MULTI-PERIOD OPTIMIZATION OF PAVEMENT MANAGEMENT SYSTEMS

A Dissertation

by

JAEWOOK YOO

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2004

Major Subject: Industrial Engineering

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ABSTRACT

Multi-Period Optimization of Pavement Management Systems. (May 2004)

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The purpose of this research is to develop a model and solution methodology for selecting and scheduling timely and cost-effective maintenance, rehabilitation, and reconstruction activities (M & R) for each pavement section in a highway network and allocating the funding levels through a finite multi-period horizon within the constraints imposed by budget availability in each period, frequency availability of activities, and specified minimum pavement quality requirements. M & R is defined as a chronological sequence of reconstruction, rehabilitation, and major/minor maintenance, including a "do nothing" activity. A procedure is developed for selecting an M & R activity for each pavement section in each period of a specified extended planning horizon. Each activity in the sequence consumes a known amount of capital and generates a known amount of effectiveness measured in pavement quality. The effectiveness of an activity is the expected value of the overall gains in pavement quality rating due to the activity performed on a highway network over an analysis period. It is assumed that the unused portion of the budget for one period can be carried over to subsequent periods.

Dynamic Programming (DP) and Branch-and-Bound (B-and-B) approaches are combined to produce a hybrid algorithm for solving the problem under consideratioin. The algorithm is essentially a DP approach in the sense that the problem is divided into smaller subproblems corresponding to each single period problem. However, the idea of fathoming partial solutions that could not lead to an optimal solution is incorporated within the algorithm to reduce storage and computational requirements in the DP frame using the B-and-B approach.

The imbedded-state approach is used to reduce a multi-dimensional DP to a one-dimensional DP. For bounding at each stage, the problem is relaxed in a Lagrangean fashion so that it separates into longest-path network model subproblems. The values of the Lagrangean multipliers are found by a subgradient optimization method, while the Ford-Bellman network algorithm is employed at each iteration of the subgradient optimization procedure to solve the longest-path network problem as well as to obtain an improved lower and upper bound. If the gap between lower and upper bound is sufficiently small, then we may choose to accept the best known solutions as being sufficiently close to optimal and terminate the algorithm rather than continue to the final stage.

To my family

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I would like to give honor to God through whom all things are possible including empowering me to complete the demanding task of obtaining this Ph.D.

I also want to express my sincere appreciation and gratitude to Dr. Alberto Garcia-Diaz, the chairman of my advisory committee, for his inspiring guidance and encouragement throughout the research as well as for his suggestion of the research topic. His broad knowledge and deep understanding of the field were instrumental in initiating this research topic and were a great help in accomplishing this research effort. Special thanks should be extended to Dr. César O. Malavé, Dr. Amarnath Banerjee, and Dr. Moo-Hyun Kim for serving on my advisory committee.

I wish to thank my wife, Soyoung, my son, Seok-Min, and my daughter, Seung-A for their endurance during my study in this university. Finally, I would like to acknowledge the endless love and support of my parents, Mr. Wonyeol Yoo and Mrs. Jungja Yoo-Yoon.

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CHAPTER I

INTRODUCTION

I.1. Motivation

As pavements continue to deteriorate, they become structurally deficient and functionally obsolete. For this reason, there is a need for generating both timely and cost-effective M & R strategies. In the face of limited resources, selecting and scheduling efficient M & R programs has become the major concern of many highway agencies in managing their networks.

Most estimates show the funds currently being spent on roads are inadequate, indicating the need to spend available funds more effectively. According to the highway statistics prepared by the Federal Highway Administration (FHWA), $65.2 billion was spent on highway M & R by all units of government for fiscal year 2000. According to the recent district and county statistics provided by Texas Department of Transportation (TxDOT), TxDOT spent $3.2 billion in M & R expenditures for Texas highway facilities in fiscal year 2001. Given the great expenditures on highway management, the development of an optimization model for M & R scheduling and fund allocation that maximizes pavement quality in highway networks over a multi-period planning horizon is critical.

Limited funds should be allocated to pavement work in the most cost-effective manner. Typical considerations in the selection of M & R activities and in the computation of related cost and effectiveness are (1) pavement material (flexible or rigid) and age; (2) service conditions of pavement; (3) highway type (US, state highway, farm-to-market); and (4) stochastic variations of pavement service conditions, load

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capacity, and traffic characteristics over time. The magnitude and complexity of the process for organizing, recommending, and scheduling these M & R activities in a network of pavements has been the principal motivation for developing rational and systematic methodologies, known as Pavement Management Systems (PMS).

The decisions regarding the selection of pavement improvement activities would be more effective and consistent if they were considered on a system-wide basis along a mid-to-longterm planning horizon. Some of the most important limitations of current optimization procedures in most PMS are as follows: (1) only relatively short planning periods (3-5 years) are considered; (2) the effects of future resource supplies such as budget and frequency of activities on the present management schedule are not considered; and (3) state-of-the art mathematical techniques are not effectively used.

I.2. Problem definition

The purpose of this research is to develop a model and solution methodology for selecting and scheduling timely and cost-effective M & R activities for each pavement section in a highway network and to allocate the funding levels through a finite multi-period horizon within the constraints imposed by budget availability in each period, frequency availability of activities, and specified minimum pavement quality requirements.

Specifically, the measure of effectiveness to be maximized is the total volume of pavement benefit derived from the entire set of selected strategies over the multi-period planning horizon. It is assumed that the following information is given: (a) the group of pavement sections to be considered; (b) the M and R cost and treatment effect associated with each action on each pavement section in each period; (c) the available budgets for each period; (d) the availability activities on pavement sections over the multi-period planning horizon regarding a life time of these activities; and (e) the minimum pavement quality level requirements and serviceable pavement quality levels for pavement sections in the multi-period planning horizon. It is also assumed that the unused portion of the budget for one period can be carried over to the subsequent periods.

It should be noted that the stochastic variation of pavement quality over time is a typical consideration in the selection of an activity in the computation of effectiveness. It could be handled by a transition probability matrix, but, in this research it is assumed that this information is known. A study outlining specific procedures for generating these data was conducted by Butt et al. (1987).

I.3. Objectives and Contributions

The focus of this dissertation is to formulate and solve an optimization model for programming M & R strategies in a PMS along a multi-period planning horizon. To accomplish this goal, the following specific objectives will be considered:

- the conversion of the original model to a resource-constrained longest-path network problem
- the application of a dynamic programming approach to solve for management solutions
- the design of a problem-specific branch-and-bound procedure
- the application of Lagrangean relaxation and subgradient optimization procedures to obtain lower and upper bounds on the remaining stages at each stage in the dynamic programming model
- the computerized sample runs of the proposed procedure.

The most significant contributions of the research developed in this dissertation are:

- a computationally efficient solution procedure for multi-period planning problems obtained by combining dynamic programming and branch-and-bound procedures, and exploiting the computational efficiency of network algorithms at each iteration in a subgradient optimization procedure

- an optimization procedure that allows for the selection and the scheduling of M & R activities for each section in a highway network
- the capability of the model to allow pavement managers to make more consistent and effective decisions regarding the allocation of limited funds in each period as well as the frequency of activities
- the demonstration that more wholistic (i.e. system-wide and multi-period) considerations are critical to improving current methods in M & R and PMS

I.4. Organization of Dissertation

This dissertation consists of six chapters. The introduction has provided the motivation for the research, the definition of the problem, the research objectives, and the expected contributions. The second chapter is a brief review of related past work on Pavement Management Systems and the mathematical tools used in this dissertation. Chapter III presents the mathematical formulation of the problem and the overall solution approach. Chapter IV provides the development of each procedural component of the proposed solution methodology. Chapter V presents implementation and computational results. Finally the summary, conclusions of the work, and directions for further research are presented in Chapter VI. In addition to the six chapters, computer implementation and data generation procedure are provided in the appendices.

CHAPTER II

LITERATURE REVIEW

II.1. Introduction

The development of systematic optimization approaches for Pavement Management System (PMS) has received increasing attention in the last few decades. The basic framework of these approaches is the utilization of mathematical programming techniques. Integrality of the decision variables has shifted most the research efforts toward the use of integer programming techniques.

In this chapter, the literature on the application of optimization approaches to PMS will be discussed first. Since the proposed methodology in this research is for solving multi-dimensional binary knapsack problems, the second part of the review will be devoted to that area.

II.2. Literature on the Application of Optimization Approaches to PMS

Three common mathematical techniques employed by optimization procedures for PMS are linear programming (LP), integer programming (IP), and dynamic programming (DP). In the rest of this section a brief review of the available literature on the application of optimization approaches to PMS will be discussed according to these three mathematical techniques.

II.2.1. Linear Programming

Golabi et al. (1982) developed a PMS for the state of Arizona to produce optimal maintenance policies for each mile of the 7,400 miles network of highways. This optimization system employed a combination of Markov prediction modeling and LP, with minimization of total cost used as the objective function in the LP model. Utilizing 'fraction of network' as decision variables in the LP formulation results in the loss of exact location information. It has been subsequently enhanced and used as an analysis tool many times by Wang et al. (1993,1994,1995) and Liu (1996). Since the original development a number of other PMS's have adopted the same basic formulation such as Alaska, Kansas, and Portugal (Alviti et al., 1994, Golabi, 2002).

Grivas et al. (1993) presented an LP model for planning period and budget allocation involved in network-level pavement management. The LP was formulated to model interactions between economic and engineering factors in an effective manner. It enabled decisions about the type of treatment, timing, and magnitude of work to be made simultaneously. In the mode, both project- and network-level constraints can be imposed to develop a pavement managememt that meets specified requirements on condition and budget. The developed methodology has been implemented as part of the New York State Thruway Authority's PMS.

Mbwana and Turnquist (1996) developed a new formulation of a network-level PMS using models based on a Markov decision process, utilizing Markov transition probabilities for pavement condition modeling and including the identification of specific network links in the optimization. The incorporation of specific links into the model allowed easier translation of network-level policies to project-level decisions than had previously been possible. This formulation also allowed the easy incorporation of user and agency costs as well as a variety of other specific constraints on the solution.

Theodorakopoulos et al. (2002) developed a decision support system to assist pavement management agencies in M & R planning and implementation in the road network of Greece. Optimal strategy selection in the network level was obtained via LP model which aims to minimize agency costs subject to constraints related to the desirable pavement condition over the network and planning horizon. In the project level, decisions about M & R project structure, planning and resource allocation were provided.

II.2.2. Integer Programming

Chen et al. (1992) applied on IP model in the Oklahoma Department of Transportation for strategic planning of pavement rehabilitation and maintenance, which provided a valuable tool for the highway agencies to manage the network properly. In this application, the overall effectiveness of all selected maintenance and rehabilitation projects is maximized in the 0-1 integer linear programming, which is subject to the constraints of minimum pavement serviceability, available budget, and resource suppliers. However, integer programming becomes computationally intensive and unreasonably long if it is applied to a large scale road network, in particular if multi-period decisions of pavement preservation strategies are considered.

Li et al. (1998) developed a cost-effectiveness-based integer programming on a year-by-year basis for the preservation of deteriorated pavements in a road network with the constraints of budget limitations and a required pavement serviceability levels. The objective of the optimization system was to select the most effective M & R projects for each programming year.

Fwa et al. (2000) developed a genetic-algorithm-based procedure for solving multi-objective network level pavement maintenance programming problems. The concepts of Pareto optimal solution set and rank-based fitness evalutation for selecting an optimal solution were adopted. IP formulation and development of the solution algorithm were described and demonstrated with a numerical example problem in which a hypothetical network level pavement maintenance programming analysis were performed for two- and three-objective optimization, respectively.

The performance of the genetic algorithms is affected by the method used to handle the many constraints present in the formulation of resource allocation problems like the network pavement maintenance problem. Chan et al. (2001) proposed a method that is based on prioritized allocation of resources to maintenance activities and the maximum utilization of resources. It was demonstrated that the genetic algorithm with the prioritized resource allocation method outperforms the traditional genetic algorithm.

II.2.3. Dynamic Programming

The PAVER system was originally developed by the U.S. Army Corps of Engineers and has been in existence for more than twenty years. In 1987, the use of Markov chain prediction models in DP formulation began to take shape when Butt et al. (1987) published a paper on the application of Markov chain to pavement performance prediction. Soon afterwards Feighan et al. (1988) showed how to use DP for optimization using this Markov model although this had not been fully implemented at the time. Since then a number of papers concerning PAVER or similar formulation have been published (Butt et al., 1994, Feighan et al., 1989a, 1989b).

Another DP formulation that has appeared in the literature on a number of occasions is Chua et al.'s 'Dynamic Decision Model' (1993). Many formulations using Markov chain prediction models only allow transition matrices where probabilities are dependent only on the current state regardless of history and are thus time invariant. PAVER allows different matrices for different broad stages and has been described as time variant, but Chua et al.'s formulation (1993) has gone a step further in that overlays of the pavement are tracked by a structure vector which dictates reference to a different transition matrix depending on the current structure of the pavement. While this is similar it is considerably more limited because it has to be an integer in the sigmoidal modeling for restoration and deterioration and is incapable of taking partial repair such as patching into account. Additionally, a number of different condition variables can be tracked by making use of a condition vector.

II.3. Solution Procedure for Multi-Dimensional Binary Knapsack Problems

In this research, the optimal allocation of resources in a highway maintenance and rehabilitation system over a multi-period planning horizon can be formulated as a multi-dimensional binary knapsack problem with alternative selection constraints and precedence-feasibility constraints. These side constraints and the characteristic of the objective function transform the formulation model into a resource-constrained longest-path problem equivalent to the original problem. One approach for finding an optimal solution to this type of model is through the use of DP and B-and-B techniques.

In the rest of this section a brief review of the available technical literature in the computational area of DP and B-and-B algorithms will be presented. After this review, the particular algorithms developed for solving multi-dimensional binary knapsack type problems will be reviewed, and summarizing remarks will be described.

Nemhauser and Wolsey (1988) explained an almost complete line of algorithms for a variety of 0-1 integer programming problems including knapsack problems. Algorithms give an exact solution or an approximate solution. Exact solution procedures are basically one of two types: an implicit enumeration approach or a polyhedral approach. Heuristics for an approximate solution include primal heuristics, dual approaches, and heuristics employing relaxation techniques such as linear programming relaxation, surrogate relaxation, and Lagrangean relaxation to get information about the optimal solution.

Implicit enumeration approaches are based on the techniques of DP and B-and-B. Since the enumeration is basically of exponential time, the number of decision variables makes this approach difficult. Therefore, efficient algorithms usually employ a reduction scheme in which a sensitivity analysis is conducted to set as many variables as possible equal to their optimal values before initiating the implicit enumeration.

DP is a well-known approach for the optimization of a separable function which provides a global optimal solution even in the case of nonconvex programming problems. However, the use of this powerful technique for discrete variable problems is limited by its excessive computer storage and computational requirements. These computational problems become more severe whenever: 1) the state variables are defined by a vector of more than three dimensions and 2) the states are in a low dimension form, but the number of discontinuities of the states grow exponentially in the algorithmic process.

Considerable research has been devoted to overcoming the problem of dimensionality in DP techniques. A significant attempt for reducing the dimensionality of state variables is found in the approach of Morin and Marsten (1976a) who developed an algorithmic procedure for solving multi-dimensional DP problems by searching over an imbedded state space. The idea behind an imbedded state approach is to find the integer lattice points which cause a jump in the values of the return function. The points of discontinuities are then checked for feasibility and based upon this information sets of infeasible solutions can be eliminated. Feasible points are checked for dominancy and the dominated points are eliminated. As a result, a set of feasible and efficient solution points is defined as an imbedded state, and a search is performed over these points in a stagewise manner.

As mentioned earlier, DP approaches tend to require excessive storage space and this makes the algorithm very inefficient when the problem is multi-dimensional or the value of the right-hand side is quite large. B-and-B techniques tend not to be effected by this disadvantage and this is why the most efficient enumeration algorithms for more complicated problems are based on branch-and bound.

In B-and-B algorithms it is essential to have a design that is well-balanced between bounding schemes, branching rules, and heuristics for improving feasible solutions. When a B-and-B procedure fails, it is usually because of too many nodes in the search tree or too much computing time at each node. The size of the search tree largely depends on the branching rules and the tightness of the bounds.

It is often the case that obtaining an exact solution may not be necessary. For instance, a solution close to optimal would be sufficient when the parameters of the model are only expectations of returns. In computational complexity most 0-1 problems are in the class of NP-complete, an approximate solution may be satisfactory, especially for large problems when restrictions on computational times are made. Moreover, even if an exact method is applicable, the first step for the method is usually to obtain a good starting feasible solution by use of heuristics.

The employment of relaxation involves a tradeoff between the bound strength and calculation speed. LP and Lagrangean relaxations are widely used for bounding. The LP relaxation is not considered in this dissertation. The strength of bounds depends upon the choice of constraints to be relaxed in Lagrangean relaxation [see Geoffrion (1974) and Fisher (1981) for a general theory of Lagrangean relaxation; see Fisher (1985) for a practical guide to the Lagrangean relaxation with many examples and illustrations; see Handler and Zang (1980) and Beasley and Christofides (1989) for the Lagrangean relaxation with a resource-constrained shortest-path problem].

Morin and Marsten (1976b, 1978) have also demonstrated how the B-and-B method can be implemented in DP for reducing the storage and computational requirements. The use of the simple yet effective techniques of B-and-B to eliminate states in DP algorithms is a general approach in the sense that it can be applied to all finite dynamic programs. Both the idea of B-and-B and the imbedded state approach have been incorporated with the separation and initial fathoming provided by DP to produce a hybrid DP/B-and-B algorithm.

Dyer et al. (1995) developed a hybrid DP/B-and-B algorithm to solve the multiple choice knapsack problems. Lagrangean duality was used in a computationally efficient manner to compute tight bounds on every active node in the search tree. The use of Lagrangean duality also enabled the use of a reduction procedure to reduce the size of the problem for the enumeration phase.

CHAPTER III

MODEL AND SOLUTION APPROACH

III.1. Introduction

Multi-period optimization problems of PMS's can be formulated as a multi-dimensional binary knapsack models with alternative selection and precedence-feasibility constraints. Since the practical problems are large, the algorithm must be computationally efficient as well as feasible. This chapter is organized in three sections. The mathematical model formulation is presented in the first section; the overall conceptual approach to solving the model is described in the second; and a brief summary of the chapter is provided in the last section

III.2. The Model

Suppose that there are I pavement sections in a system, T periods in the planning horizon, and J actions for each pavement section in each period to be considered. Let e_{ijt} represent the effectiveness of alternative j for pavement section i in period t, c_{ijt} the cost of alternative j for pavement section i in period t, B_t the budget available for period t, N_{ij} the maximum number of times alternative j can be used on pavement section i in the planning period horizon, PQ_{it} the pavement quality level of pavement section i in period t, Dp_j the treatment effect of alternative j, s the serviceable pavement quality level such that if the pavement quality level is above this level in any time period t, then pavement section i is not considered for a maintenance in that particular period t, m the minimum pavement quality level, and M the maximum pavement quality level. The decision variable x_{ijt} is equal to 1 if alternative j for pavement section i in period t is selected, and it is equal to 0 otherwise.

The problem of generating a sequence of interrelated M & R strategies over a fixed planning time period for each pavement section so as to maximize the overall highway pavement quality level while not exceeding budget availability in each period, as well as not exceeding frequency availability of actions for pavement sections over the planning periods, under the assumption that unused budget portions in a period are carried over to subsequent periods, can be formulated as a multi-dimensional 0-1 knapsack problem with alternative selection and precedence-feasibility constraints.

Problem (*P*)

$$\max \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ijt} x_{ijt} \qquad (1)$$

s.t

$$\sum_{i=1}^{I} \sum_{j=1}^{J} c_{ijt} x_{ijt} \le B_t \qquad \text{for all } t \qquad (2)$$

$$\sum_{t=1}^{T} x_{ijt} \le N_{ij} \qquad \text{for some } i, j \qquad (3)$$

$$\sum_{j=1}^{J} x_{ijt} = 1 \qquad \text{for all } i, t \qquad (4)$$

$$PQ_{it} \ge m \qquad \text{for all } i, t \qquad (5)$$

$$PQ_{it} \ge s \quad \Rightarrow \sum_{j=2}^{J} x_{ijt} = 0 \qquad \text{for all } i, t \qquad (6)$$

$$PQ_{i,t-1} + \sum_{j=1}^{J} Dp_j x_{ijt} \le M \qquad \text{for all } i, t \qquad (7)$$

$$x_{ijt} \in \{0,1\} \qquad \text{for all } i, j, t \qquad (8)$$

In the formulation of **Problem (*P*)** the objective function (1) maximizes the total effectiveness for the system; the budget constraint set (2) indicates that the capital consumption by the selected alternatives can not exceed the available budget in each period; the frequency constraint set (3) ensures that some alternatives on some pavement sections can not be taken more than the available frequency regarding the lifetimes of the alternatives over the multi-period planning horizon; the alternative selection constraint set (4) forces the problem to choose one and only one strategy for each pavement section in any time period. (Note: Strategy '1' stands for 'do-nothing' activity.); the constraint set (5) is used to eliminate any alternative strategy that does not meet the minimum pavement quality level requirements for a pavement section in a period; the constraint set (6) ensures that a pavement section is not considered for a maintenance if its condition is better than a predefined serviceable pavement quality level in a given period; the constraint set (7) makes the alternatives infeasible when treatment effects make the pavement quality exceed the maximum pavement quality level; and the constraint set (8) imposes the integrality of the decision variables.

There are three types of constraints imposed on this problem: resource, alternative selection, and precedence-feasibility. The resource constraints consist of constraint sets (2) and (3). Constraint set (4) is an alternative selection constraint. Constraint sets (5), (6), and (7) are associated with precedence-feasibility constraints.

Example 1. For illustrating the problem definition, consider an example with two pavement sections, three maintenance alternatives (in which alternative 1 is do nothing, alternative 2 is minor maintenance, and alternative 3 is major maintenance), and three planning periods. It is assumed that pavement qualities for each pavement section are 3.9 and 2.6 respectively, the minimum pavement quality m is 2.5, the serviceability pavement quality s is 4.0, and the maximum pavement quality M is 5.0. It assumed that the transition probability matrix for each section is known, the major maintenance cannot be taken more than 1 time for 3 planning periods for each section, and that the budgets for the three planning periods are 20, 22, and 24 respectively. The required data for this example is given in Table III.1.

Using matrix notation, **Problem (*P*)** can be reformulated as $\max\{RX : AX \le b, X \in W\}$, where R is the total effectiveness for the system, (A, b) is for the knapsack constraint set, and W = {X: alternative selection constraint set, precedence-feasibility constraint set, and 0-1 integrality constraint set}.

R is calculated using a transition probability matrix and a state vector. Details of the calculation procedure for objective function are discussed in Appendix B. $X = (x_{111}, x_{121}, x_{131}, x_{211}, x_{221}, x_{231}, x_{112}, x_{122}, x_{132}, x_{212}, x_{222}, x_{232}, x_{113}, x_{123}, x_{133}, x_{213}, x_{223}, x_{233})^T$. Matrix A and vector b are obtained from Table III.1:

$$A = \begin{bmatrix} 0 & 9 & 15 & 0 & 12 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 18 & 0 & 15 & 24 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 22 & 0 & 18 & 29 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 20 \\ 22 \\ 24 \\ 1 \\ 1 \end{bmatrix}$$

Table III.1. An Example Data Set

Action	Cost in period 1		Cost in period 2		Cost in period 3		Treat Effect	Frequency
	Sect 1	Sect 2	Sect 1	Sect 2	Sect 1	Sect 2		
1	0	0	0	0	0	0	0	-
2	9	12	11	15	13	18	0.5	-
3	15	20	18	24	22	29	1	1
Bedget	20		22		24		-	-

Figure III.1 shows an overall frame of the formulation of **Problem (*P*),** where $\hat{X}_t$ is an M & R strategy for a highway network in period t; $\hat{X}_t = (X_{1t}, X_{2t}, ..., X_{It})$; $X_{it} = (x_{i1t}, x_{i2t}, ..., x_{iJt})$; and R_t is a return (effectiveness) of strategy $\hat{X}_t$.

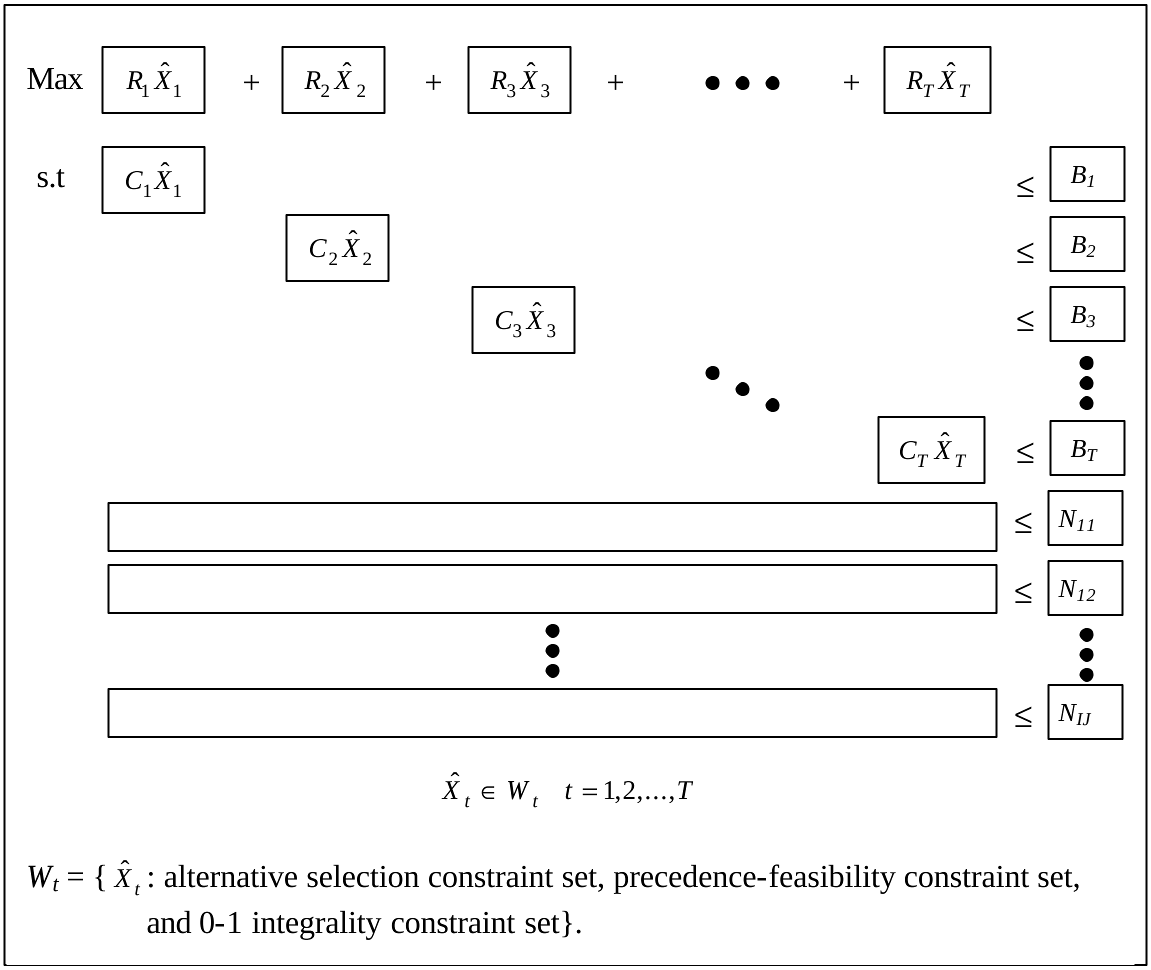

Figure III.1. Overall Frame of Formulation

III.3. Solution Approach

DP and B-and-B approaches are combined to produce a hybrid algorithm for solving the problem formulated as a multi-dimensional 0-1 knapsack problem with alternative selection and precedence-feasibility constraint. The algorithm is essentially a DP approach in the sense that the problem is divided into smaller subproblems corresponding to each single period problem. However, the idea of fathoming partial solutions that could not lead to an optimal solution is incorporated within the algorithm to reduce storage and computational requirements in the DP using the B-and-B approach. The feature of the hybrid algorithm is its capability of reducing the state-space which otherwise would present an obstacle in solving multi-dimensional DP problems. This is due to the use of the imbedded-state approach, which reduces a multi-dimensional DP to a one-dimensional DP (Morin and Marsten, 1976a). Other reductions are made through fathoming the state-space and subsequent elimination of the state-space, which tends to eliminate inferior solutions compared to the predetermined lower bound or updated lower bound.

Due to alternative selection, precedence-feasibility, and an integrality constraint set, the original problem is transformed to a resource-constrained longest-path network model. A Lagrangean relaxation of the resource-constrained longest-path problem (RCLPP) into an unconstrained longest-path problem is developed, providing an initial lower and upper bound for the objective function as well as a lower and upper bound for bounding tests at each stage of DP. At each stage, the lower and upper bound are also updated and are used for termination and fathoming criteria. The relaxed problem can be solved by using a subgradient optimization procedure, while a network algorithm (Ford-Bellman) is employed at each iteration of the subgradient optimization procedure to solve the longest-path network problem as well as to obtain an improved lower and upper bound. If the gap of the lower and upper bound is in predetermined parameter e or the improved lower bound is optimal, then the procedure is terminated rather than continuing to stage T. Otherwise, the DP approach for a single period problem is conducted to identify feasible solutions to the next period problem corresponding to the next stage in the multi-period DP.

Feasible solutions that are dominated by any other feasible solutions are eliminated. Efficient solutions that are not dominated are then obtained. By performing a bounding test, the efficient partial solutions that cannot lead to a solution that has a lower bound better than the best known bound are fathomed. Lagrangean relaxation and subgradient optimization procedures are applied to the remaining problem in order to perform the bounding process at the current stage. Then the survivors are used to generate potential solutions for the next stage.

Figure III.2 shows the overall conceptual approach of the proposed methodology. The proposed approach can be divided into two procedures: DP and B-and-B. The DP procedure consists of steps 4 and 5, and B-and-B procedure steps 1, 2, 3, 6, and 7. Step 6 is a repetition of step 2 and 3. A brief description of each major component of the methodology is provided as follows.

Step 1. Reformulation

The alternative selection, precedence-feasibility, and integrality constraint set transform **Problem (*P*)** into a RCLPP equivalent to the original **Problem (*P*)**.

Step 2. Lagrangean Relaxation

The RCLPP is relaxed in a Lagrangean fashion by dualizing the budget constraint set in each period and the frequency constraint set over a multi-period planning horizon so that the relaxed problem is decomposable into subproblems, one subproblem per pavement section.

Step 3. Subgradient Optimization

The value of the Lagrangean multipliers, which gives the least upper bound for **Problem (*P*)**, is obtained by subgradient optimization. At each iteration of the subgradient optimization procedure, a network algorithm (Ford-Bellman) is employed to solve the longest-path network problem as well as to obtain an improved lower and upper bound for **Problem (*P*)**. If the improved lower bound is optimal or the gap of the lower and upper bound are in predetermined parameter e, then the procedure is terminated: Otherwise, Step 4 is performed.

Step 4. Single Period DP

DP approach is conducted to identify feasible solutions to a single period problem corresponding to a stage in multi-period DP making use of the imbedded-state approach.

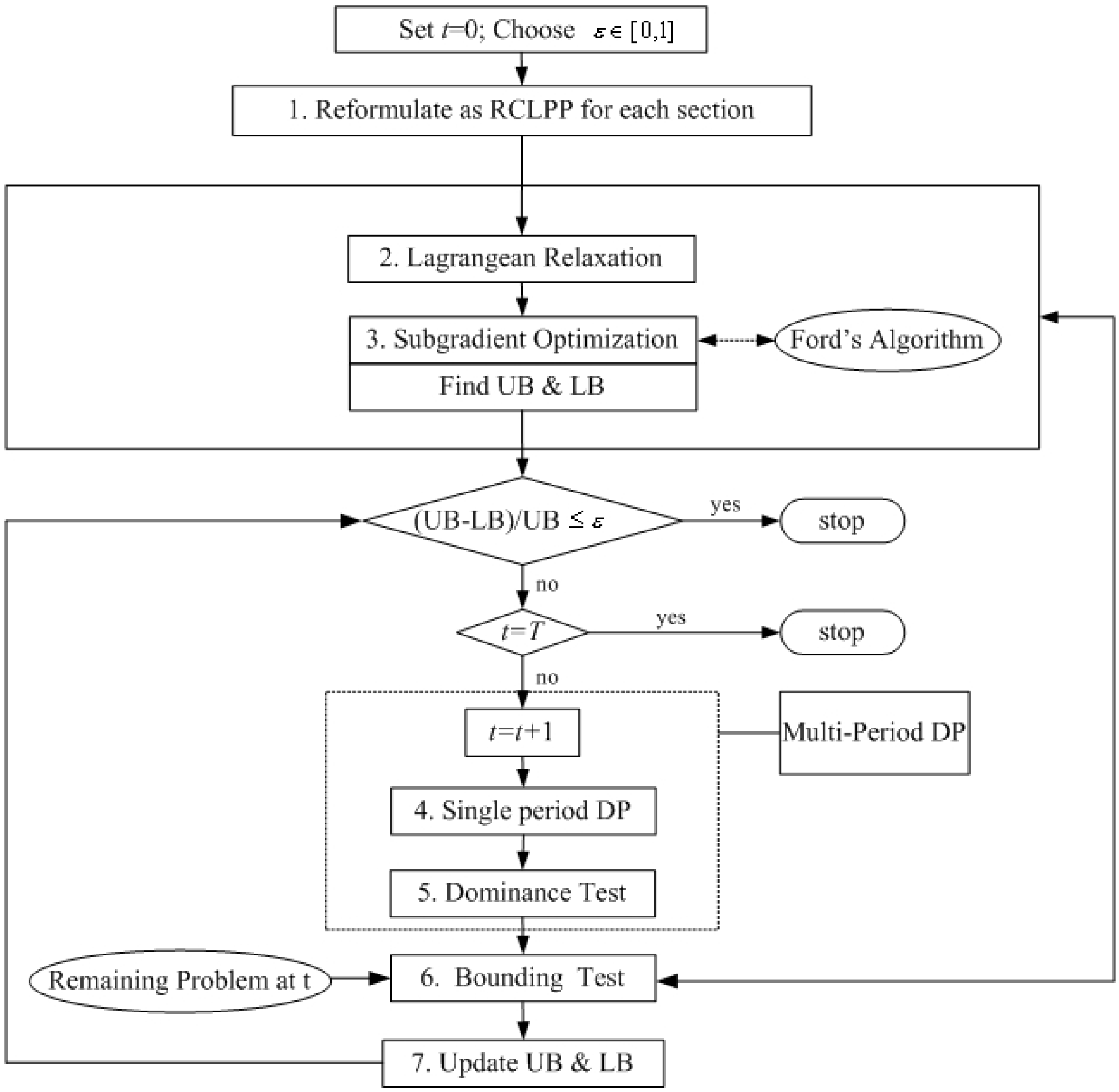

Figure III.2. Overall Conceptual Approach

Step 5. Dominance Test

By dominance testing, the feasible solutions that are dominated by any other feasible solutions are eliminated. The efficient solutions, which are not dominated by any other feasible solution, are obtained.

Step 6. Bounding Test

By bounding testing, the efficient partial solutions, which cannot lead to a solution that is better than the incumbent, are fathomed. The survivors at the stage *t*, which are not eliminated by bounding test, are obtained and used to generate potential solutions to the next stage.

Step 7. Update Upper Bound (UB) and Lower Bound (LB)

The UB and LB at stage *t* are updated if it is available.

III.4. Summary

Multi-period optimization of PMS's is formulated as a multi-dimensional binary knapsack problem with alternative selection and precedence-feasibility constraints, and a solution approach is outlined. The approach is a hybrid DP/B-and-B procedure with imbedded Lagrangean relaxation. Relaxation and fathoming criteria, which are fundamental to B-and-B, are incorporated within the separation and fathoming provided by the DP framework in order to provide the hybrid DP/B-and-B algorithm.

Detailed descriptions of each step of the solution approach (including resource-constrained longest-path network representation, Lagrangean relaxation, subgradient optimization, multi-period DP, single period DP, imbedded state space approach, and B-and-B procedure) are presented in Chapter IV.

CHAPTER IV

DEVELOPMENT OF SOLUTION PROCEDURES

IV.1. Introduction

The detailed procedures of the algorithm proposed in chapter III are presented in this chapter. The approach is a hybrid algorithm combining DP and B-and-B. The remaining portion of this chapter consists of eight additional sections. Section IV.2 describes and discusses a resource-constrained longest-path network representation of **Problem (*P*)**; Section IV.3 presents Lagrangean relaxation and some theoretical results; Section IV.4 covers the subgradient optimization procedure; Section IV.5 presents dynamic programming for multiple periods; Section IV.6 describes single-period dynamic programming; Section IV.7 presents the imbedded state space approach; Section IV.8 describes B-and-B techniques; and Section IV.9 provides a brief summary of the chapter. A small hypothetical example will be considered to illustrate each step of the proposed methodology doing the way.

IV.2. Resource-Constrained Longest-Path Network Representation

Because of the alternative selection, precedence-feasibility, and integrality constraints, it is easy to model **Problem (*P*)** as an RCLPP. A network model of **Problem (*P*)** is shown in Figure IV.1. The network model for pavement section i is constructed in a stagewise fashion, where each stage corresponds to a value of period t, and there are a total of T stages. Variables considered for network generation at each stage t consist of the variables in the corresponding alternative selection constraint set.

For example, at stage t variables $x_{i1t}, x_{i2t}, \ldots, x_{iJt}$ are considered. Node s_i is a source node. A feasible set of variables in an alternative selection constraint set is determined

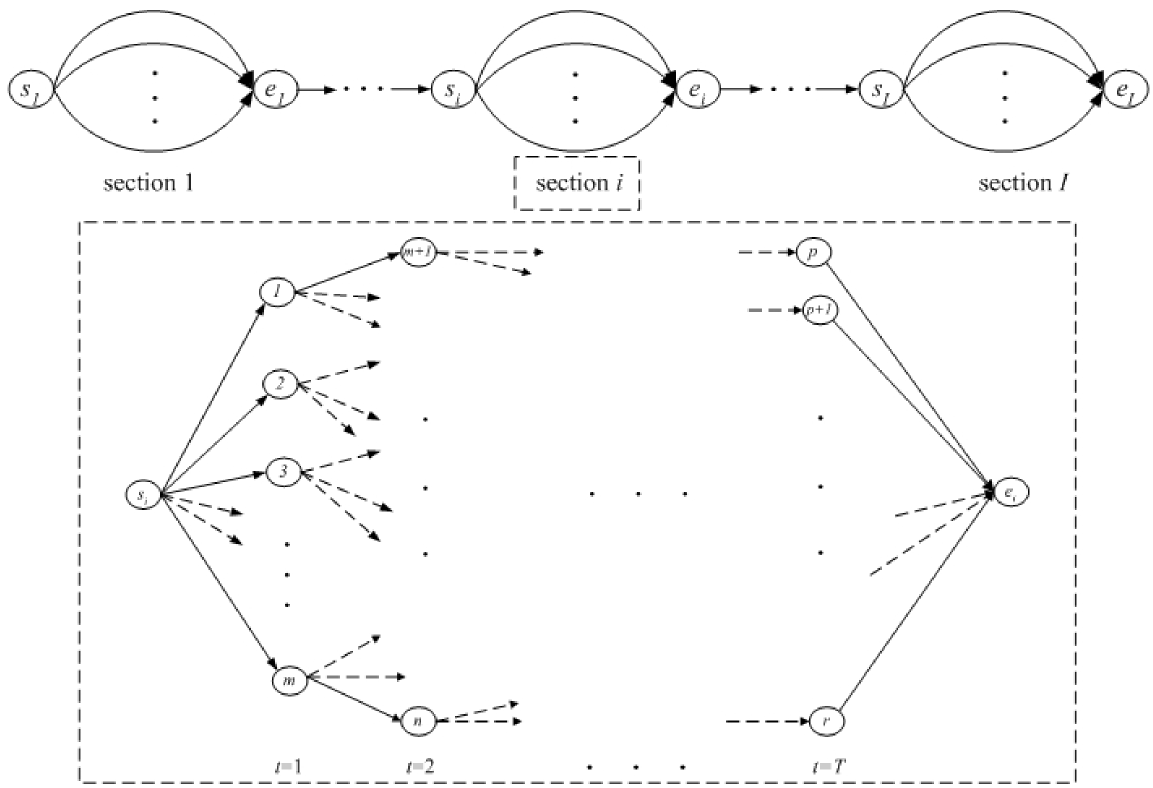

Figure IV.1. A Network Model for Pavement Sections

from the precedence-feasibility constraints set with period t=1, and arcs are added for each feasible strategy x_{ij1} from the source node to nodes 1, 2, . . . , m, where m is the number of feasible strategies in stage 1 ($m \leq J$). This set of nodes is considered to represent stage 1. A feasible set of strategies at stage 2 is again determined at each of the nodes 1, 2, . . . , m, and more arcs and nodes are generated for each of these nodes to represent stage 2. This process is continued until stage T is reached. Arcs emanating from each node in stage T are converged to a single node e_i which is defined as the sink node. The arc lengths are calculated from the objective function for the corresponding value of alternative j and period t on pavement section i. This calculation is possible because each e_{ijt} is a function of the strategies employed at previous stages on a path from node s_i to a particular node.

Even if there were only four or five strategies feasible at each stage, the number of nodes and arcs rapidly increases beyond computational limitations. The number of arcs and nodes can be reduced by the following method. Suppose at some node n at stage t strategy j is feasible and an arc is emanated from node n to some other node $q > n$. Node q is at a stage t+1 by the previously described, but if only strategy 1 (do nothing) were feasible for node q, the corresponding effective coefficient $e_{i,1,t+1}$ would be added to the length of the arc from node n to node q, and at this point node q is moved into stage t+2. This process is repeated until node q reaches a stage t such that a strategy other than just strategy 1 is feasible or node q reaches state T. This procedure is always applicable if there are precedence-feasibility constraints in the problem.

The length of the longest-path from source node s_i to node e_i (satisfying resource constraints) is the optimal solution to the subproblem corresponding to pavement section i, and the corresponding solution can be obtained from the arcs and nodes on this path. A similar network is generated for each subproblem corresponding to each pavement section in the highway network. The network models for each subproblem are linked sequentially such that e_i is connected to s_{i+1} for i = 1, 2, …, I-1. Then the longest-path from source s_1 to sink node e_I (satisfying resource constraints) is an optimal solution to **Problem (*P*)**.

Example 2. Consider the problem given in Example 1 in section III.2. As assumed in Example 1, the current pavement qualities for each pavement section are 3.9 and 2.6 respectively, the minimum pavement quality is 2.5, the serviceability pavement quality is 4.0, and the maximum pavement quality is 5.0. It is also assumed that the transition probability matrices for these two pavement sections are as follows.

$$P_1 = \begin{bmatrix} 0.717 & 0.283 & 0 & 0 & 0 & 0 \\ 0 & 0.727 & 0.273 & 0 & 0 & 0 \\ 0 & 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0.664 & 0.336 & 0 \\ 0 & 0 & 0 & 0 & 0.692 & 0.308 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0.366 & 0.634 & 0 & 0 & 0 & 0 \\ 0 & 0.715 & 0.285 & 0 & 0 & 0 \\ 0 & 0 & 0.97 & 0.03 & 0 & 0 \\ 0 & 0 & 0 & 0.814 & 0.186 & 0 \\ 0 & 0 & 0 & 0 & 0.574 & 0.426 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The arc lengths are calculated from the objective function for the corresponding value of alternative j and period t on pavement section i using the state vector and transition probability matrix. (Details of the calculation procedure for objective functions are discussed in Appendix A.) The arcs are added for each feasible strategy determined from the precedence-feasibility constraint set in each period.

The resource-constrained longest-path network model for this example is also shown in Figure IV.2.

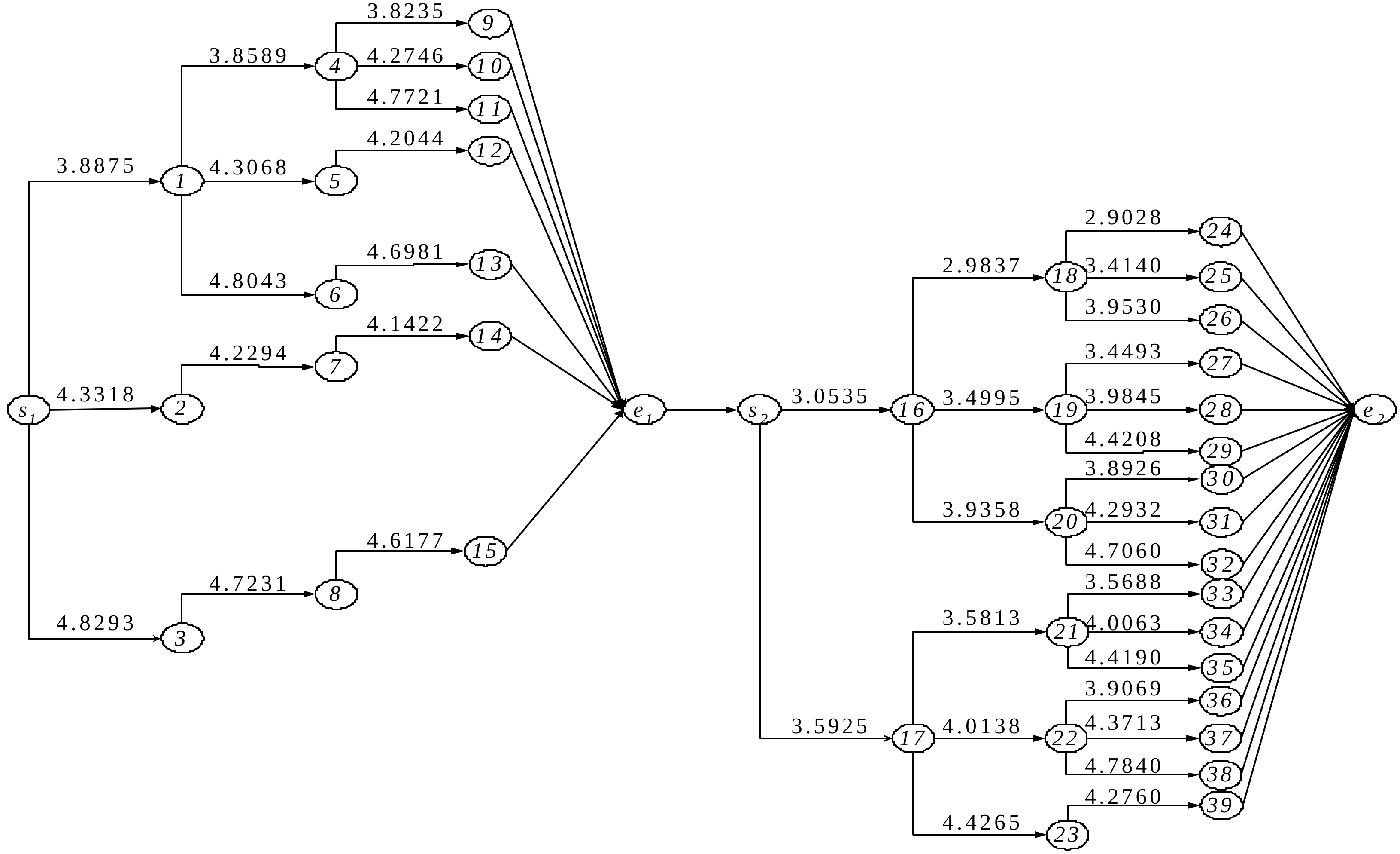

Figure IV.2. Network Representation of Example 1

IV.3. Lagrangean Relaxation

The Lagrangean relaxation approach to obtaining bounds in IP problems is the second most widely used after LP relaxation. A Lagrangean relaxation of an IP problem is obtained by removing the complicating constraints and including them in the objective function using multipliers so that the resulting problem is much easier to solve because of the special structure of the remaining constraints, and sometimes being decomposable by itself. A Lagrangean relaxation scheme is more attractive than the LP relaxation if the decomposition and continuity of the special structure can be achieved.

Using matrix notation, **Problem (*P*)** can be reformulated as

Problem (*P*)

$$\max\{ RX : AX \le b, X \in W \} \tag{9}$$

where (A, b) is for the knapsack constraint set and W = {X : alternative selection constraint, precedence-feasibility constraint, integrality constraint}. By dualizing the knapsack constraint set, the relaxed **Problem (*LR* (l))** is obtained:

Problem (*LR* (l))

$$\max\{(R - lA)X + lb : X \in W\} \tag{10}$$

where l is the vector of Lagrangean multipliers. This problem can be easily solved by decomposing it into I subproblems, one subproblem for each pavement section which represents a longest-path problem. The solution to each subproblem is obtained by applying the Ford-Bellman algorithm.

The least upper bound for **Problem (*P*)** is obtained by solving **Problem (*LD*)**.

Problem (*LD*)

$$\min\{ LR(l) : l \ge 0\} \tag{11}$$

An optimal value of the Lagrangean multipliers, l^*, is an optimal solution to the Lagragean dual, **Problem (*LD*)**, but it need not be solved optimally. Since the convergence rate of l is very slow in the neighborhood of the optimal value, a near optimal value will be satisfactory. The subgradient optimization method will be used to solve **Problem (*LD*)**. The following are some theoretical results.

Proposition 1. *If X_0 is an optimal solution to Problem ($LR(l_0)$), then*

$$s_0 = b - AX_0$$

is a subgradient of $f(l) = v(LR(l))$ at $l = l_0$.

Proof. See Proposition 4.1 of Section II.5.4 of Nemhauser and Wolsey (1988). In the statement, $v(\ ^*\ )$ represents the solution value of the problem (*).

Theorem 1. *If, for a given l ³ 0, a vector X satisfies the three conditions*

(i) X is an optimal solution to Problem ($LR(l)$),

(ii) $AX \leq b$,

(iii) $l\,(b - AX) = 0$,

then X is an optimal solution to Problem (P). If X satisfies (i) and (ii) but not (iii), then X is an x-optimal solution to Problem (P) with $x = l(b\text{-}AX)$.

Proof. See Theorem 1 of Geoffrion (1974) or Corollary 6.10 of Section II.3.6 of Nemhauser and Wolsey (1988).

IV.4. Subgradient Optimization

A review of methods for solving the Lagrangean dual, **Problem (*LD*)**, can be found in Bazaraa and Goode (1979) or Gavish (1978). Near-optimal multipliers are obtained by a subgradient optimization method or a multiplier adjustment method. The latter method is generally a specialized algorithm that exploits the structure of a particular problem.

The most popular method is subgradient optimization, because it is easy to implement and has worked well on many practical problems, especially on 0-1 IP problems (see Fisher for examples, 1985). Subgradient optimization is also considered to be a promising approach for solving the dual, **Problem (*LD*)**, especially when the relaxed problem is easy to solve. In this research the relaxed problem, **Problem (*LR* (l))**, is a longest-path network problem as mentioned before.

The subgradient method is an adaptation of the gradient method in which gradients are replaced by subgradients. Given l_0, a sequence l_p is generated by the rule

$$l_{p+1} = l_p + t_p (AX_p^* - b)^T, \tag{12}$$

where:

- l_p = the value of l at iteration p of the subgradient procedure, usually $l_0 = 0$ is the most natural choice, but in some cases other appropriate values (which are obtainable through experiments) can do better.
- t_p = the step length at iteration p, given by

$$t_p = p_p \frac{v(\mathrm{LR}(l_p)) - \mathrm{LB}}{\| AX_p^* - b \|^2}, \tag{13}$$

- X_p^* = the optimal solution to Problem (LR (l_p)),
- p_p = a scalar satisfying $0 < p_p \le 2$,
- v (LR (l_p)) = the value of the optimal solution to Problem (LR (l_p)),
- LB = the lower bound = the value of the best known solution, and
- Every negative element of l_{p+1} must be replaced with zero because of the nonnegativity requirement of l.

Computational performance and theoretical convergenece properties of the subgradient method are discussed by Held et al (1974). The fundamental theoretical result is that the sequence $\{v(LR\ (l_p))\}$ converges to $v(LD)$ if the sequence $\{t_p\}$ converges to 0 as p approaches to ∞ and $\sum_{0 \leq p \leq k} t_p$ approaches ∞ as k approaches ∞, as has been discussed by Bazaraa and Goode (1979), Fisher (1981), and Held et al. (1974). The subgradient method guarantees convergence to the optimal, but it does not guarantee a monotone improvement of $v\ (LR\,(l))$ at every iteration.

Often p_0 = 2 and p_p is multiplied by a factor whenever $v\ (LR\ (l))$ has failed to improve in some fixed number of iterations or in some combination of the number of iterations and the rate of improvement. The most widely used multiplication factor is 0.5. Gavish and Pirkul (1991) used it whenever no improvement was made in 15 consecutive iterations for the multi-resource generalized assignment problem, but Diaby et al. (1992) used 0.8 for very-large-scale capacitated lot-sizing whenever the improvement was not more than 0.5% in 3 consecutive iterations. These rules have worked well empirically, even though they do not guarantee to satisfy the sufficient condition given above for optimal convergence.

The procedure is terminated upon obtaining an optimal solution or upon reaching a predetermined iteration limit. If $v(LR\,(l_p)) = LB$, then the best known solution is optimal. The condition $v\ (LR\ (l_p)) - LB < 1$ can be applied when the objective function coefficients of **Problem (*P*)** are integers. Again, Gavish and Pirkul (1991) terminated the procedure if the total number of iterations = 500 or the number of iterations without improvement = 75, while Diaby et al. (1992) terminated the procedure if the improvement was not more than 0.5% in 40 iterations.

The subgradient optimization algorithm used in this dissertation is delineated as follows. Since the subproblem structure is straightforward, the subgradient algorithm can be run for a large number of iterations in order to ensure the convergence of the Lagragean multipliers to a near-optimal value.

Subgradient Optimization Procedure

(i) Choose $l_0 \geq 0$, $0 < p_0 \leq 2$, $z^* =$ a large number, $X^* = 0$, and $h^* = l_0$, Let $p = 0$ and LB=0 (the value of the best known solution).

(ii) Solve Problem (LR (l_p)). Let X^*_p be the solution.

(iii) If X^* is feasible, then if v (LR (l_p)) – $LB < e$, go to (xi); otherwise, $X = X^*_p$.

(iv) If $RX > LB$, then $LB = RX$ and the best known solution = X.

(v) If $z^* > v$ (LR (l_p)), then $z^* = v$ (LR (l_p)), $X^* = X^*_p$, and $h^* = l_p$.

(vi) If the improvement of v (LR (l_p)) is not more than 1 in 10 consecutive iterations, then set p_p equal to 0.5 p_p.

(vii) If the improvement of v (LR (l_p)) is not more than 1 in 20 consecutive iterations, then go to (x).

(viii) Obtain (l_{p+1}) using Equation 12.

(ix) Set p equal to p+1. Go to (ii).

(x) Terminate with $l^* = h^*$, v (LD) = v (LR (l^*)) = z^*, X^*, and the best known solution and LB.

(xi) X^*_{p} is an optimal solution to **Problem (*P*)** and the procedure is terminated.

Example 3. The subgradient optimization procedure outlined above is applied to the problem given in Example 1, where the matrix A and vector b are given. The procedure is initialized with $p_0 = 0.25$, $l_0 = (l_1,\ l_2,\ l_3,\ l_4,\ l_5)_0 = (0, 0, 0, 0, 0)$, and $LB = 0$. The procedure terminated in 20 iterations with the best known lower bound (x_{111}, x_{121}, x_{131}, x_{112}, x_{122}, x_{132}, x_{113}, x_{123}, x_{133}, x_{211}, x_{221}, x_{231}, x_{212}, x_{222}, x_{232}, x_{213}, x_{223}, x_{233}) = (1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0). The solution value is 24.1325, obtained at iteration 15 and the best known upper bound is obtained at iteration 17 with the Lagrangean multiplier, l_{17} = (0.1107, 0.0405, 0.0227, 0.0130, 0.0150) and v (LR (l_{17})) = 25.0805.

IV.5 Dynamic Programming for Multiple Periods

Both the problem of allocating funds in each period and the problem of selecting and scheduling activities could be handled by DP. The DP model for multiple periods is developed in a compact form of the separable nonlinear multidimensional knapsack problem (NKP) as shown in **problem (*D*)**, which is equivalent to **Problem (*P*)**, and is as shown in Figure IV.3:

Problem (*D*)

$$f_T(b) = \max \sum_{t=1}^{T} R_t \hat{X}_t \tag{14}$$

$$\text{s.t} \quad \sum_{t=1}^{T} A_{rt} \hat{X}_t \le b_r \qquad 1 \le r \le T + I \times J \tag{15}$$

$$\hat{X}_t \in W_t \qquad t = 1,2,\ldots,T \tag{16}$$

where $\hat{X}_t$ is an M & R strategy for a highway network in period t; $\hat{X}_t = (X_{1t}, X_{2t}, \ldots, X_{It})$; $X_{it} = (x_{i1t}, x_{i2t}, \ldots, x_{iJt})$, R_t is a return (effectiveness) of strategy $\hat{X}_t$, A_{rt} is the amount of resource (budget and frequency of actions) r taken by strategy $\hat{X}_t$ for a highway network in period t, b_r is an available resource r, and W_t={ $\hat{X}_t$: alternative selection constraint, precedence-feasibility constraint, and 0-1 integrality constraint}.

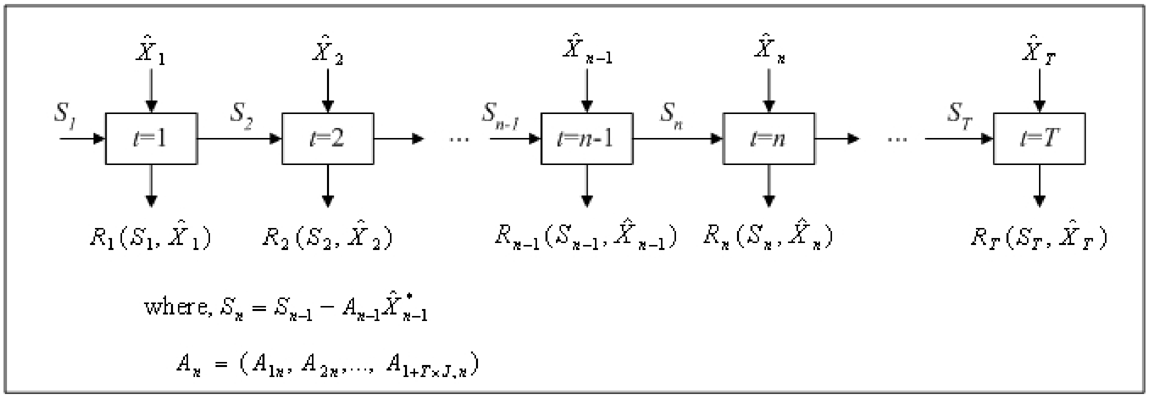

Figure IV.3. Dynamic Programming for Multi-Period

Referring to Figure IV.3, allocation of resources to a highway network using the DP approach results in the following recursive relationship:

$$f_n^*(S_n) = \max_{\hat{X}_n \in F_n} \{ f_{n-1}^*(S_{n-1}) + R_n(S_n, \hat{X}_n) \} \tag{17}$$

where $F_n = \{\hat{X}_n$; budget constraint, frequency of action constraint, alternative selection constraint, precedence/feasibility constraint, & 0-1 integrality constraint} and the state variable S_n represents the amount of resource b which is available for allocation in period n and is a $T+I\times J$ dimensional vector. The vector is divided into two groups. The first group is represented by T dimensional vector corresponding to the budget in each period. The second group is represented by $I\times J$ dimensional vector corresponding to frequency availability of actions for pavement sections.

Problem (*D*) can be decomposed into subproblems that can be considered as a single stage in the multi-period DP problem. In each single period, the feasible solutions are obtained by applying a single-period DP approach and the efficient solutions that are not dominated by any other feasible solutions are constructed by dominance testing.

IV.6. Single Period Dynamic Programming

In order to obtain feasible and efficient solutions in each period, a DP approach is applied to single period problems corresponding to single stages in a multi-period DP. A single-period DP model is developed in a compact form (nonlinear knapsack model) as shown in **Problem (D_S)** and in Figure IV.4:

Problem (D_S)

$$R_n(S_n, \hat{X}_n) = \max \sum_{i=1}^{I} R_{in} X_{in} \tag{18}$$

$$\text{s. t.} \ \sum_{i=1}^{I} A_{rin} X_{in} \le S_{rn} \qquad 1 \le r \le T + I \times J \tag{19}$$

$$X_{in} \in W_{in}, \qquad i=1, 2,\ldots, I \tag{20}$$

where X_{in} is $(x_{i1n}, x_{i2n},..., x_{iJn})$, A_{rin} is the amount of resource (budget and available frequency of actions) r taken by strategy X_{in} for pavement section i in period n; $1 \le r \le T + I \times J$, S_{rn} is an available resource r in period n, R_{in} is the return (effectiveness) of strategy X_{in}, and W_{in}={ X_{in} : alternative selection constraint, precedence/feasibility constraint, and 0-1 integrality constraint}.

Allocation of resources to a highway network using the DP approach in a single period results in the following recursive relationship:

$$f^*_{mn}(S_{mn}) = \max_{X_{mn} \in F_{mn}} \{ f^*_{m-1,n}(S_{m-1,n}) + R_{mn}(S_{mn}, X_{mn}) \} \tag{21}$$

where $F_{mn} = \{X_{mn}$; budget constraint, frequency of action constraint, alternative selection constraint, precedence/feasibility constraint, and 0-1 integrality constraints}, and the state variable S_{mn} represents the amount of resources available for allocation on pavement section m in period n. The solution to $f^*_{In}(S_{In})$ in period n is equal to the solution to $R_n(S_n, \hat{X}_n)$.

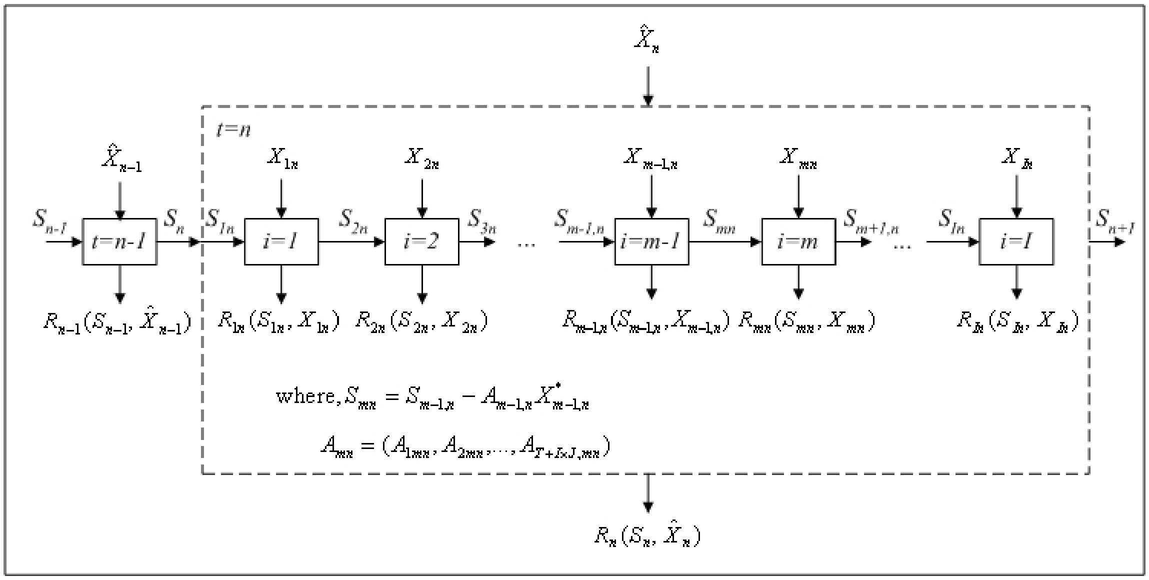

Figure IV.4. Single Period Dynamic Programming Model

IV.7. Imbedded State Space Approach

The imbedded state space approach for state reduction in DP problems is a methodology that converts a multi-dimensional state variable (vector) to a single-state variable. This is accomplished utilizing the points of discontinuity in the return function as a possible solution space. It is assumed that the return function remains constant in the consecutive points of discontinuity. This is a realistic assumption since in the case of IP, the function's value between two integer points is of no concern to decision makers. To illustrate the concept of imbedded-state, consider **Problem (*D*)** presented in section IV.5. For any $S_n = (S_{1n}, \ldots, S_{T+I\times J,n})$ in the state space

$$\overline{S} = \{S_n \in R^{T+I\times J} \mid 0 \le S_n \le b\} \tag{22}$$

we have

$$f_n^*(S_n) = \max_{\hat{X}_n \in F_n} \{ f_{n-1}^*(S_{n-1}) + R_n(S_n, \hat{X}_n)\} \tag{23}$$

for n=1,2,…,T, with the boundary condition

$$f_0^*(S_0) = 0 \qquad\qquad \forall S_0 \ge 0 \tag{24}$$

where

$$f_n^*(S_n) = \max\{\sum_{t=1}^{n} R_t \hat{X}_t \mid \sum_{t=1}^{n} A_{rt}\hat{X}_t \le S_{rn} \;\forall r \text{ and } \hat{X}_t \in W_t, t = 1, 2, ..., n\} \tag{25}$$

$$R_n(S_n, \hat{X}_n) = \max_{X_{in} \in W_{in}} \{\sum_{i=1}^{I} R_{in} X_{in} \mid \sum_{i=1}^{I} A_{rin} X_{in} \le S_{rn}, \forall r\} \tag{26}$$

and vector inequalities are taken element-wise. The solution of the functional equation for $f_T^*(b)$ and the subsequent (policy) reconstruction process to determine an optimal solution is straightforward, but the multi-dimensionality of the state space may present a serious computation problem. However, we can effect a dramatic reduction in dimensionality by exploiting the imbedded state space approach.

Theorem 2. For each n=1, 2,…,T, $R_n(S_n, \hat{X}_n)$, $f_n^*(S_n)$, and $f_{n-1}^*(S_{n-1})$ are nondecreasing step functions on $\overline{S}$. Moreover, if the respective domain sets of points of discontinuity of f_n^*, R_n and f_{n-1}^* are denoted by F_n, $\overline{R}_n$, and F_{n-1}, and F_0={0} where 0 denotes a multi-dimensional vector, then we have following recurrence relation

$$F_n \subseteq \{\overline{R}_n \otimes F_{n-1}\} \subseteq \overline{S}, \qquad n = 1, 2, \ldots, T, \tag{27}$$

where $\overline{R}_n \otimes F_{n-1}$ denotes the set obtained by forming all sums of exactly one element of $\overline{R}_n$ and exactly one element of $F_{n\text{-}1}$.

Proof. See Morin and Marsten (1976).

As an immediate consequence of the theorem we have

Corollary 1. $\exists\, z \in F_T$ such that $f_T^*(b) = f_T^*(z)$.

Therefore, we only have to calculate $f_n^*(S_n)$ for $S_n \in F_n$, and F_n can be determined recursively from $\overline{R}_n$ and $F_{n\text{-}1}$ using (27). In this calculation we can usually eliminate certain elements of $\{\overline{R}_n \otimes F_{n-1}\}$ as being either inefficient or infeasible, thereby reducing the cardinality of F_n. We have reduced a multi-dimensional DP defined on $\overline{S}$ to a one-dimensional DP defined on the sequence of imbedded state space $F_0, F_1, \ldots, F_T \subseteq \overline{S}$.

The method of generating these successive imbedded state spaces will now be described. For each $n = 1, 2, \ldots, T$, the set $\overline{R}_n$ of points of discontinuity of $R_n(S_n, \hat{X}_n)$ on $\overline{S}$ may be obtained by applying the imbedded state space approach to the single period problems corresponding to n stages in the multi-period dynamic programming model.

$$\overline{R}_n \subseteq \{g^0, g^1, \ldots, g^k\} \tag{28}$$

where

$$g^k = (A_{1n}\hat{X}_n^k, \ldots, A_{T+I\times J,n}\hat{X}_n^k), \quad k = 0, 1, \ldots, K.$$

The point g^k belongs to $\overline{R}_n$ unless $g^k \notin \overline{S}$ or unless there exists a $k' \neq k$, $0 \leq k' \leq K$, such that

$$A_{rn}\hat{X}_n^{k'} \leq A_{rn}\hat{X}_n^{k}, \qquad 1 \leq r \leq T{+}I{\times}J, \text{ and} \tag{29}$$

$$R_n\hat{X}_n^{k'} \geq R_n\hat{X}_n^{k}, \tag{30}$$

with at least one strict inequality among (29) and (30). When this is the case, we say that $g^{k'}$ dominates g^k and that g^k is inefficient. When $g^k \notin \overline{S}$ we say that g^k is infeasible. Letting $\overline{U}_n$ contain the indices of the undominated feasible points we have

$$\overline{R}_n = \{g^k \mid k \in \overline{U}_n\}, \text{ and} \tag{31}$$

$$R_n(g^k) = R_n\hat{X}_n^k \qquad \text{for } k \in \overline{U}_n \tag{32}$$

Labelling the elements of $F_{n\text{-}1}$ as $\{S_{n-1}^0, S_{n-1}^1, ..., S_{n-1}^P\}$, where $S_{n-1}^0 = 0$, gives

$$\overline{R}_n \otimes F_{n-1} = \{g^k + S_{n-1}^p \mid k \in \overline{U}_n, 0 \leq p \leq P\}. \tag{33}$$

If there exists $r \in \{1, 2, \ldots, T{+}I{\times}J\}$ such that

$$A_{rn}\hat{X}_n^k + S_{r,n-1}^p > b_r \tag{34}$$

then $g^k + S_{n-1}^p \notin \overline{S}$ and hence $g^k + S_{n-1}^p \notin F_n$. As above, we say that $g^k + S_{n-1}^p$ is infeasible if it falls outside of $\overline{S}$.

If , on the other hand, $\exists k, k' \in \overline{U}_n$ and $0 \leq p, p' \leq P$ such that

$$A_{rn}\hat{X}_n^{k'} + S_{r,n-1}^{p'} \leq A_{rn}\hat{X}_n^{k} + S_{r,n-1}^{p}, \qquad r = 1,\ldots,T{+}I{\times}J, \text{ and} \tag{35}$$

$$R_n \hat{X}_n^{k'} + f_{n-1}(S_{n-1}^{p'}) \geq R_n \hat{X}_n^{k} + f_{n-1}(S_{n-1}^{p}), \tag{36}$$

with at least one strict inequality among (35) and (36), then $g^k + S_{n-1}^p$ is dominated by $g^{k'} + S_{n-1}^{p'}$ and cannot be an element of F_n. By eliminating all infeasible and dominated points from $\{\overline{R}_n \otimes F_{n-1}\}$ we obtain F_n. For each $g^k + S_{n-1}^p \in F_n$, then we have

$$f_n^*(g^k + S_{n-1}^p) = R_n \hat{X}_n^k + f_{n-1}^*(S_{n-1}^p). \tag{37}$$

The origin belongs to F_n for all $n = 0, 1, \ldots, T$ and will be denoted S_{n-1}^0. Notice that $0 \in \overline{U}_n$ for all $n = 1, \ldots, T$ and that

$$g^0 = (A_{1n}(0), \ldots, A_{T+I\times J,n}(0)) = (0, \ldots, 0) = S_n^0 \tag{38}$$

by our assumption that $(\forall\ rn)\ A_{rn}(0) = 0$. It follows that every element of F_{n-1}, unless dominated, is also an element of F_n since $g^0 + S_{n-1}^p = S_n^p$. The following algorithm uses the feasibility and dominance tests to construct the successive imbedded state spaces and terminates with the complete family of undominated feasible solutions F_T.

DP *Algorithm*

Step 1. Set $n = 0$, $F_0 = \{g^0\}$, and $f_0(g^0) = 0$.

Step 2. Set $n = n + 1$ and $k = 0$.

Step 3. If $n > T$, stop.

Step 4. Set $P = |F_{n-1}| - 1$ and label the points of F_{n-1} as $\{S_{n-1}^0, S_{n-1}^1, \ldots, S_{n-1}^P\}$

Step 5. Set $F_n = F_{n-1}$.

Step 6. Set $k = k + 1$. If $k > K$, go to Step 2.

Step 7. If $k \notin \overline{U}_n$, go to Step 6.

Step 8. Set $p = 0$.

Step 9. If $g^k + S_{n-1}^p \notin \overline{S}$, go to Step 13. (Feasibility test)

Step 10. If $g^k + S_{n-1}^p$ is dominated by some point already in F_n, go to Step 13.

Step 11. Set $F_n = F_n \cup \{g^k + S_{n-1}^p\}$ and $f_n^*(g^k + S_{n-1}^p) = R_n \hat{X}_n^k + f_{n-1}^*(S_{n-1}^p)$.

Step 12. Set $F_n = F_n - \{\text{all points dominated by } g^k + S_{n-1}^p\}$.

Step 13. Set $p=p+1$. If $p > P$, go to Step 6. Otherwise, go to Step 9.

Example 4. Consider again the problem given in Example 1. The imbedded state space approach outlined above is applied to the first stage problem. The nonlinear knapsack model and data for the problem are as follows.

$$f_1^*(S_1) = \max\{\sum_{i=1}^{2} R_{i1} X_{i1} : \sum_{i=1}^{2} A_{ri1} X_{i1} \le S_{r1}, 1 \le r \le 5, X_{i1} \in W_{i1}, i = 1,2\}$$

where $X_{11} = (x_{111}, x_{121}, x_{131})^{\mathrm{T}}$, $X_{21} = (x_{211}, x_{221}, x_{231})^{\mathrm{T}}$, R_{11} = [3.8875, 4.3318, 4.8293], R_{21} = [2.3900, 3.0535, 3.5925],

$$A = \begin{bmatrix} 0 & 9 & 15 & 0 & 12 & 20 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } S_1 = \begin{bmatrix} 20 \\ 22 \\ 24 \\ 1 \\ 1 \end{bmatrix}.$$

Stage1. F_0 = {(0, 0, 0, 0, 0)}. $\overline{R}_1$ ={(0, 0, 0, 0, 0), (9, 0, 0, 0, 0), (15, 0, 0, 1, 0)}. $\overline{R}_1 \otimes F_0$ = {(0, 0, 0, 0, 0), (9, 0, 0, 0, 0), (15, 0, 0, 1, 0)}. None of these points are infeasible or dominated, so F_1= {(0, 0, 0, 0, 0), (9, 0, 0, 0, 0), (15, 0, 0, 1, 0)}.

Stage 2. $\overline{R}_2$ ={(12, 0, 0, 0, 0), (20, 0, 0, 0, 1)}, where, x_{211} must be zero because minimum pavement quality for pavement sections should be greater than 2.5 according to the precedence-feasibility constraint. F_1= {(0, 0, 0, 0, 0), (9, 0, 0, 0, 0), (15, 0, 0, 1, 0)}. $\overline{R}_2 \otimes F_1$ ={(12, 0, 0, 0, 0), (20, 0, 0, 0, 1), (21, 0, 0, 0, 0), (29, 0, 0, 0, 1), (27, 0, 0, 0, 0), (35, 0, 0, 1, 1)}. The points (21, 0, 0, 0, 0), (29, 0, 0, 0, 1), (27, 0, 0, 0, 0), (35, 0, 0, 1, 1) are infeasible since S_1 = (20, 22, 24, 1, 1). None of these points are dominated. At the end of the final stage, the imbedded state space approach is summarized as Table IV.1.

Table IV.1. Summary for Example 4

$\hat{X}_1 = (X_{11}, X_{21})$	S_1	$f_1^*(S_1)$
(1 0 0 0 1 0)	12 0 0 0 0	6.9410
(1 0 0 0 0 1)	20 0 0 0 1	7.4800
(0 1 0 0 1 0)	21 0 0 0 0	infeasible
(0 1 0 0 0 1)	29 0 0 0 1	infeasible
(0 0 1 0 1 0)	27 0 0 0 0	infeasible
(0 0 1 0 0 1)	35 0 0 1 1	infeasible

IV.8. Branch-and-Bound Approach

Fathoming of a partial solution by the B-and-B approach effectively eliminates nonpromising points from the state space and hence provides extensive savings in computational time and storage. This is done by incorporating elimination by bound into the DP framework.

Consider any $\tilde{X}_n = (\hat{X}_1, \hat{X}_2, ..., \hat{X}_n) \in \tilde{X}_n^e$, where $\tilde{X}_n^e$ denotes the set of efficient solutions, and let

$$b = \sum_{t=1}^{n} A_t \hat{X}_t . \tag{39}$$

We may interpret b as the resource consumption vector for the partial solution $\tilde{X}_n$. Given $\tilde{X}_n$, the remaining problem at stage n, **Problem (*R*)** is formulated below:

Problem (*R*)

$$\bar{f}^*_{n+1}(b - b) = \max \sum_{t=n+1}^{T} R_t \hat{X}_t \tag{40}$$

$$\text{s.t. } \sum_{t=n+1}^{T} A_{rt} \hat{X}_t \le b_r - b_r \quad 1 \le r \le T + I \times J \tag{41}$$

$$\hat{X}_t \in W_t \qquad n+1 \le t \le T \tag{42}$$

Thus $\bar{f}^*_{n+1}(b - b)$ is the maximum possible return from the remaining stages, given that resources b have already been consumed. For each $0 \le n \le T-1$, let UB_{n+1} be an upper bound functional for $\bar{f}^*_{n+1}(b - b)$, i.e.

$$\bar{f}^*_{n+1}(b - b) \le UB_{n+1}(b - b) \quad \text{for all } 0 \le b \le b \tag{43}$$

UB_{n+1} may be taken as the optimal value of any relaxation of **Problem (*R*)**.

Any known feasible solution of **Problem (*D*)** provides a lower bound on $f^*_T(b)$. The best of the known solutions is called the *incumbent* and its value denoted LB, so that $LB \le f^*_T(b)$. These upper and lower bounds can be used to eliminate efficient partial solutions, which cannot lead to a solution that is better than the incumbent.

That is, if $\tilde{X}_n \in \tilde{X}^e_n$ and

$$\sum_{t=1}^{n} R_t \hat{X}_t + UB_{n+1}(b - \sum_{t=1}^{n} A_t \hat{X}_t) \le LB, \tag{44}$$

then no completion of $\tilde{X}_n$ can be better than the incumbent. The survivors at stage *n* will be denoted $\tilde{X}_n^s$ where

$$\tilde{X}_n^s = \{ \tilde{X} \in \tilde{X}_n^e \mid \sum_{t=1}^{n} R_t \hat{X}_t + UB_{n+1}(b - \sum_{t=1}^{n} A_t \hat{X}_t) > LB \} . \tag{45}$$

The lower bound may be improved during the course of the algorithm by finding additional feasible solutions. Only the survivors at stage *n* are used to generate potential solutions at stage *n*+1.

As mentioned in section IV.3 and IV.4, Lagrangean relaxation and subgradient optimization techniques are applied to obtain tight bounds on **Problem (*R*)** at each stage in multi-period DP. The relaxed problem, **Problem¢ (*LR* (l))**, is formulated as

Problem¢ (*LR* (l))

$$\max\{ \sum_{t=n+1}^{T} (R_t - lA_t)\hat{X}_t + l(b - b) : \hat{X}_t \in W_t \} , \tag{46}$$

where l is the Lagrangean multiplier vector. This problem can be solved by decomposing it into I subproblems, one subproblem for each pavement section, each of which is a longest-path problem and is solved by the Ford-Bellman network flow algorithm.

The subgradient optimization method is applied to **Problem¢ (*LR* (l))** to obtain a good (near optimal) set of Lagrangean multipliers while improving feasible solutions. An integer feasible solution could be obtained through the solution process of **Problem¢ (*LR* (l))**. If this integer feasible solution is better than the currently known best solution,

i.e.
$$\sum_{t=1}^{n} R_t \hat{X}_t + UB_{n+1}(b - \sum_{t=1}^{n} A_t \hat{X}_t) > LB \tag{47}$$

then this solution becomes the best known solution or the incumbent.

At the end of stage n we know that $f_T^*(b)$ falls between LB and the global upper bound

$$UB = \max\{\sum_{t=1}^{n} R_t \hat{X}_t + UB_{n+1}(b - \sum_{t=1}^{n} A_t \hat{X}_t) \mid \tilde{X}_n \in \tilde{X}_n^s\}. \tag{48}$$

If the gap (UB-LB) is sufficiently small, then we may choose to accept the incumbent as being close to optimality in value and terminate the algorithm rather than continuing to stage T.

Example 5. In Example 3, the initial lower and upper bound are obtained with 24.1325 and 25.0805 (respectively) by applying the subgradient optimization approach to problem (LR (l)). It is presented in Example 4 that in the first stage of a multi-period DP, single-period DP is applied to $f_1^*(S_1) = \max_{\hat{X}_1 \in F_1}\{R_1(S_1, \hat{X}_1)\}$ in order to obtain all efficient solutions. Then $\hat{X}_1^1 = (1, 0, 0, 0, 1, 0)$ and $\hat{X}_1^2 = (1, 0, 0, 0, 0, 1)$ are obtained as efficient solutions with $f_1^*(12,0,0,0,0) = 6.9410$ and $f_1^*(20,0,0,0,1) = 7.48$ (respectively), where $\hat{X}_1 = (x_{111}, x_{121}, x_{131}, x_{211}, x_{221}, x_{231})$. Lagrangean relaxation and subgradient optimization approaches are applied to the remaining problems at stage 1 based on the efficient solutions $\hat{X}_1^1$ and $\hat{X}_1^2$, in which knapsack constraints are relaxed in Lagragean fashion, to get improved bounds as well as survivors. The network models for the remaining problems based on the efficient solutions $\hat{X}_1^1$ and $\hat{X}_1^2$ are shown in Figure IV.5 and Figure IV.6. The lower and upper bounds are 23.3801 and 24.1008 for the remaining problem for $\hat{X}_1^1$ as well as 24.1324 and 25.0498 for the remaining problem for $\hat{X}_1^2$.

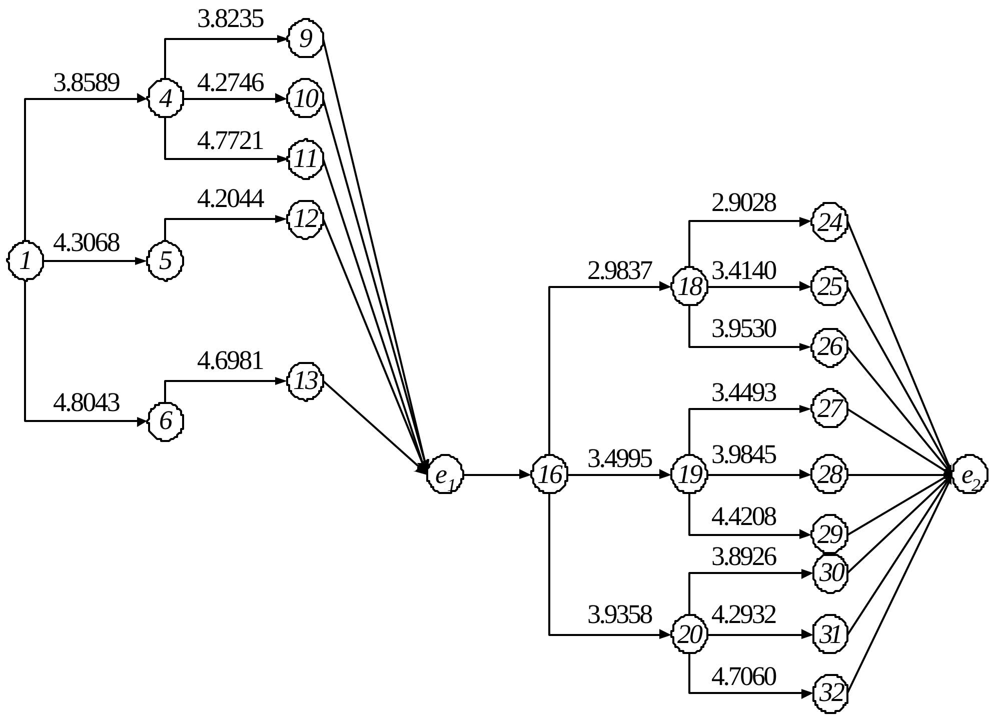

Figure IV.5. Network Representation for Remaining Problem on $\hat{X}_1^1$

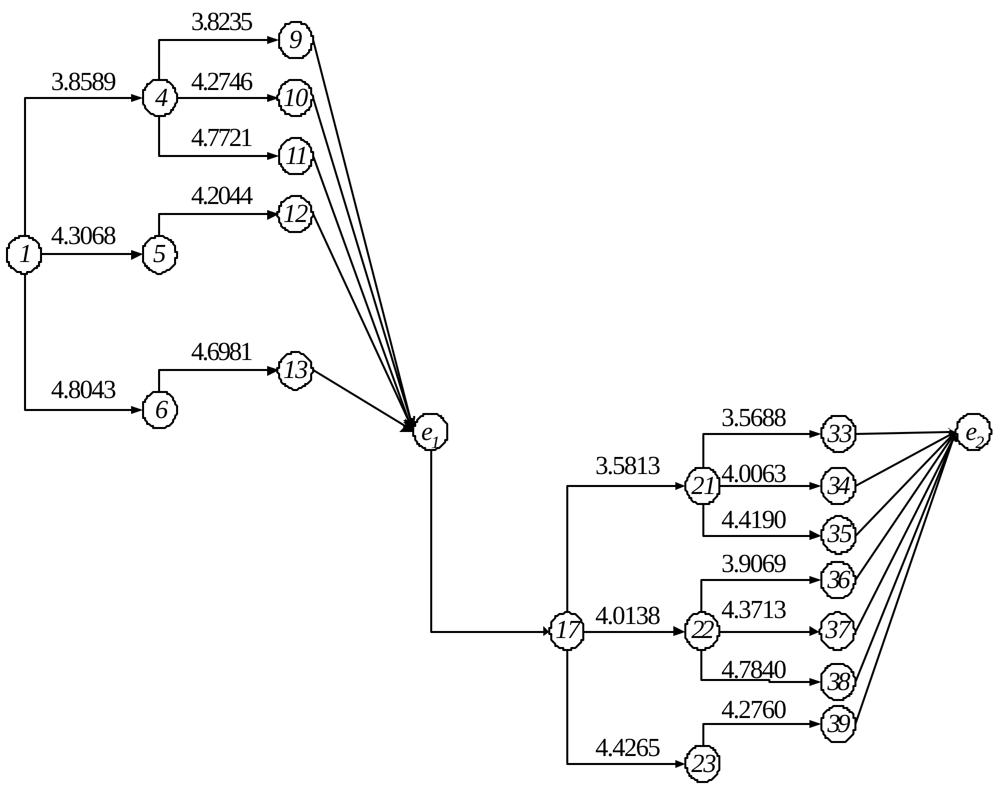

Figure IV.6. Network Representation for Remaining Problem on $\hat{X}_1^2$

By bounding test, $\hat{X}_1^1$ is fathomed and $\hat{X}_1^2$ remains the survivor. At stage 1 the lower and upper bounds are updated with 24.1324 and 25.0498 respectively. Eventually, $\hat{X}_1^2$ is used to generate potential solutions at stage 2. The procedure continues until the third stage, and is terminated with an optimal solution $\tilde{X}_3^* = (\hat{X}_1^*, \hat{X}_2^*, \hat{X}_3^*)$, where $\hat{X}_1^* = (1,0,0,0,0,1)$, $\hat{X}_2^* = (0,0,1,1,0,0)$, and $\hat{X}_3^* = (0,1,0,1,0,0)$. Figure IV.7 shows the overall solution procedure for the example.

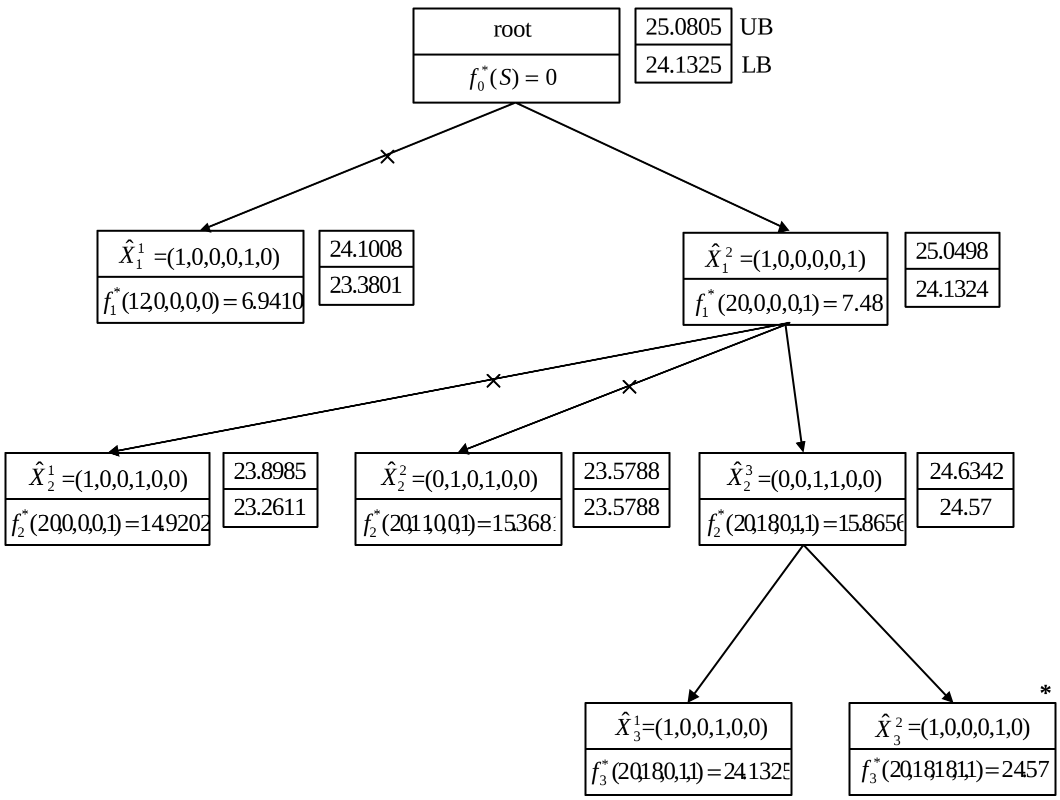

Figure IV.7. Overall Solution Procedure for Example 4

IV.9. Summary

The proposed solution approach is a hybrid algorithm in which DP and B-and-B are combined, as outlined in section III.3. The DP procedures are approaches for solving multi-period and single-period problems formulated as a multi-dimensional binary knapsack with side constraint sets and using an imbedded state space approach to reduce a multi-dimensional DP to a one-dimensional DP. The procedures for B-and-B are a resource-constrained longest-path network representation of the original problem, a Lagrangean relaxation of the problem, and a subgradient optimization to obtain near-optimal values of Lagrangean multipliers as well as lower and upper bounds.

CHAPTER V

COMPUTERIZATION AND APPLICATIONS

V.1. Introduction

All the developed procedures have been computerized and run for a range of multi-period planning PMS scenarios to test the computational performance of the proposed methodology. In order to demonstrate the application of the proposed methodology to a state department of transportation, typical data for the state of Texas has been used. All procedures were coded using the MATLAB language and executed on a personal computer with an Intel Pentium IV 3.06 GHz processor.

The problem for the application is described in Section V.2. Section V.3 presents experimentation and computational results. A brief summary of the chapter is provided in Section V.4.

V.2. Problem Description

The Texas Department of Transportation maintains a highway pavement network divided into a number of autonomous regions called districts and each district is allocated a certain fraction of the yearly state budget depending on its needs. A district contains a number of highways and these highways are further divided into management sections. Ideally a management section has uniform environmental conditions and traffic intensity through the process of segmentation.

The district maintains highways by management section rather than entire portions of the highway system. A district highway management system is defined as a system that analyzes the data on highway conditions and generates a good maintenance schedule within the constraints of resources and desired driving characteristics. Each year, district supervisors schedule maintenance for some subsets of the highway management sections in the district within the limits of resource availability. The solution approaches of this research have been applied to one such set of highway management sections.

This is a real world application and data for the problem was obtained from the 1998 Road Inventory File. The state of Texas is divided into 5 regions by climatic factors such as temperature, precipitation, evaporation, and freeze-thaw cycles. In each region, there are 25 districts which are considered for M & R activities. The districts in each region are given in Table V.1.

Table V.1. Climatic Regions and Their Districts

Region 1 East Texas	**Region 2** West Texas	**Region 3** Texas Panhandle	**Region 4** South Texas	**Region 5** North-Central Texas
Atlanta	Abilene	Amarillo	Corpus Christi	Austin
Beaumont	El Paso	Childress	Laredo	Brownwood
Houston	Odessa	Lubbock	Pharr	Bryan
Lufkin	San Angelo		San Antonio	Dallas
Paris			Yoakum	Fort Worth
Tyler				Waco
				Wichita Falls

For the sample highway maintenance problem, a total of 402.5 lane-miles of different class of highways are taken from Brazos and Robertson county in the Bryan district. The network is segmented into 40 pavement management sections. The pavement management sections are classified into two groups. The first group consists of ‘US’ and ‘State Highways’, whereas the second group consists of ‘Farm-to-Market’. Both groups of highways have asphalt pavements, but have different thickness of road base and surface. The Farm-to-Market, because of the lower traffic intensity, have a thinner base and surface asphalt layers.

There are nineteen ‘group 1’ highway sections and twenty-one ‘group 2’ highway sections, and the pavement network information data, including pavement section code, highway type, section length and lane, traffic volume, and the current PSI (Present Serviceability Index) are shown in Table V.2.

PSI is used to measure pavement section deterioration. The Markov chain approach is used to reflect the stochastic nature of individual changes in present serviceability and service life. To model the way in which the pavement deteriorates with time, it is necessary to identify the Markov probability matrix. As mentioned in Section 1.2, it is assumed that this information is known. Details for PSI and transition probabilities are provided in Appendix B.

There are a total of five alternatives to be considered for managing a highway network: (1) do nothing; (2) minor maintenance; (3) major maintenance; (4) rehabilitation; and (5) reconstruction. Table V.3 lists four standardized M & R treatment strategies for the highway network. Each of the strategies includes: (1) treatment requirements and specifications for each M & R action; (2) treatment effects in terms of raising the existing pavement condition state by a certain amount of PSI points; and (3) unit costs for implementing the M & R actions. It is assumed that application of M & R treatment strategies to highway pavement sections in groups 1 and 2, results in that highway pavement rating being set equal to the points gained by application of these strategies for respective groups of highway pavements. The highway pavement quality level resulting from application of any maintenance strategy cannot be greater than the ideal highway pavement quality or less than the minimum. If an application of any one strategy causes this to occur, the maintenance strategy is infeasible because of the constraints of desired driving requirements of highway pavement quality.

The minimum pavement quality level and the serviceable pavement quality level are defined to be 50% and 80% of the maximum quality level. These are used to determine feasible strategies in each time period. A pavement section is also not considered for maintenance scheduling if the pavement quality levels are greater than the serviceable pavement quality level. Theses constraints, along with the alternative selection constraints, are used to construct the network model for each pavement section over multiple periods.

Table V.2. Pavement Network Information Data

Pavement Management Section	Highway Type	Length (miles)	Lane	Traffic Volume AADT/Truck (%)	Current PSI
004909	US	7.5	4	224170/7.9	3.9
004912	SH	13.6	3.8	456510/11.7	2.6
005001	SH	7.9	4	573500/1.8	2.7
005002	SH	15.5	4	192080/12.6	3.2
011604	SH	11.8	3.4	424200/14.3	2.8
011605	FM	1.3	4	55700/9.2	3.3
011701	US	9.8	2.9	162500/12.3	3.4
011702	US	6.8	2	42200/16.1	2.6
021203	FM	12.9	2.7	283200/11.5	3.8
047501	SH	8	2	5020/18.5	3.2
047502	SH	21	2	13800/16.8	2.6
050601	FM	10.8	3.8	894200/4.6	4
054003	FM	11.6	2	35700/9.6	3.8
054004	FM	21.4	3	306180/4.4	4.1
054005	FM	16.6	2	4170/8.5	3.9
059901	SH	1.4	4	46500/1.9	3.7
064802	FM	4.8	2	7850/10.7	2.6
131601	FM	14.9	3	263410/6.8	3.8
131602	FM	5.4	2	920/15.3	2.5
156001	FM	10.8	2	10030/10.4	3.3
004906	US	14.5	2.8	140700/23.4	3.5
004907	US	7.1	3.4	153300/18.7	3.3
004908	US	12.2	4	332600/14.7	3.6
004914	FM	1.3	2	4050/14.2	3.8
004915	SH	4.2	2	8950/12.6	3.5
009308	US	1.3	2	3500/12.9	3.3
020409	US	8.5	2.7	60300/18.9	3.4
020501	US	9.1	2.3	56100/22.9	2.6
020502	US	17.5	2.3	111000/25	3.8
026203	FM	6.1	2	14000/9.3	3.2
026206	SH	13.3	2	24450/6.5	2.6
038204	SH	8.9	2	20150/14.9	4
054001	FM	16.9	2	26060/9.3	3.8
054002	FM	13	2	19050/9.6	4.1
054006	FM	10.5	2	3150/13.1	3.9
064801	FM	11.8	2	8700/15.4	3.7
119105	FM	5.5	2	1240/13.7	2.6
121001	FM	10.7	2	720/12.6	3.8
121002	FM	5.1	2	1020/12.4	2.5
156301	FM	11.3	2	3120/10.8	3.2

Table V.3. Treatment Types and Costs

No. of Treatment Strategy	Treatment Requirements and Specifications	Treatment Effect & Impact/ Cost ($1000/lane mile)
1. Minor Maintenance	• Crack Sealing • Joint Sealing • Surface Sealing	• Raise of the exiting PSI by 0.5 • Unit Cost: $6
2. Major Maintenance	• Concrete Pavement Restoration • Thin Asphalt Overlay	• Raise of the exiting PSI by 1.0 • Unit Cost: $60
3. Rehabilitation	• Patching • Mill and Thick Asphalt Overlay	• Raise of the exiting PSI by 1.5 • Unit Cost: $125
4. Reconstruction	• Concrete Overlay • Remove Asphalt Surface • Replace and Rework Base	• Raise of the exiting PSI by 2.0 • Unit Cost: $400

V.3. Experimentation and Computational Results

To examine the behavior of the proposed algorithm as a function of both problem size and budget availability, the following combinations are considered: (1) number of pavement sections $\in$ {20, 40}; (2) number of M & R alternatives $\in$ {3, 4}; (3) number of periods $\in$ {5, 7}; and (4) budget availability factor $\in$ {10%, 20%}. Each combination will be referred to as a problem type. Let i represent the number of pavement sections in a problem, j represent the number of M & R alternatives, t represent the number of periods, and θ represent the budget availability factor.

For any choice of i, j, and t, the available budget in each period is determined by the formula $B_t = biq$, where B_t is the budget available in period t in **Problem (P)**, and b is the average cost of the M & R alternatives in each pavement section (obtained by summing the costs of all of the M & R alternatives in a highway network and dividing by the term of multiplying the number of pavement sections and the number of M & R alternatives in each pavement section). It is assumed that the unused portion of the budget for one period can be carried over to subsequent periods and major maintenance and rehabilitation on each pavement section can not be taken more than one time over the planning periods for which the number of M & R alternatives factor is 3 and 4.

A fractional factorial design was applied to plan the experiments. In this experiment there are four factors, each with 2 levels: (1) the number of pavement sections; (2) the number of M & R alternatives; (3) the number of periods; and (4) budget availability factor. The eight experimental conditions obtained from orthogonal arrays for the experimentation ran. Table V.4 represents the orthogonal array OA (8,4,2,3): in this notation, 8 indicates the number of the run; 4 the number of the factor; 2 the number of the levels; and 3 the strength, which is the number of columns where it is guaranteed to see all the possibilities an equal number of times. The per-level combination factors are translated into problem types. The rows of the array represent the experimental conditions. The columns of the orthogonal array correspond to the different variables or factors whose effects are being analyzed. The entries in the array specify the levels at which the factors are to be applied.

Table V.4. Orthogonal Array OA (8,4,2,3)

0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The quality of solutions is measured by a gap. The gap is defined as truncated hundredths of a percent of the difference between the lower bound (the value of an optimal solution or the best known solution) and the upper bound compared to the upper bound:

$$\text{Gap} = 100\,\frac{\text{Upper Bound Value - Lower Bound Value}}{\text{Upper Bound Value}}$$

In this case study, it is assumed that the gap between lower bound and upper bound is within 2 percents for a termination rule of the proposed algorithm.

In addition to the gaps, the computational performance is measured by computation times in minutes and seconds. Computational experience for the experiment is reported in Table V.5. Table V.5 indicates that every solution for each experimental condition is within a 2 % gap and memory usage increases with increasing the problem size.

Table V.5. Computational Results for Experiment

Problem Type				Measures		Memory Usage (MegaByte)
Section(i)	Alternative(j)	Period(t)	Budget(q)	CPU time (min:sec)	Gap	
20	3	5	10 %	02:55	0.0143	0.45
20	3	7	20 %	08:10	0.0116	2.69
20	4	5	20 %	00:28	0.0171	0.78
20	4	7	10 %	24:19	0.0138	3.66
40	3	5	20 %	02:07	0.0158	0.82
40	3	7	10 %	133:09	0.0050	5.33
40	4	5	10 %	02:11	0.0112	1.55
40	4	7	20 %	19:23	0.0142	8.01

The effect of a factor is defined to be the change in response that is computation time in this experiment produced by a change in the level of the factor. This is frequently called a main effect because it refers to the primary factors of interest in the experiment. The main effect of a factor in this two-level design can be thought of as the difference between the average response at the low level of the factor and the average response at the high level of the factor. Numerically, in this experiment these are

$$E_i = \frac{(02:07+133:09+02:11+19:23)}{4} - \frac{(02:55+08:10+00:28+24:19)}{4} = 30:15\,,$$

$$E_j = \frac{(00:28+24:19+02:11+19:23)}{4} + \frac{(02:55+08:10+02:07+133:09)}{4} = -25:00\,,$$

$$E_t = \frac{(08:10+24:19+133:09+19:23)}{4} - \frac{(02:55+00:28+02:07+02:11)}{4} = 44:20\,,$$

and

$$E_J = \frac{(08:10+00:28+02:07+19:23)}{4} - \frac{(02:55+24:19+133:09+02:11)}{4} = -33:06\,.$$

That is, increasing factor i, j, t, and q from the low level to the high level causes an average response increase of 30:15, -25:00, 44:20, and –33:06, respectively.

In the range of this experiment, computation times seem to decrease with the earlier periods in finding the solution within the 2% gap and seem to increase with the increasing number of efficient solutions obtained by using DP in each period. This indicates that more efficient solutions in each period take much more time to obtain survivors (or a solution within the 2% gap) because of the number of performing bounding tests. The increasing budget level reduces the fewer variables since the number of promising M & R alternative combinations is increased. Hence, from the standpoint of problem reduction, difficulty levels seem to be affected by budget availability and data structure within budget levels.

Generally, computation time is a function of the problem size and difficulty. The problem size is exponentially proportional to the number of pavement sections and periods. Increasing the number of periods seems to increase computation time in obtaining bounds on efficient solutions (using Lagrangean relaxation and subgradient optimization methodology), while increasing the number of pavement sections tends to increase computation time in obtaining efficient solutions in each period (using DP).

The dominating factor in difficulty is likely to be budget availability and the data structure since the computation time appears to decrease or increase according to the combination of the two. Problems with some combination of the two will be harder to solve than those of larger size with the other combinations. For example, the computation time for a problem with 40 pavement sections, 3 alternatives, 7 periods, and 10% of the budget level is about 133 minutes, compared to about 19 minutes for a problem with 40 pavement sections, 4 alternatives, 7 periods, and 20% of the budget level. Figure V.1 shows the trend of computation times according to the problem size. The tendency of memory usages according to the problem size is shown in Figure V.2.

No problem was solved to optimality in the proposed algorithm. It is not possible to compare the computational results from this algorithm with those from others, since no special purpose algorithms for this type of research problem are available in the related technical literature.

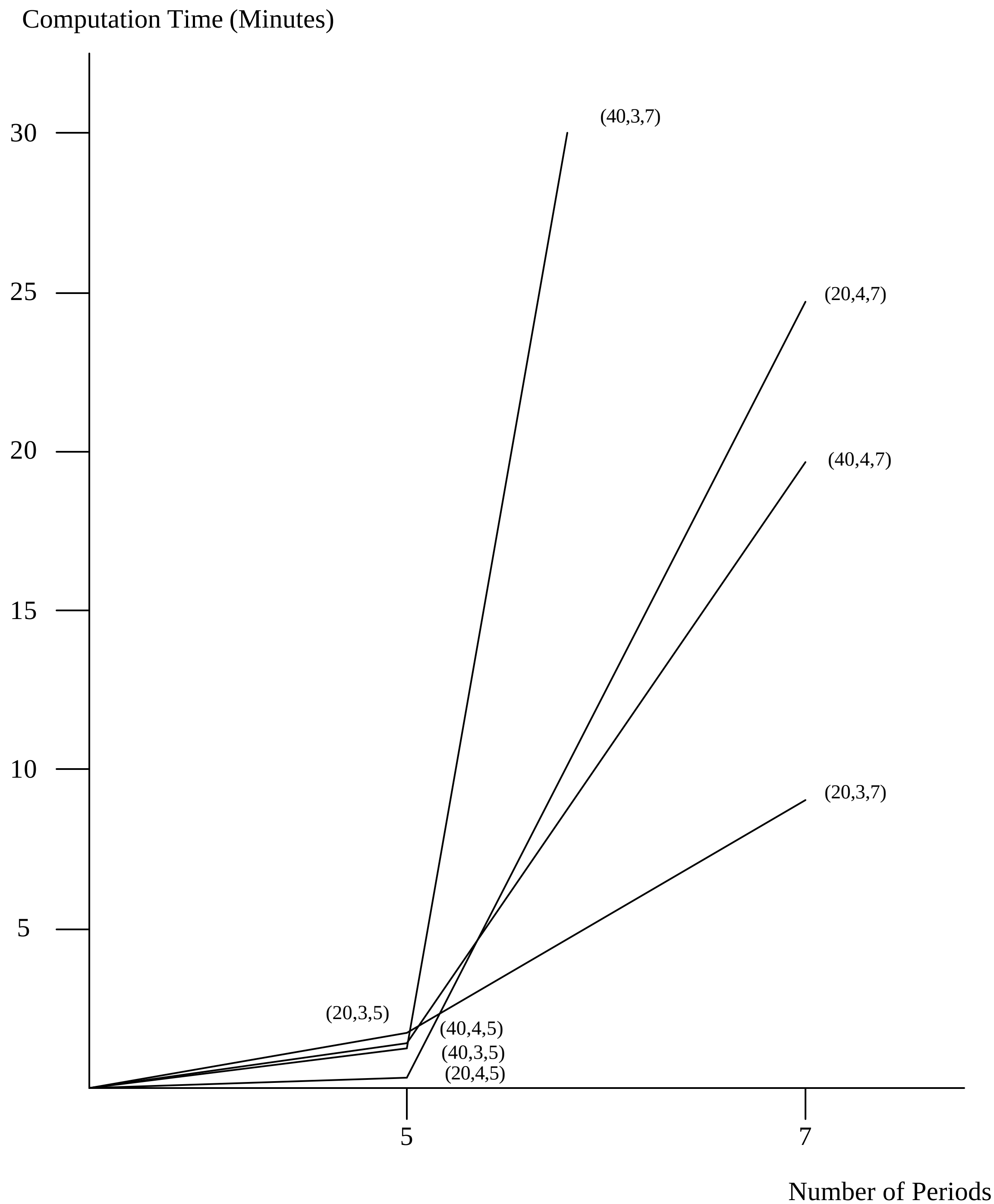

Figure V.1. Computation Time vs Number of Periods

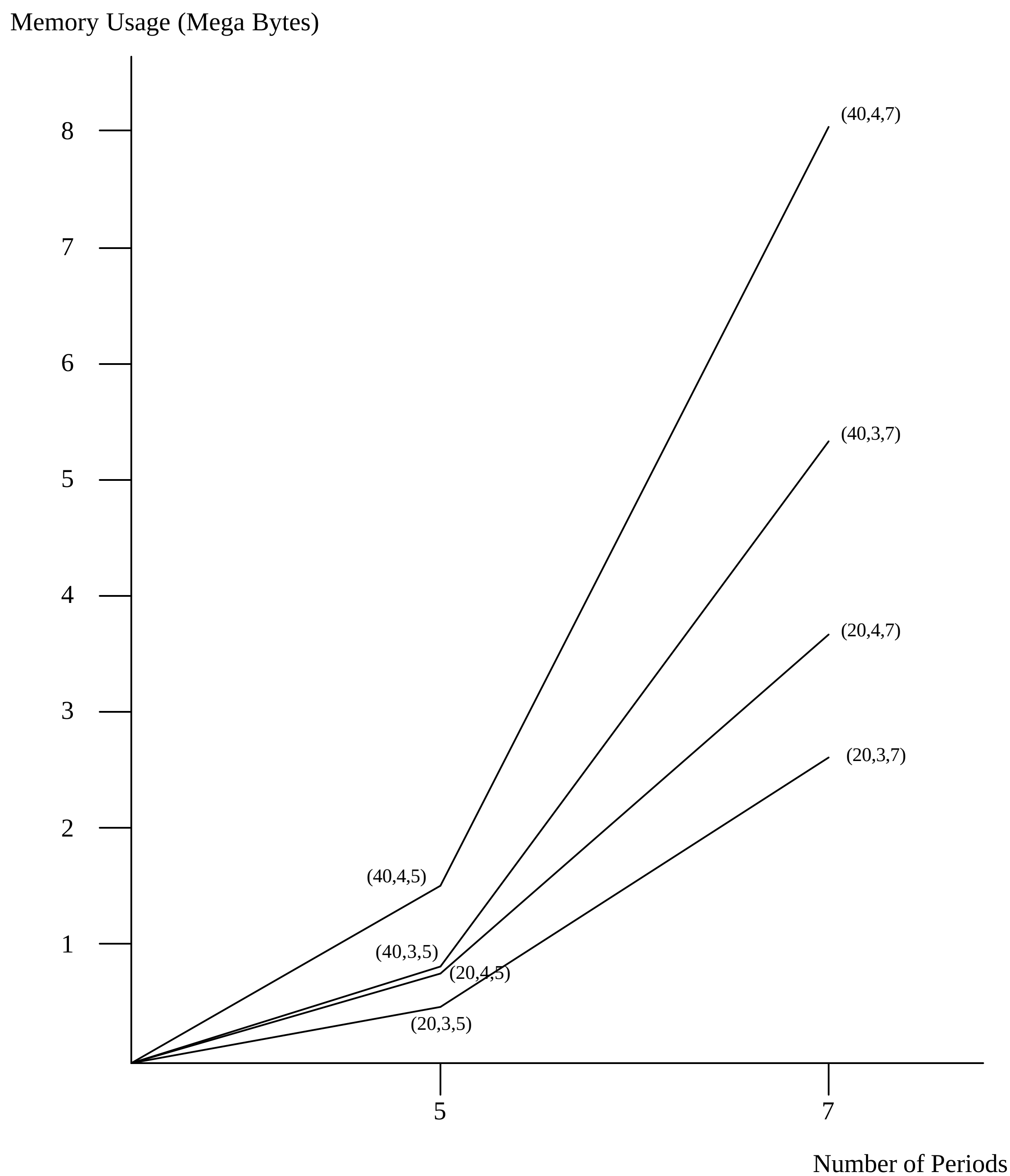

Figure V.2. Memory Usage vs Number of Periods

V.4. Summary

In this chapter, the problem for the application was described and the experiments computational results were presented. The data for the problem was obtained from 1998 Road Inventory File. The set of pavement sections used in this problem was segmented by the column of 'highway department control/section number' in 1998 Road Inventory File. All computations were conducted on an Intel Pentium 3.06 GHz process personal computer using MATLAB code.

Based on the results reported in this chapter, the following derivations can be made: (1) solutions to problems are near optimal within 2 percent maximum accuracy; (2) computation times and solution accuracy tend to increase according to the combination of the budget availability factor and the data structure as well as the increasing numbers of pavement sections, maintenance alternatives, and planning periods.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

VI.1. Summary

The purpose of this dissertation is to develop a model and solution methodology for determining the most cost-effective M & R activities for each pavement section in a highway network along a specified extended planning horizon. A pavement project is defined as a chronological sequence of M & R activities along the given horizon, including the "do-nothing" activity. Each M & R activity in a project is associated with an estimated cost and an effectiveness measure.

The problem under investigation is that of selecting and scheduling timely and cost-effective M & R activities for each pavement section in a highway network and allocating the funding levels through a finite multi-period horizon, within the constraints imposed by budget availability, frequency activities, and specified minimum pavement quality. It is assumed that the unused portion of the budget for one period can be carried over to subsequent periods.

The problem is formulated as a multi-dimensional 0-1 knapsack model with alternative selection and precedence-feasibility constraints. DP and B-and-B approaches are combined to produce a hybrid DP/B-and-B algorithm for solving the problem. The algorithm is essentially a DP approach in the sense that the problem is divided into smaller subproblems corresponding to each single period problem. The idea of fathoming partial solutions that could not lead to an optimal solution is incorporated within the algorithm to reduce storage and computational requirements.

The imbedded-state approach is used to reduce a multi-dimensional DP to a one-dimensional DP and to obtain all promising solution points in a stagewise fashion. The non-promising solution points that cannot lead to an optimal solution are eliminated by three schemes: (1) feasibility tests; (2) dominance tests; and (3) bounding tests. The feasibility test eliminates the solution space leading to an infeasible point. The dominance test is conducted to screen those solution points which consume more of the resources and provide lesser returns. The bounding test eliminates solutions in the state space that result in a return worse than the best known bound.

In order to obtain initial bounds and bounds at each stage in the DP, the original problem is transformed to a resource-constrained longest-path network model. A Lagrangean optimization methodology for solving the RCLPP is developed. For bounding tests at each stage in the DP, the Lagrangean optimization methodology applies to each of the remaining problems. In Lagrangean optimization, the values of the Lagrangean multipliers are found by a subgradient optimization method, while the Ford-Bellman network algorithm is employed at each iteration of the subgradient optimization procedure to solve the longest-path network problem as well as to obtain an improved lower and upper bound.

The proposed algorithm was implemented in the MATLAB language on an Intel Pentium IV 3.06 GHz processor. Tests for the proposed solution methodology were conducted using a typical data set for the state of Texas as well as an experimental design concept. The duality gap of the problems was sufficiently small enough (2% maximum) and the lower bound was near optimal.

VI.2. Conclusions and Contributions

In this dissertation a model and solution algorithm is developed to obtain an optimal or near-optimal solution to the problem of selecting and scheduling timely and cost-effective M & R strategies of pavement sections in a highway network and for allocating the available funding levels in each period along an extended planning horizon resulting in maximum benefits.

The model developed in this research is a multi-dimensional 0-1 knapsack problem with side constraint sets. One special property of the model is that every coefficient column and the right hand side value column of the knapsack constraints have nondecreasing elements. This property comes from the realistic assumption that the unused portion of the budget for a period can be carried over to subsequent periods.

It is not possible to compare the performance of the proposed solution algorithm with that of other algorithms since there are no other special-purpose algorithms for this kind of research. However, it can be concluded that the proposed algorithm solves multi-period optimization problems in an efficient and effective manner: (1) solutions are optimal or near-optimal within 2% maximum in accuracy; (2) computation times and solution accuracy tend to increase according to the combination of the budget availability factor, and the data structure as well as problem size.

The methodology developed in this research is considered to be a significant step in the development of multi-period optimization methodology for PMS's because of: (1) a computationally efficient solution procedure for multi-period problems obtained by combining DP and B-and-B procedures and exploiting not only the imbedded state space approach for state reduction in DP, but also the computational efficiency of network algorithms at each iteration in a subgradient optimization procedure; (2) an optimization procedure that allows the selection and scheduling of timely and cost-effective M & R activities for each pavement section in a highway network along an extended planning horizon as well as the allocation of the available funding levels in each period resulting in maximum benefits for multi-period PMS; and (3) the capability of the model to allow pavement managers to make more consistent and effective decisions regarding the allocation of limited funds in each period as well as the frequency of activities over time.

The significance of contributions also comes from the fact that the decisions regarding the lifetime of pavement improvement activities would be more effective and consistent by considering them on a system-wide basis and along a mid-to-long term planning horizon. Furthermore, the proposed procedure is general enough to be successfully and directly applied to real life PMS's.

VI.3. Recommendations for Further Research

Further extensions to the model and solution methodology can be considered as follows. The identified bottleneck in the computational performance for large-scale hard problems has been the time required for solving the resource-constrained longest-path network model at each iteration in the subgradient optimization methodology. The network modeling technique cannot be used in some cases because the size of the resulting network grows exponentially beyond computational capabilities as the number of planning periods increases. It should be an area of further research to develop techniques other than network modeling to be used in solving the network model.

Further, a heuristic device for generating an appropriate feasible solution from an infeasible integer solution (obtained using Lagrangean optimization) needs to be developed for use in the B-and-B procedure. Most of the computation time for this procedure is used for Lagrangean relaxation and subgradient optimization, which is called for every efficient solution in each stage of the DP frame. The employment of such a heuristic tool at each iteration will save a significant amount of computational efforts.

Finally, more constraints can be added to the model. For instance, other resources such as man-hours, materials, etc. can be added to expand the scope of its capacity.

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APPENDIX A

COMPUTER IMPLEMENTATIOIN

A.1. Introduction

The computer implementation of the proposed algorithm in this research is described in this appendix. Section A.2 presents the general description of the code. Section A.3 explains the program flow and some critical values of the flow control parameters to control the flow of execution in the developed code. Section A.4 discusses input data format. The main part of MATLAB code is provided in A.5.

A.2. General Description of the Code

All procedures of the algorithm were coded in the MATLAB language. The code can be executed on a computer with an Intel Pentium IV 3.06 GHz processor and 512 megabytes of main memory. The maximum problem size that has been solved by the developed code is 40 pavement sections, 4 M & R alternatives (including ‘do nothing’) for each pavement sections, and seven years in the planning period horizon.

In developing the code for the algorithm, no effort was made to optimize the code with respect to main memory use and computational requirements. In the network modeling procedure, the concept of the depth first search rule was employed to build the network. In the B-and-B procedure, the search tree was set-up by applying a dynamic programming approach and computations of bound values at nodes was conducted from the root node until the gap between the lower and upper bound is in the predetermined parameter e or the improved lower bound is optimal. The width first search rule was utilized in fathoming the efficient solutions and building the survivors with the improved bounds at each stage by bounding tests.

A.3. Flow and Flow Control Parameters

The basic steps of the code are as follows.

(1) Input the problem data. This includes the capital consumption by each alternative for each pavement section in each period, the budget available in each period, the alternative frequency available for the planning periods, transition probability matrices for each pavement section, treatment effect of alternatives, and the current pavement quality level of pavement sections.

(2) Perform the network generation to transform the original problem into an RCLPP.

(3) Obtain the initial bounds

  a) Take the Lagrangean relaxation of the RCLPP into a longest-path problem.

  b) Perform the subgradient optimization procedure.

  c) If the gap between the lower and upper bound is in e, stop; otherwise, go to (4).

(4) Perform the dynamic programming for a single period.

  a) Feasibility test

  b) Dominance test

(5) Build the remaining problems.

(6) Perform the bounding test.

  a) Perform (3.a) and (3.b) for the remaining problems.

  b) Build the survivors by bounding test.

  c) Obtain the updated bounds.

  d) If the gap is in e or the current period is the last, stop; otherwise, go to (4).

The output of the code is the lower and upper bound values and the M & R schedules for a highway network under consideration. Computational experiments showed that problems having more than a seven-year period planning horizon and more than forty pavement sections consume excessive amounts of time in performing subgradient optimization procedures and dynamic programming approaches in each period. For computational implementation within a reasonable computation time, forty pavement sections, four M & R alternatives, and seven-years planning period were considered as reasonable-sized problems in this dissertation.

A.4. Input Data Format

The input data file consists of three parts. Their forms are matrices. The first part contains the capital consumptions for each M & R alternative in each period and the frequency of alternatives for pavement sections over the planning periods. The second part includes the budget available in each period and the frequency of alternatives available over the planning period in which the frequency of the most expensive action over the period is constrained once. The transition probability matrix for each pavement section is included in the third part. Table A.1 is an example of input data for a problem that has twenty pavement sections, three M & R alternatives, and seven-year planning period.

A.5. Output

The output from the computer code is shown in Table A.2. It includes a list of M & R activities selected and scheduled for each pavement section in a highway network and the funding level allocated over the planning periods.

Table A.1. Input Data

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0 180 1799 0 306 3065 0 189 1890 0 371 3708 0 241 2414 0 30 302 0 172 1722 0 81 814 0 211 2114 0 96 961 0 251 2514 0 249 2488 0 140 1398
0 391 3909 0 199 198 0 33 333 0 57 574 0 264 2641 0 64 645 0 129 1292
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Table A.1. (Continue)

2681 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

0 2815 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0 0 2956 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0 0 0 3104 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 3259 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 3422 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 3593 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0.15 0.85 0 0 0 0
0 0.31 0.69 0 0 0
0 0 0.41 0.59 0 0
0 0 0 0.01 0.99 0
0 0 0 0 0.79 0.21
0 0 0 0 0 1

0.315 0.685 0 0 0 0
0 0.445 0.555 0 0 0
0 0 0.565 0.435 0 0
0 0 0 0.565 0.435 0
0 0 0 0 0.565 0.435
0 0 0 0 0 1

0.215 0.785 0 0 0 0
0 0.355 0.645 0 0 0
0 0 0.455 0.545 0 0
0 0 0 0.555 0.445 0
0 0 0 0 0.445 0.555
0 0 0 0 0 1

0.41 0.59 0 0 0 0
0 0.52 0.48 0 0 0
0 0 0.42 0.58 0 0
0 0 0 0.52 0.48 0
0 0 0 0 0.52 0.48
0 0 0 0 0 1

0.315 0.685 0 0 0 0
0 0.445 0.555 0 0 0
0 0 0.565 0.435 0 0
0 0 0 0.565 0.435 0
0 0 0 0 0.565 0.435
0 0 0 0 0 1

0.39 0.61 0 0 0 0
0 0.61 0.39 0 0 0
0 0 0.76 0.24 0 0
0 0 0 0.76 0.24 0
0 0 0 0 0.76 0.24
0 0 0 0 0 1

Table A.1. (Continue)

0.19	0.81	0	0	0	0
0	0.31	0.69	0	0	0
0	0	0.35	0.65	0	0
0	0	0	0.62	0.38	0
0	0	0	0	0.62	0.38
0	0	0	0	0	1

0.39	0.61	0	0	0	0
0	0.64	0.36	0	0	0
0	0	0.74	0.26	0	0
0	0	0	0.54	0.46	0
0	0	0	0	0.74	0.26
0	0	0	0	0	1

0.39	0.61	0	0	0	0
0	0.64	0.36	0	0	0
0	0	0.74	0.26	0	0
0	0	0	0.54	0.46	0
0	0	0	0	0.74	0.26
0	0	0	0	0	1

0.39	0.61	0	0	0	0
0	0.64	0.36	0	0	0
0	0	0.74	0.26	0	0
0	0	0	0.54	0.46	0
0	0	0	0	0.74	0.26
0	0	0	0	0	1

0.15	0.85	0	0	0	0
0	0.31	0.69	0	0	0
0	0	0.41	0.59	0	0
0	0	0	0.01	0.99	0
0	0	0	0	0.79	0.21
0	0	0	0	0	1

0.315	0.685	0	0	0	0
0	0.445	0.555	0	0	0
0	0	0.565	0.435	0	0
0	0	0	0.565	0.435	0
0	0	0	0	0.565	0.435
0	0	0	0	0	1

0.215	0.785	0	0	0	0
0	0.355	0.645	0	0	0
0	0	0.455	0.545	0	0
0	0	0	0.555	0.445	0
0	0	0	0	0.445	0.555
0	0	0	0	0	1

0.41	0.59	0	0	0	0
0	0.52	0.48	0	0	0
0	0	0.42	0.58	0	0
0	0	0	0.52	0.48	0
0	0	0	0	0.52	0.48
0	0	0	0	0	1

Table A.1. (Continue)

0.315 0.685 0 0 0 0
0 0.445 0.555 0 0 0
0 0 0.565 0.435 0 0
0 0 0 0.565 0.435 0
0 0 0 0 0.565 0.435
0 0 0 0 0 1

0.39 0.61 0 0 0 0
0 0.61 0.39 0 0 0
0 0 0.76 0.24 0 0
0 0 0 0.76 0.24 0
0 0 0 0 0.76 0.24
0 0 0 0 0 1

0.19 0.81 0 0 0 0
0 0.31 0.69 0 0 0
0 0 0.35 0.65 0.1 0
0 0 0 0.62 0.38 0
0 0 0 0 0.62 0.38
0 0 0 0 0 1

0.39 0.61 0 0 0 0
0 0.64 0.36 0 0 0
0 0 0.74 0.26 0 0
0 0 0 0.54 0.46 0
0 0 0 0 0.74 0.26
0 0 0 0 0 1

0.39 0.61 0 0 0 0
0 0.64 0.36 0 0 0
0 0 0.74 0.26 0 0
0 0 0 0.54 0.46 0
0 0 0 0 0.74 0.26
0 0 0 0 0 1

0.39 0.61 0 0 0 0
0 0.64 0.36 0 0 0
0 0 0.74 0.26 0 0
0 0 0 0.54 0.46 0
0 0 0 0 0.74 0.26
0 0 0 0 0 1

Table A.2 M & R Actions and Capital Consumed for Each Pavement Section in Each Period

M&R Actions and Capital Consumed for Each Pavement Section in Each Period

	1 year		2 year		3 year		4 year		5 year		6 year		7 year	
Section	M&R	Cap	M&R	Cap	M&R	Cap	M&R	Cap	M&R	Cap	M&R	Cap	M&R	Cap
1	2	180	1	0	2	198	1	0	2	219	1	0	2	241
2	2	306	1	0	2	338	2	355	2	372	2	391	2	411
3	2	189	2	198	2	208	2	219	2	230	2	241	1	0
4	1	0	2	389	1	0	2	429	2	451	2	473	2	497
5	1	0	2	253	1	0	2	279	2	293	2	308	1	0
6	2	30	2	32	2	33	2	35	1	0	3	385	1	0
7	2	172	1	0	1	0	2	199	2	209	1	0	2	231
8	2	81	2	85	2	90	1	0	1	0	2	104	1	0
9	2	211	2	222	2	233	1	0	2	257	1	0	2	283
10	2	96	2	101	2	106	1	0	1	0	1	0	2	129
11	1	0	2	264	1	0	2	291	2	306	2	321	1	0
12	2	249	2	261	2	274	2	288	2	302	2	318	1	0
13	2	140	2	147	2	154	2	162	2	170	2	178	1	0
14	1	0	2	410	1	0	2	452	2	475	2	499	2	524
15	2	199	1	0	2	219	2	230	1	0	2	254	1	0
16	2	33	2	35	2	37	2	39	1	0	3	425	1	0
17	2	57	1	0	2	63	1	0	2	70	1	0	2	77
18	2	264	1	0	2	291	1	0	2	321	1	0	1	0
19	2	64	2	68	2	71	2	75	1	0	1	0	2	86
20	2	129	2	136	2	142	1	0	1	0	1	0	2	173

A.6. Main Body of MATLAB Code

```
format compact
clear all
clc

global Efft A b set_sol T_E  PQ_origin Prob  cum_b cpu_start AM cpu_start = cputime;

% Get A matirx from input_A.txt
fid1 = fopen('input_A.txt','r');
for i = 1 : 7
   transp_A{i} = fscanf(fid1,'%d',[60,27]);
   A{i} = sparse(transp_A{i}');
end
fclose(fid1);

% Get b matrix from input_b.txt
fid1 = fopen('input_b.txt','r');
for i = 1 : 7
   transp_b(:,i) = fscanf(fid1,'%d',[27,1]);
   b = transp_b';
end
fclose(fid1);

% Get Probability matrix from input_Prob.txt
fid1 = fopen('input_Prob.txt','r');
for i = 1 : 20
   Prob(:,:,i) = fscanf(fid1,'%f',[6,6]);
   Prob(:,:,i) = sparse(Prob(:,:,i)');
end
fclose(fid1);

% cumulative budget for each period
cum_b = [b(1,1) b(1,1)+b(2,2) b(1,1)+b(2,2)+b(3,3) b(1,1)+b(2,2)+b(3,3)+b(4,4)
b(1,1)+b(2,2)+b(3,3)+b(4,4)+b(5,5) b(1,1)+b(2,2)+b(3,3)+b(4,4)+b(5,5)+b(6,6)
b(1,1)+b(2,2)+b(3,3)+b(4,4)+b(5,5)+b(6,6)+b(7,7)];

% j = 1 ; Do nothing        ;
% j = 2 ; Minor maintenance ;
% j = 3 ; Major maintenance ;

T_E = [0 .5  1]; % Treatment_Effect
```

```
% PQ_it is pavement quality of section i in period t %

PQ_origin = [3.9 2.6 2.7 3.2 3.5 2.6 3.5 3.2 2.8 3.3 3.9 2.6 2.7 3.2 3.5 2.6 3.5 3.2 2.8 3.3]
;

%Network Generation from original problem

[Efft,ini_Efft,Adj]=ng*;

Efft; % coefficients of objective function
ini_Efft; % sorted Efft for network representation
Adj; % Adjacent matrix for network representation

%Lagrangean Relaxation and Subgradient optimization

[LB, UB,Long_path_info] = Ini_subgradient(ini_Efft,Adj);

Incumb = LB; % the best known lower bound
UB; % the best known upper bound

% If the gap LB and UB is in a fixed range, then the following will be implemented.
for num_pave = 1 : 20
  for i = 1 : size(Long_path_info{1,num_pave},2)-2
    for j = 1 : size(Adj{1,num_pave},2)
      search_Adj = Long_path_info{1,num_pave}(i:i+1);
      if isempty(Adj{1,num_pave}{search_Adj(1),j}) == 0
        if search_Adj == Adj{1,num_pave}{search_Adj(1),j}
          M_and_R{1,num_pave}(i)=[j];
          PSI{1,num_pave}(i) = ini_Efft{1,num_pave}(search_Adj(1),7+j);%5 is the
                                                                   # of periods.
          break
        end
      end
    end
  end
end

%find cost for each action

consum={[zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)]
[zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)]
[zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)] [zeros(1,7)]};
```

```
for period = 1 : 7
   for num_pave = 1 : 20
      if M_and_R{num_pave}(period) == 1
         consum{num_pave}(period) = AM{period,num_pave}(period,1);
      elseif M_and_R{num_pave}(period) == 2
         consum{num_pave}(period) = AM{period,num_pave}(period,2);
      elseif M_and_R{num_pave}(period) == 3
         consum{num_pave}(period) = AM{period,num_pave}(period,3);
      elseif M_and_R{num_pave}(period) == 4
         consum{num_pave}(period) = AM{period,num_pave}(period,4);
      end
   end
end

% open an output file
fid1 = fopen('output.txt','w+');
[RowM_and_R] = size(M_and_R,2);
[ColumnM_and_R] = size(M_and_R{1,1},2);
fprintf(fid1,'%c','M&R Actions and Capital Consumed for Each Pavement Section in
Each Period');
fprintf(fid1,'\n');fprintf(fid1,'\n');
fprintf(fid1,'%c','        1 year          2 year          3 year          4 year          5 year
6 year          7 year');
fprintf(fid1,'\n');
fprintf(fid1,'%c','Section  M&R    Cap     M&R     Cap     M&R     Cap     M&R
Cap     M&R     Cap     M&R     Cap     M&R     Cap   ');
fprintf(fid1,'\n');


for k = 1 : RowM_and_R
   fprintf(fid1,'%2d',k);
   for kk = 1 : ColumnM_and_R
      fprintf(fid1,'%9d',M_and_R{1,k}(1,kk));
      fprintf(fid1,'%9d',consum{1,k}(1,kk));
   end
   fprintf(fid1,'\n');
end
fclose(fid1);

%%%%% termination rule %%%%%%
if (UB-LB)/UB < .02
   cpu_end = cputime;
   execute_time = cpu_end-cpu_start;
   finishsav;
```

```
    exit;
end

%%%%% Initialize variables %%%%%
solution=[]; % solutions for single period DP
state_var = []; % state variable of each stage in single period DP
L=[0 0 0 0 0 0 0]; % the index of nodes in search tree (7 is the # of periods)
set_sol={[] [] [] []}; % solution information of single DP;{solution, f_state, state_var, L}
red_L = [0 0 0 0 0 0 0]; % the index of nodes in search tree after bounding test

for num_period=1:7

    %find all possible Efft(objective function) for each period according to set_sol
    [all_index] = find_index(Efft,num_period,set_sol);

    m = size(all_index,1);
    n = size(set_sol,1);
    k = n-m+1;

    for i = 1 : m

        % pick a Efft among all Efft
        [index,res] = pick_index(all_index,i,k,set_sol,b,num_period);

        % Solve Single DP for each Efft from pick_index
        [solution,f_state,state_var]=SDP(res,index,num_period);

        % Make index for each solution from SDP
        [dp_index,sol,period_sol] = comp_dp_index(solution,f_state, state_var,
                                        num_period,L,set_sol,k);
        L = dp_index;
        dp_index = [];
        set_sol = sol;
        sol = [];
        sol_for_dom{i} = period_sol;% sol_for_dom is for dominance test

        % Dominace Test
        if num_period ~= 1
            [set_sol,red_L,L] = dominance_test(num_period,
                                    sol_for_dom,set_sol,red_L,L);
        end
        k = k + 1;
    end
    index = [];
```

```
sol_for_dom = []; period_sol=[];

% Bounding Test %
if num_period ~= 7
   [net_sol,temp_Adj,temp_ini_Efft,res] = remain_prob(set_sol,num_period)

   n = size (net_sol,1);
   for ind = 1 : n
      [max_lb, min_ub] =
      subgradient(temp_Adj,temp_ini_Efft,res,net_sol,ind,num_period);
      temp_lb(ind) = max_lb + net_sol{ind,3};
      temp_ub(ind) = min_ub + net_sol{ind,3};
   end
   temp_Adj = []; temp_ini_Efft = [];

   % Fathom and find elements of L that must be reduced %
   n = size(temp_ub,2);
   kk = 1;
   for ind = n : -1 : 1
      if temp_ub(ind) < Incumb;
         temp_ub(ind) = [];
         temp_lb(ind) = [];
         red_L(kk,:) = net_sol{ind,1};
         kk = kk + 1;
         m = size(set_sol,1);
         for j = m :-1: 2
            if set_sol{j,1} == net_sol{ind,1}
               set_sol(j,:) = [];
            end
         end
      end
   end
   net_sol = [];

   max(temp_lb);
   max(temp_ub);
   if Incumb < max(temp_lb)
      Incumb = max(temp_lb);
   end
   if UB > max(temp_ub)
      UB = max(temp_ub)
   end

   %terminatioin rule%
```

```
        if (UB-Incumb)/UB < .02
            cpu_end = cputime;
            execute_time = cpu_end-cpu_start;
            finishsav;
            exit;
        end

        % Reduce elements of L from L
        m = size(red_L,1);
        for j = 1 : m
            n = size(L,1);
            for jj = n : -1 : 1
                if  red_L(j,:) == L(jj,:)
                    L(jj,:) = [];
                end
            end
        end
        red_L = [];
        temp_lb = []; temp_ub = [];
    end
end
```

* : Bold texts represent functions.

APPENDIX B

DATA GENERATION PROCEDURE

B.1. Measure of Pavement Quality

Present serviceability index (PSI) is a measure of pavement surface roughness or riding comfort. It is measured on a scale between 0 and 5, with 5 being a perfectly smooth ride. PSI can be estimated for a pavement section (see Table B.1). In reality, new pavement has a PSI of 4.2 to 4.5. At the point where the pavement cannot perform in a serviceable manner, the index drops to between 2.0 to 2.5. Local roads have a terminal serviceability index (TSI) of around 2.0, while highways such as interstates and principal arterials have a TSI of 2.5 to 3.0. In this dissertation, it is assumed that pavement sections are usable until the index reaches a value of 2.5.

Table B.1. Present Serviceability Index

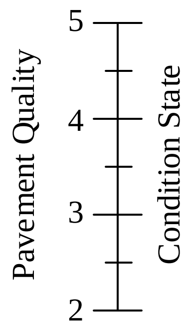
PSI	Pavement Quality Condition
5 ~ 4	Very Good
4 ~ 3	Good
3 ~ 2	Fair
2 ~ 1	Bad
1 ~ 0	Very Bad

B.2. Pavement Quality Prediction

In this section, results found in Butt, A. et al. (1987) are summarized. A pavement performance curve is a relationship between pavement quality and pavement age, which reflects the deterioration and level of service of a pavement section. These curves can be used to estimate the remaining service life of a pavement section as well as to estimate a measure of effectiveness of a pavement improvement activity. Two approaches are generally used for obtaining the curves.

The statistical approach describes the average behavior of pavement sections using regression functions. However, when the regression function is used to predict service life, it is hard to obtain the service life for individual pavement sections that currently have a different condition rating in the same age. The Markov chain approach reflects the stochastic nature of individual changes in condition rating and service life. In this dissertation, the Markov chain approach was used to predict the future pavement quality of sample pavement sections

The Markov chain is based on a definition of the state of pavement quality and the probability of pavement quality from one state to another. The assumption that the pavement condition rating does not drop more than one in a single year is generally made. Thus, the pavement will either stay in its current state or move to the next lower state in one year. Figure B.1 shows the schematic representation of condition state, pavement quality, and the transition probability matrix for a particular pavement section. The transition probability matrix has a diagonal structure, where p_i is the probability that a pavement in condition state i will remain in that condition state after one year and $q_i = 1-p_i$. Since a pavement section is considered to be usable until its pavement quality reaches 2, a state of 1 is defined to represent an absorbing state. The pavement condition cannot transit from this state unless repairs are performed.



$$
\begin{array}{c}
6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1
\end{array}
\begin{bmatrix}
p_6 & q_6 & 0 & 0 & 0 & 0 \\
0 & p_5 & q_5 & 0 & 0 & 0 \\
0 & 0 & p_4 & q_4 & 0 & 0 \\
0 & 0 & 0 & p_3 & q_3 & 0 \\
0 & 0 & 0 & 0 & p_2 & q_2 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Figure B.1. Diagram of pavement quality, condition state, probabilities

Pavement condition state at any period t can be represented by a state vector S(t). This is a vector of probabilities that a pavement section will be one state at the beginning of period t. The state vector for any period t, S(t) is obtained by multiplying the initial state vector S(0) by the transition probability matrix P_i for pavement section i raised to the power of t. Thus,

$$
\begin{aligned}
S(1) &= S(0) * P_i \\
S(2) &= S(1) * P_i = S(0) * P_i^2 \\
\vdots \quad & \quad \vdots \quad\quad\quad \vdots \\
S(t) &= S(t\text{-}1) * P_i = S(0) * P_i^t
\end{aligned}
$$

Using this state vector and the Markov transition probabilities, the future condition of the road at any time t can be predicted.

B.3. Generation of a Set of Standardized M & R Treatment Alternatives

An improvement activity is selected to correct any identified deficiencies using the optimization methodology developed in this dissertation. For the application, a set of five standardized pavement M & R treatment strategies has been developed for use in the network optimization. As shown in Figure B.2, these five M & R treatment strategies are: (1) Do Nothing; (2) Minor Maintenance; (3) Major Maintenance; (4) Rehabilitation; and (5) Reconstruction. Each of the five treatment strategies is defined by pavement maintenance action, work content, unit cost and treatment effect on the existing pavement.

The minimum level of PSI for all the pavements in the network was chosen to be 2.5. The unit cost for each of the five treatments is based on the information of average pavement construction and maintenance costs in Texas. In this application, the cost for each treatment activity is 0, 6, 60, 125, and 400 dollars per lane mile.

In addition, the treatment effect of an M & R action on improvement of the existing pavement quality is defined in terms of raising the existing pavement PSI up to a certain amount. In other words, after implementation of a maintenance action, the PSI will be rising to a higher level, depending on which maintenance strategy is selected. For instance, if Minor Maintenance is selected for year t, then a rise of 0.5 units of PSI can be obtained in that year, and there should be a small jump in that year on the performance prediction curves. Alternatively, if a Rehabilitation treatment (i.e., strategy 4) is selected in year t, then the PSI of the pavement will be increased by 1.5 units in that year. Following the PSI jump point, where a treatment action is applied, a new deterioration model, which reflects the improved pavement structure by the treatment, should be established to predict the pavement deterioration in year $t+1$. The procedure is repeated in each consecutive year until the entire analysis period is completed for the integrated performance prediction.

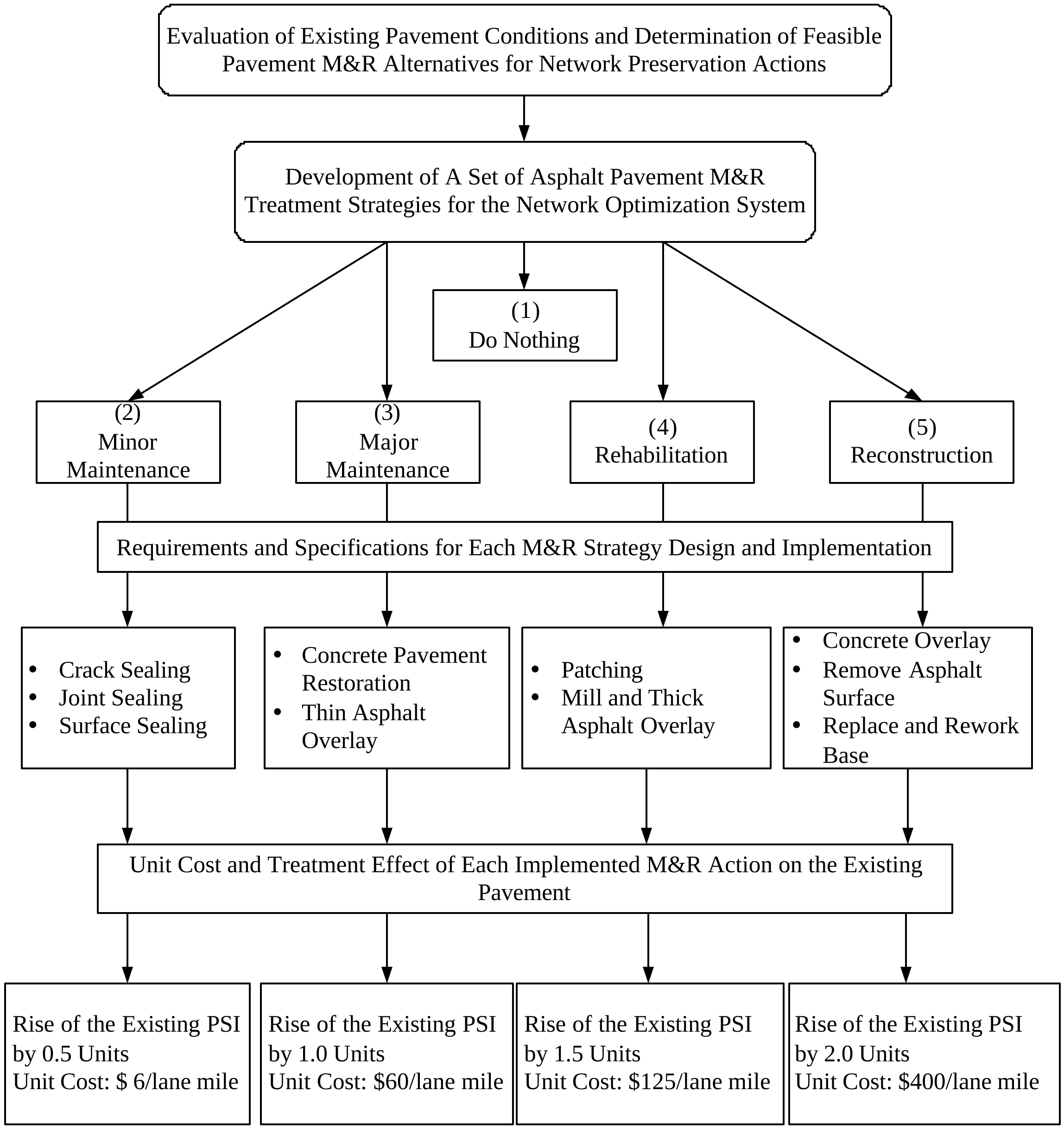


Figure B.2. Generation of Five Standardized Asphalt M & R Treatment Strategies

B.4. Effectiveness of Improvement Activities

The effectiveness measure of pavement improvement activities can be used as the objective criterion. As described in Section B.2, performance curves are obtained by a Markov chain approach. A pavement improvement activity moves up the performance curves and extends the remaining service life of improved pavement. The objective function value for an M & R strategy set applied to a particular pavement section is determined from the pavement quality level curves in the following manners. Suppose an M & R strategy set for a pavement section is as shown in Table B.2. (Note: alternative 1 is a ‘do nothing’ and all other strategy numbers correspond to the different M & R alternatives)

Table B.2. An M & R Strategy for a Particular Pavement Section

Period	1	2	3	4	5	6	7	8	9	10
M & R Strategy	1	1	1	1	*j*	1	1	*k*	1	1

The corresponding pavement quality level curves for this M & R strategy set are shown in Figure B.3. As illustrated in Section III.2, *M* represents the maximum pavement quality level, *m* the minimum pavement quality level, and *s* the serviceable pavement quality level such that if the pavement quality level is above this level in any time period *t*, then pavement section *i* is not considered for maintenance in that particular period *t*.

The objective function value, called the total effectiveness for a given M & R strategy set, is the sum of areas in the corresponding pavement quality level curves for the given M & R strategy set. For example, the objective function coefficient for a strategy ‘*j*’ in period 5, e_{ij5}, is the area under the pavement quality level curve in period 5 (the shaded area in Figure B.3). It is assumed that a pavement quality level curve is linear piecewise. As an illustration, suppose that the pavement quality level at the beginning of period 5 is 3.75 and transition probability matrix for the pavement section is P_1 given in the example in section IV.2. Then the pavement quality level at the end of period 5 (3.725) is obtained by multiplying the state vector corresponding to the pavement quality level at the beginning of period 5, [0 0 1 0 0 0], and transition probability matrix P_1 and taking the expectation of the resulting state vector. Therefore, the objective function coefficient for a strategy '*j*' in period 5, e_{ij5}, is 3.7325, the area under the pavement quality level curve in period 5 (the shaded area in Figure B.3).

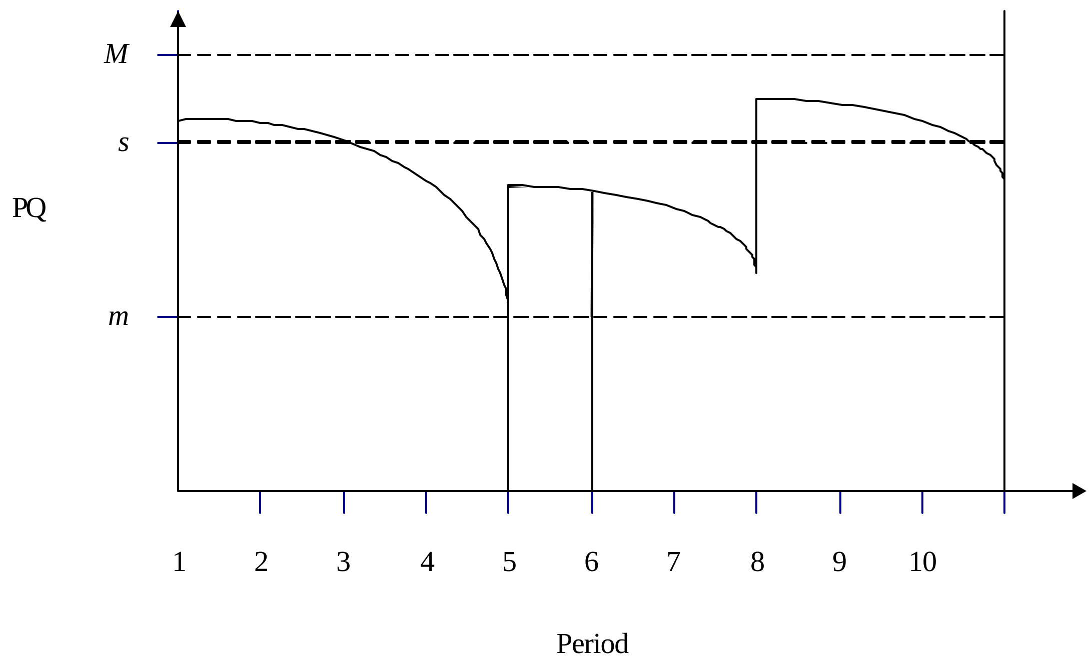


Figure B.3. Pavement Quality Level Curves of an M & R Strategy Set for a Section

VITA

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