STRATEGIC FUNDRAISING FOR PUBLIC GOODS AND SERVICES

A Dissertation

by

PIRUZ SABOURY

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Chair of Committee,	Silvana Krasteva
Committee Members,	Jonathan Meer
	Marco A. Palma
	Ragan Petrie
Head of Department,	Timothy J. Gronberg

August 2020

Major Subject: Economics

ABSTRACT

In this dissertation, I study two pro-social markets and employ the theory of signaling games to understand how socially motivated institutions strategically choose fundraising mechanisms to convey private information to social investors in non-transparent environments. My findings explain two empirical observations about the charitable giving market and the microfinance market.

First, I focus on leadership giving in charitable fundraising. While existing theory predicts that matching leadership gifts raise more voluntary contributions for public goods than seed money, recent experiments find otherwise. I reconcile the two by studying a model of leadership giving with incomplete information about the quality of public good. Both the fundraising scheme and the lead donor's contribution size may signal quality to donors. I show that if the lead donor is informed about quality, she will convey information to downstream donors through the size of her contribution. Thus, the charity will have no signaling concerns and opts for a matching gift to mitigate free-riding. However, when the lead donor's information is limited, her contribution is less informative, and a high-quality charity utilizes the fundraising scheme to convey information. In particular, a high-quality charity uses seed money as a costly signal to convince donors of high quality. Therefore, seed money becomes a stronger signal of quality and induces higher expected contributions.

I then turn to the microfinance market, where the commercialization trend in a once entirely non-profit industry, has triggered much debate. While some argue that the transition to a for-profit sector is a necessary step towards efficiency, others believe that profit-seeking results in mission drift, i.e., a diversion away from the original mission of helping the poor. I explain this polarization using a model where micro-lending costs are increasing in poverty, and there are two types of social investors: Rawlsians whose goal is to help the poorest of the poor and utilitarians who aim to maximize consumer surplus. With

unobservable costs, commercialization signals low costs, which appeals to utilitarians but dissuades Rawlsians due to the wealthier borrowers associated with low costs. Therefore, with a predominately Rawlsian investor pool, all MFIs (microfinance institutions) operate as non-profits. However, when utilitarian preferences prevail, low-cost MFIs who serve the marginal poor, offer high repayment to investors to separate from high-cost MFIs. The latter, who operate in highly impoverished communities, remain non-profit. Hence, utilitarian investors divide the microfinance mission between for-profits who raise more funds and serve the marginal poor, and non-profits who carry on the more costly task of serving the poorest of the poor.

ACKNOWLEDGMENTS

My deepest gratitude goes to my advisor, Dr. Silvana Krasteva, for giving me her support all the way through my PhD journey and advocating for my success. I also express my deepest appreciation to my committee members, Dr. Jonathan Meer, Dr. Marco A. Palma, and Dr. Ragan Petrie for their care, support, and flexibility.

I am especially thankful to Dr. Adel Varghese and the Human Behavior Laboratory at Texas A&M University, led by Dr. Marco A. Palma, for their great support during my years in the PhD program.

I am also very thankful to the faculty and staff of the Department of Economics, especially Dr. Timothy J. Gronberg and Ms. Teri Tenalio for their efforts to make my PhD journey possible.

My endless thanks to my wife, my mother, my father, and my sister, whose support made this process possible.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

This work was supported by a dissertation committee consisting of Dr. Silvana Krasteva (chair), Dr. Jonathan Meer, and Dr. Ragan Petrie of the Department of Economics and Dr. Marco A. Palma of the Department of Agricultural Economics.

The analyses depicted in Section 2 and Appendix A were conducted in part by Dr. Silvana Krasteva of the Department of Economics.

All other work conducted for the dissertation was completed by the student independently.

Funding Sources

My graduate study was supported by an assistantship from the Department of Economics of Texas A&M University, an assistantship from the Human Behavior Laboratory at Texas A&M University, and a research fellowship from the Institute for Humane Studies at George Mason University.

TABLE OF CONTENTS

Page

1. INTRODUCTION

Roman aqueducts, medieval hospitals, and caravansaries along the silk road are just a few examples demonstrating the fact that charity and provision of public goods and services have been a part of human societies since ancient times. However, the presence of a large number of non-government organizations with pro-social missions backed by private philanthropists is, for the most part, a phenomenon that gradually evolved during modern times. As the number of such organizations increased throughout the 20th century, the focus of economists shifted from studying public goods as a source of market failure to seeing their provision as a new type of "market." In this market, charitable organizations and socially motivated firms "sell" public goods and services to altruistic donors and social investors. Moreover, due to increased market size, an increasing level of attention has been drawn to the quality and impact of public goods, and social services. This trend has given rise to several charity watchdogs such as CharityWatch, Charity Navigator, and GiveWell over the last three decades. The attention to quality and impact is also reflected in the increasing use of new terminology for charity such as "impact investing" and "social business" that imply a more demanding and less forgiving attitude in the area of social spending. In response, socially motivated organizations have increasingly felt the need to credibility communicate their impact to donors and social investors.

This dissertation employs information economics and, more specifically, the theory of signaling games to study pro-social institutions' fundraising in non-transparent environments. This work specifically focuses on how such institutions make strategic use of solicitation mechanisms to signal otherwise unobservable information about their social impact to potential donors and social investors. Each of the main sections investigates a specific pro-social market and discusses how signaling by socially motivated institutions can explain a gap between existing theories and empirical evidence.

Section 2 focuses on leadership giving in charitable fundraising, which has garnered significant attention in the charitable giving literature (e.g., Vesterlund, 2003; Andreoni, 2006; List and Lucking-Reiley, 2002; Rondeau and List, 2008; Huck et al., 2015).[1] It refers to a fundraising strategy by charities which entails soliciting first a wealthy donor and announcing his or her donation to encourage others' giving. Most commonly, leadership gifts are in the form of an unconditional lump sum donation called "seed money" or a promise of matching small donations by a fixed ratio called "matching gift."

Theoretical studies suggest that a matching gift should encourage more donations than seed money since the former strategy reduces free-riding (e.g., Varian, 1994; Guttman, 1978; Danziger and Schnytzer, 1991). Recent experimental studies, however, find otherwise. In particular, Rondeau and List (2008), Karlan et al. (2011), Huck and Rasul (2011), and Huck et al. (2015) find evidence that a seed money announcement attracts more donors and increases total contributions, while a matching gift announcement has a weak or even adverse effect on contributions.

The section reconciles this ostensible discrepancy between the theory and the experimental evidence by studying quality signaling by charities through the choice of fundraising schemes. The theoretical argument relies on the observation that the choice of the fundraising scheme–seed money or matching gift–conveys information to donors about the quality of the charity. I find that seed money, which is seemingly the less effective strategy, can be used by the charity to credibly signal higher quality. This explains the significant increase in giving upon an announcement of a seed money gift and the weak response to a matching gift.

To demonstrate this finding, I present a theoretical model of leadership giving in a

[1] On the theory front, other related studies include Varian (1994), Guttman (1978), Danziger and Schnytzer (1991), Admati and Perry (1991), Andreoni (1998), Romano and Yildirim (2001), Bag and Roy (2011), Krasteva and Yildirim (2013), and Gong and Grundy (2014). There is also a large experimental literature that includes both lab experiments (e.g., Eckel and Grossman, 2003; Potters et al., 2005; Güth et al., 2007; Potters et al., 2007; Eckel and Grossman, 2006a,b; Eckel et al., 2007) and field experiments in a variety of settings (e.g., Silverman et al., 1984; Frey and Meier, 2004; Meier and Frey, 2004; Soetevent, 2005; Meier, 2007; Falk, 2007; Alpizar et al., 2008; Croson and Shang, 2008; Shang and Croson, 2009; Karlan and List, 2007; Eckel and Grossman, 2008; Karlan et al., 2011; Huck and Rasul, 2011; Adena and Huck, 2017).

large economy with incomplete information about the quality of the public good provided by a charity. Both the fundraising scheme employed by the charity and the contribution decision by the lead donor may signal the charity's quality to subsequent donors. The charity solicits optimally for a matching gift if the lead donor is informed about the quality of the public good. Intuitively, an informed lead donor conveys quality information to downstream donors through the size of her contribution. As a result, the charity has no signaling concerns and opts for a matching gift because it mitigates the free-riding incentives among donors and leads to higher contributions. The preference for matching, however, reduces when information acquisition by the lead donor is costly and thus limited. In this case, the lead donor's contribution is less informative. Hence, a high-quality charity utilizes the fundraising scheme to convey information. In particular, the charity uses seed money as a costly signaling device to convince donors of its high quality. As a result, seed money becomes a strong signal of quality and induces higher expected contributions by donors. This finding is consistent with experimental data, where seed money is associated with higher donations relative to a matching gift.

Section 3 turns to the microfinance market that since the 1980s has grown rapidly to become a large industry.[2] Once limited to a few non-profits, microfinance has not only grown but also witnessed a surge of for-profits.[3] Dieckmann et al. (2007) state that microfinance has provided an increasingly attractive venue for socially responsible investment.

This "commercialization" trend has triggered much debate centered on the future direction of the industry. One side, with the support from the existing theories (e.g., Ghosh and Van Tassel, 2011, 2013; Karaivanov, 2018) argues that pursuing profits induces efficiency and forces high-cost institutions out of the microfinance market. Thus, they interpret for-profits' entry as a sign of the industry's health and success. The other side of the debate, however, believes that profit-seeking leads to "mission drift." The latter

[2] According to Khamar (2015), the global loan portfolio of microfinance reached $92.4 billion in 2015 (8.6% annual growth), with 116.6 million borrowers around the world (13.5% annual growth).

[3] For example, for-profit institutions formed 81% of the Indian microfinance market in 2009 (Parameshwar et al., 2010).

term refers to a diversion of microfinance away from its original mission of alleviating poverty. This is in line with empirical studies (e.g., Cull et al., 2007, 2011) that find for-profit MFIs to perform, on average, worse in measures of outreach to the poor, compared to non-profits.[4]

This section offers a novel theoretical explanation for this polarization by exploring signaling by MFIs (microfinance institutions) in the funding process through the choice of profit status. The theory presented is based on the fact that MFIs vary in the poverty level of their clients that, in turn, results in a variation in costs. I show that high repayments to social investors burden poor borrowers but can also function as a signal of low costs and lead to more fundraising. Therefore, MFIs with wealthier clientele (the marginal poor) may use such a signal and become for-profit. However, MFIs that operate in highly impoverished communities and incur high costs cannot afford this signal and remain non-profit.

I demonstrate this finding by modeling microfinance as a sequential game with information asymmetry. The model has two distinguishing features. First, the costs of micro-lending are increasing in poverty. Second, social investors have one of two distinct welfare goals: helping the poorest of the poor (Rawlsian) or maximizing consumer surplus (utilitarian). I show that when costs are unobservable, commercialization signals low costs, which appeals to utilitarian investors but discourages Rawlsian investors due to the wealthier borrowers that are associated with low costs. Therefore, so long as the Rawlsian philosophy is dominant, all MFIs operate as non-profits. However, once utilitarian preferences take over, MFIs who serve the marginal poor offer high repayments to social investors as a credible signal of low costs and raise more funds compared to their high-cost counterparts. The latter who serve the extremely poor cannot mimic the high repayments offered by for-profits and hence operate as non-profits revealing their high costs. I conclude that the commercialization trend in microfinance is driven by the effort

[4]Morduch (2000) refers to this polarization as "the microfinance schism" and offers a thorough discussion of it.

to attract funding from utilitarian social investors. This, in turn, divides the microfinance mission. For-profits attract more resources into the industry and serve the marginal poor while non-profits carry on the more costly task of serving the poorest of the poor. This result is consistent with the empirical findings of the microfinance industry and suggests that both types of MFI have an essential role in helping the poor.

2. INFORMATIVE FUNDRAISING: THE SIGNALING VALUE OF SEED MONEY AND MATCHING GIFTS

2.1 Overview

This section focuses on leadership giving in charitable fundraising and studies quality signaling by charities through the choice of seed money and matching gift fundraising schemes. I reconcile the existing theory prediction that a matching gift scheme should be more effective in fundraising than a seed money scheme with the seemingly contradictory experimental evidence that donors tend to give more under a seed money scheme.

My model features a large economy with altruistic donors, in which the charity is privately informed about the quality of the public good that it provides.[1] Donors possess limited information about the quality of the public good. This setting is consistent with the state of the non-profit sector in the USA that features significant quality heterogeneity among existing non-profits and limited information among donors about how non-profits use their donations.[2] In the base model, this quality is binary, but I extend the analysis in Subsection 2.6.1. The charity chooses its fundraising mechanism to maximize donations. In particular, the charity chooses whether to solicit the lead donor for seed money or a matching gift. Subsequently, given this fundraising strategy, the lead donor decides whether to acquire costly information about the public good's quality before making a donation decision. Under leadership giving, the information acquired not only benefits the lead donor directly, as it results in more informed giving, but it enables the lead donor

[1]While the main analysis focuses on purely altruistic donors, Subsection 2.6.3 extends the analysis to impure altruism in line with Andreoni (1988, 1990).

[2]For instance, Charity Navigator, the largest charity rating agency in the USA, has classified close to one-third of rated charities in years 2007-2010 as having exceptionally poor or poor performance (Yörük, 2016). At the same time, in a survey, Neighbor et al. (2015) find that almost half of donors are not sure of how their money is used by charities. At first glance, this lack of information might be attributed to the donors' lack of interest. However, the survey findings suggest otherwise. It reveals that donors want nonprofits to be clearer regarding the charitable services that their money provides (Neighbor et al., 2015). Thus, the lack of information can be explained by findings such as the fact that donors often are unsure where to begin, don't find their desired information available, and are under time pressure (Neighbor et al., 2015).

to signal the quality of the public good to downstream donors through the size of her contribution.

In general, both the fundraising strategy and the size of the lead donor's gift may convey information about the charity's quality. These two signaling channels interact with each other to form the charity's and the lead donor's equilibrium strategies. In particular, the lead donor acquires information only if the value of information exceeds the cost. The value of information, however, varies not only with the prior quality distribution, but also with the equilibrium fundraising strategy. Thus, multiplicity of equilibria may arise with varying degree of information acquisition.

Considering first the two extremes of fully informed and fully uninformed equilibria, I establish that these types of equilibria cannot explain the use of seed money. The first extreme of fully informed lead donor causes the charity to rely on the lead donor to reveal the charity's quality to subsequent donors through the size of her donation. This eliminates the signaling concern of the high quality charity when choosing her optimal fundraising scheme. Consequently, consistent with the existing theoretical literature, I find that in the absence of signaling considerations, the charity optimally chooses the matching scheme, as it alleviates the free-riding problem present in public good provision. The other extreme of fully uninformed lead donor also fails to explain the success of seed money over matching. This is because in the absence of quality verification by the lead donor, the low quality charity can costlessly imitate the high quality charity's fundraising strategy. This makes it impossible for the high quality charity to separate from its low quality counterpart. Thus, donors fail to learn any useful information from the fundraising scheme or the lead donor's contribution amount and as a result all charity types and all schemes on the equilibrium path raise the same amount of money.

In order to explain the successful use of seed money, I turn to partially informed equilibria with seed money on the equilibrium path. I refer to such equilibria as *SPI* (seed-partial info) equilibria. Note that under partial information acquisition by the lead donor, imitation by the low quality charity is not costless any longer since there is some possibility of verification. Moreover, seed money is the less efficient scheme as it results in lower overall contributions compared to matching for any fixed quality level. Consequently, seed money is used as a costly quality signal. In particular, I show that in *every SPI* equilibrium, the high quality charity chooses seed money fundraising more frequently compared to the low quality charity, causing seed money to emerge as a signal of higher quality. Intuitively, as the lead donor becomes less reliable at signaling quality, the high quality charity engages in costly signaling through the fundraising scheme by choosing to solicit for seed money.

This theoretical analysis establishes a plausible mechanism by which leadership giving may convey information to donors. Subsection 2.6 demonstrates the robustness of seed money as a signal of higher quality in richer economic environments that include large number of quality types (Subsection 2.6.1), the possibility of opting out of leadership giving (Subsection 2.6.2), the presence of warm-glow motivations for giving (Subsection 2.6.3), and the availability of alternative information channels (Subsection 2.6.4). Subsection 2.6.1 reveals that while the likelihood of seed money fundraising is not necessarily monotonically increasing in the charity's quality, seed money is associated with higher expected quality in *every SPI* equilibrium. Subsection 2.6.3 establishes that altruistic motives for giving are important for incentivizing quality signaling via the fundraising scheme since strong warm-glow motivations for giving cause donors to become less sensitive to the scheme choice, thus making both schemes equally attractive for the charity. In general, my analysis suggests that the use of seed money as a costly signaling device persists in the presence of sufficiently strong altruistic motives for giving and significant quality uncertainty that is not fully resolved by the lead donor's information acquisition strategy or alternative information channels. This suggests that seed money fundraising is likely a more attractive fundraising strategy for newer charities with significant public good component, who are striving to establish quality reputation among donors.

In the following subsections, I discuss the relevant literature then present the model and findings. Subsection 2.2 provides a review of the relevant literature. Subsection 2.3 describes the theoretical model. Subsection 2.4 considers the benchmark case of complete information and describes how the fundraising schemes rank in terms of total contributions. Subsection 2.5 presents the full model with information asymmetry and endogenous information acquisition, and discusses the possibility of signaling through the fundraising scheme. Subsection 2.6 presents a few extensions to the base model. A summary of the results is provided in Subsection 2.7.

2.2 Related Literature

This theoretical model builds upon a large theoretical literature. Early theoretical work on private provision of public goods, such as Warr (1983) and Bergstrom et al. (1986), have focused on simultaneous contributions. They show the equivalence of the non-cooperative equilibrium from the simultaneous contributions game to the Lindahl equilibrium. Admati and Perry (1991) expand the analysis to a mechanism of alternating sequential contributions towards a threshold public good. They find that this can lead to an inefficient outcome. Similarly, Varian (1994) considers sequential fundraising and finds that it results in lower public good provision compared to simultaneous contributions due to donors' incentives to free-ride on earlier contributions. However, the possibility of donors subsidizing each others' contributions can alleviate this problem (e.g. Guttman, 1978; Danziger and Schnytzer, 1991). The implication of these findings is that a matching gift is more effective at encouraging contributions by downstream donors compared to a seed money gift.

In the context of complete and symmetric information, the use of seed money can be rationalized by the presence of threshold public good or other-regarding preferences. In particular, Andreoni (1998) shows that charities can use seed money to avoid zero-contributions equilibrium, in which no donor contributes due to an expectation that the threshold will not reached. Romano and Yildirim (2001) show that other-regarding preferences can give rise to upward-sloping best response functions, making sequential fundraising more effective than simultaneous fundraising. In the context of standard altruistic preferences, Gong and Grundy (2014) illustrate the possibility of matching raising less donations than seed money due to the lead donors' reluctance to offer high match ratios. They show that a necessary (but not sufficient) condition for this is that donors' marginal utility of the public good responds elastically to changes in the level of the public good. Such elastic response exacerbates the free-riding problem at high match ratios and may sometimes result in a very small matching gift and a lower fundraising amount compared to seed money. While this finding provides an alternative explanation for the desirability of seed money, it is limited to environments with a relatively small number of donors and an elastic marginal utility of the public good, which is violated in a wide range of commonly-used preferences (e.g. CES utility functions).[3] Instead, I focus on an alternative environment with many donors and standard inelastic preferences and find that the lower effectiveness of seed money is in fact an advantage under asymmetric information as it allows seed money to emerge as a costly signal of quality and consequently raise more funds compared to matching.

There is sparse theoretical literature that has considered incomplete information about the public good. Bag and Roy (2011) show that when donors have independent private valuations for the public good, free-riding incentives could diminish with sequential giving and thus sequential contributions might result in higher total donations compared to simultaneous ones. Krasteva and Yildirim (2013) consider an independent value threshold public good, in which each donor can choose whether to contribute informed or uninformed. They find that announcing seed money discourages informed giving while a matching gift encourages it. However, in both studies, the independence of donors' val-

[3] In Subsection 2.4, I show that as the donor population grows, matching must eventually (weakly) outperform seed money irrespective of the elasticity level. Thus, the preference for seed money is limited to an environment with a small donor pool. Moreover, as Gong and Grundy (2014) illustrate, elastic marginal utility of the public good is a necessary, but not sufficient condition for seed money to result in higher overall donations than a matching gift. Thus, even with an elastic utility and a small donor pool, matching may still emerge as the dominant scheme.

uations precludes the possibility of signaling via the scheme choice or the contribution amount by the lead donor. In this respect, the closest papers to this work are Vesterlund (2003) and Andreoni (2006).

Similar to my model, Vesterlund (2003) and Andreoni (2006) consider the use of seed money as a signaling device to convey the charity's quality. They demonstrate that leadership gifts in the form of seed money may result in larger total donations compared to simultaneous contributions since seed money enables the lead donor to signal the charity's quality to subsequent donors. However, an important distinction between these papers and my work is that they only allow for a seed money leadership scheme and ignore the possible signaling value of a matching gift. By enabling charities to choose between seed money and matching, I allow them to use the structure of the leadership gift itself to convey quality information to donors. In particular, such quality signaling through the scheme becomes an important tool of information transmission when acquiring information about the charity's quality is costly for donors.

In the realm of experimental studies, Silverman et al. (1984), Frey and Meier (2004), Soetevent (2005), Croson and Shang (2008), and Shang and Croson (2009) find that donors respond positively to information about other donors' gift, and Güth et al. (2007) show the positive impact of leadership gifts in particular. Furthermore, field experiments by List and Lucking-Reiley (2002), and Landry et al. (2006) demonstrate that both the likelihood and the size of donations significantly increase with the seed money amount. More interestingly, Potters et al. (2005) find that when some donors are informed and others are not, sequential contributions are likely to emerge endogenously, with more informed donors choosing to contribute first. All of these findings support the theory of seed money having signaling value. Potters et al. (2007) confirm this in an experiment that compares sequential contributions with an informed lead donor to simultaneous contributions.[4]

[4] Other related empirical work (e.g., Khanna and Sandler, 2000; Okten and Weisbrod, 2000; Andreoni and Payne, 2003, 2011) study the impact of government grants on private contributions. They find mixed results, which could be attributed to the differential impact of government grants on the fundraising effort by charities.

The impact of matching gifts has also been studied experimentally. Eckel and Grossman (2003), Meier and Frey (2004), Eckel and Grossman (2006a), Eckel and Grossman (2006b), Eckel et al. (2007), Falk (2007), and Eckel and Grossman (2008) find evidence in support of matching gifts being effective in boosting donations. However, Meier (2007) illustrates that this effect is short-lived and reverses in the long run. Moreover, Karlan and List (2007) find that while downstream donors respond positively to an announcement of a matching gift, they are unresponsive to an increase in the match ratio. More recently, Karlan et al. (2011) find little response to a matching gift and Adena and Huck (2017) find a negative response by donors.

All of the above studies focus on seed money or matching in isolation, but some of the recent literature directly compares the two schemes in the field. For example, Alpizar et al. (2008), Rondeau and List (2008), Huck and Rasul (2011), and Huck et al. (2015) find in a variety of field settings that knowledge of others' lump sum contribution amounts increases individual donations, but a reduction in the price of giving via a match (or reciprocal gift) has little impact. Moreover, Rondeau and List (2008) compare the effectiveness of both in the field and in a subsequent threshold public good lab experiment with complete information. They find that seed money is more effective in the field relative to the lab and conjecture that this is due to the signaling value of the leadership gift in the field where donors' knowledge of the public good is likely limited. In contrast, the returns from the public good are known in the lab and thus the leadership gift conveys no signaling benefits. My analysis confirms this intuition and provides a theoretical foundation for the above experimental findings.

2.3 Model Description

A single charity, C, aims to maximize the amount of money raised, G, to a continuous public good. The quality of the public good q takes two values- $q \in \{q_l, q_h\}$ with $0 < q_l < q_h$. The prior quality distribution is denoted by $\pi = \{\pi_l, \pi_h\}$ where $\pi_h \in (0, 1)$ stands for the probability of high quality.

On the contributors' side, the economy consists of a large set of donors, $\mathcal{D}$. Donors are characterized by their wealth $\in \{w_1, w_2, ..., w_I\}$ with $w_i > w_{i+1}$ for all $i \in [1, I-1]$. Moreover, $t_i \in (0, \infty)$ denotes the number of donors of wealth w_i. A donor of wealth w_i has the following preference over private and public consumption:

$$u_i(g_i, G, q) = h(w_i - g_i) + qv(G) \tag{2-1}$$

where $h'(\cdot) > 0, h''(\cdot) < 0, v'(\cdot) > 0, v''(\cdot) < 0$. Moreover, I assume that $qv'(0) > h'(w_1)$ and $\lim_{G\to\infty} qv'(G) = 0$, which jointly ensure positive and finite provision of the public good for all quality realizations.

The charity fundraises by employing leadership giving, in which it first solicits a lead donor, denoted by L, followed by simultaneous solicitation of all remaining donors, denoted by F. I let $w_L = w_1$ so that the lead donor belongs to the richest individuals in the economy. This is consistent with Andreoni (2006) who finds that the wealthiest individuals have the strongest incentives to become leaders in charitable campaigns. The leadership gift scheme, Z, chosen by C, can take the form of either seed money, S, or matching gift, M. Under S, L makes an unconditional contribution g_L^S that is publicly announced prior to the follower donors' contribution decisions. Under M, L commits to a match ratio m, which is publicly announced, and results in a contribution $g_L^M = m \sum_{i \in F} g_i^M = mG_F^M$ by L. To simplify the exposition, I denote the lead donor's choice by d_L^Z where $d_L^S = g_L^S$ and $d_L^M = m$.

The timing of the game is as follows. First, C privately observes q and publicly commits to a fundraising scheme $Z \in \{S, M\}$. Then, it solicits L for a donation. L privately decides whether to learn q at cost k and then publicly makes her contribution decision d_L^Z. All follower donors then observe Z and d_L^Z, and simultaneously choose their individual donations g_i^Z for $i \in F$.

The following subsections provide the equilibrium characterization of the above game.

I focus on a large economy, but my findings extend to any size economy, in which matching raises more donations than seed money for a given quality of the public good.[5] In particular, Subsection 2.4 presents a benchmark with observable quality and shows that a sufficiently large economy guarantees that matching always (weakly) dominates seed money. This is a foundational result that informs my analysis of unobservable quality, presented in Subsection 2.5.

2.4 Observable Quality

Given publicly observable quality q and fundraising scheme Z, each follower donor chooses her donation to maximize her payoff given by eq. (2-1). A contributing donor equates the marginal cost and benefit of donating, resulting in

$$h'(w_i - g_i^Z) = qv'(G^Z)\,(1 + m\mathbb{1}_M) \tag{2-2}$$

where $\mathbb{1}_M = 1$ for $Z = M$ and 0 otherwise. It is evident from eq. (2-2) that matching increases the marginal benefit of giving relative to seed money. Thus, for the same amount of total giving, i.e. $G^M = G^S$, i contributes more under the matching scheme as long as $m > 0$.

The concavity of $h(\cdot)$ and $v(\cdot)$ implies that higher anticipated G^Z reduces incentives to give and ensures the uniqueness of the equilibrium contributions by the follower donors.[6] Moreover, the decrease in the marginal value of the public good as a response to higher G^Z implies that each wealth "type" w_i has a drop-out level $G_i^{Z,0}(q, m\mathbb{1}_M)$ of the public good above which i becomes a non-contributor. $G_i^{Z,0}(q, m\mathbb{1}_M)$ solves

$$qv'(G_i^{Z,0})\,(1 + m\mathbb{1}_M) - h'(w_i) = 0 \tag{2-3}$$

[5] My focus on a large economy is consistent with the size of the charitable giving market in the USA, in which 68% of contributions come from individual donations (National Philanthropic Trust, 2019).

[6] See Bergstrom et al. (1986) for proof of equilibrium existence and uniqueness.

Note that $G_i^{M,0}(q,m) > G_i^{S,0}(q)$ for any $m > 0$ as long as these drop-out levels are finite. The condition $\lim_{G\to\infty} qv'(G) = 0$ ensures that this is indeed the case for both schemes and all quality levels.[7] Moreover, $G_i^{Z,0}(q, m\mathbb{1}_M)$ is strictly decreasing in i for both schemes, which allows me to employ the Andreoni-McGuire algorithm (see Andreoni and McGuire, 1993) to derive the aggregate best response of the follower donors, $G_F^Z(q, d_L^Z)$, to a leadership gift d_L^Z. In particular, consistent with Yildirim (2014), let C^Z denote the set of contributing donors and G_F^Z-their total giving. Then if $C_i^Z = \{1, 2, ..., i\} \subseteq C^Z$, the short-fall in provision by C_i^Z is

$$\Delta_i^Z(q, G_F^Z, d_L^Z) = G_F^Z - \sum_{j=1}^{i} t_j \left[w_j - \phi \left(qv'(G_F^Z + g_L^Z)(1 + m\mathbb{1}_M) \right) \right] \tag{2-4}$$

where by definition $g_L^M = mG_F^M$ and $\phi(\cdot) = [h']^{-1}(\cdot)$ is strictly decreasing in its argument. Therefore, $\Delta_i^Z(q, G_F^Z, d_L^Z)$ is strictly increasing in G_F^Z. Thus, the Andreoni-McGuire algorithm uniquely pins down the set of contributing donors and their equilibrium donation amount $G_F^Z(q, d_L^Z)$. The following lemma extends the equilibrium characterization by Yildirim (2014) to include the possibility of matching[8].

Lemma 2-1. *Given d_L^Z, let $\Delta_i^{Z,0}(q, d_L^Z) = \Delta_i^Z(q, G_i^{Z,0}(q, m\mathbb{1}_M) - g_L^Z, d_L^Z)$ and C^Z denote the equilibrium set of contributors. Then, $i \in C^Z$ if and only if $\Delta_i^{Z,0}(q, d_L^Z) > 0$ and $i \in C^Z$ implies that $j \in C^Z$ for all $j < i$. Moreover, given $C^Z = \{1, 2, ..., e\}$, $G_F^Z(q, d_L^Z)$ uniquely solves $\Delta_e^Z(q, G_F^Z, d_L^Z) = 0$.*

Lemma 2-1 reveals that donor i becomes a contributor only if the follower donors with higher wealth than i fall short of providing the necessary contribution (i.e. $G_i^{Z,0}(q, m\mathbb{1}_M) - g_L^Z$) to reach i's drop-out level, $G_i^{Z,0}(q, m\mathbb{1}_M)$. Moreover, the contribution incentives are

[7] Yildirim (2014) provides a weaker condition for a finite drop-out level under seed money of $\frac{d}{dg_i} u_i(0, G, q) \leq 0$ for some G. Instead, this condition ensures that provision is finite under both seed money and matching. I discuss the possibility of infinite drop-out levels in Subsection 2.6.3 in the presence of warm-glow motivations for giving.

[8] The proof of Lemma 2-1 is analogous to the proof of Proposition 2 in Yildirim (2014) and thus omitted here.

decreasing in i. Thus, deriving the equilibrium contributor set, C^Z, boils down to finding the highest i with $\Delta_i^0(q, d_L^Z) > 0$. Given C^Z, the equilibrium condition $\Delta_e(q, G_F^Z, d_L^Z) = 0$ requires that $G_F^Z(q, d_L^Z)$ eliminates any short-fall in contributions among C^Z, which precludes profitable deviation to higher giving by any of the contributing donors.

From eq. (2-4), it is evident that the size of the leadership gift has an impact on the follower donors' contributions. In particular, I focus on the effect of d_L^Z in a large economy, in which the set of contributing donors grows infinitely large. In particular, I let $\mathcal{D}_n$ denote the n-replica economy, in which the donor population of each wealth type w_i is replicated n times. Then, the following Proposition describes the equilibrium response to the leadership gift g_L^Z as n approaches infinity.

Proposition 2-1. *Let* $\epsilon_v(G) = \frac{-v''(G)G}{v'(G)}$. *Then, the follower donors' equilibrium response,* $G_F^Z(q, d_L^Z)$, *to* d_L^Z *for* $Z = \{S, M\}$ *is as follows:*

a) $G_F^S(q, g_L^S)$ *is strictly decreasing in* g_L^S *whenever the contributor's set is non-empty* ($C^S \neq \emptyset$), *while the total contributions* $G^{S,L}(q, g_L^S) = G_F^S(q, g_L^S) + g_L^S$ *are strictly increasing in* g_L^S. *Moreover,* $\lim_{n\to\infty} G^{S,L}(q, g_L^S) = G_1^{S,0}(q)$ *with* $\frac{dG_1^{S,0}(q)}{dg_L^S} = 0$.

b) $G_F^M(q, m)$ *is strictly increasing in* m *if and only if* $\epsilon_v(G^{M,L}) < 1$ *where* $G^{M,L}(q, m) = (1 + m)G_F^M(q, m)$ *denotes the total contributions. Moreover,* $G^{M,L}(q, m)$ *is strictly increasing in* m *and* $\lim_{n\to\infty} G^{M,L}(q, m) = G_1^{M,0}(q, m)$, *with* $G_1^{M,0}(q, 0) = G_1^{S,0}(q)$ *and* $\frac{dG_1^{M,0}(q,m)}{dm} > 0$.

Part a) of Proposition 2-1 highlights the free-riding incentives present under seed money. As pointed out by Andreoni (1988) and more recently by Yildirim (2014), these incentives are exacerbated in the limit economy with seed money converging to the highest drop-out level $G_1^{S,0}(q)$. Since $G_1^{S,0}(q)$ is independent of the size of the leadership gift g_L^S, seed money is ineffective at increasing total contributions in a large economy. In contrast, part b) reveals that the follower donors' response to increasing m is positive as long as the marginal value of the public good is inelastic to an increase in total contributions $G^{M,L}$, i.e. $\epsilon_v(G^{M,L}) < 1$. Intuitively, an increase in m has two opposing effects. On the positive side, m reduces the effective price of giving and as a result increases the follower donors' marginal willingness to contribute. On the negative side, higher m also increases $G^{M,L}$ for a fixed giving by the follower donors, G_F^M. This, in turn, reduces the marginal willingness to contribute due to the free-riding incentives. The inelastic response to increasing $G^{M,L}$ ensures that the positive effect dominates the negative.[9] However, irrespective of which effect dominates, the overall impact of higher m is an increase in total giving $G^{M,L}(q,m)$. Moreover, in contrast to seed money, $G^{M,L}(q,m)$ is responsive to higher match ratios even in the limit economy. This is because, as revealed by eq. (2-3), matching increases the individual drop-out levels, making donors more willing to become contributors. As a result, matching converges to strictly higher total contributions compared to seed money for any non-zero match ratio.

Turning to the lead donor's problem, d_L^Z is chosen to maximize

$$u_L(q,d_L^Z) = h\left(w_1 - G^{Z,L}(q,d_L^Z) + G_F^Z(q,d_L^Z)\right) + qv\left(G^{Z,L}(q,d_L^Z)\right) \tag{2-5}$$

Differentiating $u_L(q,d_L^Z)$ with respect to d_L^Z gives rise to the following marginal utility of giving:

$$\begin{aligned} \frac{du_L(q,d_L^Z)}{dd_L^Z} &= \left[-h'\left(w_1 - G^{Z,L}(q,d_L^Z) + G_F^Z(q,d_L^Z)\right) + qv'(G^{Z,L}(q,g_L^Z))\right]\frac{dG^{Z,L}(q,d_L^Z)}{dd_L^Z} + \\ &+ h'\left(w_1 - G^{Z,L}(q,d_L^Z) + G_F^Z(q,d_L^Z)\right)\frac{dG_F^Z(q,d_L^Z)}{dd_L^Z} \end{aligned} \tag{2-6}$$

Eq. (2-6) reveals that the follower donors' response to the leadership gift plays a crucial role in the lead donor's contribution choice. In particular, in a large economy, Proposition 2-1a) states that $\frac{dG^{S,L}(q,g_L^S)}{dg_L^S} \to 0$ as $n \to \infty$. Thus, the sign of $\frac{dG_F^S(q,g_L^S)}{dg_L^S}$ is the sole deter-

[9] Gong and Grundy (2014) show that if $\epsilon_v(G) > 1$ for some G, $G_F^M(q,m)$ may have an inverted U shape with donors reducing their donation amounts as a response to high match ratios. Consequently, the lead donor may settle for a low match. Then, in a finite economy, it is possible for matching to induce significantly lower leadership gift than seed money, resulting in lower overall donations under matching. This possibility, however, disappears in a large economy since, as stated by Propositions 2-1 and 2-2, seed money completely crowds out giving by the follower donors in the limit economy and thus is always less effective than matching in increasing public good provision.

minant of the lead donor's optimal contribution choice. Since Proposition 2-1a) reveals that $\frac{dG_F^S(q,g_L^S)}{dg_L^S} < 0$, it follows that $\lim_{n\to\infty} \frac{du_L(q,g_L^S)}{dg_L^S} < 0$ for all g_L^S. Thus, in a large economy, the lead donor has no incentives to contribute to the public good under seed money , i.e. $\lim_{n\to\infty} g_L^{S,*}(q) = 0$. This stands in contrast to the matching scheme, in which the lead donor's gift may encourage more giving by the follower donors. To see how this impacts the lead donor's willingness to give in a large economy, recall that the total contribution amount under M converges to $G_1^{M,0}(q,m)$. Moreover, since $G_1^{M,0}(q,0) = G_1^{S,0}(q)$ (by Proposition 2-1b)), for $m = 0$, eq. (2-6) reduces to

$$\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = \left[-h'(w_1) + qv'(G_1^{S,0}(q))\right] \frac{dG_1^{M,0}(q,0)}{dm} + h'(w_1) \lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm} \quad \text{(2-7)}$$

By eq. (2-3), the first term drops out, and $\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = h'(w_1) \lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm}$. Thus, the lead donor will always find it optimal to offer a positive match in a large economy as long as this induces higher giving by the follower donors. The following Proposition formalizes this finding.

Proposition 2-2. *In the limit economy ($n \to \infty$), the equilibrium total donations, $G_\infty^{Z,*}(q) = \lim_{n\to\infty} G^{Z,*}(q)$, satisfy $G_\infty^{M,*}(q) \geq G_\infty^{S,*}(q)$, with a strict inequality if $\epsilon_v(G_1^{S,0}(q)) < 1$.*

The sufficient condition provided by Proposition 2-2 implies that $\lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm} > 0$. Thus, the matching scheme outperforms seed money due to its ability to reduce free-riding incentives by the follower donors. Together, Propositions 2-1 and 2-2 reveal that under complete information about the charity's quality, the fundraiser is likely to favor the matching scheme. Therefore, to understand the use of seed money, I next turn to an environment with incomplete information about the charity's quality.

2.5 Unobservable Quality

In this subsection, I extend the analysis to the incomplete information game described in Subsection 2.3. For the remainder of the analysis, I maintain the assumption of an inelastic marginal value of the public good, i.e. $\epsilon_v(G_1^{S,0}(q)) < 1$ for all q, which guarantees the strict preference of matching over seed money in a large economy by every charity type in the absence of information asymmetry. I show that this preference causes seed money to emerge as a costly signal of quality in the presence of limited information about q.

In the last stage of the game, the follower donors make simultaneous donation decisions corresponding to $G_F^Z(q_{\mathcal{F}}^Z, d_L^Z)$, derived in Subsection 2.4, where $q_{\mathcal{F}}^Z$ denotes the follower donors' belief about the charity's quality. Given the asymmetric access to information, the lead donor's utility is given by

$$\overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{F}}^Z, d_L^Z) = h\left(w_L - G^{Z,L}(q_{\mathcal{F}}^Z, d_L^Z) + G_F^Z(q_{\mathcal{F}}^Z, d_L^Z)\right) + q_{\mathcal{L}}^Z v\left(G^{Z,L}(q_{\mathcal{F}}^Z, d_L^Z)\right) \tag{2-8}$$

The above equation captures the possibility that the lead donor and the follower donors hold asymmetric beliefs about the charity's quality. In a sequential equilibrium, donors' beliefs have to be consistent with the charity's contribution strategy Z, and the lead donor's donation decision d_L^Z. In particular, letting $\beta^Z(q_j)$ denote the probability that a charity of type q_j for $j \in \{l, h\}$ chooses scheme Z, the posterior belief of type q_j upon observing scheme Z, denoted by π_j^Z, satisfies Bayes' rule:

$$\pi_j^Z = \frac{\beta^Z(q_j)\pi_j}{\sum_{y\in\{l,h\}} \beta^Z(q_y)\pi_y} \tag{2-9}$$

Given π_j^Z, the expectation of quality in absence of any additional information is simply the posterior expected value $q_U^Z = \sum_j \pi_j^Z q_j$. However, the lead donor may also choose to learn the charity's true quality at a cost k. Therefore, the lead donor's belief $q_{\mathcal{L}}^Z$ can take one of three possible values- $\{q_l, q_U^Z, q_h\}$, denoting the cases of informed low quality, uninformed, and informed high quality, respectively. From the point of view of the follower donors, $q_{\mathcal{L}}^Z$ is the lead donor's type. Letting α^Z denote the lead donor's likelihood of information acquisition in scheme Z, the probability of type $q^Z_{\mathcal{L}}$, denoted by $\eta^Z_{\mathcal{L}}$, satisfies

$$\eta^Z_{\mathcal{L}} = \begin{cases} \pi^Z_{\mathcal{L}}\alpha^Z & \text{for } \mathcal{L} \neq U \\ 1-\alpha^Z & \text{for } \mathcal{L} = U \end{cases} \tag{2-10}$$

Note from eqs. (2-9) and (2-10) that whenever the two types of charities perfectly separate, such as $\beta^Z(q_l) = 1 - \beta^Z(q_h) = 1$, the scheme becomes perfectly informative with $q^Z_U = q_l$ and thus $q^Z_{\mathcal{L}} = q_l$ is independent of the information acquisition strategy of the lead donor. In this case, the lead donor's gift d^Z_L is not an essential tool of information transmission. In the spirit of sequential equilibrium, however, I consider equilibrium behavior that is consistent with the limit of fully mixed strategies. Therefore, the fundraising scheme choice always leaves some uncertainty about the charity's quality. As a result, the lead donor's gift d^Z_L has additional signaling value and can provide further information to the follower donors whenever the lead donor acquires information.

Turning to the choice of leadership gift d^Z_L, note that the lead donor's objective function, given by eq. (2-8), satisfies $\frac{\partial^2 \overline{u}_L(q^Z_{\mathcal{L}}, q^Z_{\mathcal{F}}, d^Z_L)}{\partial q^Z_{\mathcal{L}} \partial G^{Z,L}} = v'(G^{Z,L}) > 0$, which as shown in Lemma A-2 in Appendix A serves as the single crossing property that guarantees the existence of a separating equilibrium, in which the lead donor always reveals her type $q^Z_{\mathcal{L}}$ to the follower donors. In particular, I focus on the least costly (Riley) equilibrium, which is uniquely selected by the Cho-Kreps intuitive criterion (Cho and Kreps, 1987). The equilibrium contribution of the lead donor of type $q^Z_{\mathcal{L}}$ in the Riley equilibrium satisfies:[10]

$$\overline{d}^{Z,*}_L(q^Z_{\mathcal{L}}) = \underset{d^Z_L}{argmax} \quad \overline{u}_L(q^Z_{\mathcal{L}}, q^Z_{\mathcal{L}}, d^Z_L) \tag{2-11}$$

$$\text{s.t.} \quad \overline{u}_L(q_l, q_l, \overline{d}^{Z,*}_L(q_l)) \geq \overline{u}_L(q_l, q^Z_U, \overline{d}^{Z,*}_L(q^Z_U))$$

[10]Lemma A-2 shows that as typical in signaling games, the two constraints given by (2-11), ensuring no deviation incentives by the quality types q_l and q^Z_U of lead donor towards higher donation amounts, are the only ones that might bind in the least costly separating equilibrium.

$$\overline{u}_L(q_U^Z, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z)) \geq \overline{u}_L(q_U^Z, q_h, \overline{d}_L^{Z,*}(q_h))$$

Thus, each type $q_{\mathcal{L}}^Z$ chooses the contribution level that maximizes her utility subject to an incentive compatibility (IC) constraint ensuring that no lower quality type can profit from mimicking her contribution. As typical for the Riley solution, there is no distortion in the contribution level of the low quality type q_l. This, in turn, implies that the total contributions raised by a low quality charity coincides with the amount raised under complete information, i.e $\overline{G}^{Z,*}(q_l) = G^{Z,*}(q_l)$. Therefore, conditional on an informed lead donor, the low quality charity always raises more donations under matching. This comparison is less clear for the higher quality types, whose contribution amounts may be distorted towards higher contribution levels as a result of the IC constraints above. However, as pointed out by Andreoni (2006), in a large economy the equilibrium donations under seed money $\overline{G}^{S,*}(q_{\mathcal{L}})$ are bounded from above by the full information amount $G_1^{S,0}(q_{\mathcal{L}})$. By Lemma 2-1, any higher amount would turn all follower donors into non-contributors and thus cannot be sustained in equilibrium. This implies that in a large economy, the seed money contributions in the Riley equilibrium must necessarily fall below the matching contributions.[11] The following Lemma formalizes this statement.

Lemma 2-2. *For sufficiently large n,* $\overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$ *for* $j \in \{l, h\}$ *and* $\overline{G}^{M,*}(q_U^M) > \overline{G}^{S,*}(q_U^S)$ *for* $q_U^M \geq q_U^S$.

Lemma 2-2 establishes that in a large economy matching necessarily dominates seed money whenever the lead donor is informed about the quality or whenever matching is associated with (weakly) higher quality level. The last observation follows from the fact that the expected equilibrium contributions are increasing in the donors' belief about the

[11]It is worth pointing out that with a relatively small donor base, it is possible for contributions under seed money in the Riley equilibrium to exceed the ones under matching for a fixed donor type q, i.e. $\overline{G}^{S,*}(q) > \overline{G}^{M,*}(q)$. Intuitively, with costly quality signaling, a low type of lead donor may be more reluctant to pool with a higher type under matching since the resulting higher donation amounts by the follower donors also increase the lead donor's contribution through the match. This can make separation by the high type of lead donor less costly under matching compared to seed money and as a result lead to lower overall contributions. While this finding is consistent with the anecdotal evidence alluded to in Subsection 2.1, it is of limited scope and thus not the focus of this analysis.

charity's quality.[12] Note that the expected total contributions in scheme Z by a charity of quality q_j depend both on the expected quality q_U^Z as well as the expected likelihood of information acquisition and are given by

$$\overline{G}_E^Z(q_j, q_U^Z, \alpha^Z) = \alpha^Z \overline{G}^{Z,*}(q_j) + (1-\alpha^Z)\overline{G}^{Z,*}(q_U^Z) \tag{2-12}$$

Given the above equation, Lemma 2-2 implies that in order for seed money to be attractive for the charity, it must either be associated with a higher expected quality than matching or result in more favorable information acquisition strategy by the lead donor.

In order to understand the lead donor's information acquisition incentives, I next turn to the lead donor's value of information. In particular, let $\overline{u}_L^{Z,*}(q_{\mathcal{L}}^Z) = \overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{L}}^Z, \overline{d}_L^{Z,*}(q_{\mathcal{L}}^Z))$ denote the optimal utility of type $q_{\mathcal{L}}^Z$ from the contribution stage of the game. Anticipating this utility level, the value of informed giving for the lead donor is simply the difference between the expected informed and uninformed utility:

$$V_I^Z(\pi^Z) = \pi_h^Z \overline{u}_L^{Z,*}(q_h) + \pi_l^Z \overline{u}_L^{Z,*}(q_l) - \overline{u}_L^{Z,*}(q_U^Z) \tag{2-13}$$

The value of information depends crucially on the charity's equilibrium fundraising strategy through its effect on q_U^Z and π^Z. In particular, the following lemma points out that $V_I^Z(\pi^Z)$ is positive if and only if the two charity types (partially) pool in equilibrium, thus leaving the lead donor uncertain of the charity's quality.

Lemma 2-3. *$V_I^Z(\pi^Z)$ is continuous in $\pi_h^Z \in [0,1]$. Moreover, $V_I^Z(\pi^Z) = 0$ for $\pi_h^Z \in \{0,1\}$ and $V_I^Z(\pi^Z) > 0$ for all $\pi_h^Z \in (0,1)$.*

The value of information is always non-negative as more informed giving allows the lead donor to better tailor her contribution to the value of the public good. In the extreme case of the two charity types following a fully separating fundraising strategy, the fundraising scheme is perfectly informative, i.e. $\pi_j^Z = 1$ for some $j \in \{l, h\}$, rendering

[12] See Claim 1 in the proof of Lemma A-2.

information acquisition inconsequential. Such fully separating equilibrium, however, is rather incidental as it holds for very limited set of parameter values. Note that in such an equilibrium, it must be the case that the low quality charity chooses the matching scheme, while the high quality charity chooses seed money. Otherwise, if matching is a pure signal of high quality, by Lemma 2-2, $\overline{G}^{M,*}(q_h) > \overline{G}^{S,*}(q_l)$ and thus the low quality charity will for sure have incentives to mimic the high type and deviate to matching. Therefore, in a fully separating equilibrium, it must be the case that seed money is a pure signal of high quality. Moreover, in order to prevent deviation by either type of charity, the two schemes must raise the same amount of money ($\overline{G}^{S,*}(q_h) = \overline{G}^{M,*}(q_l)$). This makes the fully separating equilibrium rather incidental. However, the observation that both charity types must generate the same amount of equilibrium donations extends to any equilibrium with no information acquisition, as highlighted by the following Proposition.

Proposition 2-3. *(Fully uninformed equilibria) In every equilibrium with no information acquisition on the equilibrium path, i.e.,* $\alpha^{Z,*} = 0$ *for all* Z *with* $\sum_j \beta^{Z,*}(q_j) > 0$*, each scheme on the equilibrium path results in the same total donations and each charity raises the same amount of money.*

In the absence of information acquisition, the high quality charity is not able to effectively separate from the low quality charity. This is because the charity's payoff function does not satisfy the single crossing property and thus imitation by the low type is completely costless in this case. Consequently, the two charities will either pool on the same scheme or the two schemes would be equally attractive to prevent profitable deviation. Since this is not consistent with the experimental evidence alluded to in the Introduction, I instead focus on equilibria, in which information acquisition occurs with positive probability.

In order for information acquisition to take place, the value of information should be sufficiently high relative to the cost. In particular, if the value of information under matching at the prior distribution $V_I^M(\pi)$ exceeds the cost k, the other extreme case of fully informed equilibrium always exists. Such fully informed equilibrium, however, requires both charity types to pool on the matching scheme, as revealed by the following Proposition.

Proposition 2-4. *(A fully informed equilibrium) Fully informed equilibrium with* $\alpha^{Z,*} = 1$ *for all Z on the equilibrium path (i.e.* $\sum_j \beta^{Z,*}(q_j) > 0$*) exists if and only if* $V_I^M(\pi) \geq k$*. Moreover, the fully informed equilibrium is unique with* $\beta^{M,*}(q_j) = 1$ *for all* $j \in \{l, h\}$*,* $\alpha^{M,*} = 1$*, and* $\overline{G}^{M,*}(q_h) \geq G^{M,*}(q_h)$*.*

Proposition 2-4 is an immediate consequence of Lemma 2-2 that reveals the superiority of matching over seed money for a fixed quality level. Intuitively, as long as the lead donor obtains information with certainty, the high quality charity can fully rely on the lead donor to signal this quality to the follower donors through her donation choice. As a result, matching is preferred by both charity types since it incentivizes more giving. Interestingly, the amount of money raised by the high quality charity in equilibrium exceeds the amount raised under complete information ($\overline{G}^{M,*}(q_h) \geq G^{M,*}(q_h)$). This is because the lead donor's contribution is tailored to signal away from the low quality type, which may require a match that exceeds the one chosen under complete information. In this respect, limited quality transparency on the market can in fact benefit the high quality charity by increasing the lead donor's contribution amount.

Proposition 2-4 implies that the lead donor must have reduced incentives to acquire information in order for the high quality charity to find seed money attractive. However, Proposition 2-3 indicates that the other extreme of no information acquisition also does not provide strict incentives for seed money fundraising. Thus, I next turn to partial information acquisition. In particular, I focus on equilibria with partial information acquisition, in which seed money is on the equilibrium path[13]. I refer to such equilibria

[13] As typical for most signaling games, there is multiplicity of equilibria, including an equilibrium, in which seed money is off the equilibrium path due to very pessimistic beliefs about the charity's quality. For my purposes, however, the more relevant equilibria involve seed money being utilized by charities in equilibrium since it allows me to address the question of which type of charity is more likely to employ seed money fundraising.

as *SPI* (seed-partial info) equilibria. More formally, the likelihood of scheme Z emerging in equilibrium, $E[\beta^{Z,*}]$, and the corresponding expected likelihood of information acquisition, $E[\alpha^*]$, are given by

$$E[\beta^{Z,*}] = \pi_h \beta^{Z,*}(q_h) + (1-\pi_h)\beta^{Z,*}(q_l) \tag{2-14}$$

$$E[\alpha^*] = \sum_{Z\in\{S,M\}} E[\beta^{Z,*}]\alpha^{Z,*} \tag{2-15}$$

The following statement provides a formal definition of a *SPI* equilibrium.

Definition 2-1. *SPI equilibrium satisfies* $E[\beta^{S,*}] > 0$ *and* $E[\alpha^*] \in (0,1)$.

A *SPI* equilibrium requires both that seed money is chosen with positive probability by some quality type and that there is limited information acquisition on the equilibrium path. Note that limited information may arise as a result of randomization in the information acquisition strategy by the lead donor for a given scheme or the lead donor's asymmetric information acquisition strategy under the two schemes. The following Lemma provides sufficient conditions for the existence of a *SPI* equilibrium and some notable properties.

Lemma 2-4. *(Existence of a SPI equilibrium) A SPI equilibrium exists if* $V_I^S(\pi) \geq k$ *and* $\overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l)$*. Moreover, every SPI equilibrium satisfies 1)* $\beta^{S,*}(q_j) > 0$ *for all* $j \in \{l,h\}$*; 2)* $\alpha^{M,*} < 1$ *and* $\alpha^{S,*} \in (0,1)$*.*

Lemma 2-4 states that a *SPI* equilibrium exists as long as the cost of information is low relative to the value of information under seed money at the prior π (i.e, $V_I^S(\pi) \geq k$), and the prior expected quality is high enough so that the uninformed seed money fundraising at the prior is sufficiently attractive for the low type (i.e, $\overline{G}^S(E[q]) > \overline{G}^M(q_l)$). This is because, as stated by the first property, both charity types must be present in seed money. To understand the first property, note that the low type would never unilaterally choose seed money since it would perfectly reveal its quality. The high type, on the other hand, may find seed money attractive if it is perfectly revealing of its quality, but the resulting zero value of information and no quality verification by the lead donor, would make seed money also attractive for the low type. Thus, in equilibrium, both types need to utilize seed money, resulting in strictly positive value of information (Lemma 2-3).

Given the presence of both types in seed money, the second property in Lemma 2-4 requires that information acquisition is less than perfect under the matching scheme and that the lead donor strictly randomizes in her information acquisition strategy under seed money. Less than perfect information acquisition under matching ($\alpha^{M,*} < 1$) and some information acquisition under seed money ($\alpha^{S,*} > 0$) is necessary in order for the high type to consider seed money fundraising. In addition, limited information acquisition under seed money $\alpha^{S,*} < 1$ is required in order to make seed money attractive for the low type.

Lemma 2-4 establishes that with partial information acquisition, seed money cannot be a perfectly revealing signal of quality. Nevertheless, I are interested in how seed money compares to matching in conveying quality information to donors. The following Proposition delivers a sharp prediction by establishing that in any *SPI* equilibrium, seed money is a stronger signal of high quality compared to matching.

Proposition 2-5. *In every SPI equilibrium, the seed money scheme is associated with higher expected quality, i.e.* $q_U^{S,*} > q_U^{M,*}$, *and higher expected donations, i.e.* $E_j\left[\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})\right] > E_j\left[\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})\right]$, *for* $j \in \{l, h\}$.

Proposition 2-5 is consistent with the experimental evidence alluded to in the Introduction. It reveals that in every *SPI* equilibrium, seed money is associated with higher expectation of quality, which implies that it is chosen by the high quality charity more frequently that by the low quality charity. Intuitively, the attraction of seed money for the high quality charity is in its ability to signal the charity's quality more reliability. Thus, by eq. (2-12), seed money must be either associated with higher expected quality for the uninformed lead donor or induce more information acquisition by the lead donor relative to matching. However, if the benefit is coming purely from more information acquisition, such that $\alpha^{S,*} > \alpha^{M,*}$ and $q_U^{M,*} > q_U^{S,*}$, then the low quality charity would strictly prefer to fundraise for matching. This is because unlike the high type, the low type dislikes information acquisition and would find matching more attractive if it is less informative and associated with more optimistic belief regarding its type. Thus, a necessary condition for both types to find seed money attractive is for seed money to signal higher quality to donors.

An immediate consequence of the higher posterior belief under seed money (i.e. $\pi_h^{S,*} > \pi_h^{M,*}$) is that seed money raises higher expected donations relative to matching. To see this, note that since both charity types choose seed money with positive probability (by Lemma 2-4), it must be true that seed money generates at least as much expected contributions as matching for either type, i.e. $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. However, since these expected contributions are strictly increasing in the charity's quality, high contributions are more frequent under seed money relative to matching. As a result, the overall expected donations are higher under seed money.

In terms of the fundraising strategies, the *SPI* equilibrium is not unique. While both types need to be present in seed money (Lemma 2-4), this is not necessarily the case for the matching scheme. The possible equilibrium strategies vary with both types pooling on seed money, only the low type being present in matching, or each type being present in both schemes. The more interesting equilibria involve both schemes being on the equilibrium path. Thus, in the remainder of this subsection, I focus on characterizing this set of *SPI* equilibria.

For any equilibrium with strict mixing in information acquisition under Z, it must be the case that the value of information is equal to the cost. Let $(\hat{\pi}^S, \hat{\pi}^M)$ denote the pair of posterior beliefs that satisfy the following conditions:

Definition 2-2. *The set of posterior beliefs $(\hat{\pi}^S, \hat{\pi}^M)$ with corresponding expected qualities $(\hat{q}_U^S, \hat{q}_U^M)$*

satisfy:

$$C1: \quad V_I^Z(\hat{\pi}^Z) = k \quad \textit{for} \quad Z = \{S, M\}$$
$$C2: \quad \hat{\pi}_h^S > \hat{\pi}_h^M$$

In Appendix A, I show that as long as the value of information under the prior exceeds the cost for each scheme, i.e. $V_I^Z(\pi) \geq k$, there always exits a (non)degenerate strategy by the two types of charities that guarantees a pair of posterior beliefs that satisfy C1 and C2. Using this property, the following Proposition describes the equilibrium strategies by the two charities that emerge under a *SPI* equilibrium.

Proposition 2-6. *Consider SPI equilibria, in which M is on the equilibrium path.*

1) *If* $V_I^S(\pi) \geq k$ *and* $\overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l)$*, there exists an equilibrium with* $\beta^{S,*}(q_h) = 1$ *and* $\beta^{S,*}(q_l) \in (0,1)$ *satisfying* $V_I^S\left(\pi^{S,*}\right) = k$.

2) *If* $V_I^Z(\pi) > k$ *for all Z and* $\overline{G}^{S,*}(\hat{q}_U^S) > \overline{G}^{M,*}(\hat{q}_U^M)$*, there exists a fully non-degenerate equilibrium with*

$$\beta^{S,*}(q_h) = \frac{\hat{\pi}_h^S}{\pi_h}\frac{\pi_h - \hat{\pi}_h^M}{\hat{\pi}_h^S - \hat{\pi}_h^M}; \beta^{S,*}(q_l) = \frac{1-\hat{\pi}_h^S}{1-\pi_h}\frac{\pi_h - \hat{\pi}_h^M}{\hat{\pi}_h^S - \hat{\pi}_h^M} \tag{2-16}$$

where $0 < \beta^{S,*}(q_l) < \beta^{S,*}(q_h) < 1$.

Proposition 6 characterizes two types of equilibria. In the first one, only the low quality type chooses matching, making matching a sure signal of low quality, while both types are present in seed money. Note that in such an equilibrium, the low type of charity is indifferent between the two schemes and in equilibrium randomizes to make the lead donor indifferent in her information acquisition strategy under seed money. To guarantee the existence of such an equilibrium, the cost of information should be sufficiently low to ensure some information acquisition under seed money in equilibrium. Moreover, the low quality charity should raise significant donations under seed money when the lead donor is uniformed to compensate for the lower donations when she is informed.

In the second equilibrium, both charities are randomizing between matching and seed money. This equilibrium is important since it illustrates that both schemes could be used by the two charity types. Thus, neither schemes is perfectly informative, but rather in equilibrium the follower donors use both the fundraising scheme and the size of the lead donor's gift to infer information about the charity's quality. This equilibrium requires not only that seed money is sufficiently lucrative for the low type when the lead donor is uninformed, but also that uninformed donations raised under matching are low enough to make seed money an attractive option for the high type.

Overall, the analysis in this subsection illustrates that with costly information acquisition, seed money is likely used by the high quality charity to credibly signal its quality. More importantly, I illustrate that with both schemes being utilized in equilibrium, the seed money scheme is always indicative of a higher expected quality compared to the matching scheme. This is a rather strong result that provides a feasible explanation for the recent experimental findings. In the next subsection, I discuss a few extensions and variations of the model both to highlight the robustness of this finding and to inform how the signaling via the fundraising scheme is affected by factors such as the possibility of opting out of leadership fundraising, the presence of an alternative credible signal of quality, and warm-glow incentives for giving among donors.

2.6 Model Extensions and Variations

This subsection extends this model in multiple directions. Subsection 2.6.1 illustrates that the role of seed money as a signal of higher quality extends to an arbitrary finite quality distribution. Subsection 2.6.2 studies the impact of expanding the set of scheme choices by the charity to allow for no leadership giving, while Subsection 2.6.3 studies the impact of warm-glow motivations for giving on the signaling role of the fundraising scheme. Both extensions illustrate the robustness of my results to richer environments.

Subsection 2.6.4 studies the impact of an alternative information source to donors. It establishes that the presence of such information decreases the lead donor's value of information under seed money, which in turn reduces the possibility of *SPI* equilibria.

2.6.1 Multiple Quality Types

Consider an extension of the base model to finite qualities where $q \in \{q_1, q_2, ..., q_t\}$ with $t > 2$ and $q_{j-1} < q_j$ for all $j \in (2, t]$. The corresponding distribution of types $\pi = (\pi_1, \pi_2, ..., \pi_t)$ denotes the likelihood of each type prior to any action being taken by the players. The information structure and timing of the game is identical to the base model.

Analogous to the base model, the lead donor's type $q^Z_{\mathcal{L}} \in \{q_1, ...q_j, q^Z_U, q_{j+1}, ..., q_t\}$ can take $t+1$ values as it includes the possibility of the lead donor choosing to remain uninformed, where her type is the expected quality $q^Z_U = \sum_{j=1}^t \pi^Z_j q_j$. Given the probability of information acquisition, α^Z, and letting $\mathcal{L} = \{1, 2, ..., t\} \cup \{U\}$, the prior belief, $\eta^Z_{\mathcal{L}}$ is given by eq. (2-10).

Similar to the two type case, in the least costly separating equilibrium the lead donor's contribution amount is perfectly informative of her type with $\overline{G}^M(q_{\mathcal{L}}) > \overline{G}^S(q_{\mathcal{L}})$ for all $q_{\mathcal{L}}$ in a large economy. The corresponding value of information is

$$V^Z_I(\pi^Z) = \sum_{j=1}^{t} \pi^Z_j \overline{u}^{Z,*}_L(q_j) - \overline{u}^{Z,*}_L(q^Z_U) \tag{2-17}$$

It is straightforward to verify that the fully informed equilibrium exists as long as $V^M_I(\pi) \geq k$ and necessitates pooling on matching. The other extreme of fully uninformed equilibrium requires each charity and each scheme on the equilibrium path to raise the same amount of money. Thus, similar to the two-type case, I focus my analysis on *SPI* equilibria defined by Definition 2-1. The following Lemma states that in any *SPI* equilibrium, information acquisition has to be limited under matching and positive under seed money to induce seed money fundraising by some charity types.

Lemma 2-5. *Every SPI equilibrium satisfies 1) $\alpha^{M,*} < 1$ and $\alpha^{S,*} > 0$; 2) $\pi_j^{S,*} < 1$ for all $j \in \{1, ..., t\}$.*

Limited information acquisition under matching ($\alpha^{M,*} < 1$) is necessary to prevent unraveling, in which each charity type deviates to matching. To see this, note that with full information under matching, total donations under matching must exceed total donations under seed money for each charity with above average seed money quality, $q_{\mathcal{L}} > q_U^{S,*}$, since $\overline{G}^{M,*}(q_{\mathcal{L}}) > \overline{G}^{S,*}(q_{\mathcal{L}}) > \overline{G}^{S,*}(q_U^{S,*})$. Intuitively, a charity is willing to solicit for seed money only if it generates more favorable beliefs about its type under seed money. This implies that any charity above the average quality $q_U^{S,*}$ would prefer to avoid seed money. This would reduce the expected quality under seed money, causing further unraveling, in which all charities gravitate towards matching. Thus, to induce seed money fundraising, matching should be associated with less than perfect information acquisition.

Similar dynamics as the one described above would take place if there is no information acquisition under seed money. Then, by the definition of a *SPI* equilibrium, $\alpha^{M,*} > 0$. Thus, the expected giving under matching $\overline{G}_E^{M,*}(q_{\mathcal{L}}, q_U^{M,*}, \alpha^{M,*})$ is strictly increasing in $q_{\mathcal{L}}$, while the expected giving under seed money, $\overline{G}^{S,*}(q_U^{S,*})$, is uniform across the charities. This implies that the highest quality types would choose matching and thus the expected quality under matching, $q_U^{M,*}$, should exceed the one under seed money, $q_U^{S,*}$. Consequently, any type $q_{\mathcal{L}} > q_U^{S,*}$ would have strict incentives to deviate to matching, further reducing $q_U^{S,*}$ and causing all charity types to gravitate towards matching. Thus, some information acquisition under seed money ($\alpha^{S,*} > 0$) is necessary to make seed money fundraising attractive.

The second property in Lemma 2-5 follows immediately from the first one. In order for information acquisition to take place under seed money, it must be the case that the value of information is positive, which necessitates (partial) pooling, i.e. $\pi_j^{S,*} < 1$. Even though seed money is only partially informative about the charity's quality in equilibrium, the following Proposition states that it is associated with higher expected quality relative to matching.

Proposition 2-7. *In every SPI equilibrium, the seed money scheme is associated with higher expected quality, i.e.,* $q_U^{S,*} > q_U^{M,*}$.

Proposition 2-7 generalizes the main result by showing that for arbitrary discrete distribution of types, seed money on the equilibrium path must be associated with higher expected quality relative to matching. To glean more insight into the equilibrium forces that drive this result, note that the highest type present in seed money $\overline{q}^S > q_U^{S,*}$ must necessarily exceed the expected quality under match, i.e. $\overline{q}^S > q_U^{M,*}$ to prevent $\overline{q}^S$ from deviating to matching[14]. Moreover, $\overline{q}^S$ finds seed money attractive either because it leads to higher uninformed giving, implying $q_U^{S,*} > q_U^{M,*}$, or has an informational advantage over matching, $\alpha^{S,*} > \alpha^{M,*}$. However, if the advantage is coming purely from information acquisition, then the lowest type under seed money $\underline{q}^S$ must have strict incentives to deviate to matching. To see this, note that by definition $\underline{q}^S < q_U^{S,*} < q_U^{M,*}$, implying that information acquisition is never good news for $\underline{q}^S$. Thus, matching would be a more attractive option for $\underline{q}^S$ as it is both less informative and associated with more optimistic beliefs about its type. This shows that $q_U^{S,*} > q_U^{M,*}$ is necessary to prevent deviation by both the highest ($\overline{q}^S$) and the lowest ($\underline{q}^S$) type under seed money.

Unlike the two-types case, characterizing the full set of *SPI* equilibria can be challenging. In the two-types model, the higher expected quality under seed requires that the high quality charity chooses seed money more often than the low quality charity. Thus, the likelihood of choosing seed money has to be monotonically increasing quality. This monotonic relationship no longer needs to hold with multiple quality types as the following example illustrates.[15]

[14]Note that if $q_U^{M,*} > \overline{q}^S > q_U^{S,*}$, then $\overline{G}^{M,*}(q_U^{M,*}) > \overline{G}^{M,*}(\overline{q}^S) > \overline{G}^{S,*}(\overline{q}^S) > \overline{G}^{S,*}(q_U^{S,*})$.

[15]Proposition 2-7 establishes that the equilibrium posterior distribution under seed money $\pi^{S,*}$ second-order scholastically dominates the one under matching, $\pi^{M,*}$. With two types, second-order stochastic dominance implies first-order stochastic dominance as well, which in turn requires a monotonically increasing relationship between the charity's quality and the likelihood of seed money. With multiple types, second-order stochastic dominance does not require such monotonic relationship.

Example: Let $u_i(g_i, G, q) = (w_i - g_i)^{0.9} + qG^{0.1}$ with $w_1 = 2000$, and $q \in \{50, 500, 600, 700, 2000\}$ with $\pi = (0.65, 0.1, 0.05, 0.05, 0.15)$. For $k = 17.67$, the following strategies constitute a sequential equilibrium in the limit economy:

$$\beta^{S,*}(q) = (0,1,0,0,1),\ \alpha^{S,*} = 1,\ \alpha^{M,*} = 0.48$$

To verify this equilibrium, note that the posterior beliefs and expected qualities are:

$$\pi^{S,*} = (0, \frac{4}{10}, 0, 0, \frac{6}{10}) \quad , \quad \pi^{M,*} = (\frac{13}{15}, 0, \frac{1}{15}, \frac{1}{15}, 0) \quad , \quad q_U^{S,*} = 1400 \quad , \quad q_U^{M,*} = 130$$

To derive the equilibrium contributions, note that by Proposition 2-1 and eq. (2-3), $G^{S,L}(q_{\mathcal{F}}, d_L^S) = \left(\frac{.9w_1^{-.1}}{.1q_{\mathcal{F}}}\right)^{\frac{1}{-.9}}$ and $G^{M,L}(q_{\mathcal{F}}, d_L^M) = \left(\frac{.9w_1^{-.1}}{.1(m+1)q_{\mathcal{F}}}\right)^{\frac{1}{-.9}}$. The lead donor's contribution for each type $q_{\mathcal{L}}$, $\overline{d}_L^{Z,*}(q_{\mathcal{L}})$, maximizes $\overline{u}_L(q_{\mathcal{L}}, q_{\mathcal{L}}, d_L^Z)$ s.t. $\overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}-1}, \overline{d}_L^{Z,*}(q_{\mathcal{L}-1})) \geq \overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}}, \overline{d}_L^{Z,*}(\mathcal{L}))$, where $q_{\mathcal{L}-1}$ denotes the highest type below $q_{\mathcal{L}}$.[16] The numeric solution to this constrained optimization problem is[17]:

q_j	$\overline{g}_L^{S,*}(q_j)$	$\overline{G}^{S,*}(q_j)$	$\overline{m}^*(q_j)$	$\overline{G}^{M,*}(q_j)$	$\overline{G}_E^{S,*}(q_j)$	$\overline{G}_E^{M,*}(q_j)$
50	0	15.64	0.10	17.39	15.64	46.61
500	45.55	202.01	0.56	331.09	202.01	197.19
600	86.76	247.37	0.70	446.07	247.37	252.38
700	129.26	293.58	0.77	553.68	293.58	304.03
2000	610.54	942.59	0.78	1788.82	942.59	896.90
$q_U^{S,*}$	361.23	634.18	–	–	–	–
$q_U^{M,*}$	–	–	0.55	73.59	–	–

where $\overline{G}^{Z,*}(q_j) = G^{Z,L}(q_{\mathcal{F}}, \overline{d}_L^{Z,*}(q_j))$ and $\overline{G}_E^{Z,*}(q_j)$ is the equilibrium expected giving defined by eq. (2-12). Comparing $\overline{G}_E^{S,*}(q_j)$ and $\overline{G}_E^{M,*}(q_j)$ establishes $\beta^{S,*}(q) = (0,1,0,0,1)$.

[16]Lemma A-2 establishes that $\overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}-1}, \overline{d}_L^{Z,*}(q_{\mathcal{L}-1})) \geq \overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}}, \overline{d}_L^{Z,*}(\mathcal{L}))$ is the only constraint that might be binding at the optimum.

[17]This numeric solution allows for lump sum donations and match ratios to be multiples of 0.01.

Moreover, by eq. (2-17) $V_I^S(\pi^{S,*}) = 40.25 > V_I^M(\pi^{M,*}) = 17.67 = k$. Thus, the lead donor has no incentive to deviate from $\alpha^{S,*} = 1$ and $\alpha^{M,*} = 0.48$.

2.6.2 Opting Out of Leadership Giving

So far, I have assumed that the charity always chooses to reveal the lead donor's gift and thus the only decision that the charity makes is whether to ask for seed or matching leadership gift. One may wonder how the relative appeal of the two leadership schemes may change if I allow the charity to opt out of leadership giving completely. In the spirit of Vesterlund (2003), suppose that the charity can commit not to announce (N) the lead donor's contribution and instead to solicit each donor for an unconditional gift. This turns the contribution game into a simultaneous game, precluding the possibility of signaling by the lead donor and leaving the scheme choice as the only possible source of information.

It is important to note that unlike Vesterlund (2003), who allows the lead donor to donate multiple times, I model the lead donor's decision as a one-time contribution. However, this distinction becomes immaterial in a large economy. As pointed out by Vesterlund (2003), under symmetric quality information, sequential and simultaneous contributions raise the same amount of money if the lead donor is allowed to contribute multiple times. This equivalence also holds in the limit economy with purely altruistic donors. This is because, as pointed out by Proposition 2-1, the lead donors' seed money gift is completely crowded out in the limit, making seed leadership giving inconsequential under complete information. Thus, in the limit economy, the main distinction between seed money and non-announcement must come from the quality information conveyed to donors. The analysis in this subsection focuses on this case.[18]

The equivalence of seed money and non-announcement under complete information implies that matching is still the dominant scheme. Under endogenous information ac-

[18] While for the sake of brevity I focus on the limit economy, similar to the base model, it can be shown that my main insights hold in the case of a finite, but large economy.

quisition, the comparison of the three schemes is less clear since the use of a leadership scheme does not guarantee informed contributions. Similar to the main model, in the absence of information acquisition, the two charity types must raise the same amount of money under any scheme on the equilibrium path since the lack of verification makes it costless for the low type to mimic the high type. Interestingly, however, fully informed equilibrium, in which the lead donor acquires information with probability one, no longer guarantees the use of matching. Recall from Subsection 2.5 that the low quality charity favors matching over seed money if his type is fully revealed in equilibrium. Non-announcement, however, provides means for the low quality charity to pool with the high even if the lead donor chooses to acquire information. The high quality charity may also favor non-announcement if matching is associated with sufficiently pessimistic beliefs about the charity's quality, forcing it off the equilibrium path. Nevertheless, a fully informed equilibrium precludes the possibility of seed money as stated by the following Lemma.

Lemma 2-6. *In the limit economy ($n \to \infty$), any fully informed equilibrium (i.e., $\alpha^{Z,*} = 1$ for all Z on the equilibrium path) requires that seed money is chosen with zero probability (i.e. $\beta^{S,*}(q_j) = 0$ for all $j = \{l, h\}$). Moreover, the two types of charity pool either on matching ($\beta^{M,*}(q_j) = 1$ for all $j = \{l, h\}$) or non-announcement ($\beta^{N,*}(q_j) = 1$ for all $j = \{l, h\}$).*

The intuition behind Proposition 2-6 is straightforward. Given an informed lead donor, matching always dominates seed money for the low charity type. Thus, seed money can be chosen only by the high type, which in turn results in no verification under seed money. Therefore, fully informed equilibrium precludes the use of seed money.[19]

The second part of Lemma 2-6 rules out the possibility of both non-announcement and matching being on the equilibrium path at the same time in a fully informed equilibrium.

[19] Lemma 2-6 stands in contrast to Vesterlund (2003), who shows the existence of an equilibrium, in which seed money results in full information acquisition by the lead donor. The possibility of matching and the fact that the low quality is non-zero ($q_l > 0$) precludes such equilibrium in my setting since the inability of the low quality charity to pool with the high under seed money makes matching strictly more attractive for the low quality charity.

This is because, as shown in Lemma A-3 in Appendix A, the contribution amount raised by a high quality charity in the limit economy under non-announcement, $\overline{G}_{\infty}^{N,*}(q_h, q_U^N, \alpha^N)$, is bounded above by the corresponding seed money contributions, $\overline{G}_{\infty}^{S,*}(q_h)$, due to the lead donor's inability to signal the charity's quality under non-announcement. Moreover, from Proposition 2-2, I know that matching performs strictly better than seed money in a fully informed equilibrium for any quality type. It follows that the high quality charity would also strictly prefer matching over non-announcement if matching is on the equilibrium path and results in full verification. This, in turn, implies that non-announcement has to be associated with low quality, making it unattractive for the low quality charity as well. Clearly, both types choosing the matching scheme under full verification is sustainable with off-equilibrium belief that non-announcement and seed money are associated with low quality. Both types choosing non-announcement with off-equilibrium belief of low quality under matching and seed is also sustainable as long as the low quality charity raises more money by pooling with the high type under non-announcement than getting the low quality contributions under matching, i.e. $\overline{G}_{\infty}^{N,*}(q_l, q_U, 1) > \overline{G}_{\infty}^{M,*}(q_l)$.[20]

Similar to the main model, Lemma 2-6 implies that seed money should be associated with partial information acquisition in order to attract both charity types. The following Proposition establishes the possibility that seed money conveys the strongest signal of quality in equilibrium.

Proposition 2-8. *Every SPI equilibrium satisfies $q_U^{S,*} > q_U^{M,*}$. Moreover, if $V_I^S(\pi) \geq k$ and $\overline{G}_{\infty}^{S,*}(E[q]) > \overline{G}_{\infty}^{M,*}(q_l)$, there exists a SPI equilibrium, in which seed money is associated with the highest expected quality, $q_U^{S,*} > \max\{q_U^{M,*}, q_U^{N,*}\}$.*

The first part of Proposition 2-8 generalizes my main finding and establishes that the presence of non-announcement does not impact the relative quality comparison of seed

[20]In Lemma A-3, I show that $\overline{G}_{\infty}^{N,*}(q_l, q_U^N, \alpha^N) > \overline{G}_{\infty}^{S,*}(q_l)$ for $q_U^N > q_l$ since non-announcement allows the low quality charity to conceal their quality from downstream donors. This makes it possible for the low quality charity to raise more money under non-announcement relative to matching in an incomplete information setting.

money and matching gift. This is intuitive in light of the earlier discussion. The second part of the Proposition establishes the possibility that seed money is also a stronger quality signal relative to non-announcement. In fact, an equilibrium, in which non-announcement is off the equilibrium path and construed as a signal of low quality clearly meets this description. However, the comparison between non-announcement and seed money is less clear-cut and similar to Vesterlund (2003), I cannot rule out the existence of equilibria, in which non-announcement is a signal of higher quality than seed money. Such equilibrium requires that non-announcement emerges as the highest quality signal since $q_U^{S,*} > q_U^{M,*}$ holds in every *SPI* equilibrium. Since $\overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) < \overline{G}_\infty^{S,*}(q_h)$ for $q_U^N < q_h$ (see Lemma A-3), it also requires that seed money induces little verification, preventing the high quality charity from generating significant separation from the low quality charity under seed money. Intuitively, the only advantage of seed money over non-announcement in a large economy is in its signaling potential through the lead donor's gift. Thus, the lack of significant verification on the lead donor's part would remove this advantage of seed money over non-announcement, opening the possibility for non-announcement to emerge as a higher quality signal.[21]

2.6.3 Warm-glow Giving

So far, I have assumed that donors' giving is driven purely by altruistic motives. However, as contended by Andreoni (1988, 1990) many settings involve donors that exhibit "impure altruism." Thus, in this subsection, I incorporate warm-glow motives as well and show that my main findings extend as long as the altruistic motives for giving are suf-

[21] While I cannot rule out this possibility, constructing such an equilibrium is challenging. Similar to seed money, it is easy to see that non-announcement cannot be perfectly informative in equilibrium, implying that the two schemes are equally attractive for both quality types, i.e. $\overline{G}_\infty^N(q_j, q_U^{N,*}, \alpha^{N,*}) = \overline{G}_{E,\infty}^S(q_j, q_U^{S,*}, \alpha^{S,*})$ for $j = \{l, h\}$. This, in turn, requires that $\alpha^{N,*}[\overline{G}_\infty^{N,*}(q_h, q_U^{N,*}, \alpha^{N,*}) - \overline{G}_\infty^{N,*}(q_l, q_U^{N,*}, \alpha^{N,*})] = \alpha^{S,*}[\overline{G}_\infty^{S,*}(q_h) - \overline{G}_\infty^{S,*}(q_l)]$. As revealed by Lemma A-3, the gap in informed contributions between the high and the low quality charity is lower under non-announcement as it precludes information transmission to downstream donors. As a result, non-announcement should result in significantly more information acquisition than seed money. Investigating this possibility numerically using a CES utility function for donors' preferences reveals that this requires very low expected quality under seed money, making matching more attractive than seed money for the low quality charity. This, in turn, causes the *SPI* equilibrium to fail.

ficiently strong to engender finite drop-out levels for all wealth "types" in the economy. To illustrate this point, consider the following generalization of donors' preferences:

$$\tilde{u}_i(g_i, G, q) = h(w_i - g_i) + q\tilde{v}(G, g_i) \tag{2-18}$$

where $\tilde{v}_G(\cdot) > 0, \tilde{v}_{GG}(\cdot) < 0, \tilde{v}_g(\cdot) > 0$, and $\tilde{v}_{gg}(\cdot) < 0$. Thus, donor i cares not only about the total public good provision, but also about being the one providing it. I further assume that $q\tilde{v}_G(0,0) > h'(w_1)$, ensuring positive equilibrium provision; $\tilde{v}_{Gg}(\cdot) + \tilde{v}_{gg}(\cdot) < 0$, ensuring the uniqueness of the follower donors' best response function; and $\tilde{v}_{GG}(\cdot) + \tilde{v}_{gG}(\cdot) < 0$, ensuring downward sloping reaction functions, which in turn guarantees the uniqueness of the equilibrium contributions. Consistent with my main model, I continue to assume that $\lim_{G \to \infty} q\tilde{v}_G(G, 0) = 0$, which implies that altruistic motives for giving eventually vanish as provision grows[22]. Then, under common belief about q, donor i's optimal contribution amount solves:

$$h'(w_i - g_i^Z) = q\tilde{v}_G(G^Z, g_i^Z)\,(1 + m\mathbb{1}_M) + q\tilde{v}_g(G^Z, g_i^Z) \tag{2-19}$$

The right-hand side of eq. (2-19) is the sum of the marginal benefit of contributing to the public good due to altruism and warm-glow, respectively. Interestingly, only the former depends directly on the match ratio since the matching contributions are given by the lead donor and thus do not induce any additional warm-glow for the follower donors. Therefore, from the above equation, it is evident that the difference between the followers' response to matching and seed money, which is at the core of my analysis in Subsection 2.5, is driven entirely by altruistic considerations. Consequently, in order for matching to outperform seed money in a large economy, it is necessary for altruism to outweigh warm-glow considerations by donors, resulting in a finite contribution amount in the limit economy. The following Proposition provides sufficient conditions for matching to

[22]Recall from eq. (2-3), that this ensures finite drop-out level in absence of warm-glow.

strictly outperform seed money in the presence of warm-glow.

Proposition 2-9. *Given publicly observable quality q, in the limit economy (i.e., $n \to \infty$) total contributions, $\lim_{n\to\infty} \tilde{G}^{Z,*}(q) = \tilde{G}^{Z,*}_{\infty}(q)$, satisfy:*

a) $\tilde{G}^{S,*}_{\infty}(q) \leq \tilde{G}^{M,*}_{\infty}(q) < \infty$ if $\lim_{G\to\infty} q\tilde{v}_g(G,0) < h'(w_1)$.[23] *Moreover, the first inequality is strict if the elasticity $\epsilon_{\tilde{v}}(\tilde{G}^{S,0}_1(q),0) < 1$.*[24]

b) $\tilde{G}^{S,*}_{\infty}(q) = \tilde{G}^{M,*}_{\infty}(q) = \infty$ if $\lim_{G\to\infty} q\tilde{v}_g(G,0) \geq h'(w_1)$.

Part a) of Proposition 2-9 provides a sufficient condition for a finite drop-out level under both schemes. It requires that the warm-glow contribution incentives are weaker than the benefit of private consumption at high levels of public good provision. To see the necessity of this condition, recall that the drop-out level $\tilde{G}^{Z,0}_i(q, m\mathbb{1}_M)$ solves

$$q\tilde{v}_G(\tilde{G}^{Z,0}_i,0)\,(1+m\mathbb{1}_M) + q\tilde{v}_g(\tilde{G}^{Z,0}_i,0) - h'(w_i) = 0 \tag{2-20}$$

Since the altruistic motives for giving vanish as the public good provision grows, i.e. $\underset{G\to\infty}{lim}\, q\tilde{v}_G(G,0) = 0$, in order for contributions to converge to a finite level, it must be the case that the warm-glow motives for giving are also weak to dissuade giving at large levels of public good provision. Then, analogous to Subsection 2.4, finite provision in the limit economy implies that matching outperforms seed money. The comparison between the two schemes is strict as long as the lead donor finds it optimal to provide a positive match in the limit economy, which is guaranteed by the last condition in part (a) of Proposition 2-9. This strict preference for matching by the charity, in turn, sets the stage for the charity to use seed money as a costly signal of high quality. In fact, it is straightforward to verify that the analysis in Subsection 2.5 generalizes to preferences that satisfy the conditions outlined in part a).

[23] Yildirim (2014) shows that this condition is equivalent to downward sloping reaction functions in the limit economy under simultaneous contributions.

[24] Recall that the elasticity of the marginal value $\tilde{v}_G(G,g)$ is $\epsilon_{\tilde{v}}(G,g) = -\frac{(\tilde{v}_{GG}(G,g)+\tilde{v}_{gG}(G,g))G}{\tilde{v}_G(G,g)}$.

Part b) of Proposition 2-9 reveals that in the presence of strong warm-glow motivations, public good provision grows infinitely large as the donor population increases. This occurs because some wealthy donor types, driven purely by warm-glow considerations, give strictly positive individual contributions in the limit economy. As a result, total contributions increase without bound as the donor population grows. This has important implications for the charity's signaling incentives. In particular, with warm-glow as the only driving force, the form of the leadership gift does not have any effect on the followers' giving motives. Thus, matching and seed money become equivalent in a large economy and end up raising the same amount of funds. This makes signaling by the charity obsolete and thus seed money is as likely to be a signal of high quality as matching. Overall, Proposition 2-9 reveals that altruistic motives for giving play an important role in incentivizing quality signaling via the fundraising scheme in a large economy.

2.6.4 Alternative Information Source

In the base model, I assume that the scheme and the lead donor's donation amount are the only possible sources of information for donors. This stark case aims to isolate the informational impact of the scheme from other possible sources of information. In this subsection, I briefly consider the impact of alternative information sources. To capture this possibility in a simple framework, suppose that there is an alternative information channel, which is successful in reaching downstream donors with probability γ. Suppose also that the realization of γ occurs after the information acquisition decision by the lead donor.[25]

Recall from Subsection 2.5 that the *SPI* equilibrium requires patial information acquisition, which in turn implies that the value of information under seed money should

[25] Alternatively, if γ is realized prior to the lead donor's information acquisition and contribution decisions, the two subgames that start after the realization of γ would correspond to either the complete information game described in Subsection 2.4 or the incomplete information game described in Subsection 2.5. Thus, the probability of seed money fundraising will trivially decrease as γ increases since, as shown in Subsection 2.4, the charity has strict preference for matching when donors are exogenously informed about the charity's quality.

equal its cost, i.e. $V_I^S(\pi^{S,*}) = k$. However, the alternative information source available to donors should intuitively reduce the value of information for the lead donor, as it reduces the lead donors' need to signal the charity's quality to the follower donors. This is particularly salient in the limit economy, in which the lead donor's sole purpose for information acquisition under seed money is to transmit this information to the follower donors. To see this, recall from Proposition 2-1 that under seed money, any contribution amount by the lead donor is fully crowded-out, implying that the total money raised, $G_1^{S,0}(q_{\mathcal{F}})$, is only a function of the follower donors' belief about the charity's quality. Thus, given the lead donor's fully separating contribution strategy, $\overline{g}_{L,\infty}^{S,*}(q_{\mathcal{L}})$, her informed and uninformed equilibrium utilities in the limit economy are given by

$$\overline{u}_{L,\infty}^S(q_j,\gamma) = h(w_1 - \overline{g}_{L,\infty}^{S,*}(q_j)) + q_j v(G_1^{S,0}(q_j)), \tag{2-21}$$

$$\begin{aligned} \overline{u}_{L,\infty}^S(q_U^S,\gamma) &= h(w_1 - \overline{g}_{L,\infty}^{S,*}(q_U^S)) + (1-\gamma) q_U^S v(G_1^{S,0}(q_U^S)) + \\ &+ \gamma(\pi_h^S q_h v(G_1^{S,0}(q_h)) + \pi_l^S q_l v(G_1^{S,0}(q_l))). \end{aligned} \tag{2-22}$$

The above utilities capture the fact that the lead donor's gift in the large economy affects total contributions only through its impact on the follower donors' beliefs. Moreover, the uninformed lead donor's impact on total contributions is reduced by the presence of an exogenous information channel. Taking into account the binding incentive constraints for the low and the uninformed types of lead donor given by eq. (2-11), I arrive at the following observation.

Proposition 2-10. *The value of information in the limit economy under seed money is given by* $V_{I,\infty}^S(\pi^S,\gamma) = (1-\gamma)\left(q_h - q_U^S\right)\left(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S))\right)\pi_h^S$, *which is strictly decreasing in* γ.

Proposition 2-10 is a direct consequence of the lead donor's reduced value of signaling as the follower donors become more informed. This, in turn, implies that *SPI* equilibria, which require positive information value by the lead donor, become harder to sustain with more informed donor population. Moreover, the increased possibility of informed giving as a result of the alternative information channel will tilt the charity's preference in favor of the matching scheme. Consequently, consistent with my intuition, the matching scheme should become more prevalent as the reliability of the alternative information channels increases.

2.7 Chapter Summary

My analysis provides a theoretical foundation for understanding the recent empirical findings in favor of seed money fundraising. It suggests that seed money can be used as a signaling tool for high quality charities to differentiate themselves from lower quality charities. This result is rather robust since seed money emerges as a signal of higher quality in every equilibrium, in which it is utilized with positive probability and features some information acquisition by the lead donor. I show that this finding continues to hold in richer environments that include arbitrary finite number of types, the possibility of simultaneous fundraising, and the presence of warm-glow motivations for giving.

3. HOW TO INTERPRET THE COMMERCIALIZATION OF MICROFINANCE: MISSION DRIFT OF MISSION DIVISION?

3.1 Overview

"...the future of microfinance is unlikely to follow a single path... Commercial investment is necessary to fund the continued expansion of microfinance, but institutions with strong social missions, many taking advantage of subsidies, remain best placed to reach and serve the poorest customers..."

Cull et al. (2009, p. 169)

This section focuses on the microfinance market, which was once limited to a few non-profits, but has witnessed a surge of for-profit MFIs (microfinance institutions) over the last two decades. This "commercialization," trend has triggered much debate centered on the future direction of the industry. While some have interpreted for-profits' entry as a sign of the industry's health and success, others have expressed a concern that commercialization leads to profit-seeking and "mission drift." The latter term refers to a diversion of microfinance away from its original mission of alleviating poverty. By exploring an often overlooked aspect of microfinance, the funding process, I offer a novel theoretical explanation for this polarization.

My model is motivated by two empirical observations. First, studies such as Cull et al. (2007, 2011) find that commercialization correlates negatively with measures of outreach to the poor. Second, evidence such as Gonzalez (2007); Husain and Pistelli (2016), reveal that microfinance costs are increasing in poverty. In other words, non-profit MFIs tend to serve customers that are poorer and more costly than those served by more commercial MFIs. These findings point to the fact that social investors fund two distinct types of microfinance: one that targets extreme poverty and is typically non-profit, and another that is more commercial and has lower costs. Ghosh and Van Tassel (2013) present a theory that explains the investors' support of the latter type. They focus on the agency problem of cost unobservability and show that high interest rates charged by social investors filter high-cost institutions out of the market. However, this theory does not explain why investors would focus on low-cost MFIs, given that they come at the expense of MFIs that target extreme poverty. This theory also cannot explain why the latter MFIs persist in the market despite high costs.

The above observations suggest that each type of MFI, i.e., non-profit and for-profit, appeals to a different group of investors with varying degrees of concern about targeting the poorest within the poor. I present a model that captures this variation by allowing for two types of altruistic social investors. Rawlsian[1] investors favor the poorest of the poor, while utilitarian investors are not sensitive to the level of poverty and focus on costs as a result. These investors face a repayment amount per dollar offered by a socially motivated MFI with unobservable costs that are increasing in poverty.[2] The two sides play a signaling game, in which investors infer costs and poverty from the MFI's profit status (repayment offer).

I first demonstrate that in the absence of information asymmetry, commercialization is never optimal because the additional funds raised as a result of the higher returns offered to the investors are not worth the higher repayment burden imposed on the borrowers. Consequently, the MFI, regardless of costs, has no incentive to commercialize in a transparent environment. However, under information asymmetry, if the fraction of utilitarian investors is high enough, a low-cost MFI will increase repayment to send a credible signal of low costs that appeals to utilitarian investors. A high-cost MFI cannot afford this signal and chooses not to offer any repayment to investors as in the symmetric information case.

Therefore, I conclude that the combination of the high presence of utilitarian investors

[1] According to the Stanford Encyclopedia of Philosophy, John Rawls argues for fairness as a superior interpretation of justice (Wenar, 2017) than the utilitarian view, which understands justice as maximizing the collective happiness (Crimmins, 2020).

[2] This is equal to principal plus interest or the total amount that is to be paid back by the MFI to investors in the future. It is negatively correlated with MFIs' subsidy dependence and thus a measure of commercialization.

and low-cost MFI's efforts to attract more funds in a non-transparent environment drives the commercialization of microfinance. Thus, the presence of for-profits is not a sign of mission drift. The current mix of non-profits and for-profits is a "division" of the microfinance mission: for-profits serve large numbers of the marginal poor by tapping into utilitarian funds, and non-profits take up the more costly task of serving the poorest of the poor. This finding confirms the conjecture quoted at the beginning of this paper by Cull et al. (2009), based on their overview of the literature.

The intuition behind this result requires a more detailed description of the model. A socially motivated MFI and a pool of altruistic social investors play a sequential signaling game. The MFI privately knows whether it serves an extremely poor community (high-cost) or a marginally poor community (low-cost). It moves first and solicits the investors for funds by announcing a repayment amount per dollar. Investors, do not observe the MFI's type but know the distribution, and can infer more information from the repayment offer. They respond to the MFI's offer and decide how much to invest in microfinance. The MFI, in turn, uses the funds to lend to the poor and charges them the repayment promised to investors (financial costs) plus transaction costs that are increasing in poverty. The MFI's objective is to maximize consumer surplus. Each investor's utility is increasing in both private consumption and consumer surplus. A sub-group of them are Rawlsian and only value consumer surplus if the MFI serves an extremely poor community. The rest are utilitarian and value consumer surplus regardless of the poverty level. The distribution of investor types is public information. I solve for the sequential equilibrium and use the Cho and Kreps (1987) intuitive criterion to refine the set of equilibria.

I first analyze a benchmark model where poverty is observable (Subsection 3.4) to show that the MFI's first best choice is full subsidy (repayment equal to 0). Underlying this finding is the fact that most for-profit MFIs are unable to pay interest rates that can compete with those of conventional financial markets, and some even rely on subsidies of some sort.[3] This is not surprising as the microfinance movement's reason to exist is that conventional banks do not find the poor profitable. Consequently, investors incur an opportunity cost when investing in microfinance by forgoing returns from more lucrative conventional finance. In other words, investing in microfinance is a form of implicit donation out of the future value of investors' wealth with the incentive of helping the poor. The model incorporates this idea as an assumption that the net profit margin of microfinance is lower than conventional finance.[4] To see the effect, consider a given investment amount at zero repayment (full subsidy). If the MFI increases the repayment offer, and the investors want to keep their net giving (forgone future value) constant, they have to increase their investment. However, since the net yield is lower in microfinance, the amount that the MFI has to repay the investors to keep their net giving the same will exceed the extra yield that the added funds create. Therefore, the impact (marginal consumer surplus) of forgone wealth diminishes, which, in turn, reduces investors' giving incentives. Note that, the investment amount might increase but not enough to translate into more net giving.

A simple example would help clarify this point. Say, the conventional market interest rate is 10% over a year. Hence, the future value of a hundred dollars is \$110. If one invests that money in microfinance, the MFI will lend it to the poor at a transaction cost of \$10. A borrower invests the money in a micro-enterprise, that after a year yields \$115. Note that the maximum amount that the MFI can charge the poor borrower is \$115, in which case the investor will receive \$115-\$10=\$105 after costs. Consequently, the investor will be \$5 short of the future value of her money. The five dollars will be her net donation in this case. First, consider the case of zero repayment. The MFI will then only have to charge \$10 to the borrower to cover its costs, leaving him with a surplus of \$115-\$10=\$105. The net donation, in this case, is \$110. Now consider a 55% repayment. In this case, for a

[3]For example, Dieckmann et al. (2007) report that other than a few exceptions, microfinance returns on equity (with an average of around 4%) are not high enough to attract purely profit-oriented investors.

[4]The profit margin of microfinance is equal to the future value of a dollar in the hands of the poor, minus the transaction costs of lending that dollar to the poor.

\$100 loan, the MFI has to charge the borrower \$55+\$10=\$65 to cover costs and financial obligations, which leaves the borrower with a surplus of \$115-\$65=\$50. Thus, a \$100 investment translates into a donation of \$110-\$55=\$55. In order for the net giving to reach \$110, the investor has to contribute twice as before that is \$200. The consumer surplus, however, will be $2 \times \$50 = \$100$ that is \$5 less than before! This example demonstrates that as repayment increases, the reduction of impact renders forgoing future dollars less attractive to social investors. Of course, since higher repayment reduces the opportunity cost of investing, investments might increase. However, the added funds will not be high enough to outweigh the increase in borrowers' burden due to higher repayment. The result is that no repayment is optimal.

Nonetheless, despite the result form the benchmark model, the reality is that for-profit MFIs hold the lion's share of microfinance investments. In Subsection 3.5, I introduce the full model with uncertainty to explain what motivates commercialization and why social investors increase funding in response to higher repayment amounts, notwithstanding the added burden on poor borrowers. As a first step, I show that increasing repayment offer is less costly for a low-cost MFI (in a marginally poor community) because it has a higher profit margin compared to a high-cost MFI (in an extremely poor community). Thus, a high-cost MFI is more reluctant to commercialize, and a low-cost MFI can use repayment to reveal its type, which is a form of single-crossing property. The opposite, however, is not true. A high-cost MFI cannot separate from the other type, because a low-cost MFI can always mimic the repayment behavior of a high-cost MFI. As a result, the form of equilibrium (separating or pooling) depends on the signaling incentives of a low-cost MFI. That, in turn, depends on the breakdown of Rawlsian and utilitarian preferences in the investor pool.

I find that so long as the fraction of Rawlsian investors is high enough, serving the marginal poor (as opposed to the extremely poor) is not very popular. Thus, in the absence of information asymmetry, a high-cost MFI will receive more funds than a low-cost

MFI. Hence, a low-cost MFI has no incentive to expose its type and pools with the other type on the first best repayment of 0. As the fraction of Rawlsian investors drops, the investor pool's preference for tackling extreme poverty diminishes, and so does a low-cost MFI's incentive to pool. Therefore, partially separating equilibria become possible. In such equilibria, a low-cost MFI mixes between pooling with the other type and separating at a higher repayment amount. With a further drop in the fraction of Rawlsian investors, the possibility of fully pooling equilibria diminishes. Once the utilitarian fraction of the investor pool becomes high enough, the order of funding reverses, i.e., investors contribute more when they verify low costs compared to when they observe high costs. Thus, a low-cost MFI has an incentive to offer a positive repayment (commercialization) and signal its type. The single-crossing property described earlier, guarantees that a high-cost MFI is unable to afford this signal and chooses the first best, i.e., zero repayment. As a result, the game reaches a separating equilibrium where the MFI perfectly reveals its type. In summary, as the fraction of utilitarian investors increases from 0 to 1, the equilibrium changes from full pooling on zero repayment to full separation with a low-cost MFI commercializing.

The two distinguishing features of the model underly this result: 1) the variation of costs with poverty 2) the distinction between Rawlsian and utilitarian investors. The first feature is a reflection of the intrinsic characteristics of extremely poor communities, such as remoteness, low connectivity, weak infrastructure, and small loan sizes, that increase the costs of financial transactions. Thus, even though higher costs mean lower consumer surplus, they are unavoidable in alleviating extreme poverty. The second feature of the model captures the variation in investors' preferences regarding this trade-off, and in turn, determines which MFI type would be favored and receive more funds. Commercialization as a signal of low costs arises as a result of this interaction between investors' preferences and MFI costs and poverty under information asymmetry.

Separation (full or partial) in equilibrium is consistent with the presence and higher market share of for-profits in the microfinance market and the empirical finding that there is a negative correlation between financial independence and serving the poorest of the poor.[5] Therefore, this study suggests that the dominance of the utilitarian view of welfare is the underlying explanation. Consequently, the commercialization trend is not necessarily a sign of mission drift. It can alternatively be interpreted as a "mission division" where all investors are socially motivated and intend to improve welfare for the poor but have different approaches to the mission that results in different business models. MFIs that tend to be more commercial (less subsidized) attract more funds from utilitarian investors and expand microfinance while non-profits rely on subsidies to serve the poorest of the poor and make sure no one is left behind. In this picture, both non-profit and for-profit microfinance are indispensable to poverty alleviation. Hence, the microfinance community should neither sideline non-profits in the name of efficiency, nor view for-profits as drifting away from the mission of fighting poverty.

On the policy side, this study urges policymakers to pay closer attention to sources of funding for microfinance programs to ensure that they are compatible with their poverty alleviation goals. The reason is that investors' preferences can affect who benefits from their funds by affecting microfinance business models. Neglecting this point can result in a program missing its intended target. This idea is especially important given the attractiveness of microfinance as a cost-effective tool of poverty alleviation.

In the following subsections, I discuss the relevant literature then present the model and findings. Subsection 3.2 provides a review of the relevant literature. Subsection 3.3 describes the theoretical model. Subsection 3.4 analyzes a benchmark case of full information to find the first best strategy of the MFI. The full model with uncertainty is discussed in Subsection 3.5. Each sub-subsection of Subsection 3.5 discusses the game's outcome with a different distribution of preferences in the investor pool. A summary of the results is provided in Subsection 3.6.

[5]For example, Cull et al. (2018) find that on average commercial institutions make bigger loans (commonly interpreted as wealthier borrowers) and have lower costs compared to non-profits.

3.2 Related Literature

The theoretical literature on microfinance commercialization is rather sparse. In one of the earliest papers in this literature, McIntosh and Wydick (2005) show how competing against for-profit institutions undermines the mission of non-profits by limiting their ability to cross-subsidize poorer borrowers through more profitable wealthier costumers. Later, Ghosh and Van Tassel (2011) argue for the benefits of competition between microfinance institutions for funds, by showing that it causes high-cost institutions to drop out of the market. Karaivanov (2018) focuses on moral hazard and finds that demanding higher interest on social investments incentivizes MFIs to operate more efficiently. In the closest paper to my work, Ghosh and Van Tassel (2013) demonstrate that as the microfinance sector grows, it reaches a point where social investors squeeze high-cost institutions out of the market to induce efficiency, by charging high interest on their funds. I take a few steps further by presenting a richer model, which leads to a result that is more consistent with empirical observations. According to their theory, all microfinance should converge to for-profit once the sector is large enough. However, as explained earlier, both for-profits and non-profits continue to have a strong presence in the market, which is consistent with the findings presented in the current paper.

On the empirical side, studies of commercialization generally point to the social costliness of profits. For example, Cull et al. (2007) conclude that there is evidence of a trade-off between profits and impact as the more profitable institutions perform worse in measures of outreach such as average loan size and the fraction of borrowers that are women. Cull et al. (2011) also find that profit-oriented MFIs are less likely to lend to women or poorer borrowers compared to institutions with lower commercial motives. Of course, there is some empirical evidence like Caudill et al. (2009), who find that lower subsidies correspond to more cost-effectiveness over time and Cull et al. (2018) , who find that on average commercial institutions make bigger loans while having lower costs compared to non-profits. Nevertheless, there is a trade-off between lower costs and outreach. For example, Hudon et al. (2020) use a dataset containing near 500 institutions and find that only 3% of those institutions are truly profitable and at the same time, serve their social goals. The rest face a trade-off between profitability and outreach.

There is also some empirical literature on returns to investment and subsidy dependence in commercial (and non-commercial) microfinance. They generally suggest that microfinance is not very attractive from a profitability viewpoint. For example, Mersland and Strøm (2008) find that the difference between the performance of shareholder-owned MFIs and non-government MFIs is negligible. Cull et al. (2007) look at data from more than 120 microfinance institutions and find that even in their sample that contains the most mature and efficient institutions, there is some reliance on subsidy. A Study by Cull et al. (2018) later confirms this finding by using data from more than 1300 institutions to find that the industry is highly reliant on subsidies that average at $132 per borrower. They also find that most subsidies are indirect in the form of cheap capital or equity grants.

3.3 Model Description

3.3.1 The Poor

Each poor individual i needs a small loan to invest in a project (micro-enterprise, education, health, etc.). The expected outcome of the project is y_i per dollar of investment. The poor do not have the initial capital for the project and do not have access to conventional finance. Their only source of capital is microfinance (if available) and their outside option is normalized to 0. The transaction cost of micro-lending (due to remoteness of location, loan size, etc.) is c_i per dollar.

3.3.2 Poverty

There are two types of poor communities (villages, slums, or neighborhoods) each with a large set of poor borrowers. One is extremely poor while the other is marginally poor. This distinction is captured by a poverty indicator $p \in \{0,1\}$ where $p = 1$ indicates extreme poverty and $p = 0$-marginal poverty. The average outcome of projects and average transaction cost of lending in a community are functions of this indicator: $Y(p) = E(y_i|p)$ and $C(p) = E(c_i|p)$ respectively. The interest rate in conventional financial markets (the future value of \$1) is I_M. Consistent with empirical evidence, I assume that costs of microfinance make it less profitable than conventional banking.[6]

Assumption 3-1. $\forall p \quad I_M > Y(p) - C(p)$

Also, consistent with data in Gonzalez (2007) and Husain and Pistelli (2016), I assume that microfinance becomes more costly at extreme poverty. Therefore, the microfinance profit margins are higher when poverty is lower.

Assumption 3-2. $Y(0) - C(0) > Y(1) - C(1)$

3.3.3 Microfinance Institution

The MFI is set up in a marginally poor community with probability π an a extremely poor community otherwise. The MFI is privately informed about its type, while the public only knows the distribution of p. The MFI offers a repayment $I \in [0, \bar{I}(p)]$ to raise funds F. Here, $\bar{I}(p) = Y(p) - C(p)$ is the maximum possible interest rate that the MFI can charge the poor. For example $I = 0$ represents pure donations. Alternatively $I > 0$ represents a lower level of subsidy dependence and more commercialization. $I > 1$ represents positive return to investment (accounting profit). The MFI would then lend to borrowers and charge $C(p) + I$ per dollar to cover its operational costs (transaction costs) and financial obligation to the social investor (cost of capital). The MFI's goal is to maximize its social impact.

$$v(F, I, p) = [Y(p) - C(p) - I] \cdot F \tag{3-1}$$

The term in the brackets represents the average surplus gained by the poor per dollar of loan. This term multiplied by the available funds F is the total consumer surplus from

[6]If this does not hold nothing prevents conventional banks from profitably lending to the poor and there will be no need for microfinance.

the microfinance activity.

3.3.4 Social Investors

There is a pool of n altruistic social investors. Each investor i has a large initial wealth of w and chooses to invest funds f_i after observing repayment I. An investor's utility depends on both private consumption and social impact:

$$u_i(f_i, F_{-i}, I, p) = g((w - f_i)I_M + f_i I) + \phi_i(p)(Y(p) - C(p) - I)h(F) \tag{3-2}$$

$g()$ and $h()$ are increasing and concave, i.e, $g'() > 0$, $g''() < 0$, $h'() > 0$, $h''() < 0$. The first term on the right-hand side represents diminishing utility from future private consumption that is equal to returns from investment in conventional finance and microfinance. The second term represents diminishing utility from the social impact of microfinance activity. $F = \sum_{i=1}^{n} f_i$ is the aggregate funds raised and $F_{-i} = F - f_i$. $\phi_i()$ is the investor's philosophy coefficient and depends on her type. A fraction $\Phi_R \in [0, 1]$ of the population, denoted by set R, are Rawlsian. They are focused on the welfare of the poorest and do not value consumer surplus if the MFI is not located in the poorer community type. For this group and $\phi_i(p) = p$. The $\Phi_U = 1 - \Phi_R$ remaining fraction of investors is denoted by set U. They are utilitarian and value consumer surplus in either community. For this group $\phi_i(p) = 1$. Hence, in summary:

$$\phi_i(p) = \begin{cases} p & \text{if } i \in R \\ 1 & \text{if } i \in U \end{cases}$$

The distribution of types, denoted by $\Phi = (\Phi_U, \Phi_R)$, is publicly observable.

Timing of the game is as follows:

1. The investors' average philosophy is publicly observed (Φ).
2. The MFI privately observes its type (p).

3. The MFI publicly chooses repayment (I).

4. The investors simultaneously choose how much to invest in MFI (F).

5. The MFI disburses loans and payoffs realize.

In the next two subsections, I first consider a full information benchmark (Subsection 3.4) an then solve for the sequential equilibrium of the full model (Subsection 3.5). I apply the Cho-Kreps intuitive criterion for equilibrium refinement (Cho and Kreps, 1987).

3.4 Full Information

Consider the benchmark case, in which the MFI's type p is observable to the investors. Once the MFI has offered a repayment I, each investor i chooses her investment (f_i) to maximize her utility:

$$\max_{f_i} u_i(f_i, F_{-i}, I, p)$$

An investor's optimal response $f_i(F_{-i}, I, p)$ has to satisfy the corresponding first oder condition derived from the above problem and eq. (3-2):[7]

$$g'(wI_M - f_i(F_{-i}, I, p)(I_M - I))(I_M - I) \geq \phi_i(p)(Y(p) - C(p) - I)h'(F_{-i} + f_i(F_{-i}, I, p)) \tag{3-3}$$

Thus the individual best response is:

$$f_i(F_{-i}, I, p) = \left(\frac{1}{I_M - I}\right) max \left\{ wI_M - g'^{-1}\left(\frac{\phi_i(p)(Y(p) - C(p) - I)h'(F_{-i} + f_i(F_{-i}, I, p))}{I_M - I}\right), 0 \right\} \tag{3-4}$$

From here, the aggregate best response can be calculated as:

$$F(I, p, \Phi) = n\left(\frac{\Phi_U + p\Phi_R}{I_M - I}\right) max \left\{ wI_M - g'^{-1}\left(\frac{(Y(p) - C(p) - I)h'(F(I, p, \Phi))}{I_M - I}\right), 0 \right\} \tag{3-5}$$

[7]The inequality is strict only for the case of a corner solution where $f_i(F_{-i}, I, p) = 0$.

Eq. (3-5) states the investor pool's optimal level of funding in the MFI for a given repayment I. However, it is not immediately clear how this optimal investment varies with changes in repayment amount. In fact, examining eq. (3-3) reveals that for a given community type p, a repayment increase has two opposing effects: 1) it reduces the marginal cost of investment (the left-hand side) by closing the repayment gap between microfinance and conventional finance and 2) it reduces the marginal benefit of investment (the right-hand side) by reducing the consumer surplus per dollar of investment. Since it is not trivial which effect is dominant, it is not easy to determine how aggregate funding responds to changes in repayment amount.

In order to shed some light on the investors' behavior it is best to focus on subsidy instead of funding. The former is the sum of future returns that social investors "forgo" when investing in the MFI who repays bellow the markets rate I_M. A slight rearrangement eq. (3-5) results in the following:

$$F(I,p,\Phi)(I_M - I) = n\left(\Phi_U + p\Phi_R\right) max \left\{ wI_M - g'^{-1}\left(\frac{(Y(p) - C(p) - I)h'(F(I,p,\Phi))}{I_M - I}\right), 0 \right\} \tag{3-6}$$

The left-hand side of the eq. (3-6) is the investors' "subsidy" or "net giving" to the MFI, which can be shown to be decreasing in repayment I. The first step is to focus on the term $\frac{(Y(p)-C(p)-I)h'(F(I,p,\Phi))}{I_M-I}$ in the right-hand side. This represents the marginal benefit of a unit of net giving (subsidy) which is decreasing in repayment rate I. The intuition is that an increase in repayment reduces both the consumer surplus and the net giving per unit of investment, but since by Assumption 3-1 the latter is always bigger, the ratio of the two terms or the consumer surplus per unit of net giving diminishes.[8] As a result, subsidizing the MFI becomes less attractive as the MFI increases repayment to social investors. Hence, one should expect the total subsidy to be diminishing in repayment rate I, which in turn suggests that a the lowest possible repayment (a pure non-profit) has to be optimal for the MFI.

[8] The mathematical proof is given in Appendix B as part of the proof of Proposition 3-1.

The above prediction can be formally shown to hold by focusing on the MFI's problem. Anticipating the investors' response given by eq. (3-5), the MFI aims to maximize the social impact by choosing the repayment I:

$$\max_I v(F(I,p,\Phi),I,p)$$

At a first glance, the effect of increasing the repayment seems unclear. On the on hand it has both a direct cost of reducing the consumer surplus per dollar for the MFI and an indirect effect of reducing the marginal benefit of investment for the investors. On the other hand, a repayment increase reduces the marginal cost of investment. Nonetheless, the objective function can be rewritten as the product of total subsidy and the consumer surplus per unit of subsidy:

$$\max_I [\frac{Y(p)-C(p)-I}{I_M-I}] \cdot F(I,p,\Phi)(I_M-I)$$

Since both terms are decreasing in the repayment amount, the MFI's payoff will also be decreasing in the repayment I. Thus, the optimization problem has a corner solution at 0.

Proposition 3-1. *If p is observable to the social investors, the MFI's optimal repayment is 0 for both types, i.e. $I^*(1) = I^*(0) = 0$.*[9] *Moreover, let $\overline{\Phi}$ satisfy $F(0,0,\overline{\Phi}) = F(0,1,\overline{\Phi})$. For any Φ such that $\Phi_U > \overline{\Phi}_U$, $F(0,0,\Phi) > F(0,1,\Phi)$ and for any Φ such that $\Phi_U < \overline{\Phi}_U$, $F(0,0,\Phi) < F(0,1,\Phi)$.*

Proposition 3-1 reveals that in the absence of information asymmetry and signaling motives, the MFI is reluctant to commercialize. Even though this benchmark model does not apply to today's microfinance market, it can help us with understanding the early days of microfinance when the entire market comprised a handful of non-profits. The small market was more transparent and in-line with the full information benchmark.

[9] $I^*(p)$ represents the equilibrium choice of type p under information symmetry that is independent of average investor philosophy Φ.

Moreover, early MFIs were focused on the poorest of the poor, which in the context of this model corresponds to $p = 0$.[10] The resulting equilibrium is that the MFI always optimally chooses a repayment of 0 and has no incentive to further commercialize. Furthermore, it is straightforward to establish that this result is independent of Φ. I, hereon, will refer to this as the first best repayment.

In the next subsection, I will analyze the full model and demonstrate that under information asymmetry, when the utilitarian fraction of investors (Φ_U) is low enough, both types of MFI pool on the first best repayment. Conversely, when the investors are utilitarian enough, the MFI types separate with an institution in the poorer community type staying at the first best and an institution in the wealthier community type increasing repayment (commercialization).

3.5 Full Model

Under information asymmetry, once the MFI has offered repayment I, the investors form a belief about the MFI's type (p) from the repayment. Let the investors' belief about the posterior probability of $p = 1$, upon observing I, be denoted as $\eta(I)$. At this point an investor will choose how much to invest (f_i) to maximize her expected utility:

$$\max_{f_i} E_{p\sim\eta(I)} u_i(f_i, F_{-i}, I, p)$$

From the above problem and eq. (3-2), an investor's optimal response $f_i(F_{-i}, I, \eta)$ has to satisfy the following first oder condition:[11]

$$g'(wI_M - f_i(F_{-i}, I, \eta)(I_M - I))(I_M - I) \geq E_{p\sim\eta}[\phi_i(p)(Y(p) - C(p) - I)h'(F_{-i} + f_i(F_{-i}, I, p))] \tag{3-7}$$

[10] As an exmaple one can refer to the famous case of Grameen bank in its early days. Muhammad Yunus himself was both the source of funding and the manager of the bank. Thus obviously, there was no information asymmetry between the investor and the MFI. Moreover, he targeted the poorest of the poor in rural Bangladesh. (Yunus, 2007)

[11] The inequality is strict only for the case of a corner solution where $f_i(F_{-i}, I, \eta) = 0$.

From this, the individual best response is:

$$f_i(F_{-i}, I, \eta) =$$
$$\left(\frac{1}{I_M - I}\right) \quad max\left\{wI_M - g'^{-1}\left(\frac{E_{p\sim\eta}[\phi_i(p)(Y(p)-C(p)-I)h'(F_{-i}+f_i(F_{-i},I,p))]}{I_M-I}\right), 0\right\} \tag{3-8}$$

Comparing the above result with eq. (3-4) implies that each investor's response lies between her best responses under full information with $p = 0$ and $p = 1$ respectively, that is:

$$min(f_i(F_{-i}, I, 0), f_i(F_{-i}, I, 1)) \leq f_i(F_{-i}, I, \eta) \leq max(f_i(F_{-i}, I, 0), f_i(F_{-i}, I, 1))$$

However, this does not automatically extend to the aggregate best response $F(I, \eta, \Phi)$. To see why, consider the case of $\eta = 0$. In this case the marginal benefit of investment for the Rawlsian investors is 0 and eq. (3-7) holds with inequality. Thus a marginal increase in η will increase their marginal benefit, but will not affect their giving. Nonetheless, for the utilitarian investors, eq. (3-7) holds with equality and they immediately respond to a marginal increase in η by reducing their investment. As a result, the aggregate funding drops even if the pool of investors is predominantly Rawlsian. Note that this is true so long as there is at least one utilitarian investor in the pool. Therefore, even though for a low enough Φ_U the aggregate investment will eventually increase in η, the change is not monotonic.

Lemma 3-1. *The aggregate funding is decreasing in belief at $\eta = 0$, i.e. $\frac{\partial F(I,\eta,\Phi)}{\partial \eta}|_{\eta=0} \leq 0$. The inequality is strict for any $\Phi_U > 0$. Moreover, there exists $\underline{\Phi}$ such that $\underline{\Phi}_U < \overline{\Phi}_U$ and satisfies $F(0, 0, \underline{\Phi}) = F(0, \pi, \underline{\Phi})$. Then for any Φ such that $\Phi_U > \underline{\Phi}_U$, $F(0, 0, \Phi) > F(0, \pi, \Phi)$ and for any Φ such that $\Phi_U < \underline{\Phi}_U$, $F(0, 0, \Phi) < F(0, \pi, \Phi)$.*

Lemma 3-1 reveals that the aggregate best response is non-monotonic in the belief η over the intermediate range of $(\underline{\Phi}_U, \overline{\Phi}_U]$. The intuition is that when the MFI is known to be in a marginally poor community ($\eta = 0$) all the utilitarian investors find it worthwhile to invest. However, when the two types have pooled together ($\eta = \pi$) the investment incentives of the utilitarian investors diminish while the Rawlsian investors still do not find that probability high enough to invest. Thus the overall investment drops. At the other extreme, when the MFI is known to be in an extremely poor community ($\eta = 1$) all the Rawlsian investors find it worthwhile to invest and the aggregate funding goes back up and above the level with $\eta = 0$.

Since investors infer $\eta(I)$ from the repayment, the MFI's problem would be:

$$\max_{I} v(F(I, \eta(I), \Phi), I, p)$$

One can extend Proposition 3-1 and find that for a given belief held by the investors, the first best for the MFI is to set the repayment equal to 0.[12] Yet, under information asymmetry, an MFI might be able to improve the investors' belief and increase its impact by increasing the repayment. However, by Assumption 3-2, the marginal consumer surplus of an MFI in the wealthier community ($p = 1$) is higher for a given repayment, i.e. $Y(1) - C(1) - I < Y(0) - C(0) - I$ for any I. Therefore, from eq. (3-1) it can be shown that a repayment increase is less costly for the MFI when $p = 0$.

Lemma 3-2. ***(Single Crossing Property)*** *It is less costly for the MFI to increase repayment, when it is in the wealthier community, i.e. for any* $I_1 < I_2$ *such that* $F(I_1, \eta(I_1), \Phi) < F(I_2, \eta(I_2), \Phi)$,

$$v(F(I_1, \eta(I_1), \Phi), I_1, 1) \leq v(F(I_2, \eta(I_2), \Phi), I_2, 1)$$

implies

$$v(F(I_1, \eta(I_1), \Phi), I_1, 0) < v(F(I_2, \eta(I_2), \Phi), I_2, 0)$$

[12] This is Lemma B-1 in Appendix B.

Lemma 3-2 reveals that only when the MFI is located in the wealthier community it can separate itself form the other type by sending a credible signal (higher repayment). This property suggests that in any equilibrium, if more than one repayment appears on the equilibrium path, higher rates must belong to an MFI that serves a marginally poor community. This is formally stated in the following lemma.

Lemma 3-3. *In all sequential equilibria, the following hold on the equilibrium path:*

1. *The two types of MFI pool on no more than one repayment, denoted by I_p.*
2. *An MFI in a wealthier community ($p = 0$) chooses at most one other repayment, denoted by I_H.*
3. *An MFI in a poorer community ($p = 1$) chooses at most one other repayment, denoted by I_L.*
4. *The repayments on the equilibrium path satisfy $I_L < I_P < I_H$.*

Lemma 3-3 proves that not only it is less costly for an MFI in a wealthier community to increase repayments ex ante (Lemma 3-2) but also posterior probability of extreme poverty is non-increasing in repayment on the equilibrium path. Therefore, it is plausible to expect the investors to hold monotonic off-equilibrium beliefs. In other words, it is counterintuitive for the investors to observe a higher repayment and form a belief that the probability of the MFI being in an extremely poor community is higher. I use this to refine the set of equilibria to monotonic belief equilibria.

Assumption 3-3. *A monotonic belief equilibrium is an equilibrium where the investor's posterior beliefs satisfy:* $I_1 < I_2 \Rightarrow \eta(I_1) \geq \eta(I_2)$

The exact form of the equilibrium depends on whether an MFI in a marginally poor community has an incentive to separate and reveal its type. That, in turn, depends on which type of MFI would receive better funding from the investor pool, which is determined by the average philosophy Φ. In the next three subsections, I show that when the investor pool is more Rawlsian ($\Phi_U \leq \overline{\Phi}_U$), they prefer to contribute more funds to an MFI in a poorer community. Thus, an MFI in the wealthier community is better off mixing with the other type and the game ends in a pooling or partially pooling equilibrium. In contrast, a more utilitarian group ($\Phi_U > \overline{\Phi}_U$) favors an MFI in a wealthier community. Therefore, the two types separate in equilibrium.

3.5.1 Rawlsian Investor Pool

Consider a purely Rawlsian investor pool ($\Phi_U = 0$). From eq. (3-5), it can be seen that if the MFI's type were to be revealed, it would only receive funding, if the location was the poorer community type. The intuition is that the marginal benefit of investing in a wealthier community is 0 for all investors as evident in eq. (3-7). Thus, an MFI in the wealthier community has no incentive to separate. Moreover, by Lemma 3-2, an MFI in a poorer community cannot send a credible signal to separate itself. Consequently, only fully pooling equilibria are possible. Moreover, under monotonicity of beliefs as explained in Assumption 3-3, the pooling strategy cannot exceed 0 which is the MFI's first best.[13] The results is that the set of equilibria will be refined to a unique equilibrium. Moreover, by Lemma 3-1, with any investor pool that is Rawlsian enough ($\Phi_U \leq \underline{\Phi}_U$), the MFI receives more funding under prior belief than when it is known to be in a wealthier community. Hence, the MFI types cannot separate in equilibrium.

Proposition 3-2. *For all Φ such that $\Phi_U \leq \underline{\Phi}_U$, there is a unique monotonic belief sequential equilibrium. In such equilibrium, the two MFI types pool on zero repayment, i.e., $I^{**}(0, \Phi) = I^{**}(1, \Phi) = 0$.*

Proposition 3-2 states that if the investor pool is Rawlsian enough, it induces both MFI types to keep to the first best repayment of 0 without any incentive to commercialize.

[13] This is Lemma B-2 in Appendix B.

Intuitively, the Rawlsian investor's favorite type is an MFI who serves the core poor. One would expect such MFI to send a credible signal to investors and separate itself form the other type. However, any signal that is affordable for an MFI in an extremely poor community where costs are high, is also affordable for a low-cost MFI in a wealthier community. Therefore, since the latter type has an incentive to pretend to be the favored type, any separation is impossible and the game ends in a pooling equilibrium.

3.5.2 Utilitarian Investor Pool

At the other extreme, consider a purely utilitarian pool of investors ($\Phi_U = 1$). In this case, the marginal benefit of investing in an MFI (the right-hand side of eq. (3-7)) is solely dependent on costs and thus higher for a low-cost MFI. Therefore, the utilitarian investors' preferred type is an MFI in the wealthier community which gives it an incentive to separate form the other type. Lemma 3-2 guarantees that a credible signal of low-cost is possible and separating equilibrium exists. Moreover, the intuitive criterion and monotonicity of beliefs limit the set of equilibria to the least costly separating (Riley) equilibrium. Furthermore, this result can be extended to any pool of investors that is utilitarian enough ($\Phi_U > \overline{\Phi}_U$) such that an MFI in a marginally poor community receives more funding. Over this range, such MFI has an incentive to separate from an MFI in an extremely poor community.

Proposition 3-3. ***(Mission Devision Equilibrium)*** *For all Φ such that $\Phi_U > \overline{\Phi}_U$, Riley separating equilibrium is the unique monotonic belief sequential equilibrium that satisfies Cho-Kreps intuitive criterion. The MFI in the extremely poor community ($p = 1$) chooses the first best repayment of 0 while the MFI in the marginally poor community ($p = 0$) chooses a strictly higher repayment and raises more funds, i.e. $I^{**}(0, \Phi) > I^{**}(1, \Phi) = 0$ and $F(I^{**}(0, \Phi), 0, \Phi) > F(I^{**}(1, \Phi), 1, \Phi)$.*

Proposition 3-3 describes the "mission devision" equilibrium in microfinance. An MFI that is located in a wealthier community and serves the marginal poor, offers a higher repayment to signal lower costs and raise more funds. This is consistent with the higher average loan size and bigger market share of the commercial MFIs observed in data. In contrast, an MFI that is located in a poorer community and serves the core poor cannot afford the more commercial model of the other type and thus, offers a zero repayment that corresponds to a non-profit model.

3.5.3 Semi-Rawlsian Investor Pool

What happens if the pool of investors includes a significant fraction of both Rawlsian and utilitarian types? More accurately what will the equilibrium look like if $\underline{\Phi}_U < \Phi_U \leq \overline{\Phi}_U$. In this case, on the one hand, by Proposition 3-1, an MFI in a marginally poor community receives less funding when exposed than when it is believed to be in an extremely poor community. Therefore, as in the Rawlsian case, it has an incentive to pool with the other type and a fully separating equilibrium does not occur. However, on the other hand, by Lemma 3-1, it raises more funds when exposed than when it is fully pooling with the other type of MFI. Thus, a partial separation from the pool is desirable. This results in a partially separating equilibrium, where an MFI in a marginally poor community mixes between pooling with the other type and a higher repayment that separates it from the pool.

Proposition 3-4. *For all Φ such that $\underline{\Phi}_U < \Phi_U \leq \overline{\Phi}_U$, in any monotonic belief sequential equilibrium that satisfies Cho-Kreps intuitive criterion, an MFI in an extremely poor community ($p = 1$) chooses one repayment $I^{**}(1, \Phi) \geq 0$ for sure. The strategy of an MFI in a marginally poor community ($p = 0$) is pooling with the other type at $I^{**}(1, \Phi)$ with probability $\gamma \in (0, 1]$ and separating from the pool at a higher repayment $I^{**}(0, \Phi) > I^{**}(1, \Phi)$ with probability $1 - \gamma$. Moreover, a partially separating equilibrium ($\gamma < 1$) always exists.*

Proposition 3-4 states the interesting feature of the equilibria over this interim range. Even though a fully pooling equilibrium cannot be ruled out entirely, partial separation is always possible. In other words, separation incentives of an MFI in a marginally poor community strengthen with the increase in utilitarianism.

Interestingly, the monotonic belief sequential equilibria are not unique in this case, even after imposing Cho-Kreps intuitive criterion. In fact, as the fraction of utilitarian investors increases, it becomes possible for the strategy of an MFI in an extremely poor community to be a strictly positive repayment, i.e. $I^{**}(1, \Phi) > 0$. This is in contrast to the cases of utilitarian and Rawlsian investor pools, where such MFI always chooses $I^{**}(1, \Phi) = 0$. Such equilibria require off-equilibrium beliefs at low repayments that are "worse" than the pooling distribution to prevent downward deviation. Moreover, since $\Phi_U \leq \overline{\Phi}_U$, by Proposition 3-1, low poverty ($\eta = 0$) raises less funds and is a "worse" belief compared to high poverty ($\eta = 1$). Thus, at a first glance, it seems as such equilibria can only be maintained by violating Assumption 3-3. In other words, one might expect that any increase in η upon downward deviation, as required by belief monotonicity, will result into a "better" belief than the pooling distribution and provides a deviation incentive. Yet, interestingly, this is not always the case, since funding does not monotonically change with the probability of extreme poverty. In fact, while by Proposition 3-1, $\Phi_U < \overline{\Phi}_U$ implies $F(I^*(0), 0, \Phi) < F(I^*(1), 1, \Phi)$, by Lemma 3-1, $\Phi_U > \underline{\Phi}_U$ implies that $F(I^*(0), 0, \Phi) > F(I^*(0), \pi, \Phi)$. Consequently, there may exist posterior distributions that put higher probability on extreme poverty, but raise less funds than the pooling distribution. Such posterior distributions, as off-equilibrium beliefs, do not violate Assumption 3-3 and prevent downward deviation, which gives raise to equilibria where the two types of MFI (partially) pool on a positive repayment.

3.6 Chapter Summary

According to this analysis, despite a trade-off existing between repayments and social impact, positive returns in microfinance are not necessarily driven by pure profit motives. In fact, under symmetric information or when the social investors have a Rawlsian view of welfare, an MFI has no incentive to offer high repayments. It is a combination of information asymmetry and the utilitarian view of welfare on the investor side that prompts signaling by positive repayments. In such an environment, an MFI that serves a marginally poor community uses a more commercial model than an MFI in an extremely poor community. The former MFI type has lower costs due to a wealthier clientele and signals that through a positive repayment to social investors to raise more funds. The latter type, however, operates closer to a charity model, because it cannot afford high repayments due to high costs.

4. SUMMARY AND CONCLUSIONS

The findings discussed over the two main sections of this dissertation reveal that in the rather non-transparent world of pro-social markets, the organizations on the supply side (such as charities and MFIs) cannot rely on just their social message and the goodwill of donors to maximize their revenues. They need to use fundraising tools strategically and send the right signal to the donors and social investors on the demand side, who have become increasingly demanding and skeptical about both the quality of social services and the worthiness of the recipients.

In Section 2, I focused on quality signaling by charities to provide a theoretical rationale for the emerging field experimental data regarding the relative effectiveness of seed money and matching gift fundraising schemes. Moreover, my theoretical findings provide a promising avenue for future empirical work that can test the use of leadership giving in different information environments. On the experimental side, the findings in this dissertation suggest that the optimal fundraising scheme and the resulting donations vary significantly with the information available to donors. Thus, by varying the information available to donors in a lab setting, one can directly test this prediction and obtain further insight into the use of seed money and matching gift fundraising. On the empirical side, my model further suggests that newer charities may be more eager to seek seed money financing than established charities since the former are more likely to have reputation building concerns. Moreover, it predicts that donors would respond differently to an announcement of a matching gift if a charity is less established compared to a more established one. These predictions can be investigated using available market data or a field experimental setting.

In Section 3, I turned my attention to signaling of poverty by MFIs and presented a theory explaining the commercialization and subsequent polarization of the microfinance market. I concluded that deferent business models in microfinance can be a result of differences in the poverty level of the consumers and the welfare goals of social investors. The two types of microfinance divide the mission and tackle different levels of poverty, making both indispensable to poverty alleviation. Moreover, the investors' welfare philosophy can have a significant impact on the type of poor customers that benefit most from microfinance activities. Therefore, development policymakers need to pay due attention to the funding sources of microfinance. Moreover, from an empirical analysis standpoint, the negative causal relationship predicted in this dissertation between the poverty of costumers and profit orientation of MFIs, can be the basis of a testable hypothesis for future research using microfinance databases.

Finally, the findings of this dissertation demonstrate that interpreting the choices made by pro-social organizations, such as solicitation strategies or adopting a for-profit status, is not always straightforward. Thus, more theoretical, experimental, and empirical research is required to understand the behavior of pro-social organizations, which in turn affects how pro-social markets work. Developing such understanding is especially crucial from a policy standpoint due to the role that pro-social markets play in achieving social goals such as poverty alleviation and human development.

REFERENCES

Adena, Maja and Steffen Huck, "Matching donations without crowding out? Some theoretical considerations, a field, and a lab experiment," *Journal of Public Economics*, 2017, *148*, 32–42.

Admati, Anat R and Motty Perry, "Joint projects without commitment," *The Review of Economic Studies*, 1991, *58* (2), 259–276.

Alpizar, Francisco, Fredrik Carlsson, and Olof Johansson-Stenman, "Anonymity, reciprocity, and conformity: Evidence from voluntary contributions to a national park in Costa Rica," *Journal of Public Economics*, 2008, *92* (5), 1047–1060.

Andreoni, James, "Privately provided public goods in a large economy: The limits of altruism," *Journal of Public Economics*, 1988, *35* (1), 57–73.

_ , "Impure altruism and donations to public goods: A theory of warm-glow giving," *The Economic Journal*, 1990, *100* (401), 464–477.

_ , "Toward a theory of charitable fund-raising," *Journal of Political Economy*, 1998, *106* (6), 1186–1213.

_ , "Leadership giving in charitable fund-raising," *Journal of Public Economic Theory*, 2006, *8* (1), 1–22.

_ **and A Abigail Payne**, "Do government grants to private charities crowd out giving or fund-raising?," *American Economic Review*, 2003, *93* (3), 792–812.

_ **and** _ , "Is crowding out due entirely to fundraising? Evidence from a panel of charities," *Journal of Public Economics*, 2011, *95* (5), 334–343.

_ **and Martin C McGuire**, "Identifying the free riders: A simple algorithm for determining who will contribute to a public good," *Journal of Public Economics*, 1993, *51* (3), 447–454.

Bag, Parimal Kanti and Santanu Roy, "On sequential and simultaneous contributions under incomplete information," *International Journal of Game Theory*, 2011, *40* (1), 119–

145.

Bergstrom, Theodore, Lawrence Blume, and Hal Varian, "On the private provision of public goods," *Journal of Public Economics*, 1986, *29* (1), 25–49.

Caudill, Steven B, Daniel M Gropper, and Valentina Hartarska, "Which microfinance institutions are becoming more cost effective with time? Evidence from a mixture model," *Journal of Money, Credit and Banking*, 2009, *41* (4), 651–672.

Cho, In-Koo and David M Kreps, "Signaling games and stable equilibria," *The Quarterly Journal of Economics*, 1987, *102* (2), 179–221.

Crimmins, James E., "Jeremy Bentham," in Edward N. Zalta, ed., *The Stanford Encyclopedia of Philosophy*, summer 2020 ed., Metaphysics Research Lab, Stanford University, 2020.

Croson, Rachel and Jen Yue Shang, "The impact of downward social information on contribution decisions," *Experimental Economics*, 2008, *11* (3), 221–233.

Cull, Robert, Asli Demirgüç-Kunt, and Jonathan Morduch, "Financial performance and outreach: A global analysis of leading microbanks," *The Economic Journal*, 2007, *117* (517), F107–F133.

_ , _ , and _ , "Microfinance meets the market," *Journal of Economic Perspectives*, 2009, *23* (1), 167–192.

_ , _ , and _ , "Does regulatory supervision curtail microfinance profitability and outreach?," *World Development*, 2011, *39* (6), 949–965.

_ , _ , and _ , "The microfinance business model: Enduring subsidy and modest profit," *The World Bank Economic Review*, 2018, *32* (2), 221–244.

Danziger, Leif and Adi Schnytzer, "Implementing the Lindahl voluntary-exchange mechanism," *European Journal of Political Economy*, 1991, *7* (1), 55–64.

Dieckmann, Raimar, Bernhard Speyer, Martina Ebling, and Norbert Walter, "Microfinance: An emerging investment opportunity," *Deutsche Bank Research*, 2007, *19*, 1–20.

Eckel, Catherine C and Philip J Grossman, "Rebate versus matching: Does how we subsidize charitable contributions matter?," *Journal of Public Economics*, 2003, *87* (3), 681–701.

— **and** — , "Do donors care about subsidy type? An experimental study," in R Mark Isaac and Douglas D Davis, eds., *Experiments investigating fundraising and charitable contributors (Research in experimental economics, Vol. 11)*, Emerald Group Publishing Limited, Bingley, UK, 2006, pp. 157–175.

— **and** — , "Subsidizing charitable giving with rebates or matching: Further laboratory evidence," *Southern Economic Journal*, 2006, *72* (4), 794–807.

— **and** — , "Subsidizing charitable contributions: A natural field experiment comparing matching and rebate subsidies," *Experimental Economics*, 2008, *11* (3), 234–252.

— **,** — **, and Angela Milano**, "Is more information always better? An experimental study of charitable giving and Hurricane Katrina," *Southern Economic Journal*, 2007, *74* (2), 388–412.

Falk, Armin, "Gift exchange in the field," *Econometrica*, 2007, *75* (5), 1501–1511.

Frey, Bruno S and Stephan Meier, "Social comparisons and pro-social behavior: Testing "conditional cooperation" in a field experiment," *American Economic Review*, 2004, *94* (5), 1717–1722.

Ghosh, Suman and Eric Van Tassel, "Microfinance and competition for external funding," *Economics Letters*, 2011, *112* (2), 168–170.

— **and** — , "Funding microfinance under asymmetric information," *Journal of Development Economics*, 2013, *101*, 8–15.

Gong, Ning and Bruce D Grundy, "The design of charitable fund-raising schemes: Matching grants or seed money," *Journal of Economic Behavior & Organization*, 2014, *108*, 147–165.

Gonzalez, Adrian, "Efficiency drivers of microfinance institutions (MFIs): The case of operating costs," *Microbanking Bulletin*, 2007, *15*, 37–42.

Güth, Werner, M Vittoria Levati, Matthias Sutter, and Eline Van Der Heijden, "Leading by example with and without exclusion power in voluntary contribution experiments," *Journal of Public Economics*, 2007, *91* (5), 1023–1042.

Guttman, Joel M, "Understanding collective action: Matching behavior," *American Economic Review*, 1978, *68* (2), 251–255.

Huck, Steffen and Imran Rasul, "Matched fundraising: Evidence from a natural field experiment," *Journal of Public Economics*, 2011, *95* (5), 351–362.

_ , _ , and Andrew Shephard, "Comparing charitable fundraising schemes: Evidence from a natural field experiment and a structural model," *American Economic Journal: Economic Policy*, 2015, *7* (2), 326–69.

Hudon, Marek, Marc Labie, and Patrick Reichert, "What is a fair level of profit for social enterprise? Insights from microfinance," *Journal of Business Ethics*, 2020, *162* (3), 627–644.

Husain, Meraj and Micol Pistelli, "Where good intentions meet good business practice: A correlation analysis of social, operational and financial performance in microfinance," *Report, Microfinance Information Exchange (MIX), Washington, DC, USA*, 2016.

Karaivanov, Alexander, "Non-grant microfinance, incentives and efficiency," *Applied Economics*, 2018, *50* (23), 2509–2524.

Karlan, Dean and John A List, "Does price matter in charitable giving? Evidence from a large-scale natural field experiment," *American Economic Review*, 2007, *97* (5), 1774–1793.

_ , _ , and Eldar Shafir, "Small matches and charitable giving: Evidence from a natural field experiment," *Journal of Public Economics*, 2011, *95* (5), 344–350.

Khamar, Mohita, "Global outreach & financial performance benchmark," *Report, Microfinance Information Exchange (MIX), Washington, DC, USA*, 2015.

Khanna, Jyoti and Todd Sandler, "Partners in giving: The crowding-in effects of UK government grants," *European Economic Review*, 2000, *44* (8), 1543–1556.

Krasteva, Silvana and Huseyin Yildirim, "(Un) informed charitable giving," *Journal of Public Economics*, 2013, *106*, 14–26.

Landry, Craig E, Andreas Lange, John A List, Michael K Price, and Nicholas G Rupp, "Toward an understanding of the economics of charity: Evidence from a field experiment," *The Quarterly Journal of Economics*, 2006, *121* (2), 747–782.

List, John A and David Lucking-Reiley, "The effects of seed money and refunds on charitable giving: Experimental evidence from a university capital campaign," *Journal of Political Economy*, 2002, *110* (1), 215–233.

McIntosh, Craig and Bruce Wydick, "Competition and microfinance," *Journal of Development Economics*, 2005, *78* (2), 271–298.

Meier, Stephan, "Do subsidies increase charitable giving in the long run? Matching donations in a field experiment," *Journal of the European Economic Association*, 2007, *5* (6), 1203–1222.

_ **and Bruno S Frey**, "Matching donations: Subsidizing charitable giving in a field experiment," *Zurich IEER working paper no. 181*, 2004.

Mersland, Roy and Reidar Øystein Strøm, "Performance and trade-offs in microfinance organisations- Does ownership matter?," *Journal of International Development*, 2008, *20* (5), 598–612.

Morduch, Jonathan, "The microfinance schism," *World development*, 2000, *28* (4), 617–629.

National Philanthropic Trust, "Charitable Giving Statistics," Web, Retrived from `https://www.nptrust.org/philanthropic-resources/charitable-giving-statistics/` July 2019.

Neighbor, Hope, Josh Drake, Tom Eagle, Jessica Vandermark, Bill Wilkie, Robb Willer, Tim Durbin, Salim Haji, and Liz Horberg, "Money for good: Revealing the voice of the donor in philanthropic giving," *Report, Camber Collective, Seattle, USA*, 2015.

Okten, Cagla and Burton A Weisbrod, "Determinants of donations in private nonprofit markets," *Journal of Public Economics*, 2000, *75* (2), 255–272.

Parameshwar, Devyani, Neha Aggarwal, Roberto Zanchi, and Sagar Siva Shankar, "Indian MFIs: Growth for old and new institutions alike," *MicroBanking Bulletin*, 2010, *20*, 1–7.

Potters, Jan, Martin Sefton, and Lise Vesterlund, "After you- Endogenous sequencing in voluntary contribution games," *Journal of Public Economics*, 2005, *89* (8), 1399–1419.

_ , _ , and _ , "Leading-by-example and signaling in voluntary contribution games: An experimental study," *Economic Theory*, 2007, *33* (1), 169–182.

Romano, Richard and Huseyin Yildirim, "Why charities announce donations: A positive perspective," *Journal of Public Economics*, 2001, *81* (3), 423–447.

Rondeau, Daniel and John A List, "Matching and challenge gifts to charity: Evidence from laboratory and natural field experiments," *Experimental Economics*, 2008, *11* (3), 253–267.

Shang, Jen and Rachel Croson, "A field experiment in charitable contribution: The impact of social information on the voluntary provision of public goods," *The Economic Journal*, 2009, *119* (540), 1422–1439.

Silverman, Wendy K, Steven J Robertson, Jimmy L Middlebrook, and Ronald S Drabman, "An investigation of pledging behavior to a national charitable telethon," *Behavior Therapy*, 1984, *15* (3), 304–311.

Soetevent, Adriaan R, "Anonymity in giving in a natural context- A field experiment in 30 churches," *Journal of Public Economics*, 2005, *89* (11-12), 2301–2323.

Varian, Hal R, "Sequential contributions to public goods," *Journal of Public Economics*, 1994, *53* (2), 165–186.

Vesterlund, Lise, "The informational value of sequential fundraising," *Journal of Public Economics*, 2003, *87* (3), 627–657.

Warr, Peter G, "The private provision of a public good is independent of the distribution of income," *Economics Letters*, 1983, *13* (2-3), 207–211.

Wenar, Leif, "John Rawls," in Edward N. Zalta, ed., *The Stanford Encyclopedia of Philosophy*, spring 2017 ed., Metaphysics Research Lab, Stanford University, 2017.

Yildirim, Huseyin, "Andreoni–McGuire algorithm and the limits of warm-glow giving," *Journal of Public Economics*, 2014, *114*, 101–107.

Yörük, Barış K, "Charity ratings," *Journal of Economics & Management Strategy*, 2016, *25* (1), 195–219.

Yunus, Muhammad, *Banker to the poor: Micro-lending and the battle against world poverty*, PublicAffairs, New York, USA, 2007.

APPENDIX A

PROOFS FOR SECTION 2

Lemma A-1. *Let* $g_L^S < G_1^{S,0}(q)$*. Then, total donations* $G^{Z,L}(q,d_L^Z) = G_F^Z(q,d_L^Z)(1+m\mathbb{1}_M) + (1-m\mathbb{1}_M)g_L^S$ *are*

a) strictly increasing in d_L^Z *for all* Z*;*

b) strictly increasing in n *with* $\lim_{n\to\infty} G^{Z,L}(q,d_L^Z) = G_1^{Z,0}(q,m\mathbb{1}_M)$*, where* $G_1^{M,0}(q,0) = G_1^{S,0}(q)$ *and* $\frac{dG_1^{M,0}(q,m)}{dm} > 0$.[1]

Proof. **of Lemma A-1**

The proof of part a) follows a contradiction argument.[2] In particular, let $d_{L,1}^Z < d_{L,2}^Z$, where $g_{L,2}^S < G_1^{S,0}(q)$ (by assumption), and suppose that $G^{Z,L}(q,d_{L,2}^Z) \leq G^{Z,L}(q,d_{L,1}^Z)$. Letting e_1 and e_2 denote the lowest contributing wealth type under $d_{L,1}^Z$ and $d_{L,2}^Z$ respectively, note that by definition $G^{Z,L}(q,d_{L,1}^Z) < G_{e_1}^{Z,0}(q,\mathbb{1}_M m_1)$. Implicit differentiation of eq. (2-3) w.r.t. m results in

$$\frac{dG_i^{M,0}(q,m)}{dm} = -\frac{v'(G_i^{M,0})}{v''(G_i^{M,0})(1+m)} > 0 \tag{A-1}$$

since $v'(\cdot) > 0$ and $v''(\cdot) < 0$. Therefore, $G^{Z,L}(q,d_{L,2}^Z) \leq G^{Z,L}(q,d_{L,1}^Z) < G_{e_1}^{Z,0}(q,\mathbb{1}_M m_1)$ $\leq G_{e_1}^{Z,0}(q,\mathbb{1}_M m_2)$. It follows that all contributors under $d_{L,1}^Z$ should also be contributors under $d_{L,2}^Z$, which in turn implies that $e_2 \geq e_1$. Then, by Lemma 1, the equilibrium total giving by the follower donors is given by:

$$G_F^Z(q,d_{L,y}^Z) = \sum_{j=1}^{e_y} t_j \left[w_j - \phi\left(qv'(G^{Z,L}(q,d_{L,y}^Z))(1+m_y\mathbb{1}_M) \right) \right] \; for\ y = \{1,2\} \tag{A-2}$$

[1]Recall that n denotes the number of replications of the original economy $\mathcal{D}$.

[2]For $Z = S$, the proof is analogous to the proof of Proposition 3 in Yildirim (2014).

where $m_2 > m_1$. Since $\phi(\cdot)$ and $v'(\cdot)$ are both strictly decreasing in their arguments, $e_2 \geq e_1$, $G^{Z,L}(q, d^Z_{L,2}) \leq G^{Z,L}(q, d^Z_{L,1})$, and $m_2 > m_1$ for $Z = M$, eq. (A-2) requires that $G^Z_F(q, d^Z_{L,2}) > G^Z_F(q, d^Z_{L,1})$. This, however, leads to a contradiction since $d^Z_{L,2} > d^Z_{L,1}$ and $G^Z_F(q, d^Z_{L,2}) > G^Z_F(q, d^Z_{L,1})$ imply that $G^{Z,L}(q, d^Z_{L,2}) > G^{Z,L}(q, d^Z_{L,1})$ for all Z. This establishes that $G^{Z,L}(q, d^Z_L)$ is strictly increasing in d^Z_L for all Z.

Turning to part b), proving that $G^{Z,L}(q, d^Z_L)$ is increasing in n is analogous to the proof of part a). Letting $n_2 > n_1 \geq 1$, and e_2 and e_1 denote the corresponding lowest contributing wealth types, suppose that $G^{Z,L}(q, d^Z_L|n_2) \leq G^{Z,L}(q, d^Z_L|n_1) < G^{Z,0}_{e_1}(q, m\mathbb{1}_M)$. The last inequality, in turn, implies that $e_2 \geq e_1$ and the equilibrium total giving by the follower donors is given by:

$$G^Z_F(q, d^Z_L|n_y) = n_y \sum_{j=1}^{e_y} t_j \left[w_j - \phi\left(qv'(G^{Z,L}(q, d^Z_L|n_y))(1 + m\mathbb{1}_M)\right)\right] \; for \; y = \{1, 2\} \qquad \text{(A-3)}$$

Then, $n_2 > n_1$, $e_2 \geq e_1$, and $G^{Z,L}(q, d^Z_L|n_2) \leq G^{Z,L}(q, d^Z_L|n_1)$ immediately imply that $G^Z_F(q, d^Z_L|n_2) > G^Z_F(q, d^Z_L|n_1)$. This, in turn results in a contradiction since $G^{Z,L}(q, d^Z_L|n)$ is strictly increasing in $G^Z_F(q, d^Z_L|n)$, implying that $G^{Z,L}(q, d^Z_L|n_2) > G^{Z,L}(q, d^Z_L|n_1)$. Therefore, $G^{Z,L}(q, d^Z_L)$ is strictly increasing in n.

To prove that $\lim_{n\to\infty} G^{Z,L}(q, d^Z_L) = G^{Z,0}_1(q, \mathbb{1}_M m)$, first note that $\lim_{n\to\infty} G^{Z,L}(q, d^Z_L) > G^{Z,0}_1(q, \mathbb{1}_M m)$ implies that $\lim_{n\to\infty} C^Z = \emptyset$ and $G^{Z,L}(q, d^Z_L) = (1 - \mathbb{1}_M) g^S_L < G^{Z,0}_1(q, \mathbb{1}_M m)$, a contradiction. Alternatively, $\lim_{n\to\infty} G^{Z,L}(q, d^Z_L) < G^{Z,0}_1(q, \mathbb{1}_M m)$ implies that a follower of wealth w_1 contributes a strictly positive amount, that is $\lim_{n\to\infty} g^Z_1(q, d^Z_L) > 0$, which leads to $\lim_{n\to\infty} G^{Z,L}(q, d^Z_L) \geq \lim_{n\to\infty} nt_1 g^Z_1(q, d^Z_L) = \infty$, a contradiction. This establishes that $\lim_{n\to\infty} G^{Z,L}(q, d^Z_L) = G^{Z,0}_1(q, \mathbb{1}_M m)$. Finally, $G^{M,0}_1(q, 0) = G^{S,0}_1(q)$ follows immediately from eq. (2-3) and by eq. (A-1) $\frac{dG^{M,0}_1(q,m)}{dm} > 0$. □

Proof. **of Proposition 2-1**

Lemma A-1 proves the equilibrium properties of $G^{Z,L}(q, d^Z_L)$. Thus, I focus on establishing the equilibrium impact of d^Z_L on $G^Z_F(q, d^Z_L)$.

To prove part a), first note that by Lemma A-1, $g^S_{L,2} > g^S_{L,1}$ implies that $G^{S,L}(q, g^S_{L,1}) < G^{S,L}(q, g^S_{L,2}) < G^{S,0}_{e_2}(q)$, where e_i for $i = \{1,2\}$ denotes the lowest contributing wealth type under $g^S_{L,i}$. Thus, $e_2 \leq e_1$. Then, by eq. (A-2), $G^{S,L}(q, g^S_{L,2}) > G^{S,L}(q, g^S_{L,1})$ and $e_2 \leq e_1$ immediately imply that $G^Z_F(q, g^Z_{L,2}) < G^Z_F(q, g^Z_{L,1})$ since $\phi(\cdot)$ and $v'(\cdot)$ are both strictly decreasing in their arguments.

To prove part b), let $g^M_i(q,m)$ denote the solution to eq. (2-2) for a fixed m, which corresponds to total giving $G^{M,L}(q,m)$. Then, by implicit differentiation of eq. (2-2),

$$\frac{dg^M_i(q,m)}{dm} = -\phi'(qv'(G^{M,L}(q,m))(1+m))q\left(v'(G^{M,L}) + v''(G^{M,L})(1+m)\frac{dG^{M,L}(q,m)}{dm}\right) \quad \text{(A-4)}$$

Eq. (A-4) reveals that $\frac{dg^M_i(q,m)}{dm}$ is independent of w_i and thus identical across all donors with $g^M_i(q,m) \geq 0$. Let N_m denote the number of follower donors with $g^M_i(q,m) \geq 0$ given m. Then, by definition, $\frac{dG^M_F(q,m)}{dm} = N_m \frac{dg^M_i(q,m)}{dm}$. Moreover, recall that $G^{M,L}(q,m) = (1+m)G^M_F(q,m)$. Therefore, $\frac{dG^{M,L}(q,m)}{dm} = G^M_F(q,m) + (1+m)\frac{dG^M_F(q,m)}{dm} = G^F_M(q,m) + (1+m)N_m\frac{dg^M_i(q,m)}{dm}$. Substituting for $\frac{dG^{M,L}(q,m)}{dm}$ in eq. (A-4), and solving for $\frac{dg^M_i(q,m)}{dm}$ obtains:

$$\frac{dg^M_i(q,m)}{dm} = \frac{-\phi'\left(qv'(G^{M,L}(q,m))(1+m)\right)q\left(v'(G^{M,L}) + v''(G^{M,L})G^{M,L}\right)}{1+\phi'\left(qv'(G^{M,L}(q,m)(1+m)\right)qv''(G^{M,L})(1+m)^2 N_m} \quad \text{(A-5)}$$

Since $\phi'(\cdot) < 0$ and $v''(\cdot) < 0$, by eq. (A-5), $\frac{dg^M_i(q,m)}{dm} > 0$ (and thus $\frac{dG^M_F(q,m)}{dm} > 0$) if and only if $\frac{-v''(G^{M,L})G^{M,L}}{v'(G^{M,L})} = \epsilon_v(G^{M,L}) < 1$. □

Proof. **of Proposition 2-2**

Let $G^Z_\infty(q, d^Z_L) = \underset{n\to\infty}{lim} G^{Z,L}(q, d^Z_L)$ and $G^{Z,*}_\infty(q) = \underset{n\to\infty}{lim} G^{Z,L}(q, d^{Z,*}_L)$ where $d^{Z,*}_L(q)$ denotes the equilibrium value of d^Z_L. Then, by Proposition 2-1, $G^Z_\infty(q, d^Z_L) = G^{Z,0}_1(q, m\mathbb{1}_M)$. For $Z = S$, $G^S_\infty(q, g^Z_L) = G^{S,0}_1(q) = G^{S,*}_\infty(q)$ since $G^{S,0}_1(q)$ does not depend on g^S_L. By Lemma A-1, $G^M_\infty(q,m) = G^{M,0}_1(q,m) \geq G^{S,0}_1(q)$ with strict inequality for $m > 0$. To complete the proof, I show that $\lim_{n\to\infty} m^*(q) > 0$ if $\frac{-v''(G^{S,0}_1(q))G^{S,0}_1(q)}{v'(G^{S,0}_1(q))} = \epsilon_v(G^{S,0}_1(q)) < 1$. Note that $\frac{du_L(q,0)}{dm} > 0$ guarantees $m^*(q) > 0$. In the limit economy, by eq. (2-7),

$\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = h'(w_1) \lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm}$ since $G_1^{S,0}(q)$ solves eq. (2-3). Recall that by definition $G_F^M(q,m) = \frac{G^{M,L}(q,m)}{1+m}$. Since $\lim_{n\to\infty} G^{M,L}(q,m) = G_1^{M,0}(q,m)$, $\lim_{n\to\infty} G_F^M(q,m) = \frac{G_1^{M,0}(q,0)}{1+m}$. Therefore, $\lim_{n\to\infty} \frac{dG_F^M(q,m)}{dm} = \frac{1}{1+m}\frac{dG_1^{M,0}(q,m)}{dm} - \frac{G_1^{M,0}(q,m)}{(1+m)^2}$. Substituting for $\frac{dG_1^{M,0}(q,m)}{dm}$ from eq. (A-1) and evaluating at $m = 0$ obtains

$$\lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm} = G_1^{M,0}(q,0)\left(\frac{v'(G_1^{M,0}(q,0))}{-v''(G_1^{M,0}(q,0))G_1^{M,0}(q,0)} - 1\right) \tag{A-6}$$

Taking into account that $G_1^{M,0}(q,0) = G_1^{S,0}(q)$ (see Lemma A-1), eq. (A-6) implies that $\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = h'(w_1) \lim_{n\to\infty} \frac{G_F^M(q,0)}{dm} > 0$ if and only if $\epsilon_v(G_1^{S,0}(q)) < 1$. This completes the proof. □

Proof. **of Lemma 2-2**

Let $\overline{G}_\infty^{Z,*}(q^Z) = \underset{n\to\infty}{lim}\, \overline{G}^{Z,*}(q^Z)$. To prove the statement in Lemma 2-2, it suffices to show that there exists $\overline{n} < \infty$, such that $\overline{G}^{M,*}(q^M) > \overline{G}^{S,*}(q^S)$ for $q^M \geq q^S$ and $n > \overline{n}$. Note that by Proposition 2-1,

$$\overline{G}_\infty^{S,*}(q^S) = \lim_{n\to\infty} G^{S,L}(q^S, \overline{g}_L^{S,*}) = G_1^{S,0}(q^S) \tag{A-7}$$

Moreover, implicitly differentiating eq. (2-3) w.r.t. q^S for $Z = S$ results in $\frac{dG_1^{S,0}(q^S)}{dq^S} = -\frac{v'(G_1^{S,0}(q^S))}{qv''(G_1^{S,0}(q^S))} > 0$, implying that $G_1^{S,0}(q^M) > G_1^{S,0}(q^S)$.

Turning to $Z = M$, note that the optimization problem given by eq. (2-11) implies that $\overline{m}^*(q^M) \geq m^*(q^M) > 0$ (see also Claim 1 in the proof of Lemma A-2). Then, by Proposition 2-1,

$$\overline{G}_\infty^{M,*}(q^M) = G_1^{M,0}(q^M, \overline{m}^*(q^M)) \geq G_1^{M,0}(q^M, m^*(q^M)) = G_\infty^{M,*}(q^M) \tag{A-8}$$

since by eq. (A-1), $G_1^{M,0}(q^M, m)$ is strictly increasing in m. Moreover, by Lemma A-1, $G_1^{M,0}(q^M, m^*(q^M)) > G_1^{S,0}(q^M)$ since $m^*(q^M) > 0$. Thus, (A-7) and (A-8) imply that

$\overline{G}_{\infty}^{M,*}(q^M) > \overline{G}_{\infty}^{S,*}(q^M) > \overline{G}_{\infty}^{S,*}(q^S)$.

To complete the proof, it suffices that $\overline{G}^{Z,*}(q^Z)$ is continuous in n. This follows from the fact that $G_F^Z(q_{\mathcal{F}}^Z, d_L^Z)$ is continuous in n (by eq. (A-3)), implying the continuity of $\overline{u}_L(q^Z, q_{\mathcal{F}}^Z, d_L^Z)$. Therefore, $\overline{G}_{\infty}^{M,*}(q^M) > \overline{G}_{\infty}^{S,*}(q^S)$ implies that there exists $\overline{n} < \infty$ such that $\overline{G}^{M,*}(q^M) > \overline{G}^{S,*}(q^S)$ for all $n > \overline{n}$. □

Lemma A-2. *Let $\mathcal{Q}_L^Z = \{q_1, q_2, ..., q_y\}$ with $q_j > q_{j-1}$ for all $j \in \mathbb{Z} : j \in [2, y]$ denote the set of quality types of the lead donor. Moreover, the equilibrium donation $\overline{d}_L^{Z,*}(q_j)$ by each type $q_j \in \mathcal{Q}_L^Z$ satisfies*

$$\begin{aligned} &\overline{d}_L^{Z,*}(q_j) = \underset{d_L^Z}{argmax}\ \overline{u}_L(q_j, q_j, d_L^Z) \\ s.t. \quad &\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) \geq \overline{u}_L(q_j, \tilde{q}, \overline{d}_L^{Z,*}(\tilde{q})) \text{ for all } q_j, \tilde{q} \in \mathcal{Q}_L^Z \end{aligned} \tag{A-9}$$

Then, $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, \tilde{q}, \overline{d}_L^{Z,*}(\tilde{q}))$ for all $q_j \in \mathcal{Q}_L^Z$ and all $\tilde{q} \in \mathcal{Q}_L^Z \setminus \{q_j, q_{j+1}\}$.

Proof. Note that $\overline{G}^{Z,*}(q_j) = G^{Z,L}(q_j, \overline{d}_L^{Z,*}(q_j))$ and let $\overline{g}_L^{Z,*}(q_j)$ denote the equilibrium donation by L, where by definition donations satisfy $\overline{g}_L^{S,*}(q_j) = \overline{d}_L^{S,*}(q_j)$ and $\overline{g}_L^{M,*}(q_j) = \overline{d}_L^{M,*}(q_j)\overline{G}_F^{M,*}(q_j, \overline{d}_L^{M,*}(q_j))$. The proof proceeds by establishing three claims.

Claim 1: The equilibrium total donation $\overline{G}^{Z,*}(q_j)$ and the lead donor's gift $\overline{g}_L^{Z,*}(q_j)$ are strictly increasing in j.

I first establish that $\overline{G}^{Z,*}(q_j)$ is strictly increasing in j by means of a contradiction. Contrary to Claim 1, suppose that there exists $j \in [2, y]$ such that $\overline{G}^{Z,*}(q_{j-1}) \geq \overline{G}^{Z,*}(q_j)$. The incentive constraint for q_{j-1} given by eq. (A-9) requires

$$\begin{aligned} &\overline{u}_L(q_{j-1}, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1})) \geq \overline{u}_L(q_{j-1}, q_j, \overline{d}_L^{Z,*}(q_j)) \Longrightarrow \\ &q_{j-1}\left[v(\overline{G}^{Z,*}(q_{j-1})) - v(\overline{G}^{Z,*}(q_j))\right] \geq h(w_L - \overline{g}_L^{Z,*}(q_j)) - h(w_L - \overline{g}_L^{Z,*}(q_{j-1})) \end{aligned} \tag{A-10}$$

Consider first the possibility of $\overline{G}^{Z,*}(q_{j-1}) = \overline{G}^{Z,*}(q_j)$. Then, the inequality given by

(A-10) becomes $h(w_L - \overline{g}_L^{Z,*}(q_j)) - h(w_L - \overline{g}_L^{Z,*}(q_{j-1})) \leq 0$, which in turn implies that $\overline{g}_L^{Z,*}(q_j) > \overline{g}_L^{Z,*}(q_{j-1})$ in a separating equilibrium since $h(\cdot)$ is strictly increasing in its argument.[3] It then immediately follows that $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) < \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$, which violates eq. (A-9) for q_j, and contradicts $\overline{g}_L^{Z,*}(q_j)$ being an optimal solution. Therefore, $\overline{G}^{Z,*}(q_{j-1}) = \overline{G}^{Z,*}(q_j)$ cannot occur in a separating equilibrium.

Next, consider $\overline{G}^{Z,*}(q_{j-1}) > \overline{G}^{Z,*}(q_j)$. Then, $[q_j - q_{j-1}]\left[v(\overline{G}^{Z,*}(q_{j-1})) - v(\overline{G}^{Z,*}(q_j))\right]$ > 0 since $v(\cdot)$ is increasing in its argument. Therefore, by (A-10), this implies that

$$q_j\left[v(\overline{G}^{Z,*}(q_{j-1})) - v(\overline{G}^{Z,*}(q_j))\right] > h(w_L - \overline{g}_L^{Z,*}(q_j)) - h(w_L - \overline{g}_L^{Z,*}(q_{j-1})) \Longrightarrow$$
$$\overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1})) > \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$$

The last inequality violates eq. (A-9) for q_j, and again contradicts $\overline{g}_L^{Z,*}(q_j)$ being an optimal solution. Therefore, $\overline{G}^{Z,*}(q_{j-1}) > \overline{G}^{Z,*}(q_j)$ cannot occur in a separating equilibrium. This, in turn, implies that $\overline{G}^{Z,*}(q_j)$ is strictly increasing in q_j.

To establish that $\overline{g}_L^{Z,*}(q_j)$ is strictly increasing in j, note that $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$ is strictly increasing in $\overline{G}^{Z,*}(q_j)$ and strictly decreasing in $\overline{g}_L^{Z,*}(q_j)$. Therefore, $\overline{g}_L^{Z,*}(q_j) \geq \overline{g}_L^{Z,*}(q_{j+1})$ for some j would result in $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) < \overline{u}_L(q_j, q_{j+1}, \overline{d}_L^{Z,*}(q_{j+1}))$ since $\overline{G}^{Z,*}(q_{j+1}) > \overline{G}^{Z,*}(q_j)$. This, in turn, violates eq. (A-9). Therefore, $\overline{g}_L^{Z,*}(q_j)$ must be strictly increasing in j. This completes the proof of Claim 1.

Claim 2: Let $|z| \geq 1$ and $a = \frac{z}{|z|}$. Then, $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) \geq \overline{u}_L(q_j, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z}))$ for any j implies $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a}))$.

[3]For $Z = S$, it is immediately obvious that $\overline{g}_L^{S,*}(q_j) \neq \overline{g}_L^{S,*}(q_{j-1})$ in a separating equilibrium. For $Z = M$, note that $\overline{G}^{M,*}(q_j) = \overline{G}_F^{M,*}(q_j, \overline{m}^*(q_j)) + \overline{m}^*(q_j)\overline{G}_F^{M,*}(q_j, \overline{m}^*(q_j))$. Since $G_F^M(q_j, m)$ is strictly increasing in m and q_j, it follows that $\overline{g}_L^{Z,*}(q_j) = \overline{g}_L^{Z,*}(q_{j-1})$ requires that $\overline{m}^*(q_j) < \overline{m}^*(q_{j-1})$ and $\overline{G}_F^{M,*}(q_j, \overline{m}^*(q_j)) > \overline{G}_F^{M,*}(q_{j-1}, \overline{m}^*(q_{j-1}))$. This, however, results in $\overline{G}^{M,*}(q_j) > \overline{G}^{M,*}(q_{j-1})$, contradicting my conjecture of $\overline{G}^{M,*}(q_j) = \overline{G}^{M,*}(q_{j-1})$. Thus, $\overline{g}_L^{M,*}(q_j) \neq \overline{g}_L^{M,*}(q_{j-1})$.

Note that by definition,

$$\overline{u}_L(q_j, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z})) = \overline{u}_L(q_{j-z}, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z})) + (q_j - q_{j-z})v(\overline{G}^{Z,*}(q_{j-z})) \quad \text{(A-11)}$$

Then, by eq. (A-9),

$$\begin{aligned} \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) &\geq \overline{u}_L(q_{j-z}, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z})) + (q_j - q_{j-z})v(\overline{G}^{Z,*}(q_{j-z})) \\ &\geq \overline{u}_L(q_{j-z}, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a})) + (q_j - q_{j-z})v(\overline{G}^{Z,*}(q_{j-z})) \\ = \overline{u}_L(q_j, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a})) &+ (q_j - q_{j-z})\left[v(\overline{G}^{Z,*}(q_{j-z})) - v(\overline{G}^{Z,*}(q_{j-z-a}))\right] \end{aligned}$$

It then follows that $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a}))$ since $v'(G) > 0$ and by Claim 1, $\overline{G}^{Z,*}(q_j)$ is strictly increasing in q_j, implying that

$$(q_j - q_{j-z})\left[v(\overline{G}^{Z,*}(q_{j-z})) - v(\overline{G}^{Z,*}(q_{j-z-a}))\right] > 0$$

It follows by Claim 2 that all constraints with $|z| > 1$ are non-binding as they are implied by $|z| = 1$. The third, and final, claim shows that the constraint for $z = 1$ is non-binding as well, leaving $z = -1$ as the only possible binding constraint.

Claim 3: $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$ for all $j \in [2, y]$.

Let $D_j(\overline{d}_L^{Z,*}(q_j), \overline{d}_L^{Z,*}(q_{j-1})) = \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) - \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$. By eq. (A-11),

$$\begin{aligned} D_{j-1}(\overline{d}_L^{Z,*}(q_{j-1}), \overline{d}_L^{Z,*}(q_j)) &+ D_j(\overline{d}_L^{Z,*}(q_j), \overline{d}_L^{Z,*}(q_{j-1})) = \quad \text{(A-12)} \\ &= (q_j - q_{j-1})\left[v(\overline{G}^{Z,*}(q_j)) - v(\overline{G}^{Z,*}(q_{j-1}))\right] > 0 \end{aligned}$$

where strict inequality follows from Claim 1. Contrary to Claim 3, suppose that there exists $k \in [2, y]$ such that $D_k(\overline{d}_L^{Z,*}(q_k), \overline{d}_L^{Z,*}(q_{k-1})) = 0$. Thus, from eq. (A-12), it must hold that $D_{k-1}(\overline{d}_L^{Z,*}(q_{k-1}), \overline{d}_L^{Z,*}(q_k)) > 0$. I next show that there exists alternative contribution levels $\tilde{d}_L^Z(q_j)$ for all q_j that satisfy (A-9) and result in $u_L(q_j, q_j, \tilde{d}_L^Z(q_j)) \geq u_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$, with a strict inequality for $j = k$. This contradicts the optimality of $\overline{d}_L^{Z,*}(q_j)$. To establish the existence of such $\tilde{d}_L^Z(q_j)$, let $\tilde{d}_L^Z(q_j)$ satisfy the following properties:

1. for $j < k$, $\tilde{d}_L^Z(q_j) = \overline{d}_L^{Z,*}(q_j)$.

2. for $j \geq k$, I define recursively $\tilde{d}_L^Z(q_j)$ such that if $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) \leq 0$, $\tilde{d}_L^Z(q_j)$ solves $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \tilde{d}_L^Z(q_j)) = 0$; otherwise $\tilde{d}_L^Z(q_j) = \overline{d}_L^{Z,*}(q_j)$.

For $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) > 0$, it is trivial that $u_L(q_j, q_j, \tilde{d}_L^Z(q_j)) = u_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$. For $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) \leq 0$, eq. (A-12) implies that $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \overline{d}_L^{Z,*}(q_j)) > 0$. Therefore, by the continuity of $\overline{G}^{Z,L}(q_j, d_L^Z)$ and $\overline{g}_L^Z(q_j, d_L^Z)$ in d_L^Z, there exists $\tilde{d}_L^Z(q_j)$ with resulting total contributions $\overline{G}^{Z,L}(q_j, \tilde{d}_L^Z(q_j)) \in \left(\overline{G}^{Z,L}(q_{j-1}, \tilde{d}_L^Z(q_{j-1})), \overline{G}^{Z,*}(q_j)\right)$ satisfying $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \tilde{d}_L^Z(q_j)) = 0$. Moreover, by eq. (A-12), $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j-1})) > 0 = D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1}))$, implying that $\overline{u}_L(q_j, q_j, \tilde{d}_L^Z(q_j)) > \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$. This establishes that $u_L(q_j, q_j, \tilde{d}_L^Z(q_j)) \geq u_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$, with a strict inequality for $j = k$ since by construction $D_k(\overline{d}_L^{Z,*}(q_k), \tilde{d}_L^Z(q_{k-1})) = 0$.

It remains to establish that $\tilde{d}_L^Z(q_j)$ satisfies (A-9) for all q_j. By Claim 2, it suffices to show that $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j+z})) \geq 0$ for $z = \{-1, 1\}$. Consider first $z = -1$. Then, if $\tilde{d}_L^Z(q_j) = \overline{d}^{Z,*}(q_j)$, $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j-1})) \geq 0$ follows immediately for $j < k$ since $\tilde{d}_L^Z(q_{j-1}) = \overline{d}_L^{Z,*}(q_{j-1})$ and by assumption $\overline{d}_L^{Z,*}(q_j)$ satisfies eq. (A-9). For $j \geq k$, if $\tilde{d}_L^Z(q_j) = \overline{d}^{Z,*}(q_j)$, then by definition $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) > 0$. For $j \geq k$ and $\tilde{d}_L^Z(q_j) \neq \overline{d}^{Z,*}(q_j)$, $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \tilde{d}_L^Z(q_j)) = 0$ and eq. (A-12) implies $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j-1})) > 0$. Turning our attention to $z = 1$, by construction $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j+1})) = 0$ whenever $D_{j+1}(\overline{d}_L^{Z,*}(q_{j+1}), \tilde{d}_L^Z(q_j)) \leq 0$. Otherwise, if $D_{j+1}(\overline{d}_L^{Z,*}(q_{j+1}), \tilde{d}_L^Z(q_j)) > 0$, then $\tilde{d}_L^Z(q_{j+1}) = \overline{d}_L^{Z,*}(q_{j+1})$, which implies that $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j+1})) \geq 0$ since by my assumption $\overline{d}_L^{Z,*}(q_j)$ satisfies eq. (A-9). However, since I have established that $\overline{u}_L(q_j, q_j, \tilde{d}_L^Z(q_j)) \geq \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$, this implies that $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j+1})) \geq D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j+1})) \geq 0$.

This establishes that $D_k(\overline{d}_L^{Z,*}(q_k), \overline{d}_L^{Z,*}(q_{k-1})) = 0$ for some $k \in [2, y]$ contradicts the optimality of $\overline{d}_L^{Z,*}(q_k)$. Therefore, for all j, $\overline{d}_L^{Z,*}(q_j)$ must satisfy $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$. □

Proof. **of Lemma 2-3**

Recall that $\overline{u}_L^{Z,*}(q_{\mathcal{L}}^Z) = \overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{L}}^Z, \overline{d}_L^{Z,*}(q_{\mathcal{L}}^Z))$, allowing me to re-write eq. (2-13) as

$$V_I^Z(\pi^Z) = \sum_{j=\{l,h\}} \pi_j^Z \left[\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) - \overline{u}_L(q_j, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z))\right] \quad \text{(A-13)}$$

Clearly, $V_I^Z(\pi^Z)$ is continuous in π_h^Z since $\overline{d}_L^{Z,*}(q_U^Z)$ and q_U^Z are continuous functions. Moreover, if $\pi_j^Z = 1$ for some j, then $q_U^Z = q_j$. Thus, $V_I^Z(1,0) = V_I^Z(0,1) = 0$ follows immediately from $\pi_h^Z + \pi_l^Z = 1$. If $\pi_j^Z \in (0,1)$, $V_I^Z(\pi^Z) > 0$ since by Lemma A-2, $\overline{u}_L(q_l, q_l, \overline{d}_L^{Z,*}(q_l)) \geq \overline{u}_L(q_l, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z))$ and $\overline{u}_L(q_h, q_h, \overline{d}_L^{Z,*}(q_h)) > \overline{u}_L(q_h, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z))$. □

Proof. **of Proposition 2-3**

Let $\alpha^{Z,*} = 0$ for all Z. By eq. (2-12), it is immediately obvious that $\overline{G}_E^{Z,*}(q_h, q_U^{Z,*}, 0) = \overline{G}_E^{Z,*}(q_l, q_U^{Z,*}, 0)$ for all Z. Moreover, by definition $\beta^{M,*}(q_j) = 1 - \beta^{S,*}(q_j)$ with $\beta^{Z,*}(q_j) = \underset{\beta^Z}{argmax} \sum_Z \beta^Z \overline{G}_E^{Z,*}(q_j, q_U^{Z,*}, 0)$.

The linearly of the above objective function implies that $\beta^{Z,*}(q_j) \in (0,1)$ if and only if $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, 0) = \overline{G}_E^{S,*}(q_j, q_U^{S,*}, 0)$. Together with $\overline{G}_E^{Z,*}(q_h, q_U^{Z,*}, 0) = \overline{G}_E^{Z,*}(q_l, q_U^{Z,*}, 0)$, this implies that $\overline{G}_E^{M,*}(q_h, q_U^{M,*}, 0) = \overline{G}_E^{M,*}(q_l, q_U^{M,*}, 0) = \overline{G}_E^{S,*}(q_l, q_U^{S,*}, 0) = \overline{G}_E^{S,*}(q_h, q_U^{S,*}, 0)$, completing the proof. □

Proof. **of Proposition 2-4**

First, I show that $\alpha^{Z,*} = 1$ for all Z in the equilibrium path implies $\beta^{S,*}(q_j) = 0$ for all $j \in \{l, h\}$. By means of a contradiction argument, suppose that $\beta^{S,*}(q_j) > 0$ for some j. Then, $\alpha^{S,*} = 1$ implies that $V_I^S(\pi^{S,*}) \geq k > 0$. Then by Lemma 2-3, $\pi_j^{S,*} \in$

$(0,1)$ for all j, which by eq. (2-9) requires $\beta^{S,*}(q_j) > 0$ for all j. Moreover, by eq. (2-12) the expected equilibrium contributions are $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, 1) = \overline{G}^{S,*}(q_j)$. However, since $\overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l) \leq \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$, in such equilibrium $\beta^{S,*}(q_l) = 0$, which in turn implies that $\pi_h^{S,*} = 1$ and $V_I^S(\pi^{S,*}) = 0 < k$, contradicting $\alpha^{S,*} = 1$. Therefore, $\alpha^{S,*} = 1$ requires $\beta^{S,*}(q_j) = 0$ for all j.

To establish the existence of an equilibrium with $\alpha^{M,*} = 1$, note that by eq. (2-9), $\beta^{M,*}(q_j) = 1 - \beta^{S,*}(q_j) = 1$ for all j implies that $\pi^{M,*} = \pi$. Therefore, $\alpha^{M,*} = 1$ requires $V_I^M(\pi) \geq k$. No deviation incentives to $\beta^S(q_j) > 0$ for some j is guaranteed by an off-equilibrium belief $\alpha^{S,*} = 1$ since $\overline{G}^{S,*}(q_j) < \overline{G}^{M,*}(q_j)$ for all j. □

Proof. **of Lemma 2-4**

I first show the existence of a *SPI* equilibrium for $V_I^S(\pi) \geq k$ and $\overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l)$ by constructing such an equilibrium. By Lemma 2-3, $V_I^S(\pi^S)$ is continuous in π_h^S and reaches a minimum at $\pi_h^S = 1$ with $V_I^S(0,1) = 0$. This implies that there exists $\check{\pi}^S$ with $\check{\pi}_h^S > \pi_h$ such that $V_I^S(\check{\pi}^S) = k$. Consider an equilibrium with $\pi^{S,*} = \check{\pi}^S$ and $\beta^{S,*}(q_h) = 1$. Then, by eq. (2-9), $\beta^{S,*}(q_l) = \frac{\pi_h}{\pi_l}\left(\frac{1}{\check{\pi}_h} - 1\right) \leq 1$ and $\pi_l^{M,*} = 1$. It follows that $q_U^{M,*} = q_l$, and $V_I^M(\pi^{M,*}) = 0$ (by Lemma 2-3), implying that $\alpha^{M,*} = 0$. Then, by eq. (2-12), $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, 0) = \overline{G}^{M,*}(q_l)$ for all j. Since $V_I^S(\pi^{S,*}) = k$, the lead donor is indifferent in her information acquisition strategy α^S. To prevent deviation from $\beta^{S,*}(q_l)$, it suffices that $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{M,*}(q_l)$. Substituting for $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*})$ in eq. (2-12) and solving for $\alpha^{S,*}$ results in $\alpha^{S,*} = \frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{M,*}(q_l)}{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{S,*}(q_l)} \in (0,1)$ since $q_U^{S,*} > E[q]$ as a result of $\pi_h^{S,*} > \pi_h$, which implies that $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l) > \overline{G}^{S,*}(q_l)$. Moreover, there is no incentive to deviate from $\beta^{S,*}(q_h)$ since $\overline{G}_E^{S,*}(q_h, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{M,*}(q_l)$.

To establish property 1), note that if $\beta^{S,*}(q_j) = 0$ for some j then by eq. (2-9), $\pi_j^{S,*} = 0$ and thus $V_I^S(\pi^{S,*}) = 0$ with $\alpha^{S,*} = 0$ and $\alpha^{M,*} > 0$ (by Definition 2-1). Then, $q_U^{S,*} = q_{-j}$ where $q_{-j} = \{l, h\} \setminus \{j\}$ and by eq. (2-12), $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_{-j})$ for all j. If $q_j = q_h$ and $q_{-j} = q_l$, then there is strict deviation incentives to $\beta^M(q_l) = 1$ since

$\overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l)$. If $q_j = q_l$ and $q_{-j} = q_h$, $\beta^S(q_h) > 0$ implies that $\overline{G}^{S,*}(q_h) \geq \overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$ due to the fact that $\overline{G}^{M,*}(q_h) > \overline{G}^{M,*}(q_l)$ and $\alpha^{M,*} > 0$. This, in turn implies a profitable deviation to $\beta^S(q_l) = 1$. It follows that $\beta^{S,*}(q_j) = 0$ for some q_j cannot be supported as a *SPI* equilibrium and thus in any *SPI* equilibrium $\beta^{S,*}(q_j) > 0$ for all j.

To show that $\alpha^{M,*} < 1$, note that by eq. (2-12) $\overline{G}_E^{M,*}(q_h, q_U^{M,*}, 1) = \overline{G}^{M,*}(q_h) > \overline{G}^{S,*}(q_h) \geq \overline{G}_E^{S,*}(q_h, q_U^{S,*}, \alpha^{S,*})$. Therefore, $\alpha^{M,*} = 1$ results in $\beta^{S,*}(q_h) = 0$, contradicting property 1). Analogously, $\alpha^{S,*} = 1$ implies that $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, 1) = \overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l) \leq \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$, which in turn implies that $\beta^{S,*}(q_l) = 0$, contradicting property 1).

Finally, to establish that $\alpha^{S,*} > 0$, note that by Definition 2-1, $\alpha^{S,*} = 0$ implies that $\alpha^{M,*} > 0$ and $\beta^{M,*}(q_j) > 0$ for all j by eq. (2-9) and Lemma 2-3. Then, by Lemma 2-2 and eq. (2-12), $\overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$ and $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_U^{S,*})$ for all j. Since by property 1) $\beta^{S,*}(q_j) > 0$ for all j, it must be true that $\overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$, which implies that $\beta^{M,*}(q_l) = 0$, contradicting $\beta^{M,*}(q_j) > 0$ for all j. This completes the proof. □

Proof. **of Proposition 2-5**

To establish that $q_U^{S,*} > q_U^{M,*}$, by Lemma 2-2, it suffices to show that $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{M,*}(q_U^{M,*})$. By means of a contradiction, suppose that $\overline{G}^{S,*}(q_U^{S,*}) \leq \overline{G}^{M,*}(q_U^{M,*})$. By Lemma 2-4, $\beta^{S,*}(q_j) > 0$ for all j, which requires $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. Let $\alpha^{M,*} = \alpha^{S,*} + \epsilon$. Then, $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ can be re-written as

$$\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) = \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) + \epsilon \left[\overline{G}^{M,*}(q_j) - \overline{G}^{M,*}(q_U^{M,*})\right]$$

Note that $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) > \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})$ since $\overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{S,*}(q_U^{S,*})$ (by assumption), $\overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$ (by Lemma 2-2), and $\alpha^{S,*} > 0$ (by Lemma 2-4). Thus, $\overline{G}_E^{S,*}(q_h, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*})$ requires $\epsilon < 0$ since $\overline{G}^{M,*}(q_h) > \overline{G}^{M,*}(q_U^{M,*})$.

But this results in $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*}) < \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$ since $\overline{G}^{M,*}(q_l) < \overline{G}^{M,*}(q_U^{M,*})$, contradicting $\beta^{S,*}(q_l) > 0$. Therefore, $\overline{G}^{S,*}(q_U^{S,*}) \leq \overline{G}^{M,*}(q_U^{M,*})$ must hold in any *SPI* equilibrium. By Lemma 2-2, this implies that $q_U^{S,*} > q_U^{M,*}$. This, in turn, requires that $\pi_h^{S,*} > \pi_h^{M,*}$, which coupled with $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ for all j and with $\overline{G}_E^{Z,*}(q_h, q_U^{Z,*}, \alpha^{Z,*}) > \overline{G}_E^{Z,*}(q_l, q_U^{Z,*}, \alpha^{Z,*})$ for all Z results in $\sum_j \pi_j^{S,*}\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) > \sum_j \pi_j^{M,*}\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ for $j \in \{l, h\}$. □

Proof. **of Proposition 2-6**

1) is established in the proof of Lemma 2-4. To establish 2), note that by Lemma 2-3, $V_I^Z(\pi^Z)$ is continuous in π_h^Z and satisfies $V_I^Z(1,0) = V_I^Z(0,1) = 0$. Therefore, since $V_I^Z(\pi) > k$, there exist $\underline{\pi}_h^Z$ and $\overline{\pi}_h^Z$ satisfying $0 < \underline{\pi}_h^Z < \pi_h < \overline{\pi}_h^Z < 1$ such that $V_I^Z(\underline{\pi}_h^Z, 1 - \underline{\pi}_h^Z) = V_I^Z(\overline{\pi}_h^Z, 1 - \overline{\pi}_h^Z) = k$ for all Z. Let $\hat{\pi}_h^S = \overline{\pi}_h^S$ and $\hat{\pi}_h^M = \underline{\pi}_h^M$. Then, substituting for $\hat{\pi}_h^S$ and $\hat{\pi}_h^M$ in eq. (2-9) and taking into account that $\beta^M(q_j) = 1 - \beta^S(q_j)$ yields $\beta^{S,*}(q_j)$ for $j \in \{l, h\}$ given by eq. (2-16). Moreover, $0 < \beta^{S,*}(q_l) < \beta^{S,*}(q_h) < 1$ follows immediately from $\hat{\pi}_h^M < \pi_h < \hat{\pi}_h^S$. Lastly, I need to ensure that there are no deviation incentives from $\beta^{Z,*}(q_j)$, which requires $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ for all j. Solving for $\alpha^{M,*}$ and $\alpha^{S,*}$ yields:

$$\alpha^{M,*} = \frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{M,*}(q_U^{M,*})}{\overline{G}^{M,*}(q_h)\frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{S,*}(q_l)}{\overline{G}^{S,*}(q_h) - \overline{G}^{S,*}(q_l)} + \overline{G}^{M,*}(q_l)\frac{\overline{G}^{S,*}(q_h) - \overline{G}^{S,*}(q_U^{S,*})}{\overline{G}^{S,*}(q_h) - \overline{G}^{S,*}(q_l)} - \overline{G}^{M,*}(q_U^{S,*})} \in (0,1)$$

$$\alpha^{S,*} = \frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{M,*}(q_U^{M,*})}{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{S,*}(q_h)\frac{\overline{G}^{M,*}(q_U^{M,*}) - \overline{G}^{M,*}(q_l)}{\overline{G}^{M,*}(q_h) - \overline{G}^{M,*}(q_l)} - \overline{G}^{S,*}(q_l)\frac{\overline{G}^{M,*}(q_h) - \overline{G}^{M}(q_U^{M,*})}{\overline{G}^{M,*}(q_h) - \overline{G}^{M,*}(q_l)}} \in (0,1)$$

where $\alpha^{Z,*} \in (0,1)$ follows immediately from the following equilibrium property established by Lemma 2-2 and Proposition 2-5:

$$\overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l) < \overline{G}^{M}(\hat{q}_U^M) < \overline{G}^{S,*}(\hat{q}_U^S) < \overline{G}^{S,*}(q_h) < \overline{G}^{M,*}(q_h).$$

This completes the proof. $\square$

Proof. **of Lemma 2-5**

First, I establish property 2). Note that if $\pi_j^{S,*} = 1$ for some j, then $\pi_{-j}^{S,*} = 0$ where $-j \in \{1, 2..., t\} \setminus \{j\}$ and $q_U^{S,*} = q_j$. Consequently, from eq. (2-17), $V_I^S(\pi^{S,*}) = 0$ with $\alpha^{S,*} = 0$ and $\alpha^{M,*} > 0$ (by Definition 2-1). Then, by eq. (2-12), $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_j)$ for all j. If $j = 1$, then there is a strict deviation incentive to $\beta^M(q_1) = 1$ since $\overline{G}^{S,*}(q_1) < \overline{G}^{M,*}(q_1) \leq \overline{G}_E^{M,*}(q_1, q_U^{M,*}, \alpha^{M,*})$. If $j > 1$, then $\beta^S(q_j) > 0$ implies that $\overline{G}^{S,*}(q_j) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_{j-1}, q_U^{M,*}, \alpha^{M,*})$ due to the fact that $\overline{G}^{M,*}(q_j) > \overline{G}^{M,*}(q_{j-1})$ and $\alpha^{M,*} > 0$. This, in turn, implies a profitable deviation to $\beta^S(q_{j-1}) = 1$, contradicting $\pi_{j-1}^{S,*} = 0$ (by eq. 2-9). Thus, it follows that in any *SPI* equilibrium, $\pi_j^{S,*} < 1$ for all j.

To show that $\alpha^{M,*} < 1$, consider the contrary- $\alpha^{M,*} = 1$. Then, for any type q_j satisfying $\beta^S(q_j) > 1$ and $q_j \geq q_U^{S,*}$, eq. (2-12) and Lemma 2-2 imply that $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) = \overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j) \geq \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})$. This results in strict deviation incentives to $\beta^M(q_j) = 1$. Thus, in every *SPI* equilibrium, $\alpha^{M,*} < 1$.

Finally, to establish that $\alpha^{S,*} > 0$, by means of a contradiction suppose that $\alpha^{S,*} = 0$, which by Definition 2-1 implies that $\alpha^{M,*} > 0$. Consider type q_j such that $\beta^{S,*}(q_j) > 0$ and $q_j \geq q_U^{S,*}$. By Definition 2-1, the existence of such type is guaranteed. Note that $\alpha^{S,*} = 0$ and $\beta^{S,*}(q_j) > 0$ imply that $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. Moreover, since $\overline{G}^{Z,*}(q_j)$ is strictly increasing in q_j, $q_j \geq q_U^{S,*}$ implies that $\overline{G}^{S,*}(q_j) \geq \overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. The last inequality requires $q_j > q_U^{M,*}$. To see this, note that by Lemma 2-2, $\overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$ and $q_j \leq q_U^{M,*}$ implies that $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) \geq \overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$, which contradicts the earlier inequality. Thus, $q_j > q_U^{M,*} = \sum_j \pi_j^{M,*} q_j$. This, in turn, implies the existence of a quality type $q_i < q_U^{M,*}$ with $\pi_i^{M,*} > 0$, which by eq. (2-9) requires $\beta^{M,*}(q_i) > 0$. However, $\alpha^{M,*} > 0$ implies that $\overline{G}_E^{M,*}(q_i, q_U^{M,*}, \alpha^{M,*}) < \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) \leq \overline{G}^{S,*}(q_U^{S,*}) = \overline{G}_E^{S,*}(q_i, q_U^{S,*}, \alpha^{S,*})$, contradicting the optimality of

$\beta^{M,*}(q_i) > 0$. Thus, $\alpha^{S,*} = 0$ and $\alpha^{M,*} > 0$ leads to a profitable deviation by some $q_i < q_U^{M,*}$ and thus cannot be supported in a *SPI* equilibrium. This proves that $\alpha^{S,*} > 0$ in every *SPI* equilibrium. $\square$

Proof. **of Proposition 2-7**

Analogous to Proposition 2-5, in order to establish that $q_U^{S,*} > q_U^{M,*}$, by Lemma 2-2, it suffices to show that $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{M,*}(q_U^{M,*})$.[4] By a contradiction argument, suppose that $\overline{G}^{S,*}(q_U^{S,*}) \leq \overline{G}^{M,*}(q_U^{M,*})$. Since $\pi_j^{S,*} < 1$ for all j (Lemma 2-5), by eq. (2-9) there exist at least two quality types, denoted by $q_{\hat{l}}$ and $q_{\hat{h}}$, such that $\beta^{S,*}(q_j) > 0$ for all $j \in \{\hat{l}, \hat{h}\}$ and $q_{\hat{l}} < q_U^{S,*} < q_{\hat{h}}$. Moreover, for all $j \in \{\hat{l}, \hat{h}\}$, $\beta^{S,*}(q_j) > 0$ implies $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. If $q_U^{M,*} \geq q_{\hat{h}}$ then by Lemma 2-2, $\overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{M,*}(q_{\hat{h}}) > \overline{G}^{S,*}(q_{\hat{h}}) > \overline{G}^{S,*}(q_U^{S,*})$, which implies $\overline{G}_E^{M,*}(q_{\hat{h}}, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{S,*}(q_{\hat{h}}, q_U^{S,*}, \alpha^{S,*})$, a contradiction. Thus $q_{\hat{h}} > q_U^{M,*}$. If $q_U^{M,*} \leq q_{\hat{l}}$ then $\overline{G}^{M,*}(q_{\hat{l}}) \geq \overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{S,*}(q_{\hat{l}})$. Since by Lemma 2-5 $\alpha^{S,*} > 0$, this gives $\overline{G}_E^{M,*}(q_{\hat{l}}, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{S,*}(q_{\hat{l}}, q_U^{S,*}, \alpha^{S,*})$, a contradiction. Thus, $q_{\hat{l}} < q_U^{M,*} < q_{\hat{h}}$. Let $\alpha^{M,*} = \alpha^{S,*} + \epsilon$. Then, $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ can be rewritten as

$$\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) = \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) + \epsilon \left[\overline{G}^{M,*}(q_j) - \overline{G}^{M,*}(q_U^{M,*})\right]$$

From here, the proof is analogs to that of Proposition 2-5. The assumption, $\overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{S,*}(q_U^{S,*})$ and $\alpha^{S,*} > 0$ (by Lemma 2-5) imply $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) > \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})$. Therefore, $\overline{G}_E^{S,*}(q_{\hat{h}}, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_{\hat{h}}, q_U^{M,*}, \alpha^{M,*})$ requires $\epsilon < 0$. However, this results in $\overline{G}_E^{S,*}(q_{\hat{l}}, q_U^{S,*}, \alpha^{S,*}) < \overline{G}_E^{M,*}(q_{\hat{l}}, q_U^{M,*}, \alpha^{M,*})$, a contradiction. Thus, in any *SPI* equilibrium, $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{M,*}(q_U^{M,*})$. $\square$

Lemma A-3. *For $Z = N$, let $q_{\mathcal{L}}^N \in \{q_l, q_U^N, q_h\}$ denote the leader's quality type, $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)$ $= \lim_{n\to\infty} \overline{G}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)$ denote the total contributions in the limit economy given $q_{\mathcal{L}}^N$ and*

[4]It is straightforward to establish that Lemma 2-2 extends to the case of multiple quality types.

the posterior expected quality q_U^N*, and* $C_\infty^N = \lim_{n\to\infty} C^N$ *denote the equilibrium contributor's set in the limit economy. Then, in the limit economy (*$n \to \infty$*):*

a) any donor with $w_i < w_1$ *is a non-contributor, i.e.* $i \notin C_\infty^N$ *for any* $i > 1$*;*

b) the total equilibrium donations satisfy $\overline{G}_\infty^{S,*}(q_l) \leq \overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N) \leq \overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) \leq \overline{G}_\infty^{S,*}(q_h)$*, with strict inequalities for* $q_U^N \in (q_l, q_h)$*. Moreover,* $\overline{G}_\infty^{S,*}(q_l) = \overline{G}_\infty^{N,*}(q_l, q_l, \alpha^N)$ *and* $\overline{G}_\infty^{S,*}(q_h) = \overline{G}_\infty^{N,*}(q_h, q_h, \alpha^N)$*.*

Proof. **of Lemma A-3**

Analogous to $Z = \{S, M\}$, let $\eta_{\mathcal{L}}^N$ denote the posterior belief by the follower donors of type $q_{\mathcal{L}}^N$ upon observing $Z = N$. Then, given a conjecture about others' contributions $\overline{G}_{-i}^N$, a contributing donor i maximizes

$$E_{q_{\mathcal{L}}^N}[u_i(g_i, \overline{G}_{-i}^N, q_{\mathcal{L}}^N)] = h(w_i - g_i) + \sum_{\mathcal{L}} \eta_{\mathcal{L}}^N q_{\mathcal{L}}^N v\left(\overline{G}_{-i}^N + g_i\right) \quad \text{(A-14)}$$

Therefore, i's giving for $Z = N$ satisfies[5]

$$h'(w_i - \overline{g}_i^{N,*}) = \sum_{\mathcal{L}} \eta_{\mathcal{L}}^N q_{\mathcal{L}}^N v'\left(\overline{G}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)\right) \quad \text{(A-15)}$$

Analogous to $Z = \{S, M\}$, $\overline{g}_i^{N,*}(q_U^N, \alpha^N)$ is (weakly) increasing in w_i. Therefore, $i \in C^N$, implies that $i - 1 \in C^N$. To establish a), note that $i \in C_\infty^N$ for $i > 1$, implies that $\overline{g}_{1,\infty}^{N,*}(q_U^N, \alpha^N) > 0$ and $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}, q_U^N, \alpha^N) \geq \lim_{n\to\infty} nt_1 \overline{g}_{1,\infty}^{N,*}(q_U^N, \alpha^N) = \infty$. This, in turn, leads to a contradiction since $\lim_{n\to\infty} qv'(G) = 0$ and by eq. (A-15) this implies that $\overline{g}_{i,\infty}^{N,*}(q_U^N, \alpha^N) = 0$ for all i and thus $C_\infty^N = \emptyset$. Therefore, $i \notin C_\infty^N$ for any $i > 1$.

To establish part b), note that $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}, q_U^N, \alpha^N) = \overline{G}_{F,\infty}^{N,*}(q_U^N, \alpha^N) + \overline{g}_{L,\infty}^{N,*}(q_{\mathcal{L}}, q_U^N, \alpha^N)$, where the first term denotes the follower donors' total contributions, which are indepen-

[5]Note that there is a one-to-one relationship between π_h^N and q_U^N, allowing me to express the optimal equilibrium contributions $\overline{G}_\infty^{N,*}(q_j, q_U^N, \alpha^N)$ as a function of q_U^N.

dent of the unobservable type $q_{\mathcal{L}}$, and the second term denotes the lead donor's contribution. If $1 \in C^N_\infty$, then analogous to the argument above, $\lim_{n\to\infty} \overline{g}^{N,*}_{1,\infty}(q^N_U, \alpha^N) = 0$, which by eq. (A-15) implies the following equilibrium condition:

$$\begin{aligned} h'(w_1) &= \alpha^N \left(\pi^N_l q_l v'(\overline{G}^{N,*}_\infty(q_l, q^N_U, \alpha^N)) + \pi^N_h q_h v'(\overline{G}^{N,*}_\infty(q_h, q^N_U, \alpha^N))\right) + \\ &+ (1-\alpha^N) q^N_U v'(\overline{G}^{N,*}_\infty(q^N_U, q^N_U, \alpha^N)), \end{aligned} \quad \text{(A-16)}$$

where I have used eq. (2-10) to substitute for $\eta^N_{\mathcal{L}}$. Since the lead donor's contribution is unobservable for $Z = N$, $\overline{g}^{N,*}_{L,\infty}(q^N_{\mathcal{L}}, q^N_U, \alpha^N)$ maximizes eq. (2-1) resulting in

$$h'(w_1 - \overline{g}^{N,*}_{L,\infty}(q^N_{\mathcal{L}}, q^N_U, \alpha^N)) \geq q^N_{\mathcal{L}} v'(\overline{G}^{N,*}_\infty(q^N_{\mathcal{L}}, q^N_U, \alpha^N)) \quad \text{(A-17)}$$

where inequality in eq. (A-17) is strict if and only if $h'(w_1) > q^N_{\mathcal{L}} v'(\overline{G}^{N,*}_\infty(q^N_{\mathcal{L}}, q^N_U, \alpha^N))$. Moreover, by eq. (A-17), $q^N_{\mathcal{L}} v'(\overline{G}^N_{F,\infty}(q^N_U, \alpha^N) + \overline{g}^N_L) > \hat{q}^N_{\mathcal{L}} v'(\overline{G}^N_{F,\infty}(q^N_U, \alpha^N) + \overline{g}^N_L)$ for any $q^N_{\mathcal{L}} > \hat{q}^N_{\mathcal{L}}$ and thus $h''(\cdot) < 0$ implies that $\overline{g}^{N,*}_{L,\infty}(q^N_{\mathcal{L}}, q^N_U, \alpha^N) \geq \overline{g}^{N,*}_{L,\infty}(\hat{q}^N_{\mathcal{L}}, q^N_U, \alpha^N)$ and also $\overline{G}^{N,*}_\infty(q^N_{\mathcal{L}}, q^N_U, \alpha^N) \geq \overline{G}^{N,*}_\infty(\hat{q}^N_{\mathcal{L}}, q^N_U, \alpha^N)$, with strict inequality if $\overline{g}^{N,*}_{L,\infty}(q^N_{\mathcal{L}}, q^N_U, \alpha^N) > 0$.

To compare $\overline{G}^{N,*}_\infty(q_j, q^N_U, \alpha^N)$ and $\overline{G}^{S,*}_\infty(q_j)$ for $j = \{l, h\}$, recall from eq. (A-7) that $\overline{G}^{S,*}_\infty(q_j) = G^{S,0}_1(q_j)$, where $G^{S,0}_1(q_j)$ solves eq. (2-3). Since $h''(\cdot) < 0$, by eq. (2-3) and eq. (A-17), it follows that $\overline{g}^{N,*}_{L,\infty}(q_j, q^N_U, \alpha^N) > 0$ implies

$$q_j v'(\overline{G}^{N,*}_\infty(q_j, q^N_U, \alpha^N)) = h'(w_1 - \overline{g}^N_{L,\infty}(q_j, q^N_U, \alpha^N)) > h'(w_1) = q_j v'(\overline{G}^{S,*}_\infty(q_j)) \quad \text{(A-18)}$$

Since $v''(\cdot) < 0$, the above inequality results in $\overline{G}^{N,*}_\infty(q_j, q^N_U, \alpha^N) < \overline{G}^{S,*}_\infty(q_j)$.

To establish that $\overline{G}^{N,*}_\infty(q_h, q^N_U, \alpha^N) < \overline{G}^{S,*}_\infty(q_h)$ for $q^N_U < q_h$, by (A-18) it suffices to show that $\overline{g}^{N,*}_{L,\infty}(q_h, q^N_U, \alpha^N) > 0$. Since $\overline{g}^{N,*}_{L,\infty}(q_h, q^N_U, \alpha^N) \geq \overline{g}^{N,*}_{L,\infty}(q_e, q^N_U, \alpha^N)$ for $e \in \{l, U\}$, $\overline{g}^{N,*}_{L,\infty}(q_h, q^N_U, \alpha^N) = 0$ implies by eq. (A-16) that $\overline{G}^{N,*}_\infty(q_{\mathcal{L}}, q^N_U, \alpha^N)$ is independent of $q_{\mathcal{L}}$ and α^N and solves $h'(w_1) = q^N_U v'(\overline{G}^{N,*}_\infty(q^N_U))$ for all $q_{\mathcal{L}}$. However, by eq. (A-17), this results in a contradiction since $h'(w_1) < q_h v'(\overline{G}^{N,*}_\infty(q^N_U))$ for $q^N_U < q_h$ implies a profitable deviation to $\overline{g}^N_{L,\infty}(q_h, q^N_U, \alpha^N) > 0$. Therefore, for $q^N_U < q_h$, $\overline{g}^N_{L,\infty}(q_h, q^N_U, \alpha^N) > 0$ and by (A-18), $\overline{G}^{N,*}_\infty(q_h, q^N_U, \alpha^N) < \overline{G}^{S,*}_\infty(q_h)$. For $q^N_U = q_h$, $\overline{g}^{N,*}_{L,\infty}(q_h, q_h, \alpha^N) = 0$ and $h'(w_1) = q_h v'(\overline{G}^{N,*}_\infty(q_h, q_h, \alpha^{N,*}))$ satisfies eqs. (A-16) and (A-17), which by (A-18) results in $\overline{G}^{N,*}_\infty(q_h, q^N_U, \alpha^N) = \overline{G}^{S,*}_\infty(q_h)$.

To establish that $\overline{G}^{N,*}_\infty(q_l, q^N_U, \alpha^N) > \overline{G}^{S,*}_\infty(q_l)$ for $q^N_U > q_l$, note that by eq. (2-3) and (A-18), $\overline{G}^{N,*}_\infty(q_l, q^N_U, \alpha^N) \leq \overline{G}^{S,*}_\infty(q_l)$ requires that $q_l v'(\overline{G}^{N,*}_\infty(q_l, q^N_U, \alpha^N)) = h'(w_1 - \overline{g}^N_{L,\infty}(q_l, q^N_U, \alpha^N))$. Since $\overline{g}^{N,*}_{L,\infty}(q_\mathcal{L}, q^N_U, \alpha^N)$ is increasing in $q_\mathcal{L}$, this implies that eq. (A-17) holds with equality for all $q_\mathcal{L}$. Substituting for eq. (A-17) into eq. (A-16) results in

$$\begin{aligned} h'(w_1) &= \alpha^N \left(\pi^N_l q_l h'(w_1 - \overline{g}^{N,*}_{L,\infty}(q_l, q^N_U, \alpha^N)) + \pi^N_h h'(w_1 - \overline{g}^{N,*}_{L,\infty}(q_h, q^N_U, \alpha^N)) \right) + \\ &+ (1 - \alpha^N) h'(w_1 - \overline{g}^{N,*}_{L,\infty}(q^N_U, q^N_U, \alpha^N)). \end{aligned}$$

For $q^N_U > q_l$, requiring $\pi^N_h > 0$, the above equation is violated since $\overline{g}^{N,*}_{L,\infty}(q_h, q^N_U, \alpha^N) > \overline{g}^{N,*}_{L,\infty}(q^N_U, q^N_U, \alpha^N) > 0$. Therefore, for $q^N_U > q_l$, $\overline{G}^{N,*}_\infty(q_l, q^N_U, \alpha^N) \leq \overline{G}^{S,*}_\infty(q_l)$ cannot be supported in equilibrium, implying that $\overline{G}^{N,*}_\infty(q_l, q^N_U, \alpha^N) > \overline{G}^{S,*}_\infty(q_l)$. For $q^N_U = q_l$, $\pi^N_h = 0$ and $\overline{g}^{N,*}_{L,\infty}(q^N_U, q^N_U, \alpha^N) = \overline{g}^{N,*}_{L,\infty}(q_l, q^N_U, \alpha^N) = 0$ solve eq. (A-16) and eq. (A-17) with equalities, resulting in $\overline{G}^{N,*}_\infty(q_l, q_l, \alpha^N) = \overline{G}^{S,*}_\infty(q_l)$.

Finally, to complete the proof, I establish that $1 \notin C^N_\infty$ cannot be sustained in equilibrium. In that case, by part a), $C^N_\infty = \emptyset$, which in turn implies $\overline{G}^{N,*}_{F,\infty}(q^N_U, \alpha^N) = 0$. Then, eq. (A-17) holds with equality for all $q_\mathcal{L}$ and thus $\overline{g}^{N,*}_{L,\infty}(q_\mathcal{L}) > 0$ for all $q_\mathcal{L}$ since $h'(w_1) > q_\mathcal{L} v'(0)$. It follows that $h'(w_1) < q_\mathcal{L} v'(\overline{G}^{N,*}_\infty(q_\mathcal{L}))$ for all $q_\mathcal{L}$, which by eq. (A-15) implies that $\overline{g}^N_{1,\infty}(q^N_U, \alpha^N) > 0$, contradicting $C^N_\infty = \emptyset$. Therefore, $1 \in C^N_\infty$. □

Proof. **of Lemma 2-6**

To prove that $\beta^{S,*}(q_j) = 0$ for all j, analogous to the proof of Proposition 2-4, suppose by means of a contradiction that $\beta^{S,*}(q_j) > 0$ for some j. This, in turn, implies that $\beta^{S,*}(q_j) > 0$ for all j since $\alpha^{S,*} = 1$ requires $V^S_I(\pi^{S,*}) \geq k > 0$, which by Lemma 2-3 necessitates $\pi_h^{S,*} \in (0,1)$. Moreover, by eq. (2-12), $\overline{G}_{E,\infty}^{S,*}(q_l, q_U^{S,*}, 1) = \overline{G}_\infty^{S,*}(q_l)$. However, this results in a profitable deviation to $\beta^S(q_l) = 0$ since $\overline{G}_\infty^{S,*}(q_l) < \overline{G}_\infty^{M,*}(q_l) \leq \overline{G}_{E,\infty}^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$. Therefore, $\beta^{S,*}(q_j) > 0$ for some j cannot be supported in equilibrium, implying that $\beta^{S,*}(q_j) = 0$ for all j.

To complete the proof, note that $\beta^{M,*}(q_j) > 0$ for some q_j, requires $\alpha^{M,*} = 1$ in a fully informed equilibrium. Then, by eq. (2-12) and Lemmas 2-2 and A-3, $\overline{G}_{E,\infty}^{M,*}(q_h, q_U^{M,*}, 1) = \overline{G}_\infty^{M,*}(q_h) > \overline{G}_\infty^{S,*}(q_h) \geq \overline{G}_\infty^{N,*}(q_h, q_U^{N,*}, 1)$, implying that $\beta^{N,*}(q_h) = 0$. Then, if $\beta^{N,*}(q_l) > 0$, it follows by eq. (9) that $\pi_l^{N,*} = 1$. By Lemmas 2-2 and A-3 this implies $\overline{G}_\infty^{N,*}(q_l, q_l, 1) = \overline{G}_\infty^{S,*}(q_l) < \overline{G}_\infty^{M,*}(q_l)$, contradicting $\beta^{N,*}(q_l) > 0$. Therefore, $\beta^{M,*}(q_j) > 0$ for some q_j implies $\beta^{N,*}(q_j) = 0$ for all q_j. Conversely, $\beta^{N,*}(q_j) > 0$ for some q_j requires $\beta^{M,*}(q_j) = 0$. Since $\beta^{S,*}(q_j) = 0$ for all j, this implies that $\beta^{N,*}(q_j) = 1$ for all j. Such equilibrium is possible as long as $\overline{G}_\infty^{N,*}(q_l, q_U, 1) > \overline{G}^{M,*}(q_l)$ and is supported with an off-equilibrium belief of $\pi_h^{M,*} = 0$. □

Proof. **of Proposition 2-8**

Proving that $q_U^{S,*} > q_U^{M,*}$ is analogous to the proof of Proposition 2-5. Similar to Lemma 2-4, I first establish that $\beta^{S,*}(q_j) > 0$ for all q_j and $\alpha^{S,*} > 0$. In particular, $\beta^{S,*}(q_j) > 0$ follows from the fact that $\beta^{S,*}(q_j) = 0$ for some j implies that $V_I^S(\pi^{S,*}) = 0$, resulting in $\alpha^{S,*} = 0$. This, in turn, implies $q_U^{S,*} = q_{-j}$ where $q_{-j} = \{l, h\} \setminus \{j\}$ and by eq. (2-12), $\overline{G}_{E,\infty}^{S,*}(q_{-j}, q_{-j}, 0) = \overline{G}_\infty^{S,*}(q_{-j})$. If $-j = l$, then there is a profitable deviation to $\beta^{S,*}(q_l) = 0$ since $\overline{G}_\infty^{S,*}(q_l) < \overline{G}_\infty^{M,*}(q_l) \leq \overline{G}_{E,\infty}^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$. If $-j = h$, then $\beta^{S,*}(q_h) > 0$ implies that $\overline{G}_\infty^{S,*}(q_h) \geq \overline{G}_{E,\infty}^{\tilde{Z},*}(q_h, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*}) > \overline{G}_{E,\infty}^{\tilde{Z},*}(q_l, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*})$ for $\tilde{Z} = \{M, N\}$ since $\overline{G}_\infty^{\tilde{Z},*}(q_h) > \overline{G}_\infty^{\tilde{Z},*}(q_l)$ for all $\tilde{Z}$. This, in turn implies a profitable deviation to $\beta^S(q_l) = 1$. Consequently, in any *SPI* equilibrium $\beta^{S,*}(q_j) > 0$ for all $j = \{l, h\}$.

To establish $\alpha^{S,*} > 0$, note that $\alpha^{S,*} = 0$ implies by eq. (2-12) that $\overline{G}_{E,\infty}^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}_\infty^{S,*}(q_U^{S,*})$ for all $j = \{l, h\}$. Moreover by Definition 2-1, $\alpha^{\tilde{Z},*} > 0$ for some $\tilde{Z}$, implying that $\beta^{\tilde{Z},*}(q_j) > 0$ for all j. However, $\overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_{E,\infty}^{\tilde{Z},*}(q_h, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*}) > \overline{G}_{E,\infty}^{\tilde{Z},*}(q_l, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*})$. The last strict inequality implies a profitable deviation to $\beta^S(q_l) = 1$, implying that $\alpha^{S,*} = 0$ cannot be supported in equilibrium. Consequently, in any *SPI* equilibrium $\alpha^{S,*} > 0$. Given $\alpha^{S,*} > 0$ and $\beta^{S,*}(q_j) > 0$ for all $j = \{l, h\}$, establishing that $q_U^{S,*} > q_U^{M,*}$ is identical to the proof of Proposition 2-5 and thus omitted here.

To establish the existence of a *SPI* equilibrium with $q_U^{S,*} > \max\{q_U^{N,*}, q_U^{M,*}\}$, consider the equilibrium constructed in the proof of Lemma 2-4 with $\beta^{N,*}(q_j) = 0$ for all j and an off-equilibrium belief of $\pi_h^{N,*} = 0$, implying $q_U^{N,*} = q_l$. Then, it is straightforward to verify that $V_I^N(0,1) = 0$, resulting in $\alpha^{N,*} = 0$. Then, by Lemma A-3, $\overline{G}_\infty^{N,*}(q_l, q_l, 0) = \overline{G}_\infty^{S,*}(q_l) \leq \overline{G}_{E,\infty}^{Z,*}(q_j, q_U^{Z,*}, \alpha^{Z,*})$ for $Z = \{S, M\}$ implying no profitable deviation to N. □

Proof. **of Proposition 2-9**

Let $\tilde{G}_\infty^{Z,*}(q) = \lim_{n\to\infty} \tilde{G}^{Z,*}(q)$. To establish part a), I first show that $\lim\limits_{\tilde{G}\to\infty} qv_g(G, 0) < h'(w_1)$ is necessary and sufficient for the existence of $\tilde{G}_i^{Z,0}(q, m\mathbb{1}_M) < \infty$ for all i that solves eq. (2-20). Recall $v_{GG}(\cdot) < 0$, and $v_{GG}(\cdot) + v_{gG}(\cdot) < 0$, resulting in $qv_{GG}(G, 0)\,(1 + m\mathbb{1}_M) + qv_{gG}(G, 0) < 0$. Then, since $h''(\cdot) < 0$, implicit differentiation of eq. (2-20) results in $\frac{\partial \tilde{G}_i^{Z,0}(q,m\mathbb{1}_M)}{\partial w_i} = \frac{h''(w_i)}{qv_{GG}(\tilde{G}_i^{Z,0},0)(1+m\mathbb{1}_M)+qv_{gG}(\tilde{G}_i^{Z,0},0)} > 0$. Thus, it suffices to establish that $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M) < \infty$. Note that $qv_G(0,0)\,(1 + m\mathbb{1}_M) + qv_g(0, 0) - h'(w_1) > 0$ since $qv_G(0, 0) > h'(w_1)$. Then, $qv_{GG}(G, 0)\,(1 + m\mathbb{1}_M) + qv_{gG}(G, 0) < 0$ implies that there is at most one value $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M)$ that satisfies eq. (2-20). Moreover, $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M) < \infty$ exists if an only if the left-hand side of eq. (2-20) turns strictly negative for some value of G, i.e. $\lim\limits_{G\to\infty} qv_G(G, 0)\,(1 + m\mathbb{1}_M) + qv_g(G, 0) - h'(w_1) < 0$. Since by assumption, $\lim\limits_{G\to\infty} qv_G(G, 0) = 0$, this condition reduces to $\lim\limits_{G\to\infty} qv_g(G, 0) - h'(w_1) < 0$.

Given $\tilde{G}_i^{Z,0}(q, m\mathbb{1}_M) < \infty$ for all i, proving that $\tilde{G}_\infty^{S,*}(q) \leq \tilde{G}_\infty^{M,*}(q)$ is analogous to the proof of Proposition 2. First, by a symmetric argument to the proof of Lemma A-1, it is straightforward to establish that $\mathop{lim}\limits_{n\to\infty} \tilde{G}^{Z,L}(q, d_L^Z) \to \tilde{G}_1^{Z,0}(q, m\mathbb{1}_M)$. Moreover, note that by eq. (2-20), $\tilde{G}_i^{M,0}(q, 0) = \tilde{G}_i^{S,0}(q)$ and implicit differentiation of the same equation results in

$$\frac{d\tilde{G}_i^{M,0}(q,m)}{dm} = \frac{\tilde{v}_G\left(\tilde{G}_i^{M,0}(q,m),0\right)}{-\left(\tilde{v}_{GG}\left(\tilde{G}_i^{M,0}(q,m),0\right)(1+m)+\tilde{v}_{gG}\left(\tilde{G}_i^{M,0}(q,m),0\right)\right)} > 0 \qquad \text{(A-19)}$$

since $qv_{GG}(G,0)\,(1+m\mathbb{1}_M) + qv_{gG}(G,0) < 0$. Thus, $\tilde{G}_\infty^{S,*}(q) \leq \tilde{G}_\infty^{M,*}(q,m)$, with strict inequality whenever $m > 0$. To show that $\tilde{m}^*(q) > 0$, analogous to the proof of Proposition 2, it suffices to show that $\frac{d\tilde{u}_L(q,0)}{dm} > 0$. The objective function of the lead donor for $Z = M$ is given by

$$\tilde{u}_L(q,m) = h(w_1 - \tilde{G}^{Z,L}(q,m) + \tilde{G}_F^Z(q,m)) + q\tilde{v}\left(\tilde{G}^{Z,L}(q,m), \tilde{G}^{Z,L}(q,m) - \tilde{G}_F^Z(q,m)\right) \qquad \text{(A-20)}$$

where $\tilde{G}_F^Z(q,m)$ denotes the follower donors' aggregate best response for a fixed m.[6] In order for the lead donor to choose $m > 0$, it must be true that $\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} > 0$. Recall that for $m = 0$, $\tilde{G}_1^{M,0}(q,0) = \tilde{G}_F^M(q,0) = \tilde{G}_1^{S,0}(q)$. Thus, differentiating (A-20) w.r.t. m and evaluating it at $m = 0$ gives rise to

$$\begin{aligned}\lim_{n\to\infty}\frac{d\tilde{u}_L(q,0)}{dm} &= \left[-h'(w_1) + q\tilde{v}_G\left(\tilde{G}_1^{S,0}(q),0\right) + q\tilde{v}_g\left(\tilde{G}_1^{S,0}(q),0\right)\right]\frac{d\tilde{G}_1^{M,0}(q)}{dm} + \\ &+ \left[h'(w_1) + q\tilde{v}_g\left(\tilde{G}_1^{S,0}(q),0\right)\right]\lim_{n\to\infty}\frac{d\tilde{G}_F^M(q,0)}{dm} \qquad \text{(A-21)}\end{aligned}$$

By eq. (2-20), the first term in the above equation equals 0. Moreover, since $h'(\cdot) > 0$ and $\tilde{v}_g(\cdot) > 0$, $\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} > 0$ if and only if $\lim_{n\to\infty} \frac{d\tilde{G}_F^M(q,0)}{dm} > 0$. Since it holds that $\lim_{n\to\infty} \tilde{G}_F^M(q,m) = \frac{\tilde{G}_1^{M,0}(q,m)}{1+m}$, differentiating with respect to m gives $\lim_{n\to\infty} \frac{d\tilde{G}_F^M(q,m)}{dm} = \frac{1}{1+m}\frac{d\tilde{G}_1^{M,0}(q,m)}{dm} - \frac{\tilde{G}_1^{M,0}(q,m)}{(1+m)^2}$. Substituting for $\frac{d\tilde{G}_1^{M,0}(q,m)}{dm}$ from eq. (A-19) and evaluating at

[6] Similar to the base model, taking into account that $\tilde{G}_i^{Z,0}(q, m\mathbb{1}_M) < \infty$ for all i and that the individual best response for a contributing follower, $\tilde{g}_i^{Z,*}(q,G)$, uniquely solves eq. (2-19), I can employ the Andreoni-McGuire algorithm to uniquely pin down $\tilde{G}_F^Z(q, d_L^Z)$ (see Yildirim, 2014).

$m = 0$ obtains

$$\lim_{n\to\infty} \frac{d\tilde{G}_F^M(q,0)}{dm} = \tilde{G}_1^{S,0}(q) \left[\frac{\tilde{v}_G\left(\tilde{G}_1^{S,0}(q),0\right)}{-\left(\tilde{v}_{GG}\left(\tilde{G}_1^{S,0}(q),0\right) + \tilde{v}_{gG}\left(\tilde{G}_1^{S,0}(q),0\right)\right)\tilde{G}_1^{S,0}(q)} - 1 \right] \tag{A-22}$$

From the above equation, it is immediately evident that $\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} > 0$ if and only if $\frac{-\left(\tilde{v}_{GG}(\tilde{G}_1^{S,0}(q),0)+\tilde{v}_{gG}(\tilde{G}_1^{S,0}(q),0)\right)\tilde{G}_1^{S,0}(q)}{\tilde{v}_G(\tilde{G}_1^{S,0}(q),0)} = \epsilon_{\tilde{v}}\left(\tilde{G}_1^{S,0}(q),0\right) < 1$, implying that $\tilde{m}^*(q) > 0$ and $\tilde{G}_\infty^{S,*}(q) < \tilde{G}_\infty^{M,*}(q)$.

To establish part b), note that from the proof of part a), $\lim_{G\to\infty} qv_g(G,0) - h'(w_1) \geq 0$ implies that $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M) = \infty$. Since $\tilde{G}_i^{Z,0}(q,\mathbb{1}_M) \leq \tilde{G}_1^{Z,0}(q,\mathbb{1}_M)$ for all i, it must be the case that $1 \in C$. Otherwise, if $1 \notin C$, then $C = \emptyset$, implying that $\tilde{G}_\infty^{Z,*}(q) = g_L^{Z,*} \leq \infty$. This, in turn, implies that the individual contribution by the wealthiest follower type, $\tilde{g}_1^{Z,*}(q)$, must satisfy $\lim_{n\to\infty} \tilde{g}_1^{Z,*}(q) > 0$ since $\tilde{G}_1^{Z,0}(q,\mathbb{1}_M) > \tilde{G}_\infty^{Z,*}(q)$, contradicting $1 \notin C$. Given $1 \in C$, if $\tilde{G}_\infty^{Z,*}(q) < \infty$, then analogous to the above argument, $\lim_{n\to\infty} \tilde{g}_1^{Z,*}(q) > 0$, which in turn results in a contradiction since $\tilde{G}_\infty^{Z,*}(q) \geq \lim_{n\to\infty} nt_1\tilde{g}_1^{Z,*}(q) = \infty$. Therefore, this establishes that $\tilde{G}_\infty^{Z,*}(q) = \infty$ if $\lim_{G\to\infty} qv_g(G,0) - h'(w_1) \geq 0$ and completes the proof. □

Proof. **of Proposition 2-10**

First, note that the lead donor's informed payoff is $\overline{u}_{L,\infty}^S(q_j,\gamma) = \overline{u}_L(q_j, q_j, \overline{g}_{L,\infty}^{S,*}(q_j))$, while the her uninformed payoff can be expressed as

$$\overline{u}_{L,\infty}^S(q_U^S,\gamma) = \overline{u}_L(q_U^S, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) + \gamma\left(\pi_h^S q_h v(G_1^{S,0}(q_h)) + \pi_l^S q_l v(G_1^{S,0}(q_l)) - q_U^S v(G_1^{S,0}(q_U^S))\right)$$

Therefore, taking into account that $q_U^S = \sum_j \pi_j^S q_j$ for $j \in \{l,h\}$, analogous to eq. (A-13), $V_{I,\infty}^S(\pi^S,\gamma)$ can be expressed as

$$V_{I,\infty}^S(\pi^S,\gamma) = \tag{A-23}$$
$$\sum_{j=\{l,h\}} \pi_j^Z \quad \left[\overline{u}_L(q_j, q_j, \overline{g}_{L,\infty}^{S,*}(q_j)) - \overline{u}_L(q_j, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) - \gamma q_j\left(v(G_1^{S,0}(q_j)) - v(G_1^{S,0}(q_U^S))\right)\right]$$

To account for the incentives constraints in eq. (2-11), recall from Proposition 2-1 that $\frac{dG_1^{S,0}(q)}{dg_L^S} = 0$ and thus if q is common knowledge, $g_{L,\infty}^S(q) = \lim_{n\to\infty} g_L^{S,*}(q) = 0$ for all q. Thus, the incentives constraints must be binding with $\overline{g}_{L,\infty}^S(q_{\mathcal{L}}) = \lim_{n\to\infty} \overline{g}_L^{S,*}(q_{\mathcal{L}}) > 0$ for $q_{\mathcal{L}} \in \{q_U^S, q_h\}$ where $q_U^S > q_l$.[7]. These binding constraints for the uninformed and the high type of lead donor are given by

$$\overline{u}_L(q_l, q_l, \overline{g}_{L,\infty}^{S,*}(q_l)) = \overline{u}_L(q_l, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) + \gamma q_l \left(v(G_1^{S,0}(q_l)) - v(G_1^{S,0}(q_U^S)) \right)$$
$$\overline{u}_L(q_U^S, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) = \overline{u}_L(q_U^S, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) - \gamma q_U^S \left(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S)) \right) \quad \text{(A-24)}$$

Note that eq. (A-24) implies that the summation in eq. (A-24) for $j = l$ reduces to 0. To simplify the remaining expression, note that by definition,

$$\overline{u}_L(q_h, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) = \overline{u}_L(q_U^S, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) + (q_h - q_U^S) v(G_1^{S,0}(q_U^S)) \quad \text{(A-25)}$$

Substituting for the incentive constraint given by eq. (A-24) in eq. (A-25) and accounting for $\overline{u}_L(q_U^S, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) = \overline{u}_L(q_h, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) - (q_h - q_U^S) v(G_1^{S,0}(q_h))$, results in

$$\overline{u}_L(q_h, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) = \overline{u}_L(q_h, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) - \left(q_h - (1-\gamma) q_U^S \right) \left(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S)) \right) \quad \text{(A-26)}$$

Finally, substituting for $\overline{u}_L(q_h, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S))$ given by the above equation in eq. (A-24) and simplifying results in $V_{I,\infty}^S(\pi^S, \gamma) = (1-\gamma)(q_h - q_U^S)(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S)))$, which completes the proof. □

[7]Recall that $q_U^S = q_l$ implies that $(\pi_l^S, \pi_h^S) = (1, 0)$ and thus $V_{I,\infty}^S((1,0), \gamma) = 0$.

APPENDIX B

PROOFS FOR SECTION 3

Proof. **of Proposition 3-1**

The aggregate funding from eq. (3-5) can be rewritten as

$$F(I,p,\Phi) = n\left(\frac{\Phi_U + p\Phi_R}{I_M - I}\right)\left(wI_M - g'^{-1}(X)\right)$$

where $X = \frac{(Y(p)-C(p)-I)h'(F(I,p,\Phi))}{I_M - I}$. Thus the marginal funding is:

$$\frac{\partial F(I,p,\Phi)}{\partial I} = \frac{F(I,p,\Phi)}{I_M - I} + n\left(\frac{\Phi_U + p\Phi_R}{I_M - I}\right)\left(-g'^{-1\prime}(X)\frac{\partial X}{\partial I}\right) \tag{B-1}$$

It is straight forward that if $\frac{\partial F(I,p,\Phi)}{\partial I} < 0$ then $\frac{\partial X}{\partial I} < 0$. Also if $\frac{\partial F(I,p,\Phi)}{\partial I} \geq 0$ then:

$$\frac{\partial X}{\partial I} = \frac{(Y(p)-C(p)-I)h''(F(I,p,\Phi))\frac{\partial F(I,p,\Phi)}{\partial I}(I_M - I) + (Y(p)-C(p)-I_M)h'(F(I,p,\Phi))}{(I_M - I)^2} < 0 \tag{B-2}$$

Moreover the MFI's payoff is

$$v(F(I,p,\Phi),I,p) = n(Y(p)-C(p)-I)\left(\frac{\Phi_U + p\Phi_R}{I_M - I}\right)\left(wI_M - g'^{-1}(X)\right)$$

thus the marginal effect of repayment I is:

$$\frac{\partial v(F(I,p,\Phi),I,p)}{\partial I} \tag{B-3}$$
$$= n\left(\frac{\Phi_U + p\Phi_R}{I_M - I}\right)\quad\left[\left(\frac{Y(p)-C(p)-I_M}{I_M - I}\right)(wI_M - g'^{-1}(X)) - (Y(p)-C(p)-I)g'^{-1\prime}(X)\frac{\partial X}{\partial I}\right]$$

which by eq. (B-2) implies $\frac{\partial v(F(I,p,\Phi),I,p)}{\partial I} < 0$. Thus, the MFI's problem has a corner solution at $I^* = 0$. This completes the first part of the proposition.

At $p = 1$ funding is independent of Φ and has to satisfy:

$$F(I,1,\Phi) = \left(\frac{n}{I_M - I}\right)\left(wI_M - g'^{-1}\left(\frac{(Y(1) - C(1) - I)h'(F(I,1,\Phi))}{I_M - I}\right)\right)$$

At $p = 0$ funding satisfies:

$$F(I,0,\Phi) = \left(\frac{n\Phi_U}{I_M - I}\right)\left(wI_M - g'^{-1}\left(\frac{(Y(0) - C(0) - I)h'(F(I,0,\Phi))}{I_M - I}\right)\right)$$

Comparing the two and noting that both $h'()$ and $g'^{-1}()$ are decreasing functions reveals that for any I and any Φ, $F(I,0,(0,1)) = 0 < F(I,1,\Phi) < F(I,0,(1,0))$. Therefore, there exist $\overline{\Phi}_U$ such that $F(0,0,\overline{\Phi}) = F(0,1,\overline{\Phi})$. Moreover, since both $h'()$ and $g'^{-1}()$ are decreasing functions, $F(I,0,\Phi)$ is increasing in Φ_U for any I. This completes the proof.

□

Proof. **of Lemma 3-1**

The aggregate best response can be calculated as:

$$F(I,\eta,\Phi) = \left(\frac{\Phi_R n}{I_M - I}\right) max\left\{wI_M - g'^{-1}\left(\eta X_1\right),0\right\} + \left(\frac{\Phi_U n}{I_M - I}\right) max\left\{wI_M - g'^{-1}\left((1-\eta)X_0 + \eta X_1\right),0\right\} \tag{B-4}$$

where $X_0 = \frac{(Y(0)-C(0)-I)h'(F(I,\eta,\Phi))}{I_M - I}$ and $X_1 = \frac{(Y(1)-C(1)-I)h'(F(I,\eta,\Phi))}{I_M - I}$. Consider $\bar{\eta}$ such that for $\Phi_U = 0$, $wI_M - g'^{-1}\left(\bar{\eta}X_1\right) = 0$. Since $F(I,\eta,\Phi)$ is increasing in Φ_U , it is straightforward that for any $\Phi_U > 0$, $wI_M - g'^{-1}\left(\bar{\eta}X_1\right) < 0$. Therefore, for any $\eta \in [0,\bar{\eta}]$ eq. (B-5) simplifies to: $F(I,\eta,\Phi) = \left(\frac{\Phi_U n}{I_M - I}\right)\left(wI_M - g'^{-1}\left((1-\eta)X_0 + \eta X_1\right)\right)$. As a result:

$$\frac{\partial F(I,\eta,\Phi)}{\partial \eta} = \frac{\left(\frac{\Phi_U n}{I_M-I}\right)\left(-g'^{-1'}\left((1-\eta)X_0 + \eta X_1\right)\right)\left(\frac{(Y(1)-C(1)-Y(0)+C(0))h'(F(I,\eta,\Phi))}{I_M-I}\right)}{1 - \left(\frac{\Phi_U n}{I_M-I}\right)\left(-g'^{-1'}\left((1-\eta)X_0 + \eta X_1\right)\right)\left(\frac{[(1-\eta)(Y(0)-C(0)-I)+\eta(Y(1)-C(1)-I)]h''(F(I,\eta,\Phi))}{I_M-I}\right)} \leq 0$$

and the inequality is strict for any $\Phi_U > 0$. This completes the proof of the first part of the lemma. In order to prove the second part, I will consider two cases separately.

Case 1: If $\pi \leq \bar{\eta}$ then, from part 1, for all $\eta \leq \bar{\eta}$, at $\Phi_U = 0$: $F(I, \eta, \Phi) = 0$ and $\frac{\partial F(I,\eta,\Phi)}{\partial \eta} = 0$. Therefore it is straightforward that if $\pi \leq \bar{\eta}$, then $\underline{\Phi}_U = 0$ satisfies $F(0, 0, \underline{\Phi}) = F(0, \pi, \underline{\Phi})$. Moreover, since by part 1, for any $\eta \leq \bar{\eta}$ at any $\Phi_U > 0$, $\frac{\partial F(I,\eta,\Phi)}{\partial \eta} < 0$, then for any $\Phi_U > \underline{\Phi}_U = 0$, $F(0, 0, \Phi) > F(0, \pi, \Phi)$.

Case 2: If $\pi > \bar{\eta}$, then by part 1, for any $\Phi_U > 0$, $\frac{\partial F(I,\eta,\Phi)}{\partial \eta} < 0$ at any $\eta \leq \bar{\eta}$. As a result for any $\Phi_U \leq \overline{\Phi}_U$, since by Proposition 3-1, $F(0, 0, \Phi) \leq F(0, 1, \Phi)$, there exists $\eta_0(\Phi) \in (\bar{\eta}, 1]$ such that it solves $F(0, 0, \Phi) = F(0, \eta_0(\Phi), \Phi)$. It is also straightforward that $\eta_0(\overline{\Phi}) = 1$. Moreover, from previous case $lim_{\Phi_u=0}\eta_0(\Phi) \to \bar{\eta}$. Therefore, for any $\pi \in (\bar{\eta}, 1)$ there exist $\underline{\Phi}$ such that $\underline{\Phi}_U < \overline{\Phi}_U$ and satisfies $\eta_0(\underline{\Phi}) = \pi$. Then for any Φ such that $\Phi_U > \underline{\Phi}_U$, $F(0, 0, \Phi) > F(0, \pi, \Phi)$ and for any Φ such that $\Phi_U < \underline{\Phi}_U$, $F(0, 0, \Phi) < F(0, \pi, \Phi)$. □

Lemma B-1. *For any given posterior belief (η) MFI's payoff is decreasing in repayment, i.e.* $\frac{\partial v(F(I,\eta,\phi),I,p)}{\partial I} < 0$.

Proof. **of Lemma B-1**

Consider the aggregate best response as given by eq. (B-5). There are two possible cases.

Case 1: If $wI_M \geq g'^{-1}(X_1)$ then since $g'^{-1}()$ is positive and decreasing:

$$\frac{\partial F(I, \eta, \Phi)}{\partial I} = \left(\frac{\Phi_R n}{(I_M - I)^2}\right)\left(wI_M - g'^{-1}(X_1)\right) - \left(\frac{\Phi_R n}{I_M - I}\right) g'^{-1\prime}(X_1)\left(\frac{\partial X_1}{\partial I}\right)$$

$$+\left(\frac{\Phi_U n}{(I_M - I)^2}\right)\left(wI_M - g'^{-1}(X_0 + X_1)\right) - \left(\frac{\Phi_U n}{I_M - I}\right) g'^{-1\prime}(X_0 + X_1)\left(\frac{\partial X_0}{\partial I} + \frac{\partial X_1}{\partial I}\right) \quad \text{(B-5)}$$

It is straight forward that if $\frac{\partial F(I,\eta,\Phi)}{\partial I} < 0$ then $\frac{\partial X_p}{\partial I} < 0$ for all p. Also if $\frac{\partial F(I,p,\Phi)}{\partial I} \geq 0$

then:

$$\frac{\partial X_p}{\partial I} = \eta_p \times \tag{B-6}$$
$$\frac{(Y(p) - C(p) - I)h''(F(I,\eta,\Phi))\frac{\partial F(I,\eta,\Phi)}{\partial I}(I_M - I) + (Y(p) - C(p) - I_M)h'(F(I,\eta,\Phi))}{(I_M - I)^2} < 0$$

The MFI's payoff is $v(F(I,\eta,\Phi), I, p) = (Y(p) - C(p) - I)F(I,\eta,\Phi)$. Thus from eq. (B-5) the marginal effect of repayment I is:

$$\frac{\partial v(F(I,\eta,\phi), I, p)}{\partial I} = -F(I,\eta,\Phi) + (Y(p) - C(p) - I)\frac{\partial F(I,p,\phi)}{\partial I} =$$
$$\left(\frac{\Phi_R n}{I_M - I}\right)\left[\left(\frac{Y(p) - C(p) - I_M}{I_M - I}\right)\left(wI_M - g'^{-1}(X_1)\right) - (Y(p) - C(p) - I)g'^{-1\prime}(X_1)\left(\frac{\partial X_1}{\partial I}\right)\right]$$
$$+ \left(\frac{\Phi_U n}{I_M - I}\right)\left[\left(\frac{Y(p) - C(p) - I_M}{I_M - I}\right)\left(wI_M - g'^{-1}(X_0 + X_1)\right)\right.$$
$$\left. - (Y(p) - C(p) - I)g'^{-1\prime}(X_0 + X_1)\left(\frac{\partial X_0}{\partial I} + \frac{\partial X_1}{\partial I}\right)\right]$$

which by eq. (B-7) implies $\frac{\partial v(F(I,\eta,\phi),I,p)}{\partial I} < 0$.

Case 2: If $wI_M < g'^{-1}(X_1)$ then only utilitarian investors invest thus:

$$\frac{\partial F(I,\eta,\Phi)}{\partial I} = \left(\frac{\Phi_U n}{(I_M - I)^2}\right)\left(wI_M - g'^{-1}(X_0 + X_1)\right) - \left(\frac{\Phi_U n}{I_M - I}\right)g'^{-1\prime}(X_0 + X_1)\left(\frac{\partial X_0}{\partial I} + \frac{\partial X_1}{\partial I}\right)$$

The solution is analogous to case 1. □

Proof. **of Lemma 3-2**

From eq. (3-1) $v(F(I_1, \eta(I_1), \Phi), I_1, 1) \leq v(F(I_2, \eta(I_2), \Phi), I_2, 1)$ can be written as:

$$(Y(1) - C(1) - I_1)F(I_1, \eta(I_1), \Phi) \leq (Y(1) - C(1) - I_2)F(I_2, \eta(I_2), \Phi) \tag{B-7}$$

Since $F(I_1, \eta(I_1), \Phi) < F(I_2, \eta(I_2), \Phi)$ and by Assumption 3-2, $Y(1) - C(1) < Y(0) - C(0)$, eq. (B-7) implies:

$$(Y(0) - C(0) - I_1)F(I_1, \eta(I_1), \Phi) < (Y(0) - C(0) - I_2)F(I_2, \eta(I_2), \Phi)$$

which completes the proof. □

Proof. **of Lemma 3-3**

To prove part 1, assume by the means of contradiction that both types of MFI offer two repayments $I_1 < I_2$ with positive probability. Thus, it must be true that an MFI with $p = 1$ is indifferent between these two strategies, i.e. $v(F(I_1, \eta(I_1), \Phi), I_1, 1) = v(F(I_2, \eta(I_2), \Phi), I_2, 1)$. However, by Lemma 3-2, this implies $v(F(I_1, \eta(I_1), \Phi), I_1, 0) < v(F(I_2, \eta(I_2), \Phi), I_2, 0)$ which contradicts MFI with $p = 0$ mixing the two strategies I_1 and I_2.

To prove parts 2 and 3, assume by the means of contradiction that an MFI of type p chooses two repayments with positive probability $I_1 \neq I_P$ and $I_2 \neq I_P$ such that $I_1 < I_2$. By part 1, the other MFI type will not choose either of these two with positive probability. Therefore, consistent posterior belief has to be $\eta(I_1) = \eta(I_2) = p$. By Proposition 3-1, $v(F(I_1, p, \Phi), I_1, p) > v(F(I_2, p, \Phi), I_2, p)$ which contradicts the MFI mixing the two strategies I_1 and I_2.

In order to prove part 4, I will consider two possible cases.

Case 1: If the equilibrium is (partially) pooling then I_P has to be on the equilibrium path. Assume by the means of contradiction that on the equilibrium path $I_H < I_P$. Thus, an MFI with $p = 0$ will be indifferent between these two repayments, that is $v(F(I_H, \eta(I_H), \Phi), I_H, 0) = v(F(I_P, \eta(I_P), \Phi), I_P, 0)$. However, since by Lemma 3-2 this implies $v(F(I_H, \eta(I_H), \Phi), I_H, 1) > v(F(I_P, \eta(I_P), \Phi), I_P, 1)$, an MFI with $p = 1$ has an incentive to deviate. Hence, $I_H > I_P$ and by a symmetric argument, $I_L < I_P$.

Case 2: If the equilibrium is separating then I_P is not on the equilibrium path. Assume by the means of contradiction that on the equilibrium path $I_H < I_L$. Thus, an MFI with $p = 1$ (weakly) prefers I_L over I_H, i.e. $v(F(I_H, \eta(I_H), \Phi), I_H, 1) \leq v(F(I_L, \eta(I_L), \Phi), I_L, 1)$. But, since by Lemma 3-2 this implies $v(F(I_H, \eta(I_H), \Phi), I_H, 0) < v(F(I_L, \eta(I_L), \Phi), I_L, 0)$, an MFI with $p = 0$ has an incentive to deviate. Hence, $I_H > I_L$. This completes the proof. □

Lemma B-2. *In all monotonic belief sequential equilibria, an MFI in an extremely poor community always chooses a repayment of 0 when it is not pooling with an MFI in a marginally poor community. That is $I_L = 0$ if it is on the equilibrium path.*

Proof. **of Lemma B-2** Assume by the means of contradiction that $I_L > 0$, then consistent belief is $\eta(I_L) = 1$. Monotonicity of beliefs thus requires, $\eta(0) = 1$. But this gives an MFI in an extremely poor community an incentive to deviate, since by Proposition 3-1 $I^*(1) = 0$ and $v(F(I^*(1), 1, \Phi), I^*(1), 1) > v(F(I_L, 1, \Phi), I_L, 1)$. □

Proof. **of Proposition 3-2**

Step 1: I will prove that an MFI in an extremely poor community will not unilaterally choose any repayment with positive probability. By the means of contradiction if it does, by Lemmas 3-3 and B-2 it is $I_L = 0$, which implies $\eta(0) = 1$. In this case for any strategy $I > 0$ that an MFI in a marginally poor community chooses with positive probability, by Lemma B-1 $v(F(I, \eta(I), \Phi), I, 0) < v(F(0, \eta(I), \Phi), 0, 0)$. However, since $\Phi_U \leq \underline{\Phi}_U$, $v(F(0, \eta(I), \Phi), 0, 0) < v(F(0, \eta(0), \Phi), 0, 0)$. Thus an MFI in a marginally poor community has an incentive to deviate.

Step 2: I will prove that the two MFI types will not pool on any positive repayment. By the means of contradiction if they do, by Lemma 3-3 it has to be a unique repayment $I_P > 0$. Then, for both types (any $p \in \{0, 1\}$), by Lemma B-1 $v(F(I_P, \eta(I_P), \Phi), I_P, p)$ $< v(F(0, \eta(I_P), \Phi), 0, p)$. Moreover, by step 1 only an MFI in a marginally poor community might partially separate, which implies $\eta(I_P) \geq \pi$. Additionally, monotonicity of beliefs requires $\eta(0) \geq \eta(I_P) \geq \pi$. However, since $\Phi_U \leq \underline{\Phi}_U$ it must be true that $v(F(0, \eta(I_P), \Phi), 0, p) \leq v(F(0, \eta(0), \Phi), 0, p)$. Thus both MFI types have an incentive to deviate.

Step 3: I will prove that an MFI in a marginally poor community will not unilaterally choose any repayment with positive probability. By the means of contradiction if it does, by Lemma 3-3 it has to be a unique repayment $I_H > 0$, which implies $\eta(I_H) = 0$. Moreover, by Lemma B-1 $v(F(I_H, \eta(I_H), \Phi), I_H, 0) < v(F(0, \eta(I_H), \Phi), 0, 0)$. Additionally, by step 1 and 2, the two types partially pool on $I_P = 0$, which implies $\eta(0) > \pi$. However, $\Phi_U \leq \underline{\Phi}_U$ implies $v(F(0, \eta(I_H), \Phi), 0, 0) < v(F(0, \eta(0), \Phi), 0, 0)$. Thus an MFI in a marginally poor community has an incentive to deviate.

Step 4: By steps 1 to 3 and Lemma 3-3, the only remaining monotonic belief sequential equilibrium is fully pooling on zero repayment, which implies $\eta(0) = \pi$. Consider the off equilibrium belief that for all $I > 0$, $\eta(I) = \pi$. This is a monotonic belief structure. Moreover, by Lemma B-1 neither type has an incentive to deviate. Thus, such equilibrium exists and is the unique (in strategies) monotonic belief sequential equilibrium of the game. □

Proof. **of Proposition 3-3**

Step 1: I will prove that the two MFI types will not pool on any repayment. By the means of contradiction, if they do, by Lemma 3-3 it has to be a unique repayment I_P and the posterior belief will be $\eta(I_P) \in (0, 1)$. Moreover, $\Phi_U > \overline{\Phi}_U$ implies that $F(I_P, 0, \Phi) > F(I_P, 1, \Phi)$ which in turn gives $F(I_P, 0, \Phi) > F(I_P, \eta(I_P), \Phi)$. Additionally, the payoff of an MFI in an extremely poor community is 0 at $\bar{I}(1)$, i.e., $v(F(\bar{I}(1), 0, \Phi), \bar{I}(1), 1) = 0$. Therefore, there always exists a repayment $I_D \in (I_P, \bar{I}(1))$ such that $v(F(I_D, 0, \Phi), I_D, 1) = v(F(I_P, \eta(I_P), \Phi), I_P, 1)$. Then, by Lemma 3-2 $v(F(I_D, 0, \Phi), I_D, 0) > v(F(I_P, \eta(I_P), \Phi), I_P, 0)$. This provides an MFI in a marginally poor community with an incentive to deviate since the equilibrium belief is $\eta(I_D) = 0$. The reason is that since by Lemma B-1 an MFI's payoff is decreasing in repayment for a given belief, any $I \geq I_D$ is equilibrium dominated for an MFI with $p = 1$ and the intuitive criterion imposes the belief $\eta(I_D) = 0$.

Step 2: The separating equilibrium is unique because by Lemma B-2 in any separating equilibrium the two MFI types will choose repayments that satisfy $I^{**}(0,\Phi) > I^{**}(1,\Phi) = 0$. Moreover, by Lemma B-1 an MFI's payoff is decreasing in repayment for a given belief, therefore, an MFI in a marginally poor community will choose the least costly signal that satisfies:

$$v(F(I^{**}(0,\Phi),0,\Phi),I^{**}(0,\Phi),1) = v(F(0,1,\Phi),0,1)$$

It is straightforward that such equilibrium can be supported by the following beliefs:

$$\eta(I) = \begin{cases} 1 & \text{if } I < I^{**}(0,\Phi) \\ 0 & \text{if } I \geq I^{**}(0,\Phi) \end{cases}$$

and that an MFI in a marginally poor community raises more funds: $F(I^{**}(0,\Phi),0,\Phi) > F(I^{**}(1,\Phi),1,\Phi)$. □

Proof. **of Proposition 3-4**

Step 1: I will prove that an MFI in an extremely poor community will not unilaterally choose any repayment with positive probability. By the means of contradiction if it does, by Lemmas 3-3 and B-2 it can only by a repayment of $I_L = 0$, which implies $\eta(0) = 1$. Moreover, for any strategy $I > 0$ that an MFI in a marginally poor community chooses with positive probability, by Lemma B-1 $v(F(I,\eta(I),\Phi),I,0) < v(F(0,\eta(I),\Phi),0,0)$. Additionally, since $\Phi_U \leq \overline{\Phi}_U$, $v(F(0,\eta(I),\Phi),0,0) < v(F(0,\eta(0),\Phi),0,0)$. Thus, an MFI in a marginally poor community has an incentive to deviate. As a result, by Lemma 3-3, an equilibrium can only entail an MFI in an extremely poor community choosing a repayment $I_P \geq 0$ and an MFI in a marginally poor community fully or partially pooling with it. In the latter case, by Lemma 3-3, it will mix between I_P with some probability

$\gamma \in (0,1)$ and one other repayment $I_H > I_P$ with probability $1-\gamma$. Furthermore, $\gamma = 1$ corresponds to full pooling.

Step 2: I will construct a partially separating equilibrium as described in step 1 such that the two MFI types partially pool at $I_p = 0$. Since $\underline{\Phi}_U < \Phi_U \leq \overline{\Phi}_U$, by Proposition 3-1 and Lemma 3-1, $F(0,\pi,\Phi) < F(0,0,\Phi) \leq F(0,1,\Phi)$. Hence, there exists $\eta^* > \pi$ such that $F(0,\eta^*,\Phi) = F(0,0,\Phi)$. There also exists $\varepsilon > 0$ such that $\eta^* - \varepsilon > \pi$ and $F(0,\eta^* - \varepsilon,\Phi) < F(0,0,\Phi)$. Moreover, since $F(\bar{I}(0),0,\Phi) = 0$, there exists $I_H \in (0,\bar{I}(0))$ such that $v(F(I_H,0,\Phi),I_H,0) = v(F(0,\eta^* - \varepsilon,\Phi),0,0)$. Additionally, since $\eta^* - \varepsilon > \pi$, there exists $\gamma^* = \frac{\pi(1-\eta^*+\varepsilon)}{(\eta^*-\varepsilon)(1-\pi)} \in (0,1)$ and if an MFI in a marginally poor community pools with the other type at $I_P = 0$ with probability γ^*, then the posterior belief in the pool is $\eta(I_P) = \eta^* - \varepsilon$. As a result, $v(F(I_H,0,\Phi),I_H,0) = v(F(0,\eta(0),\Phi),0,0)$ and by Lemma B-1, under the following beliefs:

$$\eta(I) = \begin{cases} \eta^* - \varepsilon & \text{if } I < I_H \\ 0 & \text{if } I \geq I_H \end{cases}$$

an MFI in a marginally poor community will not have an incentive to deviate from a mixed strategy of choosing $I_P = 0$ with probability γ^* and I_H otherwise. Furthermore, by Lemma 3-2 $v(F(I_H,0,\Phi),I_H,1) < v(F(0,\eta(0),\Phi),0,1)$ and thus by Lemma B-1, an MFI in an extremely poor community will not have an incentive to deviate from choosing $I_P = 0$. This completes the proof. □