EMPIRICAL STUDY ON THE KOREAN TREASURY AUCTION FOCUSING ON THE REVENUE COMPARISON IN MULTIPLE VERSUS SINGLE PRICE AUCTION

A Dissertation

by

BOO-SUNG KANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2004

Major Subject: Economics

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Approved as to style and content by:

Steven Puller (Chair of Committee)	Qi Li (Member)
William S. Neilson (Member)	Kishore Gawande (Member)
	Leonardo Auernheimer (Head of Department)

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ABSTRACT

Empirical Study on the Korean Treasury Auction

Focusing on the Revenue Comparison

in Multiple versus Single Price Auction. (December 2004)

Boo-Sung Kang, B.A., Seoul National University;

M.A., Seoul National University

Chair of Advisory Committee: Dr. Steven Puller

This dissertation pursues to find an answer empirically to the question of the revenue ranking between the multiple price auction and the single price auction. I also attempt to get empirical clues in terms of the efficiency ranking between the two. Under the assumptions of symmetric bidders and private independent value (PIV), I derive the optimal bidding conditions for both auction formats. Following the structural model estimation approach, I estimate the underlying distribution of market clearing price using the nonparametric resampling strategy and recover the bidders' unknown true valuations corresponding to each observed bid point. With these estimated valuations of the bidders, I calculate what the upper bound of the revenue would have been under the Vickery auction to perform the counterfactual revenue comparison with the actual revenue. I find that, ex-post, the multiple price auction yields more revenue to the Korean Treasury than the alternative. I also investigate the efficiency ranking by comparing the number of bids switched and the amount of surplus change which would occur when the bidders are assumed to report their true valuations as their bids. I find that the multiple price auction is also superior to the alternative in efficiency which supports the current theoretical prediction. Finally, I investigate the robustness of my model and empirical results by relaxing the previous assumptions. I, first, extend the model and estimation to the case of asymmetric bidders where the bidders are divided into two groups based on their size. It shows that the model and estimation framework are still valid and that the empirical findings are very similar to the symmetric case. I also test for the presence of common value (CV) component in the bidders' valuation function. I propose the simple regression model adopting the idea of the policy experimental approach. I obtain quite an inconclusive result in general but find some evidence supporting PIV for relatively higher bid prices while supporting CV for lower bid prices.

DEDICATION

Glory be to the Father, and to the Son, and to the Holy Spirit

This dissertation is gratefully dedicated to:

my parents, Chan-Joong Kang and Sun-Hwa Lim;

my parents-in-law, Han-Chang Kim and Kyung-Hee Kim;

my wife, Mee-Jung;

my sons, Kyu-Nam and Kyu-Min;

and the memory of my sister, Chun-Hee Kang

ACKNOWLEDGMENTS

I would like to express my first and special appreciation to Dr. Steven Puller, my advisor, for his hearty encouragement, inspiring guidance, and precious suggestions throughout the works of this dissertation. I also would like to thank the members of my dissertation committee, Dr. William S. Neilson, Dr. Qi Li, and Dr. Kishore Gawande for their valuable comments and Dr. Hae-shin Hwang for his constant encouragement during my graduate years. My thanks are also extended to Dr. Ali Hortaçsu who is the author of my main reference and has encouraged me with many helpful and practical suggestions.

Finally, I would like to express my sincere gratitude to all those who have prayed for me.

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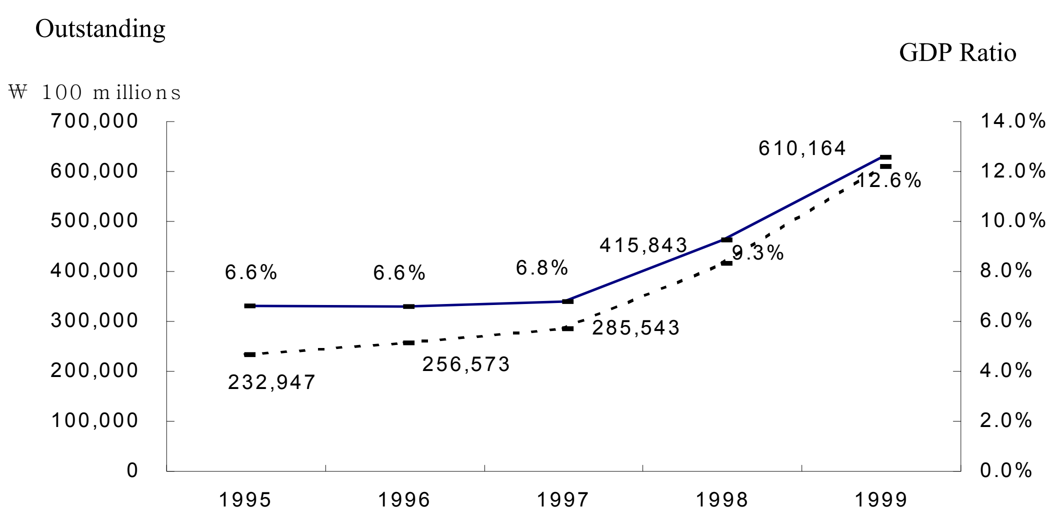
CHAPTER I

INTRODUCTION

Until the economic crisis that broke out in 1997 and the resulting slowdown in economic activities, the fiscal condition in Korea had been maintained fairly healthy under the operating principle of balanced budget. However, following the crisis, the government's fiscal condition took a dramatic downturn with sharp increases in budget deficits projected to last for many years. The volume of Treasury Bonds (TB) being issued was significantly increased to finance the budget deficit.



1) 40 trillion Korean Won(KW) is approximately 33 billion US $(exchage rate 1US$=1,200KW)

FIGURE 1. Domestic TB Outstanding & GDP Ratio

This dissertation follows the style and format of the *American Economic Review*.

Figure 1 shows the increase of TB outstanding and its ratio to GDP. Consequently with a dramatic increase in TB issuance, it became a crucial concern of the Korean government to raise the required level of funding in a cost effective manner. In order to reduce the funding cost, which also means maximization of revenue given a cost level, the Korean government has been implementing for the various policy improvements and institutional changes. One policy instrument is the market design of auctioning TB. In August 2000, the Korean government decided to alter the auction mechanism from the multiple price auction (discriminatory or "pay-as-bid" auction) to the single price auction (uniform price auction).

Figure 2 depicts the difference in the payment rules for the two auction formats. In the multiple price auction, Treasury acts as the perfectly discriminating monopolist by awarding the security to the highest competitive bidder and working down through the competitive bids until the entire amount is sold, which is similar to first degree price discrimination of monopolist. In this auction bidders may submit multiple price-quantity pairs as their bids, which trace out bid functions on the price-quantity plane. Treasury aggregates individual bid functions and finds where the aggregate bid function meets supply. The winning bidders pay the price bid for each unit. On the other hand, in the single price auction winning bids are determined in the same manner as in the multiple price auction, but every winning bidder pays the same price equal to the lowest winning bid (or market clearing price) rather than their bid price.

In making this conversion the Korean government might follow the conventional belief that under the single price auction revenue can increase by inducing more active or aggressive participation by the potential bidders.

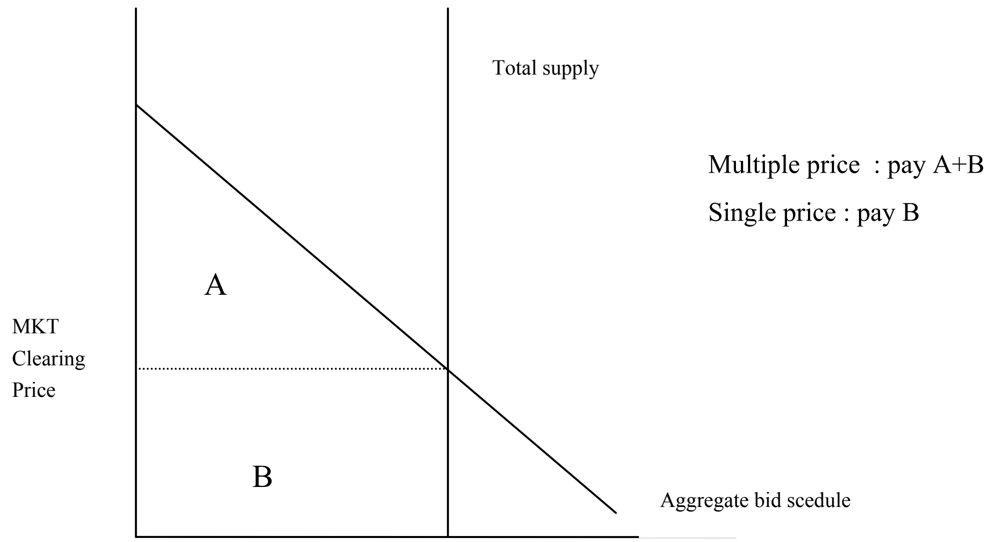


FIGURE 2. Pricing Rules of Two Different Auctions

The basis of this argument is the following: bidders will attempt to maximize their profit from the auction, which can be defined as the difference between the true(or marginal) value of the security and the actual bid, by shading down their bids in the auction relative to their assessment of the true value of the security. In the multiple price auction winning bidders have to pay their bids, the incentive for bid-shading tends to be larger than in the single price auction. This reasoning basically follows the standpoints of conventional theoretical view which was suggested by earlier researchers such as Milton Friedman (1960).

However, many earlier researchers tended to consider that the two mechanisms used in Treasury auction were analogies corresponding to 1st price auction and 2nd price auction in the single item auction respectively. Therefore they argued that single price auction could yield more revenue and more efficient outcomes than multiple price auction like 2nd price auction could do. [1]

This incorrect conjecture resulted from not considering the distinctive characteristics of the multi-unit auction. Treasury auction is a multi-unit or divisible good auction where bidders have demand for multiple units of the good. Bidders submit their bid function in the form of a demand schedule, and they can choose both a bid price and a demanded quantity at that price. Hence the quantity is also a choice variable and we have to consider the possibility that each bidder's quantity choices can affect the determination of the market clearing price. Therefore, the results of the single item auction cannot be applied to multi unit auction, and furthermore, when considering the common value, theoretical comparison becomes even more complicated and difficult.

[1] There are four common auction techniques to sell a single item, i.e., the first-price auction, the Dutch auction, the second-price auction, and the English auction The English (ascending, open, oral) auction is conducted by an auctioneer who calls successively higher prices until only one willing bidder remains, and the object is given to him at the last price which is called by auctioneer. The Dutch(descending) auction, which has been used to sell flowers for export in Holland, is conducted by an auctioneer who initially calls for a very high price and then continuously lowers the prices until some bidder stops the auction and claims the flowers for that price. The first-price auction is a sealed-bid auction in which the buyer making the highest bid claims the object and pays the amount he has bid. The second-price auction is a sealed-bid auction in which the buyer making the highest bid claims the object, but pays only the amount of the second highest bid.

Recently, the incorrect attempt to connect the single item auction and the multi unit auction described above has been criticized and the fallacy can be summarized by the following statement.

> Most public debate about the relative merits of these two alternatives has been confused by an imperfect analogy between single unit and multi unit auctions……They incorrectly posit that uniform auction inherits the same attractive truth-telling and efficiency attributes as 2nd price auction and infer that uniform auction generate greater expected seller revenue. [2]

Existing theory does not provide expected revenue ranking predictions for multi-unit auctions, and seems to conclude that the revenue ordering in the multiple unit auction is generally ambiguous between the multiple price and single price auction.

To increase understanding, I will explain the possible trade-off between the revenue from the two auction mechanisms graphically. That is, the revenue trade off depends on the amount of bid-shading each bidder decides to undertake. To explain this trade off, I will use the individual bid function and residual supply function for one particular bidder which can be calculated by subtracting the aggregate bid function of all other bidders from the total supply. The market clearing price is at the point where individual bid function intersects the residual supply function.

In Figure 3, in the multiple price auction, the bidder pays the area of OAE_MC, while in the single price auction the bidder pays the rectangle of OBE_SD so that the revenues from both auctions can be ranked by comparing the size of OAE_MC and

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[2] See Ausubel & Crampton(2002)

OBE_SD. In a rough sense, the auctioneer's revenue in single price auction is more sensitive to the marginal units, whereas it is more affected by infra marginal units in multiple price auction.

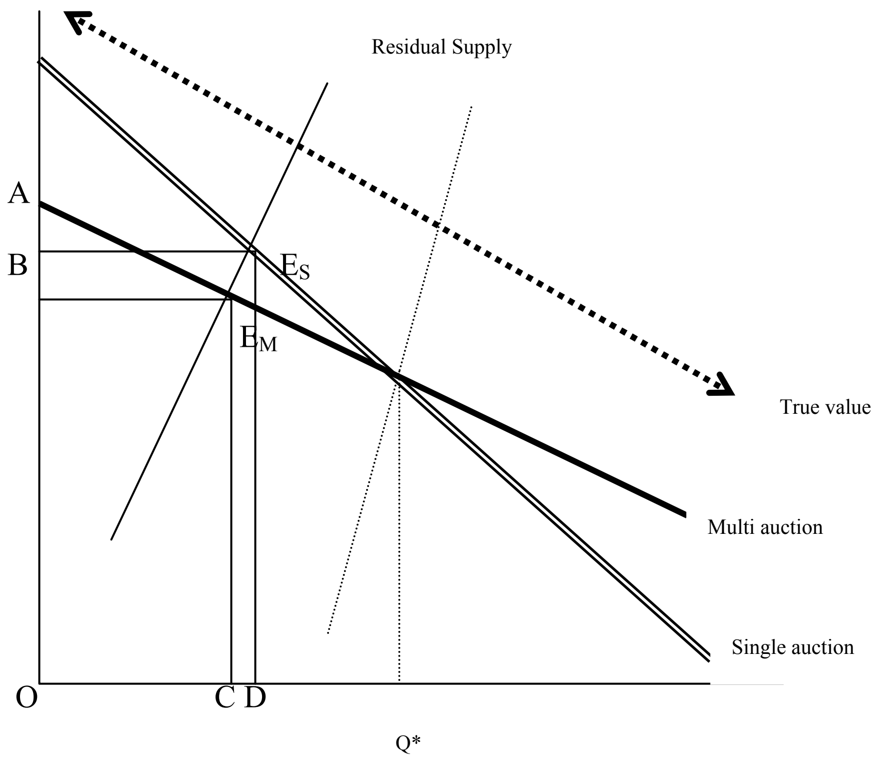

FIGURE 3. Revenue Comparison in Individual Bidder Level

Note that I depict the actual bid functions under both auction formats lying below the true valuation line, which means the bidder shades his bid in both cases. In addition, I draw the bid function of the single price auction to be steeper than that of multiple price auction considering the possibility that a different type of bid reduction may appear according to the auction format. The most of recent literature predicts that in the single price auction, the bidder has more incentive to lower the market clearing price due to its pricing rule than in the multiple price auction.[3] In general, we cannot tell which auction format will yield more revenue before comparing the areas of the revenues, which, in turn, depend on bidders' shading behavior and the location of the market clearing price. The amount by which a bidder shades his bid relative to his valuation depends on where he believes the market clearing price, or the residual supply function will lie. Observe that in this graph, if residual supply is larger than q^*, then the revenue comparison is in vain because the multiple price auction always yields more revenue.

Therefore, we need a different model to analyze the multi-unit auction such as the Treasury auction from that to analyze single item auction case. To my knowledge a complete theoretical model for the multiple unit good auction has not been developed and so it is an open question as to what the optimal auction scheme is.

Separate from the efforts to make theoretical predictions, there have been also many attempts of empirical approaches to compare the revenues using actual auction data.

The empirical studies on this matter have proceeded in two different ways. One is "policy experiments" and the other is "structural model approach". In "policy experiments", researchers choose a kind of base price[4] which is thought to reflect the

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[3] However, since in the multi-unit auction many equilibria may exist, and the steeper bidding strategy may be one of the equilibria, we cannot conclude this is always the case.

[4] Usually when-issued market rates or resale market rates are used as the base price. When-issued trading occurs during the period between the auction announcement date and the actual issue date of

true value of Treasury bond to be auctioned and get the differential[5] between the auction price and this value. This differential indicates how much different the auction price is from its true value so that if this gap gets larger then economic rents to the Treasury are smaller. Next, they regress the differential on the dummy variable representing the change in auction mechanism and some control variables. If the coefficient of this dummy variable is significant then the mechanism change has an effect on the government revenue.

On the other hand, "structural model approach" estimates the components in the equilibrium condition which is derived from solving a certain theoretical model directly. The common feature of the structural model approach is that while ones primary interests are the parameters that characterize the distribution of TB valuations one can observe only bids that are related to but not identical to these latent variables. In this circumstance, if a researcher knows a structural model that defines a mapping between these valuations and observed bids, and relies on the hypothesis that the observed bids are equilibrium bids, then he can estimate the distribution of valuations using observed bid data.

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security. Prior to the Treasury's scheduled auction date for a given security, dealers and investors may either take long positions or short positions in the Treasury security to be auctioned. When-issued market performs various functions. First of all, the trading in this market gives potential bidders the demand for the security to be auctioned and generates more information about the depth and the diversity of the participants i.e. price discovery role. Secondly, this market also provides price and quantity insurance.

[5] The differential is "auction spread" which is defined as r_A- r_s (r_A : weighted average yield of winning bids, r_s : yield in when-issued market). If this spread is positive then we conclude that the revenue of auction is small.

In case of single unit auction with private independent value paradigm, the structural model is simply given by this relationship.

$b_i = s(v_i, F)$

where v_i *: private value of bidder i,* $F(.)$ *: the distribution of* v_i*,*

$s(.)$ *: optimal strategy function,* b_i *: bid of bidder i*

Since private values are unobserved and random, bids are naturally random with a certain type of distribution, say $G(.)$, that is determined by the structural elements of the model. The equilibrium bid distribution, $G(.)$ depends on the underlying distribution $F(.)$ in two ways: (i) through the unobservable v_i, which is drawn from $F(.)$, and (ii) through the equilibrium strategy, which is characterized by the function $s(.)$. If the structural model is identified, which means that $F(.)$ can be determined from observed bids by imposing some theoretical restrictions on $G(.)$, and that $s(.)$ is invertible, then we can recover v_i and $F(.)$ by estimates of $G(.)$ using only observed bids.

This logic can also be applied to the auction for the multi-unit goods such as TB auction. In TB auction, one usually estimates the distribution of market clearing price and recovers the implied true value.

Both approaches have their own merits and shortcomings. In case of the "policy experiment approach", it is closer to the ideal experiment one would like to see and its procedure has easily understandable intuition. In addition, from the practical view point, it can be applied to the case in which the aggregate data are only available. However, it heavily relies on the assumption that bidder's true valuations for the securities are accurately reflected in resale markets. Thus if the researcher cannot control for factors that may have changed between the close of the auction and the start of trade in the resale market, the results may lead to incorrect conclusions. Also, it cannot analyze the distributional aspects of policy change when using the individual bid data. Practically this method cannot be applied to countries which do not have the experience of using both auction formats. In contrast, the "structural model approach" is specifying a full model of bidding so that it can measure richer implications about the strategic behaviors at an individual bidder level and the distributional aspects of the auction game. However, because it heavily relies on the theoretical model, the results depend critically upon theoretical assumptions.

In this paper I will attempt to compare the revenue ranking empirically between the two auction mechanisms in selling TB with the Korean auction data. Considering that the TB is basically multi unit good, I will try to set up a more appropriate model to capture this aspect. As the empirical method, I will demonstrate how to apply the structural model approach to this case, and how to use non parametric approaches to estimate relevant unknown parameters.

This dissertation is organized as follows: in CHAPTER II, I review the previous literatures on both theoretical and empirical side. In CHAPTER III, I summarize the rules and institutional details of the Korean Treasury auction and describe my data set. CHAPTER IV sets up the theoretical model required to perform the structural model estimation with the discussions about the relevant assumptions. In CHAPTER V, I discuss the estimation method in detail which I use. CHAPTER VI presents the empirical findings for both multiple price auction period and single price auction period, and attempts to evaluate which auction format is superior in terms of the revenue and efficiency. CHAPTER VII deals with the robustness of my model and empirical framework when I relax some of the previous assumptions. I extend my analysis to asymmetric bidders' case and present the empirical findings under asymmetries. Then, I attempt to test whether the common value component exists in the underlying value structure of bidder. Finally CHAPTER VIII provides concluding remarks.

CHAPTER II

REVIEW OF LITERATURE

2.1 Theoretical Literatures

In the study of the auction for a single indivisible item the well known “Revenue Equivalence Theorem” has been established by Vickery (1961). That is, if the risk neutral bidders adopt strategies which constitute a non cooperative equilibrium in the PIV paradigm, then the expected selling price is the same for all four mechanisms.[6] After this many researchers have tried to suggest the revenue ranking under more general circumstances and found that when one or some of the above assumptions are relaxed, particular auction forms emerge as being superior.[7]

However, this famous theorem does not hold in divisible or multi-unit auctions as in the case of the Treasury auction even under same conditions as the case of single

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[6] Because on average the sale will be at the lowest price at which supply equals demand and the expected revenue generated for the seller by the mechanism is precisely the expected value of the object to the second highest evaluator for all four mechanisms.

[7] For the risk-averse bidders, Holt (1980) show that the seller will strictly prefer the Dutch or first-price auction to the English or second-price. Milgrom and Weber(1982) generalizes the assumption on the valuation structure in which both PIV and CV components are involved, so called “correlated value or affiliated value”, and shows that when bidders are risk neutral, the four common single auction mechanisms can be ranked as the English auction, the second-price auction, and the Dutch which yields same revenue as the first-price auction.

unit auction. Furthermore, when the assumptions are relaxed, theoretical comparison becomes more complicated and difficult.

The first attempt to set the theoretical model for multi-unit auction is found in Smith(1966).[8] He modeled the case that both bid price and quantity are choice variables with the assumption that each bidder associate a subject probability with each possible value for the minimum successful bid, and using the numerical example, he showed single price auction may yield more revenue than multiple price auction. Although he was the first researcher to recognize the unique characteristic of multi-unit auction explicitly, he made the problem similar to the single unit auction model by assuming that the bidder can bid for only one unit or can submit only one price. Harris and Raviv(1981) tried to generalize Smith's model by allowing the change in bidder's risk averse but maintained the same assumption of bidding for one unit only as Smith's. They argue that the revenue equivalence between the two auction formats holds if bidders are risk neutral, but multiple price auction yields more revenue if bidders are risk averse.

The problem of "single bid"(or unit demand) assumption was criticized first by Scott and Wolf(1979). They showed that multiple bids are optimal under the multiple

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[8] The earlier comments on the Treasury Auction were given by Friedman, D. Carson and A. Brimmer in early 60s. Friedman and Carson have independently suggested that the Treasury should abandon the policy of price discrimination in weekly treasury auction bills and substitute a simulated purely competitive auction in which all bids are filled at a uniform market clearing price. Brimmer, however, challenged this view, arguing in favor of price discrimination on the ground that efficient "…resource allocation should be subordinated to the minimization of interest cost to the Treasury…". See Smith (1966)

price auction so that a single bid assumption was not an appropriate assumption in the multi-unit auction. However, in setting the expected profit function they did not reflect the possibility that strategic interactions of bidders may affect the market clearing price so that bidders are assumed to be a price taker. Nautz(1995) also worked with multiple bids but still with a price taker's assumption. However, his contribution is that he analyzed a more realistic case, i.e., the discrete space of bids instead of the continuous choice sets. Though the optimal condition which he derived for single price auction was imperfect due to the price taker's assumption, his basic setting for analyzing the discrete case will be adopted when I set up my own theoretical model.

The literatures which have been reviewed up to now have limits to fully deal with the characteristics of multi-unit auction because they assumed either that bidder can submit only one bid or that bidders are price takers. Therefore their suggestion is also very similar to the result from the study of a single item auction, which is that the multiple price auction and the single price auction could be viewed as multi-unit extension of 1st price auction and 2nd price auction in single item auction case. Consequently, they provide that in the multiple price auction all bidders would shade their bids by the same amount regardless of whether they possess very different degrees of market power and that in the single price auction bidders tend to submit their true value as their bids.

The explicit consideration for these important properties overlooked by earlier researchers was taken by Wilson(1979). He considered the downward sloping bid functions and more importantly, bidders' strategic behavior by introducing the concept of the "residual supply". First, he assumes that each bidder *i* has been able to obtain the information about the value that is summarized in an estimate or statistics s_i. For a bidder participating in a share auction, a strategy is described by a function $x(p;s_i)$ of both the price *p* and his information s_i. Under symmetric equilibria, an optimal strategy can be characterized in the following way. If each one other than bidder *i* is using the strategy *x*, then when *i* uses a strategy *y* the sale price p^* will be that price for which

$$\Sigma_j x(p^*;s_j) + y(p^*;s_i)=1$$

Note that the bidders must consider the fact that one can bid for only the residual supply which is defined by total supply minus other's demand and the possibility that they can affect p^* by choosing a different bid schedule.

Recognizing the strategic behaviors through the concept of the residual supply and using some numerical examples, he shows that in share auction(or multi-unit auction) the seller may experience a considerable reduction in revenue compared to single unit auction. Regarding the comparison of the two auction formats, he gave a very brief and intuitive mention that the change of auction format does not affect the revenue because the bidders will respond to this maneuver by altering their strategies which have the exactly same allocation and payment as before the change.[9]

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[9] Suppose the sale price and the optimal bid function in single price auction are *p* and $x(p)$ and those in multiple price auction are q, $q(y)$. Then each bidder will receive the same allocation and pay the same amount to the seller if his two strategies in the two types auctions are related by the equation, $p(x)x=\int^x q(y)q$. Differentiating this equation with respect to x yields the relationship $q(x)=p(x)+xp'(x)$. Thus, from his strategy for one auction, he can easily derive a corresponding strategy for an alternative. The consequence for the seller is that the sale price is reduced by converting to multiple price auction but his revenue remains unchanged.

Since Wilson's setup is mostly accepted in the recent literatures to deal with multi-unit auction including the Treasury auction, I will also follow his basic setup and take the "residual supply" as the key feature to assess the probability distribution of market clearing price.

After Wilson, many counterarguments to the earlier findings which are basically similar to the predictions in the single unit auction have been suggested. Back and Zender(1993) pointed out that bidders' marginal cost depends on other's strategies due to involving the residual supply. They showed that the single price auction may be much worse than multiple price auction in terms of revenue because in the single price auction bidders tend to submit steeper demand schedules so that the marginal cost to a bidder may increase by forming steeper residual supply curve. Engelbrecht-Wiggans and Kahn(1998a and 1998b), using the case of two units to be sold, show that bidders shade the highest bid more and lowest bid less in a multiple price auction while the shading behavior is reversed in single price auction. They conclude that both auction formats are not efficient and the revenue equivalence between them are broken down.

More recently, Ausubel (1997) and Ausubel and Crampton (2002) criticized again the imperfect analogy between the single unit and multi unit auction and called the thought of no bid shading and efficient allocation in the single price auction as "Single Price Auction Fallacy". They extended Back and Zender and Engelbrecht-wiggans and Kahn's results to an entire set of equilibria and the interdependent value environment and argued that the outcomes from single price auction are not efficient due to the existence of bid shading(demand reduction) and the differential bid shading over quantity. Since in the single price auction bid on later units can be pivotal, and bidders try to shade more for later units to pay less on the earlier units.

A simple example of how this yields inefficiency is illustrated in Figure 4.[10] In the figure, there are only two bidders. Each bidder's marginal true valuation is drawn by a solid line but their actual bid schedules are drawn by dashed and dotted lines respectively. We see that there is differential bid shading over the bid quantity. Now, suppose that total supply of the auction is *Q* and that market clearing price is set at "mcp" line. Then bidder 2 will win *q2* and bidder 1 will *q1* such that *q1+q2=Q*. However, since bidder 2's true valuation is higher than bidder 1's at all quantities up to total supply *Q*, the efficient allocation should allocate the whole amount of *Q* to bidder2. Thus this mechanism is not efficient in the sense that one who values more will not get the good.

However in the multiple price auction this incentive is not very strong, they tend to shade bids by similar amounts and an efficient outcome can be more possible. This does not mean that multiple price auction is always efficient or should be preferred to the single price auction. Instead they find that generally the multiple price auction is not efficient at its equilibrium.

[10] This graph combines two graphs depicted in Ausubel and Crampton(2002) p24.

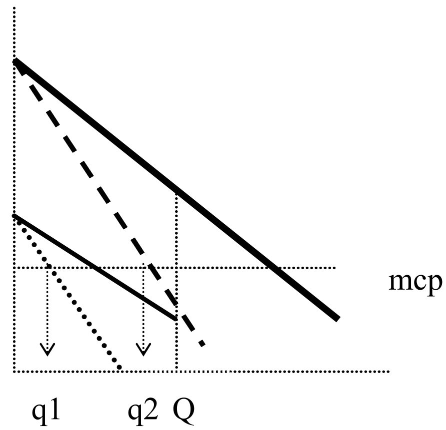

FIGURE 4. Illustration of Differential Bid Shading

Regarding the revenue comparison, they concluded the inherent ambiguity in revenue ranking between the two formats by suggesting several counterexamples which yield contradictory revenue rankings. They also suggest several theorems about the relationship between efficiency and the revenue maximization. To be summarized, with symmetric bidders and flat demands, the revenue maximization coincides with efficiency, that is, the revenue maximizing auction awards all quantity to the buyers who value them the most. However, this does not generalize to the case of downward-sloping demand curves.[11] Whether the revenue maximizing assignment distorts the efficient assignment depends on the distribution of uncertainty. Their conclusion is that

[11] Maskin & Riley(1989) determine the optimal selling procedure in the case of downward sloping demand and they find that it typically does not result in an efficient assignment.

for all distributions other than the exponential distribution, there is a conflict between revenue maximization and efficiency with downward sloping demands.

In conclusion, theoretical studies for multi-unit auction get to the following points:

Firstly, contrary to earlier studies, the single price auction reveals a certain type of demand(or bid) reductions. Secondly, the type of bid reduction may be different from each other. More specifically, more shading for higher bid price is usual in the multiple price auction whereas, there is more shading for lower price in the single price auction. Thirdly, due to different types of bid shading, the revenue ranking between the two auction mechanisms is inherently ambiguous. Finally, the single price auction may more severely distort efficiency.

As I mentioned earlier, with keeping these findings in mind, I will follow the Wilson's setup and utilize Nautz's idea for the extension to discrete strategy space when constructing my theoretical model. I will also try to find empirical evidence for these theoretical predictions with the Korean Treasury auction data.

2.2 Empirical Literatures

Due to the complexity involved in the multi-unit auction environment, the theoretical approaches have not provided any conclusive results yet. Naturally considerable endeavor to find answers to the questions addressed have been devoted from the empirical side, as well. There are two distinctive empirical approaches to compare auction revenues between the multiple price auction and the single price auction.

2.2.1 Policy Experiment

This approach is to investigate the coefficient of dummy variable for the change of an auction mechanism in a carefully designed regression model which sets the variable indicating bidders' markup or profit from the auction as a dependent variable. If the coefficient is significant, then this indicates that the switch of the auction format affects the bidders' bid price and consequently seller's auction revenue. Also the sign tells which auction format is preferable regards to revenue.

To take this approach, the researcher should have the aggregate data from both auction formats. He also has to construct the appropriate dependent variable that captures the economic rents acquired by the bidders well. This variable has been usually the differential between the weighted average bid price of the auction and when-issued market rate or resale market rate of the Treasury bonds. This implies that when-issued or resale market rate is considered as the true value of the bond to be auctioned, and consequently this approach makes the common value assumption for the underlying bidders' value structure. In addition, this approach needs to carefully consider what kinds of independent variables should be included in the regression model to control possible effects of the sources other than the switch of auction format on the dependent variable.

Many researchers have tried to investigate the revenue changes with this approach, but they have reported different results across countries or data periods. Umlauf(1993) analyzed the Mexican 30 day Treasury Bill auction results which consisted of 181 multiple price auctions and 26 single price auctions from August 1986 to May 1991. He used the weighted average resale market price as a benchmark of true value and calculated the auction profit taking the ratio of this price to the weighted average bid prices of the auctions. Higher auction profit means that bidders obtain the bills at a relatively cheaper price than at resale market price. As the control variables, he included the overnight lending market rate, the number of bidders, the variance of bid prices, the total quantity for sale, and the dummy variable for the single price auction. Since he cared about that the six largest bidders accounted for an immense number of shares in the auction purchases in the Mexican Treasury auction market, he included the number of bidders and the variance of bid prices to capture the degree of competition and collusion.[12] He found that the coefficient of dummy for single price auction was -2.44, and interpreted the result as the single price auction having superior revenue to the multiple price auction.

Simon(1994) used the results of auction for 15~30 year maturity Treasury bonds in U.S. The data covered 1973 to 1976 in which 6 single price auctions and 10 multiple price auctions were held. As the dependent variable, he constructed the bidders' "markup" rate measured by the average winning rates minus when-issued market rate

[12] A wider dispersion of bid prices suggests failure of large bidders to rig their bids collectively and more competitive auction.

at 1PM of the date of auction. Since he used the rates submitted without converting these to the prices higher "markup" means actually "markdown" in terms of bid price. It might be notable that he included a deterministic time trend to guard the possibility of omitting variables which are trending over the sample period. In his regression, the coefficient of dummy for single price auction was 0.08 which implied that "markup" is 8 basis points higher in the single price auction than the multiple price auction. Therefore he argued that the revenue from the single price auction decreased compared to the multiple price auction.

Nyborg and Sundaresan(1996)[13] and Malvey and Archibald(1998) also analyzed U.S. data but they did not find statistically significant difference in the revenue between the two auction formats.

2.2.2 Structural Model Approach

As I briefly mentioned in CHAPTER I, the "policy experiment approach" has several shortcomings. The most important one is that it cannot analyze the strategic behavior occurring in the bidder level and the distributional aspects of the individual

[13] In fact, they were more interested in the effect of the auction format on the pattern of information release at different times in the when-issued market. If the pre-auction trading is more active, then the information about the value of the Treasury bill to be auctioned which bidders have in mind is more likely to be revealed so that it may help bidders to avoid the winner's curse. They found that the pre-auction trading was more active in single price auction than in multiple price auction.

bid data. Hence with this approach we cannot obtain richer empirical information for the predictions made in the recent theoretical papers.

Recently the "structural model approach" has been highlighted as an alternative to the "policy experimental approach". In fact, in the case of the single unit auction, there have been more a than a few attempts to estimate using this approach both parametrically and nonparametrically. However for multi-unit auction case, very few studies can be found due to the complexity of problems.

The common feature of the structural model approach is to derive the optimal bid condition which is mapping the observed bid data to the unknown but concerned variables by solving the certain theoretical model. Then we try to estimate the components of this condition which in most cases would be the distribution of the unknown variables. Using these estimated distributions, we can estimate or recover the unknown variables.

The first attempt of applying this approach to the Treasury auction is found in Heller and Lengwiler(1998). They analyzed the Swiss Treasury market using the theoretical model which had been suggested by Nautz(1995) and concluded that the shift to the single auction improved the Treasury's revenue in Swiss. However, as we have known from the previous section, Nautz's model made a price taker assumption so that the derived optimal condition was not the right form when we consider bidders' strategic interactions.

More recently, Hortaçsu(2002) has accomplished immense development in both the theoretical model and the empirical methodology. He constructed a more complete model reflecting the strategic characteristics by adopting Wilson(1979)'s setup and derived the optimal condition for multiple price auction. He also extended the the method of nonparametric estimation suggested by Gurre et al.(2000) to multi-unit auction case to estimate bidders' true valuations. He applied this method to the Turkish 13-week Treasury bill auctions. The data analyzed covered 25 auctions which held under the multiple price auctions from Oct. 1991 to Oct. 1993. His *ex-post* revenue comparison showed the revenue loss in the single price auction and *ex-ante* revenue comparison could not reject the revenue equality hypothesis.

Since I couldn't find any further advance after Hortaçsu(2002) in dealing with the multi-unit auction, I will take his attainments as the starting point of this paper and attempt to extend his ideas and methods to the single price auction.

CHAPTER III

THE KOREAN TREASURY AUCTION AND DATA

3.1 The Korean Treasury Auction Market

The Korean Treasury has auctioned Treasury bonds since 1994. However, before 1998, the infrastructure of the Treasury bond market was not fully developed so that the auction system operated imperfectly. For example, there existed a minimum cut off price which was internally fixed by the Treasury before the auction, and an implicit assignment for the remaining quantities which were not fulfilled in an auction was executed. Therefore a true type of an auction is said to have begun in the middle of 1999 when the minimum cut off price was abolished and the primary dealer system was introduced.

As for the auction mechanism, the multiple price auction system had been adopted until July 2000,[14] but this was completely converted to the single price auction as of August 2000.

The maturities of TB to be auctioned are diversified to 1, 3, 5, 7 and 10 years. All of them are coupon bearing bonds. The interest according to their coupon is paid quarterly. Among them the 3-year TB makes up 40~50% of the issuing amount. The

[14] Although not many, there were a few cases that single price auction system was applied to sell the long-term bonds like 7 and 10 years maturity bonds.

auction is held regularly. The auction date is fixed by the maturity date; usually a 3 year bond is auctioned on the Monday of 2nd week of every month and the other maturities in the 1st or 3rd week of every month. The auction amount is pre announced around 3~5 days prior to the auction date.

After the Treasury introduced the primary dealer system, the participants in the auction market are restricted to 28~38 financial institutions which are designated as Primary Dealer(PD) or the approved candidates of PD. All of PDs are either commercial banks or security houses(or brokerage firms), and the portion of each group in PD group is around 50:50. The long-term investors such as pension funds, investment trust companies, and insurance companies can not be designated as PDs so that they are prohibited from submitting the bids directly to the auction. They do constitute an important and potential demand for TB by placing their orders onto PDs.

Each bidder is asked to specify a yield and a quantity demanded at that yield. Yields have to be specified up to the second decimal place. With a simple calculation method, yields can be converted to prices as the amount a bidder is willing to pay for one unit of TB with a face value of 10000 Korean Won(KW) and a certain coupon accruing the periodic interest payments.[15] In all the multiple price auctions and some single price auctions, the coupon rates were set to the quantity weighted average of the

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[15] To convert yield to price, I use a conventional way which is widely used in the market. The arithmetic formula is given by:

$$price = \sum_{i=1}^{11} \frac{it}{\left(1 + \frac{yield}{4}\right)^{i}} + \frac{10000 + it}{\left(1 + \frac{yield}{4}\right)^{12}}$$

where it=quarterly interest payment

winning yields after the auction. In these cases, with setting this yield to be 10000 KW, the bidding yields are converted to the bid prices in the same way.

Quantities are specified in terms of the face value of TB the bidder wants to buy. The minimum quantity increment which a bidder is allowed to submit is 0.1 million KW(around $83,000). In practice the bidders increase their bid quantities by the 5 billion Korean Won(around $4 million). The maximum number of yield-quantity pairs is restricted up to 5. Bids are submitted by 3:00 pm on Monday and auction results are announced at 4~4:30 pm on the same day. The date of settlement is Wednesday.

As for secondary markets for trading TB, there are two distinguishable markets. One is the exchange market managed by Korean Stock Exchange(KSE) and the other is the conventional over-the-counter(OTC) market. The exchange market by KSE was established in May of 1999. This market uses a fully computerized trading system and an individual auction method at multiple price quotes. These two markets are complimentary to each other and contribute to revitalizing the TB trade and deepening the secondary market for TB.

3.2 Data

The data is for 1, 3, 5, and 10 year maturity TB auctions which were held betweens September 1999 and April 2002. Except for 3 year maturity TB, the time span which the data covers is severely unbalanced between the multi price auction period and the single price auction period. Especially, in the case of 1 or 10 year TB, the data for one of the two different auction formats are not available. In addition, the portion of 3 year maturity TB in the auction market possesses the largest volume and the participation in 3 year maturity TB auction is more active than the 5 year maturity TB. Therefore, in order to analyze both multi price auction and single price auction data, I will only use the 3 year maturity TB auction data.

Table 1 describes the summary statistics of my data set. According to the auction format used, there were 10 multiple price auctions and 20 single price auctions which makes a total number of 30 auctions.[16]

The original data consists of the quantity of TB supplied, pairs of yield and quantity submitted by each bidder, the market clearing yield for each auction, and the TB trading yield in the secondary market for the same period.

As I mentioned before, since bidders bid in terms of yield, I converted bid yields to price in the unit of the Korean Won while setting the market clearing yield for each auction to be 10000 KW. A 0.01%p in bid yield represents an increase or decrease by about 3 KW in terms of price. Quantities are also normalized by the ratio to the total supply of each auction so that total supply of each auction is normalized to be 1.

Regarding the number of bidders, i.e., primary dealers, during the sample period 38 PDs appeared in the auction, but every PD did not always participate in the auction. There have been a few cases of either some were newly designated as PD or some were disqualified as a PD because they quit their business or were merged with other

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[16] I drop 2 auctions from original sample, one from multiple price auction period and the other from single price auction, due to the lack of appropriate data.

institutions. There are six bans and six security houses among the 38 bidders who enter into or exit from the group of PDs in the middle of the sample period.[17]

My data set consists of a total 2,499 bid points which are pairs of bid price and quantity. The statistics for bid prices, bid quantities, and other indicators of data are summarized in Table 2.

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[17] The bidders who showed discontinuity in participation are as follows: (I use bidder code number instead of the name of institution)

- banks : #7(~2000.5), #9(~2000.11), #10(~2000.9), #15(~2000.9), #17(2000.10~), #18(2002.1~)
- security houses : #33(2000.10~), #34~ #38(2002.1~)

TABLE 1 — Summary Statistics on 30 Auctions, Sep. 1999~Apr. 2002

Date	Auction Format	# of bidders	# of bid points submitted	Total Supply (Bill KW)	Total bidding Amount (Bill KW)	Market Clearing Yield (%)
9/13/1999	Multi	28	107	1200	2535	9.39
10/11/1999	Multi	31	111	1357	3565	8.39
11/15/1999	Multi	28	108	1185	2238	8.37
1/17/2000	Multi	29	107	764	2275	9.58
2/14/2000	Multi	30	115	1228	4230	8.99
3/13/2000	Multi	29	97	664	1910	9.04
4/10/2000	Multi	30	104	840	3038	8.83
5/8/2000	Multi	27	86	762	1920	8.84
6/12/2000	Multi	23	67	600	976	8.61
7/10/2000	Multi	25	63	586	1140	7.92
8/14/2000	Single	23	74	600	1349	7.70
9/18/2000	Single	24	57	900	1260	8.15
10/9/2000	Single	23	80	900	1935	7.84
11/13/2000	Single	25	82	950	2310	7.00
1/8/2001	Single	26	74	750	1895	6.00
2/5/2001	Single	24	48	770	1170	5.39
3/12/2001	Single	23	39	500	770	6.10
4/2/2001	Single	26	64	800	1710	6.60
5/7/2001	Single	25	59	600	1350	6.52
6/4/2001	Single	26	70	400	1430	6.10
7/2/2001	Single	27	88	400	1800	5.86
8/6/2001	Single	26	79	700	1990	5.38
9/3/2001	Single	24	69	850	1790	5.03
10/8/2001	Single	26	95	890	2380	4.40
11/7/2001	Single	26	73	750	1590	4.88
12/3/2001	Single	25	81	1100	1830	5.65
1/7/2002	Single	27	118	1200	4200	6.10
2/4/2002	Single	24	65	400	1400	5.94
3/4/2002	Single	23	77	500	1740	5.92
4/1/2002	Single	24	77	570	1760	6.44

TABLE 2 — Summary Statistics on Bidding Data at Each Auction

Date	Bid amont (Bill KW)			Winning amount (Bill KW)			Avg. #of Bid Points	Avg. win Bid Points	Bid price(KW)		
	Mean	Max	Min	Mean	Max	Min			Mean*	Max	Min
9/13/1999	90.5	300	10	57.0	300	5	3.82	2.67	9996.74	10028.50	9945.86
10/11/1999	115.0	350	10	52.2	203.1	7.2	3.74	2.08	9996.22	10010.52	9971.15
11/15/1999	79.9	290	10	47.4	200.3	8.6	3.86	2.24	9998.76	10010.52	9973.76
1/17/2000	78.4	180	20	28.3	70.7	2.1	3.79	2.07	9996.34	10012.91	9974.24
2/14/2000	141.0	385	20	45.5	145	5	4.13	2.33	9998.76	10010.42	9989.59
3/13/2000	65.9	150	10	26.6	115	2	3.59	2.20	9997.95	10007.81	9976.62
4/10/2000	101.3	340	20	36.5	210	2	3.70	1.83	9997.60	10007.83	9984.35
5/8/2000	71.1	165	10	34.6	72	1	3.63	2.27	9998.53	10005.22	9989.57
6/12/2000	42.4	180	10	28.9	108.7	1	3.00	2.60	10002.10	10020.97	9973.86
7/10/2000	45.6	120	10	27.9	100	1	2.64	2.05	9997.27	10015.89	9731.74
8/14/2000	58.7	130	15	29.1	81	5	3.22	2.00	9999.43	10066.66	9973.48
9/18/2000	52.5	170	10	42.9	156	7	2.38	1.90	10001.10	10026.42	9908.17
10/9/2000	84.1	220	10	40.7	184	5	3.61	2.05	9999.69	10015.92	9984.11
11/13/2000	92.4	210	10	45.2	137	9	3.36	1.86	9996.59	10018.82	9946.47
1/8/2001	72.9	220	10	41.4	100	10	2.92	1.83	10005.24	10276.98	9975.49
2/5/2001	48.8	160	10	36.7	100	10	2.00	1.43	10008.31	10052.46	9969.77
3/12/2001	33.5	80	10	21.7	60	5	1.78	1.26	10005.58	10054.62	9891.77
4/2/2001	65.4	240	10	38.1	160	10	2.62	2.05	9987.18	10067.80	9892.61
5/7/2001	54.0	150	10	28.6	110	5	2.56	1.52	9999.35	10046.11	9948.76
6/4/2001	56.2	150	10	21.1	50	5	2.77	1.32	9996.59	10027.27	9975.53
7/2/2001	67.8	140	10	33.3	90	10	3.33	1.58	9987.29	10016.41	9953.67
8/6/2001	76.9	220	10	38.9	195	5	3.15	1.56	9997.23	10022.06	9967.02
9/3/2001	74.6	180	10	40.5	180	10	3.04	2.05	9999.97	10036.06	9966.84
10/8/2001	92.7	180	20	35.6	90	10	3.85	1.84	9993.69	10028.01	9960.94
11/7/2001	61.2	140	10	30.0	80	10	2.92	1.88	9997.66	10036.15	9966.76
12/3/2001	73.2	180	10	45.8	115	10	3.32	2.38	10002.18	10030.21	9986.30
1/7/2002	161.5	500	30	57.1	240	5	4.52	2.10	9994.36	10030.00	9953.84
2/4/2002	59.6	150	10	28.6	80	5	2.79	1.93	9995.83	10024.60	9970.03
3/4/2002	75.7	150	10	29.4	85	5	3.52	1.88	9994.97	10024.61	9972.74
4/1/2002	73.3	140	10	35.6	80	10	3.33	5.00	9993.84	10024.41	9970.26

* quantity weight average bid price = $\sum_{i=1}^{N}\sum_{k=1}^{K}(b_{ikt} \times q_{ikt}) / \sum_{i=1}^{N}\sum_{k=1}^{K} q_{ikt}$

where (b_{ikt} , q_{ikt}): bidder i's k^{th} bid price and bid amount(increment) at auction t

CHAPTER IV

THE BIDDING MODEL WITH PIV SETTING AND SYMMETRIC BIDDER

4.1 Discussion about the Assumptions

In order to perform the structural model estimation, we should have a theoretical model which reflects the auction environment well. In this CHAPTER, I set up the model and solve for the tractable optimal condition which will be the object of estimation.

Since the auction problem involves asymmetry of information[18] between the seller and the bidders and among the bidders as its crucial element, it fits well into the non cooperative game situation under incomplete information. If we model our auction problem in this way, the next thing to do is to find out the optimal strategy or bid function which constitutes the Bayesian Nash Equilibrium.

However, when one tries to design the auction model, one must be cautious of which assumptions should be imposed on the model setting. The important considerations which have been usually taken into account are the underlying valuation structure of bidder, symmetric or asymmetric bidders, and bidders' attitude toward

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[18] That is, the seller does not know any bidders' valuation of the item for sale and one bidder does not know other bidders' valuation either.

risk. In addition, some researchers pointed out the case of uncertain supply, stochastic participation problem and the matter of collusion among bidders.

Henceforth, considering the institutional characteristics and the market environment in Korea, I will try to rationalize the assumptions which I make in modeling the Korean TB auction.

First of all, regarding the possibility of uncertain supply and stochastic participation, I will treat both total supply(Q) and the number of bidders(N) as common knowledge which is fixed exogenously. As for the total supply, since this amount is announced before the auction as common knowledge to all bidders in Korea and the Treasury does not exercise any kind of ex post cancellation of competitive bids, we can think of the total supply as not being random. In the case of the number of participants, the bidders are restricted to the Primary Dealers designated by the Treasury as mentioned before, and we can see very stable participation from the number of bidders across time in Table 1, and this can also be assumed to be not random. However, many potential bidders do exist who are not PDs but demand TB with non trivial amounts such as for pension funds and insurance companies. Since these long term investors have to acquire TB through PDs more fluctuation in number of bids from Table 1 may indicate the variation in demand by these institutions. Since the accurate information about the participation of the potential bidders at each individual auction is not available I will adopt the assumption of a fixed number of bidders as was usually made in the previous literatures.

Regarding the bidders' risk attitude, the main risk that bidders are facing in the auction is the risk of being forced to rely on secondary market to satisfy demand.[19] Therefore, if the value of the individual bids is relatively small with respect to a bidder's total assets, a likely scenario, risk neutrality appears as a reasonable assumption. This argument seems to be the case in the Korean Treasury auction because the TB is just one financial instrument among the various financial goods for financial institutions to invest in such as stocks, corporate bonds, foreign exchange, and many other financial derivatives. The bidders' size of funds allocated to buy TB may not be a big portion of their total assets. Of course, when bids are solely for fulfilling some type of a reserve requirement, then a bidder may be sensitive to the risk. However, since bidders' demands are a mixture of many different types so as to be very hard to be clarified, I will simply assume a risk-neutral bidder.[20]

The assumption of symmetric bidders implies that bidders are entirely alike in their characteristics, that is, whatever is an optimal bidding strategy for anyone bidder uses the same optimal strategy in preparing his bid. In a theoretical model, symmetry is represented by the assumption that every bidder has the same distribution about a certain latent value such as the unknown true value of an item for sale or a signal which each bidder perceives before the auction.

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[19] Because bidders can buy securities in the secondary market, the risk may be evaluated by (value-$[p^{auction}-p^{2nd\ mkt}]$).

[20] There are a few papers that model risk aversion in the single unit auction but not in the multi unit auction. In particular, the papers focusing on TB auction is usually assuming risk neutral bidder. In single unit auction, when bidders are risk averse, they have an incentive to raise the probability of winning by marginally increasing their bid, hence the seller can do strictly better with risk averse bidders.

Whether this assumption is valid for the Korean auction situation may be questionable because the bidders are composed of two different types of financial institutions, which are the banks and the security houses. However, for the time being I will assume symmetry so that I can focus on finding a symmetric Nash equilibrium for my model. This will provide a benchmark model. In CHAPTER VII, I will attempt to reinvestigate the validity of the symmetry assumption and to extend the model and the estimation by considering the possibility of asymmetry among the bidders.

The most important and controversial assumption is the underlying value structure. In most applications of auction theory, researchers typically choose to work within one of two very different theoretical settings. One is the private (independent) value paradigm(PIV) and the other is the common value paradigm(CV). In the PIV setting, each risk-neutral bidder knows the value of the object to himself, but does not know the value of the object to the other bidders. The values are modeled as being independently drawn from some continuous distribution. In contrast, CV setting assumes that the value of the object to the various bidders can be regarded as equal, but the bidders may have differing estimates of the common value.

Milgrom and Weber(1982) provided the general functional formula which captured these two paradigms.[21] Each bidder possesses some information concerning the object for sale; let $X=(X_1, \ldots, X_n)$ be a vector, the components of which are the real-valued informational variables(or value estimates, signals, or bidder's type) available to each bidder only, i, and $S=(S_1, \ldots, S_m)$ be a vector of additional real-valued variables

[21] see Milgrom and Weber(1982), p1093, pp1097-1098.

available to all the bidders commonly. The actual value of the object to bidder *i* may be denoted by $V_i=v_i(S, X)$. Then PIV is represented by the case of m=0 and each $V_i=X_i$, while CV is the case of m=1 and each $V_i=S_1$. Usually PIV also assumes the commonly known distribution function of X_i, F_i so that V_i is assumed to be drawn the distribution function F_i. In CV, if V is the unobserved true common value, then the bidders' perceived value V_i, $i=1,\dots,n$, are independent draws from some probability distribution $G(S|V)$ which is already known to all the bidders.

Which value paradigm is most appropriate to the Korean TB auction situation? Through the literature survey, it seems that the assumption chosen depends on the purpose and methodology of the study and the consideration of the simplicity of the model. Overall, the empirical papers using the policy experimental approach adopted the assumption of CV while those using the structural model approach chose the PIV paradigm.[22] In the case of the theoretical papers, the assumptions vary by authors.

[22] The theoretical papers can be divided as following according to the value structure assumption.

PIV	CV	IDV
Haris & Raviv(81), Maskin & Riley(89), Nautz(95), Engelbrecht-Wiggans & Kahn(98a and 98b)	Smith(66), Scott & Wolf(79), Wilson(79), Back & Zender(93)	Ausbel & Crampton (02)

In case of the empirical papers,

CV	PIV
Umlauf(93)*, Nyborg & Sundaresen(96)*, Malvey & Archibaid(98)*, Simon(94)*	Heller & Lengwiler(98)** Hortacsu(02)**

*Policy Experimental papers / **Structural Model papers

The value structure may be determined by many sources but one of the most important factors may be the nature of the objects being auctioned.[23] In the case of the TB, the characteristic of the objects, i.e., bonds to be issue, can vary by the bidders' purpose in buying them. That is, if bidders participate in the auction to get the bonds for the purpose of holding them up to the maturity, then the underlying value structure can be assumed to be close to the PIV setting. If their purpose is to obtain the bonds is to trade them in the secondary market, then the value of the bonds to be auctioned may be governed by a CV setting because every bidder must consider the forecasted value of the price in the secondary market which is unknown but possibly common to all the bidders before the auction.[24] However we cannot reject the possibility of a more general value structure because bidders can participate in the Treasury auction with both purposes at the same time.

Unfortunately, the way to test of the underlying value structure has not been developed for multi unit auction models. Therefore, instead of proceeding with more detailed discussions about the value structure, I will assume a PIV setting following the convention of the previous papers dealing with the structural model estimation. However I will attempt to test the existence of common value components with scrutinizing the demand structure of the bidders more carefully in CHAPTER VII to reinvestigate the validity of PIV assumption in the Korean market.

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[23] In general, the PIV is more applicable to auctions for non-durable consumer goods. However the CV may apply to the auction for oil, gas and mineral rights on a certain tract of land where the value of the object depends on the unknown amount of recoverable reserve.

[24] This logic is usually found in the policy experimental approach literatures.

When we assume PIV, the possible sources of private information for the value of TB could be[25]:

1) Each bidder may have a different reserve requirement for TB or a different availability of liquidity which is not known to their rivals.
2) Each bidder may have a unique business relationship with his own customers who place the orders for purchasing the TB.
3) Each bidder may have a different expectation or a different forecasting capability about the future resale price of the TB.

Related to the value structure, there is one other thing to be considered. Since the TB is a divisible good, the valuation may be the function of quantities demanded. That is whether each bidder has a constant marginal value over the total quantities demanded by the bidder i, q_i, or bidders' marginal values are smoothly decreasing with quantity. These are called a "flat demand" assumption and a "downward-sloping demand" assumption respectively. The flat demand assumption was discussed in earlier literatures in which bidders possessed unit demands, and in particular it provided a generalization of Milgrom and Weber's(1982) model of an auction for a single object.[26] However we discussed the limits of a unit demand model in CHAPTER II,

[25] Hortaçsu (2002) pointed out 1) and 3) as possible sources of private information.

[26] See Ausbel and Crampton(2002). They first adopted flat demand assumption to simplifies their analysis and found even if the valuation is constant the bid may be reduced at the later demanded units under single price auction scheme.

and it might result in the fallacy of imperfect analogy of the single unit auction. Furthermore, if the flat demand makes sense, the bidder is more likely to submit only a single bid or demands an identical fraction of the total supply. However this expectation is easily rejected in my sample statistics in CHAPTER III.[27] Therefore, I will assume a downward sloping demand, that is, the first derivative of value function with respect to quantity is weakly negative.

Finally, as for the actual demand or bid function submitted by bidders, various types of assumptions may be imposed. Previous researchers applied one or two among the three assumptions. Those are unit-demand model, demand schedule model, and lumpy bidding or discrete model.[28]

Unit-demand model assumes that each bidder bids for only one of the several units and can only choose only the prices. Most of the results from this model are extensions of the theories from the single-unit auction and no conclusions can be drawn as to the strategic role of quantity choices. Demand schedules model assumes that bidders submit schedules starting at the different prices at which they wish to pay for the various quantities. Furthermore it assumes perfectly divisible price-quantity decisions. These models always induce regular (value-monotonic) allocations.

In the real world, a TB auction usually has the smallest price increment and a minimum bid quantity restriction. Bidders can place several units of demand at one price. This phenomenon may arise endogenously, but may also originate in

[27] From Table 2, we find that average number of bids used by bidders is 3~4 and that quantities by the bidders show wide variation.

[28] See Tenario(1997)

exogenously imposed minimum quantities. Therefore, it is natural that bidders submit a finite number of price-quantity pairs and do lumpy bidding which translates into several units demanded at one price. This results in "step-wise" bid functions instead of continuous functions.[29]

Step-wise bid function is also plausible in the Korean situation. The Korean treasury imposes a minimum price increment which is up to two decimal points in terms of percentage. The minimum bid quantity is around 0.1 million dollars and the bidder can increase the bid amount by the unit of 0.1 million dollars. There is a maximum quantity restriction where only 30% of the total supply can be purchased by one bidder.

Although the step-wise bid functions are more realistic, in order to get a more comprehensive understanding and to help recognize the distinction by the demand

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[29] Different equilibrium outcomes may arise depending upon the assumption of the continuous or discrete bid functions. Fabra, et al.(2004), in their analysis of electricity auction markets with step bid functions, found that there exists a unique, competitive outcome in the single price auction independent of the number of admissible steps in each supplier's bid function, so as long as this number is finite. In fact, Wilson(1979) and Back and Zender(1993) showed that in the single price auction with continuous bid functions, some of the equilibria may be extremely collusive-like which yield very low revenues for the auctioneer because a bidder's incentive to price more aggressively is offset by the large decrease in price that is required to capture an increment in quantity when bidders offer very steep bid functions at the equilibria. However, when bidders are limited to a finite number of price-quantity bids, a positive increment in quantity can always be obtained by just slightly undercutting the price of a rival's unit. Hence the collusive-like equilibria found in the continuous auction cannot be implemented. With this finding, they raised some doubt on the relevance of applying the continuous share auction model to electricity markets in which participants are limited to a small number of offer prices per generating unit.

function, I will start to deal with the continuous case first and then extend the analysis to the discrete case.

4.2 The Model with PIV and Symmetric Bidder-Continuous Case

First, I will work with the continuous space of strategies which deal with both the bid price and the bid quantities which are assumed to be continuous variables. I will also assume that the bid schedule or the bidder's strategy function is strictly decreasing to quantities and that the bid function is differentiable in the continuous case. I will basically follow the approach and the notations adopted by Hortaçsu(2002) using the share auction setup of Wilson(1979).

Let the total supply be Q and the number of bidders be N(denoted by $i = 1,\ldots,N$, $N \geq 2$) which are commonly known to bidders. Bidders are assumed to be symmetric in the sense that optimal bid for one bidder is also optimal to others. Also bidders are assumed to be risk neutral.

Let v_i be the true valuation(or demand) for TB of bidder i, t_i be the private signal only known to i, and s be commonly known signal among bidders. Then generally bidder i's marginal valuation function is given by $v_i = v_i\ (q, t_i, s)$ with $v_q \leq 0$ and t_i & s may be correlated. However, since I restrict my concern to the PIV and symmetric bidders, the valuation function $v_i\ (q, t_i, s)=v(q, t_i)$.

In addition, I denote submitted demand schedule of bidder *i*, by $y_i(p)$ which is assumed to be strictly decreasing, differentiable and to constitute Bayesian Nash Equilibrium. Then $y^{-1}_i(q)$ represents the bid function of *i*.

At market clearing price(MCP), p^c

$$y_i(p^c) = Q - \sum_{j \neq i}^{N} y_j(p^c)$$

Define the distribution function of MCP conditional on submitting $y_i(p)$.

$$H(p, y_i(p)) = \Pr\{y_i(p) \leq Q - \sum_{j \neq i}^{N} y_j(p)\} \; = \Pr\{p^c \leq p \mid y_i(p)\}$$

4.2.1 Multiple Price Auction[30]

Bidder's expected profit maximization problem will be:

$$\text{(1)} \qquad \max_{y(.)} \int_0^\infty \{\int_0^{y(p^c)} [v(q, t_i) - y^{-1}(q)] dq\} dH(p^c, y(p^c))$$

Note that I have dropped the *i* subscript from the demand function.

Let the term in the { } be $\pi(y(p))$, surplus from winning $y(p)$, then

$$\frac{d\pi}{dp} = \frac{d\pi}{dq} \cdot \frac{dq}{dp} = [v(y(p), t_i) - y^{-1}(y(p))] y'(p)$$

And, using integrating by parts, we get

$$\int_0^\infty \pi(y(p)) dH(p, y(p)) = \pi(y(p)) H(p, y(p)) \Big|_0^\infty - \int_0^\infty H(p, y(p)) d\pi(y(p))$$

[30] The derivation section is based upon Hortaçsu(2002) p32.

Since if $p=\infty$ $\pi(y(\infty))=0$ and if $p=0$ $H(0)=0$, equation (1) becomes:

(2) $$\max_{y_i(.)} -\int_0^\infty H(p,y(p))[v(y(p),t_i)-p]y'(p)dp$$

Observe that the integrand is a function of *p, y* and *y'*, denote it by *F(p,y,y')*. The Euler equation which is a necessary condition for optimality is given by

$$F_y = \frac{d}{dp}F_{y'}$$

Therefore, the Euler condition for the differential equation (2) is given by

(3) $$v(y_i(p),t_i) = p + \frac{H(p,y_i(p))}{H_p(p,y_i(p))}$$

4.2.2 Single Price Auction

Since the change in the auction format affects the bidders' optimal behaviors resulting in a different distribution of MCP, I will use a different notation for the bid function and distribution of MCP denoted by *x(p)* and *I(p,x(p))* respectively.

Bidder's expected profit maximization problem in single price auction will be:

(4) $$\max_{x_i(.)} \int_0^\infty \{\int_0^{x(p)} v(q,t_i)dq - px(p)\}dI(p,x(p))$$

In the same way, let the fraction of { } be *π(x(p))* which is surplus from winning *x(p)* units, then

$$\frac{d\pi}{dp} = [v(x(p),t_i) - p]x'(p) - x(p)$$

Using the same boundary conditions and integrating by parts, (4) becomes

(5) $$\max_{x(.)} - \int_0^\infty I(p,x(p))\{[v(x(p),t_i) - p]x'(p) - x(p)\}dp$$

After solving equation (5), we get (6) as Euler condition for the single price auction.

(6) $$v(x_i(p),t_i) = p - \frac{I_x(p,x_i(p))}{I_p(p,x_i(p))} \cdot x(p)$$

4.3 The Model with PIV and Symmetric Bidder-Discrete Case

Now, instead of a continuous bid function where both prices and quantities are perfectly divisible, I will consider the discrete bid function more explicitly. Following the Nautz(1995) and Hortaçsu(2002) analysis, I will maintain perfect divisibility of the quantities but restrict prices to lie on a discrete grid. The set of possible prices are given by

$$p_0 < p_1 < \cdots < p_{K+1}$$

Then bid vector submitted by bidder i takes the form of a series of quantities specified for each of these prices.

$$\vec{y}_i : \{ y_{io} \geq y_{i1} \geq \cdots \geq y_{iK+1} \}$$

Note that I impose the monotonicity constraints on quantity bid vector, $y_{ik} \geq y_{ik+1}$ to obtain decreasing bid function and account for the unobserved bid points.

After all bids are submitted, the Treasury determines the market clearing price by aggregating the quantity bids for each point on the price grid and finding the price at which the total demand falls just short of the total supply:

$$p_{k^*} : k^* = \min\{ k : \sum_{i=1}^{N} y_{ik} \leq Q \}$$ [31]

If we define the MCP in this way, we do not have to care about a rationing problem when several tied bids exist at the market clearing price. Similar to the continuous case, define the probability that the MCP is below p_k, conditional on the bid vector of bidder i, $\vec{y}_i$, to be: $H(p_k, \vec{y}_i)$

However, since bidders do not always place different quantities for all price increments, let $\{ y_{ik1} > y_{ik2} > \cdots > y_{ikL} \}$ be the set of L "observed" quantity bids

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[31] Hortaçsu(2002) also defined the MCP in this way, and mentioned that his conversation with the participants in the Turkish auction market indicated that the rationing was not a big concern. My experience with those in the Korean market tells me that this is the case in Korea as well. However, when I compare the revenue or surplus for the comparison, to make it be more accurate I will calculate the amount after rationing, applying the actual rationing rule.

that the bidder i submits on the price points $\{p_{k1} < p_{k2} < \cdots < p_{kL}\}$ which is a subset of the entire price grid above.

For the time being the value function is assumed to be continuous and differentiable. The distribution of market clearing price is assumed to be continuous and differentiable with respect to the quantity. I will continue to use different notations for the bid vector and the distribution of MCP in multi price auction and single price auction which are denoted by $\vec{y}_i$, *H(.)* for multi price auction and $\vec{x}_i$, *I(.)* for single price auction respectively. Since bidders are assumed to be symmetric, I will derive the optimal condition by dropping subscript *i*.

4.3.1 Multiple Price Auction[32]

The expected payoff of a risk neutral bidder who submits the bid vector $\vec{y} : \{ y_o \geq y_1 \geq \cdots \geq y_{k+1} \}$ will be:

$$(7) \qquad \sum_{k=1}^{K} [\Pr\{ p_{k-1} \leq MCP \leq p_k , given\ \vec{y}\}] \times \{\text{profit on bids above } p_{k-1}\}$$

$$= \sum_{k=1}^{K} [H(p_k, \vec{y}) - H(p_{k-1}, \vec{y})] \times \sum_{j=k}^{K} (\int_{y_{j+1}}^{y_j} v(q, t_i) dq - p_j (y_j - y_{j+1}))$$

[32] The derivation in this part is also basically same as that in Hortaçsu(2002), p33 but I corrected the sign of one term in his FOC which should have been reversed.

Note that most of the products with *H(p_k)* and all the products with *H(p_k)* in the next adding term cancel out which only $H(p_k)\times(\int_{y_{k+1}}^{y_k} v(q,t_i)dq - p_k(y_k - y_{k+1}))$ is left over.

Using this fact and adding the monotonicity constraints,[33] we can rewrite the Lagrangian as:

(8) $$L = \sum_{k=1}^{K} H(p_k, \vec{y})(\int_{y_{k+1}}^{y_k} v(q,t_i)dq + p_k(y_k - y_{k+1})) + \lambda_k(y_k - y_{k+1})$$

The first-order conditions for a maximum are for each j :[34]

(9) $$\begin{aligned} &H(p_k)[v(y_k,t_i) - p_k] + \frac{\partial H(p_k)}{\partial y_k}(\int_{y_{k+1}}^{y_k} v(q,t_i)dq + p_k(y_k - y_{k+1}) + \lambda_k = \\ &H(p_{k-1})[v(y_k,t_i) - p_{k-1}] - \frac{\partial H(p_{k-1})}{\partial y_k}(\int_{y_k}^{y_{k-1}} v(q,t_i)dq + p_{k-1}(y_{k-1} - y_k) + \lambda_{k-1} \end{aligned}$$

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[33] Since the Korean Treasury restricts the maximum bid points up to 5, one may wonder if a number of bid points should be formalized as a constraint in the model. However, the average number of bid points utilized per bidder is 3, so I assume that the constraint on the number of bid points is not binding.

[34] As for the sign of $\frac{\partial H(p_k)}{\partial y_j}$ *and* $\frac{\partial H(p_{k-1})}{\partial y_j}$, both have negative signs because increasing ones quantity at a particular price point either increase the market clearing price or leaves it the same, so that the probability that the market clearing price is below a given price becomes less or equal than what it was.

Suppose that we observe strictly increasing quantity bid at every possible price point, then the monotoicity constraints are not binding, that is $\lambda_k = \lambda_{k-1} = 0$. Hence we can solve this equation for $v(y_k, t_i)$ and we get:

(10) $$v(y_k, t_i) = p_k + \frac{H(p_{k-1})[p_k - p_{k-1}]}{H(p_k) - H(p_{k-1})} - \frac{(B_k + A_k)}{H(p_k) - H(p_{k-1})}$$

where $$A_k = \frac{\partial H(p_k)}{\partial y_k}\left(\int_{y_{k+1}}^{y_k} v(q, t_i)dq + p_k(y_k - y_{k+1}\right),$$

$$B_k = \frac{\partial H(p_{k-1})}{\partial y_k}\left(\int_{y_k}^{y_{k-1}} v(q, t_i)dq + p_{k-1}(y_{k-1} - y_k\right)$$

However we can see distinct quantity bids at only a subset of the possible price points, $\{y_{ik1} > y_{ik2} > \cdots > y_{ikL}\}$. We can interpret that the monotonicity constraint at the observed quantity bid point is not binding, i.e. $\lambda_k=0$, and at the unobserved points the constraint is binding, i.e. $\lambda_k>0$ which means $y_k=y_{k+1}$.

For example, suppose that we observe only (p_2, y_2) and (p_5, y_5) but don't observe (p_3, y_3), (p_4, y_4). If we add the first-order conditions for k=3,4,5, we see that the Lagrange multipliers, λ_3 and λ_4 cancel out from successive equations, and that most of the integral terms conveniently vanish using the facts of $y_3=y_4=y_5$ and that $\frac{\partial H}{\partial y_j}$ is the same across consecutive equations.

Hence the left over is:

$$(11) \quad \begin{aligned} &H(p_5)[v(y_5,t_i) - p_5] + \frac{\partial H(p_5)}{\partial y_5}\left(\int_{y_6}^{y_5} v(q,t_i)dq + p_5(y_5 - y_6)\right) \\ &= H(p_2)[v(y_5,t_i) - p_2] - \frac{\partial H(p_2)}{\partial y_5}\left(\int_{y_5}^{y_2} v(q,t_i)dq + p_2(y_2 - y_5)\right) \end{aligned}$$

After rearranging the terms and solving for $v(y_5, t_i)$ we get:

$$(12) \quad \begin{aligned} &v(y_5,t_i) = p_5 + \frac{H(p_2)[p_5 - p_2]}{H(p_5) - H(p_2)} \\ &- \frac{[\frac{\partial H(p_2)}{\partial y_5}\left(\int_{y_5}^{y_2} v(q,t_i)dq + p_2(y_2 - y_5)\right) + \frac{\partial H(p_5)}{\partial y_5}\left(\int_{y_6}^{y_5} v(q,t_i)dq + p_5(y_5 - y_6)\right)]}{H(p_5) - H(p_2)} \end{aligned}$$

Note that this is entirely expressed by the observed bids, (p_2, y_2) and (p_5, y_5).

If we rewrite the above equation using general notations of the observed bids, $\{y_{ik1} > y_{ik2} > \cdots > y_{ikL}\}$ and $\{p_{ik1} < p_{ik2} < \cdots < p_{ikL}\}$

$$(13) \quad v(y_{km},t_i) = p_{km} + \frac{H(p_{km-1})[p_{km} - p_{km-1}]}{H(p_{km}) - H(p_{km-1})} - \frac{(B_{km} + A_{km})}{H(p_{km}) - H(p_{km-1})}$$

where $A_{km} = \frac{\partial H(p_{km})}{\partial y_{km}}\left(\int_{y_{km+1}}^{y_{km}} v(q,t_i)dq - p_{km}(y_{km} - y_{km+1})\right)$,

$$B_{km} = \frac{\partial H(p_{km-1})}{\partial y_{km}}\left(\int_{y_{km}}^{y_{km-1}} v(q,t_i)dq - p_{km-1}(y_{km-1} - y_{km})\right)$$

In order to get the value of $v(y_{km}, t_i)$ we need to know the exact form of $v(q, t_i)$. Therefore we will assume that the marginal valuations are given by a step function which assume constant values, $v(y_{km})$ on (y_{km+1}, y_{km}). Figure 5 illustrates our step functional marginal valuation.

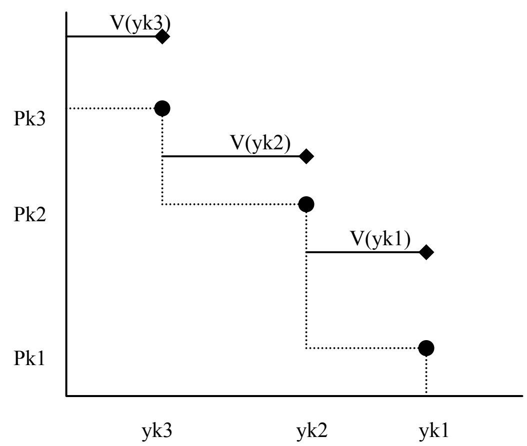

Figure 5. Illustration of Step Valuation Function

Then the integrals in A_{km} and B_{km} can be evaluated and these terms become:

$$A_{km} = \frac{\partial H(p_{km})}{\partial y_{km}} \{(v(y_{km}) - p_{km})(y_{km} - y_{km+1})\}$$

$$B_{km} = \frac{\partial H(p_{km-1})}{\partial y_{km}} \{(v(y_{km-1}) - p_{km-1})(y_{km-1} - y_{km})\}$$

Now, let's consider the boundary conditions.

Firstly the boundary condition for the bid with the lowest price (p_{k1}, y_{k1}) :

I assume $y_{k1}=y_0$. This is plausible because the bidder is also willing to accept q_{k1} units at the minimum price grid, p_0. Without loss of generality, we can assume $p_0=0$ and $H(p_0)=0$.

Hence:

$$v(y_{k1}, t_i) = p_{k1} - \frac{\frac{\partial H(p_{k1})}{\partial y_{k1}} \{(v(y_{k1}) - p_{k1})(y_{k1} - y_{k2})\}}{H(p_{k1})} \tag{14}$$

Secondly the boundary condition for the bid with the highest price (p_{kL}, y_{kL}) :

I assume "unobserved bid" for price p_{kL+1}, $y_{kL+1}=0$ because the bidder is not willing to accept any units of the bond for prices above p_{kL}.

So:

$$v(y_{kL}, t_i) = p_{kL} + \frac{H(p_{kL-1})[p_{kL} - p_{kL-1}]}{H(p_{kL}) - H(p_{kL-1})} - \frac{[\frac{\partial H(p_{kL-1})}{\partial y_{kL}}\{(v(y_{kL-1}) - p_{mL-1})(y_{mL-1} - y_{mL})\} + \frac{\partial H(p_{kL})}{\partial y_{kL}}\{(v(y_{kL}) - p_{kL})y_{mL}\}]}{H(p_{kL}) - H(p_{kL-1})} \tag{15}$$

Hence, given estimates of $H(p_{km})$ and $\frac{\partial H(p_{km})}{\partial y_{km}}$, we have a recursive set of L linear equations which we can solve for the "steps" of the marginal valuation function.

4.3.2 Single Price Auction

The expected payoff of a risk neutral bidder in single price auction is given by:

$$\sum_{k=1}^{K}[I(p_k, \vec{x}) - I(p_{k-1}, \vec{x})](\int_0^{x_k} v(q, t_i)dq - p_k x_k) \tag{16}$$

Note that

$$\int_0^{x_k} v(q, t_i)dq - p_k x_k = \int_0^{x_{k-1}} v(q, t_i)dq - p_{k-1}x_{k-1} - \int_{x_k}^{x_{k-1}} v(q, t_i)dq - p_k x_k + p_{k-1}x_{k-1}$$

Using this fact, we can rewrite the expected payoff as:

$$(17) \qquad \sum_{k=1}^{K} I(p_k, \vec{x})\left(\int_{x_{k+1}}^{x_k} v(q,t_i)dq + p_{k+1}x_{k+1} - p_k x_k\right)$$

Therefore the Lagrangian of the objective function of a bidder with adding the monotonicity constraints can be written as:

$$(18) \qquad L = \sum_{k=1}^{K} I(p_k, \vec{x})\left(\int_{x_{k+1}}^{x_k} v(q,t_i)dq + p_{k+1}x_{k+1} - p_k x_k\right) + \lambda_k (x_k - x_{k+1})$$

The first-order conditions for a maximum are for each k :

$$(19) \qquad \begin{aligned} &I(p_k)[v(x_k,t_i) - p_k] + \frac{\partial I(p_k)}{\partial x_k}\left(\int_{x_{k+1}}^{x_k} v(q,t_i)dq + p_{k+1}x_{k+1} - p_k x_k\right) + \lambda_k = \\ &I(p_{k-1})[v(x_k,t_i) - p_k] - \frac{\partial I(p_{k-1})}{\partial x_k}\left(\int_{x_k}^{x_{k-1}} v(q,t_i)dq + p_k x_k - p_{k-1}x_{k-1}\right) + \lambda_{k-1} \end{aligned}$$

Suppose that we can observe strictly increasing quantity bid at every possible price point, since $\lambda_k = \lambda_{k-1} = 0$ we can solve this equation for $v(x_k, t_i)$ and we get:

(20)

$$v(x_k,t_i) = p_k - \frac{\left[\frac{\partial I(p_{k-1})}{\partial x_k}\left(\int_{x_k}^{x_{k-1}} v(q,t_i)dq + p_k x_k - p_{k-1}x_{k-1}\right) + \frac{\partial I(p_k)}{\partial x_k}\left(\int_{x_{k+1}}^{x_k} v(q,t_i)dq + p_{k+1}x_{k+1} - p_k x_k\right)\right]}{I(p_k) - I(p_{k-1})}$$

With the same arguments on observed quantity bids as a subset of the possible price points in the multiple price auction case, we can derive the FOC which is expressed in terms of only observed bid points.

For example, suppose that we observe only *(p_2, x_2)* and *(p_5, x_5)* but don't observe *(p_3, x_3)* , *(p_4, x_4)*. Let the first order conditions for k=3,4,5, be [1], [2], and [3] respectively. If we get [1]-[2]+[3] and use the fact of $x_3=x_4=x_5$, then we see that the Lagrange multipliers, λ_3 and λ_4 cancel out from successive equations, and that most of the integral terms conveniently vanish.

Hence the left over is:

$$\begin{aligned}
& I(p_5)[v(x_5,t_i)-p_5]+\frac{\partial I(p_5)}{\partial x_5}(\int_{x_6}^{x_5} v(q,t_i)dq + p_6x_6 - p_5x_5) \\
& + I(p_3)[v(x_5,t_i)-p_3]-I(p_4)[v(x_5,t_i)-p_4]= \\
& I(p_2)[v(x_5,t_i)-p_3]-\frac{\partial I(p_2)}{\partial x_5}(\int_{x_5}^{x_2} v(q,t_i)dq + p_3x_5 - p_2x_2) \\
& - I(p_3)[v(x_5,t_i)-p_4]+I(p_4)[v(x_5,t_i)-p_5]
\end{aligned}$$

After rearranging the terms we get:

$$\begin{aligned}
& (I(p_3)-I(p_2))(v(x_5,t_i)-p_3)-(I(p_4)-I(p_3))(v(x_5,t_i)-p_4) \\
& + (I(p_5)-I(p_4))(v(x_5,t_i)-p_5) \\
& = -[\frac{\partial I(p_2)}{\partial x_5}(\int_{x_5}^{x_2} v(q,t_i)dq + p_3x_5 - p_2x_2)+\frac{\partial I(p_5)}{\partial x_5}(\int_{x_6}^{x_5} v(q,t_i)dq + p_6x_6 - p_5x_5)]
\end{aligned}$$

Now we assume that at the unobserved bid point *j*, *$I(p_j)-I(p_{j-1})=0$*, which implies *$I(p_2)=I(p_3)$* in this case.[35] This follows because a bidder need not place bid x_3 at higher price p_3 if he cannot increase his chance of winning at the higher price. This implies $x_3=x_4$. In the same context, we can conclude *$I(p_2)=I(p_3)=I(p_4)$*.

Using this equality, the first two terms in the left-hand side of the above equation vanish and *$I(p_4)$* in the third term can be replaced by *$I(p_2)$*.

Therefore, we get:

[35] Nautz provides more detailed discussion for this argument. See Nautz(1995) p304.

(21)

$$v(x_5,t_i) = p_5 - \frac{[\frac{\partial I(p_2)}{\partial x_5}(\int_{x_5}^{x_2} v(q,t_i)dq + p_3x_5 - p_2x_2) + \frac{\partial I(p_5)}{\partial x_5}(\int_{x_6}^{x_5} v(q,t_i)dq + p_6x_6 - p_5x_5)]}{I(p_5) - I(p_2)}$$

Since $p_3x_5 - p_2x_2 = x_5(p_3 - p_2) - p_2(x_2 - x_5)$ and $p_6x_6 - p_5x_5 = x_6(p_6 - p_5) - p_5(x_5 - x_6)$, if we rewrite the above equation using general notations of the observed bids, $\{x_{ik1} > x_{ik2} > \cdots > x_{ikL}\}$ and $\{p_{ik1} < p_{ik2} < \cdots < p_{ikL}\}$ with letting p_3-p_2 be Δp, then we can finally get:

(22) $$v(x_{km},t_i) = p_{km} - \frac{[\frac{\partial I(p_{km})}{\partial x_{km}} x_{km+1}(p_{km+1} - p_{km}) + \frac{\partial I(p_{km-1})}{\partial x_{km}} x_{km}\Delta p]}{I(p_{km}) - I(p_{km-1})} - \frac{(B_{km} + A_{km})}{I(p_{km}) - I(p_{km-1})}$$

where $A_{km} = \frac{\partial I(p_{km})}{\partial x_{km}}(\int_{x_{km+1}}^{x_{km}} v(q,t_i)dq - p_{km}(x_{km} - x_{km+1})$,

$$B_{km} = \frac{\partial I(p_{km-1})}{\partial x_{km}}(\int_{x_{km}}^{x_{km-1}} v(q,t_i)dq - p_{km-1}(x_{km-1} - x_{km})$$

Note that this is entirely expressed by the observed bids, (p_{km}, x_{km}), (p_{km-1}, x_{km-1}), and price increment, Δp.

If we assume that the marginal valuation is a step function same as before, then the integrals in A_{km} and B_{km} can be evaluated and these terms become:

$$A_{km} = \frac{\partial I(p_{km})}{\partial x_{km}}\{(v(x_{km}) - p_{km})(x_{km} - x_{km+1})\}$$

$$B_{km} = \frac{\partial I(p_{km-1})}{\partial x_{km}}\{(v(x_{km-1}) - p_{km-1})(x_{km-1} - x_{km})\}$$

After considering the boundary conditions, we can have a recursive set of L linear equations similar to the case of multiple price auction.

The boundary condition for the bid with the lowest price (p_{k1},x_{k1}) :

Using $x_{kl}=x_0$, $p_0=0$ and $I(p_0)=0$, we get:

(23) $$v(x_{k1},t_i) = p_{k1} - \frac{[\frac{\partial I(p_{k1})}{\partial x_{k1}} x_{k2}(p_{k2}-p_{k1}) + (v(x_{k1})-p_{k1})(x_{k1}-x_{k2})]}{I(p_{k1})}$$

Next, the boundary condition for the bid with the highest price (p_{kL},x_{kL}) :

Using the implied bid for price p_{kL+1}, $x_{kL+1}=0$, we get:

(24)

$$v(x_{kL},t_i) = p_{kL} - \frac{[\frac{\partial I(p_{kL-1})}{\partial x_{kL}} x_{kL}\Delta p + \frac{\partial I(p_{kL-1})}{\partial x_{kL}}(v(x_{kL-1})-p_{kL-1})(x_{kL-1}-x_{kL}) + \frac{\partial I(p_{kL})}{\partial q_{kL}}(v(x_{kL})-p_{kL})x_{kL}]}{I(p_{kL})-I(p_{kL-1})}$$

4.4 Comparison of FOCs Between the Continuous and the Discrete Case

Now, let's compare the discrete version of FOCs with the continuous case. We will write down the FOCs again. The FOCs in the continuous case are given by:

multiple price auction: $v(y_i(p),t_i) = p + \dfrac{H(p,y_i(p))}{H_p(p,y_i(p))}$

single price auction: $v(x_i(p),t_i) = p - \dfrac{I_x(p,x_i(p))}{I_p(p,x_i(p))} \cdot x(p)$

The FOCs in the discrete case are as followings:

multiple price auction:

$$v(y_{km}, t_i) = p_{km} + \frac{H(p_{km-1})[p_{km} - p_{km-1}]}{H(p_{km}) - H(p_{km-1})} - \frac{(B_{km} + A_{km})}{H(p_{km}) - H(p_{km-1})}$$

single price auction :

$$v(x_{km}, t_i) = p_{km} - \frac{[\frac{\partial I(p_{km})}{\partial x_{km}} x_{km+1}(p_{km+1} - p_{km}) + \frac{\partial I(p_{km-1})}{\partial x_{km}} x_{km} \Delta p]}{I(p_{km}) - I(p_{km-1})} - \frac{(B_{km} + A_{km})}{I(p_{km}) - I(p_{km-1})}$$

where A_{km} and B_{km} are as defined in the previous section

If we ignore the correction terms of A_{km} and B_{km}, we can see that the both FOCs in discrete case are naturally analogous to those in the continuous case.

In the next CHAPTER, when I calculate the marginal valuations of bidders that correspond to each price-quantity pair submitted, I will try to test all three types of FOCs, which are the continuous version, the discrete version without the correction terms and the discrete version with the correction terms.

The reason for testing the continuous version of FOCs is to see how much difference the results would yield when we simply assume the continuous strategy space despite the discrete strategy space nature of the real auction environment. This attempt may give us the chance to verify Fabra, et al.(2004)'s argument which pointed out the possibility that the continuous bid function and the discrete bid function may yield different outcomes.

The purpose of implementing both the discrete version without the correction terms and that with the correction terms is to check the effect of correction terms. Hortaçsu(2002) who performed the structural estimation for multiple price auction in the Turkish Treasury auction market, found that the correction terms derived in the discrete case were negligible in almost all instances. Hence he argued that in most practical settings where the quantity of objects for sale are very large, the FOC without correction term can be used as the estimating equation without much loss of accuracy. I will try to verify if this conjecture is valid for the Korean Treasury auction and the single price auctions.

Regarding the FOCs derived up to now, I should note that there might be another question about the existence and uniqueness of equilibrium bid functions. In fact, many researchers have pointed out the possibility of multiple equilibria, but the general and formal works for this matter have not been found yet.[36] However, as Hortaçsu (2002) addressed, regardless of which equilibrium is selected among possible equilibria, the equilibrium bid functions must obey the FOCs derived.

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[36] Regarding the uniqueness of equilibrium in multiunit auction, Ausubel and Crampton (2002) mentioned that "In Particular we know from Wilson (1979) and subsequent papers that when the items are infinitely divisible, a vast multiplicity of equilibria is probably inherent to *the uniform price auction* and, in any case, the calculation of equilibria may be difficult." On the other hand, as for the multiple price auction, Hortaçsu (2002) said that "…whether a given set of model primitives(the vector of marginal valuation functions and joint distribution of signals) leads to a unique set of equilibrium bid functions. Unlike the independent private value first-price auction, there are no results regarding the uniqueness of the Bayesian Nash equilibrium of *a discriminatory auction*."

CHAPTER V

THE EMPIRICAL FRAMEWORK

5.1 Nonparametric Identification

As pointed out in the literature surveys, I have to acknowledge that the "Policy Experimental Approach" has a limit when it comes to analyze the interaction of the bidders' strategic behaviors which get inevitably involved in the auction game, and affects the outcome of the revenue. Therefore this paper will basically pursue the "Structural Model Approach" instead of the "Policy Experiment".[37]

Regarding the identification problem of the structural model estimation, Guerre et al. (2000)[38] solved the identification problem for the first price single-unit auction with the PIV and symmetric bidders. Their theorem states that there exist a unique distribution of underlying value $F(v_i)$ such that $G(b_i)$ is the distribution of equilibrium

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[37] However, in case of testing the underlying value structure under the environment of multi-unit or divisible good auction, the applicable structural model approach does not yet exist to my knowledge. Hence, to deal with this matter, I will utilize the idea and method suggested by the policy experiment approach in CHAPTER 7.

[38] See Guerre et al. (2000) , p527-531. For the reference, the equilibrium condition is given by:

$$v_i = \xi(b_i, G, I) \equiv b_i + \frac{1}{I-1}\frac{G(b_i)}{g(b_i)} \ ,$$

where v_i: private value, b_i: observed bid, I: # of bidders,

$G(b)$, $g(b)$, $F(v)$, $f(v)$: cdf and pdf of bid and value resectively,

satisfying $g(b)/G(b)=(1/s'(v))f(v)/F(v)$ and $s(v)=b$

bids if and only if (i) bids are independent and identically distributed, and (ii) the inverse of equilibrium strategy is strictly increasing over the support of bids. Therefore if one can estimate the distribution of observed bids $G(b_i)$ the distribution $F(.)$ as well as bidder's v_i are identified.

Using the insight of Guerre et al., Hortaçsu(2002) built the identification strategy for the structural model estimation in the Treasury auction.

Note that the first order condition which was derived under private independent value(PIV) paradigm in CHAPTER IV is expressed entirely by the observed bid price, bid quantity and the distribution of market clearing price. Therefore, if we make the assumption that the bid functions we observe in the data are indeed generated by a Bayesian Nash equilibrium of the auction game, and if we can estimate the latent distribution of market clearing price which is uniquely determined by observed bid prices and bid quantities, then marginal valuations, $v(q(p,t_i),t_i)$, corresponding to each point on the bid function, $q(p,t_i)$, can be identified.

To see more clearly, let us define the following distribution:[39]

(25) $$G_i(p,q) = \Pr\{q(p,t_i) \le Q - \sum_{j \ne i}^{N} q(p,t_j)\}$$

which is the probability that a demand of quantity is less than the residual supply faced by bidder i at price p. Observe that the distribution defined is simply the probability that MCP is less than p, i.e., the distribution of market clearing price, and also

[39] See Hortaçsu(2002) p8~9.

corresponds to the distribution of bid, $G(b_i)$ in Geurre et al. (2000). Because we assumed that the observed bid points are satisfied with the equilibrium condition, this probability can be estimated for all (p,q) pairs if the joint distribution of observed $q(p,t_j)$, $j \neq i$ can be estimated from the data.

Then, the following equalities (26)-(28) hold, and all components of the first order condition are identified from the data so that the marginal valuation corresponding to each bid point are identified.[40]

(26) $$H(p, q(p,t_i)) = G_i(p,q)\,|_{q=q(p,t_i)}$$

(27) $$\frac{\partial}{\partial p} H(p, q(p,t_i)) = \frac{\partial}{\partial p} G_i(p,q)\,|_{q=q(p,t_i)}$$

and

(28) $$\frac{\partial}{\partial q} H(p, q(p,t_i)) = \frac{\partial}{\partial q} G_i(p,q)\,|_{q=q(p,t_i)}$$

The next step is to estimate the distribution of $G_i(p,q)$. This can be done either parametrically or nonparametrically. In the case of the single unit auction, the parametric approach assumes that $F(.)$ follows a specific distribution such as lognormal or exponential distribution so that $F(.)$ is parameterized by a vector of unknown

[40] In addition, Hortaçsu(2002) emphasized the point that this identification argument did not assume anything about the symmetry of bidder. It implies that, in the extended model considering asymmetry, the structural model is also identified if the *i.i.d*. assumption between and within the groups holds and the necessary condition doesn't change. I will give more discussion about this in CHAPTER 7.

parameters. Then it estimates the unknown parameters using a likelihood-based approach.[41]

For our case with PIV setting, since the residual supply that a bidder *i* faces is a function of *N-1* signals, $(t_1, t_2, \ldots t_{N-1})$, which are random variables, we can simulate the gamut of residual supply curves with the assumptions on the distribution of signals and the mapping of the signal to bid quantities. That is, we can generate random draw of the *N-1* element signal vector of the competing bidders from known $F(t)$, and we can evaluate *N-1* equilibrium opponents' bids corresponding to these signals.

However, the distribution of bidders' signals is not observed by the researcher. More importantly, the differential equation characterizing the Bayesian Nash equilibrium strategy cannot be solved explicitly in many auction models or, when this can be done, that the solution is highly nonlinear in the latent distribution, and the exact mapping from the signals to the equilibrium bid vectors is generally not available in a closed form. Thus the parametric approach is usually associated with many priori assumptions and immense computational burden.

For these reasons, the considerable efforts to develop non parametric estimations have been made to circumvent this difficulty which is resulted from the parametric approach. However, most of the non parametric structural model estimation analyses are found in the single unit auction environment[42]. The examples applied to the multi

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[41] According to the survey paper by Hendricks and Paarsch (1995), the parametric approaches can be divided into the method of maximum likelihood proposed by Donald and Paarsch (1991) and the method of simulated non-linear least squares proposed by Laffont, Ossard, and Vuong (1995).

[42] For example, Guerre et al. (2000), Li, Perrigne, and Vuong (2000, 2002).

unit auction environment are seldom found except for the method suggested by Hortaçsu (2002).

Therefore, I will follow the nonparametric strategy developed by Hortaçsu to estimate the main component of my structural model which is the distribution of the market clearing price. In the next section, I will introduce his strategy and discussion related to the asymptotic property of the estimation in detail.

5.2. Resampling Method to Estimate *H(.)*(or *I(.)*)

Hortaçsu(2002) suggested the following procedure simplifying the estimation problem:

1) Fix bidder i among total N bidders in auction t.

2) From the sample of N bid vectors, draw a random sample of N-1 with replacement, giving equal probability of 1/N.[43]

3) Construct the residual supply function generated by N-1 "resampled" bid vectors.

4) Intersect with bidder i's bid to find the market clearing price.

5) Repeat B times for each bidder and generate B market clearing prices conditional on $q_i(p)$ vector.

6) Estimate H(.) by counting the frequency with which a given p_k remained above the market clearing prices generated above.

[43] the number of possible cases : N^{N-1}

A set of point estimates for marginal valuations rationalizing each price-quantity pair observed in one auction yields perfectly competitive outcome in this auction where bidders reveal their true value. Aggregating these estimate across the bidders and finding the intersection of this aggregated schedule with total supply, we can calculate the market clearing price and revenue which is considered to be the ideal competitive outcome in the auction.

The resampling procedure has definite advantages in terms of its minimal use of distributional assumptions, but heavily relies on the assumption that bid functions are *i.i.d*. Another practical concern with the resampling procedure is the lack of precise estimates in the case where the number of bidders *N* is small.

Regarding the consistency problem, Hortaçsu provided the asymptotic properties of the resampling estimator.(His Proposition 1)[44] He suggested two kinds of properties for the resampling estimator. The first one is for the case when using all the auction data across different points of time. That is, if the bids in different auctions are generated from the same distribution, then as *NT*(number of bidders times number of auctions) goes to infinity, the resampling estimator $\hat{\mathrm{H}}^{R}(p_k, \vec{y}_i)$ converges to $H(p_k, \vec{y}_i)$ almost surely.

This property is basically when the auctions occur in a static environment. However, in the TB auction, it is plausible to expect that the economic environment surrounding the bidders changes from auction to auction. Because the auction data is almost on a monthly basis, there might be lots of uncontrollable factors between the

[44] See Hortacsu(2002) p12~p13 and Appendix 8.3~8.5 for related proofs.

auctions. Considering this, he also suggested the second property for the case when we use data from a single auction as following:

> *If we use data from a single auction, that is, set T=1, but let the number of bidders, N, go to infinity, then* $\hat{\mathrm{H}}^R(p_k, \vec{y}_i)$ *is a consistent estimator of* $H(p_k, \vec{y}_i)$ *if* $E\,\vec{y}(s)$ *can be estimated consistently at a rate faster than* $\sqrt{N}$ *. This can be done by a two step procedure in which we first compute* $\hat{\mathrm{H}}^R(p_k, \vec{y}_i)$ *using data from a single auction, and then correct this estimate using an estimator of* $E\,\vec{y}(s)$ *that uses data across auctions (hence convergence is at rate* $\sqrt{NT}$ *where T can be taken to infinity independent of N).*

He noted that since the market clearing price is a statistic that aggregates bidders' signals, the resampling algorithm can be interpreted as "bootstrapping" the market clearing price about its observed value. Thus the standard bootstrap consistency results suggest that, as the number of bidders in the auction increase, the distribution of the bootstrapped market clearing price around the realized value of the market clearing price is a consistent estimate of the distribution of the realized market clearing price about its population mean.

Although he pointed out the necessity of the procedure of correction to get the desired consistency, he showed through Monte-Carlo experiment that even without the above additional correction related to the convergence rate of the population mean, the resampling algorithm that uses data from a single auction performed quite well.

5.3 Estimation of Derivatives

Since Hortaçsu analyzed only multiple price auction data and estimated the unknown true value by using the discrete version of FOC without correction terms, he didn't have to care about the estimation for the first derivatives of the MCP distribution function with respect to price and quantity, H_p(or I_p) and H_y(or I_x).

However, in my analysis to attempt to use all of the three types of FOCs which has been derived in the previous CHAPTER, we need to estimate the derivatives to completely recover the true value.

5.3.1 Estimation of H_p(or I_p)

In order to utilize the continuous version of FOC directly, we have to estimate the first derivative of the distribution of MCP to price.[45] Since each bidders bid function is given by a step function in our data, the residual supply constructed by the resampled bid vectors will also be a step function so that the resulting market clearing prices should lie on a finite discrete price grid. Figure 6 illustrates this situation.

Therefore, to get the MCPs dispersed continuously so as to estimate continuous type of H_p, we need to have a continuous functional form of either individual bid vectors or residual supply. In the case of the individual's bid vector, there are at the most up to 5 steps because maximum number of bids per bidder is restricted to 5.

[45] Note that we don't have to have this derivative in using discrete version of FOC

Making the bid vector be continuous may introduce new errors and result in excessive smoothness. Hence, I will smooth out residual supply and intersect them with individual step bid function. This procedure is depicted in Figure 7 and Figure 8.

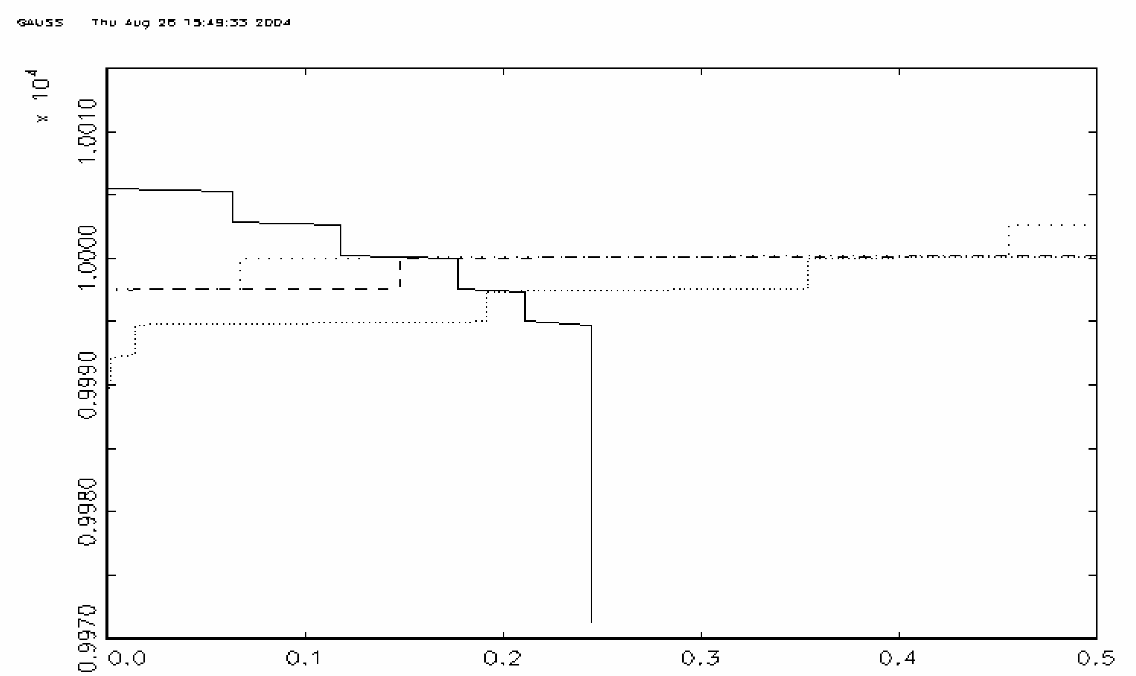


FIGURE 6. Illustration of Discrete Distribution of MCP

To obtain smooth residual supply curves, I use nonparametric kernel estimation for the regression function. Let the price quantity pairs on the residual supply be $\{(p_1,q_1), (p_2,q_2),\ldots, (p_K,q_K)\}$, then the formula is given by:[46]

---

[46] Wolak(2003) suggested a slightly different formula, which is $RS(p) = \sum_{k=1}^{K} q_k K(\frac{p_k - p}{h})$ (where, q_k : bid increment at price p_k, $K(.)$: standard normal cdf.), but I found that both ways yielded very similar results. In addition, as the smoothing parameter, I chose the value by ad-hoc method which is standatd in nonparametric structural estimation.

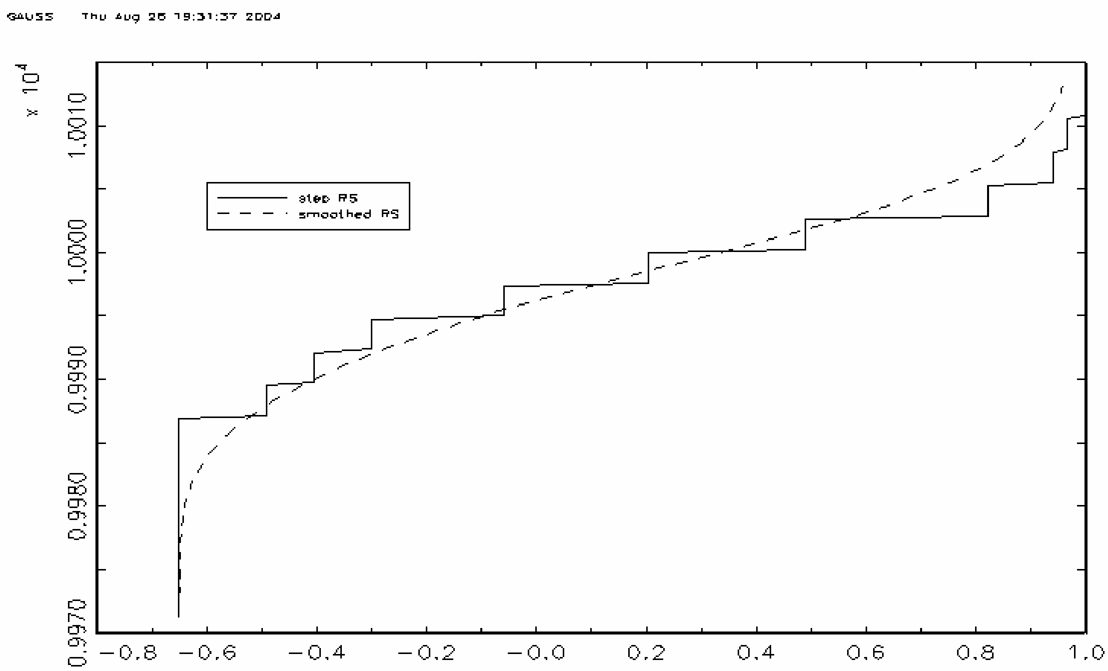


FIGURE 7. Smoothing the Residual Supply

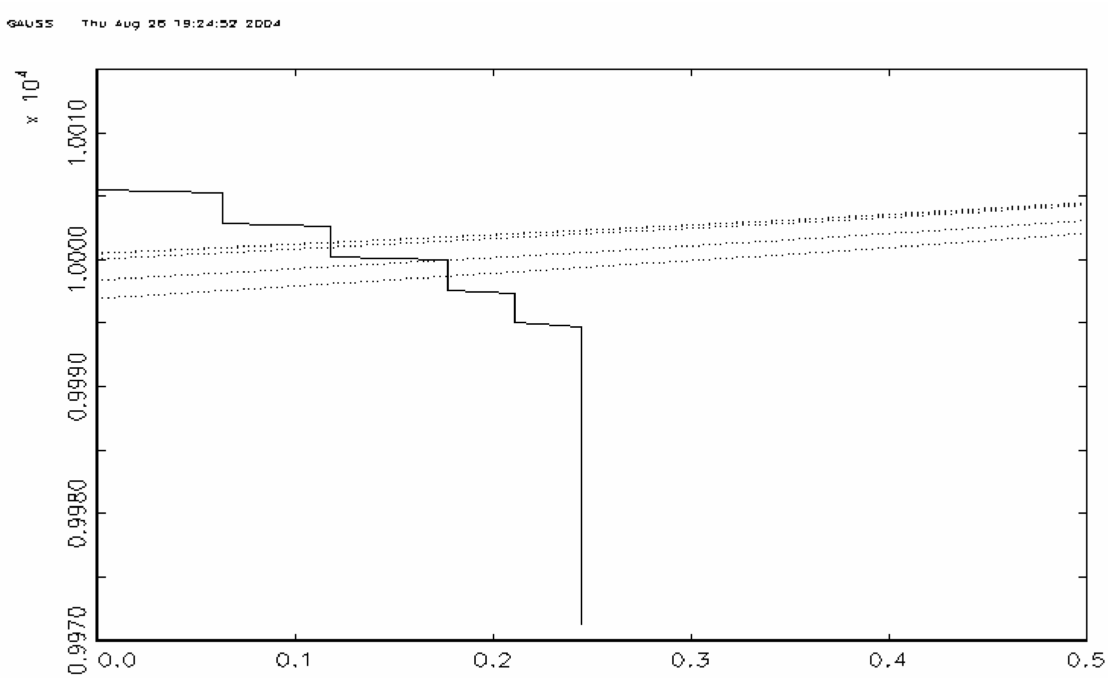


FIGURE 8. Intersection of Step Bid and Smoothed Residual Supply

$$RS(p) = \frac{\sum_{k=1}^{K} q_k K(\frac{p_k - p}{h})}{\sum_{k=1}^{K} K(\frac{p_k - p}{h})}$$

where *K(.):* standard normal kernel function,

h: smoothing parameter(or bandwidth)

If we smooth bid functions, then we can get continuously spread MCPs. With these *B* resampled MCPs, I estimate *H(p,y(p))* by the same frequency method, i.e, $\frac{\{\#ofMCPs \leq p \mid y(p)\}}{B}$.

To estimate H_p, we can evaluate the following amount for "small" *dp*: [47]

$$[H(p, y(p)) - H(p+dp, y(p))]/dp = [\Pr\{p^c \leq p \mid y(p)\} - \Pr\{p^c \leq p+dp \mid y(p)\}]/dp$$

However more formally, we can estimate this using nonparametric kernel estimation. That is, with *i.i.d*. data $\{X_1, X_2, \ldots, X_n\}$ randomly drawn form distribution *F(x)*, pdf *f(x)=dF/dx* can be estimated nonparametrically using the following formula:

$$f(x) = \frac{1}{nh} \sum_{i=1}^{n} K(\frac{X_i - x}{h})$$

Since $H_p(p, y(p)) = \frac{\partial}{\partial p} \Pr\{p^c \leq p \mid y(p)\}$,i.e., the density function of *H(p,y(p))* when *y(p)* is submitted, so that in our case, *H(.)* is corresponding to *F(.)*, and *Hp(.)* is to *f(.)*, our formula will be:

[47] Since *H(.)* is the probability distribution *conditional on* a certain bidder's bid vector, the change in price must reflect the change in *y(p)*. However, because our resampled MCPs have been obtained by intersecting a certain fixed individual bid vector, in the difference of *H(.)* between two price points, this effect is already included.

$$\hat{H}_p(p, y(p)) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{P_i^c - p}{h})$$

where $P_i^c = \{ P_1^c, P_2^c, \ldots, P_B^c \}$ is B MCPs which are obtained by the resampling, with # of iteration B

Figure 9 displays the estimated H_p of bidder #29 at auction #3 by both kernel estimation and calculating incremental change for "small" *dp*. We can see the results of the incremental changes which are represented by x are located along the line estimated by the kernel function so that both methods are not much different from each other in a practical sense. Throughout I will use the kernel estimation results of H_p to recover the true value.

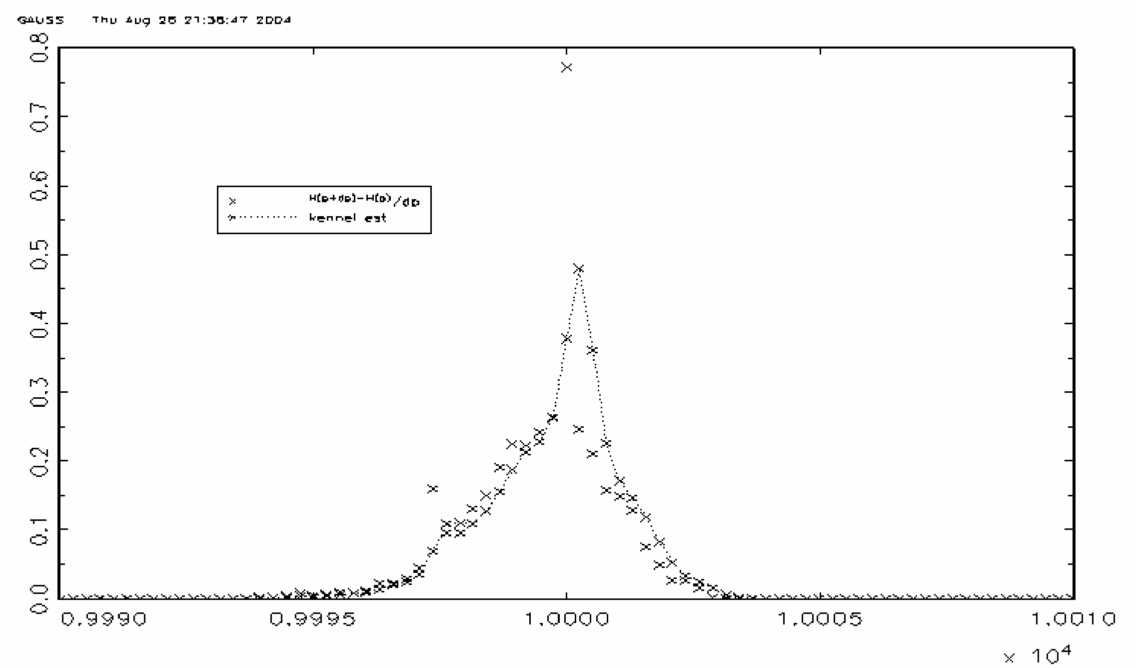


FIGURE 9 − Comparison of Kernel Estimation and Calculation of Incremental Change of H_p

5.3.2 Estimation of H_y(or I_x)

Except for the cases using the continuous version of FOC and the discrete version of FOC without correction terms in multiple price auction, we need to estimate the first derivative of MCP distribution with respect to quantity, $\frac{\partial H(p_k)}{\partial y_k}$. Especially, to use the discrete version of FOCs with correction terms $\frac{\partial H(p_{k-1})}{\partial y_k}$ also need to be estimated. [48]

To estimate this, I use the fact that the distribution of MCP can be represented as the probability distribution of the sum of *N-1* bid quantities which are *i.i.d.* random variables. That is:

$$(26) \qquad H(p_k, y_i(p)) = \Pr\{p_{k^*} \le p_k \mid y_i(p)\} = \Pr\{\sum_{j \ne i}^{N} y_j(p_k) \le Q - y_i(p_k)\}$$

In words, given price point p_k the probability that MCP is less than or equal to this price is same as the probability that sum of the competitors' bid amount at p_k is less than or equal to total supply minus *i*'s own bid amount at p_k.

Let the residual supply faced by bidder *i* at price p_k be:

$RS(p_k) = Q - \sum_{j \ne i}^{N} y_j(p_k)$.

Then we can rewrite the distribution of MCP:

---

[48] $\frac{\partial H(p_k)}{\partial y_k}$ can be interpreted by the shift in the distribution of MCP due to a change in *y(p)*.

$$\Pr\{\sum_{j\neq i}^{N} y_j(p_k) \leq Q - y_i(p_k)\} = \Pr\{RS(p_k) \geq y_i(p_k)\}$$

Let $F_i(.,y_i(p)), f(.)$ denote the cdf and the pdf of *RS(p) conditional on* $y_i(p)$, then $H(p,y(p))=1- F_i(.,y_i(p))$.

Hence,

$$H_y(p, y(p)) = \frac{\partial}{\partial y}(1 - \Pr\{RS(p) \leq y(p)\}) = \frac{\partial}{\partial y}\{1 - F(RS, y(p)\} = -f(RS, y(p))$$

Since we can calculate 5000 residual supply amount at each price from our 5000 resampled residual supply functions, we can estimate $-f(RS)$ at fixed price point by the kernel estimation.

The formula is given by:

$$\hat{H}_y(p, y(p)) = -\frac{1}{nh}\sum_{i=1}^{n} K(\frac{RS_i(p) - y(p)}{h})$$

Alternatively, by the same way as the case of *Hp* we can calculate $\frac{H(y, p_k) - H(y + \Delta y, p_k)}{\Delta y}$ for "small" Δy after obtaining $H(y \mid p = p_k)$ by counting the frequency with which a given y remained below the residual supplies at the given price p_k.

Figure 10 displays the estimated H_y[49] of bidder #30 at auction #27 by both ways suggested above. I show examples at three different price levels where this bidder placed a bid quantity. In calculating incremental change I used minimum bid amount as "small" Δy. We can see the results from incremental change are distributed along

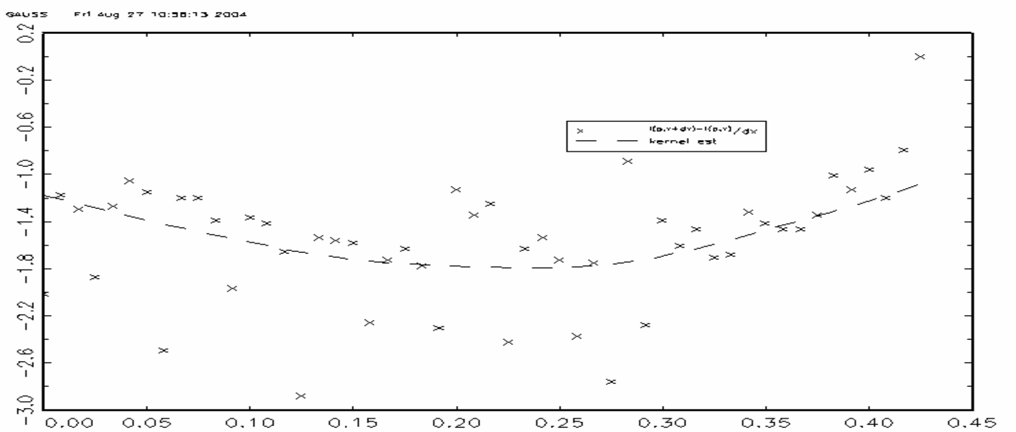
[49] More precisely, according to my notation, this is I_x.

the line estimated by kernel function which is as similar as H_p case. Henceforth I will use the result of kernel estimation from the next section on.

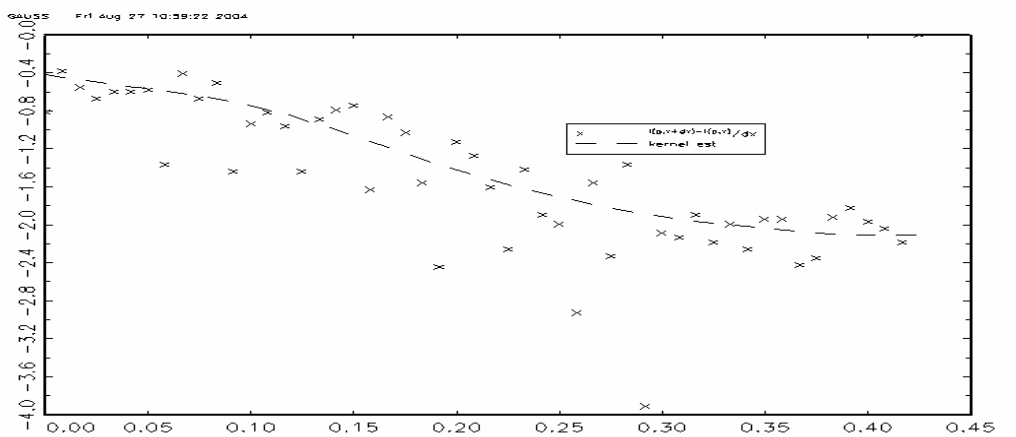
We can also estimate $\frac{\partial H(p_{k-1})}{\partial y_k}$ by the same way as that used in estimating $\frac{\partial H(p_k)}{\partial y_k}$, that is, either by calculating $[H(P_{k-1}|y_k)-H(P_{k-1}|y_k+dy)]/dy$ or by doing kernel estimation for the residual supplies at price P_{k-1} .

With the estimates of MCP distribution and its derivatives evaluated at the specific point of bid price and quantity we can compute the shading amount and the unknown true value.

1) p_k=10000KW



2) p_k=10002.7KW



3) p_k=9997.3KW

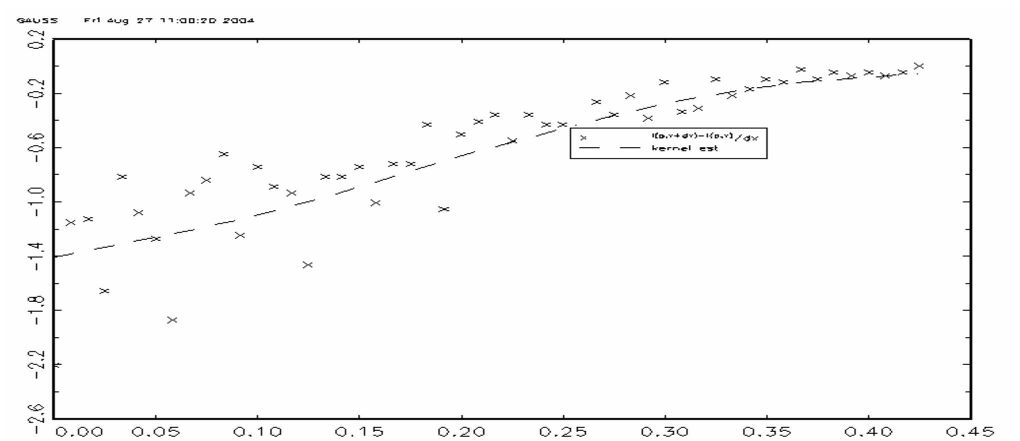


FIGURE 10. Comparison of Kernel Estimation and Calculation of Incremental Change of I_x

CHAPTER VI

EMPIRICAL FINDINGS

6.1 Revenue Comparison Method

The main purpose of this paper is to establish a revenue ranking between the two auction formats for selling TB. As I pointed out in CHAPTER V, underlying economic environment varies from auction to auction and there are many uncontrollable factors which can affect the auction outcome. Therefore, I will perform the empirical work auction by auction, without pooling the whole data or the data falling into a certain period when the same auction format was used.

To compare the revenues in this auction by auction strategy, the counterfactual comparison is the most effective way to do that. However, both auction formats produce certain types of “demand reduction” or “bid shading”, but we do not have any explicit functional form which can relate optimal bidding in the multiple price auction to that of the single price auction. Hence we can never directly estimate the outcome of the single price auction to that in which the multiple price auction format was implemented or vice versa. This means that direct counterfactual comparison is not possible in this analysis.

Therefore, I will attempt to do “indirect” counterfactual comparison using the outcome of the Vickery auction which can be estimated once we recover true valuations. To see why this idea is reasonable, we need to understand the working procedure of the Vickery auction.[50]

Same as the other two conventional auction formats, bidders in the Vickery auction also simultaneously and independently submit sealed bids consisting of demand functions to the auctioneer. The auctioneer apportions units to the highest bidders but does not assign payment according to the single price nor the multiple price formula. Instead, the payment rule is that a bidder who wins the K items pays the amount of the k^{th} *highest rejected bid other than his own* for the k^{th} item($k=1,\ldots,K$) in the case of M indivisible items. In the case of a perfectly divisible items, the payment, P_i of bidder i who wins $y_i(p^c)$ is given by:

$$P_i = y_i(p^c)p^c - \int_{p^{-i}}^{p^c} \{1 - y_{-i}(r)\}dr$$

where p^c : the market clearing price

p^{-i} : the market clearing price if bidder i had been absent from the auction

$y_{-i}(.)$: the aggregate demand of all other bidders

The payment of bidder i, P_i, in the Vickery auction is depicted in Figure 11. It is the area under the $\{1-y_{-i}(p)\}$ curve, from 0 to $y_i(p^c)$, which is shaded in the figure.

Note that each bidder's payment is independent of his own bids conditional on the number of units won. Therefore, in the Vickery auction bidders do not have any incentive for bid reduction and consequently truth-telling is an equilibrium which makes the Vickery auction especially conducive to efficiency.

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[50] See Ausubel (1997)

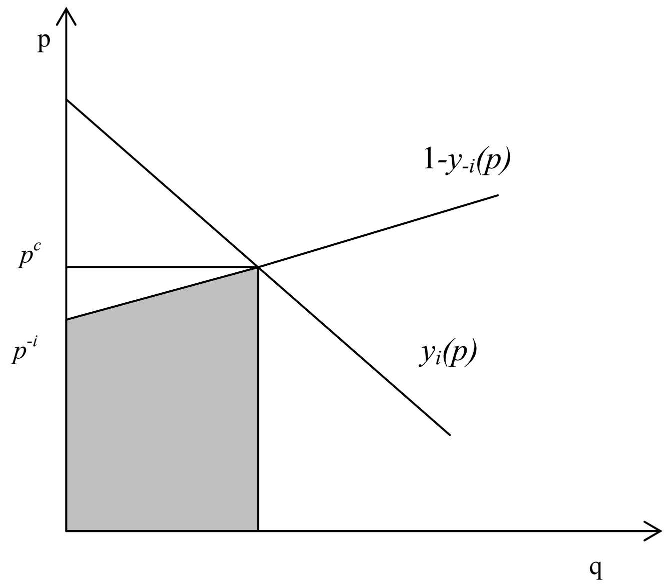
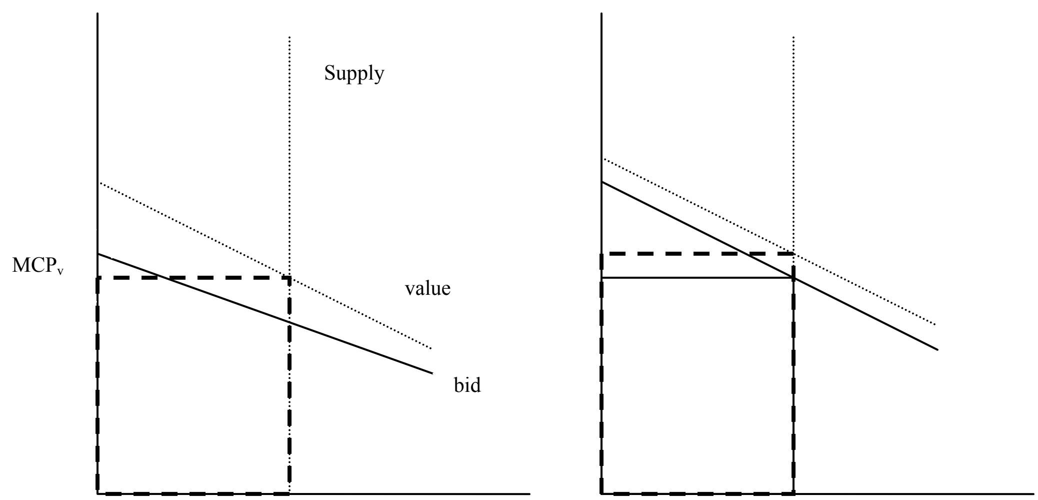


FIGURE 11. Payment Rule in Vickery Auction

In our analysis, if the true valuations corresponding to observed bid points are estimated, the intersection of the aggregate true valuation schedule and total supply yields the perfectly competitive MCP of this auction (denoted by MCPv), in which bidders reveal their true valuation and the rectangle formed by the intersection of these two schedule yields an upper bound to the revenue from the Vickery auction. Therefore, we can perform “indirect” counterfactual revenue comparison by comparing this upper bound of the revenue in the Vickery auction(denoted by Rv) to the actual revenues from two conventional auction formats(denoted by Ra).

Figure 12 illustrates the way of counterfactual comparison for the two auction formats. In the figure, note that the upper bound of the revenue in the Vickery auction is always larger than the actual revenue in single price auction, while one may be larger or smaller than the other in the multiple price auction.

To see the amount of the revenue change, I calculated a revenue loss for each auction by percentage terms (Ra-Rv)/Rv. Again, note that the revenue loss in the single price auction is always negative but that in the multiple price auction may be positive or negative. Thus, if the revenue loss in the multiple price auction is significantly positive, then we can conclude that the multiple price auction format is superior to the single price auction in terms of revenue.



* shaded area : Ra / area surrounded dashed line : Rv

FIGURE 12. Counterfactual Comparison for Two Auction Formats

As a supplementary indicator, I will also check the change in MCP. We know that regardless of the auction format bid shading exists, but the degree of bid reduction along the price grid may be different according to the auction format. The theoretical prediction is that the bid reduction is higher as the bid price is larger in the multiple price auction but that at lower bid prices, it is larger in the single price auction. If MCPv of the single price auction is quite larger than that of the multiple price auction, then the above conjecture may be supported. This can be an indicator of whether revenue comparison would be meaningful. If there is not a significant difference of MCP in the multiple price auction, then the revenue loss in the multiple price auction is more likely to be positive so that the multiple price auction may yield more revenue than the single price auction.

6.2 Multiple Price Auction

In order to show the process more clearly, I will present the results for the third auction in my data of Nov. 15, 1999. In this auction there were 28 bidders(14 banks and 14 security houses) for what amounted to 1184.9 billion KW – about 10 billion U.S. dollars. A total of 108 bids were submitted and 56 bids(52%) were successful. The cutoff yield was 8.37%. When I converted the yield to price by setting this cutoff yield to be 10000 KW, the range of bid prices were from 9971.14 KW to 10013.15 KW in which the number of price grids is 17. To demonstrate bidder-level procedure, I will focus on bidder #29, who submitted 5 price-quantity pairs totaling to a demand of 29 billion KW.

As discussed in section 5.2, I generated a random drawing of 27 bid vectors from the sample of 28 bid vectors with replacement, giving equal probability of 1/28 to each bid vector in the original sample. I performed 5000 iterations of this resampling procedure to generate 5000×27 "resampled" bid vectors and 5000 residual supply curves. By intersecting these 5000 residual supply curves with bidder #29's bid function I found 5000 market clearing prices. From these 5000 resampled market clearing prices, I constructed the distribution of market clearing price by counting the frequency with which a given price level remained above the market clearing prices.

Figure 13 shows the probability distribution of market clearing price conditional on bidder #29's bid vector. This probability distribution is a discrete version of cdf when using the step functional bid vector and the step functional residual supplies. I marked bidder #29's bid points with a star(x). We see that all bids except for one at the lowest price lie within the support of market clearing price distribution.

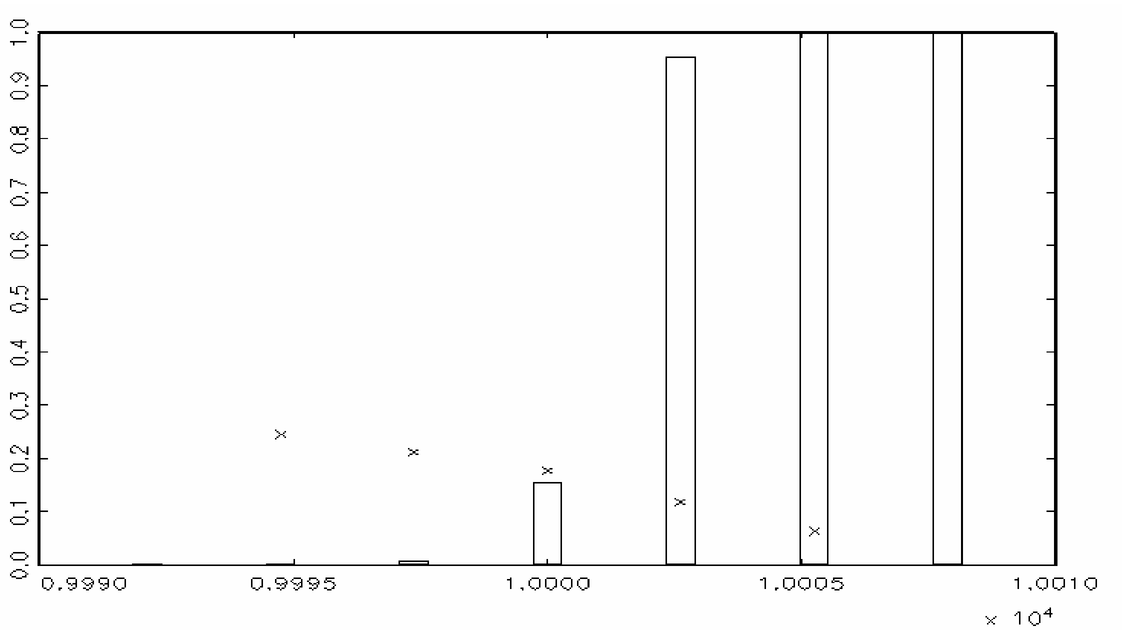


FIGURE 13. Bidder #29's Bids and H(p,y(p))

Figure 14 displays the continuous version of *H(.)* and $H_p(.)$ which are depicted by a solid line and dotted line with the discrete version of *H(.)* in one graph. Note that the continuous version is the outcome from the intersection of step functional bid vectors and kernel smoothed residual supply curves. In this auction, the continuous version of *H(.)* is a little higher than the discrete version of that at a certain price.

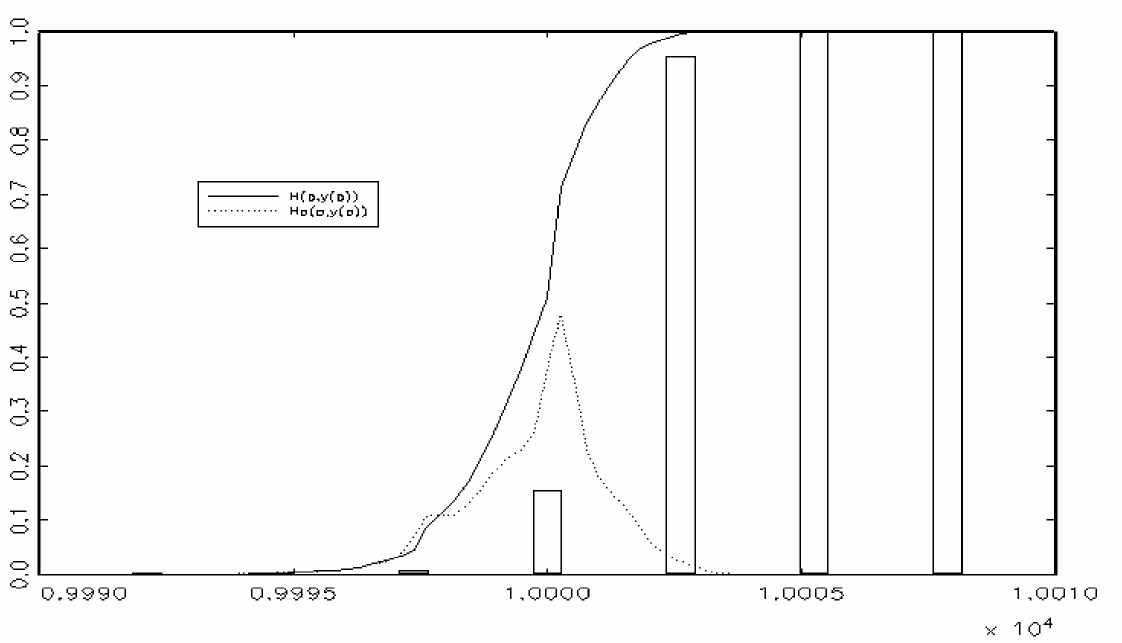
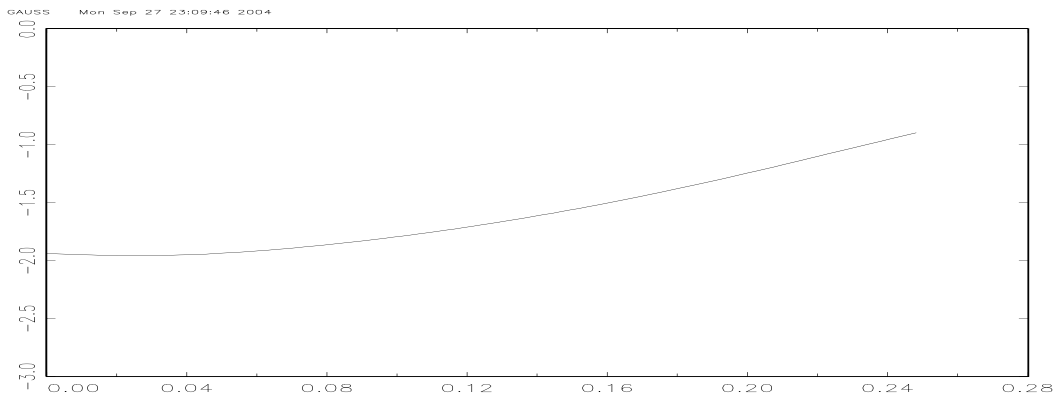


FIGURE 14. Discrete Version of H(.) and Continuous Version of H(.) & Hp(.)

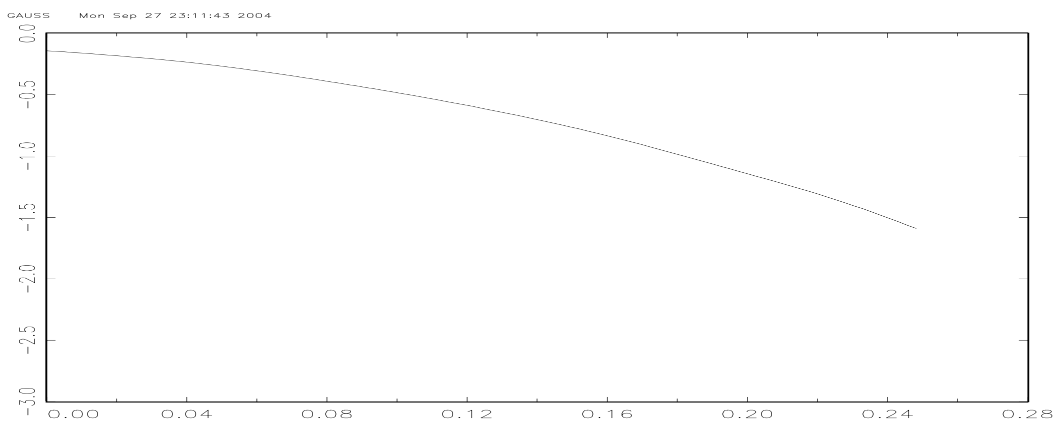
In order to apply the discrete FOCs with correction terms, we also need the estimates of H_y. Figure 15 displays the estimation results of H_y at three different price levels. Here, I depict only results from the kernel estimation.

Using these market clearing price distribution and FOCs of multiple price auction, I can recover the marginal valuations of bidder #29 that correspond to his own price-quantity pairs submitted.

1) p_k**=10000KW**



2) p_k**=10002.6KW**



3) p_k**=9997.4KW**

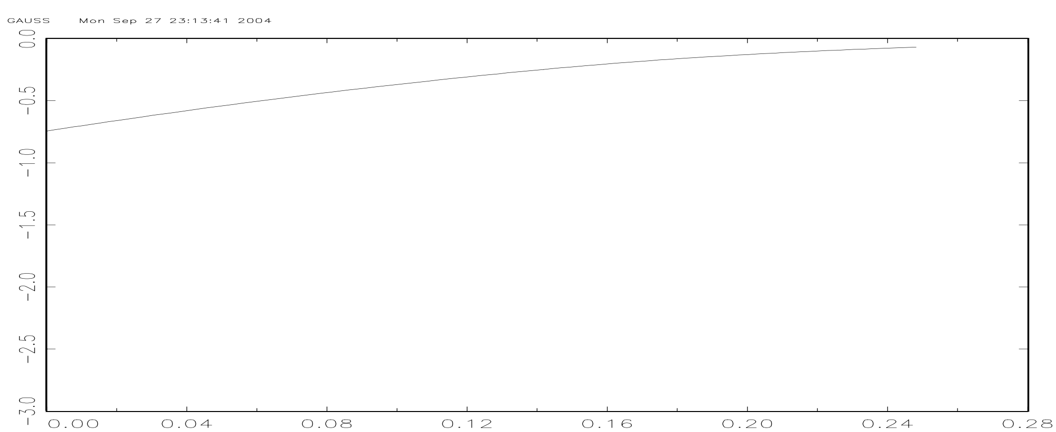


FIGURE 15. Kernel Estimation of H_y

Table 3 shows the estimated marginal valuations of bidder #29. In this table, I present all the results from the three kinds of FOCs. We see that there is not much difference between the two discrete versions of FOCs, but the version with the correction terms yields a little higher valuation than that without the correction terms. This is due to the fact that the sign of the whole correction terms is positive.

TABLE 3 − Marginal Valuation of Bidder #29

Bid price	Quantity	Marginal Valuation				
		Dis. FOC w/o correct	Dis. FOC w/ correct	(SEjack)	Cont. FOC	(SEjack)
10005.26	0.063	10058.11	10058.32	708.12	-	-
10002.63	0.118	10003.13	10003.19	1.15	10043.22	1333.4
10000.00	0.177	10000.19	10000.27	0.36	10001.35	1.69
9997.37	0.211	9997.45	9997.60	0.45	9998.02	0.36
9994.74	0.245	-	-	-	9995.26	0.87

In addition, the biggest difference between the discrete and continuous FOCs occurs at the price of 10002.63 KW. Quite a big shading at this price in the continuous case is due to a relatively bigger probability of $H(.)$ and smaller H_p than the discrete case which might be related to the fact that overall distribution of MCP in the continuous case is shifted to left compared to that of the discrete case.

In the case of the discrete version of FOCs, I cannot recover the marginal value for the lowest bid price. This is either because I assumed $H(p_0)$=0, for $p_0<p_{k1}$(the lowest observed bid price) so that $H(p_{k1-1})$ in the numerator of FOC becomes zero or because at sufficiently lower price, as we saw in the shape of the distribution of MCP, usually $H(p_k)-H(p_{k-1})$ in the denominator of FOC takes zero value so as to make it

impossible to get the value. Similarly, in the case of the continuous version of FOCs, at the highest bid price, we cannot recover the true value because H_p, denominator, might be zero at this price.

Regarding the efficiency measure for our estimated valuation, it is usually known to be difficult to calculate the standard error for the resampling method because both the consistency proof for valuation estimates does not yield the asymptotic formula for this variance and it is a non-linear function in *H(.)*.[51]

Thus, I use the "jackknife to bootstrap" method suggested by Efron(1992) to compute the standard errors for my estimates of the marginal valuation. He suggested a two step methodology to compute the "jackknife to bootstrap" standard error for the bootstrap statistic $\hat{\gamma}(x)$.[52] First, we need to compute the jackknife influence function(denoted by $u_i\{\hat{\gamma}\}$) by calculating the deleted point values, $\hat{\gamma}_{(i)} = \hat{\gamma}(x_{(i)})$ where $x_{(i)}$ is the data set remaining after deletion of the i^{th} data point. The estimate of the jackknife influence function, $\tilde{u}_i\{\hat{\gamma}\}$, is given by:

$$\tilde{u}_i\{\hat{\gamma}\} = (n-1)(\bar{\gamma}_{()} - \hat{\gamma}_{(i)})$$

where, $\bar{\gamma}_{()} \equiv \sum_i \hat{\gamma}_{(i)} / n$

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[51] See Hortcsu(2002) p41.

[52] See Efron(1992) pp88-90. According to Efron's notation, this bootstrap statistic is result from this setting: Let a random variable *T(x, F)* be a function of *x* and *F*. Let *[T(X, F)]* indicate the probability distribution of *T(X, F)*, for $X=\{X_1,\ldots,X_n\}$, an *i,i,d* sample from *F*, and let *Φ[T(X,F)]* be some functional of this distribution. Finally set *γ(F)≡ Φ[T(X,F)]* , and define the bootstrap statistic $\hat{\gamma}(x) \equiv \gamma(\hat{F})$, where $\hat{F}$ is the empirical probability distribution. With this definition, the bootstrap statistic is equivalent to $\hat{\gamma}(x) \equiv \phi[T(x^*, \hat{F})]$.

Then the estimate of the "jackknife to bootstrap" standard error, $se_{jack}\{\hat{\gamma}\}$, is:

$$s\widetilde{e}_{jack}\{\hat{\gamma}\} = [\sum_i \widetilde{u}_i\{\gamma\}^2 / n(n-1)]^{1/2}$$

It is known that usually $\widetilde{u}_i\{\hat{\gamma}\} \to u_i\{\hat{\gamma}\}$ and $s\widetilde{e}_{jack}\{\hat{\gamma}\} \to se_{jack}\{\hat{\gamma}\}$ as number of iteration(*B*) goes to infinity.

If we carefully look at the above algorithm, we can find that the main factors for our setting in estimating bidders' valuation have a one to one relationship with those of Efron's setting. That is, *X→y(p)*, *F→underlying distribution of signal or residual supply*, *T(x, F)→market clearing price, [T(x, F)]→distribution of market clearing price, H(.), Φ[T(X,F)]=* $\hat{\gamma}(x)$*→estimated valuation.* Therefore, we can calculate the "jackknife to bootstrap" standard error for our valuation estimates using the same formula as Efron's.

Let the estimate for the marginal valuation of bidder *i* for y_{ik} units of TB be $\hat{v}(y_{ik})$ and let the bootstrap estimate over a set of resamples that do not contain the *i*'s bid vector be $\hat{v}_{(i)}(y_{ik})$. Then, the estimate of jackknife influence function, $\widetilde{u}_i\{\hat{v}(y_{ik})\}$, and estimate of "jackknife to bootstrap" standard error, $s\widetilde{e}_{jack}\{\hat{v}(y_{ik})\}$ are:

$$\widetilde{u}_i\{\hat{v}(y_{ik})\} = (n-1)(\overline{v}_0 - \hat{v}_{(i)}(y_{ik}))$$

where, $\overline{v}_0 \equiv \sum_i \hat{v}_{(i)}(y_{ik}) / n$

$$s\widetilde{e}_{jack}\{\hat{v}(y_{ik})\} = [\sum_i \widetilde{u}_i\{\hat{v}(y_{ik})\}^2 / n(n-1)]^{1/2}$$

$$= [(n-1)^2 \sum_i (\overline{v}_0 - \hat{v}_{(i)}(y_{ik})^2 / n(n-1)]^{1/2}$$

$$= [\frac{n-1}{n}\{\sum_i (\bar{v}_{()} - \hat{v}_{(i)}(y_{ik}))^2\}]^{1/2}$$ [53]

In Table 3, I reported the se_{jack} of recovered marginal valuations for bidder #29. As the shading amount is quite larger, the standard errors also tend to be larger.

There is one practical concern that we must consider to aggregate the true marginal valuations. As we have seen before, there are some points where we cannot recover the true value, and therefore, need to use some values for these missing values for a more complete revenue comparison.

Since it is extremely hard to know the exact form of true value function in the multi-unit auction environment, I will try to circumvent this problem by setting some restrictions on the valuation structure. One restriction is that the valuation function is weakly decreasing to the quantity, and the other is that at a lower price range where the losing bids are located, bidders are assumed to not shade.

Hence if I fail to recover their true value for the winning bid, I will assign $max(p_k, v_{k-1})$ at that point, where p_k is the bid price, and v_{k-1} is the recovered value at the next lower bid price. Also, if the missing value occurred at a losing bid, I will just assign its bid price as its true value.

Figure 16 depicts the marginal valuation curve of bidder #29 after filling in the missing values according to the steps described above. I now display the case of the

[53] Hortcsu(2002) also calculated "jackknife to bootstrap" standard error for his marginal valuation, but he used $\frac{N}{N-1}$ in his formula.

continuous version of FOC and the case of discrete version of FOC with the correction terms.

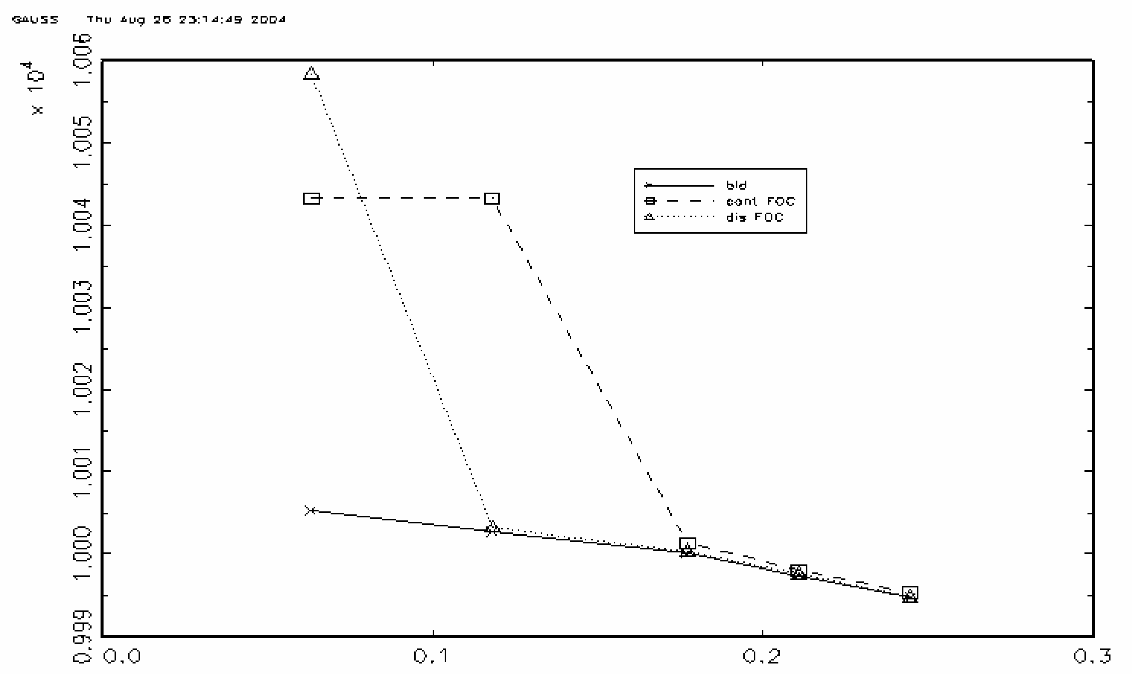

FIGURE 16. Marginal Valuation Curve of Bidder #29

We see that the marginal value for the higher bid is larger which implies that bidders may value initial units very highly. In addition, the point estimates of the marginal value seem to decline with quantity.

I repeat this analysis for every bidder in this auction. This gives me 28 bidders' point estimates of their marginal valuations rationalizing each price quantity pair observed in this auction. I aggregate these estimated marginal valuations across bidders, and find the intersection of this aggregate schedule with the total supply which is normalized to 1. This yields the (hypothetical) Vickery auction outcome, in which bidders reveal their true marginal valuations.

Figure 17 displays the aggregate bid function and the estimated aggregate marginal valuation functions which are resulting from the continuous FOC and the discrete FOC with the correction terms.

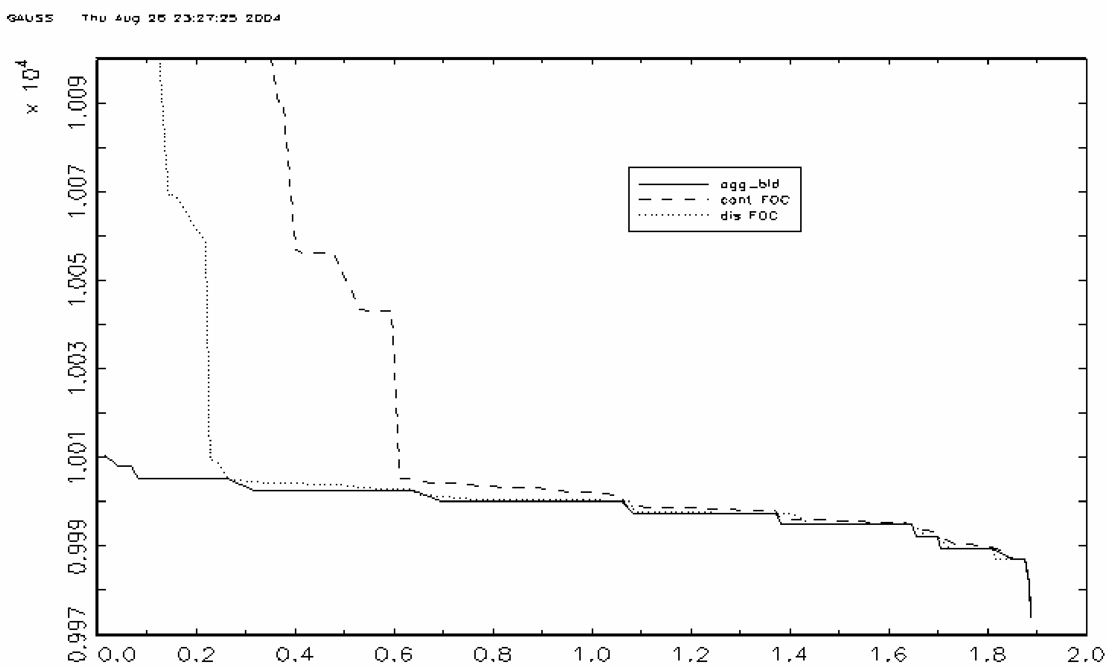

FIGURE 17. Aggregate Bid Function and Aggregate Value Function

To see the difference more clearly between them around the market clearing price, I have narrowed down the scale of the two axes in Figure 18.

From Figure 18, we can see that the aggregate marginal values lie above the actual bid price at each bid amount point up to the point where the aggregate bid amount exceeds one. We also see that at the equilibrium where the aggregate schedules meet the vertical line which is drawn at the point where the quantity equals to one, the market clearing price of the marginal value schedule (MCPv) is slightly higher than the actual market clearing price which is normalized to 10000.00KW. The MCPv of the continuous FOC case is 10002.04 KW while that of the discrete FOC with correction terms is 10000.27. In the discrete case, the MCPv is almost same as the actual market clearing price.

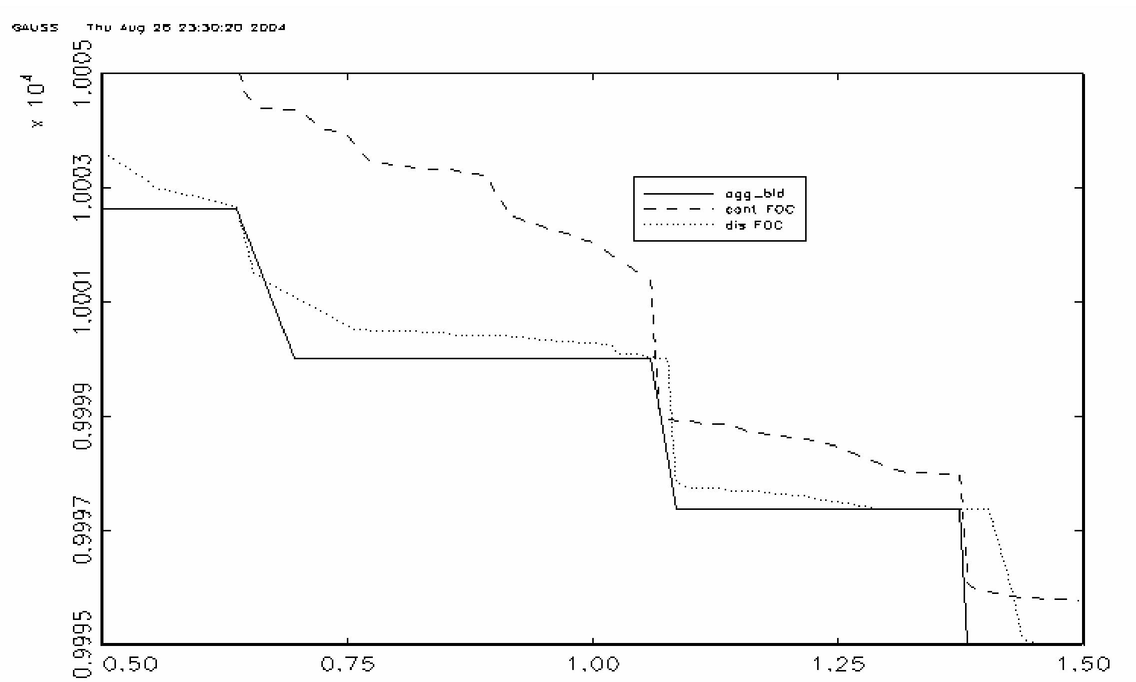

FIGURE 18. Zoom in Aggregate Bid Function and Aggregate Value Function

As depicted in Figure 12, the revenue of the multiple price auction is given by the area under the aggregate schedule up to the total supply. The upper bound to the Vickery auction revenue, Rv, would be 987.62 million US dollar in the continuous case and 987.44 million US dollar in the discrete case as opposed to 987.67 million US dollar of the actual revenue. The revenue loss is 0.005% and 0.023% which means that the Vickery auction causes a revenue loss at this auction.

I performed the same procedure for the other nine 3-year TB auctions. Table 4 and Table 5 summarize the results for every multiple price auctions.

TABLE 4 − Results of Multiple Price Auctions : MCPv

Date	Con. FOC	Dis. FOC (w/ correction)	Dis. FOC (w/o correction)
9/13/1999	10007.44	10002.59	10001.91
10/11/1999	10001.31	10000.00	10000.00
11/15/1999	10002.04	10000.27	10000.11
1/17/2000	10000.60	10000.00	10000.00
2/14/2000	10000.36	10000.00	10000.00
3/13/2000	10000.76	10000.00	10000.00
4/10/2000	10000.80	10000.00	10000.00
5/8/2000	10001.18	10000.00	10000.00
6/12/2000	10004.05	10000.00	10002.62
7/10/2000	10001.79	10000.02	10002.65

* Actual market clearing price are normalized to 10000 KW

TABLE 5 − Results of Multiple Price Auctions : Revenue

Date	Actual Revenue (Ra, $mill)	Vickery Revenue (Rv, $mill)			Revenue loss(%) (=[Ra-Rv]/Rv)		
		(1)	(2)	(3)	(1)	(2)	(3)
9/13/1999	1001.259	1000.74	1000.26	1000.19	0.051	0.100	0.107
10/11/1999	1131.546	1131.31	1131.17	1131.17	0.020	0.033	0.033
11/15/1999	987.672	987.62	987.44	987.43	0.005	0.023	0.025
1/17/2000	637.333	637.12	637.08	637.08	0.033	0.039	0.039
2/14/2000	1023.630	1023.37	1023.33	1023.33	0.025	0.029	0.029
3/13/2000	553.402	553.29	553.25	553.25	0.020	0.027	0.027
4/10/2000	700.042	699.97	699.92	699.92	0.010	0.018	0.018
5/8/2000	635.111	635.08	635.00	635.00	0.006	0.017	0.017
6/12/2000	500.408	500.20	500.00	500.13	0.041	0.082	0.055
7/10/2000	488.585	488.42	488.33	488.46	0.034	0.051	0.025

(1) Continuous FOC / (2) Discrete FOC w/correction terms / (3) Discrete FOC w/o correction terms

From Table 4, except for the 1st and the last auction, the MCPv is not quite different from the actual market clearing price in the multiple price auction. It can be interpreted that in the multiple price auction, there is no strong incentive to shade bids around MCP. In Table 5, we can see that all revenue losses are positive numbers which suggests that the multiple price auction outperforms the Vickery auction in terms of revenue.

Since the revenue losses seem to be very small, one might be wondering whether the amount of revenue loss calculated above is significant or not. In fact, the observed bids and estimated marginal valuations are random variables so that the auction revenue is also a random variable. Therefore, if we can construct a confidence interval for the difference between Rv and Ra, the question in regards to the significance of the revenue loss may be answered. I will attempt to do that by the bootstrap procedure.

First, I constructed 10,000 resamples of the pair of actual bids and estimated marginal valuations for each auction. In each resample there are N_t actual bid vectors and N_t marginal valuations vectors drawn randomly from the original set of bids and the set of estimated marginal valuations vectors respectively.

With these resamples, I calculated the market clearing price and revenue for each pair of resample and obtained 10,000 differences of Rv and Ra. The mean value of these resampled revenue difference gives us the mean estimates of *ex-ante* revenue difference. We can also construct the confidence interval for the revenue difference. After sorting these 10,000 differences in an ascending order, we can obtain the 95% bootstrap confidence interval by finding the 2.5% percentile and 97.5% percentile.

To test whether the revenue difference is significant is the same as to test $H_0 : Ra\text{-}Rv=0$. Hence if zero lies within this 95% interval we cannot reject the null hypothesis, but if not, we can conclude that the revenue difference is significantly different from zero. Table 6 summarizes the test result.

TABLE 6 − Test for Revenue Difference(Ho:Ra-Rv=0)

Date	Con. FOC	Dis. FOC (w/ correction)	Dis. FOC (w/o correction)
9/13/1999	no reject	no reject	Reject
10/11/1999	no reject	reject	Reject
11/15/1999	no reject	reject	Reject
1/17/2000	no reject	reject	Reject
2/14/2000	reject	reject	Reject
3/13/2000	no reject	reject	reject
4/10/2000	no reject	reject	reject
5/8/2000	reject	reject	reject
6/12/2000	no reject	reject	reject
7/10/2000	no reject	reject	reject

From Table 6, the amounts of *ex-post* difference which we obtained, turn out to be significant at almost of all auctions in the discrete case.

6.3 Single Price Auction

Similar to the previous section, I will start by showing the results for one of the single price auctions which was held on January 7, 2002(auction code #27). In this auction there were 27 bidders (11 banks and 16 security houses) for what amounted to 1,200 billion Korean won – about 10 billion U.S. dollars. A total of 118 bids were submitted, and 37% of the bids were successful. The cutoff yield was 6.10%. The range of bid prices were from 9953.83 KW to 10032.73 KW in which the number of price grids is 30.

I will focus on bidder #30 who submitted 5 price-quantity pairs totaling a demand of 10% of the total supply. Using the same resampling procedure as before, I constructed 5000 residual supply curves and 5,000 resampled market clearing prices. With this resample, I estimated the distribution of market clearing price. Figure 19 shows the distribution of the market clearing price conditional on bidder #30's bid vector.

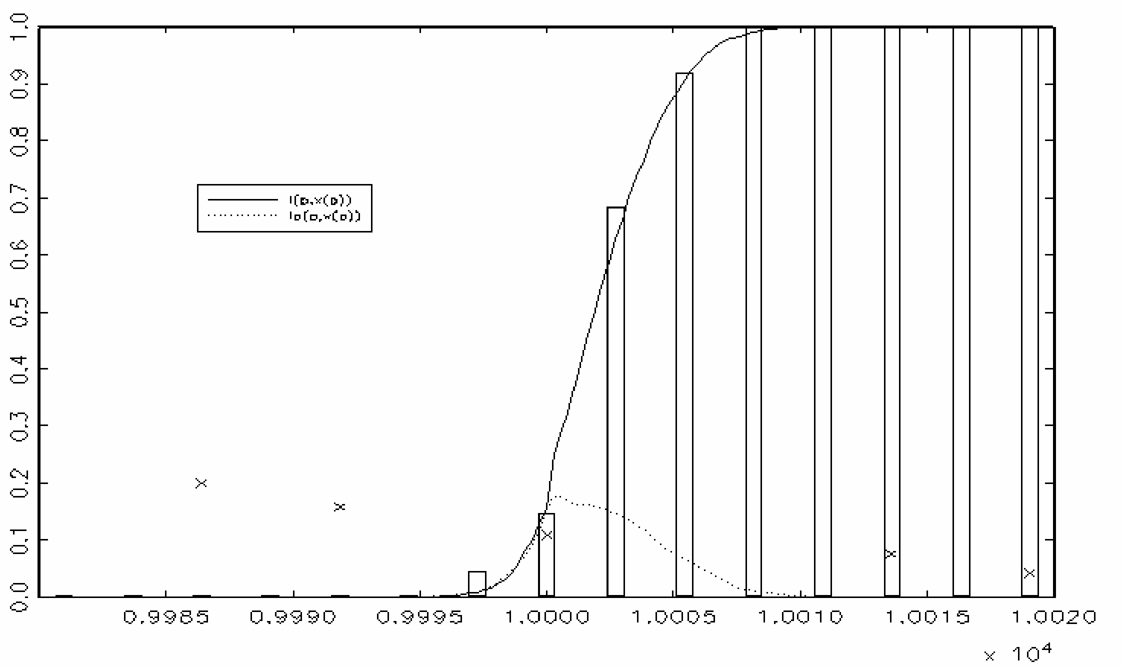

FIGURE 19. The Distribution of MCP with Bidder #30's Bid Vector

In Figure 19, the distribution of MCP for the discrete case is depicted by a bar-shape and that of the continuous case is shown by a solid and dotted line. In this auction, the distribution of the continuous case is slightly shifted to the right compared to that of the discrete case. In the case of the derivative of *I(.)* with respect to quantity, I_x, refer to the results depicted in Figure 10.

I calculated the marginal valuations for bidder #30 corresponding to each price-quantity pair submitted using the first-order conditions derived in section 4.3. Table 7 summarizes the true valuations of bidder #30 and their standard error.

TABLE 7 – Marginal Valuation of Bidder #30

Bid price	Quantity	Marginal Valuation				
		Dis. FOC w/o correct	Dis. FOC w/ correct	(SEjack)	Cont. FOC	(SEjack)
10005.44	0.008	10005.47	10005.60	0.15	10005.45	0.04
10002.72	0.016	10002.76	10002.78	0.03	10002.92	0.02
9994.55	0.033	9997.60	9999.41	23.21	9994.91	0.14
9991.83	0.066	-	-	-	-	-
9989.12	0.100	-	-	-	-	-

We cannot recover the true value for the last lowest two bids because $I(p_k) - I(p_{k-1})$ or I_x are zero.

After filling in the missing values using the method suggested in the previous section, the valuation function of bidder #30 is drawn in Figure 20.

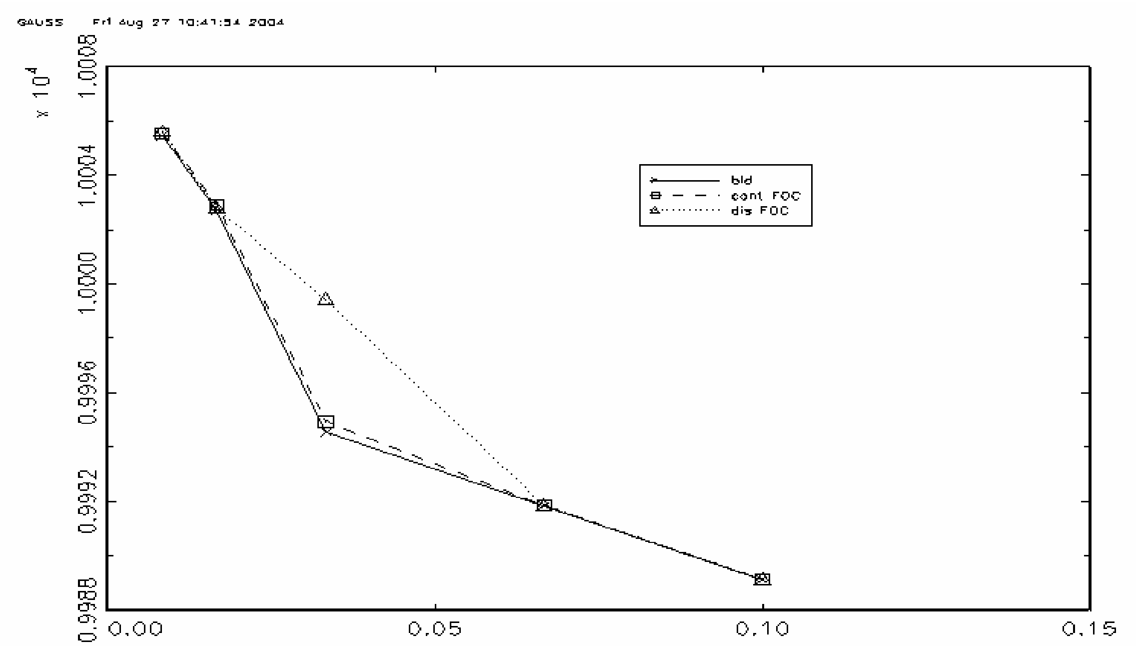

FIGURE 20. Marginal Valuation Curve of Bidder #30

I repeated this analysis for every bidder in this auction. Then, I aggregated these estimated marginal valuations across the bidders, and found the intersection of the aggregate schedule with total supply.

This yields the perfectly competitive outcome for this auction, in which bidders reveal their true marginal valuations under the single price auction. Figure 21 depicts the aggregate bid and aggregate marginal valuations schedule and Figure 22 shows the difference around MCP more closely.

In this auction, the MCPv of the continuous FOC case is 10000.51 KW while that of the discrete FOC with correction terms is 10002.04. In the continuous case, the MCPv is almost the same as the actual market clearing price.

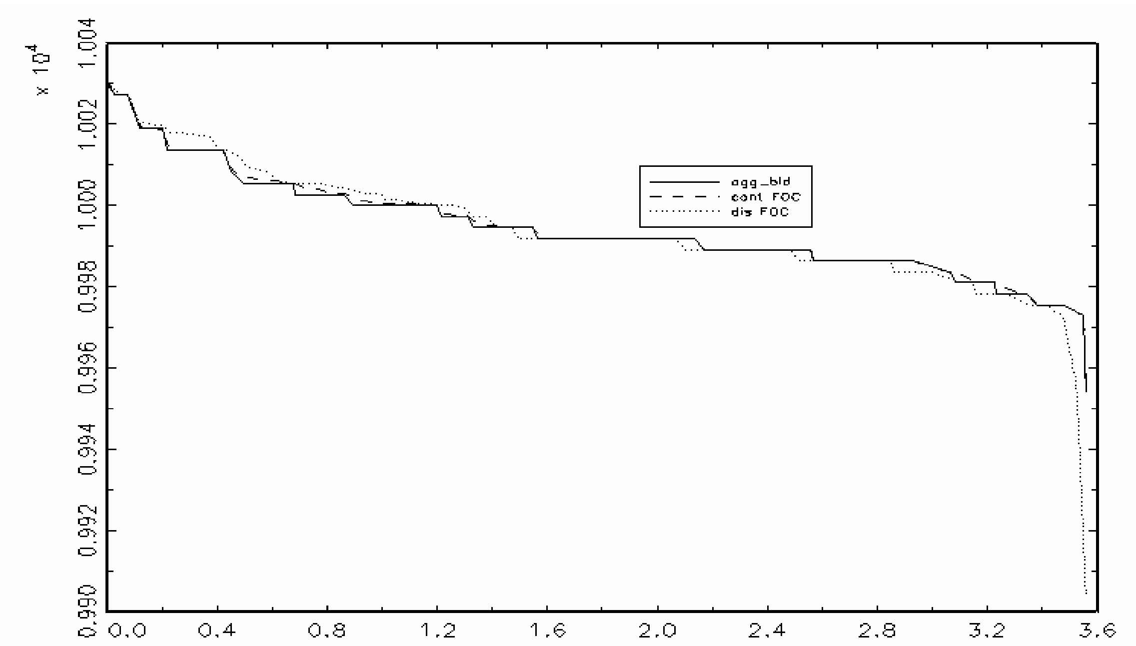

FIGURE 21. Aggregate Bid and Aggregate Valuation Schedule

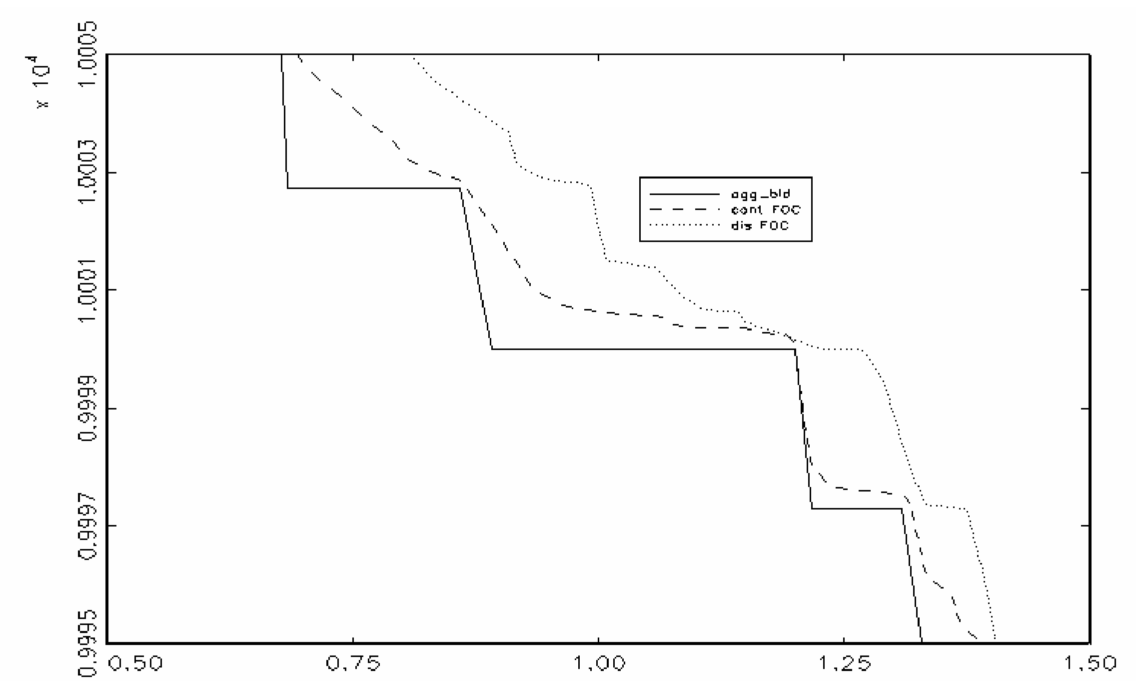

FIGURE 22. Zoom in Aggregate Bid and Aggregate Valuation

As depicted in Figure 12, the revenue for the single price auction is given by the rectangle formed by the intersection of the aggregate schedule and total supply. The upper bound of the Vickery auction revenue, $Rv_{,}$ would be 1000.05 million US dollar in the continuous case and 1000.20 million US dollar in the discrete case as opposed to 1000.00 million US dollar of actual revenue. The revenue losses are -0.005% and -0.020% respectively which means that the revenue for the single price auction is less than that of the Vickery auction.

I performed the same procedure for the other 19 single price auctions. Table 8 and Table 9 summarize the results for every auction.

Table 8 − Results of Single Price Auctions : MCPv

Date	Con. FOC	Dis. FOC (w/ correction)	Dis. FOC (w/o correction)
8/14/2000	10000.23	10003.57	10001.85
9/18/2000	10001.16	10000.00	10000.93
10/9/2000	10000.35	10002.61	10002.35
11/13/2000	10000.10	10002.17	10000.29
1/8/2001	10000.21	10002.73	10003.21
2/5/2001	10000.83	10002.75	10002.75
3/12/2001	10000.20	10002.72	10002.75
4/2/2001	10000.84	10002.70	10004.07
5/7/2001	10000.87	10002.72	10004.45
6/4/2001	10003.15	10002.70	10002.74
7/2/2001	10001.16	10002.51	10002.73
8/6/2001	10002.06	10002.30	10002.77
9/3/2001	10000.39	10000.79	10002.78
10/8/2001	10000.31	10002.16	10001.18
11/7/2001	10000.30	10000.00	10001.26
12/3/2001	10000.66	10001.62	10001.20
1/7/2002	10000.51	10002.04	10002.98
2/4/2002	10000.78	10002.02	10002.54
3/4/2002	10000.84	10003.37	10002.80
4/1/2002	10000.41	10002.73	10002.71

TABLE 9 − Results of Single Price Auctions : Revenue

Date	Actual Revenue (Ra, $mill)	Vickery Revenue (Rv, $mill)			Revenue loss(%) (=[Ra-Rv]/Rv)		
		(1)	(2)	(3)	(1)	(2)	(3)
8/14/2000	500.000	500.01	500.18	500.09	-0.002	-0.036	-0.019
9/18/2000	750.000	750.09	750.00	750.07	-0.012	0.000	-0.009
10/9/2000	750.000	750.03	750.20	750.18	-0.004	-0.026	-0.023
11/13/2000	791.667	791.67	791.84	791.69	-0.001	-0.022	-0.003
1/8/2001	625.000	625.01	625.17	625.20	-0.002	-0.027	-0.032
2/5/2001	641.667	641.72	641.84	641.84	-0.008	-0.028	-0.028
3/12/2001	416.667	416.68	416.78	416.78	-0.002	-0.027	-0.027
4/2/2001	666.667	666.72	666.85	666.94	-0.008	-0.027	-0.041
5/7/2001	500.000	500.04	500.14	500.22	-0.009	-0.027	-0.045
6/4/2001	333.333	333.44	333.42	333.42	-0.032	-0.027	-0.027
7/2/2001	333.333	333.37	333.42	333.42	-0.012	-0.025	-0.027
8/6/2001	583.333	583.45	583.47	583.50	-0.021	-0.023	-0.028
9/3/2001	708.333	708.36	708.39	708.53	-0.004	-0.008	-0.028
10/8/2001	741.667	741.69	741.83	741.75	-0.003	-0.022	-0.012
11/7/2001	625.000	625.02	625.00	625.08	-0.003	0.000	-0.013
12/3/2001	925.000	925.06	925.15	925.11	-0.007	-0.016	-0.012
1/7/2002	1000.000	1000.05	1000.20	1000.30	-0.005	-0.020	-0.030
2/4/2002	333.333	333.36	333.40	333.42	-0.008	-0.020	-0.025
3/4/2002	416.667	416.70	416.81	416.78	-0.008	-0.034	-0.028
4/1/2002	475.000	475.02	475.13	475.13	-0.004	-0.027	-0.027

(1): Continuous FOC / (2): Discrete FOC w/correction terms / (3): Discrete FOC w/o correction terms

I also tested whether the revenue difference is significantly different from zero using the bootstrap procedure. I can reject the null hypothesis in the continuous case and the discrete case without correction terms, but I cannot in the discrete case with correction terms.

Compared to the result for the multiple price auction, we can see that the MCPv is higher than in the multiple price auction. This suggest that there is more of a shading incentive in the bids around the market clearing price.

In the case of the revenue loss rates, as I pointed out in the section 6.1, the signs for them are negative and their absolute sizes are not much different from that of the multiple price auction. Since most of them are very small values we need to check the significance of those differences. Table 10 summarizes these test results. Interestingly, the result of the discrete FOC case shows that we cannot reject the null hypothesis of equivalence of Rv and Ra in the lots of the auctions even though the results of the continuous FOC case is ambiguous. This result implies that the revenue from the single price auction is likely to be indifferent from the upper bound of revenue from the Vickery auction.

TABLE 10 − Test for Revenue Difference(Ho:Ra-Rv=0)

Date	Con. FOC	Dis. FOC (w/ correction)	Dis. FOC (w/o correction)
8/14/2000	Reject	No reject	Reject
9/18/2000	Reject	No reject	No reject
10/9/2000	Reject	No reject	Reject
11/13/2000	Reject	reject	Reject
1/8/2001	Reject	No reject	Reject
2/5/2001	Reject	No reject	No reject
3/12/2001	Reject	No reject	No reject
4/2/2001	No reject	No reject	No reject
5/7/2001	Reject	No reject	Reject
6/4/2001	Reject	No reject	No reject
7/2/2001	No reject	No reject	No reject
8/6/2001	No reject	No reject	Reject
9/3/2001	No reject	No reject	Reject
10/8/2001	Reject	No reject	No reject
11/7/2001	No reject	No reject	Reject
12/3/2001	Reject	No reject	No reject
1/7/2002	Reject	No reject	Reject
2/4/2002	Reject	No reject	Reject
3/4/2002	Reject	No reject	No reject
4/1/2002	Reject	reject	no reject

6.4 Conclusion in Revenue Ranking

To see the difference in MCPv and the revenue loss between the two auction formats more clearly, I calculated the mean values which are summarized in Table 11. Since the total supply in an auction varies greatly from 400 billion KW to 1357 billion KW during the sample period, I divided the auctions to where the total supply is over 800 bill KW and those below 800 bill KW, and calculated the mean for each group considering the possible effects of the amount of total supply on the outcomes.

TABLE 11 − Mean Value of MCPv and %Revenue Ross

		Format	Con. FOC	Dis. FOC (w/ correction)	Dis. FOC (w/o correction)
MCPv	>800	multi(5)	10002.39	10000.57	10000.40
		single(12)	10000.54	10001.76	10001.97
	<800	multi(5)	10001.67	10000.00	10001.05
		single(8)	10000.92	10002.51	10002.72
	Total	Multi(10)	10002.03	10000.29	10000.73
		Single(20)	10000.77	10002.21	10002.42
Revenue Loss(%)	>800	multi(5)	0.023	0.041	0.042
		single(12)	-0.005	-0.018	-0.020
	<800	multi(5)	0.027	0.043	0.033
		single(8)	-0.009	-0.025	-0.027
	total	Multi(10)	0.025	0.042	0.038
		Single(20)	-0.008	-0.022	-0.024

*the number in ( ) is the number of auctions fallen into the group

First of all, the average of MCPv in the single price auction is higher than that of the multiple price auction, except the continuous FOC case.[54] This finding supports

[54] However, in the results of continuous case, since MCPv in the auction #1 is quite

our expectation for the shading incentive around MCP which is much stronger in the single price auction than in the multiple price auction.

Obviously, the percentage of revenue loss which is compared to the potential Vickery auction revenue tells us that the multiple price auction outperforms the single price auction in terms of revenue because every mean value in the multiple price auction has a positive sign. As we saw in the results for the test of revenue equivalence, the difference in Rv and Ra seemed to be significant.

Comparing Figure 17 and Figure 21 which depict the aggregate bid and the aggregate valuation schedule, we can expect that the total bid shading amount from the bidders' true valuation is likely to be larger in the multiple price auction than in the single price auction. That is, the single price auction may induce bidders to submit their bids more closely to their true value than the multiple price auction. However, even though larger bid shading may occur in the multiple price auction, the multiple price auction is more advantageous to the Korean Treasury in terms of the revenue.

6.5 Discussion of the Efficiency Matter

The efficient auction mechanism is the mechanism which puts items in the hands of those who value them the most. In the case of auctions for single, indivisible item, it is well known that the second-price sealed-bid auction and the English auction induce

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big(10007.4KW) compared to others we need to consider the fact that this value may distort the total mean value and the mean for the bigger total supply group.

buyers to bid sincerely so that efficient outcomes are always possible. Under the first-price sealed-bid auction, bidders shade their bids relative to their true values, but efficiency is still possible when there are symmetric bidders who adopt symmetric strategies.

However, in the circumstances of the multi-unit auction in which the items for sale are divisible and bidders can demand multiple units, there is no general conclusion about efficiency ranking between the multiple price auction and the single price auction.

Earlier researchers who assumed each bidder demands only a single unit among the several units suggested a similar prediction to that of the results for the single-unit auction. That is, in the multiple price auction there exists bid shading whereas in the single price auction bidders bid their true value which means that there is no bid reduction. Therefore, the single price auction may be superior to the multiple price auction in efficiency.[55] As I pointed out in the earlier part of this paper, this conclusion was misleading by an imperfect analogy between single-unit and multi-unit auctions.

After researchers began to consider the case where bidders desired multiple units explicitly, different predictions about efficiency from the above have been suggested. Engelbrecht-Wiggans and Kahn(1998a)[56] concluded that both multiple price auctions and single price auctions are not efficient because there exists bid shading in both auction formats. More specifically, they pointed out that in the multiple price auction

[55] Harris and Raviv(81)

[56] They analyzed the case where N+1 bidders demand up to two identical units.

there is the tendency for the higher bids to be shaded more significantly while in the single price auction the tendency is reversed which means the shading becomes more significantly for the lower valued units. Back and Zender(93) predicted that the multiple price auction may be more efficient because relatively high inframarginal bids are costless in a single price auction but they are costly in a multiple price auction. Therefore, the multiple price auction induces bidders to submit flatter demand curves, which in turn stimulates greater price competition at the margin.

Most recently, Ausubel and Crampton(2002) analyzed a more general case where bidders submit a continuous downward sloping demand and suggested that the efficiency of the multiple price auction may exceed that of the single price auction. The intuition for that derives from the fact that the multiple price auction is not subject to bid shading that is increasing in quantity because a bid for an additional unit in the multiple price auction has no effect on the price which is paid for earlier units. So it is possible for bidders with similar marginal valuations at different quantities to be shading their bids by similar amounts. However, in the single price auction a bidder's bid on a later unit will be pivotal to decide his payment so that he has an incentive to bid less than his true value on later units in order to reduce the price he will pay on the earlier units. Therefore, bidders with identical marginal valuations may shade their bids by different amounts at different quantities.

In conclusion, the general opinion about efficiency in the multi unit auction is that since both auction formats result in bid shading, no obvious efficiency ranking is possible, though some researchers support that the multiple price auction may be superior to the single price auction in efficiency. Hence, we have to view this matter as an empirical question that depends on the actual nature of demands.

As appropriate indices that can indicate the efficiency change, I suggest two candidates. First of all, considering the definition of efficiency and the characteristic of multiple bids per one bidder in the TB auction, how many bids are switched from a winning to a losing bid when ordering the actual bids by their estimated true valuations may be a possible indication of efficiency. If some bids which win under true valuation ordering are switched to lose under actual bid ordering, and the number of switches is larger in one auction format, say A, than in the other format B, it implies that the possibility of failing to allocate the items to the bidder who values them more is higher in format A.

Next candidate for efficiency measure is to compare the winners' total valuations under true valuation ordering[True winners' total valuations] with actual winners' total valuations which I will name by TWV and AWV respectively. If the gap between TWV and AWV is bigger in format A, it means that the amount of potential surplus which the winners can obtain is reduced more in A or that format A realizes more loss of valuations. This measurement gives us the information about the amount of efficiency loss differently from the switched number of bids.

Table 12, 13 and 14 summarize these two indicators for each auction. From Table 12, we can see the number of switched bids is greater in the single price auction than in the multiple price auction. Also, the quantity change corresponding to the switched bids is quite larger in the single price auction.

Table 12 − Efficiency Implication 1 : Bid Switching

Format	Date	Con. FOC		Dis. FOC (w/ correction)		Dis. FOC (w/o correction)	
		Num.	Qt.(%)	Num.	Qt.(%)	Num.	Qt.(%)
Multi Price	9/13/1999	0	0.00	2	1.67	2	1.67
	10/11/1999	0	0.00	0	0.00	0	0.00
	11/15/1999	0	0.00	1	1.69	0	0.00
	1/17/2000	0	0.00	0	0.00	0	0.00
	2/14/2000	0	0.00	0	0.00	0	0.00
	3/13/2000	0	0.00	0	0.00	0	0.00
	4/10/2000	0	0.00	0	0.00	0	0.00
	5/8/2000	0	0.00	0	0.00	0	0.00
	6/12/2000	0	0.00	0	0.00	0	0.00
	7/10/2000	0	0.00	0	0.00	0	0.00
Single Price	8/14/2000	0	0.00	5	16.60	2	8.33
	9/18/2000	0	0.00	1	2.22	3	7.78
	10/9/2000	0	0.00	5	8.89	6	8.33
	11/13/2000	0	0.00	7	31.58	6	16.84
	1/8/2001	0	0.00	2	5.33	4	12.00
	2/5/2001	0	0.00	5	11.69	4	11.69
	3/12/2001	0	0.00	1	2.00	1	2.00
	4/2/2001	0	0.00	4	10.00	2	5.00
	5/7/2001	1	5.00	3	13.33	4	21.67
	6/4/2001	2	10.00	1	2.50	2	10.00
	7/2/2001	1	7.50	1	2.50	1	7.50
	8/6/2001	0	0.00	2	8.57	3	7.14
	9/3/2001	0	0.00	2	8.24	2	12.94
	10/8/2001	0	0.00	6	11.24	5	8.99
	11/7/2001	0	0.00	3	5.33	6	10.67
	12/3/2001	0	0.00	4	13.51	6	10.81
	1/7/2002	3	5.00	2	2.50	4	9.17
	2/4/2002	0	0.00	2	7.50	3	25.00
	3/4/2002	0	0.00	3	8.00	3	8.00
	4/1/2002	0	0.00	4	14.04	4	14.04

* Switched bids which are winning when truth-telling but losing when actual bidding

(number of bids and corresponding incremental bid quantities measured by % of total supply)

TABLE 13 − Efficiency Implication 2 : Surplus Change(mill US$)

Format	Date	Con. FOC		Dis. FOC (w/ correction)		Dis. FOC (w/o correction)	
		TWV	AWV	TWV	AWV	TWV	AWV
Multi Price	9/13/1999	1457.438	1457.438	1039.670	1039.598	1039.274	1039.242
	10/11/1999	1134.640	1134.640	1132.526	1132.524	1132.521	1132.520
	11/15/1999	994.603	994.603	990.870	990.865	990.857	990.855
	1/17/2000	637.796	637.796	637.686	637.685	637.686	637.685
	2/14/2000	1025.597	1025.597	1026.488	1026.488	1026.488	1026.488
	3/13/2000	555.606	555.606	554.032	554.032	554.031	554.031
	4/10/2000	873.687	873.687	704.762	704.762	704.663	704.663
	5/8/2000	635.150	635.150	640.418	640.418	640.417	640.417
	6/12/2000	608.645	608.645	558.419	558.399	558.167	558.123
	7/10/2000	756.150	756.150	496.082	496.038	496.084	496.075
Single Price	8/14/2000	500.429	500.429	500.646	500.544	500.509	500.476
	9/18/2000	750.604	750.604	750.593	750.576	750.590	750.547
	10/9/2000	750.440	750.440	750.697	750.441	750.438	750.406
	11/13/2000	791.974	791.974	792.585	792.033	792.014	791.983
	1/8/2001	625.640	625.640	625.998	625.578	625.741	625.648
	2/5/2001	643.105	643.105	643.124	642.983	643.771	642.976
	3/12/2001	417.284	417.284	417.334	417.275	417.313	417.270
	4/2/2001	667.975	667.975	670.036	667.790	667.894	667.777
	5/7/2001	500.565	500.565	500.675	500.610	500.701	500.582
	6/4/2001	333.687	333.687	333.751	333.667	333.671	333.649
	7/2/2001	333.712	333.712	333.786	333.781	333.787	333.696
	8/6/2001	584.109	584.109	584.084	584.003	584.022	583.956
	9/3/2001	709.274	709.274	709.533	709.378	709.361	709.246
	10/8/2001	742.583	742.583	742.538	742.459	742.503	742.411
	11/7/2001	625.725	625.725	625.805	625.693	625.679	625.658
	12/3/2001	925.773	925.773	926.009	925.853	925.851	925.787
	1/7/2002	1001.259	1001.259	1001.182	1001.111	1001.175	1001.032
	2/4/2002	333.571	333.571	333.628	333.584	333.588	333.564
	3/4/2002	417.060	417.060	417.203	417.074	417.069	417.043
	4/1/2002	475.859	475.859	475.509	475.351	475.376	475.335

TWV : the sum of winners' valuation under truth-telling = the area under aggregate marginal valuation function

AWV : the sum of actual winners' valuations

TABLE 14 − Efficiency Implication 3: TWV-AWV(mill US$)[57]

Format	Date	Con. FOC	Dis. FOC (w/ correction)	Dis. FOC (w/o correction)
Multi Price	9/13/1999	0.007	0.073	0.031
	10/11/1999	0.007	0.001	0.001
	11/15/1999	0.010	0.004	0.001
	1/17/2000	0.004	0.001	0.001
	2/14/2000	0.004	0.000	0.000
	3/13/2000	0.002	0.000	0.000
	4/10/2000	0.007	0.000	0.000
	5/8/2000	0.002	0.000	0.000
	6/12/2000	0.007	0.019	0.044
	7/10/2000	0.001	0.043	0.009
	(Mean)	0.005	0.014	0.009
Single Price	8/14/2000	0.001	0.102	0.033
	9/18/2000	0.005	0.017	0.043
	10/9/2000	0.002	0.256	0.032
	11/13/2000	0.001	0.553	0.031
	1/8/2001	0.000	0.419	0.094
	2/5/2001	0.000	0.140	0.794
	3/12/2001	0.000	0.059	0.044
	4/2/2001	0.003	2.247	0.116
	5/7/2001	0.001	0.065	0.119
	6/4/2001	0.026	0.084	0.021
	7/2/2001	0.002	0.005	0.091
	8/6/2001	0.002	0.080	0.065
	9/3/2001	0.001	0.155	0.115
	10/8/2001	0.001	0.078	0.092
	11/7/2001	0.000	0.112	0.022
	12/3/2001	0.001	0.157	0.064
	1/7/2002	0.281	0.071	0.143
	2/4/2002	0.003	0.043	0.024
	3/4/2002	0.001	0.129	0.026
	4/1/2002	0.001	0.157	0.042
	(Mean)	0.017	0.246	0.101

[57] In calculating TWV and AWV, for the bids that tied at the market clearing price I rationed the demands exceeding total supply by tied bidders' incremental bid quantities at that price following the actual rationing rule. Therefore, in case of the auction without any bid switched, we can see somewhat small amount of surplus change due to the change in the composition of tied bids.

From Table 14, the average difference of TWV and AWV are over 10 times larger in the single price auction than in the multiple price auction. These results seem to support the recent argument about efficiency in the multi-unit auction, which is, the multiple price auction may be better than the single price auction in efficiency.

CHAPTER VII

THE ROBUSTNESS OF MODEL ASSUMPTIONS

In this CHAPTER, I reinvestigate two assumptions which I postponed in earlier discussions in CHAPTER IV. First, I will discuss a way to deal with certain kinds of asymmetries among bidders within our resampling procedure, and compare the empirical results when asymmetries are considered and the previous results with a symmetric bidder. Then I will investigate the underlying value structure of the Korean Treasury auction market by testing whether a common value component is present. I will also see if it is stronger compared to a private value. This is to see if my analysis under the PIV setting is adequate to explain the real Korean situation.

7.1 Asymmetries among Bidders

It is well known that if bidders are not symmetric in an auction the outcome would be different from if we assume symmetric bidders. In the case of the single unit auction, there are many literatures to account for the asymmetries under both the PIV setting and the CV setting.[58] They are mainly dealing with the comparison of the first

[58] An example of an asymmetric bidding situation in the single unit auction arises in government procurement when both domestic and foreign firms submit bids and for some reason, there are systematic cost differences between domestic and foreign firms. Another example can be found in antiques auctions in which bidders can be classified into dealers and collectors.

price auction and the English auction, and conclude that asymmetry breaks down the Revenue Equivalence between the benchmark auction models.[59]

However, in multi unit auction cases, though some policy experimental researches attempted to explain the asymmetries by introducing the bidder's type dummy in a reduced form of the regression model, it is very hard to find the literatures which consider asymmetry in both theoretical papers and empirical papers using the structural model approach.

In this section, I try to recognize whether my data reveal any type of asymmetries among the bidders, and I try to modify my current resampling method to cope with the existence of asymmetries. I will also perform the empirical estimation for the revenue and efficiency comparison with relaxing the symmetry assumption.

Since it is not possible to extend the resampling strategy in a way to control for bidder-level fixed effects, I will restrict my concern to the case where the sample can be broken down to a small number of *i.i.d* sub-samples.

When we consider grouping the data, one practical concern still remains. That is "what is the proper number of groups?". If it is too large, then the number of elements in each group gets smaller so that it makes the problem similar to a bidder-level fixed effects. If it is small but the number of elements in each group is too uneven(eg.

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[59] Maskin & Riley(85) conjecture that, with asymmetric bidders, a first price auction yields a higher expected revenue than an English auction if the bidders have similar valuation distributions but with different supports. McAfee & McMillan(87) argue that when bidders are asymmetric, the first price sealed bid auction yields a different price from the English auction, and the Revenue Equivalence breaks down.

group1=2 and group2=26), the randomness of the resamples from smaller groups will disappear. Therefore, the proper number of groups should be small and the number of elements of each group should be allocated as evenly as possible. In this case, dividing into 2 or 3 groups seems to be best considering that the range of total number of bidders is 23 to 31.

7.1.1 The Existence of Asymmetries in My Data

Keeping in mind the consideration about the proper number of groups, I expect two possible kinds of asymmetries. First of all there may be asymmetry between banks and security houses. In Korea, these two groups are thought to be different from each other in terms of funding capability and the activity in the TB market. Banks are generally superior to the security houses in their funding capability and banks are believed to participate in the auction to hold TBs, while security houses want to trade them in the secondary market. This makes the grouping quite obvious. Hence the asymmetry between these two is worth being tested.

The other possibility is the asymmetry between big bidders and small bidders. The sources of this asymmetry may come from the difference in their ability to acquire information. We can think of the big bidders as having advanced business networks compared to the small bidders. This lets the big bidders acquire more accurate information which is required to make better expectations of the auction outcome.[60]

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[60] Umlauf(93), in his study of Mexican T-bill auction, found evidence that large bidders' ex post

However, the groupings according to size do not have an absolute criterion so that we must be very careful when selecting the cutting line between the groups. There are many possibilities for choosing a criterion to classify the size of the bidders. For example, amount of assets, the amount of capital, and the amount of revenue, etc. However, these criteria may not always indicate how actively the bidders play in the Treasury market. Therefore, I used the performances within the auction market itself to classify the size of the bidders by the total bid amounts and the total winning amounts by each bidder.

In order to divide the bidders by size, I choose the following way:

1) Ranked firms by their total bid quantities in a multiple price auction period and in a single price auction period, respectively.

2) Found the point of the biggest gap between one and the next.

3) Used a ranking by total winning quantities as a supplementary criterion. That is, I ranked firms by total winning quantities again. If the firm belonging to the big group in terms of total bid amounts is still found in the big group in terms of winning quantities, it was classified as a big bidder, but if not, it was removed from the big bidder group.

Before grouping according to the method above, we still need to consider that bidders were not identical for all of the auctions during the sample period. Some firms remained on PDs' list at all times, but others entered or exited. Therefore, to maintain

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profits exceeded those of small bidders, and explained this by saying that small bidders bid solely on the basis of public information. Hendricks & Porter and Engelbrecht-Wiggans, Milgrom & Weber(83) dealt with this kind of information asymmetries in the single unit auction environment.

the consistency for the pool of bidders, I get rid of 2 banks and 6 security houses which showed long discontinuity in participation from the data for a multiple price auction period, and 5 banks and 5 security houses for a single price auction period. Thus finally we have totally 30 bidders(16 banks, 14 security houses) for a multiple price auction and 28 bidders(13 banks, 15 security houses) for a single price auction.

For a multiple price auction period, I found a big decrease between the 12^{th} and 13^{th} bidder which amounted to 93 billion KW, and first I choose 12 firms as big bidders. However, after applying the criterion of the winning amount, I removed five firms from the original 12 firms so that finally I have 7 firms as big bidders. In the case of the single price auction period, I found a big gap between the 13^{th} and 14^{th} bidder which amounted to 250 billion KW and chose the top 13 firms as big bidders. Among them one firm was dropped by the winning amount criterion so that 12 firms were chosen as big bidders for a single price auction period.[61]

Now, I attempted to figure out which classification is more reasonable to capture asymmetries by looking at some summary statistics. Table 15 and Table 16 show the mean values of several characteristics between the classified groups. Form Table 15, it is difficult to find significant differences between the banks and the security firms except for few items such as average bid quantities, the number of bids in a single price auction period and a bid spread measured by the difference between maximum

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[61] Big bidders' code: multiple price auction : #3, #6, #9, #23, #27, #29, #30 / single price auction : #3, #6, #8, #19, #21, #22, #23, #24, #27, #28, #29, #30

bid and MCP in a single price auction period.[62] In contrast, we do see that more of a distinction between the big and small firms under both of the auction formats in Table 16.

Table 15 − Comparison b/w Banks and Security Firms

	Multiple price auction		Single price auction	
	Bank	Security	Bank	Security
Participation ratio(%)	90.62	95.71	78.57	87.66
Avg. Bid quant. (% of total supply)	8.17	10.17	8.62	11.86
Avg. Win quant. (% of total supply)	3.46	3.64	3.81	4.30
# of bids	3.35	3.84	2.53	3.33
# of winning bids	1.78	1.94	1.30	1.52
Max bid–Min bid(KW)	11.27	11.34	12.07	17.53
Max bid–MCP(KW)	2.59	4.13	5.76	7.33

Table 16 − Comparison b/w Big and Small Bidders

	Multiple price auction		Single price auction	
	Big	Small	Big	Small
Participation ratio(%)	95.71	92.17	95.41	74.70
Avg. Bid quant. (% of total supply)	13.80	7.67	13.36	8.14
Avg. Win quant. (% of total supply)	5.62	2.92	5.19	3.27
# of bids	4.01	3.45	3.54	2.51
# of winning bids	2.24	1.74	1.70	1.21
Max bid–Min bid(KW)	11.27	11.31	19.31	11.78
Max bid–MCP(KW)	4.28	3.02	8.73	5.05

[62] Both the average of bid quantities and winning quantities of security firms are larger than those of banks. This may reflect the fact that the number of banks among the big bidders is relatively small for both auction periods, which are 3/7 for multiple price auction period and 3/12 for single price auction period.

Therefore, the classification of big and small firms is more likely to capture the distinction between groups than that of the banks and security firms. Also, in Table 16, we can see that big firms seem to use a greater number of bids and more widely spread bids, especially in the range of above MCP versus the small firms. This may reflect that big firms are likely to be stronger in collecting potential demand using their business networks, and that they are likely to participate in the auction more aggressively due to certain factors such as to maintain their market share.

In conclusion, I decided to choose the classification of big and small firms in the analysis of asymmetries.

7.1.2 Effects of Introducing Asymmetries in the Previous Model

To summarize, the model analyzed in the previous CHAPTER assumes that each bidder has his own private signal t_i regarding his value from winning a given quantity of the TB which is assumed to be drawn from the commonly known distribution of signals across the bidders, $F(t)$, with the marginal distribution of bidder i, $F_i(t)$. In addition, bidder i's marginal value from winning q unite of TB is given by the marginal valuation function, $v_i(q, t_i)$. Symmetric assumption was imposed by dropping the subscript i in either $F_i(t)$ or $v_i(q, t_i)$. That is, we assumed that there is one common distribution F from which the bidders draw their signals, or that the form of the marginal valuation function is identical across the bidders.

However, in the previous CHAPTER, instead of estimating the signal distribution or identifying the explicit functional form of the optimal bid function or the valuation function, we estimated the distribution of market clearing price by the construction of a residual supply using the observed bid vectors. In doing that, we implicitly assumed that others' optimal bid strategies, $q^*_j(p, t_j)$, are the results of their own signals drawn from a common distribution among all bidders, $F(.)$, even though we do not know the exact mapping between the signal and optimal bid function. Further we assumed that their observed bid vectors are their optimal bid functions.

Now, suppose that the asymmetry is introduced between group1($N1$) and group2($N2$). If we assume that signals. where the members of each group draw their signals independently from the distribution $F_1(.)$ and $F_2(.)$ respectively, and two distributions are independent of each other, then we can interpret the residual supply as sum of total $N-1$ random variables which are composed of $N1-1$ random variables drawn from $F_1(.)$ and $N2$ drawn from $F_2(.)$ independently.

Considering the setting and solving of a bidder's profit maximization problem in the previous CHAPTER, though the construction of a residual supply becomes [sum of ($N1-1$) bid vectors + sum of $N2$ bid vectors] if bidder i belongs to group1, this doesn't change the definition of $H(.)$ and the form of FOC which we derived under symmetric assumption

The validity of the resampling method is also maintained with the asymmetries because the observed $q(p)$ is assumed to be the outcome of optimal strategic behavior even though the underlying mapping now becomes *(t_i, $F_1(t)$, $F_2(t)$)→$q(p,t_i)$* instead of *(t_i, $F(t)$)→$q(p,t_i)$*.

To see this more clearly in our model, I provide a simple example with a few assumptions about the distributions.

Suppose that there are n_1 type 1 bidders and n_2 type 2 bidders and type i bidders draw their signals from F_i which is a normal distribution with mean $\bar{t}_i$ and variance σ_i^2. Also suppose that their optimal bid function is linear in price and the signals, that is:

$$y_i(p,t_i) = a_i + b_i p + ct_i$$

If the bidder belongs to type 1, his market clearing condition is:

$$1 = y_1 + (n_1 - 1)a_1 + (n_1 - 1)b_1 p + c\sum_{j\in 1} t_j + n_2 a_2 + n_2 b_2 p + c\sum_{j\in 2} t_j \quad {}_i$$

Then the CDF of the MCP will be:

$$H(p,y) = \Pr\{K \le 1 - y_1\}$$

where $K = (n_1 - 1)a_1 + (n_1 - 1)b_1 p + c\sum_{j\in 1} t_j + n_2 a_2 + n_2 b_2 p + c\sum_{j\in 2} t_j \quad {}_i$

Since $(\sum_{j\in 1} t_j + \sum_{j\in 2} t_j)$ is distributed with $N((n_1 - 1)\bar{t}_1 + n_2\bar{t}_2, (n_1 - 1)\sigma_1^2 + n_2\sigma_2^2)$,

$$H(p,y) = \Phi\left(\frac{\dfrac{1 - y_1 - (n_1 - 1)a_1 - (n_1 - 1)b_1 p - n_2 a_2 - n_2 b_2 p}{c} - \mu}{\sigma}\right)$$

where
$$\mu = (n_1 - 1)\bar{t}_1 + n_2\bar{t}_2$$
$$\sigma = \sqrt{(n_1 - 1)\sigma_1^2 + n_2\sigma_2^2}$$

Hence, in order to maximize their expected profit, type 1 and type 2 have to solve the same form of FOC as the previous one but they must consider the different composition of the distributions for the market clearing price according to their types.

7.1.3 Modification of the Resampling Procedure

In order to apply the resampling method to the asymmetric bidder case, we need to modify the resampling procedure slightly according to which group a bidder belongs to using the following:

1. Fix bidder i
2. If i belongs to $N1$ group, draw $N1$-1 bid vectors from $N1$ bid vectors by giving the same probability $1/N1$, and draw $N2$ bid vectors from $N2$ bid vectors by giving the same probability $1/N2$
3. With these resampled bid vectors($N1$-1+$N2$), construct the residual supply faced by bidder i and intersect i's actual bid schedule to find market clearing price.
4. Repeat the above procedure B times, and find B market clearing prices.
5. Estimate $H(.)$ and related derivatives for bidder i who belongs to $N1$
6. Do the same procedure for bidder who is in $N2$ group

The remaining steps to compare the revenue or the efficiency after resampling the are same as before.

7.1.4 Empirical Result with Asymmetries Assumption

To highlight the difference from the symmetric case, I present only the results of the discrete version of FOCs with correction terms.[63] Table 17 summarizes the change in revenue terms and Table 18 displays the change in the efficiency implication.

Note that the figures of symmetric case are slightly different from those using the same FOCs shown in the previous CHAPTER. This is because, in this CHAPTER, I excluded some bidders who showed long discontinuity in participating in the auction for each auction period. Especially, in auction #27(1/7/2002) and auction #30(4/1/2002) many bid points were removed from the original data set which were 20 and 13 bid points respectively, so we need to be careful with these two auctions.

If I assume the existence of asymmetries between big and small firms, the results are not much different from where I simply assumed symmetry. The mean values of MCPv and the revenue loss rate do not change much for both auction formats. The efficiency results under asymmetries also have the same interpretation as those under symmetries.

[63] In the case of the other two versions of FOCs, I got qualitatively similar results.

TABLE 17 − Changes by Asymmetries in Revenue Comparison

	Date	MCPv		Revenue loss(%) (=[Ra-Rv]/Rv)	
		Asym.	Sym.	Asym.	Sym.
MULTI	9/13/1999	10000.88	10002.59	0.117	0.100
	10/11/1999	10000.00	10000.00	0.033	0.033
	11/15/1999	10000.18	10000.27	0.024	0.023
	1/17/2000	10000.00	10000.00	0.039	0.039
	2/14/2000	10000.00	10000.00	0.029	0.029
	3/13/2000	10000.00	10000.00	0.027	0.027
	4/10/2000	10000.00	10000.00	0.018	0.018
	5/8/2000	10000.00	10000.00	0.018	0.017
	6/12/2000	10000.21	10000.00	0.080	0.082
	7/10/2000	10002.65	10000.02	0.025	0.051
	(Mean)	10000.39	10000.28	0.041	0.042
SINGLE	8/14/2000	10002.84	10002.70	-0.028	-0.027
	9/18/2000	10000.00	10000.00	0.000	0.000
	10/9/2000	10002.65	10002.81	-0.027	-0.028
	11/13/2000	10001.31	10002.17	-0.013	-0.022
	1/8/2001	10000.00	10001.29	0.000	-0.013
	2/5/2001	10002.75	10002.75	-0.028	-0.028
	3/12/2001	10002.72	10002.72	-0.027	-0.027
	4/2/2001	10000.00	10002.77	0.000	-0.028
	5/7/2001	10004.34	10002.72	-0.043	-0.027
	6/4/2001	10003.02	10002.70	-0.030	-0.027
	7/2/2001	10002.73	10002.51	-0.027	-0.025
	8/6/2001	10002.14	10002.30	-0.021	-0.023
	9/3/2001	10001.84	10000.79	-0.018	-0.008
	10/8/2001	10002.80	10002.16	-0.028	-0.022
	11/7/2001	10000.01	9999.99	0.000	0.000
	12/3/2001	10001.50	10001.62	0.021	-0.016
	1/7/2002*	9997.69	10000.00	-0.004	-0.027
	2/4/2002	10001.34	10001.29	-0.013	-0.013
	3/4/2002*	10001.83	10001.93	-0.046	-0.047
	4/1/2002*	9998.90	9998.99	-0.016	-0.017
	(Mean)	10001.52	10001.71	-0.017	-0.021

The actual MCP in the auctions with * are 9997.2 KW due to the exclusion of some bidders who didn't participate the auction continuously.

TABLE 18 − Changes by Asymmetries in Efiiciency

	Date	# of Bid Switch		Quant. Change of Bid Switch(%)		TWV-AWV (mill US$)	
		Asym	Sym.	Asym.	Sym.	Asym.	Sym.
MULTI	9/13/1999	2	2	1.67	1.67	0.068	0.073
	10/11/1999	0	0	0.00	0.00	0.001	0.001
	11/15/1999	0	1	0.00	1.69	0.002	0.004
	1/17/2000	0	0	0.00	0.00	0.001	0.001
	2/14/2000	0	0	0.00	0.00	0.000	0.000
	3/13/2000	0	0	0.00	0.00	0.000	0.000
	4/10/2000	0	0	0.00	0.00	0.000	0.000
	5/8/2000	0	0	0.00	0.00	0.000	0.000
	6/12/2000	0	0	0.00	0.00	0.016	0.044
	7/10/2000	1	0	1.71	0.00	0.127	0.043
	(Mean)			0.34	0.34	0.021	0.017
SINGLE	8/14/2000	6	5	16.67	15.00	0.189	0.033
	9/18/2000	1	1	2.22	2.22	0.014	0.014
	10/9/2000	6	6	12.22	12.22	0.137	0.140
	11/13/2000	6	7	22.11	31.58	0.923	0.553
	1/8/2001	1	1	1.30	1.30	0.089	0.077
	2/5/2001	5	5	11.69	11.69	0.119	0.140
	3/12/2001	1	1	2.00	2.00	0.194	0.059
	4/2/2001	1	5	1.25	13.75	0.043	0.129
	5/7/2001	3	3	13.33	13.33	0.229	0.065
	6/4/2001	1	1	2.50	2.50	0.099	0.084
	7/2/2001	1	1	2.50	2.50	0.006	0.005
	8/6/2001	2	2	4.29	8.57	0.059	0.080
	9/3/2001	3	2	9.41	8.24	0.445	0.155
	10/8/2001	6	6	12.36	11.24	0.489	0.078
	11/7/2001	4	3	6.67	5.33	0.022	0.112
	12/3/2001	4	4	5.40	13.51	0.068	0.157
	1/7/2002*	5	6	8.33	15.00	0.163	0.280
	2/4/2002	2	2	7.50	7.50	0.078	0.042
	3/4/2002*	3	3	8.00	8.00	0.080	0.077
	4/1/2002*	4	4	12.28	12.28	0.275	0.227
	(Mean)			8.10	9.89	0.186	0.125

7.2 The Presence of Common Values

As I pointed out in CHAPTER IV, the underlying value structure may be a very critical assumption in designing the optimal auction mechanism because the structure of the expected pay-off functions and the equilibrium strategies may vary depending upon the assumption of the value paradigm. In addition, the PIV and CV should be interpreted as extreme cases. Real world auction situations are likely to contain both aspects, simultaneously.

Focusing on a TB auction, Bikhchandani and Huang (1993)[64] and Commack (1991)[65] argued that whenever bidders buy an object for resale rather than for personal consumption, the common value assumption is reasonable and that in a Treasury auction, the CV is more appropriate because the value of each bidder is the resale price which is common but unknown to all the bidder.[66] However, they didn't exclude the possibility of a more general value structure either. As Bikhchandani & Huang pointed out, in the Treasury securities markets, the primary dealers' private information comes from two sources. First, primary dealers have their own forecasts of the movements of the term structure of interest rates. Second, which is more important thing, before the

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[64] See Bikhchandani and Huang (1993).

[65] Cammack(1991), "The appropriate model for the treasury auction is the CV, where the true of the good being auctioned is the same for all bidders but the true value is unknown and bidders have different information about it."

[66] Actually, a similar idea to this can be found in Smith(66). He divided his models into three types according to whether or not the bidders consider the resale price though he didn't mention the term of the valuation.

auction institutional buyers such as pension funds and insurance companies place orders for Treasury securities to be auctioned with primary dealers. This conveys information about the demand for these securities. Therefore, the two incentives may exist to purchase the securities at the same time so that the valuation may also be ruled by both the PIV and the CV structure.

However, in the cases of an auction for multi-unit item, as I mentioned, the complete development in terms of both the theoretical model and the empirical methodology for this matter has not been accomplished. To my knowledge, there is no literature dealing with the multi-unit case regarding the test to see if a CV component exists. Also, the nonparametric "structural model estimation" such as that demonstrated in this paper cannot be applied to the case where a CV component is present because this method critically hinges on the assumption that bid functions are *i.i.d*. If there is a CV component, the bidders' valuations should be correlated.

Considering the above limitation in availability of complete examples for the value structure in the multi-unit auction case, in this section, I will just attempt to test the presence of a CV component using basically the idea which has been developed in single unit auction literatures instead of estimating the structural model under the presence of a CV component. I think that this attempt itself is meaningful because it may give us some useful information to the question, "Was the PIV assumption in the Korean auction market very bad?" and it is the first attempt to test the presence of CV in the multi-unit auction environment.

7.2.1 Theoretical Prediction of Bid Behavior to Value Structure

Under the existence of common values, bidders are susceptible to the winner's curse. It occurs in this way: assume for simplicity that one indivisible object is for sale and that bidders use similar rules-of-thumb to calculate their bids based on their estimates of the true underlying value. In a common value auction with many bidders, even though each bidder's estimate is unbiased, some estimates will be high and some low. As a result, the highest bidder (the winner) is usually the one who is the most optimistic about the true value. Or to put it another way, upon winning, the successful bidder then learns something striking: all the other bidders had lower estimates of the true value. A bidder who fails to take this factor into account may easily bid too high, thus winning the auction but discovering he bid too high given the realized value of the good. This phenomenon is called the winner's curse. [67]

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[67] There is the concept of the "Loser's Curse" which was introduced in Pesendorfer and Swinkels(1997). In the literatures of common value auction, it has been one of the biggest concerns of the researchers whether the auction mechanism can aggregate the information that is diffused through the economy, which becomes whether the bid price converges in probability to true common value. Their object is also to show the condition for the price convergence to the true value. They worked with the auction allocating k identical objects to n bidders instead of selling just single item. However, since they restricted each bidder to obtain only one object, the structure of auction is same as the single item auction. With this setting, they proved the convergence with interaction of winner's curse effect and loser's curse effect. Winning with a bid b conveys the negative information that at most k of the other bidders submitted bid above b upon receiving their signals. This is the winner's curse. However losing with a bid b conveys the positive information that at most n-k of the other bidders chose to bid *below b*. They called this "Loser's Curse". Therefore they established that if both k and n-k go to infinity, then two effects balance to yield the equilibrium price which converges to the true value. However, because

The winner's curse has several implications for optimal bidding strategies.[68] First, as the number of bidders increase (and other factors are held constant), it is usually optimal to bid more conservatively. This is because the highest of, say ten estimates is likely to be much greater than the highest of two estimates. Thus the winner's curse is reinforced as the number of bidders increases and, despite the increased competitive pressure, it often causes bidders to shade their bids below their estimates of true underlying value by a greater amount. Nevertheless, the effect of having more bidders tends to outweigh the incentives for bidding conservatively, in that the highest bids and the selling price increase as the number of bidders increases. On the other hand, the reverse change of the bidding behavior may be possible. That is, initially more competition due to the increase in number of bidders makes the bidder more aggressive, but with more competition, bidders must also recognize the winner's curse effect and hence bid functions ultimately decrease in competition. Therefore, in general, the expected value of the winning bid at auctions within the common value paradigm will have no predictable relationship with the number of bidders.[69] However under the private value paradigm, since there is no winner's curse effect and bidders consider only competition effect, the winning bid will increase monotonically with the number of bidders.

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the main purpose of this section is testing the existence of CV and it is not available to predict the effect of the "Loser's Curse" on our auction environment, I will ignore this effect in my analysis to simplify the problem.

[68] Bikhchandani & Huang(1993)

[69] See Paarsch(1990) p.198.

Second, as the uncertainty about the value of the object decreases, the winner's curse is weaker. For example, if a bidder can assess the true value to within, say, plus or minus 10 percent, then the highest estimate will tend to be much lower than if bidders can assess the true value only to within plus or minus 50 percent. As uncertainty decreases, bidders recognize that they have less to worry about from the winner's curse. They bid less conservatively, and the selling price typically increases. Naturally, in private value paradigm, the bidders don't have to infer the true value of the object so that the bid is not affected by the degree of the uncertainty.

To summarize graphically, the predicted relationships between bid prices and the number of bidders or the uncertainty in the value of the item to be auctioned can be depicted in Figure 23.

7.2.2 Empirical Framework to Test the Value Structure

Although deciding which paradigm applies in a specific situation is of considerable interest to those using auctions, there were few papers which developed an empirical framework within which to examine this matter. For the auction of a single good, Paarsch(1992) designed and applied the empirical framework to data from a sample of tree planting contract auctions held in the province of British Colombia, Canada in an attempt to decide between PIV and CV, but for the multiple unit auction this kind of analysis does not exist to my knowledge.

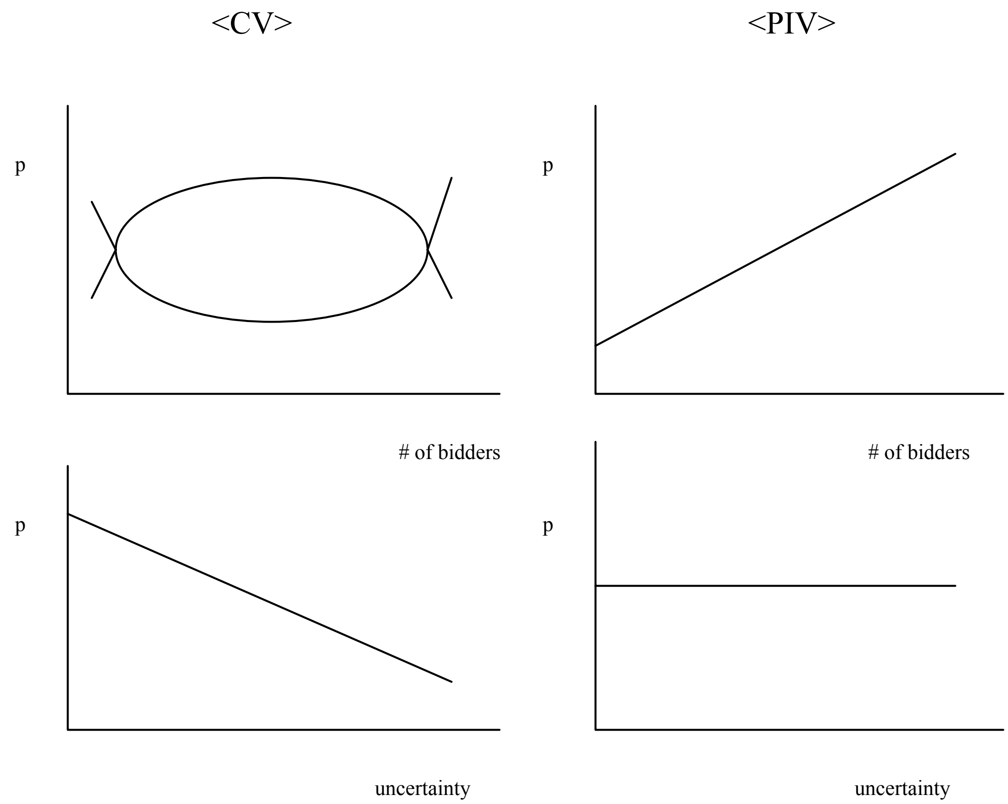

FIGURE 23. Comparison of Bid Behaviors between CV and PIV

Therefore, instead of developing a rigorous model to test, I will follow the idea which was developed in the "Policy Experimental Approach". That is, I constructed a simple regression model which is able to check the relationship between the bid price as a dependent variable and the independent variables such as the number of bidders and the uncertainty variable to see how strongly these relevant coefficients support the theoretical predictions of either a pure CV model or a pure PIV model which were discussed in the section 7.2.1.

Hypotheses to test

First of all, I will investigate the behavior of the representative bid price of each bidder at each auction according to the change in two key independent variables to see general relationship between them.

Since a TB auction is for multi-unit items and each bidder can submit multiple bids, I will attempt to decompose each bidder's bid points by scrutinizing the sources of demand for the Treasury securities. Then, I will test a more clarified hypothesis resulting from this discussion of demand. The sources of demand for the Treasury securities can be classified into three categories. They are the reserve requirement, the orders from customers, and the arbitrage requirement.

Reserve requirement: This is the demand for holding the securities up to its maturity. All financial institutions have their own investment schedule which distributes their available funds into risk-free assets and risky assets. In order to maintain the soundness of their portfolios they must have a certain inner standard about the portion of risk-free assets among total assets in their possession. Furthermore after the Asian crisis in 1997, the Korean government enforced the regulations on the soundness of financial institutions and imposed the obligation of maintaining a certain ratio of risk-free assets on them following the international standard.

Orders from customers: In the Korean treasury auction market, the banks and security houses that are designated as primary dealers can participate in the auction as bidders so that other types of institutional buyers like pension funds and insurance companies should buy the bonds to be auctioned via the primary dealers. Since these institutions are usually considered to be long-term investors these demands are likely for the purpose of holding. The primary dealers who want to buy the bonds on behalf of their customers may have a responsibility to fulfill their orders to sustain business relationships with them.

Arbitrage requirement: This is the demand to get marginal gain through trading the bonds which they acquire at the auction. This demand may be less urgent than the first two sources and the expectation of the secondary market rate would be important when the bidders decide their bidding strategy.

Relating these sources to the matter of the value structure, the natural conjecture is that the first two sources of demand are likely to be governed by the PIV and the last one, arbitrage requirement, be governed by the CV paradigm. In case of the reserve requirement, since the degree of urgency to fulfill the requirement may be different across the bidders due to the difference in the size of available funds and the status of their portfolios, it is reasonable to think that the value of this portion would be placed individually and independently. The values of the orders from the customers are actually decided by the customers, not by the bidders though they discuss the possible bid prices with each other before the auction. If we accept the fact that the customers are long-term investors and that they want to buy the bonds for the purpose of holding rather than trading, the value can be thought to be governed by PIV. Naturally, it is plausible that the value for the arbitrage requirement are more likely to be formed by

CV because the bidders have to expect the movement of the market rate in the future which is common but unknown to all bidders.

In order to make these conjectures according to the classification of demand sources be testable in my regression model, we need to relate these three kinds of demands to their bid prices. For this reason, I divided the three sources into two portions again, saying, “MUST BUY” and “MAY BUY”. Since the first two sources of demands need to be fulfilled more urgently than the third one, I call the first two demands as “MUST BUY” portion and the last one as “MAY BUY” portion.

In the case of “MUST BUY” portion, the bidder has to increase the probability of winning so we will place the higher bid price to this portion, while the other portion has less incentive to place a higher bid. Since the Korean government restricts the number of bids to five points we can think of that the relatively higher bid prices may be for the “MUST BUY” portion and the relatively lower bid price may be for the “MAY BUY” portion.

Therefore, the hypothesis related to the demand characteristics is that the MUST BUY portion which is represented by relatively higher(or winning) bids is likely to follow the PIV paradigm, while the MAY BUY portion which is represented by relatively lower(or losing) bids is likely to follow the CV paradigm.

Model and Variables

To see the effect of the number of bidders and the uncertainty on bid price, I suggest the following simple regression model:

$$
\begin{aligned}
BIDPRICE_{it} = C &+ B_1*TNUMBID_t + B_2*SQTNUMBID_t + B_3*UNCERTAIN_t \\
&+ \alpha_1*INTEREST_t + \alpha_2*TSUPPLY_t \\
&+ \delta_1*AUCDUMMY_t + \delta_2*BANKDUMMY_i + \delta_3*BIGDUMMY_i \\
&+ ERROR_{it}
\end{aligned}
$$

$BIDPRICE_{it}$:

In this section I will use the same data set as before which covers the results for the 3-year TB auctions during Sep. 1999~ Apr. 2002. However, in order to run the regression model for this analysis we need to pool the data which was held at different times in a certain way. Unlikely to the estimation for each individual auction, when pooling the data, it is very important to consider how to normalize the bid price to obtain consistency over the whole sample period to be analyzed. Note that in the Korean TB auction, the bidders submit yield bids and these yields are fluctuated according to the current yield in the resale market at the time of the auction. In Figure 24, we can see that the yields of TB in the resale market had a variation from 4% to 10% during the sample period.

Furthermore, considering that the multiple bids are submitted by one bidder, to see each bidder's reaction to the number of bidders or uncertainty we need to have a certain type of a representative bid price for each bidder at a certain auction. To test my hypothesis that the “MUST BUY”, where the bidders assign the higher bid price is governed by PIV and “MAY BUY”, where bidders assign the lower bid is governed by CV, I also have to construct the bid price representing these two portions.

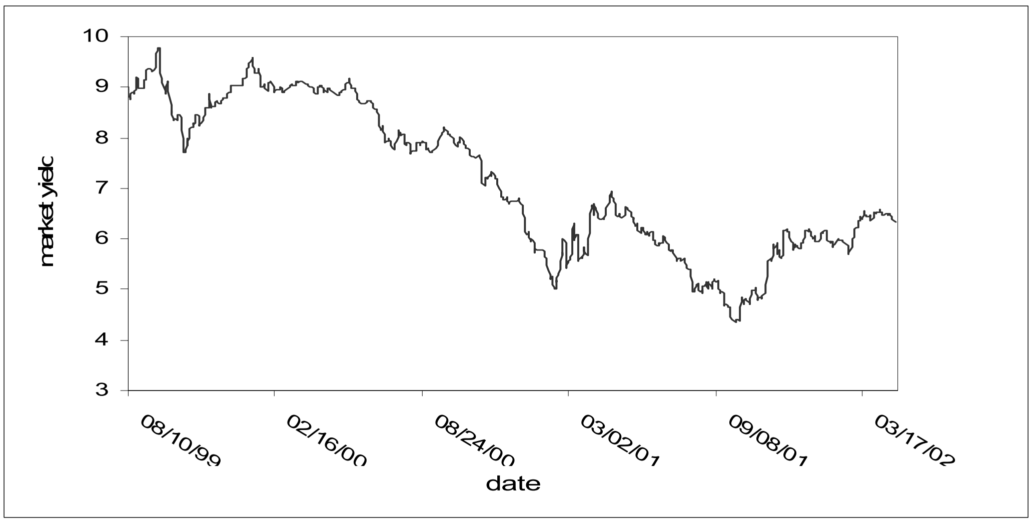

FIGURE 24. Trends of Resale Market Rate

First of all, to obtain the consistency for the bid price over the sample period, I calculated the average yield weighted by the probability of each bid yield in the distribution of all yields submitted at that auction.[70] Then I subtracted each bid yield from this weighted average yield so that the resulting value indicates how much higher or lower each bid is compared to the average bid price or representative bid price for that auction. For example, suppose that 10 bid yields are submitted by bidders at a

[70] I also thought about the average yield weighted by incremental bid quantity. However, there were some cases where the relatively big quantity placed at lower bid price so that the quantity weighted average may be biased to lower. Thus I decided to use the probability weighted average.

certain auction, say, one 7.01%, one 7.02%, two 7.03%, three 7.04%, two 7.05%, and one 7.06%. Then the probability distribution assigned to each bid yield is {1/10, 1/10, 2/10, 3/10, 2/10, and 1/20} and the average yield weighted by these probabilities is 7.037%. The final bid prices in my data set will be 0.027, 0.017, 0.007, -0.003, -0.013, and -0.023. Note that a relatively big positive number implies that a bidder submitted a relatively higher bid price to average bid price and that a relatively small positive or a big negative number implies that a bidder submitted a lower bid price relative to average bid price. Finally, to express this value in terms of basis point[71] I multiplied these converted bid prices by 100.

After the normalization of the bid price, I constructed several dependent variables to test my hypotheses. Firstly, to see the general results I constructed $WGTBID_{it}$ as the representative bid price of bidder *i* at auction *t,* measured by the probability weighted average of bidder *i*'s bids at auction *t*. Secondly, to see the bid behavior for the "MUST BUY" portion, I used two variables which are the maximum bid (MAXBID) of each bidder and the probability weighted average of winning bids (WINBID). Finally, to see the bid behavior for the "MAY BUY" portion, I used the probability weighted average of losing bids of each bidder (LOSEBID) and the minimum bid (MINBID) as a dependent variable.[72]

$TNUMBID_t$: the number of bidders at auction *t*.

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[71] 1 basis point corresponds to 0.01%p.

[72] In case that every bids submitted by one bidder are losing bids, I treat she doesn't have MAXBID. Similarly, if every bids are winning bids, then she doesn't have MINBID.

The first natural candidate for this variable is the number of bidders to participate in the auction *t*. However, as I pointed out, there are only 38 financial institutions which are designated as primary dealers and are allowed to participate in the auction. Furthermore, by the contract between the Treasury and the individual institutions, in order to maintain the position of primary dealer they are required to participate in the auction continuously and to purchase a certain minimum amount of the TB in the auction market throughout the whole year. Therefore, the number of bidders may not represent the number of real participants. One more thing to be considered is the fact that a real demander can be different from the primary dealers because many investors who want to purchase bonds but are not PDs such as pension funds and insurance companies exist. Therefore I will use the total number of bid points as the proxy of this variable instead of the number of bidders.[73]

Table 19 shows the summary statistics and Figure 25 depicts the trends for these two variables. We can see that the number of bidders doesn't have enough variation to use as a variable but is highly correlated with the total number of bids.

TABLE 19 − Statistics of #Bids and #Bidders

Variable	Mean	Std. Dev.	Min	Max	Obs
# of bidders	25.9	2.309	23	31	30
# of bids	83.3	21.817	40	124	30
Correlation coefficient : 0.83					

[73] Actually, the number of bids may also have a limit because the total number of bids submitted by each bidder is restricted up to five.

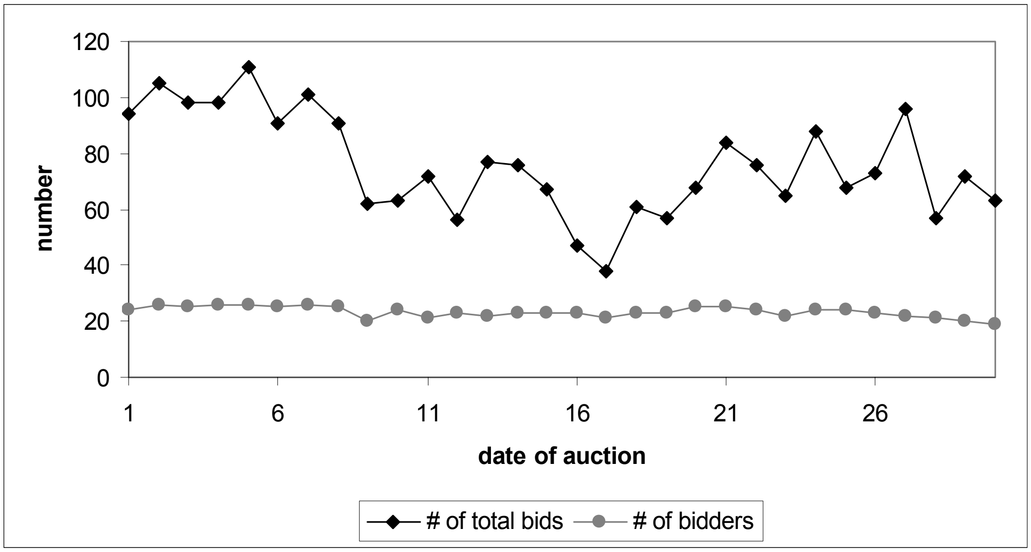

FIGURE 25. Trends of # Bids and # Bidders

$SQTNUMBID_t$: The square of total number of bids

This is included to capture the parabolic shape of the relationship between BIDPRICE and TNUMBID.

$UNCERTAIN_t$: The proxy for the uncertainty about the value of the object

A prediction of auction theory is that bidders adjust for the winner's curse by bidding at higher markups in terms of yield over his true valuation in the presence of increased uncertainty about the market value of the bonds they purchase. Adopting Simon(1992)'s idea, I will use the standard deviation of the market yields, calculated from the observations at the close of business for 10 days before the auctions as proxy for this variable.

$INTEREST_t$: The proxy for the (short term) trend of resale market rate

Spreads between the true valuation and actual bid should include a premium for the risks involved in bidding at auctions. These risks involved in TB auction may be classified into three types of risks.

The first one is the winner's curse if there exists a CV component. Secondly, there is the quantity risk which means the risk of not being awarded securities. This is important especially for dealers bidding to cover short positions in the auctioned security. Finally, the interest rate risk may be involved. That is, if the interest rate in the resale market goes up which means the price of the TB goes down, the bidders who purchased TB at the auction have to burden the capital loss. This is the most common consideration which the bidders are taking into account when they are preparing for their bids. Thus, the movement of interest rates in the resale market has nontrivial effect on the decisions of bid price.

In order to control the effect of relatively short term interest rates on bid price, I linearly fitted the trend of resale market rates for 30 days before the auction and used the coefficient of that as the proxy for the short term interest rate trend. Note that since the interest rate has a reciprocal relationship with TB price, the positive value of the coefficient implies a decrease in TB price.

$TSUPPLY_t$: The total supply of each auction

The total supply varies during the sample period as we see in Figure 26. Though the amount to be auctioned is announced before the auction, the size of the total supply may affect bidders' bidding strategies. For example, when a pretty big amount is auctioned the bidders expect a supply shock in the secondary market in the near future which has an effect of lowering the price. The bidders take this expectation into account when deciding their bid prices.

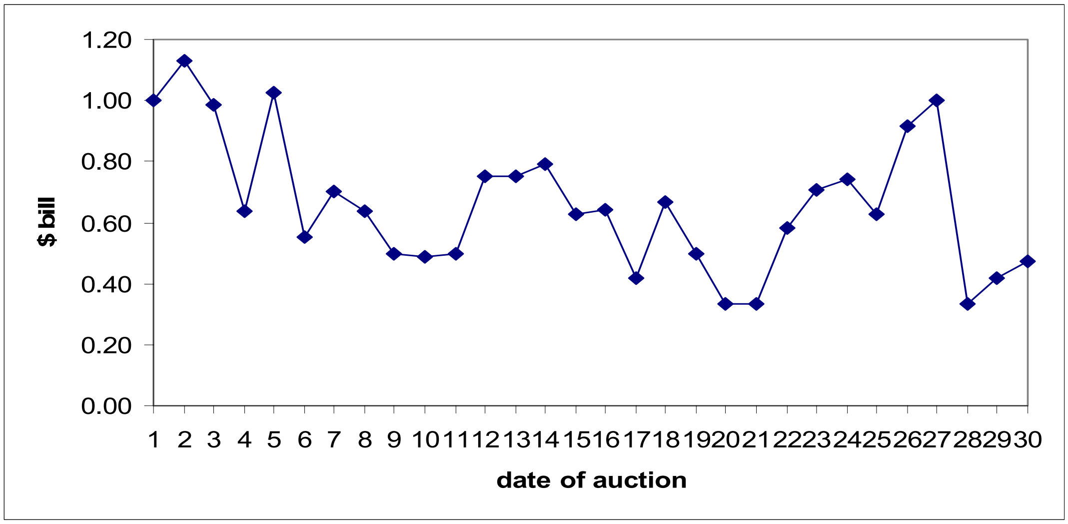

FIGURE 26. Total Supply of TB (bill US$)

$AUCDUMMY_t$: The dummy for the change of auction format

This is to control the fixed effect due to the switch of the auction format from the multiple price auction to the single price auction in Aug. 2000. This dummy takes on values of one for multiple price auctions and zero for single price auctions. The coefficient of the dummy indicates the average change in BIDPRICE attributable to multiple price auction scheme.

$BANKDUMMY_i$: The dummy variable for banks and security houses

This dummy variable takes on values of one for bidders of banks and zero for bidders of security houses.

$BIGDUMMY_i$: The dummy variable for big and small bidders

As I discussed in the section 7.1, there are a few distinctions in bid behavior between big and small bidders. However, in the previous section, the big bidders in the multiple price auction period are not the same as those in the single price auction period. Hence, here, I designated the bidders who were classified as the big bidder for both auction periods. There are 6 bidders whose bidder codes are #3, #6, #23, #27, #29, and #30.

The summary statistics of the variables in the model are in Table 20.

TABLE 20 − Summary Statistics of the Variables

Variable	Mean	Std. Dev.	Min	Max	Obs
WGTBID	.197	3.397	-31.481	18.506	777
MAXBID	4.461	5.454	-3.008	98.166	645
WINBID	2.761	3.152	-2.835	25.518	645
LOSEBID	-2.715	3.220	-34.997	3.298	593
MINBID	-4.105	5.866	-101.951	3.644	593
TNUMBID	84.837	21.756	40	124	777
SQTNUMBID	7670.212	3786.512	1600	15376	777
UNCERTAIN	.0983	.0619	.0335	.2936	777
INTEREST	-.0059	.0289	-.0857	.0640	777
TSUPPLY	.6690	.2209	.3333	1.1311	777

Regression results

Before seeing the empirical results, I will summarize the sign and significance of the relevant coefficients for testing my hypotheses predicted by theory.

If the Treasury auction is governed by the CV paradigm and the marginal function of TNUMBID is concave which means the competition effect dominates the winner's curse effect, the coefficients of TNUMBID and SQTNUMBID should be significantly positive and significantly negative respectively and *d(BIDPRICE)/d(TNUMBID)* should show the change of sign from plus to minus. However, if the marginal function of TNUMBID is convex which means the winner's curse effect dominates the competition effect first, then expected sign of TNUMBID and SQTNUMBID will be reverse. In case of the coefficient of UNCERTAIN should be negative and significant. However, if the underlying paradigm is PIV, then TNUMBID should be significantly positive with insignificant coefficient of SQTNUMBID or there is no change in sign of *d(BIDPRICE)/d(TNUMBID)* if the coefficient of SQTNUMBID is significant. Also the coefficient of UNCERTAIN should be insignificant.

The hypothesized coefficients for all regressors are summarized in Table 21.

TABLE 21 − Hypothesized Coefficients for Regression Analysis

Regressor	Expected estimate	Rationale
TNUMBID	(PIV) + & sig. (CV) + & sig. or - & sig.	
SQTNUMBID	(PIV) insig. (CV) - & sig. or + & sig.	
UNCERTAIN	(PIV) insig. (CV) - & sig.	
INTEREST	Negative	Reduce bid price when TB yield goes up(TB price goes down)
TSUPPLY	Negative	Expect excess supply of TB in the market after big sale
AUCDUM	Uncertain	
BANKDUM	Uncertain	
BIGDUM	Uncertain	

First of all, I ran five regressions for each of the five dependent variables. The regression results are shown in Table 22.

TABLE 22 − Results for Five Dependent Variable

Dep. Var	WGTBID	MAXBID	WINBID	LOSEBID	MINBID
TNUMBID	-.096*	.076	-.075**	.301*	.326*
	(.044)	(.074)	(.040)	(.051)	(.104)
SQTNUMBID	-.0005*	-.0003	-.0006*	-.001*	-.001*
	(.0002)	(.0004)	(.0002)	(.0002)	(.0006)
UNCERTAIN	1.902	9.283*	5.352*	-5.051*	-8.043**
	(2.146)	(3.651)	(1.992)	(2.107)	(4.281)
INTEREST	1.802	14.21*	5.398	-21.26*	-13.99**
	(4.276)	(7.182)	(3.918)	(4.227)	(8.587)
TSUPPLY	-.852	-.193	-2.949*	-3.904*	-2.087
	(.759)	(1.356)	(.740)	(.717)	(1.456)
AUCDUMMY	-.801*	-3.246*	-2.887*	-.766*	-.393
	(.339)	(.595)	(.324)	(.320)	(.650)
BANKDUM.	.654*	.349	.893*	-.324	-.087
	(.244)	(.422)	(.230)	(.234)	(.475)
BIGDUMMY	-.452	-.901**	-.589*	.320	-.615
	(.292)	(.501)	(.273)	(.265)	(.539)
CONS	4.492*	1.207	6.695*	-16.38*	-19.57*
	(1.995)	(3.292)	(1.796)	(2.335)	(4.745)
R^2	.034	.076	.176	.256	.074
Obs	777	645	645	593	593

Std. Dev in parenthesis / * significant at the 5% level or higher / ** significant at the 10% level

To look at the coefficients of the relevant variables for the existence of CV more closely, I resummarize these estimates only in Table 23.

TABLE 23 − Summary of the Results for Concerned Variables

	TNUMBID (B1)	SQTNUMB (B2)	Dy/dx x=38	Dy/dx x=76	Dy/dx x=111	UNCERTAIN (B3)
WGTBID	-.096*	.0005*	-.058	-.020	.015	1.902
MAXBID	.076	-.0003	.053	.030	.009	9.283*
WINBID	-.075**	.0006	-.029	-.016	.058	5.352*
LOSEBID	.301*	-.0011*	.217	.133	.056	-5.051*
MINBID	.326*	-.0013*	.227	.128	.037	-8.043**
Expected (CV)	+ & sig. (- & sig.)	- & sig. (+ & sig.)	+ to – (or – to +)			- & sig.
Expected (PIV)	+ & sig	insig.	+ for all supports			insig.

* significant at 5% level, *dy/dx* $=B_1+2*B_2*x$, *where* x*=TNUMBID*, y*=BIDPRICE*

1) Regression of WGTBID

The estimates of the coefficients of TNUMBID and SQTNUMBID are negative and positive respectively, and both are significant. The coefficient of TNUMBID, -.096, is interpreted that the bid price decrease by around 0.1 basis point due to the increase of 1 bid point. That implies that there may be a convex relationship between the bid price and the number of bids. This convex type is validated by the change in *dy/dx* along the support of TNUMBID from minus to plus slope. This may imply that as the number of bidders increases the winner's curse effect outweighs the competition effect the first time but gradually the competition effect grows and outweighs the winner's curse effect eventually. This result supports the pure CV paradigm.

However, the coefficient of UNCERTAIN is 1.9 which means that there is 2 basis points increase in bid price due to one standard deviation increase in the resale market rates. Since the mean of UNCERTAIN is 0.09, overall response of bid price to UNCERTAIN is 0.2 bp which corresponds to 0.002%p. Thus, although the sign of that is different from what we expected, but the amount tells us that the bidder is not very sensitive to the volatility in the resale market rates. Statistical significance also shows this to be insignificant. Furthermore, the coefficient of INTEREST is insignificant so that my conjecture may be confirmed once again. These findings support the PIV paradigm.

Therefore, the result is inconclusive regarding whether underlying value structure is governed by pure CV or pure PIV.

2) Regression of MAXBID and WINBID

If my hypothesis that relatively higher bid prices may follow PIV characteristics, then the coefficient of TNUMBID should be significantly positive but those of SQTNUMBID and UNCERTAIN should not be significant.

In the case of the behavior of bid prices with respect to the number of bids, the result of regression with MAXBID shows a clue to support PIV because the coefficient of SQTNUMBID is not significant and the slope of that is positive over the whole support of TNUMBID. However, the result of the regression with WINBID is similar to that of the regression with WGTBID though the coefficient of QTNUMBID is not significant.

The coefficients of UNCERTAIN have somewhat unusual values which are significantly positive in both regressions. Also, the coefficients of INTEREST have positive estimates which are different from the theoretical prediction. These results support neither PIV nor CV. It is very hard to find a reasonable explanation for these results but there might be some noise which cannot be captured by the researcher.

Therefore, the results are not quite obvious but they seem to support PIV weakly.

3) Regression of LOSEBID and MINBID

Both results give us relatively strong suggestion for CV. Since the coefficients of TNUMBID are significantly positive and those of SQTNUMBID are significantly positive the relationship shows a kind of a concave shape though the slope doesn't change to negative clearly at the end point of TNUMBID support. Also, the coefficients of UNCERTAIN and INTEREST shows the behaviors which are predicted in pure CV paradigm.

Therefore, my hypothesis that relatively lower bid prices are likely to have strong CV characteristics is validated.

4) Interpretation of other regressors

TSUPPLY : Every regressions yields negative coefficients though the level of significance varies across the regressions. This result validates my conjecture that when the supply is relatively big, bidders tend to participate in the auction more conservatively with the expectation of an excess supply in the TB market.

AUCDUMMY : Most of the regressions report significantly negative numbers which means that the bid prices in the single price auction are about 0.8 basis point higher than those in the multiple price auction on average.

BANKDUMMY : In case of the weighted average bid price, the maximum bid, and the weighted average winning bids, banks submit higher bid prices than security firms. However, in case of lower bid prices, security firms are likely to submit higher bids than banks.

BIGDUMMY : In most ranges of bid prices except for the weighted average of loosing bids, the big bidders submit relatively lower bid prices than small bidders. This result suggests that big bidders might have higher capability in aggregating various information and forecasting the market clearing price so that they seldom submit quite high bid prices to win.

5) Conclusion

The overall results do not seem to be enough to make a conclusive decision between pure PIV and pure CV. However, the regressions with decomposed data set of bid points supports my hypothesis that PIV appears strongly in higher bid prices whereas CV in lower bid prices.

Since in any case we couldn't reject the possibility of the existence of the CV component, these findings suggest that we need to develop a more general model which can encompass various types of the value structure such as interdependent value structure having both PIV and CV components. As I mentioned before, the benchmark model for this applicable both theoretically and empirically has not been developed yet, so I want this to remain for future research.

CHAPTER VIII

CONCLUSION

In this dissertation, I empirically evaluated the performance of the two representative auction mechanisms, the multiple price auction vs. the single price auction used to sell the Treasury bonds. Though this topic has been dealt with by many researchers for over 40 years, general conclusions have not been made yet due to the strategically very complex settings of divisible or multi-unit good auctions.

In order to make my research contributable to the academic efforts which have been devoted up to now, I have clarified the formula of the optimal bidding condition in the single price auction. Although the analytical characterization of strategic equilibria for the single price auction was given by Wilson(1979), Engelbrecht-Wiggans and Kahn(1998b) and Ausubel and Crampton(2002), their conditions were derived under the assumption of perfectly divisible price-quantity choice set or under the case of two units for sale. However, I have derived it under the discrete strategy space which restricts the prices to lie on a discrete grid reflecting the real institutional setting of the Treasury auction. Furthermore, since this condition is entirely expressed by the observed bid points it is also well applicable to the structural model estimation of the unknown true valuation.[74]

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[74] In the case of the multiple price auction, I used the formula derived by Hortaçsu(2002).

Regarding the empirical method, I followed the resampling by bootstrap which Hortaçsu(2002) has developed to analyze the Turkish Treasury auction. Since the parametric estimation of the structural model for multi-unit auctions is extremely hard to be implemented, his method is very helpful to avoid the risk to rely on many uncertain distributional assumptions and the enormous computational burden. However, to apply this method to the single price auction as well, I had to estimate the derivatives of the distribution of the market clearing price. I have developed the estimation techniques for those and extended the applicability of his resampling method. In addition, to compare the efficiency in both auction formats, I suggested the reasonable and estimable indicators.

The main findings of this paper contribute to theoretical and practical debates surrounding the choice of auction format. Firstly, I found that the multiple price auction yielded more revenue than the single price auction in Korea. This result suggests that the conventional belief that the single price auction may raise more auction revenue than the multiple price auction is not always true. Thus, policy makers who are considering change in the auction format should be careful of "the fallacy of imperfect analogy" conceptualized by Ausubel and Crampton(2002). Secondly, the counterfactual Vickery auction revenues were less than actual revenues from every multiple price auction in my sample. As Maskin and Riley (1989), and Ausubel and Crampton (2002) point out, the optimal selling mechanism to maximize the revenue does not result in an efficient assignment in case of the multi-unit auction with downward-sloping demand curves. This implies that inefficient mechanisms may yield more revenue to the Treasury. Since the Vickery auction yields efficient outcome, my findings support their predictions. Thirdly, in the efficiency ranking, my empirical results suggest that the single price auction may produce more inefficiency. This finding empirically supports the recent theoretical prediction that the single price auction may be less efficient than the multiple price auction because of the differential bid reduction behavior. Finally, we found some evidence that assuming the continuity in choice set regardless of the real environment of discreteness might yield different results from explicitly taking this discreteness into consideration. Especially, in the multiple price auction, we obtained somewhat different estimates of the true valuations and revenue losses between from the continuous version of FOCs and from the discrete version of FOCs.

However this paper suggests avenues for future research. This paper has not formally considered the possibility that changing the auction mechanism may affect bidders' participation decisions. Market participants in the Korean auction market claim that the single price auction might induce more participation due to less bid preparation costs. Though I utilize this implication in testing the existence of a CV component, I assumed the number of bidders to be fixed and known to every bidder for my analysis. Therefore, for a more complete comparison of the two mechanisms, it may be worthwhile to consider the effect of endogenous bidder participation. As I pointed out in the discussion of the robustness problem, the assumption of PIV may not be the case in the Korean Treasury auction. The broader consideration of the value structure including the affiliated value paradigm may strengthen the validity of the analysis. However, when the bidders' values or signals about the values are correlated so that the *i.i.d* assumption is broken down, the nonparametric resampling method is not feasible. Therefore without the development of empirical methods applicable to more general divisible good auctions it is unclear how to deal with the general value structures. Lastly, more careful investigation into the bidders' risk attitude may suggest possible extensions of the current model. In the single unit auction setting, many attempts to explain the effect of risk attitudes have been attempted,[75] but explicit consideration has not been modeled in the multi-unit auction setting. These issues are fruitful for future research.

[75] Holt(80) found that when the bidders are risk averse, the revenue equivalence does not hold.

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