MIXED PHASE IMPACT ON FIREBALL EVOLUTION

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by

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ABSTRACT

Mixed Phase Impact on Fireball Evolution

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In the evolution of fireballs produced by Ulra-Relativistic High Energy Collisions, the introduction of a first-order phase transition between Quark Gluon Plasma (QGP) and hadronic matter phases was predicted to extend the fireball lifetime. In this paper, this prediction was tested through use of a thermal simulation of an isentropic fireball, at a collision energy of $\sqrt{s}$ = 4.7GeV.

It has been previously established that there is a strong correlation between fireball lifetimes and integrated dilepton yields, through use of an earlier version of this simulation. After updating the simulation to find the fireball acceleration based on its equation of state, an effective implementation of a first-order phase transition was obtained, and this implementation's impact on integrated dilepton yields is then studied. It is confirmed that the correlation between integrated dilepton yield and lifetime was rather accurately conserved.

After reintroducing the EoS-based acceleration, the transition's effects on fireballs with collision energies other than 4.7GeV are also to be evaluated. Through combination of all of these results, the previously smooth trend between fireball lifetime and collision energy was seen to break at lower collision energies. As collision energy is decreased, at low energies, rather than continuing to decrease as is the case at high energies, the lifetime can be seen to increase again. This increase is explained as the effect of introducing a mixed-phase between QGP and hadronic matter, and is experimentally testable through finding the integrated dilepton yields.

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Contributors

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Thanks also go to my friends and colleagues and the department faculty and staff for making my time at Texas A&M University a great experience.

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All other work conducted for the thesis was completed by the student independently.

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NOMENCLATURE

QGP	Quark Gluon Plasma
HM	Hadronic Matter
EoS	Thermodynamic Equations of State
s	Entropy Density, $(\frac{\text{Total Entropy}}{\text{Total Volume}})$
p	Pressure
T	Temperature
ϵ	Energy Density
β	Flow Velocity, typically taken at surface

1. INTRODUCTION

1.1 What is QGP?

As its name indicates, QGP is a plasma comprised primarily of quarks and gluons, though, before diving into a description of QGP, it is useful to describe what these two things are themselves.[1]

Rather than working backwards up a long chain of particles, it is more natural to begin with the atom, which contains a nucleus orbited by electrons. The nucleus of an atom is a cluster of neutrons and protons, the two most common examples belonging to a family of particles known as "baryons". Historically, protons and neutrons were believed to be elementary particles; later, they were believed to actually be the same particle with a property called "isospin" distinguishing the two. However, following years of explosive growth and discovery in the field of particle physics, symmetries in the newly expanded family of baryons led to the acceptance of "quark theory" to describe these particles.

Quark theory explains that baryons are not elementary particles, but rather are a collection of three quarks (which are elementary particles). These quarks come in six "flavors" (up, down, strange, charm, bottom, top) as well as having antimatter forms (antiquarks). It also explains that mesons, another family of particles discovered alongside baryons, are a quark-antiquark pair. In order to obey quantum mechanics, these quarks must also have some additional property, which is found in "color".

Despite the name, color is most analogous to a charge, taking values of red, green, or blue (anti-red, anti-green, or anti-blue for antiquarks). Color drives interactions between quarks through the strong force, which is mediated through gluons (which are named so as they "glue" together baryons). However, baryons must contain exactly one red, one green, and one blue quark, mesons must contain a red and an anti-red (or similar for green or blue), and antibaryons must contain exactly one anti-red, one anti-green, and one anti-blue antiquark. These combinations are all color neutral, and no particle has yet been observed with a net color. No discovery has been made of a singular, deconfined quark, which is where QGP comes into relevance.

In QGP, quarks and gluons are deconfined and free to move independently. In a typical plasma, nuclei and electrons decouple, as the interactions between them are shielded by other nuclei and electrons. Similarly, in QGP, the color charge is shielded by free quarks and gluons, resulting in a plasma.

1.2 QGP History

In the late 1970s, theories of a new state of matter known as a "Quark Gluon Plasma" were first created[2]. As explained above, in this theoretical state of matter, extremely high energies are present and allow for elementary quarks and gluons to overcome the strong force, specifically the binding in hadrons. The resultant matter has the unique existence of *deconfined* quarks, rather than the *confined* quarks present in hadronic matter, such as mesons and baryons. Another key phenomenon seen in QGP is mass production. At these extreme energies, most of the phenomena of physics obeyed by our everyday world are no longer effective, leaving this subject as a highly interesting area of modern research.

In 2000, high energy particle collisions first indicated the existence of this state of matter[3], heightening the relevance of this area of inquiry. Relatively little was known of this matter's behavior, although after this point, theory was finally testable.

1.3 Relevance of QGP Study

It is the very nature of physics to seek the unknown, and question the known. As QGP presents physicists with a bounty of questions to ask, it is only natural to dig deeper.

Beyond this primal desire to know more, the relevance of studying QGP can be found in the Big Bang. Within the Big Bang theory, the early universe (from around 10^{-10} to 10^{-6} seconds after creation) was composed entirely of QGP. As this energetic early universe expanded, it cooled down, eventually forming hadronic matter and, much later, the universe of today. This transition in particular, from QGP to hadronic matter, is not fully understood and is the focus of this thesis.

One of the major questions posed of the Big Bang theory is the existence of more matter than antimatter. As both quarks and antiquarks are present within QGP, one possible cause of this imbalance could be the existence of a first-order phase transition between QGP and hadronic matter. Much like water freezing into ice, another first-order phase transition, the presence of both states of matter would allow for the breaking of some symmetries, which, in turn, would explain how our universe composed of matter exists. While there are other possible explanations for our matter-antimatter imbalance, the main, proven-to-exist explanations we have today do not sufficiently answer this question.

1.4 Past Studies of QGP

1.4.1 Modeling Heavy Ion Collisions and QGP behavior

The experiments needed to study QGP require extremely high collision energies, producible at a limited number of particle colliders around the world. Computer models are a common way to theoretically study "fireballs", or the aftermath of a heavy-ion collision (starting with QGP and including the transition to hadronic matter). I will divide these simulations into two groups, dependent on the behavior of the fireball that they focus most heavily on. These two groups are "hydrodynamic models" and "thermodynamic models".

Hydrodynamic models, such as that used by Hung and Shuryak[4], allow for an in-depth understanding of the internal evolution of the fireball. This allows for great leaps in theory, but as can be seen from the cited paper, has limited ability to produce easily experimentally tested data.

Thermodynamic models, such as that used by Rapp and van Hees[5], excel at replicating and predicting experimental data. However, they are not as effective at predicting the exact hydrodynamics of the fireball. Most notably, these models allow for straightforward calculation of radiation spectra produced by the fireball, while also confirming that they can be used to find fireball lifetimes and temperatures. Radiation spectra are observed experimentally for QGP analysis, so use of a thermodynamic model to predict theoretical radiation might be used to confirm the presence of a first-order phase transition.

1.5 Dilepton Radiation Analysis

While it is straightforward to find the lifetime of a fireball in a simulation/model, measuring the lifetime of the fireballs produced experimentally directly is not feasible, as such lifetimes are on the order of tens of fm/c (the time required for light to travel one femtometer, 10^{-15}m), or $\approx 10^{-23}$ seconds. A promising method for finding proof of a first-order phase transition within fireballs is through changes in fireball lifetimes, thus, making this task a highly relevant one to this research.

This research was inspired by work done by Ralf Rapp and Hendrik van Hees[5] which showed their is a strong correlation between integrated dilepton-yields and fireball lifetimes. Thanks to this, in this research we aimed to show not only that fireball lifetime was predicted to increase due to a first-order phase transition, but that this increased lifetime is accompanied by an identical increase in integrated dilepton yield.

2. METHODS

To recap, in high energy nuclear physics, the collision of two atomic nuclei is believed to result in the production of a hot, dense state of matter known as a quark-gluon plasma (QGP). This state of matter is extremely short-lived, and its internal nature is vastly different from typical matter in our everyday world, so it is very difficult to study the behavior of this substance. It is, however, a highly important area of inquiry due to the fact that such high density and high temperature matter is what comprised the early universe. As such, in order to better understand the origins of our universe, we must first develop our understanding of how QGP behaves, which is what our research aims to accomplish.

More specifically, our research focuses on the behavior of expanding QGP near its transition into hadronic matter. At large baryon concentration (large μ_b, i.e. at a quark over antiquark excess), this transition has been theorized to take place as a first-order phase transition, and to test this theory, we study the effects of a mixed phase existing between QGP and hadronic matter in the aftermath of high energy collisions. The system of the two nuclei after colliding (including QGP, transition, and hadronic matter) is referred to as a fireball, due to its high temperature, high energy, and rapid expansion. These fireballs have a lifetime that is known (in higher energy collisions) to increase as collision energy is increased. However, in lower energy collisions, still with enough energy to form QGP, the fireball lifetime is expected to increase as collision energy is decreased, assuming the existence of a mixed phase. In our research, we make the assumption of a mixed phase's existence, and thus we find the increase that can be expected if a mixed phase does exist.

To measure the lifetime of a fireball in an experiment, the relevant calculation is finding the integrated dilepton yield [5]. The simulation we worked with in this research, developed by Rapp and his collaborators, has been previously shown to have a highly accurate dilepton production in comparison to experimental data [1]. Through use of this simulation, a parameter study was initially conducted to find the most relevant parameters to dilepton production and fireball lifetime.

Initially, these parameters were free to be set by the user, and chosen in agreement with experimental constraints. As the inclusion of a mixed phase would have the primary effect of modifying these parameters, it was necessary to find the parameters our project needed to focus on.

2.1 Parameter Study

The fireball is assumed to result from a central collision, and therefore its shape is approximated to be cylindrical[6]. Additionally, the simulation is built off the assumption of isentropic expansion (S_{tot} = contant) giving the following set of equations.

$$\frac{s}{n_b} = \text{constant} \tag{Eq. 2.1}$$

$$s = \frac{S_{tot}}{V_{FB}} \tag{Eq. 2.2}$$

$$V_{FB}(t) = \pi r_\perp(t)^2 z(t) \tag{Eq. 2.3}$$

Where s the entropy density, n_b the baryon number density, S_{tot} the total entropy, and $V_{FB}(t)$ the total volume of the fireball at time t. Also, $r_\perp(t)$ and $z(t)$ are the radius and length, respectively, of the fireball cylinder at time t. These were found, with constant radial acceleration ($a_\perp$), as

$$r(t) = r(0) + \frac{\sqrt{1 + a_\perp^2 t^2} - 1}{a_\perp} \tag{Eq. 2.4}$$

and

$$z(t) = z(0) + v_z t \tag{Eq. 2.5}$$

assuming constant longitudinal velocity (v_z). A relativistic acceleration formula was used for Eq. 2.4.

The primary parameters needed to be studied were the transverse and longitudinal acceleration of the fireball. The dilepton production is dependent on fireball surface velocity and volume (which due to the isentropic nature of the fireball can be used to find temperature and other thermodynamic properties of the fireball through known equation of state). Following all of this, modifications of the accelerations of the fireball thus impact the dilepton production. Within a reasonable range of values, the graphs below show the dilepton production of the fireballs as a function of acceleration parameters used.

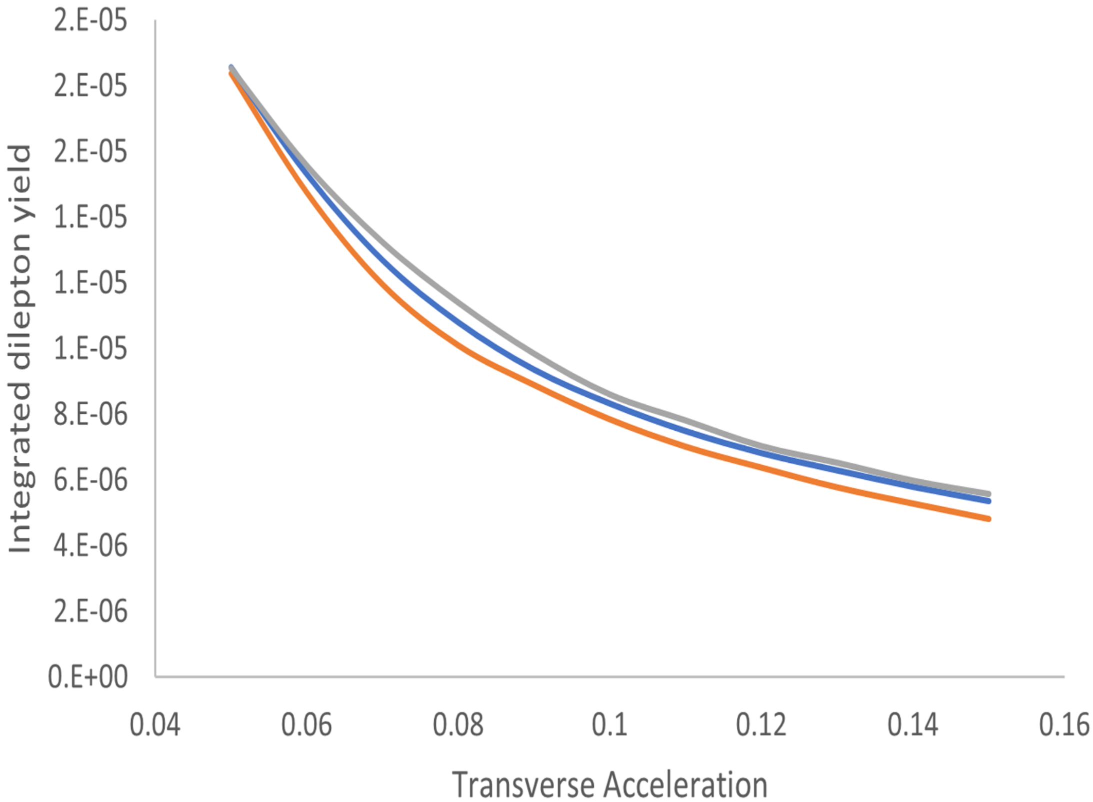

Figure 2.1: This figure shows the dilepton yield as a function of input transverse acceleration. For these data points, a longitudinal acceleration of 0 was assumed, with an effective longitudinal velocity (twice the velocity of each end of the cylinder) of 0.6, 0.8, and 1, for the orange, blue, and grey lines, respectively. Accelerations are given in c^2/fm while velocities are given in c.

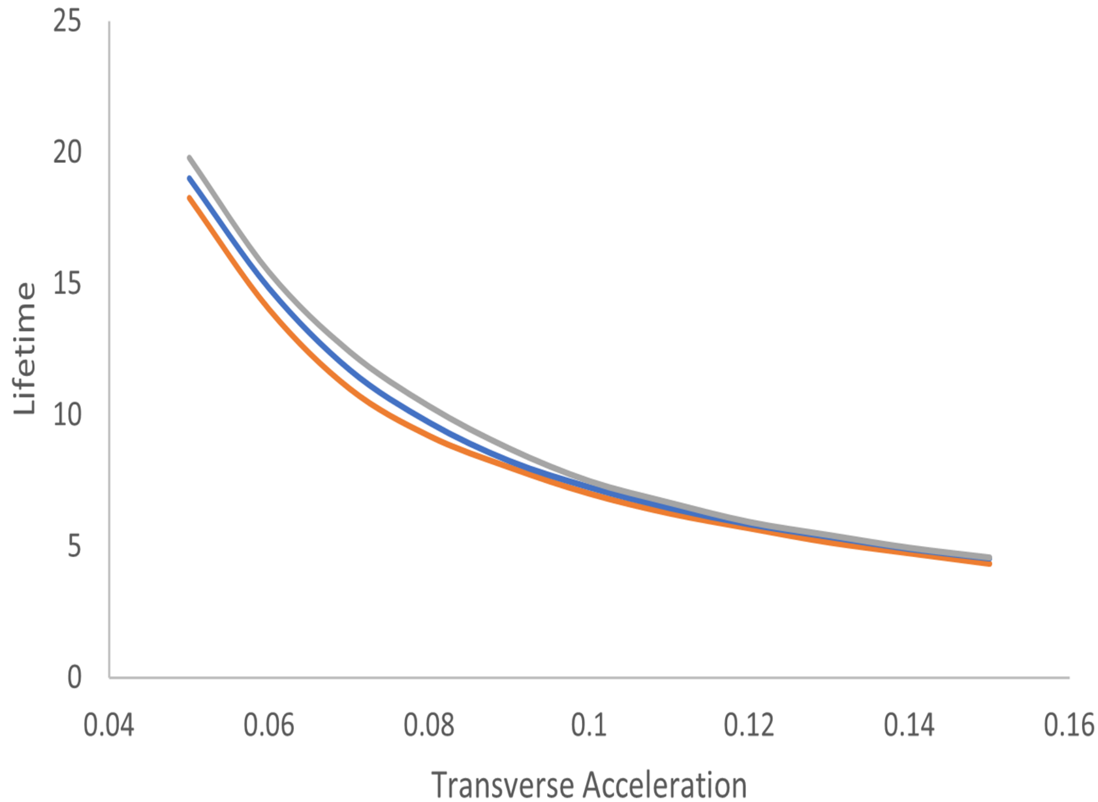

Figure 2.2: Using the same key as in Fig. 2.1, and taking the same assumption, this figure shows the simulated fireball time as a function of input transverse acceleration. The primary purpose of this plot is to confirm the relation between integrated dilepton yield and fireball lifetime still holds within the range measured, and therefore obeys the same trend. Accelerations are still given in c^2/fm, while lifetime is given in units of fm/c.

From Figures 2.1 and 2.2, it is clear that the transverse acceleration is a highly relevant parameter, and modifications of it result in significant alteration to the fireball's integrated dilepton yield and lifetime. Meanwhile, it is also clear that while there is some effect in dilepton yield and fireball lifetime from altering the longitudinal expansion velocity, these effects are minor and for our purposes negligible.

After analysis of the above figures, it was decided to focus solely on the impact of mixed phase inclusion on the transverse acceleration of the fireball. It is also worth noting, mixed phase inclusion is expected to effectively lower this transverse acceleration, increasing fireball lifetime. Additionally, lower collision energy fireballs have a lower transverse acceleration already, resulting in a stronger impact of mixed phase inclusion on the final integrated dilepton yield. This conclusion makes sense as fireball lifetime is ended when thermal freeze-out is achieved, which is dependent on an effective (Doppler blue-shifted) temperature[6]. This effective temperature is dependent on the squared flow-velocity, which requires more time, at low accelerations, to reach the experimentally determined value (from final-state hadron spectra).

3. RESULTS

This chapter will focus on the equation of state (EoS) of the fireball; specifically, how and why it was implemented into the existing simulation, and the original results obtained for its values.

3.1 EoS-based acceleration

Following the parameter study, the fireball acceleration's previous origins based on user input was deemed unsatisfactory for the goals of this project. In its place, we derived a formula for transverse acceleration based on the fireball's internal equation of state. This was done through solving relativistic expansion equations, with the assumptions of: no external forces, constant entropy, and linearly distributed pressure (p), energy density (ϵ), and flow velocity. In the end, we obtained an acceleration of p/ϵ. A similar result was also used previously by Hung and Shuryak [4], in a hydrodynamic context.

3.1.1 Computational, Time-Step-Based Relativistic Kinematics

As this thermodynamics-based acceleration is clearly non-constant Eq. 2.4 is no longer applicable for finding the radius. Following along with basic special relativity, we arrive at the equation

$$\frac{\partial(\gamma_{rest} v)}{\partial t} = A(t) \qquad \text{(Eq. 3.1)}$$

where $A(t)$ is an arbitrary proper acceleration, v is velocity, and γ is the usual Lorentz factor[7]. For this equation, the frame used must be the rest frame, to allow for $\tau = t$. Discretizing and rearranging this equation leads to

$$\gamma_1 v_1 = A_0 \Delta t + \gamma_0 v_0 \qquad \text{(Eq. 3.2)}$$

where notably γ_1 and γ_0 are found in different frames, to satisfy the conditions required of the above equation.

Solving this leads to an incredibly long expression that will be omitted here, but v_1 can be solved for analytically (after substituting in $\gamma_1 = 1/\sqrt{1 - v_1^2}$). This provides a step-based solution to the problem of varying acceleration, as v_0 can repeatedly be set as the velocity from the past iteration of the loop, and $r_\perp(t)$ can be solved for similarly to how $z(t)$ is solved (as the frame is repeatedly changed to near rest, relativistic effects are negligible in each step).

3.1.2 EoS Derivations

In order to implement this into the simulation, the EoS already present in the simulation needed to be expanded. The basis of all thermodynamics calculations were the grand canonical ensemble,

$$p = \epsilon - Ts - \sum \mu_i n_i \qquad \text{(Eq. 3.3)}$$

where μ_i are the chemical potentials of all internal baryons and mesons, n_i are the number densities of the internal baryons and mesons, s is entropy density, and T is temperature. As the simulation is built upon the idea of isentropic expansion, and baryon number must be conserved, $S_{tot}/N_b^{net} = s/n_b$ is a conserved quantity. Taking a known relation of temperature to both entropy density and μ_i, pressure was straightforward to find through Eq. 3.4, and energy density in turn from Eq. 3.3.

$$dp = sdT + \sum n_i d\mu_i \qquad \text{(Eq. 3.4)}$$

To allow for flexibility in the simulation, this was solved for p and ϵ as functions of s for both the QGP and HM phases individually. In the mixed phase, the pressure, temperature, and baryon chemical potential are held constant, with the energy density changing to maintain Eq. 3.3. Pressure was obviously found through integrating the expression in Eq. 3.4, and as dT and $d\mu_i$ are all zero throughout the mixed phase, the hadronic pressure at the critical temperature was used as a constant to find the pressure on the QGP side. Mathematically (using T' to avoid repeating the integrated variable),

$$p_{QGP}(T') = \int_{T_{ch}}^{T'} sdT + p_{ch} \qquad \text{(Eq. 3.5)}$$

where p_{ch} is the pressure on the hadronic side of the EoS taken at $T = T_{ch}$.

Initially, the initial relationship between temperature and entropy density were configured such that there was a smooth transition between the hadronic matter phase and the QGP phase, with no mixed phase or 1st-order phase transition. To introduce the phase transition, the relations were changed on both sides such that the critical entropy density on the QGP side was raised and the critical entropy density on the hadronic side was lowered. In between these two values the temperature was held constant at the critical temperature, which was unchanged from the initial 157MeV. On the hadronic side this change was slowly increased starting from the chemical freeze-out (at $T = T_{chem}$)until the phase transition. On the QGP side, the change was constant, as a scalar change in the entropy at any given temperature.

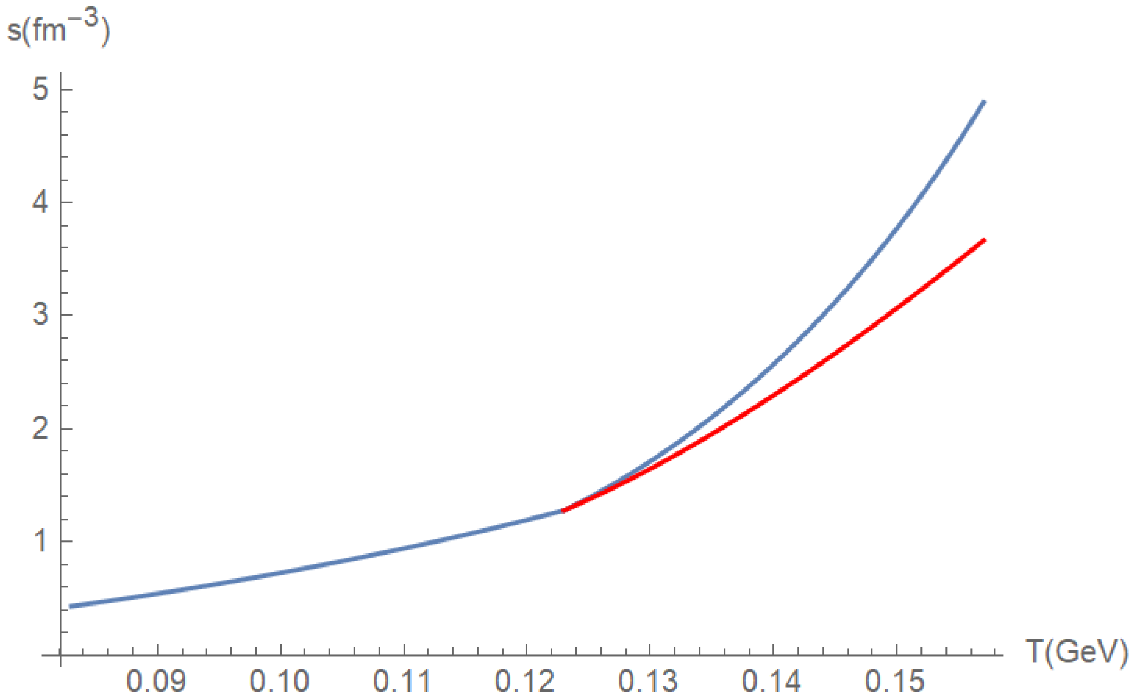

Figure 3.1: This figure shows the change to entropy as a function of temperature implemented to the *hadronic* side of the simulation. The data without the phase transition adjustments can be seen in blue, while the new data with the phase transition can be seen in red.

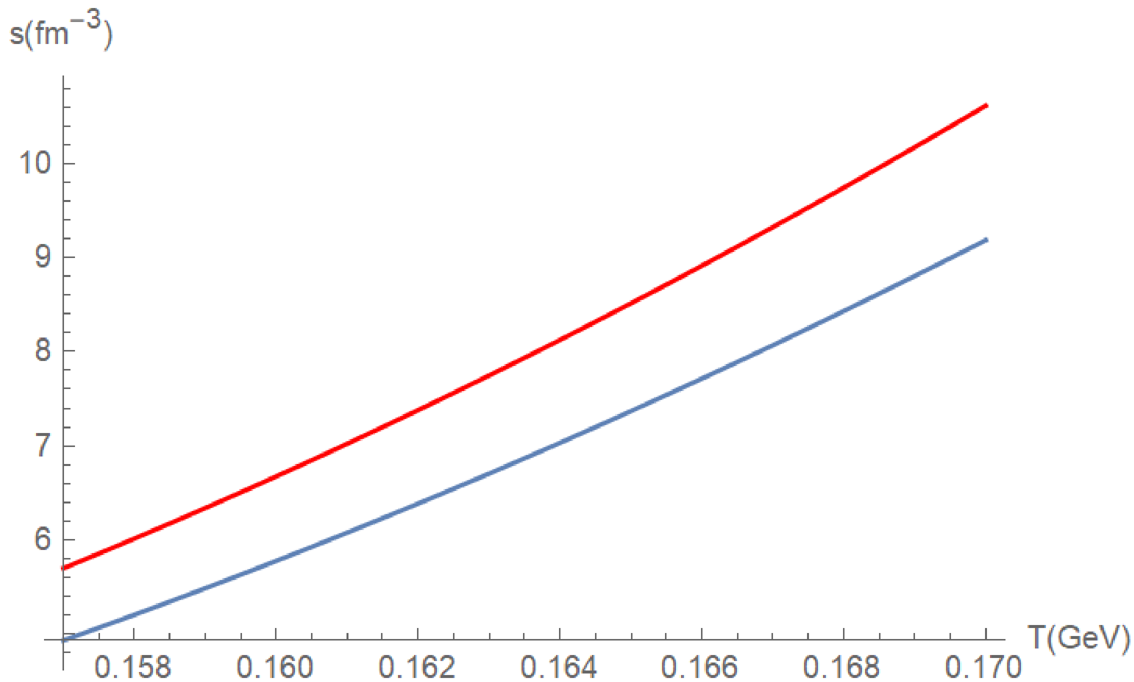

Figure 3.2: This figure shows the change to entropy as a function of temperature implemented to the *QGP* side of the simulation. The data without the phase transition adjustments can be seen in blue, while the new data with the phase transition can be seen in red.

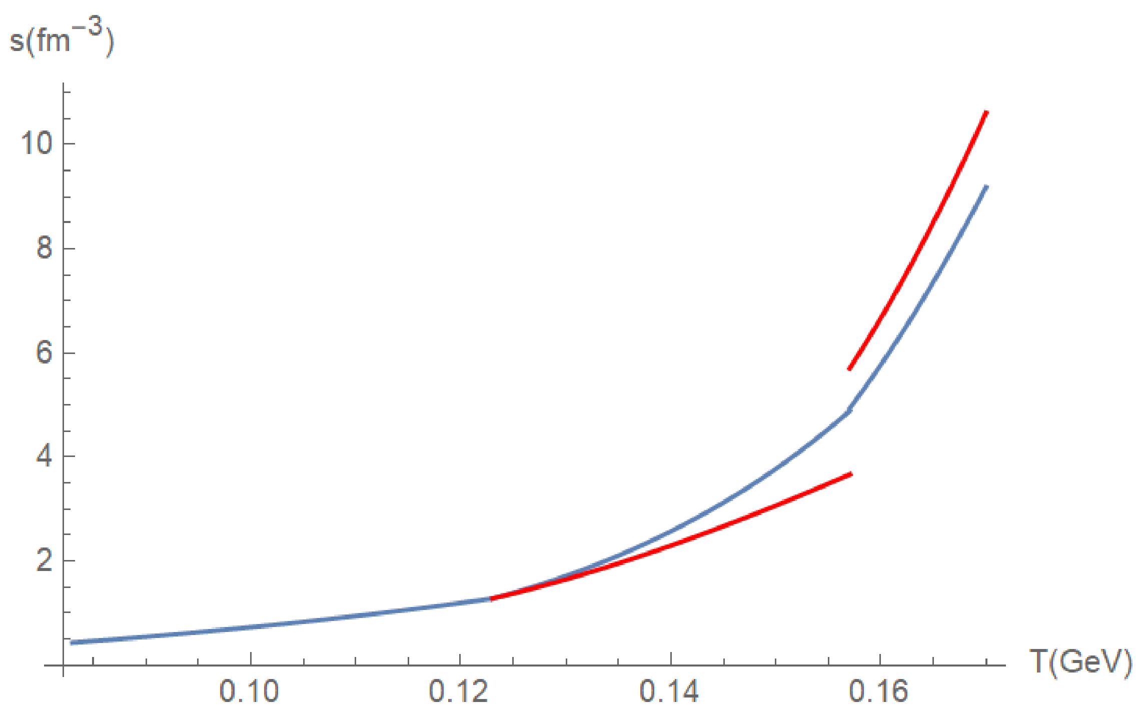

Figure 3.3: This figure shows the change to entropy as a function of temperature implemented to the full simulation. The data without the phase transition adjustments can be seen in blue, while the new data with the phase transition can be seen in red.

As can be seen from Figure 3.3 the introduced discontinuity at 0.157GeV, the critical temperature, allows for temperature to be held constant while entropy density is decreased due to expansion. It is worth noting that, with these adjustments, the full EoS calculation should be split into four sections: hadronic below chemical freeze-out, hadronic above chemical freeze-out, mixed phase, and QGP. All but the first of these sections are altered by the introduction of the phase transition (and the mixed phase effectively non-existent before this introduction).

Using the above data and equation Eq. 3.4, the pressure can be found as a function of entropy density (rather than temperature, as is standard, to allow for visibility of the mixed phase). After this, equation Eq. 3.3 can be used to find ϵ, and acceleration (p/ϵ).

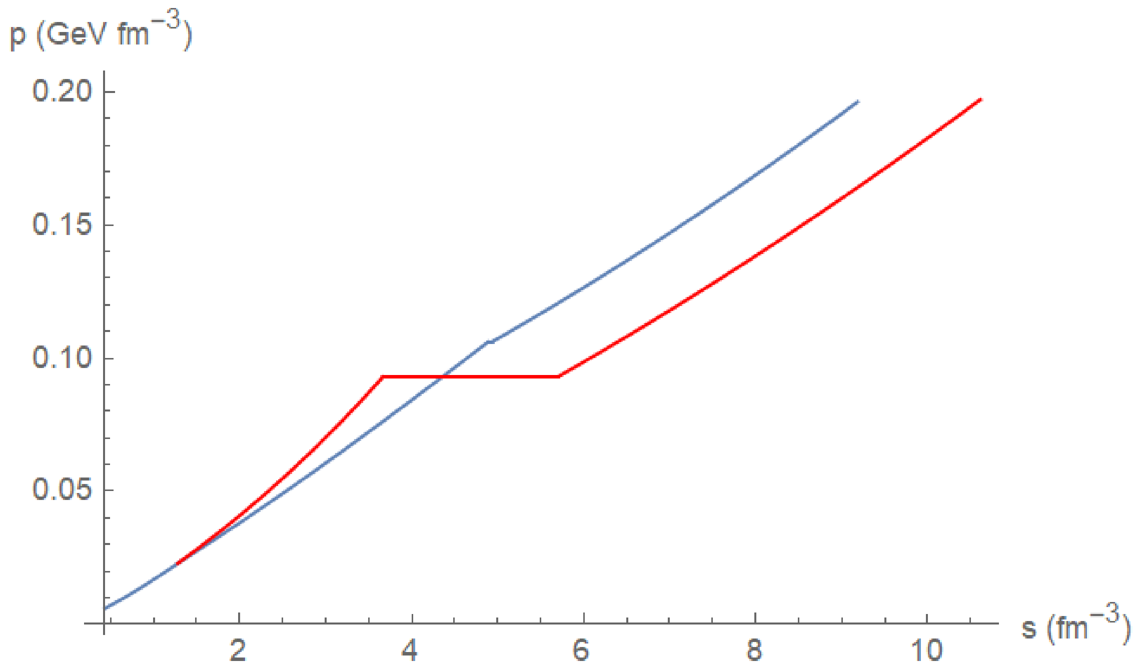

Figure 3.4: This figure shows the pressure of the system throughout its evolution. The data before phase transition introduction is shown in blue; the data after phase transition introduction is shown in red.

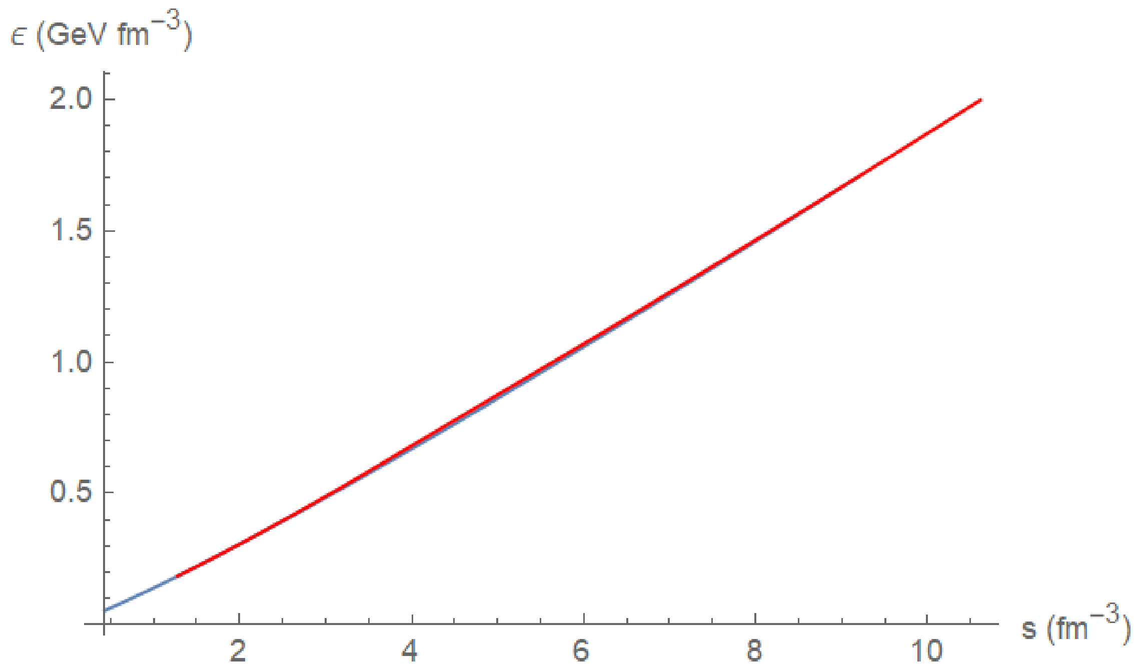

Figure 3.5: This figure shows the energy density of the system throughout its evolution. The data before phase transition introduction is shown in blue; the data after phase transition introduction is shown in red. The data is aligned such that the two line happen to be on top of each other, but both do extend to $s > 10(\mathrm{fm}^{-3})$

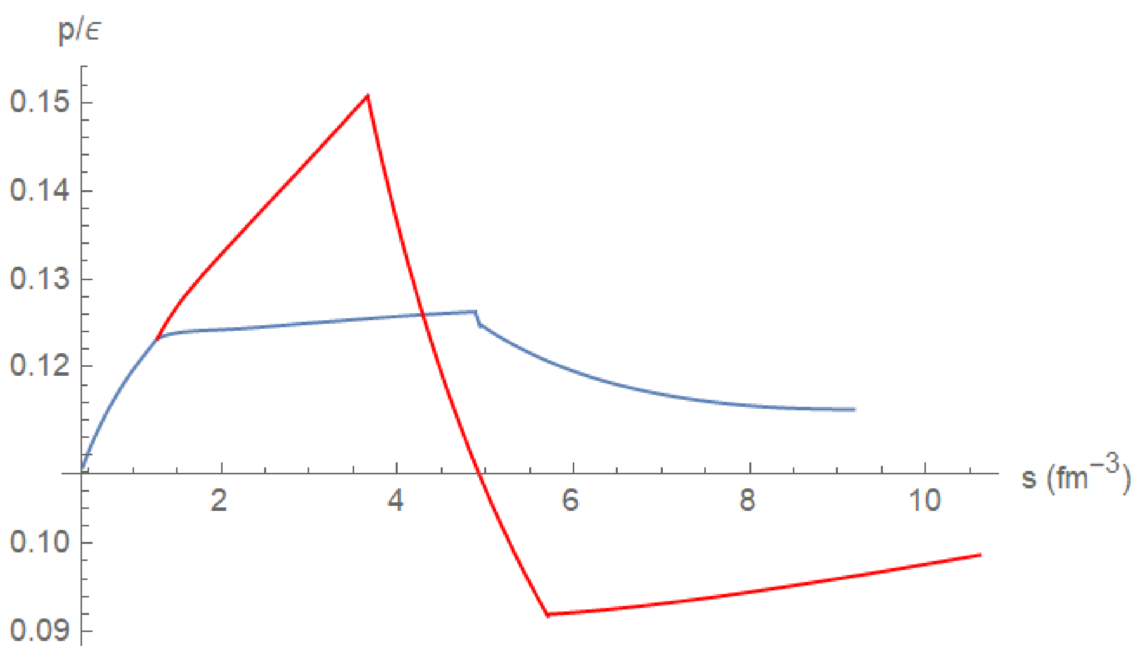

Figure 3.6: This figure shows the acceleration (p/ϵ) of the system throughout its evolution. The data before phase transition introduction is shown in blue; the data after phase transition introduction is shown in red.

It is worth paying a bit more attention to the fact that, on the hadronic side of the EoS, introducing a phase transition actually increases the acceleration of the system (as can be seen in Figure 3.7); however, the average acceleration throughout the fireball is balanced out by decreased acceleration in the QGP phase. Due to the relativistic nature of the expansion, the increase in velocity change over one time step (found from Equation Eq. 3.2) falls off for larger accelerations. Due to this, it is likely that the time-average acceleration of the fireball will be lower, providing the expected result (this cannot be fully confirmed until implemented into the code). With this lower average acceleration, Figures 2.1 and 2.2 predict a larger integrated dilepton yield and a longer fireball lifetime.

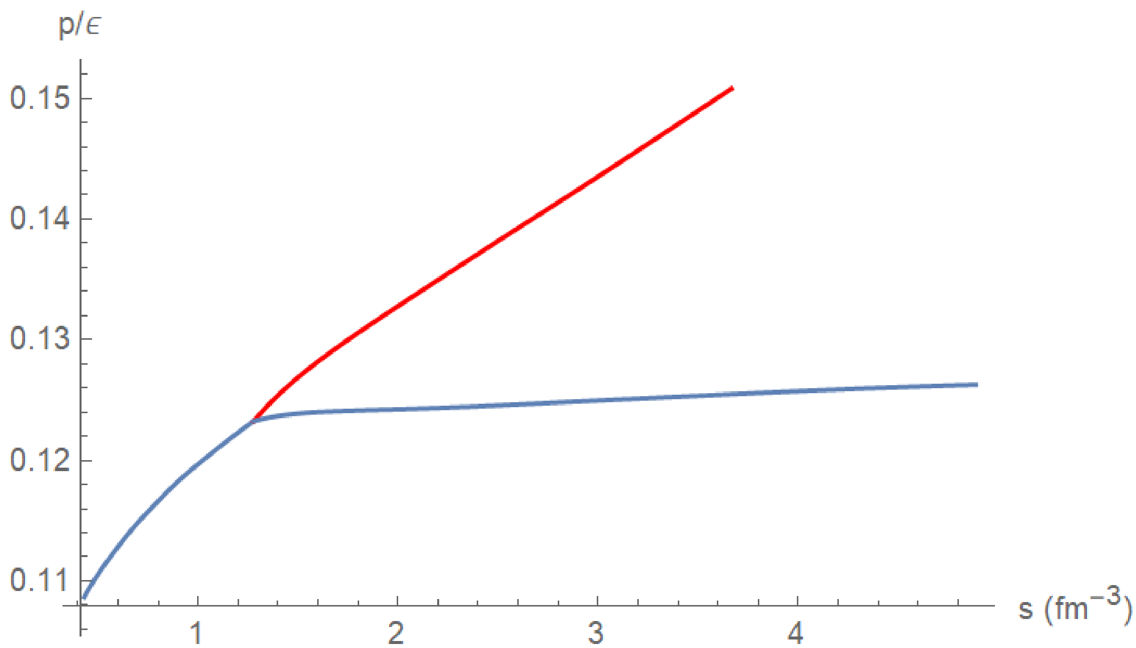

Figure 3.7: This figure shows acceleration (p/ϵ) in only the hadronic side of the EoS. In blue is the data before the introduction of the phase transition, and in red is the data after the introduction.

Figure 3.7 highlights this increased acceleration mentioned above. Additionally, while the continuation of the red line (after introduction of the phase transition) does seem to be a more satisfying and continuous trend, the smoothness of this continuation seems to be coincidental. In modifying the relation between entropy and temperature, the scalar quantity used was mostly arbitrarily chosen, and was chosen long before the generation of this data. Therefore, the trend, while visually appealing, is treated as coincidence.

However, the lining up of data in Figure 3.5 is not treated as coincidental at all. While the work has not been done analytically to prove so, Figure 3.5 clearly shows that the scaling provided to entropy held an equal impact to energy density.

4. CONCLUSION

4.1 Next Steps

After publication of this thesis, work will be underway on this project to implement the newly derived EoS data into the code of the simulation, and to find the integrated dilepton yields (and fireball lifetimes) both with and without the phase transition. From here, similar changes (with recalculated EoS extensions due to differing s/n_b) can be implemented into other simulations with different $\sqrt{s}$ collision energies to look for a trend. Without the transition, the fireball lifetimes are expected to decrease while $\sqrt{s}$ decreases. However, following our parameter study and our derivations of thermodynamics-based acceleration, we expect to see a significant increase to fireball lifetime within a small range of lower collision energies.

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