N_1 - F\left(a_m\right)} \int_{a_m}^{1} rdf\left(a\right) = \frac{N_1 \mu q^{\gamma}\left(a_m\right)^{\delta}}{N_1 - F\left(a_m\right)} \int_{a_m}^{1} a^{\delta} da \\ &= \frac{\mu q^{\gamma}\left(a_m\right)^{\delta}}{\left(1-a_m\right)\left(\delta+1\right)} \left(1-\left(a_m\right)^{\delta+1}\right) \end{aligned} \tag{3.25}$$

When we have $aw=49{,}252$, $a_m=0.598$, $q=0.964$, $\gamma=0.4$, and $\delta=0.6$, we will have μ=5.7. In our baseline model, we use a uniform distribution of students' abilities. In our alternative distribution, we use a beta distribution for the students' abilities. Since it is impossible to find out the student ability distribution by states, we use the preliminary SAT/National Merit Scholarship Qualifying Test (PSAT/BMSQT) as a

Table VI. Student Ability Distribution by States: Verbal Score in PSAT

	US		California		Texas		Florida		Colorado	
Score	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
75-80	10,646	0.8	1,408	1.03	829	0.69	316	0.63	158	0.80
70-74	23,956	1.8	2,363	1.72	1,902	1.59	1,037	2.05	326	1.65
65-69	54,550	4.1	5,482	4.00	3,959	3.30	2,373	4.69	858	4.35
60-64	106,518	8.0	10,206	7.45	7,567	6.31	4,435	8.77	1,707	8.65
55-59	168,634	12.6	16,179	11.81	12,334	10.28	6,664	13.18	2,736	13.87
50-54	226,913	17.0	20,959	15.30	17,766	14.81	8,829	17.47	3,589	18.19
45-49	271,297	20.3	26,925	19.65	22,965	19.15	9,972	19.73	4,222	21.40
40-44	225,410	16.8	23,218	16.95	22,293	18.59	7,985	15.80	3,214	16.29
35-39	117,362	8.8	13,477	9.84	13,295	11.08	3,991	7.89	1,454	7.37
30-34	80,638	6.0	10,101	7.37	10,146	8.46	2,857	5.65	915	4.64
25-29	32,396	2.4	3,817	2.79	4,539	3.78	1,330	2.63	365	1.85
20-24	19,450	1.5	2,868	2.09	2,356	1.96	762	1.51	188	0.95
TOTAL	1,337,770		137,003		119951		50551		19732	
MEAN	48.3		47.6		46.4		48.8		49.3	
STD. EV.	10.5		11		10.9		10.7		10	

Notes: The source is from http://www.collegeboard.com/research/html/2001_psat_pdf_jr.html.

proxy for the student ability distribution. PSAT/BMSQT is a standardized test, which gives the practice for SAT, but can be qualified for National Merit Scholarship programs.[7] Table VI shows the verbal scores of PSAT/BMSQT for those seniors in high schools who took this test. We concentrate on four states, because it is enough to show how student ability distribution is different by states in terms of mean and standard deviation. Compared between Texas and the other three states, the means are statistically different as we easily calculate *t* value for differences in mean between two states from Table VI. Using Table VI, we draw distributions of student scores by four states in Figure 15. The distribution of PSAT score looks like

[7] Refer to http://www.collgeboard.com.

normal distribution, but in our model, the ability is ranged from zero to one. In the United States, the sample mean, and the sample standard distribution gives the possible range of ability from zero to one, we use the beat distribution as an alternative distribution to the deviation are 48.3 and 10.5, respectively. We change the scale into 0.483 and 0.105, and hence, we calculate *p,* and *q* from (3.14) so that p=10.46, and q=11.19. Note that different combinations of these two shape parameters give a variety of different distribution of students' abilities. Using the U.S. example, we show our alternative distribution, beta distribution in Figure 16. As shown in Figure 16, the beta distribution replicates the U.S. distribution of PSAT scores closely. We check the possible value for political weight to have meaningful equilibrium values, which mean that we have some interior solutions for the state government. No corner solution exists when $w \geq 0.9$. We set $w=0.95$ in our baseline model, which implies that the state government equally views among those who have no direct benefit from university education and those who send their children to the university. The households have same degree of satisfaction from consuming private good or from consuming education good, because the return from education is an annualized future income. Therefore, we set $\alpha=\beta=0.5$. We assume more relative importance of student's own ability than the quality of university, so that γ=0.4, and δ=0.6. In the same way, in the quality production function, we assume more relative importance of educational expenditure than the average student ability, so that λ=0.4, and κ=0.6.

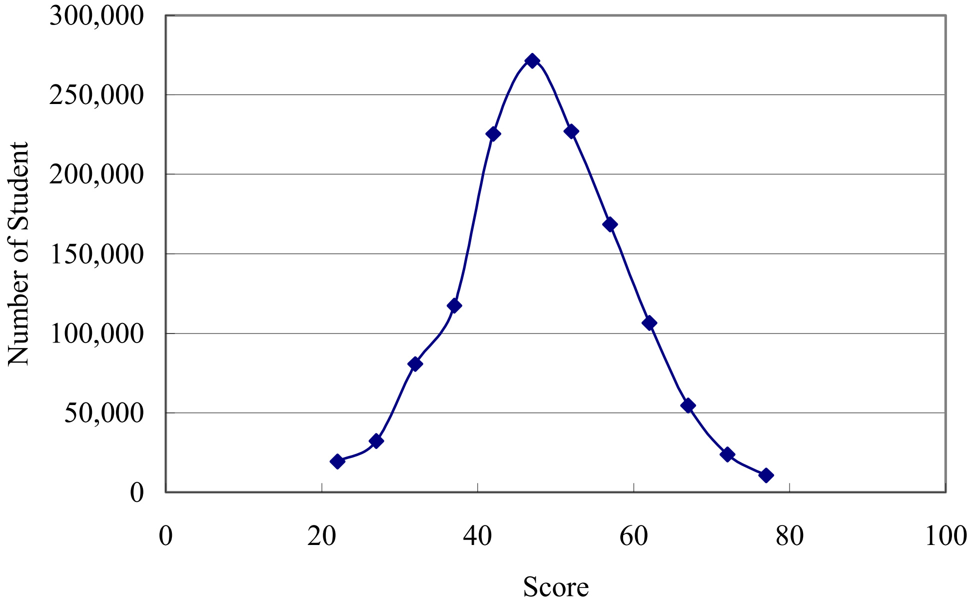

Figure 15. Student Ability Distribution in U.S.: Verbal Score in PSAT

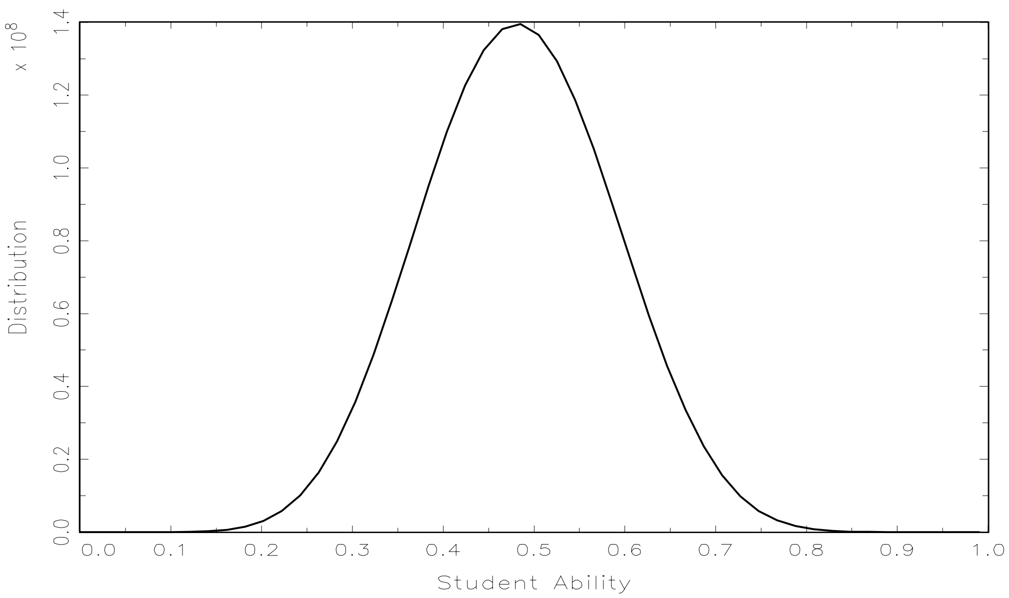

Figure 16. The Beta Distribution, where p=10.46, q=11.19, N1=38,022,115

III.2.3 Comparative Statics in State Government's Optimization

In order to have more intuition for overall equilibrium, we will take a deeper look at the state government's optimization in the case of a uniform distribution. In order to check the effect of change in marginal ability on the optimal solutions of the state government, consider the following comparative statics as derived from Chapter II.

$$\frac{de}{da_m} = \frac{-A_{11}\left\{AMG_{ea_m} + f\left(a_m\right)V_{2x}\left(x_{11}\right)\right\} + V_{2xx}\left(x_0\right)V_{2xx}\left(x_{11}\right)\theta hf\left(a_m\right)}{D} \tag{3.26}$$

Since the second term of the numerator in (3.26) is positive, the effect of an increase in a_m on the educational expenditure depends on the first term of the numerator. We know that

$$\begin{aligned} AMG_{ea_m} &= \frac{N_1 \sigma e^{\sigma-1}}{2^{\nu}}\left(1+a_m\right)^{\nu-1}\left\{\nu\left(1-a_m^{1+\alpha\delta}\right)-\left(1+\alpha\delta\right)\left(1+a_m\right)a_m^{\alpha\delta}\right\} \\ &= AMG_e \frac{1}{\left(1-a_m\right)\left(1-a_m^{1+\alpha\delta}\right)}\left\{\nu\left(1-a_m^{1+\alpha\delta}\right)-\left(1+\alpha\delta\right)\left(1+a_m\right)a_m^{\alpha\delta}\right\} \end{aligned} \tag{3.27}$$

The terms in the brace are illustrated in Figure 17, where the first term is downward sloping and the second term is upward sloping. Since $\upsilon<1<2(1+\alpha\delta)$, there is a unique value a_m^* such that

$$AMG_{ea_m} > 0 \text{ if } a_m < a_m^* \tag{3.28}$$

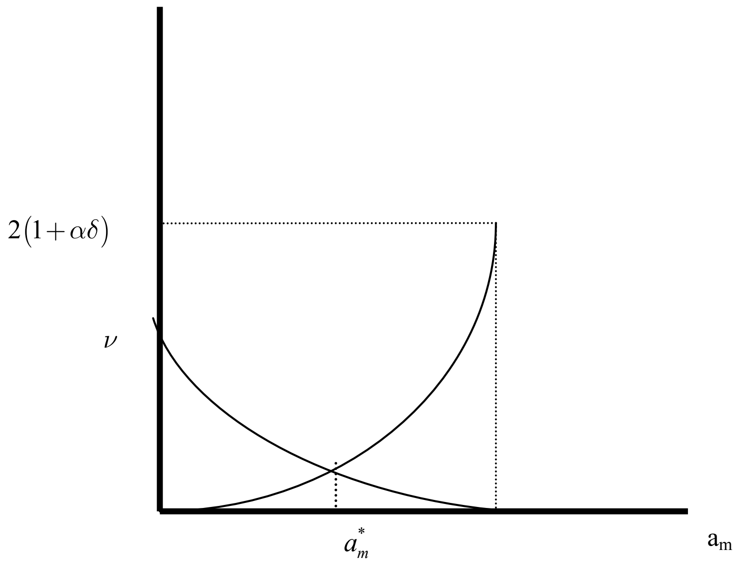

Figure 17. AMG_{ea_m}

and vice versa. Now, we will check the sign of the following term

$$AMG_{ea_m} + f(a_m)V_{2x}(x_{11}) \tag{3.29}$$

Substituting first-order condition $AMG_e=N_1(1-a_m)V_{2x}(x_{11})$, we have

$$\begin{aligned} &AMG_{ea_m} + f(a_m)V_{2x}(x_{11}) = \\ &= \frac{N_1(1-a_m)V_{2x}(x_{11})}{(1+a_m)\left(1-a_m^{1+\alpha\delta}\right)}\left\{\nu(1-a_m)\left(1-a_m^{1+\alpha\delta}\right)+(1+a_m)\Psi\right\} \end{aligned} \tag{3.30}$$

where

$$\Psi=\left(1-a_m^{1+\alpha\delta}\right)-(1+\alpha\delta)(1-a_m)a_m^{\alpha\delta}>0$$

To show $\Psi>0$, we use the fact that $\Psi=1$ for $a_m=0$ and $\Psi=0$ for $a_m=1$, and Ψ is a monotonically decreasing in a_m. The last fact is from

$$\begin{aligned} \frac{\partial\Psi}{\partial a_m} &= -(1+\alpha\delta)a_m^{\alpha\delta} - (1+\alpha\delta)\left\{-a_m^{\alpha\delta} + \alpha\delta(1-a_m)a_m^{\alpha\delta-1}\right\} \\ &= -(1+\alpha\delta)\alpha\delta(1-a_m)a_m^{\alpha\delta-1} < 0 \end{aligned} \tag{3.31}$$

Therefore, it is true that

$$AMG_{ea_m} + f(a_m)V_{2x}(x_{11}) > 0 \tag{3.32}$$

Therefore, from (3.26), the effect of an increase in a_m on educational expenditure is always positive. Graphically, as marginal ability increases, the EL_e line shifts upward. On the other hand, since this specific function belongs to the additively separable utility function, as marginal ability increases, the EL_h line shifts to the left. As marginal ability increases, the educational expenditure will always increase, but the head tax may increase or decrease. Using uniform distribution function, we confirm this analysis about the effect of change in marginal ability on the educational expenditure in Figure 18.

In order to know more about the effect of change in marginal ability on the head tax, consider the following comparative statics as derived in Chapter II

$$\frac{dh}{da_m} = \frac{-wV_{2xx}(x_{11})\left\{AMG_{ee}\theta_m h + AMG_{ea_m} + f(a_m)V_{2x}(x_{11})\right\}}{D} \tag{3.33}$$

Again, the effect of change in a_m on head tax depends on the numerator. We know

$$\begin{aligned} AMG_{ee}\theta_m h &= \left(\frac{\sigma-1}{e}AMG_e\right)\frac{N}{N_1(1-a_m)^2}h \\ &= \frac{\sigma-1}{e}N_1(1-a_m)V_{2x}(x_{11})\frac{N}{N_1(1-a_m)^2}h \\ &= -N_1V_{2x}(x_{11})\frac{N(1-\sigma)}{N_1(1-a_m)}\frac{h}{e} = -N_1V_{2x}(x_{11})(1-\sigma)\frac{\theta h}{e} \end{aligned} \tag{3.34}$$

where we used second FOC. Therefore, we can write

$$AMG_{ea_m} + f(a_m)V_{2x}(x_{11}) + AMG_{ee}\theta_m h = N_1(1-\sigma)V_{2x}(x_{11})\left\{\Phi_1 + \Phi_1 - \sigma\right\} \tag{3.35}$$

where

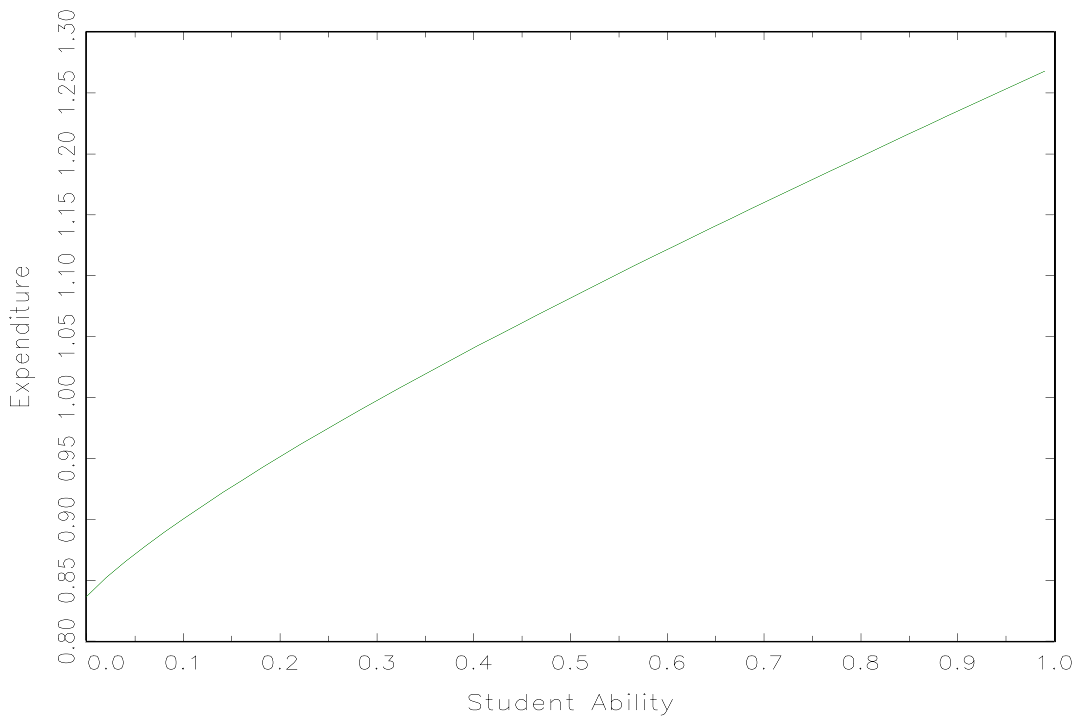

Figure 18. The Effect of an Increase in a_m on Expenditure
: Uniform Distribution of Student Ability

$$\rho = \frac{\theta h}{e}, \quad \Phi_1 = \frac{\nu(1-a_m)}{(1-\sigma)(1+a_m)}, \quad \Phi_2 = \frac{\Psi}{(1-\sigma)\left(1-a_m^{1+\alpha\delta}\right)} \tag{3.36}$$

Note that the EL_e line indicates that $0 \leq \sigma \leq 1$ at any solution. Therefore, a sufficient condition for (3.35) to take a positive value is either $\Phi_1 \geq 1$ or $\Phi_2 \geq 1$ or $\Phi = \Phi_1 + \Phi_2 > 1$. Since the numerator of Φ_1 is a decreasing function of a_m and its denominator is increasing in a_m, there is a unique value of a_m such that

$$\Phi_1 \overset{\geq}{\underset{<}{-}} 1 \quad \text{as } a_m \frac{\leq \nu + \sigma - 1}{> 1 + \nu - \sigma} \tag{3.37}$$

If $\upsilon + \sigma < 1$, then Φ_1 takes a value less than one for all positive a_m. However, if there is a sufficiently strong increasing returns to scale ($\lambda+\kappa>1$) in the quality production function and the marginal utility of education quality is high, (i.e., $\alpha\gamma$ is large), then Φ_1 will take a value greater than one for a low marginal ability.

A similar analysis shows

$$\Phi_2 \gtrless 1 \text{ as } (1+\alpha\delta)(1-a_m)a_m^{\alpha\delta} \lessgtr \sigma\left(1-a_m^{1+\alpha\delta}\right) \tag{3.38}$$

Define the left hand side function in (3.38) as LHS^1 and the right hand side function as RHS^1. Since LHS^1 is zero when a_m is zero or one, and LHS^1 is maximum value when $a_m=\alpha\delta/(1+\alpha\delta)$. RHS^1 is a decreasing function of a_m. Note that the RHS^1curve is flatter than the LHS^1 curve at a_m=1, because

$$abs\left(\left.\frac{\partial \mathrm{P}HS^1}{\partial a_m}\right|_{a_m=1}\right) = (1+\alpha\delta) > abs\left(\left.\frac{\partial LHS^1}{\partial a_m}\right|_{a_m=1}\right) = \sigma(1+\alpha\delta) \tag{3.39}$$

Therefore, there is a unique value of marginal ability below which Φ_2 takes a value greater than one as shown in Figure 19.

Now, consider both Φ_1 and Φ_2 terms.

$$\Phi = \Phi_1 + \Phi_2 = \frac{\nu(1-a_m)}{(1-\sigma)(1-a_m)} + \frac{\Psi}{(1-\sigma)\left(1-a_m^{1+\alpha\delta}\right)}$$

$$= \frac{1}{(1-\sigma)}\left(\frac{\nu(1-a_m)}{(1+a_m)} + 1 - \frac{(1+\alpha\delta)(1-a_m)a_m^{\alpha\delta}}{1-a_m^{1+\alpha\delta}}\right) \tag{3.40}$$

The sufficient condition for $\Phi>1$ is

$$LHS^2 \equiv \sigma + \frac{\nu(1-a_m)}{1+a_m} > \frac{(1+\alpha\delta)(1-a_m)a_m^{\alpha\delta}}{1-a_m^{1+\alpha\delta}} \equiv RHS^2 \tag{3.41}$$

Note that LHS^2 is a monotonically decreasing function of a_m with a value $\upsilon+\sigma$ at $a_m=0$ and σ at $a_m=1$. RHS^2 is monotonically increasing in a_m with a zero value at

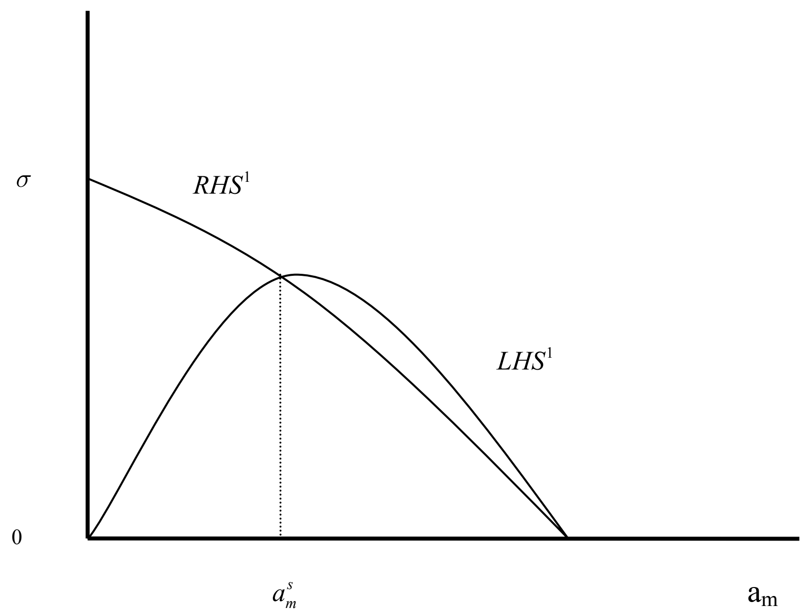

Figure 19. Unique Value of Marginal Ability: $\Phi_2 \geq 1$

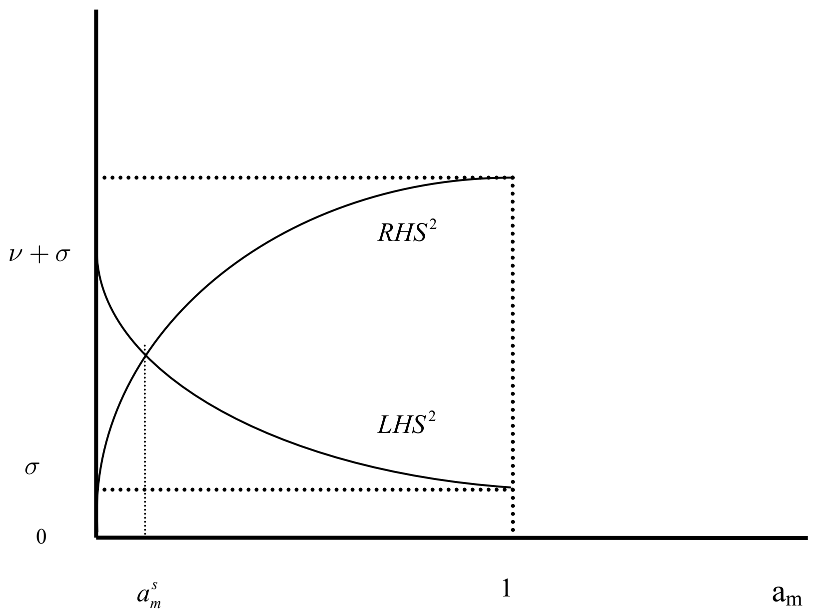

Figure 20. Unique Value of Marginal Ability: $\Phi > 1$

$a_m=0$ and one at $a_m=1$. Figure 20 illustrates that there is a unique a_m below which Φ>1. Therefore, when the marginal ability is low, the equation (3.35) will take a positive value. If a_m is high and the solved head tax is high, the equation (3.35) can take a negative value. If there is a sufficiently large increasing returns to scale in the quality production function and the marginal utility of educational quality is high, when a_m is low, as the marginal ability increases, the head tax will increase from (3.33). That is, the numerator of dh/da_m is positive so that the solution value of head tax will increase as marginal ability increases. Otherwise, we have a negative effect of change in marginal ability on the head tax. We confirm this analysis using simulation as shown in Figure 21. As a_m rises, head tax increases before $a_m=0.14$, and decreases beyond $a_m=0.14$.

The effect of change in a_m on tuition, which is derived in Chapter II is

$$\frac{dt}{da_m}=\frac{\begin{bmatrix}-C_{1y}\left\{AMG_{ea_m}+f(a_m)V_{2x}(x_{11})-V_{2xx}(x_{11})\theta hf(a_m)\right\}\\+\left\{\left(N_{11}V_{2xx}(x_{11})+\theta AMG_{ee}\right)\left(wV_{2xx}(x_{11})\theta_m h\right)\right\}-\theta_m h\end{bmatrix}}{D} \tag{3.42}$$

The effect of an increase in a_m on tuition is indeterminate, in general. Our simulation result for the relationship between tuition and marginal ability is shown in Figure 22. To the opposite of the effect on the head tax, as a_m rises, tuition decreases before $a_m=0.14$, and increases after $a_m=0.14$. The reason why the graph is sloped upward beyond $a_m=0.14$ is that the second term of the numerator dominates the summation of the first term and the third term.[8]

The effect of change in a_m on subsidy is

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[8] In general, C_{1y} is indeterminate, but positive in our Simulation.

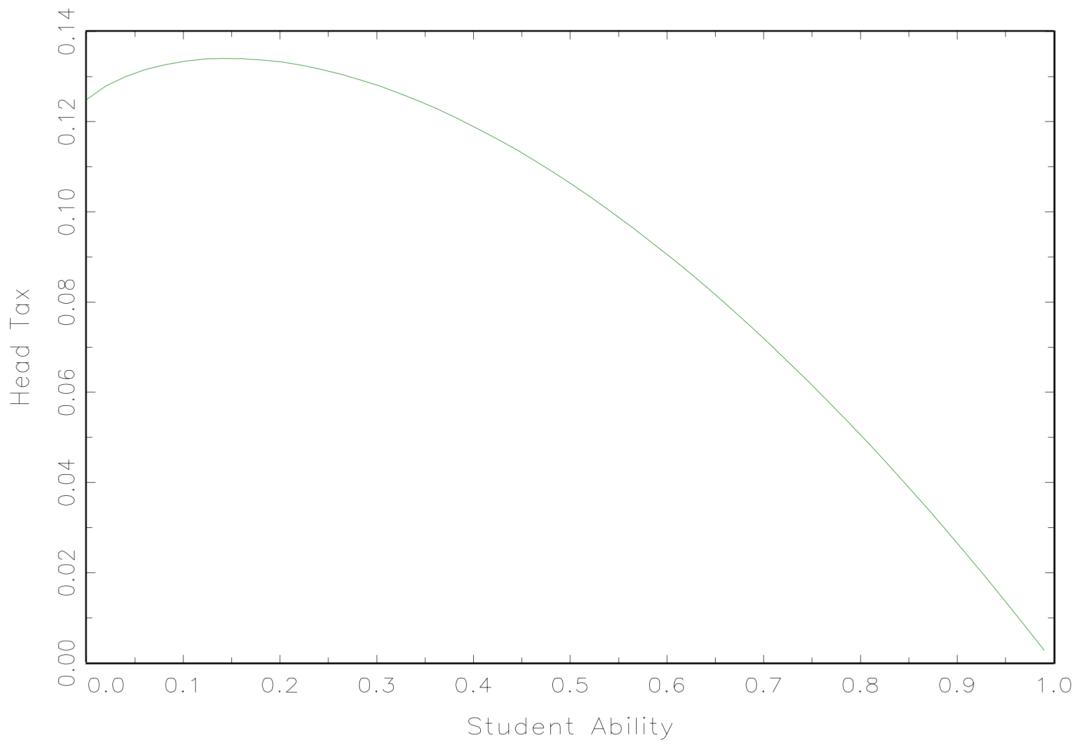

Figure 21. The Effect of an Increase in Marginal Ability on Head Tax:
Uniform Distribution of Student Ability

$$\frac{ds}{da_m} = \theta_m h + \theta \frac{dh}{da_m} \tag{3.43}$$

where $\theta_m = \theta h/(1-a_m)$. Since the first term in (3.43) is positive, and the second term is positive at a certain value of a_m, we expect that the effect of an increase in a_m is positive before that certain value of a_m. Beyond this value of a_m, it depends on which term of (3.43) is bigger. From our simulation, the effect of an increase in a_m on

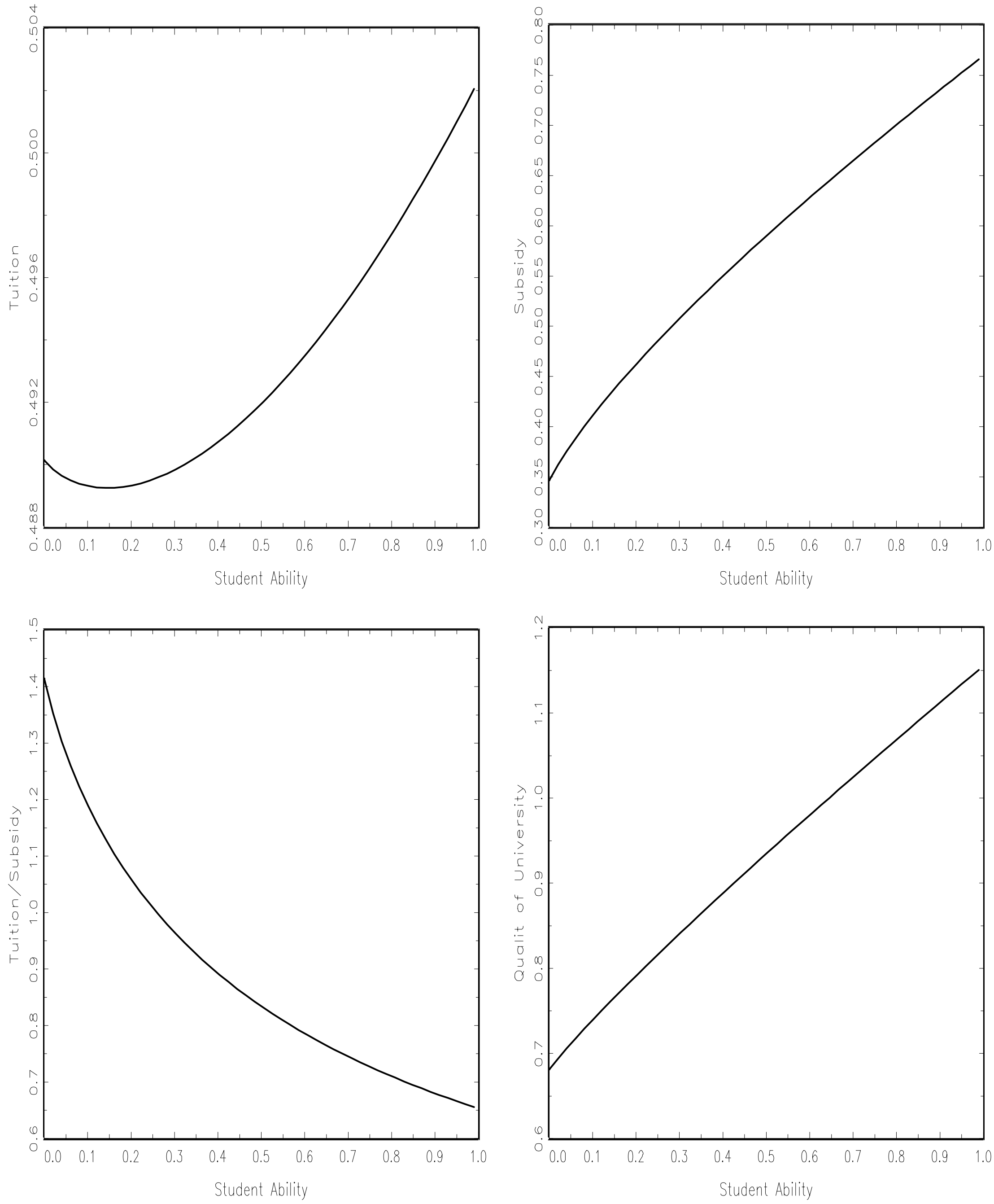

Figure 22. The Effect of an Increase in a_m on Tuition, Subsidy, Tuition /Subsidy Ratio, and Quality of University: Uniform Distribution of Student Ability

subsidy is always positive as shown in Figure 22.

The effect of an increase in a_m on tuition/subsidy ratio depends on how fast tuition or subsidy rises as marginal ability rises. In Figure 22, we draw the graph for tuition/subsidy ratio. The effect of an increase in a_m on tuition/subsidy ratio is negative, because the level of tuition does not increase much, but the subsidy rises much more quickly as a_m rises.

The effect of an increase in income on the educational expenditure is

$$\frac{de}{dy} = \frac{NV_{2xx}\left(x_{11}\right)V_{2xx}\left(x_0\right)}{D} > 0 \tag{3.44}$$

Therefore, regardless of what kind of ability distribution is, with our specific form of additively separable functions, as income rises, the educational expenditure will always rise. We confirm this by considering different income levels. The effect of an increase in income on head tax is

$$\frac{dh}{dy} = \frac{AMG_{ee}C_{1y} + N_{11}V_{2xx}\left(x_0\right)V_{2xx}\left(x_{11}\right)}{D} \tag{3.45}$$

From our simulation, we observe that there is almost no effect of an increase in income on head tax. This implies that C_{1y} is positive. The effect of an increase in income on tuition is positive, because

$$\frac{dt}{dy} = \frac{-\theta C_{1y}AMG_{ee}}{D} \tag{3.46}$$

which also implies that C_{1y} is positive.

Instead of using uniform distribution of students' ability, we use the beta distribution to check what the effect of an increase in a_m on expenditure, head tax,

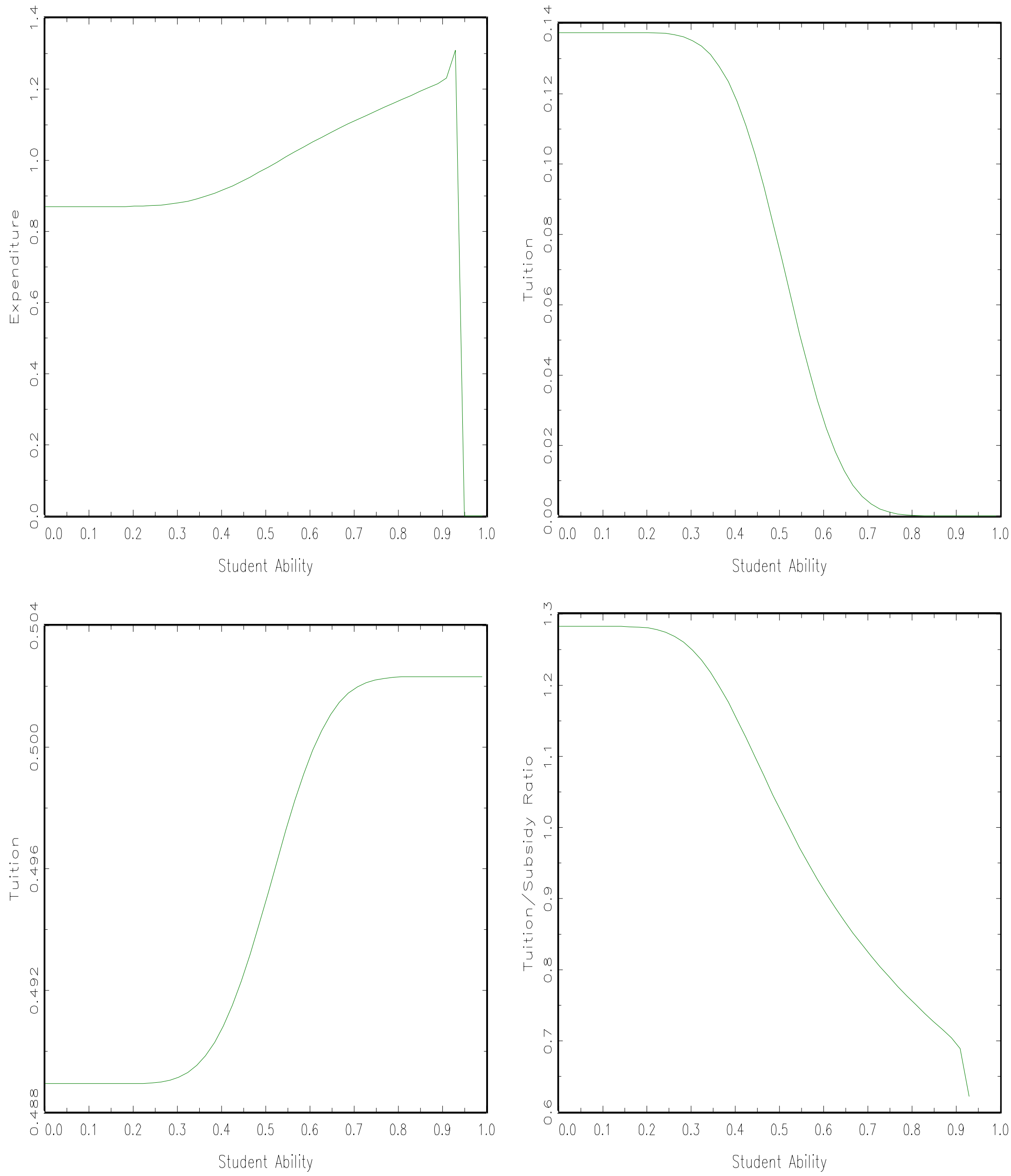

Figure 23. The Effect of an Increase in a_m on Expenditure, Head Tax, Tuition, and Tuition /Subsidy Ratio: Beta Distribution of Student Ability

tuition, and tuition/subsidy ratio is. This is shown in Figure 23. We can say that the effect of an increase in a_m is not much different from the case of a uniform distribution. We observe that as a_m rises, the educational expenditure also increases, except some ranges between a_m=0.95 and a_m=1. The reason why expenditure decreases as a_m rises is that change in aggregate marginal gain from an increase in expenditure becomes negative, and that the distribution values in the ranges between a_m=0.95 and a_m=1 are almost zeros. The rate of rise of expenditure in the case of beta distribution is lower than that of uniform distribution. The same pattern is observed for tuition, subsidy, and quality of university. Regarding the effect of an increase in a_m on head tax, we do not observe an increase in head tax up unlike the case of uniform distribution. Tuition/subsidy ratio decreases much more slowly than the uniform distribution case.

III.3 Simulation Result: Overall Equilibrium

Given a uniform distribution of student ability, we investigate the effect of change in income on marginal ability, expenditure, tuition, tuition/subsidy ratio, and quality of university. For ten different state median incomes, we show the simulation results in Table VII. From our baseline model, in U.S. expenditure, tuition, and tuition/subsidy ration, respectively, are \$11,209, \$4,934, and 0.79 from our simulation, which are higher than the real data for Texas from Table IV. In overall equilibrium, as income rises, more students will attend the university, as shown that there is a decrease in the marginal ability from Table VII. The effect of an increase in income on the educational expenditure is positive as derived in (3.44). We can confirm that as income increases, the expenditure rises in Table VII. The effect of an increase in income on the head tax is ambiguous as shown in (3.45). Note that in our

Table VII. Change in Income : Uniform Distribution

States	Income Level	a_m	h	e	t	t/s	q	r
Oklahoma	43,138	0.61	840	10,138	4,124	0.69	0.93	41,177
Florida	44,829	0.61	855	10,361	4,287	0.71	0.94	41,261
South Dakota	45,043	0.61	857	10,389	4,308	0.71	0.94	41,265
Texas	46,757	0.61	871	10,611	4,474	0.73	0.95	41,341
North Carolina	46,973	0.61	873	10,638	4,495	0.73	0.95	41,344
New York	52,799	0.60	918	11,365	5,058	0.80	0.99	41,573
Indiana	52,962	0.60	919	11,386	5,074	0.80	0.99	41,592
Washington	53,153	0.60	920	11,408	5,093	0.81	0.99	41,588
Colorado	53,632	0.60	923	11,467	5,139	0.81	0.99	41,617
California	53,807	0.59	925	11,487	5,156	0.81	0.99	41,612
U.S.	51,518	0.60	909	11,209	4,934	0.79	0.98	41,526

simulation, $C_{1y}>0$. Therefore, as income increases, the dominance of the second term of (3.45) over the first term becomes no longer true, so that the effect of an increase in income on the head tax may be negative as shown in Table VII. The effect of an increase in income on tuition is positive if $C_{1y}>0$, which is true in our simulation, as shown in (3.46). It is straightforward to know that the quality of university will increase as income rises, because there is no change in student input, but an increase in educational expenditure. Tuition/subsidy ratio rises as income increases, because tuition rises faster than subsidy. The annualized income of the marginal ability student rises as income increases, because quality of university increases with no change in the marginal ability.

Since each state differs widely in location, industry, and resource, the degree of attraction to college education will be different. According to Goldin and Katz (1998), the state government regards the public universities as the main organizations

Table VIII. Change in Reservation Wage Income: Uniform Distribution

Reservation Wage	a_m	h	e	t	t/s	q	r
30,000	0.49	1,212	10,739	4,905	0.84	0.93	36,188
31,000	0.52	1,173	10,839	4,909	0.83	0.94	37,426
32,000	0.54	1,132	10,938	4,913	0.82	0.95	38,653
33,000	0.57	1,087	11,040	4,917	0.80	0.96	39,917
34,000	0.59	1,041	11,139	4,922	0.79	0.97	41,147
35,000	0.62	990	11,242	4,926	0.78	0.99	42,415
36,000	0.64	938	11,344	4,932	0.77	1.00	43,673
37,000	0.67	883	11,446	4,937	0.76	1.01	44,922
38,000	0.69	826	11,549	4,943	0.75	1.02	46,186
39,000	0.72	764	11,653	4,948	0.74	1.03	47,465
40,000	0.75	699	11,761	4,955	0.73	1.04	48,784
41,000	0.78	634	11,863	4,961	0.72	1.06	50,024

to improve the economic development of the states. Borjas and Ramsey (1995) provide estimating return wage differential among college graduates and high school graduate for the 44 metropolitan areas. Averaging log wage differential into the state levels, we have 0.47 for California, 0.5 for Florida, 0.42 for North Carolina, and 0.46 for Texas. In U.S., college graduates earned 46.6 percent more than high school graduates. Since the annual wage income of high school graduate was \$34,260, we have \$49,970 for the wage of college graduates. There are two ways to do the comparative statics of return function. One way is to change the wage of high school graduates. The other way is to have a change in the constant term in the Cobb-Douglas return function which implies a change in wage return of college graduates, but no change in the wages of the high school graduates.

In Table VIII, we show the effect of a change in the wage of high school

Table IX. Change in μ: Uniform Distribution

μ	a_m	h	e	t	t/s	q	r
5	0.84	305	8,032	3,335	0.71	0.85	42,208
5.1	0.81	363	8,065	3,331	0.70	0.85	42,100
5.2	0.79	419	8,099	3,328	0.70	0.84	41,997
5.3	0.76	474	8,133	3,324	0.69	0.84	41,883
5.4	0.73	527	8,167	3,321	0.69	0.84	41,780
5.5	0.71	578	8,203	3,317	0.68	0.83	41,693
5.6	0.69	627	8,239	3,314	0.67	0.83	41,605
5.7	0.67	676	8,276	3,311	0.67	0.83	41,517
5.8	0.65	723	8,313	3,308	0.66	0.83	41,433
5.9	0.63	769	8,349	3,305	0.66	0.83	41,331
6	0.61	814	8,387	3,302	0.65	0.82	41,261
6.1	0.59	858	8,425	3,299	0.64	0.82	41,179

graduates. As reservation wage increases, the option of college attendance becomes less attractive so that the marginal ability will increase. With an increase in the marginal ability, we know that from state optimization, expenditure rises, head tax decreases, and tuition increases, except much lower marginal ability. Note that subsidy increases, because the number effect dominates the tax effect. Therefore, tuition/subsidy ratio increases. Because of both higher marginal ability and more expenditure, quality of university increases, as shown in Table VIII. The second way to apply wage differential to our model is to change the constant term. In Table IX, we show the effect of change in reservation wage on the equilibrium. As reservation wage increases, the college education becomes more attractive so that more students will attend the university, because the less ability student will become marginal

Table X. Change in w: Uniform Distribution

w	a_m	h	e	t	t/s	q	r
0.90	0.73	144	11,053	9,761	7.55	1.00	47,103
0.91	0.70	280	11,081	8,808	3.88	1.00	45,924
0.92	0.67	437	11,107	7,846	2.41	0.99	44,796
0.93	0.65	614	11,132	6,877	1.62	0.99	43,696
0.94	0.62	810	11,152	5,902	1.12	0.98	42,602
0.95	0.60	1,025	11,171	4,923	0.79	0.98	41,538
0.96	0.58	1,258	11,186	3,940	0.54	0.97	40,480
0.97	0.55	1,506	11,202	2,956	0.36	0.97	39,480
0.98	0.53	1,771	11,211	1,970	0.21	0.96	38,438
0.99	0.51	2,050	11,222	984	0.10	0.96	37,455

student. Unlike change in reservation wage income, the change in the reservation wage affects state government directly. In (3.9), we know that aggregate marginal gain from expenditure will increase. Therefore, the government will increase educational expenditure. Given marginal ability, for the state government optimization, we know that the EL_e shifts upward, but the EL_h does not shift. Therefore, as the reservation wage increases, we observe that both expenditure and head tax increase. From Table IX, we observe that tuition will decrease, because otherwise less able student will not attend the university, even though the return to education gives some incentive to attend the university. As we can see from Table IX, the return from college education for the marginal ability student becomes less as the reservation wage increases. Even though it is not easy for us to quantify political weight, we can investigate the role of political considerations on the optimal choice of funding instruments. In Table X, we report the effect of an increase in the political

Table XI. Change in Income: Beta Distribution

States	Income Level	a_m	h	e	t	t/s	q	r
Oklahoma	43,138	0.63	158	9,653	4,191	0.77	0.84	40,337
Florida	44,829	0.63	168	9,850	4,354	0.79	0.85	40,395
South Dakota	45,043	0.63	168	9,877	4,375	0.80	0.85	40,422
Texas	46,757	0.63	178	10,076	4,541	0.82	0.86	40,474
North Carolina	46,973	0.63	180	10,096	4,562	0.82	0.86	40,474
New York	52,799	0.62	210	10,740	5,127	0.91	0.89	40,681
Indiana	52,962	0.62	211	10,760	5,143	0.92	0.89	40,699
Washington	53,153	0.62	212	10,779	5,162	0.92	0.89	40,693
Colorado	53,632	0.62	214	10,831	5,208	0.93	0.89	40,716
California	53,807	0.62	216	10,848	5,225	0.93	0.89	40,708
U.S.	51,518	0.62	203.11	10,603	5,003	0.89	0.88	40,649

weight. When w increases, the state government values those enrolled households relatively more than the non-enrolled households. Therefore, tuition decreases and tax increases so that tuition/subsidy ratio decreases. More students are enrolled at the university in equilibrium.

So far, we assume that the ability distribution is uniform. With a beta distribution of students' ability, we investigate the effect of change in income. As explained before, using PSAT score distribution of U.S., we investigate the effect of an increase in the median income in Table XI. Given the same median income, change in distribution of students' abilities from uniform distribution to beta distribution brings higher marginal ability, because the average student ability increases less in the beta distribution than the uniform distribution. For the state government's optimization, because of beta distribution, the aggregate marginal gain from expenditure will be smaller than the uniform distribution. Therefore, the educational expenditure is in this beta distribution case is smaller than the head tax in

Table VIII. To the opposite, tuition will be higher than the uniform distribution. Therefore, tuition/subsidy is bigger than the uniform distribution. Given the beta distribution, the effect of an increase in income on marginal ability, expenditure, head tax, tuition, tuition/subsidy ratio, and quality of university is similarly explained as the uniform distribution.

From our simulation, we learn that differences in median income can explain why we have differences in the mix of funding. The higher median income will bring higher tuition/subsidy ratio and higher university quality. The wage differential between college graduates and high school graduates also explain the differences in the mix of funding. Tuition/subsidy ratio is higher with the bigger wage differential. Different political weight of state government can explain the mix of funding in public higher education. The higher political weight on the college enrollees results in the lower tuition/subsidy ratio. Different distribution of students' abilities also explains the mix of funding across states.

CHAPTER IV

CONCLUSION

McPherson and Schapiro (2003) point out that over the past 60 years user charge finance has gradually replaced tax financing in higher education. Furthermore, we observe more divergence in the relative usage of user charge to tax finance across states. Still, the between-states inequality dominates the within-state inequality in terms of tuition/ subsidy ratio. This dissertation has tried to give a theoretical foundation for the relative use of general state subsidies (tax finance) and tuition (user charge finance) in the state financing of higher education. As mentioned in the literature review in Chapter I, there are few articles dealing with the simultaneous use of both methods of financing methods. We develop a model which yields the mixed financing methods in the equilibrium public finance of a private good. Another contribution is to study the comparative statics of the model. Both analytical and numerical simulation comparative statics results were obtained.

In this study, we only consider the State Regime in which the state government chooses tuition, tax, and expenditure and the state university simply is treated as a passive agent. The state government is assumed to take the marginal student ability as given. Therefore, the model resembles the competitive market analysis. The households who have a child decide whether or not to enroll their children at the university, taking head tax, tuition, and quality of university as given. In the household equilibrium, their perceived quality of university is equal to the actual quality of university.

The first first-order condition for the state government's optimization shows how to redistribute the income among the types of households. The second first-order condition deals with the allocation problem in the economy. Note that holding tax constant, a change in tuition is equivalent to change in expenditure by the state budget constraint. The state government affects the public good, i.e. the quality of the university, directly. The solution to the allocation problem leads to a modified Samuelson condition.

Combining the two first-order conditions, we show that under certain conditions, we have an interior solution of both head tax and expenditure. We then derive the effect of change in political weight and in median income on head tax, tuition, and expenditure. Since it is impossible for us to do more comparative statics, in Chapter III, we use a simulation method to derive our comparative statics. Using a uniform distribution of students' abilities, we study the effect of an increase in income, the effect of a change in wage differential between college graduates and high school graduates, and the effect of a change in political weight. As the median income rises, both tuition/subsidy ratio and university quality increase, and marginal ability decreases. As college wage differential increases, tuition/subsidy ratio, university quality, and marginal ability decrease. As the state government views those enrolled-households more importantly than those non-enrolled households, tuition/subsidy ratio, university quality, and marginal ability decrease.

For empirical work on higher education funding, our model suggests that a simultaneous equation model is required. Holding expenditure constant, Lowry (2001) estimates a system of four equations: state appropriation, tuition, spending on research, and spending on public service. Using 428 public universities in all 50 states, interestingly, Lowry (2001) tries to test for the effect of differences in financial autonomy of universities. We have several hypotheses from our theory. One of the hypotheses is that an increase in the median income raises tuition/subsidy ratio, but (almost) no change in quality of university. That is, recession may bring a financial stress for the university, but no decrease in quality of university. Furthermore, when the households expect that college wage differential between college graduates and high school graduates increases, we predict that expenditure increases, tuition decreases, and tuition/subsidy ratio decreases.

Theoretically, in our future research we may allow income to be heterogeneous in order to find out the effect of change in income distribution on our endogenous variables. Since we assume that the state government takes the minimum ability as given, we may expand our model so that it allows the government to know the household demand curve for entry. In this case, the government will decide head tax, tuition, and expenditure subject to the additional marginal household behavioral constraint. Note that the state government has to consider how many households will send their children to the university when it decides its choice variables.

Finally, we consider only the State Regime in which state government decides everything and the public university is passive. We can consider the University Regime in which state government decides head tax, and the public university decides user charge and expenditure. We may view the university as quality maximizing institution following Epple, Romano, and Sieg (2001). We have to develop the game theoretical model in order to consider the strategic interaction between state government and university.

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APPENDIX

GAUSS PROGRAM FOR SIMULAION

```
output file=c:\Gauss4.0\Simul\kimout    reset;
format /m1/rd 15,12;

alpha=0.5; beta=0.5;
lambda=0.4; kappa=1-lambda;
gam=0.4; delta=1-gam;
mu=5.7;
r0=3.43;
w=0.95;

om=w^(1/(1-beta));
sig=alpha*kappa*gam;
nu=alpha*lambda*gam;
ad=alpha*delta;

TN=105480101;
N1=38022115;
nn1=TN/N1;
ncase=1;
na=2000; achng=0.99/(na-1);
a=seqa(0, achng,na);
ncase=11;
emat=zeros(na,ncase);hmat=emat;tmat=emat;am_mat=emat;tsr=emat;subs=ematt;

/***Income Change***/
vecy={4.3138, 4.4829, 4.5043,4.6757,4.6973,5.2799,5.2962,5.3153,5.3632,5.3807,5.1518};
lcase=1; do while lcase<=ncase;
y=vecy[lcase];


/***Uniform Distriubtion Function***/
@p=1;q;@

/****Beta Distribution Function***/
x=0.483;
s=0.105;
q=(1-x)*(x*(1-x)/s^2-1);
p=x*(x*(1-x)/s^2-1);

proc g(a);
retp(a^(p-1).*(1-a)^(q-1));
endp;

x1=1|0;
B=intquad1(&g,x1);

fa = N1/B*a^(p-1).*(1-a)^(q-1);

Famc=zeros(na,1);
```

```
i=1; do while i<=na;
            am=a[i];
      x2=am|0;
            Famc[i] =N1/B*intquad1(&g,x2);
            i=i+1;
endo;

avg=zeros(na,1);
i=1;do while i<=na;
            am=a[i];
      N11=N1-Famc[i];
      x3=1|a[i];
      temp1=intquad1(&u,x3);
            avar=(1/N11)*temp1;
      avg[i,1]=avar;
i=i+1;
endo;


proc u(x);
retp(N1/B*x^p.*(1-x)^(q-1));
endp;

proc v(x);
retp(N1/B*x^(p+ad-1).*(1-x)^(q-1));
endp;

iam=1; do while iam<=na;
      am=a[iam];
      N11=N1-Famc[iam];
      x4=1|a[iam];
      temp2=intquad1(&v,x4);
      temp3=mu^alpha*avg[iam]^nu*sig*temp2;
      temp4=beta*N11;
      tau = (temp3 ./ temp4)^(1/(beta-1));
      theta=TN/N11;

/* Finding the Optimal Values for State Government */
x1=0; x2=y;
tol    = 1e-5 ;
maxit=20;

fmid=(om+theta-1).*tau.*x2^((1-sig)/(1-beta)) + om*x2 - theta*om*y;
f=(om+theta-1).*tau.*x1^((1-sig)/(1-beta)) + om*x1 - theta*om*y;

if (f*fmid .ge 0); print " root is outside of the boundary"; goto return1; endif;
rtbis=x1; dx=x2-x1;

j=1; do while j<=maxit;
dx=dx*0.5;
xmid=rtbis+dx;
fmid=(om+theta-1).*tau.*xmid^((1-sig)/(1-beta)) + om*xmid+om-theta*om*y;
if (fmid .le 0.0); rtbis=xmid; endif;
if(abs(dx) .lt tol); goto return1; endif;
j=j+1; endo;

return1:
```

```
emat[iam,lcase]=xmid;
temp5=(xmid-(1-om)*y)/(om+theta-1);
if temp5<0; hmat[iam,lcase]=0; else; hmat[iam,lcase]=temp5; endif;
tmat[iam,lcase]=emat[iam,lcase]-theta*hmat[iam,lcase];

/* Finding Households' Equilbirum:am value*/
z1=0; z2=1;
rtbis2=z1; dz=z2-z1;

j=1; do while j<=maxit;
dz=dz*0.5;
zmid=rtbis2+dz;
N11=N1-Famc[j];
temp6=(avg[j])^nu*(zmid)^ad;
temp7=((y-hmat[iam,lcase])^beta+r0^alpha-(y-hmat[iam,lcase]-
tmat[iam,lcase])^beta)/(mu^alpha*emat[iam,lcase]^sig);
fmid=temp6-temp7;
if (fmid .le 0.0); rtbis2=zmid; endif;
if(abs(dz) .lt tol); goto return2; endif;
j=j+1; endo;

return2:
am_mat[iam,lcase]=zmid;

iam=iam+1; endo;

/***Finding Overall Equilibrium***/

tol2=1e-3;

i=1; do while i<=na;
am_gap=a[i]-am_mat[i,lcase];
if(abs(am_gap) .lt tol2); goto return3; endif;
i=i+1; endo;

return3:
am_e=a[i];
h_e=hmat[i,lcase];
e_e=emat[i,lcase];
t_e=tmat[i,lcase];
N11=N1-Famc[i];
q_e=(avg[i])^lambda*e_e^kappa;
r_e=mu*(q_e)^gam*(am_e)^delta;
s_e=TN/N11*h_e;
tsr=t_e/s_e;

print am_e~h_e~e_e~t_e~tsr~q_e~r_e;

lcase=lcase+1; endo;
end;
```

VITA

Name	Dokoan Kim
Address	609-1 Dukchun-2 Dong Buk-Gu Busan, South Korea 616-817 Phone: (51)335-1607 E-mail: pdk335@hanmail.net
Education	Ph.D., Economics, Texas A&M University, August 2003. M.A., Economics, George Washington University, May 1996. B.A., Economics, Busan National University, February 1993.
Doctoral Dissertation	Taxes, User Charges and the Public Finance of College Education
Fields of Specialization	Public Finance Econometrics Industrial Organization