EXTERNAL POWER SUPPLY TO A CONDUCTIVE PLANET VIA ELECTROMAGNETIC INDUCTION

An Undergraduate Research Scholars Thesis

by

JACOB P. COPELAND

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This project did not require approval from the Texas A&M University Research Compliance & Biosafety office.

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External Power Supply to a Conductive Planet via Electromagnetic Induction

Jacob P. Copeland
Department of Geology & Geophysics
Texas A&M University

Faculty Research Advisor: Dr. Mark Everett
Department of Geology & Geophysics
Texas A&M University

The magnetic field variability of a young solar-type star delivers a time-varying magnetic flux to its hosted planets. In the case of early Earth prior to the establishment of the magnetosphere, the associated electromotive force from a magnetically active young Sun would have facilitated the transfer of substantial inductive power, making it available to a nascent geodynamo during its poorly-understood initiation process. Herein a simple mechanism which creates favorable conditions during the powering-up phase of a planetary dynamo is considered. Constraints on the amplitude of stellar-derived magnetic-flux variability are herein determined in order for a given amount of power to be transferred to a planetary body during an epoch of dynamo initiation. Astrophysical observations of the magnetic variability of young solar-type stars are accumulating but much uncertainty remains, even in our solar system case, as to the amount of inductive power transferred from the young Sun to any of its hosted planets. The shielding of a planetary interior by an early-developing magnetosphere could greatly reduce the effectiveness of this type of solar inductive power transfer as a means to initiate and/or reinforce dynamo action.

DEDICATION

To my parents, who have demonstrated to me in the truest sense faithfulness, dedication, integrity, and perseverance.

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Contributors

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1. INTRODUCTION

Within the interior of the Earth, a global magnetic field is produced. At the time of writing, dynamo theory is used to explain the production of the global magnetic field. Fluid motions in the outer core make up the geodynamo. While the Earth has maintained a magnetic field since at least 3.4 Ga [Selkin et al., 2007, Biggin et al., 2011, Tarduno et al., 2014, Brenner et al., 2020], this dynamo regime may be significantly younger and Earth's inner core may have cooled much more recently. The timing of crystallization of the inner core is intertwined with the thermal conductivity of the core. Higher conductivity implies a faster cooling rate, which implies that the threshold for the crystallization of the inner core happened sooner rather than later [Davies et al., 2015]. The increasing likelihood of late core crystallization has led to proposals of alternative power sources to sustain the geomagnetic field. It is clear the Earth has had a dynamo within its interior for a large part of its history, but the origin of the magnetic field is poorly understood. Herein, the ability of a young star to provide the favorable conditions for dynamo generation to a planet is considered.

The results of this study may be applied to any exoplanetary study, and not just those of our solar system. However, using what is already known of the Earth-Moon system greatly helps put the results into perspective and to understand how other, similar planets may have acquired a magnetic field.

The young sun was much more magnetically active than it is today [Folsom et al., 2017]. It is important to review star formation and solar system evolution. A star system begins with a nebular dust cloud. Upon the gravitational collapse of the dust cloud, matter clumps together like dust bunnies and begins the process of star formation. Eventually, the clump of matter becomes so dense and rotates so quickly that it forms a proto-star. As the star progresses into being a true star and begins the "spin-down" process [Skumanich, 1972], the star sheds angular momentum via the stellar wind.

The origin of Earth's magnetic field is poorly understood. This is true for any planet, not just Earth. At present, proposals for the origin of the magnetic field are inconclusive. The origins of a magnetic field differ from the evolution of a magnetic field. The mechanism that generates a magnetic field, a dynamo, requires an initial "kick-off," whereas the growth of a magnetic field requires the dynamo action to be already present. Most dynamo studies assume an initial small seed field that grows into a larger, complex geodynamo.

This project asks, "What is the amount of inductive power transferred to a bare hosted planet as a function of the time-variability and intensity of the magnetic field of a young, solar-type star?" This question is important for two principal reasons potentially bearing on the origin of planetary self-sustaining magnetic fields. The first is to better understand the effect of magnetic fields on the formation and subsequent evolution of the Earth and other members of the solar system; the second is to advance the state of knowledge on the role of magnetic fields in the evolution of exoplanet systems.

In the current literature, papers studying the magnetic fields of planets without a magnetosphere are abundant, but very few focus on the possible mechanisms leading to the initiation of a self-sustaining magnetic field. My project acknowledges this knowledge gap and I hope to provide some insight into early Earth's magnetic field generation. An important potential implication of my work is to provide constraints on the habitability of exoplanets and extrasolar systems.

To address the project goals, I calculate the power dissipated within a young planet subjected to an external time-varying magnetic field originating in a host solar-type star. Abundant astrophysical evidence shows that young solar-type stars are considerably more magnetically active than older stars of the same type, so that early Earth likely received much larger doses of electromotive force (which is proportional to time-variation of magnetic flux threading the planet) compared with the amount received at Earth by the less-active present-day Sun. Accordingly, a relatively large amount of power might have been transferred depending on the magnitude and timescale of the external magnetic field variation. The peak power for the idealized scenarios I have studied compares favorably with current estimates of the power required to sustain the present-day geodynamo [Driscoll and Du, 2019].

It is well-known that terrestrial planets are differentiated roughly into distinct layers. Earth, for example, has a solid inner core composed primarily of metal, a liquid outer core of similar composition, a dense solid mantle of rocky silicate material, a ductile asthenosphere, and a rigid lithosphere that includes the outermost crust. In the rotating outer core, geodynamo action is sustained by the interaction of liquid-metal motions with the geomagnetic field [Elsasser, 1956]. The geomagnetic field protects inhabitants of Earth against the solar wind and gamma-ray bursts. The current state of the solar system, the Sun, and their magnetic fields are well-known compared with our lack of knowledge of the early solar system. Given that we cannot go back in time to observe the state of the early solar system, we must rely on proxies to provide information. Before the Sun formed, the matter that eventually coalesced into the solar system was essentially a rotating cloud of dust and gas. The dust and gas began to collapse, eventually forming a dense, rotating proto-star at the center of an accretionary disk. The proto-star likely exhibited a rapid rotation rate and a large magnetic field strength characterized by potentially chaotic temporal fluctuations. This chaotic nature is exemplified by the jets produced by these T Tauri stars and the flip-flopping of their magnetic field behavior [Dyda et al., 2015]. As the proto-star evolves, its rotation rate begins to slow down, angular momentum loss is reduced, and magnetic field intensity is also reduced. Little is known about this stage of star evolution, however substantial evidence supports a direct relationship between rotational velocity and magnetic field strength [Folsom et al., 2017].

1.1 Feasibility

The evolution of the magnetic field intensity of a solar-type star follows a distinctive inverse power-law as a function of age. During the early epochs of its formation, the proto-star rotates rapidly, generating a large, highly variable magnetic field. With increasing age, the spin-down of the star causes the magnetic field to weaken. This process is particularly evident as the star transitions from proto-star to star. It is thus reasonable to assume that the early Sun was more magnetically active than it is today. It is widely believed that the Moon was formed by a collision between Earth and a Mars-sized impactor. Radiometric dating estimates vary by a range of ages for the timing of the Moon-forming event, but generally center around $\sim$120 Myr after formation of the solar system. At that time, large and frequent solar-sourced magnetic disturbances are a distinct possibility. Treating the Earth as a homogeneous conducting sphere, a storm of modest magnetic intensity can transfer more than 1 TW of power to the Earth, which according to recent estimates is roughly the amount needed to sustain the present-day geodynamo. And for storms of greater, more realistic intensity like those shown in the figures below, they satisfy the power threshold of 12 TW seen in [Driscoll and Du, 2019].

The magnetic evolution of a star follows an inverse power-law as it ages [Skumanich, 1972, Folsom et al., 2017]. The most widely accepted model for star and solar system formation is the nebular hypothesis. This hypothesis supposes that star formation begins with the gravitational collapse of a nebula, i.e. a rotating cloud of cosmic dust and gas. The forming star accretes matter from the co-rotating cloud, growing in size as the nebular material falls into the proto-star. Star type varies greatly and the majority of potentially habitable exoplanets orbit G, M, or K-type stars. The Sun is a G-type main-sequence star. Young solar-type stars (those like the Sun) and the early Sun have vigorous magnetohydrodynamic dynamos [Guinan and Engle, 2008]. Dynamo action is likely because of the rapid rotation of the young star [Guinan and Engle, 2008, Folsom et al., 2017]. As the star ages and spins-down because of the angular momentum loss due to the shedding of solar wind particles, the surface magnetic field intensity decreases [Weber and Davis Jr., 1967, Folsom et al., 2017].

A series of classic papers by Sonnett considered various physical mechanisms that can heat asteroids. One of these mechanisms is similar to the one that I am investigating; however, the source of the electromotive force that induces the heating is different. In Sonnett's papers, T-Tauri solar wind is ultimately responsible for providing the necessary conditions for asteroid heating. Additionally, the time-scale over which the planetary body was exposed to the external excitation was vastly different in Sonnett's studies. T-Tauri stars are only $\sim$4-10 Myr old, whereas the epoch that I am investigating (immediately following the Moon-forming event) is $\sim$120 Myr after solar-system formation.

This project is of exceptional current interest due to it's applicability to exoplanetary studies. My project is relevant to studies of the evolution and habitability of other star systems, and also acknowledges the current lack of understanding about viable mechanisms to initiate a planetary dynamo. Additionally, this project is of interest because there are solar-system planets that, for some reason, lack an intrinsic magnetic field and/or a magnetosphere. Mercury is so close to the Sun that any developing magnetosphere is quickly eroded by the powerful solar wind. It is widely thought that Venus should have a stable dynamo, given how similar in size it is to Earth. However, the only magnetic field present at Venus is the one that is induced in its ionosphere by the magnetic field carried along with the solar wind.

1.2 Types of Stars and Their Evolution

There are many types of stars from the intensely hot O type to the cool M type. The Earth is located in the habitable zone of a G-type main-sequence star. The Sun is a yellow dwarf, on the hotter end of the G-type spectrum.

The star type determines many of the characteristics of a star system. The star type affects, for example, whether the planets orbiting it have intrinsic magnetic fields. Also, if the star were an extremely dense neutron star, it is reasonable to assume that no habitable planets could form. Any such planet would need to be at a sufficient distance and rapidly orbiting to escape the gravitational attraction of the neutron star, in which case little to no radiative heat energy would reach it.

Compared to our knowledge of the solar system and the Sun, we know little about other systems, planets, and stars. However, with this in mind, stars appear to share some common fundamental characteristics. These characteristics include but are not limited to magnetic activity, rotation, and evolutionary history. In a classic paper, [Skumanich, 1972] observed for several young solar-type stars a distinct power-law decrease of rotation speed as a function of age. Although it has since been shown that many older stars do not obey a Skumanich-type power law, the original rule has generally proven reliable as a first approximation of star evolution.

In this thesis, I chose to look at solar-type stars for several reasons. First, in the context of exoplanetary search, a planetary system that orbits a star similar to our own immediately stands out for its potential habitability. Second, there are certain difficulties with theoretical explanations as to how Earth acquired its magnetic field. The most widely accepted concept is that the geomagnetic field grew spontaneously from an arbitrarily prescribed seed field. This concept does have some support from numerical dynamo simulations. There have been several other mechanisms proposed but most are seriously flawed and none are completely satisfactory. Most of the current research has overlooked the origin of the magnetic field in favor of understanding its evolution, sustenance, and dynamics. The question of its origin remains therefore largely uncertain.

1.3 Why Earth and not Mars or Venus?

A common question asked is why Earth has an intrinsic planetary-scale magnetic field sourced from its deep interior, while the twin planet Venus does not. Venus and Earth are similar in size and composition. However, the Venusian atmosphere is very different from that of Earth, and the planet is much closer to the Sun. Until the origin of the geomagnetic field is better understood, progress will be slow in understanding the magnetic fields of other planets. Earth is unique in our solar system insofar as it is a moderately sized terrestrial planet with active tectonism, a strong magnetic field, a stable atmosphere, and a rapid rotation speed. The formation of the Moon, with its comparatively large radius $0.25R_E$, may play a crucial role in understanding the origin of the geomagnetic field.

Approximately 120 Myr after the formation of the solar system, it is widely believed that a Mars-sized impactor struck early Earth. The impact vaporized the impactor, led to the formation of the Moon, and disrupted ongoing internal processes within early Earth. The Sun at this time was likely much more active than the present day. The Sun was rotating rapidly, generating a comparatively intense magnetic field perhaps with frequent superflares [Maehara et al., 2012]. The early Earth under these conditions could have been subjected to large magnetic-field temporal variations due to super-flares and other solar mass ejection events. In such a case, sizeable amounts of inductive power could have been transferred from the Sun to the Earth in the epoch immediately following the Moon-forming event.

The evolutionary history of Venus is poorly understood; however, if the distance from the host star is relatively unimportant compared to planetary size or composition, then perhaps a late giant impact is necessary to acquire a magnetic field. Mars, by contrast, did have a planetary-scale intrinsic magnetic field in its early history but it has long been shut off. Once planetary dynamo action shuts off, theoretical arguments suggest that it is difficult to be re-started [Cattaneo and Hughes, 2022].

1.4 Solar System Evolution

The first 100 Myr after the formation of the solar system (defined as the beginning of accretion onto the proto-Sun) were chaotic and are only beginning to be understood. Meteorites can be used to understand the interplanetary environment during this period. Additionally, meteorites have been used to understand details about the solar nebula. Star formation begins with the gravitational collapse of a nebular dust cloud. The collapse increases rotation, with material accreting toward the center at an ever-increasing pace. Eventually, this mass forms a proto-star. In the case of solar-type stars, this type of proto-star is referred to as a T-Tauri type star. T-Tauri stars rotate very rapidly and carry an intense, highly variable magnetic field. Thus, early solar system activity was likely to be chaotic. Eventually, the star begins its spin-down and sheds angular momentum via the expulsion of a stellar wind. As the star spins down, planets form even while their bombardment continues. During this period in solar-system history, the Sun rotated at 10 times the velocity it does today. Young solar-type stars, like the early Sun, have "vigorous" magnetohydrodynamic dynamos [Guinan and Engle, 2008]. These young stars exhibit large star-spot regions covering 5-30% of their surfaces. This is an important observation because it shows that superflares were likely more common in the early solar system, affecting much more of the solar system due to the large sun-spot area. Magnetic field lines emanate from the spot, and affect a vast swath of space as they project outward into interplanetary space.

2. METHODS

In order to determine the power generated in a young bare planet, multiple parameters are necessary before any calculations are made. What is the original magnetic field intensity? By what timescale does the magnetic field decay? What is the electrical conductivity of the material in the planet? What is the radius of the planet? After these are specified, calculations can be done with Gfortran programs on the computer. My calculations rely heavily on the inverse Laplace Transform, which is necessary to transform the referenced equations into usable equations in the time domain, since my scenario is time-dependent, not frequency-dependent.

Herein I calculate the instantaneous Ohmic dissipation in a uniform conductive sphere that is subject to an idealized, storm-like, time-varying magnetic field excitation of the form

$$B(t) = B_1 u_0(t) \mathrm{exp}\left(\frac{-t}{\tau}\right) \tag{1}$$

where $u_0(t)$ is the unit step function and τ is the decay time-scale of the "storm" excitation. The direction of the exciting magnetic field $B(t)$, which has an initial amplitude B_1, is arbitrary for now.

It is well-known that, in the frequency domain wherein the exciting magnetic field is of the time-harmonic form $B_1 \mathrm{exp}\,(-\mathrm{i}\omega t)$, the electric field inside the sphere at radius r is purely azimuthal with respect to the magnetic field direction and is given by

$$E_\phi(r, \omega) = \frac{3\mathrm{i}\omega B_1}{2}\left[\frac{j_1(kr)}{k j_0(ka)}\right] \mathrm{sin}\theta \tag{2}$$

where θ is colatitude and $k = \sqrt{\mathrm{i}\omega\mu_0\sigma}$ is the characteristic propagation constant of the sphere. The parameters μ_0, σ are respectively the magnetic permeability of free space and the electrical conductivity of the sphere. The angular frequency of the external magnetic field is ω.

In equation (2), the quantities

$$j_1(kr) = \frac{\sin(kr)}{k^2r^2} - \frac{\cos(kr)}{kr}; \tag{3}$$

$$j_0(ka) = \frac{\sin(ka)}{ka}; \tag{4}$$

are spherical Bessel functions.

The transient storm-time response is most easily evaluated in the Laplace domain. The Laplace transform of the excitation function in equation (1) is [Everett and Martinec, 2003]:

$$\mathcal{L}\left\{u_0(t)\text{exp}\left(\frac{-t}{\tau}\right)\right\} = \frac{s\tau}{1+s\tau} \tag{5}$$

where $s = \mathrm{i}\omega$ is the Laplace variable. In the Laplace domain, the electric field is

$$e_\phi(r,s) = \frac{3\mathrm{s}B_1}{2}\left[\frac{j_1(\kappa r)}{\kappa j_0(\kappa a)}\right]\text{sin}\theta \tag{6}$$

where $\kappa = \sqrt{\mu_0\sigma\mathrm{s}}$.

The transient storm-time response of the sphere is the inverse Laplace transform of the s-domain product of the excitation function and the electric field, namely

$$E_\phi^{ST}(r,t) = \mathcal{L}^{-1}\left\{e_\phi^{ST}(r,s)\right\} = \mathcal{L}^{-1}\left\{\frac{s\tau}{1+s\tau}\frac{3\mathrm{s}B_1}{2}\frac{j_1(\kappa r)}{\kappa j_0(\kappa a)}\text{sin}\theta\right\} \tag{7}$$

The inverse Laplace transform in the previous equation is readily calculated using the Heaviside expansion theorem (see Everett and Martinec 2003). To apply the theorem, first recognize that $e_\phi^{ST}(r,s)$ can be written as a quotient

$$e_\phi^{ST}(r,s) = \frac{P(s)}{Q(s)} \tag{8}$$

where

$$P(s) = 3s^2\tau B_1 j_1(\kappa r)\sin\theta; \tag{9}$$

$$Q(s) = 2(1 + s\tau)\kappa j_0(\kappa a) \tag{10}$$

The Heaviside expansion theorem prescribes the following formula for computing the transient response

$$E_\phi^{ST}(r,t) = \sum_{n=1}^{\infty} \frac{P(s_n)}{Q'(s_n)} \exp(-s_n t) \tag{11}$$

where the set $\{s_n\}_{n=1}^{\infty}$ comprise the zeroes of the equation $Q(s) = 0$. The denominator in (8) is

$$Q'(s_n) = \left.\frac{\partial Q}{\partial s}\right|_{s=s_n} \tag{12}$$

Using the equation (10) it is easy to show that the zeroes are

$$s_n = \frac{n^2\pi^2}{\mu_0\sigma a^2}, n = 1, 2, ..., \infty \tag{13}$$

Moreover, elementary differentiation of equation (10) results in

$$Q'(s_n) = \frac{2\tau}{a}\sin(\kappa_n a) + \sqrt{\mu_0\sigma}\left\{\frac{1}{\sqrt{s_n}} - \tau\sqrt{s_n}\right\}\cos(\kappa_n a) \tag{14}$$

where $\kappa_n = \sqrt{\mu_0\sigma s_n}$.

Once equation (8) has been evaluated, the transient storm-time response $E_\phi^{ST}(r,t)$ is known. The Joule heating density $J_h(r,t)$ in W/m^3 throughout the sphere is then

$$J_h(r,t) = \frac{1}{2}\sigma\left[E_\phi^{ST}(r,t)\right]^2 \tag{15}$$

and finally the total Ohmic dissipation $D_{\mathrm{ohm}}(t)$ in [W] inside the sphere, at some time t after storm onset, is the integration of the Joule-heating density throughout the sphere,

$$D_{\mathrm{ohm}}(t) = \frac{8\pi}{3} \int_0^a r^2 J_h(r,t) \mathrm{d}r \tag{16}$$

3. RESULTS

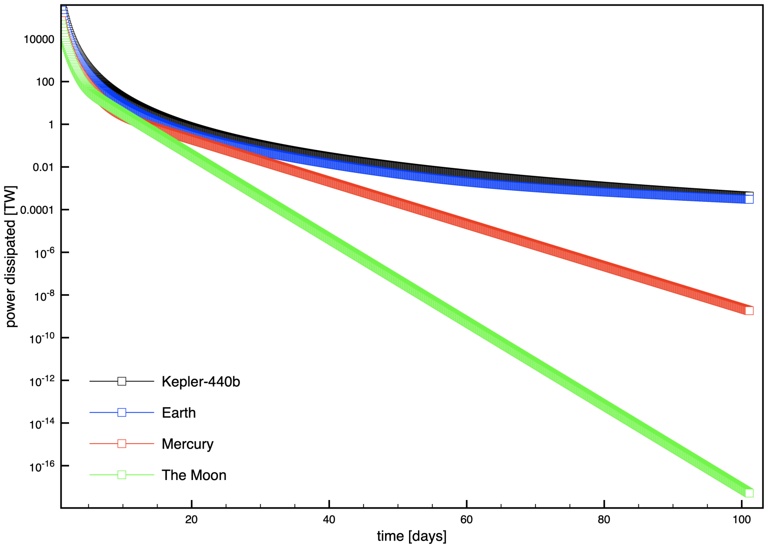

Figure 1: Power versus Time due to change in Radii

In Figure 1, the changes in power dissipated throughout the sphere due to a change in radius of the sphere is calculated. The radii are those of the Moon, Mercury, Earth, and Kepler-440b. 1737.4 km, 2439.7 km, 6371 km, and 11850.06 km, respectfully. The parameters for Figure 1 include $B_1 = 50,000\,\mathrm{nT}$, $\sigma = 1\,\mathrm{S/m}$, and $\tau = 30\,\mathrm{days}$.

In Figure 2, the changes in power dissipated throughout the sphere due to a change in the storm decay constant (τ) is calculated. The τ range from 1 day to 100 days. The parameters for Figure 2 include $B_1 = 50,000\,\mathrm{nT}$, $a = 6371\,\mathrm{km}$, and $\sigma = 1\,\mathrm{S/m}$.

In Figure 3, the changes in power dissipated throughout the sphere due to a change in the magnetic field strength is calculated. The magnetic field strengths range from 100 nT to 100,000 nT. The parameters for Figure 3 are $a = 6371\,\mathrm{km}$, $\sigma = 1\,\mathrm{S/m}$, $\tau = 30\,\mathrm{days}$.

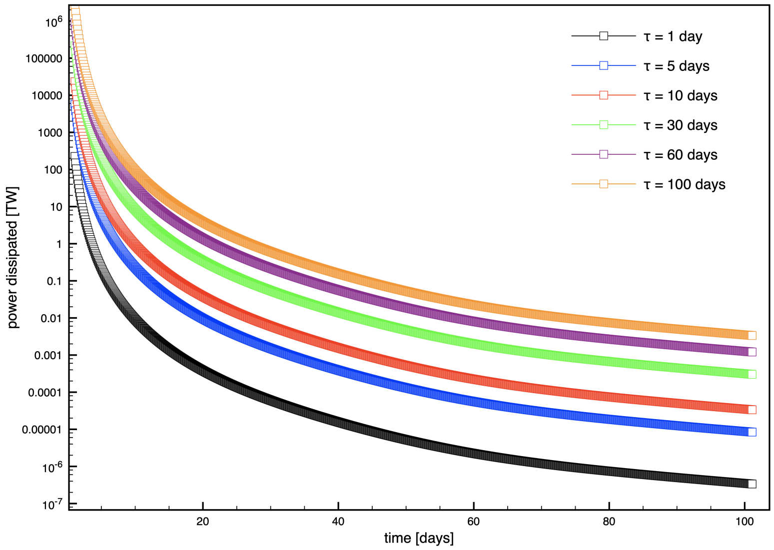

Figure 2: Power versus Time due to change in tau

In Figure 4, the changes in power dissipated throughout the sphere due to a change in conductivity is calculated. The conductivity values range from 0.1 S/m to 100 S/m. The parameters for Figure 4 include $\tau = 30 \text{ days}$.

The Earth is believed to have been impacted by a Mars-size impactor around 120 Myr after formation of the solar system. This impact formed the moon, and left the earth as an undifferentiated, partially homogeneous sphere. There were likely hot and cold spots within the earth, with a global convection current driven by the temperature and density differences between the impact site and the rest of the planet. Not enough has been done to understand this complex evolutionary history, however, it is plausible that a magnetic field could have been sustained in the early mantle of the earth. This alternative dynamo may have been what causes the anomalous early magnetic behavior of earth. A problem with this is what led to the transition between this dynamo and the outer-inner core regime we see today. In addition to the hot and cold spots driving convection, more regional magnetic fields could have been induced by infalling crustal material forced down into the mantle.

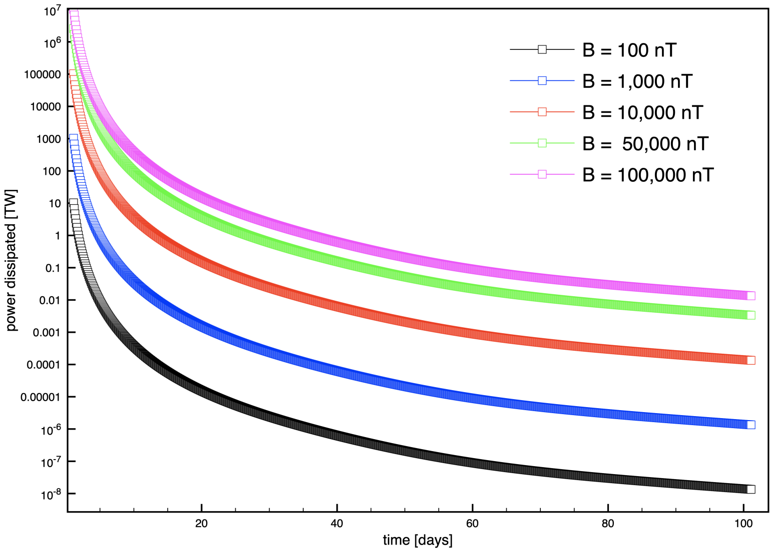

Figure 3: Power versus Time due to change in magnetic field

Additionally, this undifferentiated state of the earth is prime for power dissipation by the sun, given that there is no present magnetosphere to fight with and the sphere is approximately homogeneous.

Peak power transferred by the time-varying magnetic field of the host star to the hosted planet does not depend on the planetary conductivity. Exposure time spent above a prescribed threshold level of a planet to star-transferred power increases with conductivity. For a given star-planet system of fixed orbital radius R, the product of conductivity divided by the decay rate (or equivalently multiplied by the characteristic frequency ω of the external magnetic field variation is constant: this is an example of electromagnetic similarity in which the product uniquely determines the EM response. Sufficient power for maintaining the present-day geodynamo is available over timespans of up to millions of years depending on the magnetic activity of the young host star. More power is dissipated in planets of larger radius because they present a larger cross-section to the external magnetic flux lines; however, the skin depth makes up a larger fraction of the radius of smaller planets so eddy currents penetrate more deeply into the latter. The external magnetic-field variation from the hosting star can be of the size of present-day geomagnetic storms but requires significantly longer duration, on the order of years and higher, or multiple successive storms of shorter duration, in order to transfer an amount of power comparable to that required to operate the present-day geodynamo.

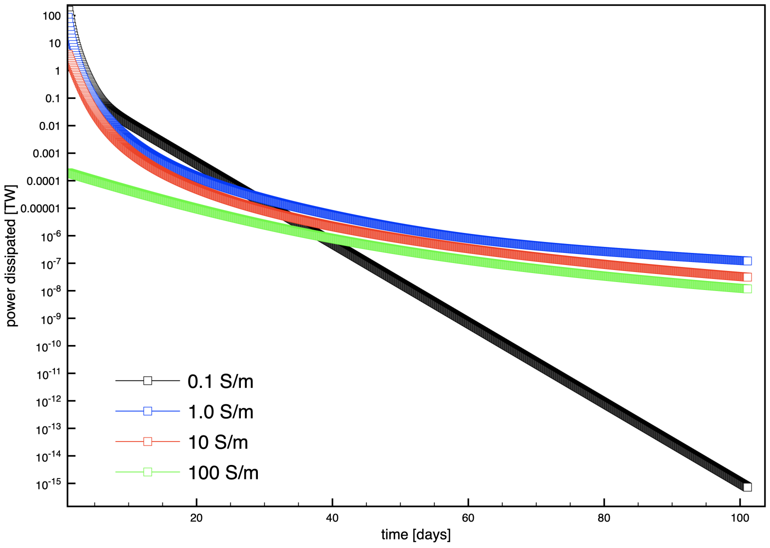

Figure 4: Instaneous Power dissipated with respect to time

Notice in Figure 3 that the majority of the plots stay above 0.1 TW for the majority of the time the storm dissipates power in the sphere. This is significant as the necessary power threshold is exceeded for sustaining a core dynamo. The heat flow according to older literature is satisfied by this power threshold. Looking at newer literature, however, shows a larger power of around 12 TW necessary [Driscoll and Du, 2019].

4. DISCUSSION

Electromagnetic induction heating of a highly conductive sphere is a classic problem in physics and engineering. The same problem is also relevant to mineral exploration geophysics and buried metal detection, although in those applications it is the secondary magnetic field generated by induced eddy currents, rather than the heating aspects, that is of primary importance. Most, if not all, of the engineering applications are concerned with the heating induced by an external time-harmonic magnetic field. A time-varying flux of magnetic field threading the sphere generates eddy currents in the interior that heat the sphere via ohmic dissipation (i.e. resistive heating). Induction heating may be an important driver of convection in a planet's rotating liquid interior. One must also consider the fact that electromagnetic induction, via the electromotive force, drives electric currents in a predominantly azimuthal direction (i.e. in a toroidal, or "doughnut-type" geometry) that is favorable for maintaining dynamo action. It is important also to recognize that the induced currents are concentrated in the outer few skin depths of the sphere.

The sun is thought to have been more magnetically active in the early solar system than it is at presnt-day, due to its early rapid rotation and subsequent spin down. Astrophysical observations on solar-type stars suggests that rapid early rotation, followed by spin down, causes the average stellar magnetic field intensity to follow an inverse power law as a function of age [Folsom et al., 2017].

Due to intense magnetic activity of a young, rapidly rotating host star such as the Sun, early fluctuations of magnetic field shed in association with the copious angular momentum loss are likely to be much more common and intense than later fluctuations that are shed as the star ages and its rotation slows down. These early fluctuations can be modeled, at the location of the hosted planet, as a sudden rise in external magnetic field followed by an exponential decay, similar to a present-day geomagnetic storm [Everett and Martinec, 2003]. The precise shape of the time variation however is not as important as the initial amplitude and decay time.

When a conducting sphere is subjected to an electromotive force brought about by an externally-sourced fluctuation in magnetic flux, azimuthal (toroidal) eddy currents are generated in the interior, and Ohmic (resistive) heat is dissipated throughout the outermost skin depths of the sphere. A recent paper modeled the inductively-transferred power dissipated in a natural satellite, calculated in nearly the same way as the scenario considered herein [Chyba et al., 2021]. The present calculations differ from those of Chyba et al. insofar that in my project the induced azimuthal currents due to the external magnetic-field fluctuation are explicitly time-dependent, whereas the magnetic field variation in the Chyba et al. scenario is time-harmonic and hence more idealized.

To better understand the time-dependence of power dissipation in the Earth or an exoplanet, with concomitant implications for the generation of an intrinsic planetary magnetic field, the inductive power transferred by the hosting star is herein calculated in the time domain. This is conveniently done by converting the time-harmonic expression for the induced azimuthal electric field from [Bex and Greuel, 1993] into the Laplace domain using the Laplace variable $s = i\omega$, multiplying the resulting expression with the Laplace transform of the external excitation waveform, and then taking the inverse Laplace transform. This procedure is equivalent to convolving the time-domain impulse response with a geomagnetic storm-like excitation.

4.1 Application to Exoplanetary Studies

These results are helpful in the study of exoplanetary systems. The primary issue with studying exoplanets is that they are not visible nor are they visitable. However, scientists hope to identify habitable exoplanets in the search of extraterrestrial life and future homes for spacefaring civilizations. It is assumed that life requires a planet with a magnetosphere to develop, although this assumption is currently under scrutiny (Dr. Claire Nichols, personal communication, December 14, 2022). However, for the time being, it is fair to assume the need for a strong magnetosphere in exoplanets, especially in the face of terraforming efforts. At present, the magnetic fields of exoplanets are not measurable due to the need for clean, high-resolution signals using techniques such as Zeeman-splitting. These signals are disrupted by stellar contamination.

Until exoplanetary magnetic fields are measurable, a new way to determine the possible magnetic fields of exoplanets is necessary. Here in this project, I present a method that constrains the ability of a planet to acquire a magnetic field. Based on the age of the star and planet and other compositional characteristics of the planet, the planet may be able to acquire a magnetic field from its host star. This method identifies the amount of power that could be dissipated in the planet, and not the magnetic field generated. An area for future consideration would take this project one step further, and consider whether a dynamo actually will form from the magnetic impetus of a host star. Future projects would determine mechanisms for magnetic field generation relying on the assumption that a star provided the necessary magnetic flux to generate satisfactory power in the planet.

So far, it is unclear whether planets actually acquire a magnetic field from their star. It is clear, however, that this is possible, and a solution to the problem of the origin of magnetic fields. In the pursuit of habitable star systems, knowledge of the evolution of the target star system is important. In addition to understanding the conditions of the exoplanets in another system, the distribution of materials and how the other planets interact with each other are critical. In the event of intelligent alien life, or space-faring civilization, resources are key. The collection of failed planetesimals, meteorites, satellites, and other leftover debris from star formation contains necessary materials for interstellar travel. As the field of study (exoplanetary research) grows, so will our knowledge of these star system characteristics.

4.2 Application to the Solar System

Of all the planets in the Solar System, not a single one's magnetic field origin is understood. Mercury appears to have a dynamo but is stripped of any magnetosphere it produces by the solar wind. Venus does not produce a dynamo (despite being a similar size to the Earth) and has a slight magnetosphere produced by interactions between its atmosphere and the solar wind. Earth has a strong magnetic field and core dynamo regime. Mars had a dynamo at one point but lost it. None of the terrestrial planets other than Earth have any sizeable moons. It is questionable as to whether a terrestrial planet requires a significantly large moon such as the Earth's moon to maintain a dynamo, or whether it requires a well-timed massive impact such as the moon-forming impact. It is possible the scenario proposed by this thesis could also have helped to initiate the magnetic fields of the gas giants. These planets, however, still have not fully understood histories and may have migrated from the inner to the outer solar system. There are many papers – [Ida et al., 2018, Carter and Stewart, 2020, Ida et al., 2013, Dodson-Robinson et al., 2009] for example – that consider the scenarios of gas giant formation. The mechanism that caused dynamo initiation in Jupiter, Saturn, Neptune, and Uranus may vary greatly or be due to a similar mechanism as the Earth's.

4.3 Parameters of the Planet Under Consideration

In defining the parameters for testing the mathematical aspects of the project, realistic values were necessary. The independent variables were the decay timescale, the original applied magnetic field intensity, the electrical conductivity of the planet, and the duration of the storm. The state of the young sun and the planet in which power is dissipated are of primary importance when determining the parameters. It has been well-established thus far in this thesis that the young sun was more active than it is today. By activity, I mean that sunspot duration and magnetic field intensity are both greater. In selecting the magnetic field intensities to calculate, it was best to choose a moderate magnetic field intensity for the majority of calculations. 50,000 nT was chosen due to the surface magnetic field strengths of young-solar type stars, and how much that magnetic field intensity scales upwards in the event of a storm. Of the stars in the sample considered by [Folsom et al., 2017], those in the 120 Myr age range had strong magnetic fields, ranging from approximately 300 Gauss to 20 Gauss (the equivalent of 30,000,000 nT to 2,000,000 nT). Compare that to the average surface magnetic field of the Sun today at 1 Gauss (100,000 nT). Moderate storms today can be of the strength 500 nT. Therefore, comparing this ratio of typical intensity to what a storm in the early solar system may have been, 50,000 nT is a favorable strength.

In determining the conductivity of the planet, the majority of the planet will be rocky and solid. Therefore, the planet will have more resistive electrical conductivities compared to a conducting fluid. For the outer core, it is reasonable to test conductivities of 10^3 to 10^5 S/m. However, the eddy currents generated in the sphere only penetrate down to a certain skin depth according to the equation $\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$. The larger the planet, the smaller the fraction of the planet's total radius that receives power, and therefore and liquid core would not receive significant power. Additionally, the composition of the planet, and how well-differentiated the planet is may vary greatly. For the Earth, any power generated in the proto-Earth prior to the Moon-forming event was likely insignificant. If a self-sustaining dynamo had begun because of any interactions between the planet and the early Sun's magnetic field, it would have been disrupted when a Mars-size impactor (Theia) hit proto-Earth. It is possible that the mantle of the Earth could have remained unmixed following impact [Nakajima and Stevenson, 2015]. Whether the mantle remained unmixed or became partially mixed following the impact, the electrical conductivities of the rock that is affected by the magnetic storms may be around 1 S/m [Velímsky et al., 2006].

4.4 Using Meteorites as Proxies for Early Solar System Behavior

Meteorites have been used extensively to understand the early solar system. Many meteorites (and asteroids) are supposed to be the remnants of failed planetesimals. Certain classes of asteroids are assumed to be the cores of failed planets or actually make up the cores of other planetary objects and satellites in the solar system (Enceladus with a chondritic core, for example) [Sekine et al., 2015]. The asteroids and meteorites are "frozen" in time since their last major melting/heating event. Asteroids such as Allende have been the subject of many papers in the search for reliable paleo recorders in the hopes of better understanding the Solar System's history. Allende unfortunately was found to be mostly unreliable for understanding the ambient field of the early solar system [O'Brien et al., 2020] although the topic of this unreliability is still up for discussion [Carporzen et al., 2011].

Meteorites are even capable of providing information predating our solar system. The Semarkona meteorite recorded the ambient magnetic field intensity of the solar nebula [Fu et al., 2014]. This is useful in understanding the conditions that cause the formation of certain types of solar systems. However, in the scope of this thesis, this is not as important as the conditions of the early Solar System. Unfortunately, few to no meteorites have been found to provide the conditions of the early solar system. A difficulty in using meteorites to understand the conditions of the early solar system is that it is unknown where the meteorite recorded the magnetic field. In the case of this project, it would be most helpful if a meteorite formed near the Earth, or another terrestrial planet, as that would help to understand the general conditions of the time period being investigated. If such meteorites are found, it will be a great help in future calculations and constraints on possible mechanisms of magnetic field generation in a planet driven by the Sun.

4.5 Revisiting the Earth as a "type-planet"

By a type-planet, I mean using the Earth as a benchmark to judge all other habitable planet evolutionary tracks. There are multiple possibilities regarding Earth. The first is that the early Sun had little to no effect on the generation of a dynamo within the Earth. This would mean that, with respect to the behavior of the Sun, the timing of a major impact such as the one that formed the moon is unimportant. The second is that the timing of the moon-forming impact was critical in the generation of a magnetic field in the Earth. In [Folsom et al., 2017], there is a large discrepancy in magnetic field strengths for stars of the approximate age 120 Myr. It is unclear whether this is due to the stars sampled just varying, or whether there is an evolutionary reason for the great discrepancy in behavior. If this time in a young solar-type star's evolution does vary greatly, it could mean the timing of a large impact is critical. The third scenario is that the early Sun does cause/stimulate magnetic field generation in the Earth, however, a large impact is unnecessary. This, however, reprises the question, "Then why don't Venus and Mars have a proper geodynamo?"

Depending on what actually caused the Earth to obtain a self-sustaining dynamo will determine whether the Earth can be a "type-planet." If the moon-forming impact was the critical factor to dynamo generation, then similar planets will need a similar evolutionary history. If a similar evolutionary history is necessary, then many more constraints may be placed on the characteristics of a "habitable" planet, and narrow the search for such planets a great deal. The importance of "Critical Timing" will be explored below.

4.6 Impact Scenarios and the Possibility of "Critical Timing"

The above section briefly explored the idea of "Critical Timing" - an idea that assumes special evolutionary events must occur within a certain time frame for a specific type of planet to form. In the case of the Earth, according to current knowledge, the following are the prerequisites for such a planet to form (assuming critical timing). A solar-type star must form, and the evolutionary track of this star must be more chaotic around 120 Myr after formation. At this time, a planet such as Earth is struck by a Mars-size impactor at an oblique angle, causing disturbance to the interior of the planet. The disturbed and now partially undifferentiated Earth is now susceptible to magnetic flux from a superflare of the young Sun. A large sunspot, producing many solar storms on the surface of the young Sun sends out magnetic flux and interacts with the disturbed Earth. Power is dissipated within the planet, and a dynamo forms (likely of an alternative mechanism compared to the modern-day inner core outer core driven dynamo).

4.7 Future Work

Future areas for consideration vary widely, but the most immediate direction to test next is the mechanism that could drive dynamo generation. Here in this thesis, it was shown that enough power to sustain today's geodynamo is supplied to a hosted planet by storms of a young solar-type star. However, this does not mean a dynamo will form. The next steps are to explore the mechanism that would produce a dynamo in a planet, and whether this is viable for the origin of the Earth's magnetic field. This work is also useful for terraforming efforts in the far future. Future research building off the concepts explored in this thesis could lead to the ability to artificially start a dynamo within a "dead" planet such as Mars. Also, the electromagnetic induction heating explored in this thesis is the same mechanism that can be used in asteroid mining efforts to heat asteroids and separate out the components of the melt. That is, applying an external magnetic field would heat the interior of the asteroid and melt it.

5. CONCLUSION

Conventional models of the geodynamo presume that a small seed field is originally present in the local environment of early Earth. The seed field, of arbitrary, unspecified provenance is amplified, leading to the self-sustained magnetic field generation with the magnitude and behavior we observe today. The conventional model has proven to be enduring, leading to remarkable predictions and insight into Earth's evolution. Here, we have explored another mechanism involving external time-varying magnetic flux for lending impetus to geodynamo generation. This mechanism does not require a seed field and moreover is consistent with astrophysical observations on young solar-type stars.

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