FAULT DETECTION OF MULTIVARIABLE SYSTEM USING ITS DIRECTIONAL PROPERTIES

A Thesis

by

AMIT PANDEY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2004

Major Subject: Mechanical Engineering

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Approved as to style and content by:

Suhada Jayasuriya (Chair of Committee)	Shankar Bhattacharyya (Member)
Alexander Parlos (Member)	Dennis O'Neal (Interim Head of Department)

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ABSTRACT

Fault Detection of Multivariable System Using Its Directional Properties.

(December 2004)

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A novel algorithm for making the combination of outputs in the output zero direction of the plant always equal to zero was formulated. Using this algorithm and the result of MacFarlane and Karcanias, a fault detection scheme was proposed which utilizes the directional property of the multivariable linear system. The fault detection scheme is applicable to linear multivariable systems. Results were obtained for both continuous and discrete linear multivariable systems. A quadruple tank system was used to illustrate the results. The results were further verified by the steady state analysis of the plant.

To the ALMIGHTY Who Chose Me to Do This Work

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NOMENCLATURE

The symbols used for the continuous time system are defined as follows

$U(t)$	Input vector for both plants P and P'
$x(t)$	State variable vector for both plants P and P'
$y(t)$	Output vector for plant P
$y'(t)$	Output vector for plant P'
g	Input zero direction of the plant P
g'	Input zero direction of the plant P'
x_0	State zero direction of the plant P
x'_o	State zero direction of the plant P'
v	Output zero direction of the plant P
v'	Output zero direction of the plant P'
z	Transmission zero of the plant P
z'	Transmission zero of the plant P'

where the continuous time plants P and P' are defined by (2.5) and (2.6) respectively.

For the discrete time system the following symbols, as defined below, are used

$U(k)$	Input vector for both plants P and P'
$x(k)$	State variable vector for both plants P and P'
$y(k)$	Output vector for plant P
$y'(k)$	Output vector for plant P'
g	Input zero direction of the plant P
g'	Input zero direction of the plant P'
x_0	State zero direction of the plant P
x'_o	State zero direction of the plant P'
v	Output zero direction of the plant P
v'	Output zero direction of the plant P'

q Transmission zero of the plant P

q' Transmission zero of the plant P'

where the discrete time plants P and P' are described by (5.5) and (5.6) respectively

CHAPTER I

INTRODUCTION

Ever since the first stone tool was invented man has always been concerned about the condition of the machines he uses. For the major part of the human history the only way to learn about the malfunctions and their locations was by the five human senses for example touching to feel heat or vibration, smelling for fumes from overeating etc. This approach is still in use. Measuring devices called sensors were introduced later on to detect the state of the system. However with every passing day the importance of product quality, safety and reliability is increasing in the industrial processes. A simple temperature sensor malfunctioning lead to the loss of seven highly talented astronauts and billions of dollar worth Columbia space shuttle. With the advent of feedback control system the presence of faults in the plant or the sensor have become even more potentially devastating. The feedback may multiply a small fault manifolds. Hence the importance of a reliable faults detecting mechanism.

1.1 Terminology

Before moving further it is advisable to exactly define the terms related to fault detection which will be used again and again in this work. Isermann and Balle (2000) in [1] presented the definitions of terms commonly used in the fault detection and diagnosis field. These definitions were reviewed and discussed at SAFEPROCESS 2000 conference. Few of those definitions are provided below:

This thesis follows the style of *IEEE Transactions on Control Systems Technology*.

Fault: an unpermitted deviation of at least one characteristic property or parameter of the system from acceptable/usual/standard condition.

Failure: a permanent interruption of a system's ability to perform a required function under specified operating conditions.

Fault Detection: determination of faults present in a system and time of detection.

Fault Isolation: determination of kind, location and time of detection of a fault. It follows fault detection.

Fault Identification: determination of size and time-variant behavior of a fault. It follows fault isolation.

Fault Diagnosis: determination of kind, size, location and time of a fault. It follows fault detection and includes fault isolation and identification.

Reliability: ability of a system to perform a required function under stated conditions, within a given scope, during a given period of time. It is measured in mean time between failures.

Safety: ability of a system to not cause danger to persons or equipment or the environment.

Availability: probability that a system or equipment will operate satisfactorily and effectively at any point in time.

1.2 Types of Faults

Gertler (1998) [1] discusses the work of Basseville and Nikiforov (1993) and Isermann (1997) who gave the following three criteria for the classification of faults [1].

a) Classification based on location in the physical system: Depending on whether the fault is located in the sensor, actuator or in one of the components we have the sensor fault, actuator fault or the component fault respectively. In a linear system sensor fault results in a changed C and D matrices, the actuator fault result in a changed B and D matrices and the component fault results in the changed A matrix.

b) Classification based on mathematical properties: Depending on whether the faults are additive or multiplicative in nature we have the additive faults and the multiplicative fault.

c) Classification based on the time behavior characteristics: if there is an abrupt change from the nominal value to the faulty value then it is called abrupt fault. If there is a gradual change from the nominal value to the faulty value then it is called it is called incipient fault. If the fault term changes from the nominal value to the faulty value and returns to the nominal value after a short period of time then it is called intermittent fault.

Fault detection and diagnosis systems implement the following tasks:

1) Fault detection, that is, the indication that something is going wrong in the monitored system;
2) Fault isolation, that is, the determination of the exact location of the fault (the component which is faulty)
3) Fault identification, that is, the determination of the magnitude of the fault.

The fault isolation and fault identification tasks are referred together as fault diagnosis. The detection performance of the diagnostic technique is characterized by a number of important and quantifiable benchmarks namely fault sensitivity – the ability to detect faults of reasonably small size, reaction speed – the ability of the technique to detect faults with reasonably small delay after their arrival and robustness – the ability of the technique to operate in the presence of noise, disturbances and modeling errors, with few false alarms. Isolation performance is the ability of the diagnostic system to distinguish faults depends on the physical properties of the plant, on the size of faults, noise disturbances and model errors, and on the design of the algorithm. The tasks to be performed in the in the fault detection and diagnosis can be shown by the following diagram

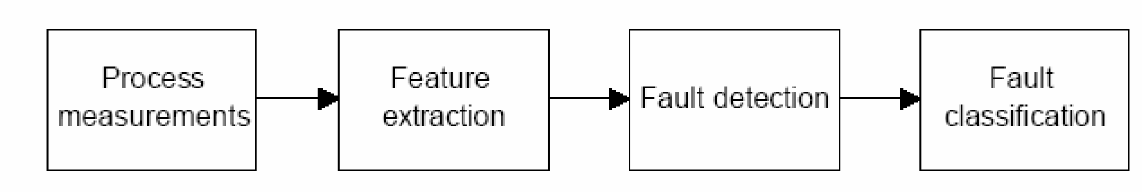

Figure 1.1: The fault detection and isolation task

1.3 Approaches to the Fault Detection and Diagnosis

Fault detection schemes can be broadly classified into two main categories depending on the plant's operating condition, namely: 1) off-line detection schemes in which the plant is investigated offline, and 2) online detection schemes, where the plant is operational. Of these, online schemes, although difficult to develop, are preferable because many faults occur only when the plant is running and also because it provides an opportunity to take on-line real-time corrective measures and maintain a healthy operation of the plant. A schematic diagram is shown in Fig. 1.1

Fault detection and isolation methods can also be classified into two major groups namely Model-Based Methods and Model-Free Methods. The former utilize the mathematical model of the plant and the latter do not utilize the mathematical model of the plant. A brief description is as follows:

1.3.1 Model-Free Method

This fault detection and isolation method does not use the mathematical model of the plant range from physical redundancy to logical reasoning. Some of the prominent model-free methods are as follows:

1) *Physical Redundancy.* In this approach multiple sensors are installed to measure the same physical quantity. Difference between the measurements indicates a sensor fault. One of the drawbacks of the physical redundancy method is that it leads to extra hardware costs and extra weights.
2) *Special Sensors.* Sometimes special sensors may be installed explicitly for detection and diagnosis.

3) *Limit Checking.* In this method plant measurements are compared by computer to preset limits. When the threshold quantity is exceeded it indicates a fault. Generally there are two levels of limits, the first serving for pre-warning while the second triggers an emergency reaction. One of the drawbacks of the limit checking method is that the test threshold should be set quite conservatively in order to take into account the normal input variations. Also, the effect of a single component fault may propagate to many plant variables setting off a confusing multitude of alarms.
4) *Spectrum Analysis.* Analysis of the spectrum of the measured plant variables may also be used for detection and isolation. Most plant variables also exhibit a typical frequency spectrum under normal operating conditions. Any deviation from this is an indication of the abnormality. Some type of faults may also have their own characteristic signature in the spectrum, facilitating fault isolation.
5) *Logical Reasoning.* Logical reasoning techniques form a broad class which is complementary to the methods outlined above, in that they are aimed at evaluating the symptoms obtained by the detection hardware or software. The system may process the information presented by the detection hardware/software or may interact with a human operator inquiring from him about the particular symptoms and guiding him through the entire logical process.

1.3.2 Model Based Methods

Model based fault detection and diagnosis utilizes an explicit mathematical model of the monitored plant. The mathematical description of the plant is in differential equations or equivalent transformed representations. Stages of model based fault detection and diagnosis are shown in Fig. 1.2.

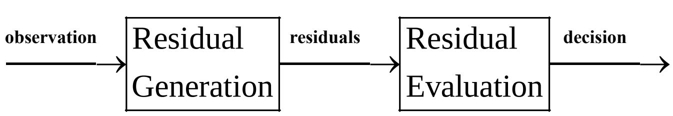

Figure 1.2: Stages of model-based fault detection and diagnosis

According to Isermann & Balle [1] there are three basic categories of model-based fault detection and diagnosis:

1) *System Identification and Parameter Estimation.* In this method process parameters are estimated using a system identification technique on input/output measurements. The estimated values are compared with the nominal parameter set. The difference is called the residue and is used for fault identification.
2) *State and Output Observer.* In this model an observer, often a Kalman filter is used to estimate the system's state variables and reconstruct the system outputs. The residual, defined as the difference between the real and the estimated output, can be used as a fault indicator. A special class of observer-based approach is the multiple-model estimation approach.
3) *Residual Generation.* In this approach first of all primary residuals are formed as the difference between the actual plant outputs and those predicted by the model. The primary residuals are then subjected to a linear transformation to obtain the desired fault-detection and isolation properties such as sensitivity to faults.

Figure 1.3 describes the model-based fault detection using parameter estimation and residual generation. Here x is the state variable and θ is the parameter variable. The hat denotes the estimated values.

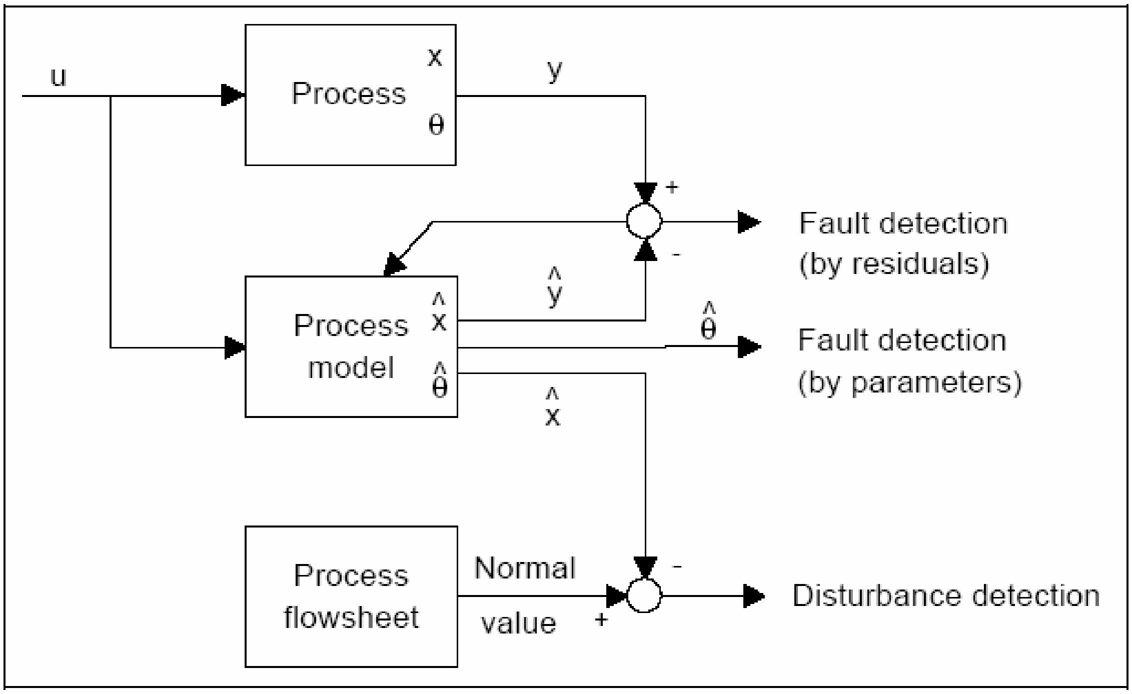

Figure 1.3: Use of estimation for diagnosis of faults and disturbances

1.3.3 Other Methods

When a process is too complex to be modeled analytically and signal analysis does not yield an unambiguous diagnosis then fault detection is done through some other approaches such as artificial intelligence, logic models etc. Some of the approaches are as described below:

1) *Logic Models.* In this approach a description of the system in the form of logical propositions about the relations between the system components and the observations available is developed. These descriptions are called logic models. Reiter (1987) in [1] developed a general theory of diagnosis for system with logic models. However the formulation of logical models suitable for analysis by Reiter's method is not always possible.
2) *Digraph Method.* In this method relationship between the variables is coded as a signed directed graph also called the digraph. Powerful results from graph theory are used to analyze the interrelations in the system. One of the advantages of this approach is that detailed modeling is not needed. Therefore they can be applied to poorly known systems with relatively little effort.

3) *Probabilistic Methods.* If we consider a long time of operation of the plant then the occurrence of faults and disturbances is a stochastic process. In this method the probability is used to find the most likely diagnosis compatible with the available information about the state of the system.

Apart from the above described methods some other notable methods include the artificial neural network approach and the fuzzy logic approach.

1.4 Brief Description of Previous Work

According to Gertler [1] R.K. Mehra and J. Peschon and Allan Willsky (1976 and 1986) were among the first few who started using Kalman filter for fault detection [1]. Gertler also discusses the works of Lund (1992) used multiple Kalman filters to discriminate between two or more process models and Alessandri et al, (1999b) who used sliding-mode observers for the purpose of residual generation in fault diagnosis for unmanned underwater vehicles [1]. Alessandri et al, compared performances obtained using sliding-model observer and extended Kalman filter approaches for residual generation. A special class of observer-based approach is the multiple-model estimation approach which was described by Rong Li also mentioned by Gertler in his book [1]. Also mentioned in [1] are the works of Isermann (1993) who used the system identification techniques to determine process parameters which are used for fault detection. Other major contributors in the field of parameter estimation mentioned in [1] include A. Rault (1984) G. C. Goodwin (1991) [1].

A brief description of the signal based method was given by Gustafson (2000). Other source of information for this method is in the paper by Isermann and Balle (2000) [1]. Rojas-Guzman and Kramer [2] use probability to find the most likely fault based on the available information about the state of the system. An alternative approach to fault detection and diagnosis that has received considerable interest in recent years is based on the use of multivariate statistical techniques (Wise and Gallagher 1996, Macgregor, 1995) [1]. This idea is motivated by the univariate statistical process control method. Frank (1990) gave detailed information about the use of fuzzy logic for fault detection

[1]. The artificial neural networks approach was taken by Koppen-Seliger and Frank [1]. Neural networks based methods for fault diagnosis have received considerable attention over the last few years. Their learning and interpolation capabilities have led to several successful implementations over various processes (Venkatasubramanian and coworkers, 1989, 1993, 1994) [1].

Reiter (1987) developed logic models for systems and used them in diagnosis. Forbus (1984) and Kuipers (1987) used signed directed graph (digraph) for detecting faults [1]. Various other methods and variations of the above described methods have been used for fault detection and isolation but to the best knowledge none of the fault detection and isolation schemes have used the multivariable zeros and zero-directions.

1.5 Motivation for the Present Work

Considerable amount of effort has been applied in developing the design methodologies such as H_∞, μ and QFT . This has resulted in a knowledge base which is sufficient to solve the feedback design problems of the multi-input multi-output (MIMO) systems to a satisfactory level. However in none of the previous efforts the directional properties of the MIMO systems such as the transmission zeros, input zero direction, output zero direction etc was utilized. Neither were the directional properties of MIMO systems utilized in the various previously developed popular fault detection and isolation techniques of MIMO systems. As a first attempt towards fully utilizing the directional properties of MIMO systems the present work aims at developing a novel online fault-detection scheme for linear MIMO systems was developed based on multivariable zeros and zero directions.

CHAPTER II

ZEROING OF OUTPUTS IN OUTPUT-ZERO DIRECTIONS

2.1 Introduction

The concept of zeros and the zero directions of a system has been the subject of lot of research in the last three decades. [1] gives an interesting discussion of the notable works done by Amin and Hassan (1988); El-Ghazawi et al; Emami-Naeini and Van Dooren (1982); Hewer and Martin (1984); Latawiec (1988); Lataweic et al (1999); Misra et al (1994); Owens (1977); Sannuti and Saberi (1987); Tokarzewski (1996 and 1998) and Wolovich (1973). MacFarlane and Karcanias, 1976 [3] presented their own definition of zeros. This also led to a number of different definitions of transmission zeros and they are not necessarily equivalents. Davison and Wang [4] discussed the properties of the transmission zeros [4]. Schrader and Sain [5] provided and comprehensive survey about the different types of zeros. The classification of different zeros into following three main groups by Tokarzewski is discussed in details in [1]:

a) Those originating from the Rosenbrock's approach and related to the Smith-Mcmillan form. Some of the notable works in this field mentioned in [1] are by Amin and Hassan, ; Emami-Naeini and Van Dooren, 1982; MacFarlane and Karcanias, 1976; Misra et al, 1994; Sannuti and Saberi, 1987; Wolovich, 1973, Rosenbrock, 1970.
b) Those connected with the concept of state-zero and input-zero directions introduced in MacFarlane and Karcanias, 1976.
c) Those employing the notions of inverse systems. Notable works in this field discussed in [1] are by Lataweic, 1998; Lataweic et al, 1999.

Few of the widely known types of zeros are as follows:

1) *Invariant Zeros:* The set of the zeros of the invariant polynomials of the system matrix $P(s)$ are called the system invariant zeros.
2) *Transmission Zeros:* The zeros of the system transfer function matrix $G(s)$ are called the transmission zeros. If a system is completely controllable and completely observable, then the set of invariant zeros and transmission zeros are the same.
3) *Decoupling Zeros.* Decoupling zeros were introduced by Rosenbrock, (1970) [1] and are associated with the situation were some free modal motion of the system state, of exponential type, is uncoupled from the system's input or output. The decoupling zeros are further classified into two categories namely the output decoupling zeros and the input decoupling zeros. Sometimes some decoupling zeros satisfy the properties the both the input decoupling and output decoupling zeros and are called input-output decoupling zeros.
4) *System Zeros.* Roughly speaking the set of system zeros is the set of transmission zeros plus the set of decoupling zeros. The exact relationship involved is given by the following set equality

$$\left\{\text{system zeros}\right\} = \left\{\begin{matrix}\text{input-decoupling zeros, output-decoupling} \\ \text{zeros,transmission zero}\end{matrix}\right\} - \left\{\begin{matrix}\text{input-output} \\ \text{decoupling} \\ \text{zero}\end{matrix}\right\}$$

The relationship between system zeros, invariant zeros and transmission zeros is shown in Figure 2.1

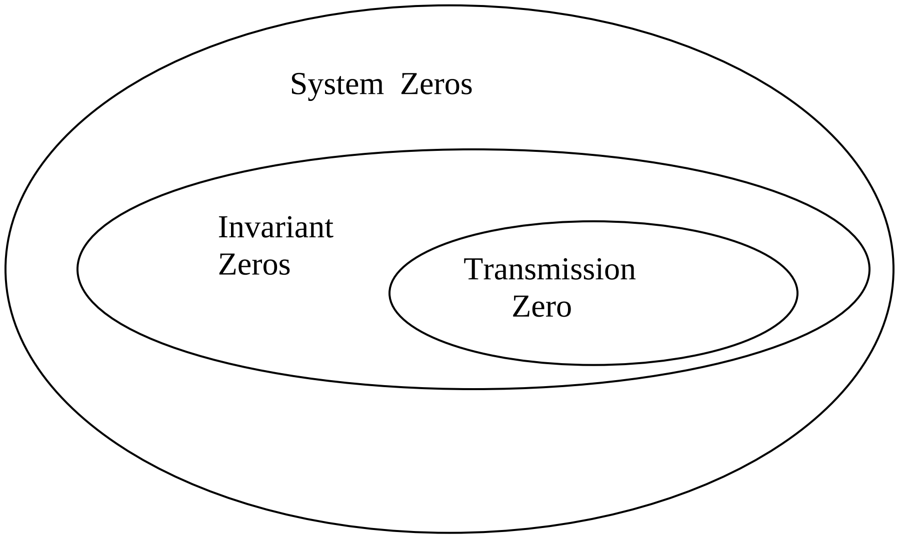

Figure 2.1: Relationship between system zeros, invariant zeros and transmission zeros

The relationship between the transmission zeros, decoupling zeros and the invariant zeros is shown in the Figure 2.2

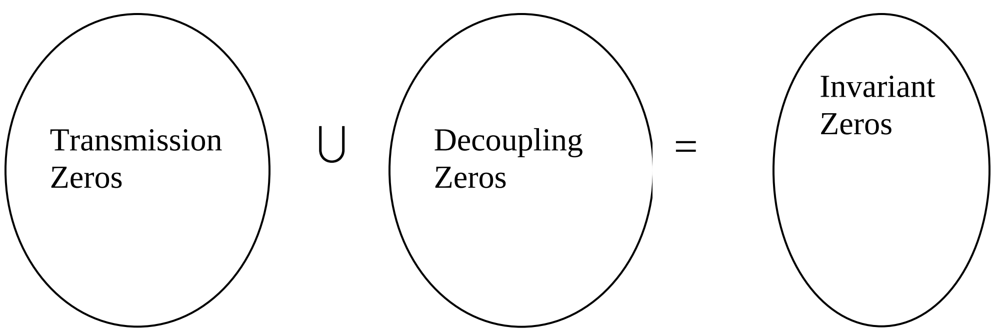

Figure 2.2: Relation between transmission, decoupling and invariant zeros

When the system is fully controllable and observable then the transmission zeros and the invariant zeros are the same. If the system is not fully controllable and observable then under those circumstances there are some zeros called the decoupling zeros which belong to the invariant zeros but do not belong to the transmission zeros.

However throughout this present work the zeros refer to transmission zero satisfying the definitions provided by the MacFarlane and Karcanias in 1976. One fundamental difference between SISO and the MIMO system is the presence of directional properties in the MIMO system. The input zero direction and the output zero direction are two such directional properties. Again the definitions provided by MacFarlane and Karcanias are followed.

2.2 Definitions, Problem Setup and Assumptions

Before proceeding further it will useful to provide some definitions of the terms which will be used in the rest of this chapter.

2.2.1 Definitions

For a linear system defined as

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{2.1}$$

with n states, m inputs and r outputs the polynomial system matrix $P(s)$ is defined as

$$P(s) = \begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix} \tag{2.2}$$

MacFarlane and Karcanias [3] defined the transmission zeros are the values $s = z$ for which $P(s)$ loses rank. The state zero vector, x_0 and the input zero direction, g are defined as the solution to the following equation.

$$\begin{bmatrix} zI - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.3}$$

The output zero direction v is defined as follows

$$\begin{bmatrix} x_v & v \end{bmatrix} \begin{bmatrix} zI - A & -B \\ C & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \tag{2.4}$$

2.2.2 Transmission-blocking Theorem of MacFarlane and Karcanias [3]

A necessary and sufficient condition for an input $u(t) = g\exp(zt)1(t)$ to yield a rectilinear motion in the state space $x(t) = x_0 \exp(zt)1(t)$ and to be such that $y(t) \equiv 0$ for $t \geq 0$ is that

$$\begin{bmatrix} zI - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

It is a well known fact that in the steady state each output of the plant goes to zero when the input is applied in the input zero direction. Also if the plant is in steady state then the combination of outputs in the output zero direction is always zero. MacFarlane and Karcanias showed that output zeroing property can be obtained even when the plant is not in the steady state. In the following sections it has been proved that the zeroing of the output combination in the output zero direction is also possible for the non-steady state of the plant.

2.2.3 Problem Formulation of the zeroing of output in output zero direction

Consider a plant P defined by the following equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2.5}$$

with n state variables, m inputs and r outputs. Now if v is the output zero direction of the plant P then taking the combination of outputs in the output zero direction can be described by following block diagram

$$U(s) \longrightarrow \boxed{G(s) = C\left(sI - A\right)^{-1} B} \xrightarrow{y} \boxed{v} \longrightarrow \text{output combination in } v \text{ direction}$$

which can be further simplified to

$$U(s) \longrightarrow \boxed{G'(s) = vC\left(sI - A\right)^{-1} B} \longrightarrow \text{output combination in } v \text{ direction}$$

Thus the problem of zeroing the output combination in output zero direction of plant P can be reduced to the problem of output zeroing of the plant P' which is defined as follows

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= vCx\end{aligned} \tag{2.6}$$

where A,B and C are the system matrices of the original plant, P and v is the output zero direction of the original plant P. At first glance the solution to this problem seems very obvious because the transmission zero and input zero direction of P' can be calculated using the following equation

$$\begin{bmatrix} z'I - A & -B \\ vC & 0 \end{bmatrix}\begin{bmatrix} x'_0 \\ g' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.7}$$

and then from the output zeroing result of MacFarlane and Karcanias we can send the input signal of the form $g'e^{z't}$ with initial state vector equal to x'_0 in order to get the output of the plant P' always equal to zero or in other words get the combination of the outputs of the plant P in the output zero direction of P, always equal to zero. However the problem is not as trivial as it seems. It should be noted that the number of outputs for the plant P is one whereas the number of inputs to the plant P is m. Davison and Wang [4] showed that if the number of inputs and outputs are not same for almost all (A,B,C) triples the system has no transmission zeros. Hence there is a need to approach this problem in an alternative way.

Let the kth column of the B matrix be denoted by b_k. Let z_k be the transmission zero corresponding to the kth input channel and is defined as the value $s = z_k$ for which the following matrices loses its rank

$$\begin{bmatrix} sI - A & -b_k \\ vC & 0 \end{bmatrix} \tag{2.8}$$

Let g_k and x_{0k} be the input zero direction and state zero vector respectively corresponding to the kth input channel and they are found by the following equation

$$\begin{bmatrix} z_k I - A & -b_k \\ vC & 0 \end{bmatrix}\begin{bmatrix} x_{0k} \\ g_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.9}$$

Notice that existence of z_k is guaranteed (Kouvaritakis and MacFarlane, 1976 [8],[9]) for almost all cases since the number of output and input for the plant is equal (i.e. one).

2.3 Main Result

If the input to the plants P and P' is given by

$$u(t) = \left[g_1 \, \mathrm{e}^{z_1 t} \quad g_2 \, \mathrm{e}^{z_2 t} \quad .... \quad g_k \, \mathrm{e}^{z_k t} \, ... \quad g_m \, \mathrm{e}^{z_m t} \right]^T \tag{2.10}$$

for all $t \geq 0$ then the following result holds.

Theorem 2.1: For previously defined plants P and P' and input $u(t)$ the state vector for both the plants is given by

$$x(t) = e^{tA} \left(x(0) - \sum_{k=1}^{m} x_{0k} \right) + \sum_{k=1}^{m} x_{0k} e^{z_k t} \tag{2.11}$$

The output of the plant P' is given by

$$y'(t) = vCe^{tA} \left(x(0) - \sum_{k=1}^{m} x_{0k} \right) \tag{2.12}$$

and the output to the plant P is given by

$$y(t) = Ce^{tA} \left(x(0) - \sum_{k=1}^{m} x_{0k} \right) + C \sum_{k=1}^{m} x_{0k} e^{z_k t} \tag{2.13}$$

where $x(0)$ is the initial state vector for both the plants P and P' since the state vector for both P and P' is same for all time (change in the output matrix has no effect on the state variables).

Proof: The generalized solution for state vector for P and P' is given by

$$x(t) = e^{tA} x(0) + \int_0^t e^{A(t-\tau)} BU(\tau) \, d\tau \tag{2.14}$$

Substituting for $u(\tau)$ we get

$$x(t) = e^{tA} x(0) + \int_0^t e^{A(t-\tau)} \left(\sum_{k=1}^{m} b_k g_k e^{z_k \tau} \right) d\tau \tag{2.15}$$

For the kth input channel we have the following relations from (2.9)

$$(z_k I - A) x_{0k} = b_k g_k \tag{2.16}$$

$$vCx_{0k} = 0 \tag{2.17}$$

Substituting (2.16) in (2.15) we get

$$\begin{aligned} x(t) &= e^{tA}x(0) + \sum_{k=1}^{m}\int_{0}^{t} e^{A(t-\tau)}\left(z_k I - A\right)x_{0k}e^{z_k\tau}d\tau \\ &= e^{tA}x(0) + e^{tA}\sum_{k=1}^{m}\int_{0}^{t} e^{\left(z_k I - A\right)\tau}\left(z_k I - A\right)x_{0k}d\tau \\ &= e^{tA}\left(x(0) - \sum_{k=1}^{m}x_{0k}\right) + \sum_{k=1}^{m}x_{0k}e^{z_k t} \end{aligned} \tag{2.18}$$

Now,

$$y'(t) = vCx(t) \tag{2.19}$$

Substituting (2.17) and (2.18) in (2.19) we get

$$y'(t) = vCe^{tA}\left(x(0) - \sum_{k=1}^{m}x_{0k}\right)$$

Now output to the plant P is given by

$$y(t) = Cx(t) \tag{2.20}$$

Substituting (2.18) in (2.20) we get

$$y(t) = Ce^{tA}\left(x(0) - \sum_{k=1}^{m}x_{0k}\right) + C\sum_{k=1}^{m}x_{0k}e^{z_k t}$$

The above results can be generalized as follows.

Theorem 2.2: For previously defined plants P and P' and input $u(t)$ defined as

$$u(t) = \left[\alpha_1 g_1 e^{Z_1 t} \quad \ldots \quad \alpha_k g_k e^{Z_k t} \quad \ldots \quad \alpha_m g_m e^{Z_m t}\right]^T \tag{2.21}$$

where α_k is a scalar, the state vector for both the plants is given by

$$x(t) = e^{tA}\left(x(0) - \sum_{k=1}^{m} \alpha_k x_{0k}\right) + \sum_{k=1}^{m} \alpha_k x_{0k} e^{z_k t} \tag{2.22}$$

The output of the plant P' is given by

$$y'(t) = vCe^{tA}\left(x(0) - \sum_{k=1}^{m} \alpha_k x_{0k}\right) \tag{2.23}$$

and the output to the plant P is given by

$$y(t) = Ce^{tA}\left(x(0) - \sum_{k=1}^{m} \alpha_k x_{0k}\right) + C\sum_{k=1}^{m} \alpha_k x_{0k} e^{z_k t} \tag{2.24}$$

where $x(0)$ is the initial state vector for both the plants P and P' since the state vector for both P and P' is same for all time (change in the output matrix has no effect on the state variables).

Proof: The proof is similar to the proof of the previous theorem.

The generalized solution for state vector for P and P' is given by

$$x(t) = e^{tA}x(0) + \int_0^t e^{A(t-\tau)} BU(\tau)\, d\tau \tag{2.25}$$

Substituting for $u(\tau)$ we get

$$x(t) = e^{tA}x(0) + \int_0^t e^{A(t-\tau)} \left(\sum_{k=1}^{m} \alpha_k b_k g_k e^{z_k \tau}\right) d\tau \tag{2.26}$$

For the kth input channel we have the following relations from (2.9)

$$(z_k I - A)x_{0k} = b_k g_k \tag{2.27}$$

$$vCx_{0k} = 0 \tag{2.28}$$

Substituting (2.27) in (2.26) we get

$$x(t)=e^{tA}x(0)+\sum_{k=1}^{m}\int_{0}^{t}e^{A(t-\tau)}\alpha_k\left(z_kI-A\right)x_{0k}e^{z_k\tau}d\tau$$

$$=e^{tA}x(0)+e^{tA}\sum_{k=1}^{m}\int_{0}^{t}e^{(z_kI-A)\tau}\alpha_k\left(z_kI-A\right)x_{0k}d\tau$$

$$=e^{tA}\left(x(0)-\sum_{k=1}^{m}\alpha_k x_{0k}\right)+\sum_{k=1}^{m}\alpha_k x_{0k}e^{z_kt}$$

(2.29)

Now,

$$y'(t)=vCx(t) \tag{2.30}$$

Substituting (2.28) and (2.29) in (2.30) we get

$$y'(t)=vCe^{tA}\left(x(0)-\sum_{k=1}^{m}\alpha_k x_{0k}\right)$$

Now output to the plant P is given by

$$y(t)=Cx(t) \tag{2.31}$$

Substituting (2.29) in (2.31) we get

$$y(t)=Ce^{tA}\left(x(0)-\sum_{k=1}^{m}\alpha_k x_{0k}\right)+C\sum_{k=1}^{m}\alpha_k x_{0k}e^{z_kt}$$

Lemma 2.1: In the results of Theorem 2.1 and Theorem 2.2 if we substitute $x(0)=\sum_{k=1}^{m}x_{0k}$ and $x(0)=\sum_{k=1}^{m}\alpha_k x_{0k}$ respectively, in both the cases we get $y'(t)=0$ for all $t\ge 0$. It should be noted that even though the output of plant P is non-zero yet the output of the plant P' is zero for the above initial condition. In other words even though the components of the output of the plant P are non-zero yet their combination in the output zero direction of P is zero. This useful result will be used to obtain the combination of outputs of the original plant P in its output zero direction equal to zero.

Remark 2.1: Let $\mathbf{U} \in \mathbf{R}^m, \mathbf{X} \in \mathbf{R}^n, \mathbf{Y} \in \mathbf{R}^r$ be the input vector space, state vector space and the output vector space for the plant P respectively then

$$\mathbf{U} = span\left(\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \quad \cdots \quad \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \right)$$

and $\mathbf{X} = span\left(x_{01} \quad x_{02} \quad \cdots \quad x_{0m} \right)$ if $x(0) = \sum_{k=1}^{m} \alpha_k x_{0k}$.

Thus the relationship between the input space, state space and the output space for the zeroing of the output combination of plant P in its output zero direction can be shown by the geometrical relationships in Figure 2.3

Using the Lemma 2.1 an algorithm to obtain a set of input signals and the corresponding initial state vector is presented below such that the combinations of output components of plant P in the output zero direction of plant P is always zero. The steps are as follows:

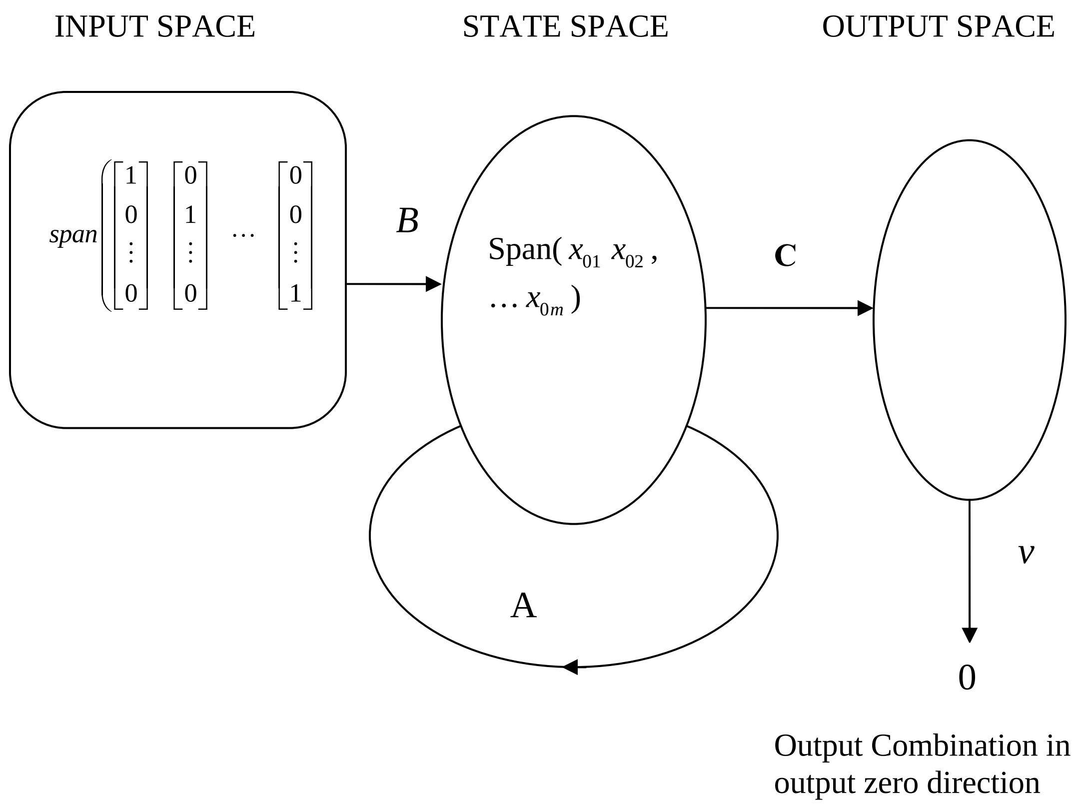

Figure 2.3: Geometrical relationships between input, output and state spaces of plant P for the zeroing of output combination in output zero direction

Step 1: Find the transmission zero, output zero direction, input zero direction and state zero vector of the plant P using (2.2), (2.3) and (2.4).

Step 2: If b_k is the kth column of the B matrix then find the transmission zero z_k , input zero direction g_k and state zero vector using x_{0k} corresponding to kth input channel using (2.8) and (2.9).

Step 3: Set the initial condition of the plant P as follows

$$x(0) = \sum_{k=1}^{m} \alpha_k x_{0k}$$

Step 4: Use $u(t)$ defined by (2.21) as the input to the plant P.

Remark 2.2: Theorem 2.2 helps to upscale or downscale the input values for each input channel. Thus even though the $g_k e^{z_k t}$ may not lie in normal range of u_k yet by careful selection of α_k we can bring it into the normal range of u_k.

CHAPTER III

USE OF OUTPUT ZEROING THEOREM FOR FAULT DETECTION

In Chapter II it was shown that it is possible to make the combination of outputs in the output zero direction equal to zero irrespective of time for some special class of inputs. In the present chapter the results derived in the previous chapter and the output zeroing result of MacFarlane and Karcanias [3] will be used for the fault detection in linear continuous time MIMO plants.

3.1 Novel Fault Detection Scheme Using Multivariable Zeros and Zero-Directions

Based on Theorem 2.1, Theorem 2.2 and Lemma 2.1 below is a test to find the faulty column of the transfer function matrix $G(s)$ of plant P.

3.1.1 Column Test

If the input to the plant P and its initial conditions are given by $u(t)=\begin{bmatrix}0 & \ldots & 0 & g_k e^{z_k t} & \ldots. & 0\end{bmatrix}$ and $x(0)=x_{0k}$ then the combination of the outputs in the output zero direction should be zero. A non-zero value indicates that the elements of the plant transfer function matrix corresponding to the kth input channel (i.e. the kth column of $G(s)$) have changed.

Based on the output zeroing result of McFarlane and Karcanias [3] stated in Chapter II the following Lemma is derived.

Lemma 3.1: Let z , x_0 and g be the transmission zero, state zero vector and the input zero direction of the plant respectively. Then for input $U(t)=ge^{zt}$ and initial condition $x(0)=x_0$ the non-zero value of the kth output indicates that the kth row of the transfer function matrix is faulty.

Proof: For the given input and initial condition all the outputs should be identically zero according to MacFarlane and Karcanias. Since the kth output depends only on the kth row of $G(s)$ therefore the non-zero kth output indicates faulty kth row of $G(s)$.

Using Lemma 3.1 the following test for finding the faulty rows of the plant transfer function matrix of plant P is obtained.

3.1.2 Row Test

For input $u(t) = ge^{zt}$ and initial condition $x(0) = x_0$ for the plant P the non-zero value of the kth output indicates that the kth row of the transfer function matrix is faulty.

Using the row test and the column test in conjunction on the plant transfer function matrix $G(s)$ we can pin-point the faulty element of the plant transfer function matrix. Suppose using the row test we find that the ith row of $G(s)$ is faulty and using the column test we find that the kth column of $G(s)$ has faults then we have the scenario as show in Figure 3.1

$$\begin{array}{cl} \begin{bmatrix} & \vdots & \\ \cdots\cdots\cdots & \boxed{g_{ik}} & \cdots\cdots \\ & \vdots & \end{bmatrix} & \text{i-th faulty row} \\ \text{k-th faulty column} & \end{array}$$

Figure 3.1: Faulty ith row and faulty kth column

Thus if we have only one faulty row and only one faulty column then we can easily deduce that only one element of the plant transfer function matrix is faulty. Thus if the ith row and kth column are faulty then we can easily deduce that the g_{ik} element of plant transfer function matrix is faulty.

3.2 An Illustrative Example

The above results are now illustrated using a quadruple tank system. The system has four stable poles and two multivariable zeroes. A complete description of the system, derivation of the non-linear model using mass balance and Bernoulli's equation and the linearized model was given by Johansson [7]. The outputs are the voltages from level measurement devices and the inputs are the input voltages to the pump.

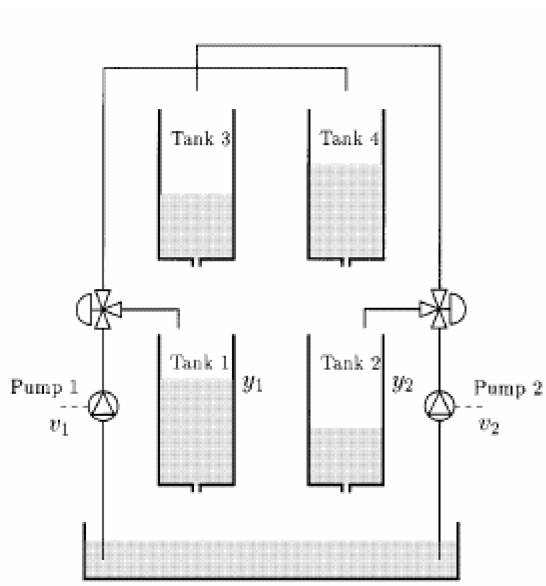

Figure 3.2: Schematic representation of quadruple-tank system [7].

A schematic diagram of the process is shown in Fig. 3.2. The process inputs are v_1 and v_2 and the outputs are y_1 and y_2. Mass balances and Bernoulli's law yield

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2) k_2}{A_3} v_2$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1) k_1}{A_4} v_1$$

where

A_i	Cross-section of Tank i
a_i	Cross-section of the outlet hole
h_i	Water level

The voltage applied to Pump is v_i and the corresponding flow is $k_i v_i$. The parameters $\gamma_1, \gamma_2 \in (0,1)$ are determined from how the valves are set prior to an experiment. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and the flow to Tank 4 is $(1-\gamma_1) k_1 v_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted by g. The measured level signals are $k_c h_1$ and $k_c h_2$. The parameter values are following:

A_1, A_3 [cm^2]	28
A_2, A_4 [cm^2]	32
a_1, a_3 [cm^2]	0.071
a_2, a_4 [cm^2]	0.057
k_c [V/cm]	0.50
g [cm/s^2]	981

After linearizing about a particular operating point we have the following system matrices

$$A = \begin{bmatrix} -0.0161 & 0 & 0.0435 & 0 \\ 0 & -0.0111 & 0 & 0.0333 \\ 0 & 0 & -0.0435 & 0 \\ 0 & 0 & 0 & -0.0333 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0628 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix}$$

3.2.1 Verification of Theorem 2.1

Now the transmission zeros corresponding to the first and second input channels found using (2.8) are $z_1 = -0.0594$ and $z_2 = -0.0333$. Using (2.9) the corresponding input directions and state zero vectors are given by

$$g_1 = [0.3827] \quad g_2 = [0.0675] \quad x_{01} = [-0.7367 \quad 0.3168 \quad 0.000 \quad -0.4587]^T$$

$$x_{02} = [-0.8042 \quad 0.3458 \quad 0.3182 \quad -0.3577]^T$$

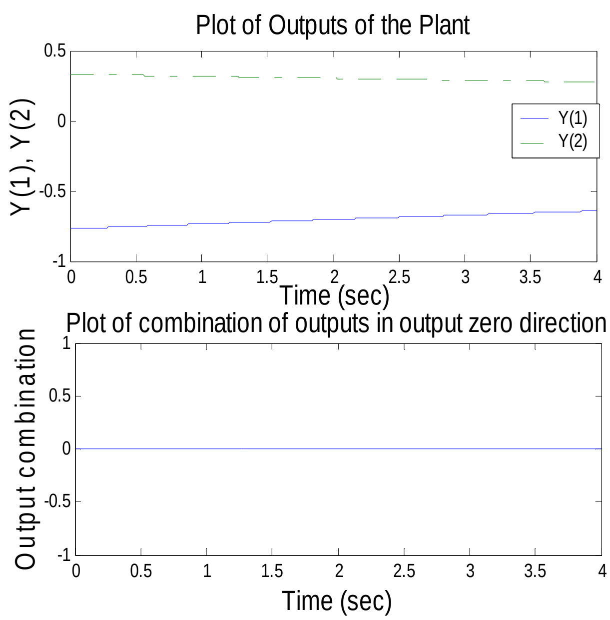

Figure 3.3: Plant outputs and their combination in output zero direction

For $u(t) = \left[0.3827e^{-0.0594t} \quad 0.0675e^{-0.0333t}\right]^T$ and initial condition $x(0) = x_{01} + x_{02}$ we get the outputs as shown in Figure 3.3. The results of Theorem 2.1 are verified by plots of Figure 3.3. The plant transfer function matrix of the plant is given by

$$G(s) = \begin{bmatrix} \dfrac{2.6}{1+62s} & \dfrac{1.5}{(1+23s)(1+62s)} \\ \dfrac{1.4}{(1+30s)(1+90s)} & \dfrac{2.8}{(1+90s)} \end{bmatrix} \tag{3.1}$$

Now let us introduce some faults in the second column of $G(s)$ by changing the second column of the B matrix. Note that the changes to $G(s)$ can be made by changing either A, B or C matrices however changing second column of B only changes the second column of $G(s)$. Let the new B matrix be given as

$$B = \begin{bmatrix} 0.0833 & 0.5 \\ 0 & 0.0628 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix}$$

A and C matrices remain same. Then the new transfer function matrix is given by

$$G_{changed}(s) = \begin{bmatrix} \dfrac{2.6}{1+62s} & \dfrac{356.50s+16.98}{(1+23s)(1+62s)} \\ \dfrac{1.4}{(1+30s)(1+90s)} & \dfrac{2.8}{(1+90s)} \end{bmatrix} \tag{3.2}$$

It can be noticed that $(1,2)$ element of $G(s)$ has changed.

3.2.2 Column Test

We will use the column test to identify the faulty column of $G(s)$. Now the transmission zeros corresponding to the first and second input channels found using (2.8) are $z_1 = -0.0594$ and $z_2 = -0.0333$. Using (2.9) the corresponding input directions and state zero vectors are given by

$g_1 = [0.3827]$ $g_2 = [0.0675]$ $x_{01} = [-0.7367 \quad 0.3168 \quad 0.000 \quad -0.4587]^T$

$x_{02} = [-0.8042 \quad 0.3458 \quad 0.3182 \quad -0.3577]^T$

For input signal $u(t) = \begin{bmatrix} 0 & 0.0675e^{-0.0333t} \end{bmatrix}^T$ and the initial condition $x(0) = x_{02} = [-0.8042 \quad 0.3458 \quad 0.3182 \quad -0.3577]^T$ the combination of outputs in output zero direction is shown in Figure 3.4.

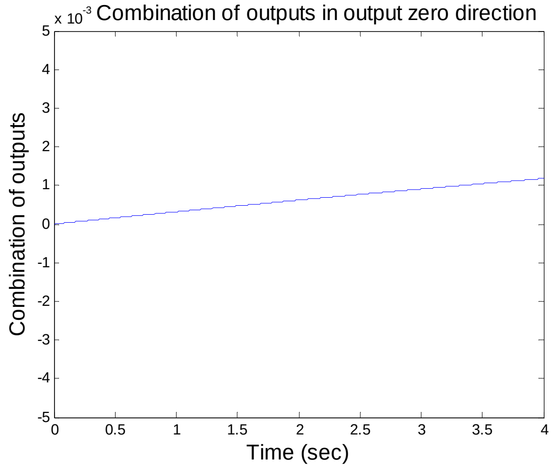

Figure 3.4: Combination of outputs in the output zero direction for the second column

Using the column test and Figure 3.4 we conclude that the fault lies in the second column of the transfer function matrix. This result is verified by looking at the changed $G(s)$. By following a similar procedure for the first input channel it is concluded that there is no fault in the first column of the $G(s)$. For this case the combination of outputs in output zero direction is shown in Figure 3.5.

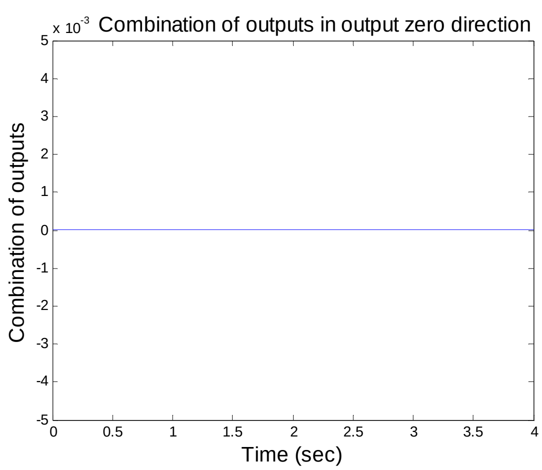

Figure 3.5: The combination of outputs in output zero direction for the first column

Thus we conclude that the fault lies only in the second column of the plant transfer function matrix.

3.2.3 Row Test

Now for our system the transmission zero, state zero vector and the input zero direction are as follows

$z = -0.0594$

$x_0 = \begin{bmatrix} 0 & 0.000 & 0.7506 & 0.4699 \end{bmatrix}^T$

$g = \begin{bmatrix} -0.3920 & -0.2494 \end{bmatrix}^T$

For $u(t) = \begin{bmatrix} -0.3920 & -0.2494 \end{bmatrix}^T e^{-0.0594t}$ and $x(0) = \begin{bmatrix} 0 & 0.000 & 0.7506 & 0.4699 \end{bmatrix}^T$ the outputs are shown in Figure 3.6.

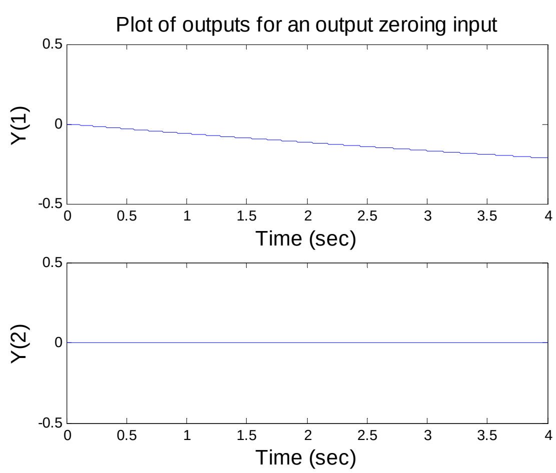

Figure 3.6: Outputs of the continuous time plant for an output zeroing input

From Figure 3.6 it is clear that the first row (using the row test) of the plant transfer function matrix is faulty.

Since in this case there is only one faulty row and one faulty column we can straightaway conclude that the fault lies in $(1,2)$ element of plant transfer function matrix. This matches with the result obtained by comparing the transfer function matrices given in . (3.1) and (3.2).

3.3 Steady State Analysis

From the Final Value Theorem we have $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$ where $Y(s)$ the Laplace transform of stable $y(t)$. Thus for an input of the form $u(t) = [\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_m]^T 1(t)$, where $1(t)$ denotes a unit step function, the steady state output is given by

$$y_{ss} = G(0)[\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_m]^T \tag{3.3}$$

Lemma 3.1: If $u(t) = [0 \quad \dots \quad \alpha_k \quad \dots 0]^T 1(t)$ then the ith steady state output is given by $y_{ss,i} = G_{ik}(0)\alpha_k$ where $G_{ik}(0)$ is the (i,k) element of $G(0)$. Thus if actual ith steady state output is different from $G_{ik}(0)\alpha_k$ then it can be concluded that the (i,k) element of the plant transition matrix $G(s)$ is faulty.

Using the transmission zeros and the zero directions of the quadruple-tank system we concluded that G_{12} is faulty. Now since our plant has all the poles in the left half plane we can corroborate our previous conclusion using steady state analysis.

Let the input be $u(t) = [0 \quad 1]^T 1(t)$ then for the defective plant we get the output plot as shown in Figure 3.7.

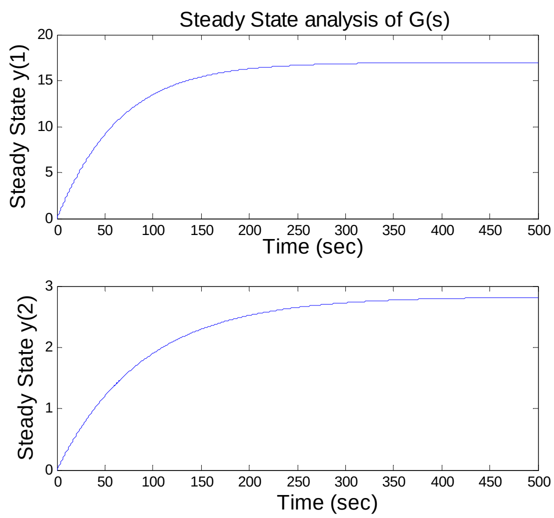

Figure 3.7: Steady state analysis of the second column

Similarly for $u(t)=\begin{bmatrix}1 & 0\end{bmatrix}^T 1(t)$ we get Figure 3.8 for steady state output of the plant.

$$G_{ideal}(0)=\begin{bmatrix}2.6 & 1.5\\ 1.4 & 2.8\end{bmatrix} \tag{3.4}$$

From Figure 3.7 and Lemma 3.1 we conclude that G_{22} has not changed whereas G_{12} is faulty. From Figure 3.8 we conclude that both G_{11} and G_{12} have no faults. This conclusion is the same as the one arrived using transmission zeros and zero directions.

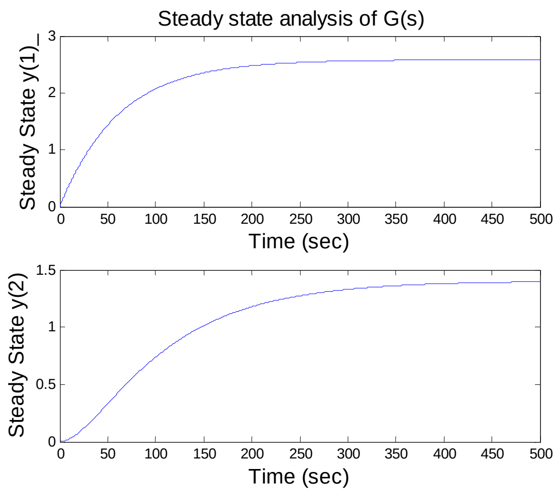

Figure 3.8: Steady state analysis of the first column

CHAPTER IV

FURTHER RESULTS FOR FAULT DETECTION USING ZERO AND ZERO DIRECTIONS

In the previous chapter novel fault detection scheme for MIMO continuous time system using transmission zeros and zero directions, was developed. The results were also verified by the steady state analysis of the system. However it was assumed that the system has at most one defective column and one defective row. In this chapter the scheme will be generalized to multiple defective rows and defective columns. Some other results are also discussed in this chapter.

4.1 Extension to Multiple Faulty Rows and Columns

If we have only one faulty row and only one faulty column then we can easily deduce that only one element of the plant transfer function matrix is faulty. Thus if the i^{th} row and k^{th} column are faulty then we can easily deduce that the g_{ik} element of plant transfer function matrix is faulty.

Deductions is still easy for the following two cases 1) one faulty row and more than one faulty columns 2) more than one faulty rows and one faulty column. For the first case (Figure 4.1) the only possibility which satisfies the result of the row test and the column test is that the g_{ik} and g_{pk} elements are the defective elements. For the second case (Figure 4.2) the only possibility is that g_{ik} and g_{pk} are the defective elements of the plant transfer function matrix.

$$
\begin{array}{cl}
\begin{bmatrix}
 & \vdots & \\
\cdots\cdots & \boxed{g_{ik}} & \cdots\cdots \\
 & \vdots & \\
\cdots\cdots & \boxed{g_{pk}} & \cdots\cdots \\
 & \vdots &
\end{bmatrix}
&
\begin{array}{l}
\\
\text{i-th faulty row} \\
\\
\text{p-th faulty row} \\
\\
\end{array}
\\
\text{k-th faulty column} &
\end{array}
$$

Figure 4.1: Deduction for the case in which there is multiple faulty rows and single faulty column

$$
\begin{array}{cl}
\begin{bmatrix}
 & \vdots & & \vdots & \\
\cdots\cdots & \boxed{g_{ik}} & \cdots\cdots & \boxed{g_{il}} & \cdots\cdots \\
 & \vdots & & \vdots &
\end{bmatrix}
&
\begin{array}{l}
\\
\text{i-th faulty row} \\
\\
\end{array}
\\
\text{k-th and l-th faulty columns} &
\end{array}
$$

Figure 4.2: Deduction for the case in which there is single faulty row and multiple faulty columns

However the deduction becomes difficult if both multiple faulty rows and columns exist as shown in Figure 4.3

$$
\begin{array}{cl}
\begin{bmatrix}
 & \vdots & & & \\
\cdots\cdots & \boxed{g_{ik}} & \cdots\cdots & \boxed{g_{il}} & \cdots\cdots \\
 & \vdots & & \vdots & \\
\cdots\cdots & \boxed{g_{jk}} & \cdots\cdots & \boxed{g_{jl}} & \cdots\cdots \\
 & \vdots & & \vdots &
\end{bmatrix}
&
\begin{array}{l}
\\
\text{i-th faulty row} \\
\\
\text{j-th faulty row} \\
\\
\end{array}
\\
\text{k-th faulty col} \quad \text{l-th faulty col} &
\end{array}
$$

Figure 4.3: The case in which there are both multiple faulty rows and multiple faulty columns

Suppose using the row test we found that the ith and jth rows are faulty and similarly using the column test we found out that the kth and lth columns are faulty. For the above combination of faulty rows and columns we can have the following possibilities:

(1) $g_{ik}, g_{il}, g_{jk}, g_{jl}$ are the faulty elements of the plant transfer function matrix

(2) g_{ik}, g_{jk}, g_{jl} are the faulty elements of the plant transfer function matrix

(3) g_{il}, g_{jk}, g_{jl} are the faulty elements of the plant transfer function matrix

(4) g_{ik}, g_{il}, g_{jl} are the faulty elements of the plant transfer function matrix

(5) g_{ik}, g_{il}, g_{jk} are the faulty elements of the plant transfer function matrix

Therefore we see that in the first possibility all the elements where the faulty rows and columns intersect are defective. However in the last four possibilities only three of the total four intersection points are defective. In order to find out which of the above five possibilities is the real status of the plant we will take the “help” of some faultless row of the plant transition matrix. Suppose using the row test it has been found out that the pth row of $G(s)$ is without any faults. The situation is shown in Figure 4.4

$$
\begin{array}{cl}
\left[\begin{array}{ccccc}
 & \vdots & & \vdots & \\
\cdots & \boxed{g_{ik}} & \cdots & \boxed{g_{il}} & \cdots \\
 & \vdots & & \vdots & \\
\cdots & \boxed{g_{pk}} & \cdots & \boxed{g_{pl}} & \cdots \\
 & \vdots & & \vdots & \\
\cdots & \boxed{g_{jk}} & \cdots & \boxed{g_{jl}} & \cdots \\
 & \vdots & & \vdots &
\end{array}\right] &
\begin{array}{l}
 \\
\text{i-th row (faulty)} \\
 \\
\text{p-th row (faultless)} \\
 \\
\text{j-th row (faulty)} \\

\end{array} \\
\begin{array}{cc}\text{k-th faulty col.} & \text{l-th faulty col.}\end{array} &
\end{array}
$$

Figure 4.4: Using the faultless row for finding the faulty elements

Now consider a plant with the transfer function matrix as following

$$G_i(s) = \begin{bmatrix} g_{ik} & g_{il} \\ g_{pk} & g_{pl} \end{bmatrix} \tag{4.1}$$

It can be shown that the state space realization of this transfer function matrix is

$$A_i = A;\ \ B_i = \begin{bmatrix} b_k & b_l \end{bmatrix};\ C_i = \begin{bmatrix} c_i \\ c_p \end{bmatrix} \tag{4.2}$$

where c_i and c_p are the ith and the pth rows of the C matrix of plant P and b_k and b_l are the kth and the lth columns of the B matrix of plant P. If we measure only the ith and pth output of plant P and use the kth and the lth input channel of plant P for input keeping the input to the rest channels equal to zero, then it is same as the plant described by the plant transfer function matrix $G_i(s)$. Now we can calculate the transmission zeros, input zero directions and output zero direction of this new plant and perform the row test and the column test. However since the second column of the new plant is faultless hence a column test is sufficient to find whether the elements g_{ik} and g_{il} are faulty. Similarly we can construct a plant with transfer function matrix

$$G_j(s) = \begin{bmatrix} g_{pk} & g_{pl} \\ g_{jk} & g_{jl} \end{bmatrix} \tag{4.3}$$

The state space realization of the plant above plant transfer function matrix is

$$A_j = A;\ \ B_j = \begin{bmatrix} b_k & b_l \end{bmatrix};\ C_j = \begin{bmatrix} c_p \\ c_j \end{bmatrix} \tag{4.4}$$

The above plant can be visualized as the plant P whose pth and jth outputs are only measured and which has non-zero inputs to only its kth and lth input channels. Again by performing the column test we can find whether the elements g_{jk} and g_{jl} are faulty.

In case when it is not possible to find a row without faults in the plant transfer function matrix of P then we can take the help of faultless column of $G(s)$. Thus if we have the scenario as shown in Figure 4.5 then we can take the help of faultless mth column to find which elements out of g_{il}, g_{ik}, g_{jl} and g_{jk} are faulty.

$$\begin{array}{cc} \begin{bmatrix} & \vdots & & \vdots & & \vdots & \\ \cdots\cdots & \boxed{g_{ik}} & \cdots\cdots & \boxed{g_{il}} & \cdots\cdots & \boxed{g_{im}} & \cdots\cdots \\ & \vdots & & \vdots & & \vdots & \\ \cdots\cdots & \boxed{g_{jk}} & \cdots\cdots & \boxed{g_{jl}} & \cdots\cdots & \boxed{g_{jm}} & \cdots\cdots \\ & \vdots & & \vdots & & \vdots & \end{bmatrix} & \begin{array}{l} \\ \text{i-th faulty row} \\ \\ \text{j-th faulty row} \\ \\ \end{array} \end{array}$$

k-th col.(faulty) l-th col.(faulty) m-th col.(faultless)

Figure 4.5: k^{th} and l^{th} columns are faulty and m^{th} column is without any faults

In other words we can perform the row test on the following transfer function matrices to find the faulty elements

$$G_k(s) = \begin{bmatrix} g_{ik} & g_{im} \\ g_{jk} & g_{jm} \end{bmatrix} \;;\; G_l(s) = \begin{bmatrix} g_{il} & g_{im} \\ g_{jl} & g_{jm} \end{bmatrix} \tag{4.5}$$

4.2 Extension of Theorem 2.1 and Theorem 2.2 to the non-proper systems

In Chapter II, Theorem 2.1 and Theorem 2.2 were derived to make the combination of outputs in the output zero direction equal to zero irrespective of time. However the main assumption was that the system was proper, that is $D = 0$. In the following work corresponding versions of Theorem 2.1 and Theorem 2.2 are derived for non-proper systems $(D \neq 0)$.

Let a linear non-proper system P be defined by the following equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + \left\{ D_i \frac{d^i}{dt^i} + D_{i-1} \frac{d^{i-1}}{dt^{i-1}} + \cdots + D_1 \frac{d^1}{dt^1} + D_0 \right\} u \end{aligned} \tag{4.6}$$

with n states, m inputs and r outputs.

Let another linear non-proper plant P' be defined by the following equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= vCx + v\left\{ D_i \frac{d^i}{dt^i} + D_{i-1} \frac{d^{i-1}}{dt^{i-1}} + \cdots + D_1 \frac{d^1}{dt^1} + D_0 \right\} u \end{aligned} \tag{4.7}$$

where v is the output-zero direction of plant P and is defined by the following equation

$$\begin{bmatrix} x_v & v \end{bmatrix} \begin{bmatrix} zI - A & -B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \tag{4.8}$$

Here $D = D_i z^i + D_{i-1} z^{i-1} + \cdots + D_0$ (4.9)

and z is the transmission zero of the plant P .

Let $d_{i,k}, d_{i-1,k} \cdots d_{0,k}$ be the k-th column of $D_i, D_{i-1} \cdots D_0$ respectively. Let z_k be the transmission zero corresponding to the k-th input channel and is defined as the value of z_k at which the following matrix loses its rank

$$\begin{bmatrix} z_k I - A & -b_k \\ vC & vd_k \end{bmatrix}$$

where d_k is defined as follows

$$d_k = z_k^i d_i + z_k^{i-1} d_{i-1} \cdots + z_k^1 d_1 + z_k^0 d_0 \tag{4.10}$$

Let g_k and x_{0k} be the input zero direction and state zero vector respectively corresponding to the k[th] input channel and they are found by the following equation

$$\begin{bmatrix} z_k I - A & -b_k \\ vC & vd_k \end{bmatrix} \begin{bmatrix} x_{0k} \\ g_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.11}$$

Notice that existence of z_k is guaranteed (Kouvaritakis and MacFarlane, 1976 [8], [9]) for almost all cases since the number of output and input for the plant is equal (i.e. one).

4.3 Main Results

If the input to the plants P and P' is given by

$$u(t) = \begin{bmatrix} g_1\, e^{z_1 t} & g_2\, e^{z_2 t} & \ldots & g_k\, e^{z_k t} \ldots & g_m\, e^{z_m t} \end{bmatrix}^T \tag{4.12}$$

for all $t \geq 0$ then the following result holds.

Theorem 4.1: For previously defined plants P and P' and input $u(t)$ the state vector for both the plants is given by

$$x(t) = e^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k} \right) + \sum_{k=1}^{m} x_{0k} e^{z_k t} \tag{4.13}$$

The output of the plant P' is given by

$$y'(t) = vCe^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) \tag{4.14}$$

and the output to the plant P is given by

$$y(t) = Ce^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) + \sum_{k=1}^{m}\left(Cx_{0k} + d_k g_k\right)e^{z_k t} \tag{4.15}$$

where $x(0)$ is the initial state vector for both the plants P and P' since the state vector for both P and P' is same for all time (change in the output matrix has no effect on the state variables).

Proof: The generalized solution for state vector for P and P' is given by

$$x(t) = e^{tA}x(0) + \int_0^t e^{A(t-\tau)}BU(\tau)\,d\tau \tag{4.16}$$

Substituting for $U(\tau)$ we get

$$x(t) = e^{tA}x(0) + \int_0^t e^{A(t-\tau)}\left(\sum_{k=1}^{m} b_k g_k e^{z_k \tau}\right)d\tau \tag{4.17}$$

For the kth input channel we have the following relations from (4.11)

$$\left(z_k I - A\right)x_{0k} = b_k g_k \tag{4.18}$$

$$vCx_{0k} + vd_k g_k = 0 \tag{4.19}$$

Substituting (4.18) in (4.17) we get

$$\begin{aligned} x(t) &= e^{tA}x(0) + \sum_{k=1}^{m}\int_0^t e^{A(t-\tau)}\left(z_k I - A\right)x_{0k}e^{z_k\tau}\,d\tau \\ &= e^{tA}x(0) + e^{tA}\sum_{k=1}^{m}\int_0^t e^{(z_k I - A)\tau}\left(z_k I - A\right)x_{0k}\,d\tau \\ &= e^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) + \sum_{k=1}^{m} x_{0k}e^{z_k t} \end{aligned}$$

(4.20)

Now,

$$y'(t) = vCx(t) + v\left\{D_i \frac{d^i}{dt^i} + D_{i-1}\frac{d^{i-1}}{dt^{i-1}} + \cdots + D_1 \frac{d^1}{dt^1} + D_0\right\}u \tag{4.21}$$

Substituting (4.19) and (4.20) in (4.21) we get

$$\begin{aligned} y'(t) &= vCe^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) + vC\left(\sum_{k=1}^{m} x_{0k}e^{z_k t}\right) + v\left(\sum_{k=1}^{m} d_k g_k e^{z_k t}\right) \\ &= vCe^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) + v\sum_{k=1}^{m}\left(Cx_{0k} + d_k g_k\right)e^{z_k t} \\ &= vCe^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) \end{aligned}$$

Now output to the plant P is given by

$$y(t) = Cx(t) + \left\{D_i \frac{d^i}{dt^i} + D_{i-1}\frac{d^{i-1}}{dt^{i-1}} + \cdots + D_1 \frac{d^1}{dt^1} + D_0\right\}u \tag{4.22}$$

Substituting (4.20) in (4.22) we get

$$\begin{aligned} y(t) &= Ce^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) + C\sum_{k=1}^{m} x_{0k}e^{z_k t} + \sum_{k=1}^{m} d_k g_k e^{z_k t} \\ &= Ce^{tA}\left(x(0) - \sum_{k=1}^{m} x_{0k}\right) + \sum_{k=1}^{m}\left(Cx_{0k} + d_k g_k\right)e^{z_k t} \end{aligned}$$

The above results can be generalized as follows.

Theorem 4.2: For previously defined plants P and P' and input $U(t)$ defined as

$$U(t) = \left[\alpha_1 g_1 e^{Z_1 t} \quad \ldots \quad \alpha_k g_k e^{Z_k t} \quad \ldots \quad \alpha_m g_m e^{Z_m t}\right]^T \tag{4.23}$$

where α_k is a scalar, the state vector for both the plants is given by

$$x(t) = e^{tA}\left(x(0) - \sum_{k=1}^{m} \alpha_k x_{0k}\right) + \sum_{k=1}^{m} \alpha_k x_{0k} e^{z_k t} \tag{4.24}$$

The output of the plant P' is given by

$$y'(t) = vCe^{tA}\left(x(0) - \sum_{k=1}^{m}\alpha_k x_{0k}\right) \tag{4.25}$$

and the output to the plant P is given by

$$y(t) = Ce^{tA}\left(x(0) - \sum_{k=1}^{m}\alpha_k x_{0k}\right) + \sum_{k=1}^{m}\alpha_k \left(Cx_{0k} + d_k g_k\right)e^{z_k t} \tag{4.26}$$

where $x(0)$ is the initial state vector for both the plants P and P' since the state vector for both P and P' is same for all time (change in the output matrix has no effect on the state variables).

Proof: The proof is similar to the proof of the previous theorem.

4.4 Tests for Diagnosing Faults in A and C Matrices

In the earlier section we gave a detailed discussion on how to locate the faulty elements of the plant transfer function matrix. A fault in $G(s)$ indicates that there is fault in some o of the system matrices (A and C) but still we cannot say using the tests described in the previous section which of the system matrices are faulty. We present here a set of two tests- one each for A and C , to find the faulty system matrices.

4.4.1 Test for A Matrix

Let the representation of plant P given by (2.1) be minimal. Let $\lambda \in \rho(A)$. Then there exists an eigenvector $x_{eig} \in \mathbf{C}^n$ such that $(\lambda I - A)x_{eig} = 0$. Now if the input $u(t) \equiv 0$ and initial condition is $x(0) = x_{eig}$ then the state vector is given by $x(t) = x_{eig}e^{\lambda t}$. Let h be a vector orthogonal to x_p . It can be easily seen that the combination of states in the direction of h is always zero. Assuming that all the states are measurable we can detect an occurrence of fault in the A matrix by combining the measured states in the direction of h and noting whether the combination is zero or not.

4.4.2 Test for the C Matrix

Suppose that all the states are measurable. The fault in the pth row of C matrix, c_p can be found by comparing the pth component of the output vector and the quantity $c_p x$ where x is the measured state vector. A difference in the values shows the presence of fault in the pth row of C.

CHAPTER V

ZEROING OF OUTPUTS OF DISCRETE TIME SYSTEMS IN THE OUTPUT-ZERO DIRECTIONS

In Chapter II a theorem for zeroing the outputs in the output-zero direction for a continuous time system was derived. The theorem provided a method to generate a special class of inputs corresponding to which the combination of outputs of a continuous time plant in its output-zero direction is zero irrespective of time. However in the real world most of the continuous time models are discretized to make them compatible for use with microprocessors and digital signal processors. In this chapter similar results for the discrete-time system will be derived.

5.1 Definitions, Problem Setup and Assumptions

Before proceeding further it will useful to provide some definitions of the terms which will be used in the rest of this chapter.

5.1.1 Definitions

For a linear system defined as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{5.1}$$

with n states, m inputs and r outputs the polynomial system matrix $P(z)$ is defined as

$$P(z) = \begin{bmatrix} zI - A & -B \\ C & 0 \end{bmatrix} \tag{5.2}$$

Here z is the z-transform variable. z has the same role in discrete time system as s has in the continuous time system. The transmission zeros are the values $z = q$ for which $P(z)$ loses rank. The state zero vector, x_0 and the input zero direction, g are defined as the solution to the following equation.

$$\begin{bmatrix} qI-A & -B \\ C & 0 \end{bmatrix}\begin{bmatrix} x_0 \\ g \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5.3}$$

The output zero direction v is defined as follows

$$\begin{bmatrix} x_v & v \end{bmatrix}\begin{bmatrix} qI-A & -B \\ C & 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \tag{5.4}$$

5.1.2 Transmission-blocking Theorem for Discrete-time by Tokarzewski (1999)

The transmission blocking problem as formulated by Isidori, 1995 [1] is as follows: find all pairs $\left(x_0, u(k)\right)$, consisting of an initial state $x_0 \in R^n$ and a real-valued input vector sequence $u_0(k), k = 0,1,2.....$, such that the corresponding output $y(k)$ is identically zero for all $k = 0,1,2..$ In 1999 Tokarzewski [6] came up with a solution to this problem. If $q \in C$ is a transmission zero of plant P then the input

$$u(k)=\begin{cases} g & \text{for k = 0} \\ gq^k & \text{for k = 1,2...} \end{cases}$$

applied to P at the initial condition $x(0) = x_0$ yields the solution to the state equation of the form

$$x(k)=\begin{cases} x_0 & \text{for k = 0} \\ \mathrm{x}_0 q^k & \text{for k = 1,2,..} \end{cases}$$

and the system response $y(k) = 0$ for $k = 0,1,2,..$

It is a well known fact that in the steady state each output of the plant goes to zero when the input is applied in the input zero direction. Also if the plant is in steady state then the combination of outputs in the output zero direction is always zero. MacFarlane and Karcanias showed for continuous time plants and Tokarjewski [6] showed for discrete time plants that output zeroing property can be obtained even when the plant is not in the steady state. In the following sections it has been proved that the zeroing of the output combination in the output zero direction is also possible for the non-steady state of the discrete time plants.

5.1.3 Problem Formulation of the Zeroing of Output in Output Zero Direction:

Consider a plant P defined by the following equations

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{5.5}$$

with n states, m inputs and r outputs. Now if v is the output zero direction of the plant P then taking the combination of outputs in the output zero direction can be described by following block diagram

$$U(z) \longrightarrow \boxed{G(z) = C(zI - A)^{-1} B} \xrightarrow{y} \boxed{v} \longrightarrow \text{output combination in } v \text{ direction}$$

which can be further simplified to

$$U(z) \longrightarrow \boxed{G'(z) = vC(zI - A)^{-1} B} \longrightarrow \text{output combination in } v \text{ direction}$$

Thus the problem of zeroing the output combination in output zero direction of plant P can be reduced to the problem of output zeroing of the plant P' which is defined as follows

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= vCx(k) \end{aligned} \tag{5.6}$$

where A, B and C are the system matrices of original plant P and v is the output zero direction of the original plant P. At first glance the solution to this problem seems very obvious because the transmission zero and input zero direction of P' can be calculated using the following equation

$$\begin{bmatrix} q'I - A & -B \\ vC & 0 \end{bmatrix} \begin{bmatrix} x_0' \\ g' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5.7}$$

and then from the output zeroing result of Tokarzewski [6] we can send the input signal of the form $g'(q')^k$ ($k = 0,1,2...$) with initial state vector equal to x_0' in order to get the output of the plant P' always equal to zero or in other words get the combination of the outputs of the plant P in the output zero direction of P, always equal to zero. However the problem is not as trivial as it seems. It should be noted that the number of outputs for the plant P is one whereas the number of inputs to the plant P is m. Davison and Wang [4] showed that if the number of inputs and outputs are not same for almost all (A, B, C)

triples the system has no transmission zeros. Hence there is a need to approach this problem in an alternative way.

Let the j^th column of the B matrix be denoted by b_j. Let q_j be the transmission zero corresponding to the j^th input channel and is defined as the value $z = q_j$ for which the following matrices loses its rank

$$\begin{bmatrix} zI - A & -b_j \\ vC & 0 \end{bmatrix} \tag{5.8}$$

Let g_j and x_{0j} be the input zero direction and state zero vector respectively corresponding to the j^th input channel and they are found by the following equation

$$\begin{bmatrix} q_j I - A & -b_j \\ vC & 0 \end{bmatrix}\begin{bmatrix} x_{0j} \\ g_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5.9}$$

Notice that existence of q_j is guaranteed for almost all cases since the number of output and input for the plant is equal (i.e. one).

5.2 Main Result

If the input to the plants P and P' is given by

$$u(k) = \begin{bmatrix} g_1 q_1^k & g_2 q_2^k & .... & g_j q_j^k .... & g_m q_m^k \end{bmatrix}^T \tag{5.10}$$

for all $k = 0,1,2.....$ then the following result holds.

Theorem 5.1: For previously defined plants P and P' and input $u(k)$ the state vector for both the plants is given by

$$x(k) = A^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right) + \sum_{j=1}^{m} q_j^k x_{0j} \tag{5.11}$$

The output of the plant P' is given by

$$y'(k) = vCA^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right) \tag{5.12}$$

and the output to the plant P is given by

$$y(k) = CA^k\left(x(0) - \sum_{j=1}^{m} x_{0j}\right) + C\sum_{j=1}^{m} x_{0j}q_j^k \tag{5.13}$$

where $x(0)$ is the initial state vector for both the plants P and P' since the state vector for both P and P' is same for all k (change in the output matrix has no effect on the state variables).

Proof: The generalized solution for state vector for P and P' is given by

$$x(k) = A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}Bu(l) \tag{5.14}$$

Substituting for $u(l)$ we get

$$\begin{aligned} x(k) &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(b_1g_1q_1^l + b_2g_2q_2^l + \dots\dots + b_mg_mq_m^l\right) \\ &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\sum_{j=1}^{m} b_jg_jq_j^l\right) \end{aligned} \tag{5.15}$$

For the j[th] input channel we have the following relations from (5.9)

$$\left(q_jI - A\right)x_{0j} = b_jg_j \tag{5.16}$$

$$vCx_{0j} = 0 \tag{5.17}$$

Substituting (5.16) in (5.15) we get

$$\begin{aligned} x(k) &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\sum_{j=1}^{m} b_jg_jq_j^l\right) \\ &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\sum_{j=1}^{m}\left(q_jI - A\right)x_{0j}q_j^l\right) \\ &= A^k x(0) + \underbrace{\sum_{l=0}^{k-1} A^{k-l-1}\sum_{j=1}^{m} q_j^{l+1}x_{0j}}_{I} - \sum_{l=0}^{k-1} A^{k-l}\sum_{j=1}^{m} q_j^l x_{0j} \end{aligned}$$

In I by doing change of variable $(l+1) \to l$ we get

$$x(k) = A^k x(0) + \sum_{l=1}^{k} A^{k-l} \sum_{j=1}^{m} q_j^l x_{0j} - \sum_{l=0}^{k-1} A^{k-l} \sum_{j=1}^{m} q_j^l x_{0j}$$
$$= A^k x(0) + \sum_{j=1}^{m} q_j^k x_{0j} - A^k \sum_{j=1}^{m} x_{0j} + \left(\sum_{l=1}^{k-1} A^{k-l} \sum_{j=1}^{m} q_j^l x_{0j} - \sum_{l=1}^{k-1} A^{k-l} \sum_{j=1}^{m} q_j^l x_{0j} \right) \qquad (5.18)$$
$$= A^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right) + \sum_{j=1}^{m} q_j^k x_{0j}$$

Now,

$$y'(k) = vCx(k) \qquad (5.19)$$

Substituting (5.17) and (5.18) in (5.19) we get

$$y'(k) = vCA^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right) + vC \sum_{j=1}^{m} q_j^k x_{0j}$$
$$= vCA^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right)$$

Now output to the plant P is given by

$$y(k) = Cx(k) \qquad (5.20)$$

Substituting (5.18) in (5.20) we get

$$y(k) = CA^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right) + C \sum_{j=1}^{m} x_{0j} q_j^k$$

The above results can be generalized as follows.

Theorem 5.2: For previously defined plants P and P' and input $u(k)$ defined as

$$u(k) = \begin{bmatrix} \alpha_1 g_1 q_1^k & \alpha_2 g_2 q_2^k & \ldots & \alpha_j g_j q_j^k \ldots & \alpha_m g_m q_m^k \end{bmatrix}^T \qquad (5.21)$$

where α_j is a scalar, the state vector for both the plants is given by

$$x(k) = A^k \left(x(0) - \sum_{j=1}^{m} \alpha_j x_{0j} \right) + \sum_{j=1}^{m} \alpha_j q_j^k x_{0j} \qquad (5.22)$$

The output of the plant P' is given by

$$y'(k) = vCA^k\left(x(0) - \sum_{j=1}^{m}\alpha_j x_{0j}\right) \tag{5.23}$$

and the output to the plant P is given by

$$y(k) = CA^k\left(x(0) - \sum_{j=1}^{m}\alpha_j x_{0j}\right) + C\sum_{j=1}^{m}\alpha_j x_{0j} q_j^k \tag{5.24}$$

where $x(0)$ is the initial state vector for both the plants P and P' since the state vector for both P and P' is same for all k (change in the output matrix has no effect on the state variables).

Proof: The proof is similar to the proof of the previous theorem. The generalized solution for state vector for P and P' is given by

$$x(k) = A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1} Bu(l) \tag{5.25}$$

Substituting for $u(l)$ we get

$$\begin{aligned} x(k) &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\alpha_1 b_1 g_1 q_1^l + \alpha_2 b_2 g_2 q_2^l + ... + \alpha_m b_m g_m q_m^l\right) \\ &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\sum_{j=1}^{m}\alpha_j b_j g_j q_j^l\right) \end{aligned} \tag{5.26}$$

For the j^{th} input channel we have the following relations from (5.9)

$$(q_j I - A)x_{0j} = b_j g_j \tag{5.27}$$

$$vCx_{0j} = 0 \tag{5.28}$$

Substituting (5.27) in (5.26) we get

$$
\begin{aligned}
x(k) &= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\sum_{j=1}^{m} \alpha_j b_j g_j q_j^l\right) \\
&= A^k x(0) + \sum_{l=0}^{k-1} A^{k-l-1}\left(\sum_{j=1}^{m} \alpha_j \left(q_j I - A\right) x_{0j} q_j^l\right) \\
&= A^k x(0) + \underbrace{\sum_{l=0}^{k-1} A^{k-l-1} \sum_{j=1}^{m} \alpha_j q_j^{l+1} x_{0j}}_{I} - \sum_{l=0}^{k-1} A^{k-l} \sum_{j=1}^{m} \alpha_j q_j^l x_{0j}
\end{aligned}
$$

In I by doing change of variable $(l+1) \rightarrow l$ we get

$$
\begin{aligned}
x(k) &= A^k x(0) + \sum_{l=1}^{k} A^{k-l} \sum_{j=1}^{m} \alpha_j q_j^l x_{0j} - \sum_{l=0}^{k-1} A^{k-l} \sum_{j=1}^{m} \alpha_j q_j^l x_{0j} \\
&= A^k x(0) + \sum_{j=1}^{m} \alpha_j q_j^k x_{0j} - A^k \sum_{j=1}^{m} \alpha_j x_{0j} + \left(\sum_{l=1}^{k-1} A^{k-l} \sum_{j=1}^{m} \alpha_j q_j^l x_{0j} - \sum_{l=1}^{k-1} A^{k-l} \sum_{j=1}^{m} \alpha_j q_j^l x_{0j}\right) \\
&= A^k \left(x(0) - \sum_{j=1}^{m} \alpha_j x_{0j} \right) + \sum_{j=1}^{m} \alpha_j q_j^k x_{0j}
\end{aligned}
\tag{5.29}
$$

Now,

$$
y'(k) = vCx(k) \tag{5.30}
$$

Substituting (5.28) and (5.29) in (5.30) we get

$$
\begin{aligned}
y'(k) &= vCA^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right) + vC \sum_{j=1}^{m} q_j^k x_{0j} \\
&= vCA^k \left(x(0) - \sum_{j=1}^{m} x_{0j} \right)
\end{aligned}
$$

Now output to the plant P is given by

$$
y(k) = Cx(k) \tag{5.31}
$$

Substituting (5.29) in (5.31) we get

$$
y(k) = CA^k \left(x(0) - \sum_{j=1}^{m} \alpha_j x_{0j} \right) + C \sum_{j=1}^{m} \alpha_j x_{0j} q_j^k
$$

Lemma 5.1: In the results of Theorem 5.1 and Theorem 5.2 if we substitute $x(0)=\sum_{j=1}^{m} x_{0j}$ and $x(0)=\sum_{j=1}^{m} \alpha_j x_{0j}$ respectively, in both the cases we get $y'(k)=0$ for all $k \geq 0$. It should be noted that even though the output of plant P is non-zero yet the output of the plant P' is zero for the above initial condition. In other words even though the components of the output of the plant P are non-zero yet their combination in the output zero direction of P is zero. This useful result will be used to obtain the combination of outputs of the original plant P in its output zero direction equal to zero.

Remark 5.1: Let $\mathbf{U} \in \mathbf{R}^m, \mathbf{X} \in \mathbf{R}^n, \mathbf{Y} \in \mathbf{R}^r$ be the input vector space, state vector space and the output vector space for the plant P respectively then

$$\mathbf{U} = span\left(\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\\vdots\\0\end{bmatrix} \quad \cdots \quad \begin{bmatrix}0\\0\\\vdots\\1\end{bmatrix}\right)$$

and $\mathbf{X} = span\begin{pmatrix}x_{01} & x_{02} & \cdots & x_{0m}\end{pmatrix}$ for $x(0)=\sum_{j=1}^{m} \alpha_j x_{0j}$.

Thus the relationship between the input space, state space and the output space for the zeroing of the output combination of plant P in the output zero direction of plant P can be shown by the geometrical relationships in Figure 5.1

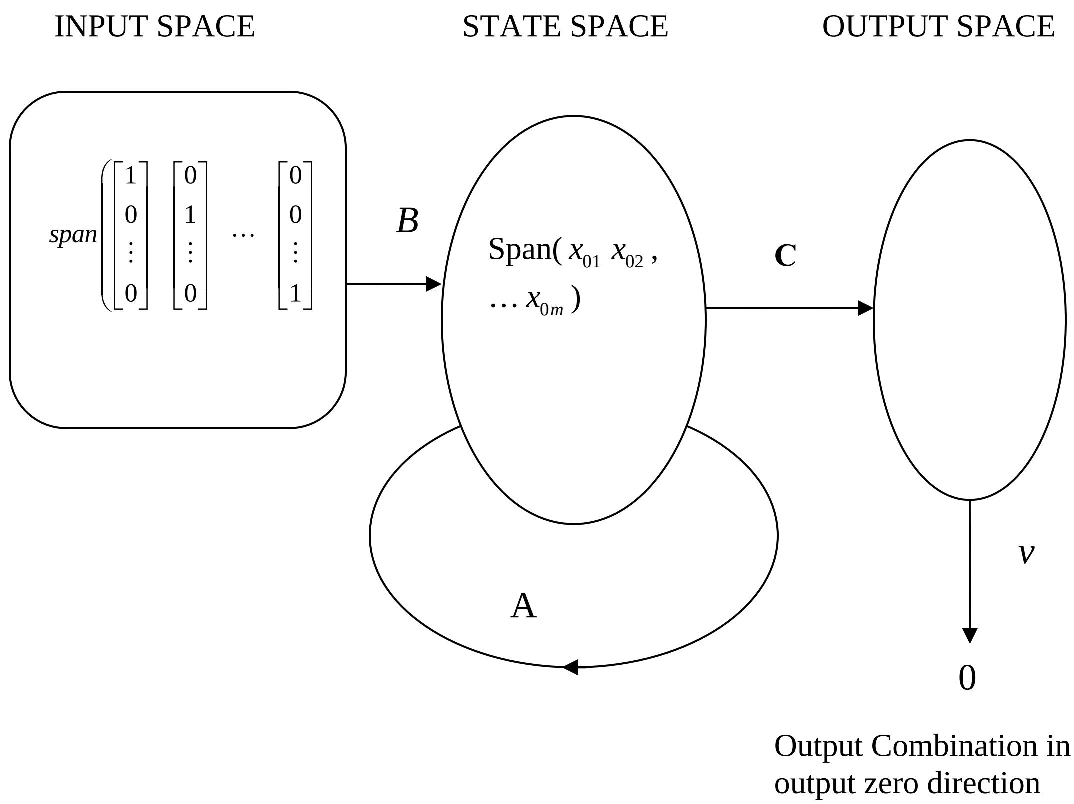

Figure 5.1: Geometrical relationships between input, output and state spaces of discrete plant P for the zeroing of output combination in output zero direction

Using the Lemma 5.1 an algorithm to obtain a set of input signals and the corresponding initial state vector such that the combinations of output components of the discrete time plant P in the output zero direction of plant P is always zero, is presented below. The steps are as follows:

Step 1: Find the transmission zero, input zero direction, output zero direction and state zero vector of the plant P using (5.2), (5.3) and (5.4).

Step 2 : If b_j is the jth column of the B matrix then find the transmission zero q_j, input zero direction g_j and state zero vector using x_{0j} corresponding to jth input channel using (5.8) and (5.9).

Step 3: Set the initial condition of the plant P as follows

$$x(0)=\sum_{j=1}^{m}\alpha_j x_{0j}$$

Step 4: Use $u(k)$ defined by (5.21) as the input to the plant P.

Remark 5.2: Theorem 5.2 helps us to upscale or downscale the input values for each input channel. Thus even though the $g_j q_j^k$ may not lie in normal range of u_j yet by careful selection of α_j we can bring it into the normal range of u_j.

CHAPTER VI

USE OF OUTPUT ZEROING THEOREM FOR DISCRETE TIME SYSTEM FOR FAULT DETECTION

In Chapter V it was shown that it is possible to make the combination of outputs of discrete time systems in the output zero direction equal to zero for all $k = 0,1,2...$ for some special class of inputs. In the present chapter the results derived in the previous chapter and the output zeroing result of Tokarzewski [6] will be used for the fault detection in linear continuous time MIMO plants.

6.1 Novel Fault Detection Scheme for Discrete Time Systems

Based on Theorem 5.1, Theorem 5.2 and Lemma 5.1 below is a test to find the faulty column of the transfer function matrix $G(z)$ of plant P.

6.1.1 Column Test

If the input to the plant P and its initial conditions are given by $u(k) = \begin{bmatrix} 0 & ... & 0 & g_j q_j^k & .... & 0 \end{bmatrix}$ and $x(0) = x_{0j}$ then the combination of the outputs in the output zero direction should be zero. A non-zero value indicates that the elements of the plant transfer function matrix corresponding to the jth input (i.e. the jth column of $G(z)$) channel has changed.

Based on the output zeroing result of Tokarzewski [6] stated before the following Lemma can be stated.

Lemma 6.1: Let q, x_0 and g be the transmission zero, state zero vector and the input zero direction of the plant respectively. Then for input $u(k) = gq^k$ for all $k \geq 0$ and initial condition $x(0) = x_0$ the non-zero value of the ith output indicates that the ith row of the transfer function matrix is faulty.

Proof: For the given input and initial condition all the outputs should be identically zero according to Tokarzewski [6]. Since the ith output depends only on the ith row of $G(z)$ therefore the non-zero ith output indicates faulty ith row of $G(z)$.

Using Lemma 6.1 we get the following test for finding the faulty rows of the plant transfer function matrix of plant P.

6.1.2 Row Test

For input $u(k) = gq^k$ and initial condition $x(0) = x_0$ for the plant P the non-zero value of the ith output indicates that the ith row of the transfer function matrix is faulty.

Using the row test and the column test in conjunction on the plant transfer function matrix $G(z)$ we can pin-point the faulty element of the plant transfer function matrix. Suppose using the row test we find that the ith row of $G(z)$ is faulty and using the column test we find that the jth column of $G(z)$ has faults then we have the scenario as show in Figure 6.1

$$\begin{array}{cl} \begin{bmatrix} & \vdots & \\ \cdots\cdots\cdots\cdots & \boxed{g_{ij}} & \cdots\cdots \\ & \vdots & \end{bmatrix} & \text{i-th faulty row} \\ j\text{-th faulty column} & \end{array}$$

Figure 6.1: Faulty ith row and faulty jth column

Thus if we have only one faulty row and only one faulty column then we can easily deduce that only one element of the plant transfer function matrix is faulty. Thus if the ith row and jth column are faulty then we can easily deduce that the g_{ij} element of plant transfer function matrix is faulty.

6.2 An Illustrative Example

The above results are now illustrated using a discrete time model of the quadruple tank system discussed in the Chapter IV. The discrete time model was obtained from the continuous time model using time step $Ts = 0.1$ second. We have the following system matrices after discretization.

$$A = \begin{bmatrix} 0.0984 & 0 & 0.0043 & 0 \\ 0 & 0.0989 & 0 & 0.0033 \\ 0 & 0 & 0.0957 & 0 \\ 0 & 0 & 0 & 0.0967 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0083 & 0 \\ 0 & 0.0063 \\ 0 & 0.0048 \\ 0.0031 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix}$$

6.2.1 Verification of Theorem 5.1

Now the transmission zeros corresponding to the first and second input channels found using (5.8) are $q_1 = 0.0986$ and $q_2 = 0.0967$. Using (5.9) the corresponding input directions and state zero vectors are given by

$g_1 = [-0.0220]$ $g_2 = [0.0675]$ $x_{01} = [-0.9178 \quad 0.3947 \quad -0.000 \quad -0.0358]^T$

$x_{02} = [-0.8042 \quad 0.3458 \quad 0.3182 \quad -0.3577]^T$

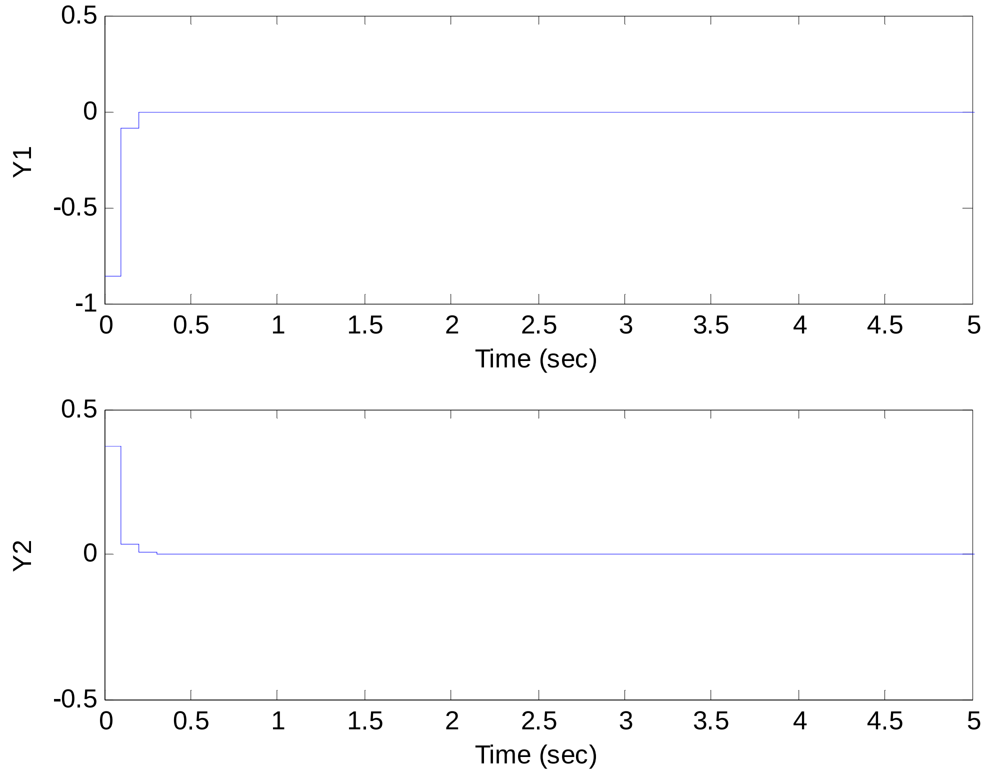

Figure 6.2: Plant outputs

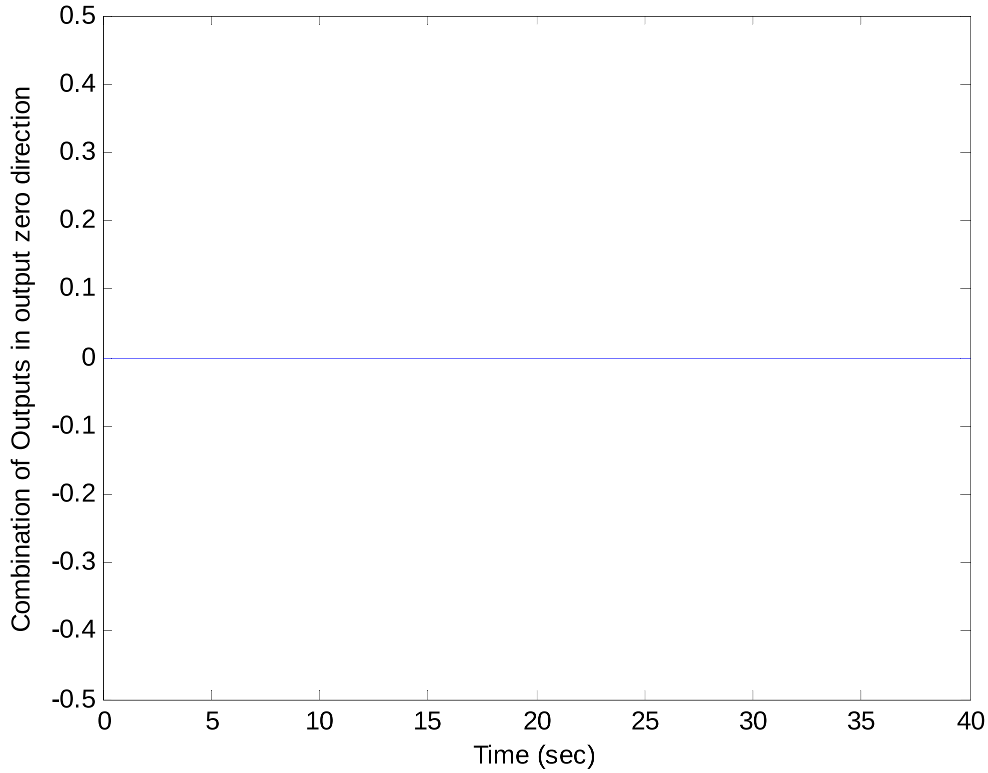

Figure 6.3: Plot of combination of outputs of discrete time plant

For $u(k) = \left[-0.0220(0.0986)^k \quad 0.0675(0.0967)^k\right]^T$ and initial condition $x(0) = x_{01} + x_{02}$ we get the outputs as shown in Figure 6.2. The results of Theorem 2.1 are verified by plots of Figure 6.3. The plant transfer function matrix of the plant is given by

$$G(z) = \begin{bmatrix} \dfrac{2.58}{620z-61} & \dfrac{0.74}{(620z-61)(115z-11)} \\ \dfrac{1.40}{(300z-29)(900z-89)} & \dfrac{2.82}{(900z-89)} \end{bmatrix} \tag{6.1}$$

Now let us introduce some faults in the second column of $G(z)$ by changing the second column of the B matrix. Note that the changes to $G(z)$ can be made by changing either A, B or C matrix however changing second column of B only changes the second column of $G(z)$. Let the new B matrix be given as

$$B = \begin{bmatrix} 0.0083 & 0.5 \\ 0 & 0.0062 \\ 0 & 0.0047 \\ 0.0031 & 0 \end{bmatrix}$$

A and C matrices remain same. Then the new transfer function matrix is given by

$$G(z) = \begin{bmatrix} \dfrac{2.58}{620z-61} & \dfrac{17825z-1704.3}{(620z-61)(115z-11)} \\ \dfrac{1.40}{(300z-29)(900z-89)} & \dfrac{2.82}{(900z-89)} \end{bmatrix} \tag{6.2}$$

It can be noticed that $(1,2)$ element of $G(z)$ has changed.

6.2.2 Column Test

We will use the column test to identify the faulty column of $G(z)$. Now the transmission zeros corresponding to the first and second input channels found using (5.8) are $q_1 = 0.0986$ and $q_2 = 0.0967$. Using (5.9) the corresponding input directions and state zero vectors are given by

$g_1 = [-0.0220]$ $g_2 = [0.0675]$ $x_{01} = [-0.9178 \quad 0.3947 \quad -0.000 \quad -0.0358]^T$

$x_{02} = [-0.8042 \quad 0.3458 \quad 0.3182 \quad -0.3577]^T$

For input signal $u(k) = \begin{bmatrix} 0 & 0.0675(0.0967)^k \end{bmatrix}^T$ and the initial condition $x_{02} = [-0.8042 \quad 0.3458 \quad 0.3182 \quad -0.3577]^T$ the combination of outputs in output zero direction is shown in Figure 6.4

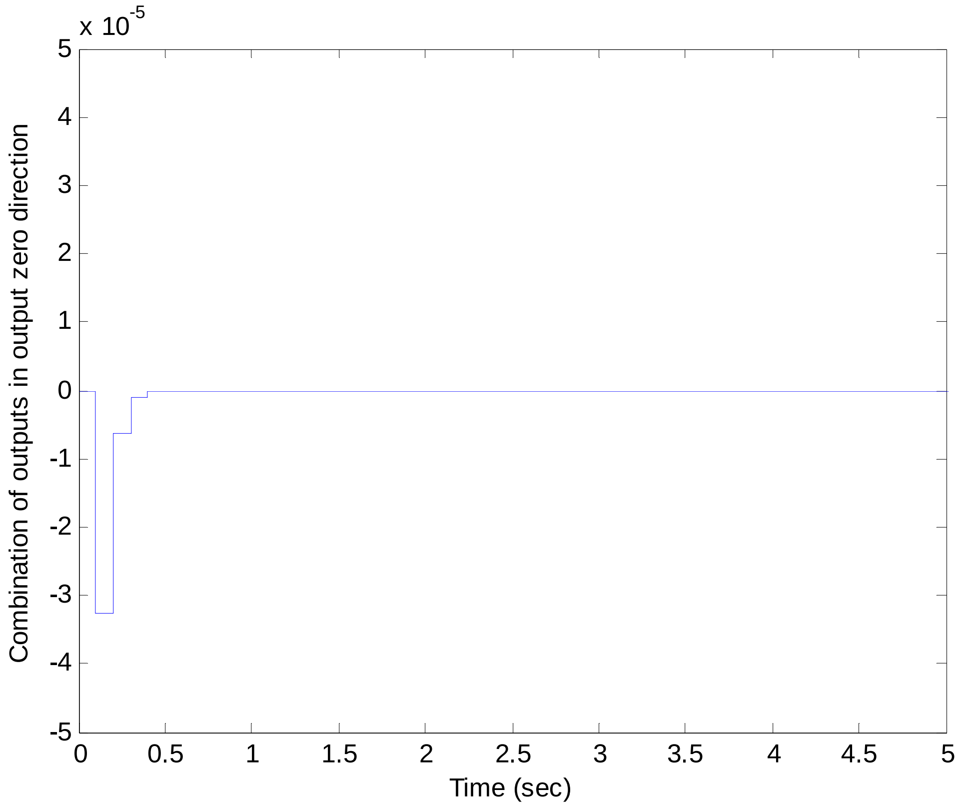

Figure 6.4: Combination of outputs in the output zero direction for the second column of the discrete time system

Using the column test and Figure 6.4 we conclude that the fault lies in the second column of the transfer function matrix. This result is verified by looking at the changed $G(s)$. By following a similar procedure for the first input channel it is concluded that there is no fault in the first column of the $G(s)$. For this case the combination of outputs in output zero direction is shown in Figure 6.5

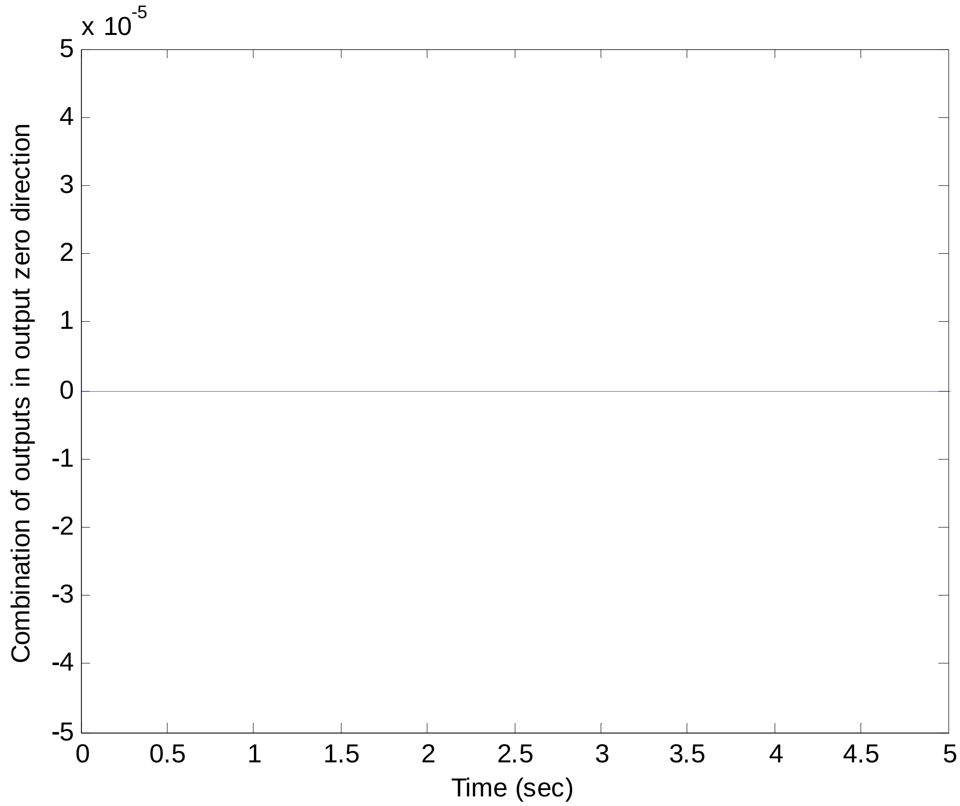

Figure 6.5: The combination of outputs in output zero direction for the first column of the discrete time system

Thus we conclude that the fault lies only in the second column of the plant transfer function matrix.

6.2.3 Row Test

Now for our system the transmission zero, state zero vector and the input zero direction are as follows

$z = 0.0941$

$x_0 = \begin{bmatrix} 0 & -0.000 & -0.7506 & -0.4699 \end{bmatrix}^T$

$g = \begin{bmatrix} 0.3920 & 0.2494 \end{bmatrix}^T$

For $u(k) = \begin{bmatrix} 0.3920 & 0.2494 \end{bmatrix}^T (0.0941)^k$ and $x(0) = \begin{bmatrix} 0 & -0.000 & -0.7506 & -0.4699 \end{bmatrix}^T$ the outputs are shown in Figure 6.6

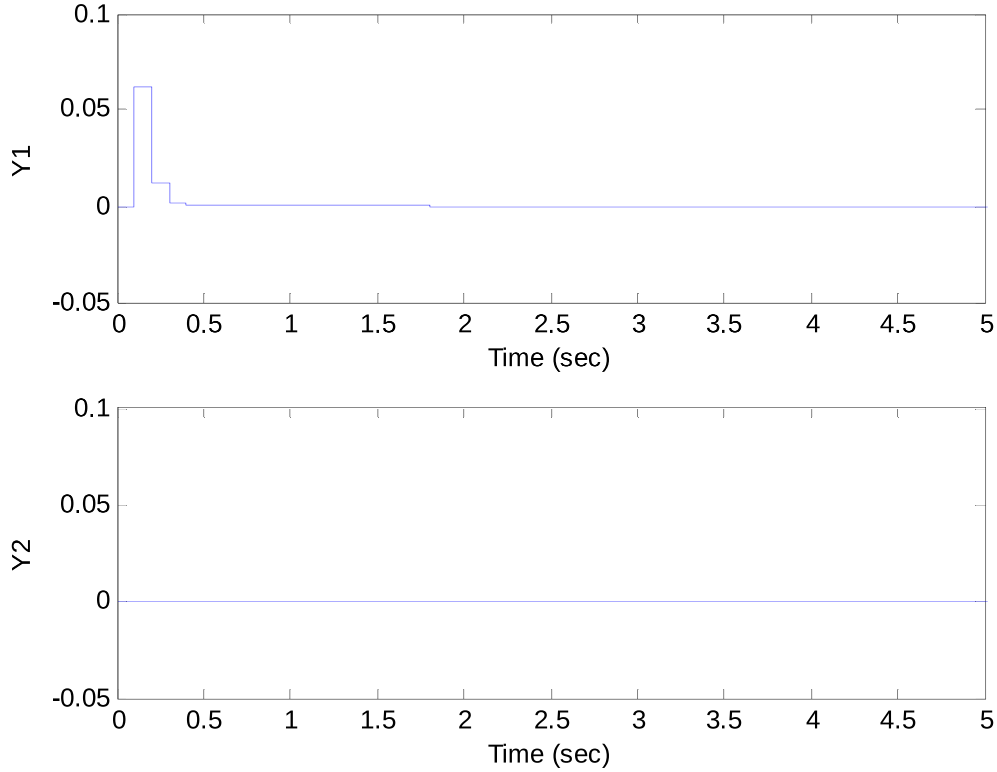

Figure 6.6: Outputs of the discrete time plant for an output zeroing input

From Figure 6.6 it is clear that the first row (using the row test) of the plant transfer function matrix is faulty.

Since in this case the there is only one faulty row and one faulty column we can straightaway conclude that the fault lies in $(1,2)$ element of plant transfer function matrix. This matches with the result obtained by comparing the transfer function matrices given in . (6.1) and (6.2).

6.3 Steady State Analysis

From the Final Value Theorem we have $\lim_{k\to\infty} y(k) = \lim_{z\to 1}\left(1-z^{-1}\right)Y(z)$ where $Y(z)$ the z-transform of stable is $y(k)$. Thus for an input of the form $u(k) = \begin{bmatrix}\alpha_1 & \alpha_2 & \ldots & \alpha_m\end{bmatrix}^T 1(k)$, where $1(k)$ denotes a unit step function, the steady state output is given by

$$y_{ss} = G(0)\begin{bmatrix}\alpha_1 & \alpha_2 & \ldots & \alpha_m\end{bmatrix}^T \tag{6.3}$$

Lemma 6.2: If $u(k) = \begin{bmatrix}0 & \ldots & \alpha_j & \ldots 0\end{bmatrix}^T 1(k)$ then the ith steady state output is given by $y_{ss,i} = G_{ij}(0)\alpha_j$ where $G_{ij}(0)$ is the (i, j) element of $G(0)$. Thus if actual ith steady state output is different from $G_{ij}(0)\alpha_j$ then it can be concluded that the (i, j) element of the plant transition matrix $G(z)$ is faulty.

Using the transmission zeros and the zero directions of the quadruple-tank system we concluded that G_{12} is faulty. Now since our plant has all the poles inside the unit circle we can corroborate our previous conclusion using steady state analysis.

Let the input be $u(k) = \begin{bmatrix}1 & 0\end{bmatrix}^T 1(k)$ then for the defective plant we get the output plot as shown in Figure 6.7.

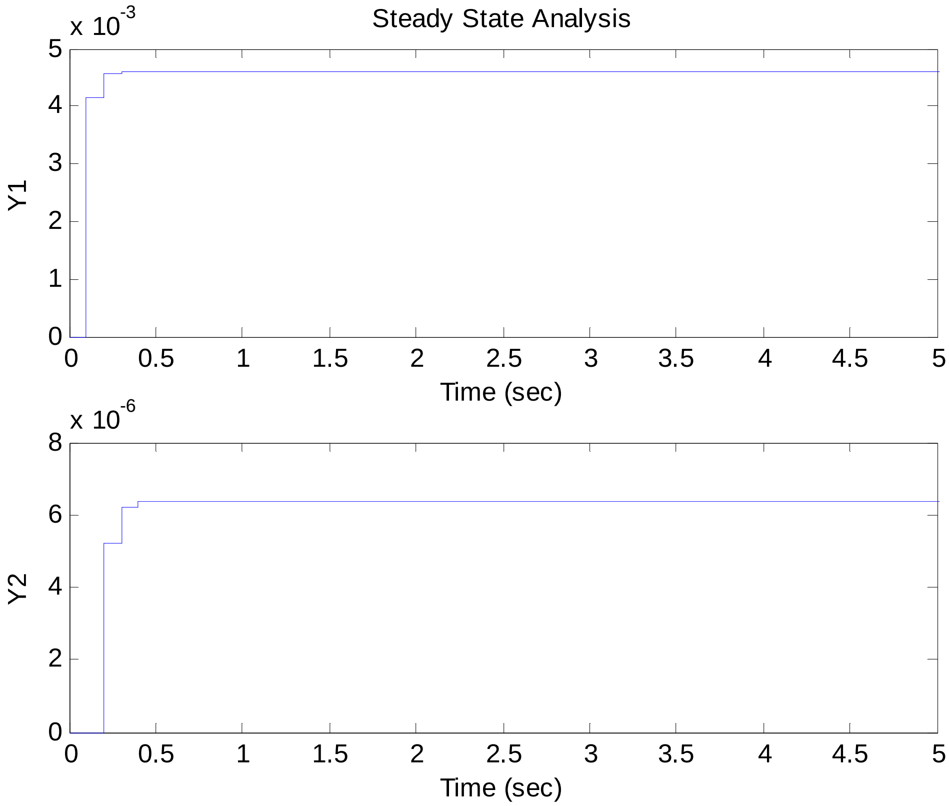

Figure 6.7: Steady state analysis of the first column of the discrete time system

Similarly for $u(k)=\begin{bmatrix}0 & 1\end{bmatrix}^T 1(k)$ we get Figure 6.8 for steady state output of the plant.

$$G_{ideal}(0)=\begin{bmatrix}-0.0423 & 0.0011\\ 0.0005 & -0.0317\end{bmatrix} \tag{6.4}$$

From Figure 6.7 and Lemma 6.2 we conclude that both G_{11} and G_{21} have not changed. From Figure 6.8 we conclude that G_{12} is faulty whereas G_{22} has no faults. This conclusion is the same as the one arrived using transmission zeros and zero directions.

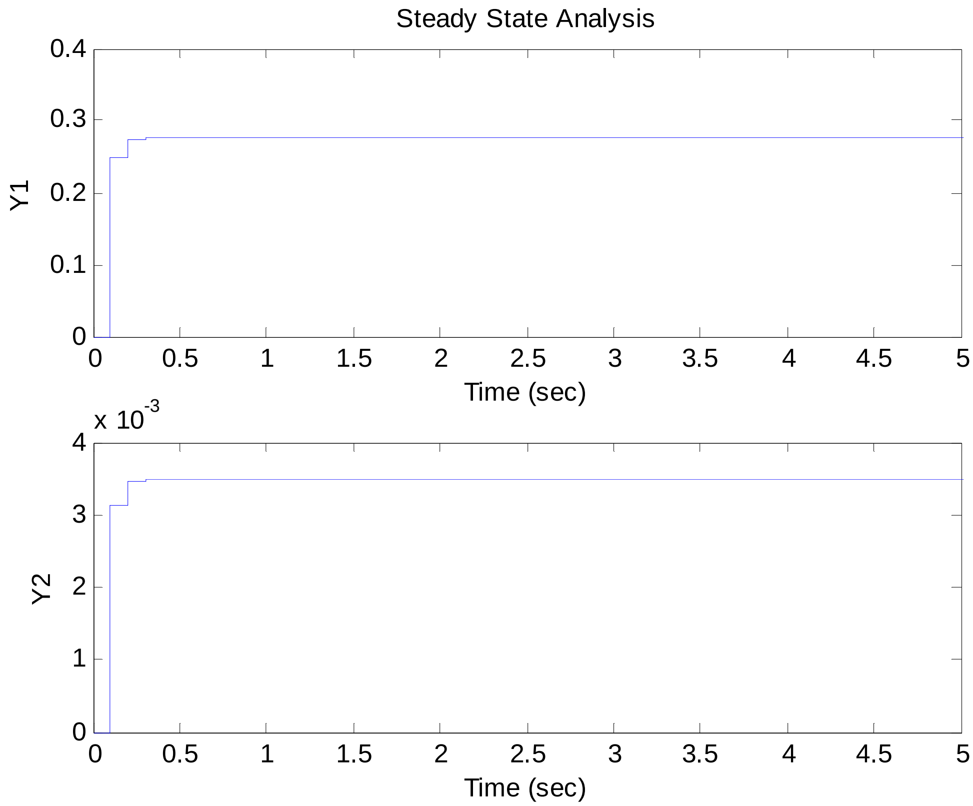

Figure 6.8: Steady state analysis of the second column of the discrete time system

6.4 Extension of Fault Detection Results to System with Multiple Faulty Rows and Columns

The results obtained for finding the faulty element of transfer function matrix with the help of row test and the column test can be extended to transfer function matrices with multiple faulty rows and columns. The method is very similar to the method described for the continuous time system described in Chapter IV.

6.5 Tests for Diagnosing Faults in A and C Matrices

In the earlier section we gave a detailed discussion on how to locate the faulty elements of the plant transfer function matrix. A fault in $G(z)$ indicates that there is fault in some of the system matrices (A and C) but still we cannot say using the tests described in the previous section which of the system matrices are faulty. We present here a set of three tests- one each for A and C, to find the faulty system matrices of a discrete time system.

6.5.1 Test for A Matrix

Let the representation of plant P given by (2.1) be minimal. Let $\lambda \in \rho(A)$. Then there exists an eigenvector $x_{eig} \in \mathbf{C}^n$ such that $(\lambda I - A)x_{eig} = 0$. Now if the input $u(k) \equiv 0$ for $k \geq 0$ and initial condition is $x(0) = x_{eig}$ then the state vector is given by $x(k) = x_{eig}\lambda^k$. Let h be a vector orthogonal to x_{eig}. It can be easily seen that the combination of states in the direction of h is always zero. Assuming that all the states are measurable we can detect an occurrence of fault in the A matrix by combining the measured states in the direction of h and noting whether the combination is zero or not.

6.5.2 Test for the C Matrix

Suppose that all the states are measurable. The fault in the $\mathrm{p^{th}}$ row of C matrix, c_p can be found by comparing the $\mathrm{p^{th}}$ component of the output vector and the quantity $c_p x$ where x is the measured state vector. A difference in the values shows the presence of fault in the $\mathrm{p^{th}}$ row of C.

CHAPTER VII

SUMMARY AND FUTURE WORK

Multivariable plants are different from single variable plants in that they have directional properties. In other words the MIMO systems behave differently for different direction of inputs. Similarly the output measurements are different in different output direction. A novel online fault detection scheme for linear systems using multivariable zeros and zero directions (input and output) was presented. The scheme is a model based online fault diagnosis scheme. We could locate the faulty elements of the plant transfer function matrix using the row test and column test. The linearity of the system is a precondition for the applicability of this scheme. The plant may have more than one faulty element. The scheme was illustrated on a quadruple-tank system.

Recently fault detection and isolation of non-linear system have generated a lot of interest. [10] discusses an observer-based fault detection and isolation for nonlinear systems. Garcia and Frank, (1997) [11] used observer based FDI for nonlinear system. Hammouri *et al*. [12] extended the geometric approach FDI to nonlinear systems. Though the present work deals with only linear systems there is scope of extending this work to the non-linear systems. Recently some work on non-linear zeros has been done. There is a possibility of using the properties of non-linear zeros to detect and isolate faults present in a non-linear plant.

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APPENDIX

Continuous Time Simulation Code for MATLAB

```
clc;
clear;
close all;
T1=62;
T2=90;
T3=23;
T4=30;
A1=28;
A2=32;
A3=28;
A4=32;
y1=0.70;
y2=0.60;
k1=3.33;
k2=3.35;
kc=0.50;


A=[-1/T1    0      A3/(A1*T3)   0
   0      -1/T2     0          A4/(A2*T4)
   0       0       -1/T3        0
   0       0         0          -1/T4 ];
eig(A);


B=[ (y1*k1)/A1    0
   0        (y2*k2)/A2
   0         (1-y2)*k2/A3
   (1-y1)*k1/A4    0];

C=[kc  0    0    0
   0  kc   0    0 ];

sys=ss(A,B,C,0);
% sys=tf(sys)
z=zero(sys);
a=z(1);

P=[a*eye(4)-A     -B
   C             zeros(2,2)];
```

```
X=null(P);

X0=[X(1)
   X(2)
   X(3)
   X(4)];

g=[ X(5)
   X(6)];

t=0:0.01:4;

for i=1:401
   U(i,1)=g(1)*exp(a*t(i));
   U(i,2)=g(2)*exp(a*t(i));
end
%lsim(sys,U,t,X0)
%This verifies the MacFarlane and Karcanias theorem

% Lets find the output zero direction
X1=null(P');

X01=[X1(1)
    X1(2)
    X1(3)
    X1(4)];

V=[ X1(5) X1(6)];  % This is the output zero direction



% Now lets find the zero corresponding to the first input
B1=B(:,1);
sys1=ss(A,B1,V*C,0);
z1=zero(sys1);
a1=z1(1);

% Now lets find the input zero direction and state zero direction
%corresponding to the first input
P1=[a1*eye(4)-A     -B1
   V*C            zeros(1,1)];
Xa=null(P1);

Xoa=[Xa(1)
    Xa(2)
    Xa(3)
```

```
     Xa(4)];

ga=[Xa(5)];

t=0:0.01:4;

for i=1:401
    U1(i)=ga*exp(a1*t(i));

end
%figure
% lsim(sys1,U1,t,Xoa)
% This plot verifies Theorem 2.1 for the first input

% Now lets find the zero corresponding to the second input
B2=B(:,2);
sys2=ss(A,B2,V*C,0);
z2=zero(sys2);
a2=z2(1)

%Now lets find the input zero direction and state zero direction
%corresponding to the second input

P2=[a2*eye(4)-A     -B2
    V*C             zeros(1,1)];
Xb=null(P2);

Xob=[Xb(1)
     Xb(2)
     Xb(3)
     Xb(4)]
 gb=[Xb(5)]

t=0:0.01:4;

for i=1:401
    U2(i)=gb*exp(a2*t(i));

end

%figure
% lsim(sys2,U2,t,Xob)
% This plot verifies Theorem 2.1 for the second input

U3=[U1' U2'];
Xo3=Xoa + Xob;
```

```
sys3=ss(A,B,V*C,0);

% figure
% [y3,t3,x3]=lsim(sys,U3,t,Xo3);
%  The plot verifies Theorem 2.1 the for 2 inputs

% [y4,t4,x4]=lsim(sys3,U3,t,Xo3);
%
% subplot(2,1,1)
% plot(t3,y3(:,1),t3,y3(:,2))
% subplot(2,1,2)
% plot(t4,y4)


B=[ (y1*k1)/A1    0
    0       (y2*k2)/A2
    0        (1-y2)*k2/A3
    (1-y1)*k1/A4    0]


% Lets introduce some fault in G(s) by changing the B matrix


delB=[0 0.5
    0 0
    0 0
    0 0];

Bprime= B+ delB;


% Applying the COLUMN TEST
figure
sys5=ss(A,Bprime,V*C,0);

for i=1:401
    U5(i,1)=ga*exp(a1*t(i));
    U5(i,2)=0;
end
[y5,t5,x5]=lsim(sys5,U5,t,Xoa);
plot(t5,y5)


% Applying the ROW TEST
figure
sys6=ss(A,Bprime,C,0)
```

```
[y6,t6,x6]=lsim(sys6,U,t,X0);
subplot(2,1,1)
plot(t6,y6(:,1))
subplot(2,1,2)
plot(t6,y6(:,2))
```

Discrete Time Simulation Code for MATLAB

```
clc;
clear;
close all;
T1=62;
T2=90;
T3=23;
T4=30;
A1=28;
A2=32;
A3=28;
A4=32;
y1=0.70;
y2=0.60;
k1=3.33;
k2=3.35;
kc=0.50;


A=[-1/T1    0      A3/(A1*T3)   0
    0      -1/T2     0          A4/(A2*T4)
    0       0       -1/T3        0
    0       0         0          -1/T4 ];

B=[ (y1*k1)/A1    0
    0        (y2*k2)/A2
    0         (1-y2)*k2/A3
    (1-y1)*k1/A4    0];

C=[kc  0    0    0
    0  kc   0    0 ];

Ts = 0.1;

A = Ts*(A+eye(4));
eig(A)

B = Ts*B;

C = C;

sys = ss(A,B,C,0,Ts);

z = zero(sys);
```

```
a=z(1);

P=[a*eye(4)-A     -B
   C            zeros(2,2)];

X=null(P);

X0=[X(1)
   X(2)
   X(3)
   X(4)]

g=[ X(5)
   X(6)]

t=0:0.01:4;

for i=1:401
   U(i,1)=g(1)*a^(i-1);
   U(i,2)=g(2)*a^(i-1);
end
%lsim(sys,U,[],X0)
% The is above plot verifies the Tokarjewski Theorem

% Finding the Output Zero direction of the plant
X1=null(P');

X01=[X1(1)
    X1(2)
    X1(3)
    X1(4)];

V=[ X1(5) X1(6)]  % This is the output zero direction


B1=B(:,1);
sys1=ss(A,B1,V*C,0,Ts);
z1=zero(sys1);
a1=z1(2);

% Finding the input zero direction and state zero direction corresponding to
% to the first input
P1=[a1*eye(4)-A     -B1
   V*C            zeros(1,1)];
Xa=null(P1);
```

```
Xoa=[Xa(1)
     Xa(2)
     Xa(3)
     Xa(4)];
 ga=[Xa(5)];

 for i=1:401
    U1(i)=ga*a1^(i-1);
end
%lsim(sys1,U1,[],Xoa,Ts)
% The plot verifies Theorem 5.1 for the first input

%Finding the zero corresponding to the second input
B2=B(:,2);
sys2=ss(A,B2,V*C,0,Ts);
z2=zero(sys2);
a2=z2(1);

%Finding the input zero direction and state zero direction corresponding to
% to the second input
P2=[a2*eye(4)-A     -B2
    V*C             zeros(1,1)];
Xb=null(P2);

Xob=[Xb(1)
     Xb(2)
     Xb(3)
     Xb(4)];

gb=[Xb(5)];

 for i=1:401
    U2(i)=gb*a2^(i-1);
 end

 %lsim(sys2,U2,[],Xob,Ts)
 % The above plot verifies Theorem 5.1 for the second input

U3=[U1' U2'];
Xo3=Xoa + Xob;
sys3=ss(A,B,V*C,0,Ts);

% %lsim(sys3,U3,[],Xo3,Ts);
% [y3,t3,x3]=lsim(sys,U3,[],Xo3,Ts);
% The above plot verifies Theorem 5.1 for 2 inputs
```

```
% [y4,t4,x4]=lsim(sys3,U3,[],Xo3,Ts);
%  figure
% subplot(2,1,1)
% stairs(t3,y3(:,1));
% subplot(2,1,2)
% stairs(t3,y3(:,2))
%
% figure
%
% stairs(t4,y4)

% Lets introduce some fault in G(z) by changing the B matrix
delB=[0 0.5
   0 0
   0 0
   0 0];

Bprime= B+ delB

% s=sym('s');
% H=C*inv(s*eye(4)-A)*Bprime

% % Applying the COLUMN TEST
% figure
% sys5=ss(A,Bprime,V*C,0,Ts);
%
% for i=1:401
%     U5(i,1)=ga*(a1)^(i-1);
%     U5(i,2)=0;
% end
% [y5,t5,x5]=lsim(sys5,U5,[],Xoa,Ts);
% stairs(t5,y5)

%
% % Applying the ROW TEST
% figure
% sys6=ss(A,Bprime,C,0,Ts)
% [y6,t6,x6]=lsim(sys6,U,[],X0,Ts);
% subplot(2,1,1)
% stairs(t6,y6(:,1))
% subplot(2,1,2)
% stairs(t6,y6(:,2))
%

% STEADY STATE ANALYSIS
```

```
sys7=ss(A,Bprime,C,0,Ts);

for i=1:401
    U7(i,1)=0;
    U7(i,2)=1;
end
X0=zeros(4,1);
[y7,t7,x7]=lsim(sys7,U7,[],X0,Ts);
subplot(2,1,1)
stairs(t7,y7(:,1));
subplot(2,1,2)
stairs(t7,y7(:,2));

H=C*inv(-A)*B
```

VITA

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