NON-DATA AIDED DIGITAL FEEDFORWARD TIMING ESTIMATORS FOR LINEAR AND NONLINEAR MODULATIONS

A Thesis

by

PRADEEP KIRAN SARVEPALLI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2003

Major Subject: Electrical Engineering

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ABSTRACT

Non-Data Aided Digital Feedforward

Timing Estimators for Linear and Nonlinear Modulations. (August 2003)

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We propose to develop new non-data aided (NDA) digital feedforward symbol timing estimators for linear and nonlinear modulations, with a view to reducing the sampling rate of the estimators. The proposed estimators rely on the fact that sufficient statistics exist for a signal sampled at the Nyquist rate. We propose an ad hoc extension to the timing estimator based on the log nonlinearity which performs better than existing estimators at this rate when the operating signal-to-noise ratio (SNR) and the excess bandwidth are low. We propose another alternative estimator for operating at the Nyquist rate that has reduced self-noise at high SNR for large rolloff factors. This can be viewed as an extension of the timing estimator based on the square law nonlinearity. For continuous phase modulations (CPM), we propose two novel estimators that can operate at the symbol rate for MSK type signals. Among the class of NDA feedforward timing estimators we are not aware of any other estimator that can function at symbol rate for this type of signals. We also propose several new estimators for the MSK modulation scheme which operate with reduced sampling rate and are robust to carrier frequency offset and phase offset.

To my Parents and my brother Bobby

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There have been many people to whom I am indebted and who have been instrumental in my finishing this thesis. I would first like to acknowledge my advisor, Dr. Erchin Serpedin, for his patience, for the willingness to wait when results were not forthcoming and for waiting through all the false alarms and all the ideas that led to nowhere. I am also thankful for his critical and painstaking review of my proposals and thesis which have rendered them of a better quality. Among my colleagues I would like to thank Kai Shi for the time spent in discussing and critically evaluating the ideas that I bounced off him. I have benefited much from his practical approach to research. Thanks to Jun, Janath, Yik-Chung for their positive and encouraging attitude to my hesitant steps in research. Thanks also to my friends Ashish, Ali, Deepak, Nandan and Vishal because of whom there was a cheerful and lively environment at home and work. Also I am grateful to Jais for his long distance support when his own commitments were pressing upon him. My family, though now far from this place, have been behind me all through this venture. For their long-suffering patience and willingness to let go when they did not agree with me, I cannot thank them enough. In a document where every word is weighed with caution and every statement expected to be rigorously justified and Reason to be the sole torch, it is almost an embarrassment to acknowledge one's gratitude to God. Yet to remain silent is just as difficult. In all the times past He has been "my refuge and strength, an ever-present help". Though the last to be acknowledged I owe everything to Him.

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CHAPTER I

INTRODUCTION

A. Symbol Timing Recovery

Symbol timing recovery is an important task in all digital communication receivers, and it is necessary for the optimal performance of the receiver. The received signal is corrupted by noise and is subject to a delay due to transmission across the channel. From the point of view of detection of the transmitted data it is necessary to know this delay. Imperfect knowledge of this delay affects the bit error rate (BER) of the receiver. In digital communications, a symbol is transmitted once every T seconds. This is called the symbol rate. At the receiving end the optimum receiver consists of a matched filter at the front end. In digital receivers, we will sample the signal and digitize it. The signal-to-noise ratio (SNR) at the output of the matched filter is time varying. Therefore it is critical to sample at the instant of maximum SNR. The SNR is maximum when the sampling instants coincide with that of the transmitter's clock. The intersymbol interference (ISI) is also minimum at these instants. Since in most communication systems the clock is not transmitted separately this will mean that we will have to extract the symbol transitions from the received data. Mathematically, we can represent this as follows

$$r(t) = s(t-\tau) + n(t),$$

where $s(t)$ is the transmitted signal, $r(t)$ is the received signal and $n(t)$ is the noise due to the channel and τ is the delay. The delay is normalized to be within the range

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$(-T/2, T/2]$. (Our model is rather simple and we have not included effects due to phase offset and multiplicative noise and other such effects). Symbol timing recovery (STR) involves estimating this delay so that we can sample at the optimal sampling instants. It can therefore be considered as a problem of parameter estimation.

There are many schemes for performing STR. They can be classified as being feedback or feedforward. One general feature of feedback schemes is that they take longer time to acquire the information about the timing delay. In situations where the delay information needs to be available with little latency this is highly undesirable. Feedforward schemes are preferred in such situations. Most feedforward schemes usually operate on a block of data and provide an estimate of the delay [1, 2, 3]. (Schemes which operate on a single block of data are called one shot estimators [4, p. 334-335]). Feedback schemes on the other hand estimate by using a tracking loop. This leads to a problem when the initial delay is near $\pm T/2$. Since these values are equivalent, feedback schemes oscillate around these two stable values being unable to converge. This phenomenon is often called hangup and is typical in feedback based estimators. The delay in the channel is usually not constant and keeps changing. Any estimator that needs to track this change whether feedback or feedforward will face a problem called cycle slip. This happens when the delay changes across the boundaries ($\pm T/2$). This leads to a temporary loss of synchronization and the estimator needs to re-synchronize. Such problems are more severe in feedback estimators which by their construction are tracking the delay. In feedforward estimators the problem can be decoupled and handled separately as in [1]. Solutions exist for the feedback estimators also.

Another classification of the estimators is also possible depending on whether the estimator makes use of the data transmitted i.e., the information symbols. If the estimation technique makes use of the transmitted data then it is called data aided and non-data aided otherwise. (Actually there is another class called decision directed but we can consider that as a special case of data aided and we will treat them as being data aided). Data aided schemes are more frequently used in conjunction with feedback estimators than with feedforward ones. In such schemes there is the freedom to use preambles or else use the tentative decisions of the demodulator. The latter approach is preferred when we do not want to send any preambles. In feedforward case we prefer to use a preamble if it is to be data aided. In burst type of communications this is going to affect the throughput adversely. So non-data aided estimators are preferred in such cases. When we compare data aided and non-data aided schemes usually the data aided estimators perform better than the non-data aided estimators [4, 5]. Often the complexity of non-data aided schemes is lesser than that of data aided ones. Also there is more robustness to the non-data aided schemes. Quite often it is possible to achieve as good a performance as the data aided ones. In this thesis we will focus on the subclass of estimators that are non-data aided and feedforward. In the next section, we will explain the perspective of our approach to this problem and henceforth we will confine ourselves to this class of estimators.

B. Motivation and Problem Definition

We will assume the following signal model. Later in the thesis we will modify it to make it more detailed. Once again let $s(t), r(t), n(t)$ refer to the transmitted signal, received signal and noise, respectively. Then we can write

$$r(t) = s(t - \epsilon T) + n(t), \tag{1.1}$$

where T is the symbol period and ϵ is the normalized timing delay. Our goal is to form an estimate $\hat{\epsilon}$. Clearly ϵ is within the range $(-0.5, 0.5]$. The received signal, usually after matched filtering is sampled at a rate P/T, where P is the oversampling ratio. Typically most of the estimators assume an oversampling ratio $P \geq 4$. This ensures that the signal does not alias and also certain nonlinear operations performed on the signal by the timing recovery circuit will not cause aliasing. (Often this translates to increased computational complexity).

But strictly speaking according to Shannon's sampling theorem, sampling at $P = 2$ contains sufficient statistics for all subsequent signal processing. Here we are assuming that the signal is band-limited. So the existence of sufficient statistics is in itself strong motivation for us to look at estimators that work at lower sampling rates. It may be that lower sampling rates might make the algorithms more complex and we might end up with greater complexity. Or their performance may not be as good as the ones at higher rates. The primary motivation for this work is the fact that sufficient statistics exist for a minimally sampled signal and we can form estimators at that rate. From a practical perspective our estimators must also be simple in order to retain the advantage of lower oversampling rates or at most they should not compromise too much on performance with respect to the estimators that assume higher sampling rates. There is one more reason why we should be interested in designing estimators of lower sampling rates. It is interesting to note that among the class of feedback timing estimators there exist estimators that require only one sample per symbol [6, 7]. However such schemes for the feedforward case are comparatively unknown[1].

[1]There have been two exceptions to this. Reference [8] claims that timing recovery can be done at the symbol rate. However, the input is still sampled at the Nyquist rate. Also, the details of the estimator are not fully clear with respect to its performance. It has not been compared to any existing estimator or the known theoretical limits. It has been claimed in [9] that timing recovery at sub-symbol rates is possible. But this approach is suboptimal and requires large packet length and also the performance is reasonable only when the packet length is around 1000 symbols. We are more interested in packets of very short length (around 100 symbols). Moreover the algorithm presented in [9] presents significant self-noise even at very large packet lengths.

This is rather strange because given a packet of data sampled at rate P/T, the same information is available to both the feedback and feedforward estimators. We would expect that complete equivalence would exist between the feedforward and feedback scenarios with respect to extracting this information. So from a purely theoretical perspective, it would be interesting to lower the sampling rate of the feedforward estimators.

Finally, from a system point of view it is useful to investigate estimators at lower rates. Typically we have an analog to digital converter (ADC) at the front end of the receiver that needs to work at the same speed as the oversampling rate. Any reduction in this rate will relax the design constraints on the ADC. But it might make the design of the anti-aliasing filter a little more constrained.

C. Outline of the Thesis

In the next chapter we will review the background of this work. We will look at the signal models for linear and nonlinear modulations and also briefly review some relevant estimators. Following that we will look at some extensions to existing estimators for linear modulations. We propose an ad hoc estimator that operates at an oversampling ratio of 2 for the logarithmic nonlinearity. The proposed estimator performs better than the existing estimators at this rate in the low SNR and low excess bandwidth regime. We also propose one more new estimator that can be considered as an extension of the scheme in [1] for operating at Nyquist rate. Among the Nyquist rate estimators it has lower self-noise when the excess bandwidth is high. In Chapter IV we consider estimators for the nonlinear modulations. In particular, we concentrate on the MSK type modulations. We propose two new estimators that can operate at the symbol rate. We consider two alternate estimators which are robust to frequency offset. Their performance is comparable to existing estimators. To conclude our primary goal in this thesis will be aimed at coming up with new estimators for timing recovery that are non-data aided and feedforward in nature and well suited for burst communications. Our main emphasis will be to make them operate with minimal oversampling.

CHAPTER II

BACKGROUND

In this chapter we present the signal models used for derivation of the estimators and review some of estimators which we intend to extend and improve. We shall also establish the notation to be used for the rest of the thesis. But first we shall consider some performance measures for estimators.

A. Performance Measures for Estimators

Since we are concerned with the problem of estimation it is useful to review some performance measures for the estimators that we plan to design. These will help us to compare different estimators and make an evaluation of estimators with respect to standard benchmarks.

1. Bias: The first most important property of any estimator is that it should be unbiased. Bias is defined as difference between the true value and the mean value of the estimate

$$\text{bias} = \epsilon - E[\hat{\epsilon}], \tag{2.1}$$

   where ϵ is the true value of the parameter to be estimated (in our case the timing delay) and $\hat{\epsilon}$ is the estimate. Bias increases the error of the estimate in certain ranges, consequently it alters the overall error performance defined in terms of the mean squared error (MSE).

2. Mean Squared Error: The second figure of merit that we are interested in is the error in the estimator. There are many measures of the error but the one commonly used is the mean squared error. One main reason for this is that it

is directly related to the SNR. This is defined as

$$MSE(\hat{\epsilon}) = E\left[(\hat{\epsilon} - \epsilon)^2\right]. \tag{2.2}$$

Another related measure is the variance of the estimator defined as

$$\text{var}(\hat{\epsilon}) = E\left[(\hat{\epsilon} - E[\hat{\epsilon}])^2\right]. \tag{2.3}$$

MSE is related to the variance of the estimator as follows

$$MSE(\hat{\epsilon}) = \text{var}(\hat{\epsilon}) + (\epsilon - E[\hat{\epsilon}])^2. \tag{2.4}$$

Clearly this indicates that bias increases the MSE of the estimator. Also we see that we should be careful in our use of variance as a performance measure if the estimators are unbiased only asymptotically. This is because when the estimates are made with small number of samples there will be more error due to the bias which is not completely removed. With unbiased estimators, MSE reduces to the variance of the estimator.

3. Lower Bounds on Performance: We frequently need to know if the estimators we designed can be improved further or if their performance is the best that we can achieve. In order to do this we need to establish bounds for the estimators telling us what can be achieved and what cannot be. Therefore, it is common to compare the estimator performance with respect to the variance or MSE with some bounds that are derived in estimation and detection theory. The most often used is the Cramer Rao Lower Bound (CRLB). This is defined as follows [10]

$$\text{var}(\hat{\epsilon}) = \frac{1}{E\left[\left\{\frac{\partial \ln p(r|\epsilon)}{\partial \epsilon}\right\}^2\right]}, \tag{2.5}$$

where $p(r|\epsilon)$ is the probability distribution of the received signal given ϵ. Frequently this bound is not easy to calculate analytically, therefore a modified bound is often used to evaluate the performance of the estimator. This is the Modified Cramer Rao Bound (MCRB) developed in [11]. Apart from its ease of calculation with respect to the CRLB, it turns out to be the same as the CRLB in many cases. One problem with MCRB is that it gives a pessimistic picture of the estimator performance when the CRLB and the MCRB differ greatly. In this thesis, we will refer to the MCRB for performance evaluation. For a linear modulation with pulse shape $g(t)$, this bound is defined as follows [11, 12, p. 65]

$$MCRB(\epsilon) = \frac{1}{8\pi^2\psi L(E_s/N_o)}, \tag{2.6}$$

$$\psi = T^2\frac{\int_{-\infty}^{\infty} F^2|G(F)|^2dF}{\int_{-\infty}^{\infty}|G(F)|^2dF}, \tag{2.7}$$

where $G(F)$ is the Fourier transform of the pulse $g(t)$, L is the number of symbols used for estimation, E_s is the energy per symbol and N_o is the noise spectral density. For the spectral root raised cosine pulse (RCOS) [12, p. 12] with an excess bandwidth factor of β, this is given by

$$MCRB(\epsilon) = \frac{1}{8\pi^2 L(E_s/N_o)}.\frac{1}{1/12+\beta^2(1/4-2/\pi^2)}. \tag{2.8}$$

The MCRB for the CPM signals is as follows

$$MCRB(\epsilon) = \frac{1}{8\pi^2\psi L(E_s/N_o)}, \tag{2.9}$$

$$\psi = E[I_k^2]h^2T\int_{-\infty}^{\infty} g^2(t)dt, \tag{2.10}$$

where I_k are the data symbols and h is the modulation index. These will be the bounds used for performance analysis throughout this thesis. For the CPM signals we will focus on the MSK type modulations ($h = 1/2$).

B. Signal Model for Linear Modulations

We will assume the following complex base band signal model for our subsequent discussion on the linear modulations. Again let $s(t), r(t)$ and $n(t)$ represent the transmitted signal, received signal and noise, respectively. Then we can write

$$s(t) = \sum_k I_k g(t - kT), \tag{2.11}$$

$$r(t) = e^{j2\pi f_e t/T} s(t - \epsilon T) + n(t), \tag{2.12}$$

where f_e is the normalized carrier frequency error. (This is zero in case of baseband signals). We will now make the following assumptions:

1. The data symbols I_k are i.i.d. and that their variance $E[I_k I_k^*] = 1$.

2. The pulse $g(t)$ is assumed to be band-limited to $\pm(1+\beta)/2T$. In our discussion we will assume that the pulse is RCOS with excess bandwidth factor β.

3. We assume that n(t) is circular Gaussian with spectral density N_o.

C. Review of Some Existing Non-Data Aided Feedforward Schemes

The basic structure of these schemes is usually as shown in Fig. 1.

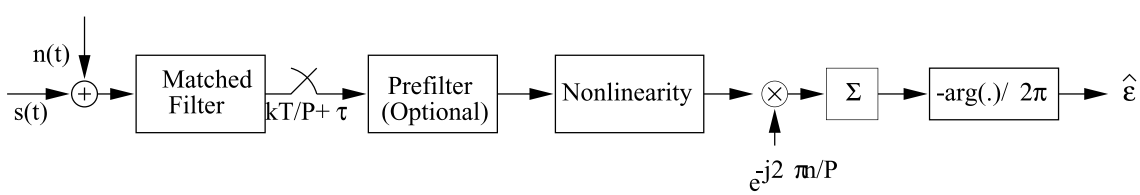

Fig. 1. Block Diagram of a Typical Feedforward Symbol Timing Recovery Estimator

The signal is usually filtered by a matched filter and then passed through a nonlinearity. The output of the nonlinearity contains a spectral line at the symbol rate or a harmonic of it. The phase of this spectral line contains information about the delay. Taking into account the matched filtering we obtain the following

$$x_c(t) = r(t) * g(t) + n(t) * g(t) \tag{2.13}$$

$$= \sum_k I_k h_c(t - kT - \epsilon T) + v(t), \tag{2.14}$$

where $v(t)$ is the filtered noise and $h_c(t)$ is the convolution of the transmit and receive filters (including any optional prefilter). Discretization gives us the following model

$$x(n) := x_c(nT/P), \tag{2.15}$$

$$h(n) := h_c(nT/P - \epsilon T), \tag{2.16}$$

where the notation := stands for *is defined as*. Now we can write a general form for the schemes that employ the nonlinearity as follows [15]

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\left\{\sum_{n=0}^{PL-1} F(x(n))e^{-j2\pi n/P}\right\}, \tag{2.17}$$

where $F(.)$ is some nonlinearity and L is the number of transmitted symbols used for estimation. The most commonly employed nonlinearities are square law (SLN), absolute value (AVN), logarithmic function (LOGN) and fourth power (FLN). Each of the estimators have their advantages at different operating conditions or system requirements. The square law is preferred for its low complexity and ease of implementation. The absolute and log nonlinearity are preferred when the operating SNR and the excess bandwidths are low. The fourth law seems preferable when the operating range is medium SNR. However, these are not optimal in terms of performance with respect to the MSE.

One thing which we must note is that these estimators involve increasing of the signal bandwidth at the output of the nonlinearity. So in order to recover the timing information it is necessary that we do not alias after the nonlinear operation. An analysis of the timing recovery operation with a general nonlinearity in [13, 14] showed that the nonlinearity can be represented as a sum of even powers of the input signal. All of them contain an even power (≥ 2) of the input signal and consequently the bandwidth at the output of the nonlinearity at least doubles. So the sampling needs to take care of this. In all the above estimators the spectral line we are interested in is the frequency component at $1/T$. So we can allow aliasing beyond $1/T$. If we consider the square law estimator, then assuming an excess bandwidth factor β, the sampling rate needs to be at least $(2+\beta)/T$ to prevent aliasing of the component at $1/T$ at the squarer output. Typically the oversampling ratio is chosen to be 4 since it is not so easy to realize any arbitrary multiple of symbol rate. However, the existence of the symbol rate feedback timing estimators implies that estimators that use an oversampling of 4 are redundant in the sense of sufficient statistics. Before we design estimators at symbol rate, we will start with estimators at Nyquist rate. The estimators proposed in [15, 16] showed one way how this can be achieved. All the previous estimators exploited the existence of the term x^2 in their Taylor series expansion. In a sense these estimators were exploiting the periodicity of the autocorrelation function but only that at lag zero. The new estimators exploit the lags other than zero to achieve this.

1. Errors in Estimators

We will illustrate in detail the performance measures we mentioned earlier by considering the performance of the square law estimator. The simulations are done with a QPSK modulation and with $L = 100$ symbols. Each data point is obtained by running 1,000 Monte Carlo runs. The excess bandwidth factor is 0.5 and $P = 4$. The simulation results are shown in Fig. 2. As pointed out in [1] there are three

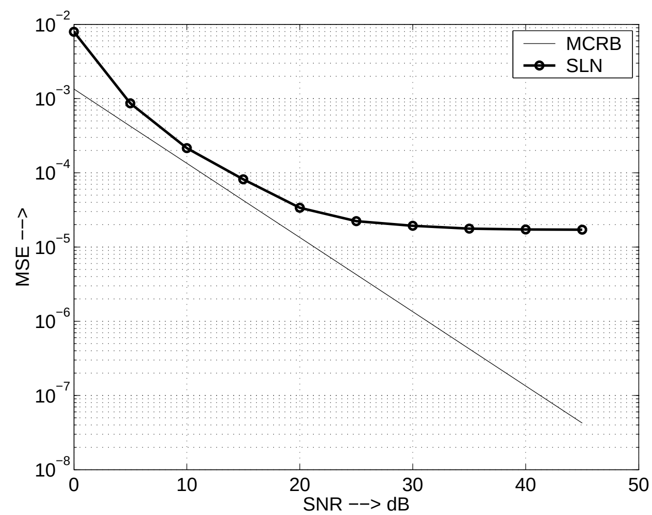

Fig. 2. MSE vs SNR of SLN for $P = 4$

sources of errors in the estimates we form. These errors dominate in different regions. In the low SNR regime the errors are dominated by the noise. Observe the error floor in the high SNR regions. This remains even with increasing SNR. This error floor is due to the randomness of data. This is responsible for the variation in the estimates of the autocorrelation. This is called self-noise and cannot be made zero no matter how high the SNR is made. There are two things we can do to reduce this self-noise, one is to preprocess the data, the other is to increase the estimation length because asymptotically the self-noise is zero for this estimator without any preprocessing other than the matched filter [1]. We will look at the former approach while dealing with some of the estimators we propose. The third component is due to the cross correlation between the signal and the noise. And as expected it dominates the errors in the mid SNR range where both contribute. The qualitative terms low, mid and high SNR are somewhat estimator dependent and will be clear when we look at their MSE performance.

2. Extension of the Square Law Timing Recovery Using Prefilters

As pointed out in the previous section the square law timing recovery suffers from self-noise. It was recognized that this self-noise arising from the data sequence modulating the transmit pulse can be made to vanish in the case of analog synchronizers by appropriately filtering the received signal after the matched filter [17]. The condition that ensures zero self-noise was related to the overall filter response of the signal as follows

$$H_c(1/2T + F) \quad = \quad H_c^*(1/2T - F), \tag{2.18}$$

where $H_c(F)$ is the Fourier transform of $h_c(t)$. The filter that is used effectively shapes the overall pulse to have conjugate symmetry around $1/2T$. This was later extended to the digital synchronizers with rigor in [18]. We will illustrate this graphically to make it clear. Assuming that the raised cosine pulse is used for signaling, we have the overall response at the output of the matched filter as shown in Fig. 3. The frequency is normalized so that $1/2T$ corresponds to 0.5. Observe that the response is not symmetric at $1/2T$. The prefilter $H_{pref}(F)$ that needs to be used for achieving this self-noise free condition is very simple to design and can be derived from the overall response as follows

$$H_{pre}(F) \quad = \quad \frac{1}{2}\left[H_c(F - 1/T) + H_c(F + 1/T)\right]. \tag{2.19}$$

If we consider filtering the received signal by the following filter $H_{pre}(F)$, (this is nothing but the overall response shifted to $\pm 1/T$), then we observe that the new response has a symmetry around $1/2T$. We will make use of these concepts of prefiltering to a great extent later in our work. Within a scaling factor the symmetric pulse that we obtain after prefiltering in case of the RCOS is shown in Fig. 4.

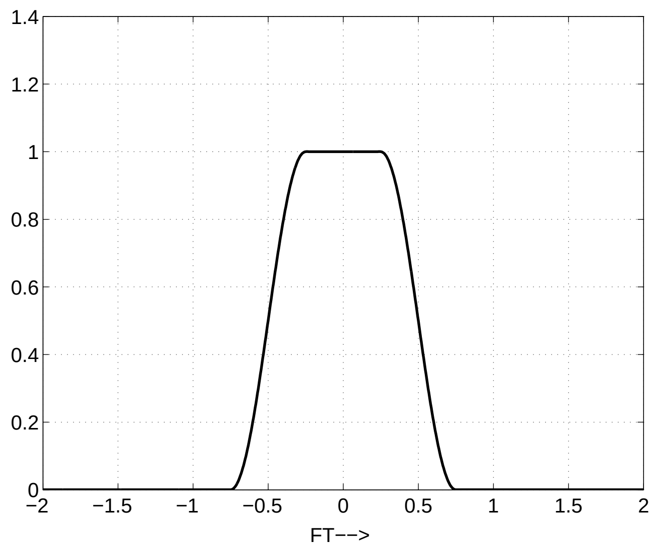

Fig. 3. Raised Cosine Pulse ($P = 4$)

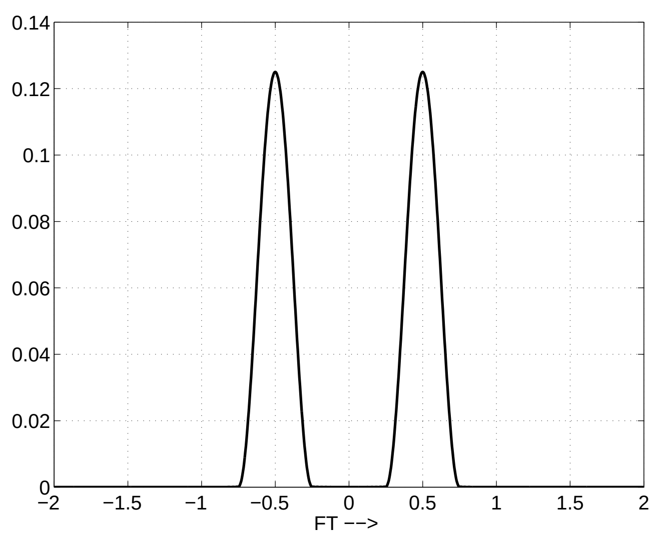

Fig. 4. Prefiltered Pulse That Has Symmetry Around $1/2T$ ($P = 4$)

3. Maximum Likelihood Estimator for Timing Recovery

In [12, p. 398-402] an estimator that was derived according to maximum likelihood principles was presented. We will consider this briefly since this estimator can operate at $P = 2$ and we intend to make use of this in our work indirectly. The estimator is given by the following

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\left\{\sum_{n=0}^{PL-1} y(n)z(n)\right\}, \tag{2.20}$$

$$y(n) = r(n)e^{-j\pi n/P}, \tag{2.21}$$

$$z(n) = \sum_{k=0}^{PL-1} r^*(k)q(n-k)e^{-j\pi k/P}, \tag{2.22}$$

$$Q(F) = G(F-1/2T)G^*(F+1/2T), \tag{2.23}$$

where G(F) is the Fourier transform of the transmit pulse. A block diagrammatic representation of this is shown in Fig. 5. Note that there is no need for matched filtering in this scheme (indicated by the use of $r(n)$ instead of $x(n)$). We shall refer to this estimator as ML estimator.

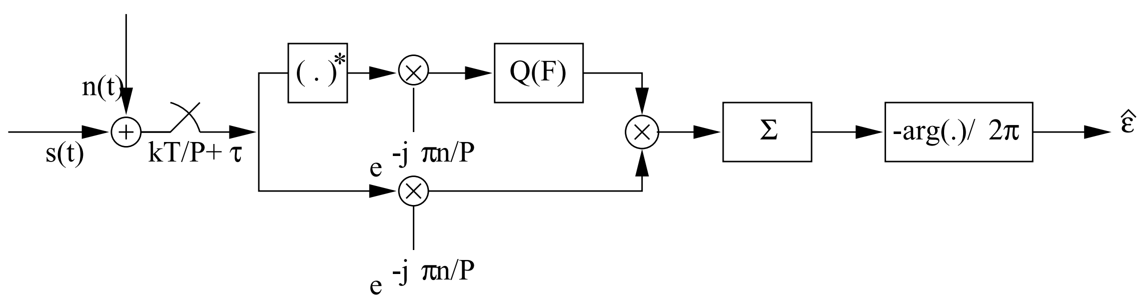

Fig. 5. Implementation of ML Estimator

D. Cyclostationary Framework for Feedforward Estimators [19, 20]

In many of the estimators that we deal with, a feature that we consistently exploit is the cyclostationarity of the received signal. For this reason we shall outline briefly the cyclostationary work originally proposed in [19] and later extended in [20].

A signal is said to be cyclostationary if it exhibits periodicity in its mean and correlation and all other higher order moments . However, we shall be more interested in a relaxed definition of cyclostationarity called wide sense cyclostationarity (WSC). A signal $x(n)$ is said to be wide sense cyclostationary if its mean $(m_x(n))$ and the autocorrelation function $(r_x(n;\tau))$ are periodic

$$m_x(n) := E[x(n)] = E[x(n+K)], \tag{2.24}$$

$$r_x(n;\tau) := E[x^*(n)x(n+\tau)] = E[x^*(n+K)x(n+K+\tau)], \tag{2.25}$$

where K is the period of the mean and the autocorrelation functions. (The signals we deal with have this period same and equal to the oversampling ratio P). Being periodic $r_x(n;\tau)$ can be expanded into a Fourier series. The coefficients of the Fourier series are given by

$$\begin{aligned} R_x(k;\tau) &:= \frac{1}{P}\sum_{n=0}^{P-1} r_x(n;\tau)e^{-j2\pi kn/P} \\ &= \frac{1}{P}\sum_{n} h^*(n)h(n+\tau)e^{-j2\pi kn/P}, \end{aligned} \tag{2.26}$$

where the last step comes from the substitution of the definitions of $r_x(n;\tau)$ and the signal model we have assumed. $R_x(k;\tau)$ is termed the cyclic correlation at cycle k and lag τ. (The cycles k have a one to one correspondence with the harmonics of the signal at the output of the nonlinearity). Estimates of the cyclic correlation coefficients are given by [20]

$$\hat{R}_x(k;\tau) = \frac{1}{PL}\sum_{n=0}^{PL-\tau-1} x^*(n)x(n+\tau)e^{-j2\pi kn/P}, \tau \geq 0. \tag{2.27}$$

We will consider the square law estimator in detail to understand a few aspects of the estimators. Using the cyclostationary framework we can write the square law estimator (SLN) as [19]

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\left\{\hat{R}_x(1;0)\right\}. \tag{2.28}$$

It was also shown in [20] that this can be related to the Fourier transform of $h_c(t)$ as follows

$$R_x(1;0) = \frac{1}{P}\int_{-1/2}^{1/2} H(f)H(f+1/P)df \tag{2.29}$$

$$= \frac{e^{-j2\pi\epsilon}}{T}\int_{-P/2T}^{P/2T} H_c(F)H_c(F+1/T)dF, \tag{2.30}$$

where $H(f)$ is the discrete time Fourier transform of $h(n)$ and $H_c(F)$ is the continuous time Fourier transform of $h_c(t)$. Taking the transmitted pulse as the RCOS pulse then the above equation can be graphically represented as shown in Fig. 6, where we have assumed $\beta = 0.5$ and $P = 4$. Let us note a few things about this. First the magnitude of the spectral line at cycle $k = 1$ (or equivalently $1/T$) depends on the nonzero overlap of $H(f)$ and $H(f + 1/P)$ which in turn is proportional to the excess bandwidth of the pulse. So timing recovery becomes difficult for lower rolloff factors. Secondly this is valid only if $P \geq 2 + \beta$. For $P = 2$ there is aliasing and the estimator needs to be modified. The modified estimator for $P = 2$ is as follows

$$\hat{\epsilon} = \frac{1}{2\pi}\arg\left(b\hat{R}_x(1;0) + j\hat{R}_x(1;1)\right) \tag{2.31}$$

$$= \frac{1}{2\pi}\arg\left(b\sum_{n=0}^{2L-1}|x(n)|^2e^{-j\pi n} + j\sum_{n=0}^{2L-2}\mathrm{Re}\{x^*(n)x(n+1)\}e^{-j\pi n}\right), \tag{2.32}$$

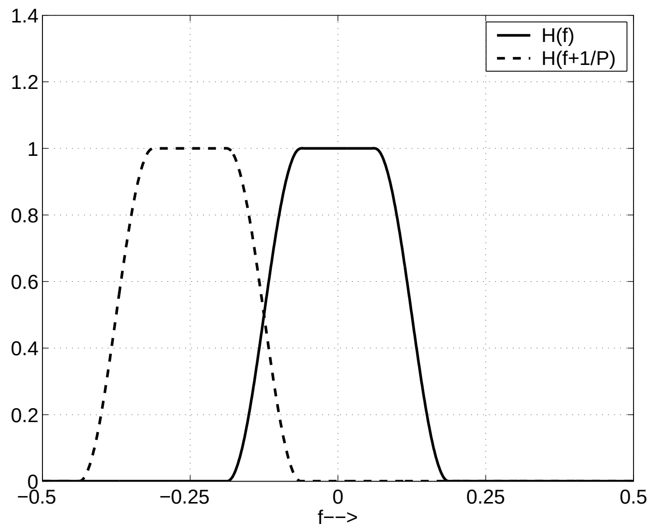

Fig. 6. $H(f)$ and $H(f+1/P)$ for P=4

where $\hat{R}_x(k;l)$ are the estimates of the cyclic correlations of the signal $x(n)$ at the cycle k and lag l, L is the number of symbols and b is a bias correction factor that depends on the transmit pulse shape. The bias correction factor is related to the pulse shape as follows [16]

$$b = G(1)/G(0), \tag{2.33}$$

$$G(\tau) = \frac{2}{T}\int_{-1/2T}^{1/2T} H_c(F+1/2T)H_c(F-1/2T)e^{j\pi\tau FT}dF. \tag{2.34}$$

Once again we see that the timing recovery operation depends on the excess bandwidth. We shall refer to this estimator as Wang's estimator.

E. Signal Model for Nonlinear Modulations

We also propose to look at estimators for continuous phase modulation (CPM) in this thesis. We shall use the following model for CPM signals. Once again $s(t)$, $r(t)$ are transmitted and received signals, respectively. Assume that $n(t)$ is circular Gaussian noise and I_k is a zero mean, i.i.d. sequence with $E[I_k I_k^*] = 1$. Then we have

$$\begin{aligned} s(t) &= \sum_k e^{j\phi(t;I)}, &(2.35)\\ \phi(t) &= \sum_k I_k q(t-kT), &(2.36)\\ r(t) &= s(t-\epsilon T)+n(t), &(2.37)\end{aligned}$$

where $q(t)$ is the phase pulse [4, p. 187] of the CPM signal. It is a little more difficult to classify the schemes in case of CPM signals so it must be kept in mind that the following is only a rough outline. The first category involves transforming the CPM signal such that we can access the phase and then design estimators that operate on the phase. The second class involves approximating the CPM signal by a sum of linearly modulated signals. We shall look briefly at some of the schemes in both these areas and later propose some alternate implementations, not necessarily an improvement over the existing schemes.

F. Review of Some Existing Schemes for CPM Signals

The feedforward schemes mentioned for the linear modulations will not work straightaway for the CPM signals because the information bearing signal is also present in the phase. We must use some additional techniques before we can extend the methods studied before. Specifically we will focus on the MSK type modulations.

1. Square Law Timing Recovery for GMSK

First we shall consider the extension of the square law nonlinearity based method to GMSK signals [21, 22]. It was shown in [23] that the CPM signal can be represented as a superposition of linear modulations. For a CPM signal with a partial response of L periods, this representation is as follows

$$r(t) = \sum_k \sum_{i=0}^{2^L-1} a_{i,k} C_i(t - kT - \epsilon T). \tag{2.38}$$

This in turn can be approximated by the dominant terms in the expansion. It was shown in [22] that nearly all the energy is present in the primary term associated with $C_0(t)$ for some important MSK type modulations. Then we can write

$$r(t) \approx \sum_k a_{0,k} C_0(t - kT - \epsilon T), \tag{2.39}$$

where $a_{0,k} = \exp(j\pi h \sum_{l=-\infty}^{k} I_l)$. For MSK type signals we can write a recursive relation for the $a_{0,k}$ as

$$a_{0,k} = j a_{0,k-1} I_k. \tag{2.40}$$

Using the above relation $s(t)$ can be written as a sum of two linearly modulated components which are in quadrature with each other.

$$r(t) = \sum_k b_{2k+1} C_0(t - 2kT - T - \epsilon T) + j \sum_k b_{2k} C_0(t - 2kT - \epsilon T),$$

where $b_{2k+1} = -b_{2k-1} I_{2k} I_{2k+1}$ and $b_{2k} = -b_{2k-2} I_{2k-1} I_{2k}$. It has been pointed out in [21] that this can be regarded as an OQPSK modulation and that we can extract the timing information from each of the channels separately. Note that the inphase and quadrature components are at a rate $1/2T$ and since the digital estimators give the normalized delay with respect to the rate, these estimates cannot be used straightaway. We need to combine the estimates from both the channels appropriately. The complete scheme is as follows. First we treat this as a linear modulation and perform matched filtering using $C_0(t)$ as the matched filter. The signal after the matched filter can be written as

$$x(t) = \sum_k b_{2k+1} h_c(t - 2kT - T - \epsilon T) + j \sum_k b_{2k} h_c(t - 2kT - \epsilon T),$$

where $h_c(t) = C_0(t)*C_0(t)$. We will define $x_I(t)$ and $x_Q(t)$ as the inphase and quadrature components of $x(t)$ and $x_I(n)$, $x_Q(n)$ as their discrete time versions, respectively.

$$x_I(n) := x_I(nT/P), \tag{2.41}$$

$$x_Q(n) := x_Q(nT/P). \tag{2.42}$$

Then we can extract the timing delay in each channel as follows

$$\hat{\epsilon_I} = -\frac{1}{2\pi}\arg\left\{\sum_{n=0}^{PL-1} |x_I(n)|^2 e^{-j2\pi/2P}\right\}, \tag{2.43}$$

$$\hat{\epsilon_Q} = -\frac{1}{2\pi}\arg\left\{\sum_{n=0}^{PL-1} |x_Q(n)|^2 e^{-j2\pi/2P}\right\}, \tag{2.44}$$

$$\hat{\epsilon} = \begin{cases} 2\hat{\epsilon_Q} & \text{if } |\hat{\epsilon_Q}| \le 0.25 \\ 2\hat{\epsilon_I} & \text{if } |\hat{\epsilon_I}| \le 0.25 \\ 2\hat{\epsilon_I} \text{ or } 2\hat{\epsilon_Q} & \text{else .} \end{cases} \tag{2.45}$$

A few things need to be pointed out here because they can cause some confusion. First note that the range of ϵ_I and ϵ_Q is $(-0.5, 0.5]$. So $2|\epsilon_I|$ can be greater than 0.5 whereas $\epsilon \in (-0.5, 0.5]$, therefore we consider only the estimate that is less than 0.25 in magnitude. A detailed relation between the two channel phases is found in [21]. A block diagram representation of this scheme is shown in Fig. 7.

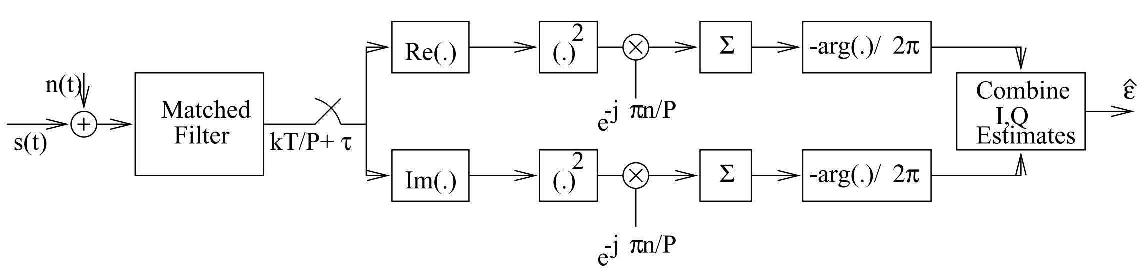

Fig. 7. Implementation of [21]

2. Lambrette and Meyr Scheme [3]

This scheme is based on transforming the CPM signal so that we can operate on the phase of the CPM signal. It was derived for MSK type of signals. In this scheme there is no need for matched filtering and we can directly operate on the received signal after anti-alias filtering. An analytic representation of the estimate is shown below

$$\begin{aligned} \hat{\epsilon} &= -\frac{1}{2\pi}\arg\left\{\sum_{k=0}^{P-1} y_k(n)e^{-j2\pi k/P}\right\}, & (2.46) \\ y_k(n) &= \sum_{n=0}^{L-1} \left|\arg\left[r(nT+T+kT/P)r^*(nT+kT/P)\right]\right|. & (2.47) \end{aligned}$$

Instead of the absolute value nonlinearity the square law nonlinearity can also be used in the above estimator. It was shown in [3] that the AVN performs better than the SLN. Both forms of the estimator require $P \geq 4$. We shall refer to this scheme as LM estimator.

3. Morelli and Vitetta Scheme

Another scheme that is based on approximating the CPM signal as a linear signal is the scheme proposed in [24]. In this reference the log likelihood function is derived for the approximated CPM signal and the timing estimator is derived as the value of ϵ that maximizes the log likelihood function. This estimate is given by

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\sum_{k=0}^{P-1}\sum_{n=0}^{L-1}(-1)^n x^2(nT+kT/P)e^{-j2\pi k/P} \qquad (2.48)$$

This estimator also requires an oversampling of 4. This scheme is highly sensitive to carrier frequency error f_e, but in its absence, the performance is very close to the MCRB.

G. Summary

To conclude, in this chapter we have briefly reviewed the following concepts. First we considered the performance measures for the symbol timing estimators. Then we reviewed the signal models for linear and CPM signals and some known timing recovery schemes. The notation for further work has also been established. Here we will summarize it.

H. Notation

P oversampling ratio

$s(t)$ Transmitted signal

ϵ Unknown timing delay which has to be estimated

T Symbol rate

T_s Sampling period

$r(n)$ Received signal that is sampled at rate T/P

$x(n)$ Output of the matched filter that is sampled at T/P

$r_x(n;\tau)$ Autocorrelation function of $x(n)$ at n and lag τ

$R_x(k;\tau)$ Cyclic correlation coefficient at cycle k and lag τ

$\hat{R}_x(k;\tau)$ Estimates of the cyclic correlation coefficient at cycle k and lag τ

L Number of symbols transmitted (or number of symbols used for estimation)

$h_c(t)$ Convolution of the transmit and receive filters

$h(n)$ Sampled version of $h_c(t)$

$H_c(F)$ Continuous time Fourier transform of $h_c(t)$

$H(f)$ Discrete time Fourier transform of $h(n)$

CHAPTER III

SYMBOL TIMING ESTIMATORS FOR LINEAR MODULATIONS

In this chapter we look at some extensions to existing estimators for the timing recovery of linear modulations. As mentioned earlier one of our goals is to reduce the sampling rate of the estimators. We propose an extension to the LOGN [2] estimator. Proceeding along similar lines we propose another estimator that operates at $P = 2$ and which can be considered as an extension of the square law estimator.

A. Proposed Estimator for Logarithmic Nonlinearity (P=2)

We will continue to make use of the signal model proposed in the previous chapter. To recapitulate $x(n)$ is the output of the matched filter sampled at T/P. The logarithmic estimator [2] is given as

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\left\{\sum_{n=0}^{PL-1} \log(1 + (\frac{Es}{N_o})^2|x(n)|^2)e^{-j2\pi n/P}\right\}, \tag{3.1}$$

This estimator was derived according to the maximum likelihood principles with some approximations. The approximations were valid for the low SNR regime. We expect it therefore to be the optimal estimator in this range. Our goal is to extend this to a lower sampling rate and retain the attractive property of good performance when the SNR and excess bandwidth are low. Our extension of this estimator is rather ad hoc and at first sight we do not have rigorous reasons to hope that this approach will work. It turns out that the proposed estimator achieves a performance better than the timing estimators [16, 12, p.398-402] in the lower SNR regime, in the presence of small data lengths, and pulses with reduced rolloff factors.

1. Derivation of Estimator

The logarithmic nonlinearity is known to be very intractable for analysis, therefore our approach will be heuristic and supported only by computer simulations. As mentioned earlier most of the nonlinearities used for timing recovery contain only even powers of the signal in their Taylor series expansion. Therefore, the bandwidth of the signal at the output of the nonlinearity increases, potentially causing aliasing. For $P = 2$ this is always true. However, the aliasing effects can be properly taken into account by exploiting lags of the signal correlation other than zero. An estimator based on this idea was first proposed in [15] which was later extended in [20] to make it unbiased. The unbiased estimator has the following form [20]

$$\hat{\epsilon} = \frac{1}{2\pi}\arg\left\{b\sum_{n=0}^{2L-1}|x(n)|^2 e^{-j\pi n} + j\sum_{n=0}^{2L-2}\mathrm{Re}\{x^*(n)x(n+1)\}e^{-j\pi n}\right\}, \quad (3.2)$$

$$G(\tau) = \frac{2}{T}\int_{-1/2T}^{1/2T} H_c(F-1/2T)H_c(F+1/2T)e^{j\pi\tau TF}dF \quad (3.3)$$

where $b = G(1)/G(0)$ is a bias correction factor that depends on the pulse shape, and $H_c(F)$ is the continuous time Fourier transform of $h_c(t)$. Guided by the form of the estimator for $P = 2$ we conjecture that we may be able to extend the LOGN estimator for $P = 2$ as follows:

$$\begin{aligned}\hat{\epsilon} &= \frac{1}{2\pi}\arg\left\{b\sum_{n=0}^{2L-1}\log(1+k|x(n)|^2)e^{-j\pi n}\right.\\ &\left.+\; j\sum_{n=0}^{2L-2}\log(1+k\mathrm{Re}\{x^*(n)x(n+1)\})e^{-j\pi n}\right\}, \quad (3.4)\end{aligned}$$

where b is still given by the same equation as in (3.3). The question however is how to decide on the scaling factor k. First let us consider the Taylor series expansion of $\log(1 + x)$.

$$\log(1+x) = x - x^2/2 + x^3/3 - \cdots + (-x)^n/n + \cdots \,. \quad (3.5)$$

For $|x| << 1$ we expect that the right hand side of (3.5) will reduce to x and therefore the proposed estimator (3.4) will reduce to the estimator [20]. However, that would not be of any use, so we expect that k would not be very small. We arrived at the initial value of k by trial and error and testing with simulations. The expansion (3.5) is quite accurate when $|x| \leq 0.1$. So we would expect k to scale the maximum values of $x^2(n)$ and $x^*(n)x(n+1)$ to ≈ 0.1. It turns out that to achieve this, $k \approx 0.1$. Admittedly this is not rigorous but we can provide an understanding for it. From the Taylor series expansion we see that the estimator makes use of the second order moment (autocorrelation) as well as the higher order moments. If properly weighted these moments can improve the estimate. The small value indicates that contribution from higher moments is weighed lesser than the second order moment since aliasing of spectral components will certainly occur at $P = 2$. So when we consider the expansion of the proposed estimator we see that the contribution from the fourth moment is weighed by k justifying our intuition that higher order moments should be given less weight. We can actually suggest one more improvement for this estimator by recognizing that aliasing is dependent on the excess bandwidth. So to a first order we might be able to improve the estimate by modifying the constant k linearly with β as $k = (2 - \beta)/10$. The justification for this being that as we alias lesser at lower β we can increase the contribution from the fourth order moment.

Finally, we can think of an alternative approach to getting this constant. Instead of the logarithmic nonlinearity if we consider forming an estimate based on an optimally weighted linear combination of the second and the fourth order moments, we should end up with the same performance as the proposed estimator. However, this optimization is also very complex and difficult to carry out. But we did verify that with the constants used the performance of the weighted estimator is almost the same as the proposed estimator. A similar approach of using weighted moments for estimation has been tried before for $P \geq 4$ in [25].

2. Simulations

The performance of timing recovery schemes [16, 12, p. 398-402] and the proposed estimator have been evaluated via computer simulations assuming a QPSK modulation with $E_s := E|I_k|^2 = 1$ and L=100 symbols. The mean squared error (MSE) of the proposed estimator and that of [20, 12] is plotted versus SNR. The MSE is averaged over ϵ varying from $(-0.4, 0.4)$, and over a number of 1,000 Monte-Carlo runs. We can observe that the proposed estimator is better than [20] at lower SNR and lower rolloff factors. At higher rolloff factors, the square law performs better than the proposed estimator since increased aliasing occurs in the new estimator due to the presence of the higher order correlations. The proposed estimator does not perform as well as the LOGN estimator at $P \geq 4$ though. The simulation results are shown in Fig. 8 and 9.

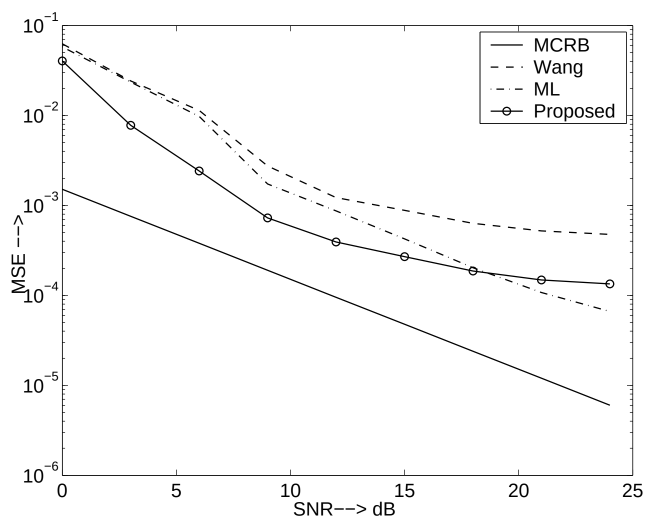

Fig. 8. MSE vs SNR for the Proposed Estimator and [16, 12, p. 398-402] with $\beta = 0.1$

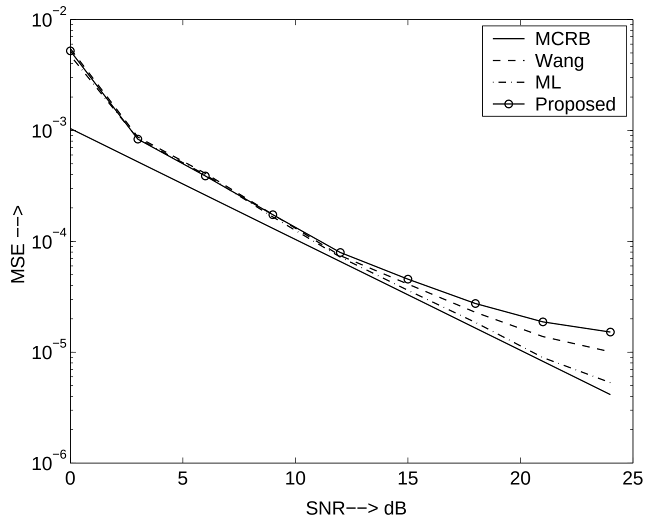

Fig. 9. MSE vs SNR for the Proposed Estimator and [16, 12, p. 398-402] with $\beta = 0.9$

B. A New Two Sample Feedforward Estimator

1. Derivation of Estimator

In this section we propose an alternative method to extend the square law nonlinearity based estimator. Unlike [15, 20] it does not need to make use of the nonzero lags of the cyclic correlation to achieve this. The key idea behind the proposed estimator can be easily understood when we look at the frequency domain interpretation of the cyclic correlation coefficients when there is no aliasing and when there is aliasing. The aliasing refers to the aliasing occurring in $R_x(1;\tau)$. For the square law estimator when there is no aliasing we have [20]

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\left\{\hat{R}_x(1;0)\right\}, \quad (3.6)$$

$$\begin{aligned} R_x(k;\tau) &= \frac{1}{P}\int_{-1/2}^{1/2} H(f)H(f+k/P)e^{j2\pi(f+k/P)\tau}df \quad (3.7) \\ &= \frac{e^{-j2\pi k\epsilon}}{T}\int_{-P/2T}^{P/2T} H_c(F)H_c(F+k/T)e^{j2\pi\tau TF/P}dF. \end{aligned}$$

Consider the graphical representation of the above equation as shown in Fig. 10. When there is no aliasing the frequency responses of $H(f)$ and $H(f+1/P)$ for $|f| < 1/2$ are given by

$$H(f) = \frac{1}{T_s} H_c(f/T_s) e^{-i2\pi f \epsilon P}, \quad (3.8)$$

$$H(f+1/P) = \frac{1}{T_s} H_c(\frac{f+1/P}{T_s}) e^{-i2\pi f \epsilon P}. \quad (3.9)$$

Since we are assuming that the pulse $h_c(t)$ is band-limited to $\pm(1+\beta)/2T$, $R_x(1;0)$ is

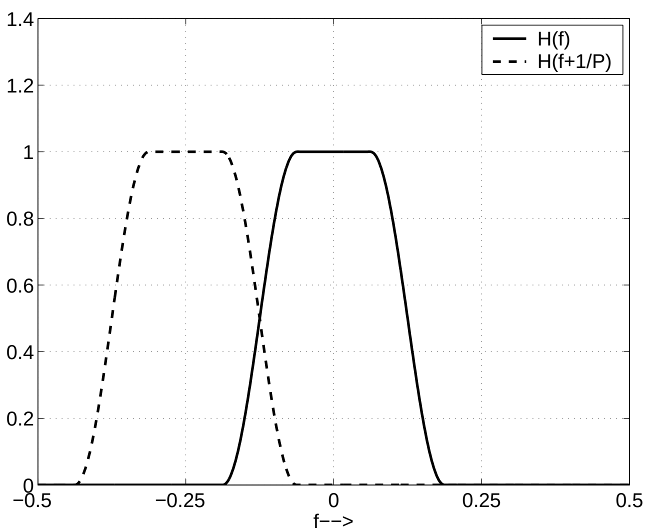

Fig. 10. $H(f)$ and $H(f+1/P)$ for P=4

defined by the nonzero overlap between $H(f)$ and $H(f+1/P)$ which is the frequency range given by $[-(1+\beta)/2P, -(1-\beta)/2P]$ (or equivalently $[-(1+\beta)/2T, -(1-\beta)/2T]$ if we consider $H_c(F)$). With real symmetric pulses $h_c(t)$, $H(f)$ is real and even and it can be shown that the integral in the equation (3.7) is real. So this does not contribute to the phase of $R_x(1;0)$. The overall product $H(f)H(f+1/P)$ is shown in Fig. 11. Also note that this overlap region has a bandwidth of β/P. When the

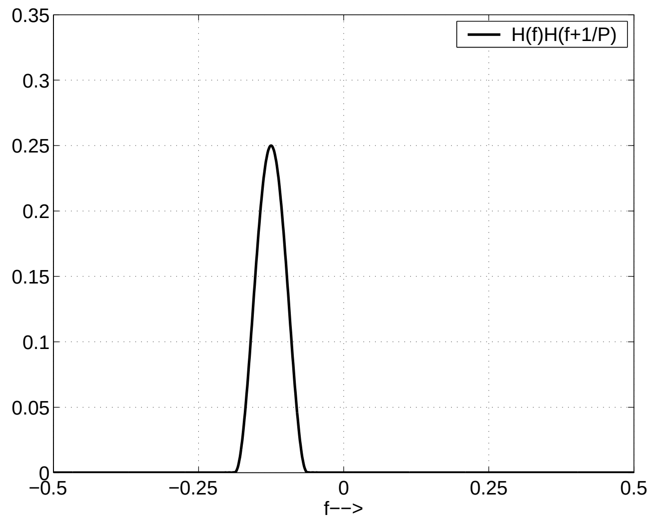

Fig. 11. $H(f)H(f+1/P)$ for P=4

signal aliases i.e., when $P = 2$

$$H(f+1/2) \;=\; \frac{P}{T}\left(H_c(F+1/T)e^{-j2\pi\epsilon} + H_c(F-1/T)e^{j2\pi\epsilon}\right)e^{-j4\pi f\epsilon}. \quad (3.10)$$

So there is an extra contribution to $R_x(1;0)$ when P=2. Pictorially, this can be seen in Fig. 12. There is one more region where the product $H(f)$ and $H(f+1/2)$ is nonzero. This time it consists of the regions $[-(1+\beta)/2P, -(1-\beta)/2P]$ and $[(1-\beta)/2P, (1+\beta)/2P]$. This can be seen in Fig. 13. But the information necessary for timing recovery is already contained in the first term of (3.10). So all we need to do is prevent the other term from coming into the integral. In other words we just need to do some anti-alias filtering. Once again from the Fig. 13 we see that this term occurs due to the portion from $(1-\beta)/2P < f < (1+\beta)/2P$. So we can have a filter removing this term having a passband of $[-(1+\beta)/2P, -(1-\beta)/2P]$. Then we are dealing with the same term as in the conventional square law estimator when there is no aliasing and we can use the argument of the cyclic correlation of the filtered signal

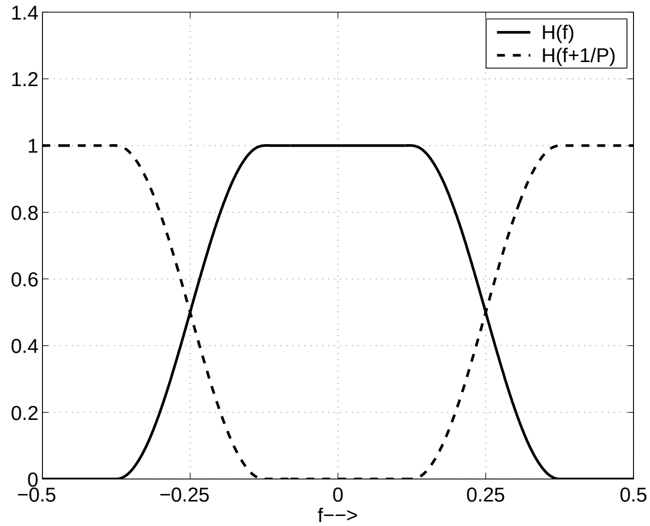

Fig. 12. $H(f)$ and $H(f+1/P)$ for P=2

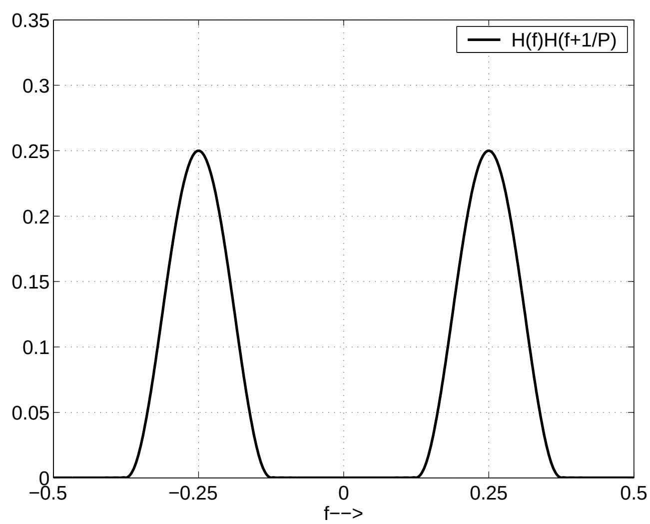

Fig. 13. $H(f)H(f+1/P)$ for P=2

to get the timing delay. To be more precise with the digital anti-alias filter (DAF) the correlation of the filtered signal $R_{x,f}(1;0)$ is given as

$$R_{x,f}(1;0) = \frac{1}{2}e^{-j2\pi\epsilon}\int_{-1/2T}^{1/2T} H_c(F)H_c(F+1/T)dF, \tag{3.11}$$

which is the same as the integral in the non-aliased case. This means the new estimator is given by

$$\hat{\epsilon} = -\frac{1}{2\pi}\arg\{\hat{R}_{x,f}(1;0)\}, \tag{3.12}$$

where $R_{x,f}$ is the correlation of the signal after anti-alias filtering. (Strictly speaking this is not the autocorrelation anymore). A block diagrammatic representation of this estimator is shown in Fig. 14. Note that the anti-aliasing filter must be present before

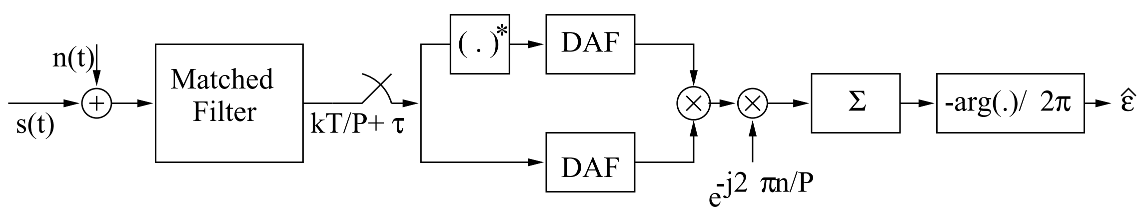

Fig. 14. Implementation of the Proposed Estimator at $P = 2$

the multiplication and this makes two such filters necessary since the signal needs to be conjugated in one branch. The conjugation operation cannot be interchanged with the DAF which is complex. Also note that the DAF is complex because it is not symmetric around the origin in the frequency domain.

2. Simulation Results

The proposed estimator was simulated and its performance is comparable to the $P = 2$ estimator of [16]. It has however a little lower self-noise. The probable reason for this is that in [16] we first alias and then attempt to recover the timing delay whereas in the proposed estimator we prevent the aliasing from happening. However, we notice that self-noise is still a problem with this estimator. The simulation results are shown in Fig. 15.

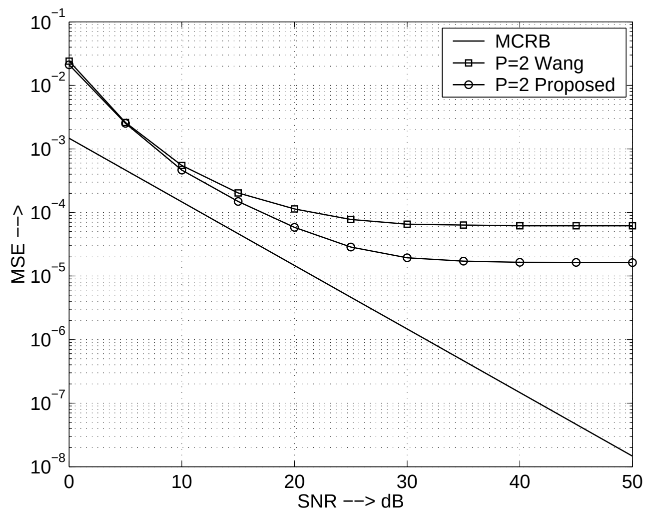

Fig. 15. MSE of the Proposed Estimator $\beta = 0.25$

3. Use of Prefilters to Improve the Performance

The previous scheme can be made very effective by using the idea of prefilters. It was shown in [18] that if we shape the overall pulse response to have symmetry around $1/2T$ (or equivalently $1/2P$) then the estimator based on square law will be self-noise free. We now realize that the same result holds true for the case of $P = 2$ in the proposed estimator. This is because once we prevent aliasing, the cyclic correlation coefficient is same as in the no aliasing case (for which the self-noise result was derived). The only change in implementation is that we now have a prefilter after the matched filter before we do the anti-alias filtering. The location of the prefilter does not matter so much in this case because it is real and the conjugation operation will not affect it.

This estimator is somewhat similar in form to the estimator proposed in [12, p. 398-402]. However there is significant difference between the proposed estimator and ML estimator proposed in [12] both in concept and in performance. The ML estimator was derived using maximum likelihood principles and some approximations. Whereas here we have used a cyclostationary framework to derive the estimator. Also in that case the overall pulse does not have the required symmetry around $1/2T$ and consequently there is self-noise.

4. Simulation Results with Prefilter

Here we present the simulation results for a QPSK signal with the following parameters. The length of the transmitted sequence is 100, the timing delay ϵ is averaged over $(-0.4, 0.4)$ and 500 Monte Carlo runs are performed for each SNR data point. The results with $\beta = 0.1$ are shown in Fig. 16. In this case there is no difference between [20] (also prefiltered) and the proposed estimator. However when we have $\beta = 0.9$ we observe the proposed estimator has lower self-noise and is much more closer to the MCRB. This is shown in Fig. 17.

5. Drawbacks

It will be clear from the previous section that the proposed estimator is more complex than [16, 12, p398-402]. However as we pointed earlier we were not so much interested in lowering the complexity as much as attempting to explore the alternatives of timing recovery at lower sampling rate.

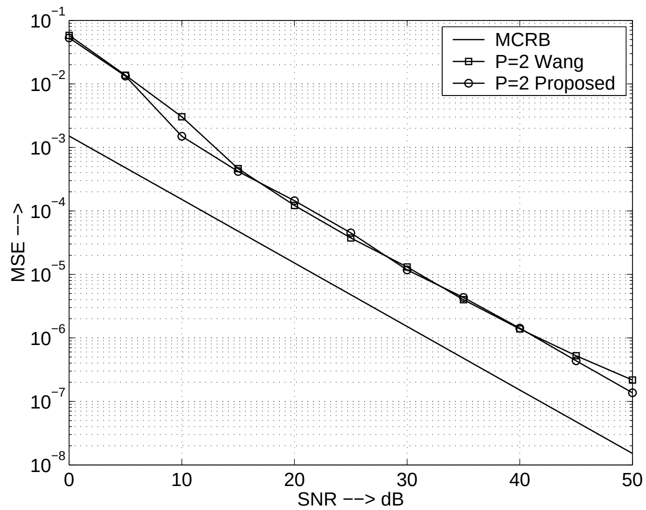

Fig. 16. MSE of the Proposed Estimator $\beta = 0.1$

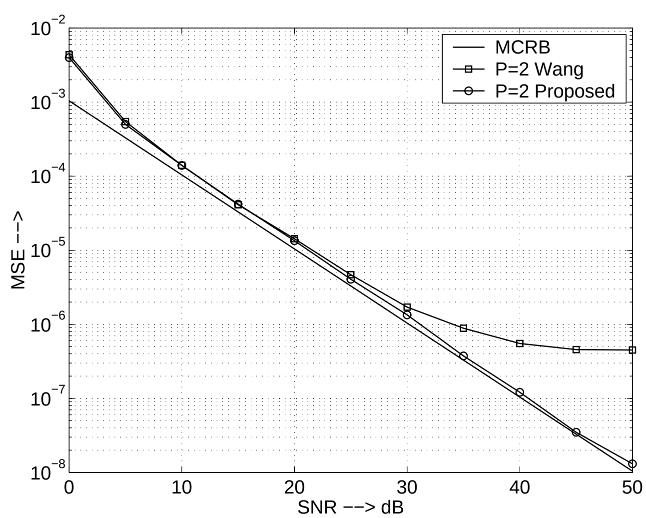

Fig. 17. MSE of the Proposed Estimator $\beta = 0.9$

C. Summary

To summarize in this chapter we have proposed two new timing estimators for linear modulations. Both of them can operate at the Nyquist rate. They can be considered as extensions to the SLN and LOGN estimators. The proposed logarithmic estimator is the best estimator at low SNR and low excess bandwidths among all Nyquist rate estimators. The proposed extension to SLN offers reduced self-noise performance for large excess bandwidths when compared to other estimators at this rate.

CHAPTER IV

SYMBOL TIMING ESTIMATORS FOR NONLINEAR MODULATIONS

In this chapter we propose two new non data-aided estimators for symbol timing recovery of GMSK signals. Typically most of the feedforward estimators require to sample the input signal at a rate at least twice as large as the symbol rate. The estimators we propose can operate at the symbol rate and serve to illustrate the idea that sufficient statistics exist for a signal sampled at symbol rate. They are effective in the case of narrow band signals. We also propose two new estimators for MSK modulation. They offer alternative methods for timing recovery with comparable performance to the existing estimators. Most of the estimators are derived using the cyclostationary framework. The performance of the estimators is evaluated through simulations.

A. Symbol Rate Estimator for GMSK Signals

Before we proceed with the derivation of the estimator, we will give some heuristic arguments to give the idea behind the scheme. It will be remembered that in Chapter II we mentioned that the GMSK signal can be approximated as an OQPSK modulation. The key thing to be noticed in that approximation was that each of the channels in the OQPSK is at a lower rate. Let us consider this approximation once again to clarify this claim

$$r(t) = \sum_k b_{2k+1} C_0(t - 2kT - T - \epsilon T) + j \sum_k b_{2k} C_0(t - 2kT - \epsilon T). \qquad (4.1)$$

If we observe closely we see that if we sample at a rate P/T, since the modulating symbols b_k change once in $2T$, we oversample at twice the rate $(2P/T)$ effectively. This means that cyclostationarity is induced even in a signal sampled at symbol rate.

Further it implies that we can recover the timing delay using the estimators that operate at $P = 2$ on a linear modulation. We must be careful though because the $P = 2$ estimators presented so far are under the assumption of i.i.d. data and also band-limited signals. Here it is not true because the b_k are not i.i.d. We will continue to make the approximation that the signal is band-limited.

1. Derivation of Estimator

Here we will derive the estimator for the case when the signal is oversampled by $P = 1$. The development makes use of the cyclostationary framework of [16]. Let $s(t)$ be the complex envelope of the base band signal. Using Laurent's expansion [23] we can write this as

$$r(t) = \sum_k \sum_{i=0}^{2^L-1} a_{i,k} C_i(t - kT - \epsilon T), \quad (4.2)$$

$$\approx \sum_k a_{0,k} C_0(t - kT - \epsilon T). \quad (4.3)$$

For MSK type signals we can write the following recursive relation for $a_{0,k}$

$$a_{0,k} = j a_{0,k-1} I_k. \quad (4.4)$$

We can further simplify this by recognizing that $a_{0,k}$ alternately are real and imaginary valued. This leads to the following

$$s(t) = \sum_k b_{2k+1} C_0(t - 2kT - T - \epsilon T) + j \sum_k b_{2k} C_0(t - 2kT - \epsilon T), \quad (4.5)$$

where $b_{2k+1} = -b_{2k-1} I_{2k} I_{2k+1}$ and $b_{2k} = -b_{2k-2} I_{2k-1} I_{2k}$. Note that all the $C_i(t)$ are real and symmetric. (A question might arise here as to an inherent ambiguity of phase. Whether the inphase channel has the delay of $T + \epsilon T$ or the quadrature channel. For now we will assume that the inphase has the excess delay and proceed. The ambiguity will get resolved when we derive the expression for ϵ). Let us represent the real and imaginary parts of the received signal as $x(t)$ and $y(t)$, respectively. Using these relations we can write the output of the matched filter as $r(t) * C_0(t) = x(t) + jy(t)$ and the discrete time model becomes $x(n) := x(nT), y(n) := y(nT), h(n) := h_c(nT - T - \epsilon T), h_c(t) = C_0(t) * C_0(t)$. Then we can define the autocorrelation sequences of the signals $x(n)$ and $y(n)$ as follows

$$\begin{aligned}
r_x(n;\tau) &= E[x^*(n)x(n+\tau)] \\
&= E\left[\left(\sum_l b_{2l+1}h^*(n-2l)\right)\left(\sum_m b_{2l+2m+1}h(n+\tau-2l-2m)\right)\right] \\
&= E\left[\sum_l\sum_m b_{2l+1}b_{2l+2m+1}h^*(n-2l)h(n+\tau-2l)\right] \\
&= \sum_l\sum_m E\left[b_{2l+1}b_{2l+2m+1}\right]h^*(n-2l)h(n+\tau-2l-2m) \qquad (4.6) \\
r_y(n;\tau) &= E[y^*(n)y(n+\tau)] \\
&= \sum_l\sum_m E\left[b_{2l}b_{2l+2m}\right]h^*(n+1-2l)h(n+1+\tau-2l-2m). \qquad (4.7)
\end{aligned}$$

Here we need to find the autocorrelation of the sequence b_k. Under the assumption that $I_k \in \{\pm 1\}$ it is seen that $b_k \in \{\pm 1\}$ and that $E[b_k b_k^*] = 1$. As to $E[b_{2l+1}^* b_{2l+2m+1}]$ without loss of generality we can consider $m > 0$. We can write

$$\begin{aligned}
b_{2l+1}b_{2l+2m+1} &= |b_{2l+1}|^2 \prod_{k=2l+2}^{2l+2m+1}(-1)^m I_k \qquad (4.8) \\
&= \prod_{k=2l+2}^{2l+2m+1}(-1)^m I_k \\
\Rightarrow E[b_{2l+1}b_{2l+2m+1}] &= E\left[\prod_{k=2l+2}^{2l+2m+1}(-1)^m I_k\right] \\
&= \prod_{k=2l+2}^{2l+2m+1}(-1)^m E[I_k] \\
&= 0 \text{ because } I_k \text{ are i.i.d} \qquad (4.9)
\end{aligned}$$

So we can write $r_x(n;\tau)$ as follows

$$r_x(n;\tau) \;=\; \sum_l h^*(n-2l)h(n+\tau-2l). \tag{4.10}$$

We can see that $r_x(n;\tau)$ is periodic with period 2. Consider

$$\begin{aligned} r_x(n+2;\tau) &= \sum_l h^*(n+2-2l)h(n+2+\tau-2l) \\ &= \sum_l h^*(n-2(l-1))h(n+\tau-2(l-1)) \\ &= \sum_l h^*(n-2l)h(n+\tau-2l) \\ &= r_x(n;\tau) \end{aligned}$$

where the last equation is obtained by a change of index. Since $r_x(n;\tau)$ is periodic we can expand and write a Fourier series for it

$$r_x(n;\tau) \;=\; \sum_{n=0}^{1} R_x(k;\tau)e^{j2\pi kn/2}, \tag{4.11}$$

$$\begin{aligned} R_x(k;\tau) &= \frac{1}{2}\sum_{n=0}^{1} r_x(n;\tau)e^{-j2\pi kn/2} \\ &= \frac{1}{2}\sum_{n=0}^{1}\sum_l h^*(n-2l)h(n+\tau-2l)e^{-j2\pi kn/2} \\ &= \frac{1}{2}\sum_n h^*(n)h(n+\tau)e^{-j2\pi kn/2}, \end{aligned} \tag{4.12}$$

the last step is obtained by replacing $n-2l$ by n and combining the two summations into one. Now we can use Parseval's relation to transform this summation into frequency domain as follows

$$R_x(k;\tau) \;=\; \frac{1}{2}\int_{-1/2}^{1/2} H^*(f)H(f+k/2)e^{j2\pi(f+k/2)\tau}df, \tag{4.13}$$

where $H(f)$ is the Fourier transform of the discrete time sequence $h(n)$. Let $H_c(F)$ be the continuous time Fourier transform of $h_c(t)$. Under the approximation that

$H_c(F)$ is band-limited to $1/2T$, $H(f)$ is related to $H_c(F)$ as follows

$$\begin{aligned} H(f) &= \frac{1}{T} H_c(f/T) e^{-j2\pi(f/T)(T+\epsilon T)} \\ &= \frac{1}{T} H_c(f/T) e^{-j2\pi f(1+\epsilon)}, \\ H(f+1/2) &= \frac{1}{T}\left[H_c(\frac{f-1/2}{T}) e^{-j2\pi(f-1/2)(1+\epsilon)} + H_c(\frac{f+1/2}{T}) e^{-j2\pi(f+1/2)(1+\epsilon)} \right], \\ R_x(1;\tau) &= \frac{1}{T^2}\int_{-1/2}^{1/2} H_c^*(\frac{f}{T}) \left[H_c(\frac{f-1/2}{T}) e^{j\pi(1+\epsilon)} + H_c(\frac{f+1/2}{T}) e^{-j\pi(1+\epsilon)} \right] \\ &\quad e^{j2\pi(f+1/2)\tau} df \\ &= \frac{1}{T}\int_{-1/2T}^{1/2T} H_c^*(F) \left[H_c(F-1/2T) e^{j\pi(1+\epsilon)} + H_c(F+1/2T) e^{-j\pi(1+\epsilon)} \right] \\ &\quad e^{j2\pi(FT+1/2)\tau} dF. \end{aligned}$$

Next we use the fact that $h_c(t) = C_0(t) * C_0(t)$ is a real and even function therefore its Fourier transform is real and even. With this and the assumption that $H_c(F)$ is band-limited to $1/2T$ we can further manipulate the above equations as follows

$$\begin{aligned} R_x(1;\tau) &= \frac{1}{T}\int_{-1/2T}^{1/2T} H_c(F+1/4T) H_c(F-1/4T) e^{j\pi(1+\epsilon)} e^{j2\pi(FT+1/2)\tau} e^{j2\pi(1/4)\tau} dF \\ &+ \int_{-1/2T}^{1/2T} H_c(F-1/4T) H_c(F+1/4T) e^{-j\pi(1+\epsilon)} e^{j2\pi(FT+1/2)\tau} e^{-j2\pi(1/4)\tau} dF. \end{aligned}$$

Next we make use of the fact that $H_c(F)$ is real and even. This means that $H_c(F - 1/4T)H_c(F + 1/4T)$ is even (and real). From this it follows that

$$R_x(1;\tau) = (-1)^\tau \frac{2}{T} \cos\left(2\pi(\frac{1+\epsilon}{2} + \frac{\tau}{4})\right) \int_{-1/2T}^{1/2T} H_c(F+1/4T) H_c(F-1/4T) e^{j2\pi FT\tau} dF.$$

We can now write an expression for ϵ as follows

$$\begin{aligned} R_x(1;0) &= \frac{2}{T} \cos\left(2\pi(\frac{1+\epsilon}{2})\right) \int_{-1/2T}^{1/2T} H_c(F+1/4T) H_c(F-1/4T) dF, \\ R_x(1;1) &= \frac{2}{T} \sin\left(2\pi(\frac{1+\epsilon}{2})\right) \int_{-1/2T}^{1/2T} H_c(F+1/4T) H_c(F-1/4T) e^{j2\pi FT} dF, \\ \tan\left(2\pi(\frac{1+\epsilon}{2})\right) &= \frac{R_x(1;1)G(0)}{R_x(1;0)G(1)} \end{aligned}$$

$$G(\tau) = \frac{2}{T}\int_{-1/2T}^{1/2T} H_c(F+1/4T)H_c(F-1/4T)e^{j2\pi\tau FT}dF, \quad (4.14)$$

$$\tan(\pi\epsilon) = \frac{R_x(1;1)G(0)}{R_x(1;0)G(1)}. \quad (4.15)$$

Going through the same derivation for $R_y(1;\tau)$ we get the following equations

$$\begin{aligned}
R_y(1;0) &= \frac{2}{T}\cos\left(2\pi(\frac{\epsilon}{2})\right)\int_{-1/2T}^{1/2T} H_c(F+1/4T)H_c(F-1/4T)dF,\\
R_y(1;1) &= \frac{2}{T}\sin\left(2\pi(\frac{\epsilon}{2})\right)\int_{-1/2T}^{1/2T} H_c(F+1/4T)H_c(F-1/4T)e^{j2\pi FT}dF,\\
\tan\left(2\pi(\frac{\epsilon}{2})\right) &= \frac{R_x(1;1)G(0)}{R_x(1;0)G(1)},\\
\tan(\pi\epsilon) &= \frac{R_y(1;1)G(0)}{R_y(1;0)G(1)}, \quad (4.16)
\end{aligned}$$

where $G(0)$,$G(1)$ are defined as in (4.14). Note that even though there was an excess delay in one of the channels in the final expression for ϵ it has the same form whether evaluated from the inphase or the quadrature. However that does not mean they are the same. It will be seen that the signs of the cyclic correlation are different in each case. We form an estimate for ϵ by combining the estimates from both the channels. So finally we end up with the following estimator

$$\begin{aligned}
\hat{\epsilon} &= \frac{1}{2\pi}\text{atan}(\frac{\hat{R}_x(1;1)G(0)}{\hat{R}_x(1;0)G(1)}) + \frac{1}{2\pi}\text{atan}(\frac{\hat{R}_y(1;1)G(0)}{\hat{R}_y(1;0)G(1)}), \quad (4.17)\\
\hat{R}_x(1;0) &= \frac{1}{L}\sum_{n=0}^{L-1}|x(n)|^2(-1)^n, \quad (4.18)\\
\hat{R}_x(1;1) &= \frac{1}{L}\sum_{n=0}^{L-2}\text{Re}\{x^*(n)x(n+1)\}(-1)^n, \quad (4.19)\\
\hat{R}_y(1;0) &= \frac{1}{L}\sum_{n=0}^{L-1}|y(n)|^2(-1)^n, \quad (4.20)\\
\hat{R}_y(1;1) &= \frac{1}{L}\sum_{n=0}^{L-2}\text{Re}\{y^*(n)y(n+1)\}(-1)^n. \quad (4.21)
\end{aligned}$$

This concludes our derivation of the estimator for $P = 1$. Simulation results are shown in Fig. 19. It will be observed that the proposed estimator has self-noise. So we made some further modifications to the estimator which are discussed in the next section.

2. Use of Prefilters to Improve the Performance

We mentioned earlier that prefilter based schemes can be used to improve the performance of the estimator for $P = 2$ [18]. However, these results cannot be applied directly. Note that the prefilter is shifted to $1/T$ in case of the linear modulations. When we sample at the symbol rate this is the same as the overall response and obviously it is not what we are looking for (because it does not give rise to any symmetry). The symmetry condition gets modified for $P = 1$. The modified condition for reduction in self-noise is that the overall pulse should have symmetry around $1/4T$. The prefilter (up to a scale factor) needed to make the pulse symmetric around $1/4T$ is given as follows

$$H_{pre}(F) = H_c(F - 1/2T) + H_c(F + 1/2T), \tag{4.22}$$

$$H_c(F) = |C_0(F)|^2, \tag{4.23}$$

where $C_0(F)$ is the frequency response of $C_0(t)$. The prefilter is shown in Fig. 18. Also note that the resulting response is now symmetric at $1/4T$. $G(0), G(1)$ need to be recalculated with the prefilter. Actually, these results are applicable to other MSK type signals where we can approximate the signal as an OQPSK signal. For instance, for MSK modulations, timing recovery is possible at the symbol rate. However, it may not be practical in the sense that for MSK signal the bandwidth is slightly greater than $1/2T$ which means that signal recovery may not be possible if we sample at the symbol rate and we may be forced to sample at a higher rate in which case we can afford to use an estimator with higher sampling rate. However, in theory there is no problem in applying this scheme to other MSK type modulations. We also note that

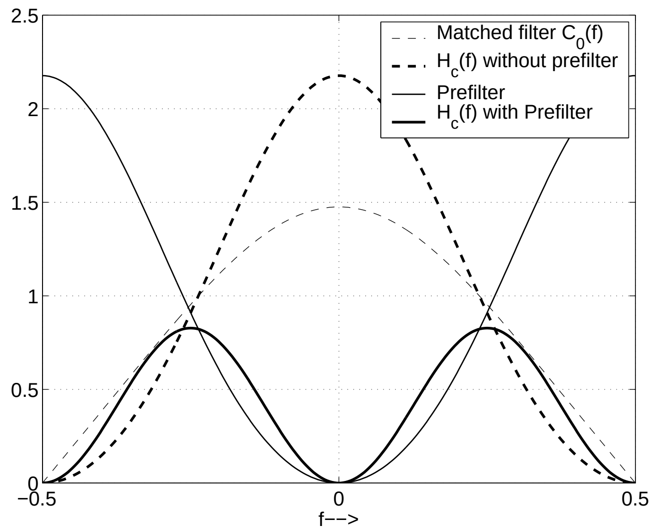

Fig. 18. Frequency Responses

the prefilter is not always required to improve the performance. In case of MSK the improvement is marginal as can be seen in Fig. 20. This is because the spectrum of MSK is not band-limited to $1/2T$.

3. Simulation Results

Here we present the simulation results for a GMSK signal with the following parameters. We assume $BT = 0.3$, the length of the transmitted data to be $L = 100$ symbols. In each case the mean squared error (MSE) is averaged over $\epsilon \in (-0.4, 0.4)$. The performance of the new estimator with and without prefilter is shown in Fig. 19. We also show the simulations for MSK in Fig. 20 to corroborate the previous statements.

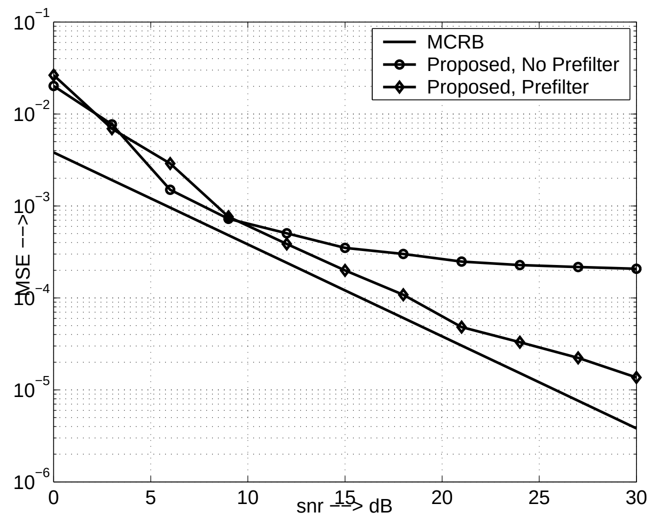

Fig. 19. MSE vs SNR of the Proposed Estimator for GMSK with and without Prefilter

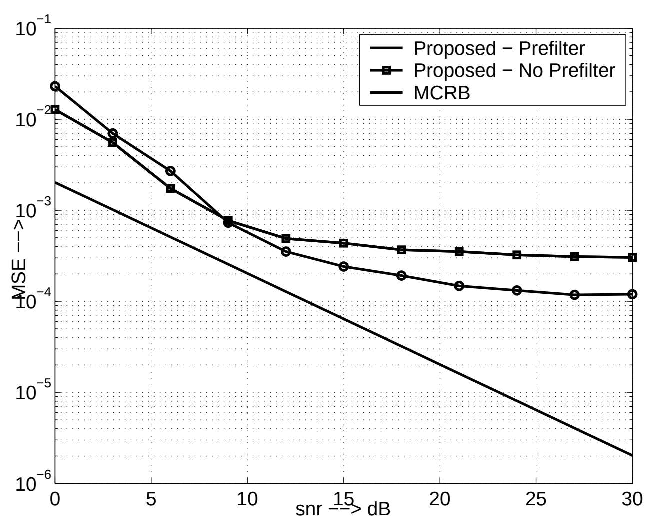

Fig. 20. MSE vs SNR of the Proposed Estimator for MSK with and without Prefilter

B. Alternate Estimator at Symbol Rate

As pointed out in the last section the inphase and the quadrature components of the GMSK signal are oversampled twice with respect to the received signal. So we can use other estimators at $P = 2$ derived for the linear modulations as the basis for deriving new estimators. The estimator just proposed can be considered as an extension of [15, 16] to nonlinear modulations. We can also consider the ML estimator [12] at $P = 2$ or the estimator we proposed in Chapter III. We will be rather brief on this since the main idea is common to the previous scheme. We are just using an alternate scheme that can recover timing at $P = 2$. It will be recalled that the ML timing estimator proposed in [12] can operate at $P = 2$. This was reviewed in Chapter II. The modifications to this scheme will be similar to the previous scheme. The prefilter we use in one of the branches in the ML estimator is of the form $G(F - 1/2T)G^*(F + 1/2T)$. Here this will translate to $C_0(F - 1/4T)C_0^*(F + 1/4T)$.

The basic idea is illustrated in the Fig. 21. The block "P=2 STE" refers to a

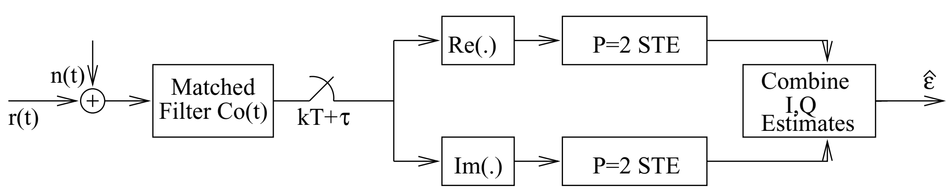

Fig. 21. Implementation of P=1 Estimators

symbol timing estimator at $P = 2$. This can be any of the $P = 2$ estimator discussed so far for linear modulations.

C. Further Extensions to the Square Law Based Estimator for GMSK Signals

1. Use of Prefilters in the Original Scheme for $P \geq 2$

Since it was established that prefilters improve the performance of the estimators with square law recovery in the linear modulations we can also improve [21] simply by prefiltering. We tested this idea and found it to be true in case of the GMSK modulations. In fact as BT becomes smaller in the GMSK modulations the prefilter becomes more effective in improving the scheme of [21].

2. Use of the ML Estimator

We can also improve the performance of [21] by the ML scheme explained in Chapter II and in the previous section. For $P = 2$ this makes the performance very close to the MCRB as shown in Fig. 22.

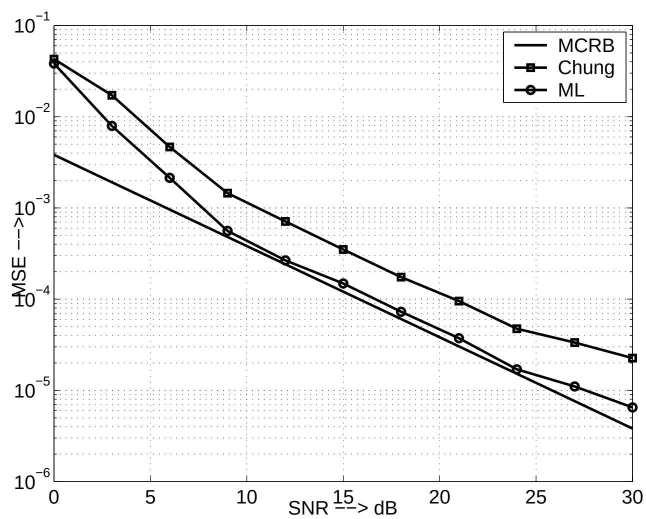

Fig. 22. MSE of the ML Estimator for GMSK at P=2

3. Drawbacks of the Proposed Estimators

The proposed estimator however has two problems which may be severe under some conditions. Firstly it is not tolerant of carrier frequency offset. Secondly it is sensitive to phase offset. However, unlike the error in frequency it can withstand some amount of phase offset.

D. Two New Symbol Timing Estimators for MSK Modulations

In [3] an interesting method of symbol timing recovery for MSK type signals was proposed. This method can be considered as an extension of the timing recovery schemes for linear modulation schemes to CPM signals. Here we look at the same scheme in a slightly broader perspective which is more useful in that it allows us to take advantage of the timing recovery schemes in linear modulations. Again consider the complex envelope of a CPM signal this time with phase offset ϕ_o and carrier frequency error f_e:

$$r(t) = e^{j(\phi(t-\epsilon T;I)+\phi_o+2\pi f_e t)}, \tag{4.24}$$

$$\phi(t) = 2\pi h \sum_k I_k q(t-kT), \tag{4.25}$$

$$\begin{aligned} x(t) &:= r(t)r^*(t-mT) \\ &= e^{j(\phi(t-\epsilon T;I)-\phi(t-mT-\epsilon T;I)+2\pi f_e mT)}, \end{aligned} \tag{4.26}$$

$$y(t) := \frac{1}{2\pi h}\arg\{x(t)\} \tag{4.27}$$

$$\begin{aligned} &= \frac{1}{2\pi h}\{\phi(t-\epsilon T;I)-\phi(t-mT-\epsilon T;I)+2\pi f_e mT\} \\ &= \sum_k I_k q(t-kT-\epsilon T) - \sum_k I_k q(t-kT-mT-\epsilon T) + \frac{f_e mT}{h}. \end{aligned} \tag{4.28}$$

Let us now define another pulse $p_m(t)$ as follows

$$p_m(t) := q(t) - q(t-mT). \tag{4.29}$$

$$\text{Then } y(t) \;=\; \sum_k I_k p_m(t - kT - \epsilon T) + \frac{f_e m T}{h} \tag{4.30}$$

We can see $y(t)$ is a linear modulation with the pulse shape given by $p_m(t)$. Now we can extend almost all the schemes that we know for linear modulations for performing symbol timing recovery. We are not however guaranteed any performance gains because we do not have as much control over the pulse shape as we did in the case of linear modulations. This will become clear when we examine some specific cases. We will consider MSK signals and perform feedforward timing recovery for oversampling ratios $P = 2$ and $P = 4$. Defining as usual the discrete time model $y(n) := y(nT/P), p_m(n) := p_m(nT/P - \epsilon T)$, we have

$$y(n) \;=\; \sum_j I_j p_m(n - jP) + \frac{f_e m T}{h}. \tag{4.31}$$

Consider the autocorrelation of the signal $y(n)$ given by $r_y(n;\tau) = E[y^*(n)y(n+m)]$, assuming that I_k are zero mean, i.i.d. and $E[I_k^* I_k] = 1$, we obtain

$$r_y(n;\tau) \;=\; \sum_j p_m^*(n - jP)p_m(n + \tau - jP) + \frac{f_e^2 m^2 T^2}{h^2}. \tag{4.32}$$

As can be seen this signal is periodic in n with period P and we can expand it into a Fourier series or equivalently we can write the cyclic correlation coefficients

$$\begin{aligned} R_y(k;\tau) &= \frac{1}{P}\sum_{n=0}^{P-1}\left(\sum_j p_m^*(n - jP)p_m(n + \tau - jP) + \frac{f_e^2 m^2 T^2}{h^2}\right) e^{-j2\pi kn/P} \\ &= \frac{1}{P}\sum_n p_m^*(n)p_m(n+\tau)e^{-j2\pi kn/P} + \frac{f_e^2 m^2 T^2}{h^2}\delta(k). \end{aligned} \tag{4.33}$$

Clearly we can see that the error due to the frequency comes in as a dc term and if we look at any other harmonic we will be able to relate the phase of the harmonic to the timing delay ϵT. Therefore we will have an estimator that is robust to frequency offset. Here we have two possible approaches to calculating the cyclic correlation coefficients. In the previous schemes we had worked in the frequency domain. This is particularly efficient when the signal is band-limited. Sometimes when the pulse is time limited and the time domain waveform is given in closed form expression, it is more convenient to work in the time domain itself. We will now illustrate the latter approach using MSK signals. For MSK the pulse $q(t)$ is defined as

$$q(t) = \begin{cases} 0 & t \leq -T/2 \\ \frac{t+T/2}{2T} & |t| \leq T/2 \\ \frac{1}{2} & t > T/2. \end{cases} \tag{4.34}$$

Taking $m = 1$ we have $p_1(t)$ purely real and defined as

$$p_1(t) = \begin{cases} 0 & t \leq -T \\ \frac{1}{2}(1 - |t|/T) & |t| \leq T \\ 0 & t > T. \end{cases} \tag{4.35}$$

1. Proposed Estimator at P=2

For $P = 2$ and $m = 1$ we can have at most 5 samples and therefore evaluation of the summation in (4.33) for calculating the cyclic coefficients becomes rather simple. We do not have to deal with complex integrals here

$$\begin{aligned} R_y(k;\tau) &= \frac{1}{P}\sum_n p_1(n)p_1(n+\tau)e^{-j2\pi kn/P} && (4.36) \\ &= \frac{1}{2}\sum_n (-1)^{kn} p_1(n)p_1(n+\tau), \\ R_y(1;\tau) &= \frac{1}{2}\sum_n (-1)^n p_1(n)p_1(n+\tau). && (4.37) \end{aligned}$$

$p(t)$ is simply the triangular waveform and if the pulse is delayed and sampled we get $p_1(n)$. $p(t)$ and its delayed version are shown in the Fig. 23. Now the samples of the discrete time pulse $p_1(n)$ are given in Table I. Using these values we can calculate

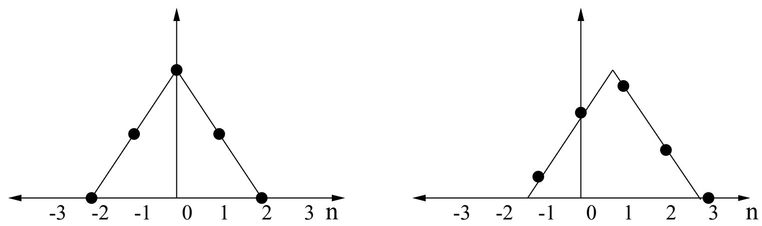

Fig. 23. $p(t)$ without Delay and Its Delayed Version

Table I. $p_1(n)$ for $P = 2$

n	-2	-1	0	1	2
$p_1(nT/2)$	$\frac{\|\epsilon\|}{4} - \frac{\epsilon}{4}$	$\frac{1}{4} - \frac{\epsilon}{2}$	$\frac{1}{2}(1 - \|\epsilon\|)$	$\frac{1}{4} + \frac{\epsilon}{2}$	$\frac{\|\epsilon\|}{4} + \frac{\epsilon}{4}$

the cyclic correlation coefficients for $k = 1$ and $\tau = 0, 1$ as follows

$$R_y(1;0) = \frac{1}{16}(1 - 4|\epsilon|), \tag{4.38}$$

$$R_y(1;1) = \frac{3\epsilon}{16}(1 - 2|\epsilon|). \tag{4.39}$$

Using these two relations we can form an estimate for ϵ in different ways. The following estimators are suggested

$$\hat{\epsilon} = \text{sign}(\hat{R}_y(1;1)) \left(\frac{1 - 16\hat{R}_y(1;0)}{4} \right), \tag{4.40}$$

$$\hat{\epsilon} = \frac{32}{3} \frac{\hat{R}_y(1;1)}{1 + 16\hat{R}_y(1;0)}. \tag{4.41}$$

2. Simulation Results

The following simulations were done for MSK modulation with $L = 100$ and MSE is averaged over $\epsilon \in (-0.4, 0.4)$. Each data point was obtained by running 500 Monte Carlo runs. Both the original scheme of [3] and the proposed estimator (4.40) at

$P = 2$ are shown. It will be seen that the LM scheme is better at higher SNRs. The other estimator (4.40) performance is a little worse. The results are shown in Fig. 24.

3. Proposed Estimator at P=4

We can also derive another estimate for the timing delay ϵ based on the cyclic correlations which is an alternative to the estimator of [3] at $P = 4$. We essentially follow the same method as for $P = 2$, i.e., work in the time domain for calculating the cyclic coefficients. As before the pulse $p_1(t)$ is a triangular pulse and has a period of $2T$. With an oversampling of $P = 4$ we have 9 samples. The values of these samples when $p_1(t)$ is delayed by ϵT are given in Table II.

The cyclic correlation at the cycle $k = 1$ and a lag of $\tau = 0$ is given by

$$R_y(k;\tau) = \frac{1}{P}\sum_{n=-5}^{5} p_1(n)^2 e^{-j2\pi kn/P}, \tag{4.42}$$

$$R_y(1;0) = \frac{1}{4}\sum_{n=-5}^{5} (-j)^n p_1(n)^2 \tag{4.43}$$

$$= \begin{cases} \frac{1}{32}\{(1-4|\epsilon|) + j2(2\epsilon - 1)\} & 0.25 \le \epsilon \\ \frac{1}{32}\{(1-4|\epsilon|) - j4\epsilon\} & |\epsilon| \le 0.25 \\ \frac{1}{32}\{(1-4|\epsilon|) + j2(2\epsilon + 1)\} & \epsilon \le -0.25. \end{cases} \tag{4.44}$$

It might seem that this is not in an easy form to form an estimate. But actually we can observe that the imaginary part of the $R_y(1;0)$ always has an opposite sign to that of the true value of ϵ. Therefore, once we know the sign from the imaginary part of $\hat{R}_y(1;0)$, we can evaluate the magnitude of ϵ from the real part of $\hat{R}_y(1;0)$. So the proposed estimator takes the form

$$\hat{\epsilon} = -\text{sign}\{Im(\hat{R}_y(1;0))\}\frac{\{1 - 32Re(\hat{R}_y(1;0))\}}{4}, \tag{4.45}$$

$$\hat{R}_y(1;0) = \frac{1}{L}\sum_{n=0}^{4L-1} |y(n)|^2(-j)^n. \tag{4.46}$$

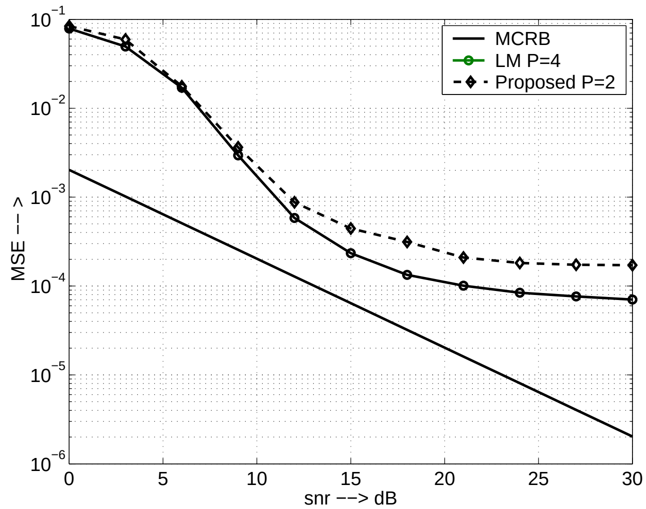

Fig. 24. MSE vs SNR for (4.40) and LM

Table II. $p_1(n)$ for $P = 4$

n	$p_1(nT/4)$
-5	$\frac{\lvert\epsilon+0.25\rvert}{4} - \frac{\epsilon+0.25}{4}$
-4	$\frac{\lvert\epsilon\rvert}{4} - \frac{\epsilon}{4}$
-3	$\frac{\lvert\epsilon-0.25\rvert}{4} - \frac{\epsilon-0.25}{4}$
-2	$\frac{1}{4} - \frac{\epsilon}{2}$
-1	$\frac{1-\lvert\epsilon+0.25\rvert}{2}$
0	$\frac{1}{2}(1 - \lvert\epsilon\rvert)$
1	$\frac{1-\lvert\epsilon-0.25\rvert}{2}$
2	$\frac{1}{4} + \frac{\epsilon}{2}$
3	$\frac{\lvert\epsilon+0.25\rvert}{4} + \frac{\epsilon+0.25}{4}$
4	$\frac{\lvert\epsilon\rvert}{4} + \frac{\epsilon}{4}$
5	$\frac{\lvert\epsilon-0.25\rvert}{4} + \frac{\epsilon-0.25}{4}$

4. Simulation Results

The proposed estimator was simulated to evaluate its performance for MSK modulation with $L = 100$ symbols and 500 Monte Carlo runs. As always the MSE is averaged over ϵ. The results are shown in Fig. 25. It will be seen that the proposed estimator does not perform as well as the original scheme proposed in [3]. However, the performance degradation is not too much and it demonstrates an alternative means to form estimates.

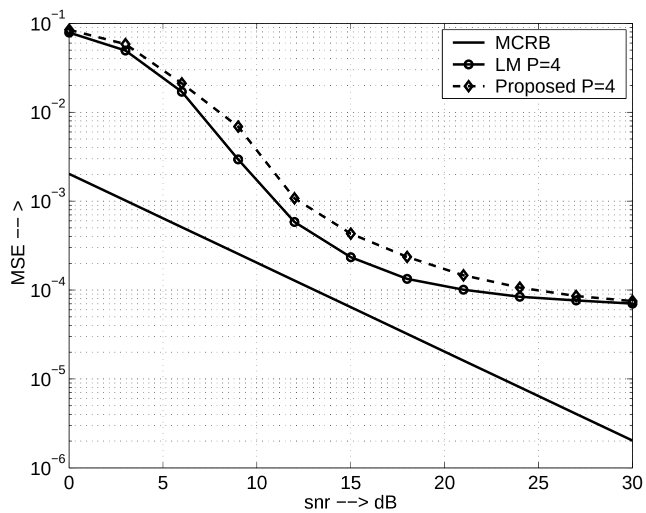

Fig. 25. MSE vs SNR for (4.45) and LM

E. Summary

In this chapter we have looked at various new timing estimators for MSK type modulations. We proposed two new estimators which are the only ones to function at symbol rate among the class of non-data aided feedforward timing estimators for CPM signals. We also suggested a few more extensions to the scheme proposed in [21]. We proposed two new estimators for the MSK modulation scheme based on the cyclostationary statistics of the phase of the signal. They are robust to carrier frequency error and phase offset. Their performance is comparable to the existing estimators.

CHAPTER V

CONCLUSIONS

A. Summary of the Thesis

To summarize much of the work in this thesis was based on exploring the idea of minimal rate feedforward non-data aided timing estimators. The idea that a signal sampled at Nyquist rate contains sufficient statistics was pushed to its logical conclusion in terms of the rate required for an estimator. And we derived some new estimators some which were ad hoc and others based on cyclostationary framework. For linear modulation schemes, we developed two new estimators with somewhat increased complexity but with improved performance among the class of existing Nyquist rate estimators. For nonlinear modulations we were able to derive novel estimators at the symbol rate which are among the only ones known for feedforward timing recovery of CPM signals. Also we proposed two new estimators for MSK modulations both of which are robust to frequency and phase offsets.

B. Suggestions for Further Work

One of the drawbacks of the estimators was their increased complexity. While we successfully demonstrated various alternatives to estimation of the timing delay, from a practical perspective we would also be interested in reducing the complexity of the estimators. It would be interesting to see if there are lower complexity estimators at lower rates. While we were able to come up with symbol rate estimators for the nonlinear modulations we were unable to do so for the linear case. However, we conjecture that there exist symbol rate feedforward timing estimators for the linear modulations too. It would be an interesting problem to explore. Another area which can be worked on is the analysis of MSE of the estimators. We have relied on the computer simulations to a great deal to evaluate the performance of the estimators we proposed. It would be interesting to corroborate these results by theory. Also we note that the asymptotic analysis of estimators that make use of the sign function should prove to be an interesting problem since most of the estimators that we encounter are not. We have started working on it but there are still issues to be figured out.

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VITA

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