THE EFFECTS OF DIVERGENT PRODUCTION ACTIVITIES WITH MATH INQUIRY AND THINK ALOUD OF STUDENTS WITH MATH DIFFICULTY

A Dissertation

by

HIJA PARK

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

May 2004

Major Subject: Educational Psychology
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ABSTRACT

The Effects of Divergent Production Activities with Math Inquiry and Think Aloud of Students with Math Difficulty. (May 2004)

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The present study was designed to investigate the effects of divergent production activities with math inquiry and think aloud strategy of students with math difficulty. Multiple baseline across behaviors design was replicated across four participants. This research also investigated relationships between the interventions and creativity scores by employing pretest and posttest design as measured by the Torrance Tests of Creative Thinking (TTCT). The results varied with the participants and the interventions. The overall mean in all three treatments increased a mild degree based on descriptive statistics. All four participants showed a drastic variability in math problem solving. T-test results from the TTCT showed that there were significant differences in both fluency and flexibility scores. Elaboration and originality scores appeared unaffected since the instructions were mean in all three treatments increased to a mild degree based on descriptive statistics. All four primarily involved in fluency and flexibility creativity constructs. Originality was excluded as a measure and elaboration was not instructed intensely enough in the program.
Conclusively, 5th grade students with math difficulty improved both think aloud and math inquiry scores based on visual/statistical inspection of Mean+Trend difference analysis. It was found that only math inquiry intervention was effective to a mild degree for three out of four participants. Fluency and flexibility scores increased as a result of divergent production activities, however not enough to say that overall creativity is fostered directly by the program. Issues emerged out of the math problem solving in terms of controlling variability and developing content materials for the instruction. Further research is needed to ascertain the effects of multiple interventions on students with math difficulty. Replications are needed to expand the findings to the development of viable instructions. Future study is also needed to use varied math inquiry skills and think aloud strategy in order to improve both problem solving ability and creativity associated with mathematics.
To my husband, Sang-yun Bae
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I am indebted to so many people for my being here today with honor and pride. They have all made it possible for me to complete my education and dissertation research and most of all, to support and trust in me all the time no matter what. I would like to express my sincere gratitude for always being there for me in every possible way:

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CHAPTER I
INTRODUCTION

Education in general can be said to be a process or a set of processes that aim to develop the potential of each and every student. Educators and practitioners strive to meet students’ individual needs and goals in daily practice, hoping to help all students achieve their potential. Some students experience problems in academic areas. Problems in the subject matter of mathematics emerge at the elementary level and tend to progressively worsen through secondary school levels to adult age (Miller & Mercer, 1998). Roughly up to one out of three or four students in the elementary level fall below average achievement in mathematics (National Assessment of Educational Progress: NAEP, 2001). Facilitating students who are achieving in all subject areas except math is more likely if they are provided with effective, interesting, and practical math instruction. Varied mathematics instruction should be sought to alleviate the problems of students with math difficulty. Due to the lack of research on multiple instructions with divergent production activities, instructional/learning possibilities should be examined either divergent production activities alone or other strategies combined with, for examples, think aloud and inquiry skills.

Statement of the Problem

The U.S. Department of Education (2002) reported that around 13 percent of the total

This dissertation follows the style of The Journal of Educational Research.
enrollment in public schools, kindergarten through 12th grade, was served in federally supported programs for the disabled. Half of the disabled were categorized as under the “specific learning disabilities” category. Students with learning disabilities exhibit problems with mathematics (Scheid, 1990) and this population has been growing steadily in numbers and percentages. Unlike the identified students in special education, it is not clear whether students with mild math difficulty received intervention programs or any other type of assistance in the classroom. As Miller and Mercer (1998) have pointed out, the lack of understanding learner characteristics (information processing, cognitive/ metacognitive, language disability, and social/emotional perspectives) by their teachers often aggravates students’ independent growth in terms of academic achievement since understanding students’ strengths and weaknesses in many aspects affect math teaching strategies. The National Assessment statistics showed that students in general are alarmingly weak in mathematical problem-solving skills (NCES, 2001). For instance, data from the National Assessment of Educational Progress statistics (NAEP, 2001) indicate that nationwide, 33 percent of the fourth graders are below basic mathematics achievement with 23 percent in the state of Texas.

The National Council of Teachers of Mathematics (NCTM) has led the mathematics reform movement in the 1990s (NCTM, 1989, 1991, 1995), which core is a paradigm shift in mathematics instruction for all students from skill-based instruction to problem-solving learning of active-learners facilitated by teachers’ guidance and questioning. Five goals proposed by NCTM (1989) were that students should: learn to value mathematics; become confident in their ability to do mathematics; become
mathematical problem solvers; learn to communicate mathematically; and learn to reason mathematically. NCTM (1989) supported the curriculum standards that focus on reasoning and communication skills that emphasize creative problem solving and divergent thinking ability. The standards were established to promote activity based inquiry for teaching and learning mathematics (Rivera, 1998). The instructions should be provided students with math difficulty with materials emphasizing on problem solving and inquiry skills as well. Students with math difficulty should receive any alternative assistance to alleviate their problem within the school system. Therefore, there are at least two reasons why students with math difficulty should be provided with educational intervention. First, students with math difficulty will be affected in the long-term. Second, special assistance is likely given to labeled or identified students as learning disabilities. Accordingly, there is an important need to find techniques to assist students with mild deficits in mathematics.

Research in mathematics intervention for students with learning disabilities has primarily focused in the areas of computation and word problems (Bottage & Hasselbring, 1993; Marzola, 1985; Montague & Bos, 1986; Smith & Lovitt, 1975 &1976; Shunk, 1985) as specific mathematical deficiencies. As Rivera (1998) documented, mathematical interventions were largely focused on cognitive or metacognitive strategies (Desoete et al, 2001; Marzola, 1985; Montague, 1996; Montague & Bos, 1986). Research relevant to both divergent production and mathematics has been conducted with an average ability student population through various grade levels. Some researchers investigated the relationship between divergent
production and mathematics (Baer, 1993; Dirkes, 1978; Perry & Stacey, 1994) and the effects of divergent production activities or creative program on the students (Carman, 1992). Some others studied the possibilities to enhance mathematical creativity by employing a variety of divergent production problems or by instructing problem solving skills (Balka, 1974a; Haylock, 1984; Jensen, 1973). Still little research has been conducted on the effectiveness of divergent production intervention programs in mathematics. Program models for students with math difficulty have not provided yet convincing evidence on long-term effectiveness. In this regard, divergent production activities need to be reexamined for the effects on students with math difficulty and mathematical creativity. According to Baer (1993), there still exists a debate on whether divergent thinking plays a significant role in creative performance. Some creativity training programs have demonstrated contradictory conclusions on the success of promoting creative thinking. Although problem solving skills have often been used for students with learning disabilities in order to ease mathematical learning difficulties (Cawley et al., 1979), many studies have been conducted on average performing students from regular classrooms. In an attempt to benefit students with short term and minimal mathematics difficulty and do not receive special education, the effects of the divergent production activities need to be investigated and effective instruction should be invented.

Another existing problem area is math inquiry skills. Both question writing and finding missing information were used to measure creative ability as a result of divergent production activities (Balka, 1974a; Haylock, 1984; Jensen, 1973). Studies
revealed that divergent problem writing is positively correlated with mathematical creativity and higher math performance. Research has not been reported yet on the effects of question writing when math problem-solving instruction is provided. Should inquiry as a necessary skill for critical thinking (Glasser, 1966) be combined with strategy training for students with math difficulty? What kind of outcomes might be produced as a result of the special instruction? Would it be effective in improving students’ math performance and increasing students’ mathematical creativity? Researchers have not attempted to study the use of divergent inquiry activities and the effects of the training on questioning skills in mathematics. Findings from relevant studies may imply the likelihood that math question writing can be effective when incorporated into divergent production activities. Further research is needed to develop an exemplary instruction program and its assessment instrument on behalf of students confronting obstacles to learn mathematics.

Cognitive and metacognitive strategies have been provided to train students with and without LD (Desoete et al, 2001; Montague, 1996; Montague & Bos, 1986). It was found that the strategy training programs were effective, and students improved their problem solving ability in mathematics. However, it is uncertain that the effects of think aloud still remain consistent when multiple treatments are brought together to a single subject when provided with a short-term instruction. There is no information available on whether multiple interventions in mathematics have been conducted with students experiencing math difficulty. Think aloud has been known to be a helpful strategy for school children with learning disabilities. Therefore, if the think aloud
strategy is taught to students with mild math difficulty, it would be expected to increase mathematical performance, since students’ deficit is less likely to interrupt students’ learning mathematics.

To recapitulate, three primary issues remain unsubstantiated. First, there is no information on how multiple divergent production treatments that consist of metacognitive strategies, inquiry skills, and problem solving skills are arranged in order to impact students’ math performance. Second, there are no established measurement scales that can reflect adequate sensitivity to growth of students in the program. Third, divergent production models have shown conflicting results on its effectiveness to foster creativity, and the relationship between divergent production and mathematics needs further investigation. Future applied research is needed to assess the effects of divergent production activities as a varied investigation of multiple baseline across behaviors design for multiple participants. When divergent production activities are specifically combined with three selected instructions such as metacognitive strategy, inquiry skills, and math problem solving ability, it is expected to contribute to mathematical performance and eventually mathematical creativity.

Purpose of the Study

Due to the lack of findings of multiple interventions in the research literature for students with math difficulty, the purpose of this study was to extend the literature on mathematical creativity by attempting to identify a functional relationship between each intervention and divergent production. Three interventions used for the investigation are: think aloud strategy, math inquiry skills, and math problem-solving
ability. This study assessed the effects of the divergent production activities and the participants’ math performance as measured by researcher developed summative scores on the above three target behaviors. This study was also designed to determine if there were any significant differences between the pretest and posttest of four creative abilities as measured by *Torrance Tests of Creative Thinking* (Torrance, 1974).

**Research Questions**

The research questions underlying the investigation were as follows.

1. Do participants improve their math performance on the three dependent variables (think aloud, math inquiry, and math problem solving) as measured by the researcher developed summative scales as a result of the instructions?

2. If students demonstrate improvement, is there any evidence to show the program effectiveness among the three divergent production activities (think aloud strategy, math inquiry, and math problem solving) as measured by the researcher developed summative scales?

3. Are there any significant differences in the scores between the pretest and the posttest of all participants as measured by the *Torrance Tests of Creative Thinking* (Torrance, 1974)?

**Definition of Terms**

*Students with Math Difficulty (S/MD).* This is not a label for a special subset of children. Students with math difficulty (S/MD) is a group of students who are achieving in other subject areas while they are having a lack of knowledge and skills in learning mathematics, compared to at or above average students in mathematics. The
deficits in math are not severe enough to be identified as learning disabilities using a screening test. S/MD can be interchangeably used with low achiever in math (LAM) in this study, and S/MD shows difficulty completing assignments and producing the amount of work usually expected (ACCG, 2003).

_Divergent Production (DP)._ Guilford (1967) defined divergent production (DP) as the generation of information from given information, where the emphasis is upon variety and quantity of output. Fluency, flexibility, originality, and elaboration are considered four divergent production abilities that contribute to the more complex construct of creativity. In this study, DP is redefined limited to three abilities: fluency, flexibility, and elaboration only. These three constructs are imbedded in DP activities based on math content by using three different strategies, i.e., think aloud, math inquiry, and problem solving.

_Think Aloud (TA)._ Think aloud is a metacognitive strategy defined as an active, reflective process directed toward one’s own cognitive activity (Flavell, 1976). Think aloud includes self-instruction, self-question, and self-monitor. Students recall strategies that they know, apply them appropriately to the given situation, and monitor their problem solving activities. While working on the task, students are encouraged to talk aloud about how to solve the problem in a tone audible to the observer. TA in this study is redefined as self-instruction and self-question and, therefore, TA strategy is taught and assessed limited to these two scaling categories.

_Math Inquiry (MI)._ Inquiry has been considered as a necessary skill for students to be productive individuals (Glasser, 1966; McCollum, 1978; Suchman, 1962). In this
study, math inquiry is defined as asking the questions in writing on the worksheet and gathering information to solve the given problems using appropriate verbal expressions and mathematical/everyday life terms. MI instruction encourages students to fill in the missing mathematical information or to write down as many questions and statements as possible about the given situation.

**Mathematical Problem Solving (PS).** NCTM (2000) suggests that problem solving in elementary level means engaging in a task for which the solution method is unknown in advance. In this study, math problem solving is defined as when students draw on their knowledge in order to find many and varied solutions and, therefore, problem solving instruction helps develop students’ own mathematical understandings and methods through the intervention process.

**Mathematical Creativity (MC).** Krutetskii (1976) defined mathematical creativity as the ability to leave the patterned stereotyped means of solving a problem and find a few different ways of solving it. Cornish and Wines (1980) defined creativity in mathematics as extending patterns, rearranging models, transforming familiar conventions in practical situations, and predicting effects. Based on the definitions above, this study redefines mathematical creativity as fluent and flexible thinking in math problem solving.

**Limitations**

There would be at least three limitations in this study. First, the reactive nature of the subject’s responses (Foster, 1986) is a possible limitation due to the nature of the single subject research methodology and the short-term instruction. It is likely that
participants show reactivity causing problems in establishing basal data and the interpretation of contracted gains. If the individual is aware of being observed on a particular target behavior, he or she can alter the behavior or performance as a response to being observed (Richards et al., 1999). Second, multiple treatments interaction, or interference is likely to occur during the study, threatening the internal validity of the gains (Krishef, 1991). This may be caused when two or more treatments are given in relatively quick order. Carry-over effects can influence the results that are obtained on other behaviors. Third, the study would not allow strong generalizability due to the nature of the single subject research design and the eight week-short term interventions (Barlow & Hersen, 1984).
CHAPTER II
REVIEW OF THE LITERATURE

This study focused on the effects of divergent production activities with math inquiry and the think aloud strategy on students with math difficulty. This chapter will introduce theoretical frameworks of the study by reviewing the literature closely related to five research areas: (1) students with math difficulty, (2) divergent production, (3) think aloud, (4) math inquiry, and (5) mathematical creativity. The literature will help imply where this study should focus and how the suggestions from the literature are related to students with math difficulty.

Students with Math Difficulty

Students with math difficulty is not a label for a special subset of children. It is a group of students who are achieving in other subject areas while they are demonstrating a lack of knowledge and skills in learning mathematics compared to students at or above average achievement in mathematics. The terms, students with math difficulty (S/MD) and low achiever in math (LAM) can be interchangeably used since both groups belong to the same level based on the math achievement and their learning characteristics. Ginsburg (1998) defined students with mathematics learning disabilities as those who have normal intellectual abilities but unusually severe problems only in learning mathematics and who are unlikely to profit from sound instruction. Unlike students with learning disabilities (S/LD), students with math difficulty (S/MD) or low achiever in mathematics (LAM) show difficulty completing
assignments and producing the amount of work usually expected (ACCG, 2003). Their deficits in math are not severe enough to be identified as learning disabilities using a screening test. It is suggested that grades assigned on the quality of performance rather than the quantity is helpful for this group. For example, if given 100 math problems during a timed test, the grade should be determined by the percentage correct of the total number of problems actually attempted (ACCG, 2003).

Miller and Mercer (1998) documented four learner characteristics: information processing, cognitive/metacognitive, language disability, and social/emotional perspectives. In information processing perspective, learners’ math performance may be affected by attention deficits, visual-spatial deficits, auditory-processing difficulties, memory problems, or motor disabilities. In cognitive and metacognitive perspectives, students with learning disabilities lack awareness of the skills, strategies, and resources that are needed to perform a task and tend to fail to use self-regulatory mechanisms. In language disabilities perspective, language skills become very important to math achievement because math symbols are an expression of numerical language concepts. For example, Smith (1994) reported that reading difficulties interfere with word problems solving ability among many students with learning disabilities. In social and emotional characteristics, studies show the significant relationships between mathematics and self-esteem. For instance, repeated failure in math was believed to lower self-esteem and produce emotional passivity in mathematical learning (Cherkes-Julkowski, 1985; Patten, 1983). It was found that mental ability and self-efficacy had strong direct effects on mathematical problem-solving performance of 329 high school
students (Pajares & Kranzler, 1995). Ability also had a strong direct effect on self-efficacy, which mediated the indirect effect of ability and level on performance. Their results also supported Bandura's (1986) finding that self-efficacy can mediate between the sources of its creation and subsequent outcomes. Students tend to experience strong influence through their judgments of their own capabilities to do specific tasks in a way that their motivation and behavior are affected. Therefore, teachers should understand learner characteristics in order not to interfere with students’ independent development in terms of academic achievement (Miller & Mercer, 1998).

General characteristics found in students with mathematical learning disabilities are not synonymous with those found in students with math difficulty, although these two groups have much in common. Kavale et al. (1994) claimed that students with learning disabilities have rather differential learning characteristics from low achievers so that they can be clearly differentiated. Understanding learner characteristics of students with learning disabilities also can be helpful in that the implications from the literature are conducive to develop intervention programs for low achievers in math or students with mild math difficulty. As aforementioned, the transition from low deficit to severe deficiency exists throughout all school levels (Miller & Mercer, 1998). Educators need to find out where instruction and remedial intervention for students with math difficulty should focus and what instruction they need to implement in order to improve the deficits of students in mathematics.

Behrend (1994) provided individualized education programs (IEP) to examine the problem solving processes of five second and third grade students with learning
disabilities. Students received individual interviews and were encouraged to share their own strategies during group sessions. Both independent and assisted problem solving abilities were assessed by a measure based on *Cognitively Guided Instruction* (Fennema & Carpenter, 1985). Behrend found that all students in the study benefited from utilizing their own problem solving strategies and did not need to be taught specific strategies.

As seen in the preceding review, students with math difficulty differ from students with mathematics learning disabilities. Students with math difficulty typically find their mathematical ability lies between learning disabilities (LD) and achieving or average performing students. While those who identified as LD receive special program and extra attention from the educational system, it is uncertain that students with math difficulty have been provided any treatment. Little information or statistics has been available in how educators have taught S/MD in classroom levels using systematic remedial instruction. Therefore, more consideration must be given to the yet unlabeled but potentially to-be-math LD, since this negligible low degree of the lag in math can be found elsewhere in the classroom and become deteriorated in-depth and spread to other subject areas. Problems with mathematics usually begin in the elementary school and continue to develop through elementary into adulthood (Miller & Mercer, 1998). It is imagined that special attention plays a critical role to recover deficits in mathematics and reinforces their self-confidence in math problem solving, which could contribute to both a higher level of mathematical creativity and mathematical performance.
Divergent Production (DP)

Creativity has attracted enormous attention and has been studied in a variety of ways by researchers, particularly since Guilford (1950) addressed the importance of creativity at the American Psychological Association. Divergent production has been one of the major areas in the creativity field, along with factors such as fluency, novelty, flexibility, synthesizing ability, analyzing ability, reorganization or redefinition of already existing ideas, complexity, evaluation, originality, elaboration, problem solving and such (Guilford, 1967; Runco, 1999; Torrance, 1974; Torrance & Safer, 1999). Since the term divergent production was Guilford’s own for divergent thinking (Runco, 1999), both divergent production and divergent thinking can be interchangeably used in this study.

Guilford (1967) defined divergent production as the generation of information from given information, where the variety and quantity of output is emphasized. Four components are considered as important divergent production abilities that contribute to creativity are fluency, flexibility, originality, and elaboration. Each was defined by Guilford (1959) as follows:

Fluency of thinking is being able to think well and effortlessly in the given problem situation; flexibility as being able to easily abandon old ways of thinking and adopt new ones; originality as coming up with ideas that are statistically unusual; and elaboration as being able to fill in details given a general scheme. (p. 142)
Torrance (1974) defined creativity as:

A process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on; identifying the difficulty; searching for solutions, making guesses, or formulating hypotheses about the deficiencies: testing and retesting these hypotheses and possibly modifying and retesting them; and finally communicating the results. (p. 8)

Based on his definition of creativity, Torrance attempted to find ways to test for divergent production. He developed and validated the *Torrance Tests of Creative Thinking* (1974, 1990) with four measuring variables: fluency, flexibility, originality, and elaboration. Further, Torrance and his colleague Safter (1999) invented materials and methods that can be used to facilitate creative abilities and the production of creative solutions.

Balka (1974b) selected six criteria for measuring creative ability in mathematics from a list of 25 general creativity criteria. The six criteria consist of four divergent and two convergent aspects determined by mathematics teachers, mathematics educators, and mathematicians. The sixth criterion was to assess the ability to split general mathematical problems into specific subproblems through divergent thinking activities. For example, he asked junior high school students to solve a problem which geometric figures given are alike and why they are alike: square, rectangle, parallelogram, rhombus, and equilateral triangle.

Dirkes (1978) reviewed the literature on the relationship between learning and divergent production to find supporting evidence for classroom instruction. From the
previous studies, she found that divergent production and the learning of students are related as creative thinking ability and problem solving are associated. In her exploratory activity, Dirkes asked four 7-year old students to brainstorm on a mathematical situation that was beyond their competence. She found that the response lists get longer and transfer increases with practice. This assures that divergent production provides students with an opportunity to generate many and varied ideas that can be used to increase academic achievement. She suggested some guidelines for divergent thinking to help the learning-disabled or ill prepared student with simple but a challenging subject matter. It is suggested that divergent thinking can be used to direct students by facilitating complex learning where sequencing is inadequate.

Carman (1992) studied seventh grade students randomly selected from clusters of students and gave them a short mathematical problem-solving ability test. Odyssey of the Mind (OM) is a creativity training program in which team members solve complex, open-ended problems and present their solutions at a competition. Students were taught essential creativity skills such as fluency, flexibility, elaboration, and problem solving through the program. After comparison was made between the students who participated in OM and those who did not, Carman found that participation on an OM team was associated with a mathematical problem-solving ability and teacher/student ratings of problem-solving ability. Although participants were not taught directly how to solve the problem situations, this was not surprising since the teachers and coaches worked with participants by encouraging their leadership and motivating their problem solving. It may indicate that mathematical
problem-solving ability can increase with creativity skills acquired through divergent problem solving training experiences.

Baer (1993) clarified that divergent thinking theory of creativity suggests that creativity will be enhanced by considering three facets of ideas: many ideas; a wide range of ideas; unusual ideas. Divergent thinking has been a major component in most creativity training programs. As Baer stated (1993), creativity training programs made mixed claims for their success in promoting creative thinking. Some appeared to support the divergent thinking theory of creativity. Others seemed to contradict that divergent thinking plays a significant role in creative performance. Baer seemed reasonable to conclude this way in that the programs that have yielded successful results show only students’ improvement in the specific kind of problem solving taught in the course. It may imply that that these improvements in specific task areas cannot necessarily be generalized to extend to general creative thinking ability. Divergent thinking or divergent production is not synonymous with creativity. Divergent thinking training programs have resulted in substantial increases in divergent test scores, but may have not shown similar results in general creativity test. To assure that divergent thinking training can improve creative performance on a variety of tasks, supporting evidence should be found in the effects of the intervention on creativity. Further study is needed to develop comprehensive divergent production activities that can improve creativity in mathematics.

Baer (1993) presented the results of five studies on creative performance across task domains. Among these, the first two studies contain mathematics closely
related to the creative performance of individuals on several tasks to determine if the creative performance of individuals on one task is predictive of their creative performance on other tasks. In the first study, eighth grade participants were given five creativity tests, which involve verbal and math domains. All students scored above average as measured by the California Achievement Test in reading and mathematical achievement. In the second study, fourth grade participants were given two writing tests: a mathematical word problem and a mathematical equation. The word-problem-creating test asked students to write an interesting and original math word problem including all needed information so that it can be solved by someone else. An example written by a student is “Once there were 20 flying dogs. 10 landed. How many left? 12 more flew in. How many flying now?” (p.111). In the equation-creating test, students were asked to write an interesting, original equation using given examples of a few equalities. An example is “2+2=2+2; [9/3][2/6]=[2/3][9/6]” (p.50). It was found that mathematical skill as measured by mathematical IQ and mathematical achievement tests contributed to creativity in mathematical-domain tasks. It was also found that word problem creating tests involve both mathematical and verbal skills. Domain-independent creative thinking skills like divergent thinking appeared insignificant in creative performance across a variety of task domains as Baer confirmed in verbal and mathematical task domains.

Perry and Stacey (1994) studied the use of taught and invented methods of math problem solving. They administered four subtraction questions in vertical format to students attending an academically oriented private school for boys. A three item
assessment of lateral thinking was given to the students. Items were either lateral thinking or divergent thinking closely related to lateral thinking. Students were asked to show their work and to explain in writing the methods they used. It was found that students used more invented methods at grade 12 than grade 7. Students in the below average achievement group were twice more likely to use invented methods than those in the above average group. There was found no difference in the lateral thinking scores between students using taught and invented methods.

As seen in the preceding review, it appears to suggest that divergent production activities can facilitate students with complex math problem solving: invented materials and methods can be effectively used for students in the below average achievement. Specific mathematical problems can be taught using divergent thinking activities. Learning-disabled or ill prepared student can be helped with simple but challenging math problem following creativity training. Mathematical skills were found to contribute to creativity in mathematical-domain tasks. It may imply that divergent production can possibly affect students’ math learning and transfer to other subject areas. It is hypothesized that game-like or puzzle-like math problems would be favorable to students with math difficulty. Therefore, divergent production activities could be reconsidered as a way of supplementing students with short term and mild math deficit.

Think Aloud (TA)

Think aloud is a way of metacognitive strategies. It can be described as verbal mediation training that children think out loud when they think about the task and
solve the problem. In general, metacognition has been defined as an active, reflective process directed toward one’s own cognitive activity (Berardi-Coletta et al., 1995; Kluwe, 1982) and “one’s knowledge concerning one’s own cognitive processes and products or anything related to them” (Flavell, 1976, p.232). Metacognition consists of an awareness of skills and strategies needed to perform a task effectively as well as the ability to use self-regulatory mechanisms to ensure the successful completion of the task (Baker, 1982). Think aloud as a metacognition strategy has been believed to increase self direction and autonomy of the learner both in academic and social behavior.

Meichenbaum and Goodman (1971) trained children from a special education classroom on modifying their classroom behavior and improving cognitive test results by verbal mediation activity. In their study, self-instruction is defined as speaking the directions or process statements for problem solving, personal affirmations, accuracy, neatness checks, or other statements that are designed to regulate behavior.

Camp and Bash (1981) developed the think aloud program, which combined cognitive training and social problem solving through verbal mediation. Based on the research by Meichenbaum and Goodman (1971), think aloud was initially designed for young aggressive boys. Later, Camp and Bash (1981) developed the think aloud program for regular elementary classrooms. Four self-instruction questions are basically used to promote children’s problem solving skills: “1) what is my problem? Or what am I supposed to do? 2) What is my plan? Or how can I do it? 3) Am I using my plan? and 4) How did I do?” (p. 17).
Montague and Bos (1986) investigated the effect of cognitive strategy training on verbal math problem solving. They selected six students aged between 15 and 19 from small-group remediation classes for students with learning disabilities. Training involved an eight-step strategy designed to enable students to read, understand, carry out, and check verbal math problems encountered in math curriculum. Strategy acquisition training was three 50-minute sessions. Students were asked to read the problem aloud, paraphrase the problem aloud, visualize the problem, complete the given statements aloud, complete the given statements aloud for hypothesizing, write the estimates, show the calculation, and finally, do self-check. Montague and Bos found that five of the six students made substantial progress in solving verbal math problems after receiving cognitive strategy training in terms of the time reduction to complete the test as well as the utilization of the strategy. Cognitive strategy has been frequently used for students with learning disabilities along with think aloud. It may imply that the cognitive strategy can be an alternative effective learning strategy for students with mild deficit to improve the verbal math problem solving. Students are encouraged to talk to themselves about how to solve the problem. While working on the task, students are trained to talk in a tone audible to the observer.

Montague (1996) presented an alternative assessment procedure of mathematical problem solving that included both cognitive and metacognitive strategies. He addressed in the model the three metacognitive strategies that good problem solvers use: self-instruction, self-question, and self-monitor. He pointed out that students recall the strategies they know about, apply them appropriately,
monitor their thinking during problem solving in order to seek solutions in the problem situation.

Pressley (1986) noted that metacognitive strategies are used to organize information, represent it accurately, execute the solution, and check the problem solving process. Self-instruction prompts the individual to participate actively in the behavior change process. Self-question is used to validate the process of thinking and redirect the behavior. Self-question includes monitoring questions and evaluating statements that are used for self-judging the answer: Am I following the steps? Or is this answer correct?

Desoete et al. (2001) examined the relationship between metacognition and mathematical problem solving on third graders with average intelligence. All participants attended general elementary school without reading or mathematics learning disabilities. They completed three types of tests individually outside the classroom setting, which are a standardized test on mathematics (the KRT), a reading fluency test (the EMT), and two metacognitive tests (the MAA and the MSA). Factor analysis and multivariate analysis of variance (MANOVA) were conducted to process the data. Metacognition explained 66 to 67 percent of the common variance and differentiated between average and above-average mathematical problem solvers. The findings confirmed that metacognition is important both in the initial or fore-thought phase and in the final or self-reflection phase of mathematical problem solving as well.

The above studies may imply that think aloud can be helpful for students with math difficulty in enhancing self-efficacy and improving their academic performance.
When learners believe that think aloud strategy is a useful tool, they are likely to seek the solutions using this strategy with self-confidence and self-assertion (Davis, 1986) and become more independent in solving given problems on their own. In this regard, students’ own initiative and deliberately designed math instruction together can induce students to learn more within specific domains such as mathematics (Montague, 1996). Think aloud is assumed to be a more helpful strategy for schoolchildren with math difficulty to increase mathematical performance since these students are in need of self-confidence and self-efficacy.

*Math Inquiry (MI)*

The skills of inquiry have been emphasized as one of the necessary skills for students to acquire in becoming productive individuals (Glasser, 1966; McCollum, 1978; Suchman, 1962). Glasser (1966) found that inquiry or discovery learning is important to critical thinking since inquiry skill prepares students to confront problems, generate, and test ideas for themselves. When students are given a relatively unguided trial and error problem, they apply their own structure using inquiry skills. McCollum (1978) explained that the process of inquiry begins with specific data, requires concept clarification, and then reaches generalizations. He viewed inquiry as a necessary process to become a productive and self-generative learner. Suchman (1962) noted that the teacher should provide the inquiry situations that the learner controls. The major processes of inquiry involve four skills: describing, explaining, predicting, and choosing. The processes of inquiry have been applied to find answers by using given information. Ellis and Alleman-Brooks (1977) provided the inquiry skills checklist for
the evaluation of student’s inquiry learning, which includes specific skills such as asking questions, drawing maps, making observations, making graphs, drawing sketches, making measurements, and recording data. Sorenson et al. (1996) affirmed that inquiry processes help students develop their own strategies in order to seek out information and solve problems in all subject areas. Some of the fundamental inquiry skills include observing, classifying, inferring, predicting, and measuring. These specific skills can be adopted in developing an assessment to measure math inquiry skills in terms of divergent production.

Jensen (1973) studied the relationships between mathematical creativity and numerical aptitude, and mathematical achievement in relation to computation and problem-solving. Mathematical creativity was measured by the divergent production tests in mathematics devised by Jensen. She presented students the problem situations such as written form, graphic form, and chart form and asked them to write down all the questions that they could think of from the given graph. She found moderately high correlations among the constructs and recommended the possibility of mathematical creativity as a supplementary evidence of a student’s mathematical performance. Jensen (1973) viewed creativity as the ability to give numerous, different, and applicable questions when presented with a mathematical situation in written, graphic or chart form. Jensen’s study suggests two possibilities: One is how math instruction is designed to train inquiry skills. The other is how possibly the effect of the instruction can be assessed. Math inquiry skills could be trained using divergent production activities if problems were properly presented in a question writing form. Students
should have ample opportunities to practice questioning to find solutions.

Balka (1974b) selected six criteria for measuring creative ability in mathematics. His fifth criterion was to measure the ability to sense missing pieces from a given mathematical situation and to ask questions in order to fill the missing part through divergent thinking. He gave the students a paragraph long piece of information, for example, a U.S. Agriculture report then directed them to make up as many questions as they could in the given mathematical situation. Students were only to generate questions that could be answered from given information in the paragraph. They were not asked to solve the problems that they wrote. Each question would score two points, one point each for fluency and flexibility. For example, a question like “how much does an average American family spend for groceries in one year?” (p. 635) would award one point since the answer is not given but can be calculated from the given information. Balka (1974a) found that students with high flexibility scores perceive adjustments and changes in a given mathematical situation and make the most of their capabilities in the given situation. He also pointed out the lack of research in the development and use of creativity instruments in mathematics for future study.

Haylock (1984) constructed a number of tests to assess the ability of divergent production. One of tests was called problem posing which a “scattergram” graph asks students to make up as many and varied questions as they can from the given information. The other test was called redefinition. For example, the students were asked to write down as many different statements of what the two numbers 16 and 36 have in common. Students were required either to practice continual redefinition or to
make up questions. Math inquiry training can utilize samples similar to Haylock’s study for developing inquiry skills and divergent thinking skills.

As discussed above, the literature indicates that inquiry skills can be important for schoolchildren that contribute to both mathematical creativity and mathematical performance. It also implies that math inquiry can be taught and assessed through divergent production activities in the regular classes. Previous studies suggest that some exemplary samples can be used for assessing students’ math inquiry skills. In this study, math inquiry was defined as questioning skills in mathematics through divergent production activities in pursuit of as many and varied solutions. Questions were devised in order for students to make out as many questions or statements as they could ask in the given situation.

Mathematical Creativity (MC)

Relatively little consideration has been given to the creativity concept within mathematical education. As in the preliminary search by the researcher in September 2002, the Educational Resources Information Center (ERIC) database held 2,426 articles limited to English language and in journals format in the area of creativity using the descriptor, “creativity” since 1966. Only 44 articles (less than 2 %) among them were found on mathematics research in the area of creativity using descriptors of “creativity” and “mathematics.” Perhaps one explanation would be that school mathematics was thought to be associated with convergent thinking rather than divergent thinking. Hudson (1966) found that students were explicitly or implicitly required in mathematics to find the single best solution to each given problem. Perhaps
the emphasis on convergent thinking in mathematics may be attributed to the following factors: the nature of mathematics, the perceptions of school teachers, or the assessment instruments. However, a number of mathematical educators have seen that divergent production tasks are potentially relevant to the assessment of mathematical ability, since divergent thinking allows flexibility in solving math problems (Balka, 1974a; Hollands, 1972; Krutetskii, 1969; Wood, 1968).

Krutetskii (1969) viewed mathematical creativity as varied approaches to find the solution to a problem in easy and flexible ways. Mathematical creativity appeared to facilitate finding a few different ways of solving the problem and encouraged resistance to use stereotyped ways of solving it. Krutetskii (1976) commented that schoolchildren display mathematical creativity of “the independent formulation of uncomplicated mathematical problems, finding ways and means of solving these problems, the invention of proofs and theorems, the independent deduction of formulas, and finding original methods of solving nonstandard problems.” (p. 68). It is clearly seen that several traits of creative thinking were included in his definition such as independent, different/ varied, inventive, and original in mathematical activity. Krutetskii (1976) also stressed that flexibility significantly comprises mathematical ability in schoolchildren and mathematical material should be mastered independently and creatively for mathematically gifted students during school instruction. One might conjecture that flexibility would be one of the attributes that needs to be extended for fostering mathematical ability for other students, rather than being focused on the mathematically gifted.
Some findings from the literature are in conflict regarding creativity training and mathematics. Hiatt’s study (1970) supported that divergent production tasks in mathematics do measure some aspect of mathematical ability which were not assessed by conventional attainment tests. Evans (1964) reported significant positive correlations between divergent production in mathematical situation and arithmetic achievement, mathematics attainment, and general creativity. Meyer (1970) found no gains when a mathematics program was instructed in a creative approach to the first grade schoolchildren. Evidence was found in Haylock’s (1987a) study on the effects of divergent thinking training program to mathematical performance. No significant correlations were found in mathematics achievement tests with mathematical divergent production tests (Mainville, 1972; Baur, 1970).

Haylock (1984) investigated mathematical creativity in school children aged 11-12 years old to identify some aspects of creativity and its significant characteristics which the students might show in mathematics. To determine whether rigidity and fixation played roles during doing mathematics and how students perform divergent production in mathematics, students were given three types of puzzles like problems in both numerical and spatial domains. Haylock (1984) reported that children may show a fixation in mathematics, and the fixation may provide some self-restriction that may cause them to fail to solve the problem. The study showed some limiting effect on performance in mathematical creativity tests in that only the group within the very high achieving band revealed the largest variation in overcoming fixation (OF) and divergent production (DP) scores. It was found that OF/DP scores were positively
correlated with *Category Width* as measured by a modification of Pettigrew’s (1958) test and negatively with anxiety towards mathematics as assessed by a questionnaire based on Wallach and Kogan’s (1965) *Mathematics Attitude Inventory*. It suggests that the students in the higher level of mathematics attainment would be more likely to have abilities to overcome fixations and to demonstrate divergent thinking. This may indicate that the fixation would restrict divergent production in mathematics and hinder facilitating mathematical attainment by inappropriately narrowing down the possibilities and restricting the range of elements relevant to the problem.

Haylock (1987b) stated that mathematical creativity in schoolchildren must be defined in the areas of both mathematics and creativity, whichever more emphasis is weighted. Haylock seemed to put the equal emphasis on both mathematics and creativity in assessing mathematical creativity of school children. Mathematical creativity can be rephrased as creativity in school mathematics. Since math problems in his study were solved using divergent thinking processes, the students were asked to redefine the given situation as many and different subsets as possible.

Bibby (2002) studied teachers’ perceptions of creativity and logic in primary school mathematics using semi-structured interviews. The logic meant by the teachers here is the systematic, step-by-step conception of logic rather than the logical reasoning of deduction, argument, or justification. All the teachers in the study believed that problem solving area was the place where mathematics could be creative, while almost all the teachers saw creativity in mathematics as problematic. Creativity in mathematics was thought not to be the flexible mathematical creativity of
association, but rather the logical creativity of algorithmic knowledge. Bibby concluded that the teachers showed severely limited understanding of the potential for creativity within mathematics.

The preceding review leads to questions about whether divergent production can be an important component or strategy for schoolchildren to contribute to both mathematical creativity and mathematical performance. This current study favors Haylock’s viewpoint that mathematical creativity can be measured by divergent production tests. No evidence was found yet to support directly mathematical creativity of students with math difficulty. Divergent production implies feasibility as to fostering creativity only for mathematically gifted under the school instruction (Krutetskii, 1976). Other studies claim that students may benefit from creativity instruction which raises fluency, flexibility, originality, and inventiveness as an objective of mathematics (Hollands, 1972; Wood, 1968). Kieren (1997) suggested that research practices place emphases on the mechanisms of student’s mathematical thinking and on acting with students in doing mathematics. This may indicate that mathematics education should be widely open for varied hands-on activities, divergent thinking problems, and invented methods by the learners in order to empower learners’ own problem solving ability. Conceivably, this current study would obtain some results based on suggestions from the literature. By introducing multiple divergent production instructions combined with math inquiry skills and the think aloud strategy, students with math difficulty are expected to improve their math performance and eventually develop their creativity potential.
CHAPTER III
METHODS

This chapter introduces the research methods used in (1) the research design for the study, (2) identification of participants, (3) experimental context, (4) development of materials, (5) experimental procedures, and (6) data collection/analysis. The study was conducted by using a multiple baseline across behaviors design (Barlow & Hersen, 1984), along with a pretest/posttest design (Gall et al., 1996). The effects of instruction on the target behaviors of think aloud, math inquiry, and problem solving were measured by both the author’s *Divergent Production Scales* (DPS) and the *Torrance Tests of Creative Thinking* (Torrance, 1974).

*Design*

This researcher integrated two research methods. The principal method was a multiple baseline across behaviors design. The supplementary method was a pretest/posttest design.

*Experimental single subject research design.* The advantages of single subject research are that it is evidence-based methodology used in practical settings and that it provides directly observed gains as scientifically acceptable evidence (Kazdin, 1992; Lundervold & Belwood, 2000). A single subject design is commonly used in educational settings to test the effectiveness of the intervention (Swassing & Amidon, 1991). This researcher employed a multiple baseline across behaviors design, which, in general, is used in three cases: when a single participant has more than one problem;
when two or more participants have a similar type of problem; or when there are two
or more settings that involve the same problem (Krishef, 1991; Richards et al, 1999).
This researcher then replicated the multiple baseline design on four participants who
were functionally similar. Each student was observed across three target behaviors.
One baseline observation is terminated and then the first treatment begins on the
behavior, while the other two behaviors are being observed, so that they will have
more extended baseline periods. Therefore, two other behaviors are under time-lagged
conditions (Krishef, 1991). Multiple baseline across behaviors design has three
strengths in terms of validity. First, external validity can be enhanced because the
treatment is applied to several behaviors of the same person in the same settings. The
term external validity is used interchangeably with generalizability or transferability
since results of the study can be transferred to different environments, different types
of instruction, or different students. By having more than a single participant under
more than one behavior or setting, a multiple baseline design can increase its external
validity (Gall et al., 1996). Second, it helps control the threat to internal validity such
as history (Krishef, 1991). Internal validity refers to the degree to which you can claim
that the effects or gains of the study are due to the program or treatment. History threat
comes from any information, directly or indirectly, related to the ongoing program if a
participant received any experience or information that affects the results other than the
treatment (Kazdin, 1982). This multiple baseline design allows more reasonable
explanation as to why some data points at a certain time of the study displayed a lot of
bounce. History threat to internal validity can be controlled through careful observation
of the participant and monitoring of experimental procedures and measurements. Third, this design assures that effects can be attributed to the intervention rather than to various extraneous variables since both direct and systematic replications under identical conditions over several participants’ cases have been completed (Kazdin, 1982).

In this study, baseline phase (A) provided information about the subjects’ current level of the behavior as measured by Divergent Production Scales (DPS) designed by the researcher prior to the intervention procedures. The intervention phase (B) was composed of three types of instructions, and began when the stability of baseline was evident for each dependent variable (Kazdin, 1982). Therefore, each subject had differing lengths of the baseline and intervention across three behaviors. The follow-up data was collected twice, on week one and week three. The interventions were terminated in order to see a cause-and-effect relationship between the instructions and the student’s ability to produce ideas divergently with math inquiry and think aloud on math problem.

*Pretest/posttest design.* This researcher adopted a supplementary method as measured by *Torrance Tests of Creative Thinking* (Torrance, 1974). As in Harkow’s study (1996), TTCT was used for measuring the effect of multiple interventions in the short term. Each student who participated in the study was administered a 30 minute test before initiating the probe and immediately after terminating the instructions. This pretest/posttest was intended to detect any statistically significant differences between two means, to ensure the effects resulted from the instructions (Gall et al., 1996). A
pretest/posttest design provides additional information for the researcher to assist in the interpretation of results gained by the single subject research. Combining more than two methods may enrich the implications by looking at the gains of the study at several different angles.

Participants

Four elementary students with math difficulty participated in the study. Participants lived in a small suburban city in the Southwestern United States. The selection process began with referrals from the math teacher and the learning development specialist at the school, based on the following criteria. Each participant 1) was a fifth grader; 2) was currently identified as having below average achievements in math; 3) referred by their mathematics teacher; 4) had never been in a divergent production training; and 5) received parental permission and gave assent to participate in the proposed study.

Student #1. This student was an 11-year-old Caucasian female. The following descriptions were given by her math teacher about her typical school behavior relating to math class:

Student #1 did not care about schoolwork. She was capable of doing better than she did. She got tutoring during the day. While her homework was always done correctly, she could not do the same problem in class. It seemed someone sat down with her one-on-one at home and helped her with the work, or did it for her. I took daily grades on homework. Journals obviously brought her grades up. If grades had been only taken on tests, the grade would have lowered, for example, to a C or D. At the time, she had problems at home between her parents and her test scores declined last year.

Her Texas Assessment of Academic Skills (TAAS) score on math last year (2002) supported a description of math difficulty. She passed 40 percent of the total
objectives of understanding concepts and 66 percent of problem solving objectives. This accounted for more than 30 percent of the failure to meet mastery at the fourth grade math level. Her math teacher reported that she was easily distracted during the instruction and assessment as well. She had difficulty sustaining attention to steps in problem solving (e.g., probability). She also showed some memory problems. For instance, she often performed poorly on review lessons and failed to retain math facts.

Student #2. This student was an 11-year-old Caucasian female. The following observations were provided by her math teacher:

Student #2 tried hard on math problems, but information she learned in class did not stay any longer. One day she could do it, and the next she could not. She has been tested for learning disabilities, however, she was not qualified as a student with learning disability. She was in counseling and took medicine for depression at the time of the study due to her parents’ problems in their relationship. Mom had remarried and had a new baby. Student #2 was the oldest of four kids. This seemed to affect her studying and test scores. Test scores from last year showed problems in math.

Like student #1, her TAAS score on math last year showed that she fell short by 20 percent of the total objectives of understanding concepts and 66 percent of problem solving objectives. This accounted for 25 percent of failure to meet the objective mastery at the fourth grade math level. According to the teacher’s description and the investigator’s observation, her math performance seemed germane to her emotional variables (e.g., depression), which could negatively affect in mathematical learning, along with other factors such as anxiety, low self-esteem, and passivity (Pajares & Kranzler, 1995). She also showed some memory problems. For instance, she often failed to recall math facts and use new strategies. She appeared to need extra help and hard work on her problems in math.
Student #3. This student was an 11-year-old Caucasian male. His math teacher observed him in the math class:

Although student #3 tried hard, he did not solve problems since information that he acquired did not stay in his brain. He attended pre-first the year between kindergarten and first grade but he still had gaps. He had never been tested for learning disabilities. He was in tutoring like student #4. Student #3 lived with both parents. Test scores from last year showed that he had math difficulty.

As reported by his math teacher, student #3 seemed to have very much understanding of the lesson in the beginning, but did not retain the knowledge for the next lesson. He obviously showed memory problems. For instance, he had difficulty solving multi-step problems such as probability and counting a stack of blocks. His TAAS math score from last year also supported that he seriously lacked understanding of math concepts by 80 percent of the objective mastery and 66 percent in problem solving. This accounted for 50 percent of failure to meet the objective mastery at the fourth grade math level. His TAAS score appeared to demand of him great efforts in math in order to achieve the fifth grade level.

Student #4. This student was a 10-year-old Hispanic male. The following were the observations and information collected by his math teacher:

Student #4 had difficulty with math last year. He had a personal conflict with his math teacher, therefore, he didn't try hard. At the time of being in the study, he did much better than other students in the group. He had become more confident about his math abilities and was trying harder. Like other participants in the study, his grades would have been a C or D if homework and journals had not brought it up. Although student #4 made a lot of progress this year, he still had gaps in math performance. He got tutoring. He lived with both parents and had no problems with family. Since he was absent on TAAS test last year, his math score was unavailable.

Student #4 had no reference as to TAAS score. He seemed interested in math again since he participated in the study. His avoidance behavior and poor performance in
math were believed not to come from the math subject itself (e.g., texts or materials) but from the bad relationship with his math teacher last year. He showed some visual-spatial problems. For instance, he had difficulty drawing shapes such as combined polygons.

As described above, all participants for the research had math difficulty, which were not screened either by the letter grade inclusive of all performances or by learning disability tests. However, they all had explicit difficulties to certain degree in understanding the mathematical concepts and in solving math problems judged by the math teacher and TAAS math scores alone. They appeared to have their dependency on the teacher for extra help in understanding concepts and solving problems correctly.

*Context*

The research school was situated five miles away from the center of a small southwestern town dominantly populated with middle class Anglo Americans. The total enrollment of the school was 467 students, whose proportion was 73 percent Anglo Americans, 13 percent Hispanic and African Americans, respectively, and .5 percent Asian Americans. This school can be called a medium size elementary school with a well provided educational environment. Primarily instruction was conducted in the office of the instructor who was a learning development specialist at the school. The office was used to administer a variety of testing instruments to students at the school. It was equipped with two large rectangular tables in a quiet, spacious, and well enough organized setting to give the participants individual lessons without noticeable distraction. At times the video library was used for instruction and assessment due to either unavailability of the office or for reduction of the travel time from their
classrooms. The video library was closer for the students to access, located right in the hallway across the classrooms, while the office was down the hall around the corner. On only one assessment day when neither of the rooms was available, the session was conducted in a small cozy corner in the library which was used for reading.

**Measurements**

*Test materials.* Two measurements were used in the study. The first measure, the *Divergent Production Scales* (DPS) developed by the researcher, was used weekly for assessments on each behavior during the intervention and two sessions for the follow-up phase. The DPS consisted of three summative scales of behaviors, i.e., think aloud (TA), math inquiry (MI), and problem solving (PS) (Appendix E). The DPS consisted of the material that reflected the mathematics standards of problem solving for the fifth graders guided by Texas Educational Agency (TEA, 2002). Subscales of DPS were composed of two to three variables which characterize the significant aspects of creativity. Think aloud (TA) and math inquiry (MI) were assessed for 5 minutes, respectively and problem solving (PS) for 10 minutes.

The second measure was the *Torrance Tests of Creative Thinking* (Torrance, 1974). The participants were given the 30 minute TTCT, Figural Form A at the beginning and the alternate Form B following the withdrawal of the intervention program to see if the treatment affected each student’s divergent production. The TTCT (Torrance, 1974) provided four different measures of divergent production ability: fluency flexibility, originality, and elaboration. In brief for both verbal and figural forms of TTCT, fluency was the number of ideas, flexibility was the number of
different kinds of ideas, and originality was the unusualness of ideas scored based on infrequency of appearance of ideas. Elaboration was details of ideas counted only on figural test.

*Development of materials.* The materials used for instruction and assessment in this study involved a wide range of researcher-made instruments. The worksheets were developed either by revising problems used in the previous studies (Balka, 1974a; Dirkes, 1978; Evans, 1964; Haylock, 1984; Jensen, 1973; Krutetskii, 1976; Lean & Clements, 1981; Inoue, 1999), or by creation based on a tentative model of divergent production with math problem solving by the investigator. Materials used for assessing divergent production are presented in Appendix G.

*Dependent Variables*

Data measured by *Divergent Production Scales* (DPS) were reported as three summative scores of think aloud (TA), math inquiry (MI), and problem solving (PS) respectively. There were two scoring categories in TA, self-direct and self-question. There were three scoring categories in MI, fluency, flexibility, and elaboration and two in PS such as fluency and flexibility. Each category is defined as a variable of DPS as follows.

*Summative scores of think aloud (TA).* Think aloud included self-instruction and self-question. During five minute assessment time, students recalled strategies they knew about and applied them appropriately to the given problem. When the subjects performed with self-instructing talk in a tone audible to the rater during on-task,
occurrences were tallied. For example, one occurrence is “I will try another shape” or “my estimate answer is 33” in the computation situation. Self-question refers to asking questions oneself regarding what s/he is doing. Examples are: do I need this?; what should I do?; or is this right answer? If the verbalization were relevant to the task and if the subject directed his/her behavior to solve the problem or question himself/herself to check the answer, it was counted as one occurrence.

**Summative scores of math inquiry (MI).** Math inquiry was defined as asking the questions in written down fashion on the worksheet and gathers information to solve the given problems using appropriate verbal expressions and mathematical/everyday life terms. Students were given a mathematical situation and asked to fill in the missing mathematical information by making inquiries, or the students wrote down as many questions as possible about the situation so that the problem could be solved and answered from the given information. The summative scores of math inquiry were determined by adding the number of relevant questions written on the worksheet in order to solve the problem (fluency), the number of questions or statement made to get the information in different categories/ways (flexibility), and the degree of how detailed the questions/ statements to solve problems (elaboration). Elaboration measured how detailed the responses were in the process of solving the problem. Examples are: drawing more lines; illustrating the solution with graphs or figures; or combining two questions into one complex statement and such.
Summative scores of math problem solving (PS). Divergent production activities were presented to the participants as a form of math problem solving, which meant engaging in a task for which the solution method was not known in advance. In order to find a solution, students drew on their knowledge through this process and developed new mathematical understandings. The summative scores of PS were determined by the composite scores of two variables: Fluency is defined as being able to think well and effortlessly in the given problem situation (Guilford, 1959). Fluency measured how many responses the student could think of for the given situation, i.e., the number of responses for a given problem (fluency). Flexibility is defined as being able to easily abandon old ways of thinking and adopt new ones (Guilford, 1959). Flexibility measured how many categories of responses the participant could think of by shifting thinking from one way into different ways of thinking. It is the number of different categories of responses (flexibility).

Experimental Procedures

Pre-consultation. Before baseline data were collected, consultation with the instructor/rater was conducted to determine whether the material for the instruction had the appropriate syntax difficulty level for participants and to see whether the questions in the DPS assessment had face validity.

Baseline phase (A). The researcher started with observing each subject’s three basal measures at the same time each session to establish baseline norms prior to the intervention procedures. Baseline phase for each student was dependent on his/her own schedule of maintaining a minimum of three stable data points (Barlow & Hersen,
1973). The investigator observed the subject for twenty minutes and rated the three behaviors on the worksheet. The baseline data were collected until one of the three target behaviors was stable enough to begin the instruction of Divergent Production Activities.

*Intervention phase (B1/B2/B3).* Once a baseline was established as stable, the first intervention (B1) began on that stable behavior of three behaviors under experimental control, whichever it turned stable first. The other two behaviors were observed for additional baseline data until they were stable. The intervention (B2) was given on the second behavior, when it was stable. When the third behavior was stable, the intervention (B3) was given on it. Therefore, each subject had a running baseline until the final intervention was implemented. Each intervention (B1/B2/B3) of divergent production activities were think aloud (TA), math inquiry (MI), and problem solving (PS). The participants were given the maximum 20 minute-intervention three times a week. Total intervention lasted for eight weeks. As proceeded with sessions, the first intervention B1 was set to think aloud (TA), B2 of math inquiry (MI), and B3 of problem solving (PS) for all participants. For instruction time, problem solving was allotted ten minutes. Both TA and MI were given five minutes, respectively.

*The follow up phase.* Follow-up checking was conducted week one (7th day for all) and week three (21st day for two of the participants) after all interventions were withdrawn, similar to the study by Brown (1979) on the 11th and 25th day. If it indicated that the child had retained skills, it would suffice to say that there would be a cause-and-effect relationship between the student’s performance and the instruction.
**Procedural Reliability**

Interrater reliability was checked to ensure the agreement of the raters. Agreement was calculated by both point-by-point agreement (Richards et al., 1999) and Kappa reliability (Barlow & Hersen, 1984). Kappa reliability is an index of rater reliability that reflects the amount of agreement beyond chance, which is more conservative than the percent agreement since it controls for chance (Barlow & Hersen, 1984). The two hour-training session consisted of the following activities: raters studied the definitions of think aloud, math inquiry, and math problem solving for this study; raters received training on measuring the variables of *Divergent Production Scales* (DPS); the observers trained on recording and scoring. On 50 percent of the sessions, this researcher observed the instructor conducting a session.

**Data Collection and Analysis**

The data were collected and scored primarily by the researcher and instructor during the study. The data were analyzed visually and graphically using Mean + Trend Difference Model (NCSS, 2000). This model measures the differences among phases of both mean and trend among phases since a change in trend was predicted in the data. The effect size was assessed among the phases. The t-test also was used to test for the significance of the difference between two means or scores. If there were a statistical significance between the pretest and the posttest of *Torrance Tests of Creative Thinking* (Torrance, 1974), it can be sufficient to say that the results of the study were meaningful and the changes in participants were attributed to the intervention.
CHAPTER IV
RESULTS

The purpose of the study was to investigate the effects of divergent production activities including math inquiry and think aloud strategy for students with math difficulty. This chapter presents the results visually as a series of multiple A-B designs with follow-up stacked on top of one another (Richards, et al., 1999). The effect sizes were statistically reported among the phases and interrater reliability data were compared. The t-tests were reported to provide the statistical difference between two means of the pretest and posttest as measured by the Torrance Tests of Creative Thinking (Torrance, 1974).

Criteria for Evaluation of Results

Experimental single subject research in general aims to determine the effects of the study in terms of two characteristics: the improvement of participant’s behavior and the effectiveness of the program. In judging the improvement of the participant, visual inspection is used to look at the variability of the data, change of mean level, and rate of improvement. Variability of the data can be established by the effect size represented as R-squared among phases. Since effect sizes alone are not sufficiently significant to conclude that participants substantially improved over the three interventions, further analysis was conducted through visual and statistical procedures in order to obtain better technical results. To determine the subject’s improvement, the charts showed a visual mean shift. Substantial change of the mean level can also add
some credibility in determining student improvement. The rate of improvement is determined by the trend indicating either improving or deteriorating among phases. In evaluating the effectiveness of the program, visual inspection measures the changing trend and the gap at the beginning of the instruction phase. Increasing trend for positive behaviors or desirable performance indicates that the program was effective on the individual. The noticeable intercept gap at the onset of the instruction following baseline phase can add some evidence that the intervention has impacted on the participant’s behavior.

Data of the summative scores from three interventions were reported for each participant, phase, and instruction. Since this study employed multiple baseline across behaviors design, three intervention charts appeared visually as stacked on top of one another for the expedient comparison of each participant’s multiple behaviors (Figures 1-4). Think Aloud chart is placed on top, Math Inquiry in the middle, and Problem Solving at the bottom. All students participated in the study were exposed to the same treatment with three phase conditions: baseline (A), intervention (B), and follow-up. Only intervention phase differed from one another depending on both the type of instructions and the baseline stability of each student. Mean and trend differences for each participant were reported by instruction and phase in Table 1 in order to provide numerical feel for the figures. Effect sizes are presented in Table 2 by participant and instruction type. T-test results were also reported in Table 3 for pretest and posttest of the results of the TTCT. Overall mean differences and standard deviations for four creativity measures were reported along with statistical significance.
Data Reported on Reliability

Reliability between raters for TA, MI and PS behaviors were assessed on 31 percent of the sessions. Simple percent agreement of interrater reliability was 81 percent using point-by-point agreement calculations which is acceptable degree (Hersen & Barlow, 1976). Kappa reliability was also assessed for controlling for chance level and so that the reliability could be more robust beyond chance. Kappa test reliability came out around 63 percent which indicates good agreement between the raters. When Kappa reliability falls between .40 and .75, we could say that the observations were done fair to good in terms of reliability (Fleiss, 1981). The difference between the simple percent agreement and Kappa reliability is 18 percent which indicates chance alone.

Data on Dependent Measures Reported by Participant

Student #1. The summative scores for student #1 are presented graphically in Figure 1 and numerically in Table 1. The first instruction was think aloud strategy. When the participant was in baseline phase, summative scores of TA ranged from 2 to 4 with a mean of 3. When exposed to the instruction phase, the student’s summative score of TA gradually increased from 3 to 7 with a mean of 4.63. When the instruction was withdrawn, student #1 was followed up one week later. Student #1 scored 4 on the follow-up which slightly increased from the basal level, but it also decreased slightly from the intervention phase. The data points on charts were not actual scores that the participant earned but the derived summative scores from performance level. Since several scores were consolidated into one derived score, small and unimportant
amount of increase in this study could imply substantial differences in the student among phases.

Second instruction was math inquiry. When the participant was in baseline phase, summative scores of MI ranged from 5 to 6 with a mean of 5.5. When exposed to instruction phase, the student’s MI score increased two points to 7 with a mean of 6.2. After withdrawn from the instruction, student #1 scored 7, which still showed a maintained improvement. It also showed some increase by 1.5 from the baseline phase.

Third instruction was divergent problem solving. When the participant was in baseline phase, summative scores of PS ranged from 2 to 7 with a mean of 4.38. Unlike the first two TA and MI behaviors, PS data points revealed a lot of bounce. However, during the instruction phase, the student’s score was stable with no variability and with a mean of 7. After withdrawn from the instruction, Student #1 scored 5 which fairly decreased by two points from the instruction phase, although slightly higher than the basal level.

The R-squared value of each behavior for all participants is presented in Table 2. R-squared refers to the effect size of the mean and trend differences between phases. Student #1 showed moderate R-squares of .636 in think aloud (TA) and .70 in math inquiry (MI) but a mild R-squared of .43 in problem solving (PS). As seen in the Figure 1, no instruction showed a large intercept difference between phases A and B. No significant trend difference was found in all three instructions. Slopes appeared to increase gradually as seen in TA and MI and then there was an abrupt stability arrived in PS instruction.
Figure 1. Divergent production interventions on student #1
Table 1
Mean and Trend Differences by Instruction and Phase

<table>
<thead>
<tr>
<th></th>
<th>Baseline (A)</th>
<th></th>
<th>Intervention (B)</th>
<th></th>
<th>Follow-up (1&lt;sup&gt;st&lt;/sup&gt;/2&lt;sup&gt;nd&lt;/sup&gt;)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range (L-H)</td>
<td>Mean</td>
<td>Trend</td>
<td>Range (L-H)</td>
<td>Mean</td>
<td>Trend</td>
</tr>
<tr>
<td>S1</td>
<td>2-4</td>
<td>3</td>
<td>+</td>
<td>3-7</td>
<td>4.63</td>
<td>+</td>
</tr>
<tr>
<td>S2</td>
<td>2-4</td>
<td>2.67</td>
<td>+</td>
<td>3-8</td>
<td>5.25</td>
<td>+</td>
</tr>
<tr>
<td>TA</td>
<td>S3</td>
<td>2-3</td>
<td>2.67</td>
<td>+</td>
<td>3-6</td>
<td>4.5</td>
</tr>
<tr>
<td>S4</td>
<td>3</td>
<td>3</td>
<td>.</td>
<td>4-8</td>
<td>4.88</td>
<td>.</td>
</tr>
<tr>
<td>S1</td>
<td>5-6</td>
<td>5.5</td>
<td>.</td>
<td>5-7</td>
<td>6.2</td>
<td>+</td>
</tr>
<tr>
<td>S2</td>
<td>3-6</td>
<td>4.67</td>
<td>.</td>
<td>7-11</td>
<td>8.8</td>
<td>+</td>
</tr>
<tr>
<td>MI</td>
<td>S3</td>
<td>3-6</td>
<td>4.5</td>
<td>-</td>
<td>6-9</td>
<td>7.2</td>
</tr>
<tr>
<td>S4</td>
<td>4-5</td>
<td>4.83</td>
<td>-</td>
<td>5-7</td>
<td>6</td>
<td>+</td>
</tr>
<tr>
<td>S1</td>
<td>2-7</td>
<td>4.38</td>
<td>+</td>
<td>7</td>
<td>7</td>
<td>.</td>
</tr>
<tr>
<td>S2</td>
<td>2-8</td>
<td>3.75</td>
<td>+</td>
<td>7-8</td>
<td>7.33</td>
<td>-</td>
</tr>
<tr>
<td>PS</td>
<td>S3</td>
<td>4-10</td>
<td>8.13</td>
<td>+</td>
<td>7-13</td>
<td>10</td>
</tr>
<tr>
<td>S4</td>
<td>3-9</td>
<td>4.88</td>
<td>.</td>
<td>4-10</td>
<td>7.33</td>
<td>+</td>
</tr>
</tbody>
</table>

Note. Range (L-H): lowest score and highest score within the phase; +: increasing trend; -: decreasing trend; .: no slope in trend; 1<sup>st</sup>/2<sup>nd</sup>: first follow-up check and second follow-up check; TA: think aloud; MI: math inquiry; PS: problem solving; S1: student #1; S2: student #2; S3: student #3; S4: student #4.
### Table 2
**Analysis of Variance by Participant**

<table>
<thead>
<tr>
<th>Student #1</th>
<th>Instruction</th>
<th>Sum of Square</th>
<th>Total (Adjusted)</th>
<th>R-squared</th>
<th>F-ratio</th>
<th>P</th>
<th>Power (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TA</td>
<td>15.05</td>
<td>23.67</td>
<td>.636</td>
<td>2.094</td>
<td>.197</td>
<td>.348</td>
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<tr>
<td>MI</td>
<td>6.23</td>
<td>8.92</td>
<td>.699</td>
<td>2.784</td>
<td>.122</td>
<td>.451</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>16.76</td>
<td>38.92</td>
<td>.431</td>
<td>.907</td>
<td>.532</td>
<td>.167</td>
<td></td>
</tr>
<tr>
<td>Student #2</td>
<td>TA</td>
<td>24.13</td>
<td>34.77</td>
<td>.694</td>
<td>3.174</td>
<td>.082</td>
<td>.553</td>
</tr>
<tr>
<td>MI</td>
<td>83.92</td>
<td>94</td>
<td>.893</td>
<td>11.661</td>
<td>.003*</td>
<td>.988</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>74.45</td>
<td>105.23</td>
<td>.707</td>
<td>3.385</td>
<td>.071</td>
<td>.583</td>
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<tr>
<td>Student #3</td>
<td>TA</td>
<td>13.02</td>
<td>17.67</td>
<td>.737</td>
<td>3.366</td>
<td>.086</td>
<td>.531</td>
</tr>
<tr>
<td>MI</td>
<td>28.73</td>
<td>40.92</td>
<td>.702</td>
<td>2.829</td>
<td>.119</td>
<td>.457</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>42.13</td>
<td>77.67</td>
<td>.543</td>
<td>1.423</td>
<td>.337</td>
<td>.244</td>
<td></td>
</tr>
<tr>
<td>Student #4</td>
<td>TA</td>
<td>14.35</td>
<td>27.08</td>
<td>.530</td>
<td>1.579</td>
<td>.281</td>
<td>.293</td>
</tr>
<tr>
<td>MI</td>
<td>7.15</td>
<td>11.23</td>
<td>.637</td>
<td>2.457</td>
<td>.136</td>
<td>.442</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>42.51</td>
<td>96</td>
<td>.443</td>
<td>1.113</td>
<td>.432</td>
<td>.214</td>
<td></td>
</tr>
</tbody>
</table>

*.* p < .05

**Note.** TA: Think Aloud; MI: Math Inquiry; PS: Problem Solving; p: probability level.

# Student #2. The summative scores for student #2 are presented in Figure 2. The first instruction was the think aloud strategy. When the participant was on baseline phase, the summative scores ranged from 2 to 4 with a mean of 2.67. When exposed to TA instruction phase, the student’s score gradually increased from 3 to 8 with a mean of 5.25. Student #1 was followed up two times after withdrawn from the instruction. Measurements were taken one week and three weeks later. Student #2 scored 5 and 6 respectively, which increased noticeably by twice the basal level and slightly higher than the intervention.

Second instruction was math inquiry. When the participant was in baseline phase, the summative scores ranged from 3 to 6 with a mean of 4.67. When exposed to
MI instruction phase, the student’s score increased from 7 to 11 with a mean of 8.8. After withdrawn from the instruction, student #2 scored 7 and 12, respectively. Second follow-up revealed an abrupt increase although the absence of instruction time passed for three weeks. Student #2 showed a maintenance effect of instruction.

Third instruction was divergent problem solving. When the participant was in baseline phase, summative scores of PS ranged from 2 to 8 with a mean of 3.75. Compared to the first two behaviors, the data points revealed a lot of variability similar to students #1. When exposed to PS instruction, the student’s score ranged from 7 to 8 with a mean of 7.33. After withdrawn from the PS instruction, student #1 scored 8 and 11 respectively, which showed some improvement even more when in the second follow-up check. The mean increased by almost twice than the baseline data. As seen in the Figure 1, no summative scores showed a large intercept difference between phases A and B. However, MI instruction showed a slight intercept difference between phase A and B and so did PS except for the summative score of the session eight. The participant reported a deteriorating trend during PS instruction. There was a large trend difference in MI instruction. As presented in Table 2, student #2 showed moderate R-squares of .70 in TA and .71 in PS, and a high effect size of .89 in MI.
Figure 2. Divergent production interventions on student #2

Figure showing divergent production interventions on student #2 with data points and trend lines for TA, MI, and PS Scores across baseline, intervention, and follow-up sessions.
Student #3. The summative scores for student #3 are presented in Figure 3. The first instruction was think aloud strategy. When the participant was in baseline phase, the summative scores ranged from 2 to 3 with a mean of 2.67. When exposed to TA instruction phase, the student’s score of TA ranged from 3 to 6 with a mean of 4.5. After withdrawn from the instruction, student #1 was followed up one week later. Student #3 scored 6 which increased slightly from the instruction phase but considerably higher than the baseline.

Second instruction was math inquiry. When the participant was in baseline phase, summative scores of MI ranged from 3 to 6 with a decreasing trend and a mean of 4.5. The student showed a large intercept difference at the beginning of the instruction with a gradual increasing trend. Scores ranged from 6 to 9 with a mean of 7.2. After withdrawn from the instruction, student #3 scored 7, appearing to retain the gains from the instruction.

Third instruction was divergent problem solving. When the participant was in baseline phase, the summative score of PS ranged from 4 to 10 with a mean of 8.13. Compared to the first two behaviors, the data points demonstrated a lot of variability. When exposed to PS instruction phase, the student showed improvement on divergent problem solving performance with a mean of 10. There was no intercept difference. After withdrawn from the PS instruction, student #3 scored 9 which demonstrating instruction gains. Overall, student #3 showed a similar trend of effect sizes as student #1. As seen in the Figure 3, MI instruction showed a large intercept difference between phases A and B. PS instruction phase showed a large trend difference.
Figure 3. Divergent production interventions on student #3

[Graph showing data points and trend lines for TA, MI, and PS Scores across Baseline, Intervention, and Follow-Up sessions.]
As presented in Table 2, R-squares for student #3 resulted in moderate effect sizes of .74 in TA, .70 in MI respectively. A mild effect size of .54 resulted from PS instruction.

**Student #4.** The summative scores for student #4 are presented in Figure 4 and Table 1. The first instruction was think aloud strategy. In baseline phase, the student had stable scores with a mean of 3. During the TA instruction, scores ranged from 4 to 8 with a mean of 4.875. After withdrawn from the instruction, student #4 was followed up twice. One week and three weeks later, student #4 scored 5 and 7. The participant maintained the gains from TA instruction and continued to show an increase in the follow-up phase.

Second instruction was math inquiry. In MI baseline phase, the participant was almost stable with scores of 5 points. When entered MI instruction phase, the student’s scores showed some variables with a range of 5 to 7 and with a mean of 6. When withdrawn from the instruction, scores dropped down to the basal level. This MI instruction showed a typical case of the A/B/A withdrawal design in that the gains from the intervention were not sustained.

Third instruction was divergent problem solving. Throughout all phases, the data contained a lot of variability, compared to the TA and MI. In PS baseline phase, data ranged from 3 to 9 with a mean of 4.875. When PS instruction was introduced, the student’s score ranged from 4 to 10 with a mean of 7.33. After withdrawn from the PS instruction, student #4 scored 6 and 11 respectively on the two follow-ups. As seen in the Figure 4, TA and MI instruction showed some intercept difference between phases
A and B. No large trend difference was found in all three instructions.

**Figure 4. Divergent production interventions on student #4**
As presented in Table 2, student #4 demonstrated moderate R-squares both in TA and MI of .53 and .63 respectively, and a mild R-squared of .44 in PS.

*Data on TTCT Reported by Creativity Measures*

The t-tests of TTCT for all participants are presented in Table 3. The four fifth graders with math difficulty were administered a TTCT Figural Form A before the instructions was introduced, and a TTCT Figural Form B after the instructions were withdrawn. As seen in Table 3, mean differences between pretest and posttest were produced by large amount for fluency and elaboration measures. Contrarily, mean differences between pretest and posttest for both flexibility and originality measures only increased by a small amount. In terms of statistical significance of pretest and posttest, t-test indicates that there was a significant difference (p=.027) in two mean scores for fluency. T-test indicates that there was a significant difference (p=.002) in two mean scores for flexibility. There is no statistical significance found for originality and elaboration.
Table 3
*t-test of Pretest and Posttest of TTCT*

<table>
<thead>
<tr>
<th>Measures of TTCT</th>
<th>Paired Differences</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td></td>
</tr>
<tr>
<td>Fluency</td>
<td>20.50</td>
<td>10.08</td>
<td>4.066</td>
</tr>
<tr>
<td>Flexibility</td>
<td>4.75</td>
<td>.96</td>
<td>9.922</td>
</tr>
<tr>
<td>Originality</td>
<td>1.00</td>
<td>3.46</td>
<td>-.577</td>
</tr>
<tr>
<td>Elaboration</td>
<td>18.00</td>
<td>12.88</td>
<td>2.794</td>
</tr>
</tbody>
</table>

**. p< .01
*. p< .05

Note. TTCT: Torrance Tests of Creative Thinking; p: probability level.
CHAPTER V
CONCLUSIONS AND IMPLICATIONS

This chapter includes a thorough discussion of the conclusions and implications of the study. Since this study was conducted using a single subject research design, the discussion also includes a section on interrater reliability for internal validity (McCormick, 1990) and a section on external validity. Internal validity was achieved to some degree by complying with instruction procedures and scoring guidelines. External validity was established to some degree by conducting identical and systematic replications across four participants. This chapter will discuss some of the possible future research directions based on the findings from this study.

Summary

All participants improved over three performances. Summative mean scores increased to a mild degree based on descriptive statistics. The effect sizes varied from mild to high degree for all participants. Three of the four students demonstrated improvements on both think aloud and math inquiry. Three of the students’ scores yielded a mild effect size for problem solving activities. However, all of the participants appeared to have a lot of variability in their summative scores of PS performance. Therefore, it is unreasonable to conclude that students improved their problem solving scores as a result of instruction. Only TA and MI instructions appeared to demonstrate a small but significant improvement.
Three of the participants provided evidence supporting the MI intervention. Program effectiveness was evaluated by examining intercept gap differences between phases A and B and looking into the trends. Although immediate and observable changes did not occur at the onset of the TA intervention, think aloud seemed to have a gradual effect on students. In addition, student #4 showed evidence that TA instruction was effective. No participant showed a large intercept gap at the onset of PS intervention. Student #1 demonstrated no intercept difference across all three interventions.

On the follow-up analysis, three students with math difficulty maintained the gains after the MI instruction was withdrawn. Only student #4 maintained the gains from TA instruction. Student #1 did not perform well on TA and MI and failed to establish a stable performance on PS. Student #2 maintained gains from all three instructions. Student #3 retained TA and MI strategies, while student #4 did TA and PS during the follow-up phase. Since each participant showed variable changes, no confidence was confirmed that systematic change was attributed to the instruction.

T-test results provide support for the conclusions that intervention helped participants achieve some facets of creativity on fluency and flexibility scores. Fluency and flexibility appeared either to be readily teachable constructs to the instructor or easily learnable to students. Students improved greatly on the means after instructions. The results on fluency and flexibility were included in the discussion. However, originality and elaboration should not be discussed in this chapter because the study did not intend to examine these two constructs. Unsurprisingly, none of the
participants demonstrated any significant difference or improvement on elaboration and originality as measured by the TTCT. The findings suggest that there was improvement in the participants in terms of some aspects of creativity as a result of multiple divergent instructions (TA, MI, and PS). Divergent production activities seemed to help to raise some aspects of creativity as measured by TTCT. Regardless of summative scores from the multiple baseline across three behaviors, divergent production activities seemed to help to raise some aspects of creativity as measured by TTCT.

In conclusion, the experiment was carried carefully in an effort to hold internal and external validity. Satisfied levels of consistency were obtained during the course of study. Participants showed improvement based on mean scores over three interventions. In terms of mean changes and effect sizes, students demonstrated limited improvement in both think aloud and math inquiry. Only MI program proved effective to a limited degree. Students improved both fluency and flexibility scores and significant differences were found between two mean scores as measured by TTCT. However, multiple instructions did not seem to provide supporting evidence to assure that the total program was effective. Problem solving instruction was problematic presumably due to the varied difficulty level within the assessment content.

**Strengths of the Study**

This study obtained interrater reliability indicating a 63 percent of good agreement beyond chance as determined by Kappa reliability (Fleiss, 1981). Reliability data indicated reasonable during observations and assessments.
Internal validity was achieved to certain degree by controlling for threats from extraneous variables. First, multiple intervention interference would threaten this study since multiple treatments were introduced in relatively quick order without systematically monitoring the treatment (Krishef, 1991). The participants may have possibly been influenced by the interaction of the three consecutive instructions based on the different strategies. Second, multiple baseline across subjects design has the possible selection and mortality threats to internal validity since finding three or more subjects with similar problems may not be easy (Krishef, 1991). Losing some of the participants during the study could possibly make the comparison difficult. This study deliberately set the criteria for subject selection in order to reduce these threats. Third, instructions and observation were monitored more than 50 percent of the time by the researcher on random basis. The researcher advised the instructor strictly to follow the guidelines during observing and scoring. The instructor was encouraged on all sessions to comply with control procedures in order to increase the internal validity of the study. For example, the instructor was told 1) to use the same instruction protocol, the amount of praises, the repetitions of verbal/nonverbal directives, and 2) to abide by assigned time for each session and procedures. Raters adhered strictly to the scoring guidelines for sustaining interrater reliability. Frequent information exchange between the instructor and the researcher made the research progress run smoothly and efficiently.

External validity was established by direct and systematic replications across participants (Kazdin, 1982). Within one experimental setting, four students were taught the same procedures through one-on-one instruction by the same single instructor.
Single subject research design does not hold strong generalizability due to its design nature, nonetheless the extension of the multiple A-B designs with follow-up can increase its generalizability to some degree. Four sample cases were utilized to improve its generalizability.

*Limitations of the Study*

Experimental single subject research originally purports to measure individuals’ behaviors repeatedly using assessments after individually tailored and delivered instructions. Interpretation of the data and further transferability to other cases tend to be limited to the specifically defined circumstances and participants. Nevertheless, the significance of this single subject research exists in its rarity or exceptionality since each case carries unique needs and demands for special assistance in practical settings. To some intervention studies, the results seemed to rest simply upon the art of teaching. Skillful and highly trained instructors were thought to produce either more gains or more reliable results. Controlling for a variety of threats to internal validity may contribute to divergent production performance. Therefore, caution is advised when considering the impacts of the divergent production activities (think aloud, math inquiry, and math problem solving).

First, all participants expectedly showed reactivity (Foster, 1986) especially in the PS baseline phase. When participants are exposed relatively new and noble materials or experience, they are likely to alter their responses abruptly. Reactive performance caused variability that contracted gains and generated inconsistency of the results.
Second, due to the eight week intervention period and the nature of the design, this study did not have stronger generalizability although replications to multiple cases added some credibility to the inference that the treatment was effective. Establishing a basal level and evaluating intervention in mathematics are recommended to run more sessions than in this study in order to assure the sensitivity of growth and detect the change over time.

Third, instructional materials were believed to contain some variability in terms of difficulty level because students showed varied interests and fluctuating performance depending on problem types. Standardized interpretation would not be feasible for this reason. Conclusive remarks were not be easily drawn with regards to the impact of the intervention and the improvement of the participants. Further research is needed to control inherent variability of instruction/assessment materials, especially in math problem solving.

Fourth, the treatment interference among three instructions was problematic and unable to eliminate completely. The participants could have possibly been influenced by the consecutive instructions which constituted three different strategies at any time during the study. However, it was difficult to say which treatment or which combination of the treatments brought about the gains. Selecting dependent variables should be in balance between being not completely independent and nor significantly interrelated. Caution is advised when using similar assessment material repeatedly. Some alternatives would be either to use multiple probe techniques or longer intervals between assessments.
**Implications for Future Research**

This study provides some support for increased creativity as measured by TTCT as a result of the multiple instructions on divergent production activities in math along with think aloud and math inquiry. Math inquiry skills enhanced students’ divergent production ability in mathematics and expectedly affected mathematics performance. Future studies are needed to explore the possibility of math inquiry in order to develop programs that foster mathematics performance of students with mild math difficulty. Further research is needed to develop TA intervention since think aloud seemed to have a gradual effect on students. TA instruction, although no immediate and observable intercept gaps occur, is expected to become an effective program for students with math difficulty if instructed in more intensive and long-term fashion.

Measuring scales should be cautiously designed to have adequate sensitivity to growth over time so that the progress of the participant can be graphically observable. This is particularly important when the research is carried out on cognitive/intellectual performance due to its gradual rates of change.

Fluency and flexibility scores increased following divergent production interventions in mathematics. Due to the measuring difficulty of originality, this study excluded originality and included elaboration only on math inquiry assessment. Inference from the results should be limited to the changes of some aspects of creativity rather than divergent production ability.
Replication of the study is recommended to provide supplementary activities and assessment instruments. There is no question that more cases support the conclusions of the study. Further study is needed to expand training programs on different behaviors or populations who experience different levels of math difficulty. An extended baseline and prolonged interventions are recommended for ample effects. Intensive instructional program would be necessary for the participants’ substantial improvement as well. Future research is suggested to develop scales and activities of divergent math problem solving that are less variable and more sensitive to the growth of mathematical creativity. Scales should be spread out to reflect improvement of the performance.

Further information is needed to determine if the application of multiple creative programs can possibly enhance students’ understanding and thinking of the solutions in given math problems in many and different ways. Efforts to supplement findings of the study are needed to develop feasible but differential instructional programs for students with math difficulty in order to help achieve better mathematics performance as well as mathematical creativity.
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Dear Parents:

At this time, we would like to invite your child to be the part of the following project. This project will involve your child’s classroom teacher and teacher aide. This program involves a teacher instructing your child for about ten to twenty minutes a day, three times a week and working with your child’s researcher to check on the results of the task on the fourth day of the week. It is expected that this divergent production program in math should help with creative thinking and, hopefully, improve math problem solving skills. Thus, we are hoping to obtain your permission and cooperation for your child’s participation in this voluntary program.

This program has three parts. Part 1 involves assessing your child’s creativity level before the intervention. Part 2 involves training in different divergent production activities each week. These include math problem solving, math inquiry, and think aloud tasks. Part 3 involves assessing your child’s creativity level after the instruction. The testing and monitoring of the program will be done by a teacher (instructor), and the principal investigator.

To ensure that the information we collect is not misused in any manner, your child will be identified by a number instead of his/her name. The information from this program will be used to complete a dissertation and may be shared through articles in professional publications.

If, at any time during the study, you should wish to withdraw your child’s participation, you may do so after discussion with one of our program consultants. However, participation is completely voluntary. If you would like your child to participate in this program, sign the attached Parental/Guardian Consent Form and return the form to the teacher with your child. You may keep this letter for your information.

We are hoping that you will help us by allowing your child to participate in our divergent production program. Thank you very much for your time and consideration in this program.
APPENDIX B
Parental/Guardian Consent Form

Project Title: The Effects of Divergent Production Activities with Math Inquiry and the Think Aloud Strategy of Students with Math Difficulty

Parental/Guardian Consent Form

Researcher: Hija Park, Graduate Student
Intelligence, Creativity, and Giftedness
Dept. of Educational Psychology
Texas A&M University

My child has been asked to participate in a ten week study that looks at how learning to think in creative ways will help students with math difficulties when they are learning math. I understand that my child will participate in an instructional program that lasts for about ten weeks. During the intervention my child will take 10 to 20 minute a day, three times a week lessons and a 20 minute task assessment weekly. For the first two weeks before beginning the intervention, my child will be observed and after the intervention my child will be checked two times after a ten week instruction. My child will be administered a 30 minute creativity test before and after the program (Torrance tests of Creative Thinking, Figural Forms A&B).

The study will be conducted during the spring of 2003 in my child’s elementary school in the Bryan Independent School District. The students will be referred by their teacher. Parent and student permission will be obtained. Students may quit the study at any point by telling the teacher their intent. Quitting the study will not affect my child’s school grades. Student information will be held in strictest confidence and no names or personal information will be shown anywhere in the researcher’s information. Divergent production activities will not cause any harm to the students during or after participation in the activities.

Date ____________ Initial _______
The information obtained from this study will be used to complete a doctoral dissertation and in journal articles. The results of the research will provide a better understanding of intervention techniques and instructional insight for teachers, researchers, and parents of gifted students with learning disabilities.

I have read and understand the explanation provided to me. I voluntarily agree to allow my child to participate in this study. I have been given a copy of this consent form.

Signature of parent/guardian _________________________ Date _______________

Printed name of student ______________________________

I agree to conduct and report this study according to the described terms.

Signature of researcher ______________________________ Date _______________

This research study has been reviewed and approved by the Institutional Review Board Human Subjects in Research, Texas A&M University. For research-related problems or questions regarding subjects’ rights, the Institutional Review Board may be contacted through Dr. Michael W. Buckley, Director of Support Services, Office of Vice President for Research and Associate Provost for Graduate Studies at (979) 845-8585.

For more information about this study, you may contact:

Researcher: Dr. William R. Nash or Hija Park
Dept. of Educational Psychology Dept. of Educational Psychology
Texas A&M University Texas A&M University
College Station, Texas 77843-4225 College Station, Texas 77840
(979)845-1893 (979)862-9244.
email: wnash@neo.tamu.edu hija-park@neo.tamu.edu
APPENDIX C
Student Assent Form

Project Title: The Effects of Divergent Production Activities with Math Inquiry and the Think Aloud Strategy of Students with Math Difficulty

Student Assent Form

I have been asked by Hija Park from Texas A&M University to participate in a study that looks at how learning to think in creative ways will help me when I am learning math. I understand that this study will last fourteen weeks. During this time I will be doing a variety of activities using creative thinking and math. I understand that I will be taking a test on creative thinking at the beginning and the end of the study.

I understand that I will be given a number instead of my name so that no one will know who I am anywhere in the paper. I may quit the study at any time by telling the teacher. I understand that my school grades will not be affected by my choice to participate or not participate in the study.

I have read and understand this project. I have had my questions answered and agree to participate in this study.

Printed name of student ________________________________

Signature of student ________________________________ Date _____________

I agree to conduct this study as it is described.

Signature of researcher ________________________________ Date _____________

Date ____________   Initial _______
This research study has been reviewed and approved by the Institutional Review Board Human Subjects in Research, Texas A&M University. For research-related problems or questions regarding subjects’ rights, the Institutional Review Board may be contacted through Dr. Michael W. Buckley, Director of Support Services, Office of Vice President for Research and Associate Provost for Graduate Studies at (979) 845-8585.

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(979)845-1893.
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Dept. of Educational Psychology
Texas A&M University
College Station, Texas 77840
(979)862-9244.
hija-park@neo.tamu.edu
APPENDIX D

Teacher Consent Form

Project Title: The Effects of Divergent Production Activities with Math Inquiry and the Think Aloud Strategy of Students with Math Difficulty

Teacher Consent Form

Researcher: Hija Park, Graduate Student
Intelligence, Creativity, and Giftedness
Dept. of Educational Psychology
Texas A&M University

I have been asked to participate in a study that looks at the effects of divergent production activities with the think aloud strategy on the mathematical creativity of students with math difficulties. I understand that the study will be conducted during the spring of 2003 in my school in the Bryan Independent School District. I understand the following responsibilities and agree to participate voluntarily in the study for ten weeks.

- identify students who meet the criteria for the study.
- receive training on intervention and observation.
- observe the student and collect baseline data.
- administer a 30 minute creativity test to the student before and after the program (Torrance tests of Creative Thinking, Figural Forms A&B).
- teach 10 to 20 minute lessons, three days a week for about eight week intervention period for the student.
- collect the worksheets and pass them to the researcher.

I am aware that I have the right to withdraw from the study at any point and it is my responsibility to inform the investigator of my withdrawal from the study.

Date ____________   Initial _______
I have read and understand the project. I have been adequately informed of the procedures and responsibilities. I agree to participate in this study. The information obtained from this study will be used to complete a doctoral dissertation and will be publish in journal articles.

Signature of teacher ____________________________ Date _______________

Printed name of teacher ________________________________

I agree to conduct and report this study according to the described terms.

Signature of researcher ________________________________ Date _______________

This research study has been reviewed and approved by the Institutional Review Board Human Subjects in Research, Texas A&M University. For research-related problems or questions regarding subjects’ rights, the Institutional Review Board may be contacted through Dr. Michael W. Buckley, Director of Support Services, Office of Vice President for Research and Associate Provost for Graduate Studies at (979) 845-8585.

For more information about this study, you may contact:

Researcher: Dr. William R. Nash or Hija Park
Dept. of Educational Psychology Dept. of Educational Psychology
Texas A&M University Texas A&M University
College Station, Texas 77843-4225 College Station, Texas 77840
(979)845-1893. (979)862-9244.
e mail: wnash@neo.tamu.edu hija-park@neo.tamu.edu

Date ____________   Initial _______
### APPENDIX E

**Divergent Production Scales (DPS)**

**Summative scale for Think Aloud (TA): 5 minutes**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Tally</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-instruct</td>
<td>None-2</td>
<td>3-5</td>
<td>6-8</td>
<td>9-11</td>
<td>12-14</td>
<td>15-17</td>
<td>18-20</td>
<td>21+</td>
<td></td>
</tr>
<tr>
<td>Self-question</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. Summative scale= a 16 point scale (from least divergent TA=2 to most divergent TA=16)*

**Summative scale for Math Inquiry (MI): 5 minutes**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluency</td>
<td>None-3</td>
<td>4-7</td>
<td>8-11</td>
<td>12-15</td>
<td>16-19</td>
<td>20+</td>
</tr>
<tr>
<td>Flexibility</td>
<td>None-2</td>
<td>3-5</td>
<td>6-8</td>
<td>9-11</td>
<td>12-14</td>
<td>15+</td>
</tr>
<tr>
<td>Elaboration</td>
<td>None (1)</td>
<td>Barely noticeable(2)</td>
<td>Noticeable(3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. Summative scale= a 15 point scale (from least divergent MI=3 to most divergent MI=15)*

**Summative scale for Math Problem Solving (PS): 10 minutes**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluency</td>
<td>None -5</td>
<td>6-10</td>
<td>11-15</td>
<td>16-20</td>
<td>21-25</td>
<td>26-30</td>
<td>31-35</td>
<td>36+</td>
</tr>
<tr>
<td>Flexibility</td>
<td>None -3</td>
<td>4-7</td>
<td>8-11</td>
<td>12-15</td>
<td>16-19</td>
<td>20-23</td>
<td>24-27</td>
<td>28+</td>
</tr>
</tbody>
</table>

*Note. Summative scale= a 16 point scale (from least divergent PS=2 to most divergent PS=16)*
APPENDIX F
Guidelines for Divergent Production Program

I. Introduction

The “Divergent Production Activities” have been developed for the instructor and observers of the study to investigate the effects of the multiple instructions implemented with students with math difficulty. Detailed guidelines provide the standardized procedures of the instruction and observations in the course of the study so that the study can enhance internal validity by controlling the expected threats. Guidelines also provide information to help future researchers replicate the results and eventually make this study have stronger generalizability.

II. Intervention Form

1. The 10, 5, and 5 minute worksheets of math problem solving, math inquiry, and think aloud.
2. Instruction and practice (two-thirds of the time)
3. Instructor – student interaction (one-third of the time)

III. Setting up the Divergent Production Activities

1. Selection criteria for the participants
   1) Identifying eligible students
   2) Having math difficulties as judged by their teachers
   3) Teacher recommendations
   4) Have never participated in divergent production training
2. Selecting the Intervention materials
   Divergent production and think aloud will be trained using worksheets and instructor-student interaction. Divergent production materials used in the intervention will be compiled that are modified from several activities developed by previous studies and by the investigator. The materials will be tried out before being used in the intervention to see whether or not they meet the grade level and the approximate time to complete one worksheet. The content and difficulty level of mathematical divergent production activities should meet the fourth grade mathematics standards of Texas Assessment of Academic Skills (TAAS, 1998).

3. Procedure to instruct problem solving
   The instructor explains the directions and the student works on the problem (for approximately 6 minutes). The instructor plays a role as a guide or a director by answering questions, commenting on the work, and encouraging the participant with a verbal praise (about 4 minutes).
1) No instant correction of answers or negative comments is given to the student.
2) If the student completes the worksheet or the assigned time is over, the instructor tells the subject comments like “good job” and collects the worksheet.
3) During the ensuing session, the paper is returned to the student with verbal comments, not numerical scores.

4. Procedure to instruct math inquiry

1) The instructor explains the directions and the student works on the problem (at least 3 minutes). The instructor plays a role as a guide or a director by answering questions, commenting on the work, and encouraging the student with a verbal praise (about 2 minutes).
2) No instant correction of answers or negative comments is given to the student.
3) If the student completes the worksheet or the assigned time is over, the instructor tells the subject comments like “good job” and collects the worksheet.
4) During the following session, the paper is returned to the student with verbal comments not numerical scores.

5. Procedure to instruct think aloud

1) The instructor explains the directions and the student practices the example problem (at least 3 minutes). The instructor plays a role as a monitor by observing the subject working on the task, and by encouraging the student to talk out while thinking of solutions (about 2 minutes).
2) Correction of the answers or any hints can be given if the student asks for it.
3) If the student completes the worksheet or the assigned time is over, the instructor tells the subject comments like “good job” and collects the worksheet.
4) During the next session, the paper is returned to the student with verbal comments not numerical scores.

IV. Instruction Protocols

*Example interactions for the PS, MI, & TA instructions*

**PS:** Problem solving is to think a problem in getting the answer in many different ways. You think of as many ideas as you can. You also think of as many different ideas as possible for a limited time.
Today we’re going to have a new problem.
What is your problem?
  ■ My problem is _____.
How can you do it?
  ■ First, I’ll read the problem carefully two times.
  ■ Then I look at the problem to find what information I have.
  ■ I try one and draw my idea.
You have more than one idea, don’t you?
  ■ I can think of other ways.
Very good, let’s try the next one.
  ■ I did the second one.
What are your answers? Are they different or similar?
Can you think of other ways to look at this problem differently?
Keep interacting when the time is up.
What are the important points when you do problem solving?
  ■ Think differently than the previous one I did.
  ■ Think of as many ideas as possible.
  ■ Think of other ways when I am stuck in the problem.
You did great!

MI: Math Inquiry is a useful skill to ask questions in solving a given problem and find missing information. You think of as much information and many questions as you can. Think of as many different ideas as possible for a limited time. You can also use other data such as statistics, graph, drawing, and charts when explaining your idea.

Today we’re going to have a new problem.
What is your problem?
  ■ My problem is _____.
How can you do it?
  ■ First, I read the problem carefully two times.
  ■ Then I look at the problem to find what information I have.
  ■ I try one and write my idea.
What questions (information) can you ask (learn) using the given problem?
(When the student seems to be stuck in the problem), you have more than one idea, don’t you?
  ■ I can think of other ways.
Very good, let’s try the next one.
  ■ I did the second one.
Are the answers different or similar?
Can you use any other ways when you make your own question?
Can you think of some other things to make your question more meaningful and clear? For example, you can describe your information further using other means like graph, drawing, and sketches.
Can you think of other ways to look at this problem differently?
Keep interacting when the time is up.

What are the important points when you do math inquiry?
- Think differently from the previous one I did.
- Think as many questions as possible.
- Think elaborately as much as you can.
- Describe the details of your solution.
- Think other ways when I am stuck in the problem.

You did great!

You learned a lot about thinking more of the solutions.

**TA:** Thinking aloud is one way of teaching someone how to do something. I will teach you how to think and solve the problem. You also check the answers you got by asking yourself questions. Repeating instructor’s talk is sometimes necessary.

Today we’re going to have a new problem.

What problem do you have today?
- My problem is _____.

Good.

How can you do it?
- How can I do it?
- First, I’ll read the problem carefully two times.

Great.
- Then I look at the problem to find what information I have.

What information do you have?
- I have _____ and ________.

What do you need to do?
- I need to figure out ________________ first.

Can you check the answer if it is correct?
- I can do __________ to check the answer.

Very good. Then what do you do next?
- I need to figure out ________________.

Do you think you need to check if you got the right answer?
(repeat this procedure until all is done.)

Let me hear you think out loud.
- You don’t need to look at me and talk to me.
- You talk to yourself and you ask yourself just loud enough.
- I must hear you what you think.
- The instructor encourages the student to think again by asking him/herself.

Let me hear you think out loud.

Keep interacting when the time is up.

What are the important points when you do think aloud?
- Think out loud enough and always check your thinking.
- Think other ways or go back to the previous step when I am stuck in the problem.
You did great!
You learned a lot about Think Aloud skills.

V. Assessment Protocol

PS assessment protocol

Today, I’m going to see how you are doing on the Problem Solving question. Remember you have learned some problem solving strategies and I want you to use those skills this time. Make the most of your 5 minute time to think of plenty of new ideas.

Open your folder and read the problem.
Do you understand the problem?
Okay, good. Let’s begin!

MI assessment protocol

Today, I’m going to see how you are doing on the Math Inquiry problem. Remember you have learned some Math Inquiry strategy this week. So this time, I want you to use those skills in solving this problem. You need to write down questions (information) that you can tell from the given problem. Make the most of your 5 minute time to produce lots of ideas.

Open your folder and read the problem.
Do you understand the problem?
Okay, good. Let’s begin

TA assessment protocol

Today, I’m going to see how you are doing on the Think Aloud problem. Remember you have learned some Think Aloud strategy this week. So this time, I want you to use those skills in solving this problem. You need to keep talking about what you have in your head. Once you start working on it, I will not say, like “tell me what you’re thinking” or “let me hear what you think.” Make the most of your 5 minute time to think aloud.

Open your folder and read the problem.
Do you understand the problem?
Okay, good. Let’s begin!
VI. Instructor Training

Instructor must be in one two hour training session with the researcher on how the instruction should be taught and directed for students’ math performance and mathematical thinking in the given problem solving situation. The divergent production training proceeds as follows through instructor–researcher discussion.

1. Instructor provides and assembles materials:
   Timer, Worksheet, Pencil, and Eraser

2. First, during the instruction-practice section, instructor distributes the worksheet. Problem solving is 10 minutes. At the beginning, tell the student the following:

   Today, you will have only one problem you can work with. You will have chance to explore and have fun with it for about 6 minutes. You will find yourself so good at thinking up new ideas and solving problems in many different but interesting ways. Now, let’s look at the problem and read it carefully. Let’s work on the problem first until the time is called.

   Let the student begin to work on the task.

3. Second, at the interaction section, tell the student the following:

   If you have any questions that come up while you are working on the problem, feel free to ask or talk about it.

   Showing the example of the problem to the student, the instructor can say something like “Can we think of it this way? What do you think of the solution? These kinds of questions can prompt the student’s thinking more differently next time than this time. The student discusses with the instructor for 4 minutes.

4. Wrapping up the session of the day, the instructor collects the worksheet from the subject and informs the next meeting schedule. The instructor says thanks and gives some encouragement to the student. Before the next instruction session, the instructor provides the subject with feedback on the work. The copy of the original worksheet with the feedback will be returned to the student. The instructor scores the worksheet of the observation and gives it to the researcher.

VII. Observation and Scoring

1. Observation and scoring will take place once a week basis for the same amount of the time instructed and practiced, i.e., problem solving for ten minutes, math inquiry for five minutes, and think aloud for five minutes. Observers other than the instructor observe and score the work and turn the recording form in to the investigator.
2. Each example of the scoring variables are as follows:
   1) PS-fluency
   2) PS-flexibility
   3) MI-fluency
   4) MI-flexibility
   5) MI-elaboration
   6) TA-instruct
   7) TA-question

3. Example for Scoring Practice

   Problem: By joining dots on a nine dots centimeter square grid with straight lines, draw as many different sizes of a polygon area as you can find which have an area of 2 cm².

   Example of Problem Solving
   1) PS-fluency: The observer simply counts how many polygons of an area of 2 cm² were completed. If the shape is formed but the area is not correct, it is not counted as a case.
   2) PS-flexibility: The observer counts the number of different categories. For example, if the shape and size of the two polygons are the same but only the positions are different, it is counted as one category.

   Example of Math Inquiry
   3) MI-fluency: The observer simply counts the number of all relevant questions to solve the problem. Unless it is irrelevant, each case is counted.
   4) MI-flexibility: The observer counts the number of different questions to be asked. For example, the subject asks the length once, then asks the formula of the volume. That is counted two categories.
   5) MI-elaboration: The observer finds some evidence to illustrate or describe the solution further. For example, when presented maps, graphs, sketches, or data, then elaboration is scored.

   Example of Think Aloud

   6) TA-instruct: All statements out loud other than statements about questioning themselves and checking the answers are counted self-instruction. Each sentence makes one occurrence. Such short notions like “ok”, “now”, or “good” is not counted since they do not convey the meaning of the next step of behavior direction concretely.
   7) TA-question: The interrogative statements made by the subject to get the solution are counted. The subject asks him/herself if he/she is in the right
track of solving the problem. Examples are like “what will I need to find out?” or “should I try one higher?” when guessing the answer in computation.
Think aloud while you are working on the cross number puzzle. You may use any number that makes the sum of any straight line equal. You may use the space below the puzzle if you need.
Think aloud while you are working on the problem. If you put your hand into a pouch, how often would you expect to pick up a red marble? Inside the pouch, there are 2 blue marbles, 3 red marbles, and 4 green marbles. You cannot see inside the pouch when you pick up a marble. You may use the picture below to figure out the answer.
TA Assessment 3

Think/talk aloud while you are working on the problem. Find how many unit cubes are there in the stack of a big cube.
TA Assessment 4

Think aloud while you are working on the problem. Find the area of the figure. One square represents one square unit.
TA Assessment 5

Think/talk aloud while you are working on the problem. How many cubes are needed to completely fill the box?

![Diagram of a box with annotations for 9, 3, and 6 cube units]
TA Assessment 6

Think aloud while you are finding as many paths as you can. Any path you find in the number grid must make a sum of 12 and the path consists of four neighboring numbers. Draw a line to show each path you find.

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<th>4</th>
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TA Assessment 7

Think/talk aloud while you are working on the table. Find the answers in the blanks. The number of legs of two ducks equals to that of one dog.

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<thead>
<tr>
<th>Dogs</th>
<th>1</th>
<th>2</th>
<th>15</th>
<th>100</th>
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<tr>
<td>Ducks</td>
<td>2</td>
<td>4</td>
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Think/talk aloud while you are solving the problem.
Find the area of the figure. One small square represents one square unit.

Answer: _______________________________
TA Follow Up 1

Suppose you spend your day as in the table below. Think out loud while you are working on the problem to find out what fraction of the day do you spend on Homework?
TA Follow Up 2

Think/talk aloud while you are working on the problem. How many cube units are there in this stack?

Answer: _______________________________
MI Assessment 1

List as much information as you can tell from the given number line.

- ▶️
- ▶️
- ▶️
- ▶️
- ▶️
- ▶️
- ▶️
- ▶️
MI Assessment 2

Write down as many questions as you can think of that numbers 18 and 28 have in common.

✧

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MI Assessment 3

There are many shapes below. Write as many questions about how these shapes can be sorted.

- 
- 
- 
-
MI Assessment 4

Write down as many questions as you can ask from the given graph. You can title the graph whatever you want to make your own questions.

- 
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MI Assessment 5

Write down as many questions as you can think of from the given problem. Each square is one square unit.
MI Follow Up 1

What information can you tell about this grid? List as many possible questions as you can think of.

- ____________________________
- ____________________________
- ____________________________
- ____________________________
- ____________________________
MI Follow Up 2

There are many shapes below. Write as many questions about how these shapes can be sorted.

- 
- 
- 
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PS Assessment 1

Draw the figure that has an area of 5 square units on the grid as many different shapes as you can think of.
PS Assessment 2

By using straight lines, connect points indicated by a dot. Draw as many shapes that are evenly spaced as you can think of.

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PS Assessment 3

Using four triangles below, draw as many different arrangements as you can think of. All triangles must be connected to each other in any possible ways.
PS Follow Up 1

Draw as many different arrangements as you can think of by cutting the shape into three pieces as drawn below.
PS Follow Up 2

By joining dots on a nine dot square grid with straight lines, draw as many shapes as you can.

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   . . .   . . .   . . .
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   . . .   . . .   . . .
   . . .   . . .   . . .
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   . . .   . . .   . . .
   . . .   . . .   . . .
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College of Education
Department of Educational Psychology
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HIGHLIGHTS OF QUALIFICATIONS

- Five years of research experiences in intelligence, creativity, and giftedness field with qualitative/quantitative methods.
- Co-authoring and co-publishing experiences.
- Strong teaching experience in elementary education.

EDUCATION

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Fall, 2002-Present      Graduate assistant. Educational Psychology, Texas A&M University, College Station, TX
Summer-Fall. 2002       Translator, TAMU Children’s Center Family Handbook, College Station, TX
June, 2001 & July. 2002 Camp Counselor, Youth Adventure Program, Practicum in Educating Gifted and Talented, College Station, TX
September.1986-June. 1999 Teaching, Elementary Schools, Inchon, Korea