STRING STABILITY OF VEHICLE PLATOONS WITH HETEROGENEITY IN TIME HEADWAY

A Thesis

by

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ABSTRACT

Vehicle platooning is a concept in which a group of vehicles follow a lead vehicle and travel together in a coordinated formation. Various control policies have been suggested for the longitudinal control of vehicles in a vehicle platoon. Constant Time Headway Policy (CTHP) employs a desired inter-vehicular distance proportional to the velocity of the vehicle known as time headway. Previous studies have focused on finding the minimum employable time headway that would guarantee string stability in the presence of disturbances. However, those studies have assumed homogeneous parameters for all vehicles in the platoon. This study investigates the effects of heterogeneity in time headway on string stability of vehicle platoons.

In this study, the error propagation transfer function for a platoon with heterogeneous time headways is presented. It is found that, owing to the stability of this error propagation transfer function, the minimum employable time headway for heterogeneous case has to be greater than $\frac{\tau_o}{k_a}$, which is higher than that of the homogeneous case given by $\frac{2\tau_o}{1+k_a}$. A sufficient condition for string stability is presented, using which string stability of the platoon can be guaranteed if the headway values are monotonically decreasing from the head to the tail of the platoon. However, using this sufficient condition does not provide any insights on how different combinations of headways affect string stability of the vehicle platoon.

Finally, to investigate the evolution of spacing errors for other combinations of headways, the transfer function between error of a vehicle and the lead vehicle acceleration is derived. From this error transfer function, it is found that, for a finite range of time headways, the maximum spacing error of a vehicle for any combination of headways can be bounded irrespective of the size of the platoon.

DEDICATION

To my parents,

Ankem Subba Rao and Ankem Radha,

and my professor,

Dr. Swaroop Darbha.

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1. INTRODUCTION

Automobiles have become a prominent part of human life, and the number of automobiles on the roads is increasing day by day. Hence, over the past two or three decades, several researchers have started focusing on developing innovative technologies that could increase safety, enhance driver's comfort, reduce traffic congestion, and make automobiles more fuel efficient. Vehicle platooning is one among such technologies.

1.1 Vehicle Platoon

Vehicle platooning is a method of grouping vehicles in closed formations with the main objective of increasing road capacity. Every vehicle platoon consists of a lead vehicle, and all the other vehicles follow the lead vehicle by maintaining a desired spacing between them. A typical setup of vehicle platoon is shown in Figure 1.1.

Figure 1.1: A typical vehicle platoon setup[1]

Vehicle platooning has many potential benefits:

- The vehicles in a platoon can react much faster than humans, thereby improving safety, and mitigating collisions.
- The vehicles can travel at tighter spaces, and can accelerate/brake smoothly which increases the road capacity, and reduces traffic jams, thereby increasing highway throughput.
- The vehicles in a platoon can travel at constant speeds with less accelerating and braking, resulting in lesser fuel consumption. Also, when vehicles travel in tight spaces, drafting takes

[1] car icon source: https://icons8.com

place due to which the aerodynamic drag for the following vehicles reduces significantly which can boost the fuel efficiency for the following vehicles.

- Truck platooning can cut down the CO_2 emissions by 7% to 10% for the following vehicles, and by 1% to 8% for the lead vehicle, depending on the inter-vehicular spacing [1].

Due to the aforementioned benefits of vehicle platooning, researchers from both industry and academia have started working on developing platooning strategies. A brief overview of some of the important platooning projects is given in [2, 3]. The development of platooning strategies depends on various factors. It depends on whether the goal is to have a vehicle platoon with a single vehicle type or a mixed one. It also depends on whether the vehicles are desired to be automated in longitudinal direction, or in both longitudinal and lateral directions. Additionally, it depends on the type of information that a vehicle has access to, which depends on its on-board sensors and communication systems, and infrastructure capabilities.

This study deals with only the longitudinal control of the vehicle in a vehicle platoon. The main objective of the longitudinal controller is to maintain the desired spacing between vehicles in a platoon at all times. The desired spacing to be maintained is dictated by the type of spacing policy employed in the controller design. The most important spacing policies can be classified as Constant Spacing Policy (CSP) and Variable Spacing Policy (VSP). In a CSP, the desired spacing between the vehicles is independent of the speed of the vehicle and is a constant. Various versions of CSP are discussed in [4]. CSP employed with on-board information could lead to string instability [5]. It requires the information of lead vehicle for string stability. In a VSP, the desired spacing varies as some function of velocity of the vehicle under control. Among VSPs, the most important one is Constant Time Headway Policy (CTHP).

CTHP based controllers employ a desired inter-vehicular distance proportional to the velocity of the vehicle, and the constant of proportionality is called time headway h. In CTHP, decreasing the time headway helps in increasing traffic capacity and fuel efficiency. But if the headway is too small, any disturbances in the platoon may cause the platoon to become unstable and collisions may occur. Hence, it is important to determine the minimum safe employable time headway in case of CTHP based controllers.

Adaptive Cruise Control (ACC) is an ADAS feature that can help maintain the traffic flow stability, and it typically employs a Constant Time Headway Policy (CTHP) [6] to maintain desired spacing. When ACC is activated by the driver, the vehicle cruises at a constant speed set by the driver unless a vehicle is detected in its path, at which point the vehicle shifts to maintain a desired spacing from the vehicle ahead. ACC uses the information from its on-board sensors such as radars or lidars to maintain the desired spacing from the vehicle ahead. It was shown that stability of the platoon can be guaranteed just by using on-board sensors' information by employing CTHP [7].

A more advanced version of ACC is Cooperative Adaptive Cruise Control (CACC) which apart from utilizing the information obtained from on-board sensors, also uses the on-board communication systems to obtain information regarding its predecessor [8]. Dedicated Short Range Communication (DSRC) is used to exchange information between vehicles. It was shown in [9] that the safe employable headway range can be reduced by a significant factor by utilizing preceding vehicle's acceleration information which can be obtained through vehicle to vehicle communication. Hence, in vehicle platooning, using CACC over ACC can help reduce the time headway while guaranteeing the stability.

Majority of the work on CTHP assumed homogeneous parameters for all the vehicles in the platoon [4, 7, 9, 10]. But, in practice, a vehicle platoon can consist of vehicles of different makes and models. Hence, it is important to study how heterogeneity in vehicle parameters affects string stability. Some of the previous work on heterogeneous platoons can be found in [11–16]. A clear intuition on how the heterogeneity in time headway affects the string stability has not been provided by any of the previous studies.

In this work, we investigate the CTHP policy by considering heterogeneity in time headway only. Hence, we assume that all the other parameters in the vehicle platoon are homogeneous. This is reasonable if one were to consider a string of vehicles of same make and model, and a finite values of time headways are given as an option for the drivers to choose.

1.2 Thesis Outline

The following is a brief outline of how the reminder of the thesis is structured.

In Chapter 2, the vehicle plant model used for the controller design, the notion of string stability and the method used to analyze it, and a brief review of previous results associated with CTHP for homogeneous case are presented.

In Chapter 3, the error transfer function with respect to error of the predecessor vehicle and lead vehicle acceleration are presented, and the mathematical observations related to string stability are summarised in the form of lemmas and theorems.

In Chapter 4, the procedure for selecting controller parameters is presented, and the results from the numerical simulations for various cases that corroborate with mathematical results presented in Chapter 3 are presented and discussed.

Finally, in Chapter 5, conclusions from this study are presented, and suggestions for future work are given.

2. VEHICLE MODEL AND CONTROL POLICY

This chapter briefly reviews vehicle models for automatic vehicle following applications. Section 2.1 discusses the plant model used for the analysis of spacing control policies. Section 2.2 is concerned with definitions of string stability and robust string stability, and a most commonly employed method for analyzing string stability. Section 2.3 discusses Constant Time Headway Policy, and the previous results pertaining to it.

2.1 Vehicle Plant Model

The first step in control design for vehicle platoons is to develop or consider an individual vehicle model that reflects the actual vehicle dynamics. The longitudinal control architecture [17] usually consists of an upper level controller and a lower level controller as shown in Figure 2.1.

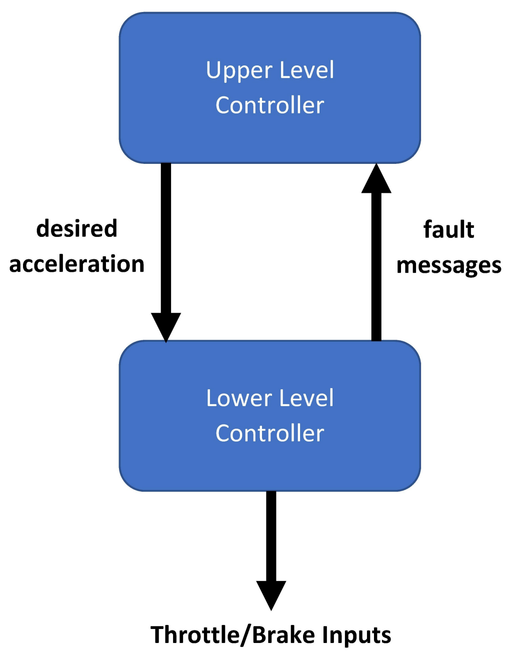

Figure 2.1: Vehicle Longitudinal Control Architecture

The upper level controller determines the desired acceleration for each vehicle based on the spacing policy, while the lower level controller determines the required throttle/brake inputs necessary to track the desired acceleration based on vehicle dynamic models, vehicle specific parameters and nonlinear controller synthesis techniques. Simplified vehicle models have been used by researchers in the past [4, 17, 18] to focus on the control design problem at the upper level so that the desired spacing is maintained while guaranteeing the stability. The plant model considered for a lower level controller for a front wheel drive vehicle is presented below. The following are the assumptions made for the aforementioned model:

1. The longitudinal tire slip is negligible.
2. The torque converter is locked, and the drive axle is rigid.
3. The brakes obey first order dynamics.
4. The ideal gas law holds in the intake manifold, and the temperature of intake manifold is constant.

From the first assumption, the longitudinal velocity of the vehicle v can be related to the angular velocity of the wheels ω_w as

$$v = r_{eff}\omega_w, \tag{2.1}$$

where r_{eff} is the effective radius of rotating tire. Owing to the second assumption, the engine speed ω_e, and the wheel speed ω_w can be related through the transmission gear ratio R_g as

$$\omega_w = R_g\omega_e. \tag{2.2}$$

Using (2.1) and (2.2), the longitudinal velocity of the vehicle v is related to the engine speed ω_e as

$$v = R_g r_{eff}\omega_e. \tag{2.3}$$

By differentiating the above equation with time, the relationship between longitudinal acceleration of the vehicle $\dot{v}$ and the angular acceleration of the engine $\dot{\omega_e}$ can be obtained, and is given as

$$\dot{v} = R_g r_{eff} \dot{\omega}_e. \tag{2.4}$$

The longitudinal equation of motion of the vehicle can be written as

$$m_v \dot{v} = F_x - R_x - F_a. \tag{2.5}$$

where, m_v is the mass of the vehicle, F_x is the longitudinal tire force on all the wheels, R_x is the rolling resistance on all the wheels, and F_a is the aerodynamic drag on the vehicle. Substituting (2.3) in the above equation gives

$$m R_g r_{eff} \dot{\omega}_e = F_x - R_x - F_a. \tag{2.6}$$

The wheel dynamics with I_w as the wheel moment of inertia, T_{br} as the total brake torque, and T_w as the total torque at the driven wheels is given by

$$I_w \dot{\omega}_w = T_w - T_{br} - r_{eff} F_x. \tag{2.7}$$

By using (2.2) and (2.6), the above equation can be written as

$$T_w = I_w R_g \dot{\omega}_e + m R_g r_{eff}^2 \dot{\omega}_e + r_{eff} R_x + r_{eff} F_a + T_{br}. \tag{2.8}$$

The transmission dynamics with I_t as the transmission shaft moment of inertia, T_t as the turbine torque, and ω_t as the turbine speed is given by

$$I_t \omega_t = T_t - R_g T_w. \tag{2.9}$$

Again, since the torque converter is locked; $\omega_e (= \omega_p) = \omega_t$ and $T_t = T_p$. Hence, by using (2.8), the above equation can be written as

$$T_p = (I_t + I_w R_g^2 + m R_g^2 r_{eff}^2)\dot{\omega}_e + R_g r_{eff}(R_x + F_a) + R_g T_{br}. \tag{2.10}$$

Finally, the engine dynamics with the engine moment of inertia I_e, the net torque from engine T_{net}, and the pump torque T_p is given by

$$I_e \dot{\omega}_e = T_{net} - T_p. \tag{2.11}$$

By using (2.10), the above equation can be written as

$$T_{net} = (I_e + I_t + I_w R_g^2 + m R_g^2 r_{eff}^2)\dot{\omega}_e + R_g r_{eff}(R_x + F_a) + R_g T_{br}. \tag{2.12}$$

Let us define the total resistance force $F_r = (R_x + F_a)$, and effective moment of inertia of the vehicle as $I_v = (I_e + I_t + I_w R_g^2 + m R_g^2 r_{eff}^2)$. By using these definitions, and (2.1), (2.12) can be written as

$$T_{net} - R_g T_{br} = \frac{I_v}{R_g r_{eff}} \dot{v} + R_g r_{eff} F_r. \tag{2.13}$$

The desired acceleration/deceleration determined by the upper level controller is used to calculate the desired engine/brake torque by using the above equation. The brake dynamics can be modeled as first order system with actuator time lag τ_{br} and the commanded brake torque T_{brc}, and is given below

$$\tau_{br} \dot{T}_{br} + T_{br} = T_{brc}. \tag{2.14}$$

The above brake model is used to determine the commanded brake torque to attain the desired brake torque. The torque from the engine is attained through the throttle input α. The net torque from the engine T_{net} is a complex nonlinear function of engine speed ω_e and the manifold pressure P_m. Since we assumed that the ideal gas law holds in the intake manifold, the mass of air in the intake manifold m_a is related to the manifold pressure P_m as

$$P_m = \frac{m_a R T_m}{V_m}. \tag{2.15}$$

In the above equation, R is the gas constant for air, T_m is the intake manifold temperature which is assumed to be ambient temperature, and V_m is the volume of the intake manifold, which is a constant. The mass of air in the manifold can be obtained by applying the principle of conservation of mass for air entering the manifold $m_{a,in}$ and the air leaving the manifold $m_{a,out}$. The rate of air entering the manifold is a function of the throttle angle α and the manifold pressure P_m. The rate of air leaving the manifold is a function of engine speed ω_e and the manifold pressure P_m. Thus, the rate of change of mass of air in the manifold $\dot{m}_a$ is given as,

$$\dot{m}_a = \dot{m}_{a,in}(\alpha, P_m) - \dot{m}_{a,out}(\omega_e, P_m). \tag{2.16}$$

Often, engine maps are provided by the engine manufacturers in which the functions $T_{net}(\omega_e, P_m)$, $m_{a,in}(\alpha, P_m)$ and $m_{a,out}(\omega_e, P_m)$ are available in the form of look-up tables. If the maps are not provided by the manufacturers, it can be obtained experimentally. These engine maps can be used to determine the required throttle angle for producing the desired net torque from engine T_{net}. Thus, for the nominal maneuvers of a vehicle, the desired acceleration of the vehicle can be obtained by appropriate throttle/brake inputs, and by considering the total torque

$$T_{net} - R_g T_{br} = R_g r_{eff} F_r + \frac{I_v}{R_g r_e f f} u_i, \tag{2.17}$$

where u_i is the desired acceleration provided by the upper level controller, the equation (2.13) for i^{th} vehicle reduces to second order model given by

$$\ddot{x}_i = u_i, \tag{2.18}$$

where x_i is the position of i^{th} vehicle and u_i is the control input for i^{th} vehicle at the upper level.

The above model is used in designing the upper level controller. As the goal is to maintain the desired spacing between the vehicles while guaranteeing stability, the above input-output model makes it easy to design a controller that achieves the goal without having to consider all the complex nonlinearities and the heterogeneity that exists between vehicles in the platoon. This model has been used by several researchers in the past and as mentioned in [19], this model is reasonable because of the following reasons:

1. Feedback linearization is typically employed in the lower level controller design rendering the model for upper level controller design to be linear and homogeneous.
2. Most vehicle maneuvers do not require braking or acceleration inputs to attain their limit.
3. Past experience with the platooning experiments that used this model has been satisfactory.

Now, it is important to note that, due to the limitations of actuation bandwidth, there might be a parasitic lag associated with tracking the desired acceleration or deceleration. This parasitic lag may be simply modeled as first order lag to (2.18), and is given as

$$\tau \dddot{x}_i + \ddot{x}_i = u_i \tag{2.19}$$

where τ is the parasitic actuation lag, the exact value of which is unknown, but an upper bound τ_o can be determined. Thus, $\tau \in [0, \tau_o]$.

2.2 String Stability

A vehicle platoon is string stable if the spacing errors caused by external disturbances doesn't propagate upstream from vehicle to vehicle in a platoon. The following definition is from [4]:

Definition 1: *A platoon is string stable if, given* $\gamma > 0$, $\exists\, \delta > 0$ *such that whenever,*

$$max\Big[\|e_i(0)\|_\infty, \|\dot{e}_i(0)\|_\infty, \sum_{j=1}^{i} \|e_j(0)\|_\infty, \sum_{j=1}^{i} \|\dot{e}_j(0)\|_\infty\Big] < \delta \implies \underset{i}{sup}\|e_i\|_\infty < \gamma$$

An extension to the above definition, robust string stability, given by [18] is,

Definition 2: *In the presence of parasitic time lag τ, a platoon is robustly string stable if the vehicles are string stable according to Definition 1 for all $\tau \in [0, \tau_o]$.*

Although various methods have been used to investigate string stability, in this work, a frequency domain method has been used. A detailed analysis of this method can be found in [4,17,18]. According to this method, the string stability is guaranteed if the following conditions are satisfied,

1. The transfer function between error of the i^{th} vehicle and $(i-1)^{th}$ vehicle in the platoon given by $H_i(s)$ satisfies the condition

$$\|H_i(j\omega)\|_\infty \leq 1. \tag{2.20}$$

2. The impulse response function $h_i(t)$ corresponding to the transfer function $H_i(s)$ does not change sign, i.e.,

$$h_i(t) \geq 0, \forall t \geq 0. \tag{2.21}$$

The above conditions are sufficient for string stability. In this work, the first condition is often referred to as magnitude condition, and the second condition is referred to as the condition for non-negativity of impulse response.

2.3 Constant Time Headway Policy

In Constant Time Headway Policy (CTHP), the desired following distance is proportional to the velocity of the vehicle. The proportionality constant in this policy is known as time headway denoted by h. The desired acceleration or the control input u_i for CTHP is defined as

$$u_i = k_{ai}\ddot{x}_{i-1} - k_{vi}(\dot{x}_i - \dot{x}_{i-1}) - k_{pi}(x_i - x_{i-1} + d_i + h_i v_i). \tag{2.22}$$

In the above equation x_i and x_{i-1} are the positions of i^{th} and $(i-1)^{th}$ vehicles with respect to a fixed ground frame respectively. d_i, h_i are the standstill distance, time headway for i^{th} vehicle respectively, and k_{pi}, k_{vi}, k_{ai} are the gains for i^{th} vehicle. The spacing error in this case is defined as

$$e_i = x_i - x_{i-1} + d_i + h_i v_i. \tag{2.23}$$

Figure 2.2 shows the vehicles maintaining the desired spacing (i.e., zero error) in case of CTHP Policy. Also, it is important to note that if the error is negative, the vehicles are closer than the desired spacing, and if the error is positive, the vehicles are farther than desired spacing.

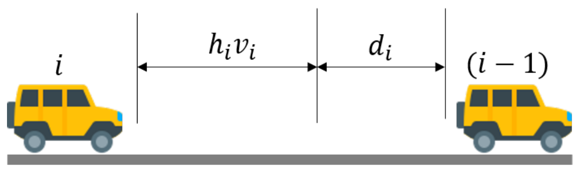

Figure 2.2: Desired Spacing in Constant Time Headway Policy

If all the controller parameters are homogeneous for all the vehicles in the platoon, i.e., if the control input is defined as

$$u_i = k_a \ddot{x}_{i-1} - k_v(\dot{x}_i - \dot{x}_{i-1}) - k_p(x_i - x_{i-1} + d + hv_i),$$

then the transfer function $H_i(s)$ between the spacing error of i^{th} vehicle and $(i-1)^{th}$ vehicle is same for all the vehicles, and is given by [9, 18]

$$\frac{E_i(s)}{E_{i-1}(s)} = H(s) = \frac{k_a s^2 + k_v s + k_p}{\tau s^3 + s^2 + (k_v + k_p h)s + k_p}. \tag{2.24}$$

The conditions for the controller parameters for the homogeneous platoon case to satisfy the string stability conditions given by (2.20) and (2.21) are given by [9, 18]. The following theorem is from [9]:

Theorem 1. *(a)* $k_a \geq 0$ *and* $\|H(\tau; j\omega)\|_\infty \leq 1$ *for all* $\tau \in [0, \tau_o]$ *implies* $k_a \in [0, 1)$ *and* $h \geq \frac{2\tau_o}{1+k_a}$.
(b) Given any $k_a \in [0, 1)$ *and* $h \geq \frac{2\tau_o}{1+k_a}$, *there exists* $k_p, k_v > 0$ *such that* $H(\tau; s)$ *is stable and* $\|H(\tau; j\omega)\|_\infty \leq 1$ *for all* $\tau \in [0, \tau_o]$.

The above theorem states that, one can find the gains $k_p, k_v > 0$ for error attenuation, if and only if the time headway is greater than or equal to minimum employable time headway $h_{min} = \frac{2\tau_o}{1+k_a}$ and the acceleration gain $k_a \in [0, 1)$.The results from this theorem will be used later to investigate string stability of a vehicle platoon with heterogeneity in time headway.

3. HETEROGENEITY IN TIME HEADWAY

In this chapter, the analysis of Constant Time Headway Policy with heterogeneity in time headway is discussed. In Section 3.1, the spacing error transfer function for the heterogeneity case is presented. In Section 3.2, the conditions to achieve robust string stability are discussed. Finally, in Section 3.3, the error transfer function of a vehicle with respect to lead vehicle acceleration, and the results deduced from it are presented.

3.1 Error Transfer Function

In this study, the heterogeneity is assumed to be present in time headway only. Thus, all other controller parameters are assumed to be homogeneous for all vehicles in the platoon. The control input for the i^{th} vehicle u_i with time headway h_i is given by

$$u_i = k_a \ddot{x}_{i-1} - k_v(\dot{x}_i - \dot{x}_{i-1}) - k_p(x_i - x_{i-1} + d + h_i v_i). \tag{3.1}$$

The error in this case is defined as

$$e_i = x_i - x_{i-1} + d + h_i v_i. \tag{3.2}$$

The transfer function between the error of i^{th} vehicle and error of $(i-1)^{th}$ vehicle is given as

$$\frac{E_i(s)}{E_{i-1}(s)} = K_i(s) H_i(s), \tag{3.3}$$

where

$$K_i(s) = \frac{s(k_a h_i - \tau) + (k_v h_i + k_a - 1)}{s(k_a h_{i-1} - \tau) + (k_v h_{i-1} + k_a - 1)}, \tag{3.4}$$

$$H_i(s) = \frac{k_a s^2 + k_v s + k_p}{\tau s^3 + s^2 + (k_v + k_p h_i)s + k_p}. \tag{3.5}$$

The derivation for the error transfer function given by (3.3) is given in Apppendix A.

3.2 String Stability

As mentioned in Section 2.2, for the vehicle platoon to be string stable, according to the frequency domain method, both the magnitude condition and the condition for non-negativity of impulse response have to be satisfied. The magnitude condition for the heterogeneity case is discussed in Section 3.2.1, and the condition for non-negativity of impulse response has been discussed in Section 3.2.2.

3.2.1 Magnitude Condition

The vehicle platoon with heterogeneity in time headway has to satisfy the following magnitude condition for string stability:

$$\|K_i(jw)H_i(jw)\|_\infty \leq 1. \tag{3.6}$$

It is difficult to determine the desired parameter set by considering the product of both the transfer functions. A subset of the desired parameter set can be obtained if both the transfer functions satisfy the conditions $\|H_i(jw)\|_\infty \leq 1$ and $\|K_i(jw)\|_\infty \leq 1$. Hence, string stability can be guaranteed by satisfying the following sufficient condition:

$$\|H_i(jw)\|_\infty \leq 1; \|K_i(jw)\|_\infty \leq 1. \tag{3.7}$$

3.2.2 Analysis of transfer function $H_i(s)$

The conditions for $H_i(s)$ to be stable and $\|H_i(j\omega)\|_\infty \leq 1$ in this case can be obtained from Theorem-1 by restating the theorem as following lemma:

Lemma 1. *For any* $k_a \in [0, 1)$*, and* $h_{min} \geq \frac{2\tau_o}{1+k_a}$*, there exists* $k_p, k_v > 0$ *such that for all* i*, and for all* $\tau \in [0, \tau_o]$*,*

1. $H_i(\tau; s)\ is\ stable,\ and$

2. $\|H_i(\tau; j\omega)\|_\infty \leq 1.$

Proof. From Theorem 1, we know that, $\|H(\tau; h; j\omega)\|_\infty \leq 1$ if and only if

$$k_a \in [0, 1);\ h \geq \frac{2\tau_o}{1 + k_a}.$$

Hence, the minimum time headway h_{min} in the vehicle platoon with heterogeneity in time headway must satisfy

$$h_{min} \geq \frac{2\tau_o}{1 + k_a}.$$

With the above minimum employable time headway, the stability of $H_i(\tau; s)$ is readily satisfied if $k_p, k_v > 0$ [9]. Now it has to be shown that one can find a set of gains $k_p, k_v > 0$ such that $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all i, and for all $\tau \in [0, \tau_o]$. From (3.5)

$$\|H_i(j\omega)\|^2 = \frac{(k_p - k_a\omega^2)^2 + k_v^2\omega^2}{(k_p - \omega^2)^2 + \omega^2(k_v + k_p h_i - \tau\omega^2)^2}.$$

From the above equation, for $\|H_i(j\omega)\|_\infty \leq 1$, we have

$$(k_p - k_a\omega^2)^2 + k_v^2\omega^2 \leq (k_p - \omega^2)^2 + \omega^2(k_v + k_p h_i - \tau\omega^2)^2.$$

Simplifying the above equation, we get

$$\tau^2\omega^4 + \omega^2[(1 - k_a^2) - 2\tau(k_p h_i + k_v)] + (k_p h_i + k_v)^2 - k_v^2 - 2k_p(1 - k_a) \geq 0. \tag{3.8}$$

The above equation is a bi-quadratic polynomial of the form $ax^2 + bx + c$, where

$x = \omega^2;\ a = \tau^2;\ b = (1 - k_a^2) - 2\tau(k_p h_i + k_v);\ \&\ c = (k_p h_i + k_v)^2 - k_v^2 - 2k_p(1 - k_a)$.

For (3.8) to be satisfied for all $\omega,\ \tau \in [0, \tau_o]$, we need either

$$a \geq 0;\ c \geq 0;\ and\ 4ac - b^2 \geq 0, \tag{3.9}$$

or

$$a \geq 0;\ c \geq 0;\ and\ b \geq 0. \tag{3.10}$$

Since $\tau \geq 0$, we have $a \geq 0$. For $c \geq 0$ to be satisfied, the gains k_p, k_v must belong to the set given by

$$S_1 = \big\{(k_p, k_v) \mid k_p > 0; k_v > 0; \frac{k_p}{p_1} + \frac{k_v}{p_2} \geq 1\big\}, \tag{3.11}$$

where

$$p_1 = \frac{2(1 - k_a)}{h_{min}^2}; p_2 = \frac{(1 - k_a)}{h_{min}}.$$

For sufficiency, let us consider the set that satisfies $b \geq 0$. The set is given by

$$S_2 = \big\{(k_p, k_v) \mid k_p > 0; k_v > 0; \frac{k_p}{p_3} + \frac{k_v}{p_4} \leq 1\big\}, \tag{3.12}$$

where

$$p_3 = \frac{(1 - k_a^2)}{2\tau_o h_{max}};\ p_4 = \frac{(1 - k_a^2)}{2\tau_o}.$$

Since $h_{min} \geq \frac{2\tau_o}{1+k_a}$, we have $p_4 \geq p_2$, and hence, $S_1 \cap S_2 \neq \phi$. Thus if the gains are chosen such that $(k_p, k_v) \in S_1 \cap S_2$, then for $k_a \in [0, 1)$ and $h_{min} \geq \frac{2\tau_o}{1+k_a}$, $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all $i,\ \tau \in [0, \tau_o]$. □

3.2.3 Analysis of transfer function $K_i(s)$

For the transfer function $K_i(s)$, the conditions are different for CACC and ACC.

(a) Transfer Function $K_i(s)$ with CACC:

For $K_i(s)$ to be stable, the denominator of $K_i(s)$ should be Hurwitz which gives the following condition for $k_a \in (0, 1)$,

$$h_i := \{x \mid x < min\Big(\frac{\tau_{min}}{k_a}, \frac{1-k_a}{k_v}\Big) \vee x > max\Big(\frac{\tau_{max}}{k_a}, \frac{1-k_a}{k_v}\Big)\}.$$

Since τ_{min} is zero and the headway h_i cannot be less than zero, considering $\tau_{max} = \tau_o$, the above condition reduces to,

$$h_i > max\Big(\frac{\tau_o}{k_a}, \frac{1-k_a}{k_v}\Big).$$

The above condition can be satisfied by assuming a $k_a \in (0, 1)$ and by choosing minimum headway and velocity gain respectively such that

$$h_{min} > \frac{\tau_o}{k_a};\ k_v > \frac{1-k_a}{h_{min}}. \tag{3.13}$$

(b) Transfer Function $K_i(s)$ with ACC:

For ACC, when $k_a = 0$, the stability condition for $K_i(s)$ is given by,

$$k_v < \frac{1}{h_{max}}. \tag{3.14}$$

From (3.4),

$$\begin{aligned} \|K_i(j\omega)\|^2 &= \frac{\omega^2(k_a h_i - \tau)^2 + (k_v h_i + k_a - 1)^2}{\omega^2(k_a h_{i-1} - \tau)^2 + (k_v h_{i-1} + k_a - 1)^2}, \\ \implies \|K_i(j\omega)\|^2_\infty &= max\Big(\frac{(k_a h_i - \tau)^2}{(k_a h_{i-1} - \tau)^2}, \frac{(k_v h_i + k_a - 1)^2}{(k_v h_{i-1} + k_a - 1)^2}\Big). \end{aligned} \tag{3.15}$$

With the condition (3.13) or (3.14), it is clear that,

$$h_i \leq h_{i-1} \implies \|K_i(j\omega)\|_\infty \leq 1,$$

and for,

$$h_i > h_{i-1} \implies \|K_i(j\omega)\|_\infty > 1.$$

At this point, we define another lemma summarizing the results pertaining to transfer function $K_i(s)$.

Lemma 2. *The transfer function $K_i(\tau; s)$ is stable and $\|K_i(\tau; jw)\|_\infty \leq 1$ if and only if*

1. $h_{min} > \dfrac{\tau_o}{k_a},\ k_v > \dfrac{1-k_a}{h_{min}},\ if\ k_a \in (0,1),$
2. $k_v < \dfrac{1}{h_{max}},\ if\ k_a = 0, and$
3. $h_i \leq h_{i-1},\ \forall i.$

Using Lemmas 1 and 2, the sufficient conditions satisfying magnitude condition (3.6) can be obtained, and are summarized by following theorem:

Theorem 2. *For a given vehicle platoon with heterogeneity in time headway, there exists $k_p, k_v > 0$ such that $K_i(\tau; s)H_i(\tau; s)$ is stable and $\|K_i(\tau; j\omega)H_i(\tau; j\omega)\|_\infty \leq 1$ for all i, for all $\tau \in [0, \tau_o]$ if*

1. $h_{min} \geq 2\tau_o\ if\ k_a = 0,$
2. $h_{min} > \dfrac{\tau_o}{k_a}\ if\ k_a \in (\dfrac{1}{2}, 1),\ and$
3. $h_i \leq h_{i-1}.$

Proof. To prove this theorem, we prove the sufficient condition given by (3.7).

Let us first consider the transfer function $H_i(s)$. Let h_{min} and h_{max} be the minimum and maximum time headways respectively. For $k_a \in [0, 1)$, if the minimum headway $h_{min} \geq \frac{2\tau_o}{1+k_a}$, from Lemma 1 we should be able to find a $k_p, k_v > 0$ such that $\|H_i(\tau; j\omega)\|_\infty \leq 1$, for all

$i,\ \tau \in [0, \tau_o]$. From Lemma 2, we have that for $K_i(s)$ to be stable and $\|K_i(j\omega)\| \leq 1$,

$$h_{min} > \frac{\tau_o}{k_a},\ k_v > \frac{1-k_a}{h_{min}},\ if\ k_a \in (0,1), \tag{3.16}$$

$$k_v < \frac{1}{h_{max}},\ if\ k_a = 0, \tag{3.17}$$

$$h_i \leq h_{i-1},\ \forall i. \tag{3.18}$$

(a) For ACC:

For $k_a = 0$, the stability of transfer function $K_i(s)$ does not place any bound on the minimum time headway, and hence, for this case, $h_{min} = 2\tau_o$ from Lemma 1. Again, for this case, the sets S_1 and S_2 given by (3.11) and (3.12) respectively can be refined to a single set S by including the condition given by (3.17). The new set S given by

$$S = \big\{(k_p, k_v) \mid k_p > 0; k_v < \frac{1}{h_{max}}; \frac{k_p}{p_1} + \frac{k_v}{p_2} \geq 1;\ \frac{k_p}{p_3} + \frac{k_v}{p_4} \leq 1\big\} \tag{3.19}$$

is a feasible set.

(b) For CACC:

It is important to note that for $k_a \in (0, \frac{1}{2})$, CACC has higher bound compared to ACC. Hence, to benefit from CACC, $k_a \in (\frac{1}{2}, 1)$, and for $k_a \in (\frac{1}{2}, 1)$

$$\frac{\tau_o}{k_a} > \frac{2\tau_o}{1+k_a},$$

and hence, $k_p, k_v > 0$ can still be found such that $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all $i,\ \tau \in [0, \tau_o]$. The sets S_1 and S_2 given by (3.11) and (3.12) respectively can be refined to a single set S' by including the condition given by (3.16). The new set S' is given by

$$S' = \big\{(k_p, k_v) \mid k_p > 0; k_v > p_2; \frac{k_p}{p_1} + \frac{k_v}{p_2} \geq 1;\ \frac{k_p}{p_3} + \frac{k_v}{p_4} \leq 1\big\}. \tag{3.20}$$

is also a feasible set.

Finally, for $\|K_i(j\omega)\|_\infty \leq 1$, the condition given by (3.18) $h_i \leq h_{i-1}$ needs to be satisfied. Hence if $h_i \leq h_{i-1}$, for a given k_a, and with appropriate minimum time headway h_{min}, one can find k_p, k_v such that

$$\|H_i(jw)\|_\infty \leq 1; \|K_i(jw)\|_\infty \leq 1 \implies \|K_i(jw)H_i(jw)\|_\infty \leq 1.$$

□

It is important to note that the stability conditions for $K_i(s)$ given by (3.13) constrain the conditions for satisfying $\|K_i(jw)\|_\infty \leq 1$ by requiring the headways to be in decreasing order. But this doesn't indicate that the actual string stability condition (3.6) can be satisfied if and only if the headways are in decreasing order. Thus, the conditions given by Theorem 2 are sufficient but not necessary to satisfy (3.6).

In practice, a driver could use any time headway that he feels comfortable with among the options offered to him, and the constraint to have time headways in decreasing order in a platoon limits those options. Hence, it is important to investigate how the spacing errors evolve for other combinations of headways. Deriving the error transfer function with respect to lead vehicle acceleration for all the vehicles in the platoon helps in this regard, and is discussed in Section 3.3.

3.2.4 Non-negativity of impulse response

The conditions for the impulse response $k_i(t) * h_i(t)$ of the transfer function $K_i(s)H_i(s)$ to be non-negative is analytically difficult to solve since it involves a fourth order transfer function with multiple parameters. One way to verify the non-negativity of impulse response condition is by plotting the impulse response of error transfer function for every possible pair of headways for the desired controller parameter set by iterating the parasitic lag τ from zero to τ_o in very small intervals, and determine those values for which impulse response is non-negative for $\tau \in [0, \tau_o]$. This is not a reliable method, and hence, further work is needed in this section.

3.3 Transfer function between error of i^{th} vehicle and lead vehicle acceleration

The transfer function between error of the i^{th} vehicle and the lead vehicle acceleration is derived in Appendix-A, and is given as

$$\frac{E_i(s)}{A_l(s)} = L_i(s) = G_i(s) \prod_{j=1(i\neq 1)}^{i-1} H_j(s), \tag{3.21}$$

where

$$G_i(s) = \frac{s(k_a h_i - \tau) + (k_v h_i + k_a - 1)}{\tau s^3 + s^2 + (k_v + k_p h_i)s + k_p}. \tag{3.22}$$

The error of the first vehicle with respect to lead vehicle acceleration is given by

$$\frac{E_1(s)}{A_l(s)} = L_1(s) = G_1(s) \tag{3.23}$$

From (3.21), if we choose the gains (k_a, k_v, k_p) and minimum time headway h_{min} such that the transfer functions $K_i(s)$ and $H_i(s)$ are stable, and $\|H_i(j\omega)\|_\infty \leq 1$, the magnitude of error transfer function of any vehicle in the platoon with respect to lead vehicle acceleration is bounded by

$$\underset{i}{sup}\|L_i(j\omega)\|_\infty \leq \underset{i}{sup}\|G_i(j\omega)\|_\infty \tag{3.24}$$

for finite values of time headways. Using the above result, we state the following theorem that relates the peak magnitude of transfer function from lead vehicle acceleration to error in maintaining the desired spacing in the vehicles.

Theorem 3. *For a given vehicle platoon with finite values of headway, there exists $k_p, k_v > 0$ such that irrespective of the size of the platoon, the maximum spacing error of a vehicle for any combination of headways is bounded if*

1. $h_{min} \geq 2\tau_o \; for \; k_a = 0, \; or$

2. $h_{min} > \frac{\tau_o}{k_a} \; for \; k_a \in (\frac{1}{2}, 1).$

Proof. The proof for this theorem is simple. From Theorem 2, we know that for an appropriate choice of k_a and minimum time headway h_{min}, we can find gains $k_p, k_v > 0$ such that the error transfer function $K_i(\tau; s)H_i(\tau; s)$ is stable and $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all $i,\ \tau \in [0, \tau_o]$.

From (3.18), we know that the transfer function between error of the i^{th} vehicle and the lead vehicle acceleration $A_l(s)$ is given as

$$\frac{E_i(s)}{A_l(s)} = L_i(s) = G_i(s) \prod_{j=1(i\neq 1)}^{i-1} H_j(s).$$

Since $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all $i,\ \tau \in [0, \tau_o]$ can be obtained by appropriate choice of (k_a, k_p, k_v, h_{min}), from the above equation, it is clear that the magnitude of the transfer function between error of the i^{th} vehicle and lead vehicle acceleration is bounded, and hence, the maximum error will also be bounded. For a finite set of headways $h_i \in [h_{min}, h_{max}]$, the bound on the magnitude of transfer function is given by (3.25). □

Remark 1: In this study, although heterogeneity is considered only in time headway, since we have $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all $i,\ \tau \in [0, \tau_o]$, the above theorem would also apply if heterogeneity in parasitic time lag exists as well. In this case, the bound on the magnitude of transfer function is given by

$$\underset{i}{sup}\|L_i(j\omega)\|_\infty \leq \underset{\tau,i}{sup}\|G_i(j\omega)\|_\infty.$$

On the other hand, Theorem 2 would not apply if heterogeneity in parasitic time lag exists. because the magnitude condition $\|K_i(\tau, j\omega)\| \leq 1$ cannot be guaranteed.

4. NUMERICAL SIMULATIONS

In this chapter, the design procedure for CTHP controller with heterogeneity in time headway is provided in Section 4.1, and the results from numerical simulations are discussed in Section 4.2.

4.1 Design Procedure for CTHP Controller

In this section, a CTHP controller is designed with a numerical example according to Theorem 3. Below are the steps for choosing gains of a CTHP controller with heterogeneity in time headway such that the maximum error of a vehicle in any combination is bounded:

1. The maximum parasitic lag has to be determined first. For simulations, $\tau_o = 0.5\ s$ is chosen.

2. In the second step, since the minimum headway is dependent on k_a, the value of k_a has to be chosen such that the desired minimum headway is above the bound $\frac{\tau_o}{k_a}$. $k_a = 0.85$ is chosen for this example.

3. In the third step, the desired minimum time headway has to be chosen such that $h_{min} > \frac{\tau_o}{k_a}$. $h_{min} = 0.6\ s$ is chosen for this example.

4. Finally, the gains k_p and k_v have to be chosen such that $\|H_i(\tau; j\omega)\|_\infty \leq 1$ for all i, for all $\tau \in [0, \tau_o]$, and conditions for the stability of $K_i(s)$ satisfy. $k_p = 4$ and $k_v = 0.6$ have been chosen from the feasible set.

The feasible set for k_p and k_v can be found using either (3.9) or (3.10) combined with the stability condition for $K_i(s)$ given by (3.16). By using the conditions given by (3.9), it can be easily shown that if we find a feasible set of gains k_p, k_v for h_{min}, then the same gains would work for all the other headways. Hence, the knowledge of minimum headway is sufficient for finding out the gains. But the condition (3.9) needs to be verified for $\tau \in [0, \tau_o]$. The gains that satisfy (3.9) and (3.16) for minimum headway $h_{min} = 0.6\ s$ and $\tau = 0.5\ s$ have been plotted, and are shown in Figure 4.1.

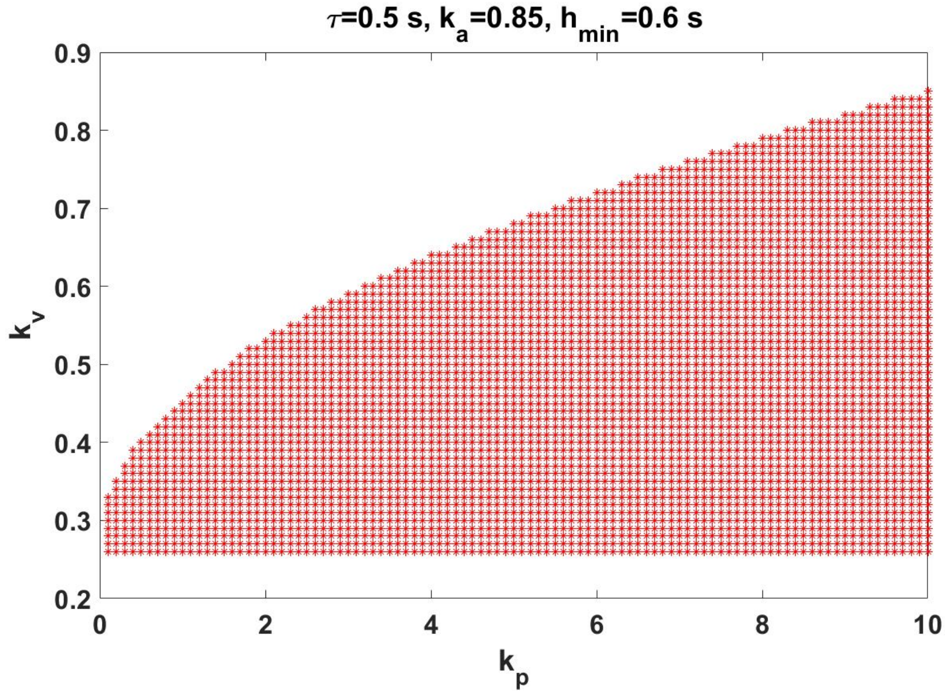

Figure 4.1: Set of gains k_p and k_v

All the simulations were performed with zero initial error, i.e., the initial positions of the vehicles are chosen such that each vehicle maintains the desired spacing initially. Although, taking different standstill distances wouldn't change the behaviour of evolution of spacing errors, $d = 5\ m$ was chosen for all the vehicles in the platoon. Also, an initial velocity of $v = 20\ m/s$, and initial acceleration $a = 0\ m/s^2$ were considered for all the vehicles in the platoon. Finally, a parasitic lag of $\tau = 0.5s$ was used for all the simulations. The final controller parameters chosen for the simulations are

$$k_a = 0.85,\ k_v = 0.6,\ k_p = 4, \&\ h_{min} = 0.6\ s. \tag{4.1}$$

4.2 Results from numerical simulations

In the subsequent sections the results from the simulations for various cases are provided and discussed. All the simulations were performed with the vehicle model given by (2.19) in MATLAB. For simulations, a vehicle platoon size of six vehicles i.e., five following vehicles $N = 5$ is considered. The headway set $h = \{0.60,\ 0.90,\ 1.20,\ ,1.50,\ 1.80\}\ s$ was chosen for simulations.

Since the gains were chosen such that $\|H_i(j\omega)\|_\infty \leq 1$ for all i, the same can be seen from the plot of frequency response of transfer function $H(s)$ for different headways shown as Figure 4.2.

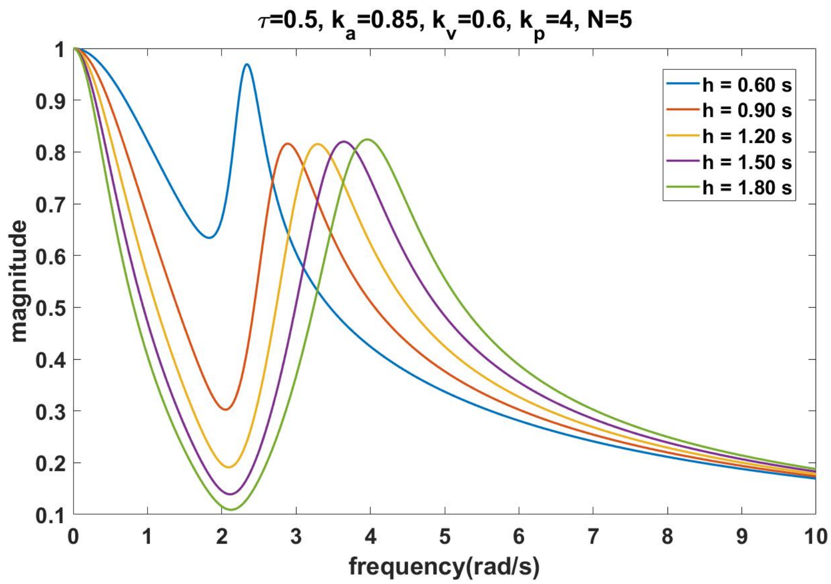

Figure 4.2: Frequency response of transfer function $H(s)$ for various headways

In Section 4.2.1, the results from simulations that have been performed with the guaranteed string stable case; the case in which the headway value decreases from the head of the platoon to the tail of the platoon are presented. In Section 4.2.2, the results from simulations for the case in which the headway value increases from the head to the tail of the platoon are presented. Finally, in Section 4.2.3, results from simulations that were performed for large platoon sizes with randomly selected headways are presented.

4.2.1 Time headways in descending order

In this section, the simulations were performed with the time headway values in descending order. Thus, the first vehicle has the maximum headway value of $h_{max} = 1.80\ s$ and the last vehicle in the platoon has the minimum headway value of $h_{min} = 0.60\ s$ for this case.. From Theorem 2, we know that the sufficient condition given by (3.7) must be satisfied if the headways are in descending order. Thus for this case,

$$\|H_i(jw)\|_\infty \le 1; \|K_i(jw)\|_\infty \le 1.$$

It has been already shown from Figure 4.2 that $\|H_i(j\omega)\|_\infty \le 1$ for all i. Figure 4.3 shows the frequency of response of transfer function $K_i(s)$ for the present case, and it can be seen that $\|K_i(jw)\|_\infty \le 1$ for all i as expected for this case. Since $\|H_i(j\omega)\|_\infty \le 1$ and $\|K_i(j\omega)\|_\infty \le 1$ for

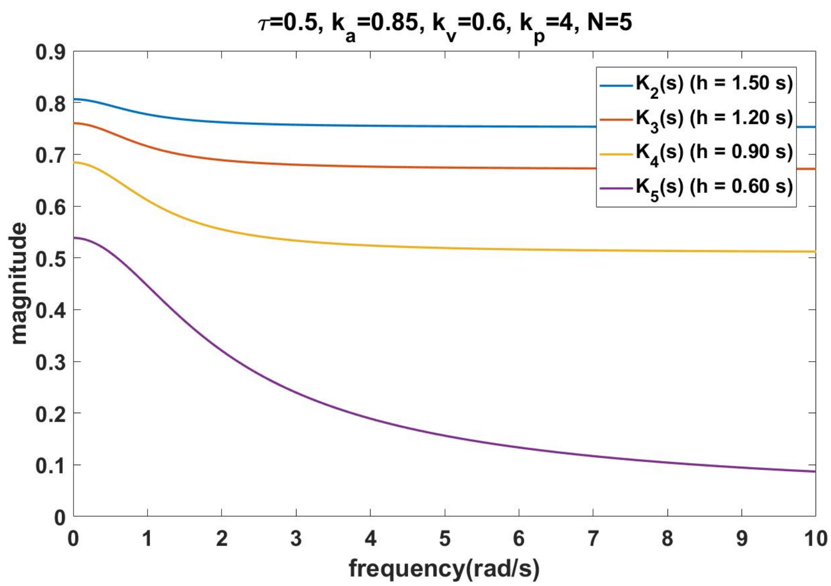

Figure 4.3: Frequency response of transfer function $K_i(s)$ for headways in descending order

all i, the magnitude of the error transfer function given by $K_i(s)H_i(s)$ should also be less than one at any given frequency which can be seen in Figure 4.4.

Now, to study the evolution of spacing errors in time domain, the simulations have been performed with the lead vehicle velocity and acceleration profiles as shown in Figure 4.5. Figure 4.6 shows the evolution of spacing errors for the case with headway values in descending order. From the plot, it can be seen that the error attenuates from the head to the tail of the vehicle platoon as expected owing to Theorem 2. Hence, for a given $k_a \in (\frac{1}{2}, 1)$ and $h_{min} > \frac{\tau_o}{k_a}$, one can find the

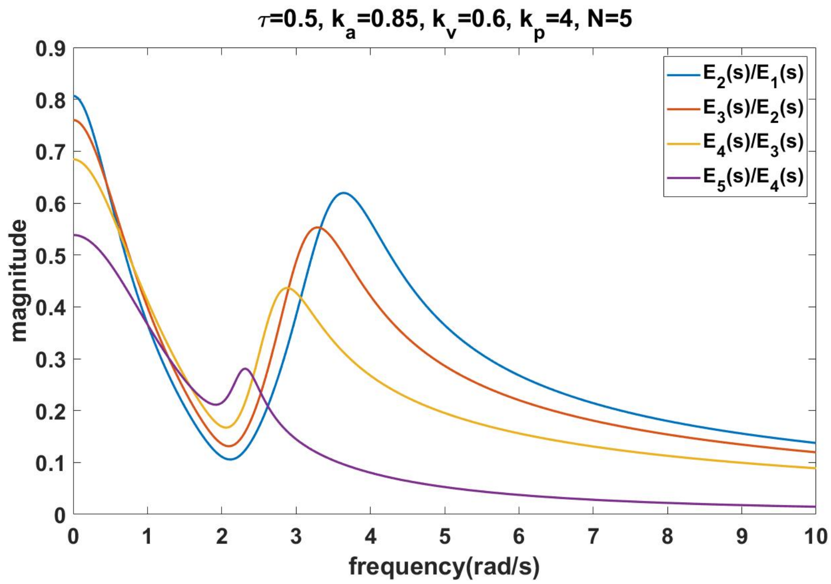

Figure 4.4: Frequency response of error transfer function $K_i(s)H_i(s)$ for headways in descending order

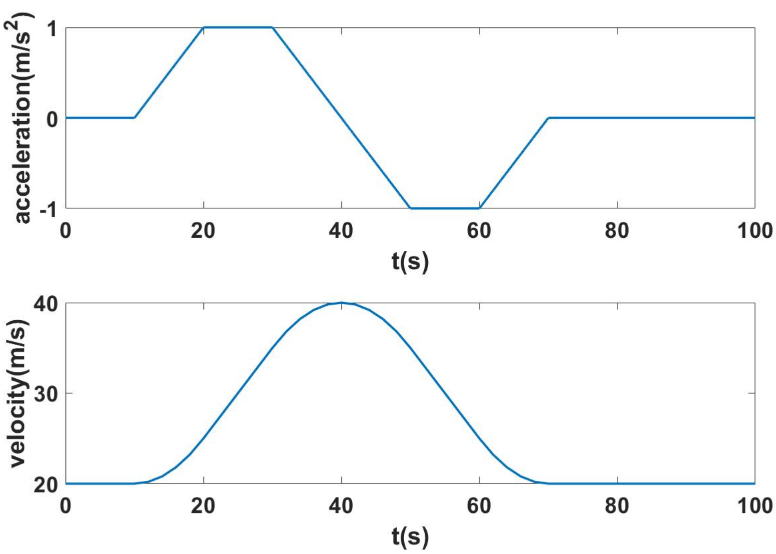

Figure 4.5: Lead vehicle velocity and acceleration profiles

gains $k_p, k_v > 0$ such that the vehicle platoon with headways in descending order is always string stable.

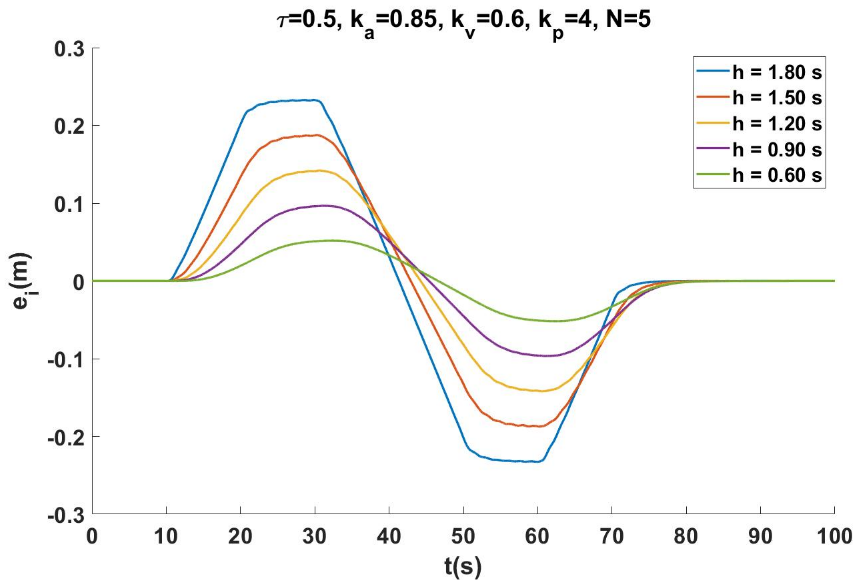

Figure 4.6: Evolution of spacing errors for headways in descending order

4.2.2 Time headways in ascending order

In this section, the simulations were performed with the time headway values in ascending order. In this case, the first vehicle in the platoon has the minimum headway $h_{min} = 0.60\ s$, and the last vehicle in the platoon has the maximum headway $h_{max} = 1.80\ s$.

It is important to note that for this case the magnitude of transfer function $K_i(s)$ is greater than one at all frequencies. But since the gains have been chosen such that $\|H_i(j\omega)\|_\infty \leq 1$ for all i, it might be possible that the magnitude of the error transfer function $K_i(s)H_i(s)$ could still be less than one at some or all frequencies for some or all vehicles. Figure 4.7 and 4.8 shows the frequency response of transfer function for $K_i(s)$ and the error transfer function $K_i(s)H_i(s)$ respectively for the headways in ascending order case. As expected, the magnitude of the transfer function $K_i(s)$ is greater than one at all frequencies, and the magnitude of the error transfer function $K_i(s)H_i(s)$ for this case is less than one at some frequencies for only some vehicles in the platoon. Figure 4.9 shows the evolution of the spacing errors for the headways in ascending order case with the same lead vehicle maneuver given by Figure 4.5.

For this case, although the errors amplify from the head to the tail in the platoon, the interesting

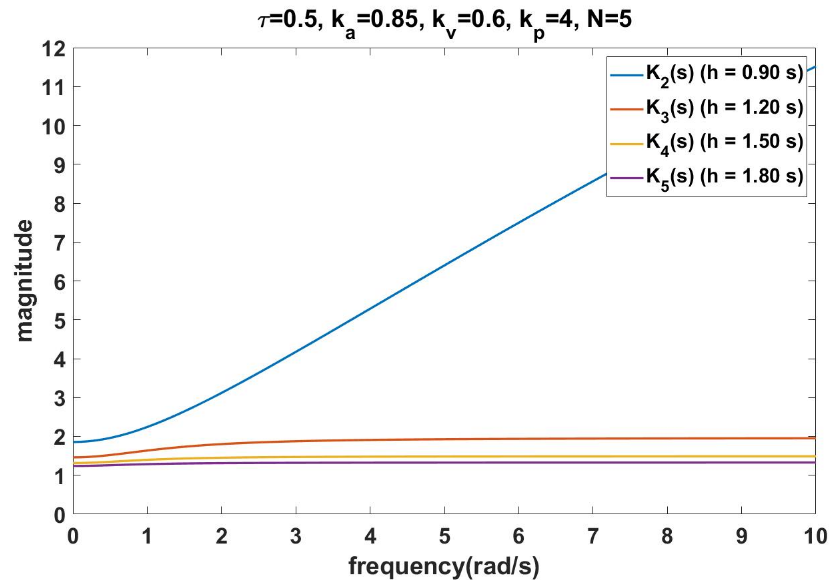

Figure 4.7: Frequency response of transfer function $K_i(s)$ for headways in ascending order

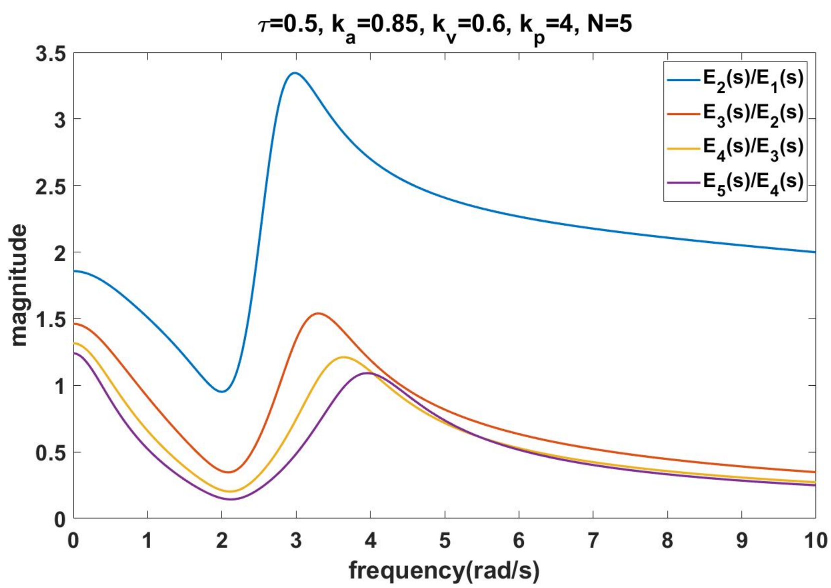

Figure 4.8: Frequency response of error transfer function $K_i(s)H_i(s)$ for headways in ascending order

point to note here is that the maximum error i.e., the spacing error corresponding to the last vehicle in the platoon is less than that of the maximum error in the case with time headways in descending order in which the error is attenuating from head to tail in the platoon. This can be explained by

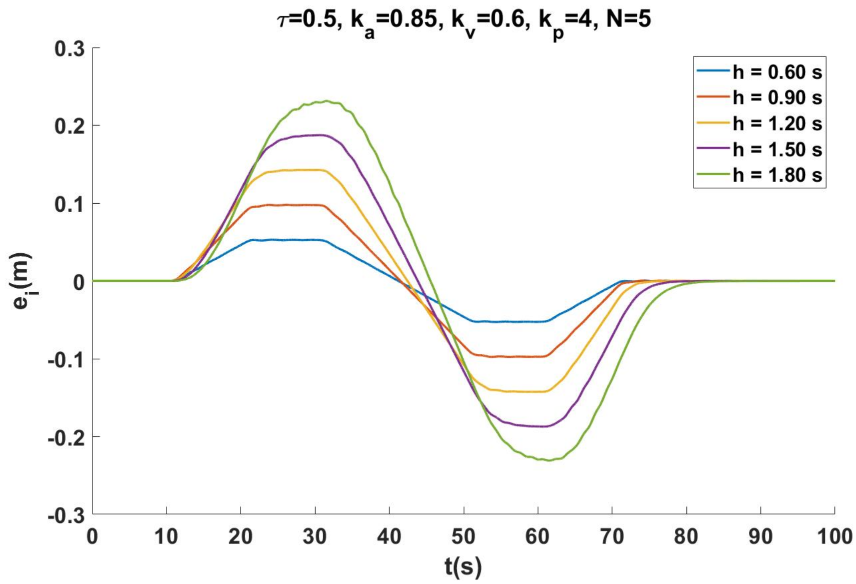

Figure 4.9: Evolution of spacing errors for headways in ascending order

using Theorem 3. For the gains that have been chosen for the simulations, the maximum spacing error for any vehicle for any combination of headways is bounded according to 3. Thus, although the errors amplify for the case with headways in ascending order, the maximum error is bounded, and in this case that bound could be the maximum error corresponding to the case with headways in descending order.

4.2.3 Time headways in random order

As stated earlier, one of the main applications of having heterogeneity in time headway is to allow the driver to choose a headway that he would be comfortable with. Hence, in practical applications, the headways could be in random order as opposed to descending or ascending order for which the simulations have been performed. Also, in the previous simulations, it was assumed that each vehicle had a different headway, and again in practice, different drivers may prefer the same headway. Hence, in this section, the simulations were performed for three cases given below:

(a) Multiple vehicles having same headway and vehicles arranged such that the headways are in descending order.

(b) Multiple vehicles having same headway and vehicles arranged such that the headways are in ascending order.

(c) Multiple vehicles having same headway and vehicles arranged in a random order.

The same controller parameters used in the previous simulations given by (4.1) were used. According to Theorem 3, for the gains that have been chosen for performing the simulations, the maximum spacing error of a vehicle for any combination has to be bounded, and hence, it is expected that the maximum error in all the three aforementioned cases will be bounded. Figures

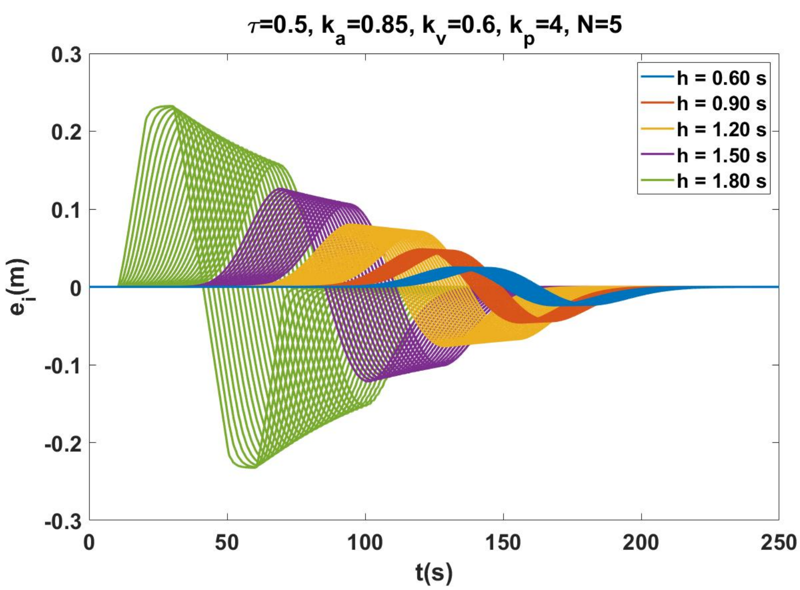

Figure 4.10: Evolution of spacing errors for case (a)

4.10, 4.11 and 4.12 show the evolution of spacing errors with time for all the three cases. The same lead vehicle maneuver given by Figure 4.5 has been used for all the three cases. As expected, the maximum error seems to be bounded in all the three cases. Hence, if the gains are chosen appropriately, the heterogeneity in time headway might not affect the stability of the vehicle platoon.

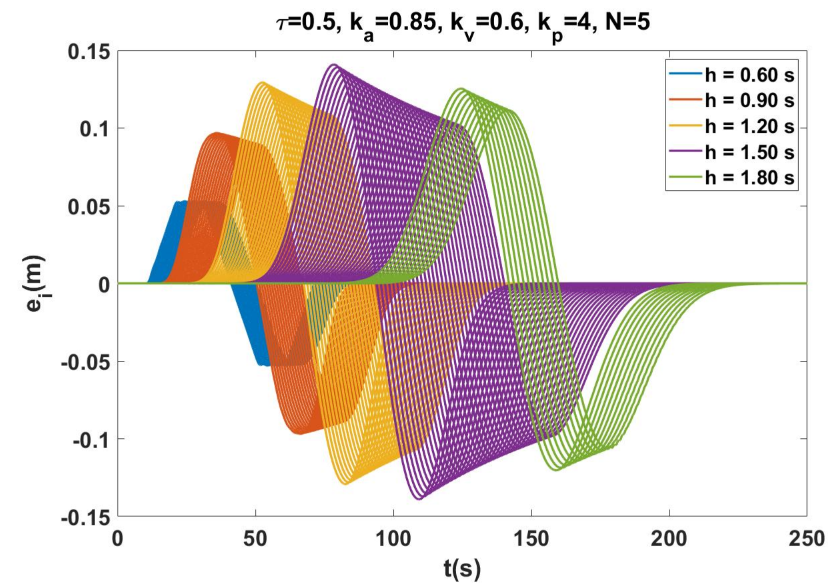

Figure 4.11: Evolution of spacing errors for case (b)

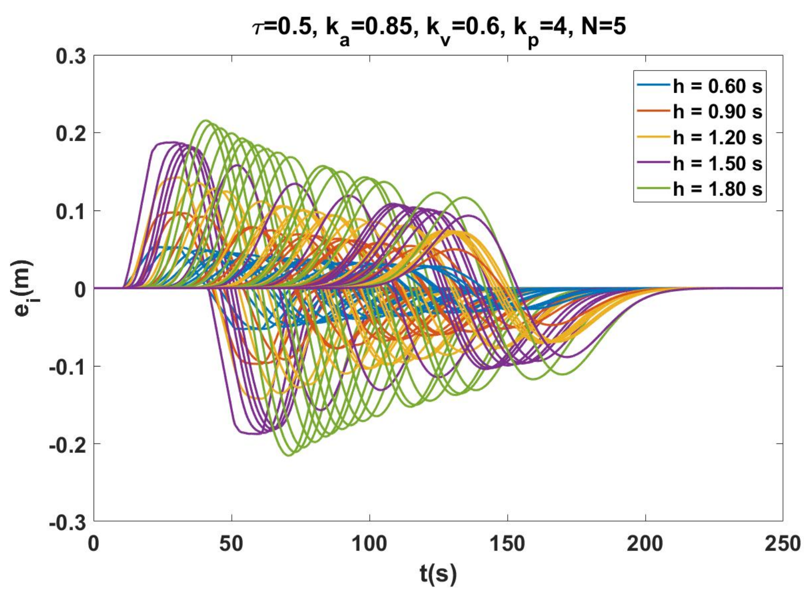

Figure 4.12: Evolution of spacing errors for case (c)

5. CONCLUSIONS AND FUTURE WORK

This study investigated the effects of heterogeneity in time headway on string stability of vehicle platoons. For the error to be bounded, the minimum employable time headway should be greater than $\frac{\tau_o}{k_a}$. This bound in case of heterogeneous headways is higher than that of the homogeneous case in which the minimum employable time headway should be greater than or equal to $\frac{2\tau_o}{1+k_a}$. A sufficient condition for string stability was defined, and using this condition, string stability can be guaranteed only for a platoon with monotonically decreasing headways. To study how other combinations of headways affect the stability, the transfer function between error of a vehicle in the platoon and the lead vehicle acceleration is derived. From the expression of this transfer function, it is found that the maximum spacing error of a vehicle for any combination of headways can bounded irrespective of the size of the platoon for a finite set of headways. Hence, it is concluded that string stability can be achieved even in the presence of heterogeneity in time headway. Numerical simulations were also performed which corroborated with the mathematical observations.

Possible extensions to this work can be:

- This work considered heterogeneity in time headway only. Future work can investigate on how heterogeneity in all controller parameters will affect the string stability of vehicle platoons.

- The control law used in this work utilizes only predecessor's information to maintain the desired spacing. Using r immediate predecessors and r^{th} predecessor information has shown to reduce the time headway in case of homogeneous platoons [9, 18]. Future work can focus on including r predecessors and r^{th} predecessor information in the presence of heterogeneity, and verify if the minimum employable time headway can be reduced.

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APPENDIX A

ERROR TRANSFER FUNCTION

Consider the i^{th} vehicle in a vehicle platoon of size N. The vehicle model in the presence of parasitic time lag is given as

$$\ddot{x}_i = a_i,$$
$$\tau \dddot{x}_i + \ddot{x}_i = u_i. \tag{A.1}$$

For CTHP, the control input for the i^{th} vehicle with heterogeneity in time headway is as given below:

$$u_i = k_a \ddot{x}_{i-1} - k_v(\dot{x}_i - \dot{x}_{i-1}) - k_p(x_i - x_{i-1} + d_i + h_i v_i). \tag{A.2}$$

$$\implies \tau \dddot{x}_i + \ddot{x}_i = k_a \ddot{x}_{i-1} - k_v(\dot{x}_i - \dot{x}_{i-1}) - k_p(x_i - x_{i-1} + d_i + h_i v_i). \tag{A.3}$$

The error is given by

$$e_i = x_i - x_{i-1} + d + h_i \dot{x}_i. \tag{A.4}$$

Let v be the initial velocity of all the vehicles in the platoon, and l_i be the initial position of the i^{th} vehicle in the platoon. Thus, the initial conditions are as given below

$$x_i(0) = l_i;\ \dot{x}_i(0) = v;\ \ddot{x}_i(0) = 0. \tag{A.5}$$

Now let us define

$$y_i = x_i - l_i - vt. \tag{A.6}$$

$$\implies \dot{x}_i = \dot{y}_i + v;\ \ddot{y}_i = \ddot{x}_i;\ \dddot{y}_i = \dddot{x}_i. \tag{A.7}$$

$$\implies e_i = y_i - y_{i-1} + h_i\dot{y}_i + (l_{i-1} - l_i + d + h_i v). \tag{A.8}$$

The initial position of i^{th} vehicle can be considered such that the initial error is zero. Therefore if

$$l_i = l_{i-1} + d + h_i v,$$

$$\implies e_i = y_i - y_{i-1} + h_i\dot{y}_i. \tag{A.9}$$

Also, from (A.5) and (A.6)

$$y_i(0) = 0;\ \dot{y}_i(0) = 0;\ \ddot{y}_i(0) = 0. \tag{A.10}$$

Using (A.6) and (A.7), (A.3) can be written as

$$\tau\dddot{y}_i + \ddot{y}_i = k_a\ddot{y}_{i-1} - k_v(\dot{y}_i - \dot{y}_{i-1}) - k_p(y_i - y_{i-1} + h_i v_i). \tag{A.11}$$

Applying Laplace transform to the above equation gives

$$Y_i(s) = \frac{k_a s^2 + k_v s + k_p}{\tau s^3 + s^2 + (k_v + k_p h_i)s + k_p} Y_{i-1}(s). \tag{A.12}$$

Applying Laplace transform to (A.9) gives

$$E_i(s) = Y_i(s) - Y_{i-1}(s) + h_i s Y_i(s)$$

$$\implies Y_i(s) = \frac{E_i(s) + Y_{i-1}(s)}{(1 + h_i s)}. \tag{A.13}$$

Substituting (A.13) in (A.12) gives

$$\frac{E_i(s)}{Y_{i-1}(s)} = \frac{(k_a s^2 + k_v s + k_p)(1 + h_i s)}{\tau s^3 + s^2 + (k_v + k_p h_i)s + k_p} - 1$$
$$\implies \frac{E_i(s)}{Y_{i-1}(s)} = \frac{s^3(k_a h_i - \tau) + s^2(k_v h_i + k_a - 1)}{\tau s^3 + s^2 + (k_v + k_p h_i)s + k_p}. \tag{A.14}$$

Defining

$$H_i(s) = \frac{H_{n,i}(s)}{D_i(s)} = \frac{k_a s^2 + k_v s + k_p}{\tau s^3 + s^2 + (k_v + k_{pi} h_i)s + k_p}, \tag{A.15}$$

$$G_i(s) = \frac{G_{n,i}(s)}{D_i(s)} = \frac{s(k_a h_i - \tau) + (k_v h_i + k_a - 1)}{\tau s^3 + s^2 + (k_v + k_p h_i)s + k_p} \tag{A.16}$$

$$\implies E_i(s) = s^2 G_i(s) Y_{i-1}(s) \tag{A.17}$$

$$\implies E_{i-1}(s) = s^2 G_{i-1}(s) Y_{i-2}(s). \tag{A.18}$$

Dividing (A.17) by (A.18) and using (A.12) gives

$$\frac{E_i(s)}{E_{i-1}(s)} = \frac{G_i(s)}{G_{i-1}(s)} H_{i-1}(s) \tag{A.19}$$

$$\implies \frac{E_i(s)}{E_{i-1}(s)} = \frac{G_{n,i}(s)}{D_i(s)} \frac{D_{i-1}(s)}{G_{n,i-1}(s)} \frac{H_{n,i-1}(s)}{D_{i-1}(s)}. \tag{A.20}$$

Since $H_{n,i}(s) = H_{n,i-1}(s)$, the above equation becomes

$$E_i(s) = \frac{G_{n,i}(s)}{G_{n,i-1}(s)} H_i(s) E_{i-1}(s) \tag{A.21}$$

$$\implies E_i(s) = K_i(s) H_i(s) E_{i-1}(s). \tag{A.22}$$

where

$$K_i(s) = \frac{s(k_a h_i - \tau) + (k_v h_i + k_a - 1)}{s(k_a h_{i-1} - \tau) + (k_v h_{i-1} + k_a - 1)}. \tag{A.23}$$

Since the initial conditions are zero, from (A.17), the transfer function between the error of the

i^{th} vehicle and the acceleration of the preceding vehicle is given by

$$E_i(s) = G_i(s)A_{i-1}(s), \tag{A.24}$$

where, $A_{i-1}(s)$ is the acceleration of the $(i-1)^{th}$ vehicle in Laplace domain. Thus, the error of the first vehicle $E_1(s)$ can be written in terms of lead vehicle acceleration $A_l(s)$ as

$$E_1(s) = G_1(s)A_l(s). \tag{A.25}$$

Using (A.25) and (A.19), the error of the i^{th} vehicle in terms of the lead vehicle acceleration $A_l(s)$ can be written as

$$E_i(s) = G_i(s) \prod_{j=1(i\neq 1)}^{i-1} H_j(s)A_l(s). \tag{A.26}$$