

TWO ESSAYS ON MONETARY POLICY UNDER THE TAYLOR RULE

A Dissertation

by

JEONG EUI SUH

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2004

Major Subject: Economics

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## ABSTRACT

Two Essays on Monetary Policy Under the Taylor Rule. (August 2004)

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In this dissertation, two questions concerning monetary policy under the Taylor rule have been addressed. The first question is on, under the Taylor rule, whether a central bank should be responsible for both bank supervision and monetary policy or whether the two tasks should be exercised by separate institutions. This is the main focus of Chapter I. The second question is on whether the Taylor rule plays an important role in explaining modern business cycles in the United States. The second question has been covered by Chapter II.

The implications of the first chapter can be summarized as follows: (i) it is inevitable for the central bank to have a systematic error in conducting monetary policy when the central bank does not have a bank supervisory role; (ii) without a bank supervisory role, the effectiveness of monetary policy cannot be guaranteed; (iii) because of the existence of *conflict of interests*, giving a bank supervisory role to the central bank does not guarantee the effectiveness of monetary policy, either; (iv) the way of setting up another government agency, bank regulator, and making the central bank and the regulator cooperate each other does not guarantee the effectiveness of monetary policy because, in this way, the systematic error in conducting monetary policy cannot be eliminated; (v) in the view of social welfare, not in the view of the effectiveness of monetary policy, it is better for the central bank to keep the whole

responsibility or at least a partial responsibility on bank supervision.

In the second chapter, we examined the effect of a technology shock and a money shock in the context of an RBC model incorporating the Taylor rule as the Fed's monetary policy. One thing significantly different from other researches on this topic is the way the Taylor rule is introduced in the model. In this chapter, the Taylor rule is introduced by considering the relationship among the Fisher equation, Euler equation and the Taylor rule explicitly in the dynamic system of the relevant RBC model. With this approach, it has been shown that, even in a flexible-price environment, the two major failures in RBC models with money can be resolved. Under the Taylor rule, the correlation between output and inflation appears to be positive and the response of our model economy to a shock is persistent. Furthermore, the possibility of an existing liquidity effect is found. These results imply that the Taylor rule does play a key role in explaining business cycles in the United States.

To My Mother, Jeong Wook Kwon

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## CHAPTER I

## INTRODUCTION

In this dissertation, two questions concerning monetary policy under the Taylor rule have been addressed. In recent years, it has been widely accepted that the so-called Taylor rule is a more realistic description of how the Fed policy operates since the Fed definitely perceives the nominal interest rate as its primary policy instrument. The first question is on, under the Taylor rule, whether a central bank should be responsible for both bank supervision and monetary policy or whether the two tasks should be exercised by separate institutions. This is the main focus of Chapter I. The second question is on whether the Taylor rule plays an important role in explaining modern business cycles in the United States. If the answer to this question is no, then, the Taylor rule is not practically meaningful, *i.e.*, researches on the first question can not have any significant meaning, either. Hence, these two questions are closely related to each other. The second question has been covered by Chapter II.

The first topic has newly attracted increased academic attention. This is because, generally speaking, there is a trend among central banks to retreat from bank supervisory functions. As the trend made itself apparent during 1990s, a concern about the effectiveness of monetary policy naturally followed. The main point is that if a central bank got rid of the bank supervisory role, it may lose information-related synergies between bank supervision and monetary policy. However, despite the importance, this topic has not yet received an extensive treatment in the economic literature,

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This dissertation follows the style and format of the *Journal of Business & Economic Statistics*.

with only a few recent papers devoted to the issue. This might result from the fact that nobody had to worry about the loss of information-related synergies between bank supervision and monetary policy since bank supervision had been mainly performed by central banks around the world until the late 1990s. Most of the existing studies have either focused on providing arguments in favor of (or against) one of the two solutions, or compared the macroeconomic performance of countries where monetary policy and bank supervision are combined with that of countries where the two functions are formally separated. No regulatory arrangement has been shown to be clearly superior. However, some recent developments occurred in many countries around the globe seem to indicate that the idea of assigning the two functions to different agencies is becoming more popular in practice.

Among those recent studies on this topic, Peek *et al.* (1999) showed a very important implication when evaluating the structure of the central bank. Unlike the current trend surrounding central banking systems, the authors argued with empirical evidence that the conduct of monetary policy requires full access to bank supervisory information. Specifically, if a central bank cannot access to the confidential component of the bank supervisory information, there will be a systematic error in making economic forecasts which is absolutely necessary for monetary policy decision-making. Direct interpretation of the findings of Peek *et al.* (1999) suggests there will be a systematic error in conducting monetary policy if the central bank does not have the ability to access bank supervisory information or if the central bank does not have any bank supervisory role. The main purpose of the first chapter is to show a possible explanation about why there should be a systematic error and how the central bank will be well served to have a bank supervisory role. With a simple game-theoretic approach, it is shown that it is inevitable for the central bank to have a systematic

error in conducting monetary policy when the central bank does not have a bank supervisory role because commercial banks also profit-maximize just like other firms. It is also shown that either giving a direct bank supervisory role to the central bank or setting up a bank regulator separate from the central bank is not good enough to guarantee the effectiveness of monetary policy. Additionally, based on the social welfare criteria, it will be argued that the central bank should take the whole responsibility or at least a partial responsibility for bank supervision, which goes against the current trend surrounding central banking systems.

The second topic is related to one of the most popular literature in economics. Real business cycle (RBC) models have been surprisingly successful at accounting for several non-monetary features of U.S. business cycles, but they have been less successful once money is involved. The second chapter examines the impact of technology and money shocks in a flexible-price general equilibrium business cycle model with no frictions except money under a cash-in-advance constraint and the assumption that the central bank follows the Taylor rule.

One thing significantly different from other researches on this topic is the way the Taylor rule is introduced in the model. In this dissertation, the Taylor rule is introduced by considering the relationship among the Fisher equation, the Euler equation and the Taylor rule in the dynamic system of the relevant RBC model. This approach can be justified based on the reason that, if the Fed follows the Taylor rule strictly, and if we assume that the Fed always achieves its target nominal interest rate, then, the nominal interest rate is determined prior to the real interest rate. Without the Taylor rule, the real interest rate is determined by the Euler equation and the nominal interest rate is subsequently determined by the Fisher equation. With this approach, it has been shown that, even in a flexible-price environment, the

two major failures in RBC models with money can be resolved. Under the Taylor rule, the correlation between output and inflation appears to be positive and the response of our model economy to a shock is persistent. Furthermore, the possibility of an existing liquidity effect is found.

## CHAPTER II

## ARE CENTRAL BANKS ALL-MIGHTY?

## DO THEY STILL NEED A BANK SUPERVISORY ROLE?

## 1. Introduction

The question of whether a central bank should be responsible for both bank supervision and monetary policy or whether the two tasks should be exercised by separate institutions, has recently received renewed attention. This might be a natural evolution of academic curiosity in that the debate surrounding central bank independence has been nearly settled with the conclusion that central bank independence is a good thing for sound macroeconomic stability. While the importance of central bank independence has been attracting a lot of academic attention (see, for example, Cukierman (1992), Alesina and Summers (1993), Posen (1995), and Fuhrer (1997)), the bank supervisory role of central bank has not acquired much attention until recently.

There is another reason, which is much more important, for this topic to have newly attracted increased academic attention. Although roughly three-quarters of OECD countries assign their central banks either total or shared responsibility for bank supervision, many of these countries are reviewing those responsibilities. Generally speaking, there is a trend among central banks to retreat from bank supervisory functions. In 1997, a comprehensive institutional reform assigned responsibility for bank supervision and regulation in the United Kingdom to the Securities and Investment Board, which was reconstituted as the Financial Services Authority (FSA), a single financial supervisory authority. These functions were previously conducted

by the Bank of England (see Di Noia and Di Giorgio (1999)). In the Euro zone, the principle of separating monetary policy and bank supervision responsibilities has been formally established in the statute of the European Central Bank (ECB). It empowers the ECB to set out and conduct monetary policy in the Euro area, but leaves the responsibility for bank supervision with the national authorities (see Padoa Schioppa (1999)). Among the European Union countries, Luxembourg created a single financial supervisory authority (the Commission de Surveillance du Secteur Financier) in 1998 and entrusted with supervisory responsibilities extending to all financial intermediaries and markets. There are no institutional relationships or formal bilateral cooperation arrangements in place with the Banque centrale du Luxembourg. In Germany, Ireland, Finland and Austria, there have been government proposals during the period of 1999 to 2001 arguing for the adoption of UK-like arrangements. In case of Germany, although an extensive cooperation between the new single authority and the Deutsche Bundesbank was called for in the government proposal, the effects on the involvement of the latter in bank supervision are still unclear. In the other European countries, according to those proposals, national central banks will be set outside of bank supervisory responsibility. In Asia, the Bank of Korea and the Bank of Japan have lost all kinds of bank supervisory responsibilities to a newly established financial supervisory authorities in 1998, while they have received enhanced central bank independence. In the case of Korea, the Bank of Korea kept the whole bank supervisory responsibilities before the new agency was created. In the case of Japan, banks treated the 'suggestions' of the Bank of Japan as binding regulations although the bank supervisory responsibility was on the Ministry of Finance before the new agency was created. In the United States, although the Federal Reserve does not appear to be going to lose its partial responsibility for bank regulation and supervision, since 1994 a series of bills before Congress have proposed consolidating all bank



supervisory responsibilities under a new single federal regulator separate from the Federal Reserve (see Shull (1993) and Coffee (1995)).

As the trend made itself apparent during 1990s, a concern about the effectiveness of monetary policy naturally followed. As Goodhart and Schoenmaker (1995) explained, the function of bank supervision had been mainly performed by the central banks ever since a central banking system was introduced. Until the trend showed up during the 1990s, nobody seemed to worry about the possibility that the effectiveness of monetary policy may be lowered because of the non-existence of bank supervisory roles in central banks' hands. The main point is that if a central bank got rid of the bank supervisory role, it may lose information-related synergies between bank supervision and monetary policy. As Peek *et al.* (1999) argued, there are two main reasons one might expect bank supervisory information to contribute to effective monetary policy. First, problems in the banking sector may serve as an early indicator of deteriorating conditions in the macro-economy generally. Alternatively, the information could provide advance notice of changes in bank lending behavior, which would affect the macro-economy to the extent that the lending view is operative (see, for example, Bernanke and Gertler (1995); Hubbard (1995); Kashyap and Stein (1994a, 1994b); and Kashyap, Stein, and Wilcox (1993)). There is also another concern about increased systemic risk on the banking industry after central banks lost their bank supervisory role. This concern, however, does not seem to attract much attention because of the expansion of deposit insurance systems across almost all major countries. In fact, activation of deposit insurance systems across countries for the purpose of financial system stability has played the main role in motivating the current trend surrounding central banking systems.

As we will see in the next section, despite the importance, this topic has not yet

received an extensive treatment in the economic literature, with only a few recent papers devoted to the issue (see Heller (1991); Goodhart and Schoenmaker (1995); Haubrich (1996); Di Noia and Di Giorgio (1999); Peek *et al.* (1999); Bini Smaghi and Gros (2000)). This might result from the fact shown above that nobody had to worry about the loss of information-related synergies between bank supervision and monetary policy since bank supervision had been mainly performed by central banks around the world until the late 1990s. Indeed, as Di Noia and Di Giorgio (1999) indicated, a sound theoretical analysis is still on the research agenda. Most of the existing studies have either focused on providing arguments in favor of (or against) one of the two solutions, or compared the macroeconomic performance of countries where monetary policy and bank supervision are combined with that of countries where the two functions are formally separated. No regulatory arrangement has been shown to be clearly superior. As we saw earlier in this section, however, some recent developments occurred in many countries around the globe seem to indicate that the idea of assigning the two functions to different agencies is becoming more popular in practice.

Among those recent studies on this topic, Peek *et al.* (1999) showed a very important implication when evaluating the structure of the central bank. Unlike the current trend surrounding central banking systems, the authors argued with empirical evidence that the conduct of monetary policy requires full access to bank supervisory information. Furthermore, for the central bank to exploit this important source of information in the conduct of monetary policy, it must have timely and reliable data. Specifically, if a central bank cannot access to the confidential component of the bank supervisory information, there will be a systematic error in making economic forecasts which is absolutely necessary for monetary policy decision-making. As the authors

indicated, their paper might be the first serious one, without depending on the lender-of-last-resort stories, in favor of the view that central banks need a direct role in bank supervision for effective monetary policy in terms of the information-related synergies between bank supervision and monetary policy. Since the implication of their paper plays a key role in this paper, a more detailed content of their paper will be presented later as part of a literature survey.

Direct interpretation of the findings of Peek *et al.* (1999) suggests there will be a systematic error in conducting monetary policy if the central bank does not have the ability to access bank supervisory information or if the central bank does not have any bank supervisory role. The main purpose of this paper is to show a possible explanation about why there should be a systematic error and how the central bank will be well served to have a bank supervisory role. With a simple game-theoretic approach, it is shown that it is inevitable for the central bank to have a systematic error in conducting monetary policy when the central bank does not have a bank supervisory role because commercial banks also profit-maximize just like other firms. This is because when an economy has a shock and the shock affects the interest rate, the reaction of the central bank should affect the profit of commercial banks. Assuming a situation where the central bank should depend on commercial banks to get information about real economy status, there is an incentive for commercial banks to lie to the central bank in order to maximize their profit. Following Poole (1970), and Caplin and Leahy (1996), it is assumed that policy makers in the central bank are ignorant of shocks occurring in the real economy. Also, following Barro and Gordon (1983), it is assumed that central banks following an interest rate rule work to minimize a given loss function. In the same game-theoretic framework, it is also shown that either giving a direct bank supervisory role to the central bank or setting

up a bank regulator separate from the central bank is not good enough to guarantee the effectiveness of monetary policy. Additionally, based on the social welfare criteria, it will be argued that the central bank should take the whole responsibility or at least a partial responsibility for bank supervision, which goes against the current trend surrounding central banking systems.

The paper proceeds as follows. In the next section, a literature survey follows. The survey comprises the traditional economic debate in favor of (or against) combining bank supervision and monetary policy within the central bank, the possible reasons why we should see the current trend surrounding central banking systems for now, and the contents of those recent papers in this topic. The third section includes a game-theoretic approach. It addresses the following questions: Can a central bank keep the effectiveness of monetary policy when it does not have any bank supervisory role? How can a bank supervisory role help a central bank improve the effectiveness of monetary policy? When there is another government agency specializing in bank supervision, does a central bank still need to have a direct bank supervisory role? The final section concludes.

## 2. Literature Survey

### 2.1. Economic Debate on Combining Bank Supervision and Monetary Policy

As Di Noia and Di Giorgio (1999) indicated, the issue of whether it is better to combine or separate the functions of bank supervision and monetary policy was broadly discussed by the traditional literature on central banking (from Bagehot (1999) on). However, only recently has a more careful investigation been undertaken. I will summarize, first, the traditional economic debate on both the advantages and

disadvantages of having bank supervision assigned to the monetary policy agency, based on Jordon (1996), Tuya and Zamalloa (1994), Goodhart and Schoenmaker (1995), Haubrich (1996), Di Noia and Di Giorgio (1999), Bini Smaghi and Gros (2000), and Lannoo (1999). Next, I will provide some possible reasons for the current trend surrounding central banking systems.

### 2.1.1. Traditional Arguments Against Combining Bank Supervision and Monetary Policy

The major disadvantage in combining monetary policy and bank supervision responsibilities is based on a potential *conflict of interests*. This is based on the possibility that bank supervisory concern about the fragility of the banking system might lead the central bank to pursue a more accommodating monetary policy than warranted for the pursuance of price stability. If the central bank is responsible for bank supervision and is concerned with the stability of financial institutions, it may be tempted to create central bank liquidity with the view of avoiding a financial instability which one or more financial institutions in difficulty experiencing losses could trigger. This situation may occur especially if the underlying macroeconomic conditions require the central bank to tighten monetary policy, for instance to counter inflationary pressures.

The second type of problem arises from the fact that negative experiences in the resolution of banking crises may jeopardize the credibility of the central bank, even with respect to its monetary policy responsibilities. Goodhart and Schoenmaker (1995) suggest, for instance, that the credibility of the Bank of England has been impaired after the BCCI crisis. Market participants may interpret the emergence of crises as a failure of the supervisory capacity of the central bank. The latter

may thus be induced to cover up such a failure through the injection of central bank liquidity. If market participants anticipate that the central bank with bank supervisory responsibilities may be more prone to use the monetary lever, the problem of moral hazard may be exacerbated. Moral hazard may also increase as a result of the lack of transparency arising from the fact that the central bank has two objectives which are not clearly distinguished and which may at times conflict.

Finally, a more general point is that the cyclical effects of micro (bank supervisory) and macro (monetary) policy tend to conflict. Monetary policy is supposed to be counter-cyclical, while the effect of regulation, *e.g.* capital adequacy requirements, tends to be pro-cyclical. As an example, consider a period of economic slowdown, in which the banks' non-performing assets are likely to increase. The bank supervisor will require higher provisions for possible loan losses and put pressure on banks to improve the quality of their portfolios. The banks' implementation of these supervisory recommendations would hence result in tighter credit in the course of a recession. Monetary policy should instead be expansionary, and would call for a temporary reduction in the minimum capital asset ratio, so as to provide more funds to the economy and speed up its recovery. On the other side, a tight monetary policy may have an undesired impact on bank solvency, as higher interest rates increase the risk of loan defaults in the banking system.

### 2.1.2. Traditional Arguments for Combining Bank Supervision and Monetary Policy

The main argument for combining the functions of monetary and supervisory management within the central bank is linked to the central bank's concern for the systemic stability of the financial system. By doing bank supervision, which includes all on and off-site surveillance of the safety and soundness of individual institutions,

the central bank can acquire micro-information. In this sense, the combination is particularly needed in times of financial crises, when only direct supervision can deliver the essential information of time. The informal inside information on how managers react and what strategies they pursue simply cannot be duplicated by reading reports or consulting with other agencies. Hence, a central bank supervising the banking system might know more precisely if a bank asking it for credit, in its role as lender-of-last-resort, is insolvent or just having a minor problem in liquidity. If the central bank does not have access to micro-information, the lender-of-last-resort function is difficult to perform and mistakes can easily be made.

Another argument is that the central bank will be able to acquire valuable insights into the overall state of the economy by being involved in the supervision and regulation of banks and, hence, to get information-related synergies. Combining bank supervision and monetary policy allows the central bank to consider the broader consequences of supervision. As the Federal Reserve Board Chairman Alan Greenspan said in testimony before the Senate Committee on Banking, Housing, and Urban Affairs, 1994:

.... Indeed, a single regulator with a narrow view of safety and soundness and with no responsibility for the macroeconomic implications of its decisions would inevitably have a long-term bias against risk-taking and innovation. It receives no plaudits for contributing to economic growth through facilitating prudent risk-taking, but it is severely criticized for too many bank failures. The incentives are clear ....

Combining the two functions has also been argued in view of payment system stability. Payment system is a key channel for the potential spread of contagion risk. As Goodhart and Schoenmaker (1992) point out, the central banks' growing awareness of the liquidity and the credit risks in net and gross (with un-collateralized overdrafts) settlements systems have initiated several risk reduction policies: the most

direct measure used to be, and still is, to control access and monitor the participants (see Di Noia and Di Giorgio (1999)). For the central bank to be able to prevent the potential spread of contagion, since the central bank should play a key role in managing the payment system, the central bank has to be able to distinguish risky banks from solvent banks, which would not be possible without a bank supervisory role.

There are also other arguments. Among them, the independence and expertise argument highlights the quality of the contribution central banks can make to financial stability. Independence of supervisory authority from political interference is important for effective supervision. This is particularly true in some emerging countries, where so-called policy loans, *i.e.*, loans granted under formal or informal pressure from governmental authorities, are still a reality. In general, the laws, regulations or acts of the public administration may interfere with the entrepreneurial choices of financial intermediaries. In such cases, when the intermediaries get into trouble because of the wrong incentives they were given, pressure to bail them out might be very high. Central bank independence might shelter supervision from undue external interference, as well from the risk of regulatory capture by the supervised entities. An additional argument for combining monetary policy and bank supervision is that central banks are generally recognized as sources of excellent research and analysis on the banking and financial system. They have gained a wealth of knowledge on the structure and performance of the domestic financial system over time, which is continually renewed through their active presence in financial markets.



### 2.1.3. Possible Reasons of the Current Trend Surrounding Central Banks

The traditional debate on how to assign two different functions, monetary policy and bank supervision, has had as long of a history as the central banking system itself. However, we have never seen such a big change in the structure of central banking systems, at least as far as the way of the two functions were assigned is concerned, until the current trend that occurred in the late 1990s. Hence, there should be some reasons, which were not covered in the traditional debate, and which are motivating enough for the current trend to occur. In this part, I summarize those possible reasons for the current trend.

The most notable change surrounding central banks is the expansion of depositor insurance systems. Although the depositor insurance system has been criticized, especially in the US due to an increase in moral hazard concerns, deposit insurance systems have been rapidly expanded in new countries over the last two decades. According to Goodhart and Schoenmaker (1995), among 24 well-developed and mid-developed countries, only eight countries had established deposit insurance systems before 1980, while ten countries have established such systems since 1980. Furthermore, all EU countries were required to introduce a common scheme of minimum insurance levels for most depositors, beyond which individual member states can extend protection levels, if they want. This reflects, I believe, the significant contribution of Diamond and Dybvig (1983) and subsequent studies, which showed clearly that depositor insurance systems are more effective in preventing banking crises over the central banking system. This also reflects the reality that as the scale of funding necessary in some banking crises (*e.g.* in the USA, Scandinavia, Japan, and Korea) had gone far beyond the sums which the central bank can provide from its own resources, the rescues have been increasingly financed by the tax-payer (see Goodhart and Schoenmaker (1995)).

The expansion of depositor insurance systems around the world must have affected what we think about how to assign the two functions. As it is shown in the above traditional debate, the central bank's lender-of-last-resort function for financial system stability has been the main concern for the proponents arguing that the two functions should be combined. Although the depositor insurance system has been well established, the lender-of-last-resort function of the central bank should have its own usefulness considering the fact that it is the most efficient way to prevent financial disturbances when some banks have only a problem of liquidity and not a problem of failure. However, it could be a good reason for legislators to wonder if the central bank should keep bank supervisory responsibilities in place, given a possible better system in preventing banking crises.

Another reason is a significant change in the way central banks conduct monetary policy. During 1970s and 1980s, most major central banks, except the Fed, had depended on monetary targeting policy. Those central banks, however, have abandoned the monetary targeting policy since it became clear that the stable relationship between aggregate money (*e.g.*  $M_1$  and  $M_2$ ) and real economic variables (*e.g.* inflation and GDP) were broken down. After giving up the monetary targeting policy, those central banks moved to either inflation targeting policy or interest rate targeting policy. As Goodhart (1999) stated, there was a time when a vocal segment of the academic community advocated a notably different operating mechanism, of monetary base control, but that debate has faded. We need to note that, under a monetary targeting policy regime, there is a necessity that the central bank should keep a close relationship with commercial banks. This can be clearly shown by the well-known money multiplier

$$M = B \left[ \frac{(C/D) + 1}{(C/D) + (R/D)} \right], \quad (2.1)$$

where  $B = C + R$ ,  $M = C + D$ ,  $M$  is money,  $B$  is monetary base,  $C$  is currency,  $R$  is bank reserves, and  $D$  is deposit. What (2.1) implies is that the central bank needs to keep monitoring bank reserves and deposits, and, thus, to also keep aggregate money targeted. However, under an inflation targeting policy regime or interest rate targeting policy regime, the central bank does not need to care about keeping targets on aggregate money, and, therefore, the central bank does not have to keep a close relationship with commercial banks. This is so because the central bank just needs to keep the interest rate as targeted regardless of how the aggregate money demand changes or the components in money multiplier vary. This argument cannot be an absolute reason for the central bank to give up the bank supervisory role. It is clear, however, that at least the change in monetary policy regime lowered the necessity for the central bank to keep a close relationship with commercial banks.

Here, it is necessary to mention that inflation targeting policy and interest rate targeting policy are basically the same if we assume that it is the interest rate not inflation which the central bank can directly control, and that inflation is a function of the interest rate. Because, under this assumption, both policies are based on interest rate manipulation. Christiano and Rostagno (2001) make it clear that under a pure Taylor rule, the central bank chooses aggregate money to achieve the following rate of interest:

$$R_t = \tilde{R} \left( \frac{\pi_{t+1}}{\tilde{\pi}} \right)^{\alpha \frac{\tilde{\pi}}{\tilde{R}}}, \quad (2.2)$$

where  $\tilde{R} = \frac{\tilde{\pi}}{\beta} - 1$ ,  $\tilde{\pi} \geq \beta$ ,  $\alpha > 0$ ,  $1 > \beta > 0$  are all parameters. Under this monetary policy rule, the supply of money is determined by the market. Note that, considering the relationship of  $\tilde{R} = \frac{\tilde{\pi}}{\beta} - 1$ , picking up an inflation target directly means picking an interest rate target. In fact, they refer to  $\tilde{\pi}$  and  $\tilde{R}$  as the inflation rate and the nominal interest rate under the target inflation steady state.

The conglomeration of financial services also contributes to the current trend. The conglomeration argument rests on the evidence that closer linkages are gradually developing between banks, securities companies, asset managers and insurance companies, while the traditional distinction between different financial contracts is blurring, so that different types of intermediaries actually compete in the same markets. Under these conditions, sectoral supervisors might be less effective in monitoring overall risk exposures in large and complex financial groups, and differences in sectoral rules or practices might alter the *level playing field* between competing intermediaries. Tools for coordinating the different sectoral authorities, such as committees, memoranda of understanding, joint board participation, etc. could alleviate the problem and have proven successful in many countries.

The conglomeration argument, however, is often related to the goal of achieving more effective supervisory structures and limiting the burden that regulation imposes upon intermediaries. Financial groups with many lines of business would avoid reporting and paying for the supervision exercised by different authorities, thereby minimizing the costs of compliance (and, perhaps, of lobbying) and the risks of conflicting supervisory assessments. It is not surprising, therefore, that the financial industry frequently supports reforms introducing a single supervisory agency. For example, in testimony before the Committee on Banking, Housing and Urban Affairs of the United States Senate, March 9, 1994, Donald Howard, former Citicorp Chief Financial Officer stated:

.... If the Federal Reserve were removed from the bank regulation process, would it have any significant impact on its ability to conduct monetary policy? My answer is: Clearly it would not .... It is clear that the primary role of the Federal Reserve is the conduct of monetary policy. That role is so important, I believe carrying out that function should not be diluted even slightly by having any of its resources diverted to any other function, including bank regulation. ....

If supervisory responsibilities over the whole financial sector have to be assigned to a single authority, the central bank would not be the most obvious candidate. The central bank traditionally plays a role in bank supervision, *i.e.*, in the monitoring of counter-parties, who are an essential component in the transmission of monetary policy. However, their *natural jurisdiction* seldom encompasses securities firms, and almost never insurance companies.

There is also a political reason that affected the current trend. The reason is that, if the central bank keeps the traditional role as a bank supervisory authority as well, the central bank could have too much power. This argument takes into consideration the fact that, in almost all major countries, the central bank independence has been significantly enhanced. Attributing to independent central bank bank regulatory and supervisory tasks as well, especially if extended to the whole financial sector, might be considered detrimental to the system of *checks and balances* on which democracies rely in order to avoid potential abuse in the performance of public functions.

## 2.2. Literature Survey

Despite the fact that this debate has a very long history, there has been none which treated the relationship between monetary policy and bank supervision in a theoretical framework. There has also been almost nothing which treated the topic in a serious empirical framework until those recent studies appeared during 1990s. Generally speaking, except Peek *et al.* (1999), all studies seem to support the idea that the two functions of monetary policy and bank supervision should be assigned to different agencies.

Naturally, the empirical evidence comparing the relative performance of systems where the central bank exercises supervisory responsibilities with those where a sep-

arate authority exercises supervision is scarce. Goodhart and Schoenmaker (1995) examine the issue from the point of view of the effectiveness of bank supervision in avoiding banking crises. On the basis of a sample made of 104 banking crises, the authors conclude that there is no evidence that one system has performed better than the other.

Concerning the inflation performance of the two systems, Heller (1991) finds that countries where the central bank is also responsible for bank supervision have recorded on average a significantly higher rate of inflation. Goodhart and Schoenmaker (1995) obtain a similar result. These authors suggest, however, that this result may not be considered as a proof of a conflict of interest between monetary policy and bank supervision, but possibly of the fact that central banks with combined responsibilities are generally less independent. The correlation between the supervisory responsibility of the central bank and inflation performance would thus be a proxy of the (negative) relation between central bank independence and inflation.

Bini Smaghi and Gros (2000) examine whether the involvement of the central bank in supervisory activities has a significant impact on inflation performance, independently from the degree of central bank independence. The empirical results show that there is a positive and statistically significant relationship between the exercise of supervisory responsibilities and inflation performance in industrial countries. The exercise of prudential supervision by the central bank has instead no significant effect on growth performance or variability.

Di Noia and Di Giorgio (1999) find that the inflation rate is considerably higher and more volatile in countries where the central banks acts as a monopolist in bank supervision than in countries where this responsibility is assigned to another agency or to more than one agency (including the central bank). They limit their analysis

to OECD countries, which are divided into two groups, according to whether or not their central bank acts as a monopolist in bank supervision.

Peek *et al.* (1999) show that, without using confidential bank supervisory information, there should be a systematic economic forecast error, and that the policy-making body, the Federal Open Market Committee (FOMC) does use the confidential bank supervisory information in making policy. In fact, they show a very important implication when evaluating the structure of the central bank. Unlike the current trend surrounding central banking systems, and the other related studies' implications, the authors argued with empirical evidence that the conduct of monetary policy requires full access to bank supervisory information.

Since the implication of their paper plays a key role in this paper, more detailed content of their paper will be presented below. First, using panel data estimation, they tested whether the Greenbook, which is official economic forecast made by the Federal Reserve staff, reflects bank supervisory information. Specifically, they estimated the following equation to compare the forecast in the Greenbook and those of other private economic forecasting agencies:

$$X_{t+i} = \alpha_0 + \alpha_1 X_{j,t+i}^e + \alpha_2 P_t + \alpha_3 GB * P_t + \varepsilon_{j,t+i}, \quad (2.3)$$

where  $j$  represents the Greenbook and other private economic agencies,  $i$  is one-, two-, three-, and four-quarter-ahead forecasts,  $X_{t+i}$  is the actual value ( $X$ ) of inflation rate and unemployment rate in quarter  $t+i$ ,  $X_{j,t+i}^e$  is a forecasted value ( $X^e$ ) by  $j$  for the quarter  $t+i$  as of time  $t$ , and  $P_t$  is proxy variable for confidential bank supervisory information, and  $GB$  is a (0,1) dummy variable that has a value of one for Greenbook forecasts. The proxy variable is the percentage of bank assets held by banks with a supervisory composite CAMEL rating of 5.

The CAMEL ratings are intended to reflect different degrees of bank health, with examiners rating each bank according to its Capital, Asset, Management, Earnings, and Liquidity (CAMEL). The ratings range from 1, indicating no significant examiner concerns, to a rating of 5, indicating examiners believe that the bank has a high probability of failure. Because an announcement by a regulator that a bank has a high probability of failure could be extremely detrimental to the institution, individual bank CAMEL ratings are highly classified. Another key point of the equation is that the forecasted value is included as an explanatory variable. In order to interpret this, we need to see the fact that it should be presumed that the forecasted values must have been estimated based on all possible information which is helpful in making economic forecasts. In this sense,  ${}_tX_{j,t+i}^e$  can be also considered as a proxy variable for all possible information which can explain the actual values.

Note that an interactive term composed of the product of  $P_t$  and  $GB$  is included in the equation. The estimated coefficient on this interactive term can be used to measure the extent to which the Greenbook outlook incorporates the confidential bank supervisory information differently than do the private forecasts. Also, note that if the Greenbook forecasts do not differ from private forecasts in the degree to which the supervisory information is incorporated,  $\alpha_3$  in equation (2.3) would not differ significantly from zero. On the other hand, if the supervisory information is fully utilized by the Greenbook, the estimated value of  $\alpha_3$  would be significant and equal to  $-\alpha_2$ .

The estimation results are striking. While  $\alpha_2$  is significantly different from zero,  $\alpha_3$  is insignificant for each of the four forecast horizons. This means that the confidential bank supervisory information does have something to do with actual economic values, and that economic forecasts made by even Federal Reserve staff do have a



systematic error because they failed to incorporate the confidential bank supervisory information into economic forecasting. In fact, an increase in the proxy variable for confidential bank supervisory information account for an underestimation of the unemployment rate, and for an overestimation of the inflation rate.

They also tested whether the FOMC decisions reflect supervisory information. Using a multinomial Logit estimation, they estimated the following equation:

$$prob(V)_{i,t} = \lambda_{1,i} + \lambda_{2,i}EF_t + \lambda_{3,i}P_t + \mu_{i,t}, \quad (2.4)$$

where  $i$  means easing or tightening policy,  $EF$  is economic forecasts taken from the Greenbook. Note that equation (2.4) generates the probability of easing or tightening of monetary policy, relative to leaving policy unchanged. The test results of this equation is also striking in that  $\lambda_{3,i}$ , the coefficient of the proxy variable for confidential bank supervisory information, is both an economically and a statistically significant determinant of monetary policy, which means that the FOMC does utilize the confidential bank supervisory information in policy making.

Finally, they tested whether the FOMC decisions take “too-big-to-fail” into consideration in order to check whether the Fed needs to have a *direct* bank supervisory role. Specifically, they added a variable of  $LP_t$  measuring the percentage of CAMEL4-rated assets among the 50 largest banks. The idea is that sometimes the bank regulator needs to consider “too-big-to-fail” in a practical sense. Specifically, “too-big-to-fail” may manifest itself in a reluctance by bank regulator to provide the lowest rating, CAMEL5, on the largest banks. Hence, sometimes CAMEL4 could actually mean CAMEL5, especially for large banks. However, in order to guarantee the effectiveness of monetary policy, the FOMC should understand which CAMEL4-rated

banks are actually CAMEL5-rated banks. They estimated the following equations:

$$X_{t+i} = \alpha_0 + \alpha_{1t}X_{j,t+i}^e + \alpha_2LP_t + \alpha_3P_t + \varepsilon_{j,t+i}, \quad (2.5)$$

$$prob(V)_{i,t} = \lambda_{1,i} + \lambda_{2,i}EF_t + \lambda_{3,i}LP_t + \lambda_{4,i}P_t + \mu_{i,t}. \quad (2.6)$$

Test results show that  $\alpha_2$ ,  $\alpha_3$ ,  $\lambda_3$ , and  $\lambda_4$  are all significantly different from zero. This means that both bank supervisory information on  $P_t$  and  $LP_t$  do have something to do with actual economic values, and that the FOMC utilize both pieces of information in policy making. Based on these results, they argued that the Fed should keep a *direct* role in bank supervision because, without a direct role, it is almost impossible to distinguish this subtle difference even though it can access to bank supervisory information.

### 3. Model: A Game-theoretic Analysis

Direct interpretation of the findings of Peek *et al.* (1999) is that there should be a systematic error in conducting monetary policy if the central bank does not have an ability to access to the bank supervisory information, or, more aggressively, if the central bank does not have any bank supervisory role. The main purpose of this paper is to show a possible explanation about why there should be a systematic error, and why there should not be if the central bank has a bank supervisory role. To address this question, I will adopt a game-theoretic approach focusing on the interaction between the central bank and a representative commercial bank. The pioneering papers in the literature on monetary policy games are Kydland and Prescott (1977) and Barro and Gordon (1983). Subsequent studies are summarized in Rogoff (1989) and Benjamin Friedman (1993) (see Caplin and Leahy (1996)). Basically, I will follow the ideas these papers suggest. However, following Poole (1970), I return to

the old idea that policy makers are ignorant of contemporary shocks, *i.e.*, they do not understand whether the shock is permanent or temporary, while incorporating the more recent notion that private agents react to policy rules.

After setting up the environment of the economy, we are going to study three different cases depending on how the responsibilities of bank supervision are assigned. The reason we have three different cases is that, if the central bank does not have a bank supervisory role, then, there should be another government agency specializing in bank supervision. Hence, depending on the relationship between the two authorities, we have the following different cases: *(i) When the central bank does not have a bank supervisory role, and the central bank and another government agency specializing in bank supervision do not cooperate with each other.* Examples are the United Kingdom, Luxembourg, Japan, and South Korea. *(ii) When the central bank has a bank supervisory role, i.e., the central bank is also the bank supervisory authority.* The United States belongs to this case. *(iii) When the central bank does not have a bank supervisory role, and the central bank and another government agency specializing in bank supervision do cooperate each other, i.e., the two authorities are expected (or obliged) to cooperate with each other as far as the objective of each authority can be kept safe.* The best example is the former Bundesbank. Let me emphasize the distinction between cases *(i)* and *(iii)*. In the case of *(i)*, the two authorities do not care about each other, *i.e.*, the two authorities only pursue their own policy goals without regarding the other's policy performances.

The first case will be the main focus of this paper, *i.e.*, why there should be a systematic error in conducting monetary policy. In the second and third cases, we will investigate if two practically available ways are good enough to eliminate the systematic error and, thus, to guarantee the effectiveness of monetary policy. One way

is to give a direct bank supervisory role to the central bank. The other way is to set up another government agency specializing in bank supervision, *i.e.*, bank regulator. Finally, based on the findings from the first case, we will seek some implications on the current trend surrounding the central banking systems.

### 3.1. Environment

There is a central bank whose objective is to minimize the following Barro-Gordon type loss function at every period  $t$ , given as

$$\min_{i_t} L_t = a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2, \quad (2.7)$$

where  $a > 0$  and  $b > 0$  are parameters,  $\pi_t(i)$  is inflation rate,  $i_t$  is nominal interest rate,  $\bar{\pi}$  is the central bank's inflation target associated with its interest rate target,  $y_t$  is aggregate output, and  $\bar{y}$  is the economy's natural output. Note that it is assumed that the inflation rate is a function of the interest rate. In fact, this is the only difference in the loss functions between Barro and Gordon (1983) and this paper. The central bank can control inflation directly in Barro and Gordon (1983) while the central bank can only control the interest rate in this paper. Also note that, according to the definitions in Rudebusch and Svensson (1999), the central bank in this model follows a *flexible* inflation targeting policy. In their paper, they defined *strict* inflation targeting as referring to the situation where only inflation enters the loss function, while *flexible* inflation targeting allows other goal variables. However, we already know that inflation targeting policy and interest rate targeting policy are basically the same if we assume that it is the interest rate not inflation which the central bank can directly control, and that inflation is a function of the interest rate. Hence, it is clear that, in our model, the central bank follows an interest rate

target policy regime. Considering the recent changes in monetary policy regimes across almost all major central banks in the world, this assumption captures reality. Furthermore, in our model, it is assumed that the central bank can exactly control the inflation rate through interest rate manipulation. Also, we assume that there is no time-inconsistency problem in conducting monetary policy, and that the economy does not have any banking crises.

There is a representative commercial bank whose objective is profit maximization. We assume that its objective function is given by

$$\max_{k_{t+1}, n_t, a_t, l_t} \Pi_t = f(k_{t+1}, n_t, a_t, l_t : i_t, k_t), \quad (2.8)$$

where  $\Pi_t$  is profit at period  $t$ , and  $k_t$ ,  $n_t$ ,  $a_t$ , and  $l_t$  are, respectively, bank capital, labor employed, bank assets, and bank liability at period  $t$ . It is also assumed that there is perfect competition in banking industry. Note that the profit function in (2.8) has a very general form. Especially,  $a_t$  and  $l_t$  should be interpreted to include all kinds of future terms which should be discounted suitably. The reason the term  $k_{t+1}$  is included is that, at the time of  $t$ , the control variable is  $k_{t+1}$  not  $k_t$  as far as capital is concerned. More specifically, it could be set up as a simplified version given by

$$\max_{B_t, n_t, k_{t+1}} \Pi_t = i_t B_t - W_t n_t - P_t(k_{t+1} - k_t), \quad (2.9)$$

where  $B_t$  is bank lending,  $W_t$  is wage,  $P_t$  is the price of capital. Here, we assume that the only asset is bank lending and there is no liability. Factor markets are assumed to be complete.

There is only one shock in each period. The shock occurs at the beginning of each period. The shock could be permanent or temporary. When a shock occurs, it is assumed that, following Poole (1970), the central bank does not understand whether

the shock is permanent or temporary, while the commercial bank does know what it is. The central bank, however, is assumed to have the ability to find out what the new steady state values are for inflation and aggregate supply at the time when there is a permanent shock, if and only if it can acquire adequate information. Also, it is assumed that the shock will reveal itself next period so that the central bank can surely understand what happened in the previous period. These assumptions also reflect a reality. Since commercial banks keep contacting continuously with the real sector on business, it is reasonable for us to assume that the representative commercial bank in the model understands what is going on in the real sector. Also, some risky commercial banks themselves could be a shock to the economy. However, when there is a shock, the central bank can only see resultant movements on the interest rate, inflation rate, and aggregate output. It takes time for policy makers to get adequate economic data. Another important assumption is that, when a shock occurs, the shock should affect aggregate output. This assumption is important as will be shown later.

Aggregate output is given by a Lucas-type aggregate supply function of the form

$$y_t = \bar{y} + (\pi_t - \pi_t^e). \quad (2.10)$$

Furthermore, it is assumed that the Fisher equation always holds in this economy<sup>1</sup>.

Hence,

$$\pi_t^e = i_t - r_t, \quad (2.11)$$

where  $r_t$  is real interest rate,  $i_t$  is nominal interest rate, and  $\pi_t^e$  is the expected inflation

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<sup>1</sup>In fact, we might not need this assumption since, as Romer (2002) indicated, equation (2.11) is considered the *Fisher identity*. By definition, the real interest rate is the difference between the nominal interest rate and expected inflation, *i.e.*,  $r \equiv i - \pi^e$ . Also, note that the Fisher equation is a direct implication of a standard representative agent model with either money-in-utility or cash-in-advance.

rate.

It is important to understand the role of Fisher equation in this model. As shown above, the central bank is assumed to be able to exactly control inflation through manipulating the nominal interest rate. This means the inflation rate should change when the central bank changes the nominal interest rate. Hence, under rational expectations, it needs to be the case that, when the central bank changes the nominal interest rate, not only the inflation rate but the expected inflation rate should change. In other words, in the context of the current model, it would be very natural that we should think that any changes in the nominal interest rate,  $i_t$ , will somehow affect expected inflation<sup>2</sup>. Hence, we need a set-up which can show how expected inflation evolves along the changes in the nominal interest rate. The Fisher equation is the most well-known and acceptable relationship between the nominal interest rate and expected inflation. Note that, by this assumption, we can specify how expected inflation adjusts to any changes in the nominal interest rate. Also, note that equation (2.11) means that, if the central bank changes the nominal interest rate,  $i_t$ , then expected inflation should change by the same amount as the nominal interest rate changed because the central bank does not have the ability to change the real interest rate,  $r_t$ .

Therefore, as time goes by, the economy behaves as follows: (i) at the beginning of each period, there occurs a shock; (ii) the central bank is required to react to the shock through manipulating interest rate,  $i_t$ , just after the shock occurred because

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<sup>2</sup>Barro and Gordon (1983) show the existence of time inconsistency in monetary policy by assuming that, at time  $t$ , the policy maker considers expected inflation,  $\pi_t^e$ , as given. Unlike Barro and Gordon (1983), in the current model, the central bank policy makers are not allowed to consider expected inflation as given. This is because it should be the case that they understand how expected inflation adjusts to the changes in nominal interest rate. This supports one of our assumptions that, in this economy, there is no time inconsistency problem.

its goal is given by (2.7); *(iii)* since both *(i)* and *(ii)* occur at the beginning of each period,  $\pi_t^e$  is also formed at the beginning of the period of  $t$ , by all economic agents, after watching the shock and the central bank's reaction; *(iv)* then, the economic performances are governed by (2.10).

Finally, it is assumed that the central bank and the commercial bank keep talking to each other every period about the state of the economy, and that the commercial bank is the only source for the central bank to ask about the state of the economy. This assumption reflects the reality that central banks are, especially in modern times, considered as an important market participant communicating with other private financial intermediaries, and that commercial banks are still important as the main part of a monetary transmission mechanism.

### 3.2. When the Central Bank Does Not Have a Bank Supervisory Role

In this first case, the problem we are facing is clear. Notice that, since the central bank has the ability to control the inflation rate exactly through interest rate manipulation, if the central bank can acquire accurate information about the state of the economy, *i.e.*, about the particular shock, the central bank can always exercise the optimal policy. Hence, the whole problem rests on whether the representative commercial bank has an adequate incentive to tell the truth about the shock to the central bank.

#### 3.2.1. Reaction Function of the Central Bank

Assume that, originally, the economy is in a steady state. This means that, before a shock occurs, the inflation rate and aggregate output stay at those desired values of  $\bar{\pi}$  and  $\bar{y}$ , and, hence, the value of the central bank's loss function is zero.



Now, when there is a shock, the problem of the central bank is given as minimizing the loss function subject to (2.10) and (2.11)<sup>3</sup>. Hence,

$$\min_{i_t} L_t = a [\pi_t(i_t) - \bar{\pi}]^2 + b [\pi_t(i_t) - i_t + r_t]^2. \quad (2.12)$$

The resulting first-order condition is

$$\frac{\partial L_t}{\partial i_t} = 2a(\pi_t(i_t) - \bar{\pi}) \frac{\partial \pi_t}{\partial i_t} + 2b(\pi_t(i_t) - i_t + r_t) \left( \frac{\partial \pi_t}{\partial i_t} - 1 \right) = 0. \quad (2.13)$$

If we simplify the first-order condition, we will get

$$\frac{\partial \pi_t}{\partial i_t} [2a(\pi_t(i_t) - \bar{\pi}) + 2b(\pi_t(i_t) - i_t + r_t)] = 2b((\pi_t(i_t) - i_t + r_t)). \quad (2.14)$$

Finally, since  $\pi_t(i_t) - i_t + r_t = y_t - \bar{y}$ , we have

$$\frac{\partial \pi_t}{\partial i_t} \left[ \frac{a(\pi_t(i_t) - \bar{\pi})}{b(y_t - \bar{y})} + 1 \right] = 1. \quad (2.15)$$

The above equation of (2.15) can be rewritten without time subscripts, since all time subscripts refer to the current period. Then, we have

$$\frac{\partial \pi}{\partial i} \left[ \frac{a(\pi - \bar{\pi})}{b(y - \bar{y})} + 1 \right] = 1. \quad (2.16)$$

Equation (2.16) defines the reaction function of the central bank. To understand the reaction function more clearly, it might be helpful to reformulate it as following

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<sup>3</sup>Let me emphasize again the role of the Fisher equation in this model. By incorporating the constraint of the Fisher equation, we can capture the effect of the central bank's manipulating nominal interest rate on expected inflation. Assuming that the Fisher equation always holds, it should be the case that, if the central bank changes the nominal interest rate,  $i_t$ , then expected inflation should be changed as well. Note that, without this constraint, *i.e.*,

$$\min_{i_t} L_t = a [\pi_t(i_t) - \bar{\pi}]^2 + b [\pi_t(i_t) - \pi_t^e]^2,$$

we cannot say anything how expected inflation will change when the nominal interest rate,  $i_t$ , changes.

equation

$$\Delta\pi\left[\frac{a(\pi - \bar{\pi})}{b(y - \bar{y})} + 1\right] = \Delta i. \quad (2.17)$$

Note that, once the economy had a shock, and the inflation rate and aggregate output deviated from the original steady state values, the term of  $\left[\frac{a(\pi - \bar{\pi})}{b(y - \bar{y})} + 1\right]$  became a constant at the time when the shock occurred. Also, note that the changes in  $\pi_t$  and  $y_t$  are known to everybody after a shock. Hence, if the central bank understands how the shock affected the economy's steady state values, *i.e.*,  $\bar{\pi}$  and  $\bar{y}$ , the central bank can exactly pick up how much it needs to change the inflation rate,  $\Delta\pi$ . Therefore, the central bank's task is determined as picking up  $\Delta i$ , *i.e.*, just manipulating the interest rate which is enough to achieve the desired  $\Delta\pi$ , taking the constant term into consideration.

Considering the well-known fact that the steady state inflation rate is determined by the money growth rate, it may be wondered how the steady state inflation rate can change without any changes in the money growth rate. It can be understood clearly, however, if we remind the relationship in Christiano and Rostagno (2001), *i.e.*,  $\tilde{R} = \frac{\tilde{\pi}}{\beta} - 1$ . If the Fed follows a monetary aggregate targeting policy, then,  $\tilde{\pi}$  is fixed because  $\tilde{\pi}$  is determined by the money growth rate. Hence, when there is a permanent change in  $\beta$ ,  $\tilde{R}$  should change permanently in order to get the equilibrium recovered. However, if the Fed follows an interest rate targeting policy, then,  $\tilde{R}$  is fixed so that  $\tilde{\pi}$  will change permanently when there is a permanent change in  $\beta$ . In the current model, the Fed follows a strict interest rate targeting policy. Therefore, when there is a permanent shock,  $\tilde{\pi}$  should change permanently.

Also, it may be wondered how the inflation rate can be determinate if the Fed can change the nominal interest rate without depending on any other variables. This is the price-level indeterminacy problem which is often noted as a potential problem

with an interest rate targeting policy. However, if we admit that it is the nominal money supply that the Fed can actually control to achieve the target nominal interest rate, then, as McCallum (1986) indicated, the price-level indeterminacy problem will not arise. Suppose that the economy reached to a new steady state after a permanent shock raised the steady state inflation rate while the steady state nominal interest rate remained unchanged. And suppose that the Fed does not like the new inflation rate and it wants to lower the inflation rate to the previous. If the Fed has the ability to change the nominal interest rate without depending on any other variables, then, the Fed can get it done easily by lowering the nominal interest rate. However, if the Fed has to depend on increasing the nominal money supply to lower the nominal interest rate, then, the Fed could not get the inflation rate lowered. In fact, the inflation rate will explode. This is because, as we saw above, since the steady state inflation rate is determined by the money growth rate, if the Fed wants to lower the inflation rate below the steady state, it will cause an increase of the money growth rate, *i.e.*, the inflation rate.

There are two important things we need to note. First, the reaction function has a striking similarity to the conventional Taylor rule. Under the conventional Taylor rule, the level of the nominal interest rate is determined by the current level of two variables, the rate of inflation and measure of the output gap, the deviation of actual output from potential, so:

$$i_t = a + b_1\pi_t + b_2(y - y^*)_t, \quad (2.18)$$

where  $a$  is the equilibrium real interest rate (usually about 2% or 3%). To understand this argument, we need to look at equation (2.17). Again, note that the term of

$[\frac{a(\pi-\bar{\pi})}{b(y-\bar{y})} + 1]$  is a constant. Let us denote the constant term as  $c$ , then, we have

$$c\Delta\pi = \Delta i, \quad (2.19)$$

which can be rewritten as

$$i = i^* + c(\pi - \pi^*), \quad (2.20)$$

where  $i^*$  and  $\pi^*$  are steady state interest and inflation rates, respectively. In fact, if we impose two of the main assumptions, the aggregate supply function and the Fisher equation, into the conventional Taylor rule in (2.18), we can derive exactly the same result shown by (2.20).

Second, when there is a shock, the aggregate output should be affected. If not, the central bank can never achieve its target even though it increases (or decreases) the interest rate to infinity. This can be easily verified through equation (2.17). Suppose, after a shock, the inflation rate deviated from its steady state, but aggregate output remained at the steady state. Then, according to (2.17), we need an infinite increase (or decrease) in the interest rate. This point might have a relationship with a rising criticism of the Taylor rule (see Benhabib, Schmitt-Grohe and Uribe (2001a, 2001b, 2002a, 2002b), and Christiano and Rostagno (2001)) because the central bank's reaction function given by (2.17) is, in effect, a modified Taylor rule as we saw above. However, I will not discuss this point more deeply because it is not related to the topic of this paper, *i.e.*, the relationship between monetary policy and bank supervision. Actually, since we assume that the central bank can exactly control the inflation rate through interest rate manipulation, this kind of phenomenon is precluded by assumption. Hence, to secure the assumption that the central bank can exactly control the inflation rate through interest rate manipulation, we need another assumption that, when there is a shock, it should affect aggregate output.

Table 1. Central Bank's Reaction

state of the economy	$c > 0$		$c < 0$	
	after a shock	$\Delta\pi > 0$	$\Delta\pi < 0$	$\Delta\pi > 0$
central bank's reaction	$\Delta i > 0$	$\Delta i < 0$	$\Delta i < 0$	$\Delta i > 0$

Now, the reaction of the central bank to a shock can be summarized by the following proposition:

**Proposition 1** *In an optimal reaction of the central bank to a shock,  $\Delta\pi$  and  $\Delta i$  will have a positive relation if  $[\frac{a(\pi-\bar{\pi})}{b(y-\bar{y})} + 1] > 0$ , and  $\Delta\pi$  and  $\Delta i$  will have a negative relation if  $[\frac{a(\pi-\bar{\pi})}{b(y-\bar{y})} + 1] < 0$ .*

In equation (2.17), if the constant term  $[\frac{a(\pi-\bar{\pi})}{b(y-\bar{y})} + 1]$  is positive, then, when the central bank needs to decrease the inflation rate, *i.e.*,  $\Delta\pi$  is negative, the central bank should decrease the interest rate, *i.e.*,  $\Delta i$  should be negative. On the contrary, if the constant term  $[\frac{a(\pi-\bar{\pi})}{b(y-\bar{y})} + 1]$  is negative, then, when the central bank needs to decrease the inflation rate, *i.e.*,  $\Delta\pi$  is negative, the central bank should increase the interest rate, *i.e.*,  $\Delta i$  should be positive. Denoting the constant term  $[\frac{a(\pi-\bar{\pi})}{b(y-\bar{y})} + 1]$  as  $c$ , this optimal reaction of the central bank to a shock can be summarized by Table 1.

### 3.2.2. Reaction Function of the Commercial Bank

We need to note that how the reaction of the central bank affects the commercial bank purely depends on the commercial bank's profit function. Let us consider the general form profit function of the commercial bank, *i.e.*,

$$\max_{k_{t+1}, n_t, a_t, l_t} \Pi_t = f(k_{t+1}, n_t, a_t, l_t : i_t, k_t). \quad (2.21)$$

Note that, since the interest rate is the control variable of the central bank, the interest rate is considered as a parameter in the view of the commercial bank. Therefore, the effect of the central bank's reaction on the commercial bank's profit can be calculated simply by applying the envelope theorem. The important thing in analyzing the reaction of the commercial bank is that the effect of the central bank's reaction on the commercial bank's profit should be either positive or negative,  $\frac{\partial \Pi_t}{\partial i_t} > 0$  or  $\frac{\partial \Pi_t}{\partial i_t} < 0$ , *i.e.*, it cannot be both simultaneously. Considering the nature of the envelope theorem, it is natural that we should have this conclusion. For example, if we assume, more specifically, a simple form of profit function as we set up in (2.9),

$$\max_{B_t, n_t, k_{t+1}} \Pi_t = i_t B_t - W_t n_t - P_t(k_{t+1} - k_t), \quad (2.22)$$

the effect of interest rate change on the commercial bank's profit should be always positive, *i.e.*,

$$\frac{\partial \Pi_t}{\partial i_t} = B_t > 0, \quad (2.23)$$

such that when the interest rate increases, the commercial bank's profit always increases.

By assumption, when there is a shock, the commercial bank exactly knows how the shock affects the economy, *i.e.*, it fully understands whether the shock is permanent or temporary. Furthermore, it should be the case that the commercial bank fully understands how the central bank reacts to a shock. Now, note that there could be a temporary shock or a permanent shock. Assume that, when there is a shock, both the inflation rate and aggregate output deviate from their previous steady state values. Then, there will be eight different cases. Also, assume that, when there is a permanent shock, not only  $\bar{\pi}$  and  $\bar{y}$  but  $\pi$  and  $y$  move together in the same direction. However, when there is a permanent shock,  $\pi$  and  $y$  are assumed to move less than

$\bar{\pi}$  and  $\bar{y}$ . This means that, when there is a permanent shock at  $t$ ,  $(\pi - \bar{\pi}) < 0$  and  $(y - \bar{y}) < 0$  if  $\bar{\pi}$  and  $\bar{y}$  increase due to a permanent shock. These assumptions are just for analytical convenience since, by these assumptions, we can reduce the number of cases to analyze. In fact, these assumptions do not affect the general conclusion. Meanwhile, it is needless to say that only  $\pi$  and  $y$  will change when there is a temporary shock. Then, the reaction of the commercial bank is summarized as following proposition:

**Proposition 2** *Assume that, when there is a shock, both the inflation rate and aggregate output deviate from their previous steady state values. Also, assume that, when there is a permanent shock, not only  $\bar{\pi}$  and  $\bar{y}$  but  $\pi$  and  $y$  move together in the same direction as their related steady state variables. [I] Then, if  $\frac{\partial \Pi_t}{\partial i_t} > 0$ , the dominant strategy for the commercial bank's reaction is that (i), if  $c > 0$ , telling the central bank that the shock is permanent if the inflation rate goes up after a shock, and telling the central bank that the shock is temporary if the inflation rate goes down after a shock, and that (ii), if  $c < 0$ , vice versa. [II] If  $\frac{\partial \Pi_t}{\partial i_t} < 0$ , the dominant strategy for the commercial bank's reaction is the opposite of the case where  $\frac{\partial \Pi_t}{\partial i_t} > 0$ .*

Assume that for the commercial bank,  $\frac{\partial \Pi_t}{\partial i_t} > 0$ . Note that there are eight cases since we adopted the assumptions in the above proposition. Then, the optimal reaction of the commercial bank can be shown in Table 2.

Here, (i) denotes the central bank's optimal reaction if it understands what happened in the real sector, and (ii) denotes the commercial bank's optimal reaction. Look at case (1). In this case,  $c = [\frac{a(\pi - \bar{\pi})}{b(y - \bar{y})} + 1]$  should be positive. Note that the movements of  $\pi$  and  $y$  at the time of a shock are known to everybody. Hence, we don't have to worry about the possibility of  $c < 0$ . Since it is a temporary shock, if the central bank understands the fact that  $(\pi - \bar{\pi}) > 0$ , *i.e.*, if it understands that

Table 2. Commercial Bank's Reaction When  $\frac{\partial \Pi_t}{\partial i_t} > 0$ 

state of the economy after a shock		$c > 0$		$c < 0$	
		$\Delta\pi > 0$	$\Delta\pi < 0$	$\Delta\pi > 0$	$\Delta\pi < 0$
temporary shock					
(1) $\pi \uparrow, y \uparrow$	(i)	--	$\Delta i < 0$	--	--
	(ii)	tell a lie		--	
(2) $\pi \uparrow, y \downarrow$	(i)	--	$\Delta i < 0$	--	$\Delta i > 0$
	(ii)	tell a lie		tell the truth	
(3) $\pi \downarrow, y \uparrow$	(i)	$\Delta i > 0$	--	$\Delta i < 0$	--
	(ii)	tell the truth		tell a lie	
(4) $\pi \downarrow, y \downarrow$	(i)	$\Delta i > 0$	--	--	--
	(ii)	tell the truth		--	
permanent shock					
(5) $\bar{\pi} \uparrow, \bar{y} \uparrow$	(i)	$\Delta i > 0$	--	--	--
	(ii)	tell the truth		--	
(6) $\bar{\pi} \uparrow, \bar{y} \downarrow$	(i)	$\Delta i > 0$	--	$\Delta i < 0$	--
	(ii)	tell the truth		tell a lie	
(7) $\bar{\pi} \downarrow, \bar{y} \uparrow$	(i)	--	$\Delta i < 0$	--	$\Delta i > 0$
	(ii)	tell a lie		tell the truth	
(8) $\bar{\pi} \downarrow, \bar{y} \downarrow$	(i)	--	$\Delta i < 0$	--	--
	(ii)	tell a lie		--	



it needs to decrease the inflation rate, *i.e.*,  $\Delta\pi < 0$ , then, its optimal policy will be characterized as  $\Delta i < 0$ . However, it goes directly against the commercial bank's interest. The commercial bank understands how the central bank reacts to a shock. The dominant strategy for the commercial bank is to lie to the central bank. Note that, in case (5), when there is a permanent shock, the optimal policy will be  $\Delta i > 0$ . Therefore, in order to maximize its profit, the commercial bank will lie to the central bank just like the shock was permanent.

Now, look at case (2). In this case, if  $c > 0$ , the subsequent story is exactly the same as the above case of (1). However, if  $c < 0$ , the subsequent story is exactly the opposite to the above case (1). Note that, at the time when a shock occurs, the central bank does not understand if the value of  $c$  is positive or negative. All other cases can be understood based on the same logic. Also, note that, if  $\frac{\partial \Pi_t}{\partial i_t} < 0$ , the reaction of the commercial bank will be exactly the opposite of Table 2.

### 3.2.3. Equilibrium

It should be the case that the central bank understands what the commercial bank will reveal about a relevant shock. Therefore, there is no reason for the central bank to believe what the commercial bank tells about the shock. The rational behavior of the central bank, then, will be conducting monetary policy based on its own expectation on the shock.

**Proposition 3** *Assume that the central bank is risk averse. Then, the central bank's reaction to a shock in equilibrium is summarized as*

$$\Delta i = E[c\Delta\pi] = E[\Delta\pi[\frac{a(\pi - \bar{\pi})}{b(y - \bar{y})} + 1]]. \quad (2.24)$$

Suppose that the economy is at the case of (1), (4), (5) or (8) in Table 2. If the probability that this economy has a permanent shock is  $q$ , then, the reaction of the central bank will be

$$\Delta i = E[c\Delta\pi] = q[\Delta\pi_p c_p] + (1 - q)[\Delta\pi_t c_t], \quad (2.25)$$

where  $\Delta\pi_p$  and  $c_p$  are the associated values under a permanent shock of each case, and  $\Delta\pi_t$  and  $c_t$  are the associated values under a temporary shock of each case. Suppose that the economy is at the case of (2), (3), (6) or (7) in Table 2. If the probability that this economy has a positive  $c$ , *i.e.*,  $c > 0$ , is  $r$ , then, the reaction of the central bank will be

$$\begin{aligned} \Delta i = E[c\Delta\pi] &= r \{q[\Delta\pi_p c_p] + (1 - q)[\Delta\pi_t c_t]\} \\ &+ (1 - r) \{q[\Delta\pi_p c_p] + (1 - q)[\Delta\pi_t c_t]\}. \end{aligned} \quad (2.26)$$

The point is that there should be a systematic error in conducting monetary policy. The amount of the systematic error in conducting monetary policy will be the difference between (2.17) and (2.25) or (2.26).

### 3.3. When Central Bank Has a Bank Supervisory Role

Now, assume that the central bank has a bank supervisory role. The meaning of this assumption is that the central bank can impose an appropriate penalty on the commercial bank when it is found that the commercial bank told a lie to the central bank about the shock. Since we assumed that the shock reveals itself after one period, there is no reason for the commercial bank to tell a lie to the central bank no matter what the shock is.

Note that, since the shock reveals itself after one period, in the case where the

commercial bank lied, the central bank can exactly calculate how much the commercial bank earned by telling a lie. Hence, it is always possible for the central bank to impose a penalty such that the penalty is bigger than or equal to the amount the commercial bank earned by telling a lie, *i.e.*,

$$Penalty \geq \left[ \frac{\partial \Pi_t}{\partial i_t} \right],$$

which means that telling the truth is always a dominant strategy for the commercial bank.

**Proposition 4** *Suppose that the shock reveals itself after a certain period. The central bank can eliminate the systematic error if it keeps a bank supervisory role which involves imposing penalties against those commercial banks which told a lie to the central bank about the shock.*

Here, the important thing is that we are assuming that the central bank can impose penalties if it has a bank supervisory role, and that it cannot impose penalties if it does not have a bank supervisory role. Note that, if the central bank can impose penalties *exactly* against those commercial banks which lied to the central bank *without* a bank supervisory role, it could be a better solution than giving a bank supervisory role to the central bank. Considering the basic functions and objectives of monetary policy and bank supervision, however, this does not seem to be reality from the view of practical assignment of the two functions.

The assumption that *the central bank can impose penalties against the commercial bank only if it has a bank supervisory role* can be justified by the two following points. First, in a normative view, the two functions are based on different fields. Monetary policy is a *macroeconomic* policy while bank supervision is a *microeconomic* policy. Monetary policy is supposed to affect the whole economy in an *indiscriminating*

manner. Bank supervision is supposed to treat each bank individually. Note that the reason bank supervision is called for is that all governments provide some form of a safety net to the banking system, whether it is explicit or implicit, they need to take steps to limit the moral hazard and adverse selection that the safety net creates (see Mishkin (2000)). Otherwise, banks will have such a strong incentive to take on excessive risks that the safety net may do more harm than good and promote banking crises rather than prevent them. Bank supervision, in which the supervisor establishes regulations to prevent excessive risk taking and then monitors banks to see that they are complying with these regulations, is thus needed to ensure the safety and soundness of the banking system. Therefore, it is impossible for the central bank to impose penalties against commercial banks in a *discriminating* manner without a bank supervisory role. Second, in a practical view, without a bank supervisory role, the central bank cannot have the ability to identify which banks did tell a lie to itself, *i.e.*, the central bank cannot identify *true lies*. Without a bank supervisory role, the central bank cannot have an adequate tool to monitor the commercial bank. Therefore, since it cannot check how the commercial bank's balance sheet changes in a timely manner, even after the shock revealed itself, the central bank may not be sure if a particular bank really told a lie. For example, there could be a situation in which a particular commercial bank told the truth to the central bank but, later, along the changing situation, it could be considered a liar.

However, there rises another problem by giving a bank supervisory role to the central bank. By giving a bank supervisory role to the central bank, we can guarantee to eliminate the systematic error. However, it does not mean that it guarantees the effectiveness in conducting monetary policy. This is so because, by definition, the effectiveness is accomplished by minimizing the loss function of the central bank

given by (2.7), which is shown below again:

$$\min_{i_t} L_t = a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2. \quad (2.27)$$

The difficulty is that, if the central bank has a bank supervisory role, its objective loss function *might* also be transformed to reflect the objective of bank supervision, *i.e.*, banking system stability.

Taking the risk of too much simplification, suppose the loss function is transformed as below:

$$\min_{i_t, o_t} L_t = a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2 + f [s_t(o_t) - \bar{s}]^2, \quad (2.28)$$

where  $f > 0$  is a parameter,  $s_t$  is a variable representing banking system stability,  $\bar{s}$  is the steady state value of the variable, and  $o_t$  is a control variable affecting  $s_t$ . Note that, as far as the two control variables are not related to each other, the central bank can achieve both objectives successfully. However, if there is some relation between the two control variables, then, the value of the loss function in (2.27) should be higher when the central bank's objective is given by (2.28). Hence, there is a trade-off. In fact, this point reflects so-called *conflict of interests* which has been argued in the traditional debate. Here, the point is that the way of giving a direct bank supervisory role to the central bank cannot guarantee the effectiveness of monetary policy.

#### 3.4. When There Is Another Government Agency Specializing in Bank Supervision

Suppose there is another government agency specializing in bank supervision such as a bank regulator. Hence, the central bank has no bank supervisory role. Note that it could be the best solution if there were no problem in cooperation between the central bank and the bank regulator. The central bank can always achieve its objective

represented by (2.27) without having any systematic error in conducting monetary policy if the bank regulator provides any necessary information for monetary policy in a timely manner, and if the bank regulator imposes any necessary penalties against the commercial bank as the central bank wants. Meanwhile, the bank regulator can focus on its own objective function without worrying about the effectiveness of monetary policy.

However, this best solution is not feasible as the following proposition summarizes:

**Proposition 5** *Let  $V_1$  denote the value of the loss function in (2.27) when the central bank can access to any kind of bank supervisory information without any restriction, and without any bank supervisory role. Let  $V_2$  denote the value of the loss function in (2.27) when the central bank does not have a bank supervisory role because there is another government agency specializing in bank supervision. Then,  $V_1 < V_2$ .*

The proof of this proposition purely depends on the objective function of the bank regulator. We can think of two extreme cases about the objective function of the bank regulator. One case is that it is the same as that of the *commercial* bank. The other case is that it is the same as that of the *central* bank. Note that, however, the objective function of bank regulator cannot be the same as that of the central bank. If they are the same, we would not have to establish two different government agencies. Suppose, as an extreme case, the objective function of bank regulator is the same as that of the commercial bank, the profit maximization of the commercial bank. Then, the equilibrium is exactly the same as what we saw in (2.24). If and only if the objective function is the same as that of the central bank, can we have  $V_1 = V_2$ . However, we already know that this cannot be the case.

If the economy remains stable, the values of  $V_1$  and  $V_2$  could be similar. However, once the economy becomes unstable, the problem could also get bigger, resulting in a much bigger  $V_2$  than  $V_1$ . As Greenspan indicated, a single regulator with a narrow view of banking system stability and with no responsibility for the macroeconomic implications of its decisions would inevitably have a long-term bias against risk-taking and innovation. Note that, for a single bank regulator with no responsibility for the macroeconomic consequences, the incentives are clear because it receives no pay-offs for contributing to economic growth through facilitating prudent risk-taking, but it is severely criticized for too many bank failures.

The point is that the way of setting up another government agency specializing in bank supervision cannot guarantee the effectiveness of monetary policy, either. However, there is a distinct difference between the two cases. In the first case, the systematic error can be eliminated while the problem comes from a conflict of interests. In the second case, the problem stems from the fact that the systematic error cannot be fully eliminated.

### 3.5. Combination vs. Separation

It has been shown that both ways, *i.e.*, giving a direct bank supervisory role to the central bank and setting up a bank regulator, are not good enough to guarantee the effectiveness of monetary policy. We could compare relative effectiveness between those two ways, but it does not seem to be meaningful because it is almost impossible to get sizable results. Limitations come from two sources. First, setting up a specific objective function for a bank regulator is difficult. Second, measuring the values of  $V_1$  and  $V_2$  is actually impossible.

Therefore, in discussing which is better between combining or separating the two

functions, the only available way seems to compare the two ways in the view of social welfare, not in the view of the effectiveness of monetary policy. In other words, it is about comparing the conflict-of-interests problem and the systematic error problem in conducting monetary policy in the view of social welfare. Considering the two following propositions, it can be argued that the central bank should take the whole responsibility or at least a partial responsibility on bank supervision, directly against the current trend surrounding central banking systems. Look back at the transformed loss function in (2.28). The key point on how to assign the two functions, monetary policy and bank supervision, is how much the two control variables,  $i_t$  and  $o_t$ , in (2.28) are related to each other.

**Proposition 6** *Let  $W_1$  denote the social welfare when the central bank has a bank supervisory role, and let  $W_2$  denote the social welfare when the central bank does not have a bank supervisory role because there is another government agency specializing in bank supervision. If the two control variables in (2.28) are never correlated with each other, and the commercial bank's profit is included in  $s_t$ , then,  $W_1 = W_2$ .*

Since the two control variables are not correlated, we have the following result:

$$\begin{aligned} & \min_{i_t, o_t} \{a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2 + f[s_t(o_t) - \bar{s}]^2\} \\ &= \min_{i_t} \{a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2\} + \min_{o_t} f[s_t(o_t) - \bar{s}]^2. \end{aligned} \quad (2.29)$$

This means that it is indifferent whether the central bank has a bank supervisory role or there is another government agency specializing in bank supervision. Notice that, by assumption that the two control variables are not correlated, the interest rate cannot affect banking system stability, including the commercial bank's profit. The assumption that the two control variables are not correlated implies that the interest rate does not affect the commercial bank's profit if the profit is included in



the category of banking system stability. Therefore, there need not be a systematic error like we saw in (2.24). However, it should be the case that, if the commercial bank's profit is not included in  $s_t$ , then,  $W_1 > W_2$ .

**Proposition 7** *Let  $W_3$  denote the social welfare when the central bank has a bank supervisory role, and let  $W_4$  denote the social welfare when the central bank does not have a bank supervisory role because there is another government agency specializing in bank supervision. If the two control variables in (2.28) are correlated with each other, then,  $W_3 > W_4$ .*

Since the two control variables are correlated, we have the following result:

$$\begin{aligned} & \min_{i_t, o_t} \{a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2 + f[s_t(o_t) - \bar{s}]^2\} \\ & < \min_{i_t} \{a [\pi_t(i_t) - \bar{\pi}]^2 + b [y_t - \bar{y}]^2\} + \min_{o_t} f[s_t(o_t) - \bar{s}]^2. \end{aligned} \quad (2.30)$$

It can be shown clearly if we assume  $i_t$  and  $o_t$  are perfectly correlated, then,  $i_t = o_t$ . In this case, since the bank regulator has actually no ability to achieve its goal while the central bank does not care about banking system stability,  $W_4$  should be lower than  $W_3$ . Note that this relationship holds whether the correlation is positive or negative. Suppose that  $i_t$  and  $o_t$  are perfectly negatively correlated, then,  $i_t = -o_t$ . Considering the fact that all components have quadratic forms, it is easy to see that the above relationship still holds. Also, note that, in this case, the inequality holds regardless of whether the commercial bank's profit is included in the category of banking system stability. However, it is clear that, the more correlated those two control variables are, the bigger value of loss function we will have, and, therefore, the welfare difference will be larger between the two cases because of the existence of systematic error in conducting monetary policy. Notice that  $W_1$  and  $W_3$  incorporate the conflict-of-interests problem while  $W_2$  and  $W_4$  do the systematic error problem in

conducting monetary policy.

Goodhart and Schoenmaker (1995) gives some clues on this argument:

.... These examples suggest that the potential for conflict between regulatory and monetary objectives depends to some large extent on the structure of the banking and financial systems. The more such a system involves intermediaries financing maturity mismatch positions through wholesale markets in a competitive milieu, the greater such dangers of conflict are likely to be. .... It is, therefore, at least possible to argue that where such conflicts really become important (in an open, competitive, market-driven system), they have to be internalized within a single authority to obtain an efficient resolution. Where such conflicts have been less pressing, because of a differing structure, e.g., in Germany and Japan, it is easier to maintain the luxury of a separation of responsibilities. One of the reasons why such separation may be regarded as a luxury is that the function of regulation has rarely received plaudits from the public or the politicians. ....

The possible contribution of this paper concerning this point is that this paper clarified the arguments of Goodhart and Schoenmaker (1995) using the comparison between the conflict-of-interests problem and the systematic error problem in conducting monetary policy in the view of social welfare.

#### 4. Conclusion

This paper begins with the observation that, as Peek *et al.* (1999) shows, there is a systematic error in making monetary policy if the central bank does not have any bank supervisory role. Based on this observation, this paper investigates why there should be a systematic error in making monetary policy. This task has been addressed using a simple game-theoretic approach. In this approach, the most critical assumption is that there is an asymmetric information about state of the economy between the central bank and commercial bank. As Caplin and Leahy (1996) indicated, monetary policy makers must steer a course between inflation and unemployment, and they

must do so with only limited information concerning the state of the economy and its reaction to the tools of policy.

The implications of this paper can be summarized as follows: (i) it is inevitable for the central bank to have a systematic error in conducting monetary policy when the central bank does not have a bank supervisory role; (ii) without a bank supervisory role, the effectiveness of monetary policy cannot be guaranteed; (iii) because of the existence of *conflict of interests*, giving a bank supervisory role to the central bank does not guarantee the effectiveness of monetary policy, either; (iv) the way of setting up another government agency, bank regulator, and making the central bank and the regulator cooperate each other does not guarantee the effectiveness of monetary policy because, in this way, the systematic error in conducting monetary policy cannot be eliminated; (v) in the view of social welfare, not in the view of effectiveness of monetary policy, it is better for the central bank to keep the whole responsibility or at least a partial responsibility on bank supervision.

The most notable point among the results is that, if the central bank does have a bank supervisory role, the bank supervisory role acts as a truth-telling mechanism when commercial banks communicate with the central bank about true state of the economy. In a situation in which the central bank is ignorant of a shock and commercial banks are aware of the shock, commercial banks should be considered as the most important source for the central bank to find out the current state of the economy. Without a bank supervisory role as a truth-telling mechanism, the central bank cannot avoid a systematic error in conducting monetary policy, especially when it follows an interest rate targeting policy. Although giving a bank supervisory role to the central bank does not guarantee the effectiveness of monetary policy, i.e., it is not an optimal solution *in terms of the effectiveness of monetary policy*, it is the

optimal solution *in terms of social welfare*, among practically available options for institutional set-up arranging the responsibilities of conducting monetary policy and bank supervision.

These implications have significant meaning on the current trend surrounding central banking systems around the world. As we saw in the introduction, many countries have been considering whether bank supervisory responsibilities are an important function for the central bank or are better assigned to a separate government agency. The results in this paper indicate that the current trend surrounding central banking systems around the world is not on the right track. These implications may be even more important in many other countries, particularly developing countries, since, in these countries, the credit markets are usually bank-centered, and the banking industry structures are often far from perfect competition. Lack of competition in the banking industry will strengthen the market power of commercial banks, and, under this situation, if the central bank does not have any bank supervisory role, a selfish profit maximizing behavior of commercial banks could significantly lower the effectiveness of monetary policy.

## CHAPTER III

BUSINESS CYCLES UNDER THE TAYLOR RULE:  
AN ANSWER TO SOME UNANSWERED QUESTIONS

## 1. Introduction

Real business cycle (RBC) models have been surprisingly successful at accounting for several non-monetary features of U.S. business cycles. As Christiano (1991) indicated, however, they have been less successful once money is involved. Serious works with a flexible-price general equilibrium business cycle model under no frictions except money include King and Plosser (1984), Cooley and Hansen (1989), Cooley and Hansen (1995), and Walsh (2003).

The most important failures in RBC models with money can be summarized as following: First, as Cooley and Hansen (1995) indicated, the inflation rate in an RBC model economy is negatively correlated with output, in contrast to what is observed in U.S. time series; Second, RBC models with money cannot reproduce the so-called *liquidity effect*, which suggests that an increase in money supply lowers both nominal and real interest rates and raises the inflation rate, output and investment, at least temporarily. According to Romer (1996), the conventional explanation of the liquidity effect is that monetary expansions reduce the real interest rate. This is because an increase in output requires a decline in the real interest rate. If the decline in the real interest rate is large enough, it more than offsets the effect of the increase in expected inflation. While many empirical studies including VAR literature support this fact, RBC models with money imply the opposite, *i.e.*, a rise in interest rates and the inflation rate, and a fall in investment and output.

Naturally, many economists tried to adopt frictions other than money into an RBC model to capture monetary business cycle features. Two popular monetary models are sticky price and limited participation models. Unfortunately, however, it is said that those models adopting frictions other than money achieved only mild success. As investigated in Christiano and Evans (1997), sticky price models fail to produce a drop in the nominal interest rate following an expansionary monetary policy shock. In contrast, limited participation models can generate the expected responses to a monetary disturbance, but only under some implausible parameters. Moreover, considering money neutrality in a long-term horizon and the fact that business cycle data is measured in a long-term horizon, it can be argued that adopting frictions other than money into an RBC model could be doubtful because those frictions could matter only in a short-term horizon.

This paper examines the impact of technology and money shocks in a flexible-price general equilibrium business cycle model with no frictions except money under a cash-in-advance constraint and the assumption that the central bank follows an interest rate rule proposed by Taylor (1993). In recent years, it has been widely accepted that the so-called Taylor rule is a more realistic description of how the Fed policy operates since the Fed definitely perceives the nominal interest rate as its primary policy instrument. Particularly, a forward-looking interest rate rule has been adopted based on Clarida and Gali (2000).

One thing significantly different from other researches on this topic is the way the Taylor rule is introduced in the model. In this paper, the Taylor rule is introduced by considering the relationship among the Fisher equation, Euler equation and the Taylor rule in the dynamic system of the relevant RBC model. This approach can be justified based on the reason that, if the Fed follows the Taylor rule strictly, and

if we assume that the Fed always achieves its target nominal interest rate, then, the nominal interest rate is determined prior to the real interest rate. Without the Taylor rule, the real interest rate is determined by the Euler equation and the nominal interest rate is subsequently determined by the Fisher equation. With this approach, it has been shown that, even in a flexible-price environment, the two major failures in RBC models with money can be resolved. Under the Taylor rule, the correlation between output and inflation appears to be positive and the response of our model economy to a shock is persistent. Furthermore, the possibility of an existing liquidity effect is found.

There have not been many papers incorporating the Taylor rule in the context of RBC modelling under a flexible-price environment. Among others, Dittmar *et al.* (2003) and Chen (2003) tried similar approaches to this paper. Giving up the cash-in-advance constraint, these two papers based on a shopping-time constraint introducing money in their models. Dittmar *et al.* (2003) succeeded in showing inflation persistence, but failed in showing a liquidity effect. Chen (2003) argued that his result showed a persistent liquidity effect. However, when the Fed takes an expansionary policy, his result shows a rise in the real interest rate and output and a fall in the inflation rate and the nominal interest rate, *i.e.*, correct movements in output and the nominal interest rate but incorrect movements in the real interest rate and the inflation rate.

## 2. Model

### 2.1. Basic Structure

The dynamic general equilibrium framework of Walsh (2003), which is a modified model of Cooley and Hansen (1989), is basically adopted in this paper. We consider a closed economy model which is populated by a large number of identical agents. The utility function of the representative agent is given as

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-h_t)^{1-\psi}}{1-\psi} \right], \quad (3.1)$$

with  $0 < \beta < 1$ . Here,  $c_t$  is real consumption in period  $t$ , while  $h_t$  is hours worked in period  $t$  so that  $1 - h_t$  is equal to leisure time. The parameters  $\Phi$ ,  $\Psi$  and  $\psi$  are restricted to be positive. The representative agent faces the cash-in-advance constraint

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} \equiv a_t, \quad (3.2)$$

and the budget constraint

$$c_t + k_t + b_t + m_t \leq y_t + (1 - \delta)k_{t-1} + \left( \frac{1 + i_{t-1}}{\Pi_t} \right) b_{t-1} + a_t, \quad (3.3)$$

where  $M_{t-1}$  is per capita currency carried over from period  $t - 1$  to period  $t$ ,  $P_t$  is the price level in period  $t$ ,  $T_t$  is per capita nominal lump-sum transfer equal to  $(e^{\mu_t} M_{t-1} - M_{t-1})$  where  $\mu_t$  is the money growth rate in period  $t$ ,  $m_t = \frac{M_t}{P_t}$  is the real money balance in period  $t$ ,  $k_{t-1}$  is per capita physical capital in period  $t$  determined in period  $t - 1$ ,  $B_{t-1}$  is the agent's holdings of nominal one-period bonds and  $b_{t-1} = B_{t-1}/P_t$ ,  $y_t$  is per capita income in period  $t$ ,  $i_t$  is the nominal interest rate in period  $t$ , and  $\Pi_t = 1 + \pi_t$  is the gross inflation rate in period  $t - 1$ . Following Cooley and Hansen (1995), we will only consider equilibria in which (3.2) and (3.3) hold with



equality.

As mentioned above, per capita money holding,  $M_t$ , is assumed to grow at the rate  $e^{\mu_t} - 1$  in period  $t$ . That is,

$$M_t = e^{\mu_t} M_{t-1}, \quad (3.4)$$

where  $\mu_t$  is revealed at the beginning of period  $t$ . In order to specify the behavior of the growth rate of the nominal money supply,  $\mu_t$ , let  $u_t$  be equal to the deviation of money growth around the steady state, *i.e.*,  $u_t = \mu_t - \bar{\mu}$ . Then,  $u_t$  is assumed to evolve according to

$$u_t = \xi_1 \hat{i}_t + \xi_2 \hat{y}_t + \varphi_t, \quad (3.5)$$

where  $\hat{i}_t = I_t - \tilde{I}$ ,  $I_t = 1 + i_t$  is the gross nominal interest rate in period  $t$ ,  $\tilde{I}$  is the steady state gross nominal interest rate,  $\hat{y}_t = \ln y_t - \ln \tilde{y}$ ,  $\tilde{y}$  is the steady state per capita income,  $\xi_1$  is the interest rate elasticity of real money balance demand,  $\xi_2$  is the income elasticity of real money balance demand, and

$$\varphi_t = \eta \varphi_{t-1} + \epsilon_t,$$

where the random variable  $\epsilon$  is a white noise innovation with  $\sigma_\epsilon^2$ .

This is the first difference between Walsh (2003) and this paper. In Walsh (2003),  $u_t$  is assumed to evolve as

$$u_t = \eta u_{t-1} + \varkappa z_{t-1} + \nu_t, \quad (3.6)$$

where  $0 < \eta < 1$  and  $0 < \varkappa < 1$ . The random variable  $\nu$  is a white noise innovation with variance  $\sigma_\nu^2$ . This means that the real money balance is determined endogenously in this paper while it is determined exogenously in Walsh (2003).

As will be clear when we discuss monetary policy, we need this change because

we want to incorporate the Taylor rule explicitly in our model. By nature of an interest rate targeting policy regime, the central bank does not care about changes in aggregate money under an interest rate targeting policy regime. Note that this feature is adequately captured in this setup. Although the central bank sets up a money growth rate in advance, it does not have any desire to keep this pre-determined money growth rate. If the economy deviates from a steady state,  $\mu_t$  will be different from  $\bar{\mu}$  until the economy gets back to a steady state so that the nominal interest rate and per capita income may stay on the steady state path. Hence, as indicated in Christiano and Rostagno (2001), money market equilibrium is purely determined by money demand in this model.

The representative agent's decision problem is characterized by the value function

$$V(k_{t-1}, b_{t-1}, a_t) = \max \left\{ \frac{c_t^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-h_t)^{1-\psi}}{1-\psi} + \beta E_t V(k_t, b_t, a_{t+1}) \right\}, \quad (3.7)$$

where the maximization is with respect to  $\{c_t, h_t, b_t, k_t, m_t\}$  and is subject to the constraints (3.2) and (3.3).

The economy's technology is given by a Cobb-Douglas constant returns to scale production function, expressed in per capita terms as

$$y_t = e^{z_t} k_{t-1}^\theta h_t^{1-\theta}, \quad (3.8)$$

where  $0 < \theta < 1$ . The technology shock,  $z_t$ , consists of a single persistent component that evolves according to the law of motion

$$z_t = \rho z_{t-1} + \varepsilon_t, \quad (3.9)$$

where  $0 < \rho < 1$ . The random variable  $\varepsilon$  is normally distributed with mean zero and standard deviation  $\sigma_\varepsilon$ . The portion of output that is not consumed is invested

in physical capital as

$$k_t = (1 - \delta)k_{t-1} + x_t, \quad (3.10)$$

where  $\delta$  is the depreciation rate with  $0 < \delta < 1$  and  $x_t$  is investment in period  $t$ .

The second difference between this paper and Walsh (2003) is that the central bank follows the Taylor rule explicitly in this paper. Note that equation (3.6) implies that the Fed in Walsh (2003) follows a monetary aggregate targeting policy. Specifically, it is assumed that the Fed follows one of the two types of Taylor rule shown below. The first rule is based on the seminal paper of Taylor (1993) such that

$$\hat{i}_t = \omega_1 (E_t \widehat{\pi}_{t+1}) + \omega_2 (\hat{y}_t), \quad (3.11)$$

where the parameter values of  $\omega_1$  and  $\omega_2$  are set as 1.5 and 0.5, respectively, and  $E_t \widehat{\pi}_{t+1} = E_t \pi_{t+1} - \tilde{\pi}$  where  $\pi_t$  is the inflation rate in period  $t$  and  $\tilde{\pi}$  is the steady state inflation rate. The two monetary policy parameter values set up here are well supported by empirical studies such as in Ball (1997). Hereafter, the model based on the first policy rule is called Model 1.

The second rule is a modification of the first rule based on a Lucas-type aggregate supply function of the form,

$$y_t = \tilde{y} + (\pi_{t+1} - E_t \pi_{t+1}),$$

which implies

$$\hat{i}_t = \omega_3 (E_t \widehat{\pi}_{t+1}), \quad (3.12)$$

where the value of parameter  $\omega_3$  is set as 1.3, accordingly.

Notice that when the Lucas-type aggregate supply function holds, based on equa-

tion (3.11), we have the following at the period  $(t - 1)$ :

$$\widehat{i}_{t-1} = \omega_1 (\widehat{\pi}_t) + \omega_2 (\widehat{y}_{t-1}),$$

which can be expressed as

$$\begin{aligned} \widehat{i}_{t-1} &= \omega_1 (\pi_t - \widetilde{\pi}) + \omega_2 (\pi_t - E_{t-1}\pi_t) \\ &= \omega_1 (\pi_t - \widetilde{\pi}) + \omega_2 (\pi_t - i_{t-1} + r_{t-1}) \\ &= (\omega_1 + \omega_2) \pi_t - \omega_1 \widetilde{\pi} - \omega_2 i_{t-1} + \omega_2 r_{t-1} \\ &= (\omega_1 + \omega_2) \pi_t - \omega_1 \widetilde{\pi} - (\omega_2 \widetilde{\pi} - \omega_2 \widetilde{\pi}) - \omega_2 i_{t-1} + \omega_2 r_{t-1} \\ &= (\omega_1 + \omega_2) \pi_t - (\omega_1 + \omega_2) \widetilde{\pi} + \omega_2 \widetilde{\pi} - \omega_2 i_{t-1} + \omega_2 r_{t-1} \\ &= (\omega_1 + \omega_2) \widehat{\pi}_t + \omega_2 (\widetilde{\pi} - i_{t-1} + r_{t-1}) \\ &= (\omega_1 + \omega_2) \widehat{\pi}_t + \omega_2 \left( \widetilde{\pi} - (i_{t-1} - \widetilde{i}) - \widetilde{i} + (r_{t-1} - \widetilde{r}) + \widetilde{r} \right) \\ &= (\omega_1 + \omega_2) \widehat{\pi}_t + \omega_2 \left( (\widetilde{\pi} - \widetilde{i} + \widetilde{r}) - \widehat{i}_{t-1} + \widehat{r}_{t-1} \right) \\ &= (\omega_1 + \omega_2) \widehat{\pi}_t + \omega_2 \left( -\widehat{i}_{t-1} + \widehat{r}_{t-1} \right). \end{aligned}$$

Therefore, if we ignore  $\omega_2 (\widehat{r}_{t-1})$ , then, we have

$$\widehat{i}_{t-1} = \left( \frac{\omega_1 + \omega_2}{1 + \omega_2} \right) \widehat{\pi}_t.$$

By advancing one period ahead, we have

$$\widehat{i}_t = \left( \frac{\omega_1 + \omega_2}{1 + \omega_2} \right) E_t \widehat{\pi}_{t+1} \approx 1.33 * E_t \widehat{\pi}_{t+1}.$$

The policy rule in (3.12), hence, assumes that the Fed makes its monetary policy decision fully taking the aggregate supply function into consideration. Suh (2004) showed that, when a central bank follows an interest rate targeting policy in an economy where the Lucas-type aggregate supply function and the Fisher equation

hold, the optimal reaction function of the central bank is determined as the policy rule in (3.12). Hereafter, the model based on the second policy rule is called Model 2.

## 2.2. Dynamic General Equilibrium System

The dynamic general equilibrium system consists of first-order conditions derived from the optimization problem of a representative agent described in the previous section, and other various related equations governing the evolution of relevant variables.

$$y_t = e^{z_t} k_{t-1}^\theta h_t^{1-\theta} \quad (3.13)$$

$$y_t = c_t + x_t \quad (3.14)$$

$$k_t = (1 - \delta)k_{t-1} + x_t \quad (3.15)$$

$$R_t = f_{k,t} + 1 - \delta = \theta \frac{E_t y_{t+1}}{k_t} + 1 - \delta \quad (3.16)$$

$$\lambda_t = \beta E_t R_t \lambda_{t+1} \quad (3.17)$$

$$\Psi (1 - h_t)^{-\psi} = (1 - \theta) \left( \frac{y_t}{h_t} \right) \lambda_t \quad (3.18)$$

$$\lambda_t = \beta E_t \left( \frac{1 + i_t}{\Pi_{t+1}} \right) \lambda_{t+1} \quad (3.19)$$

$$\lambda_t = \beta E_t \left( \frac{m_{t+1}^{-\Phi}}{\Pi_{t+1}} \right) \quad (3.20)$$

$$m_t = \frac{e^{\mu_t} m_{t-1}}{\Pi_t} \quad (3.21)$$

$$c_t = m_t \quad (3.22)$$

Equations from (3.13) to (3.16) represent the production function, the resource constraint, the investment evolution equation, and the definition of marginal product of capital. Equations from (3.17) to (3.20) represent the four first-order conditions

derived from the utility maximization problem for the representative agent, which are the Euler equation, the labor-leisure choice condition, the Fisher equation, and the marginal utility of consumption, respectively. Equation (3.21) shows the evolution of the real money balance, and equation (3.22) is the cash-in-advance constraint.

Taking log-linearization on the above dynamic general equilibrium system, we have the following log-linearized dynamic general equilibrium system. Note that we have denoted  $\widehat{a}_t = \ln a_t - \ln \widetilde{a}$ , where  $\widetilde{a}$  is the steady state value, except  $\widehat{i}_t = I_t - \widetilde{I}$ ,  $\widehat{r}_t = R_t - \widetilde{R}$ , and  $\widehat{\pi}_t = \pi_t - \widetilde{\pi}$ , where  $R_t$  is the gross real interest rate and  $\widetilde{R}$  is the steady state gross real interest rate. Therefore,  $\widehat{i}_t$ ,  $\widehat{r}_t$  and  $\widehat{\pi}_t$  are measured as a percentage rate, and other variables are measured as a percentage deviation around the steady state.

$$\widehat{y}_t = \theta \widehat{k}_{t-1} + (1 - \theta) \widehat{h}_t + z_t \quad (3.23)$$

$$0 = \frac{\widetilde{y}}{\widetilde{k}} \widehat{y}_t - \frac{\widetilde{c}}{\widetilde{k}} \widehat{m}_t - \delta \widehat{x}_t \quad (3.24)$$

$$\left(\frac{\widetilde{y}}{\widetilde{k}}\right) \widehat{y}_t = \left(\frac{\widetilde{c}}{\widetilde{k}}\right) \widehat{m}_t + \widehat{k}_t - (1 - \delta) \widehat{k}_{t-1} \quad (3.25)$$

$$\widehat{r}_t = \theta \left(\frac{\widetilde{y}}{\widetilde{k}}\right) \left(E_t \widehat{y}_{t+1} - \widehat{k}_t\right) \quad (3.26)$$

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \widehat{r}_t \quad (3.27)$$

$$\widehat{y}_t + \widehat{\lambda}_t = \left(1 + \psi \frac{\widetilde{h}}{1 - \widetilde{h}}\right) \widehat{h}_t \quad (3.28)$$

$$\widehat{\lambda}_t = \widehat{i}_t - E_t \widehat{\pi}_{t+1} + E_t \widehat{\lambda}_{t+1} \quad (3.29)$$

$$\widehat{\lambda}_t = -E_t (\Phi \widehat{m}_{t+1} + \widehat{\pi}_{t+1}) \quad (3.30)$$

$$\widehat{m}_t = \widehat{m}_{t-1} + \xi_1 \widehat{i}_t + \xi_2 \widehat{y}_t - \widehat{\pi}_t + \varphi_t \quad (3.31)$$

$$\widehat{c}_t = \widehat{m}_t \quad (3.32)$$

The above log-linearized dynamic general equilibrium system is exactly the same

as in Walsh (2003), except equation (3.31), where the two elasticity terms are not included. Note that, in Walsh's (2003) model, we have ten endogenous variables ( $k_t$ ,  $m_t$ ,  $y_t$ ,  $c_t$ ,  $x_t$ ,  $h_t$ ,  $\lambda_t$ ,  $i_t$ ,  $r_t$ ,  $\pi_t$ ) and two exogenous variables ( $z_t$  and  $\varphi_t$ ). This means that the above ten equations derived based on necessary first-order conditions and various related equations are good enough to solve this dynamic general equilibrium system.

However, in order to achieve our main goal, we have to put either equation (3.11) or (3.12) into the above log-linearized dynamic general equilibrium system. Since the number of variables has to be equal to the number of equations, this means that we have to eliminate one equation out of the system. In this paper, this task has been done based on the relationship among the Taylor rule, the Fisher equation and the Euler equation. Justification is based on the role of equation (3.27), which regulates the evolvement of the real interest rate, and (3.29), which allows an interaction among interest rates and the expected inflation rate, in the original model. Without the Taylor rule, the gross real interest rate is determined by equation (3.27). The inflation rate is determined by (3.30). So, the nominal interest rate is subsequently determined by equation (3.29). However, if the Fed follows the Taylor rule strictly, and if we assume that the Fed always achieves its target nominal interest rate, then, the sequence for interest rate determination goes the opposite way. That is, the inflation rate is determined first by (3.29), and the nominal interest rate is determined by either equation (3.11) or (3.12). Although the gross real interest rate is still determined by the equation (3.27) basically, it should interact with the inflation rate and the nominal interest rate in the Fisher equation, subsequently.

Notice that there are many ways to eliminate one equation from the system because we want to derive two equations out of three equations. Three different ma-

nipulations have been tried and, as will be shown later, the third way of manipulations has been adopted to study the dynamic properties of our models.

The first manipulation has been done as follows: Note that, in the case of Model 2, the Fisher equation (3.29) can be written as

$$0 = \widehat{i}_t - \omega_3 E_t \widehat{\pi}_{t+1} + \omega_3 E_t \widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1} + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t.$$

If the Fed always achieves its goal as below, then,

$$0 = \widehat{i}_t - \omega_3 E_t \widehat{\pi}_{t+1}, \quad (3.33)$$

the below equation should be the case so that the Fisher equation holds

$$0 = (\omega_3 - 1) E_t \widehat{\pi}_{t+1} + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t.$$

Then, using the Euler equation (3.27), we can rewrite the above equation as

$$0 = (\omega_3 - 1) E_t \widehat{\pi}_{t+1} - \widehat{r}_t. \quad (3.34)$$

Here, equation (3.34) is, in fact, a modified Fisher equation incorporating the Euler equation. After all, we can replace equations (3.33) and (3.34) with equations (3.27) and (3.29) in the system. Unfortunately, however, our model economies appear to be too volatile under this manipulation. It turns out that the volatility comes from the fact that we did, in effect, eliminate the Euler equation from the system. It is understood that, since the Euler equation plays a key role in consumption-smoothing, the first manipulation actually eliminates a smooth adjustment of the whole system to a shock. This result implies that we need to keep the Euler equation as it is when we study the dynamic properties of our models.

Secondly, the Fisher equation is simply replaced with the Taylor rule while



keeping the Euler equation as it is. Under this manipulation, however, our model economies appear to be divergent. This might be natural because, without the Fisher equation, the gross real interest rate and nominal interest rate are determined separately without interacting with each other. Note that, in this manipulation, the gross real interest rate is determined only by equation (3.27) while the nominal interest rate is determined only by either equation (3.11) or (3.12). This result implies that we need to put the Taylor rule into the system while keeping the role of the Fisher equation.

Therefore, the third manipulation has been done as follows: Note that, in the case of Model 1, the Taylor rule can be written as

$$E_t \widehat{\pi}_{t+1} = \frac{1}{\omega_1} \widehat{i}_t - \frac{\omega_2}{\omega_1} \widehat{y}_t.$$

Putting the above equation into the Fisher equation, we have

$$\begin{aligned} 0 &= \widehat{i}_t - E_t \widehat{\pi}_{t+1} + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t \\ &= \widehat{i}_t - \left( \frac{1}{\omega_1} \widehat{i}_t - \frac{\omega_2}{\omega_1} \widehat{y}_t \right) + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t. \end{aligned}$$

Hence, finally, we have the following modified Fisher equation

$$0 = \left( \frac{\omega_1 - 1}{\omega_1} \right) \widehat{i}_t + \frac{\omega_2}{\omega_1} \widehat{y}_t + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t. \quad (3.35)$$

Note that, as far as the Fed strictly follows the Taylor rule and it always achieves its target nominal interest rate, then, the above result should be the case. Also, note that this manipulation does not affect any other variables in the system. In order to study the dynamic properties, the original Fisher equation (3.29) has been replaced with equation (3.35). As will be shown later, under this manipulation, our model economies behave quite similar to what is observed in the U.S. data. From now on,

we are going to investigate the dynamic properties of our model economies based on this manipulation.

In solving the above dynamic general equilibrium system, we will apply the solution method suggested by Uhlig (1999). A detailed procedure to apply the Uhlig method is presented in the Appendix A (Log-linearization) and Appendix B (Solution Method of Uhlig (1999)).

### 2.3. Calibration

The parameters used to study dynamic properties are listed in Table 3.

Table 3. Calibrated Parameters

Parameter	$\theta$	$\delta$	$\beta$	$\Phi$	$\psi$	$\bar{\mu}$	$\sigma_\varepsilon$	$\sigma_\epsilon$	$\xi_1$	$\xi_2$
Value	.36	.019	.989	2	1	.015	.007	.0089	-.05	.5

The first five parameter values are based on Walsh (2003). The next three parameter values are taken from Cooley and Hansen (1995). The two parameter values on interest rate and income elasticity of real money balance came from Ball (2001). These parameters imply the steady state values in Table 4.

Table 4. Steady State Values

Parameter	$\tilde{R}$	$\tilde{y}/\tilde{k}$	$\tilde{c}/\tilde{k}$	$\tilde{h}/\tilde{k}$	$\tilde{I}$	$\tilde{\Pi}$
Value	1.011	0.084	0.065	0.021	1.026	1.015

### 3. Simulation Results

Simulations have been done under three different situations: technology shocks with exogenous money, technology shocks with endogenous money, and money shocks with endogenous money. The simulation results are to be compared with the U.S. data based on Cooley and Hansen (1995), and the simulation results of Walsh (2003) and Cooley and Hansen (1995), which are shown in Table 5.

Table 5. Comparison of Simulation Results When  $\rho = .95$

	Data		CH (1995)		Walsh (2003)	
	s.d.	Corr.	s.d.	Corr.	s.d.	Corr.
Output	1.72	1.00	1.69	1.00	1.033	1.00
Consumption	0.86	.77	0.42	.85	0.385	.97
Hours	1.59	.86	1.35	.99	0.221	.86
Investment	8.24	.91	5.83	.99	3.286	1.00
Inflation	0.57	.34	0.26	-.58	0.902	-.14
Real Rate	-	-	-	-	0.014	.85
Nominal Rate	0.66	.10	0.39	-.02	0.006	.83

The first simulation is just done to highlight the difference in results between when the real money balance is determined exogenously and when it is determined endogenously. As will be shown later, since our model economies behave similar to what is observed in the U.S. data with endogenous money, the third simulation with money shocks is done only with endogenous money. The two first simulations have been taken with  $\rho = .95$ . The simulation under money shocks with endogenous money has been done with  $\eta = .50$  following Cooley and Hansen (1995) and Walsh (2003).

### 3.1. Technology Shocks with Exogenous Money

We begin our simulations with technology shocks assuming the real money balance is determined exogenously. Technically, this means that we use the following equation rather than (3.31)

$$\widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t + \varphi_t.$$

Other features of our models remain unchanged. The simulation results of our models are shown in Table 6.

Table 6. Simulation with Technology Shocks and Exogenous Money

	Model 1		Model 2	
	s.d.	Corr.	s.d.	Corr.
Output	1.0302	1.00	1.0302	1.00
Consumption	0.3821	0.97	0.3821	0.97
Hours	0.2173	0.86	0.2173	0.86
Investment	3.2828	1.00	3.2828	1.00
Inflation	0.2743	-0.46	0.2743	-0.46
Real Rate	0.0142	0.85	0.0142	0.85
Nominal Rate	0.9943	-1.00	0.0615	0.85

Regarding these results, there are two points we need to note. First, both simulation results are quite similar to that in Walsh (2003). However, in the case of Model 1, the correlation between the nominal interest rate and output appears to be negative. Another thing is that the standard deviation of the nominal interest rate is substantially increased in both cases compared to Walsh (2003) although the

standard deviation of almost all variables appear to be less than what we observe in the U.S. data.

Second, in the case of Model 2, the fluctuation of the nominal interest rate is bigger than that of the real interest rate, while both of them are smaller than in U.S. data. Furthermore, the magnitude of correlation value in the nominal interest rate is the same as in the real interest rate. In the case of model 1, we do not have this result. These are, in fact, well expected from the beginning. Note that, in the case of Model 2, equation (3.35) is expressed as

$$0 = \left( \frac{\omega_3 - 1}{\omega_3} \right) \widehat{i}_t + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t,$$

which is equal to the below equation considering equation (3.27)

$$0 = \left( \frac{\omega_3 - 1}{\omega_3} \right) \widehat{i}_t - \widehat{r}_t. \quad (3.36)$$

By this equation, the relationship between the nominal interest rate and the real interest rate is tightly linked. Note that the Taylor rule is behind this tight linkage. Since the expected inflation rate in the Taylor rule is determined prior to interest rates, the magnitude of correlation value in both interest rates should be the same. Also, note that, in the case of model 2, we set up the parameter value of  $\omega_3$  as 1.3. This brings two effects: First, when the expected inflation rate varies, the nominal interest rate fluctuates more than the real interest rate; Second, the sign of correlation value in both interest rates should be the same. Note that, in the case of Model 2, if we had set up the parameter value of  $\omega_3$  less than unity, the two interest rates could not have had the same sign in terms of correlation with output.

Impulse responses are shown in Figure 1 and Figure 2. As we saw in Table 6, impulse responses of all variables except the nominal interest rate are the same in

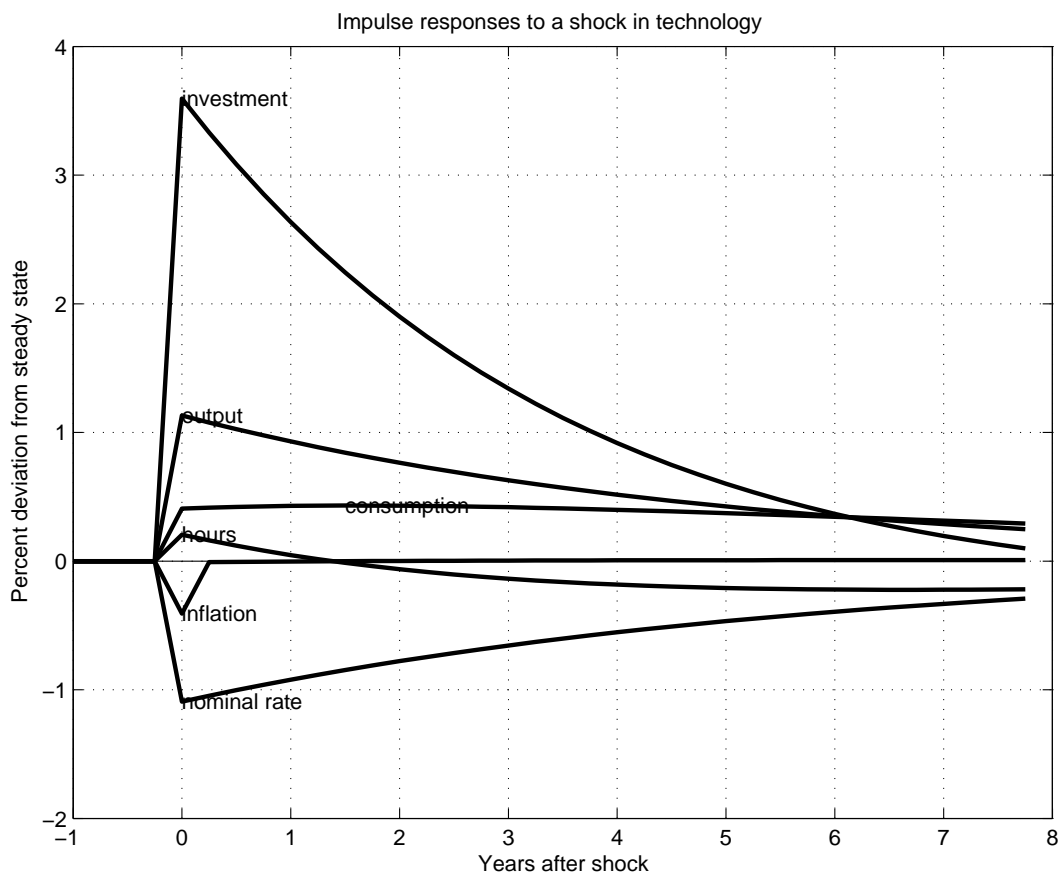


Figure 1. Impulse Responses to Technology Shocks in Model 1 with Exogenous Money

both cases. One notable thing is that the impulse responses are quite persistent. This result is, in fact, also similar to that in Walsh (2003).

### 3.2. Technology Shocks with Endogenous Money

Next, assuming the real money balance is determined endogenously, a simulation with technology shocks has been done. Table 7 shows the simulation results.

Regarding these results, the most important thing is that the correlation between inflation and output appears to be positive in both cases. Furthermore, in the case of

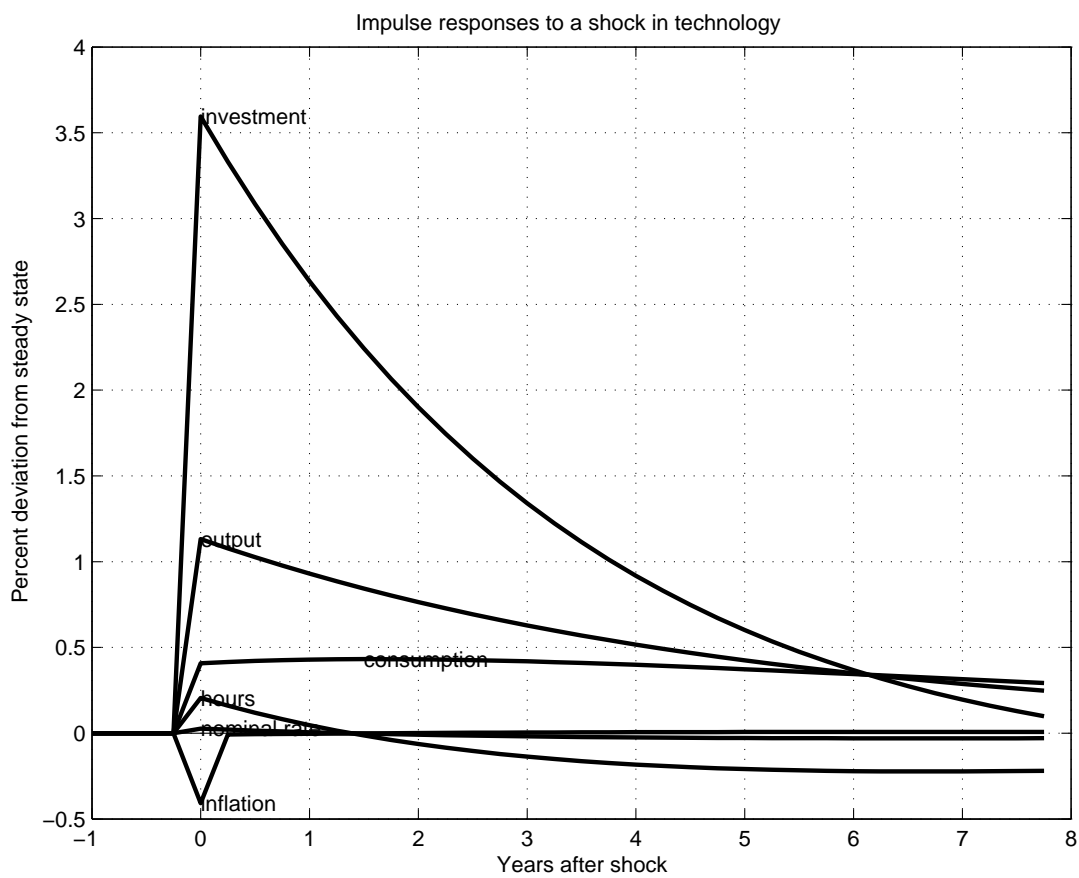


Figure 2. Impulse Responses to Technology Shocks in Model 2 with Exogenous Money

Table 7. Simulation with Technology Shocks and Endogenous Money

	Model 1		Model 2	
	s.d.	Corr.	s.d.	Corr.
Output	0.9345	1.00	0.9423	1.00
Consumption	0.2083	0.90	0.2224	0.91
Hours	0.1254	0.29	0.1290	0.38
Investment	3.4922	1.00	3.4725	1.00
Inflation	0.4542	0.97	0.4114	0.95
Real Rate	0.0121	0.75	0.0123	0.76
Nominal Rate	0.9074	-1.00	0.0533	0.76

Model 2, all correlation terms appear to be positive like the U.S. data. It turns out that this result comes from the fact that we assumed an endogenous money process in our model.

Technically, this result of positive correlation between inflation and output can be understood in two ways. First, note that, from equations (3.30) and (3.31), we have

$$\begin{aligned}\widehat{\lambda}_t &= -E_t(\Phi\widehat{m}_{t+1} + \widehat{\pi}_{t+1}) \\ &= -\Phi E_t(\widehat{m}_t + \xi_1\widehat{i}_{t+1} + \xi_2\widehat{y}_{t+1} + \varphi_{t+1}) - (1 - \Phi) E_t(\widehat{\pi}_{t+1}).\end{aligned}$$

Any variation in the expected future real money balance holding will bring an adjustment to the expected inflation rate. Also, note that we assumed the parameter value of  $\Phi$  to be 2. Hence, given  $\widehat{\lambda}_t$ , as far as  $E_t(\xi_2\widehat{y}_{t+1})$  is positive after a technology shock,  $E_t(\widehat{\pi}_{t+1})$  should be positive as well, *i.e.*, we should have a positive correlation



between expected output (or future real money balance holding) and the expected inflation rate in period  $(t + 1)$ . Second, note that, in Walsh (2003), the evolution of the real money balance is assumed to be

$$\widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t + \varphi_t, \quad (3.37)$$

and, in our models,

$$\widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t + \xi_1 \widehat{i}_t + \xi_2 \widehat{y}_t + \varphi_t. \quad (3.38)$$

Note that, considering  $\widehat{c}_t = \widehat{m}_t$ , in Walsh (2003), there should be a negative correlation between consumption and inflation in period  $t$ . Hence, whenever consumption (or production) increases, the inflation rate decreases, and vice versa. This means that, as far as the correlation between output and consumption is positive, there is a negative correlation between output and inflation in the current period. However, in our models, the correlation between output and inflation (or output and consumption) has to be positive after a technology shock in period  $t$ . That is, if output increases after a technology shock, then, inflation and consumption should increase as well.

Economic intuition of this finding has a significant meaning in the literature. According to Cooley and Hansen (1989, 1991), higher inflation increases the demand for leisure because inflation represents a tax on the purchases of goods. One effect of inflation, then, is to reduce the supply of labor. This then reduces output, consumption, investment, and the steady state capital stock. Therefore, in Walsh (2003), we need a decrease in inflation in order to have an increase in labor supply (hours worked) and, thus, output and consumption.

However, in our models, labor supply increases after a technology shock although inflation increases as well. This result came from the fact that the technology shock brought an increase in labor productivity. Even if the representative agent under-

stands that he needs to pay some inflation tax, he is still willing to increase labor supply if he can increase his consumption due to increased productivity. In other words, although there does exist the *partial* negative correlation between inflation and labor supply (and, thus, between inflation and output) as Cooley and Hansen (1989, 1991) indicated, if the positive correlation between consumption (or output) and labor supply is large enough, then, the *overall* resulting correlation between consumption (or output) and inflation could appear to be positive. In fact, the simulation results in our models imply that the positive effect of increased output on consumption offsets enough the negative effect of increased inflation on consumption, resulting in a positive correlation between consumption (or output) and inflation.

There is one more thing we need to note: the correlation values of the nominal interest rate, the real interest rate and the inflation rate are bigger than in the U.S. data while the correlation value of hours worked is smaller. This might imply that the speed of price adjustment in our model economies is much faster than in the real economy. If the speed of price adjustment were significantly slow in our model economies, *i.e.*, the correlation value of the inflation rate were lower than what we saw in Table 7, then, the correlation values of the nominal interest rate and the real interest rate might also be lower, which could lead a higher correlation value of hours worked.

Impulse responses are shown in Figure 3 and Figure 4. One thing we need to note is that the impulse response of hours worked in both cases shows that hours worked quickly decreases after the shock. This would imply the negative effect of increased inflation discussed above. However, it turns out that the response of hours worked clearly depends on the degree of persistence in the technology shocks. In fact, under  $\rho = .90$ , the impulse response of hours worked significantly shifts up so

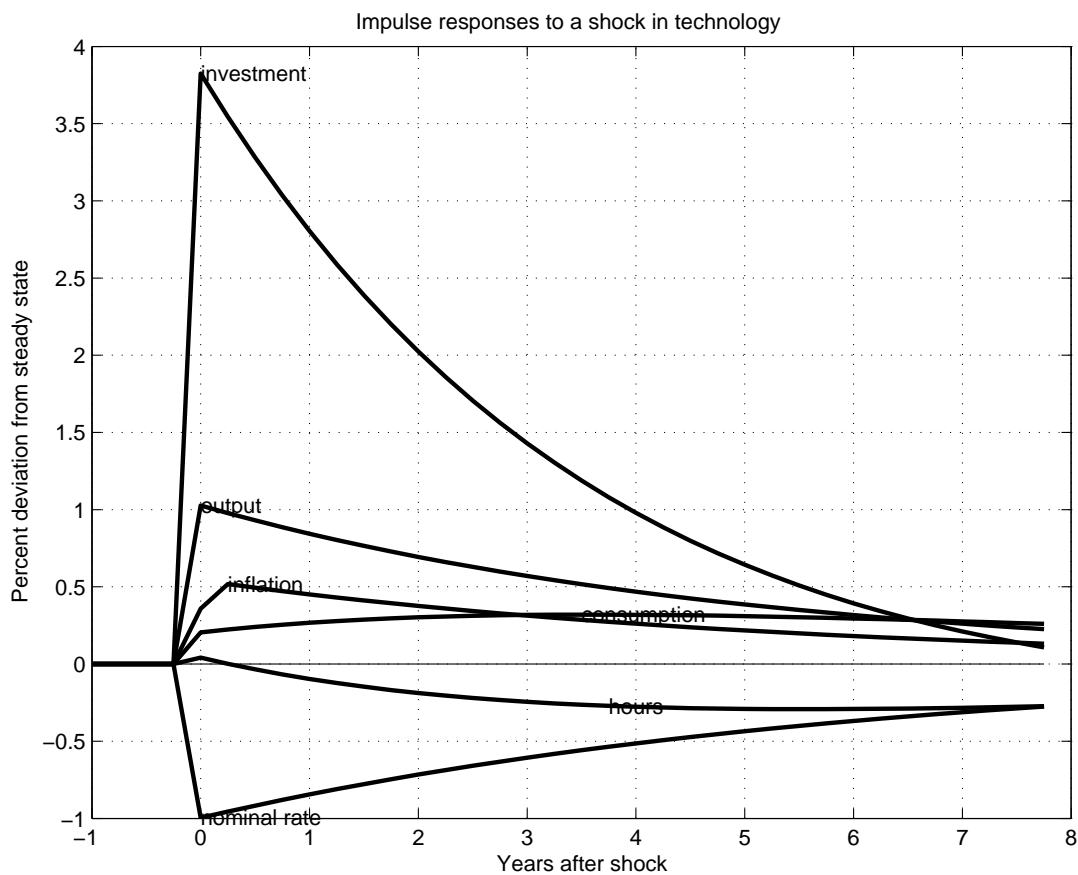


Figure 3. Impulse Responses to Technology Shocks in Model 1 with Endogenous Money

that the variable goes down slightly below the steady state one and a half years later after the shock occurred, while the impulse responses of other variables remain almost unchanged compared to when  $\rho = .95$ .

### 3.3. Money Shocks with Endogenous Money

Here, a simulation with money shocks and with endogenous money is done. Money shocks are given to our model economies in the form of changing the money

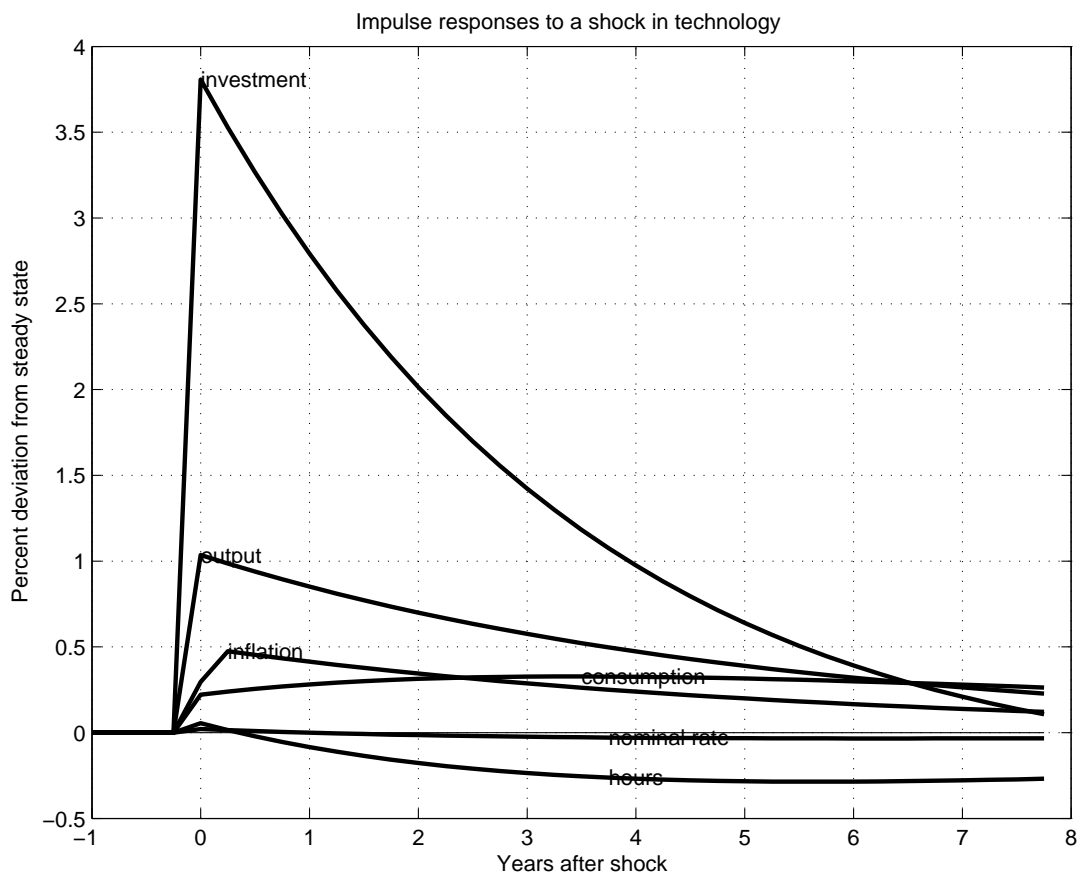


Figure 4. Impulse Responses to Technology Shocks in Model 2 with Endogenous Money

growth rate as shown in equation (3.31). In this simulation, the key point lies on whether our model can reproduce the so-called liquidity effect rather than matching the U.S. business cycle data.

Note that, although the Fed strictly follows the Taylor rule, a monetary policy shock is given in our models in the form of changing the money growth rate, not in the form of changing the nominal interest rate directly. This is based on the assumption that the Fed needs to change the nominal money supply if it wants to change the nominal interest rate. While addressing the price-level indeterminacy problem which

is often noted as a potential problem with an interest rate targeting policy, McCallum (1986) indicated that the nominal money supply is the actual instrument to control the nominal interest rate. Also note that, in this set-up, if the Fed wants to boost the economy, the Fed injects some nominal money into the economy while keeping the Taylor rule strictly<sup>1</sup>.

Impulse responses are shown in Figure 5 and Figure 6. Since Model 2 generates a better result, the simulation has been done only based on Model 2. The responses suggest the possibility of an existing liquidity effect. All variables of inflation, investment, real and nominal interest rates show correct movements as the liquidity effect would suggest.

In Walsh (2003), the nominal interest rate goes up rather than down after money shocks. It is argued that the main effect of money shocks is to increase the expected inflation rate and raise the nominal interest rate because prices have been assumed to be perfectly flexible. However, Model 2 which is also under a perfectly flexible-

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<sup>1</sup>This behavior of the Fed does not contradict. For example, Suh (2004) showed that, if aggregate output is given by a Lucas-type aggregate supply function, if the Fisher equation holds, and if the Fed's objective is to minimize the following Barro-Gordon type loss function at every period  $t$ , where  $a > 0$  and  $b > 0$ ,

$$\min_{i_t} L_t = a [\pi_t(i_t) - \tilde{\pi}]^2 + b [y_t - \tilde{y}]^2,$$

then, the optimal reaction function of the Fed is expressed as the following modified Taylor rule, where  $\omega > 0$ ,

$$\hat{i}_t = \omega(\hat{\pi}_t).$$

This means that the Taylor rule is determined in the process of minimizing the given loss function. Hence, the Fed will need no monetary shocks if both  $\pi_t$  and  $y_t$  stay near the steady state. Only if  $\pi_t$  or  $y_t$  deviates from the steady state, the Fed needs to change the nominal interest rate to affect  $\pi_t$  or  $y_t$ . Therefore, in this sense, a monetary policy shock, which is done through increasing (or decreasing) the nominal money supply to change the nominal interest rate, will be needed if and only if the Taylor rule requires that the Fed do it. In other words, a monetary policy shock, an increase (or decrease) in the nominal money supply, is done to get the Taylor rule held on, but not to defy the Taylor rule.

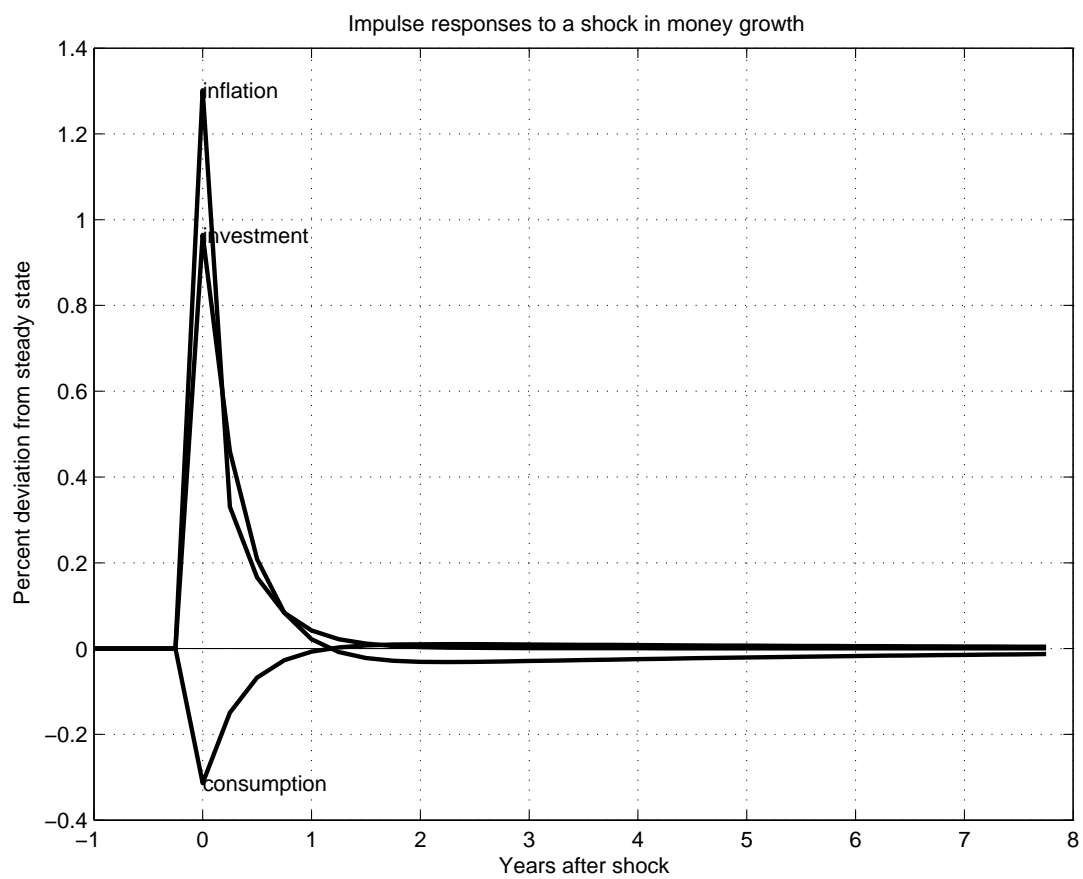


Figure 5. Impulse Responses to Money Shocks in Model 2 (I)

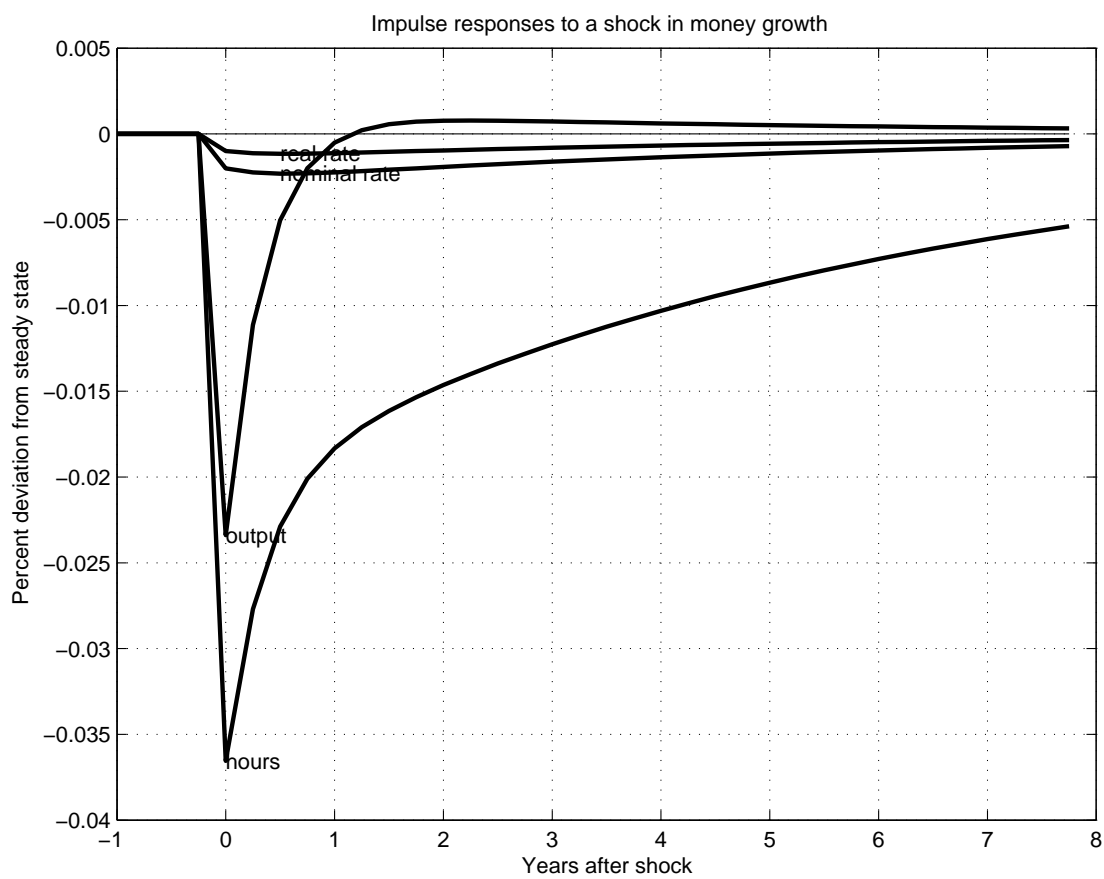


Figure 6. Impulse Responses to Money Shocks in Model 2 (II)

price environment shows a different result. It turns out that this result comes from incorporating the Taylor rule into our model. Note that, by incorporating the Taylor rule, we have equation (3.36) rewritten below

$$0 = \left( \frac{\omega_3 - 1}{\omega_3} \right) \widehat{i}_t - \widehat{r}_t. \quad (3.39)$$

Without the Taylor rule, we would have the original form of the Fisher equation as

$$0 = \widehat{i}_t - E_t \widehat{\pi}_{t+1} - \widehat{r}_t. \quad (3.40)$$

Under equation (3.40), when the expected inflation rate changes after money shocks, the resulting correlation between the nominal interest rate and real interest rate can be either positive or negative depending on the interaction among those three variables. However, under equation (3.39), the correlation between the nominal interest rate and real interest rate should be positive whatever happens in the expected inflation rate. Notice that the real interest rate is basically determined by the Euler equation, prior to the nominal interest rate. Therefore, whenever the real interest rate decreases, the nominal interest rate will also decrease as shown in Figure 6, resulting in a different movement in the nominal interest rate compared to Walsh (2003).

Output and, thus, consumption show, however, incorrect movements. This can be understood by the argument of Cooley and Hansen (1989, 1991), *i.e.*, higher inflation increases the demand for leisure because inflation represents a tax on the purchases of goods. Note that, when there is a technology shock resulting in an increase in productivity, there is a possibility that hours worked will increase if the positive effect of increased output offsets enough the negative effect of the increased inflation rate, as we saw in Figure 3 and Figure 4. However, when there is a money shock without any increases in productivity, there can be only negative effect of



increased inflation under rational expectations. Therefore, as Cooley and Hansen (1989, 1991), this will decrease hours worked and, thus, output shown in Figure 6.

#### 4. Concluding Remarks

In this paper, we examined the effect of a technology shock and a money shock incorporating the Taylor rule as the Fed's monetary policy. One thing significantly different from other researches on this topic is the way the Taylor rule is introduced in the model. In this paper, the Taylor rule is introduced by considering the relationship among the Fisher equation, Euler equation and the Taylor rule explicitly in the dynamic system of the relevant RBC model. With this approach, it has been shown that, even in a flexible-price environment, the correlation between output and inflation can be positive, and the existence of the liquidity effect is possible.

There have not been many papers incorporating the Taylor rule in the context of RBC modelling under a flexible-price environment. Among others, Dittmar *et al.* (2003) and Chen (2003) tried similar approaches to this paper. Dittmar *et al.* (2003) succeeded in showing inflation persistence, but failed in showing the liquidity effect. Chen (2003), after an expansionary monetary policy, showed a rise in the real interest rate and output and a fall in the inflation rate and the nominal interest rate, *i.e.*, correct movements in output and the nominal interest rate but incorrect movements in the real interest rate and the inflation rate.

The difference in simulation results among this paper, Dittmar *et al.* (2003) and Chen (2003) might result from three reasons. First, Dittmar *et al.* (2003) and Chen (2003) used different set-ups of the Taylor rule from this paper. Chen (2003), based

on his own estimation, used different coefficients of  $\omega_1$  and  $\omega_2$  as follows:

$$\widehat{i}_t = 0.4904 * (E_t \widehat{\pi}_{t+1}) + 0.2435 * (\widehat{y}_t).$$

Dittmar *et al.* (2003) assumed that the Fed targets the period  $(t + 1)$  nominal interest rate based on the realized inflation rate and output in period  $t$  as the following set-up shows:

$$\widehat{i}_{t+1} = 1.5 * (\widehat{\pi}_t) + 0.125 * (\widehat{y}_t).$$

Second, as noted earlier, in this paper, the Taylor rule is introduced by considering the relationship among the Fisher equation, Euler equation and the Taylor rule explicitly. Dittmar *et al.* (2003) and Chen (2003) did not look at this relationship in setting up their dynamic models. Third, there is a theoretical difference in analyzing the effect of a monetary policy shock. Chen (2003) gives a monetary policy shock to his model in a different way such that the Fed can change the nominal interest rate directly without any prior changes in the real money balance. Dittmar *et al.* (2003) do not show detailed simulation results of a monetary policy shock while noting that their model does not produce the liquidity effect. Besides, the model of Chen (2003) has a more complicated structure than that of this paper because the model of Chen (2003) includes the government fiscal policy explicitly.

Although our models achieved a mild success in resolving the two problems in RBC models, it does have some limitations as well. Among other things, our models could not produce a correct movement in output when there is a money shock. Also, the standard deviation of almost all variables appear to be less than what we observe in the U.S. data. However, these limitations suggest a future research direction. Note that our models have worked on a very strict environment where the cash-in-advance constraint, the Taylor rule and the Fisher equation are tightly linked to each other.

Therefore, while keeping the basic approach focusing on the relationship between the Euler equation and the Fisher equation, incorporating some other ways modelling a monetary economy, like the shopping time approach, could produce a better result. Similarly, seeking a way by which we can get the tight relationship between the real and nominal interest rates a little loosened, for example, introducing some mechanism which breaks down the Taylor rule and Fisher equation temporarily, could also be a good research topic.

## CHAPTER IV

## SUMMARY

In this dissertation, two questions concerning monetary policy under the Taylor rule have been addressed. In Chapter I, the question is on, under the Taylor rule, whether a central bank should be responsible for both bank supervision and monetary policy. In Chapter II, the question is on whether the Taylor rule plays an important role in explaining modern business cycles in the United States.

The first chapter begins with the observation that, as Peek *et al.* (1999) shows, there is a systematic error in making monetary policy if the central bank does not have any bank supervisory role. Based on this observation, this chapter investigates why there should be a systematic error in making monetary policy. This task has been addressed using a simple game-theoretic approach. In this approach, the most critical assumption is that there is a asymmetric information about state of the economy between the central bank and commercial bank.

The implications of the first chapter can be summarized as follows: (i) it is inevitable for the central bank to have a systematic error in conducting monetary policy when the central bank does not have a bank supervisory role; (ii) without a bank supervisory role, the effectiveness of monetary policy cannot be guaranteed; (iii) because of the existence of *conflict of interests*, giving a bank supervisory role to the central bank does not guarantee the effectiveness of monetary policy, either; (iv) the way of setting up another government agency, bank regulator, and making the central bank and the regulator cooperate each other does not guarantee the effectiveness of monetary policy because, in this way, the systematic error in conducting monetary policy cannot be eliminated; (v) in the view of social welfare, not in the view of

effectiveness of monetary policy, it is better for the central bank to keep the whole responsibility or at least a partial responsibility on bank supervision.

These implications have significant meaning on the current trend surrounding central banking systems around the world. Many countries have been considering whether bank supervisory responsibilities are an important function for the central bank or are better assigned to a separate government agency. The results in the first chapter indicate that the current trend surrounding central banking systems around the world is not on the right track. These implications may be even more important in many other countries, particularly developing countries, since, in these countries, the credit markets are usually bank-centered, and the banking industry structures are often far from perfect competition. Lack of competition in the banking industry will strengthen the market power of commercial banks, and, under this situation, if the central bank does not have any bank supervisory role, a selfish profit maximizing behavior of commercial banks could significantly lower the effectiveness of monetary policy.

In the second chapter, we examined the effect of a technology shock and a money shock in the context of an RBC model incorporating the Taylor rule as the Fed's monetary policy. One thing significantly different from other researches on this topic is the way the Taylor rule is introduced in the model. In this chapter, the Taylor rule is introduced by considering the relationship among the Fisher equation, the Euler equation and the Taylor rule explicitly in the dynamic system of the relevant RBC model. This approach can be justified based on the reason that, if the Fed follows the Taylor rule strictly, and if we assume that the Fed always achieves its target nominal interest rate, then, the nominal interest rate is determined prior to the real interest rate. Without the Taylor rule, the real interest rate is determined by

the Euler equation and the nominal interest rate is subsequently determined by the Fisher equation.

With this approach, it has been shown that, even in a flexible-price environment, the two major failures in RBC models with money can be resolved. Under the Taylor rule, the correlation between output and inflation appears to be positive and the response of our model economy to a shock is persistent. Furthermore, the possibility of an existing liquidity effect is found. These results imply that the Taylor rule does play a key role in explaining business cycles in the United States.

## REFERENCES

- Alesina, A., and Summers, L. (1993), "Central Bank Independence and Macroeconomic Performance: Some Comparative Evidence," *Journal of Money, Credit and Banking*, 25, 151-162.
- Bagehot, W. (1999), *Lombard Street: A Description of the Money Market*, New York: Wiley.
- Ball, L. (1997), "Efficient Rules for Monetary Policy," Working Paper 5952, National Bureau of Economic Research, Cambridge, MA, pp. 1-20.
- (2001), "Another Look at Long-run Money Demand," *Journal of Monetary Economics*, 47, 31-44.
- Barro, R. J., and Gordon, D. B. (1983), "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics*, 12, 101-121.
- Benhabib, J., Schmitt-Grohé, S., and Uribe, M. (2001a), "Monetary Policy and Multiple Equilibria," *American Economic Review*, 91, 167-186.
- (2001b), "The Perils of Taylor Rules," *Journal of Economic Theory*, 96, 40-69.
- (2002a), "Avoiding Liquidity Traps," *Journal of Political Economy*, 110, 535-563.
- (2002b), "Chaotic Interest Rate Rules," *American Economic Review*, 92, 72-78.
- Bernanke, B. S., and Gertler, M. (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," *Journal of Economic Perspectives*, 9, 27-48.

Bini Smaghi, L., and Gros, D. (2000), *Open Issues in European Central Banking*, New York: St. Martin's Press.

Caplin, A., and Leahy, J. (1996), "Monetary Policy as a Process of Search," *American Economic Review*, 86, 689-702.

Chen, S. (2003), "Macroeconomic Policy and Taylor Rule in an Equilibrium Business Cycle Model," unpublished manuscript, University of Wisconsin, Madison, Department of Economics.

Christiano, L. J. (1991), "Modelling the Liquidity Effect of a Money Shock," *Quarterly Review, Federal Reserve Bank of Minneapolis*, 15, 3-34.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (1997), "Sticky Price and Limited Participation Models of Money: A Comparison," *European Economic Review*, 41, 1201-1249.

Christiano, L. J., and Rostagno, M. (2001), "Money Growth Monitoring and the Taylor Rule," Working Paper 8539, National Bureau of Economic Research, Cambridge, MA, pp. 1-64.

Clarida, R., and Gali, J. G. M. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115, 147-180.

Coffee, J. (1995), "Competition versus Consolidation: The Organizational Structure in Financial and Securities Regulation," *Business Lawyer*, 50, 447-484.

Cooley, T. F., and Hansen, G. D. (1989), "The Inflation Tax in a Real Business Cycle Model," *American Economic Review*, 79, 733-748.



——— (1991), “The Welfare Costs of Moderate Inflation,” *Journal of Money, Credit, and Banking*, 23, 483-503.

——— (1995), “Money and the Business Cycle,” in *Frontiers of Business Cycle Research*, ed. T. F. Cooley, Princeton, NJ: Princeton University Press, pp. 175-216.

Cukierman, A. (1992), *Central Bank Strategy, Credibility, and Independence: Theory and Evidence*, Cambridge, MA: MIT Press.

Di Noia, C., and Di Giorgio, G. (1999), “Should Banking Supervision and Monetary Policy Tasks Be Given to Different Agencies?” *International Finance*, 2, 361-378.

Diamond, D. W., and Dybvig, P. H. (1983), “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, 91, 401-419.

Dittmar, R., Gavin, W. T., and Kydland, F. E. (2003), “Inflation Persistence and Flexible Prices,” Working Paper 2001-010D, Federal Reserve Bank of St. Louis, St. Louis, MO, pp. 1-28.

Friedman, B. M. (1993), “The Role of Judgement and Discretion in the Conduct of Monetary Policy: Consequences of Changing Financial Markets,” in *Changing Capital Markets: Implications for Monetary Policy*, Federal Reserve Bank of Kansas City, Kansas City, MO, pp. 151-196.

Fuhrer, J. C. (1997), “Central Bank Independence and Inflation Targeting: Monetary Policy Paradigms for the Next Millennium?” *New England Economic Review*, Jan/Feb 1997, 19-36.

Goodhart, C. (1999), “Central Bankers and Uncertainty,” *Proceedings of the British Academy*, 101, 229-271.

Goodhart, C., and Schoenmaker, D. (1992), "Institutional Separation between Supervisory and Monetary Agencies," *Giornale degli Economisti e Annali di Economia*, 51, 353-439.

——— (1995), "Should the Functions of Monetary Policy and Banking Supervision be Separated?" *Oxford Economic Papers*, 47, 539-560.

Haubrich, J. G. (1996), "Combining Bank Supervision and Monetary Policy," *Economic Commentary, Federal Reserve Bank of Cleveland*, Nov. 1996, 1-5.

Heller, R. (1991), "Prudential Supervision and Monetary Policy," in *International Financial Policy: Essays in Honor of J. Pollak*, eds. J. Frenkel and M. Goldstein, Washington, D.C.: International Monetary Fund, pp. 277-318.

Hubbard, R. G. (1995), "Is There a Credit Channel for Monetary Policy?" *Review, Federal Reserve Bank of St. Louis*, 77, 63-77.

Jordon, J. L. (1996), "The Future of Banking Supervision," *Economic Commentary, Federal Reserve Bank of Cleveland*, April 1996, 1-4.

Kashyap, A. K., and Stein, J. C. (1994a), "Monetary Policy and Bank Lending," in *Monetary Policy*, ed. N. Gregory Mankiw, Chicago: University of Chicago Press, pp. 221-256.

——— (1994b), "The Impact of Monetary Policy on Bank Balance Sheets," Working Paper 4821, National Bureau of Economic Research, Cambridge, MA, pp. 1-61.

Kashyap, A. K., Stein, J. C., and Wilcox, D. W. (1993), "Monetary Policy and Credit Conditions: Evidence from the Composition of External Finance," *American Economic Review*, 83, 78-98.

King, R. G., and Plosser, C. I. (1984), "Money, Credit, and Prices in a Real Business Cycle," *American Economic Review*, 74, 363-380.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988), "Production, Growth, and Business Cycles I: The Basic Neoclassical Model," *Journal of Monetary Economics*, 21, 195-232.

Kydland, F. E., and Prescott, E. C. (1977), "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 85, 473-491.

Lannoo, K. (1999), "Financial Supervision in EMU," in the *Forum of Financial Supervision and Policy Coordination in the EMU*, Brussels: Center for European Integration Studies, pp. 3-27.

McCallum, B. T. (1986), "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine," *Journal of Monetary Economics*, 17, 135-160.

Mishkin, F. S. (2000), "Prudential Supervision: Why Is It Important and What Are the Issues?" Working Paper 7926, National Bureau of Economic Research, Cambridge, MA, pp. 1-36.

Padoa, S. T. (1999), "EMU and Banking Supervision," *International Finance*, 2, 295-308.

Peek, J., Rosengren, E., and Tootell, G. (1999), "Is Bank Supervision Central to Central Banking?" *Quarterly Journal of Economics*, 114, 629-653.

Poole, W. (1970), "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," *Quarterly Journal of Economics*, 84, 197-216.

Posen, A. (1995), "Central Bank Independence and Disinflationary Credibility: A Missing Link?" Staff Report 1, Federal Reserve Bank of New York, pp. 1-41.

Rogoff, K. S. (1989), "Reputation, Coordination, and Monetary Policy," in *Modern Business Cycle Theory*, ed. R. Barro, Cambridge, MA: Harvard University Press, pp. 236-264.

Romer, D. (1996), *Advanced Macroeconomics*, New York: McGraw-Hill Companies, Inc.

——— (2002), *Advanced Macroeconomics* (2nd ed.), New York: McGraw-Hill Companies, Inc.

Rudebusch, G. D., and Svensson, L. E. O. (1999), "Policy Rules for Inflation Targeting," in *Monetary Policy Rules*, ed. J. B. Taylor, NBER Conference Report Series, Chicago: University of Chicago Press, pp. 203-246.

Shull, B. (1993), "How Should Bank Regulatory Agencies Be Organized?" *Contemporary Policy Issues*, 11, 99-107.

Suh, J. E. (2004), "Central Banks Are All Mighty? Don't They Need a Bank Supervisory Role Any More?," unpublished manuscript, Texas A&M University, College Station, Department of Economics.

Taylor, J. B. (1993), "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Tuya, J., and Zamalloa, L. (1994), "Issues on Placing Banking Supervision in the Central Bank," in *Frameworks for Monetary Stability*, eds. T. Balino and C. Cottarelli, Washington, D.C.: International Monetary Fund, pp. 663-690.

Uhlig, Harald (1999), “A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily,” in *Computational Methods for the Study of Dynamic Economies*, ed. A. Scott, Oxford, U.K.: Oxford University Press, pp. 30-61.

Walsh, C. E. (2003), *Monetary Theory and Policy* (2nd ed.), Cambridge, MA: MIT Press.

## APPENDIX A

## LOG-LINEARIZATION

## Non-stochastic Balanced Growth Path

Note that, in our production function,  $y_t = e^{z_t} k_{t-1}^\theta h_t^{1-\theta}$ , there is no term representing permanent technological variations, which is usually restricted to be in labor productivity, *i.e.*, we have no labor-augmenting technology term. Hence, in a steady state, all real variables should show no growth. Since variations in  $z_t$  is assumed to be temporary, we can ignore it for our investigation of steady state growth.

Hence, we have

- From (3.18),

$$\Psi (1 - \tilde{h})^{-\psi} = (1 - \theta) \left( \frac{\tilde{y}}{\tilde{k}} \right) \left( \frac{\tilde{k}}{\tilde{h}} \right) \tilde{\lambda}$$

Hence, by (A.2) and (A.5),

$$(1 - \tilde{h})^{-\psi} (\tilde{h})^\Phi = \left( \frac{\beta}{\tilde{\Pi}} \right) \left( \frac{1 - \theta}{\Psi} \right) \left( \frac{\tilde{y}}{\tilde{k}} \right)^{\frac{\Phi - \theta}{1 - \theta}} \left( \frac{\tilde{c}}{\tilde{k}} \right)^{-\Phi} \quad (\text{A.1})$$

- From (3.20),

$$\tilde{\lambda} = \beta \left( \frac{\tilde{c}^{-\Phi}}{\tilde{\Pi}} \right) \quad (\text{A.2})$$

- From (3.17),

$$\tilde{\lambda} = \beta \tilde{R} \tilde{\lambda}$$

Hence, we have

$$\tilde{R} = \tilde{f}_k + 1 - \delta = \frac{1}{\beta} \quad (\text{A.3})$$

- From (3.16) and (A.3),

$$\tilde{R} = \theta \frac{\tilde{y}}{\tilde{k}} + 1 - \delta$$

Hence,

$$\frac{\tilde{y}}{\tilde{k}} = \frac{1}{\theta} (\tilde{R} - 1 + \delta) \quad (\text{A.4})$$

- From (3.13),

$$\tilde{y} = \tilde{k}^\theta \tilde{h}^{1-\theta}$$

Therefore, we have

$$\frac{\tilde{h}}{\tilde{k}} = \left( \frac{\tilde{y}}{\tilde{k}} \right)^{\frac{1}{1-\theta}} \quad (\text{A.5})$$

- From (3.15),

$$\tilde{k} = (1 - \delta)\tilde{k} + \tilde{x}$$

Therefore, we have

$$\frac{\tilde{x}}{\tilde{k}} = \delta \quad (\text{A.6})$$

- From (3.22),

$$\tilde{c} = \tilde{m} \quad (\text{A.7})$$

- From (3.14),

$$\tilde{y} = \tilde{c} + \tilde{x}$$

Therefore, by (A.6), we have

$$\frac{\tilde{c}}{\tilde{k}} = \frac{\tilde{y}}{\tilde{k}} - \delta \quad (\text{A.8})$$

- From (3.21),

$$\tilde{m} = \frac{e^{\bar{\mu}} \tilde{m}}{\tilde{\Pi}}$$

Therefore,

$$\tilde{\Pi} = e^{\bar{\mu}} \quad (\text{A.9})$$

## Log-linearization

Denoting  $\widehat{a}_t = \ln a_t - \ln \widetilde{a}$ , and defining  $\widehat{i}_t = I_t - \widetilde{I}$ ,  $\widehat{r}_t = R_t - \widetilde{R}$  and  $\widehat{\pi}_t = \Pi_t - \widetilde{\Pi} = \pi_t - \widetilde{\pi}$ , we have

- Log-linearization of (3.20):

$$\lambda_t = \beta E_t \left( \frac{m_{t+1}^{-\Phi}}{\Pi_{t+1}} \right)$$

Hence,

$$\left[ \begin{array}{l} \lambda_t = \beta E_t \left( \frac{m_{t+1}^{-\Phi}}{\Pi_{t+1}} \right) \\ \lambda_t = \beta E_t \left( \frac{m_{t+1}^{-\Phi}}{1 + \pi_{t+1}} \right) \\ \widetilde{\lambda} \left( 1 + \widehat{\lambda}_t \right) = \beta E_t \left( \frac{\widetilde{m}_{t+1}^{-\Phi}}{\widetilde{\Pi}} \right) \left( \frac{1 - \Phi \widehat{m}_{t+1}}{1 + \pi_{t+1}} \right) \\ \left( 1 + \widehat{\lambda}_t \right) \approx E_t \left( 1 - \Phi \widehat{m}_{t+1} - \widehat{\pi}_{t+1} \right) \end{array} \right]$$

Therefore, approximately, we have

$$\widehat{\lambda}_t = -E_t \left( \Phi \widehat{m}_{t+1} + \widehat{\pi}_{t+1} \right) \tag{A.10}$$

- Log-linearization of (3.18):

$$\Psi (1 - h_t)^{-\psi} = (1 - \theta) \left( \frac{y_t}{h_t} \right) \lambda_t$$

Defining  $l_t = 1 - h_t$ , we have

$$\Psi l_t^{-\psi} = (1 - \theta) \left( \frac{y_t}{1 - l_t} \right) \lambda_t$$



Noting that  $\tilde{l}(1 + \widehat{l}_t) = 1 - \tilde{h}(1 + \widehat{h}_t)$ , hence,  $\widehat{l}_t = -\left(\frac{\tilde{h}}{\tilde{l}}\right)\widehat{h}_t$ , we have

$$\left[ \begin{array}{l} \Psi\tilde{l}^{-\psi}\left(1 - \psi\widehat{l}_t\right) = (1 - \theta)\left(\frac{\tilde{y}\tilde{\lambda}}{1 - \tilde{l}}\right)\left(\frac{1 + \widehat{y}_t}{1 + \widehat{h}_t}\right)\left(1 + \widehat{\lambda}_t\right) \\ \left(1 + \psi\left(\frac{\tilde{h}}{\tilde{l}}\right)\widehat{h}_t\right) = \left(\frac{1 + \widehat{y}_t}{1 + \widehat{h}_t}\right)\left(1 + \widehat{\lambda}_t\right) \\ \left(1 + \psi\left(\frac{\tilde{h}}{\tilde{l}}\right)\widehat{h}_t\right) = \left(1 + \widehat{y}_t - \widehat{h}_t + \widehat{\lambda}_t\right) \\ \left(1 + \psi\left(\frac{\tilde{h}}{\tilde{l}}\right)\right)\widehat{h}_t = \widehat{y}_t + \widehat{\lambda}_t \end{array} \right]$$

Hence, we have

$$\left(1 + \psi\left(\frac{\tilde{h}}{1 - \tilde{h}}\right)\right)\widehat{h}_t = \widehat{y}_t + \widehat{\lambda}_t \quad (\text{A.11})$$

- Log-linearization of (3.17):

$$\left[ \begin{array}{l} \lambda_t = \beta E_t R_t \lambda_{t+1} \\ \tilde{\lambda}\left(1 + \widehat{\lambda}_t\right) = \beta \tilde{R} \tilde{\lambda} E_t (1 + r_t) \left(1 + \widehat{\lambda}_{t+1}\right) \\ \left(1 + \widehat{\lambda}_t\right) \approx E_t \left(1 + \widehat{r}_t + \widehat{\lambda}_{t+1}\right) \end{array} \right]$$

Hence,

$$\widehat{\lambda}_t = \widehat{r}_t + E_t \widehat{\lambda}_{t+1} \quad (\text{A.12})$$

- Log-linearization of (3.19):

$$\left[ \begin{array}{l} \lambda_t = \beta E_t \left(\frac{1 + i_t}{\Pi_{t+1}}\right) \lambda_{t+1} \\ \lambda_t = \beta E_t \left(\frac{1 + i_t}{1 + \pi_{t+1}}\right) \lambda_{t+1} \\ \tilde{\lambda}\left(1 + \widehat{\lambda}_t\right) = \beta E_t \left(\frac{\tilde{l}}{\Pi}\right) \left(\frac{1 + i_t}{1 + \pi_{t+1}}\right) \tilde{\lambda}\left(1 + \widehat{\lambda}_{t+1}\right) \\ \left(1 + \widehat{\lambda}_t\right) = E_t \left(\frac{1 + i_t}{1 + \pi_{t+1}}\right) \left(1 + \widehat{\lambda}_{t+1}\right) \\ \left(1 + \widehat{\lambda}_t\right) \approx E_t \left(1 + \widehat{i}_t - \widehat{\pi}_{t+1} + \widehat{\lambda}_{t+1}\right) \end{array} \right]$$

Hence,

$$\widehat{\lambda}_t = \widehat{i}_t - E_t \widehat{\pi}_{t+1} + E_t \widehat{\lambda}_{t+1} \quad (\text{A.13})$$

- Log-linearization of (3.16):

$$\left[ \begin{array}{l} R_t = \theta \frac{E_t y_{t+1}}{k_t} + 1 - \delta \\ R_t = \theta \left( \frac{\tilde{y}}{\tilde{k}} \right) \left( \frac{1 + E_t \widehat{y}_{t+1}}{1 + \widehat{k}_t} \right) + 1 - \delta \end{array} \right]$$

Therefore,

$$\widehat{r}_t = \theta \left( \frac{\tilde{y}}{\tilde{k}} \right) \left( E_t \widehat{y}_{t+1} - \widehat{k}_t \right) \quad (\text{A.14})$$

In order to apply the Uhlig method, we need to eliminate expectations from equation (A.14). Using (A.11) and (A.16) to eliminate  $\widehat{h}_t$  between them, we obtain an expression for  $\widehat{y}_t$  such that

$$\widehat{y}_t = \frac{\left( 1 + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right) \right) \left( \theta \widehat{k}_{t-1} + z_t \right) + (1 - \theta) \widehat{\lambda}_t}{\theta + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right)}$$

Advancing this one period, taking expectations, and using (A.12) and (3.9), we have

$$E_t \widehat{y}_{t+1} = \frac{\left( 1 + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right) \right) \left( \theta \widehat{k}_t + \rho z_t \right) + (1 - \theta) \left( \widehat{\lambda}_t - \widehat{r}_t \right)}{\theta + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right)}$$

So, (A.14) can be expressed as

$$\widehat{r}_t = \theta \left( \frac{\tilde{y}}{\tilde{k}} \right) \left( \frac{\left( 1 + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right) \right) \left( \theta \widehat{k}_t + \rho z_t \right) + (1 - \theta) \left( \widehat{\lambda}_t - \widehat{r}_t \right)}{\theta + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right)} - \widehat{k}_t \right)$$

Finally, we have

$$\widehat{r}_t = \theta \left( \frac{\tilde{y}}{\tilde{k}} \right) \left( \frac{(\theta - 1) \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right) \widehat{k}_t + \left( 1 + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right) \right) \rho z_t + (1 - \theta) \widehat{\lambda}_t}{\left( \theta + \psi \left( \frac{\tilde{h}}{1-\tilde{h}} \right) \right) + \theta (1 - \theta) \left( \frac{\tilde{y}}{\tilde{k}} \right)} \right) \quad (\text{A.15})$$

- Log-linearization of (3.13):

$$\left[ \begin{array}{l} y_t = e^{z_t} k_{t-1}^\theta h_t^{1-\theta} \\ \log y_t = z_t + \theta \log k_{t-1} + (1 - \theta) \log h_t \end{array} \right]$$

Noting  $\log \tilde{y} = \tilde{z} + \theta \log \tilde{k} + (1 - \theta) \log \tilde{h}$  and  $\tilde{z} = 0$ , we have

$$\hat{y} = z_t + \theta \widehat{k_{t-1}} + (1 - \theta) \hat{h}_t$$

Hence,

$$\hat{y}_t = z_t + \theta \widehat{k_{t-1}} + (1 - \theta) \hat{h}_t \quad (\text{A.16})$$

- Log-linearization of (3.15):

$$\begin{bmatrix} k_t = (1 - \delta)k_{t-1} + x_t \\ \tilde{k}e^{\hat{k}_t} = (1 - \delta)\tilde{k}e^{\widehat{k_{t-1}}} + \tilde{x}e^{\hat{x}_t} \end{bmatrix}$$

Using the property that  $e^x \approx (1 + x)$ , we have

$$\begin{bmatrix} \tilde{k} + \tilde{k}\hat{k}_t = (1 - \delta)\tilde{k} + (1 - \delta)\tilde{k}\widehat{k_{t-1}} + \tilde{x} + \tilde{x}\hat{x}_t \\ \tilde{k}\hat{k}_t = (1 - \delta)\tilde{k}\widehat{k_{t-1}} + \tilde{x}\hat{x}_t \\ \hat{k}_t = (1 - \delta)\widehat{k_{t-1}} + \frac{\tilde{x}}{\tilde{k}}\hat{x}_t \end{bmatrix}$$

Hence, by (A.6), we have

$$\hat{k}_t = (1 - \delta)\widehat{k_{t-1}} + \delta\hat{x}_t \quad (\text{A.17})$$

- Log-linearization of (3.22):

$$\begin{bmatrix} c_t = m_t \\ \tilde{c}(1 + \hat{c}_t) = \tilde{m}(1 + \widehat{m_t}) \end{bmatrix}$$

Hence,

$$\hat{c}_t = \widehat{m_t} \quad (\text{A.18})$$

- Log-linearization of (3.14):

$$\left[ \begin{array}{c} y_t = c_t + x_t \\ \tilde{y}(1 + \widehat{y}_t) = \tilde{c}(1 + \widehat{c}_t) + \tilde{x}(1 + \widehat{x}_t) \end{array} \right]$$

Hence, from (A.18) and (A.6), we have

$$\frac{\tilde{y}}{\tilde{k}} \widehat{y}_t = \frac{\tilde{c}}{\tilde{k}} \widehat{m}_t + \delta \widehat{x}_t \quad (\text{A.19})$$

- Log-linearization of (3.21):

$$\left[ \begin{array}{c} m_t = \frac{e^{\mu_t} m_{t-1}}{\Pi_t} = \frac{(1+\mu_t)m_{t-1}}{(1+\pi_t)} \\ \tilde{m}(1 + \widehat{m}_t) = \frac{e^{\bar{\mu}}(1+u_t)\tilde{m}(1+\widehat{m}_{t-1})}{\bar{\Pi}(1+\widehat{\pi}_t)} \\ (1 + \widehat{m}_t) \approx (1 + u_t + \widehat{m}_{t-1} - \widehat{\pi}_t) \end{array} \right]$$

where  $u_t = \mu_t - \bar{\mu}$ . Hence, approximately, we have

$$\widehat{m}_t = \widehat{m}_{t-1} + u_t - \widehat{\pi}_t \quad (\text{A.20})$$

## APPENDIX B

## SOLUTION METHOD OF UHLIG (1999)

- In order to apply the solution method of Uhlig (1999), first, we need to classify all variables in three groups:
  - Exogenous variables (e.g.  $z_t$  and  $\varphi_t$ ). We call  $z_t$  the vector of exogenous variables,  $k \times 1$ , that show up in the model at time  $t$ .
  - State variables (e.g.  $k_t$  and  $m_t$ ). We call the vector of state variables  $x_t$ ,  $m \times 1$ .
  - Endogenous variables (e.g.  $y_t$ ,  $c_t$ ,  $h_t$ ,  $\lambda_t$ ,  $i_t$ ,  $r_t$ ,  $\pi_t$ ,  $x_t$ ). We call the vector of endogenous variables  $y_t$ ,  $n \times 1$ .
- Second, rewrite the log-linearized model as

$$\begin{aligned}
 0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t \\
 0 &= E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t] \\
 z_{t+1} &= Nz_t + \epsilon_{t+1}; \quad E_t(\epsilon_{t+1}) = 0
 \end{aligned}$$

Some matrices must meet the following conditions:

1. Matrix  $C$  has to be of size  $(l \times n)$ , where  $l \geq n$ . Here  $l$  denotes the number of deterministic equations included in the first equation above.
2. Matrix  $C$  must be of rank  $n$ .
3. Matrix  $F$  must be of size  $(m + n - l) \times m$ .
4. Matrix  $N$  have stable eigenvalues, only.

- In our exercise, therefore, we have

$$A = \begin{bmatrix} 1 & \frac{\tilde{c}}{k} \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ \theta \left( \frac{\tilde{y}}{k} \right) \psi \left( \frac{\tilde{h}}{1-h} \right) & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \delta - 1 & 0 \\ \theta & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \delta - 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -\left( \frac{\tilde{y}}{k} \right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 - \theta & 0 & 0 & 0 & 0 & 0 \\ -\xi_2 & 0 & 0 & 0 & -\xi_1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & AA & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & BB & 0 & CC & 0 & 0 \\ \frac{\omega_2}{\omega_1} & 0 & 0 & 0 & \frac{\omega_1 - 1}{\omega_1} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix}$$

$$\begin{aligned}
D' &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -\theta \left( \frac{\tilde{y}}{\tilde{k}} \right) \rho & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
F &= \begin{bmatrix} 0 & 0 \\ 0 & \Phi \end{bmatrix} \\
G &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
H &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
J &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
K &= \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \\
L &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} \rho & 0 \\ 0 & \eta \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
AA &= - \left( 1 + \psi \left( \frac{\tilde{h}}{1 - \tilde{h}} \right) \right) \\
BB &= -\theta(1 - \theta) \left( \frac{\tilde{y}}{\tilde{k}} \right) \\
CC &= \theta + \psi \left( \frac{\tilde{h}}{1 - \tilde{h}} \right) + \theta(1 - \theta) \left( \frac{\tilde{y}}{\tilde{k}} \right)
\end{aligned}$$

- A solution is matrices  $P$ ,  $Q$ ,  $R$  and  $S$  such that

$$\begin{bmatrix} x_{t+1} = Px_t + Qz_t \\ y_t = Rx_t + Sz_t \end{bmatrix}$$

and such that  $x$  and  $y$  do not explode.

- A detailed procedure to get  $P$ ,  $Q$ ,  $R$  and  $S$  is provided by Uhlig (1999).
- However, more helpfully, Uhlig has written a Matlab code that will compute the solution for us. All we have to do is to provide it with the vectors  $x$  and  $y$ , and the matrices  $A$ ,  $B$ , ... ,  $N$ . The Matlab program can be downloaded at <http://www.wiwi.hu-berlin.de/wpol/html/toolkit.htm>.



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