POST-ACUTE CARE FACILITIES: CAPACITY PLANNING AND SELECTION PROCEDURE

A Dissertation

by

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ABSTRACT

Health care services received after discharge from an acute care are called post-acute care (PAC). These services improve patient functioning and help patients for better transition from hospitals to the community. PAC can be delivered in different settings such as long-term care (LTC). LTC is vital for people with functional limitations. In the U.S., most LTC is financed by state Medicaid programs. These are administered by states and jointly financed with state and federal funding. There are two main types of LTC delivery: institutionalized care, dominated by the nursing home industry (NHC), and outpatient care, provided through home and community based organizations (HCBS). HCBS is primarily funded through Medicaid "waiver" programs that allow states to allocate some LTC funding to non-institutionalized settings. While HCBS is the less costly option, participation is limited by capacity shortages, and many state waiver programs have long waiting lists. As the population ages, the demand for LTC is projected to grow significantly, and thus HCBS capacity problems constitute a significant policy concern. This work investigates this by formulating a bi-level stochastic game model in which a Medicaid program (the leader) specifies the size of its waiver program, and then HBCS organizations (the followers) respond by specifying their capacity, with LTC service demand being uncertain. We characterize the problem and design an approximation algorithm that exploits a piecewise linear function for computing the followers' response function to the leader's decision. We use a case study based on data from the state of Texas.

Another important question in PAC studies is how to select the best providers. In addition, it is vital to determine the factors that play roles in this decision making procedure. Acute care managers are always looking for the best PAC providers, while they are willing not to pay too much. In order to determine a set of best PAC's, a multi-objective decision making approach is developed for Post-Acute Care Provider (PACP) selection. PCAP selection, similar to other subcontracting problems, depends on multiple criteria. Besides the cost metrics, considering service coverage requirements, readmission rate, and service quality make the decision making more complicated.

The proposed approach provides the decision making procedure for acute care providers subcontracting with PAC providers. This approach includes two phases. In the first phase, providers are evaluated and assigned a comparable value based on a set of criteria. These quality metrics are used to calculate closeness coefficients of each candidate PACP for both short-stay and long-stay patients. These patient categories are determined by the Medicaid. In the second phase, using the computed coefficients, we develop a multi-objective problem that considers cost, service quality, and readmission to the hospital as objectives. The novelty of this procedure is introducing a new view toward the provider selection problem. The proposed approach is implemented for the PACP selection problem in the city of Houston, TX.

DEDICATION

I dedicate this dissertation to the memory of my loving baby, Ali, whom I lost while working on this research, and to his little brother, Amirali, who gave a new meaning to my life. I will carry your love forever in my heart.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

TABLE Page

1. INTRODUCTION

Long Term Care

One of the important problems which is barely studied in the healthcare literature is capacity and network planning for post-acute care facilities. Post-acute care can be received in a wide variety of settings including: Skilled Nursing Facility (SNF), Inpatient Rehabilitation Facility (IRF), Long-term Care Facility (LTC), and Home Health Care Agencies (HHC). Long-term care (LTC) is necessary for people with limitations in activities of daily living, e.g., bathing, toileting, walking, and in instrumental activities of daily living, e.g. housekeeping and preparing meals [1]. The need for LTC is growing rapidly due to a growing elderly population and increasing incidence of chronic disease [2, 3]. In the U.S., the number of LTC recipients is projected to increase from 15 million in 2000 to 27 million in 2050 [4]. Meanwhile, spending on LTC is projected to increase from $194 billion in 2000 to more than $340 billion by 2030 [5, 6]. Since public programs are the primary funding source for LTC, federal and state policy makers seek more efficient and affordable LTC delivery methods.

One factor driving increased LTC demand is the aging of people born from 1946 and 1964 (the post-World War II "baby boom" generation). Figure 1 depicts the impact on the elderly population from 2000 to 2050. Healthcare costs in the U.S. are expected to soar as this generation reaches senior citizen status.

There are two main types of LTC delivery: nursing home care (NHC) and home- and community-based services (HCBS). Nursing homes provide around-the-clock skilled nursing care in an institutionalized setting, while HCBS provides personal assistance in the recipient's home and community. NHC is more expensive and often provides more care than necessary, but its recipients require less hospital services [7, 8]. HCBS delivers less care but is high in patient satisfaction since patients often prefer to stay in their own homes if at all possible.

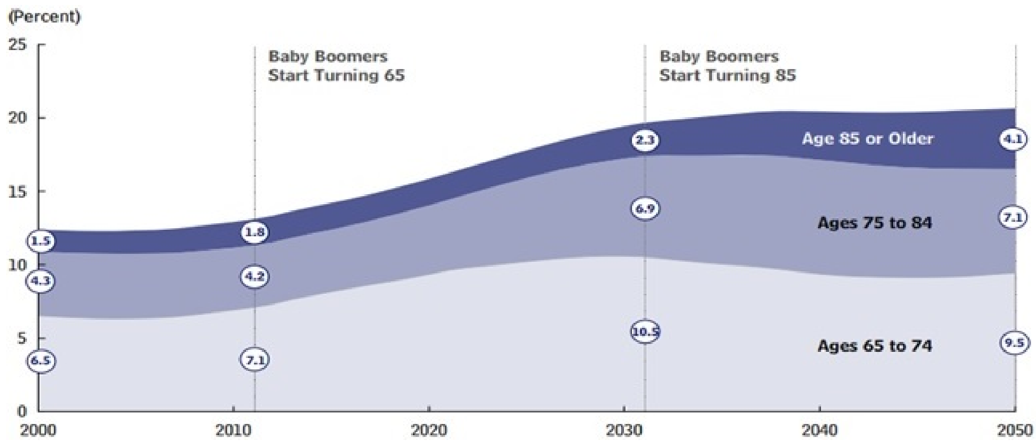


Figure 1.1: Growth in Elderly Population in the U.S.

While NHC is the main type of LTC delivery, decision makers for public programs are seeking appropriate capacity expansion of the HCBS industry as a way to deal with costs and improve social benefit.

Medicaid, a joint federal-state program, is the main funder of LTC services. Although administered by the states, federal regulations prescribe the services and service providers that can be reimbursed under Medicaid programs. In 1982, an Omnibus Budget Reconciliation Act allowed states to re-bundle Medicaid paid LTC services through the 1915 HCBS waivers. Under this law, some services traditionally provided only by nursing homes were allowed to be delivered and reimbursed through HCBS (usual standards are "waived"). Some states have adopted this approach, providing a limited number of waivers for Medicaid beneficiaries who meet nursing home admission criteria [1]. In other states, this "LTC de-institutionalization movement" has not yet taken hold and only NHC is supported, especially for the elderly [9, 10, 11, 12].

Because budgets are limited, Medicaid resources directed to HCBS are diverted from NHC, and thus state waiver programs are allowed to provide only a limited number of waiver "slots". The number of waiver slots funded is a significant decision given that it impacts all three of the major U.S. healthcare policy objectives of access, quality, and cost of care. Further, the number of waivers impacts the capacity expansion decisions of HCBS industries within a state [10]. These capacity decisions are important to Medicaid policy makers because a healthy LTC industry is essential in every state.

A stochastic Stackelberg-Nash-Cournot equilibrium model to determine capacities for HCBS providers is developed. This model consists of a leader, Medicaid who decides on the number of waivers; and followers, HCBS providers who decide on service capacities. Uncertain demand impacts both Medicaid and the HCBS decisions. To the best of our knowledge, these inter-related decisions of Medicaid waivers and LTC capacity have not been studied in a distributed decision making environment.

Post Acute Care

Post-acute care (PAC) includes rehabilitation services that beneficiaries receive after staying in an acute care hospital. Depending on the intensity of care the patient requires, treatment may include a stay in a facility, ongoing outpatient therapy, or care provided at home. Post-acute care is a growing and essential health and social service, accounting for more $2.7 trillion spent on personal health care, and, of that, almost 15% of total Medicare spending. Since 2000 PAC industry has evolved and grown substantially because of two main reasons. First and foremost, the government is placing so much pressure on hospitals about readmission. So, they need to make sure that patients receive good care after discharge from the hospital. Besides, a substantial amount of the total Medicaid budget is allocated to PAC's.

In recent years, there has been substantial progress in the relationship between acute care (AC) and post-acute care (PAC) providers and patients' transitions between them. This was precipitated by the recognition that fragmentation of care across settings is not beneficial to patients and expensive for healthcare systems [13]. There are several potential ways to reduce the impact of this fragmentation. First, AC and PAC providers may formally integrate into a single financial entity with combined legal ownership. Second, they may selectively strengthen ties with each other while remaining legally separate. Under this model, hospitals and post-acute care providers conduct a significant number of transactions with each other, sometimes with agreements or contracts to coordinate care, but they remain legally separate entities.

We study the problem of selecting a portfolio of PAC providers from hospitals' or, in a broader point of view, from an AC perspective. PAC provider portfolio selection is the process of selecting and contracting with a set of PAC providers by a specific acute care manager. This process is, indeed, strategic and so long-term decision for AC providers. A long term relationship with PAC providers is incredibly beneficial to the AC's. It offers advantages such as stability in future plans and strategies, and reliability of the quality of services provided to the patients. The standard approach taken toward the PAC selection is to select the best available to date, and probably evaluate them in periodic times. However, considering the fixed cost of contracting with a provider, this approach may not necessarily be beneficial. The contract's fixed cost may contain the cost associated with research on different providers, collecting required data, and consulting with experts. Besides, a contract needs lawyers for writing and considering the legal perspectives. Deciding on the number of years, the number of patients from each type and many other details require time and money. All these together create a fixed cost for contracting with a PAC.

To the best of our knowledge, there is no article in the literature of PAC studies, which investigates this problem from the optimization point of view. However, there are a few papers that look into this problem from a medical point of view. [14] determined the factors that influence post-acute care decisions by surveying stroke discharge planners. They conclude that nonclinical factors such as prioritization of health service referrals, patient's residence, and workforce capacity have major impact on their decisions. [15] did an extensive review on patient-related factors that guide clinical decision-making regarding to rehabilitation admission after acute stroke. The majority of PAC literature, including those published in medical journals, are related to heartstroke. We consider this problem in a broader perspective, in that we classify patients in subgroups assuming their need to specific services in PAC facilities.

There are two main things that managers in AC's look into it when deciding on selecting the best PAC's to contract with. First and foremost is providing the patients with services they need during their recovery after discharge from the hospital. PAC will agree to engage in an active and ongoing program to evaluate and modify their practice patterns and create a high degree of interdependence and cooperation within the network. The other, which recently turns to be vital, is that new healthcare legislation started using a new payment method called bundled payment (BP) system. Under BP, the insurer/public insurer (Medicare or Medicaid) only pays the pre-specified bundled payment value upfront to cover all possible services rendered to the patient within a specified time window including eventual complications in both acute and post-acute care providers. In the last decade, bundle payment attracted much attention. [16] compares the fee for services (FFS) method of payment with BP. They discuss the pros and cons of both approaches. The main disadvantage of FFS, from the patients' point of views, is that FFS provides incentives for excessive treatment intensity and results in suboptimal system payoff. They also find that assuming that providers have the option to whether accept a patient or not, BP could lead to suboptimal patient selection and treatment levels that may be lower or higher than desirable for the system, with a high level of financial risk for the provider. This risk is associated with complications of patient treatment.

[17] shows that an optimal strategy for The Centers for Medicare and Medicaid Services (CMS), under its current approach, may be to either announce a fixed threshold or keep the selection process uncertain, depending on market characteristics. They also formulate and solve the proposer selection problem as a constrained mechanism design problem, revealing

that CMS' current approach is not optimal. They present policy guidelines for government agencies pursuing bundled payment innovations.

We introduce a novel two-phase post-acute care portfolio selection procedure. In the first phase, using TOPSIS methodology, PAC providers are ranked based on a set of predetermined criteria. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method, which was originally developed by [18]. The set of criteria are generally determined by the Medicaid. An extensive survey has been conducted to calculate the importance of each criterion. A survey is distributed among experts in the field from both academia and hospitals. In the second phase, we develop a a multi-objective optimization model to determine the PAC providers with those we sign the contract. It aims to optimize contracting procedure of AC's. A portfolio of PAC providers is determined with regard to minimizing the total cost and readmission to AC, and also maximizing the quality of services, while we consider several other factors as constraints in this model such as distance.

To develop this model, we borrow the main idea from the supplier selection literature. Similar to the supplier selection problem, PAC portfolio selection is a long-term and strategic decision making problem. There are many papers in the literature that studied the problem of supplier selection or ranking the suppliers. One of the most popular approaches is ranking suppliers based on the cost associated with providing their supplies from that specific supplier. Other than that, a large portion of the literature is on multi-objective studies.

A vital aspect of PAC provider selection is defining and determining the evaluation basis's selection criteria. A multi-objective decision making approach is efficient for this problem. Determining the worth of such criteria is essential as well. To determine the importance of each factor, we did an extensive survey by around 30 experts in this field. Some PAC's may have very high credentials and operate in high efficiencies, but prove to be overqualified by most AC's, specifically when they need to pay higher for these facilities. Therefore, it is crucial to truly validate providers according to the acute care's current requirements and future vision. We address this problem by minimizing the total cost, maximizing the service quality received from these facilities, and minimizing the readmission rate the AC from them. Accordingly, a good fit is neither unacceptable nor over-qualified.

Using data from the Medicaid on cost and quality of services for PAC providers, we do analysis on impacts of different parameters of the proposed model in the state of Texas. As readmission rate from PAC's is not available, these rates are assumed. The most essential attributes, revealed by Medicaid, are some quality measures such as prehospitalization, number of patients who lose weight, number of patients who have severe pain, etc. These are all summarized in Table 1. Moreover, there are beneficial data on staff rating, which includes the average cost per patient per hour, associated with staff. We take into account all this information in preparing our survey.

The problem addressed in this paper aims to provide some guidelines for decision makers in acute care facilities towards contracting with PAC providers by ranking the providers concerning the cost of their services and the quality or level of services they provide. Besides, this would give researchers direction in this area to further investigate the factors that influence this selection.

Contributions of the work include but not limited to:

- Providing guidelines for Medicaid and HCBSs to optimally decide on the number of waivers and their capacities, respectively. The following subgoals are defined to achieve this goal:

- Capture the competing interests and distributed decision making inherent in LTC capacity planning

- Formulating this problem to optimize adherence to HCBSs quality level, with uncertainty in future demand

- Determining a set of attributes to quantify the service quality level for each PAC
- Conduct surveys from an expert panel to calculate the importance of each criterion considered by the Medicaid
- Selecting a set of PACs for a hospital, to make a contract with considering the total cost, readmission, and quality of services.
- Investigating the impact of each objective on the selected set of providers

The implications of this study can be itemized as follows:

- Medicaid has been provided by a framework to decide on the optimal number of waivers for each state. We show that Medicaid does not necessarily need to outspend its budget since part of the demand is fulfilled by LTC providers.
- LTC providers' decision is highly dependent on Medicaid's decision. LTC providers cannot make the best decision unless they are aware of Medicaid waiver provision.
- Acute care managers can easily have access to a table which includes the ranking of PAC's based on the criteria given by Medicaid.
- Attributes for determining the quality of a PAC are provided in addition to an expert derived assessment of their importance. These attributes quantify the quality measures.
- For each PAC, cost, readmission rate, and service quality measures are provided.

The rest of the dissertation is organized as follows. Section 2 reviews the relevant literature on post-acute care capacity planning. Section 3 presents the game model with its assumptions and properties and provides the solution algorithm and numerical results based on data from Texas. Section 4 presents the details of the mathematical formulation for PAC contracting problem and the methodology to solve this model, describes the case study that was the origination of this research, including the real data from the state of Texas. Computational results are provided, sensitivity analysis on different input parameters is performed, and some managerial insights are extracted. Finally, the research is concluded in Section 5, and some future directions are proposed to cover the limitations of this research.

2. LITERATURE REVIEW

In recent years, there has been rapid growth in the use of operations research (OR) in the field of healthcare resource planning [19]. Significant effort has focused on short-term or intermediate-term scheduling ([20] provides a broad overview), including personnel scheduling (e.g., see [21] and [22] for surveys on nurse scheduling, [23] for emergency physician scheduling, and [24] for resident scheduling), clinic scheduling (e.g., [25] and [26]), and facility scheduling (e.g., see [27] for an extensive review of operating room scheduling). In contrast, only a few researchers deal with strategic health care resource planning (e.g., [28] and [29] for personnel planning, and [30] for facility planning). These studies typically address strategic resource planning in acute care settings and do not specifically deal with LTC capacity planning problems.

While LTC capacity planning research is scarce, there are a few important studies. Hare et al. [31] develop a deterministic multi-state model for home and community care in British Columbia, Canada, with the objective of predicting future LTC needs. Their model incorporates both publicly-funded and non-publicly-funded LTC options. The model uses both changing age and changing health status as demographic input and is validated and tuned with the provincial-level data of British Columbia. Patrick [32] develops a Markov decision process model to determine the optimal patient flow from a hospital to LTC facilities in order to reduce hospital congestion. The model is configured using data from a Canadian hospital and shows that existing LTC capacity is insufficient to achieve satisfactory improvement on hospital census and community wait times.

Zhang et al. [33] integrate demographic and survival analysis with simulation optimization to determine the minimum LTC capacity levels to satisfy client wait time requirements over a multi-year planning horizon. Case studies based on Canadian LTC settings demonstrate improvement over current practice in terms of reduced waiting times. Cardoso et al. [34] propose a multi-objective and multi-period model for both LTC location selection and capacity planning. They use geographical and socioeconomic equity of access as their objectives and provide a case study from the Portuguese health system.

Patrick et al. [35] develop a simulation model to determine the community-based LTC capacity required to reduce waiting time for patients being discharged from hospitals in Ontario. They conclude that waiting time objectives cannot be achieved without significant increases in LTC capacity. Cardoso et al. [36] develop a stochastic MILP that helps decision makers determine and fairly distribute LTC capacity to assure equity of access across socioeconomic and geographical dimensions. They apply their model to a Portuguese jurisdiction and conclude existing capacity levels to be inadequate. Li et al. [37] study the problem of capacity planning for LTC networks. Patient flows among care settings are modeled using an open migration network, and the objective is formulated as a newsvendor type profit maximization model, with penalties being applied for violations of soft capacity constraints. The authors use the model to make several observations about capacity allocation in LTC. For example, investments in capacity resilience (e.g., cross-training or surge beds) reduces the overall LTC capacity need.

While the above papers present many good results, they all focus on centralized decision making. In fact, much of the capacity planning in U.S. healthcare systems is inherently distributed. These problems are best addressed using some form of game theoretic approach. To the best of our knowledge, there are only a handful of papers that apply game theory to study healthcare problems. Adida et al. [38] use game theory to study joint stockpiling for hospital disaster planning, and McFadden et al. [39] investigate using game models in surgical settings. Ford et al. [40] present a case study on mental care coalitions using network analysis and game theory interpretations. Finally, Kurt et al. [41] study the problem of paired kidney exchange and develop necessary and sufficient conditions for stationary equilibria.

The contributions of this study include model and solution algorithms that capture the (a) competing interests and distributed decision making inherent in LTC capacity planning, (b) interplay between a state's Medicaid waiver program and its LTC provider capacity, (c) uncertainty arising from LTC demand projections, (d) financial constraints on public LTC spending, (e) attributes which shape the quality of a PAC, and (f) impact of cost, readmission rate, and quality measures on selecting PAC's.

3. DISTRIBUTED LONG-TERM CARE CAPACITY PLANNING

3.1 Problem Description

We develop a stochastic Stackelberg-Nash-Cournot game model [42] (Table 1 defines notation). This model consists of two main players. The leader is Medicaid and the followers are HCBS providers. Stackelberg is a strategic game in which the leader moves first and then followers move sequentially [43]. Cournot competition is an equilibrium model used to formulate a game in which players compete with each other on the amount of their output, which they decide independently and simultaneously [44].

In our model, Medicaid, the leader, chooses the number of waivers, x, to maximize social benefits (we assume x to be a continuous variable). Medicaid waivers are necessary to fund LTC services in the home or community rather than in a nursing home. After the number of waivers is announced, HCBS providers, the followers, decide on their own service capacity (in patients served), which requires a portfolio of resources that might include the number of HCBS personnel, the fleet size, or the number of portable diagnostic devices. We assume Medicaid is able to make some prediction about how HCBS providers will respond to waiver announcements, which is reasonable since the federal and state governments are continually developing and maintaining significant health data infrastructure useful in predictive modeling [45].

Medicaid's objective is to improve patient access to care while maintaining care quality and controlling costs. This is captured by social benefit function, s, which specifies the benefit of total HCBS capacity in the catchment area. We assume I followers, that is, I private HCBS organizations (providers) competing for uncertain future LTC demand in the catchment area. The uncertainty in future LTC demand is captured through J demand scenarios with associated probabilities, π_j, $j = 1, \ldots, J$. We let q_{ij} represent provider i's capacity decision under scenario j; c_i represent provider i's capacity cost (assumed independent of scenario); and r_j represent the revenue per patient under scenario j (assumed independent of provider).

Table 3.1: Notation

I	Number of HCBS organizations in catchment area
J	Number of demand scenarios
π_j	Probability of demand scenario j
x	Number of waivers specified by Medicaid
s	Medicaid social benefit function
r_j	Revenue function for scenario j, in revenue per patient
q_{ij}	Capacity decision variable for provider i under scenario j
c_i	Capacity cost function of provider i

With the above specification, the leader obtains the optimal solution x^* to the Stackelberg problem:

$$x^* = \underset{x \geq 0}{\text{argmax}} \quad \{s\left(x + \sum_{j=1}^{J} \pi_j \sum_{i=1}^{I} q_{ij}(x)\right)\}, \tag{3.1}$$

and each follower i in scenario j obtains the optimal solution q_{ij}^* to the Cournot problem

$$q_{ij}^* = \underset{q_{ij} \geq 0}{\text{argmax}} \quad \{q_{ij} r_j \left(x^* + q_{ij} + \sum_{k \neq i} q_{kj}^*\right) - c_i(q_{ij})\}, \tag{3.2}$$

for $i = 1, \ldots, I$ and $j = 1, \ldots, J$.

The point $(x^*, (q_{ij}^*)_{i=1 \ldots I, j=1 \ldots J})$ is a Stochastic Stackelberg-Nash-Cournot (SSNC) equilibrium if x^* solves equation 1 and q_{ij}^* solves equation 2.

In the above formulation, the leader will determine its optimal choice x^* with consideration of the expected total service supply which is captured as $\sum_{j=1}^{J} \pi_j \sum_{i=1}^{I} q_{ij}(x)$. On the follower side, the $q_{ij}(x)$, $i = 1, \ldots, I$, $j = 1, \ldots, J$ are the joint reaction functions to the leader's choice of x in scenario j. In each scenario j, we need to solve a separate Cournot problem. For the ease of analysis, we define an aggregate reaction function in scenario j as

$$Q_j(x) = \sum_{i=1}^{I} q_{ij}(x), \quad j = 1, \ldots, J. \tag{3.3}$$

3.1.1 Assumptions

The assumptions made in the model and the associated justifications are presented as follows.

Assumption 1: The revenue function, r_j, is strictly decreasing and twice differentiable. An intuitive explanation is that as the total HCBS service capacity in a catchment area increases, the revenue that an individual HCBS organization receives for treating a patient will not increase and could decrease due to the greater supply. Further, r_j is independent of i since, as in the microeconomic context, the revenue at an equilibrium point is identical for all firms that provide a given quantity of service. This revenue function is similar to the inverse demand function used in [46]. Similarly, the social benefit function, s, is concave and twice differentiable, which implies the marginal social benefit of each additional unit of capacity is not increasing.

Assumption 2: The cost function, c_i, is convex and twice differentiable. This is a normal assumption in the Stackelberg-Nash-Cournot literature as can be seen in [47, 42, 48]. Moreover, as with the total cost function in the queuing literature, we may consider total cost as the sum of cost of service (which is strictly increasing as more customers are served) and cost of the waiting (which is strictly decreasing as more capacity is added).

Assumption 3: The number of patients treated is approximately equal to total HCBS capacity. This assumption is justified since demand for HCBS is intense and the current service supply cannot satisfy the growing demand due to population aging [49].

Assumption 4: Every organization has budgetary constraints, thus HCBS organizations cannot expand their capacities indefinitely. Let U_i be the largest possible capacity for HCBS i, so we have $q_{ij} \leq U_i$ for $i = 1, \ldots, I$ and $j = 1, \ldots, J$. This assumption is reasonable as the cost for expanding capacity is always significant. Similarly, there is an upper bound for Medicaid, U_0, on the number of waiver slots that can be provided, and thus, $x \leq U_0$.

3.1.2 Model Characterization

In this section, we explore properties of our model. These properties provide information on solution existence and uniqueness to the SSNC problem and on the continuity of $Q_j(x)$ as a function of x. Proofs are based on the aforementioned assumptions and closely follow the presentation of Sherali et al. [42]. We forego the proofs of properties 1 and 2, instead focusing on discussion, and provide a brief proof outline for property 3. Readers interested in details may refer to [42]. Property 1 ensures the existence of solutions for each of the leader's decision. This property is established by Sherali's Theorem 1 and corollary. Property 2 establishes continuity and first derivative boundaries for the aggregate reaction curve. This property is established by Sherali's Theorem 2. To prove property 1, Sherali's used assumption 1. Both assumptions 1 and 2 are used as a part of the proof for property 2.

Property 1. *For each leader's choice $x \geq 0$, there exists a unique set of quantities $[q_{1j}(x), \ldots, q_{Ij}(x)]$ satisfying the conditions in equation (2). Also, $q_{ij}(x)$ is a continuous function of x for $i = 1, \ldots, I$ and $j = 1, \ldots, J$.*

It is worth noting that at any equilibrium solution to a SSNC game, the leader has maximized its profit with clear anticipation of the reaction of the followers, while each follower has maximized its own profit given the decisions of the other followers. No follower will wish to unilaterally change its decision.

Next, we study properties of the aggregate reaction function $Q_j(x)$. The purpose is to better understand how followers react to the leader's decision.

Property 2. *For each scenario j, $j = 1, \ldots, J$, $Q_j(x)$ is a continuous function of x for*

$x \geq 0$ and satisfies

$$-1 < Q_j^+(x) < 0 \quad \textit{if} \quad Q_j(x) > 0, \tag{3.4}$$

and

$$Q_j^+(x) = 0 \quad \textit{if} \quad Q_j(x) = 0, \tag{3.5}$$

where $Q_j^+(x)$ is the right hand derivative of $Q_j(x)$ with respect to x.

Note that the existence of first order derivative for Q_j is presumed. The interpretation of this property in our context is that if the Medicaid increases the number of waivers by one, then the total number of patients whose LTC is paid by alternative sources will be decreasing, but not by more than one. It is an intuitive interpretation since increasing the number of HCBS waivers will for sure decrease the total number of patients paying for care in alternative ways.

Finally, we present the existence and uniqueness results for the stochastic Stackelberg-Nash-Cournot equilibrium. This is the most vital property of the game model. This property guarantees a unique solution to SSNC problem. Thus, we explain the important parts of its proof.

Property 3. *(i) There exists an equilibrium point $(x^*, (q_{ij}^*)_{i=1,\dots,I j=1,\dots,J})$ to the stochastic Stackelberg-Nash-Cournot problem. (ii) The equilibrium point is unique.*

Proof. (i) By property 1, $q_{ij}(x)$ is a continuous function on x, thus $s(x+\sum_{j=1}^{J} \pi_j \sum_{i=1}^{I} q_{ij}(x))$ is also a continuous function on x (by assumption 1). By assumption 4, $q_{ij} \leq U_i$ for $i = 1,\dots,I$, $j = 1,\dots,J$, the feasible set $S = \{x+\sum_{j=1}^{J} \pi_j \sum_{i=1}^{I} q_{ij}(x) \mid 0 \leq x \leq U_0\}$ is nonempty and compact over $[0, \sum_{i=0}^{I} U_i]$. Since the leader is maximizing a continuous function over a compact set, there must exist an optimal solution x^*. Further by property 1, a unique set of quantities $[q_{1j}(x^*),\dots,q_{Ij}(x^*)]$ can be obtained for a leader's choice x^* in scenario j, which proves the existence of the equilibrium point.

(ii) Uniqueness is due to the concavity of function s from assumption 1. This completes the proof of uniqueness of the equilibrium point. □

3.2 Methodology and Results

In this section, we will discuss the solution algorithm proposed to solve this problem to optimality, required data to develop a case problem, and numerical results of a case study in Texas, US.

3.2.1 Solution Algorithm

In order to solve this SSNC problem, we adapt an approximation algorithm from [50]. They proposed an efficient and easy-to-implement approximation algorithm to solve a similar stochastic Stackelberg-Nash-Cournot equilibrium problem. As this is a bi-level problem, the proposed solution algorithm contains two parts which we explain in the following.

The basic idea is to approximate $Q_j(x)$ by a piecewise linear curve that coincides with $Q_j(x)$ at each breakpoint. The leader's problem is then solved on each of these intervals with $Q_j(x)$ replaced in (1). In order to solve the leader's problem, we divide the closed interval of $[0, U_0]$ using T grid points x_t, $t = 1, \dots, T$ with $0 \leq x_1 < x_2 < \cdots < x_T \leq U_0$. We use the linear function $Q_{tj}(x)$ defined in (6) to approximate $Q_j(x)$ on $[x_t, x_{t+1}]$.

$$Q_{tj}(x) = Q_j(x_t) + \gamma_{tj}(x - x_t) \quad \text{for} \quad x_t \leq x \leq x_{t+1} \tag{3.6}$$

where

$$\gamma_{tj} = \frac{Q_j(x_{t+1}) - Q_j(x_t)}{x_{t+1} - x_t}. \tag{3.7}$$

Thus, in each interval $[x_t, x_{t+1}]$, the aggregate reaction curve is replaced by its linear approximation $Q_{tj}(x)$. We then can solve the following leader's problem.

$$\max_{x_t \le x \le x_{t+1}} \quad s\left(x + \sum_{j=1}^{J} \pi_j Q_{tj}(x)\right). \tag{3.8}$$

Let x_t^*, $t = 1, \ldots, T$, be the solution to (8). The x_t^* is the approximation of the leader's optimal output over $[x_t, x_{t+1}]$. We evaluate the x_t^* for all T grid points and find the best solution to be the leader's choice. It is easy to see that the larger the T is, the closer our approximation solution is to the real optimal solution.

The only thing remains to completely solve the SSNC problem is computation of the aggregate reaction function $Q_j(x)$ for a given x. This is done by defining a new equilibrium problem. For each scenario j, we solve the following optimization problem.

$$EP(x, Q_j): \quad \max \quad r_j(Q_j + x)\sum_{i=1}^{I} q_{ij} + \frac{1}{2} r_j'(Q_j + x)\sum_{i=1}^{I} q_{ij}^2 - \sum_{i=1}^{I} c_i(q_{ij}) \tag{3.9}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} q_{ij} = Q_j \tag{3.10}$$

$$q_{ij} \ge 0, i = 1, \ldots, I. \tag{3.11}$$

For a fixed $x \ge 0$ and $Q_j \ge 0$, each problem $EP(x, Q_j)$ involves the maximization of a strictly concave function over a nonempty, convex, and compact feasible region. Hence, there is a unique global optimum for each problem $EP(x, Q_j)$. The Q_j depends on a predetermined x, so $Q_j = Q_j(x)$. Note that the optimal Lagrange multiplier associated with the constraint (10), $\lambda^*[Q_j(x)]$, is zero [50]. Then, it can be verified that the KKT conditions for $EP(x, Q_j)$ replicate those of problem (2). The above problem (9-11) is an optimization problem with a nonlinear objective function and a linear constraint.

3.2.2 Numerical Study

In this section, we elaborate how to estimate all model parameters based on the data from publicly available databases and published studies. To estimate the total cost, we first assume that the nurses' wages are the major source of a facility cost. We consider three types of LTC workers including Registered Nurses (RN), Licensed Practical Nurses (LPN), and Aides. Aides help LTC recipients perform most basic daily tasks (e.g. dressing, feeding, and bathing). They have extensive daily contact with patients. According to the 2010-2011 Occupational Outlook Handbook published by the Department of Labor's Bureau of Labor Statistics, "LPN is a nurse who cares for people who are sick, injured, convalescent, or disabled. LPNs work under the direction of registered nurses or physicians. Experienced LPNs may supervise nursing assistants and aides, and other LPNs."

With regard to their types and the state in which they work, nurses are paid at different rates. The average pay rate for each type (according to The Prudential Insurance Company of America, 2010) is summarized in Table 2.

Table 3.2: Average Pay Rate for Nurses Per Hour ($)

RN	54
LPN	54
Aides	21

In addition, we specify the required daily nursing hours for each type of nurse from Medicaid. Then we calculate the total cost of a facility using equation (12), where W_{rn}, W_{lpn}, and W_a are the average pay rates for RN, LPN, and Aides, respectively.

$$C = RN * W_{rn} + LPN * W_{lpn} + A * W_a \tag{3.12}$$

In order to find the total cost as a function of a facility capacity, we do regression analysis which satisfies assumption 2. According to this assumption, the cost function is convex and twice differentiable. This makes sense as the smaller facilities are not necessarily cheaper; that is, they often have higher occupancy rate. Smaller facilities are usually located in rural locations. On the other hand, large facilities are also among the most expensive ones. From this, we can verify that cost function is convex and twice differentiable in shape.

In order to nullify the impact of the quality (service level) of a facility for computing the cost function, we confine our study to the facilities with the same and the highest quality measure. Medicaid has different quality criteria to rank the facilities. It also provides an overall ranking of all LTC agencies based on an integer number ranging from 1 to 5. This study is confined by LTC agencies, in the state of Texas, which have the highest overall quality measure of 5. We investigate this problem on 40 HCBS in the state of Texas.

We extract the number of hours required for each nurse type at each specific facility from Medicaid data sources. We also find the capacity of all aforementioned 40 HCBSs from the Medicaid website (medicaid.gov). Finally, a quadratic regression is done to find the cost as a function of the capacity as follows.

According to the national long-term care profit margin, which was released by the U.S. Government Accountability Office in 2012, on average, there is 6.9 percent profit for a LTC facility. Thus, we develop revenue functions which satisfy both assumption 1 and the average profitability. We use the same data for cost, but this time we do a linear regression. We come up with a decreasing function which satisfies assumption 1. We multiply it by a random number between 1.062 and 1.076 to generate revenue functions for different scenarios.

The current total capacity of the 40 HCBSs is roughly 3000 patients. We assume the upper bound of x, number of waivers provided by Medicaid, as 600 which is equal to 20 percent of the current system capacity. No penalty cost is incurred for shortage in the whole system. Further, we assume that there will be an increase in the demand for LTC, and it is normally distributed from 5 to 55 percent of the current system capacity (3000 patients). Each scenario and the associated probabilities are summarized in Table 3. The third column in Table 3 indicates the amount of increase in demand for LTC in the future. For instance, under to scenario 1, the demand is expected to increase by 150 in the next year.

Table 3.3: Scenarios and Probabilities

Scenario	Probability	Increase Demand
1	0.02	150
2	0.05	300
3	0.08	450
4	0.1	600
5	0.15	750
6	0.2	900
7	0.15	1050
8	0.1	1200
9	0.08	1350
10	0.05	1500
11	0.02	1650

We assume number of intervals for calculating x with regard to the maximum number of waivers is 6. As mentioned, there are 40 service facilities with the best quality measure in Texas according to Medicaid data source. Note that, our objective is to find the optimal number of waivers for this case. As discussed, there is no source of uncertainty for players in the second stage of the model. Followers (HCBSs), therefore, decide on their capacity after realization of the scenario as well as the leader's decision.

In order to clarify the concept, we discuss an illustrative example. We present the total system response to the choice of leader on number of waivers in Table 4. Note that each facility decides on its own capacity based on its specific cost and revenue functions. In the following table, we present the value of $Q_j(x)$ for each scenario. T(i) presents what percentage of demand will be covered by waivers. As mentioned earlier, we assume that the Medicaid does not grant more than 600 waivers.

Table 3.4: System Response ($Q_j(x)$) to Medicaid Choice of x

Scenario	T1(0)	T2(20%)	T3(40%)	T4(60%)	T5(80%)	T6(100%)
1	150	120	90	60	30	0
2	300	240	180	120	60	0
3	450	360	270	180	90	0
4	600	480	360	240	120	0
5	715	600	450	300	150	150
6	715	600	450	300	150	150
7	715	613	511	422	422	422
8	715	599	481	422	422	422
9	715	538	453	422	422	422
10	715	569	422	422	422	422
11	715	544	422	422	422	422

The following discussion clarifies the results presented in Table 4. As a case in point, the second column of Table 4 (T1) presents the case of $x = 0$. That is, Medicaid does not provide any waivers to the State (in this case for high quality LTC agencies in the State). As expected, the followers increase their capacity to meet the demand until some point. But, afterwards (in scenario 5, where demand is equal to 750), followers satisfy the demand partially. The reason is the specific revenue and cost functions assumed for this problem.

Table 5 presents the total system capacity including waivers (x) and aggregate reaction function ($Q_j(x)$) from the HCBS facilities for each scenario. For each scenario, the optimal solution for Medicaid is highlighted. As shown, for the first four scenarios, Medicaid does not provide any waivers and the demand is satisfied through HCBSs. Under scenario 1, followers increase their capacity to meet total demand, 150. Similarly, under scenario 2, followers increase their capacity to meet total demand, 300. Note that, in all cases the optimal solution for Medicaid is to provide the least number of waivers while it does not impact the total system capacity. As a case in point, for scenario 6, Medicaid prefers the solution in T3. The reason is that total system capacity is the max value for this scenario and also Medicaid provides less waivers comparing to other intervals with the same total system capacity.

Table 3.5: Total System Capacity

Scenario	T1(0)	T2(20%)	T3(40%)	T4(60%)	T5(80%)	T6(100%)
1	**150**	150	150	150	150	150
2	**300**	300	300	300	300	300
3	**450**	450	450	450	450	450
4	**600**	600	600	600	600	600
5	715	**750**	750	750	750	750
6	715	808	**900**	900	900	900
7	715	823	931	**1022**	1022	1022
8	715	839	961	**1022**	1022	1022
9	715	808	993	**1022**	1022	1022
10	715	869	**1022**	1022	1022	1022
11	715	864	**1022**	1022	1022	1022

Here we discuss on how to verify the results from the illustrative example. Results seem promising in the sense that as x increases, the extra capacity provided by the followers will decrease. This exactly verifies the second property of the model. In addition, as mentioned earlier in the paper, there is no uncertainty in the second stage of the model where followers (HCBSs) make a decision. We assume that they exactly know what the leader did. So, in case that the leader provides sufficient number of waivers, the followers will not increase their capacity.

This example illustrates how Medicaid and facilities decide on number of waivers and their incremental capacities respectively. This example is solved for each scenario and each grid point separately. It shows the interaction between these two parties as well.

The near optimal solution for leader's problem is 510, which determines the extra number of waivers. Table 6 presents the near optimal number of waivers for each interval of x. Since the only constraint we consider for Medicaid is budgetary constraint, and there is no penalty for extra number of waivers in the system, the solution is 510. Note that, for some scenarios we have more waivers than the total demand.

Table 3.6: Number of Waivers

	Int1 (0-20%)	Int2(20-40%)	Int3(40-60%)	Int4(60-80%)	Int5(80-100%)
x	120	215	328	417	510

We did the same analysis for providers whose ranking is not 5 out of 5. We consider the budgetary limit as 20% of the current total capacity. We assume the same scenarios for these cases. Table 7 presents the total capacity of providers at each level, maximum number of waivers, and near optimal solution:

Table 3.7: Number of Waivers

Service Quality	Capacity	Maximum # of Waivers	Solution
5	3000	600	510
4	4000	800	672
3	2500	500	427
2	1500	300	259

3.2.3 Sensitivity Analysis

In this section, we do sensitivity analysis on different parameters in the model. According to the table 7, as total capacity and maximum number of waivers increase, the near optimal solution increases. Now, we investigate the impact of revenue function on optimal number of waivers. We already assume the profit of a provider is about 7%. In order to analyze this impact, we consider different scenarios and compare the results. Results are provided in Table 8.

Table 3.8: Impact of Profit of a Provider on Number of Waivers

% Profit	Optimal Solution
5%	547
7%	510
10%	491
12%	476
15%	451

Figure 2 depicts this impact on number of waivers for different service quality levels.

As shown, for all levels of service quality, number of waivers decreases as net profit increased. It can be interpreted as follows. Since having more capacity in the system is more profitable for providers, Medicaid then grants less waivers. The total system capacity remains at the same optimal level.

This solution approach provides a guideline for the Medicaid on how to optimally assign its budget for waivers in each state. Medicaid has more detailed information about cost and revenue of the facilities. In addition, the solution approach gives some hints to HCBSs on how to react to the Medicaid decision on number of waivers. Note that, there might be other constraints that Medicaid encountered. These constraints may impact its decision. They

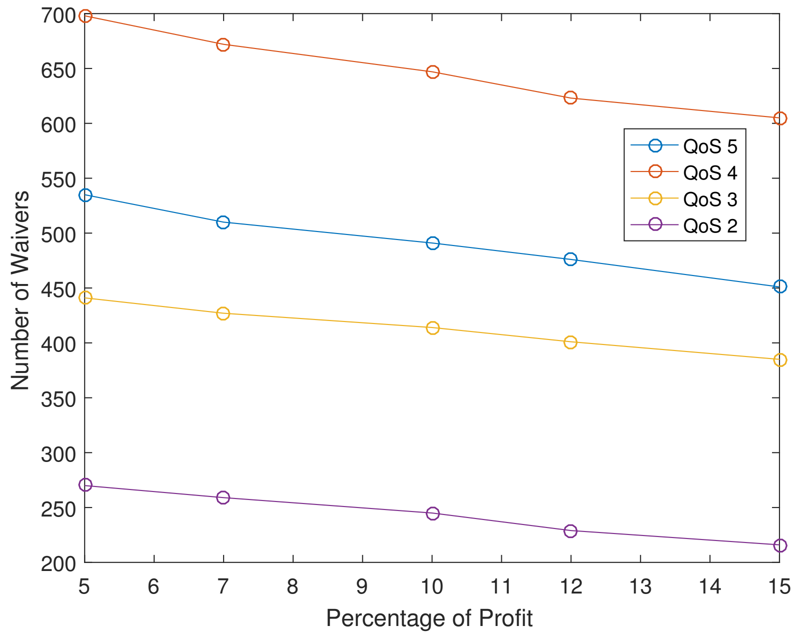


Figure 3.1: Number of Waivers for Different Percentage of Profit

only constraint that we consider in this problem for Medicaid is the budgetary constraint.

This model is easily implementable in a decision support package. The user would enter the number of facilities and their capacities, the cost and revenue functions, and the uncertain demand scenarios. Using the historical data from Medicaid, scenarios and values of the model parameters can be predicted. Medicaid would use the model on a yearly basis to decide on the number of waivers assigned to each state. HCBSs decide later based on the determined number of waivers and other facilities costs and capacities.

4. POST-ACUTE CARE PORTFOLIO SELECTION

4.1 Problem Description

In this section, we present the post-acute care provider selection problem. We consider two cases with and without uncertainty in predicting the demand. We define this problem as follows. There are two different types of patients that require post-acute services, Short-Stay Patients (SSP) and Long-Stay Patients (LSP). Patients are from different regions across the network. There is a list of pre-qualified PACP's which is denoted by $\mathbb{P}$. PACP's are evaluated in terms of a set of attributes ($\mathbb{A}$), and each attribute has an importance weight determined by the system planners. We denote the capacity of PACP i by Cap_i. There is a fixed cost of contracting with each PACP for each type of service. In case of uncertain demand, the variable cost, which is the cost of services per patient, varies across PACP's. The problem is to decide which PACP's should be considered as a part of the network and how to allocate each type of patients to PACP's to minimize the total cost while ensuring the Quality of Service (QoS) and rate of readmission.

4.1.1 Methodology

Our proposed method includes two phases for designing a post-acute care network. In the first phase of the proposed approach, we employ the TOPSIS method to take quality-related metrics into account and compute the score of each PACP for SSP and LSP services. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method originally developed by [51].

The second phase is the optimization part. We use a set of quantitative metrics that are independent of SSP and LSP services in the second phase of our proposed approach. In this phase, we develop a two-stage stochastic optimization model to take the demand uncertainty into account for PAC selection procedure and patient assignment. After that,

Table 4.1: Notation

$\mathbb{P}$	Set of pre-qualified PACP's, $i \in \mathbb{P} = \{1, \cdots, P\}$.
$\mathbb{A}$	Set of attributes, $j \in \mathbb{A} = \{A_1, \cdots, A_Q\}$.
ω_j	Importance weight for criterion j
λ_{ij}	Rating of PACP i for criterion j
$\hat{\lambda}_{ij}$	Weighted rating of PACP i for criterion j
x_i	1 if PACP i is selected to provide care services to patients; 0 otherwise.
f_i	Fixed cost for contracting with a PACP.

we develop a multi-objective optimization model in which maximizing the quality of services and minimizing the readmission rate are considered objectives other than minimizing the total cost. We use the following notation thorough the paper.

In the first phase of the proposed approach, we employ the TOPSIS decision making method to evaluate each PACP concerning providing quality services. The TOPSIS steps to calculate the closeness coefficient (CC) of each PACP-SSP and PACP-LSP are as follows. These closeness coefficients can be interpreted as indicators to show the desirability of each PACP for SSP and LSP.

4.1.1.1 Phase I. closeness coefficients

In this phase of the proposed method, we evaluate the pre-qualified PACP's based on a set of attributes. Table 4.2 presents the list of PACP evaluation attributes. There are four categories of attributes: staffing, deficiencies, quality measures for SSP, and quality metrics for LSP. The staffing and deficiencies attributes are the same for SSP and LSP. This table also shows the format of values of each attribute and the type of the attribute that is either cost (C) or benefit (B). Apparently, for the attributes of type C, lower values are more desirable, while for the attributes of type B, higher values are more desirable. We employ the TOPSIS method to compute a global score for each PACP. In the following, we describe the TOPSIS method.

Table 4.2: Post-Acute Care Provider Evaluation Attributes

Category	Code	Criterion	Values	Cost(C)/ Benefit(B)
Staffing	CST-01	Reported CNA Staffing Hours per Resident per Day	real number	B
	CST-02	Reported LPN Staffing Hours per Resident per Day	real number	B
	CST-03	Reported RN Staffing Hours per Resident per Day	real number	B
Deficiencies	CDF-01	Count of Immediate Jeopardy Deficiencies on Health Survey	integer (0-nn)	C
	CDF-02	Count of Severe Deficiencies on Health Survey	integer (0-nn)	C
	CDF-03	Count of Substandard QOC Deficiencies on Health Survey	integer (0-nn)	C
	CDF-04	Count of Administration Deficiencies	integer (0-nn)	C
	CDF-05	Count of Environmental Deficiencies	integer (0-nn)	C
	CDF-06	Count of Mistreatment Deficiencies	integer (0-nn)	C
	CDF-07	Count of Nutrition and Dietary Deficiencies	integer (0-nn)	C
	CDF-08	Count of Pharmacy Service Deficiencies	integer (0-nn)	C
	CDF-09	Count of Quality of Care Deficiencies	integer (0-nn)	C
	CDF-10	Count of Resident Assessment Deficiencies	integer (0-nn)	C
	CDF-11	Count of Resident Rights Deficiencies	integer (0-nn)	C
Quality measures for SSP	CQS-01	Percentage of short-stay residents assessed and appropriately given the pneumococcal vaccine	real number	B
	CQS-02	Percentage of short-stay residents who made improvements in function	real number	B
	CQS-03	Percentage of short-stay residents who newly received an antipsychotic medication	real number	C
	CQS-04	Percentage of short-stay residents who self-report moderate to severe pain	real number	C
	CQS-05	Percentage of short-stay residents who were assessed and appropriately given the seasonal influenza vaccine	real number	B
	CQS-06	Percentage of short-stay residents with pressure ulcers that are new or worsened	real number	C
Quality measures for LSP	CQL-01	Percentage of low risk long-stay residents who lose control of their bowels or bladder	real number	C
	CQL-02	Percentage of long-stay residents assessed and appropriately given the pneumococcal vaccine	real number	B
	CQL-03	Percentage of long-stay residents assessed and appropriately given the seasonal influenza vaccine	real number	B
	CQL-04	Percentage of long-stay residents experiencing one or more falls with major injury	real number	C
	CQL-05	Percentage of long-stay residents who have depressive symptoms	real number	C
	CQL-06	Percentage of long-stay residents who lose too much weight	real number	C
	CQL-07	Percentage of long-stay residents who self-report moderate to severe pain	real number	C
	CQL-08	Percentage of long-stay residents who were physically restrained	real number	C
	CQL-09	Percentage of long-stay residents whose ability to move independently worsened	real number	C
	CQL-10	Percentage of long-stay residents whose need for help with daily activities has increased	real number	C
	CQL-11	Percentage of long-stay residents with a urinary tract infection	real number	C
	CQL-12	Percentage of low risk long-stay residents who lose control of their bowels or bladder	real number	C

- *Weigh the attributes:* The first step is to assess the importance weight of each attribute. Readers can refer to a detailed survey of different methods for assessing the importance weight of attributes.We should note that the weight for the staffing and deficiencies attributes could be different for SSPs and LSPs.

- *Construct the decision matrix:* The decision matrix is constructed using the list of pre-qualified PACP's, attributes, the weight of each attribute, and the assessed score of each PACP in each attribute. We denote the decision matrix by $[\lambda_{ij}]_{M\times Q}$. Note that two different decision matrix must be created for SSP and LSP. The format of the decision matrix is shown below.

$$\begin{array}{c|cccccc} & A_1 & A_2 & \dots & A_j & \dots & A_Q \\ & \omega_1 & \omega_2 & \dots & \omega_j & \dots & \omega_Q \\ \hline PACP_1 & \lambda_{11} & \lambda_{12} & \dots & \lambda_{1j} & \dots & \lambda_{1Q} \\ PACP_2 & \lambda_{21} & \lambda_{22} & \dots & \lambda_{2j} & \dots & \lambda_{2Q} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ PACP_i & \lambda_{i1} & \lambda_{i2} & \dots & \lambda_{ij} & \dots & \lambda_{iQ} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ PACP_M & \lambda_{M1} & \lambda_{M2} & \dots & \lambda_{Mj} & \dots & \lambda_{MQ} \end{array}$$

- *Normalizing the decision matrix:* Since the values of attributes are in different ranges, we need to normalize the decision matrix. The are different methods to normalize the decision matrix. We use Formula (4.1) to normalize the decision matrix.

$$\hat{\lambda}_{ij} = \frac{\lambda_{ij}}{\sqrt{\sum_{i\in\mathbb{P}} \lambda_{ij}^2}} \tag{4.1}$$

- *Construct the weighted normalized decision matrix:* The weighted normalized decision matrix is computed by multiplying each score in the decision matrix using the associated attribute weight. We denote this matrix by $[\hat{\upsilon}_{ij}]_{M\times Q}$, where,

$$\hat{\upsilon}_{ij} = W_j(\cdot)\hat{\lambda}_{ij} \tag{4.2}$$

- *Determine positive-ideal solution (PIS):* The positive ideal solution is an alternative that has the best score in all attributes among the list of pre-qualified PACP's. In most cases, this alternative is a dummy and does not exist in the list of candidate PACP's. We denote the positive-ideal solution by $\Phi^+ = (\hat{v}_j^+, \cdots, \hat{v}_Q^+)$, where,

$$\hat{v}_j^+ = \begin{cases} \max_i \hat{v}_{ij}, & j \in B \\ \min_i \hat{v}_{ij}, & j \in C \end{cases} \tag{4.3}$$

- *Determine negative-ideal solution (NIS):* In contrast to PIS, the negative ideal solution has the worst score in all attributes among the candidate PACP's. We denote the negative ideal solution by $\Phi^- = (\hat{v}_j^-, \cdots, \hat{v}_Q^-)$, where,

$$\hat{v}_j^- = \begin{cases} \min_i \hat{v}_{ij}, & j \in B \\ \max_i \hat{v}_{ij}, & j \in C \end{cases} \tag{4.4}$$

- *Determine the distance of each PACP and from PIS and NIS:* We use Formula (4.5) and Formula (4.6) to calculate the distance of PACP's from PIS and NIS, respectively. One can use other distance metrics. In this paper, we set $p = 2$, but one can apply any values for p that satisfies $p \geq 1$.

$$d_i^+ = (\sum_{\Theta} (\hat{v}_{ij} - \hat{v}_j^+)^p)^{1/p} \quad \forall i \in \mathbb{P},\ p \geq 1 \tag{4.5}$$

$$d_i^- = (\sum_{\Theta} (\hat{v}_{ij} - \hat{v}_j^-)^p)^{1/p} \quad \forall i \in \mathbb{P},\ p \geq 1 \tag{4.6}$$

- *Compute Closeness Coefficients (CC):* We use the distance of each PACP from PIS and NIS to compute CC using Formula (4.7). Apparently, PACP's with higher values of CC are more desirable, that is, they have a long distance from NIS and a small

Table 4.3: Notation

N	set of PACP's, $i \in N = \{1, \cdots, n\}$
M	set of type of patients, $j \in M = \{1, \cdots, m\}$
R	set of regions, $k \in R = \{1, \cdots, r\}$
d_{ik}	The average distance of patients in region k to provider i.
A^s_{jk}	number of all patients from type j from region k under scenario s
x_{ij}	1 if the hospital contracts the PACP i for patients of type j; 0 otherwise
y^s_{ijk}	number of patients of type j who are assigned to PACP i from region k with a contract under scenario s
y_{ijk}	number of patients of type j who are assigned to PACP i from region k without a contract
f_j	fixed cost of contracting for patients of type j
Cap_i	Capacity of provider i
v_i	variable cost of service provided by PACP i under a contract
v'_i	variable cost of service provided by PACP i without a contract

distance from PIS.

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+} \tag{4.7}$$

The case that both d_i^+ and d_i^- are zero can only happen when all attributes have the same value. In that case, there is no need for extra analysis.

In the second phase of our proposed approach, CC's are used in an optimization problem to select the most desirable PACP's for SSP and LSP for some quantitative metrics, which are reflected in the constraints.

4.1.1.2 Phase II. assignment programming model

In this section, we propose two different approaches for assigning the patients to the providers. First, we present a two stage stochastic optimization method.

The objective is to minimize the total cost, which includes the fixed cost of contracting and the variable cost of services for SSP and LSP. Equation (4.8) presents the objective function of the PAC provider selection problem.

$$c(X,Y,S) := \sum_{i\in N}\sum_{j\in M} f_i x_{ij} + \mathbb{E}_S\left[\sum_{i\in N}\sum_{j\in M}\sum_{k\in R}\left(v_{ij}y^s_{ijk} + v'_{ij}y'^s_{ijk}\right)\right] \tag{4.8}$$

Constraint (4.9) is to ensure that the post-acute providers are selected in a way that the average QoS is higher than the acceptance threshold of λ. Here, QoS is defined based on the CC scores obtained from the TOPSIS method.

$$\sum_{i\in N}\sum_{j\in M}\sum_{k\in R} CC_{ij}y^s_{ijk} \geq \lambda \sum_{j\in M}\sum_{k\in R} A^s_{jk} \quad \forall s \in S \tag{4.9}$$

Another important metric is the coverage of the PAC network which is defined based on the average distance of patients between their living regions and the allocated PACP. Through Constraint (4.10), we ensure that the average distance in the network is within the threshold of α.

$$\sum_{i\in N}\sum_{j\in M}\sum_{R} d_{ik}y^s_{ijk} \leq \alpha \sum_{j\in M}\sum_{k\in R} A^s_{jk} \quad \forall s \in S \tag{4.10}$$

PACP's are capacitated, and Constraint (4.11) prevents allocating patients to a provider beyond its capacity. Apparently, this constraint incorporates both types of collaborations, i.e., with a contract or without any contracts.

$$\sum_{j\in M}\sum_{k\in R} y^s_{ijk} + y'^s_{ijk} \leq Cap_i \quad \forall i \in N, \forall s \in S \tag{4.11}$$

Constraint (4.12) ensures that all the patients are allocated to a PACP.

$$\sum_{i \in N} \sum_{j \in M} \sum_{k \in R} \left(y_{ijk}^{s} + y_{ijk}^{'s} \right) = \sum_{j \in M} \sum_{k \in R} A_{jk}^{s} \quad \forall s \in S \tag{4.12}$$

Based on Constraint (4.13), we can allocate patients with the contracting mode to a PACP only if that PACP is chosen for a collaboration under a contract.

$$y_{ijk}^{s} \leq M.x_{ij} \quad \forall i \in N, \forall j \in M, \forall k \in R, \forall s \in S \tag{4.13}$$

Finally, Constraint (4.14) defines the decision variables.

$$x_{ij} \in \{0, 1\}, y_{ijk}^{s} \in \mathbb{Z}^{+}, y_{ijk}^{'s} \in \mathbb{Z}^{+} \quad \forall i \in N, \forall j \in M, \forall k \in R \tag{4.14}$$

In the two-stage stochastic programming approach for optimization under uncertainty, the decision variables are partitioned into two sets. The first stage variables are those that have to be decided before the actual realization of the certain parameters becomes available. Subsequently, once the random events have presented themselves, further design or operational policy improvements can be made by selecting, at a certain cost, the values of the second stage or recourse variables. The objective is to choose the first stage variables in a way that the sum of first stage costs and the expected value of the random second stage or recourse costs is minimized. The reason we chose SAA is the huge number of scenarios we encounter with in this problem.

The main idea of Sample Average Approximation (SAA) approach to solving stochastic programs is as follows [52]. A sample $(\xi^1, ..., \xi^N)$ of N realizations of the random vector $\xi(\omega)$ is generated, and consequently the expected value function $E[Q(x, \xi(\omega))]$ is approximated (estimated) by the sample average function $N^{-1} \sum_{n=1}^{N} Q(x, \xi^N)$. The obtained sample average approximation

$$\text{Min } \hat{g}_N(x) := c^T x + N^{-1} \sum_{n=1}^{N} Q(x, \xi^n) \tag{4.15}$$

of the stochastic program is then solved by a deterministic optimization algorithm. This approach (and its variants) is also known under various names, such as the stochastic counterpart method and sample path optimization method.

4.2 Case Study

In this section, we elaborate the real case studied in the city of Houston, TX. We did an extensive survey analysis for criteria weighting. The case is presented in detail here. We assume SSP's are staying 20 days and LSP's are staying 50 days in average at PAC's. Total cost is calculated based on the equation (3.12) times the number of stay for each type of patient.

4.2.1 Criteria weighting

We used the expert panel methodology to evaluate the weights of the criteria. Our panel was constituted with 22 experts with related professional experience regarding the post acute care services. Twenty seven percent of the panel had a Master's degree (e.g. MA, MS, MEd), 55% with a professional degree (e.g. MD, DDS, DVM), and 18% had a Doctorate degree (e.g. PhD, EdD). Also, 27% of the experts had 1-3 years related professional experience, and 73% had more than three years related professional experience. Result from the survey is summarized in Table 4.

4.2.2 Distance

We use the equation 18 to calculate the distance between two points on the earth, where we have the latitude and longitude of the points:

$$u = sin(\frac{lat2 - lat1}{2}) \tag{4.16}$$

$$v = sin(\frac{lon2 - lon1}{2}) \tag{4.17}$$

$$d = u^2 + cos(lat1) * cost(lat2) * v^2 \tag{4.18}$$

The reason for proposing this approach to calculate the distance is that on data set provided by the Medicaid, location of the PAC's are given by the latitude and longitude, and it is more convenient.

4.2.3 Candidates

Initially, there was a list of 66 PACPs in the city of Houston, TX, out of which eight providers had some missing values in the data set. Therefore, 59 providers are final candidates for subcontracting.

Table 4.4: Weights of Criteria for PAC Provider Selection

Category	Code	Weight
Staffing	CST-01	0.1059906
	CST-02	0.1023
	CST-03	0.1659
Deficiencies	CDF-01	0.0273504
	CDF-02	0.0256
	CDF-03	0.0196
	CDF-04	0.0230
	CDF-05	0.0239
	CDF-06	0.0341
	CDF-07	0.0188
	CDF-08	0.0299
	CDF-09	0.0282
	CDF-10	0.0230
	CDF-11	0.0230
Quality measures for short-stay patients	CQS-01	0.0489429
	CQS-02	0.0611
	CQS-03	0.0509
	CQS-04	0.0550
	CQS-05	0.0530
	CQS-06	0.0795
Quality measures for long-stay patients	CQL-01	0.0266052
	CQL-02	0.0256
	CQL-03	0.0247
	CQL-04	0.0332
	CQL-05	0.0237
	CQL-06	0.0313
	CQL-07	0.0285
	CQL-08	0.0237
	CQL-09	0.0380
	CQL-10	0.0351
	CQL-11	0.0304
	CQL-12	0.0275

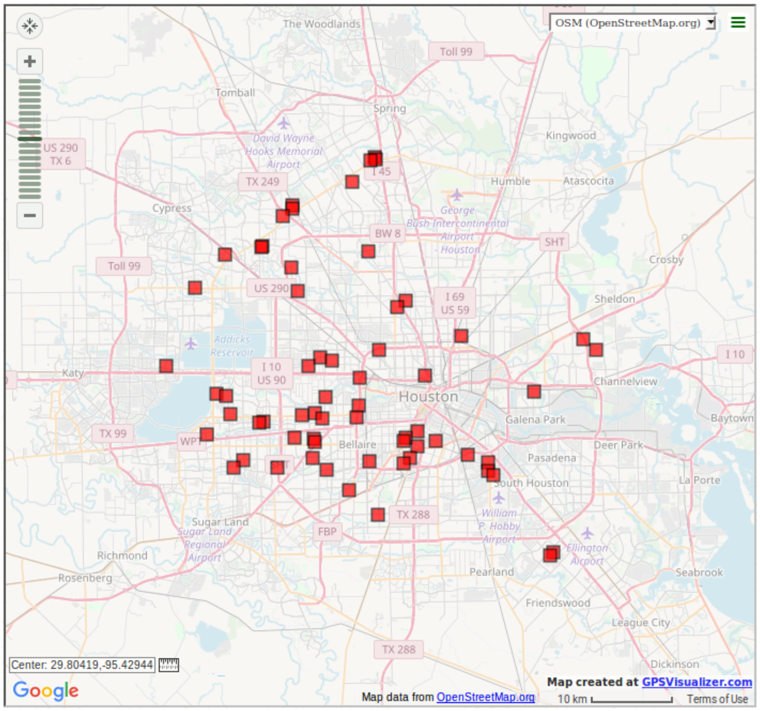


Figure 4.1: Location of Providers

4.2.4 Patients

In this study, we consider two sets of patients: LSP and SSP. It can be easily extended to further groups. The reason that we consider these types is that they are defined by the Medicaid. We generate random numbers for the number of patients in each region. Number of regions is assumed 16. Locations of patients are considered to be the center of each region. But, the exact location for each patient can be considered. Number of patients in each region is a random number.

Table 4.5: Post Acute Care Provider Selection Data Summary

Category	Code	Min.	Ave.	Max.
Staffing	CST-01	0.19023	2.389277627	3.90595
	CST-02	0.00	1.009999661	2.0125
	CST-03	0.27526	0.730885424	1.64405
Deficiencies	CDF-01	0.00	0.762711864	8
	CDF-02	0.00	1.237288136	8
	CDF-03	0.00	0.627118644	4
	CDF-04	0.00	3.355932203	9
	CDF-05	0.00	3.576271186	10
	CDF-06	0.00	0.86440678	3
	CDF-07	0.00	1.661016949	4
	CDF-08	0.00	3.847457627	9
	CDF-09	0.00	5.593220339	22
	CDF-10	0.00	2.915254237	9
	CDF-11	0.00	1.847457627	8
Quality measures for short-stay patients	CQS-01	0.712589	68.21334097	100
	CQS-02	10.795409	54.76515236	83.746721
	CQS-03	0.00	3.534053474	17.741935
	CQS-04	0.268096	11.07295705	28.070176
	CQS-05	1.408452	66.11156241	100
	CQS-06	0.00	0.923108898	3.954674
Quality measures for long-stay patients	CQL-01	0.00	8.718924898	22.222223
	CQL-02	5.853659	78.85360286	100
	CQL-03	16.580312	83.42543141	100
	CQL-04	0.00	2.305682746	6.535949
	CQL-05	0.00	2.329264169	25.874128
	CQL-06	0.00	6.564933661	22.321428
	CQL-07	0.00	4.594750831	29.358075
	CQL-08	0.00	0.15554939	1.804123
	CQL-09	7.870532	22.94619847	41.102381
	CQL-10	3.846155	19.93807131	36.90476
	CQL-11	0.00	5.109087627	30.357144
	CQL-12	27.659575	61.07706907	92.857143

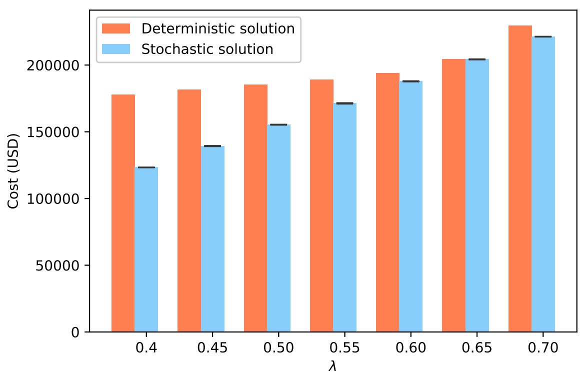
Table 4.6: Closeness Coefficients for SSP and LSP at Each PACP

Candidate	SSP	LSP	Candidate	SSP	LSP
PCAP-01	0.631772	0.749836	PCAP-31	0.554932	0.579089
PCAP-02	0.509607	0.543321	PCAP-32	0.539559	0.592645
PCAP-03	0.463016	0.532286	PCAP-33	0.487743	0.541983
PCAP-04	0.518957	0.550061	PCAP-34	0.445685	0.501755
PCAP-05	0.621651	0.735444	PCAP-35	0.548174	0.662319
PCAP-06	0.638667	0.784951	PCAP-36	0.459497	0.561767
PCAP-07	0.412889	0.488310	PCAP-37	0.649621	0.638495
PCAP-08	0.415901	0.478475	PCAP-38	0.463372	0.553105
PCAP-09	0.604897	0.621237	PCAP-39	0.579272	0.648916
PCAP-10	0.528526	0.569130	PCAP-40	0.548214	0.543270
PCAP-11	0.624184	0.707356	PCAP-41	0.557513	0.558785
PCAP-12	0.376276	0.491735	PCAP-42	0.520102	0.546624
PCAP-13	0.432560	0.486563	PCAP-43	0.576658	0.688986
PCAP-14	0.464206	0.511630	PCAP-44	0.637076	0.795583
PCAP-15	0.444365	0.552967	PCAP-45	0.523321	0.618600
PCAP-16	0.493942	0.570285	PCAP-46	0.485506	0.541934
PCAP-17	0.403669	0.518138	PCAP-47	0.433962	0.531268
PCAP-18	0.385620	0.489411	PCAP-48	0.475228	0.509586
PCAP-19	0.505476	0.584608	PCAP-49	0.579811	0.677991
PCAP-20	0.493839	0.554356	PCAP-50	0.462514	0.551948
PCAP-21	0.570863	0.698925	PCAP-51	0.523697	0.622275
PCAP-22	0.495677	0.591808	PCAP-52	0.438151	0.511048
PCAP-23	0.525434	0.512843	PCAP-53	0.578885	0.682339
PCAP-24	0.341764	0.472529	PCAP-54	0.529568	0.571210
PCAP-25	0.553146	0.599692	PCAP-55	0.596579	0.687780
PCAP-26	0.514935	0.519396	PCAP-56	0.501737	0.561529
PCAP-27	0.491137	0.454563	PCAP-57	0.481565	0.549124
PCAP-28	0.455973	0.549153	PCAP-58	0.476598	0.614772
PCAP-29	0.463227	0.522300	PCAP-59	0.494029	0.578024
PCAP-30	0.587532	0.712524			

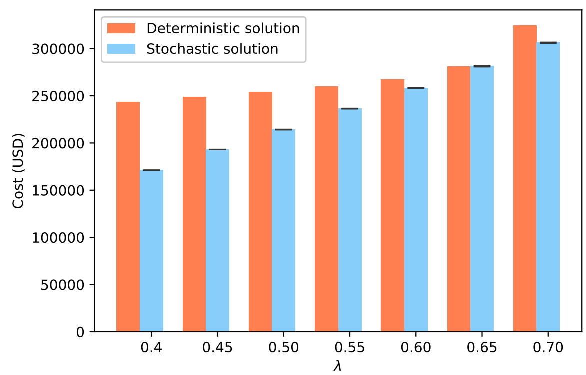
4.3 Analysis

4.3.1 Preliminary Results

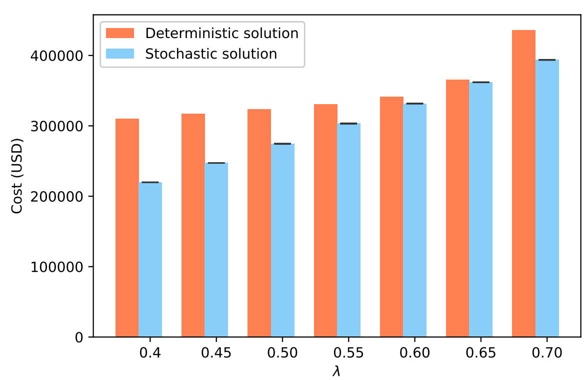
We start with presenting some preliminary results of the proposed model. Figures 4.2 through 4.5 presented cost, cost per patients, and coverage based on different values of λ. In each figure, the problem is studied for three different scenarios in terms of number of SSP and LSP.



(a) QOS-Cost(200,400)

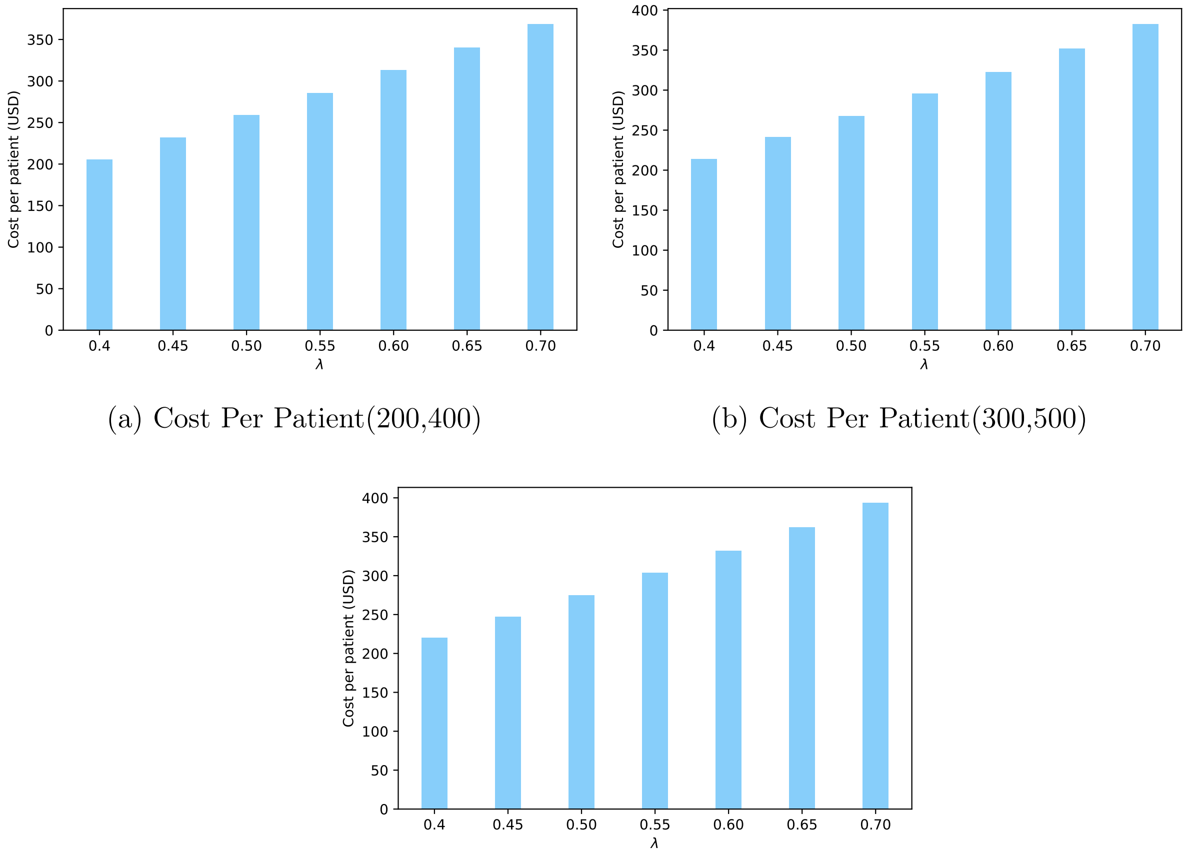


(b) QOS-Cost(300,500)



(c) QOS-Cost(400,600)

Figure 4.2: QOS-Cost



(a) Cost Per Patient(200,400)

(b) Cost Per Patient(300,500)

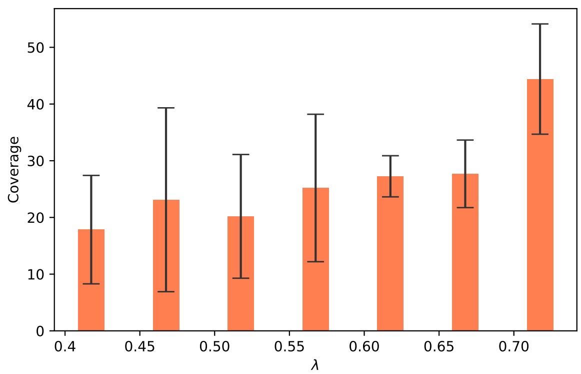
(c) Cost Per Patient(400,600)

Figure 4.3: Cost Per Patient

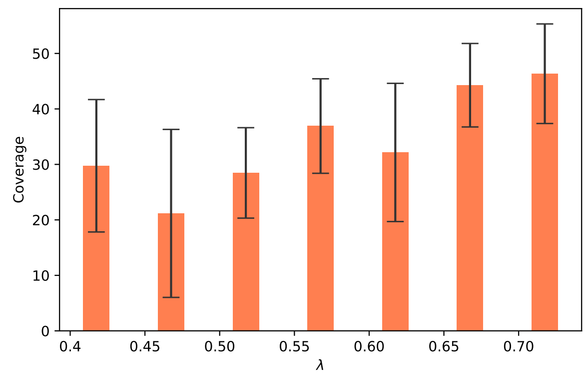
As expected, once λ increases the total cost or cost per patients increases as well. The reason is that, hospitals need to send their patients to the facility which in general charge more for providing better services. This has been checked for different combination of number of SSP and LSP. For all cases, the total cost increases by increasing the λ

Now, the impact of λ on the average distance from patients' location the PAC's is investigated.

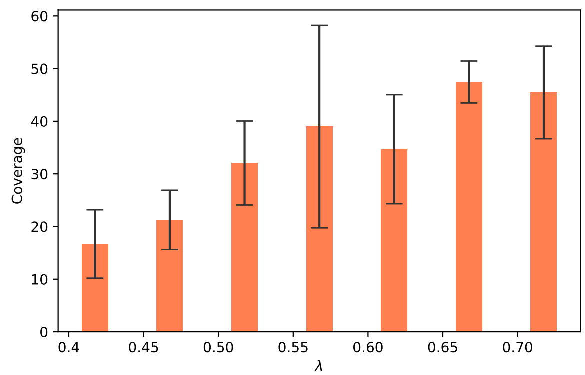
Generally, as λ increases, the average distance increases, but it is not always true. The reason is that in most cases we expect patients to drive more to get to facilities with better



(a) Distance for SSP(200,400)



(b) Distance for SSP(300,500)



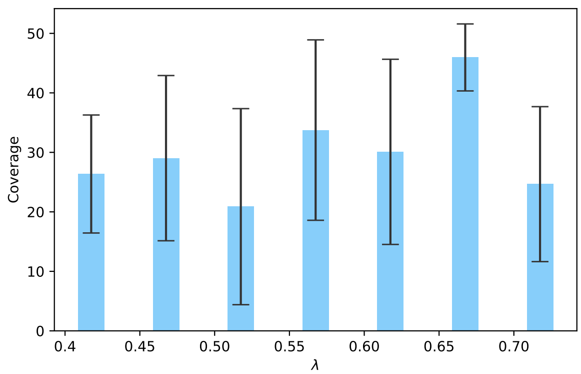
(c) Distance for SSP(400,600)

Figure 4.4: Impact of λ on Distance for SSP

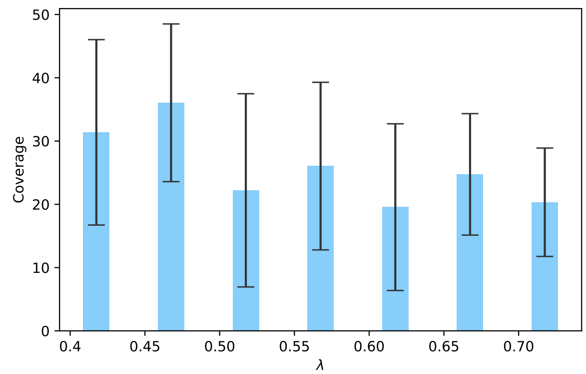
service quality, but, as the total cost is the main goal, this may not be true for all cases.

4.3.2 Readmission to Acute Care

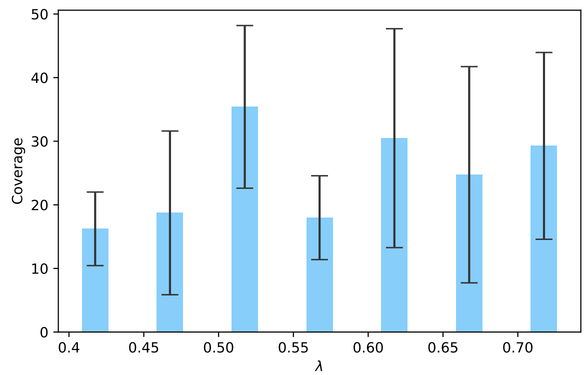
PAC is assigned after a patient is discharged from a prior hospitalization. It is expected that a patient would recover sufficiently to go back to the hospital within a reasonably short time would not happen. Nonetheless, it typically happens that some patients get readmitted to the hospital in a short period after discharge from hospitals. In the PAC sector, hospital readmission has been deployed as an indicator to represent the clinical effectiveness of the PAC services. Accordingly, it is reasonable to measure the readmission rate for a PAC facility.



(a) Distance for LSP(200,400)



(b) Distance for LSP(300,500)



(c) Distance for LSP(400,600)

Figure 4.5: Impact of λ on Distance for LSP

Readmission rate is defined as the number of patients readmitted to the hospital within a specific period after discharge from a PAC facility. The readmission rate is interpreted as an indicator of the PAC's clinical effectiveness.

Hospital readmission is also associated with a current policy enacted with the Accountable Care Act (ACA). Based on this policy, hospitals would be penalized by the Center for Medicare and Medicaid Services (CMS) if a "high-thanaverage" number of patients is readmitted to the hospital within 30 days. The 30-day window has thus given readmission a specific numerical value as a reference point. We assume a range of 0.01-0.1 for readmission

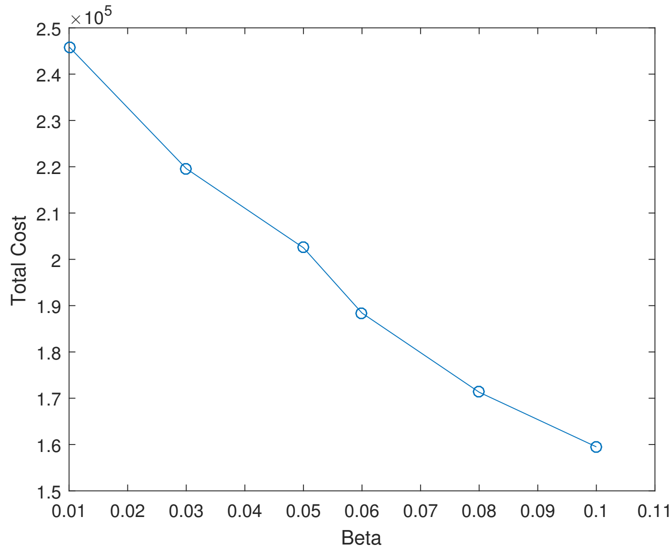


Figure 4.6: Impact of Readmission Rate

rate in facilities.

In this section, we add a new constraint (4.19) to the problem which consider the readmission rate of PAC as a criterion for decision making:

$$\sum_{i\in N}\sum_{j\in M}\sum_{R} B_i y^s_{ijk} \leq \beta \sum_{j\in M}\sum_{k\in R} A^s_{jk} \quad \forall s \in S \tag{4.19}$$

Figure 4.6 presents the impact of this new constraint on results.

As expected, by increasing the threshold for readmission rate, total cost decreases. The reason is that potentially PAC's with higher readmission rate charge their patients less.

4.4 Multi-Objective Modeling

Here we present a multi-objective optimization model for the PAC provider selection problem. The proposed model aims to incorporate the global score obtained from the TOP-

SIS method and design the PAC network that delivers quality services with minimum cost and minimum readmission rate. For this model, we are given a list of pre-qualified PACP's, and their CC's and cost for SSP and LSP. We use the notation presented in Table 3 in the formulation and description of the model.

Generally, minimizing the cost is the main objective of an AC to contract with PAC's. Recent works in the literature have shown that the length of stay and service quality of a PAC impacts the total cost of an AC. Recently, the government imposed a penalty for readmission to hospitals if it occurs less than 30 days after discharge.

Regarding the above-mentioned parameters and decision variables, we consider a new framework for patients' assignment to PAC's. This new approach proposes a multi-objective model that includes cost, service quality, and readmission rate as objectives. The mathematical model is as follows:

$$Min \quad \sum_{i \in N} \sum_{j \in M} f_i x_{ij} + \sum_{i \in N} \sum_{j \in M} \sum_{k \in R} v_{ij} y_{ijk} \tag{4.20}$$

$$Max \quad \sum_{i \in N} \sum_{j \in M} \sum_{k \in R} CC_{ij} y_{ijk} \tag{4.21}$$

$$Max \quad \sum_{i \in N} \sum_{j \in M} \sum_{k \in R} B_i y_{ijk} \tag{4.22}$$

s.t.

$$\sum_{j\in M}\sum_{k\in R} y_{ijk} \leq Cap_i \quad \forall i \in N \tag{4.23}$$

$$\sum_{i\in N}\sum_{j\in M}\sum_{k\in R} y_{ijk} = \sum_{j\in M}\sum_{k\in R} A_{jk} \tag{4.24}$$

$$y_{ijk} \leq M.x_{ij} \quad \forall i \in N, \forall j \in M, \forall k \in R \tag{4.25}$$

$$x_{ij} \in \{0,1\}, y_{ijk} \in \mathbb{Z}^+ \quad \forall i \in N, \forall j \in M, \forall k \in R \tag{4.26}$$

The proposed portfolio model is a multi-objective MIP formulation. The solution approaches to multi-objective optimization are reviewed in [53]. Considering the similar nature of the last two objective functions, the weighting method is applicable to this problem. However, the first objective function is not as the same nature as the the other two objective functions. It, therefore, should not be simply added to the other objectives using the weighting method. The model is linearized through a mixture of the weighting method and a modified lexicographic method. Using the weighting method and lexicographic method in this setting is proven efficient in supplier selection[54]. For a fair comparison of all objectives, we normalize the second and third objectives by the traditional method of:

So we define the followings:

$$\bar{B}_{ij} = B_{ij} / \sum_{j\in M} B_{ij} \quad \forall i \tag{4.27}$$

$$\bar{C}C_{ij} = CC_{ij} / \sum_{j\in M} CC_{ij} \quad \forall i \tag{4.28}$$

The first optimization problem will be as follows:

$$[P1] \quad Min \quad \sum_{i\in N}\sum_{j\in M} f_i x_{ij} + \sum_{i\in N}\sum_{j\in M}\sum_{k\in R} v_{ij} y_{ijk} \tag{4.29}$$

$$s.t. \quad Constraints(4.23) - (4.26) \tag{4.30}$$

Another important aspect in this approach is to determine the weight or importance of each criterion. Defining I_1 and I_1 ($I_1 + I_2 = 1$)as the weight of the second and third objectives respectively, we have the transformed model as:

$$[P2] \quad Min \sum_{i\in N}\sum_{j\in M}\sum_{k\in R}(I_1 \bar{B}_{ij} - I_2 \bar{C}C_{ij}) y_{ijk} \tag{4.31}$$

Finally, providers are selected by a modified lexicographic method [54]. In the traditional lexicographic method, the third objective would be considered as a secondary aim only, and can be improved, only if it does not lower the first two objectives. To provide a trade-off between risk and value, we introduce coefficient Γ into the lexicographic method. This value enables a buffer interval to reduce risk, while maintaining a Γ fraction of the initial objective value (28).

$$[P2] \quad Min \sum_{i\in N}\sum_{j\in M}\sum_{k\in R}(I_1 \bar{B}_{ij} - I_2 \bar{C}C_{ij})(y_{ijk}) \tag{4.32}$$

$$s.t. \quad Constraints(23) - (26)$$

$$\sum_{i\in N}\sum_{j\in M} f_j x_{ij} + \sum_{i\in N}\sum_{j\in M}\sum_{k\in R} v_{ij} y_{ijk} \leq \Gamma v^*(P1) \tag{4.33}$$

where $v^*(P1)$ denotes the objective value of problem(P1). [P1] and [P2] are MIP models. Considering the structure of the second problem, we cannot use decomposition algorithms. In this case, if we assume a limited number of scenarios, both models are easily solved using commercial MIP solvers. This concludes the provider election procedure introduced by this paper.

4.4.1 Results and Discussion

In this section, we present the results for the multi-objective problem. We study the impact of Γ and weights of second and third objectives (I_1 and I_2). Table 4.7 presents the results for P1 and P2 assuming different combinations of I_1 and I_2.

Readmission rate in the tables is the weighted average of the readmission rate for the selected facilities. This is the same for quality of services.

Table 4.7: Results for $x = 8$

		Cost	Readmission	Service Quality
x=8				
	P1	$1,478,235		
I1	I2			
0	1	$1,536,182	0.046	0.6
0.1	0.9	$1,556,729	0.043	0.58
0.2	0.8	$1,565,836	0.041	0.55
0.5	0.5	$1,570,033	0.037	0.53
0.8	0.2	$1,562,449	0.033	0.5
0.9	0.1	$1,558,355	0.03	0.48
1	0	$1,552,903	0.028	0.46

From table 4.8 through 4.10, we assume a fixed number of providers. This is done by adding a constraint to the model. As x increases, the total cost, service quality will be increased. But, readmission index decreases. Moving from $x = 8$ to $x = 10$, total cost increases by around 15%, but service quality index increased by approximately 13%. It is important to note that they are not changed linearly.

Table 4.8: Results for $x = 10$

		Cost	Readmission	Service Quality
x=10				
	P1	$1,546,739		
I1	I2			
0.1	0.9	$1,611,857	0.04	0.59
0.5	0.5	$1,636,914	0.035	0.55
0.9	0.1	$1,607,371	0.028	0.49

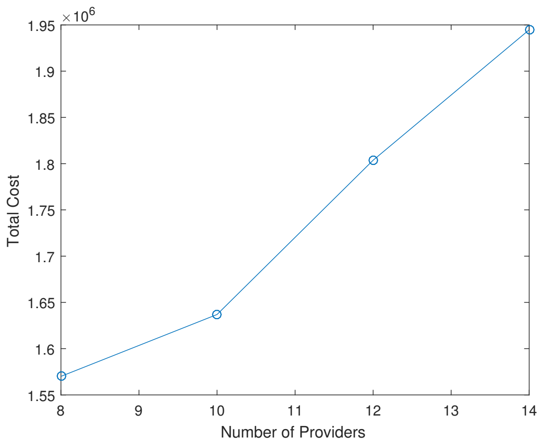
Table 4.9: Results for $x = 12$

		Cost	Readmission	Service Quality
x=12				
	P1	$1,689,316		
I1	I2			
0.1	0.9	$1,758,747	0.037	0.61
0.5	0.5	$1,803,683	0.031	0.57
0.9	0.1	$1,762,497	0.025	0.52

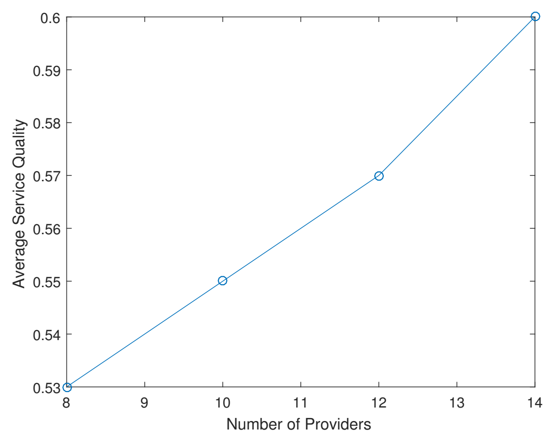
Table 4.10: Results for $x = 14$

		Cost	Readmission	Service Quality
x=14				
	P1	$1,815,291		
I1	I2			
0.1	0.9	$1,910,049	0.032	0.63
0.5	0.5	$1,944,721	0.027	0.6
0.9	0.1	$1,905,692	0.02	0.55

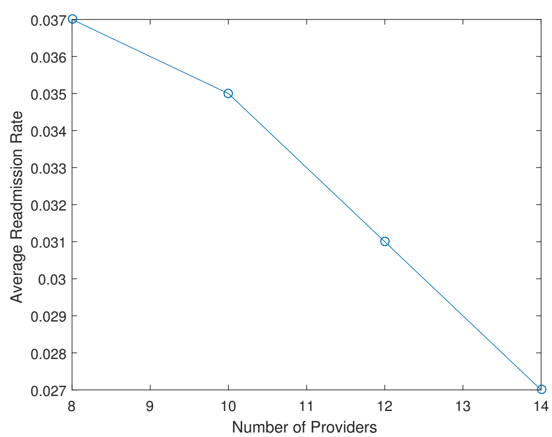
The impact of number of providers on total cost, average service quality, and average readmission rate is summarized in Figure 4.7. Total cost and average service quality increase as number of providers increases, while average readmission rate decreases.



(a) Total Cost



(b) Average Service Quality



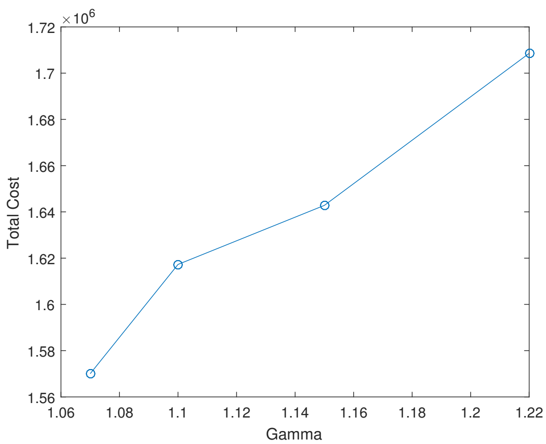
(c) Average Readmission Rate

Figure 4.7: Impact of Number of Providers

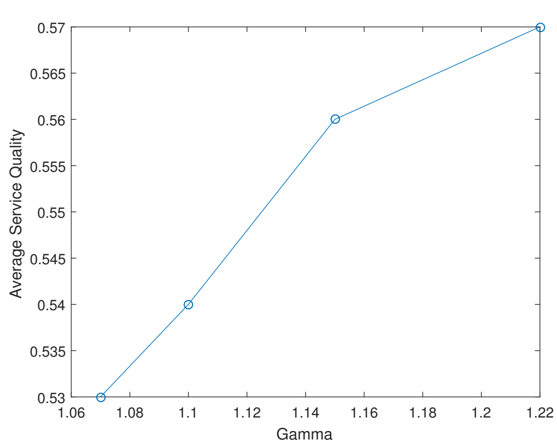
To analyze the trade-off between readmission, quality of services, and cost, we increase Γ from 1.07 to 1.22. Table 11 gives the results. When Γ=1, the traditional lexicographic method is carried out which results in the same providers chosen by [P1]. As increases, [P2] results in lower reamission rate and improvement in quality of services. Comparing Γ=1.07 to Γ=1.22 average readmission rate is decreased by roughly 10 percent, and service quality is increased by 8 percent. In other words, cost is increased by selecting higher value providers with higher service qualities, and consequently lower chance of readmission. The results are presented in table 4.11 and depicted in Figure 4.8. Total cost and average service quality increase as Gamma increases, while average readmission rate decreases.

Table 4.11: Effect of Γ on $P2$

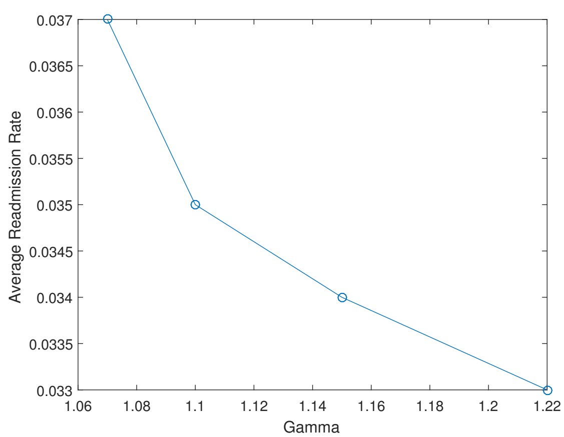
I1	I2	Cost	Readmission	Service Quality
$\Gamma = 1.07$				
0.1	0.9	$1,556,729	0.043	0.58
0.5	0.5	$1,570,033	0.037	0.53
0.9	0.1	$1,558,355	0.03	0.48
$\Gamma = 1.1$				
0.1	0.9	$1,598,362	0.042	0.6
0.5	0.5	$1,617,284	0.035	0.54
0.9	0.1	$1,599,591	0.028	0.49
$\Gamma = 1.15$				
0.1	0.9	$1,636,229	0.04	0.61
0.5	0.5	$1,642,816	0.034	0.56
0.9	0.1	$1,635,972	0.026	0.5
$\Gamma = 1.22$				
0.1	0.9	$1,696,763	0.039	0.63
0.5	0.5	$1,708,584	0.033	0.57
0.9	0.1	$1,697,360	0.025	0.52



(a) Total Cost



(b) Average Service Quality



(c) Average Readmission Rate

Figure 4.8: Impact of Gamma

We also investigate the impact of fixed cost on the optimal number of providers to contract with. Table 4.12 presents the results for the effect of fixed cost. We increase f from \$20000 to to \$90000 in increments of 10K. Comparing the case $f = 20000$ to $f = 90000$ the total cost is increased by around \$582000 while the number of providers decreased by 4. There is roughly 50% increase in total cost.

Table 4.12: Impact of Fixed Cost

Fixed Cost	Total Cost	x
20000	\$1,304,616	10
30000	\$1,400,378	9
40000	\$1,483,551	9
50000	\$1,570,033	8
60000	\$1,663,408	8
70000	\$1,742,729	7
80000	\$1,820,804	7
90000	\$1,886,145	6

5. CONCLUSIONS AND FUTURE RESEARCH

A stochastic Stackelburg-Nash-Cournot equilibrium model is proposed to study the long-term care capacity planning problem for both the public insurer and individual service providers. Game theory models have been widely used in solving strategic problems in energy and transportation fields. However, limited research has been done on the application of game theory models in health care. As healthcare system usually involves more uncertainties in the system dynamic and decision-making process; development of a conceptually valid and computationally tractable game model for solving healthcare problems poses great challenges to researchers. To the best of our knowledge, this model is the first attempt to use a game model to capture the interaction between health policy maker and individual providers as well as the competition among providers with uncertainty. We propose a framework for Medicaid to decide the optimal number of waivers for each state. It is shown in a real-case problem that Medicaid does not necessarily outspend its budget.

The model is still in its preliminary stage, and thus inevitably suffers from some limitations. To name a few, first, using an inverse demand function to capture the relationship between revenue per person and the total supply capacity still requires further justifications. Then, the competition mechanism among individual providers is not clear. In this paper, it is assumed that they decide simultaneously and independently, while there might be some cases in which some of facilities are assumed as leader for the others. Moreover, we assume capacity as the exact number of patients treated. To relax this assumption in the future research, more studies need to be done regarding the relationship between facility capacity and patient flow.

Besides, we look for a one-shot solution to this problem. In the future, researchers may extend this work to a multi-stage stochastic game and try to find the optimal solution in this new framework. This could be a very interesting problem, as in the one hand increasing capacity is a strategic decision and cannot be modified frequently. On the other, they encounter year to year increase in demand for LTC.

In addition, other researchers may extend this work by considering more accurate cost and revenue functions for the service facilities. In order to nullify the impact of quality (service level) in this problem, we restricted our case study to the facilities with highest quality measure. In the future, authors may take quality into account.

Chapter 4 introduces a novel two-phase approach to post acute care provider selection. The first phase includes evaluating the providers and assigning a comparable value based on a set of predefined criteria. We consider two types of patients which are defined by the Medicaid, short stay and long stay patients. In the second phase, we develop two different optimization method assuming whether the demand is uncertain. In order to compute the importance weight of attributes, we did an extensive survey analysis. All participants have relevant background and expertise. We consider a real-case problem in city of Houston, TX. Results are presented in detail. We investigate the impact of fixed cost on total cost and optimal number of providers.

Following this study, we can rank the PAC's in each state based on the Medicaid criteria. This can be used widely by acute care managers. Attributes and their importance weight for determining the quality of a PAC will be provided.

Future research may be carried out by introducing more constraints and decision criteria into the model. For example, readmission rate can be adjusted by the number of days a patient stays at a PAC. Any other real life constraints considered may be added to the current model to assess the effects of phase II. An interesting research direction is provider nurture versus better provider selection. A comparison may be carried out to investigate pre and post selection efforts on reaching an optimal provider portfolio, in addition to their collaboration and its results.

REFERENCES

[1] G. Smith, J. O'Keeffe, L. Carpenter, P. Doty, B. Burnwell, R. Mollica, L. Williams, *et al.*, "Understanding medicaid home and community services: A primer," 2000.

[2] K. Humes, "The population 65 Years and older: aging in America," in *The Book of the States 2005*, (New York), pp. 464–470, 2005.

[3] World Health Organization, "Lessons for long-term care policy." Produced by WHO Cross-cluster initiative on long-term care together with the WHO collaborating Center for Research on Health of the Elderly JDC-Brookdale Institute.

[4] The Lewin Group, Inc., "Elderly long-term care projections," May 2004.

[5] S. Eiken, K. Sredl, B. Burwell, and L. Gold, "Medicaid long-term care expenditures in FY 2009," *Cambridge, MA: Thomson Reuters*, 2010.

[6] D. G. Stevenson, "Planning for the future–long-term care and the 2008 election," *New England Journal of Medicine*, vol. 358, no. 19, pp. 1985–1987, 2008.

[7] P. Fox, K. Maslow, and X. Zhang, "Long-term care eligibility for people with Alzheimer's disease," *Health Care Financing Review*, vol. 20, pp. 67–85, 1999.

[8] M. C. Tonner and C. Harrington, "Nursing facility and home and community based service need criteria in the United States," *Home Health Care Services Quarterly*, vol. 22, pp. 65–83, 2003.

[9] R. H. Binstock and W. D. Spector, "Five priority areas for research on long-term care," *Health Serv Res.*, vol. 32, pp. 715–730, 1997.

[10] J. Feder, H. Komisar, and M. Niefeld, "Long-term care in the United States: an overview," *Health Affairs*, vol. 19, 2000.

[11] R. L. Kane, R. A. Kane, and W. N. Veazie, “Variation in state spending for long-term care: factors associated with more balanced system,” *Journal of Health Politics, Policy and Law*, vol. 23, pp. 363–390, 1998.

[12] A. T. Wenzlow, R. Schmitz, K. Shepperson, *et al.*, “A profile of Medicaid institutional and community-based long-term care service use and expenditures among the aged and disabled using max 2002: Final report,” *Cambridge, MA: Mathematica Policy Research, Inc*, 2008.

[13] R. M. Werner, R. T. Konetzka, M. Qi, and N. B. Coe, “The impact of medicare co-payments for skilled nursing facilities on length of stay, outcomes, and costs,” *Health Services Research*, vol. 54, no. 6, pp. 1184–1192, 2019.

[14] Z. Magdon-Ismail, A. Sicklick, R. Hedeman, J. P. Bettger, and J. Stein, “Selection of postacute stroke rehabilitation facilities: a survey of discharge planners from the northeast cerebrovascular consortium (necc) region,” *Medicine*, vol. 95, no. 16, 2016.

[15] S. Hakkennes and J. L. Keating, “Constraint-induced movement therapy following stroke: a systematic review of randomised controlled trials,” *Australian Journal of Physiotherapy*, vol. 51, no. 4, pp. 221–231, 2005.

[16] E. Adida, H. Mamani, and S. Nassiri, “Bundled payment vs. fee-for-service: Impact of payment scheme on performance,” *Management Science*, vol. 63, no. 5, pp. 1606–1624, 2017.

[17] D. Gupta and M. Mehrotra, “Bundled payments for healthcare services: Proposer selection and information sharing,” *Operations research*, vol. 63, no. 4, pp. 772–788, 2015.

[18] G.-H. Tzeng and J.-J. Huang, *Multiple attribute decision making: methods and applications*. CRC press, 2011.

[19] J. Turner, S. Mehrotra, and M. S. Daskin, "Perspectives on health-care resource management problems," in *A Long View of Research and Practice in Operations Research and Management Science*, pp. 231–247, Springer, 2010.

[20] D. Gupta and B. Denton, "Appointment scheduling in health care: Challenges and opportunities," *IIE transactions*, vol. 40, no. 9, pp. 800–819, 2008.

[21] E. K. Burke, P. De Causmaecker, G. V. Berghe, and H. Van Landeghem, "The state of the art of nurse rostering," *Journal of Scheduling*, vol. 7, no. 6, pp. 441–499, 2004.

[22] B. Cheang, H. Li, A. Lim, and B. Rodrigues, "Nurse rostering problems–a bibliographic survey," *European Journal of Operational Research*, vol. 151, no. 3, pp. 447–460, 2003.

[23] M. W. Carter and S. D. Lapierre, "Scheduling emergency room physicians," *Health Care Management Science*, vol. 4, no. 4, pp. 347–360, 2001.

[24] H. D. Sherali, M. H. Ramahi, and Q. J. Saifee, "Hospital resident scheduling problem," *Production Planning & Control*, vol. 13, no. 2, pp. 220–233, 2002.

[25] A. Ahmadi-Javid, Z. Jalali, and K. J. Klassen, "Outpatient appointment systems in healthcare: A review of optimization studies," *European Journal of Operational Research*, vol. 258, no. 1, pp. 3–34, 2017.

[26] T. Cayirli and E. Veral, "Outpatient scheduling in health care: a review of literature," *Production and Operations Management*, vol. 12, no. 4, pp. 519–549, 2003.

[27] B. Cardoen, E. Demeulemeester, and J. Beliën, "Operating room planning and scheduling: A literature review," *European Journal of Operational Research*, vol. 201, no. 3, pp. 921–932, 2010.

[28] L. S. Franz and J. L. Miller, "Scheduling medical residents to rotations: solving the large-scale multiperiod staff assignment problem," *Operations Research*, vol. 41, no. 2, pp. 269–279, 1993.

[29] D. Sinreich and O. Jabali, "Staggered work shifts: a way to downsize and restructure an emergency department workforce yet maintain current operational performance," *Health Care Management Science*, vol. 10, no. 3, pp. 293–308, 2007.

[30] P. Santibáñez, G. Bekiou, and K. Yip, "Fraser health uses mathematical programming to plan its inpatient hospital network," *Interfaces*, vol. 39, no. 3, pp. 196–208, 2009.

[31] W. Hare, A. Alimadad, H. Dodd, R. Ferguson, and A. Rutherford, "A deterministic model of home and community care client counts in British Columbia," *Health Care Management Science*, vol. 12, no. 1, p. 80, 2009.

[32] J. Patrick, "Access to long-term care: The true cause of hospital congestion?," *Production and Operations Management*, vol. 20, no. 3, pp. 347–358, 2011.

[33] Y. Zhang, M. L. Puterman, M. Nelson, and D. Atkins, "A simulation optimization approach to long-term care capacity planning," *Operations Research*, vol. 60, no. 2, pp. 249–261, 2012.

[34] T. Cardoso, M. D. Oliveira, A. Barbosa-Póvoa, and S. Nickel, "Moving towards an equitable long-term care network: A multi-objective and multi-period planning approach," *Omega*, vol. 58, pp. 69–85, 2016.

[35] J. Patrick, K. Nelson, and D. Lane, "A simulation model for capacity planning in community care," *Journal of Simulation*, vol. 9, no. 2, pp. 111–120, 2015.

[36] T. Cardoso, M. D. Oliveira, A. Barbosa-Póvoa, and S. Nickel, "An integrated approach for planning a long-term care network with uncertainty, strategic policy and equity considerations," *European Journal of Operational Research*, vol. 247, no. 1, pp. 321–334, 2015.

[37] Y. Li, Y. Zhang, N. Kong, and M. Lawley, "Capacity planning for long-term care networks," *IIE Transactions*, vol. 48, no. 12, pp. 1098–1111, 2016.

[38] E. Adida, P.-C. C. DeLaurentis, and M. Lawley, "Hospital stockpiling for disaster planning," *IIE Transactions*, vol. 43, no. 5, pp. 348–362, 2011.

[39] D. W. McFadden, M. Tsai, B. Kadry, and W. W. Souba, "Game theory: applications for surgeons and the operating room environment," *Surgery*, vol. 152, no. 5, pp. 915–922, 2012.

[40] E. W. Ford, R. Wells, and B. Bailey, "Sustainable network advantages: a game theoretic approach to community-based health care coalitions," *Health Care Management Review*, vol. 29, no. 2, pp. 159–169, 2004.

[41] M. Kurt, M. S. Roberts, A. J. Schaefer, and M. U. Unver, "Valuing prearranged paired kidney exchanges: A stochastic game approach," tech. rep., Boston College Department of Economics, 2011.

[42] H. D. Sherali, A. L. Soyster, and F. H. Murphy, "Stackelberg-Nash-Cournot equilibria: characterizations and computations," *Operations Research*, vol. 31, no. 2, pp. 253–276, 1983.

[43] H. Von Stackelberg, *Market structure and equilibrium.* Springer Science & Business Media, 2010.

[44] C. Shapiro, "Theories of oligopoly behavior," *Handbook of industrial organization*, vol. 1, pp. 329–414, 1989.

[45] C. Safran, M. Bloomrosen, W. E. Hammond, S. Labkoff, S. Markel-Fox, P. C. Tang, and D. E. Detmer, "Toward a national framework for the secondary use of health data: an american medical informatics association white paper," *Journal of the American Medical Informatics Association*, vol. 14, no. 1, pp. 1–9, 2007.

[46] R. W. Anderson, "Some theory of inverse demand for applied demand analysis," *European Economic Review*, vol. 14, no. 3, pp. 281–290, 1980.

[47] D. De Wolf and Y. Smeers, "A stochastic version of a Stackelberg-Nash-Cournot equilibrium model," *Management Science*, vol. 43, no. 2, pp. 190–197, 1997.

[48] H. Xu, "An MPCC approach for stochastic Stackelberg-Nash-Cournot equilibrium," *Optimization*, vol. 54, no. 1, pp. 27–57, 2005.

[49] D. He and R. T. Konetzka, "Public reporting and demand rationing: Evidence from the nursing home industry," *Available at SSRN*, 2013.

[50] F. H. Murphy, H. D. Sherali, and A. L. Soyster, "A mathematical programming approach for determining oligopolistic market equilibrium," *Mathematical Programming*, vol. 24, no. 1, pp. 92–106, 1982.

[51] C.-L. Hwang and K. Yoon, "Methods for multiple attribute decision making," in *Multiple attribute decision making*, pp. 58–191, Springer, 1981.

[52] A. J. Kleywegt, A. Shapiro, and T. Homem-de Mello, "The sample average approximation method for stochastic discrete optimization," *SIAM Journal on Optimization*, vol. 12, no. 2, pp. 479–502, 2002.

[53] R. T. Marler and J. S. Arora, "Survey of multi-objective optimization methods for engineering," *Structural and multidisciplinary optimization*, vol. 26, no. 6, pp. 369–395, 2004.

[54] B. Sawik, "Lexicographic and weighting approach to multi-criteria portfolio optimization by mixed integer programming," in *Financial modeling applications and data envelopment applications*, Emerald Group Publishing Limited, 2009.