CREATING A QUANTUM GATE THROUGH STRONG COUPLING OF A TWO-LEVEL SYSTEM IN A NANOCAVITY

An Undergraduate Research Scholars Thesis

by

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This project did not require approval from the Texas A&M University Research Compliance & Biosafety office.

TABLE OF CONTENTS

ABSTRACT

Creating a Quantum Gate Through Strong Coupling of a Two-Level System in a Nanocavity

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Since the dawn of human observation, we have been studying the interaction between light and the rest of the universe. From the harnessing of fire to the use of telephones, humans have been finding new ways to use light to communicate with one another. Today, our communications have transitioned to a nanolevel, and great strides have been made in the recent past within quantum computing and quantum information systems. These quantum computers are analogous to modern computers in the fact that modern computers compute using bits and logic gates, but quantum computers will transfer information using quantum gates created by a coupled two-level system. Transitioning a two-level system between states can be done in many different ways. In the past, studies would often show that a molecule with two specified energy states can transition between them based on atomic beam pulses. A more effective approach would be to use a classical or semiclassical field to transition between states. We find that quantum gates can be created through a variety of variations of a two-level system. Whether through a quantum dot, a single molecule transitioning between the electronic and vibrational frequency modes, or two molecules coupled by the cavity mode, a quantum gate can be utilized in many different applications. We show

that depending on how you set up the initial conditions you can achieve several different types of quantum gates depending on the desired results.

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1. INTRODUCTION

In recent years strong coupling of nanocavities to qubits has attracted interest in the study of Quantum Information and creating environments to form a quantum gate for computing. In this paper, Different ways will be identified to make a quantum gate in a nanocavity. The problem is not only in the creation of these quantum gates but being able to identify the state that they are in to properly use them as logic gates.

First, the problem of changing the state, using a classical electric field, of a single qubit in free space. Many problems like this have been done before starting with the Tavis Cummings problem in 1968 [1]. This can provide a proper foundation for developing further into the problem ahead. The Tavis Cummings paper can only get us so far that it deals with N many identical qubits and does not account for nonuniform qubits or a nonspacially uniform field interacting with the qubits. This problem has been identified and solved in several different ways with different assumptions to obtain a fairly analytical solution, but should still be adjusted to fit the problem that is presented here [2]. The first major adjustment is that we are changing the states of these two-level systems using photons and a classical electric field, different than the stream of atoms commonly used in older papers. The analytical solution obtained first is going to be one with exact resonance and a pulse including no dephasing or relaxation. The previous analytical solutions were all found using the Schrodinger picture, however, to find the solutions needed for the dissipation of the density matrix. When adding in dissipation the stochastic method will be used in the interaction picture of the hamiltonian.

These processes will be explained in detail on how to excite our system into dark and bright states using both quantum and classical fields. Each of these will come with advantages and disadvantages in using these methods and these will be detailed in the sections below, with the goal to get our system into a dark state with no mixing into the ground state.

Next the problem of how to excite a simple two-qubit system in a quantum field will be

addressed because this is looking like the best solution to getting our system into an entangled set of qubits. Using this method we will be able to develop our system into a series of bright and dark states. A bright state is a quickly dissipated state that is strongly coupled to its environmental radiative field, and a dark state of that which is decoupled from the system and is long-lasting in the cavity in which it is placed. With an N-qubit system, we will see how depending on which way we set up our system we are N times more likely to find our system in a dark state than in the ground state or a bright state. However, our goal is to create this with only a two-qubit gate and the probability that we find our system in a dark state decreases as the probability of finding the system in the ground state becomes greater probability.

Here we will discuss dissipation and the different methods which are commonly used and we will consider them. One that is often used in quantum systems like this is the Heisenberg-Langevin approach, but the Heisenberg equations become nonlinear and the stochastic equations of state that we use are always linear. So we switch to using the Lindbladian operators to fill the Hamiltonian of our system and address the problem of dissipation.

Now the problem or solution to be found is how to set up a photonic qubit to represent our signal qubit that will eventually reflect off of our control qubit. There are possible ways to use the field to interact with the cavity mode or use light to change the state of the control qubit and the photonic qubit which in the end will make our CNOT gate. However, the best method is still needed based on the setup of the nanocavity and the field used. Several methods that have been shown are coupling two molecules to each other as well as the cavity mode along with creating a gate from a single molecule by sending in multiple control frequencies that adjust both the electronic and the vibrational frequencies. This is one step closer to allowing quantum computers to reach their potential.

2. FORMULATION OF A SINGLE TWO-LEVEL SYSTEM IN FREE SPACE

Initially, we must identify what type of hamiltonian should be used for this problem. Of course, the simplest possible solution for the classical system would be a monochromatic electric field. However, this might not be a very practical solution. A more commonly used solution would have a pi pulse or a Gaussian, as these are what we typically find in the laser pulses used in transitioning the energy state of a two-level system. In the end, we don't want to be restricted by a specific kind of pulse so a solution for an arbitrary electric field is to be desired. This solution will also be including both dephasing and dissipation using the stochastic method within our system. All of this is to set up our transition from a single two-level system to the entangled multiple two-level systems that our gates will be created. However, without a fundamental understanding of how to properly excite the energy levels of our atoms and molecules when there is just a single qubit that we are working with then none of this will be feasible.

2.1 Monochromatic Field

While it might not be a practical solution it is vital to understand the process of being able to identify the probabilities of what state our system is in using only a monochromatic field. Most lasers you find in an experimental setting will not use a monochromatic field but can be manipulated in such a way as to mimic that of a monochromatic field with a series of pulses into our atom or molecule. Our field will consist of a single real value and will not need any field operators at the moment. Consider the trivial problem of a single two-level system with states $|1>$ and $|0>$, where the states have energy levels 0 and ω. The fermionic annihilation and creation

operators and the dipole moment operator are

$$\begin{aligned} \hat{\sigma} &= |0\rangle\langle 1| \\ \hat{\sigma}^{\dagger} &= |1\rangle\langle 0| \\ \hat{d} &= (d\hat{\sigma}^{\dagger} + d^{\star}\hat{\sigma}) \end{aligned} \tag{1}$$

which come together to form the hamiltonian in the interaction picture for a monochromatic field.

$$\hat{H}_{int} = -\hat{d} \cdot \mathbf{E} \tag{2}$$

For now, we will stay in the interaction picture and assume that there is exact resonance so that we do not need to worry about arbitrary detuning. Now we apply the Schrodinger Equation with the Hamiltonian to a wave function. This wave function will only have the states C_0 and C_1, in the ground state and first excited state respectively, since we are dealing with only a single two-level system.

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}_{int}|\Psi\rangle \tag{3}$$

From Equation 3 we will obtain

$$\begin{aligned} i\hbar(\dot{C}_0|0\rangle + \dot{C}_1|1\rangle) &= -E(d\hat{\sigma}^{\dagger} + d^{\star}\hat{\sigma})(C_0|0\rangle + C_1|1\rangle) \\ &= -E(dC_0\hat{\sigma}^{\dagger}|0\rangle + d^{\star}C_0\hat{\sigma}|0\rangle + dC_1\hat{\sigma}^{\dagger}|1\rangle + d^{\star}C_1\hat{\sigma}|1\rangle) \\ &= -E(dC_0|1\rangle + d^{\star}C_1|0\rangle) \end{aligned} \tag{4}$$

which we will get the differential equations

$$\begin{aligned} \dot{C}_0 &= \frac{i}{\hbar}Ed^{\star}C_1 \\ \dot{C}_1 &= \frac{i}{\hbar}Ed^{\star}C_0 \end{aligned} \tag{5}$$

Here we made the substitution that the Rabi frequency will be

$$\Omega_R = \frac{dE}{\hbar} \tag{6}$$

which is a function of the field itself. Now the analytical solution here is showing us that after a certain amount of time under the influence of a monochromatic field will naturally populate into the excited state, and with no dissipation, the system will remain in that state until something perturbs it.

2.2 Pi Pulse

A pi pulse is a description of pulse that is used to manipulate the state of our quantum system, whether that be one or N many qubits for our nanocavity. The term “pi” refers to the fact that the pulse has a set area designed to undergo a complete inversion of its quantum state or a 180-degree rotation in the Bloch vector that represents the state of the atom or molecule. The shape of the pulse is irrelevant to a pi pulse as long as the integral of the field over a set time is pi or pi/2. Each being equally useful in manipulating qubits

In "Optical Resonance and Two-Level Atoms" Allen and Eberly explain a pi-pulse as an EM pulse that corresponds to a particular transition to an atom or molecule. [3] A pulse of area pi flips the phase of the probability amplitude of the states, while the pi/2 pulse creates the symmetric superposition between both the upper and the lower state.

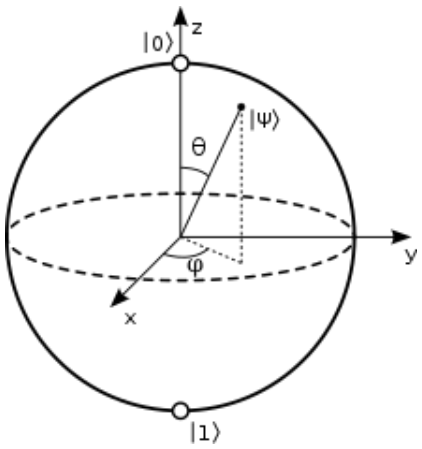

Figure 1: A physical representation of a Bloch Sphere. [4]

In addition, the book discusses how pi pulses can be used in various applications of quantum information processing, such as the implementation of quantum gates, such as the CNOT gate, and the creation of entangled states. Overall, the concept of a pi pulse is a commonly used tool in the manipulation and control of two-level quantum systems, exactly what we are trying to do. However, there is nothing that makes a pi pulse specifically unique rather it can be used with almost any type of laser.

2.3 Gaussian Pulse

One way to shape the pulses we are trying to create would be to use some form of a Gaussian curve. Here the Gaussian pulse is centered at a time of 0 and has a pulse width of τ. This here would better representation of an actual laser pulse that an experiment of this type would use. Staying in the interaction picture we will use the field function

$$E(t) = Ee^{-t^2/\tau^2} \tag{7}$$

So that will set up our state equations to be

$$C_0(t) = \frac{1}{2}e^{-i\frac{1}{2}\tau\sqrt{\pi}\Omega_R Erf(t/\tau)}(1 + e^{i\tau\sqrt{\pi}\Omega_R Erf(t/\tau)})$$
$$C_1(t) = \frac{1}{2}e^{-i\frac{1}{2}\tau\sqrt{\pi}\Omega_R Erf(t/\tau)}(-1 + e^{i\tau\sqrt{\pi}\Omega_R Erf(t/\tau)}) \tag{8}$$

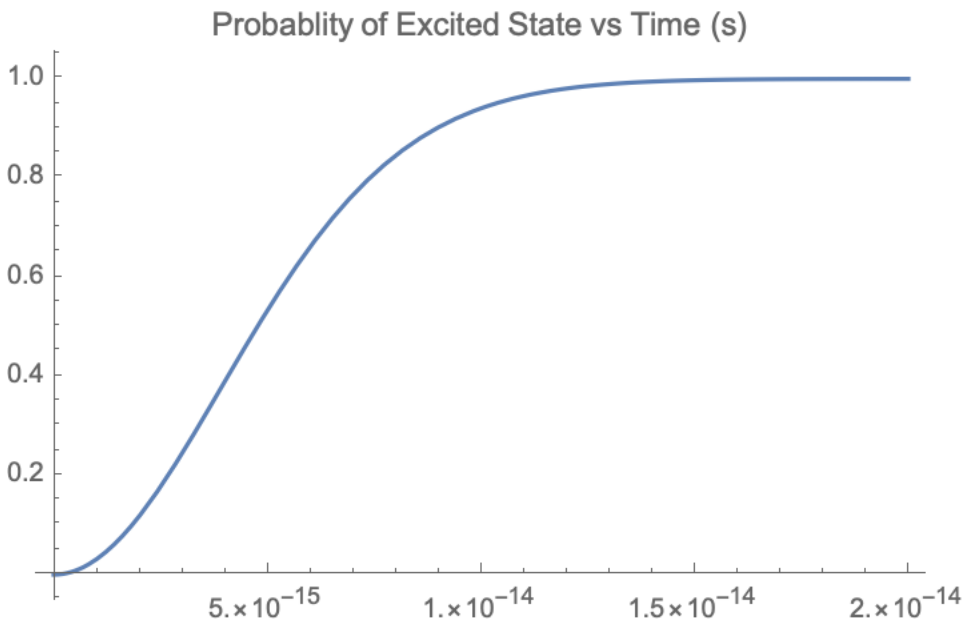

Figure 2: $|C_1|^2$ vs Time (s) graph for our gaussian pulse of the nature described above.

Using the dipole moment values of d = 6.45×10^{-18} cm StatC from Phthalocyanine Chloride Tetrasulfonic Acid and the initial condition that the system is found in the ground state, we get the probability that the excited state will be occupied as shown above.

Again here there is no dissipation in our system so the excited state will remain occupied if the pulse meets the proper conditions to raise our system to the excited state in the first place. A way to make our Gaussian equations a little more practical would be to shift the pulse over so that it is not centered on zero but perhaps a time further on. The same result should be obtained, but could better represent the time dependence of the actual Gaussian pulse.

2.4 Arbitrary Field

Now every laser is not going to have a monochromatic, pi pulse, or Gaussian pulse so we want a set of solutions that will be able to accommodate any arbitrary fields that might be used in a problem as such. We can do this by following a similar procedure to the above two while maintaining the time dependence (not spacial dependence) of our Rabi frequency. Starting off by plugging in our new field to the state equations, in the interaction picture still, we get

$$\begin{aligned} \dot{C_0} &= -i\Omega_R(t)C_1 \\ \dot{C_1} &= -i\Omega_R(t)C_0 \end{aligned} \tag{9}$$

This is very similar to the equations that we get for our monochromatic field, but now the difficulty in finding the C_1 and C_0 is that our Rabi frequency is now time-dependent upon the specific pulse that is chosen. Several methods can be used to solve a differential equation such as this one, here we will use the first integral method. By identifying our two different time-dependent variables we will get the differential equations of

$$\begin{aligned} C_0 &= \cos(\int_0^t W(t^{'})dt^{'}) \\ C_1 &= i\sin(\int_0^t W(t^{'})dt^{'}) \end{aligned} \tag{10}$$

Here this can be set up as a set of trigonometric functions that is purely dependent upon the solution to the time-dependent Rabi frequency equation from the start of the pulse to whatever arbitrary time we want to consider. Here exact resonance is what is being taken into consideration. So the difference in the energy levels exactly matches the frequency of the incoming light, or it is minuscule compared to the Rabi frequency in the equations. If arbitrary deturning were to be accounted for then there would not be any analytical solution to this problem and the only thing that could be found would be a numerical solution based on the initial conditions and the set of the cavity, incoming field, and qubits.

3. SETTING UP THE COUPLED TWO-LEVEL SYSTEM

3.1 Single-Photon Excitation

To perform quantum computations, we need to be able to create entangled states of qubits and perform operations on them. This requires the use of lasers to excite the qubits into different states, and it is essential to understand how these states interact with the laser field.

An entangled state, in this context, is where we have two or more quantum bits that are correlated in such a way that they cannot be described independently of the state of the other particles. The particles are interdependent upon each other meaning that measuring one of these particles separately will change the value of the other simultaneously. Specifically, here we will be looking at the entangled or dark states of just two qubits.

To create a quantum gate, we need two entangled two-level systems that can be excited into different states. The four possible states are $|0,0\rangle$, $|0,1\rangle$, $|1,0\rangle$, and $|1,1\rangle$, with the bright states being $|0,0\rangle$ and $|1,1\rangle$, and the entangled dark states being the superposition of $|0,1\rangle$ and $|1,0\rangle$. These could be found in either the symmetric state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ or the anti-symmetric state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. When the laser frequency is completely detuned from both energy levels of the two-level system, the two molecules in the excited states can form a coherent superposition, leading to a bright state. The bright state is characterized by strong coupling to the laser field and a fast decay through spontaneous emission.

However, when the laser frequency is resonant with the energy levels of the two-level system, the qubits can become decoupled from the laser field and experience coherent population trapping (CPT). CPT occurs when the molecule is exposed to a resonant laser field and is set into a coherent superposition of the energy levels in the qubit. This coherent superposition can lead to destructive interference in the radiation patterns within the qubit, leading to a suppressed decay of the excited state. This phenomenon leads to a long-lived dark state that does not decay with spontaneous emission.

In a multi-qubit system, such as the two entangled two-level systems required for a quan-

tum gate, there can be a collective of dark states, each of which is a coherent superposition of the individual dark states of each system. For our system, these are the symmetric state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ or the anti-symmetric state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ states, which can be considered a single dark state. The collective dark state stays long-lived, leading to slow decay and low rates of emission. This makes them ideal for use in quantum gates, as they can maintain their quantum state for longer periods of time, allowing for more complex computations to be performed.

Bright states are formed when the laser field is detuned from the qubits' energy levels, leading to strong coupling and fast decay. Dark states occur when the qubits are resonantly excited and experience CPT, resulting in a long-lived state with slow decay and low emission rates. In a multi-qubit system, there can be a collective of dark states, which are ideal for use in quantum gates as they can maintain their quantum state for longer periods of time [5].

Now the question needs to be asked of how can we get our system into either a bright state or a dark state to control our gate in the end. If this cannot be done with consistency then the whole scheme will fall apart. The methodology to do this will be using single-photon excitation in a monochromatic field. Above is discussed as an arbitrary field, but if we use a scheme called optical pumping the field of the laser will simulate that of a monochromatic field.

In the process of optical pumping the molecules or ions are excited from lower to higher energy levels by absorbing photons of a specific wavelength. If the frequency of the photons is tuned to match the frequency of the electric field the absorbed photons will then transfer energy to the molecules to simulate an electric field. Furthermore, if the light is polarized, the photons will preferentially affect molecules or qubits in that orientation, simulating that of a monochromatic field polarized in that specific direction. This allows us to maintain our Rabi frequency at a constant value.

Next a process will be used called single-photon excitation to excite one of our entangled qubits into an excited state. This however will need to use properties of a quantum field requiring new operators in the hamiltonian. The field will need to have not only energy level raising and lowering operators but also photon creation and destruction operators. The picture will also be

shifted from the interaction picture to the Schrodigner picture to get a better understanding of all that is happening in the cavity at this time. The dipole moment operation is much the same as it was above. However, the electric field, while time-independent, will have photon creation and destruction operators implemented into it.

$$\hat{E} = E(r)\hat{c} + E^{\star}(r)\hat{c}^{\dagger} \tag{11}$$

As well as the qubit operator and an additional term to the field operator after normalizing the field.

$$\begin{aligned} \hat{H}_{em} &= \hbar\omega(\hat{c}^{\dagger}\hat{c} + \frac{1}{2}) \\ \hat{H}_{a} &= W(\hat{\sigma}_0^{\dagger}\hat{\sigma}_0 + \hat{\sigma}_1^{\dagger}\hat{\sigma}_1) \end{aligned} \tag{12}$$

After all of these additions, the new total hamiltonian of our system will be

$$\hat{H} = \hat{H}_a + \hat{H}_{em} - \hat{d}\cdot(E(r)\hat{c} + E^{\star}(r)\hat{c}^{\dagger}) \tag{13}$$

Here it is assumed that the field amplitude is the same for all positions around the nanocavity. The wavefunction for this system is set up as shown.

$$\psi = \sum_{n=0}^{\infty}\sum_{p=0}^{N}\sum_{\alpha_p=0}^{C_N^p} |n\rangle|p,\alpha_p\rangle \tag{14}$$

Here we have an N-qubit system which is expanded to show that $|n\rangle$ is the boson state and $|\mathrm{p},\alpha_p\rangle$ is the fermion state. [6]

However, with multiple many photons in an N-qubit system the algebra, gets much too complicated. To simplify, only a single photon will be used to excite this two-qubit system. These single photon emitters are available to use for both single and N-qubit systems. Here the wave function will give us only four possible states. The ground state without a photon, the ground state

with the incident photon, and our two entangled states. We only care about the latter three states mentioned when it comes to finding out if our system is in a dark or bright state, but that will be explained further in the paper.

$$\psi = C_{00}|0\rangle|0_0\rangle|0_1\rangle + C_{10}|1\rangle|0_0\rangle|0_1\rangle + C_{01}|0\rangle|1_0\rangle|0_1\rangle \tag{15}$$

Using previous results from the A. Belyanin research team we get that a nontrivial analytical result can be expressed for an N-qubit system with arbitrary frequencies, still within RWA, also in the presence of dissipation and noise for both the molecules and the cavity mode. Here we will use only the interesting results that give us spontaneous long-lasting dark states. Substituting our wave function here with the total hamiltonian listed in this section we will get a set of three differential equations for our system.

$$\begin{aligned} \dot{C}_{00} &= -i\frac{\omega}{2}C_{00} \\ \dot{C}_{10} &= -i\frac{3\omega}{2}C_{10} + i\Omega_R^\star C_{01} \\ \dot{C}_{01} &= -i\frac{3\omega}{2}C_{01} + i\Omega_R^\star C_{10} \end{aligned} \tag{16}$$

From these three equations we can see that the first essentially represents what happens to our system when nothing is done to it. It will remain in the ground state and no photon will be produced. This first differential equation is completely uncoupled from the other two. If we factorize the initial condition of $\psi(0) = |1\rangle|0_0\rangle|0_1\rangle$ in which the photon mode is excited, but all of the qubits are in the ground state. We will get that our wave function will now be

$$\psi = e^{-i\frac{3\omega}{2}t}(cos(\sqrt{2}|\Omega_R|t)|1\rangle|0_0\rangle|0_1\rangle + \frac{ie^{iArg(\Omega_R)}}{\sqrt{2}}(|0\rangle|1_0\rangle|0_1\rangle + |0\rangle|0_0\rangle|1_1\rangle)) \tag{17}$$

. When squaring these values we will find out the percentage that we will find the photon in one of these states. Here we come across an issue though. After the incident photon is absorbed, there is still a chance that we find our system in the ground state. At a time of 0 we get that this is

going to be a 100 $\%$ chance, but as time goes on this will vary based on the C values for our other two states.

We can also work with the initial conditions where the qubit is excited first and not the photon mode. This will imply that our qubits will need to be individually addressed and can be done in several different ways (e.g. using a near-field nanotip with an excitation radius smaller than the distance between entangled molecules) The wave function for these new initial conditions will get the $\psi(t)$ to take on the form

$$\begin{aligned} &= e^{-i\frac{3\omega}{2}t}(\tfrac{ie^{iArg(\Omega_R)}}{\sqrt{2}}sin(\sqrt{2}|\Omega_R|t)|1\rangle|0_0\rangle|0_1\rangle \\ &\quad +(1-\tfrac{1-cos(\sqrt{2}|\Omega_R|t)}{2})(|0\rangle|1_0\rangle|0_1\rangle) \\ &\quad -\tfrac{1-cos(\sqrt{2}|\Omega_R|t)}{2}(|0\rangle|0_0\rangle|1_1\rangle) \end{aligned} \tag{18}$$

. With a higher N value of qubits, we get that the initially excited qubit will oscillate around one with a small amplitude of around 1/N and all of the other terms will do the same oscillation around zero. Meaning that the system is N times more likely to be found in the initial state than having relaxed into ground state qubits or having the photon transfer from the excited qubit to the field. This holds true even if we don't know the excited qubit, but the excitation radius must remain smaller than the distance between the qubits. On the other hand, this experiment is being done with two qubits and so we are only about two times more likely to find the system in the initial state than the other two possible states.

However, we want the smallest ground state probability that we can get after excitation. One method of doing this is called dynamic decoupling. This is used to mitigate the effects of decoherence and improve the consistency of qubit state preparation. With decoherence being a major issue in our setup, especially with the ground state and excited states, there will always be states that decay quickly back to the ground. The idea behind dynamic decoupling is to apply a series of pulses to the qubits at a specific interval that is designed to counteract the effects of the environment [7]. The design of these control pulses is critical to the technique and involves a careful balance of the environment and the pulse generator if indeed the desired effect is to

be achieved. A popular approach is to use a series of pulses based on the Hahn echo effect, which involves applying a series of pi pulses at specific intervals to cancel out the effects of the environment [8].

3.2 Dissipation

Now that we have obtained equations for our three possible states we should consider what dissipation will look like in one of these nanocavities. The standard form to add dissipation is using the master form of the density matrix [9]

$$\begin{aligned}\frac{d}{dt}\hat{\rho} &= -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \hat{L}(\hat{\rho}) \\ &= -\frac{i}{\hbar}(\hat{H}_{eff}\hat{\rho} - \hat{\rho}\hat{H}_{eff}^{\dagger}) - \hat{L}(\hat{\rho})\end{aligned} \tag{19}$$

In the Markovian models of relaxation we get that the state vector will take on the form

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{i}{\hbar}|R\rangle) \tag{20}$$

with $\langle\alpha|R\rangle = R_\alpha$. The $|R\rangle$ vector should have the statistical properties:

$$\overline{|R\rangle} = 0,\ \overline{R_\alpha(t')R_\beta(t'')} = \hbar^2\delta(t'-t'')D_{\alpha\beta},\ D_{\alpha\beta} = \langle\alpha|\hat{L}(\hat{\rho})|\beta\rangle_{\hat{\rho}\to\overline{|\psi\rangle\langle\psi|}} \tag{21}$$

One method commonly used is the Heisenberg-Langevin approach, which describes the dissipation of our system and qubits—using the Heisenberg equations of motion and the Langevin equation for environmental noise. This approach describes a set of operators that satisfy the Heisenberg equations of motion, which relate the time derivative of the operator to the commutator with the Hamiltonian of the system. With the Langevin equation, we could describe the time evolution of the environmental noise, which is explained as a random process with statistical properties. This mainly includes a dampening term that is due to the dissipation into the environment. In a sense, this is a stochastic process that treats the environmental noise as a random variable. The statistical properties of the noise can be characterized by its power spectrum, here describing the

distribution of noise power as a function of frequency [10].

However, the Heisenberg equations are nonlinear, but the stochastic equations that we have set up for the state vector are so we will move forward with the Lindbladian approach. The Lindbladian approach is used generally for open quantum systems that are coupled to their environments and thus subject to decoherence and dissipation. The Lindbladian operator is a generalized formulation of the Schrodinger equation that includes environmental effects on the system. It is a linear, first-order differential equation that describes the time evolution of the density matrix. The actual operator includes two terms: a Hamiltonian term that describes the unitary evolution of the system, and a dissipative term that accounts for the effects of the environment on the system. A sum of terms gives the dissipative term called the Lindbladian operators, which describe the specific types of interaction that cause this dissipation. Each Lindbladian operator has a corresponding rate constant, which determines the strength of the dissipative effect.

This is for the case of independent dissipative reservoirs for the field and all qubits.

$$\hat{L}(\hat{\rho}) = \sum_{j=0}^{1}[\frac{\gamma_j}{2}(\hat{\sigma}_j^\dagger\hat{\sigma}_j\hat{\rho} + \hat{\rho}\hat{\sigma}_j^\dagger\hat{\sigma}_j - 2\hat{\sigma}_j\hat{\rho}\hat{\sigma}_j^\dagger) - \frac{\mu}{2}(\hat{c}_j^\dagger\hat{c}_j\hat{\rho} + \hat{\rho}\hat{c}_j^\dagger\hat{c}_j - 2\hat{c}_j\hat{\rho}\hat{c}_j^\dagger)] \tag{22}$$

Using this Hamiltonian and the state vector above on the wave function from the single-photon excitation section. We see that our differential equations will become.

$$\begin{aligned}
\dot{C}_{00} &= -(i\frac{\omega}{2} + \gamma_{00})C_{00} - \frac{i}{\hbar}R_{00} \\
\dot{C}_{10} &= -(i\frac{3\omega}{2} + \gamma_{10})C_{10} + i\Omega_R^\star C_{01} - \frac{i}{\hbar}R_{10} \\
\dot{C}_{01} &= -(i\frac{3\omega}{2} + \gamma_{01})C_{01} + i\Omega_R^\star C_{10} - \frac{i}{\hbar}R_{01}
\end{aligned} \tag{23}$$

The relaxation constants are given by the EM field and the and are

$$\gamma_{00} = 0, \gamma_{10} = \frac{\mu}{2}, \gamma_{01} = \frac{\gamma_1}{2} \tag{24}$$

and the noise properties will be given by [6]

$$\overline{R^{\star}_{\alpha n}(t')R_{\beta m}(t'')} = \hbar^2 \delta_{nm} D_{\alpha n,\beta m} \delta(t' - t'') \tag{25}$$

$$D_{00,00} = \gamma_1 \overline{|C_{01}|^2} + \mu \overline{|C_{10}|^2}, D_{10,10} = 0, D_{01,01} = 0 \tag{26}$$

When we take equations in (23) and conserve the norm

$$\overline{|C_{00}|^2} + \overline{|C_{01}|^2} + \overline{|C_{10}|^2} = 1 \tag{27}$$

we get that the R_{10} and the R_{01} will both be 0, but the R_{00} will still remain as a noise term.

3.3 Classical Excitation

Now if we look at our system we should be able to excite our coupled qubits with just a classical field. The difference between a classical and quantum field is our photon operator. In a quantum field, we can single out photons and set them up individually with a certain wavelength and symmetry for the desired outcome, we also will be creating and absorbing photons throughout the process. However, for the classical field, we are dealing with a stream of photons, from a laser or some other source, that will cancel out the quantum effects of a single photon. The classical Hamiltonian is not going to have any quantum components so there is going to be no need for a photon operator. Also, the system is going to stay in the interaction picture with exact detuning for sake of simplicity in the fact that we will be choosing two identical qubits to be beside each other in our nanocavity.

$$\hat{H}_{int} = -\mathbf{E} \cdot \hat{d} + \hat{L}(\hat{\rho}) = -E(r)[(d\hat{\sigma}_0^{\dagger} + d^{\star}\hat{\sigma}_0) + (d\hat{\sigma}_1^{\dagger} + d^{\star}\hat{\sigma}_1)] + \hat{L}(\hat{\rho}) \tag{28}$$

Using the Lindbladian operators from the description above we can incorporate dissipation into the differential equations as shown below.

$$\begin{aligned}
\dot{C}_{00} &= -i\Omega_R^{\star} e^{-i\Delta t}(C_{01} + C_{10}) \\
\dot{C}_{01} &= -ie^{-i\Delta t}(\Omega_R^{\star} C_{11} + \Omega_R C_{00}) \\
\dot{C}_{10} &= -ie^{-i\Delta t}(\Omega_R^{\star} C_{11} + \Omega_R C_{00}) \\
\dot{C}_{11} &= -i\Omega_R e^{-i\Delta t}(C_{01} + C_{10})
\end{aligned} \tag{29}$$

Here this will excite both of our qubits into the excited or bright state ($|11\rangle$) which will eventually decay and leave us with a mixing of the ground state and entangled state. What we need to apply is a field of opposite symmetry to the cavity mode, preferably in the $|01\rangle + |10\rangle$ or $|10\rangle - |10\rangle$. These states should be separated in energy, this will require a dipole interaction between the dots though. This will help in leading us to separate the mixing of the $|00\rangle$ state into the symmetric or anti-symmetric entangled states.

3.4 Reflection

There are two main ways to reflect a quantum field after setting up our system in classical terms such as this one. The first would be to set up our nanocavity in such a way that we set our two qubits beside each other and send through the nanocavity an antisymmetric field setting up our entangled qubits in a $|01\rangle - ||10\rangle$ orientation. We would then need to send to the cavity a symmetric quantum field that will detect whether the system is in a bright state or a dark state. Here the frequency of the cavity mode μ and the cavity mode coupling strength κ must come into play to find a way to detect reflection. This method described here first is possible, but perhaps impractical as well. While it is relatively simpler to set up our quantum field to reflect in an antisymmetric field, it would be difficult to find a setup for our nanocavity in which we could truly set up our two-level system into the dark states only using a symmetric field.

The preferred method would be to send a classical symmetric field through the cavity to put our two qubits into one of the dark states. Specifically the dark symmetric state with respect to the

cavity, $|01\rangle + ||10\rangle$. This would require us to send an antisymmetric quantum field for reflection. However, we could not do this over the entirety of the cavity, but over just half of the cavity.

A question that is still needed to be answered is how we completely separate the mixing of the $|00\rangle$ state from the dark entangled states. There are several potential ways of doing this, both in the symmetric and antisymmetric regimes described above, however, the best at the moment is to consider the dipole-dipole interaction between the two qubits that are placed in close proximity. When the interaction energy between the qubits is much larger than their individual energy levels, it can result in the phenomenon of a dipole blockade. In this regime, the simultaneous excitation of both qubits is forbidden due to the dipole-dipole interaction, which effectively prevents the mixing of the $|00\rangle$ state. This helps in the actual reflection of the quantum bit because we would no longer need to find a way to incorporate the mixing of states into our coupling strength and decay rate of the cavity.

Considering this scenario: If one qubit is in the excited state, the diple-dipole interaction creates an energy shift in the other qubit that depends on its state. The shift can be either positive or negative depending on the state of the other qubit. If this shift is large enough, it can prevent the second qubit from being excited, effectively blocking the simultaneous excitation of both.

All of this to say, using the schemes described above there is still much work to be done in some of the analytical evaluations of the setup and reflection of our CNOT gate, but the foundation has been laid for this project to move forward with great speed. Following this, we will need to do certain tests with our experimental group to see if some of the assumptions we made were correct or how our analytical solutions play out in the real case, and if how we think our systems will act is how they do in practice. Hopefully, the dipole interaction can be the desired solution in clearing out the mixing of the ground state with our dark states, but in the end, only time will tell if this will fully be the case.

4. CONCLUSIONS

In this paper, we have discussed various aspects of the interaction between quantum/classical fields and a two-qubit system in a nanocavity. The paper starts by introducing the basic concepts of quantum mechanics, such as superposition and entanglement, which are essential for understanding the behaviors of our quantum systems.

Starting the fundamentals of raising and lowering a single qubit was explained. Not only this but the methods of how to do this with several different types of pulses interacting with the qubit are detailed in the beginning section. The paper then focused on the interaction between a quantum field and a two-qubit system in a nanocavity. The paper described how the two-qubit system can be coupled to a nanocavity, and how this coupling can be described using quantum Hamiltonian depending on the initial conditions and the final state of the system based on the pulse sent through the nanocavity. Also discussed in this section is the analytical solution to finding the probability of the bright and dark (entangled) states.

Furthermore, the paper explored the concept of classical excitation and reflection. Explaining how classical fields can be used to also put our system in our excited or entangled states. The interaction between the qubits can lead to the phenomenon of dipole blockade, which prevents the simultaneous excitation of both qubits. This prevents the mixing of our dark and bright states in setting up our system. Finally discussed were different methods of reflecting a quantum field using a nanocavity, using both symmetric and antisymmetric schemes.

Overall, the paper has provided a comprehensive overview of the interaction between various fields and the quantum gate. Highlighted above are some of the challenges involved in controlling and manipulating these systems with certainty.

The study of quantum gates is an exciting and rapidly developing field, with a wide range of potential applications in areas such as quantum computing and quantum information systems. The work described here provides a solid foundation for further research in this field and offers

valuable insights into the behavior of quantum systems in the presence of external fields and interactions. As research continues to explore the possibilities of quantum systems, many new and exciting discoveries will likely be made in years to come.

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